Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/38-1.2.1.9-P-x-d+e-x-^m-a+b-x+c-x^2-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [400]. This is test number [38].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (400)	0.00 (0)
Mathematica	98.25 (393)	1.75 (7)
Maple	97.00 (388)	3.00 (12)
Fricas	88.00 (352)	12.00 (48)
Giac	86.75 (347)	13.25 (53)
Maxima	72.75 (291)	27.25 (109)
Mupad	48.75 (195)	51.25 (205)
Sympy	35.50 (142)	64.50 (258)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

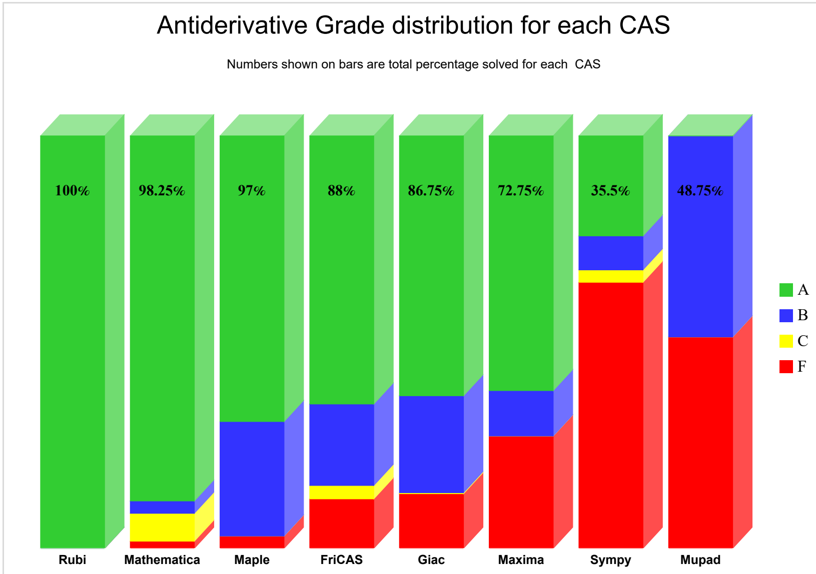
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

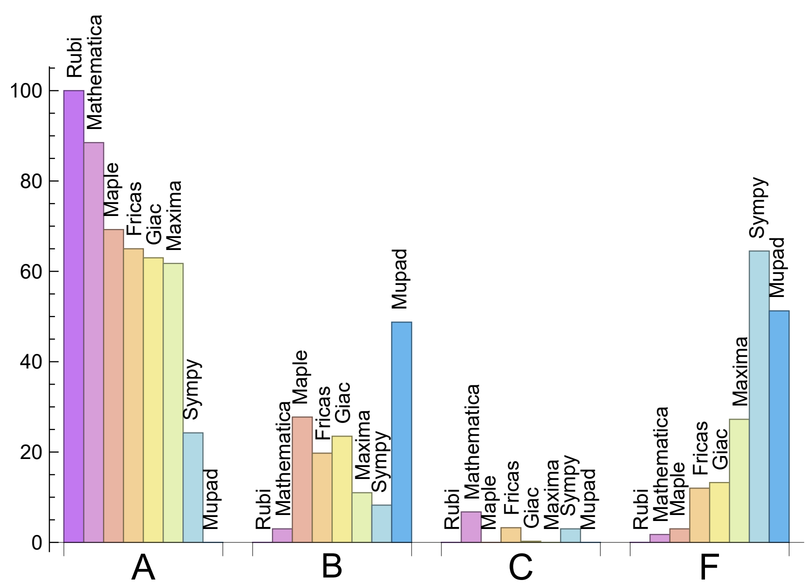
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	88.50	3.00	6.75	1.75
Maple	69.25	27.75	0.00	3.00
Fricas	65.00	19.75	3.25	12.00
Giac	63.00	23.50	0.25	13.25
Maxima	61.75	11.00	0.00	27.25
Sympy	24.25	8.25	3.00	64.50
Mupad	N/A	48.75	0.00	51.25

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	7	100.00 %	0.00 %	0.00 %
Maple	12	100.00 %	0.00 %	0.00 %
Fricas	48	25.00 %	75.00 %	0.00 %
Giac	53	49.06 %	20.75 %	30.19 %
Maxima	109	32.11 %	0.92 %	66.97 %
Sympy	258	79.46 %	20.16 %	0.39 %
Mupad	205	99.51 %	0.49 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

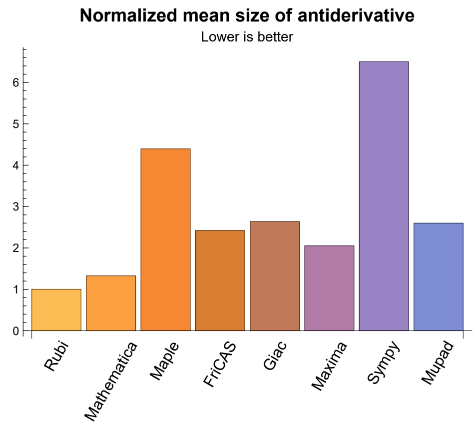
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

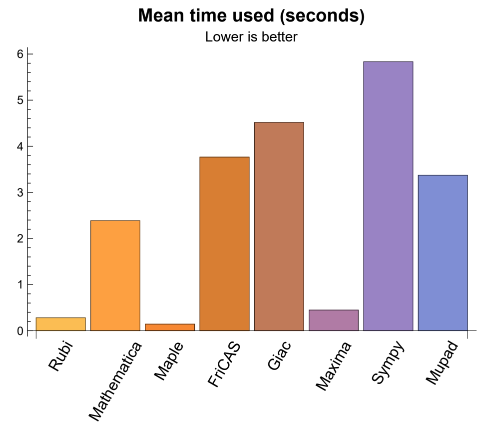
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.28	238.80	1.00	166.50	1.00
Mathematica	2.39	533.25	1.33	135.00	0.96
Maple	0.14	1990.91	4.40	197.00	1.24
Maxima	0.45	415.92	2.05	149.00	1.05
Fricas	3.77	566.74	2.42	190.50	1.38
Sympy	5.83	1996.92	6.50	220.50	1.56
Giac	4.52	899.41	2.64	191.00	1.17
Mupad	3.37	462.77	2.60	185.00	1.29

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
5
6
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
5
6
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{264, 265, 398, 399, 400\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

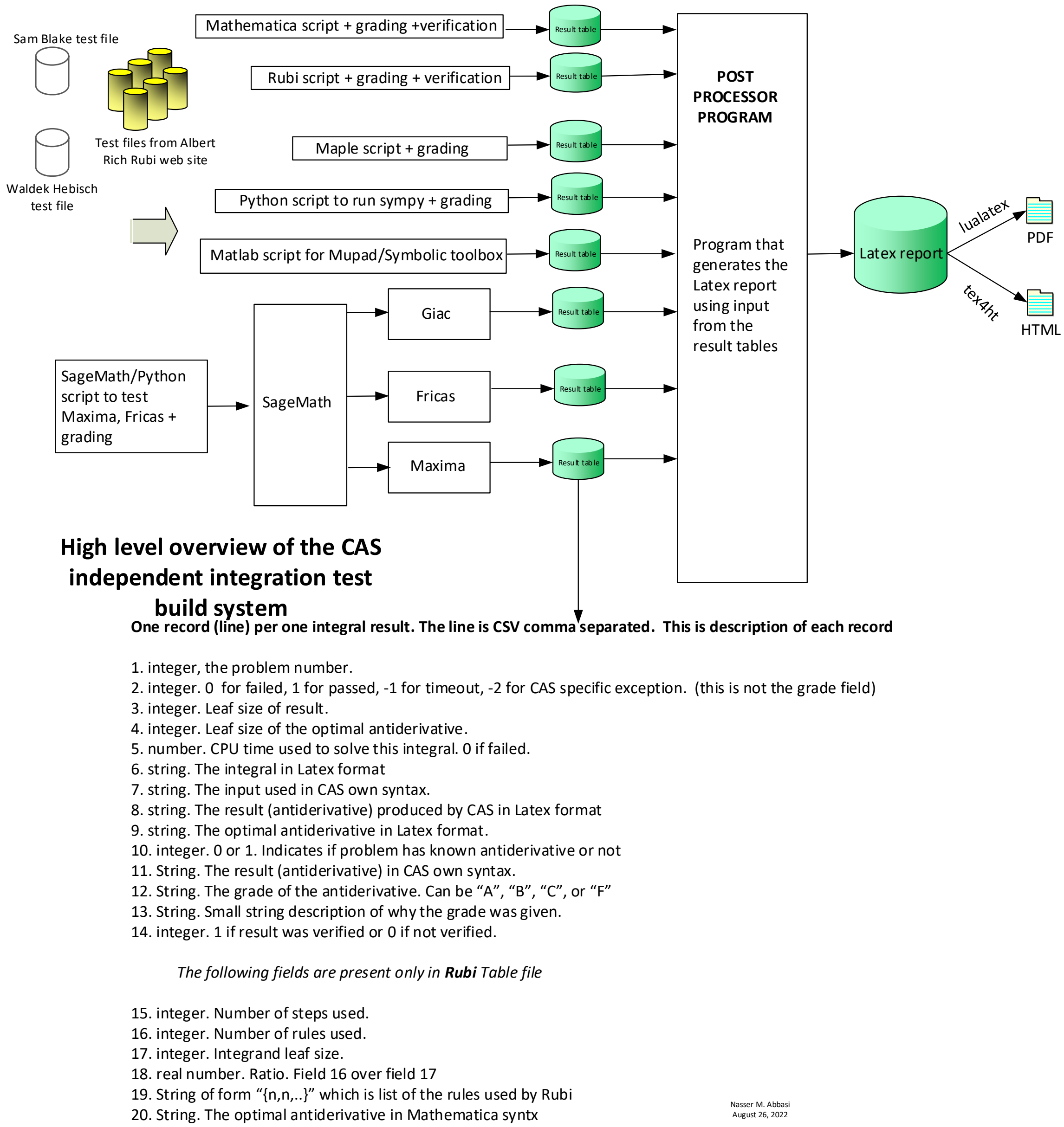
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 370, 372, 373, 374, 375, 376, 380, 381, 382, 386, 387, 388, 392, 393, 394, 398, 399, 400 }

B grade: { 39, 40, 41, 42, 114, 191, 206, 207, 278, 367, 368, 369 }

C grade: { 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 275, 366, 377, 378, 379, 383, 384, 385, 389, 390, 391, 395, 396, 397 }

F grade: { 136, 137, 138, 272, 273, 274, 371 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 134, 135, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 190, 200, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 236, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 256, 257, 275, 276, 277, 279, 283, 284, 285, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 372, 373, 374, 375, 376, 380, 381, 382, 386, 387, 388, 389, 392, 393, 394 }

B grade: { 9, 17, 39, 40, 41, 42, 64, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 99, 106, 107, 112, 113, 114, 127, 133, 147, 155, 158, 159, 178, 183, 184, 185, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 199, 201, 202, 203, 204, 205, 206, 207, 231, 232, 233, 234, 235, 237, 238, 239, 249, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 278, 280, 281, 282, 286, 287, 288, 358, 359, 360, 361, 362, 365, 366, 367, 368, 369, 377, 378, 379, 383, 384, 385, 390, 391, 395, 396, 397 }

C grade: { }

F grade: { 136, 137, 138, 139, 272, 273, 274, 370, 371, 398, 399, 400 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 88, 89, 90, 91, 93, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 140, 141, 142, 143, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 255, 256, 257, 275, 276, 277, 279, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 361, 362, 363, 364, 374, 375, 376, 380, 381, 382, 386, 387, 388, 392, 393, 394 }

B grade: { 8, 9, 16, 17, 39, 40, 41, 42, 62, 63, 64, 84, 85, 86, 87, 92, 94, 95, 96, 97, 98, 99, 106, 107, 112, 113, 114, 131, 177, 252, 253, 278, 323, 343, 358, 359, 360, 367, 368, 369, 377, 383, 389, 395 }

C grade: { }

F grade: { 6, 7, 136, 137, 138, 139, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 280, 281, 282, 283, 284, 285, 286, 287, 288, 365, 366, 370, 371, 372, 373, 378, 379, 384, 385, 390, 391, 396, 397, 398, 399, 400 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 52, 53, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 148, 149, 150, 151, 152, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 179, 180, 181, 186, 187, 188, 189, 199, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 255, 256, 257, 275, 276, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 320, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 373, 374, 375, 376, 380, 381, 382, 386, 387, 388, 392, 393, 394 }

B grade: { 31, 38, 39, 40, 41, 42, 48, 49, 50, 51, 54, 55, 57, 58, 61, 64, 65, 66, 67, 86, 87, 107, 112, 113, 114, 145, 146, 147, 155, 156, 157, 177, 178, 182, 183, 184, 185, 196, 197, 198, 232, 233, 234, 235, 236, 237, 238, 239, 253, 258, 277, 278, 310, 315, 316, 317, 318, 319, 322, 323, 360, 365, 366, 367, 368, 369, 372, 377, 378, 379, 383, 384, 385, 389, 390, 391, 395, 396, 397 }

C grade: { 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271 }

F grade: { 56, 62, 63, 82, 83, 84, 85, 92, 93, 94, 95, 96, 97, 98, 99, 106, 136, 137, 138, 139, 153, 154, 158, 159, 190, 191, 192, 193, 194, 195, 200, 201, 202, 203, 204, 205, 206, 207, 230, 231, 272, 273, 274, 370, 371, 398, 399, 400 }

2.1.6 Sympy

A grade: { 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 53, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 110, 111, 115, 118, 119, 120, 140, 141, 142, 143, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 279, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 307, 314, 321 }

B grade: { 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 116, 117, 126, 132, 144, 145, 146, 147, 148, 149, 150, 151, 156, 157, 177, 275, 276, 277, 278, 367, 368, 369 }

C grade: { 1, 2, 3, 304, 305, 306, 311, 312, 313, 318, 319, 320 }

F grade: { 4, 5, 6, 7, 8, 9, 14, 15, 16, 17, 47, 48, 49, 54, 55, 56, 57, 58, 61, 62, 63, 64, 65, 66, 67, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 107, 108, 109, 112, 113, 114, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 152, 153, 154, 155, 158, 159, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 280, 281, 282, 283, 284, 285, 286, 287, 288, 308, 309, 310, 315, 316, 317, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 6, 10, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 88, 89, 90, 91, 92, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 124, 125, 126, 127, 130, 131, 132, 133, 135, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 186, 187, 188, 189, 199, 208, 209, 210, 211, 214, 215, 216, 217, 220, 221, 222, 223, 226, 227, 228, 229, 235, 236, 240, 241, 242, 243, 246, 247, 248, 249, 252, 253, 254, 255, 279, 280, 281, 282, 284, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 334, 335, 336, 338, 344, 345, 346, 350, 351, 352, 353, 354, 358, 359, 360, 361, 365, 366, 372, 373, 374, 375, 376, 377, 380, 381, 382, 383, 386, 387, 388, 389, 392, 393, 394, 395, 396 }

B grade: { 5, 7, 9, 16, 17, 39, 40, 41, 42, 61, 62, 63, 64, 84, 85, 87, 94, 95, 97, 98, 99, 107, 112, 114, 121, 122, 123, 128, 129, 134, 147, 158, 159, 178, 184, 185, 195, 196, 197, 198, 203, 206, 207, 212, 218, 224, 232, 233, 234, 237, 239, 244, 245, 250, 251, 256, 257, 275, 276, 277, 278, 285, 286, 287, 288, 327, 328, 329, 330, 331, 332, 333, 337, 339, 340, 341, 342, 343, 347, 348, 349, 355, 356, 357, 362, 363, 364, 367, 368, 369, 379, 390, 391, 397 }

C grade: { 8 }

F grade: { 15, 83, 86, 93, 96, 106, 113, 136, 137, 138, 139, 190, 191, 192, 193, 194, 200, 201, 202, 204, 205, 213, 219, 225, 230, 231, 238, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 283, 370, 371, 378, 384, 385, 398, 399, 400 }

2.1.8 Mupad

A grade: { }

B grade: { 8, 9, 12, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 103, 104, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 180, 182, 183, 184, 185, 186, 187, 188, 189, 208, 209, 210, 236, 254, 258, 275, 276, 277, 278, 279, 284, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 367, 368, 369, 372, 373, 374, 375, 376 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 10, 11, 14, 15, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 112, 113, 114, 136, 137, 138, 139, 178, 179, 181, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 280, 281, 282, 283, 285, 286, 287, 288, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 370, 371, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	225	355	320	199	1231	197	-1
N.S.	1	1.00	0.95	1.50	1.36	0.84	5.22	0.83	-0.00
time (sec)	N/A	0.294	0.662	0.090	0.496	0.511	12.609	2.786	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	189	216	192	165	670	160	-1
N.S.	1	1.00	1.02	1.16	1.03	0.89	3.60	0.86	-0.01
time (sec)	N/A	0.143	0.582	0.108	0.496	0.690	5.777	2.668	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	124	155	109	103	343	85	-1
N.S.	1	1.00	0.99	1.24	0.87	0.82	2.74	0.68	-0.01
time (sec)	N/A	0.043	0.292	0.077	0.508	0.355	3.382	2.970	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	123	227	159	104	0	99	-1
N.S.	1	1.00	0.83	1.53	1.07	0.70	0.00	0.67	-0.01
time (sec)	N/A	0.113	0.448	0.085	0.501	0.345	0.000	2.286	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	130	287	184	180	0	311	-1
N.S.	1	1.00	0.76	1.69	1.08	1.06	0.00	1.83	-0.01
time (sec)	N/A	0.126	0.518	0.097	0.513	0.382	0.000	3.772	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	133	278	0	244	0	256	-1
N.S.	1	1.00	0.89	1.87	0.00	1.64	0.00	1.72	-0.01
time (sec)	N/A	0.113	0.626	0.095	0.000	0.361	0.000	4.480	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	130	293	0	292	0	419	-1
N.S.	1	1.00	0.66	1.49	0.00	1.49	0.00	2.14	-0.01
time (sec)	N/A	0.124	0.729	0.115	0.000	0.355	0.000	3.652	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	109	308	868	306	0	623	601
N.S.	1	1.00	0.61	1.71	4.82	1.70	0.00	3.46	3.34
time (sec)	N/A	0.127	0.756	0.093	0.332	0.379	0.000	4.121	4.670

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	144	459	1265	380	0	690	960
N.S.	1	1.00	0.62	1.96	5.41	1.62	0.00	2.95	4.10
time (sec)	N/A	0.163	0.875	0.095	0.300	0.404	0.000	3.903	5.243

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	194	391	364	167	1268	166	-1
N.S.	1	1.00	0.82	1.66	1.54	0.71	5.37	0.70	-0.00
time (sec)	N/A	0.422	0.623	0.090	0.505	0.371	11.242	3.797	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	159	286	240	135	891	131	-1
N.S.	1	1.00	0.83	1.50	1.26	0.71	4.66	0.69	-0.01
time (sec)	N/A	0.233	0.527	0.081	0.493	0.339	7.934	3.417	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	123	170	142	102	484	97	270
N.S.	1	1.00	0.86	1.19	0.99	0.71	3.38	0.68	1.89
time (sec)	N/A	0.119	0.443	0.095	0.494	0.390	3.993	3.503	5.012

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	90	109	66	68	262	52	148
N.S.	1	1.00	1.03	1.25	0.76	0.78	3.01	0.60	1.70
time (sec)	N/A	0.032	0.261	0.082	0.503	0.339	1.684	3.428	4.399

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	105	149	129	151	0	96	-1
N.S.	1	1.00	1.02	1.45	1.25	1.47	0.00	0.93	-0.01
time (sec)	N/A	0.074	0.450	0.110	0.519	0.348	0.000	3.389	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	116	195	294	211	0	0	-1
N.S.	1	1.00	0.71	1.20	1.80	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.570	0.083	0.526	0.365	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	103	308	561	231	0	336	109
N.S.	1	1.00	0.57	1.71	3.12	1.28	0.00	1.87	0.61
time (sec)	N/A	0.128	0.605	0.084	0.538	0.401	0.000	4.349	3.795

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	139	459	898	307	0	530	204
N.S.	1	1.00	0.59	1.96	3.84	1.31	0.00	2.26	0.87
time (sec)	N/A	0.153	0.716	0.088	0.553	0.402	0.000	3.618	3.776

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	173	208	217	201	208	257	242	206
N.S.	1	0.99	1.19	1.24	1.15	1.19	1.47	1.38	1.18
time (sec)	N/A	0.204	0.061	0.116	0.277	0.340	0.022	7.663	0.088

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	173	150	148	143	148	173	171	143
N.S.	1	0.99	0.86	0.85	0.82	0.85	0.99	0.98	0.82
time (sec)	N/A	0.137	0.040	0.111	0.281	0.361	0.017	5.794	3.614

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	79	85	88	97	100	80
N.S.	1	1.00	1.00	0.92	0.99	1.02	1.13	1.16	0.93
time (sec)	N/A	0.070	0.021	0.049	0.303	0.403	0.011	3.139	3.561

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	38	42	40	39
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.91	0.87	0.85
time (sec)	N/A	0.018	0.009	0.051	0.320	0.341	0.007	3.000	0.025

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	143	136	178	155	155	148	170	175
N.S.	1	0.99	0.94	1.23	1.07	1.07	1.02	1.17	1.21
time (sec)	N/A	0.150	0.052	0.085	0.283	0.336	0.237	4.000	3.623

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	151	142	172	166	234	185	240	192
N.S.	1	0.99	0.93	1.12	1.08	1.53	1.21	1.57	1.25
time (sec)	N/A	0.129	0.106	0.078	0.324	0.345	0.573	5.221	0.090

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	154	176	169	175	254	206	167	185
N.S.	1	0.99	1.13	1.08	1.12	1.63	1.32	1.07	1.19
time (sec)	N/A	0.120	0.068	0.091	0.289	0.332	2.176	4.104	0.095

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	301	335	385	361	366	445	423	332
N.S.	1	0.99	1.10	1.27	1.19	1.20	1.46	1.39	1.09
time (sec)	N/A	0.294	0.095	0.105	0.282	0.349	0.031	4.222	0.140

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	216	241	268	261	264	311	302	244
N.S.	1	1.00	1.11	1.24	1.20	1.22	1.43	1.39	1.12
time (sec)	N/A	0.195	0.064	0.108	0.288	0.364	0.025	4.268	3.724

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	144	151	161	162	180	181	140
N.S.	1	1.00	1.12	1.18	1.26	1.27	1.41	1.41	1.09
time (sec)	N/A	0.104	0.036	0.090	0.274	0.362	0.017	4.671	3.695

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	69	75	74	74	83	76	74
N.S.	1	1.00	1.03	1.12	1.10	1.10	1.24	1.13	1.10
time (sec)	N/A	0.027	0.021	0.100	0.276	0.328	0.011	3.842	0.038

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	295	285	441	368	370	359	416	422
N.S.	1	0.99	0.96	1.48	1.24	1.25	1.21	1.40	1.42
time (sec)	N/A	0.405	0.113	0.088	0.334	0.343	0.472	4.541	3.685

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	289	272	426	384	525	416	497	575
N.S.	1	0.99	0.93	1.46	1.32	1.80	1.42	1.70	1.97
time (sec)	N/A	0.320	0.190	0.091	0.280	0.366	1.239	4.648	0.121

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	292	274	399	395	580	474	397	495
N.S.	1	0.99	0.93	1.35	1.34	1.97	1.61	1.35	1.68
time (sec)	N/A	0.322	0.084	0.082	0.287	0.354	5.685	4.493	3.825

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	400	459	553	516	516	646	606	490
N.S.	1	0.99	1.14	1.37	1.28	1.28	1.60	1.50	1.21
time (sec)	N/A	0.423	0.142	0.105	0.321	0.366	0.041	4.369	4.050

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	288	329	388	374	374	447	432	343
N.S.	1	1.00	1.14	1.34	1.29	1.29	1.55	1.49	1.19
time (sec)	N/A	0.265	0.089	0.102	0.280	0.326	0.033	5.217	3.938

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	196	223	231	232	265	261	187
N.S.	1	1.00	1.16	1.32	1.37	1.37	1.57	1.54	1.11
time (sec)	N/A	0.121	0.048	0.088	0.283	0.365	0.023	4.040	0.099

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	100	111	108	108	122	111	103
N.S.	1	1.00	1.15	1.28	1.24	1.24	1.40	1.28	1.18
time (sec)	N/A	0.037	0.021	0.091	0.275	0.331	0.015	3.711	0.057

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	487	498	811	659	662	685	764	741
N.S.	1	0.99	1.02	1.66	1.34	1.35	1.40	1.56	1.51
time (sec)	N/A	0.660	0.343	0.089	0.287	0.368	0.780	3.892	3.877

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	483	641	788	679	892	748	838	1511
N.S.	1	0.99	1.32	1.62	1.40	1.84	1.54	1.72	3.11
time (sec)	N/A	0.589	0.251	0.089	0.300	0.345	2.146	3.846	3.986

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	463	438	753	690	984	816	727	1290
N.S.	1	0.99	0.94	1.62	1.48	2.11	1.75	1.56	2.77
time (sec)	N/A	0.583	0.141	0.091	0.311	0.372	11.166	4.075	3.936

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	62	76	82	78	73	111	85
N.S.	1	1.00	3.65	4.47	4.82	4.59	4.29	6.53	5.00
time (sec)	N/A	0.024	0.019	0.076	0.280	0.327	0.158	4.694	0.077

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	62	76	82	78	73	111	85
N.S.	1	1.00	3.65	4.47	4.82	4.59	4.29	6.53	5.00
time (sec)	N/A	0.012	0.010	0.066	0.273	0.357	0.159	2.769	3.836

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	90	157	160	120	153	216	252
N.S.	1	1.00	5.29	9.24	9.41	7.06	9.00	12.71	14.82
time (sec)	N/A	0.028	0.027	0.102	0.277	0.354	0.267	4.535	3.777

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	90	157	160	120	153	216	252
N.S.	1	1.00	5.29	9.24	9.41	7.06	9.00	12.71	14.82
time (sec)	N/A	0.016	0.015	0.074	0.285	0.332	0.277	3.618	0.049

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	237	223	260	242	578	1008	279	277
N.S.	1	0.99	0.93	1.08	1.01	2.41	4.20	1.16	1.15
time (sec)	N/A	0.247	0.151	0.133	0.497	0.380	5.811	3.431	3.990

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	166	155	169	160	400	638	176	181
N.S.	1	0.99	0.92	1.01	0.95	2.38	3.80	1.05	1.08
time (sec)	N/A	0.150	0.107	0.127	0.491	0.390	1.593	4.472	3.899

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	86	84	89	214	337	91	97
N.S.	1	1.00	0.92	0.90	0.96	2.30	3.62	0.98	1.04
time (sec)	N/A	0.069	0.059	0.113	0.483	0.361	0.786	2.885	3.780

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	56	47	48	125	156	48	56
N.S.	1	1.00	1.02	0.85	0.87	2.27	2.84	0.87	1.02
time (sec)	N/A	0.034	0.026	0.105	0.496	0.355	0.240	3.940	3.731

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	120	112	124	264	0	125	840
N.S.	1	1.00	0.90	0.84	0.93	1.98	0.00	0.94	6.32
time (sec)	N/A	0.105	0.070	0.134	0.490	4.382	0.000	3.723	6.490

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	188	204	252	873	0	270	1199
N.S.	1	1.00	0.88	0.95	1.18	4.08	0.00	1.26	5.60
time (sec)	N/A	0.211	0.217	0.161	0.517	20.065	0.000	4.266	6.773

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	277	317	480	1712	0	489	2500
N.S.	1	1.00	0.91	1.04	1.57	5.61	0.00	1.60	8.20
time (sec)	N/A	0.375	0.209	0.179	0.541	75.985	0.000	5.407	9.186

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	233	284	284	895	952	289	303
N.S.	1	1.00	1.08	1.31	1.31	4.14	4.41	1.34	1.40
time (sec)	N/A	0.290	0.151	0.135	0.508	0.365	28.661	6.282	4.014

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	175	187	186	623	593	184	195
N.S.	1	1.00	1.20	1.28	1.27	4.27	4.06	1.26	1.34
time (sec)	N/A	0.145	0.097	0.138	0.503	0.410	7.849	3.633	0.229

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	102	108	116	340	318	112	191
N.S.	1	1.00	1.05	1.11	1.20	3.51	3.28	1.15	1.97
time (sec)	N/A	0.048	0.066	0.104	0.496	0.394	2.816	3.751	0.139

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	65	62	195	116	60	60
N.S.	1	1.00	0.99	0.94	0.90	2.83	1.68	0.87	0.87
time (sec)	N/A	0.025	0.037	0.100	0.495	0.353	0.342	2.752	0.100

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	195	293	287	1027	0	350	1493
N.S.	1	1.00	0.86	1.30	1.27	4.54	0.00	1.55	6.61
time (sec)	N/A	0.259	0.152	0.120	0.506	20.725	0.000	3.906	7.675

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	371	320	424	593	2853	0	608	2094
N.S.	1	0.99	0.86	1.13	1.59	7.63	0.00	1.63	5.60
time (sec)	N/A	0.593	0.279	0.142	0.508	126.699	0.000	3.640	9.909

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	524	524	466	643	1007	0	0	957	2828
N.S.	1	1.00	0.89	1.23	1.92	0.00	0.00	1.83	5.40
time (sec)	N/A	1.048	0.420	0.228	0.585	0.000	0.000	3.289	14.480

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	281	333	378	1111	0	348	920
N.S.	1	1.00	1.34	1.59	1.81	5.32	0.00	1.67	4.40
time (sec)	N/A	0.185	0.169	0.148	0.523	0.365	0.000	4.417	1.767

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	175	211	222	253	803	0	254	230
N.S.	1	1.12	1.35	1.42	1.62	5.15	0.00	1.63	1.47
time (sec)	N/A	0.144	0.096	0.104	0.505	0.370	0.000	3.196	3.959

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	137	124	166	480	240	152	128
N.S.	1	1.00	1.05	0.95	1.28	3.69	1.85	1.17	0.98
time (sec)	N/A	0.066	0.070	0.103	0.506	0.354	14.854	3.869	0.150

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	90	83	98	314	156	84	88
N.S.	1	1.00	0.92	0.85	1.00	3.20	1.59	0.86	0.90
time (sec)	N/A	0.037	0.049	0.104	0.509	0.344	0.623	4.073	3.843

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	321	605	638	2306	0	715	2392
N.S.	1	1.00	0.91	1.71	1.81	6.53	0.00	2.03	6.78
time (sec)	N/A	0.434	0.278	0.203	0.529	169.841	0.000	3.382	9.900

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	571	566	498	833	1170	0	0	1107	2500
N.S.	1	0.99	0.87	1.46	2.05	0.00	0.00	1.94	4.38
time (sec)	N/A	1.305	0.479	0.185	0.554	0.000	0.000	4.656	6.657

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	753	753	672	1055	1793	0	0	1532	2500
N.S.	1	1.00	0.89	1.40	2.38	0.00	0.00	2.03	3.32
time (sec)	N/A	2.327	0.680	0.259	0.570	0.000	0.000	3.671	7.244

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	437	544	597	1859	0	636	669
N.S.	1	1.00	1.87	2.32	2.55	7.94	0.00	2.72	2.86
time (sec)	N/A	0.178	0.198	0.128	0.507	0.475	0.000	3.911	4.382

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	288	350	383	460	1380	0	475	402
N.S.	1	1.13	1.38	1.51	1.81	5.43	0.00	1.87	1.58
time (sec)	N/A	0.329	0.203	0.122	0.510	0.430	0.000	3.459	4.069

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	266	268	324	1069	0	328	287
N.S.	1	1.00	1.18	1.19	1.44	4.75	0.00	1.46	1.28
time (sec)	N/A	0.201	0.110	0.104	0.499	0.411	0.000	3.007	0.227

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	171	149	214	650	0	194	164
N.S.	1	1.00	1.04	0.90	1.30	3.94	0.00	1.18	0.99
time (sec)	N/A	0.082	0.092	0.093	0.550	0.506	0.000	3.970	3.936

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	112	100	133	430	196	109	116
N.S.	1	1.00	0.89	0.79	1.06	3.41	1.56	0.87	0.92
time (sec)	N/A	0.049	0.060	0.107	0.539	0.396	1.077	4.128	3.895

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	29	30	29	46	29	29	30
N.S.	1	1.00	0.67	0.70	0.67	1.07	0.67	0.67	0.70
time (sec)	N/A	0.032	0.014	0.096	0.521	0.396	0.040	4.372	0.035

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	27	24	23	40	20	23	23
N.S.	1	1.00	0.90	0.80	0.77	1.33	0.67	0.77	0.77
time (sec)	N/A	0.025	0.009	0.090	0.488	0.446	0.037	4.899	0.035

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	23	24	23	33	20	23	25
N.S.	1	1.00	0.79	0.83	0.79	1.14	0.69	0.79	0.86
time (sec)	N/A	0.015	0.007	0.075	0.511	0.372	0.040	4.276	0.031

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	20	10	12	14
N.S.	1	1.00	1.00	0.93	0.86	1.43	0.71	0.86	1.00
time (sec)	N/A	0.007	0.005	0.085	0.529	0.344	0.031	4.862	3.797

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	28	26	25	41	24	26	32
N.S.	1	1.00	0.90	0.84	0.81	1.32	0.77	0.84	1.03
time (sec)	N/A	0.025	0.008	0.103	0.539	0.372	0.052	4.184	0.042

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	30	34	49	31	35	38
N.S.	1	1.00	1.00	0.91	1.03	1.48	0.94	1.06	1.15
time (sec)	N/A	0.029	0.012	0.080	0.497	0.365	0.054	3.678	3.808

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	39	38	41	61	42	43	47
N.S.	1	1.00	0.87	0.84	0.91	1.36	0.93	0.96	1.04
time (sec)	N/A	0.039	0.012	0.090	0.497	0.367	0.068	4.113	0.038

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	22	12	13	12	18	8	12	12
N.S.	1	1.83	1.00	1.08	1.00	1.50	0.67	1.00	1.00
time (sec)	N/A	0.004	0.010	0.076	0.551	0.353	0.031	4.814	0.031

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	21	21	25	20	21	21
N.S.	1	1.00	1.00	0.78	0.78	0.93	0.74	0.78	0.78
time (sec)	N/A	0.008	0.008	0.083	0.495	0.340	0.035	3.285	3.830

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	387	358	358	446	890	1088	475	-1
N.S.	1	0.99	0.92	0.92	1.14	2.28	2.79	1.22	-0.00
time (sec)	N/A	0.492	1.021	0.096	0.285	0.579	17.992	4.104	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	279	245	272	311	609	738	321	-1
N.S.	1	1.00	0.88	0.97	1.11	2.18	2.64	1.15	-0.00
time (sec)	N/A	0.272	0.738	0.079	0.286	0.413	12.621	3.948	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	145	162	173	339	384	180	-1
N.S.	1	1.00	0.83	0.93	0.99	1.94	2.19	1.03	-0.01
time (sec)	N/A	0.142	0.502	0.082	0.284	0.383	5.765	3.883	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	87	113	97	192	170	87	-1
N.S.	1	1.00	0.82	1.07	0.92	1.81	1.60	0.82	-0.01
time (sec)	N/A	0.039	0.230	0.074	0.298	0.395	3.414	2.567	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	213	376	367	0	0	278	-1
N.S.	1	1.00	1.03	1.83	1.78	0.00	0.00	1.35	-0.00
time (sec)	N/A	0.232	0.736	0.107	0.352	0.000	0.000	3.592	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	303	217	861	483	0	0	0	-1
N.S.	1	0.98	0.70	2.80	1.57	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	1.071	0.108	0.363	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	295	280	1721	934	0	0	923	-1
N.S.	1	1.00	0.95	5.81	3.16	0.00	0.00	3.12	-0.00
time (sec)	N/A	0.301	1.867	0.112	0.340	0.000	0.000	3.193	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	382	2448	1782	0	0	1719	-1
N.S.	1	1.00	1.22	7.80	5.68	0.00	0.00	5.47	-0.00
time (sec)	N/A	0.289	10.534	0.109	0.374	0.000	0.000	5.389	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	312	439	3903	3416	2571	0	0	-1
N.S.	1	1.00	1.40	12.47	10.91	8.21	0.00	0.00	-0.00
time (sec)	N/A	0.250	10.863	0.084	0.434	53.462	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	432	583	6085	5811	3901	0	4212	-1
N.S.	1	1.00	1.35	14.05	13.42	9.01	0.00	9.73	-0.00
time (sec)	N/A	0.452	11.064	0.092	0.588	165.659	0.000	5.706	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	462	474	406	537	1228	1916	652	-1
N.S.	1	1.00	1.03	0.88	1.16	2.66	4.15	1.41	-0.00
time (sec)	N/A	0.673	1.439	0.079	0.298	0.656	77.036	4.770	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	345	332	320	387	851	1304	452	-1
N.S.	1	1.00	0.96	0.92	1.12	2.46	3.77	1.31	-0.00
time (sec)	N/A	0.316	1.092	0.106	0.303	0.431	49.776	5.041	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	212	197	194	216	487	768	264	-1
N.S.	1	1.00	0.92	0.91	1.01	2.29	3.61	1.24	-0.00
time (sec)	N/A	0.169	0.735	0.089	0.296	0.391	18.980	5.650	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	117	145	132	262	348	129	-1
N.S.	1	1.00	0.85	1.06	0.96	1.91	2.54	0.94	-0.01
time (sec)	N/A	0.051	0.419	0.069	0.313	0.387	10.361	4.267	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	360	643	642	0	0	551	-1
N.S.	1	1.00	1.10	1.97	1.97	0.00	0.00	1.69	-0.00
time (sec)	N/A	0.472	1.385	0.108	0.368	0.000	0.000	3.790	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	428	364	1448	717	0	0	0	-1
N.S.	1	0.99	0.84	3.35	1.66	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.554	1.574	0.119	0.339	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	488	480	361	2863	1311	0	0	1036	-1
N.S.	1	0.98	0.74	5.87	2.69	0.00	0.00	2.12	-0.00
time (sec)	N/A	0.568	2.183	0.144	0.376	0.000	0.000	5.555	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	475	469	517	4813	2431	0	0	1900	-1
N.S.	1	0.99	1.09	10.13	5.12	0.00	0.00	4.00	-0.00
time (sec)	N/A	0.515	10.827	0.122	0.407	0.000	0.000	7.554	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	511	511	575	7961	4346	0	0	0	-1
N.S.	1	1.00	1.13	15.58	8.50	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.673	11.402	0.144	0.509	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	507	507	639	10598	6673	0	0	4408	-1
N.S.	1	1.00	1.26	20.90	13.16	0.00	0.00	8.69	-0.00
time (sec)	N/A	0.530	11.377	0.093	0.565	0.000	0.000	5.120	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	403	696	16383	10749	0	0	6122	-1
N.S.	1	1.00	1.72	40.55	26.61	0.00	0.00	15.15	-0.00
time (sec)	N/A	0.357	11.552	0.095	0.752	0.000	0.000	4.676	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	531	863	24805	16286	0	0	7936	-1
N.S.	1	1.00	1.62	46.63	30.61	0.00	0.00	14.92	-0.00
time (sec)	N/A	0.546	11.613	0.104	0.923	0.000	0.000	7.840	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	142	177	166	333	510	168	-1
N.S.	1	1.00	0.85	1.05	0.99	1.98	3.04	1.00	-0.01
time (sec)	N/A	0.063	0.522	0.094	0.286	0.414	35.048	6.371	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	323	244	303	357	580	796	314	-1
N.S.	1	0.99	0.75	0.93	1.10	1.78	2.45	0.97	-0.00
time (sec)	N/A	0.395	0.785	0.094	0.300	0.576	10.862	6.662	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	222	165	217	235	387	518	206	-1
N.S.	1	1.00	0.74	0.97	1.05	1.74	2.32	0.92	-0.00
time (sec)	N/A	0.222	0.552	0.101	0.280	0.436	7.648	5.167	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	135	96	126	129	205	282	110	227
N.S.	1	0.99	0.71	0.93	0.95	1.51	2.07	0.81	1.67
time (sec)	N/A	0.105	0.413	0.087	0.275	0.394	3.832	4.330	5.171

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	64	77	62	126	150	58	107
N.S.	1	1.00	0.86	1.04	0.84	1.70	2.03	0.78	1.45
time (sec)	N/A	0.029	0.222	0.076	0.271	0.425	1.620	4.677	4.559

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	137	209	220	889	0	138	-1
N.S.	1	1.00	1.05	1.61	1.69	6.84	0.00	1.06	-0.01
time (sec)	N/A	0.108	0.480	0.107	0.301	147.241	0.000	5.832	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	177	390	422	0	0	0	-1
N.S.	1	1.00	1.05	2.32	2.51	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.917	0.092	0.314	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	224	203	810	901	1095	0	848	-1
N.S.	1	1.00	0.90	3.60	4.00	4.87	0.00	3.77	-0.00
time (sec)	N/A	0.183	1.198	0.094	0.322	3.957	0.000	2.827	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	228	256	292	354	782	0	339	-1
N.S.	1	1.00	1.12	1.28	1.55	3.41	0.00	1.48	-0.00
time (sec)	N/A	0.201	0.933	0.105	0.278	0.422	0.000	6.012	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	165	207	232	542	0	219	-1
N.S.	1	1.00	1.11	1.39	1.56	3.64	0.00	1.47	-0.01
time (sec)	N/A	0.114	0.684	0.084	0.273	0.376	0.000	5.344	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	104	118	129	282	209	116	151
N.S.	1	1.00	1.04	1.18	1.29	2.82	2.09	1.16	1.51
time (sec)	N/A	0.058	0.485	0.082	0.281	0.439	7.602	4.852	5.280

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	62	70	62	183	87	63	68
N.S.	1	1.00	1.02	1.15	1.02	3.00	1.43	1.03	1.11
time (sec)	N/A	0.023	0.353	0.068	0.288	0.363	3.500	3.893	4.332

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	147	386	457	734	0	294	-1
N.S.	1	1.00	1.07	2.80	3.31	5.32	0.00	2.13	-0.01
time (sec)	N/A	0.089	0.691	0.109	0.313	1.195	0.000	5.773	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	248	872	1092	1609	0	0	-1
N.S.	1	1.00	1.04	3.65	4.57	6.73	0.00	0.00	-0.00
time (sec)	N/A	0.262	1.443	0.096	0.391	1.713	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	372	1537	1787	2264	2893	0	1440	-1
N.S.	1	0.99	4.11	4.78	6.05	7.74	0.00	3.85	-0.00
time (sec)	N/A	0.685	11.205	0.076	0.392	5.935	0.000	6.487	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	50	105	83	68	194	48	59
N.S.	1	1.00	0.75	1.57	1.24	1.01	2.90	0.72	0.88
time (sec)	N/A	0.026	0.446	0.074	0.286	0.322	5.927	8.346	4.222

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	71	147	118	103	638	80	93
N.S.	1	1.00	0.73	1.52	1.22	1.06	6.58	0.82	0.96
time (sec)	N/A	0.037	0.514	0.091	0.282	0.346	13.365	5.270	4.281

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	92	189	153	137	1880	112	115
N.S.	1	1.00	0.72	1.49	1.20	1.08	14.80	0.88	0.91
time (sec)	N/A	0.047	0.615	0.107	0.283	0.355	26.928	2.786	4.375

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	66	79	78	60	94	54	45
N.S.	1	1.00	0.62	0.75	0.74	0.57	0.89	0.51	0.42
time (sec)	N/A	0.057	0.203	0.116	0.495	0.348	0.355	2.959	0.050

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	58	65	64	54	75	48	40
N.S.	1	1.00	0.71	0.79	0.78	0.66	0.91	0.59	0.49
time (sec)	N/A	0.052	0.165	0.092	0.487	0.368	0.224	4.969	4.099

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	51	50	49	63	44	35
N.S.	1	1.00	0.90	0.82	0.81	0.79	1.02	0.71	0.56
time (sec)	N/A	0.028	0.128	0.069	0.492	0.348	0.137	4.373	0.034

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	89	55	58	88	0	99	61
N.S.	1	1.00	1.33	0.82	0.87	1.31	0.00	1.48	0.91
time (sec)	N/A	0.042	0.238	0.138	0.501	0.352	0.000	5.262	0.189

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	92	65	65	106	0	191	68
N.S.	1	1.00	1.30	0.92	0.92	1.49	0.00	2.69	0.96
time (sec)	N/A	0.043	0.324	0.150	0.493	0.358	0.000	5.340	0.115

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	71	74	76	89	0	180	77
N.S.	1	1.00	0.92	0.96	0.99	1.16	0.00	2.34	1.00
time (sec)	N/A	0.041	0.384	0.091	0.502	0.367	0.000	4.483	0.111

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	66	79	78	76	0	54	110
N.S.	1	1.00	0.76	0.91	0.90	0.87	0.00	0.62	1.26
time (sec)	N/A	0.064	0.253	0.102	0.510	0.356	0.000	4.670	0.059

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	61	65	64	72	0	49	105
N.S.	1	1.00	0.86	0.92	0.90	1.01	0.00	0.69	1.48
time (sec)	N/A	0.053	0.229	0.080	0.505	0.342	0.000	4.471	4.067

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	56	51	50	67	114	44	100
N.S.	1	1.00	1.02	0.93	0.91	1.22	2.07	0.80	1.82
time (sec)	N/A	0.027	0.189	0.085	0.482	0.362	6.069	4.785	0.037

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	88	58	83	0	82	106
N.S.	1	1.00	0.96	1.66	1.09	1.57	0.00	1.55	2.00
time (sec)	N/A	0.036	0.357	0.092	0.503	0.332	0.000	4.012	0.136

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	98	84	103	0	168	157
N.S.	1	1.00	0.95	1.31	1.12	1.37	0.00	2.24	2.09
time (sec)	N/A	0.046	0.444	0.086	0.500	0.359	0.000	6.079	4.145

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	81	107	124	119	0	196	180
N.S.	1	1.00	0.84	1.10	1.28	1.23	0.00	2.02	1.86
time (sec)	N/A	0.085	0.479	0.093	0.506	0.351	0.000	3.612	4.169

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	64	91	105	87	0	53	212
N.S.	1	1.00	0.88	1.25	1.44	1.19	0.00	0.73	2.90
time (sec)	N/A	0.054	0.330	0.104	0.488	0.368	0.000	5.819	0.057

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	61	77	91	83	0	48	200
N.S.	1	1.00	1.02	1.28	1.52	1.38	0.00	0.80	3.33
time (sec)	N/A	0.045	0.290	0.092	0.515	0.361	0.000	4.727	0.050

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	30	51	50	40	180	25	185
N.S.	1	1.00	0.73	1.24	1.22	0.98	4.39	0.61	4.51
time (sec)	N/A	0.027	0.228	0.072	0.298	0.366	18.996	4.834	4.107

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	58	133	81	103	0	91	218
N.S.	1	1.00	0.79	1.82	1.11	1.41	0.00	1.25	2.99
time (sec)	N/A	0.047	0.599	0.087	0.499	0.335	0.000	3.983	0.135

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	91	143	107	134	0	233	270
N.S.	1	1.00	0.96	1.51	1.13	1.41	0.00	2.45	2.84
time (sec)	N/A	0.077	0.721	0.089	0.503	0.356	0.000	5.202	4.312

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	91	140	147	149	0	183	301
N.S.	1	1.00	0.78	1.20	1.26	1.27	0.00	1.56	2.57
time (sec)	N/A	0.110	0.620	0.111	0.493	0.344	0.000	2.715	4.190

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	420	417	0	0	0	0	0	0	-1
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.357	0.777	0.023	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	403	401	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.280	0.760	0.022	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.317	2.574	0.031	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	165	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.250	0.424	0.053	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	256	343	263	263	320	308	244
N.S.	1	1.00	1.01	1.35	1.04	1.04	1.26	1.21	0.96
time (sec)	N/A	0.200	0.062	0.139	0.281	0.334	0.032	3.278	0.126

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	163	223	165	165	197	187	149
N.S.	1	1.00	1.01	1.39	1.02	1.02	1.22	1.16	0.93
time (sec)	N/A	0.124	0.031	0.122	0.276	0.333	0.022	2.537	0.071

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	90	87	87	102	99	88
N.S.	1	1.00	1.00	0.94	0.91	0.91	1.06	1.03	0.92
time (sec)	N/A	0.066	0.016	0.108	0.278	0.343	0.015	3.762	4.086

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	38	42	40	39
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.91	0.87	0.85
time (sec)	N/A	0.017	0.007	0.049	0.276	0.352	0.007	4.248	0.026

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	84	82	0	265	413	78	224
N.S.	1	1.00	1.04	1.01	0.00	3.27	5.10	0.96	2.77
time (sec)	N/A	0.063	0.064	0.179	0.000	0.353	0.696	3.971	0.194

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	98	115	0	511	376	108	172
N.S.	1	1.00	0.98	1.15	0.00	5.11	3.76	1.08	1.72
time (sec)	N/A	0.046	0.054	0.115	0.000	0.357	0.677	3.871	4.533

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	160	269	0	1199	774	217	401
N.S.	1	1.00	0.99	1.67	0.00	7.45	4.81	1.35	2.49
time (sec)	N/A	0.073	0.134	0.142	0.000	0.367	1.442	4.441	4.173

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	204	510	0	2103	1224	407	698
N.S.	1	1.00	0.99	2.48	0.00	10.21	5.94	1.98	3.39
time (sec)	N/A	0.114	0.234	0.146	0.000	0.369	2.500	3.485	4.358

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	591	591	585	869	0	2072	4972	771	967
N.S.	1	1.00	0.99	1.47	0.00	3.51	8.41	1.30	1.64
time (sec)	N/A	0.929	0.373	0.253	0.000	0.758	73.110	3.855	5.462

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	345	453	0	1241	2839	426	557
N.S.	1	1.00	0.99	1.30	0.00	3.57	8.16	1.22	1.60
time (sec)	N/A	0.431	0.218	0.214	0.000	0.507	27.381	3.510	4.684

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	173	192	0	648	1265	201	273
N.S.	1	1.00	0.98	1.08	0.00	3.66	7.15	1.14	1.54
time (sec)	N/A	0.185	0.118	0.171	0.000	0.429	8.171	3.976	0.530

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	95	93	0	302	488	89	132
N.S.	1	1.00	1.03	1.01	0.00	3.28	5.30	0.97	1.43
time (sec)	N/A	0.084	0.043	0.156	0.000	0.371	1.293	3.621	0.253

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	193	179	0	609	0	204	2467
N.S.	1	1.00	0.98	0.91	0.00	3.11	0.00	1.04	12.59
time (sec)	N/A	0.209	0.127	0.160	0.000	58.360	0.000	3.888	10.450

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	281	346	0	0	0	449	2500
N.S.	1	1.00	0.89	1.09	0.00	0.00	0.00	1.42	7.91
time (sec)	N/A	0.463	0.343	0.281	0.000	0.000	0.000	3.404	14.713

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	504	629	0	0	0	1002	2500
N.S.	1	1.00	0.99	1.24	0.00	0.00	0.00	1.97	4.91
time (sec)	N/A	0.756	0.452	0.248	0.000	0.000	0.000	3.919	6.819

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	398	628	0	2690	0	540	742
N.S.	1	1.00	1.38	2.18	0.00	9.34	0.00	1.88	2.58
time (sec)	N/A	0.447	0.522	0.200	0.000	0.558	0.000	4.304	5.779

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	225	308	0	1392	1535	285	376
N.S.	1	1.00	1.26	1.73	0.00	7.82	8.62	1.60	2.11
time (sec)	N/A	0.166	0.286	0.165	0.000	0.380	28.793	5.486	5.039

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	114	133	0	632	459	125	203
N.S.	1	1.00	0.97	1.13	0.00	5.36	3.89	1.06	1.72
time (sec)	N/A	0.058	0.065	0.157	0.000	0.357	1.295	4.222	3.896

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	405	809	0	0	0	860	2500
N.S.	1	1.00	1.00	1.99	0.00	0.00	0.00	2.11	6.14
time (sec)	N/A	0.669	0.552	0.192	0.000	0.000	0.000	4.014	6.700

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	650	1345	0	0	0	1437	2500
N.S.	1	1.00	0.97	2.00	0.00	0.00	0.00	2.14	3.71
time (sec)	N/A	1.674	1.338	0.510	0.000	0.000	0.000	3.425	8.926

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	60	53	51	75	60	51	55
N.S.	1	1.00	0.97	0.85	0.82	1.21	0.97	0.82	0.89
time (sec)	N/A	0.047	0.025	0.144	0.509	0.634	0.058	5.326	0.041

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	46	46	70	54	46	48
N.S.	1	1.00	1.00	0.84	0.84	1.27	0.98	0.84	0.87
time (sec)	N/A	0.043	0.018	0.131	0.504	0.638	0.055	2.843	0.043

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	45	43	60	53	43	59
N.S.	1	1.00	1.00	0.87	0.83	1.15	1.02	0.83	1.13
time (sec)	N/A	0.026	0.014	0.118	0.500	0.529	0.055	3.744	3.838

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	39	34	32	41	41	32	35
N.S.	1	1.00	0.95	0.83	0.78	1.00	1.00	0.78	0.85
time (sec)	N/A	0.015	0.015	0.131	0.513	0.587	0.046	3.729	3.833

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	48	47	72	54	48	58
N.S.	1	1.00	1.00	0.86	0.84	1.29	0.96	0.86	1.04
time (sec)	N/A	0.038	0.019	0.107	0.510	0.697	0.069	4.893	0.100

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	55	54	85	65	55	68
N.S.	1	1.00	1.00	0.90	0.89	1.39	1.07	0.90	1.11
time (sec)	N/A	0.053	0.016	0.112	0.529	0.527	0.078	4.203	4.132

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	60	63	98	71	63	75
N.S.	1	1.00	0.97	0.88	0.93	1.44	1.04	0.93	1.10
time (sec)	N/A	0.058	0.021	0.099	0.507	0.422	0.088	4.262	0.098

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	8	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	0.80	1.00
time (sec)	N/A	0.005	0.004	0.148	0.295	0.340	0.023	3.398	0.045

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	27	27	36	27	29
N.S.	1	1.00	1.00	0.90	0.87	0.87	1.16	0.87	0.94
time (sec)	N/A	0.018	0.006	0.115	0.515	0.366	0.035	3.599	0.032

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.96	0.91	0.91	0.96	0.91	0.91
time (sec)	N/A	0.016	0.004	0.122	0.521	0.358	0.035	3.881	0.042

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	18	17	25	14	18	17
N.S.	1	1.00	0.90	0.86	0.81	1.19	0.67	0.86	0.81
time (sec)	N/A	0.008	0.007	0.097	0.284	0.454	0.021	3.578	0.042

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	14	16	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.78	0.89	0.78
time (sec)	N/A	0.010	0.004	0.119	0.288	0.560	0.036	4.190	3.922

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	12	14	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.86	1.00	0.86
time (sec)	N/A	0.010	0.004	0.104	0.285	0.456	0.037	3.283	3.851

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	31	22	21	21	22	21	17
N.S.	1	1.00	1.15	0.81	0.78	0.78	0.81	0.78	0.63
time (sec)	N/A	0.017	0.004	0.114	0.523	0.449	0.041	3.279	3.800

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	30	36	55	46	45	35
N.S.	1	1.00	1.00	0.62	0.75	1.15	0.96	0.94	0.73
time (sec)	N/A	0.029	0.027	0.132	0.515	0.362	0.040	3.645	0.111

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	19	19	15	19	17
N.S.	1	1.00	1.00	0.81	0.90	0.90	0.71	0.90	0.81
time (sec)	N/A	0.008	0.006	0.100	0.288	0.354	0.037	2.815	3.841

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	30	39	37	30	36
N.S.	1	1.00	1.00	0.87	0.77	1.00	0.95	0.77	0.92
time (sec)	N/A	0.015	0.019	0.235	0.511	0.365	0.053	3.636	3.835

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	33	33	31	11	11
N.S.	1	1.00	1.00	1.09	3.00	3.00	2.82	1.00	1.00
time (sec)	N/A	0.005	0.005	0.118	0.284	0.344	0.040	4.575	3.800

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	357	487	0	953	0	482	-1
N.S.	1	1.00	1.34	1.82	0.00	3.57	0.00	1.81	-0.00
time (sec)	N/A	0.146	1.499	0.151	0.000	0.519	0.000	3.755	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	227	370	0	605	0	297	-1
N.S.	1	1.00	1.07	1.75	0.00	2.85	0.00	1.40	-0.00
time (sec)	N/A	0.112	0.883	0.126	0.000	0.494	0.000	3.969	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	142	253	0	355	0	160	240
N.S.	1	1.00	0.90	1.61	0.00	2.26	0.00	1.02	1.53
time (sec)	N/A	0.087	0.370	0.127	0.000	0.420	0.000	4.164	4.262

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	88	138	0	203	0	84	-1
N.S.	1	1.00	0.85	1.33	0.00	1.95	0.00	0.81	-0.01
time (sec)	N/A	0.048	0.351	0.136	0.000	0.464	0.000	3.988	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	103	145	0	403	0	110	108
N.S.	1	1.00	1.05	1.48	0.00	4.11	0.00	1.12	1.10
time (sec)	N/A	0.044	0.490	0.133	0.000	0.506	0.000	4.431	4.210

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	107	259	0	242	0	193	127
N.S.	1	1.00	0.94	2.27	0.00	2.12	0.00	1.69	1.11
time (sec)	N/A	0.042	0.785	0.139	0.000	0.945	0.000	3.852	4.142

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	234	403	0	563	0	452	578
N.S.	1	1.00	1.40	2.41	0.00	3.37	0.00	2.71	3.46
time (sec)	N/A	0.058	1.758	0.124	0.000	5.283	0.000	3.548	4.525

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	410	547	0	978	0	805	1018
N.S.	1	1.00	1.86	2.49	0.00	4.45	0.00	3.66	4.63
time (sec)	N/A	0.077	3.406	0.134	0.000	14.386	0.000	4.565	5.063

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	930	927	1091	2098	0	2975	0	1702	2500
N.S.	1	1.00	1.17	2.26	0.00	3.20	0.00	1.83	2.69
time (sec)	N/A	1.658	8.385	0.135	0.000	1.453	0.000	2.975	14.702

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	584	581	654	1212	0	1881	0	1012	1881
N.S.	1	0.99	1.12	2.08	0.00	3.22	0.00	1.73	3.22
time (sec)	N/A	0.860	3.895	0.135	0.000	0.775	0.000	4.511	7.911

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	342	663	0	1043	0	495	877
N.S.	1	1.00	1.06	2.06	0.00	3.24	0.00	1.54	2.72
time (sec)	N/A	0.289	1.559	0.118	0.000	0.494	0.000	4.470	5.624

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	171	343	0	469	0	212	320
N.S.	1	1.00	0.98	1.96	0.00	2.68	0.00	1.21	1.83
time (sec)	N/A	0.098	0.063	0.123	0.000	0.458	0.000	4.209	4.240

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	318	577	0	0	0	0	-1
N.S.	1	1.00	0.99	1.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.461	1.535	0.141	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	453	1473	1097	0	0	0	0	-1
N.S.	1	0.99	3.21	2.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.637	8.383	0.171	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	446	500	2166	0	0	0	0	-1
N.S.	1	1.00	1.12	4.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.516	11.096	0.160	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	603	601	732	3090	0	0	0	0	-1
N.S.	1	1.00	1.21	5.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.873	12.312	0.132	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	497	499	813	4940	0	0	0	0	-1
N.S.	1	1.00	1.64	9.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.502	13.874	0.125	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	824	826	1633	7714	0	0	0	28857	-1
N.S.	1	1.00	1.98	9.36	0.00	0.00	0.00	35.02	-0.00
time (sec)	N/A	1.339	16.497	0.159	0.000	0.000	0.000	6.868	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1169	1166	1799	2878	0	4959	0	2977	-1
N.S.	1	1.00	1.54	2.46	0.00	4.24	0.00	2.55	-0.00
time (sec)	N/A	2.090	10.746	0.136	0.000	4.733	0.000	5.945	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	753	749	1126	1680	0	3271	0	1852	-1
N.S.	1	0.99	1.50	2.23	0.00	4.34	0.00	2.46	-0.00
time (sec)	N/A	1.159	11.251	0.148	0.000	2.191	0.000	4.550	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	598	936	0	1889	0	955	-1
N.S.	1	1.00	1.43	2.24	0.00	4.52	0.00	2.28	-0.00
time (sec)	N/A	0.363	4.273	0.135	0.000	0.816	0.000	4.928	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	290	499	0	849	0	417	-1
N.S.	1	1.00	1.23	2.11	0.00	3.60	0.00	1.77	-0.00
time (sec)	N/A	0.133	0.146	0.000	0.000	0.456	0.000	4.422	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	660	660	752	990	0	0	0	0	-1
N.S.	1	1.00	1.14	1.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.070	10.813	0.219	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	754	750	722	1853	0	0	0	0	-1
N.S.	1	0.99	0.96	2.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.447	10.816	0.193	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	824	819	692	3600	0	0	0	0	-1
N.S.	1	0.99	0.84	4.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.264	11.058	0.189	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	833	829	879	6060	0	0	0	7319	-1
N.S.	1	1.00	1.06	7.27	0.00	0.00	0.00	8.79	-0.00
time (sec)	N/A	1.329	13.682	0.165	0.000	0.000	0.000	11.488	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1097	1096	1835	10038	0	0	0	0	-1
N.S.	1	1.00	1.67	9.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.885	16.566	0.199	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1226	1223	2229	13372	0	0	0	0	-1
N.S.	1	1.00	1.82	10.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.464	16.739	0.138	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	657	660	1548	20684	0	0	0	48804	-1
N.S.	1	1.00	2.36	31.48	0.00	0.00	0.00	74.28	-0.00
time (sec)	N/A	0.729	16.798	0.157	0.000	0.000	0.000	8.045	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1062	1062	3059	31330	0	0	0	76083	-1
N.S.	1	1.00	2.88	29.50	0.00	0.00	0.00	71.64	-0.00
time (sec)	N/A	1.741	17.204	0.145	0.000	0.000	0.000	76.541	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	80	115	126	83	0	78	170
N.S.	1	1.00	0.56	0.80	0.88	0.58	0.00	0.55	1.19
time (sec)	N/A	0.080	0.433	0.170	0.497	0.353	0.000	3.952	5.541

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	75	98	109	78	0	73	153
N.S.	1	1.00	0.64	0.83	0.92	0.66	0.00	0.62	1.30
time (sec)	N/A	0.064	0.356	0.114	0.492	0.388	0.000	5.145	5.151

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	70	81	92	73	0	68	136
N.S.	1	1.00	0.75	0.87	0.99	0.78	0.00	0.73	1.46
time (sec)	N/A	0.039	0.268	0.102	0.493	0.386	0.000	4.182	4.879

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	104	95	96	115	0	126	-1
N.S.	1	1.00	1.03	0.94	0.95	1.14	0.00	1.25	-0.01
time (sec)	N/A	0.071	0.250	0.170	0.507	0.361	0.000	4.407	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	110	123	103	133	0	380	-1
N.S.	1	1.00	1.02	1.14	0.95	1.23	0.00	3.52	-0.01
time (sec)	N/A	0.072	0.407	0.171	0.521	0.336	0.000	5.498	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	111	125	114	149	0	0	-1
N.S.	1	1.00	0.97	1.09	0.99	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.436	0.161	0.514	0.333	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	90	134	155	93	0	88	-1
N.S.	1	1.00	0.57	0.85	0.98	0.59	0.00	0.56	-0.01
time (sec)	N/A	0.122	0.643	0.118	0.510	0.331	0.000	3.683	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	85	117	138	88	0	83	-1
N.S.	1	1.00	0.60	0.83	0.98	0.62	0.00	0.59	-0.01
time (sec)	N/A	0.073	0.551	0.136	0.522	0.342	0.000	4.529	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	80	100	121	83	0	78	-1
N.S.	1	1.00	0.69	0.86	1.04	0.72	0.00	0.67	-0.01
time (sec)	N/A	0.046	0.395	0.105	0.499	0.392	0.000	4.156	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	114	151	125	125	0	136	-1
N.S.	1	1.00	0.92	1.22	1.01	1.01	0.00	1.10	-0.01
time (sec)	N/A	0.082	0.384	0.162	0.515	0.375	0.000	4.207	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	121	179	132	143	0	570	-1
N.S.	1	1.00	0.92	1.37	1.01	1.09	0.00	4.35	-0.01
time (sec)	N/A	0.084	0.447	0.173	0.508	0.398	0.000	4.474	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	121	162	143	159	0	0	-1
N.S.	1	1.00	0.88	1.17	1.04	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.581	0.165	0.523	0.442	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	100	153	184	103	0	98	-1
N.S.	1	1.00	0.53	0.81	0.97	0.54	0.00	0.52	-0.01
time (sec)	N/A	0.099	0.838	0.117	0.497	0.353	0.000	4.573	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	95	136	167	98	0	93	-1
N.S.	1	1.00	0.58	0.83	1.02	0.60	0.00	0.57	-0.01
time (sec)	N/A	0.082	0.762	0.115	0.523	0.367	0.000	6.085	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	90	119	150	93	0	88	-1
N.S.	1	1.00	0.65	0.86	1.08	0.67	0.00	0.63	-0.01
time (sec)	N/A	0.057	0.634	0.105	0.511	0.345	0.000	3.568	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	124	207	154	135	0	146	-1
N.S.	1	1.00	0.84	1.41	1.05	0.92	0.00	0.99	-0.01
time (sec)	N/A	0.096	0.606	0.161	0.509	0.405	0.000	4.590	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	131	235	161	153	0	760	-1
N.S.	1	1.00	0.85	1.53	1.05	0.99	0.00	4.94	-0.01
time (sec)	N/A	0.098	0.640	0.168	0.512	0.368	0.000	6.410	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	131	199	172	169	0	0	-1
N.S.	1	1.00	0.81	1.24	1.07	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.787	0.175	0.513	0.343	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	693	692	588	1309	0	1527	0	822	-1
N.S.	1	1.00	0.85	1.89	0.00	2.20	0.00	1.19	-0.00
time (sec)	N/A	1.200	2.640	0.143	0.000	0.673	0.000	5.524	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	418	341	740	0	905	0	457	-1
N.S.	1	1.00	0.81	1.76	0.00	2.15	0.00	1.09	-0.00
time (sec)	N/A	0.579	1.281	0.134	0.000	0.519	0.000	5.044	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	178	389	0	473	0	210	-1
N.S.	1	1.00	0.80	1.74	0.00	2.12	0.00	0.94	-0.00
time (sec)	N/A	0.174	0.662	0.131	0.000	0.426	0.000	5.535	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	99	188	0	231	0	98	-1
N.S.	1	1.00	0.85	1.62	0.00	1.99	0.00	0.84	-0.01
time (sec)	N/A	0.062	0.027	0.103	0.000	0.392	0.000	4.339	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	186	293	0	0	0	0	-1
N.S.	1	1.00	1.04	1.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.723	0.149	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	239	236	485	0	0	0	0	-1
N.S.	1	0.99	0.98	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.218	1.245	0.133	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	415	1013	0	2170	0	2307	-1
N.S.	1	1.00	1.24	3.01	0.00	6.46	0.00	6.87	-0.00
time (sec)	N/A	0.392	10.806	0.157	0.000	20.105	0.000	4.254	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	502	713	1366	0	3063	0	1054	-1
N.S.	1	1.00	1.41	2.71	0.00	6.08	0.00	2.09	-0.00
time (sec)	N/A	0.670	4.078	0.162	0.000	7.396	0.000	3.214	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	288	390	775	0	1831	0	580	-1
N.S.	1	1.00	1.35	2.68	0.00	6.34	0.00	2.01	-0.00
time (sec)	N/A	0.230	2.309	0.159	0.000	6.181	0.000	4.289	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	197	411	0	919	0	271	-1
N.S.	1	1.00	1.06	2.21	0.00	4.94	0.00	1.46	-0.01
time (sec)	N/A	0.138	0.919	0.145	0.000	4.103	0.000	4.515	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	114	201	0	435	0	122	143
N.S.	1	1.00	1.03	1.81	0.00	3.92	0.00	1.10	1.29
time (sec)	N/A	0.040	0.096	0.000	0.000	0.615	0.000	4.356	4.535

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	235	547	0	1961	0	719	-1
N.S.	1	1.00	1.04	2.43	0.00	8.72	0.00	3.20	-0.00
time (sec)	N/A	0.164	1.083	0.133	0.000	11.939	0.000	3.913	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	418	474	1115	0	5320	0	0	-1
N.S.	1	0.99	1.13	2.65	0.00	12.64	0.00	0.00	-0.00
time (sec)	N/A	0.612	11.935	0.157	0.000	33.813	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	713	707	775	2268	0	10782	0	5637	-1
N.S.	1	0.99	1.09	3.18	0.00	15.12	0.00	7.91	-0.00
time (sec)	N/A	2.116	14.929	0.142	0.000	140.894	0.000	5.832	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	70	96	97	73	0	68	-1
N.S.	1	1.00	0.58	0.80	0.81	0.61	0.00	0.57	-0.01
time (sec)	N/A	0.074	0.370	0.113	0.495	0.350	0.000	5.507	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	65	79	80	68	0	63	-1
N.S.	1	1.00	0.68	0.83	0.84	0.72	0.00	0.66	-0.01
time (sec)	N/A	0.062	0.299	0.109	0.499	0.364	0.000	8.709	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	60	62	63	63	0	58	-1
N.S.	1	1.00	0.86	0.89	0.90	0.90	0.00	0.83	-0.01
time (sec)	N/A	0.037	0.190	0.107	0.494	0.387	0.000	7.060	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	93	60	67	105	0	116	-1
N.S.	1	1.00	1.19	0.77	0.86	1.35	0.00	1.49	-0.01
time (sec)	N/A	0.059	0.229	0.156	0.518	0.345	0.000	7.998	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	99	67	74	123	0	191	-1
N.S.	1	1.00	1.19	0.81	0.89	1.48	0.00	2.30	-0.01
time (sec)	N/A	0.058	0.291	0.155	0.505	0.371	0.000	3.203	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	77	74	82	96	0	204	-1
N.S.	1	1.00	0.87	0.83	0.92	1.08	0.00	2.29	-0.01
time (sec)	N/A	0.051	0.331	0.127	0.505	0.338	0.000	4.710	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	70	115	97	97	0	67	-1
N.S.	1	1.00	0.68	1.12	0.94	0.94	0.00	0.65	-0.01
time (sec)	N/A	0.076	0.491	0.110	0.505	0.370	0.000	4.124	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	65	98	80	92	0	62	-1
N.S.	1	1.00	0.79	1.20	0.98	1.12	0.00	0.76	-0.01
time (sec)	N/A	0.060	0.443	0.114	0.507	0.365	0.000	3.971	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	81	63	87	0	57	-1
N.S.	1	1.00	0.95	1.29	1.00	1.38	0.00	0.90	-0.02
time (sec)	N/A	0.036	0.363	0.116	0.502	0.355	0.000	2.794	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	70	102	64	96	0	91	-1
N.S.	1	1.00	1.13	1.65	1.03	1.55	0.00	1.47	-0.02
time (sec)	N/A	0.043	0.366	0.143	0.505	0.381	0.000	5.317	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	90	109	96	106	0	168	-1
N.S.	1	1.00	1.03	1.25	1.10	1.22	0.00	1.93	-0.01
time (sec)	N/A	0.055	0.396	0.134	0.525	0.369	0.000	3.865	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	87	111	145	126	0	223	-1
N.S.	1	1.00	0.78	0.99	1.29	1.12	0.00	1.99	-0.01
time (sec)	N/A	0.088	0.484	0.125	0.525	0.393	0.000	4.721	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	70	163	202	117	0	67	-1
N.S.	1	1.00	0.81	1.90	2.35	1.36	0.00	0.78	-0.01
time (sec)	N/A	0.063	0.681	0.134	0.512	0.383	0.000	2.622	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	65	146	185	112	0	62	-1
N.S.	1	1.00	0.96	2.15	2.72	1.65	0.00	0.91	-0.01
time (sec)	N/A	0.054	0.576	0.168	0.533	0.397	0.000	3.184	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	33	86	76	51	0	28	49
N.S.	1	1.00	0.70	1.83	1.62	1.09	0.00	0.60	1.04
time (sec)	N/A	0.029	0.406	0.175	0.311	0.363	0.000	3.922	4.196

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	158	93	126	0	101	-1
N.S.	1	1.00	0.94	1.86	1.09	1.48	0.00	1.19	-0.01
time (sec)	N/A	0.057	0.504	0.120	0.516	0.345	0.000	3.987	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	92	165	125	141	0	233	-1
N.S.	1	1.00	0.84	1.50	1.14	1.28	0.00	2.12	-0.01
time (sec)	N/A	0.092	0.566	0.116	0.517	0.339	0.000	3.637	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	105	148	174	156	0	233	-1
N.S.	1	1.00	0.78	1.10	1.29	1.16	0.00	1.73	-0.01
time (sec)	N/A	0.125	0.638	0.122	0.524	0.413	0.000	3.492	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	219	424	0	490	0	0	1089
N.S.	1	1.00	1.05	2.04	0.00	2.36	0.00	0.00	5.24
time (sec)	N/A	0.248	0.299	0.149	0.000	12.958	0.000	0.000	5.748

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	906	905	15669	19955	0	991	0	0	-1
N.S.	1	1.00	17.29	22.03	0.00	1.09	0.00	0.00	-0.00
time (sec)	N/A	1.604	33.405	0.223	0.000	0.113	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	668	668	9965	12761	0	737	0	0	-1
N.S.	1	1.00	14.92	19.10	0.00	1.10	0.00	0.00	-0.00
time (sec)	N/A	0.717	33.155	0.150	0.000	0.108	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	749	746	13240	8221	0	790	0	0	-1
N.S.	1	1.00	17.68	10.98	0.00	1.05	0.00	0.00	-0.00
time (sec)	N/A	0.902	32.898	0.176	0.000	0.117	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	712	711	8456	21038	0	1353	0	0	-1
N.S.	1	1.00	11.88	29.55	0.00	1.90	0.00	0.00	-0.00
time (sec)	N/A	0.773	32.974	0.181	0.000	0.142	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	992	989	12997	48427	0	2405	0	0	-1
N.S.	1	1.00	13.10	48.82	0.00	2.42	0.00	0.00	-0.00
time (sec)	N/A	1.171	33.446	0.207	0.000	0.217	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1363	1363	19853	88790	0	4494	0	0	-1
N.S.	1	1.00	14.57	65.14	0.00	3.30	0.00	0.00	-0.00
time (sec)	N/A	2.491	34.050	0.293	0.000	0.316	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1904	1904	29140	153623	0	7746	0	0	-1
N.S.	1	1.00	15.30	80.68	0.00	4.07	0.00	0.00	-0.00
time (sec)	N/A	3.748	35.244	0.467	0.000	0.767	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	724	724	1314	14084	0	737	0	0	-1
N.S.	1	1.00	1.81	19.45	0.00	1.02	0.00	0.00	-0.00
time (sec)	N/A	1.059	31.555	0.184	0.000	0.138	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	992	8161	0	554	0	0	-1
N.S.	1	1.00	1.78	14.65	0.00	0.99	0.00	0.00	-0.00
time (sec)	N/A	0.530	29.250	0.176	0.000	0.093	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	470	980	4251	0	441	0	0	-1
N.S.	1	1.00	2.08	9.03	0.00	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.296	28.573	0.176	0.000	0.094	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	506	772	6053	0	687	0	0	-1
N.S.	1	1.00	1.52	11.92	0.00	1.35	0.00	0.00	-0.00
time (sec)	N/A	0.414	24.688	0.168	0.000	0.099	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	684	680	1194	20481	0	1270	0	0	-1
N.S.	1	0.99	1.75	29.94	0.00	1.86	0.00	0.00	-0.00
time (sec)	N/A	0.713	29.999	0.160	0.000	0.118	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	944	942	1746	46697	0	2611	0	0	-1
N.S.	1	1.00	1.85	49.47	0.00	2.77	0.00	0.00	-0.00
time (sec)	N/A	1.342	32.924	0.206	0.000	0.200	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	510	508	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.508	1.769	0.059	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	496	494	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.395	1.449	0.059	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	590	588	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.473	3.142	0.066	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	119	36	59	75	211	141	58
N.S.	1	1.00	2.90	0.88	1.44	1.83	5.15	3.44	1.41
time (sec)	N/A	0.033	0.282	0.122	0.320	0.348	3.356	4.139	4.246

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	39	69	82	280	191	78
N.S.	1	1.00	0.74	0.85	1.50	1.78	6.09	4.15	1.70
time (sec)	N/A	0.041	0.343	0.203	0.344	0.369	58.708	3.180	4.395

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	43	51	100	123	483	314	120
N.S.	1	1.00	0.75	0.89	1.75	2.16	8.47	5.51	2.11
time (sec)	N/A	0.074	0.584	0.165	0.330	0.396	64.379	3.928	4.457

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	167	8419	1875	1764	2281	2383	2026
N.S.	1	1.00	8.35	420.95	93.75	88.20	114.05	119.15	101.30
time (sec)	N/A	0.278	0.302	0.507	0.321	0.409	0.168	5.703	4.865

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	19	18	18	20	20	18
N.S.	1	1.00	1.00	0.73	0.69	0.69	0.77	0.77	0.69
time (sec)	N/A	0.015	0.004	0.122	0.299	0.371	0.030	5.894	0.051

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	282	1161	0	711	0	330	-1
N.S.	1	1.00	0.82	3.36	0.00	2.05	0.00	0.95	-0.00
time (sec)	N/A	0.472	1.095	0.155	0.000	0.447	0.000	4.881	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	199	669	0	511	0	228	-1
N.S.	1	1.00	0.81	2.73	0.00	2.09	0.00	0.93	-0.00
time (sec)	N/A	0.248	0.735	0.148	0.000	0.408	0.000	3.913	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	139	375	0	345	0	149	-1
N.S.	1	1.00	0.79	2.12	0.00	1.95	0.00	0.84	-0.01
time (sec)	N/A	0.141	0.548	0.150	0.000	0.439	0.000	3.850	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	135	223	0	739	0	0	-1
N.S.	1	1.00	0.87	1.44	0.00	4.77	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.637	0.149	0.000	1.804	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	122	175	0	711	0	171	166
N.S.	1	1.00	0.88	1.26	0.00	5.12	0.00	1.23	1.19
time (sec)	N/A	0.132	0.752	0.155	0.000	1.464	0.000	3.793	4.460

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	170	245	0	803	0	352	-1
N.S.	1	1.00	1.07	1.54	0.00	5.05	0.00	2.21	-0.01
time (sec)	N/A	0.141	1.205	0.156	0.000	1.987	0.000	3.959	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	178	424	0	385	0	689	-1
N.S.	1	1.00	0.96	2.28	0.00	2.07	0.00	3.70	-0.01
time (sec)	N/A	0.187	0.966	0.160	0.000	2.096	0.000	3.682	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	237	749	0	545	0	1448	-1
N.S.	1	1.00	0.88	2.77	0.00	2.02	0.00	5.36	-0.00
time (sec)	N/A	0.283	1.620	0.165	0.000	10.122	0.000	4.309	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	328	1291	0	753	0	2177	-1
N.S.	1	1.00	0.88	3.48	0.00	2.03	0.00	5.87	-0.00
time (sec)	N/A	0.482	2.182	0.160	0.000	14.218	0.000	6.682	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	212	208	199	203	230	230	196
N.S.	1	1.00	0.82	0.81	0.77	0.79	0.89	0.89	0.76
time (sec)	N/A	0.163	0.027	0.122	0.340	0.329	0.027	4.689	4.204

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	136	146	145	142	158	160	137
N.S.	1	1.00	0.87	0.93	0.92	0.90	1.01	1.02	0.87
time (sec)	N/A	0.107	0.023	0.125	0.297	0.382	0.019	3.908	4.108

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	84	86	81	87	90	77
N.S.	1	1.00	1.00	0.90	0.92	0.87	0.94	0.97	0.83
time (sec)	N/A	0.070	0.009	0.048	0.305	0.341	0.012	4.409	0.047

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	34	34	37	34	34
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.88	0.81	0.81
time (sec)	N/A	0.015	0.002	0.027	0.299	0.340	0.007	4.022	0.026

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	179	249	207	212	235	228	260
N.S.	1	1.00	0.79	1.09	0.91	0.93	1.03	1.00	1.14
time (sec)	N/A	0.122	0.040	0.118	0.306	0.333	0.223	3.735	4.139

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	223	240	214	293	238	308	363
N.S.	1	1.00	0.98	1.05	0.94	1.29	1.04	1.35	1.59
time (sec)	N/A	0.126	0.057	0.113	0.351	0.348	0.450	3.961	4.181

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	204	236	221	329	248	216	297
N.S.	1	1.00	0.88	1.02	0.96	1.42	1.07	0.94	1.29
time (sec)	N/A	0.129	0.044	0.102	0.286	0.368	0.891	3.972	0.092

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	277	264	254	257	298	296	251
N.S.	1	1.00	0.71	0.68	0.65	0.66	0.76	0.76	0.64
time (sec)	N/A	0.246	0.029	0.134	0.295	0.382	0.032	4.124	4.247

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	201	186	185	180	206	206	175
N.S.	1	1.00	1.00	0.93	0.92	0.90	1.02	1.02	0.87
time (sec)	N/A	0.157	0.019	0.125	0.299	0.356	0.022	3.193	0.108

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	121	108	114	103	112	116	101
N.S.	1	1.00	1.00	0.89	0.94	0.85	0.93	0.96	0.83
time (sec)	N/A	0.103	0.012	0.114	0.307	0.405	0.015	3.965	4.173

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	45	44	44	56	44	44
N.S.	1	1.00	1.00	0.75	0.73	0.73	0.93	0.73	0.73
time (sec)	N/A	0.022	0.001	0.102	0.318	0.373	0.010	3.881	0.035

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	262	416	328	341	372	378	434
N.S.	1	1.00	0.74	1.18	0.93	0.97	1.06	1.07	1.23
time (sec)	N/A	0.209	0.084	0.111	0.315	0.374	0.343	3.382	0.079

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	342	401	335	449	393	459	939
N.S.	1	1.00	0.97	1.14	0.95	1.27	1.11	1.30	2.66
time (sec)	N/A	0.207	0.095	0.099	0.357	0.382	0.693	3.446	4.217

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	311	391	342	501	394	354	771
N.S.	1	1.00	0.88	1.10	0.97	1.42	1.11	1.00	2.18
time (sec)	N/A	0.217	0.067	0.111	0.305	0.377	1.503	4.824	0.127

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	344	382	353	539	401	345	560
N.S.	1	1.00	0.96	1.06	0.98	1.50	1.11	0.96	1.56
time (sec)	N/A	0.210	0.082	0.107	0.318	0.379	2.733	3.849	4.284

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	178	222	197	197	450	212	397
N.S.	1	1.00	0.81	1.00	0.89	0.89	2.04	0.96	1.80
time (sec)	N/A	0.115	0.081	0.262	0.541	0.365	0.738	3.196	4.182

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	130	147	140	137	303	145	223
N.S.	1	1.00	0.83	0.94	0.90	0.88	1.94	0.93	1.43
time (sec)	N/A	0.103	0.056	0.148	0.558	0.392	0.516	3.296	0.099

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	86	83	90	85	163	88	107
N.S.	1	1.00	0.87	0.84	0.91	0.86	1.65	0.89	1.08
time (sec)	N/A	0.065	0.037	0.129	0.528	0.384	0.318	4.951	0.070

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	50	44	43	43	61	43	45
N.S.	1	1.00	0.89	0.79	0.77	0.77	1.09	0.77	0.80
time (sec)	N/A	0.029	0.013	0.128	0.541	0.375	0.042	4.703	0.042

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	146	142	159	163	0	158	713
N.S.	1	1.00	0.87	0.85	0.95	0.97	0.00	0.94	4.24
time (sec)	N/A	0.125	0.075	0.237	0.583	0.435	0.000	4.622	6.389

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	233	213	274	398	0	355	312
N.S.	1	1.00	1.00	0.91	1.18	1.71	0.00	1.52	1.34
time (sec)	N/A	0.155	0.108	0.265	0.536	0.435	0.000	4.428	4.669

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	278	298	455	675	0	406	493
N.S.	1	1.00	0.88	0.94	1.44	2.13	0.00	1.28	1.56
time (sec)	N/A	0.184	0.279	0.247	0.536	0.551	0.000	3.705	4.763

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	209	214	203	333	444	206	333
N.S.	1	1.00	1.11	1.13	1.07	1.76	2.35	1.09	1.76
time (sec)	N/A	0.163	0.102	0.181	0.538	0.385	1.433	4.297	0.148

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	150	145	146	234	298	145	211
N.S.	1	1.00	1.07	1.04	1.04	1.67	2.13	1.04	1.51
time (sec)	N/A	0.128	0.078	0.149	0.523	0.359	1.212	2.626	0.114

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	96	87	96	142	165	94	115
N.S.	1	1.00	0.99	0.90	0.99	1.46	1.70	0.97	1.19
time (sec)	N/A	0.095	0.047	0.132	0.536	0.367	0.576	5.903	4.152

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	51	52	78	65	52	52
N.S.	1	1.00	0.94	0.81	0.83	1.24	1.03	0.83	0.83
time (sec)	N/A	0.038	0.025	0.130	0.527	0.343	0.061	4.345	4.148

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	186	214	281	462	0	284	330
N.S.	1	1.00	0.83	0.96	1.25	2.06	0.00	1.27	1.47
time (sec)	N/A	0.206	0.107	0.146	0.537	0.432	0.000	3.470	4.612

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	270	303	509	889	0	571	601
N.S.	1	1.00	0.86	0.97	1.63	2.84	0.00	1.82	1.92
time (sec)	N/A	0.395	0.168	0.151	0.577	0.467	0.000	4.767	4.835

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	363	400	775	1459	0	595	887
N.S.	1	1.00	0.88	0.97	1.88	3.54	0.00	1.44	2.15
time (sec)	N/A	0.553	0.253	0.234	0.549	0.629	0.000	3.977	4.940

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	209	209	213	408	469	201	299
N.S.	1	1.00	1.22	1.22	1.25	2.39	2.74	1.18	1.75
time (sec)	N/A	0.213	0.132	0.140	0.610	0.368	3.208	3.111	0.152

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	146	146	154	278	304	144	203
N.S.	1	1.00	1.09	1.09	1.15	2.07	2.27	1.07	1.51
time (sec)	N/A	0.155	0.121	0.145	0.577	0.372	1.986	3.741	4.214

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	107	91	107	163	163	97	125
N.S.	1	1.00	1.04	0.88	1.04	1.58	1.58	0.94	1.21
time (sec)	N/A	0.088	0.057	0.125	0.536	0.402	0.943	7.647	0.120

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	53	47	56	75	61	46	55
N.S.	1	1.00	0.83	0.73	0.88	1.17	0.95	0.72	0.86
time (sec)	N/A	0.031	0.029	0.110	0.536	0.336	0.069	4.551	0.049

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	282	359	537	1019	0	460	641
N.S.	1	1.00	0.86	1.09	1.63	3.10	0.00	1.40	1.95
time (sec)	N/A	0.336	0.196	0.227	0.555	0.546	0.000	4.083	4.793

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	389	473	843	1690	0	762	965
N.S.	1	1.00	0.88	1.07	1.90	3.81	0.00	1.72	2.18
time (sec)	N/A	0.725	0.338	0.146	0.557	0.657	0.000	5.058	4.989

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	80	115	126	83	0	78	170
N.S.	1	1.00	0.56	0.80	0.88	0.58	0.00	0.55	1.19
time (sec)	N/A	0.099	0.856	0.134	0.528	0.355	0.000	4.081	1.716

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	75	98	109	78	0	73	153
N.S.	1	1.00	0.60	0.79	0.88	0.63	0.00	0.59	1.23
time (sec)	N/A	0.059	0.385	0.112	0.487	0.378	0.000	2.699	0.767

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	103	127	128	125	0	129	-1
N.S.	1	1.00	0.69	0.85	0.86	0.84	0.00	0.87	-0.01
time (sec)	N/A	0.139	0.428	0.019	0.483	0.428	0.000	3.151	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	110	152	132	143	0	531	-1
N.S.	1	1.00	0.74	1.02	0.89	0.96	0.00	3.56	-0.01
time (sec)	N/A	0.141	0.493	0.011	0.512	0.397	0.000	4.237	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	110	158	143	159	0	258	-1
N.S.	1	1.00	0.73	1.05	0.95	1.05	0.00	1.71	-0.01
time (sec)	N/A	0.143	0.589	0.012	0.488	0.365	0.000	5.248	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	110	165	160	173	0	304	-1
N.S.	1	1.00	0.70	1.04	1.01	1.09	0.00	1.92	-0.01
time (sec)	N/A	0.140	0.679	0.013	0.501	0.379	0.000	4.746	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	110	167	181	189	0	327	-1
N.S.	1	1.00	0.67	1.01	1.10	1.15	0.00	1.98	-0.01
time (sec)	N/A	0.141	0.687	0.013	0.501	0.401	0.000	5.517	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	110	188	222	203	0	387	-1
N.S.	1	1.00	0.67	1.14	1.35	1.23	0.00	2.35	-0.01
time (sec)	N/A	0.140	0.794	0.016	0.504	0.364	0.000	3.767	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	86	195	250	156	0	405	-1
N.S.	1	1.00	0.51	1.15	1.48	0.92	0.00	2.40	-0.01
time (sec)	N/A	0.131	0.772	0.166	0.515	0.358	0.000	3.241	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	91	216	301	171	0	456	-1
N.S.	1	1.00	0.47	1.11	1.55	0.88	0.00	2.35	-0.01
time (sec)	N/A	0.144	1.012	0.172	0.510	0.365	0.000	3.088	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	90	134	155	93	0	88	-1
N.S.	1	1.00	0.54	0.81	0.93	0.56	0.00	0.53	-0.01
time (sec)	N/A	0.107	0.705	0.136	0.494	0.360	0.000	5.285	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	85	117	138	88	0	83	-1
N.S.	1	1.00	0.58	0.80	0.94	0.60	0.00	0.56	-0.01
time (sec)	N/A	0.065	0.569	0.147	0.501	0.346	0.000	5.208	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	113	183	157	135	0	139	-1
N.S.	1	1.00	0.66	1.06	0.91	0.78	0.00	0.81	-0.01
time (sec)	N/A	0.155	0.666	0.010	0.519	0.380	0.000	4.942	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	120	208	161	153	0	707	-1
N.S.	1	1.00	0.70	1.21	0.94	0.89	0.00	4.11	-0.01
time (sec)	N/A	0.162	0.721	0.016	0.510	0.421	0.000	4.185	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	120	214	172	169	0	268	-1
N.S.	1	1.00	0.69	1.23	0.99	0.97	0.00	1.54	-0.01
time (sec)	N/A	0.163	0.819	0.016	0.539	0.366	0.000	3.891	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	120	221	189	183	0	314	-1
N.S.	1	1.00	0.66	1.22	1.04	1.01	0.00	1.73	-0.01
time (sec)	N/A	0.159	0.862	0.022	0.521	0.388	0.000	3.746	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	120	204	210	199	0	503	-1
N.S.	1	1.00	0.64	1.09	1.12	1.06	0.00	2.68	-0.01
time (sec)	N/A	0.164	0.829	0.016	0.523	0.427	0.000	4.186	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	120	225	251	213	0	406	-1
N.S.	1	1.00	0.62	1.15	1.29	1.09	0.00	2.08	-0.01
time (sec)	N/A	0.164	0.903	0.018	0.568	0.421	0.000	4.046	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	120	246	297	229	0	452	-1
N.S.	1	1.00	0.62	1.26	1.52	1.17	0.00	2.32	-0.01
time (sec)	N/A	0.167	0.969	0.019	0.535	0.392	0.000	3.169	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	120	267	348	243	0	489	-1
N.S.	1	1.00	0.62	1.37	1.78	1.25	0.00	2.51	-0.01
time (sec)	N/A	0.162	1.206	0.023	0.527	0.442	0.000	5.372	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	70	95	96	73	0	68	-1
N.S.	1	1.00	0.58	0.79	0.80	0.61	0.00	0.57	-0.01
time (sec)	N/A	0.088	0.344	0.149	0.491	0.370	0.000	5.845	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	65	79	80	68	0	63	-1
N.S.	1	1.00	0.64	0.78	0.79	0.67	0.00	0.62	-0.01
time (sec)	N/A	0.053	0.301	0.151	0.491	0.352	0.000	3.649	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	93	92	99	115	0	119	-1
N.S.	1	1.00	0.74	0.73	0.79	0.91	0.00	0.94	-0.01
time (sec)	N/A	0.129	0.348	0.010	0.496	0.412	0.000	3.830	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	100	96	103	133	0	339	-1
N.S.	1	1.00	0.79	0.76	0.82	1.06	0.00	2.69	-0.01
time (sec)	N/A	0.126	0.511	0.010	0.504	0.370	0.000	3.679	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	100	102	114	149	0	248	-1
N.S.	1	1.00	0.78	0.80	0.89	1.16	0.00	1.94	-0.01
time (sec)	N/A	0.124	0.492	0.012	0.494	0.375	0.000	3.376	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	100	109	131	163	0	285	-1
N.S.	1	1.00	0.74	0.81	0.97	1.21	0.00	2.11	-0.01
time (sec)	N/A	0.125	0.559	0.015	0.542	0.380	0.000	3.845	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	76	116	149	125	0	164	-1
N.S.	1	1.00	0.55	0.83	1.07	0.90	0.00	1.18	-0.01
time (sec)	N/A	0.123	0.569	0.015	0.545	0.361	0.000	4.023	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	75	132	114	102	0	72	-1
N.S.	1	1.00	0.60	1.06	0.92	0.82	0.00	0.58	-0.01
time (sec)	N/A	0.094	0.519	0.011	0.549	0.371	0.000	3.830	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	70	115	97	97	0	67	-1
N.S.	1	1.00	0.68	1.12	0.94	0.94	0.00	0.65	-0.01
time (sec)	N/A	0.062	0.469	0.160	0.499	0.348	0.000	4.156	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	65	98	80	92	0	62	-1
N.S.	1	1.00	0.79	1.20	0.98	1.12	0.00	0.76	-0.01
time (sec)	N/A	0.036	0.424	0.127	0.517	0.349	0.000	4.077	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	93	148	99	149	0	118	-1
N.S.	1	1.00	0.92	1.47	0.98	1.48	0.00	1.17	-0.01
time (sec)	N/A	0.096	0.479	0.007	0.503	0.436	0.000	4.300	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	110	152	116	157	0	225	-1
N.S.	1	1.00	1.02	1.41	1.07	1.45	0.00	2.08	-0.01
time (sec)	N/A	0.099	0.553	0.011	0.510	0.385	0.000	2.988	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	76	144	149	126	0	220	-1
N.S.	1	1.00	0.68	1.29	1.33	1.12	0.00	1.96	-0.01
time (sec)	N/A	0.093	0.487	0.021	0.532	0.346	0.000	3.354	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	81	151	217	141	0	271	-1
N.S.	1	1.00	0.59	1.10	1.58	1.03	0.00	1.98	-0.01
time (sec)	N/A	0.126	0.556	0.015	0.513	0.352	0.000	4.034	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	75	180	219	122	0	71	-1
N.S.	1	1.00	0.71	1.71	2.09	1.16	0.00	0.68	-0.01
time (sec)	N/A	0.081	0.549	0.013	0.499	0.368	0.000	5.708	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	70	163	202	117	0	66	-1
N.S.	1	1.00	0.81	1.90	2.35	1.36	0.00	0.77	-0.01
time (sec)	N/A	0.050	0.675	0.012	0.515	0.355	0.000	3.845	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	65	146	185	112	0	62	-1
N.S.	1	1.00	0.96	2.15	2.72	1.65	0.00	0.91	-0.01
time (sec)	N/A	0.032	0.499	0.008	0.498	0.350	0.000	4.875	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	69	190	110	126	0	92	-1
N.S.	1	1.00	0.81	2.24	1.29	1.48	0.00	1.08	-0.01
time (sec)	N/A	0.093	0.528	0.010	0.505	0.386	0.000	5.486	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	81	194	127	141	0	206	-1
N.S.	1	1.00	0.74	1.76	1.15	1.28	0.00	1.87	-0.01
time (sec)	N/A	0.097	0.667	0.018	0.506	0.378	0.000	4.946	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	94	200	178	155	0	228	-1
N.S.	1	1.00	0.70	1.48	1.32	1.15	0.00	1.69	-0.01
time (sec)	N/A	0.130	0.627	0.012	0.505	0.361	0.000	4.368	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	91	207	246	170	0	279	-1
N.S.	1	1.00	0.57	1.29	1.54	1.06	0.00	1.74	-0.01
time (sec)	N/A	0.173	0.717	0.014	0.519	0.373	0.000	4.206	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	315	1406	0	1373	0	460	-1
N.S.	1	1.00	0.89	3.97	0.00	3.88	0.00	1.30	-0.00
time (sec)	N/A	0.244	1.917	0.011	0.000	24.428	0.000	4.555	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	319	1147	0	1385	0	482	-1
N.S.	1	1.00	0.90	3.25	0.00	3.92	0.00	1.37	-0.00
time (sec)	N/A	0.222	10.363	0.170	0.000	24.676	0.000	3.439	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	588	588	2576	5924	2267	4176	136733	10960	2500
N.S.	1	1.00	4.38	10.07	3.86	7.10	232.54	18.64	4.25
time (sec)	N/A	0.229	2.794	0.212	0.335	0.504	41.408	4.023	8.392

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	432	1476	3222	1402	2377	65193	6223	2500
N.S.	1	1.00	3.42	7.46	3.25	5.50	150.91	14.41	5.79
time (sec)	N/A	0.149	1.586	0.164	0.309	0.374	15.248	6.546	6.050

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	743	1220	785	1190	26165	3098	1425
N.S.	1	1.00	2.54	4.18	2.69	4.08	89.61	10.61	4.88
time (sec)	N/A	0.102	0.338	0.101	0.315	0.354	5.376	3.890	5.087

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	221	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.342	0.621	0.067	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.597	0.199	0.068	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	528	528	488	769	0	3708	0	635	1027
N.S.	1	1.00	0.92	1.46	0.00	7.02	0.00	1.20	1.95
time (sec)	N/A	0.842	0.692	0.245	0.000	0.428	0.000	5.367	6.175

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	765	765	754	1086	0	2646	0	982	2779
N.S.	1	1.00	0.99	1.42	0.00	3.46	0.00	1.28	3.63
time (sec)	N/A	4.675	0.471	0.286	0.000	0.639	0.000	2.750	7.261

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	99	166	177	97	0	92	221
N.S.	1	1.00	0.48	0.80	0.85	0.47	0.00	0.44	1.06
time (sec)	N/A	0.220	0.730	0.199	0.504	0.361	0.000	2.856	6.306

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	89	132	143	87	0	82	187
N.S.	1	1.00	0.54	0.80	0.86	0.52	0.00	0.49	1.13
time (sec)	N/A	0.125	0.505	0.172	0.486	0.366	0.000	2.738	6.009

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	79	98	109	77	0	72	153
N.S.	1	1.00	0.64	0.79	0.88	0.62	0.00	0.58	1.23
time (sec)	N/A	0.071	0.360	0.138	0.486	0.339	0.000	3.036	5.378

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	234	403	500	304	0	144	-1
N.S.	1	1.00	1.25	2.16	2.67	1.63	0.00	0.77	-0.01
time (sec)	N/A	0.231	0.347	0.795	0.534	0.364	0.000	3.355	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	427	1084	0	378	0	0	-1
N.S.	1	1.00	2.15	5.45	0.00	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.517	0.788	0.000	0.379	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	602	2342	0	390	0	378	-1
N.S.	1	1.00	2.83	11.00	0.00	1.83	0.00	1.77	-0.00
time (sec)	N/A	0.156	0.716	0.730	0.000	0.402	0.000	4.063	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	109	185	206	107	0	102	-1
N.S.	1	1.00	0.47	0.80	0.89	0.46	0.00	0.44	-0.00
time (sec)	N/A	0.232	0.886	0.179	0.497	0.361	0.000	3.340	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	99	151	172	97	0	92	-1
N.S.	1	1.00	0.52	0.80	0.91	0.51	0.00	0.49	-0.01
time (sec)	N/A	0.142	0.712	0.177	0.500	0.366	0.000	5.170	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	89	117	138	87	0	82	-1
N.S.	1	1.00	0.61	0.80	0.94	0.59	0.00	0.56	-0.01
time (sec)	N/A	0.082	0.518	0.147	0.486	0.369	0.000	4.226	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	254	730	535	326	0	154	-1
N.S.	1	1.00	1.21	3.48	2.55	1.55	0.00	0.73	-0.00
time (sec)	N/A	0.201	0.556	0.789	0.567	0.387	0.000	4.977	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	447	1828	0	378	0	0	-1
N.S.	1	1.00	2.01	8.23	0.00	1.70	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.701	0.775	0.000	0.384	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	636	3828	0	447	0	0	-1
N.S.	1	1.00	2.72	16.36	0.00	1.91	0.00	0.00	-0.00
time (sec)	N/A	0.200	0.808	0.802	0.000	0.442	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	89	147	148	87	0	82	-1
N.S.	1	1.00	0.48	0.79	0.80	0.47	0.00	0.44	-0.01
time (sec)	N/A	0.195	0.612	0.171	0.513	0.354	0.000	4.149	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	79	113	114	77	0	72	-1
N.S.	1	1.00	0.55	0.79	0.80	0.54	0.00	0.50	-0.01
time (sec)	N/A	0.123	0.437	0.173	0.507	0.379	0.000	4.562	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	69	79	80	67	0	62	-1
N.S.	1	1.00	0.68	0.78	0.79	0.66	0.00	0.61	-0.01
time (sec)	N/A	0.065	0.306	0.144	0.496	0.346	0.000	4.157	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	211	204	465	297	0	125	-1
N.S.	1	1.00	1.29	1.24	2.84	1.81	0.00	0.76	-0.01
time (sec)	N/A	0.136	0.305	0.774	0.521	0.375	0.000	3.925	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	352	510	0	330	0	276	-1
N.S.	1	1.00	1.98	2.87	0.00	1.85	0.00	1.55	-0.01
time (sec)	N/A	0.117	0.435	0.701	0.000	0.369	0.000	3.402	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	433	1194	0	390	0	378	-1
N.S.	1	1.00	1.91	5.26	0.00	1.72	0.00	1.67	-0.00
time (sec)	N/A	0.169	0.672	0.738	0.000	0.392	0.000	3.782	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	89	166	148	112	0	81	-1
N.S.	1	1.00	0.54	1.00	0.89	0.67	0.00	0.49	-0.01
time (sec)	N/A	0.150	0.544	0.191	0.496	0.356	0.000	3.470	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	79	132	114	102	0	71	-1
N.S.	1	1.00	0.64	1.06	0.92	0.82	0.00	0.57	-0.01
time (sec)	N/A	0.101	0.628	0.191	0.497	0.355	0.000	3.703	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	69	98	80	92	0	62	-1
N.S.	1	1.00	0.84	1.20	0.98	1.12	0.00	0.76	-0.01
time (sec)	N/A	0.053	0.416	0.160	0.487	0.356	0.000	4.900	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	199	489	777	333	0	112	-1
N.S.	1	1.00	1.20	2.95	4.68	2.01	0.00	0.67	-0.01
time (sec)	N/A	0.129	0.405	0.692	0.544	0.391	0.000	4.117	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	416	1214	0	392	0	295	-1
N.S.	1	1.00	1.93	5.65	0.00	1.82	0.00	1.37	-0.00
time (sec)	N/A	0.201	0.702	0.747	0.000	0.359	0.000	4.371	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	607	2600	0	452	0	397	-1
N.S.	1	1.00	2.43	10.40	0.00	1.81	0.00	1.59	-0.00
time (sec)	N/A	0.201	0.956	0.744	0.000	0.488	0.000	4.132	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	242	0	0	0	0	0	-1
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.301	0.025	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	236	0	0	0	0	0	-1
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.236	0.028	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	302	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.275	0.408	0.026	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [278] had the largest ratio of [75]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.00	34	0.147
2	A	6	5	1.00	32	0.156
3	A	5	5	1.00	27	0.185
4	A	5	5	1.00	34	0.147
5	A	5	5	1.00	34	0.147
6	A	5	5	1.00	34	0.147
7	A	8	6	1.00	34	0.176
8	A	4	4	1.00	34	0.118
9	A	5	4	1.00	34	0.118
10	A	7	4	1.00	34	0.118
11	A	6	4	1.00	34	0.118
12	A	5	4	1.00	32	0.125
13	A	4	4	1.00	27	0.148
14	A	4	4	1.00	34	0.118
15	A	7	5	1.00	34	0.147
16	A	4	4	1.00	34	0.118
17	A	5	4	1.00	34	0.118
18	A	2	1	0.99	25	0.040
19	A	2	1	0.99	25	0.040
20	A	2	1	1.00	23	0.043
21	A	2	1	1.00	18	0.056
22	A	2	1	0.99	25	0.040
23	A	2	1	0.99	25	0.040
24	A	2	1	0.99	25	0.040
25	A	3	2	0.99	27	0.074

Continued on next page

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	3	2	1.00	27	0.074
27	A	3	2	1.00	25	0.080
28	A	3	2	1.00	20	0.100
29	A	2	1	0.99	27	0.037
30	A	2	1	0.99	27	0.037
31	A	2	1	0.99	27	0.037
32	A	3	2	0.99	27	0.074
33	A	3	2	1.00	27	0.074
34	A	3	2	1.00	25	0.080
35	A	3	2	1.00	20	0.100
36	A	2	1	0.99	27	0.037
37	A	2	1	0.99	27	0.037
38	A	2	1	0.99	27	0.037
39	A	1	1	1.00	32	0.031
40	A	1	1	1.00	31	0.032
41	A	1	1	1.00	34	0.029
42	A	1	1	1.00	33	0.030
43	A	5	4	0.99	27	0.148
44	A	5	4	0.99	27	0.148
45	A	5	4	1.00	25	0.160
46	A	5	4	1.00	20	0.200
47	A	5	4	1.00	27	0.148
48	A	5	4	1.00	27	0.148
49	A	5	4	1.00	27	0.148
50	A	6	5	1.00	27	0.185
51	A	5	5	1.00	27	0.185
52	A	4	4	1.00	25	0.160
53	A	3	3	1.00	20	0.150
54	A	6	5	1.00	27	0.185
55	A	6	5	0.99	27	0.185
56	A	6	5	1.00	27	0.185
57	A	5	5	1.00	27	0.185
58	A	3	3	1.12	27	0.111
59	A	3	3	1.00	25	0.120
60	A	4	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	7	6	1.00	27	0.222
62	A	7	5	0.99	27	0.185
63	A	7	5	1.00	27	0.185
64	A	4	4	1.00	27	0.148
65	A	4	4	1.13	27	0.148
66	A	4	4	1.00	27	0.148
67	A	4	4	1.00	25	0.160
68	A	5	4	1.00	20	0.200
69	A	6	5	1.00	17	0.294
70	A	5	5	1.00	17	0.294
71	A	4	4	1.00	15	0.267
72	A	3	3	1.00	14	0.214
73	A	6	5	1.00	17	0.294
74	A	6	5	1.00	17	0.294
75	A	6	5	1.00	17	0.294
76	A	3	3	1.83	16	0.188
77	A	3	3	1.00	18	0.167
78	A	7	6	0.99	29	0.207
79	A	6	6	1.00	29	0.207
80	A	5	5	1.00	27	0.185
81	A	5	5	1.00	22	0.227
82	A	7	6	1.00	29	0.207
83	A	7	6	0.98	29	0.207
84	A	7	6	1.00	29	0.207
85	A	7	6	1.00	29	0.207
86	A	5	5	1.00	29	0.172
87	A	6	6	1.00	29	0.207
88	A	8	6	1.00	29	0.207
89	A	7	6	1.00	29	0.207
90	A	6	5	1.00	27	0.185
91	A	6	5	1.00	22	0.227
92	A	8	6	1.00	29	0.207
93	A	8	6	0.99	29	0.207
94	A	8	7	0.98	29	0.241
95	A	8	6	0.99	29	0.207

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	8	7	1.00	29	0.241
97	A	8	6	1.00	29	0.207
98	A	6	5	1.00	29	0.172
99	A	7	6	1.00	29	0.207
100	A	7	5	1.00	22	0.227
101	A	6	5	0.99	29	0.172
102	A	5	5	1.00	29	0.172
103	A	4	4	0.99	27	0.148
104	A	4	4	1.00	22	0.182
105	A	6	5	1.00	29	0.172
106	A	6	5	1.00	29	0.172
107	A	4	4	1.00	29	0.138
108	A	5	5	1.00	29	0.172
109	A	4	4	1.00	29	0.138
110	A	4	4	1.00	27	0.148
111	A	4	4	1.00	22	0.182
112	A	4	4	1.00	29	0.138
113	A	4	4	1.00	29	0.138
114	A	5	5	0.99	29	0.172
115	A	3	3	1.00	22	0.136
116	A	4	4	1.00	22	0.182
117	A	5	4	1.00	22	0.182
118	A	5	4	1.00	29	0.138
119	A	4	4	1.00	29	0.138
120	A	3	3	1.00	27	0.111
121	A	5	5	1.00	29	0.172
122	A	5	5	1.00	29	0.172
123	A	4	4	1.00	29	0.138
124	A	5	4	1.00	29	0.138
125	A	4	4	1.00	29	0.138
126	A	3	3	1.00	27	0.111
127	A	4	4	1.00	29	0.138
128	A	4	4	1.00	29	0.138
129	A	5	5	1.00	29	0.172
130	A	4	3	1.00	29	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	4	3	1.00	29	0.103
132	A	2	2	1.00	27	0.074
133	A	5	5	1.00	29	0.172
134	A	5	4	1.00	29	0.138
135	A	6	5	1.00	29	0.172
136	A	6	4	0.99	27	0.148
137	A	6	4	1.00	29	0.138
138	A	5	5	1.00	31	0.161
139	A	6	6	1.00	69	0.087
140	A	2	1	1.00	20	0.050
141	A	2	1	1.00	20	0.050
142	A	2	1	1.00	20	0.050
143	A	2	1	1.00	18	0.056
144	A	6	5	1.00	20	0.250
145	A	4	4	1.00	20	0.200
146	A	5	5	1.00	20	0.250
147	A	6	5	1.00	20	0.250
148	A	6	5	1.00	30	0.167
149	A	6	5	1.00	30	0.167
150	A	6	5	1.00	28	0.179
151	A	6	5	1.00	23	0.217
152	A	6	5	1.00	30	0.167
153	A	6	5	1.00	30	0.167
154	A	6	5	1.00	30	0.167
155	A	6	6	1.00	30	0.200
156	A	5	5	1.00	28	0.179
157	A	4	4	1.00	23	0.174
158	A	7	6	1.00	30	0.200
159	A	7	6	1.00	30	0.200
160	A	7	6	1.00	20	0.300
161	A	7	6	1.00	20	0.300
162	A	5	5	1.00	18	0.278
163	A	4	4	1.00	17	0.235
164	A	7	6	1.00	20	0.300
165	A	7	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	7	6	1.00	20	0.300
167	A	1	1	1.00	16	0.062
168	A	6	5	1.00	14	0.357
169	A	6	5	1.00	16	0.312
170	A	3	2	1.00	18	0.111
171	A	5	3	1.00	16	0.188
172	A	5	3	1.00	19	0.158
173	A	6	5	1.00	23	0.217
174	A	5	3	1.00	19	0.158
175	A	2	2	1.00	23	0.087
176	A	4	4	1.00	17	0.235
177	A	1	1	1.00	19	0.053
178	A	7	5	1.00	22	0.227
179	A	6	5	1.00	22	0.227
180	A	5	5	1.00	22	0.227
181	A	4	4	1.00	22	0.182
182	A	4	4	1.00	22	0.182
183	A	3	3	1.00	22	0.136
184	A	4	4	1.00	22	0.182
185	A	5	4	1.00	22	0.182
186	A	7	6	1.00	32	0.188
187	A	6	6	0.99	32	0.188
188	A	5	5	1.00	30	0.167
189	A	5	5	1.00	25	0.200
190	A	7	6	1.00	32	0.188
191	A	7	6	0.99	32	0.188
192	A	7	6	1.00	32	0.188
193	A	7	6	1.00	32	0.188
194	A	5	5	1.00	32	0.156
195	A	6	6	1.00	32	0.188
196	A	8	6	1.00	32	0.188
197	A	7	6	0.99	32	0.188
198	A	6	5	1.00	30	0.167
199	A	6	5	1.00	25	0.200
200	A	8	6	1.00	32	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	8	6	0.99	32	0.188
202	A	8	7	0.99	32	0.219
203	A	8	6	1.00	32	0.188
204	A	8	7	1.00	32	0.219
205	A	8	6	1.00	32	0.188
206	A	6	5	1.00	32	0.156
207	A	7	6	1.00	32	0.188
208	A	7	6	1.00	32	0.188
209	A	6	6	1.00	32	0.188
210	A	5	5	1.00	30	0.167
211	A	7	7	1.00	32	0.219
212	A	7	7	1.00	32	0.219
213	A	7	7	1.00	32	0.219
214	A	9	6	1.00	32	0.188
215	A	7	6	1.00	32	0.188
216	A	6	5	1.00	30	0.167
217	A	8	7	1.00	32	0.219
218	A	8	7	1.00	32	0.219
219	A	8	8	1.00	32	0.250
220	A	9	6	1.00	32	0.188
221	A	8	6	1.00	32	0.188
222	A	7	5	1.00	30	0.167
223	A	9	7	1.00	32	0.219
224	A	9	7	1.00	32	0.219
225	A	9	8	1.00	32	0.250
226	A	6	5	1.00	32	0.156
227	A	5	5	1.00	32	0.156
228	A	4	4	1.00	30	0.133
229	A	4	4	1.00	25	0.160
230	A	6	5	1.00	32	0.156
231	A	6	5	0.99	32	0.156
232	A	4	4	1.00	32	0.125
233	A	5	5	1.00	32	0.156
234	A	4	4	1.00	32	0.125
235	A	4	4	1.00	30	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	4	4	1.00	25	0.160
237	A	4	4	1.00	32	0.125
238	A	4	4	0.99	32	0.125
239	A	5	5	0.99	32	0.156
240	A	6	5	1.00	32	0.156
241	A	5	5	1.00	32	0.156
242	A	4	4	1.00	30	0.133
243	A	6	6	1.00	32	0.188
244	A	6	6	1.00	32	0.188
245	A	4	4	1.00	32	0.125
246	A	6	5	1.00	32	0.156
247	A	5	5	1.00	32	0.156
248	A	4	4	1.00	30	0.133
249	A	4	4	1.00	32	0.125
250	A	4	4	1.00	32	0.125
251	A	5	5	1.00	32	0.156
252	A	5	4	1.00	32	0.125
253	A	5	4	1.00	32	0.125
254	A	2	2	1.00	30	0.067
255	A	5	5	1.00	32	0.156
256	A	5	4	1.00	32	0.125
257	A	6	5	1.00	32	0.156
258	A	3	3	1.00	47	0.064
259	A	8	7	1.00	34	0.206
260	A	7	6	1.00	34	0.176
261	A	7	6	1.00	34	0.176
262	A	7	6	1.00	34	0.176
263	A	7	6	1.00	34	0.176
264	A	8	7	1.00	34	0.206
265	A	9	7	1.00	34	0.206
266	A	8	6	1.00	34	0.176
267	A	7	6	1.00	34	0.176
268	A	6	5	1.00	34	0.147
269	A	6	5	1.00	34	0.147
270	A	7	6	0.99	34	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	8	6	1.00	34	0.176
272	A	6	4	1.00	30	0.133
273	A	6	4	1.00	32	0.125
274	A	5	5	1.00	34	0.147
275	A	3	3	1.00	42	0.071
276	A	2	2	1.00	46	0.043
277	A	2	2	1.00	69	0.029
278	A	2	2	1.00	75	0.027
279	A	6	4	1.00	16	0.250
280	A	6	5	1.00	33	0.152
281	A	5	4	1.00	31	0.129
282	A	5	4	1.00	30	0.133
283	A	7	5	1.00	33	0.152
284	A	7	6	1.00	33	0.182
285	A	7	5	1.00	33	0.152
286	A	5	4	1.00	33	0.121
287	A	6	5	1.00	33	0.152
288	A	7	5	1.00	33	0.152
289	A	2	1	1.00	36	0.028
290	A	2	1	1.00	36	0.028
291	A	2	1	1.00	34	0.029
292	A	2	1	1.00	29	0.034
293	A	2	1	1.00	36	0.028
294	A	2	1	1.00	36	0.028
295	A	2	1	1.00	36	0.028
296	A	2	1	1.00	38	0.026
297	A	2	1	1.00	38	0.026
298	A	2	1	1.00	36	0.028
299	A	2	1	1.00	31	0.032
300	A	2	1	1.00	38	0.026
301	A	2	1	1.00	38	0.026
302	A	2	1	1.00	38	0.026
303	A	2	1	1.00	38	0.026
304	A	6	5	1.00	38	0.132
305	A	6	5	1.00	38	0.132

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	6	5	1.00	36	0.139
307	A	6	5	1.00	31	0.161
308	A	6	5	1.00	38	0.132
309	A	6	5	1.00	38	0.132
310	A	6	5	1.00	38	0.132
311	A	7	6	1.00	38	0.158
312	A	7	6	1.00	38	0.158
313	A	7	6	1.00	36	0.167
314	A	7	6	1.00	31	0.194
315	A	7	6	1.00	38	0.158
316	A	7	6	1.00	38	0.158
317	A	7	6	1.00	38	0.158
318	A	8	6	1.00	38	0.158
319	A	8	6	1.00	38	0.158
320	A	6	6	1.00	36	0.167
321	A	5	4	1.00	31	0.129
322	A	8	6	1.00	38	0.158
323	A	8	6	1.00	38	0.158
324	A	7	5	1.00	38	0.132
325	A	7	5	1.00	33	0.152
326	A	9	7	1.00	40	0.175
327	A	9	8	1.00	40	0.200
328	A	9	8	1.00	40	0.200
329	A	9	7	1.00	40	0.175
330	A	9	7	1.00	40	0.175
331	A	9	7	1.00	40	0.175
332	A	7	5	1.00	40	0.125
333	A	8	6	1.00	40	0.150
334	A	8	5	1.00	38	0.132
335	A	8	5	1.00	33	0.152
336	A	10	7	1.00	40	0.175
337	A	10	8	1.00	40	0.200
338	A	10	8	1.00	40	0.200
339	A	10	7	1.00	40	0.175
340	A	10	8	1.00	40	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	10	7	1.00	40	0.175
342	A	10	8	1.00	40	0.200
343	A	10	7	1.00	40	0.175
344	A	6	4	1.00	38	0.105
345	A	6	4	1.00	33	0.121
346	A	8	6	1.00	40	0.150
347	A	8	7	1.00	40	0.175
348	A	8	7	1.00	40	0.175
349	A	8	6	1.00	40	0.150
350	A	6	4	1.00	40	0.100
351	A	7	5	1.00	40	0.125
352	A	6	5	1.00	38	0.132
353	A	5	5	1.00	33	0.152
354	A	7	7	1.00	40	0.175
355	A	7	7	1.00	40	0.175
356	A	5	5	1.00	40	0.125
357	A	6	5	1.00	40	0.125
358	A	6	5	1.00	40	0.125
359	A	5	4	1.00	38	0.105
360	A	5	4	1.00	33	0.121
361	A	5	4	1.00	40	0.100
362	A	5	4	1.00	40	0.100
363	A	6	5	1.00	40	0.125
364	A	7	5	1.00	40	0.125
365	A	5	4	1.00	35	0.114
366	A	5	4	1.00	36	0.111
367	A	2	1	1.00	38	0.026
368	A	2	1	1.00	38	0.026
369	A	2	1	1.00	36	0.028
370	A	4	2	1.00	38	0.053
371	A	5	3	1.00	38	0.079
372	A	6	5	1.00	38	0.132
373	A	6	5	1.00	53	0.094
374	A	11	5	1.00	35	0.143
375	A	9	5	1.00	35	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	7	5	1.00	33	0.152
377	A	9	7	1.00	35	0.200
378	A	9	7	1.00	35	0.200
379	A	7	5	1.00	35	0.143
380	A	12	5	1.00	35	0.143
381	A	10	5	1.00	35	0.143
382	A	8	5	1.00	33	0.152
383	A	10	7	1.00	35	0.200
384	A	10	8	1.00	35	0.229
385	A	10	7	1.00	35	0.200
386	A	10	4	1.00	35	0.114
387	A	8	4	1.00	35	0.114
388	A	6	4	1.00	33	0.121
389	A	8	6	1.00	35	0.171
390	A	6	4	1.00	35	0.114
391	A	7	4	1.00	35	0.114
392	A	9	5	1.00	35	0.143
393	A	7	5	1.00	35	0.143
394	A	5	5	1.00	33	0.152
395	A	6	4	1.00	35	0.114
396	A	7	4	1.00	35	0.114
397	A	8	4	1.00	35	0.114
398	A	7	5	1.00	26	0.192
399	A	7	6	1.00	24	0.250
400	A	11	8	1.00	29	0.276

Chapter 3

Listing of integrals

Local contents

3.1 $\int (d+ex)^2 \left(A+Bx+Cx^2\right) \sqrt{d^2-e^2x^2}\, dx$

Optimal. Leaf size=236

$$\frac{d^2(3Cd^2+4Bde+10Ae^2)\,x\sqrt{d^2-e^2x^2}}{16e^2}-\frac{d(4Cd^2+e(7Bd+10Ae))\,(d^2-e^2x^2)^{3/2}}{15e^3}-\frac{(3Cd^2+2e(2Bd+A}{8e}$$

```
[Out] -1/15*d*(4*C*d^2+e*(10*A*e+7*B*d))*(-e^2*x^2+d^2)^(3/2)/e^3-1/8*(3*C*d^2+2*
e*(A*e+2*B*d))*x*(-e^2*x^2+d^2)^(3/2)/e^2-1/5*(B*e+2*C*d)*x^2*(-e^2*x^2+d^2
)^(3/2)/e-1/6*C*x^3*(-e^2*x^2+d^2)^(3/2)+1/16*d^4*(10*A*e^2+4*B*d*e+3*C*d^2
)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/16*d^2*(10*A*e^2+4*B*d*e+3*C*d^2)*
x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A]
time = 0.29, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1829, 655, 201, 223, 209}

$$\frac{d^4\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(10Ae^2+4Bde+3Cd^2)}{16e^3}+\frac{d^2x\sqrt{d^2-e^2x^2}\,(10Ae^2+4Bde+3Cd^2)}{16e^2}-\frac{x(d^2-e^2x^2)^{3/2}\,(2e(Ae+2Bd)+3Cd^2)}{8e^2}-\frac{d(d^2-e^2x^2)^{3/2}\,(e(10Ae+7Bd)+4Cd^2)}{15e^3}-\frac{x^2(d^2-e^2x^2)^{3/2}\,(Be+2Cd)}{5e}-\frac{1}{6}Cx^3(d^2-e^2x^2)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (d^2*(3*C*d^2 + 4*B*d*e + 10*A*e^2)*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) - (d*(4
*C*d^2 + e*(7*B*d + 10*A*e))*(d^2 - e^2*x^2)^(3/2))/(15*e^3) - ((3*C*d^2 +
2*e*(2*B*d + A*e))*x*(d^2 - e^2*x^2)^(3/2))/(8*e^2) - ((2*C*d + B*e)*x^2*(d
^2 - e^2*x^2)^(3/2))/(5*e) - (C*x^3*(d^2 - e^2*x^2)^(3/2))/6 + (d^4*(3*C*d^
2 + 4*B*d*e + 10*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int (d+ex)^2 \left(A+Bx+Cx^2\right) \sqrt{d^2-e^2x^2}\, dx = -\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \sqrt{d^2-e^2x^2}\,(-6Ad^2e^2-6de^2(Bd$$

$$= -\frac{(2Cd+Be)x^2(d^2-e^2x^2)^{3/2}}{5e} - \frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \sqrt{}}{}$$

$$= -\frac{\left(3Cd^2+2e(2Bd+Ae)\right)x(d^2-e^2x^2)^{3/2}}{8e^2} - \frac{(2Cd+Be)x^2(}{5e}$$

$$= -\frac{d\left(4Cd^2+e(7Bd+10Ae)\right)(d^2-e^2x^2)^{3/2}}{15e^3} - \frac{(3Cd^2+2e(2}{}$$

$$= \frac{d^2(3Cd^2+4Bde+10Ae^2)\,x\sqrt{d^2-e^2x^2}}{16e^2} - \frac{d(4Cd^2+e(7Bd}{}$$

$$= \frac{d^2(3Cd^2+4Bde+10Ae^2)\,x\sqrt{d^2-e^2x^2}}{16e^2} - \frac{d(4Cd^2+e(7Bd}{}$$

$$= \frac{d^2(3Cd^2+4Bde+10Ae^2)\,x\sqrt{d^2-e^2x^2}}{16e^2} - \frac{d(4Cd^2+e(7Bd}{}$$

Mathematica [A]
time = 0.66, size = 225, normalized size = 0.95

$$\frac{e\sqrt{d^2-e^2x^2}\,(C(-64d^5-45d^4ex-32d^3e^2x^2+50d^2e^3x^3+96de^4x^4+40e^5x^5)+2e(5Ae(-16d^3+9d^2ex+16de^2x^2+6e^3x^3)+B(-56d^4-30d^3ex+32d^2e^2x^2+60de^3x^3+24e^4x^4)))+15\sqrt{-e^2}\,(3Cd^6+2d^4e(2Bd+5Ae))\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{240e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(C*(-64*d^5 - 45*d^4*e*x - 32*d^3*e^2*x^2 + 50*d^2*e
^3*x^3 + 96*d*e^4*x^4 + 40*e^5*x^5) + 2*e*(5*A*e*(-16*d^3 + 9*d^2*e*x + 16*
d*e^2*x^2 + 6*e^3*x^3) + B*(-56*d^4 - 30*d^3*e*x + 32*d^2*e^2*x^2 + 60*d*e^
3*x^3 + 24*e^4*x^4))) + 15*Sqrt[-e^2]*(3*C*d^6 + 2*d^4*e*(2*B*d + 5*A*e))*L
og[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(240*e^4)
```

Maple [A]
time = 0.09, size = 355, normalized size = 1.50

method	result
risch	$-\frac{\left(-40e^5Cx^5-48Be^5x^4-96Cde^4x^4-60Ae^5x^3-120Bde^4x^3-50Cd^2e^3x^3-160Ade^4x^2-64Bd^2e^3x^2+32Cd^3e^2x^2-90Ad^2e^3x+6\right.}{240e^3}$
default	$e^2C\left(-\frac{x^3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{6e^2}+\frac{d^2\left(-\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4e^2}+\frac{d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4e^2}\right)}{2e^2}\right)+(e^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*C*(-1/6*x^3*(-e^2*x^2+d^2)^(3/2)/e^2+1/2*d^2/e^2*(-1/4*x*(-e^2*x^2+d^2)
^(3/2)/e^2+1/4*d^2/e^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arct
an((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+(B*e^2+2*C*d*e)*(-1/5*x^2*(-e^2*x
^2+d^2)^(3/2)/e^2-2/15*d^2/e^4*(-e^2*x^2+d^2)^(3/2))+(A*e^2+2*B*d*e+C*d^2)*
(-1/4*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/4*d^2/e^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/
2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))-1/3*(2*A*d*e
+B*d^2)*(-e^2*x^2+d^2)^(3/2)/e^2+d^2*A*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/
(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))
```

Maxima [A]

time = 0.50, size = 320, normalized size = 1.36

$$\frac{1}{16}Cd^6\arcsin\left(\frac{xe}{d}\right)e^{(-3)}+\frac{1}{16}\sqrt{-x^2e^2+d^2}Cd^4xe^{(-2)}+\frac{1}{2}Ad^4\arcsin\left(\frac{xe}{d}\right)e^{(-1)}+\frac{1}{8}\left(Cd^2+2Bde+Ae^2\right)d^4\arcsin\left(\frac{xe}{d}\right)e^{(-3)}-\frac{1}{8}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}Cd^2xe^{(-2)}-\frac{1}{6}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}Cx^3-\frac{1}{3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}Bd^2e^{(-2)}+\frac{1}{8}\left(Cd^2+2Bde+Ae^2\right)\sqrt{-x^2e^2+d^2}d^2xe^{(-2)}-\frac{1}{5}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}\left(2Cde+Be^2\right)x^2e^{(-2)}-\frac{2}{15}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}\left(2Cde+Be^2\right)d^2e^{(-4)}+\frac{1}{2}\sqrt{-x^2e^2+d^2}Ad^2x-\frac{2}{3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}Ade^{(-1)}-\frac{1}{4}\left(Cd^2+2Bde+Ae^2\right)\left(-x^2e^2+d^2\right)^{\frac{3}{2}}xe^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/16*C*d^6*arcsin(x*e/d)*e^(-3) + 1/16*sqrt(-x^2*e^2 + d^2)*C*d^4*x*e^(-2)
+ 1/2*A*d^4*arcsin(x*e/d)*e^(-1) + 1/8*(C*d^2 + 2*B*d*e + A*e^2)*d^4*arcsin
(x*e/d)*e^(-3) - 1/8*(-x^2*e^2 + d^2)^(3/2)*C*d^2*x*e^(-2) - 1/6*(-x^2*e^2
+ d^2)^(3/2)*C*x^3 - 1/3*(-x^2*e^2 + d^2)^(3/2)*B*d^2*e^(-2) + 1/8*(C*d^2 +
 2*B*d*e + A*e^2)*sqrt(-x^2*e^2 + d^2)*d^2*x*e^(-2) - 1/5*(-x^2*e^2 + d^2)^
(3/2)*(2*C*d*e + B*e^2)*x^2*e^(-2) - 2/15*(-x^2*e^2 + d^2)^(3/2)*(2*C*d*e +
 B*e^2)*d^2*e^(-4) + 1/2*sqrt(-x^2*e^2 + d^2)*A*d^2*x - 2/3*(-x^2*e^2 + d^2
)^(3/2)*A*d*e^(-1) - 1/4*(C*d^2 + 2*B*d*e + A*e^2)*(-x^2*e^2 + d^2)^(3/2)*x
*e^(-2)
```

Fricas [A]
time = 0.51, size = 199, normalized size = 0.84

$$-\frac{1}{240}\left(30\left(3Cd^6+4Bd^5e+10Ad^4e^2\right)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\left(64Cd^5-4\left(10Cx^5+12Bx^4+15Ax^3\right)e^5-8\left(12Cdx^4+15Bdx^3+20Adx^2\right)e^4-2\left(25Cd^2x^3+32Bd^2x^2+45Ad^2x\right)e^3+4\left(8Cd^3x^2+15Bd^3x+40Ad^3\right)e^2+\left(45Cd^4x+112Bd^4\right)e\right)\sqrt{-x^2e^2+d^2}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/240*(30*(3*C*d^6 + 4*B*d^5*e + 10*A*d^4*e^2)*arctan(-(d - sqrt(-x^2*e^2
+ d^2))*e^(-1)/x) + (64*C*d^5 - 4*(10*C*x^5 + 12*B*x^4 + 15*A*x^3)*e^5 - 8*
(12*C*d*x^4 + 15*B*d*x^3 + 20*A*d*x^2)*e^4 - 2*(25*C*d^2*x^3 + 32*B*d^2*x^2
 + 45*A*d^2*x)*e^3 + 4*(8*C*d^3*x^2 + 15*B*d^3*x + 40*A*d^3)*e^2 + (45*C*d^
4*x + 112*B*d^4)*e)*sqrt(-x^2*e^2 + d^2))*e^(-3)
```

Sympy [C] Result contains complex when optimal does not.
time = 12.61, size = 1231, normalized size = 5.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
> 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + 2*
A*d*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/
```

```
2)/(3*e**2), True)) + A*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d
**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**
2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
 > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)
) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x*
*2/d**2)), True)) + B*d**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d
**2 - e**2*x**2)**(3/2)/(3*e**2), True)) + 2*B*d*e*Piecewise((-I*d**4*acosh
(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3
/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)
), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sq
rt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5
/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + B*e**2*Piecewise((-2*d**4*sqrt(d*
*2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x*
*4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) + C*d**2
*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2
*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*
sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*
e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**
2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + 2*C*d*e*
Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 -
 e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqr
t(d**2)/4, True)) + C*e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d*
*5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e
**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(
6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)
/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2
*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*
x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

Giac [A]
time = 2.79, size = 197, normalized size = 0.83

$$\frac{1}{16}\left(3\,Cd^6+4\,Bd^5e+10\,Ad^4e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)+\frac{1}{240}\sqrt{-x^2e^2+d^2}\left(\left(2\left(\left(4\left(5\,Cxe^2+6\left(2\,Cde^9+Be^{10}\right)e^{(-8)}\right)x+5\left(5\,Cd^2e^8+12\,Bde^9+6\,Ae^{10}\right)e^{(-8)}\right)x-16\left(Cd^3e^7-2\,Bd^2e^8-5\,Ade^9\right)e^{(-8)}\right)x-15\left(3\,Cd^4e^6+4\,Bd^3e^7-6\,Ad^2e^8\right)e^{(-8)}\right)x-16\left(4\,Cd^5e^5+7\,Bd^4e^6+10\,Ad^3e^7\right)e^{(-8)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(3*C*d^6 + 4*B*d^5*e + 10*A*d^4*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) + 1/2
40*sqrt(-x^2*e^2 + d^2)*((2*((4*(5*C*x*e^2 + 6*(2*C*d*e^9 + B*e^10)*e^(-8))
*x + 5*(5*C*d^2*e^8 + 12*B*d*e^9 + 6*A*e^10)*e^(-8))*x - 16*(C*d^3*e^7 - 2*
B*d^2*e^8 - 5*A*d*e^9)*e^(-8))*x - 15*(3*C*d^4*e^6 + 4*B*d^3*e^7 - 6*A*d^2*
e^8)*e^(-8))*x - 16*(4*C*d^5*e^5 + 7*B*d^4*e^6 + 10*A*d^3*e^7)*e^(-8))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^2\,\left(C\,x^2+B\,x+A\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2*(A + B*x + C*x^2),x)
[Out] int((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2*(A + B*x + C*x^2), x)
```

3.2 $\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$

Optimal. Leaf size=186

$$\frac{d(Cd^2+e(Bd+4Ae))\,x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{(2Cd^2+5e(Bd+Ae))\left(d^2-e^2x^2\right)^{3/2}}{15e^3}-\frac{(Cd+Be)x(d^2-e^2x^2)^{3/2}}{4e^2}-$$

```
[Out] -1/15*(2*C*d^2+5*e*(A*e+B*d))*(-e^2*x^2+d^2)^(3/2)/e^3-1/4*(B*e+C*d)*x*(-e^
2*x^2+d^2)^(3/2)/e^2-1/5*C*x^2*(-e^2*x^2+d^2)^(3/2)/e+1/8*d^3*(C*d^2+e*(4*A
*e+B*d))*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*d*(C*d^2+e*(4*A*e+B*d))*x
*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A]
time = 0.14, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1829, 655, 201, 223, 209}

$$\frac{d^3\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(e(4Ae+Bd)+Cd^2)}{8e^3}+\frac{dx\sqrt{d^2-e^2x^2}\,(e(4Ae+Bd)+Cd^2)}{8e^2}-\frac{(d^2-e^2x^2)^{3/2}\,(5e(Ae+Bd)+2Cd^2)}{15e^3}-\frac{x(d^2-e^2x^2)^{3/2}\,(Be+Cd)}{4e^2}-\frac{Cx^2(d^2-e^2x^2)^{3/2}}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (d*(C*d^2 + e*(B*d + 4*A*e))*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((2*C*d^2 + 5
*e*(B*d + A*e))*(d^2 - e^2*x^2)^(3/2))/(15*e^3) - ((C*d + B*e)*x*(d^2 - e^2
*x^2)^(3/2))/(4*e^2) - (C*x^2*(d^2 - e^2*x^2)^(3/2))/(5*e) + (d^3*(C*d^2 +
e*(B*d + 4*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx = -\frac{Cx^2(d^2-e^2x^2)^{3/2}}{5e} - \frac{\int \sqrt{d^2-e^2x^2}\,(-5Ade^2-e(2Cd^2+5$$

$$= -\frac{(Cd+Be)x(d^2-e^2x^2)^{3/2}}{4e^2} - \frac{Cx^2(d^2-e^2x^2)^{3/2}}{5e} + \frac{\int (5de^2($$

$$= -\frac{(2Cd^2+5e(Bd+Ae))\left(d^2-e^2x^2\right)^{3/2}}{15e^3} - \frac{(Cd+Be)x(d^2-e$$

$$= \frac{d(Cd^2+e(Bd+4Ae))\,x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{(2Cd^2+5e(Bd+Ae}{15e^3}$$

$$= \frac{d(Cd^2+e(Bd+4Ae))\,x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{(2Cd^2+5e(Bd+Ae}{15e^3}$$

$$= \frac{d(Cd^2+e(Bd+4Ae))\,x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{(2Cd^2+5e(Bd+Ae}{15e^3}$$

Mathematica [A]
time = 0.58, size = 189, normalized size = 1.02

$$\frac{e\sqrt{d^2-e^2x^2}\,(C(-16d^4-15d^3ex-8d^2e^2x^2+30de^3x^3+24e^4x^4)-5e(-4Ae(-2d^2+3dex+2e^2x^2)+B(8d^3+3d^2ex-8de^2x^2-6e^3x^3)))+15\sqrt{-e^2}\,(Cd^5+d^3e(Bd+4Ae))\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{120e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(C*(-16*d^4 - 15*d^3*e*x - 8*d^2*e^2*x^2 + 30*d*e^3*
x^3 + 24*e^4*x^4) - 5*e*(-4*A*e*(-2*d^2 + 3*d*e*x + 2*e^2*x^2) + B*(8*d^3 +
```

```
3*d^2*e*x - 8*d*e^2*x^2 - 6*e^3*x^3))) + 15*Sqrt[-e^2]*(C*d^5 + d^3*e*(B*d
 + 4*A*e))*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(120*e^4)
```

Maple [A]
time = 0.11, size = 216, normalized size = 1.16

method	result
default	$eC\left(-\frac{x^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{5e^{2}}-\frac{2d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{15e^{4}}\right)+\left(Be+Cd\right)\left(-\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4e^{2}}+\frac{d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\,\mathrm{arct}}{}\right.}{4e^{2}}\right.$
risch	$-\frac{\left(-24e^{4}Cx^{4}-30Be^{4}x^{3}-30Cde^{3}x^{3}-40Ae^{4}x^{2}-40Bde^{3}x^{2}+8Cd^{2}e^{2}x^{2}-60Ade^{3}x+15Bd^{2}e^{2}x+15Cd^{3}xe+40Ad^{2}e^{2}+40Bd^{3}e+\right.}{120e^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*C*(-1/5*x^2*(-e^2*x^2+d^2)^(3/2)/e^2-2/15*d^2/e^4*(-e^2*x^2+d^2)^(3/2))+(
B*e+C*d)*(-1/4*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/4*d^2/e^2*(1/2*x*(-e^2*x^2+d^2)
^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))-1/3
*(A*e+B*d)*(-e^2*x^2+d^2)^(3/2)/e^2+d*A*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2
/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))
```

Maxima [A]
time = 0.50, size = 192, normalized size = 1.03

$$\frac{1}{8}\left(Cd+Be\right)d^{4}\arcsin\left(\frac{xe}{d}\right)e^{(-3)}+\frac{1}{2}Ad^{3}\arcsin\left(\frac{xe}{d}\right)e^{(-1)}-\frac{1}{5}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}Cx^{2}e^{(-1)}+\frac{1}{8}\sqrt{-x^{2}e^{2}+d^{2}}\left(Cd+Be\right)d^{2}xe^{(-2)}-\frac{2}{15}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}Cd^{2}e^{(-3)}-\frac{1}{3}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}Bde^{(-2)}-\frac{1}{4}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}\left(Cd+Be\right)xe^{(-2)}+\frac{1}{2}\sqrt{-x^{2}e^{2}+d^{2}}Adx-\frac{1}{3}\left(-x^{2}e^{2}+d^{2}\right)^{\frac{3}{2}}Ae^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(C*d + B*e)*d^4*arcsin(x*e/d)*e^(-3) + 1/2*A*d^3*arcsin(x*e/d)*e^(-1) -
 1/5*(-x^2*e^2 + d^2)^(3/2)*C*x^2*e^(-1) + 1/8*sqrt(-x^2*e^2 + d^2)*(C*d +
B*e)*d^2*x*e^(-2) - 2/15*(-x^2*e^2 + d^2)^(3/2)*C*d^2*e^(-3) - 1/3*(-x^2*e^
2 + d^2)^(3/2)*B*d*e^(-2) - 1/4*(-x^2*e^2 + d^2)^(3/2)*(C*d + B*e)*x*e^(-2)
 + 1/2*sqrt(-x^2*e^2 + d^2)*A*d*x - 1/3*(-x^2*e^2 + d^2)^(3/2)*A*e^(-1)
```

Fricas [A]
time = 0.69, size = 165, normalized size = 0.89

$$-\frac{1}{120}\left(30\left(Cd^{5}+Bd^{4}e+4Ad^{3}e^{2}\right)\arctan\left(-\frac{\left(d-\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-1)}}{x}\right)+\left(16Cd^{4}-2\left(12Cx^{4}+15Bx^{3}+20Ax^{2}\right)e^{4}-10\left(3Cdx^{3}+4Bdx^{2}+6Adx\right)e^{3}+\left(8Cd^{2}x^{2}+15Bd^{2}x+40Ad^{2}\right)e^{2}+5\left(3Cd^{3}x+8Bd^{3}\right)e\right)\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(30*(C*d^5 + B*d^4*e + 4*A*d^3*e^2)*arctan(-(d - sqrt(-x^2*e^2 + d^2
))*e^(-1)/x) + (16*C*d^4 - 2*(12*C*x^4 + 15*B*x^3 + 20*A*x^2)*e^4 - 10*(3*C
*d*x^3 + 4*B*d*x^2 + 6*A*d*x)*e^3 + (8*C*d^2*x^2 + 15*B*d^2*x + 40*A*d^2)*e
^2 + 5*(3*C*d^3*x + 8*B*d^3)*e)*sqrt(-x^2*e^2 + d^2))*e^(-3)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.78, size = 670, normalized size = 3.60

$$Ad\left(\begin{cases} -\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases}\right) + Ae\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2 = 0 \\ -\frac{(d^2-e^2x^2)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right) + Bd\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2 = 0 \\ -\frac{(d^2-e^2x^2)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right) + Be\left(\begin{cases} -\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + Cd\left(\begin{cases} -\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + Ce\left(\begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for}\ e \neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d*
*2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1
), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + A*e*P
iecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e
**2), True)) + B*d*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e*
*2*x**2)**(3/2)/(3*e**2), True)) + B*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e
**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1
+ e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*
x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*
x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1
 - e**2*x**2/d**2)), True)) + C*d*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3)
+ I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**
2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/
d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/
d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e*
*2*x**2/d**2)), True)) + C*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*
e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*
x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True))
```

Giac [A]
time = 2.67, size = 160, normalized size = 0.86

$$\frac{1}{8}\left(Cd^5 + Bd^4e + 4\,Ad^3e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\operatorname{sgn}(d) + \frac{1}{120}\,\sqrt{-x^2e^2+d^2}\,\left(\left(2\left(3\left(4\,Cxe + 5\left(Cde^6 + Be^7\right)e^{(-6)}\right)x - 4\left(Cd^2e^5 - 5\,Bde^6 - 5\,Ae^7\right)e^{(-6)}\right)x - 15\left(Cd^3e^4 + Bd^2e^5 - 4\,Ade^6\right)e^{(-6)}\right)x - 8\left(2\,Cd^4e^3 + 5\,Bd^3e^4 + 5\,Ad^2e^5\right)e^{(-6)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(C*d^5 + B*d^4*e + 4*A*d^3*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) + 1/120*sqr
t(-x^2*e^2 + d^2)*((2*(3*(4*C*x*e + 5*(C*d*e^6 + B*e^7)*e^(-6))*x - 4*(C*d^
```

```
2*e^5 - 5*B*d*e^6 - 5*A*e^7)*e^(-6))*x - 15*(C*d^3*e^4 + B*d^2*e^5 - 4*A*d*
e^6)*e^(-6))*x - 8*(2*C*d^4*e^3 + 5*B*d^3*e^4 + 5*A*d^2*e^5)*e^(-6))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)\,\left(C\,x^2+B\,x+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)*(d + e*x)*(A + B*x + C*x^2),x)
[Out] int((d^2 - e^2*x^2)^(1/2)*(d + e*x)*(A + B*x + C*x^2), x)
```

3.3 $\int (A+Bx+Cx^2) \sqrt{d^2-e^2x^2}\, dx$

Optimal. Leaf size=125

$$\frac{1}{8}\left(4A+\frac{Cd^2}{e^2}\right)x\sqrt{d^2-e^2x^2}-\frac{B(d^2-e^2x^2)^{3/2}}{3e^2}-\frac{Cx(d^2-e^2x^2)^{3/2}}{4e^2}+\frac{d^2(Cd^2+4Ae^2)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

```
[Out] -1/3*B*(-e^2*x^2+d^2)^(3/2)/e^2-1/4*C*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/8*d^2*(4
*A*e^2+C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*(4*A+C*d^2/e^2)*x*(-
e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1829, 655, 201, 223, 209}

$$\frac{d^2(4Ae^2+Cd^2)\,\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}+\frac{1}{8}x\sqrt{d^2-e^2x^2}\left(4A+\frac{Cd^2}{e^2}\right)-\frac{B(d^2-e^2x^2)^{3/2}}{3e^2}-\frac{Cx(d^2-e^2x^2)^{3/2}}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] ((4*A + (C*d^2)/e^2)*x*Sqrt[d^2 - e^2*x^2])/8 - (B*(d^2 - e^2*x^2)^(3/2))/(
3*e^2) - (C*x*(d^2 - e^2*x^2)^(3/2))/(4*e^2) + (d^2*(C*d^2 + 4*A*e^2)*ArcTa
n[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx = -\frac{Cx(d^2-e^2x^2)^{3/2}}{4e^2}-\frac{\int \left(-Cd^2-4Ae^2-4Be^2x\right)\sqrt{d^2-e^2x^2}\,dx}{4e^2}$$
$$= -\frac{B(d^2-e^2x^2)^{3/2}}{3e^2}-\frac{Cx(d^2-e^2x^2)^{3/2}}{4e^2}-\frac{\left(-Cd^2-4Ae^2\right)\int \sqrt{d^2-e}}{4e^2}$$
$$= \frac{\left(Cd^2+4Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{B(d^2-e^2x^2)^{3/2}}{3e^2}-\frac{Cx(d^2-e^2x^2)^{3/2}}{4e^2}$$
$$= \frac{\left(Cd^2+4Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{B(d^2-e^2x^2)^{3/2}}{3e^2}-\frac{Cx(d^2-e^2x^2)^{3/2}}{4e^2}$$
$$= \frac{\left(Cd^2+4Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{B(d^2-e^2x^2)^{3/2}}{3e^2}-\frac{Cx(d^2-e^2x^2)^{3/2}}{4e^2}$$

Mathematica [A]
time = 0.29, size = 124, normalized size = 0.99

$$\frac{\sqrt{d^2-e^2x^2}\left(-8Bd^2-3Cd^2x+12Ae^2x+8Be^2x^2+6Ce^2x^3\right)}{24e^2}+\frac{\sqrt{-e^2}\left(Cd^4+4Ad^2e^2\right)\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{8e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-8*B*d^2 - 3*C*d^2*x + 12*A*e^2*x + 8*B*e^2*x^2 + 6*C
*e^2*x^3))/(24*e^2) + (Sqrt[-e^2]*(C*d^4 + 4*A*d^2*e^2)*Log[-(Sqrt[-e^2]*x)
 + Sqrt[d^2 - e^2*x^2]])/(8*e^4)
```

Maple [A]
time = 0.08, size = 155, normalized size = 1.24

method	result
risch	$\frac{(6C\,e^2x^3+8B\,e^2x^2+12A\,e^2x-3C\,d^2x-8B\,d^2)\sqrt{-e^2x^2+d^2}}{24e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)A}{2\sqrt{e^2}}+\frac{d^4\arctan\left(\frac{\sqrt{e^2}}{\sqrt{-e^2x^2}}\right.}{8e^2\sqrt{e^2}}$
default	$C\left(-\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4e^2}+\frac{d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4e^2}\right)-\frac{B(-e^2x^2+d^2)^{\frac{3}{2}}}{3e^2}+A\left(\frac{x\sqrt{-e}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] C*(-1/4*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/4*d^2/e^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))-1/3*B*(-e^2*x^2+d^2)^(3/2)/e^2+A*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))

Maxima [A]
time = 0.51, size = 109, normalized size = 0.87

$$\frac{1}{8}Cd^4\arcsin\left(\frac{xe}{d}\right)e^{(-3)}+\frac{1}{8}\sqrt{-x^2e^2+d^2}\,Cd^2xe^{(-2)}+\frac{1}{2}Ad^2\arcsin\left(\frac{xe}{d}\right)e^{(-1)}-\frac{1}{4}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}Cxe^{(-2)}-\frac{1}{3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}Be^{(-2)}+\frac{1}{2}\sqrt{-x^2e^2+d^2}\,Ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")

[Out] 1/8*C*d^4*arcsin(x*e/d)*e^(-3) + 1/8*sqrt(-x^2*e^2 + d^2)*C*d^2*x*e^(-2) + 1/2*A*d^2*arcsin(x*e/d)*e^(-1) - 1/4*(-x^2*e^2 + d^2)^(3/2)*C*x*e^(-2) - 1/3*(-x^2*e^2 + d^2)^(3/2)*B*e^(-2) + 1/2*sqrt(-x^2*e^2 + d^2)*A*x

Fricas [A]
time = 0.35, size = 103, normalized size = 0.82

$$-\frac{1}{24}\left(6\left(Cd^4+4\,Ad^2e^2\right)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)e-\sqrt{-x^2e^2+d^2}\left(2\left(3\,Cx^3+4\,Bx^2+6\,Ax\right)e^4-\left(3\,Cd^2x+8\,Bd^2\right)e^2\right)\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")

```
[Out] -1/24*(6*(C*d^4 + 4*A*d^2*e^2)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)
*e - sqrt(-x^2*e^2 + d^2)*(2*(3*C*x^3 + 4*B*x^2 + 6*A*x)*e^4 - (3*C*d^2*x +
 8*B*d^2)*e^2))*e^(-4)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.38, size = 343, normalized size = 2.74

$$A\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise}\end{cases}\right)+B\left(\begin{cases}\frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0\\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise}\end{cases}\right)+C\left(\begin{cases}-\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3}+\frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}-\frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}}-\frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2
)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1),
 (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + B*Piece
wise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2)
, True)) + C*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sq
rt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**
2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asi
n(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*
sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)
)
```

Giac [A]
time = 2.97, size = 85, normalized size = 0.68

$$\frac{1}{8}\left(Cd^4+4\,Ad^2e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\operatorname{sgn}(d)-\frac{1}{24}\left(8\,Bd^2e^{(-2)}-\left(2\left(3\,Cx+4\,B\right)x-3\left(Cd^2e^2-4\,Ae^4\right)e^{(-4)}\right)x\right)\sqrt{-x^2e^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(C*d^4 + 4*A*d^2*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/24*(8*B*d^2*e^(-2
) - (2*(3*C*x + 4*B)*x - 3*(C*d^2*e^2 - 4*A*e^4)*e^(-4))*x)*sqrt(-x^2*e^2 +
 d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2), x)
```

3.4 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

Optimal. Leaf size=148

$$\frac{(Cd^2-e(Bd-2Ae))\sqrt{d^2-e^2x^2}}{2e^3}-\frac{C(d^2-e^2x^2)^{3/2}}{3e^3}+\frac{(Cd-Be)(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)}+\frac{d(Cd^2-e(Bd-2Ae))\,\mathrm{ta}}{2e^3}$$

```
[Out] -1/3*C*(-e^2*x^2+d^2)^(3/2)/e^3+1/2*(-B*e+C*d)*(-e^2*x^2+d^2)^(3/2)/e^3/(e*
x+d)+1/2*d*(C*d^2-e*(-2*A*e+B*d))*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/2*
(C*d^2-e*(-2*A*e+B*d))*(-e^2*x^2+d^2)^(1/2)/e^3
```

Rubi [A]
time = 0.11, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$,
Rules used = {1653, 809, 679, 223, 209}

$$\frac{d\,\mathrm{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(Cd^2-e(Bd-2Ae))}{2e^3}+\frac{\sqrt{d^2-e^2x^2}\,(Cd^2-e(Bd-2Ae))}{2e^3}+\frac{(d^2-e^2x^2)^{3/2}(Cd-Be)}{2e^3(d+ex)}-\frac{C(d^2-e^2x^2)^{3/2}}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] ((C*d^2 - e*(B*d - 2*A*e))*Sqrt[d^2 - e^2*x^2])/(2*e^3) - (C*(d^2 - e^2*x^2
)^(3/2))/(3*e^3) + ((C*d - B*e)*(d^2 - e^2*x^2)^(3/2))/(2*e^3*(d + e*x)) +
(d*(C*d^2 - e*(B*d - 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{C(d^2-e^2x^2)^{3/2}}{3e^3} - \frac{\int \frac{(-3Ae^4+3e^3(Cd-Be)x)\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{3e^4}$$

$$= -\frac{C(d^2-e^2x^2)^{3/2}}{3e^3} + \frac{(Cd-Be)(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)} + \frac{(Cd^2-e(Bd-2A}{}$$

$$= \frac{(Cd^2-e(Bd-2Ae))\sqrt{d^2-e^2x^2}}{2e^3} - \frac{C(d^2-e^2x^2)^{3/2}}{3e^3} + \frac{(Cd-Be)}{2e^3(}$$

$$= \frac{(Cd^2-e(Bd-2Ae))\sqrt{d^2-e^2x^2}}{2e^3} - \frac{C(d^2-e^2x^2)^{3/2}}{3e^3} + \frac{(Cd-Be)}{2e^3(}$$

$$= \frac{(Cd^2-e(Bd-2Ae))\sqrt{d^2-e^2x^2}}{2e^3} - \frac{C(d^2-e^2x^2)^{3/2}}{3e^3} + \frac{(Cd-Be)}{2e^3(}$$

Mathematica [A]
time = 0.45, size = 123, normalized size = 0.83

$$\frac{e\sqrt{d^2-e^2x^2}\,(3e(-2Bd+2Ae+Bex)+C(4d^2-3dex+2e^2x^2))+3\sqrt{-e^2}\,(Cd^3+de(-Bd+2Ae))\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{6e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (e*Sqrt[d^2 - e^2*x^2]*(3*e*(-2*B*d + 2*A*e + B*e*x) + C*(4*d^2 - 3*d*e*x +
 2*e^2*x^2)) + 3*Sqrt[-e^2]*(C*d^3 + d*e*(-(B*d) + 2*A*e))*Log[-(Sqrt[-e^2]
*x) + Sqrt[d^2 - e^2*x^2]])/(6*e^4)
```

Maple [A]
time = 0.08, size = 227, normalized size = 1.53

method	result
risch	$\frac{(2Ce^2x^2+3Be^2x-3Cdex+6Ae^2-6Bde+4Cd^2)\sqrt{-e^2x^2+d^2}}{6e^3}+\frac{d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)A}{\sqrt{e^2}}-\frac{d^2\arctan\left(\frac{\sqrt{e}}{\sqrt{-e^2x}}\right.}{2e\sqrt{e^2}}$
default	$\frac{-\frac{C\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{3e}+Be\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)-Cd\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}}{\sqrt{-e^2x^2}}\right.}{2\sqrt{e^2}}\right.}{e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(-1/3*C/e*(-e^2*x^2+d^2)^(3/2)+B*e*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^
2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))-C*d*(1/2*x*(-e^2*
x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2
))))+(A*e^2-B*d*e+C*d^2)/e^3*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2
)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))
```

Maxima [A]
time = 0.50, size = 159, normalized size = 1.07

$$\frac{1}{2}Cd^3\arcsin\left(\frac{xe}{d}\right)e^{(-3)}-\frac{1}{2}Bd^2\arcsin\left(\frac{xe}{d}\right)e^{(-2)}-\frac{1}{2}\sqrt{-x^2e^2+d^2}\,Cdxe^{(-2)}+\sqrt{-x^2e^2+d^2}\,Cd^2e^{(-3)}+Ad\arcsin\left(\frac{xe}{d}\right)e^{(-1)}+\frac{1}{2}\sqrt{-x^2e^2+d^2}\,Bxe^{(-1)}-\sqrt{-x^2e^2+d^2}\,Bde^{(-2)}-\frac{1}{3}\left(-x^2e^2+d^2\right)^{\frac{3}{2}}Ce^{(-3)}+\sqrt{-x^2e^2+d^2}\,Ae^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*C*d^3*arcsin(x*e/d)*e^(-3) - 1/2*B*d^2*arcsin(x*e/d)*e^(-2) - 1/2*sqrt(
-x^2*e^2 + d^2)*C*d*x*e^(-2) + sqrt(-x^2*e^2 + d^2)*C*d^2*e^(-3) + A*d*arcs
in(x*e/d)*e^(-1) + 1/2*sqrt(-x^2*e^2 + d^2)*B*x*e^(-1) - sqrt(-x^2*e^2 + d^
2)*B*d*e^(-2) - 1/3*(-x^2*e^2 + d^2)^(3/2)*C*e^(-3) + sqrt(-x^2*e^2 + d^2)*
A*e^(-1)
```

Fricas [A]
time = 0.35, size = 104, normalized size = 0.70

$$-\frac{1}{6}\left(6\left(Cd^3-Bd^2e+2\,Ade^2\right)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-\left(4\,Cd^2+\left(2\,Cx^2+3\,Bx+6\,A\right)e^2-3\,(Cdx+2\,Bd)e\right)\sqrt{-x^2e^2+d^2}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/6*(6*(C*d^3 - B*d^2*e + 2*A*d*e^2)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^
(-1)/x) - (4*C*d^2 + (2*C*x^2 + 3*B*x + 6*A)*e^2 - 3*(C*d*x + 2*B*d)*e)*sqr
t(-x^2*e^2 + d^2))*e^(-3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(A+Bx+Cx^2\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x), x)
```

Giac [A]
time = 2.29, size = 99, normalized size = 0.67

$$\frac{1}{2}\left(Cd^3-Bd^2e+2\,Ade^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)+\frac{1}{6}\sqrt{-x^2e^2+d^2}\left(\left(2\,Cxe^{(-1)}-3\left(Cde^3-Be^4\right)e^{(-5)}\right)x+2\left(2\,Cd^2e^2-3\,Bde^3+3\,Ae^4\right)e^{(-5)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*(C*d^3 - B*d^2*e + 2*A*d*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) + 1/6*sqrt(-x
^2*e^2 + d^2)*((2*C*x*e^(-1) - 3*(C*d*e^3 - B*e^4)*e^(-5))*x + 2*(2*C*d^2*e
^2 - 3*B*d*e^3 + 3*A*e^4)*e^(-5))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x),x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x), x)
```

3.5 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx$

Optimal. Leaf size=170

$$-\frac{(5Cd^2-2e(2Bd-Ae))\sqrt{d^2-e^2x^2}}{2de^3}-\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{de^3(d+ex)^2}-\frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)}-\frac{(5Cd^2-2e(\cdots}{\cdots}$$

```
[Out] -(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^2-1/2*C*(-e^2*x^2+d
^2)^(3/2)/e^3/(e*x+d)-1/2*(5*C*d^2-2*e*(-A*e+2*B*d))*arctan(e*x/(-e^2*x^2+d
^2)^(1/2))/e^3-1/2*(5*C*d^2-2*e*(-A*e+2*B*d))*(-e^2*x^2+d^2)^(1/2)/d/e^3
```

Rubi [A]
time = 0.13, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1653, 807, 679, 223, 209}

$$-\frac{\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(5Cd^2-2e(2Bd-Ae))}{2e^3}-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{de^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\,(5Cd^2-2e(2Bd-Ae))}{2de^3}-\frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^2,x]
```

```
[Out] -1/2*((5*C*d^2 - 2*e*(2*B*d - A*e))*Sqrt[d^2 - e^2*x^2])/(d*e^3) - ((C*d^2
- B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(d*e^3*(d + e*x)^2) - (C*(d^2 - e^2
*x^2)^(3/2))/(2*e^3*(d + e*x)) - ((5*C*d^2 - 2*e*(2*B*d - A*e))*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
```

```
|| EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx = -\frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)} - \frac{\int \frac{(e^2(Cd^2-2Ae^2)+e^3(3Cd-2Be)x)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{2e^4}$$

$$= -\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{de^3(d+ex)^2} - \frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)} - \frac{(-3e^5(C\ldots}{\ldots}$$

$$= -\frac{(5Cd^2-2e(2Bd-Ae))\sqrt{d^2-e^2x^2}}{2de^3} - \frac{(Cd^2-Bde+Ae^2)(d^2-\ldots}{de^3(d+ex)^2}$$

$$= -\frac{(5Cd^2-2e(2Bd-Ae))\sqrt{d^2-e^2x^2}}{2de^3} - \frac{(Cd^2-Bde+Ae^2)(d^2-\ldots}{de^3(d+ex)^2}$$

$$= -\frac{(5Cd^2-2e(2Bd-Ae))\sqrt{d^2-e^2x^2}}{2de^3} - \frac{(Cd^2-Bde+Ae^2)(d^2-\ldots}{de^3(d+ex)^2}$$

Mathematica [A]

time = 0.52, size = 130, normalized size = 0.76

$$\frac{\frac{e^3\sqrt{d^2-e^2x^2}\,(2e(3Bd-2Ae+Bex)+C(-8d^2-3dex+e^2x^2))}{d+ex}+(-e^2)^{3/2}\,(5Cd^2+2e(-2Bd+Ae))\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{2e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^2,x]
```

```
[Out] ((e^3*Sqrt[d^2 - e^2*x^2]*(2*e*(3*B*d - 2*A*e + B*e*x) + C*(-8*d^2 - 3*d*e*
x + e^2*x^2)))/(d + e*x) + (-e^2)^(3/2)*(5*C*d^2 + 2*e*(-2*B*d + A*e))*Log[
-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^6)
```

Maple [A]
time = 0.10, size = 287, normalized size = 1.69

method	result
risch	$\frac{(Cxe+2Be-4Cd)\sqrt{-e^2x^2+d^2}}{2e^3}-\frac{A\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+\frac{2Bd\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e\sqrt{e^2}}-\frac{5C\,d^2\arctan\left(\right.}{2}$
default	$\frac{C\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{e^2}+\frac{(Be-2Cd)\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\right.}{}\right.}{e^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] C/e^2*(1/2*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/
(-e^2*x^2+d^2)^(1/2)))+1/e^3*(B*e-2*C*d)*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1
/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/
2)))+1/e^4*(A*e^2-B*d*e+C*d^2)*(-1/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d
/e))^(3/2)-e/d*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan
((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))
```

Maxima [A]
time = 0.51, size = 184, normalized size = 1.08

$$-\frac{5}{2}\,Cd^2\arcsin\left(\frac{xe}{d}\right)e^{(-3)}+2\,Bd\arcsin\left(\frac{xe}{d}\right)e^{(-2)}+\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,Cxe^{(-2)}-2\,\sqrt{-x^2e^2+d^2}\,Cde^{(-3)}-A\arcsin\left(\frac{xe}{d}\right)e^{(-1)}-\frac{2\,\sqrt{-x^2e^2+d^2}\,Cd^2}{xe^4+de^3}+\sqrt{-x^2e^2+d^2}\,Be^{(-2)}+\frac{2\,\sqrt{-x^2e^2+d^2}\,Bd}{xe^3+de^2}-\frac{2\,\sqrt{-x^2e^2+d^2}\,A}{xe^2+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="maxima
")
```

```
[Out] -5/2*C*d^2*arcsin(x*e/d)*e^(-3) + 2*B*d*arcsin(x*e/d)*e^(-2) + 1/2*sqrt(-x^
2*e^2 + d^2)*C*x*e^(-2) - 2*sqrt(-x^2*e^2 + d^2)*C*d*e^(-3) - A*arcsin(x*e/
d)*e^(-1) - 2*sqrt(-x^2*e^2 + d^2)*C*d^2/(x*e^4 + d*e^3) + sqrt(-x^2*e^2 +
d^2)*B*e^(-2) + 2*sqrt(-x^2*e^2 + d^2)*B*d/(x*e^3 + d*e^2) - 2*sqrt(-x^2*e^
2 + d^2)*A/(x*e^2 + d*e)
```

Fricas [**A**]
time = 0.38, size = 180, normalized size = 1.06

$$-\frac{8\,Cd^3 + 4\,Axe^3 - 2\,(5\,Cd^3 + 2\,Axe^3 - 2\,(2\,Bdx - Ad)e^2 + (5\,Cd^2x - 4\,Bd^2)e)\arctan\left(-\frac{\left(d - \sqrt{-x^2e^2 + d^2}\right)e^{(-1)}}{x}\right) - 2\,(3\,Bdx - 2\,Ad)e^2 + 2\,(4\,Cd^2x - 3\,Bd^2)e + (8\,Cd^2 - (Cx^2 + 2\,Bx - 4\,A)e^2 + 3\,(Cdx - 2\,Bd)e)\sqrt{-x^2e^2 + d^2}}{2\,(xe^4 + de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(8*C*d^3 + 4*A*x*e^3 - 2*(5*C*d^3 + 2*A*x*e^3 - 2*(2*B*d*x - A*d)*e^2
+ (5*C*d^2*x - 4*B*d^2)*e)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - 2
*(3*B*d*x - 2*A*d)*e^2 + 2*(4*C*d^2*x - 3*B*d^2)*e + (8*C*d^2 - (C*x^2 + 2*
B*x - 4*A)*e^2 + 3*(C*d*x - 2*B*d)*e)*sqrt(-x^2*e^2 + d^2))/(x*e^4 + d*e^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d + ex\right)\left(d + ex\right)}\,\left(A + Bx + Cx^2\right)}{\left(d + ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(155) = 310.
time = 3.77, size = 311, normalized size = 1.83

$$-\frac{\left(8\,Cd^3\sqrt{\frac{2\,d}{xe + d} - 1}\,e^3\mathrm{sgn}\left(\frac{1}{xe+d}\right) - 8\,Bd^2\sqrt{\frac{2\,d}{xe + d} - 1}\,e^4\mathrm{sgn}\left(\frac{1}{xe+d}\right) + 8\,Ad\sqrt{\frac{2\,d}{xe + d} - 1}\,e^5\mathrm{sgn}\left(\frac{1}{xe+d}\right) - 4\,(5\,Cd^3e^3\mathrm{sgn}\left(\frac{1}{xe+d}\right) - 4\,Bd^2e^4\mathrm{sgn}\left(\frac{1}{xe+d}\right) + 2\,Ade^5\mathrm{sgn}\left(\frac{1}{xe+d}\right))\arctan\left(\sqrt{\frac{2\,d}{xe + d} - 1}\right) + \frac{\left(5\,Cd^3\left(\frac{2\,d}{xe+d} - 1\right)^{\frac{3}{2}}e^3\mathrm{sgn}\left(\frac{1}{xe+d}\right) - 2\,Bd^2\left(\frac{2\,d}{xe+d} - 1\right)^{\frac{3}{2}}e^4\mathrm{sgn}\left(\frac{1}{xe+d}\right) + 3\,Cd^3\sqrt{\frac{2\,d}{xe + d} - 1}\,e^3\mathrm{sgn}\left(\frac{1}{xe+d}\right) - 2\,Bd^2\sqrt{\frac{2\,d}{xe + d} - 1}\,e^4\mathrm{sgn}\left(\frac{1}{xe+d}\right)\right)(xe+d)^2}{d^2}\right)e^{(-6)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/4*(8*C*d^3*sqrt(2*d/(x*e + d) - 1)*e^3*sgn(1/(x*e + d)) - 8*B*d^2*sqrt(2
*d/(x*e + d) - 1)*e^4*sgn(1/(x*e + d)) + 8*A*d*sqrt(2*d/(x*e + d) - 1)*e^5*
sgn(1/(x*e + d)) - 4*(5*C*d^3*e^3*sgn(1/(x*e + d)) - 4*B*d^2*e^4*sgn(1/(x*e
 + d)) + 2*A*d*e^5*sgn(1/(x*e + d)))*arctan(sqrt(2*d/(x*e + d) - 1)) + (5*C
*d^3*(2*d/(x*e + d) - 1)^(3/2)*e^3*sgn(1/(x*e + d)) - 2*B*d^2*(2*d/(x*e + d
) - 1)^(3/2)*e^4*sgn(1/(x*e + d)) + 3*C*d^3*sqrt(2*d/(x*e + d) - 1)*e^3*sgn
(1/(x*e + d)) - 2*B*d^2*sqrt(2*d/(x*e + d) - 1)*e^4*sgn(1/(x*e + d)))*(x*e
+ d)^2/d^2)*e^(-6)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^2, x)
```

3.6 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx$

Optimal. Leaf size=149

$$\frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{(Cd^2-Bde+Ae^2)\,(d^2-e^2x^2)^{3/2}}{3de^3(d+ex)^3}-\frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^2}+\frac{(3Cd-Be)\tan^{-1}\left(\frac{}{\sqrt{d^2}}\right.}{e^3}$$

```
[Out] -1/3*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^3-C*(-e^2*x^2+d
^2)^(3/2)/e^3/(e*x+d)^2+(-B*e+3*C*d)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+2
*(-B*e+3*C*d)*(-e^2*x^2+d^2)^(1/2)/e^3/(e*x+d)
```

Rubi [A]
time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1653, 807, 677, 223, 209}

$$-\frac{(d^2-e^2x^2)^{3/2}\,(Ae^2-Bde+Cd^2)}{3de^3(d+ex)^3}+\frac{\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(3Cd-Be)}{e^3}+\frac{2\sqrt{d^2-e^2x^2}\,(3Cd-Be)}{e^3(d+ex)}-\frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^3,x]
```

```
[Out] (2*(3*C*d - B*e)*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - ((C*d^2 - B*d*e + A
*e^2)*(d^2 - e^2*x^2)^(3/2))/(3*d*e^3*(d + e*x)^3) - (C*(d^2 - e^2*x^2)^(3/
2))/(e^3*(d + e*x)^2) + ((3*C*d - B*e)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + p + 1))), x] - Dist[c*(p/(e^2*(m +
 p + 1))), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
```

```
2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx &= -\frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^2}-\frac{\int \frac{(e^2(2Cd^2-Ae^2)+e^3(3Cd-Be)x)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^4}\\
&= -\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{3de^3(d+ex)^3}-\frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^2}-\frac{(3Cd-Be
\\
&= \frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{3de^3(d+ex)^3}-\frac{C(
\\
&= \frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{3de^3(d+ex)^3}-\frac{C(
\\
&= \frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{3de^3(d+ex)^3}-\frac{C(
\end{aligned}
$$

Mathematica [A]

time = 0.63, size = 133, normalized size = 0.89

$$\frac{\frac{e\sqrt{d^2-e^2x^2}\,(Cd(14d^2+19dex+3e^2x^2)+e(Ae(-d+ex)-Bd(5d+7ex)))}{d(d+ex)^2}-3\sqrt{-e^2}\,(-3Cd+Be)\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{3e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^3,x]
```

```
[Out] ((e*Sqrt[d^2 - e^2*x^2]*(C*d*(14*d^2 + 19*d*e*x + 3*e^2*x^2) + e*(A*e*(-d +
 e*x) - B*d*(5*d + 7*e*x))))/(d*(d + e*x)^2) - 3*Sqrt[-e^2]*(-3*C*d + B*e)*
Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(3*e^4)
```

Maple [A]
time = 0.10, size = 278, normalized size = 1.87

method	result
default	$\frac{C\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}\right)}{e^3}-\frac{(Ae^2-Bde+Cd^2)\,(}{3e}$
risch	$\frac{C\sqrt{-e^2x^2+d^2}}{e^3}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)B}{e\sqrt{e^2}}+\frac{3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)Cd}{e^2\sqrt{e^2}}-\frac{2\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2}}{3e^3\left(x+\frac{d}{e}\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] C/e^3*((-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1
/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))-1/3*(A*e^2-B*d*e+C*d^2)/e^6/d/
(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+(B*e-2*C*d)/e^4*(-1/d/e/(x+d
/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-e/d*((-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(1/2))))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="maxima
")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.36, size = 244, normalized size = 1.64

$$\frac{14\,Cd^{4}-Ax^{2}e^{4}-6\,(3\,Cd^{4}-Bdx^{2}e^{3}+(3\,Cd^{2}x^{2}-2\,Bd^{2}x)e^{2}+(6\,Cd^{3}x-Bd^{3})e)\arctan\left(-\frac{\left(d-\sqrt{-x^{2}e^{2}+d^{2}}\right)e^{(-1)}}{x}\right)-(5\,Bdx^{2}+2\,Adx)e^{3}+(14\,Cd^{2}x^{2}-10\,Bd^{2}x-Ad^{2})e^{2}+(28\,Cd^{3}x-5\,Bd^{3})e+(14\,Cd^{3}+Axe^{3}+(3\,Cdx^{2}-7\,Bdx-Ad)e^{2}+(19\,Cd^{2}x-5\,Bd^{2})e)\sqrt{-x^{2}e^{2}+d^{2}}}{3\,(dx^{2}e^{5}+2\,d^{2}xe^{4}+d^{3}e^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="fricas
")
```

```
[Out] 1/3*(14*C*d^4 - A*x^2*e^4 - 6*(3*C*d^4 - B*d*x^2*e^3 + (3*C*d^2*x^2 - 2*B*d
^2*x)*e^2 + (6*C*d^3*x - B*d^3)*e)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1
)/x) - (5*B*d*x^2 + 2*A*d*x)*e^3 + (14*C*d^2*x^2 - 10*B*d^2*x - A*d^2)*e^2
+ (28*C*d^3*x - 5*B*d^3)*e + (14*C*d^3 + A*x*e^3 + (3*C*d*x^2 - 7*B*d*x - A
*d)*e^2 + (19*C*d^2*x - 5*B*d^2)*e)*sqrt(-x^2*e^2 + d^2))/(d*x^2*e^5 + 2*d^
2*x*e^4 + d^3*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(A+Bx+Cx^{2}\right)}{\left(d+ex\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**3,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**3, x)
```

Giac [A]
time = 4.48, size = 256, normalized size = 1.72

$$(3\,Cd-Be)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)+\sqrt{-x^{2}e^{2}+d^{2}}\,Ce^{(-3)}-\frac{2\left(\frac{24\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)Cd^{2}e^{(-2)}}{x}+\frac{9\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}Cd^{2}e^{(-4)}}{x^{2}}+11\,Cd^{2}-5\,Bde-\frac{12\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)Bde^{(-1)}}{x}-\frac{3\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}Bde^{(-3)}}{x^{2}}-Ae^{2}-\frac{3\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)^{2}Ae^{(-2)}}{x^{2}}\right)e^{(-3)}}{3\,d\left(\frac{\left(de+\sqrt{-x^{2}e^{2}+d^{2}}\,e\right)e^{(-2)}}{x}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] (3*C*d - B*e)*arcsin(x*e/d)*e^(-3)*sgn(d) + sqrt(-x^2*e^2 + d^2)*C*e^(-3) -
 2/3*(24*(d*e + sqrt(-x^2*e^2 + d^2)*e)*C*d^2*e^(-2)/x + 9*(d*e + sqrt(-x^2
*e^2 + d^2)*e)^2*C*d^2*e^(-4)/x^2 + 11*C*d^2 - 5*B*d*e - 12*(d*e + sqrt(-x^
2*e^2 + d^2)*e)*B*d*e^(-1)/x - 3*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*B*d*e^(-3
)/x^2 - A*e^2 - 3*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*A*e^(-2)/x^2)*e^(-3)/(d*
((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)}{{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^3, x)
```

3.7 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

Optimal. Leaf size=196

$$-\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{(Cd^2-Bde+Ae^2)\,(d^2-e^2x^2)^{3/2}}{5de^3(d+ex)^4}+\frac{(2Cd-Be)\,(d^2-e^2x^2)^{3/2}}{3de^3(d+ex)^3}-\frac{(Cd^2-Bde+Ae^2)\,(d^2}{15d^2e^3(d+ex}$$

```
[Out] -1/5*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^4+1/3*(-B*e+2*C
*d)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^3-1/15*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2
+d^2)^(3/2)/d^2/e^3/(e*x+d)^3-C*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-2*C*(-
e^2*x^2+d^2)^(1/2)/e^3/(e*x+d)
```

Rubi [A]
time = 0.12, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1651, 673, 665, 677, 223, 209}

$$-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{15d^2e^3(d+ex)^3}-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{5de^3(d+ex)^4}-\frac{C\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}+\frac{(d^2-e^2x^2)^{3/2}(2Cd-Be)}{3de^3(d+ex)^3}-\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (-2*C*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - ((C*d^2 - B*d*e + A*e^2)*(d^2
- e^2*x^2)^(3/2))/(5*d*e^3*(d + e*x)^4) + ((2*C*d - B*e)*(d^2 - e^2*x^2)^(3
/2))/(3*d*e^3*(d + e*x)^3) - ((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2)
)/(15*d^2*e^3*(d + e*x)^3) - (C*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
```

```
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + p + 1))), x] - Dist[c*(p/(e^2*(m +
 p + 1))), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \left(\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{e^2(d+ex)^4} + \frac{(-2Cd+Be)\sqrt{d^2-e^2x^2}}{e^2(d+ex)^3}\right.$$

$$= \frac{C\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{e^2} - \frac{(2Cd-Be)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^2} + \frac{(Cd^2-Bd}{}$$

$$= -\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{(Cd^2-Bde+Ae^2)\left(d^2-e^2x^2\right)^{3/2}}{5de^3(d+ex)^4} + \frac{(2Cd-B}{3de}$$

$$= -\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{(Cd^2-Bde+Ae^2)\left(d^2-e^2x^2\right)^{3/2}}{5de^3(d+ex)^4} + \frac{(2Cd-B}{3de}$$

$$= -\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{(Cd^2-Bde+Ae^2)\left(d^2-e^2x^2\right)^{3/2}}{5de^3(d+ex)^4} + \frac{(2Cd-B}{3de}$$

Mathematica [A]
time = 0.73, size = 130, normalized size = 0.66

$$-\frac{\sqrt{d^2-e^2x^2}\,(3Cd^2(8d^2+19dex+13e^2x^2)+e(d-ex)(Ae(4d+ex)+Bd(d+4ex)))}{15d^2e^3(d+ex)^3}-\frac{C\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{(-e^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(3*C*d^2*(8*d^2 + 19*d*e*x + 13*e^2*x^2) + e*(d
- e*x)*(A*e*(4*d + e*x) + B*d*(d + 4*e*x))))/(d^2*e^3*(d + e*x)^3) - (C*Log
[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(-e^2)^(3/2)
```

Maple [A]
time = 0.12, size = 293, normalized size = 1.49

method	result
default	$\frac{(A e^2-Bde+C d^2)\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^4}-\frac{\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15d^2\left(x+\frac{d}{e}\right)^3}\right)}{e^6}-\frac{(Be-2Cd)\left(-\left(x+\frac{d}{e}\right)^2 e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3e^6 d\left(x+\frac{d}{e}\right)^3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (A*e^2-B*d*e+C*d^2)/e^6*(-1/5/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(3/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2))-1/3*(B*e-2*C
*d)/e^6/d/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+C/e^4*(-1/d/e/(x+d
/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-e/d*((-(x+d/e)^2*e^2+2*d*e*(x+d/
e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima
")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(-x^2*e^2 + d^2)/(x*e + d)^4, x)
```

Fricas [A]
time = 0.36, size = 292, normalized size = 1.49

$$-\frac{24\,Cd^5 + 4\,Ax^3e^5 - 30\,(Cd^2x^3e^3 + 3\,Cd^3x^2e^2 + 3\,Cd^4xe + Cd^5)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right) + (Bdx^3 + 12\,Adx^2)e^4 + 3\,(8\,Cd^2x^3 + Bd^2x^2 + 4\,Ad^2x)e^3 + (72\,Cd^3x^2 + 3\,Bd^3x + 4\,Ad^3)e^2 + (72\,Cd^4x + Bd^4)e + (24\,Cd^4 - Ax^2e^4 - (4\,Bdx^2 + 3\,Adx)e^3 + (39\,Cd^2x^2 + 3\,Bd^2x + 4\,Ad^2)e^2 + (57\,Cd^3x + Bd^3)e)\sqrt{-x^2e^2+d^2}}{15\,(d^2x^3e^6 + 3\,d^3x^2e^5 + 3\,d^4xe^4 + d^5e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas
")
```

```
[Out] -1/15*(24*C*d^5 + 4*A*x^3*e^5 - 30*(C*d^2*x^3*e^3 + 3*C*d^3*x^2*e^2 + 3*C*d
^4*x*e + C*d^5)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (B*d*x^3 + 1
2*A*d*x^2)*e^4 + 3*(8*C*d^2*x^3 + B*d^2*x^2 + 4*A*d^2*x)*e^3 + (72*C*d^3*x^
2 + 3*B*d^3*x + 4*A*d^3)*e^2 + (72*C*d^4*x + B*d^4)*e + (24*C*d^4 - A*x^2*e
^4 - (4*B*d*x^2 + 3*A*d*x)*e^3 + (39*C*d^2*x^2 + 3*B*d^2*x + 4*A*d^2)*e^2 +
 (57*C*d^3*x + B*d^3)*e)*sqrt(-x^2*e^2 + d^2))/(d^2*x^3*e^6 + 3*d^3*x^2*e^5
 + 3*d^4*x*e^4 + d^5*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}\,(A+Bx+Cx^2)}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(176) = 352.
time = 3.65, size = 419, normalized size = 2.14

$$-C\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d) + \frac{2\left(\frac{105\left(de+\sqrt{-x^2e^2+d^2}\,e\right)Cd^2e^{(-2)}}{x} + \frac{165\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2Cd^2e^{(-4)}}{x^2} + \frac{75\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3Cd^2e^{(-6)}}{x^3} + \frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4Cd^2e^{(-8)}}{x^4} + 24\,Cd^2 + Bde + \frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)Bde^{(-1)}}{x} - \frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2Bde^{(-3)}}{x^2} + \frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3Bde^{(-5)}}{x^3} + 4\,Ae^2 + \frac{25\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2Ae^{(-2)}}{x^2} + \frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3Ae^{(-4)}}{x^3} + \frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4Ae^{(-6)}}{x^4} + \frac{5\left(de+\sqrt{-x^2e^2+d^2}\,e\right)A}{x}\right)e^{(-3)}}{15\,d^2\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -C*arcsin(x*e/d)*e^(-3)*sgn(d) + 2/15*(105*(d*e + sqrt(-x^2*e^2 + d^2)*e)*C
*d^2*e^(-2)/x + 165*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*C*d^2*e^(-4)/x^2 + 75*
```

```
(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*C*d^2*e^(-6)/x^3 + 15*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^4*C*d^2*e^(-8)/x^4 + 24*C*d^2 + B*d*e + 5*(d*e + sqrt(-x^2*e^2 +
 d^2)*e)*B*d*e^(-1)/x - 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*B*d*e^(-3)/x^2 +
 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*B*d*e^(-5)/x^3 + 4*A*e^2 + 25*(d*e + s
qrt(-x^2*e^2 + d^2)*e)^2*A*e^(-2)/x^2 + 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3
*A*e^(-4)/x^3 + 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*A*e^(-6)/x^4 + 5*(d*e +
 sqrt(-x^2*e^2 + d^2)*e)*A/x)*e^(-3)/(d^2*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e
^(-2)/x + 1)^5)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)}{{\left(d+e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^4,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^4, x)
```

3.8 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx$

Optimal. Leaf size=180

$$-\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{7de^3(d+ex)^5}+\frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^4}-\frac{(23Cd^2+e(5Bd+2Ae))(d^2-e^2x^2)^{3/2}}{35d^2e^3(d+ex)^4}-\frac{(23Cd^2 -$$

```
[Out] -1/7*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^5+C*(-e^2*x^2+d
^2)^(3/2)/e^3/(e*x+d)^4-1/35*(23*C*d^2+e*(2*A*e+5*B*d))*(-e^2*x^2+d^2)^(3/2
)/d^2/e^3/(e*x+d)^4-1/105*(23*C*d^2+e*(2*A*e+5*B*d))*(-e^2*x^2+d^2)^(3/2)/d
^3/e^3/(e*x+d)^3
```

Rubi [A]

time = 0.13, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1653, 807, 673, 665}

$$-\frac{(d^2-e^2x^2)^{3/2}(e(2Ae+5Bd)+23Cd^2)}{35d^2e^3(d+ex)^4}-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{7de^3(d+ex)^5}-\frac{(d^2-e^2x^2)^{3/2}(e(2Ae+5Bd)+23Cd^2)}{105d^3e^3(d+ex)^3}+\frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^5,x]
```

```
[Out] -1/7*((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(d*e^3*(d + e*x)^5) +
(C*(d^2 - e^2*x^2)^(3/2))/(e^3*(d + e*x)^4) - ((23*C*d^2 + e*(5*B*d + 2*A*e
))*(d^2 - e^2*x^2)^(3/2))/(35*d^2*e^3*(d + e*x)^4) - ((23*C*d^2 + e*(5*B*d
+ 2*A*e))*(d^2 - e^2*x^2)^(3/2))/(105*d^3*e^3*(d + e*x)^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
```

```
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx = \frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^4} + \frac{\int \frac{(e^2(4Cd^2+Ae^2)+e^3(3Cd+Be)x)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx}{e^4}$$

$$= -\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{7de^3(d+ex)^5} + \frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^4} + \frac{(23Cd^2+e}{}$$

$$= -\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{7de^3(d+ex)^5} + \frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^4} - \frac{(23Cd^2+e}{}$$

$$= -\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{7de^3(d+ex)^5} + \frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^4} - \frac{(23Cd^2+e}{}$$

Mathematica [A]
time = 0.76, size = 109, normalized size = 0.61

$$-\frac{(d-ex)\sqrt{d^2-e^2x^2}\,(Cd^2(2d^2+10dex+23e^2x^2)+e(5Bd(d^2+5dex+e^2x^2)+Ae(23d^2+10dex+2e^2x^2)))}{105d^3e^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^5,x]
```

```
[Out] -1/105*((d - e*x)*Sqrt[d^2 - e^2*x^2]*(C*d^2*(2*d^2 + 10*d*e*x + 23*e^2*x^2
) + e*(5*B*d*(d^2 + 5*d*e*x + e^2*x^2) + A*e*(23*d^2 + 10*d*e*x + 2*e^2*x^2
))))/(d^3*e^3*(d + e*x)^4)
```

Maple [A]
time = 0.09, size = 308, normalized size = 1.71

method	result
gosper	$-\frac{(-ex+d)\left(2A\,e^{4}x^{2}+5Bd\,e^{3}x^{2}+23C\,d^{2}e^{2}x^{2}+10Ad\,e^{3}x+25B\,d^{2}e^{2}x+10C\,d^{3}xe+23A\,d^{2}e^{2}+5B\,d^{3}e+2C\,d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{105(ex+d)^{4}d^{3}e^{3}}$
trager	$-\frac{(-2A\,e^{5}x^{3}-5Bd\,e^{4}x^{3}-23C\,d^{2}e^{3}x^{3}-8Ad\,e^{4}x^{2}-20B\,d^{2}e^{3}x^{2}+13C\,d^{3}e^{2}x^{2}-13A\,d^{2}e^{3}x+20B\,d^{3}e^{2}x+8C\,d^{4}ex+23A\,d^{3}e^{2}+5B\,d^{4}e+}{105d^{3}(ex+d)^{4}e^{3}}$
default	$\frac{(Be-2Cd)\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^{4}}-\frac{\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15d^{2}\left(x+\frac{d}{e}\right)^{3}}\right)}{e^{6}}-\frac{C\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3e^{6}d\left(x+\frac{d}{e}\right)^{3}}+\frac{(A\,e^{2}-Bde+C}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (B*e-2*C*d)/e^6*(-1/5/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/
15/d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2))-1/3*C/e^6/d/(x+d/e)^
3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+(A*e^2-B*d*e+C*d^2)/e^7*(-1/7/d/e/(x
+d/e)^5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+2/7*e/d*(-1/5/d/e/(x+d/e)^4*(-
(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e
*(x+d/e))^(3/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 868 vs. $2(166) = 332$.
time = 0.33, size = 868, normalized size = 4.82

$$-\frac{2\sqrt{-x^{2}e^{2}+d^{2}}\,Cd^{2}}{7\left(x^{4}e^{7}+4\,dx^{3}e^{6}+6\,d^{2}x^{2}e^{5}+4\,d^{3}xe^{4}+d^{4}e^{3}\right)}+\frac{\sqrt{-x^{2}e^{2}+d^{2}}\,Cd^{2}}{35\left(dx^{3}e^{6}+3\,d^{2}x^{2}e^{5}+3\,d^{3}xe^{4}+d^{4}e^{3}\right)}+\frac{2\sqrt{-x^{2}e^{2}+d^{2}}\,Cd^{2}}{105\left(d^{2}x^{2}e^{5}+2\,d^{3}xe^{4}+d^{4}e^{3}\right)}+\frac{2\sqrt{-x^{2}e^{2}+d^{2}}\,Cd^{2}}{105\left(d^{3}xe^{4}+d^{4}e^{3}\right)}+\frac{2\sqrt{-x^{2}e^{2}+d^{2}}\,Bd}{7\left(x^{4}e^{6}+4\,dx^{3}e^{5}+6\,d^{2}x^{2}e^{4}+4\,d^{3}xe^{3}+d^{4}e^{2}\right)}-\frac{\sqrt{-x^{2}e^{2}+d^{2}}\,Bd}{35\left(dx^{3}e^{5}+3\,d^{2}x^{2}e^{4}+3\,d^{3}xe^{3}+d^{4}e^{2}\right)}-\frac{2\sqrt{-x^{2}e^{2}+d^{2}}\,Bd}{105\left(d^{2}x^{2}e^{4}+2\,d^{3}xe^{3}+d^{4}e^{2}\right)}-\frac{2\sqrt{-x^{2}e^{2}+d^{2}}\,Bd}{105\left(d^{3}xe^{3}+d^{4}e^{2}\right)}+\frac{4\sqrt{-x^{2}e^{2}+d^{2}}\,Cd}{5\left(x^{3}e^{6}+3\,dx^{2}e^{5}+3\,d^{2}xe^{4}+d^{3}e^{3}\right)}-\frac{2\sqrt{-x^{2}e^{2}+d^{2}}\,Cd}{15\left(dx^{2}e^{5}+2\,d^{2}xe^{4}+d^{3}e^{3}\right)}-\frac{2\sqrt{-x^{2}e^{2}+d^{2}}\,Cd}{15\left(d^{2}xe^{4}+d^{3}e^{3}\right)}-\frac{2\sqrt{-x^{2}e^{2}+d^{2}}\,A}{7\left(x^{4}e^{5}+4\,dx^{3}e^{4}+6\,d^{2}x^{2}e^{3}+4\,d^{3}xe^{2}+d^{4}e\right)}+\frac{\sqrt{-x^{2}e^{2}+d^{2}}\,A}{35\left(dx^{3}e^{4}+3\,d^{2}x^{2}e^{3}+3\,d^{3}xe^{2}+d^{4}e\right)}+\frac{2\sqrt{-x^{2}e^{2}+d^{2}}\,A}{105\left(d^{2}x^{2}e^{3}+2\,d^{3}xe^{2}+d^{4}e\right)}+\frac{2\sqrt{-x^{2}e^{2}+d^{2}}\,A}{105\left(d^{3}xe^{2}+d^{4}e\right)}-\frac{2\sqrt{-x^{2}e^{2}+d^{2}}\,B}{5\left(x^{3}e^{5}+3\,dx^{2}e^{4}+3\,d^{2}xe^{3}+d^{3}e^{2}\right)}+\frac{\sqrt{-x^{2}e^{2}+d^{2}}\,B}{15\left(dx^{2}e^{4}+2\,d^{2}xe^{3}+d^{3}e^{2}\right)}+\frac{\sqrt{-x^{2}e^{2}+d^{2}}\,B}{15\left(d^{2}xe^{3}+d^{3}e^{2}\right)}-\frac{2\sqrt{-x^{2}e^{2}+d^{2}}\,C}{3\left(x^{2}e^{5}+2\,dxe^{4}+d^{2}e^{3}\right)}+\frac{\sqrt{-x^{2}e^{2}+d^{2}}\,C}{3\left(dxe^{4}+d^{2}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="maxima
")
```

```
[Out] -2/7*sqrt(-x^2*e^2 + d^2)*C*d^2/(x^4*e^7 + 4*d*x^3*e^6 + 6*d^2*x^2*e^5 + 4*
d^3*x*e^4 + d^4*e^3) + 1/35*sqrt(-x^2*e^2 + d^2)*C*d^2/(d*x^3*e^6 + 3*d^2*x
^2*e^5 + 3*d^3*x*e^4 + d^4*e^3) + 2/105*sqrt(-x^2*e^2 + d^2)*C*d^2/(d^2*x^2
*e^5 + 2*d^3*x*e^4 + d^4*e^3) + 2/105*sqrt(-x^2*e^2 + d^2)*C*d^2/(d^3*x*e^4
 + d^4*e^3) + 2/7*sqrt(-x^2*e^2 + d^2)*B*d/(x^4*e^6 + 4*d*x^3*e^5 + 6*d^2*x
^2*e^4 + 4*d^3*x*e^3 + d^4*e^2) - 1/35*sqrt(-x^2*e^2 + d^2)*B*d/(d*x^3*e^5
+ 3*d^2*x^2*e^4 + 3*d^3*x*e^3 + d^4*e^2) - 2/105*sqrt(-x^2*e^2 + d^2)*B*d/(
d^2*x^2*e^4 + 2*d^3*x*e^3 + d^4*e^2) - 2/105*sqrt(-x^2*e^2 + d^2)*B*d/(d^3*
x*e^3 + d^4*e^2) + 4/5*sqrt(-x^2*e^2 + d^2)*C*d/(x^3*e^6 + 3*d*x^2*e^5 + 3*
```

```
d^2*x*e^4 + d^3*e^3) - 2/15*sqrt(-x^2*e^2 + d^2)*C*d/(d*x^2*e^5 + 2*d^2*x*e
^4 + d^3*e^3) - 2/15*sqrt(-x^2*e^2 + d^2)*C*d/(d^2*x*e^4 + d^3*e^3) - 2/7*s
qrt(-x^2*e^2 + d^2)*A/(x^4*e^5 + 4*d*x^3*e^4 + 6*d^2*x^2*e^3 + 4*d^3*x*e^2
+ d^4*e) + 1/35*sqrt(-x^2*e^2 + d^2)*A/(d*x^3*e^4 + 3*d^2*x^2*e^3 + 3*d^3*x
*e^2 + d^4*e) + 2/105*sqrt(-x^2*e^2 + d^2)*A/(d^2*x^2*e^3 + 2*d^3*x*e^2 + d
^4*e) + 2/105*sqrt(-x^2*e^2 + d^2)*A/(d^3*x*e^2 + d^4*e) - 2/5*sqrt(-x^2*e^
2 + d^2)*B/(x^3*e^5 + 3*d*x^2*e^4 + 3*d^2*x*e^3 + d^3*e^2) + 1/15*sqrt(-x^2
*e^2 + d^2)*B/(d*x^2*e^4 + 2*d^2*x*e^3 + d^3*e^2) + 1/15*sqrt(-x^2*e^2 + d^
2)*B/(d^2*x*e^3 + d^3*e^2) - 2/3*sqrt(-x^2*e^2 + d^2)*C/(x^2*e^5 + 2*d*x*e^
4 + d^2*e^3) + 1/3*sqrt(-x^2*e^2 + d^2)*C/(d*x*e^4 + d^2*e^3)
```

Fricas [A]
time = 0.38, size = 306, normalized size = 1.70

$$-\frac{2\,Cd^6+23\,Ax^4e^6+(5\,Bdx^4+92\,Adx^3)e^5+2\,(Cd^2x^4+10\,Bd^2x^3+69\,Ad^2x^2)e^4+2\,(4\,Cd^3x^3+15\,Bd^3x^2+46\,Ad^3x)e^3+(12\,Cd^4x^2+20\,Bd^4x+23\,Ad^4)e^2+(8\,Cd^5x+5\,Bd^5)e+(2\,Cd^5-2\,Ax^3e^5-(5\,Bdx^3+8\,Adx^2)e^4-(23\,Cd^2x^3+20\,Bd^2x^2+13\,Ad^2x)e^3+(13\,Cd^3x^2+20\,Bd^3x+23\,Ad^3)e^2+(8\,Cd^4x+5\,Bd^4)e)\sqrt{-x^2e^2+d^2}}{105\,(d^3x^4e^7+4\,d^4x^3e^6+6\,d^5x^2e^5+4\,d^6xe^4+d^7e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="fricas
")
```

```
[Out] -1/105*(2*C*d^6 + 23*A*x^4*e^6 + (5*B*d*x^4 + 92*A*d*x^3)*e^5 + 2*(C*d^2*x^
4 + 10*B*d^2*x^3 + 69*A*d^2*x^2)*e^4 + 2*(4*C*d^3*x^3 + 15*B*d^3*x^2 + 46*A
*d^3*x)*e^3 + (12*C*d^4*x^2 + 20*B*d^4*x + 23*A*d^4)*e^2 + (8*C*d^5*x + 5*B
*d^5)*e + (2*C*d^5 - 2*A*x^3*e^5 - (5*B*d*x^3 + 8*A*d*x^2)*e^4 - (23*C*d^2*
x^3 + 20*B*d^2*x^2 + 13*A*d^2*x)*e^3 + (13*C*d^3*x^2 + 20*B*d^3*x + 23*A*d^
3)*e^2 + (8*C*d^4*x + 5*B*d^4)*e)*sqrt(-x^2*e^2 + d^2))/(d^3*x^4*e^7 + 4*d^
4*x^3*e^6 + 6*d^5*x^2*e^5 + 4*d^6*x*e^4 + d^7*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(A+Bx+Cx^2\right)}{\left(d+ex\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**5,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**5, x)
```

Giac [C] Result contains complex when optimal does not.
time = 4.12, size = 623, normalized size = 3.46

$$-\frac{1}{420}\left(\frac{\left(3\left(5\left(\frac{2d}{xe+d}-1\right)^{\frac{7}{2}}+21\left(\frac{2d}{xe+d}-1\right)^{\frac{5}{2}}+35\left(\frac{2d}{xe+d}-1\right)^{\frac{3}{2}}+35\sqrt{\frac{2d}{xe+d}-1}\right)C\,\mathrm{sgn}\left(\frac{1}{xe+d}\right)-35\left(3\left(\frac{2d}{xe+d}-1\right)^{\frac{5}{2}}+10\left(\frac{2d}{xe+d}-1\right)^{\frac{3}{2}}+15\sqrt{\frac{2d}{xe+d}-1}\right)C\,\mathrm{sgn}\left(\frac{1}{xe+d}\right)+280\left(\left(\frac{2d}{xe+d}-1\right)^{\frac{3}{2}}+3\sqrt{\frac{2d}{xe+d}-1}\right)C\,\mathrm{sgn}\left(\frac{1}{xe+d}\right)-\frac{3\left(5\left(\frac{2d}{xe+d}-1\right)^{\frac{7}{2}}+21\left(\frac{2d}{xe+d}-1\right)^{\frac{5}{2}}+35\left(\frac{2d}{xe+d}-1\right)^{\frac{3}{2}}+35\sqrt{\frac{2d}{xe+d}-1}\right)B\,\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d}+\frac{21\left(3\left(\frac{2d}{xe+d}-1\right)^{\frac{5}{2}}+10\left(\frac{2d}{xe+d}-1\right)^{\frac{3}{2}}+15\sqrt{\frac{2d}{xe+d}-1}\right)B\,\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d}-\frac{70\left(\left(\frac{2d}{xe+d}-1\right)^{\frac{3}{2}}+3\sqrt{\frac{2d}{xe+d}-1}\right)B\,\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d}-420\,C\sqrt{\frac{2d}{xe+d}-1}\,\mathrm{sgn}\left(\frac{1}{xe+d}\right)+\frac{3\left(5\left(\frac{2d}{xe+d}-1\right)^{\frac{7}{2}}+21\left(\frac{2d}{xe+d}-1\right)^{\frac{5}{2}}+35\left(\frac{2d}{xe+d}-1\right)^{\frac{3}{2}}+35\sqrt{\frac{2d}{xe+d}-1}\right)Ae^2\,\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d^2}-\frac{7\left(3\left(\frac{2d}{xe+d}-1\right)^{\frac{5}{2}}+10\left(\frac{2d}{xe+d}-1\right)^{\frac{3}{2}}+15\sqrt{\frac{2d}{xe+d}-1}\right)Ae^2\,\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d^2}\right)e^{(-4)}}{d}+\frac{4\,(23i\,Cd^2+5i\,Bde+2i\,Ae^2)e^{(-4)}\mathrm{sgn}\left(\frac{1}{xe+d}\right)}{d^3}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] -1/420*((3*(5*(2*d/(x*e + d) - 1)^(7/2) + 21*(2*d/(x*e + d) - 1)^(5/2) + 35
*(2*d/(x*e + d) - 1)^(3/2) + 35*sqrt(2*d/(x*e + d) - 1))*C*sgn(1/(x*e + d))
 - 35*(3*(2*d/(x*e + d) - 1)^(5/2) + 10*(2*d/(x*e + d) - 1)^(3/2) + 15*sqrt
(2*d/(x*e + d) - 1))*C*sgn(1/(x*e + d)) + 280*((2*d/(x*e + d) - 1)^(3/2) +
3*sqrt(2*d/(x*e + d) - 1))*C*sgn(1/(x*e + d)) - 3*(5*(2*d/(x*e + d) - 1)^(7
/2) + 21*(2*d/(x*e + d) - 1)^(5/2) + 35*(2*d/(x*e + d) - 1)^(3/2) + 35*sqrt
(2*d/(x*e + d) - 1))*B*e*sgn(1/(x*e + d))/d + 21*(3*(2*d/(x*e + d) - 1)^(5/
2) + 10*(2*d/(x*e + d) - 1)^(3/2) + 15*sqrt(2*d/(x*e + d) - 1))*B*e*sgn(1/(
x*e + d))/d - 70*((2*d/(x*e + d) - 1)^(3/2) + 3*sqrt(2*d/(x*e + d) - 1))*B*
e*sgn(1/(x*e + d))/d - 420*C*sqrt(2*d/(x*e + d) - 1)*sgn(1/(x*e + d)) + 3*(
5*(2*d/(x*e + d) - 1)^(7/2) + 21*(2*d/(x*e + d) - 1)^(5/2) + 35*(2*d/(x*e +
 d) - 1)^(3/2) + 35*sqrt(2*d/(x*e + d) - 1))*A*e^2*sgn(1/(x*e + d))/d^2 - 7
*(3*(2*d/(x*e + d) - 1)^(5/2) + 10*(2*d/(x*e + d) - 1)^(3/2) + 15*sqrt(2*d/
(x*e + d) - 1))*A*e^2*sgn(1/(x*e + d))/d^2)*e^(-4)/d + 4*(23*I*C*d^2 + 5*I*
B*d*e + 2*I*A*e^2)*e^(-4)*sgn(1/(x*e + d))/d^3)*e
```

Mupad [B]
time = 4.67, size = 601, normalized size = 3.34

$$\frac{B\sqrt{d^2-e^2x^2}}{21\,(d^3e^2+x\,d^2e^3)}-\frac{3\,B\sqrt{d^2-e^2x^2}}{7\,(d^3e^2+3\,d^2e^3x+3\,d\,e^4x^2+e^5x^3)}+\frac{2\,A\sqrt{d^2-e^2x^2}}{105\,(d^4e+2\,d^3e^2x+d^2e^3x^2)}+\frac{B\sqrt{d^2-e^2x^2}}{21\,(d^3e^2+2\,d^2e^3x+d\,e^4x^2)}-\frac{82\,C\sqrt{d^2-e^2x^2}}{105\,(d^2e^3+2\,d\,e^4x+e^5x^2)}+\frac{2\,A\sqrt{d^2-e^2x^2}}{105\,(d^4e+x\,d^3e^2)}+\frac{23\,C\sqrt{d^2-e^2x^2}}{105\,(d^2e^3+x\,d\,e^4)}-\frac{2\,A\sqrt{d^2-e^2x^2}}{7\,(d^4e+4\,d^3e^2x+6\,d^2e^3x^2+4\,d\,e^4x^3+e^5x^4)}+\frac{A\sqrt{d^2-e^2x^2}}{35\,(d^4e+3\,d^3e^2x+3\,d^2e^3x^2+d\,e^4x^3)}-\frac{2\,C\,d^2\sqrt{d^2-e^2x^2}}{7\,(d^4e^3+4\,d^3e^4x+6\,d^2e^5x^2+4\,d\,e^6x^3+e^7x^4)}+\frac{2\,B\,d\sqrt{d^2-e^2x^2}}{7\,(d^4e^2+4\,d^3e^3x+6\,d^2e^4x^2+4\,d\,e^5x^3+e^6x^4)}+\frac{29\,C\,d\sqrt{d^2-e^2x^2}}{35\,(d^3e^3+3\,d^2e^4x+3\,d\,e^5x^2+e^6x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^5,x)
```

```
[Out] (B*(d^2 - e^2*x^2)^(1/2))/(21*(d^3*e^2 + d^2*e^3*x)) - (3*B*(d^2 - e^2*x^2)
^(1/2))/(7*(d^3*e^2 + e^5*x^3 + 3*d^2*e^3*x + 3*d*e^4*x^2)) + (2*A*(d^2 - e
^2*x^2)^(1/2))/(105*(d^4*e + 2*d^3*e^2*x + d^2*e^3*x^2)) + (B*(d^2 - e^2*x^
2)^(1/2))/(21*(d^3*e^2 + 2*d^2*e^3*x + d*e^4*x^2)) - (82*C*(d^2 - e^2*x^2)^
(1/2))/(105*(d^2*e^3 + e^5*x^2 + 2*d*e^4*x)) + (2*A*(d^2 - e^2*x^2)^(1/2))/
(105*(d^4*e + d^3*e^2*x)) + (23*C*(d^2 - e^2*x^2)^(1/2))/(105*(d^2*e^3 + d*
e^4*x)) - (2*A*(d^2 - e^2*x^2)^(1/2))/(7*(d^4*e + e^5*x^4 + 4*d^3*e^2*x + 4
*d*e^4*x^3 + 6*d^2*e^3*x^2)) + (A*(d^2 - e^2*x^2)^(1/2))/(35*(d^4*e + 3*d^3
*e^2*x + d*e^4*x^3 + 3*d^2*e^3*x^2)) - (2*C*d^2*(d^2 - e^2*x^2)^(1/2))/(7*(
d^4*e^3 + e^7*x^4 + 4*d^3*e^4*x + 4*d*e^6*x^3 + 6*d^2*e^5*x^2)) + (2*B*d*(d
^2 - e^2*x^2)^(1/2))/(7*(d^4*e^2 + e^6*x^4 + 4*d^3*e^3*x + 4*d*e^5*x^3 + 6*
d^2*e^4*x^2)) + (29*C*d*(d^2 - e^2*x^2)^(1/2))/(35*(d^3*e^3 + e^6*x^3 + 3*d
^2*e^4*x + 3*d*e^5*x^2))
```

3.9 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx$

Optimal. Leaf size=234

$$-\frac{(Cd^2-Bde+Ae^2)\,(d^2-e^2x^2)^{3/2}}{9de^3(d+ex)^6}+\frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)^5}-\frac{(11Cd^2+2e(2Bd+Ae))\,(d^2-e^2x^2)^{3/2}}{42d^2e^3(d+ex)^5}-\frac{(11Cd^2+$$

```
[Out] -1/9*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^6+1/2*C*(-e^2*x
^2+d^2)^(3/2)/e^3/(e*x+d)^5-1/42*(11*C*d^2+2*e*(A*e+2*B*d))*(-e^2*x^2+d^2)^
(3/2)/d^2/e^3/(e*x+d)^5-1/105*(11*C*d^2+2*e*(A*e+2*B*d))*(-e^2*x^2+d^2)^(3/
2)/d^3/e^3/(e*x+d)^4-1/315*(11*C*d^2+2*e*(A*e+2*B*d))*(-e^2*x^2+d^2)^(3/2)/
d^4/e^3/(e*x+d)^3
```

Rubi [A]
time = 0.16, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1653, 807, 673, 665}

$$-\frac{(d^2-e^2x^2)^{3/2}\,(2e(Ae+2Bd)+11Cd^2)}{42d^2e^3(d+ex)^5}-\frac{(d^2-e^2x^2)^{3/2}\,(Ae^2-Bde+Cd^2)}{9de^3(d+ex)^6}-\frac{(d^2-e^2x^2)^{3/2}\,(2e(Ae+2Bd)+11Cd^2)}{315d^4e^3(d+ex)^3}-\frac{(d^2-e^2x^2)^{3/2}\,(2e(Ae+2Bd)+11Cd^2)}{105d^3e^3(d+ex)^4}+\frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^6,x]
```

```
[Out] -1/9*((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(d*e^3*(d + e*x)^6) +
(C*(d^2 - e^2*x^2)^(3/2))/(2*e^3*(d + e*x)^5) - ((11*C*d^2 + 2*e*(2*B*d + A
*e))*(d^2 - e^2*x^2)^(3/2))/(42*d^2*e^3*(d + e*x)^5) - ((11*C*d^2 + 2*e*(2*
B*d + A*e))*(d^2 - e^2*x^2)^(3/2))/(105*d^3*e^3*(d + e*x)^4) - ((11*C*d^2 +
 2*e*(2*B*d + A*e))*(d^2 - e^2*x^2)^(3/2))/(315*d^4*e^3*(d + e*x)^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx = \frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)^5} + \frac{\int \frac{(e^2(5Cd^2+2Ae^2)+e^3(3Cd+2Be)x)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx}{2e^4}$$

$$= -\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{9de^3(d+ex)^6} + \frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)^5} + \frac{(11Cd^2+}{}$$

$$= -\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{9de^3(d+ex)^6} + \frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)^5} - \frac{(11Cd^2+}{}$$

$$= -\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{9de^3(d+ex)^6} + \frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)^5} - \frac{(11Cd^2+}{}$$

$$= -\frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{9de^3(d+ex)^6} + \frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)^5} - \frac{(11Cd^2+}{}$$

Mathematica [A]
time = 0.87, size = 144, normalized size = 0.62

$$-\frac{(d-ex)\sqrt{d^2-e^2x^2}\,(Cd^2(4d^3+24d^2ex+66de^2x^2+11e^3x^3)+e(Ae(58d^3+33d^2ex+12de^2x^2+2e^3x^3)+Bd(11d^3+66d^2ex+24de^2x^2+4e^3x^3)))}{315d^4e^3(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^6,x]
```

```
[Out] -1/315*((d - e*x)*Sqrt[d^2 - e^2*x^2]*(C*d^2*(4*d^3 + 24*d^2*e*x + 66*d*e^2
*x^2 + 11*e^3*x^3) + e*(A*e*(58*d^3 + 33*d^2*e*x + 12*d*e^2*x^2 + 2*e^3*x^3
) + B*d*(11*d^3 + 66*d^2*e*x + 24*d*e^2*x^2 + 4*e^3*x^3))))/(d^4*e^3*(d + e
*x)^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(214) = 428$.
time = 0.10, size = 459, normalized size = 1.96

method	result
gosper	$-\frac{(-ex+d)(2Ae^5x^3+4Bde^4x^3+11Cd^2e^3x^3+12Ade^4x^2+24Bd^2e^3x^2+66Cd^3e^2x^2+33Ad^2e^3x+66Bd^3e^2x+24Cd^4ex+58Ad^3e^2+}{315(ex+d)^5d^4e^3}$
trager	$-\frac{(-2Ae^6x^4-4Bde^5x^4-11Cd^2e^4x^4-10Ade^5x^3-20Bd^2e^4x^3-55Cd^3e^3x^3-21Ad^2e^4x^2-42Bd^3e^3x^2+42Cd^4e^2x^2-25Ad^3e^3x+5}{315d^4(ex+d)^5e^3}$
default	$\frac{(Ae^2-Bde+Cd^2)\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{9de\left(x+\frac{d}{e}\right)^6}+\frac{e\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{7de\left(x+\frac{d}{e}\right)^5}+\frac{2e\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^4}-\frac{\left(-\left(x+\frac{d}{e}\right)^2}{15d}\right.}{7d}\right.}{3d}\right.}{e^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x,method=_RETURNVERBOSE)
```

```
[Out] (A*e^2-B*d*e+C*d^2)/e^8*(-1/9/d/e/(x+d/e)^6*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(3/2)+1/3*e/d*(-1/7/d/e/(x+d/e)^5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+2/7*
e/d*(-1/5/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/15/d^2/(x+d/
e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2))))+C/e^6*(-1/5/d/e/(x+d/e)^4*(-(x
+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(
x+d/e))^(3/2))+(B*e-2*C*d)/e^7*(-1/7/d/e/(x+d/e)^5*(-(x+d/e)^2*e^2+2*d*e*(x
+d/e))^(3/2)+2/7*e/d*(-1/5/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/
2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. $2(215) = 430$.
time = 0.30, size = 1265, normalized size = 5.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="maxima
")
```

```
[Out] -2/9*sqrt(-x^2*e^2 + d^2)*C*d^2/(x^5*e^8 + 5*d*x^4*e^7 + 10*d^2*x^3*e^6 + 1
0*d^3*x^2*e^5 + 5*d^4*x*e^4 + d^5*e^3) + 1/63*sqrt(-x^2*e^2 + d^2)*C*d^2/(d
*x^4*e^7 + 4*d^2*x^3*e^6 + 6*d^3*x^2*e^5 + 4*d^4*x*e^4 + d^5*e^3) + 1/105*s
qrt(-x^2*e^2 + d^2)*C*d^2/(d^2*x^3*e^6 + 3*d^3*x^2*e^5 + 3*d^4*x*e^4 + d^5*
e^3) + 2/315*sqrt(-x^2*e^2 + d^2)*C*d^2/(d^3*x^2*e^5 + 2*d^4*x*e^4 + d^5*e^
3) + 2/315*sqrt(-x^2*e^2 + d^2)*C*d^2/(d^4*x*e^4 + d^5*e^3) + 2/9*sqrt(-x^2
*e^2 + d^2)*B*d/(x^5*e^7 + 5*d*x^4*e^6 + 10*d^2*x^3*e^5 + 10*d^3*x^2*e^4 +
5*d^4*x*e^3 + d^5*e^2) - 1/63*sqrt(-x^2*e^2 + d^2)*B*d/(d*x^4*e^6 + 4*d^2*x
^3*e^5 + 6*d^3*x^2*e^4 + 4*d^4*x*e^3 + d^5*e^2) - 1/105*sqrt(-x^2*e^2 + d^2
)*B*d/(d^2*x^3*e^5 + 3*d^3*x^2*e^4 + 3*d^4*x*e^3 + d^5*e^2) - 2/315*sqrt(-x
^2*e^2 + d^2)*B*d/(d^3*x^2*e^4 + 2*d^4*x*e^3 + d^5*e^2) - 2/315*sqrt(-x^2*e
^2 + d^2)*B*d/(d^4*x*e^3 + d^5*e^2) + 4/7*sqrt(-x^2*e^2 + d^2)*C*d/(x^4*e^7
 + 4*d*x^3*e^6 + 6*d^2*x^2*e^5 + 4*d^3*x*e^4 + d^4*e^3) - 2/35*sqrt(-x^2*e^
2 + d^2)*C*d/(d*x^3*e^6 + 3*d^2*x^2*e^5 + 3*d^3*x*e^4 + d^4*e^3) - 4/105*sq
rt(-x^2*e^2 + d^2)*C*d/(d^2*x^2*e^5 + 2*d^3*x*e^4 + d^4*e^3) - 4/105*sqrt(-
x^2*e^2 + d^2)*C*d/(d^3*x*e^4 + d^4*e^3) - 2/9*sqrt(-x^2*e^2 + d^2)*A/(x^5*
e^6 + 5*d*x^4*e^5 + 10*d^2*x^3*e^4 + 10*d^3*x^2*e^3 + 5*d^4*x*e^2 + d^5*e)
+ 1/63*sqrt(-x^2*e^2 + d^2)*A/(d*x^4*e^5 + 4*d^2*x^3*e^4 + 6*d^3*x^2*e^3 +
4*d^4*x*e^2 + d^5*e) + 1/105*sqrt(-x^2*e^2 + d^2)*A/(d^2*x^3*e^4 + 3*d^3*x^
2*e^3 + 3*d^4*x*e^2 + d^5*e) + 2/315*sqrt(-x^2*e^2 + d^2)*A/(d^3*x^2*e^3 +
2*d^4*x*e^2 + d^5*e) + 2/315*sqrt(-x^2*e^2 + d^2)*A/(d^4*x*e^2 + d^5*e) - 2
/7*sqrt(-x^2*e^2 + d^2)*B/(x^4*e^6 + 4*d*x^3*e^5 + 6*d^2*x^2*e^4 + 4*d^3*x*
e^3 + d^4*e^2) + 1/35*sqrt(-x^2*e^2 + d^2)*B/(d*x^3*e^5 + 3*d^2*x^2*e^4 + 3
*d^3*x*e^3 + d^4*e^2) + 2/105*sqrt(-x^2*e^2 + d^2)*B/(d^2*x^2*e^4 + 2*d^3*x
*e^3 + d^4*e^2) + 2/105*sqrt(-x^2*e^2 + d^2)*B/(d^3*x*e^3 + d^4*e^2) - 2/5*
sqrt(-x^2*e^2 + d^2)*C/(x^3*e^6 + 3*d*x^2*e^5 + 3*d^2*x*e^4 + d^3*e^3) + 1/
15*sqrt(-x^2*e^2 + d^2)*C/(d*x^2*e^5 + 2*d^2*x*e^4 + d^3*e^3) + 1/15*sqrt(-
x^2*e^2 + d^2)*C/(d^2*x*e^4 + d^3*e^3)
```

Fricas [A]
time = 0.40, size = 380, normalized size = 1.62

$$-\frac{4\,Cd^{7} + 58\,Ax^{5}e^{7} + (11\,Bdx^{5} + 290\,Adx^{4})e^{6} + (4\,Cd^{2}x^{5} + 55\,Bd^{2}x^{4} + 580\,Ad^{2}x^{3})e^{5} + 10\,(2\,Cd^{3}x^{4} + 11\,Bd^{3}x^{3} + 58\,Ad^{3}x^{2})e^{4} + 10\,(4\,Cd^{4}x^{3} + 11\,Bd^{4}x^{2} + 29\,Ad^{4}x)e^{3} + (40\,Cd^{5}x^{2} + 55\,Bd^{5}x + 58\,Ad^{5})e^{2} + (20\,Cd^{6}x + 11\,Bd^{6})e + (4\,Cd^{6} - 2\,Ax^{4}e^{6} - 2\,(2\,Bdx^{4} + 5\,Adx^{3})e^{5} - (11\,Cd^{2}x^{4} + 20\,Bd^{2}x^{3} + 21\,Ad^{2}x^{2})e^{4} - (55\,Cd^{3}x^{3} + 42\,Bd^{3}x^{2} + 25\,Ad^{3}x)e^{3} + (42\,Cd^{4}x^{2} + 55\,Bd^{4}x + 58\,Ad^{4})e^{2} + (20\,Cd^{5}x + 11\,Bd^{5})e)\sqrt{-x^{2}e^{2} + d^{2}}}{315\,(d^{4}x^{5}e^{8} + 5\,d^{5}x^{4}e^{7} + 10\,d^{6}x^{3}e^{6} + 10\,d^{7}x^{2}e^{5} + 5\,d^{8}xe^{4} + d^{9}e^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="fricas
")
```

```
[Out] -1/315*(4*C*d^7 + 58*A*x^5*e^7 + (11*B*d*x^5 + 290*A*d*x^4)*e^6 + (4*C*d^2*
x^5 + 55*B*d^2*x^4 + 580*A*d^2*x^3)*e^5 + 10*(2*C*d^3*x^4 + 11*B*d^3*x^3 +
58*A*d^3*x^2)*e^4 + 10*(4*C*d^4*x^3 + 11*B*d^4*x^2 + 29*A*d^4*x)*e^3 + (40*
C*d^5*x^2 + 55*B*d^5*x + 58*A*d^5)*e^2 + (20*C*d^6*x + 11*B*d^6)*e + (4*C*d
^6 - 2*A*x^4*e^6 - 2*(2*B*d*x^4 + 5*A*d*x^3)*e^5 - (11*C*d^2*x^4 + 20*B*d^2
```

```
*x^3 + 21*A*d^2*x^2)*e^4 - (55*C*d^3*x^3 + 42*B*d^3*x^2 + 25*A*d^3*x)*e^3 +
 (42*C*d^4*x^2 + 55*B*d^4*x + 58*A*d^4)*e^2 + (20*C*d^5*x + 11*B*d^5)*e)*sq
rt(-x^2*e^2 + d^2))/(d^4*x^5*e^8 + 5*d^5*x^4*e^7 + 10*d^6*x^3*e^6 + 10*d^7*
x^2*e^5 + 5*d^8*x*e^4 + d^9*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(A+Bx+Cx^2\right)}{\left(d+ex\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**6,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**6, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 690 vs. 2(215) = 430.
time = 3.90, size = 690, normalized size = 2.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] 2/315*(36*(d*e + sqrt(-x^2*e^2 + d^2)*e)*C*d^2*e^(-2)/x + 144*(d*e + sqrt(-
x^2*e^2 + d^2)*e)^2*C*d^2*e^(-4)/x^2 - 84*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*
C*d^2*e^(-6)/x^3 + 504*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*C*d^2*e^(-8)/x^4 +
420*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*C*d^2*e^(-12)/x^6 + 4*C*d^2 + 11*B*d*e
 + 99*(d*e + sqrt(-x^2*e^2 + d^2)*e)*B*d*e^(-1)/x + 81*(d*e + sqrt(-x^2*e^2
 + d^2)*e)^2*B*d*e^(-3)/x^2 + 609*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*B*d*e^(-
5)/x^3 + 441*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*B*d*e^(-7)/x^4 + 945*(d*e + s
qrt(-x^2*e^2 + d^2)*e)^5*B*d*e^(-9)/x^5 + 315*(d*e + sqrt(-x^2*e^2 + d^2)*e
)^6*B*d*e^(-11)/x^6 + 315*(d*e + sqrt(-x^2*e^2 + d^2)*e)^7*B*d*e^(-13)/x^7
+ 58*A*e^2 + 1143*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*A*e^(-2)/x^2 + 2247*(d*e
 + sqrt(-x^2*e^2 + d^2)*e)^3*A*e^(-4)/x^3 + 3843*(d*e + sqrt(-x^2*e^2 + d^2
)*e)^4*A*e^(-6)/x^4 + 3465*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*A*e^(-8)/x^5 +
2625*(d*e + sqrt(-x^2*e^2 + d^2)*e)^6*A*e^(-10)/x^6 + 945*(d*e + sqrt(-x^2*
e^2 + d^2)*e)^7*A*e^(-12)/x^7 + 315*(d*e + sqrt(-x^2*e^2 + d^2)*e)^8*A*e^(-
14)/x^8 + 207*(d*e + sqrt(-x^2*e^2 + d^2)*e)*A/x)*e^(-3)/(d^4*((d*e + sqrt(
-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)^9)
```

Mupad [B]
time = 5.24, size = 960, normalized size = 4.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^6,x)
```

```
[Out] (B*(d^2 - e^2*x^2)^(1/2))/(63*(d^4*e^2 + d^3*e^3*x)) + (C*(d^2 - e^2*x^2)^(
1/2))/(135*(d^3*e^3 + d^2*e^4*x)) - (19*B*(d^2 - e^2*x^2)^(1/2))/(63*(d^4*e
^2 + e^6*x^4 + 4*d^3*e^3*x + 4*d*e^5*x^3 + 6*d^2*e^4*x^2)) + (A*(d^2 - e^2*
x^2)^(1/2))/(105*(d^5*e + 3*d^4*e^2*x + 3*d^3*e^3*x^2 + d^2*e^4*x^3)) + (2*
B*(d^2 - e^2*x^2)^(1/2))/(105*(d^4*e^2 + 3*d^3*e^3*x + d*e^5*x^3 + 3*d^2*e^
4*x^2)) - (47*C*(d^2 - e^2*x^2)^(1/2))/(105*(d^3*e^3 + e^6*x^3 + 3*d^2*e^4*
x + 3*d*e^5*x^2)) + (2*A*(d^2 - e^2*x^2)^(1/2))/(315*(d^5*e + 2*d^4*e^2*x +
 d^3*e^3*x^2)) + (11*C*(d^2 - e^2*x^2)^(1/2))/(315*(d^3*e^3 + 2*d^2*e^4*x +
 d*e^5*x^2)) - (2*A*(d^2 - e^2*x^2)^(1/2))/(9*(d^5*e + e^6*x^5 + 5*d^4*e^2*
x + 5*d*e^5*x^4 + 10*d^3*e^3*x^2 + 10*d^2*e^4*x^3)) + (A*(d^2 - e^2*x^2)^(1
/2))/(63*(d^5*e + 4*d^4*e^2*x + d*e^5*x^4 + 6*d^3*e^3*x^2 + 4*d^2*e^4*x^3))
 - (2*A*(d^2 - e^2*x^2)^(1/2))/(945*(d^5*e + d^4*e^2*x)) + (4*B*(d^2 - e^2*
x^2)^(1/2))/(315*(d^4*e^2 + 2*d^3*e^3*x + d^2*e^4*x^2)) + (2*B*d*(d^2 - e^2
*x^2)^(1/2))/(9*(d^5*e^2 + e^7*x^5 + 5*d^4*e^3*x + 5*d*e^6*x^4 + 10*d^3*e^4
*x^2 + 10*d^2*e^5*x^3)) + (37*C*d*(d^2 - e^2*x^2)^(1/2))/(63*(d^4*e^3 + e^7
*x^4 + 4*d^3*e^4*x + 4*d*e^6*x^3 + 6*d^2*e^5*x^2)) - (2*C*d^2*(d^2 - e^2*x^
2)^(1/2))/(9*(d^5*e^3 + e^8*x^5 + 5*d^4*e^4*x + 5*d*e^7*x^4 + 10*d^3*e^5*x^
2 + 10*d^2*e^6*x^3)) + (8*A*e^2*(d^2 - e^2*x^2)^(1/2))/(945*(d^5*e^3 + d^4*
e^4*x)) + (26*C*d^2*(d^2 - e^2*x^2)^(1/2))/(945*(d^5*e^3 + d^4*e^4*x)) - (B
*d*e*(d^2 - e^2*x^2)^(1/2))/(315*(d^5*e^3 + d^4*e^4*x))
```

3.10 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=236

$$-\frac{d^2(38Cd^2+45Bde+55Ae^2)\sqrt{d^2-e^2x^2}}{15e^3}-\frac{d(13Cd^2+15Bde+12Ae^2)\,x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{(19Cd^2+5e(3Bd$$

```
[Out] 1/8*d^3*(20*A*e^2+15*B*d*e+13*C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1
/15*d^2*(55*A*e^2+45*B*d*e+38*C*d^2)*(-e^2*x^2+d^2)^(1/2)/e^3-1/8*d*(12*A*e
^2+15*B*d*e+13*C*d^2)*x*(-e^2*x^2+d^2)^(1/2)/e^2-1/15*(19*C*d^2+5*e*(A*e+3*
B*d))*x^2*(-e^2*x^2+d^2)^(1/2)/e-1/4*(B*e+3*C*d)*x^3*(-e^2*x^2+d^2)^(1/2)-1
/5*C*e*x^4*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1829, 655, 223, 209}

$$\frac{d^3\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(20Ae^2+15Bde+13Cd^2)}{8e^3}-\frac{x^2\sqrt{d^2-e^2x^2}\,(5e(Ae+3Bd)+19Cd^2)}{15e}-\frac{dx\sqrt{d^2-e^2x^2}\,(12Ae^2+15Bde+13Cd^2)}{8e^2}-\frac{d^2\sqrt{d^2-e^2x^2}\,(55Ae^2+45Bde+38Cd^2)}{15e^3}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}\,(Be+3Cd)-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -1/15*(d^2*(38*C*d^2 + 45*B*d*e + 55*A*e^2)*Sqrt[d^2 - e^2*x^2])/e^3 - (d*(
13*C*d^2 + 15*B*d*e + 12*A*e^2)*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((19*C*d^2
 + 5*e*(3*B*d + A*e))*x^2*Sqrt[d^2 - e^2*x^2])/(15*e) - ((3*C*d + B*e)*x^3*
Sqrt[d^2 - e^2*x^2])/4 - (C*e*x^4*Sqrt[d^2 - e^2*x^2])/5 + (d^3*(13*C*d^2 +
 15*B*d*e + 20*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2} - \frac{\int \frac{-5Ad^3e^2-5d^2e^2(Bd+3Ae)x-5de^2\left(Cd^2+3e(Bd+Ae)\right)x^2-e^3}{\sqrt{d^2-e^2x^2}}}{5e^2}$$

$$= -\frac{1}{4}(3Cd+Be)x^3\sqrt{d^2-e^2x^2} - \frac{1}{5}Cex^4\sqrt{d^2-e^2x^2} + \frac{\int \frac{20Ad^3e^4+20d^2e^4(}{}}{}$$

$$= -\frac{\left(19Cd^2+5e(3Bd+Ae)\right)x^2\sqrt{d^2-e^2x^2}}{15e} - \frac{1}{4}(3Cd+Be)x^3\sqrt{d^2-e^2}$$

$$= -\frac{d\left(13Cd^2+15Bde+12Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{\left(19Cd^2+5e(3Bd+Ae\right.}{15e}$$

$$= -\frac{d^2\left(38Cd^2+45Bde+55Ae^2\right)\sqrt{d^2-e^2x^2}}{15e^3} - \frac{d\left(13Cd^2+15Bde+12\right.}{8e^2}$$

$$= -\frac{d^2\left(38Cd^2+45Bde+55Ae^2\right)\sqrt{d^2-e^2x^2}}{15e^3} - \frac{d\left(13Cd^2+15Bde+12\right.}{8e^2}$$

$$= -\frac{d^2\left(38Cd^2+45Bde+55Ae^2\right)\sqrt{d^2-e^2x^2}}{15e^3} - \frac{d\left(13Cd^2+15Bde+12\right.}{8e^2}$$

Mathematica [A]
time = 0.62, size = 194, normalized size = 0.82

$$\frac{-e\sqrt{d^2-e^2x^2}\left(C\left(304d^4+195d^3ex+152d^2e^2x^2+90de^3x^3+24e^4x^4\right)+5e\left(4Ae\left(22d^2+9dex+2e^2x^2\right)+3B\left(24d^3+15d^2ex+8de^2x^2+2e^3x^3\right)\right)\right)+15\sqrt{-e^2}\left(13Cd^5+5d^3e(3Bd+4Ae)\right)\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{120e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(e*Sqrt[d^2 - e^2*x^2]*(C*(304*d^4 + 195*d^3*e*x + 152*d^2*e^2*x^2 + 90*d
*e^3*x^3 + 24*e^4*x^4) + 5*e*(4*A*e*(22*d^2 + 9*d*e*x + 2*e^2*x^2) + 3*B*(2
```

```
4*d^3 + 15*d^2*e*x + 8*d*e^2*x^2 + 2*e^3*x^3)))) + 15*Sqrt[-e^2]*(13*C*d^5
+ 5*d^3*e*(3*B*d + 4*A*e))*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(120
*e^4)
```

Maple [A]
time = 0.09, size = 391, normalized size = 1.66

method	result
risch	$-\frac{\left(24 e^{4} C\,x^{4}+30 B\,e^{4} x^{3}+90 C d\,e^{3} x^{3}+40 A\,e^{4} x^{2}+120 B d\,e^{3} x^{2}+152 C\,d^{2} e^{2} x^{2}+180 A d\,e^{3} x+225 B\,d^{2} e^{2} x+195 C\,d^{3} x e+440 A\,d^{2} e^{2}+360 B}{120 e^{3}}$
default	$e^{3} C \left(-\frac{x^{4} \sqrt{-e^{2} x^{2}+d^{2}}}{5 e^{2}}+\frac{4 d^{2} \left(-\frac{x^{2} \sqrt{-e^{2} x^{2}+d^{2}}}{3 e^{2}}-\frac{2 d^{2} \sqrt{-e^{2} x^{2}+d^{2}}}{3 e^{4}}\right)}{5 e^{2}}\right)+\left(e^{3} B+3 d\,e^{2} C\right) \left(-\frac{x^{3} \sqrt{}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*C*(-1/5*x^4/e^2*(-e^2*x^2+d^2)^(1/2)+4/5*d^2/e^2*(-1/3*x^2/e^2*(-e^2*x^
2+d^2)^(1/2)-2/3*d^2/e^4*(-e^2*x^2+d^2)^(1/2)))+(B*e^3+3*C*d*e^2)*(-1/4*x^3
/e^2*(-e^2*x^2+d^2)^(1/2)+3/4*d^2/e^2*(-1/2*x/e^2*(-e^2*x^2+d^2)^(1/2)+1/2*
d^2/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))+(A*e^3+3*B
*d*e^2+3*C*d^2*e)*(-1/3*x^2/e^2*(-e^2*x^2+d^2)^(1/2)-2/3*d^2/e^4*(-e^2*x^2+
d^2)^(1/2))+(3*A*d*e^2+3*B*d^2*e+C*d^3)*(-1/2*x/e^2*(-e^2*x^2+d^2)^(1/2)+1/
2*d^2/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))-(3*A*d^2*
e+B*d^3)/e^2*(-e^2*x^2+d^2)^(1/2)+d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^
2*x^2+d^2)^(1/2))*A
```

Maxima [A]
time = 0.50, size = 364, normalized size = 1.54

$$\frac{3}{8} \left(3 C d e^{2}+B e^{3}\right) d^{4} \arcsin\left(\frac{x e}{d}\right) e^{(-5)}-\frac{1}{5} \sqrt{-x^{2} e^{2}+d^{2}}\, C x^{4} e-\frac{4}{15} \sqrt{-x^{2} e^{2}+d^{2}}\, C d^{2} x^{2} e^{(-1)}-\frac{8}{15} \sqrt{-x^{2} e^{2}+d^{2}}\, C d^{4} e^{(-3)}+A d^{3} \arcsin\left(\frac{x e}{d}\right) e^{(-1)}-\sqrt{-x^{2} e^{2}+d^{2}}\, B d^{3} e^{(-2)}-\frac{1}{4}\left(3 C d e^{2}+B e^{3}\right) \sqrt{-x^{2} e^{2}+d^{2}}\, x^{3} e^{(-2)}-\frac{3}{8}\left(3 C d e^{2}+B e^{3}\right) \sqrt{-x^{2} e^{2}+d^{2}}\, d^{2} x e^{(-4)}+\frac{1}{2}\left(C d^{3}+3 B d^{2} e+3 A d e^{2}\right) d^{2} \arcsin\left(\frac{x e}{d}\right) e^{(-3)}-3 \sqrt{-x^{2} e^{2}+d^{2}}\, A d^{2} e^{(-1)}-\frac{1}{3}\left(3 C d^{2} e+3 B d e^{2}+A e^{3}\right) \sqrt{-x^{2} e^{2}+d^{2}}\, x^{2} e^{(-2)}-\frac{2}{3}\left(3 C d^{2} e+3 B d e^{2}+A e^{3}\right) \sqrt{-x^{2} e^{2}+d^{2}}\, d^{2} e^{(-4)}-\frac{1}{2}\left(C d^{3}+3 B d^{2} e+3 A d e^{2}\right) \sqrt{-x^{2} e^{2}+d^{2}}\, x e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] 3/8*(3*C*d*e^2 + B*e^3)*d^4*arcsin(x*e/d)*e^(-5) - 1/5*sqrt(-x^2*e^2 + d^2)
*C*x^4*e - 4/15*sqrt(-x^2*e^2 + d^2)*C*d^2*x^2*e^(-1) - 8/15*sqrt(-x^2*e^2
+ d^2)*C*d^4*e^(-3) + A*d^3*arcsin(x*e/d)*e^(-1) - sqrt(-x^2*e^2 + d^2)*B*d
```

^3*e^(-2) - 1/4*(3*C*d*e^2 + B*e^3)*sqrt(-x^2*e^2 + d^2)*x^3*e^(-2) - 3/8*(3*C*d*e^2 + B*e^3)*sqrt(-x^2*e^2 + d^2)*d^2*x*e^(-4) + 1/2*(C*d^3 + 3*B*d^2*e + 3*A*d*e^2)*d^2*arcsin(x*e/d)*e^(-3) - 3*sqrt(-x^2*e^2 + d^2)*A*d^2*e^(-1) - 1/3*(3*C*d^2*e + 3*B*d*e^2 + A*e^3)*sqrt(-x^2*e^2 + d^2)*x^2*e^(-2) - 2/3*(3*C*d^2*e + 3*B*d*e^2 + A*e^3)*sqrt(-x^2*e^2 + d^2)*d^2*e^(-4) - 1/2*(C*d^3 + 3*B*d^2*e + 3*A*d*e^2)*sqrt(-x^2*e^2 + d^2)*x*e^(-2)

Fricas [A]
time = 0.37, size = 167, normalized size = 0.71

$$-\frac{1}{120}\left(30\left(13Cd^5+15Bd^4e+20Ad^3e^2\right)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\left(304Cd^4+2\left(12Cx^4+15Bx^3+20Ax^2\right)e^4+30\left(3Cdx^3+4Bdx^2+6Adx\right)e^3+\left(152Cd^2x^2+225Bd^2x+440Ad^2\right)e^2+15\left(13Cd^3x+24Bd^3\right)e\right)\sqrt{-x^2e^2+d^2}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")

[Out] -1/120*(30*(13*C*d^5 + 15*B*d^4*e + 20*A*d^3*e^2)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) + (304*C*d^4 + 2*(12*C*x^4 + 15*B*x^3 + 20*A*x^2)*e^4 + 30*(3*C*d*x^3 + 4*B*d*x^2 + 6*A*d*x)*e^3 + (152*C*d^2*x^2 + 225*B*d^2*x + 440*A*d^2)*e^2 + 15*(13*C*d^3*x + 24*B*d^3)*e)*sqrt(-x^2*e^2 + d^2))*e^(-3)

Sympy [A]
time = 11.24, size = 1268, normalized size = 5.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)

[Out] A*d**3*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 > 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2), (d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d**2), (d**2 < 0) & (e**2 < 0))) + 3*A*d**2*e*Piecewise((x**2/(2*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + 3*A*d*e**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) + I*d*x/(2*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x*sqrt(1 - e**2*x**2/d**2)/(2*e**2), True)) + A*e**3*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True)) + B*d**3*Piecewise((x**2/(2*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + 3*B*d**2*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) + I*d*x/(2*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x*sqrt(1 - e**2*x**2/d**2)/(2*e**2), True)) + 3*B*d*e**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d*

```
*2)), True)) + B*e**3*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3
*x/(8*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**
2/d**2)) - I*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1)
, (3*d**4*asin(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2))
 + d*x**3/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/
d**2)), True)) + C*d**3*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) + I*d*x/(2
*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)),
Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x*sqrt(1 - e**2*x*
*2/d**2)/(2*e**2), True)) + 3*C*d**2*e*Piecewise((-2*d**2*sqrt(d**2 - e**2*
x**2)/(3*e**4) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*
sqrt(d**2)), True)) + 3*C*d*e**2*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5)
 + 3*I*d**3*x/(8*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1
 + e**2*x**2/d**2)) - I*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2
/d**2) > 1), (3*d**4*asin(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*
x**2/d**2)) + d*x**3/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 -
 e**2*x**2/d**2)), True)) + C*e**3*Piecewise((-8*d**4*sqrt(d**2 - e**2*x**2
)/(15*e**6) - 4*d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**4) - x**4*sqrt(d**2
 - e**2*x**2)/(5*e**2), Ne(e, 0)), (x**6/(6*sqrt(d**2)), True))
```

Giac [A]
time = 3.80, size = 166, normalized size = 0.70

$$\frac{1}{8}\left(13\,Cd^{5}+15\,Bd^{4}e+20\,Ad^{3}e^{2}\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{120}\,\sqrt{-x^{2}e^{2}+d^{2}}\,\left(\left(2\left(3\left(4\,Cxe+5\left(3\,Cde^{6}+Be^{7}\right)e^{(-6)}\right)x+4\left(19\,Cd^{2}e^{5}+15\,Bde^{6}+5\,Ae^{7}\right)e^{(-6)}\right)x+15\left(13\,Cd^{3}e^{4}+15\,Bd^{2}e^{5}+12\,Ade^{6}\right)e^{(-6)}\right)x+8\left(38\,Cd^{4}e^{3}+45\,Bd^{3}e^{4}+55\,Ad^{2}e^{5}\right)e^{(-6)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(13*C*d^5 + 15*B*d^4*e + 20*A*d^3*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/
120*sqrt(-x^2*e^2 + d^2)*((2*(3*(4*C*x*e + 5*(3*C*d*e^6 + B*e^7)*e^(-6))*x
+ 4*(19*C*d^2*e^5 + 15*B*d*e^6 + 5*A*e^7)*e^(-6))*x + 15*(13*C*d^3*e^4 + 15
*B*d^2*e^5 + 12*A*d*e^6)*e^(-6))*x + 8*(38*C*d^4*e^3 + 45*B*d^3*e^4 + 55*A*
d^2*e^5)*e^(-6))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d+e\,x\right)}^3\,\left(C\,x^2+B\,x+A\right)}{\sqrt{d^2-e^2\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int(((d + e*x)^3*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2), x)
```

3.11 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=191

$$-\frac{d(4Cd^2+e(5Bd+6Ae))\sqrt{d^2-e^2x^2}}{3e^3}-\frac{(7Cd^2+4e(2Bd+Ae))\,x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{(2Cd+Be)x^2\sqrt{d^2-e^2}}{3e}$$

```
[Out] 1/8*d^2*(12*A*e^2+8*B*d*e+7*C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1/3
*d*(4*C*d^2+e*(6*A*e+5*B*d))*(-e^2*x^2+d^2)^(1/2)/e^3-1/8*(7*C*d^2+4*e*(A*e
+2*B*d))*x*(-e^2*x^2+d^2)^(1/2)/e^2-1/3*(B*e+2*C*d)*x^2*(-e^2*x^2+d^2)^(1/2
)/e-1/4*C*x^3*(-e^2*x^2+d^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1829, 655, 223, 209}

$$\frac{d^2\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(12Ae^2+8Bde+7Cd^2)}{8e^3}-\frac{x\sqrt{d^2-e^2x^2}\,(4e(Ae+2Bd)+7Cd^2)}{8e^2}-\frac{d\sqrt{d^2-e^2x^2}\,(e(6Ae+5Bd)+4Cd^2)}{3e^3}-\frac{x^2\sqrt{d^2-e^2x^2}\,(Be+2Cd)}{3e}-\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -1/3*(d*(4*C*d^2 + e*(5*B*d + 6*A*e))*Sqrt[d^2 - e^2*x^2])/e^3 - ((7*C*d^2
+ 4*e*(2*B*d + A*e))*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((2*C*d + B*e)*x^2*Sq
rt[d^2 - e^2*x^2])/(3*e) - (C*x^3*Sqrt[d^2 - e^2*x^2])/4 + (d^2*(7*C*d^2 +
8*B*d*e + 12*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2} - \frac{\int \frac{-4Ad^2e^2-4de^2(Bd+2Ae)x-e^2\left(7Cd^2+4e(2Bd+Ae)\right)x^2-4e^3(2C}{\sqrt{d^2-e^2x^2}}}{4e^2}$$

$$= -\frac{(2Cd+Be)x^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}Cx^3\sqrt{d^2-e^2x^2} + \frac{\int \frac{12Ad^2e^4+4de^3\left(4Cd^2+\right.}{}}{}$$

$$= -\frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{(2Cd+Be)x^2\sqrt{d^2-e^2x^2}}{3e} -$$

$$= -\frac{d\left(4Cd^2+e(5Bd+6Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d}}{8e^2}$$

$$= -\frac{d\left(4Cd^2+e(5Bd+6Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d}}{8e^2}$$

$$= -\frac{d\left(4Cd^2+e(5Bd+6Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d}}{8e^2}$$

Mathematica [A]
time = 0.53, size = 159, normalized size = 0.83

$$\frac{-e\sqrt{d^2-e^2x^2}\left(C\left(32d^3+21d^2ex+16de^2x^2+6e^3x^3\right)+4e\left(3Ae(4d+ex)+2B\left(5d^2+3dex+e^2x^2\right)\right)\right)+3\sqrt{-e^2}\left(7Cd^4+4d^2e(2Bd+3Ae)\right)\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{24e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(e*Sqrt[d^2 - e^2*x^2]*(C*(32*d^3 + 21*d^2*e*x + 16*d*e^2*x^2 + 6*e^3*x^3
) + 4*e*(3*A*e*(4*d + e*x) + 2*B*(5*d^2 + 3*d*e*x + e^2*x^2)))) + 3*Sqrt[-e
^2]*(7*C*d^4 + 4*d^2*e*(2*B*d + 3*A*e))*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^
2*x^2]])/(24*e^4)
```

Maple [A]
time = 0.08, size = 286, normalized size = 1.50

method	result
risch	$-\frac{(6Ce^3x^3+8Be^3x^2+16Cde^2x^2+12Ae^3x+24Bde^2x+21Cd^2ex+48Ade^2+40d^2eB+32Cd^3)\sqrt{-e^2x^2+d^2}}{24e^3}+\frac{3d^2\arctan\left(\cdots\right.}{\cdots}$
default	$e^2C\left(-\frac{x^3\sqrt{-e^2x^2+d^2}}{4e^2}+\frac{3d^2\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)}{4e^2}\right)+(e^2B+2deC)\left(\cdots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*C*(-1/4*x^3/e^2*(-e^2*x^2+d^2)^(1/2)+3/4*d^2/e^2*(-1/2*x/e^2*(-e^2*x^2+
d^2)^(1/2)+1/2*d^2/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2
))))+(B*e^2+2*C*d*e)*(-1/3*x^2/e^2*(-e^2*x^2+d^2)^(1/2)-2/3*d^2/e^4*(-e^2*x
^2+d^2)^(1/2))+(A*e^2+2*B*d*e+C*d^2)*(-1/2*x/e^2*(-e^2*x^2+d^2)^(1/2)+1/2*d
^2/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))-(2*A*d*e+B*d
^2)/e^2*(-e^2*x^2+d^2)^(1/2)+d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2
+d^2)^(1/2))*A
```

Maxima [A]
time = 0.49, size = 240, normalized size = 1.26

$$\frac{3}{8}Cd^4\arcsin\left(\frac{xe}{d}\right)e^{(-3)}-\frac{3}{8}\sqrt{-x^2e^2+d^2}\,Cd^2xe^{(-2)}+Ad^2\arcsin\left(\frac{xe}{d}\right)e^{(-1)}+\frac{1}{2}\left(Cd^2+2\,Bde+Ae^2\right)d^2\arcsin\left(\frac{xe}{d}\right)e^{(-3)}-\frac{1}{4}\sqrt{-x^2e^2+d^2}\,Cx^3-\sqrt{-x^2e^2+d^2}\,Bd^2e^{(-2)}-\frac{1}{3}\sqrt{-x^2e^2+d^2}\,(2\,Cde+Be^2)x^2e^{(-2)}-\frac{2}{3}\sqrt{-x^2e^2+d^2}\,(2\,Cde+Be^2)d^2e^{(-4)}-2\sqrt{-x^2e^2+d^2}\,Ade^{(-1)}-\frac{1}{2}\left(Cd^2+2\,Bde+Ae^2\right)\sqrt{-x^2e^2+d^2}\,xe^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] 3/8*C*d^4*arcsin(x*e/d)*e^(-3) - 3/8*sqrt(-x^2*e^2 + d^2)*C*d^2*x*e^(-2) +
A*d^2*arcsin(x*e/d)*e^(-1) + 1/2*(C*d^2 + 2*B*d*e + A*e^2)*d^2*arcsin(x*e/d
)*e^(-3) - 1/4*sqrt(-x^2*e^2 + d^2)*C*x^3 - sqrt(-x^2*e^2 + d^2)*B*d^2*e^(-
2) - 1/3*sqrt(-x^2*e^2 + d^2)*(2*C*d*e + B*e^2)*x^2*e^(-2) - 2/3*sqrt(-x^2*
e^2 + d^2)*(2*C*d*e + B*e^2)*d^2*e^(-4) - 2*sqrt(-x^2*e^2 + d^2)*A*d*e^(-1)
 - 1/2*(C*d^2 + 2*B*d*e + A*e^2)*sqrt(-x^2*e^2 + d^2)*x*e^(-2)
```

Fricas [A]
time = 0.34, size = 135, normalized size = 0.71

$$-\frac{1}{24}\left(6\left(7\,Cd^4+8\,Bd^3e+12\,Ad^2e^2\right)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\left(32\,Cd^3+2\left(3\,Cx^3+4\,Bx^2+6\,Ax\right)e^3+8\left(2\,Cdx^2+3\,Bdx+6\,Ad\right)e^2+\left(21\,Cd^2x+40\,Bd^2\right)e\right)\sqrt{-x^2e^2+d^2}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/24*(6*(7*C*d^4 + 8*B*d^3*e + 12*A*d^2*e^2)*arctan(-(d - sqrt(-x^2*e^2 +
d^2))*e^(-1)/x) + (32*C*d^3 + 2*(3*C*x^3 + 4*B*x^2 + 6*A*x)*e^3 + 8*(2*C*d*
x^2 + 3*B*d*x + 6*A*d)*e^2 + (21*C*d^2*x + 40*B*d^2)*e)*sqrt(-x^2*e^2 + d^2
))*e^(-3)
```

Sympy [A]
time = 7.93, size = 891, normalized size = 4.66

$$Ad^2\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for } d^2<0\wedge e^2<0\end{cases}\right)+2Ade\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for } e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)+Ae^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}+\frac{idx}{2e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2e^2} & \text{otherwise}\end{cases}\right)+Bd^2\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for } e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)+2Bde\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}+\frac{idx}{2e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2e^2} & \text{otherwise}\end{cases}\right)+Be^2\left(\begin{cases}-\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \text{for } e\neq 0\\ \frac{x^4}{4\sqrt{d^2}} & \text{otherwise}\end{cases}\right)+Cd^2\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}+\frac{idx}{2e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2e^2} & \text{otherwise}\end{cases}\right)+2Cde\left(\begin{cases}-\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \text{for } e\neq 0\\ \frac{x^4}{4\sqrt{d^2}} & \text{otherwise}\end{cases}\right)+Ce^2\left(\begin{cases}-\frac{3id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^5}+\frac{3id^3x}{8e^4\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{idx^3}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{3d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^5}-\frac{3d^3x}{8e^4\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{dx^3}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d**2*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2
 > 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2)
, (d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(
-d**2), (d**2 < 0) & (e**2 < 0))) + 2*A*d*e*Piecewise((x**2/(2*sqrt(d**2)),
 Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + A*e**2*Piecewise((-I
*d**2*acosh(e*x/d)/(2*e**3) + I*d*x/(2*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*
x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(
e*x/d)/(2*e**3) - d*x*sqrt(1 - e**2*x**2/d**2)/(2*e**2), True)) + B*d**2*Pi
ecewise((x**2/(2*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2,
True)) + 2*B*d*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) + I*d*x/(2*e**2*s
qrt(-1 + e**2*x**2/d**2)) - I*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**
2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x*sqrt(1 - e**2*x**2/d**2
)/(2*e**2), True)) + B*e**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e*
*4) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2))
, True)) + C*d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) + I*d*x/(2*e**2*
sqrt(-1 + e**2*x**2/d**2)) - I*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e*
*2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x*sqrt(1 - e**2*x**2/d**
2)/(2*e**2), True)) + 2*C*d*e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*
e**4) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2
)), True)) + C*e**2*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x
/(8*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/
d**2)) - I*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1),
(3*d**4*asin(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) +
 d*x**3/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d*
*2)), True))
```

Giac [A]
time = 3.42, size = 131, normalized size = 0.69

$$\frac{1}{8}\left(7\,Cd^4+8\,Bd^3e+12\,Ad^2e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\operatorname{sgn}(d)-\frac{1}{24}\sqrt{-x^2e^2+d^2}\,\left(\left(2\left(3\,Cx+4\left(2\,Cde^4+Be^5\right)e^{(-5)}\right)x+3\left(7\,Cd^2e^3+8\,Bde^4+4\,Ae^5\right)e^{(-5)}\right)x+8\left(4\,Cd^3e^2+5\,Bd^2e^3+6\,Ade^4\right)e^{(-5)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(7*C*d^4 + 8*B*d^3*e + 12*A*d^2*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/24
*sqrt(-x^2*e^2 + d^2)*((2*(3*C*x + 4*(2*C*d*e^4 + B*e^5)*e^(-5))*x + 3*(7*C
*d^2*e^3 + 8*B*d*e^4 + 4*A*e^5)*e^(-5))*x + 8*(4*C*d^3*e^2 + 5*B*d^2*e^3 +
6*A*d*e^4)*e^(-5))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2\,\left(C\,x^2+B\,x+A\right)}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int(((d + e*x)^2*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2), x)
```

3.12 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=143

$$-\frac{(2Cd^2+3e(Bd+Ae))\sqrt{d^2-e^2x^2}}{3e^3}-\frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2}-\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}+\frac{d(Cd^2+e(Bd+2Ae))}{2}$$

```
[Out] 1/2*d*(C*d^2+e*(2*A*e+B*d))*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1/3*(2*C*d
^2+3*e*(A*e+B*d))*(-e^2*x^2+d^2)^(1/2)/e^3-1/2*(B*e+C*d)*x*(-e^2*x^2+d^2)^(
1/2)/e^2-1/3*C*x^2*(-e^2*x^2+d^2)^(1/2)/e
```

Rubi [A]
time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1829, 655, 223, 209}

$$\frac{d\,\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(e(2Ae+Bd)+Cd^2)}{2e^3}-\frac{\sqrt{d^2-e^2x^2}\,(3e(Ae+Bd)+2Cd^2)}{3e^3}-\frac{x\sqrt{d^2-e^2x^2}\,(Be+Cd)}{2e^2}-\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -1/3*((2*C*d^2 + 3*e*(B*d + A*e))*Sqrt[d^2 - e^2*x^2])/e^3 - ((C*d + B*e)*x
*Sqrt[d^2 - e^2*x^2])/(2*e^2) - (C*x^2*Sqrt[d^2 - e^2*x^2])/(3*e) + (d*(C*d
^2 + e*(B*d + 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{\int \frac{-3Ade^2-e(2Cd^2+3e(Bd+Ae))x-3e^2(Cd+Be)x^2}{\sqrt{d^2-e^2x^2}}\,dx}{3e^2}$$

$$= -\frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2-e^2x^2}}{3e} + \frac{\int \frac{3de^2(Cd^2+e(Bd+2Ae))+2\ldots}{\sqrt{d^2-\ldots}}}{6e\ldots}$$

$$= -\frac{(2Cd^2+3e(Bd+Ae))\sqrt{d^2-e^2x^2}}{3e^3} - \frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx\ldots}{\ldots}$$

$$= -\frac{(2Cd^2+3e(Bd+Ae))\sqrt{d^2-e^2x^2}}{3e^3} - \frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx\ldots}{\ldots}$$

$$= -\frac{(2Cd^2+3e(Bd+Ae))\sqrt{d^2-e^2x^2}}{3e^3} - \frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx\ldots}{\ldots}$$

Mathematica [A]
time = 0.44, size = 123, normalized size = 0.86

$$\frac{-e\sqrt{d^2-e^2x^2}\,(3e(2Bd+2Ae+Bex)+C(4d^2+3dex+2e^2x^2))+3\sqrt{-e^2}\,(Cd^3+de(Bd+2Ae))\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{6e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(e*Sqrt[d^2 - e^2*x^2]*(3*e*(2*B*d + 2*A*e + B*e*x) + C*(4*d^2 + 3*d*e*x
+ 2*e^2*x^2))) + 3*Sqrt[-e^2]*(C*d^3 + d*e*(B*d + 2*A*e))*Log[-(Sqrt[-e^2]*
x) + Sqrt[d^2 - e^2*x^2]])/(6*e^4)
```

Maple [A]
time = 0.10, size = 170, normalized size = 1.19

method	result

risch	$-\frac{(2C\,e^2x^2+3B\,e^2x+3Cdex+6A\,e^2+6Bde+4C\,d^2)\sqrt{-e^2x^2+d^2}}{6e^3}+\frac{d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)A}{\sqrt{e^2}}+\frac{d^2\arctan\left(\frac{\sqrt{}}{\sqrt{-e}}\right.}{2e\sqrt{e}}$
default	$eC\left(-\frac{x^2\sqrt{-e^2x^2+d^2}}{3e^2}-\frac{2d^2\sqrt{-e^2x^2+d^2}}{3e^4}\right)+(Be+Cd)\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e}}{\sqrt{-e^2x}}\right.}{2e^2\sqrt{e^2}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*C*(-1/3*x^2/e^2*(-e^2*x^2+d^2)^(1/2)-2/3*d^2/e^4*(-e^2*x^2+d^2)^(1/2))+(B
*e+C*d)*(-1/2*x/e^2*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/e^2/(e^2)^(1/2)*arctan((e^
2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))-(A*e+B*d)/e^2*(-e^2*x^2+d^2)^(1/2)+d/(e^2
)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*A
```

Maxima [A]
time = 0.49, size = 142, normalized size = 0.99

$$\frac{1}{2}\,(Cd+Be)d^2\arcsin\left(\frac{xe}{d}\right)e^{(-3)}-\frac{1}{3}\,\sqrt{-x^2e^2+d^2}\,Cx^2e^{(-1)}-\frac{2}{3}\,\sqrt{-x^2e^2+d^2}\,Cd^2e^{(-3)}+Ad\arcsin\left(\frac{xe}{d}\right)e^{(-1)}-\sqrt{-x^2e^2+d^2}\,Bde^{(-2)}-\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,(Cd+Be)xe^{(-2)}-\sqrt{-x^2e^2+d^2}\,Ae^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(C*d + B*e)*d^2*arcsin(x*e/d)*e^(-3) - 1/3*sqrt(-x^2*e^2 + d^2)*C*x^2*e
^(-1) - 2/3*sqrt(-x^2*e^2 + d^2)*C*d^2*e^(-3) + A*d*arcsin(x*e/d)*e^(-1) -
sqrt(-x^2*e^2 + d^2)*B*d*e^(-2) - 1/2*sqrt(-x^2*e^2 + d^2)*(C*d + B*e)*x*e^
(-2) - sqrt(-x^2*e^2 + d^2)*A*e^(-1)
```

Fricas [A]
time = 0.39, size = 102, normalized size = 0.71

$$-\frac{1}{6}\left(6\left(Cd^3+Bd^2e+2\,Ade^2\right)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)+\left(4\,Cd^2+\left(2\,Cx^2+3\,Bx+6\,A\right)e^2+3\,(Cdx+2\,Bd)e\right)\sqrt{-x^2e^2+d^2}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(6*(C*d^3 + B*d^2*e + 2*A*d*e^2)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^
(-1)/x) + (4*C*d^2 + (2*C*x^2 + 3*B*x + 6*A)*e^2 + 3*(C*d*x + 2*B*d)*e)*sqr
t(-x^2*e^2 + d^2))*e^(-3)
```

Sympy [A]
time = 3.99, size = 484, normalized size = 3.38

$$Ad\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\: d^2>0\wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\: d^2>0\wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for}\: d^2<0\wedge e^2<0\end{cases}\right)+Ae\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for}\: e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)+Bd\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for}\: e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)+Be\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}+\frac{idx}{2e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\: \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2e^2} & \text{otherwise}\end{cases}\right)+Cd\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}+\frac{idx}{2e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\: \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2e^2} & \text{otherwise}\end{cases}\right)+Ce\left(\begin{cases}-\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4}-\frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \text{for}\: e\neq 0\\ \frac{x^4}{4\sqrt{d^2}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 >
0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2), (
d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d*
*2), (d**2 < 0) & (e**2 < 0))) + A*e*Piecewise((x**2/(2*sqrt(d**2)), Eq(e**
2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + B*d*Piecewise((x**2/(2*sqrt
(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + B*e*Piecewis
e((-I*d**2*acosh(e*x/d)/(2*e**3) + I*d*x/(2*e**2*sqrt(-1 + e**2*x**2/d**2))
 - I*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*
asin(e*x/d)/(2*e**3) - d*x*sqrt(1 - e**2*x**2/d**2)/(2*e**2), True)) + C*d*
Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) + I*d*x/(2*e**2*sqrt(-1 + e**2*x**
2/d**2)) - I*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1)
, (d**2*asin(e*x/d)/(2*e**3) - d*x*sqrt(1 - e**2*x**2/d**2)/(2*e**2), True)
) + C*e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2
 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True))
```

Giac [A]
time = 3.50, size = 97, normalized size = 0.68

$$\frac{1}{2}\left(Cd^3+Bd^2e+2\,Ade^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{6}\sqrt{-x^2e^2+d^2}\left(\left(2\,Cxe^{(-1)}+3\left(Cde^3+Be^4\right)e^{(-5)}\right)x+2\left(2\,Cd^2e^2+3\,Bde^3+3\,Ae^4\right)e^{(-5)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(C*d^3 + B*d^2*e + 2*A*d*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/6*sqrt(-x
^2*e^2 + d^2)*((2*C*x*e^(-1) + 3*(C*d*e^3 + B*e^4)*e^(-5))*x + 2*(2*C*d^2*e
^2 + 3*B*d*e^3 + 3*A*e^4)*e^(-5))
```

Mupad [B]
time = 5.01, size = 270, normalized size = 1.89

$$\begin{cases} \frac{2\,C\,d\,x^3+3\,B\,d\,x^2+6\,A\,d\,x}{6\,\sqrt{d^2}} & \text{if } e=0\\ \frac{A\,d\,\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{\sqrt{-e^2}}-\frac{A\,\sqrt{d^2-e^2\,x^2}}{e}-\frac{B\,d\,\sqrt{d^2-e^2\,x^2}}{e^2}-\frac{B\,x\,\sqrt{d^2-e^2\,x^2}}{2\,e}-\frac{C\,\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+e^2\,x^2\right)}{3\,e^3}-\frac{C\,d^3\,\ln\left(2\,x\,\sqrt{-e^2}+2\,\sqrt{d^2-e^2\,x^2}\right)}{2\,{\left(-e^2\right)}^{3/2}}-\frac{B\,d^2\,e\,\ln\left(2\,x\,\sqrt{-e^2}+2\,\sqrt{d^2-e^2\,x^2}\right)}{2\,{\left(-e^2\right)}^{3/2}}-\frac{C\,d\,x\,\sqrt{d^2-e^2\,x^2}}{2\,e^2} & \text{if } e\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] piecewise(e == 0, (6*A*d*x + 3*B*d*x^2 + 2*C*d*x^3)/(6*(d^2)^(1/2)), e ~= 0
, - (A*(d^2 - e^2*x^2)^(1/2))/e + (A*d*log(x*(-e^2)^(1/2) + (d^2 - e^2*x^2)
^(1/2)))/(-e^2)^(1/2) - (B*d*(d^2 - e^2*x^2)^(1/2))/e^2 - (B*x*(d^2 - e^2*x
^2)^(1/2))/(2*e) - (C*(d^2 - e^2*x^2)^(1/2)*(2*d^2 + e^2*x^2))/(3*e^3) - (C
*d^3*log(2*x*(-e^2)^(1/2) + 2*(d^2 - e^2*x^2)^(1/2)))/(2*(-e^2)^(3/2)) - (B
*d^2*e*log(2*x*(-e^2)^(1/2) + 2*(d^2 - e^2*x^2)^(1/2)))/(2*(-e^2)^(3/2)) -
(C*d*x*(d^2 - e^2*x^2)^(1/2))/(2*e^2))
```

3.13 $\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}} \, dx$

Optimal. Leaf size=87

$$-\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{(Cd^2+2Ae^2)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

```
[Out] 1/2*(2*A*e^2+C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-B*(-e^2*x^2+d^2)^(
1/2)/e^2-1/2*C*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

Rubi [A]
time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1829, 655, 223, 209}

$$\frac{(2Ae^2+Cd^2)\,\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}-\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -((B*Sqrt[d^2 - e^2*x^2])/e^2) - (C*x*Sqrt[d^2 - e^2*x^2])/(2*e^2) + ((C*d^
2 + 2*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
```

```
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx &= -\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}-\frac{\int \frac{-Cd^2-2Ae^2-2Be^2x}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}\\
&= -\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}-\frac{\left(-Cd^2-2Ae^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}\\
&= -\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}-\frac{\left(-Cd^2-2Ae^2\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-\cdots}}\right)}{2e^2}\\
&= -\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{\left(Cd^2+2Ae^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}
\end{aligned}
$$

Mathematica [A]
time = 0.26, size = 90, normalized size = 1.03

$$
\frac{(-2B-Cx)\sqrt{d^2-e^2x^2}}{2e^2}+\frac{\sqrt{-e^2}\left(Cd^2+2Ae^2\right)\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{2e^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] ((-2*B - C*x)*Sqrt[d^2 - e^2*x^2])/(2*e^2) + (Sqrt[-e^2]*(C*d^2 + 2*A*e^2)*
Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*e^4)
```

Maple [A]
time = 0.08, size = 109, normalized size = 1.25

method	result
risch	$-\frac{(Cx+2B)\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{A\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+\frac{C\,d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}$
default	$C\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)-\frac{B\sqrt{-e^2x^2+d^2}}{e^2}+\frac{A\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-1/2*x/e^2*(-e^2*x^2+d^2)^(1/2)+1/2*d^2/e^2/(e^2)^(1/2)*arctan((e^2)^(1/
2)*x/(-e^2*x^2+d^2)^(1/2)))-B*(-e^2*x^2+d^2)^(1/2)/e^2+A/(e^2)^(1/2)*arctan
((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Maxima [A]
time = 0.50, size = 66, normalized size = 0.76

$$\frac{1}{2}\,Cd^2\arcsin\left(\frac{xe}{d}\right)e^{(-3)}-\frac{1}{2}\sqrt{-x^2e^2+d^2}\,Cxe^{(-2)}+A\arcsin\left(\frac{xe}{d}\right)e^{(-1)}-\sqrt{-x^2e^2+d^2}\,Be^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*C*d^2*arcsin(x*e/d)*e^(-3) - 1/2*sqrt(-x^2*e^2 + d^2)*C*x*e^(-2) + A*ar
csin(x*e/d)*e^(-1) - sqrt(-x^2*e^2 + d^2)*B*e^(-2)
```

Fricas [A]
time = 0.34, size = 68, normalized size = 0.78

$$-\frac{1}{2}\left(2\left(Cd^2+2\,Ae^2\right)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)e+\sqrt{-x^2e^2+d^2}\,(Cx+2\,B)e^2\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(C*d^2 + 2*A*e^2)*arctan(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x)*e +
sqrt(-x^2*e^2 + d^2)*(C*x + 2*B)*e^2)*e^(-4)
```

Sympy [A]
time = 1.68, size = 262, normalized size = 3.01

$$A\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2>0\wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for}\ d^2>0\wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for}\ d^2<0\wedge e^2<0\end{cases}\right)+B\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for}\ e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)+C\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}+\frac{idx}{2e^2\sqrt{-1+\frac{e^2x^2}{d^2}}}-\frac{ix^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2e^2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 > 0)
 & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2), (d*
*2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d**2
), (d**2 < 0) & (e**2 < 0))) + B*Piecewise((x**2/(2*sqrt(d**2)), Eq(e**2, 0
)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + C*Piecewise((-I*d**2*acosh(e*x/
d)/(2*e**3) + I*d*x/(2*e**2*sqrt(-1 + e**2*x**2/d**2)) - I*x**3/(2*d*sqrt(-
1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3)
- d*x*sqrt(1 - e**2*x**2/d**2)/(2*e**2), True))
```

Giac [A]
time = 3.43, size = 52, normalized size = 0.60

$$\frac{1}{2}\left(Cd^2+2\,Ae^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{2}\,\sqrt{-x^2e^2+d^2}\,\left(Cxe^{(-2)}+2\,Be^{(-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(C*d^2 + 2*A*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/2*sqrt(-x^2*e^2 + d^2
)*(C*x*e^(-2) + 2*B*e^(-2))
```

Mupad [B]
time = 4.40, size = 148, normalized size = 1.70

$$\begin{cases} \frac{2\,C\,x^3+3\,B\,x^2+6\,A\,x}{6\,\sqrt{d^2}} & \text{if } e=0\\ \frac{A\,\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{\sqrt{-e^2}}-\frac{B\,\sqrt{d^2-e^2\,x^2}}{e^2}-\frac{C\,x\,\sqrt{d^2-e^2\,x^2}}{2\,e^2}-\frac{C\,d^2\,\ln\left(2\,x\,\sqrt{-e^2}+2\,\sqrt{d^2-e^2\,x^2}\right)}{2\,{\left(-e^2\right)}^{3/2}} & \text{if } e\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] piecewise(e == 0, (6*A*x + 3*B*x^2 + 2*C*x^3)/(6*(d^2)^(1/2)), e ~= 0, (A*l
og(x*(-e^2)^(1/2) + (d^2 - e^2*x^2)^(1/2)))/(-e^2)^(1/2) - (B*(d^2 - e^2*x^
2)^(1/2))/e^2 - (C*x*(d^2 - e^2*x^2)^(1/2))/(2*e^2) - (C*d^2*log(2*x*(-e^2)
^(1/2) + 2*(d^2 - e^2*x^2)^(1/2)))/(2*(-e^2)^(3/2)))
```

3.14 $\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=103

$$-\frac{C\sqrt{d^2-e^2x^2}}{e^3}-\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}-\frac{(Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] -(-B*e+C*d)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-C*(-e^2*x^2+d^2)^(1/2)/e^3
-(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)
```

Rubi [A]
time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1653, 807, 223, 209}

$$-\frac{\sqrt{d^2-e^2x^2}\,(Ae^2-Bde+Cd^2)}{de^3(d+ex)}-\frac{\text{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(Cd-Be)}{e^3}-\frac{C\sqrt{d^2-e^2x^2}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((C*Sqrt[d^2 - e^2*x^2])/e^3) - ((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^
2])/(d*e^3*(d + e*x)) - ((C*d - B*e)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
```

```
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{C\sqrt{d^2-e^2x^2}}{e^3} - \frac{\int \frac{-Ae^4+e^3(Cd-Be)x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$= -\frac{C\sqrt{d^2-e^2x^2}}{e^3} - \frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{de^3(d+ex)} - \frac{(Cd-Be)\int \frac{1}{\sqrt{d^2-\cdots}}}{e^2}$$

$$= -\frac{C\sqrt{d^2-e^2x^2}}{e^3} - \frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{de^3(d+ex)} - \frac{(Cd-Be)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{C\sqrt{d^2-e^2x^2}}{e^3} - \frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{de^3(d+ex)} - \frac{(Cd-Be)\tan^{-1}\left(\cdots\right.}{e^3}$$

Mathematica [A]
time = 0.45, size = 105, normalized size = 1.02

$$\frac{(-2Cd^2+Bde-Ae^2-Cdex)\sqrt{d^2-e^2x^2}}{de^3(d+ex)} + \frac{\sqrt{-e^2}\,(-Cd+Be)\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
[Out] ((-2*C*d^2 + B*d*e - A*e^2 - C*d*e*x)*Sqrt[d^2 - e^2*x^2])/(d*e^3*(d + e*x)
) + (Sqrt[-e^2]*(-(C*d) + B*e)*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/
e^4
```

Maple [A]
time = 0.11, size = 149, normalized size = 1.45

method	result
default	$\frac{-\frac{C\sqrt{-e^2x^2+d^2}}{e}+\frac{Be\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\frac{Cd\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}}{e^2}-\frac{(Ae^2-Bde+Cd^2)\sqrt{-\left(x+\frac{d}{e}\right)}}{e^4d\left(x+\frac{d}{e}\right)}$
risch	$-\frac{C\sqrt{-e^2x^2+d^2}}{e^3}+\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)B}{e\sqrt{e^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)Cd}{e^2\sqrt{e^2}}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2d}}{e^2d\left(x+\frac{d}{e}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(-C/e*(-e^2*x^2+d^2)^(1/2)+B*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2
*x^2+d^2)^(1/2))-C*d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
)-(A*e^2-B*d*e+C*d^2)/e^4/d/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
```

Maxima [A]
time = 0.52, size = 129, normalized size = 1.25

$$-Cd\arcsin\left(\frac{xe}{d}\right)e^{(-3)}+B\arcsin\left(\frac{xe}{d}\right)e^{(-2)}-\sqrt{-x^2e^2+d^2}\,Ce^{(-3)}-\frac{\sqrt{-x^2e^2+d^2}\,Cd}{xe^4+de^3}-\frac{\sqrt{-x^2e^2+d^2}\,A}{dxe^2+d^2e}+\frac{\sqrt{-x^2e^2+d^2}\,B}{xe^3+de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -C*d*arcsin(x*e/d)*e^(-3) + B*arcsin(x*e/d)*e^(-2) - sqrt(-x^2*e^2 + d^2)*C
*e^(-3) - sqrt(-x^2*e^2 + d^2)*C*d/(x*e^4 + d*e^3) - sqrt(-x^2*e^2 + d^2)*A
/(d*x*e^2 + d^2*e) + sqrt(-x^2*e^2 + d^2)*B/(x*e^3 + d*e^2)
```

Fricas [A]
time = 0.35, size = 151, normalized size = 1.47

$$-\frac{2\,Cd^3+Axe^3-2\,(Cd^3-Bdxe^2+(Cd^2x-Bd^2)e)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-(Bdx-Ad)e^2+(2\,Cd^2x-Bd^2)e+(2\,Cd^2+Ae^2+(Cdx-Bd)e)\sqrt{-x^2e^2+d^2}}{dxe^4+d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*C*d^3 + A*x*e^3 - 2*(C*d^3 - B*d*x*e^2 + (C*d^2*x - B*d^2)*e)*arctan(-(
d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (B*d*x - A*d)*e^2 + (2*C*d^2*x - B*d^
2)*e + (2*C*d^2 + A*e^2 + (C*d*x - B*d)*e)*sqrt(-x^2*e^2 + d^2))/(d*x*e^4 +
 d^2*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+Bx+Cx^2}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Giac [A]
time = 3.39, size = 96, normalized size = 0.93

$$-(Cd-Be)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\sqrt{-x^2e^2+d^2}\,Ce^{(-3)}+\frac{2\,(Cd^2-Bde+Ae^2)e^{(-3)}}{d\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(C*d - B*e)*arcsin(x*e/d)*e^(-3)*sgn(d) - sqrt(-x^2*e^2 + d^2)*C*e^(-3) +
2*(C*d^2 - B*d*e + A*e^2)*e^(-3)/(d*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/
x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\,x^2+B\,x+A}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.15 $\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=163

$$-\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{3de^3(d+ex)^2}+\frac{(2Cd-Be)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}-\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{3d^2e^3(d+ex)}+\frac{C\tan^{-1}\Big(\cdots$$

```
[Out] C*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1/3*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^
2)^(1/2)/d/e^3/(e*x+d)^2+(-B*e+2*C*d)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)-1/
3*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d^2/e^3/(e*x+d)
```

Rubi [A]
time = 0.11, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1651, 223, 209, 673, 665}

$$-\frac{\sqrt{d^2-e^2x^2}\,(Ae^2-Bde+Cd^2)}{3d^2e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}\,(Ae^2-Bde+Cd^2)}{3de^3(d+ex)^2}+\frac{C\,\mathrm{ArcTan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}+\frac{\sqrt{d^2-e^2x^2}\,(2Cd-Be)}{de^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/3*((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(d*e^3*(d + e*x)^2) + ((
2*C*d - B*e)*Sqrt[d^2 - e^2*x^2])/(d*e^3*(d + e*x)) - ((C*d^2 - B*d*e + A*e
^2)*Sqrt[d^2 - e^2*x^2])/(3*d^2*e^3*(d + e*x)) + (C*ArcTan[(e*x)/Sqrt[d^2 -
 e^2*x^2]])/e^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = \int \left(\frac{C}{e^2\sqrt{d^2-e^2x^2}} + \frac{Cd^2-Bde+Ae^2}{e^2(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{-2Cd+Be}{e^2(d+ex)\sqrt{d^2-e^2x^2}}\right)$$

$$= \frac{C\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2} - \frac{(2Cd-Be)\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e^2} + \frac{(Cd^2-Bde-}{}$$

$$= -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{3de^3(d+ex)^2} + \frac{(2Cd-Be)\sqrt{d^2-e^2x^2}}{de^3(d+ex)} + \frac{C\text{Subst}\Big(}{}$$

$$= -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{3de^3(d+ex)^2} + \frac{(2Cd-Be)\sqrt{d^2-e^2x^2}}{de^3(d+ex)} - \frac{(Cd^2-B}{3}$$

Mathematica [A]
time = 0.57, size = 116, normalized size = 0.71

$$\frac{-\frac{e\sqrt{d^2-e^2x^2}\,(-Cd^2(4d+5ex)+e(Ae(2d+ex)+Bd(d+2ex)))}{d^2(d+ex)^2}+3C\sqrt{-e^2}\,\log\left(-\sqrt{-e^2}\,x+\sqrt{d^2-e^2x^2}\right)}{3e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-((e*Sqrt[d^2 - e^2*x^2]*(-(C*d^2*(4*d + 5*e*x)) + e*(A*e*(2*d + e*x) + B*
d*(d + 2*e*x))))/(d^2*(d + e*x)^2)) + 3*C*Sqrt[-e^2]*Log[-(Sqrt[-e^2]*x) +
Sqrt[d^2 - e^2*x^2]])/(3*e^4)
```

Maple [A]
time = 0.08, size = 195, normalized size = 1.20

method	result
default	$\frac{C\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}-\frac{(Be-2Cd)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^4d\left(x+\frac{d}{e}\right)}+\frac{(A\,e^2-Bde+C\,d^2)\left(-\frac{\sqrt{-\left(x+\right.}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] C/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/e^4*(B*e-2*C
*d)/d/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/e^4*(A*e^2-B*d*e+C*d^2
)*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*
(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

Maxima [A]
time = 0.53, size = 294, normalized size = 1.80

$$C\arcsin\left(\frac{xe}{d}\right)e^{(-3)}-\frac{\sqrt{-x^2e^2+d^2}\,Cd^2}{3\,(dx^2e^5+2\,d^2xe^4+d^3e^3)}-\frac{\sqrt{-x^2e^2+d^2}\,Cd^2}{3\,(d^2xe^4+d^3e^3)}+\frac{\sqrt{-x^2e^2+d^2}\,Bd}{3\,(dx^2e^4+2\,d^2xe^3+d^3e^2)}+\frac{\sqrt{-x^2e^2+d^2}\,Bd}{3\,(d^2xe^3+d^3e^2)}-\frac{\sqrt{-x^2e^2+d^2}\,A}{3\,(dx^2e^3+2\,d^2xe^2+d^3e)}-\frac{\sqrt{-x^2e^2+d^2}\,A}{3\,(d^2xe^2+d^3e)}-\frac{\sqrt{-x^2e^2+d^2}\,B}{dxe^3+d^2e^2}+\frac{2\sqrt{-x^2e^2+d^2}\,C}{xe^4+de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] C*arcsin(x*e/d)*e^(-3) - 1/3*sqrt(-x^2*e^2 + d^2)*C*d^2/(d*x^2*e^5 + 2*d^2*
x*e^4 + d^3*e^3) - 1/3*sqrt(-x^2*e^2 + d^2)*C*d^2/(d^2*x*e^4 + d^3*e^3) + 1
/3*sqrt(-x^2*e^2 + d^2)*B*d/(d*x^2*e^4 + 2*d^2*x*e^3 + d^3*e^2) + 1/3*sqrt(
-x^2*e^2 + d^2)*B*d/(d^2*x*e^3 + d^3*e^2) - 1/3*sqrt(-x^2*e^2 + d^2)*A/(d*x
^2*e^3 + 2*d^2*x*e^2 + d^3*e) - 1/3*sqrt(-x^2*e^2 + d^2)*A/(d^2*x*e^2 + d^3
*e) - sqrt(-x^2*e^2 + d^2)*B/(d*x*e^3 + d^2*e^2) + 2*sqrt(-x^2*e^2 + d^2)*C
/(x*e^4 + d*e^3)
```

Fricas [A]
time = 0.37, size = 211, normalized size = 1.29

$$\frac{4\,Cd^4-2\,Ax^2e^4-6\,(Cd^2x^2e^2+2\,Cd^3xe+Cd^4)\arctan\left(-\frac{\left(d-\sqrt{-x^2e^2+d^2}\right)e^{(-1)}}{x}\right)-(Bdx^2+4\,Adx)e^3+2\,(2\,Cd^2x^2-Bd^2x-Ad^2)e^2+(8\,Cd^3x-Bd^3)e+(4\,Cd^3-Axe^3-2\,(Bdx+Ad)e^2+(5\,Cd^2x-Bd^2)e)\sqrt{-x^2e^2+d^2}}{3\,(d^2x^2e^5+2\,d^3xe^4+d^4e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/3*(4*C*d^4 - 2*A*x^2*e^4 - 6*(C*d^2*x^2*e^2 + 2*C*d^3*x*e + C*d^4)*arctan
(-(d - sqrt(-x^2*e^2 + d^2))*e^(-1)/x) - (B*d*x^2 + 4*A*d*x)*e^3 + 2*(2*C*d
```

```
^2*x^2 - B*d^2*x - A*d^2)*e^2 + (8*C*d^3*x - B*d^3)*e + (4*C*d^3 - A*x*e^3
- 2*(B*d*x + A*d)*e^2 + (5*C*d^2*x - B*d^2)*e)*sqrt(-x^2*e^2 + d^2))/(d^2*x
^2*e^5 + 2*d^3*x*e^4 + d^4*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/exp
(1)*(-((-i)*sageVARA*exp(1)^2+(-2*i)*sageVARB*sageVARd*exp(1)-6*sageVARC*sa
geVARd^2*atan(i)+5*i*sageVARC*sageVARd^2)/3/sageVARd^2/exp(1)^2*sign((sageV
ARx*exp(1)+sageV
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\,x^2+B\,x+A}{\sqrt{d^2-e^2\,x^2}\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2),x)
[Out] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2), x)
```

3.16 $\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=180

$$-\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}-\frac{(7Cd^2+e(3Bd+2Ae))\sqrt{d^2-e^2x^2}}{15d^2e^3(d+ex)^2}-\frac{(7Cd^2+e(3Bd}{15d^3}$$

```
[Out] -1/5*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)^3+C*(-e^2*x^2+d
^2)^(1/2)/e^3/(e*x+d)^2-1/15*(7*C*d^2+e*(2*A*e+3*B*d))*(-e^2*x^2+d^2)^(1/2)
/d^2/e^3/(e*x+d)^2-1/15*(7*C*d^2+e*(2*A*e+3*B*d))*(-e^2*x^2+d^2)^(1/2)/d^3/
e^3/(e*x+d)
```

Rubi [A]
time = 0.13, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1653, 807, 673, 665}

$$-\frac{\sqrt{d^2-e^2x^2}\,(e(2Ae+3Bd)+7Cd^2)}{15d^2e^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\,(Ae^2-Bde+Cd^2)}{5de^3(d+ex)^3}-\frac{\sqrt{d^2-e^2x^2}\,(e(2Ae+3Bd)+7Cd^2)}{15d^3e^3(d+ex)}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/5*((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(d*e^3*(d + e*x)^3) + (C
*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)^2) - ((7*C*d^2 + e*(3*B*d + 2*A*e))*Sq
rt[d^2 - e^2*x^2])/(15*d^2*e^3*(d + e*x)^2) - ((7*C*d^2 + e*(3*B*d + 2*A*e)
)*Sqrt[d^2 - e^2*x^2])/(15*d^3*e^3*(d + e*x))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
```

```
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}+\frac{\int \frac{e^2(2Cd^2+Ae^2)+e^3(Cd+Be)x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$= -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}+\frac{(7Cd^2+e(3Bd+2A}{}$$

$$= -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}-\frac{(7Cd^2+e(3Bd+2A}{15d^2e^3(d\,+}$$

$$= -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}-\frac{(7Cd^2+e(3Bd+2A}{15d^2e^3(d\,+}$$

Mathematica [A]
time = 0.61, size = 103, normalized size = 0.57

$$-\frac{\sqrt{d^2-e^2x^2}\,(Cd^2(2d^2+6dex+7e^2x^2)+e(3Bd(d^2+3dex+e^2x^2)+Ae(7d^2+6dex+2e^2x^2)))}{15d^3e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(C*d^2*(2*d^2 + 6*d*e*x + 7*e^2*x^2) + e*(3*B*d*
(d^2 + 3*d*e*x + e^2*x^2) + A*e*(7*d^2 + 6*d*e*x + 2*e^2*x^2))))/(d^3*e^3*(
d + e*x)^3)
```

Maple [A]
time = 0.08, size = 308, normalized size = 1.71

method	result
trager	$-\frac{\left(2A\,e^{4}x^{2}+3Bd\,e^{3}x^{2}+7C\,d^{2}e^{2}x^{2}+6Ad\,e^{3}x+9B\,d^{2}e^{2}x+6C\,d^{3}xe+7A\,d^{2}e^{2}+3B\,d^{3}e+2C\,d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15d^{3}e^{3}\left(ex+d\right)^{3}}$
gosper	$-\frac{\left(-ex+d\right)\left(2A\,e^{4}x^{2}+3Bd\,e^{3}x^{2}+7C\,d^{2}e^{2}x^{2}+6Ad\,e^{3}x+9B\,d^{2}e^{2}x+6C\,d^{3}xe+7A\,d^{2}e^{2}+3B\,d^{3}e+2C\,d^{4}\right)}{15\left(ex+d\right)^{2}d^{3}e^{3}\sqrt{-e^{2}x^{2}+d^{2}}}$
default	$-\frac{C\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{e^{4}d\left(x+\frac{d}{e}\right)}+\frac{\left(A\,e^{2}-Bde+C\,d^{2}\right)\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{5de\left(x+\frac{d}{e}\right)^{3}}+\frac{2e\left(-\frac{\sqrt{-\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -C/e^4/d/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+(A*e^2-B*d*e+C*d^2)/e
^5*(-1/5/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+2/5*e/d*(-1/3/d
/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)
^2*e^2+2*d*e*(x+d/e))^(1/2)))+(B*e-2*C*d)/e^4*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)
^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(166) = 332$.
time = 0.54, size = 561, normalized size = 3.12

$$-\frac{\sqrt{-x^2e^2+d^2}\,Cd^2}{5\,(dx^3e^6+3\,d^2x^2e^5+3\,d^3xe^4+d^4e^3)}-\frac{2\sqrt{-x^2e^2+d^2}\,Cd^2}{15\,(d^2x^2e^5+2\,d^3xe^4+d^4e^3)}-\frac{2\sqrt{-x^2e^2+d^2}\,Cd^2}{15\,(d^3xe^4+d^4e^3)}+\frac{\sqrt{-x^2e^2+d^2}\,Bd}{5\,(dx^3e^5+3\,d^2x^2e^4+3\,d^3xe^3+d^4e^2)}+\frac{2\sqrt{-x^2e^2+d^2}\,Bd}{15\,(d^2x^2e^4+2\,d^3xe^3+d^4e^2)}+\frac{2\sqrt{-x^2e^2+d^2}\,Bd}{15\,(d^3xe^3+d^4e^2)}+\frac{2\sqrt{-x^2e^2+d^2}\,Cd}{3\,(dx^2e^5+2\,d^2xe^4+d^3e^3)}+\frac{2\sqrt{-x^2e^2+d^2}\,Cd}{3\,(d^2xe^4+d^3e^3)}-\frac{\sqrt{-x^2e^2+d^2}\,A}{5\,(dx^3e^4+3\,d^2x^2e^3+3\,d^3xe^2+d^4e)}-\frac{2\sqrt{-x^2e^2+d^2}\,A}{15\,(d^2x^2e^3+2\,d^3xe^2+d^4e)}-\frac{2\sqrt{-x^2e^2+d^2}\,A}{15\,(d^3xe^2+d^4e)}-\frac{\sqrt{-x^2e^2+d^2}\,B}{3\,(dx^2e^4+2\,d^2xe^3+d^3e^2)}-\frac{\sqrt{-x^2e^2+d^2}\,B}{3\,(d^2xe^3+d^3e^2)}-\frac{\sqrt{-x^2e^2+d^2}\,C}{dxe^4+d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/5*sqrt(-x^2*e^2 + d^2)*C*d^2/(d*x^3*e^6 + 3*d^2*x^2*e^5 + 3*d^3*x*e^4 +
d^4*e^3) - 2/15*sqrt(-x^2*e^2 + d^2)*C*d^2/(d^2*x^2*e^5 + 2*d^3*x*e^4 + d^4
*e^3) - 2/15*sqrt(-x^2*e^2 + d^2)*C*d^2/(d^3*x*e^4 + d^4*e^3) + 1/5*sqrt(-x
^2*e^2 + d^2)*B*d/(d*x^3*e^5 + 3*d^2*x^2*e^4 + 3*d^3*x*e^3 + d^4*e^2) + 2/1
5*sqrt(-x^2*e^2 + d^2)*B*d/(d^2*x^2*e^4 + 2*d^3*x*e^3 + d^4*e^2) + 2/15*sqr
t(-x^2*e^2 + d^2)*B*d/(d^3*x*e^3 + d^4*e^2) + 2/3*sqrt(-x^2*e^2 + d^2)*C*d/
(d*x^2*e^5 + 2*d^2*x*e^4 + d^3*e^3) + 2/3*sqrt(-x^2*e^2 + d^2)*C*d/(d^2*x*e
^4 + d^3*e^3) - 1/5*sqrt(-x^2*e^2 + d^2)*A/(d*x^3*e^4 + 3*d^2*x^2*e^3 + 3*d
```

```
^3*x*e^2 + d^4*e) - 2/15*sqrt(-x^2*e^2 + d^2)*A/(d^2*x^2*e^3 + 2*d^3*x*e^2
+ d^4*e) - 2/15*sqrt(-x^2*e^2 + d^2)*A/(d^3*x*e^2 + d^4*e) - 1/3*sqrt(-x^2*
e^2 + d^2)*B/(d*x^2*e^4 + 2*d^2*x*e^3 + d^3*e^2) - 1/3*sqrt(-x^2*e^2 + d^2)
*B/(d^2*x*e^3 + d^3*e^2) - sqrt(-x^2*e^2 + d^2)*C/(d*x*e^4 + d^2*e^3)
```

Fricas [**A**]
time = 0.40, size = 231, normalized size = 1.28

$$-\frac{2\,Cd^5+7\,Ax^3e^5+3\,(Bdx^3+7\,Adx^2)e^4+(2\,Cd^2x^3+9\,Bd^2x^2+21\,Ad^2x)e^3+(6\,Cd^3x^2+9\,Bd^3x+7\,Ad^3)e^2+3\,(2\,Cd^4x+Bd^4)e+(2\,Cd^4+2\,Ax^2e^4+3\,(Bdx^2+2\,Adx)e^3+(7\,Cd^2x^2+9\,Bd^2x+7\,Ad^2)e^2+3\,(2\,Cd^3x+Bd^3)e)\sqrt{-x^2e^2+d^2}}{15\,(d^3x^3e^6+3\,d^4x^2e^5+3\,d^5xe^4+d^6e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/15*(2*C*d^5 + 7*A*x^3*e^5 + 3*(B*d*x^3 + 7*A*d*x^2)*e^4 + (2*C*d^2*x^3 +
 9*B*d^2*x^2 + 21*A*d^2*x)*e^3 + (6*C*d^3*x^2 + 9*B*d^3*x + 7*A*d^3)*e^2 +
3*(2*C*d^4*x + B*d^4)*e + (2*C*d^4 + 2*A*x^2*e^4 + 3*(B*d*x^2 + 2*A*d*x)*e^
3 + (7*C*d^2*x^2 + 9*B*d^2*x + 7*A*d^2)*e^2 + 3*(2*C*d^3*x + B*d^3)*e)*sqrt
(-x^2*e^2 + d^2))/(d^3*x^3*e^6 + 3*d^4*x^2*e^5 + 3*d^5*x*e^4 + d^6*e^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{-(-d+ex)\,(d+ex)}\,(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(166) = 332.
time = 4.35, size = 336, normalized size = 1.87

$$\frac{2\left(\frac{10\left(de+\sqrt{-x^2e^2+d^2}\,e\right)Cd^2e^{(-2)}}{x}+\frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2Cd^2e^{(-4)}}{x^2}+2\,Cd^2+3\,Bde+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)Bde^{(-1)}}{x}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2Bde^{(-3)}}{x^2}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3Bde^{(-5)}}{x^3}+7\,Ae^2+\frac{40\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2Ae^{(-2)}}{x^2}+\frac{30\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3Ae^{(-4)}}{x^3}+\frac{15\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4Ae^{(-6)}}{x^4}+\frac{20\left(de+\sqrt{-x^2e^2+d^2}\,e\right)A}{x}\right)e^{(-3)}}{15\,d^3\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(10*(d*e + sqrt(-x^2*e^2 + d^2)*e)*C*d^2*e^(-2)/x + 20*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^2*C*d^2*e^(-4)/x^2 + 2*C*d^2 + 3*B*d*e + 15*(d*e + sqrt(-x^
2*e^2 + d^2)*e)*B*d*e^(-1)/x + 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*B*d*e^(-
3)/x^2 + 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3*B*d*e^(-5)/x^3 + 7*A*e^2 + 40*
(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*A*e^(-2)/x^2 + 30*(d*e + sqrt(-x^2*e^2 + d
```

```
^2)*e)^3*A*e^(-4)/x^3 + 15*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*A*e^(-6)/x^4 +
20*(d*e + sqrt(-x^2*e^2 + d^2)*e)*A/x)*e^(-3)/(d^3*((d*e + sqrt(-x^2*e^2 +
d^2)*e)*e^(-2)/x + 1)^5)
```

Mupad [B]
time = 3.80, size = 109, normalized size = 0.61

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,C\,d^4+6\,C\,d^3\,e\,x+3\,B\,d^3\,e+7\,C\,d^2\,e^2\,x^2+9\,B\,d^2\,e^2\,x+7\,A\,d^2\,e^2+3\,B\,d\,e^3\,x^2+6\,A\,d\,e^3\,x+2\,A\,e^4\,x^2\right)}{15\,d^3\,e^3\,{\left(d+e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(2*C*d^4 + 7*A*d^2*e^2 + 2*A*e^4*x^2 + 3*B*d^3*e +
7*C*d^2*e^2*x^2 + 6*A*d*e^3*x + 6*C*d^3*e*x + 9*B*d^2*e^2*x + 3*B*d*e^3*x^2
))/(15*d^3*e^3*(d + e*x)^3)
```

3.17 $\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx$

Optimal. Leaf size=234

$$-\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4}+\frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3}-\frac{(13Cd^2+8Bde+6Ae^2)\sqrt{d^2-e^2x^2}}{70d^2e^3(d+ex)^3}-\frac{(13Cd^2+8B\cdots}{105\cdots}$$

```
[Out] -1/7*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)^4+1/2*C*(-e^2*x
^2+d^2)^(1/2)/e^3/(e*x+d)^3-1/70*(6*A*e^2+8*B*d*e+13*C*d^2)*(-e^2*x^2+d^2)^
(1/2)/d^2/e^3/(e*x+d)^3-1/105*(6*A*e^2+8*B*d*e+13*C*d^2)*(-e^2*x^2+d^2)^(1/
2)/d^3/e^3/(e*x+d)^2-1/105*(6*A*e^2+8*B*d*e+13*C*d^2)*(-e^2*x^2+d^2)^(1/2)/
d^4/e^3/(e*x+d)
```

Rubi [A]
time = 0.15, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1653, 807, 673, 665}

$$-\frac{\sqrt{d^2-e^2x^2}\,(6Ae^2+8Bde+13Cd^2)}{70d^2e^3(d+ex)^3}-\frac{\sqrt{d^2-e^2x^2}\,(Ae^2-Bde+Cd^2)}{7de^3(d+ex)^4}-\frac{\sqrt{d^2-e^2x^2}\,(6Ae^2+8Bde+13Cd^2)}{105d^4e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}\,(6Ae^2+8Bde+13Cd^2)}{105d^3e^3(d+ex)^2}+\frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/7*((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(d*e^3*(d + e*x)^4) + (C
*Sqrt[d^2 - e^2*x^2])/(2*e^3*(d + e*x)^3) - ((13*C*d^2 + 8*B*d*e + 6*A*e^2)
*Sqrt[d^2 - e^2*x^2])/(70*d^2*e^3*(d + e*x)^3) - ((13*C*d^2 + 8*B*d*e + 6*A
*e^2)*Sqrt[d^2 - e^2*x^2])/(105*d^3*e^3*(d + e*x)^2) - ((13*C*d^2 + 8*B*d*e
 + 6*A*e^2)*Sqrt[d^2 - e^2*x^2])/(105*d^4*e^3*(d + e*x))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3}+\frac{\int \frac{e^2(3Cd^2+2Ae^2)+e^3(Cd+2Be)x}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx}{2e^4}$$

$$= -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4}+\frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3}+\frac{(13Cd^2+8Bde+6A\ldots}{\ldots}$$

$$= -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4}+\frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3}-\frac{(13Cd^2+8Bde+6A\ldots}{70d^2e^3(d+\ldots}$$

$$= -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4}+\frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3}-\frac{(13Cd^2+8Bde+6A\ldots}{70d^2e^3(d+\ldots}$$

$$= -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4}+\frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3}-\frac{(13Cd^2+8Bde+6A\ldots}{70d^2e^3(d+\ldots}$$

Mathematica [A]
time = 0.72, size = 139, normalized size = 0.59

$$-\frac{\sqrt{d^2-e^2x^2}\,\left(Cd^2(8d^3+32d^2ex+52de^2x^2+13e^3x^3)+e(3Ae(12d^3+13d^2ex+8de^2x^2+2e^3x^3)+Bd(13d^3+52d^2ex+32de^2x^2+8e^3x^3))\right)}{105d^4e^3(d+ex)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^4*Sqrt[d^2 - e^2*x^2]),x]

[Out] -1/105*(Sqrt[d^2 - e^2*x^2]*(C*d^2*(8*d^3 + 32*d^2*e*x + 52*d*e^2*x^2 + 13*e^3*x^3) + e*(3*A*e*(12*d^3 + 13*d^2*e*x + 8*d*e^2*x^2 + 2*e^3*x^3) + B*d*(13*d^3 + 52*d^2*e*x + 32*d*e^2*x^2 + 8*e^3*x^3))))/(d^4*e^3*(d + e*x)^4)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(214) = 428$.
time = 0.09, size = 459, normalized size = 1.96

method	result
trager	$-\frac{(6A e^5x^3+8Bd e^4x^3+13C d^2e^3x^3+24Ad e^4x^2+32B d^2e^3x^2+52C d^3e^2x^2+39A d^2e^3x+52B d^3e^2x+32C d^4ex+36A d^3e^2+13B d^4}{105d^4(ex+d)^4e^3}$
gosper	$-\frac{(-ex+d)(6A e^5x^3+8Bd e^4x^3+13C d^2e^3x^3+24Ad e^4x^2+32B d^2e^3x^2+52C d^3e^2x^2+39A d^2e^3x+52B d^3e^2x+32C d^4ex+36A d^3e^2}{105(ex+d)^3d^4e^3\sqrt{-e^2x^2+d^2}}$
default	$\frac{(A e^2-Bde+C d^2)\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{7de\left(x+\frac{d}{e}\right)^4}+\frac{3e\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5de\left(x+\frac{d}{e}\right)^3}+\frac{2e\left(-\frac{\sqrt{-}}{}\right.}{}\right.}{}\right.}{e^6}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] (A*e^2-B*d*e+C*d^2)/e^6*(-1/7/d/e/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+3/7*e/d*(-1/5/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+2/5*e/d*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))))+(B*e-2*C*d)/e^5*(-1/5/d/e/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+2/5*e/d*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)))+C/e^4*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(209) = 418$.
time = 0.55, size = 898, normalized size = 3.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/7*sqrt(-x^2*e^2 + d^2)*C*d^2/(d*x^4*e^7 + 4*d^2*x^3*e^6 + 6*d^3*x^2*e^5
+ 4*d^4*x*e^4 + d^5*e^3) - 3/35*sqrt(-x^2*e^2 + d^2)*C*d^2/(d^2*x^3*e^6 + 3
*d^3*x^2*e^5 + 3*d^4*x*e^4 + d^5*e^3) - 2/35*sqrt(-x^2*e^2 + d^2)*C*d^2/(d^
3*x^2*e^5 + 2*d^4*x*e^4 + d^5*e^3) - 2/35*sqrt(-x^2*e^2 + d^2)*C*d^2/(d^4*x
*e^4 + d^5*e^3) + 1/7*sqrt(-x^2*e^2 + d^2)*B*d/(d*x^4*e^6 + 4*d^2*x^3*e^5 +
 6*d^3*x^2*e^4 + 4*d^4*x*e^3 + d^5*e^2) + 3/35*sqrt(-x^2*e^2 + d^2)*B*d/(d^
2*x^3*e^5 + 3*d^3*x^2*e^4 + 3*d^4*x*e^3 + d^5*e^2) + 2/35*sqrt(-x^2*e^2 + d
^2)*B*d/(d^3*x^2*e^4 + 2*d^4*x*e^3 + d^5*e^2) + 2/35*sqrt(-x^2*e^2 + d^2)*B
*d/(d^4*x*e^3 + d^5*e^2) + 2/5*sqrt(-x^2*e^2 + d^2)*C*d/(d*x^3*e^6 + 3*d^2*
x^2*e^5 + 3*d^3*x*e^4 + d^4*e^3) + 4/15*sqrt(-x^2*e^2 + d^2)*C*d/(d^2*x^2*e
^5 + 2*d^3*x*e^4 + d^4*e^3) + 4/15*sqrt(-x^2*e^2 + d^2)*C*d/(d^3*x*e^4 + d^
4*e^3) - 1/7*sqrt(-x^2*e^2 + d^2)*A/(d*x^4*e^5 + 4*d^2*x^3*e^4 + 6*d^3*x^2*
e^3 + 4*d^4*x*e^2 + d^5*e) - 3/35*sqrt(-x^2*e^2 + d^2)*A/(d^2*x^3*e^4 + 3*d
^3*x^2*e^3 + 3*d^4*x*e^2 + d^5*e) - 2/35*sqrt(-x^2*e^2 + d^2)*A/(d^3*x^2*e^
3 + 2*d^4*x*e^2 + d^5*e) - 2/35*sqrt(-x^2*e^2 + d^2)*A/(d^4*x*e^2 + d^5*e)
- 1/5*sqrt(-x^2*e^2 + d^2)*B/(d*x^3*e^5 + 3*d^2*x^2*e^4 + 3*d^3*x*e^3 + d^4
*e^2) - 2/15*sqrt(-x^2*e^2 + d^2)*B/(d^2*x^2*e^4 + 2*d^3*x*e^3 + d^4*e^2) -
 2/15*sqrt(-x^2*e^2 + d^2)*B/(d^3*x*e^3 + d^4*e^2) - 1/3*sqrt(-x^2*e^2 + d^
2)*C/(d*x^2*e^5 + 2*d^2*x*e^4 + d^3*e^3) - 1/3*sqrt(-x^2*e^2 + d^2)*C/(d^2*
x*e^4 + d^3*e^3)
```

Fricas [A]
time = 0.40, size = 307, normalized size = 1.31

$$-\frac{8\,Cd^6+36\,Ax^4e^6+(13\,Bdx^4+144\,Adx^3)e^5+4\,(2\,Cd^2x^4+13\,Bd^2x^3+54\,Ad^2x^2)e^4+2\,(16\,Cd^3x^3+39\,Bd^3x^2+72\,Ad^3x)e^3+4\,(12\,Cd^4x^2+13\,Bd^4x+9\,Ad^4)e^2+(32\,Cd^5x+13\,Bd^5)e+\left(8\,Cd^5+6\,Ax^3e^5+8\,(Bdx^3+3\,Adx^2)e^4+(13\,Cd^2x^3+32\,Bd^2x^2+39\,Ad^2x)e^3+4\,(13\,Cd^3x^2+13\,Bd^3x+9\,Ad^3)e^2+(32\,Cd^4x+13\,Bd^4)e\right)\sqrt{-x^2e^2+d^2}}{105\,(d^4x^4e^7+4\,d^5x^3e^6+6\,d^6x^2e^5+4\,d^7xe^4+d^8e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/105*(8*C*d^6 + 36*A*x^4*e^6 + (13*B*d*x^4 + 144*A*d*x^3)*e^5 + 4*(2*C*d^
2*x^4 + 13*B*d^2*x^3 + 54*A*d^2*x^2)*e^4 + 2*(16*C*d^3*x^3 + 39*B*d^3*x^2 +
 72*A*d^3*x)*e^3 + 4*(12*C*d^4*x^2 + 13*B*d^4*x + 9*A*d^4)*e^2 + (32*C*d^5*
x + 13*B*d^5)*e + (8*C*d^5 + 6*A*x^3*e^5 + 8*(B*d*x^3 + 3*A*d*x^2)*e^4 + (1
3*C*d^2*x^3 + 32*B*d^2*x^2 + 39*A*d^2*x)*e^3 + 4*(13*C*d^3*x^2 + 13*B*d^3*x
 + 9*A*d^3)*e^2 + (32*C*d^4*x + 13*B*d^4)*e)*sqrt(-x^2*e^2 + d^2))/(d^4*x^4
*e^7 + 4*d^5*x^3*e^6 + 6*d^6*x^2*e^5 + 4*d^7*x*e^4 + d^8*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**4/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(209) = 418.
time = 3.62, size = 530, normalized size = 2.26

$$\frac{2\left(\frac{56\left(de+\sqrt{-x^2e^2+d^2}\,e\right)Cd^2e^{(-2)}}{x}+\frac{168\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2Cd^2e^{(-4)}}{x^2}+\frac{140\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3Cd^2e^{(-6)}}{x^3}+\frac{140\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4Cd^2e^{(-8)}}{x^4}+8\,Cd^2+13\,Bde+\frac{91\left(de+\sqrt{-x^2e^2+d^2}\,e\right)Bde^{(-1)}}{x}+\frac{168\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2Bde^{(-3)}}{x^2}+\frac{280\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3Bde^{(-5)}}{x^3}+\frac{175\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4Bde^{(-7)}}{x^4}+\frac{105\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5Bde^{(-9)}}{x^5}+36\,Ae^2+\frac{441\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^2Ae^{(-2)}}{x^2}+\frac{630\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^3Ae^{(-4)}}{x^3}+\frac{630\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^4Ae^{(-6)}}{x^4}+\frac{315\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^5Ae^{(-8)}}{x^5}+\frac{105\left(de+\sqrt{-x^2e^2+d^2}\,e\right)^6Ae^{(-10)}}{x^6}+\frac{147\left(de+\sqrt{-x^2e^2+d^2}\,e\right)A}{x}\right)e^{(-3)}}{105\,d^4\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\,e\right)e^{(-2)}}{x}+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(56*(d*e + sqrt(-x^2*e^2 + d^2)*e)*C*d^2*e^(-2)/x + 168*(d*e + sqrt(-
x^2*e^2 + d^2)*e)^2*C*d^2*e^(-4)/x^2 + 140*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3
*C*d^2*e^(-6)/x^3 + 140*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*C*d^2*e^(-8)/x^4 +
 8*C*d^2 + 13*B*d*e + 91*(d*e + sqrt(-x^2*e^2 + d^2)*e)*B*d*e^(-1)/x + 168*
(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*B*d*e^(-3)/x^2 + 280*(d*e + sqrt(-x^2*e^2
+ d^2)*e)^3*B*d*e^(-5)/x^3 + 175*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*B*d*e^(-7
)/x^4 + 105*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*B*d*e^(-9)/x^5 + 36*A*e^2 + 44
1*(d*e + sqrt(-x^2*e^2 + d^2)*e)^2*A*e^(-2)/x^2 + 630*(d*e + sqrt(-x^2*e^2
+ d^2)*e)^3*A*e^(-4)/x^3 + 630*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*A*e^(-6)/x^
4 + 315*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*A*e^(-8)/x^5 + 105*(d*e + sqrt(-x^
2*e^2 + d^2)*e)^6*A*e^(-10)/x^6 + 147*(d*e + sqrt(-x^2*e^2 + d^2)*e)*A/x)*e
^(-3)/(d^4*((d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/x + 1)^7)
```

Mupad [B]
time = 3.78, size = 204, normalized size = 0.87

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{C}{5\,e^3}-\frac{-4\,C\,d^2+4\,B\,d\,e+3\,A\,e^2}{35\,d^2\,e^3}\right)}{{\left(d+e\,x\right)}^3}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{A}{7\,d\,e}+\frac{d\,\left(\frac{C}{7\,e^2}-\frac{B}{7\,d\,e}\right)}{e}\right)}{{\left(d+e\,x\right)}^4}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(13\,C\,d^2+8\,B\,d\,e+6\,A\,e^2\right)}{105\,d^3\,e^3\,{\left(d+e\,x\right)}^2}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(13\,C\,d^2+8\,B\,d\,e+6\,A\,e^2\right)}{105\,d^4\,e^3\,\left(d+e\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(C/(5*e^3) - (3*A*e^2 - 4*C*d^2 + 4*B*d*e)/(35*d^2*e
^3)))/(d + e*x)^3 - ((d^2 - e^2*x^2)^(1/2)*(A/(7*d*e) + (d*(C/(7*e^2) - B/(
7*d*e)))/e))/(d + e*x)^4 - ((d^2 - e^2*x^2)^(1/2)*(6*A*e^2 + 13*C*d^2 + 8*B
*d*e))/(105*d^3*e^3*(d + e*x)^2) - ((d^2 - e^2*x^2)^(1/2)*(6*A*e^2 + 13*C*d
^2 + 8*B*d*e))/(105*d^4*e^3*(d + e*x))
```

3.18 $\int (d+ex)^3 (a+cx^2) (A+Bx+Cx^2)\, dx$

Optimal. Leaf size=175

$$\frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)(d+ex)^4}{4e^5}-\frac{(ae^2(2Cd-Be)+cd(4Cd^2-e(3Bd-2Ae)))(d+ex)^5}{5e^5}+\frac{(aCe^2$$

```
[Out] 1/4*(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)*(e*x+d)^4/e^5-1/5*(a*e^2*(-B*e+2*C*d)
+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*(e*x+d)^5/e^5+1/6*(a*C*e^2+c*(6*C*d^2-e*(-
A*e+3*B*d)))*(e*x+d)^6/e^5-1/7*c*(-B*e+4*C*d)*(e*x+d)^7/e^5+1/8*c*C*(e*x+d)
^8/e^5
```

Rubi [A]
time = 0.20, antiderivative size = 173, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1642}

$$-\frac{(d+ex)^5(ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3)}{5e^5}+\frac{(d+ex)^6(aCe^2-ce(3Bd-Ae)+6cCd^2)}{6e^5}+\frac{(d+ex)^4(ae^2+cd^2)(Ae^2-Bde+Cd^2)}{4e^5}-\frac{c(d+ex)^7(4Cd-Be)}{7e^5}+\frac{cC(d+ex)^8}{8e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*(d + e*x)^4)/(4*e^5) - ((4*c*C*d^3
 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))*(d + e*x)^5)/(5*e^5) + ((6*
c*C*d^2 + a*C*e^2 - c*e*(3*B*d - A*e))*(d + e*x)^6)/(6*e^5) - (c*(4*C*d - B
*e)*(d + e*x)^7)/(7*e^5) + (c*C*(d + e*x)^8)/(8*e^5)
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^3 (a+cx^2)(A+Bx+Cx^2)\, dx &= \int \left(\frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)(d+ex)^3}{e^4}+\frac{(-4cCd^3+cd\right.\\ &= \frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)(d+ex)^4}{4e^5}-\frac{(4cCd^3-cde(3Bd\end{aligned}$$

Mathematica [A]
time = 0.06, size = 208, normalized size = 1.19

$$aAd^3x+\frac{1}{2}ad^2(Bd+3Ae)x^2+\frac{1}{3}d(ad(Cd+3Be)+A(cd^2+3ae^2))x^3+\frac{1}{4}(Bcd^3+3Acd^2e+3aCd^2e+3aBde^2+aAe^3)x^4+\frac{1}{5}(cCd^3+3cde(Bd+Ae)+ae^2(3Cd+Be))x^5+\frac{1}{6}e(3cCd^2+aCe^2+ce(3Bd+Ae))x^6+\frac{1}{7}ce^2(3Cd+Be)x^7+\frac{1}{8}cCe^3x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d^3*x + (a*d^2*(B*d + 3*A*e)*x^2)/2 + (d*(a*d*(C*d + 3*B*e) + A*(c*d^2
+ 3*a*e^2))*x^3)/3 + ((B*c*d^3 + 3*A*c*d^2*e + 3*a*C*d^2*e + 3*a*B*d*e^2 +
a*A*e^3)*x^4)/4 + ((c*C*d^3 + 3*c*d*e*(B*d + A*e) + a*e^2*(3*C*d + B*e))*x^
5)/5 + (e*(3*c*C*d^2 + a*C*e^2 + c*e*(3*B*d + A*e))*x^6)/6 + (c*e^2*(3*C*d
+ B*e)*x^7)/7 + (c*C*e^3*x^8)/8
```

Maple [A]
time = 0.12, size = 217, normalized size = 1.24

method	result
norman	$\frac{c e^3 C x^8}{8}+\left(\frac{1}{7} B c e^3+\frac{3}{7} d e^2 c C\right) x^7+\left(\frac{1}{6} A c e^3+\frac{1}{2} B c d e^2+\frac{1}{6} C a e^3+\frac{1}{2} C c d^2 e\right) x^6+\left(\frac{3}{5} d e^2 c A+\frac{1}{5} B\right.$
default	$\frac{c e^3 C x^8}{8}+\frac{(B c e^3+3 d e^2 c C) x^7}{7}+\frac{((a e^3+3 c d^2 e) C+3 B c d e^2+A c e^3) x^6}{6}+\frac{((3 d e^2 a+c d^3) C+(a e^3+3 c d^2 e) B+3 d e^2 c A) x^5}{5}+$
gosper	$\frac{1}{8} c e^3 C x^8+\frac{1}{7} x^7 B c e^3+\frac{3}{7} x^7 d e^2 c C+\frac{1}{6} x^6 A c e^3+\frac{1}{2} x^6 B c d e^2+\frac{1}{6} x^6 C a e^3+\frac{1}{2} x^6 C c d^2 e+\frac{3}{5} x^5 d e^2 c$
risch	$\frac{1}{8} c e^3 C x^8+\frac{1}{7} x^7 B c e^3+\frac{3}{7} x^7 d e^2 c C+\frac{1}{6} x^6 A c e^3+\frac{1}{2} x^6 B c d e^2+\frac{1}{6} x^6 C a e^3+\frac{1}{2} x^6 C c d^2 e+\frac{3}{5} x^5 d e^2 c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c*e^3*C*x^8+1/7*(B*c*e^3+3*C*c*d*e^2)*x^7+1/6*((a*e^3+3*c*d^2*e)*C+3*B*
c*d*e^2+A*c*e^3)*x^6+1/5*((3*a*d*e^2+c*d^3)*C+(a*e^3+3*c*d^2*e)*B+3*d*e^2*c
*A)*x^5+1/4*(3*a*d^2*e*C+(3*a*d*e^2+c*d^3)*B+(a*e^3+3*c*d^2*e)*A)*x^4+1/3*(
d^3*a*C+3*a*d^2*e*B+(3*a*d*e^2+c*d^3)*A)*x^3+1/2*(3*A*a*d^2*e+B*a*d^3)*x^2+
d^3*a*A*x
```

Maxima [A]
time = 0.28, size = 201, normalized size = 1.15

$$\frac{1}{8} Ccx^8e^3+\frac{1}{7}\left(3 Ccde^2+Bce^3\right)x^7+\frac{1}{6}\left(3 Ccd^2e+3 Bcde^2+Cae^3+Ace^3\right)x^6+Aad^3x+\frac{1}{5}\left(Ccd^3+3 Bcd^2e+Bae^3+3\left(Cae^2+Ace^2\right)d\right)x^5+\frac{1}{4}\left(Bcd^3+3 Bade^2+3\left(Cae+Ace\right)d^2+Aae^3\right)x^4+\frac{1}{3}\left(3 Bad^2e+\left(Ca+Ac\right)d^3+3 Aade^2\right)x^3+\frac{1}{2}\left(Bad^3+3 Aad^2e\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*C*c*x^8*e^3 + 1/7*(3*C*c*d*e^2 + B*c*e^3)*x^7 + 1/6*(3*C*c*d^2*e + 3*B*
c*d*e^2 + C*a*e^3 + A*c*e^3)*x^6 + A*a*d^3*x + 1/5*(C*c*d^3 + 3*B*c*d^2*e +
 B*a*e^3 + 3*(C*a*e^2 + A*c*e^2)*d)*x^5 + 1/4*(B*c*d^3 + 3*B*a*d*e^2 + 3*(C
*a*e + A*c*e)*d^2 + A*a*e^3)*x^4 + 1/3*(3*B*a*d^2*e + (C*a + A*c)*d^3 + 3*A
*a*d*e^2)*x^3 + 1/2*(B*a*d^3 + 3*A*a*d^2*e)*x^2
```

Fricas [A]
time = 0.34, size = 208, normalized size = 1.19

$$\frac{1}{5} Ccd^3x^5+\frac{1}{4} Bcd^3x^4+\frac{1}{2} Bad^3x^2+\frac{1}{3}\left(Ca+Ac\right)d^3x^3+Aad^3x+\frac{1}{840}\left(105 Ccx^8+120 Bcx^7+168 Bax^5+140\left(Ca+Ac\right)x^6+210 Aax^4\right)e^3+\frac{1}{140}\left(60 Ccdx^7+70 Bcdx^6+105 Badx^4+84\left(Ca+Ac\right)dx^5+140 Aadx^3\right)e^2+\frac{1}{20}\left(10 Ccd^2x^6+12 Bcd^2x^5+20 Bad^2x^3+15\left(Ca+Ac\right)d^2x^4+30 Aad^2x^2\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/5*C*c*d^3*x^5 + 1/4*B*c*d^3*x^4 + 1/2*B*a*d^3*x^2 + 1/3*(C*a + A*c)*d^3*x
^3 + A*a*d^3*x + 1/840*(105*C*c*x^8 + 120*B*c*x^7 + 168*B*a*x^5 + 140*(C*a
+ A*c)*x^6 + 210*A*a*x^4)*e^3 + 1/140*(60*C*c*d*x^7 + 70*B*c*d*x^6 + 105*B*
a*d*x^4 + 84*(C*a + A*c)*d*x^5 + 140*A*a*d*x^3)*e^2 + 1/20*(10*C*c*d^2*x^6
+ 12*B*c*d^2*x^5 + 20*B*a*d^2*x^3 + 15*(C*a + A*c)*d^2*x^4 + 30*A*a*d^2*x^2
)*e
```

Sympy [A]
time = 0.02, size = 257, normalized size = 1.47

$$Aad^3x + \frac{Cce^3x^8}{8} + x^7\left(\frac{Bce^3}{7} + \frac{3Ccde^2}{7}\right) + x^6\left(\frac{Ace^3}{6} + \frac{Bcde^2}{2} + \frac{Cae^3}{6} + \frac{Ccd^2e}{2}\right) + x^5\cdot\left(\frac{3Acde^2}{5} + \frac{Bae^3}{5} + \frac{3Bcd^2e}{5} + \frac{3Cade^2}{5} + \frac{Ccd^3}{5}\right) + x^4\left(\frac{Aae^3}{4} + \frac{3Acd^2e}{4} + \frac{3Bade^2}{4} + \frac{Bcd^3}{4} + \frac{3Cad^2e}{4}\right) + x^3\left(Aade^2 + \frac{Acd^3}{3} + Bad^2e + \frac{Cad^3}{3}\right) + x^2\cdot\left(\frac{3Aad^2e}{2} + \frac{Bad^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*d**3*x + C*c*e**3*x**8/8 + x**7*(B*c*e**3/7 + 3*C*c*d*e**2/7) + x**6*(A
*c*e**3/6 + B*c*d*e**2/2 + C*a*e**3/6 + C*c*d**2*e/2) + x**5*(3*A*c*d*e**2/
5 + B*a*e**3/5 + 3*B*c*d**2*e/5 + 3*C*a*d*e**2/5 + C*c*d**3/5) + x**4*(A*a*
e**3/4 + 3*A*c*d**2*e/4 + 3*B*a*d*e**2/4 + B*c*d**3/4 + 3*C*a*d**2*e/4) + x
**3*(A*a*d*e**2 + A*c*d**3/3 + B*a*d**2*e + C*a*d**3/3) + x**2*(3*A*a*d**2*
e/2 + B*a*d**3/2)
```

Giac [A]
time = 7.66, size = 242, normalized size = 1.38

$$\frac{1}{8}Ccx^8e^3 + \frac{3}{7}Ccdx^7e^2 + \frac{1}{2}Ccd^2x^6e + \frac{1}{5}Ccd^3x^5 + \frac{1}{7}Bcx^7e^3 + \frac{1}{2}Bcdx^6e^2 + \frac{3}{5}Bcd^2x^5e + \frac{1}{4}Bcd^3x^4 + \frac{1}{6}Cax^6e^3 + \frac{1}{6}Acx^6e^3 + \frac{3}{5}Cadx^5e^2 + \frac{3}{5}Acdx^5e^2 + \frac{3}{4}Cad^2x^4e + \frac{3}{4}Acd^2x^4e + \frac{1}{3}Cad^3x^3 + \frac{1}{3}Acd^3x^3 + \frac{1}{5}Bax^5e^3 + \frac{3}{4}Badx^4e^2 + Bad^2x^3e + \frac{1}{2}Bad^3x^2 + \frac{1}{4}Aax^4e^3 + Aadx^3e^2 + \frac{3}{2}Aad^2x^2e + Aad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*C*c*x^8*e^3 + 3/7*C*c*d*x^7*e^2 + 1/2*C*c*d^2*x^6*e + 1/5*C*c*d^3*x^5 +
 1/7*B*c*x^7*e^3 + 1/2*B*c*d*x^6*e^2 + 3/5*B*c*d^2*x^5*e + 1/4*B*c*d^3*x^4
+ 1/6*C*a*x^6*e^3 + 1/6*A*c*x^6*e^3 + 3/5*C*a*d*x^5*e^2 + 3/5*A*c*d*x^5*e^2
 + 3/4*C*a*d^2*x^4*e + 3/4*A*c*d^2*x^4*e + 1/3*C*a*d^3*x^3 + 1/3*A*c*d^3*x^
3 + 1/5*B*a*x^5*e^3 + 3/4*B*a*d*x^4*e^2 + B*a*d^2*x^3*e + 1/2*B*a*d^3*x^2 +
 1/4*A*a*x^4*e^3 + A*a*d*x^3*e^2 + 3/2*A*a*d^2*x^2*e + A*a*d^3*x
```

Mupad [B]
time = 0.09, size = 206, normalized size = 1.18

$$x^3\left(\frac{A\,c\,d^3}{3} + \frac{C\,a\,d^3}{3} + A\,a\,d\,e^2 + B\,a\,d^2\,e\right) + x^6\left(\frac{A\,c\,e^3}{6} + \frac{C\,a\,e^3}{6} + \frac{B\,c\,d\,e^2}{2} + \frac{C\,c\,d^2\,e}{2}\right) + x^4\left(\frac{A\,a\,e^3}{4} + \frac{B\,c\,d^3}{4} + \frac{3\,B\,a\,d\,e^2}{4} + \frac{3\,A\,c\,d^2\,e}{4} + \frac{3\,C\,a\,d^2\,e}{4}\right) + x^5\left(\frac{B\,a\,e^3}{5} + \frac{C\,c\,d^3}{5} + \frac{3\,A\,c\,d\,e^2}{5} + \frac{3\,C\,a\,d\,e^2}{5} + \frac{3\,B\,c\,d^2\,e}{5}\right) + A\,a\,d^3\,x + \frac{C\,c\,e^3\,x^8}{8} + \frac{a\,d^2\,x^2\,\left(3\,A\,e + B\,d\right)}{2} + \frac{c\,e^2\,x^7\,\left(B\,e + 3\,C\,d\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*(d + e*x)^3*(A + B*x + C*x^2),x)

[Out] x^3*((A*c*d^3)/3 + (C*a*d^3)/3 + A*a*d*e^2 + B*a*d^2*e) + x^6*((A*c*e^3)/6
+ (C*a*e^3)/6 + (B*c*d*e^2)/2 + (C*c*d^2*e)/2) + x^4*((A*a*e^3)/4 + (B*c*d^
3)/4 + (3*B*a*d*e^2)/4 + (3*A*c*d^2*e)/4 + (3*C*a*d^2*e)/4) + x^5*((B*a*e^3
)/5 + (C*c*d^3)/5 + (3*A*c*d*e^2)/5 + (3*C*a*d*e^2)/5 + (3*B*c*d^2*e)/5) +
A*a*d^3*x + (C*c*e^3*x^8)/8 + (a*d^2*x^2*(3*A*e + B*d))/2 + (c*e^2*x^7*(B*e
 + 3*C*d))/7
```

3.19 $\int (d+ex)^2 (a+cx^2) (A+Bx+Cx^2) \, dx$

Optimal. Leaf size=175

$$\frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)(d+ex)^3}{3e^5} - \frac{(ae^2(2Cd-Be)+cd(4Cd^2-e(3Bd-2Ae)))(d+ex)^4}{4e^5} + \frac{(aCe^2}{}$$

```
[Out] 1/3*(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)*(e*x+d)^3/e^5-1/4*(a*e^2*(-B*e+2*C*d)
+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*(e*x+d)^4/e^5+1/5*(a*C*e^2+c*(6*C*d^2-e*(-
A*e+3*B*d)))*(e*x+d)^5/e^5-1/6*c*(-B*e+4*C*d)*(e*x+d)^6/e^5+1/7*c*C*(e*x+d)
^7/e^5
```

Rubi [A]
time = 0.14, antiderivative size = 173, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{1642\}$

$$-\frac{(d+ex)^4(ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3)}{4e^5} + \frac{(d+ex)^5(aCe^2-ce(3Bd-Ae)+6cCd^2)}{5e^5} + \frac{(d+ex)^3(ae^2+cd^2)(Ae^2-Bde+Cd^2)}{3e^5} - \frac{c(d+ex)^6(4Cd-Be)}{6e^5} + \frac{cC(d+ex)^7}{7e^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*(d + e*x)^3)/(3*e^5) - ((4*c*C*d^3
 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))*(d + e*x)^4)/(4*e^5) + ((6*
c*C*d^2 + a*C*e^2 - c*e*(3*B*d - A*e))*(d + e*x)^5)/(5*e^5) - (c*(4*C*d - B
*e)*(d + e*x)^6)/(6*e^5) + (c*C*(d + e*x)^7)/(7*e^5)
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \int (d+ex)^2 (a+cx^2)(A+Bx+Cx^2) \, dx &= \int \left(\frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)(d+ex)^2}{e^4} + \frac{(-4cCd^3+c\ldots}{} \right. \\ &= \frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)(d+ex)^3}{3e^5} - \frac{(4cCd^3-cde(3B\ldots}{} \end{aligned}$$

Mathematica [A]
time = 0.04, size = 150, normalized size = 0.86

$$aAd^2x + \frac{1}{2}ad(Bd+2Ae)x^2 + \frac{1}{3}(Acd^2+aCd^2+2aBde+aAe^2)x^3 + \frac{1}{4}(Bcd^2+2Acde+2aCde+aBe^2)x^4 + \frac{1}{5}(cCd^2+2Bcde+Ace^2+aCe^2)x^5 + \frac{1}{6}ce(2Cd+Be)x^6 + \frac{1}{7}cCe^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d^2*x + (a*d*(B*d + 2*A*e)*x^2)/2 + ((A*c*d^2 + a*C*d^2 + 2*a*B*d*e + a
*A*e^2)*x^3)/3 + ((B*c*d^2 + 2*A*c*d*e + 2*a*C*d*e + a*B*e^2)*x^4)/4 + ((c*
C*d^2 + 2*B*c*d*e + A*c*e^2 + a*C*e^2)*x^5)/5 + (c*e*(2*C*d + B*e)*x^6)/6 +
 (c*C*e^2*x^7)/7
```

Maple [A]
time = 0.11, size = 148, normalized size = 0.85

method	result
default	$\frac{c\,e^{2} C\,x^{7}}{7}+\frac{\left(c\,e^{2} B+2 c d e C\right) x^{6}}{6}+\frac{\left(\left(a\,e^{2}+c\,d^{2}\right) C+2 B c d e+A c\,e^{2}\right) x^{5}}{5}+\frac{\left(2 a d e C+\left(a\,e^{2}+c\,d^{2}\right) B+2 c d e A\right) x^{4}}{4}+\frac{(a\,d^{2} C+2 a d e B+}{3}$
norman	$\frac{c\,e^{2} C\,x^{7}}{7}+\left(\frac{1}{6} c\,e^{2} B+\frac{1}{3} c d e C\right) x^{6}+\left(\frac{1}{5} A c\,e^{2}+\frac{2}{5} B c d e+\frac{1}{5} a C\,e^{2}+\frac{1}{5} C c\,d^{2}\right) x^{5}+\left(\frac{1}{2} c d e A+\frac{1}{4} B a\,e^{2}+\right.$
gosper	$\frac{1}{7} c\,e^{2} C\,x^{7}+\frac{1}{6} x^{6} c\,e^{2} B+\frac{1}{3} x^{6} c d e C+\frac{1}{5} x^{5} A c\,e^{2}+\frac{2}{5} x^{5} B c d e+\frac{1}{5} x^{5} a C\,e^{2}+\frac{1}{5} x^{5} C c\,d^{2}+\frac{1}{2} x^{4} c d e A+$
risch	$\frac{1}{7} c\,e^{2} C\,x^{7}+\frac{1}{6} x^{6} c\,e^{2} B+\frac{1}{3} x^{6} c d e C+\frac{1}{5} x^{5} A c\,e^{2}+\frac{2}{5} x^{5} B c d e+\frac{1}{5} x^{5} a C\,e^{2}+\frac{1}{5} x^{5} C c\,d^{2}+\frac{1}{2} x^{4} c d e A+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*c*e^2*C*x^7+1/6*(B*c*e^2+2*C*c*d*e)*x^6+1/5*((a*e^2+c*d^2)*C+2*B*c*d*e+
A*c*e^2)*x^5+1/4*(2*a*d*e*C+(a*e^2+c*d^2)*B+2*c*d*e*A)*x^4+1/3*(a*d^2*C+2*a
*d*e*B+A*(a*e^2+c*d^2))*x^3+1/2*(2*A*a*d*e+B*a*d^2)*x^2+a*d^2*A*x
```

Maxima [A]
time = 0.28, size = 143, normalized size = 0.82

$$\frac{1}{7}\,Ccx^{7}e^{2}+\frac{1}{6}\,\left(2\,Ccde+Bce^{2}\right)x^{6}+\frac{1}{5}\,\left(Ccd^{2}+2\,Bcde+Cae^{2}+Ace^{2}\right)x^{5}+Aad^{2}x+\frac{1}{4}\,\left(Bcd^{2}+Bae^{2}+2\,\left(Cae+Ace\right)d\right)x^{4}+\frac{1}{3}\,\left(2\,Bade+\left(Ca+Ac\right)d^{2}+Aae^{2}\right)x^{3}+\frac{1}{2}\,\left(Bad^{2}+2\,Aade\right)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/7*C*c*x^7*e^2 + 1/6*(2*C*c*d*e + B*c*e^2)*x^6 + 1/5*(C*c*d^2 + 2*B*c*d*e
+ C*a*e^2 + A*c*e^2)*x^5 + A*a*d^2*x + 1/4*(B*c*d^2 + B*a*e^2 + 2*(C*a*e +
A*c*e)*d)*x^4 + 1/3*(2*B*a*d*e + (C*a + A*c)*d^2 + A*a*e^2)*x^3 + 1/2*(B*a*
d^2 + 2*A*a*d*e)*x^2
```

Fricas [A]
time = 0.36, size = 148, normalized size = 0.85

$$\frac{1}{5}\,Ccd^{2}x^{5}+\frac{1}{4}\,Bcd^{2}x^{4}+\frac{1}{2}\,Bad^{2}x^{2}+\frac{1}{3}\,\left(Ca+Ac\right)d^{2}x^{3}+Aad^{2}x+\frac{1}{420}\,\left(60\,Ccx^{7}+70\,Bcx^{6}+105\,Bax^{4}+84\,\left(Ca+Ac\right)x^{5}+140\,Aax^{3}\right)e^{2}+\frac{1}{30}\,\left(10\,Ccdx^{6}+12\,Bcdx^{5}+20\,Badx^{3}+15\,\left(Ca+Ac\right)dx^{4}+30\,Aadx^{2}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/5*C*c*d^2*x^5 + 1/4*B*c*d^2*x^4 + 1/2*B*a*d^2*x^2 + 1/3*(C*a + A*c)*d^2*x
^3 + A*a*d^2*x + 1/420*(60*C*c*x^7 + 70*B*c*x^6 + 105*B*a*x^4 + 84*(C*a + A
*c)*x^5 + 140*A*a*x^3)*e^2 + 1/30*(10*C*c*d*x^6 + 12*B*c*d*x^5 + 20*B*a*d*x
^3 + 15*(C*a + A*c)*d*x^4 + 30*A*a*d*x^2)*e
```

Sympy [A]
time = 0.02, size = 173, normalized size = 0.99

$$Aad^2x + \frac{Cce^2x^7}{7} + x^6\left(\frac{Bce^2}{6} + \frac{Ccde}{3}\right) + x^5\left(\frac{Ace^2}{5} + \frac{2Bcde}{5} + \frac{Cae^2}{5} + \frac{Ccd^2}{5}\right) + x^4\left(\frac{Acde}{2} + \frac{Bae^2}{4} + \frac{Bcd^2}{4} + \frac{Cade}{2}\right) + x^3\left(\frac{Aae^2}{3} + \frac{Acd^2}{3} + \frac{2Bade}{3} + \frac{Cad^2}{3}\right) + x^2\left(Aade + \frac{Bad^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*d**2*x + C*c*e**2*x**7/7 + x**6*(B*c*e**2/6 + C*c*d*e/3) + x**5*(A*c*e*
*2/5 + 2*B*c*d*e/5 + C*a*e**2/5 + C*c*d**2/5) + x**4*(A*c*d*e/2 + B*a*e**2/
4 + B*c*d**2/4 + C*a*d*e/2) + x**3*(A*a*e**2/3 + A*c*d**2/3 + 2*B*a*d*e/3 +
 C*a*d**2/3) + x**2*(A*a*d*e + B*a*d**2/2)
```

Giac [A]
time = 5.79, size = 171, normalized size = 0.98

$$\frac{1}{7}Ccx^7e^2 + \frac{1}{3}Ccdx^6e + \frac{1}{5}Ccd^2x^5 + \frac{1}{6}Bcx^6e^2 + \frac{2}{5}Bcdx^5e + \frac{1}{4}Bcd^2x^4 + \frac{1}{5}Cax^5e^2 + \frac{1}{5}Acx^5e^2 + \frac{1}{2}Cadx^4e + \frac{1}{2}Acdx^4e + \frac{1}{3}Cad^2x^3 + \frac{1}{3}Acd^2x^3 + \frac{1}{4}Bax^4e^2 + \frac{2}{3}Badx^3e + \frac{1}{2}Bad^2x^2 + \frac{1}{3}Aax^3e^2 + Aadx^2e + Aad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/7*C*c*x^7*e^2 + 1/3*C*c*d*x^6*e + 1/5*C*c*d^2*x^5 + 1/6*B*c*x^6*e^2 + 2/5
*B*c*d*x^5*e + 1/4*B*c*d^2*x^4 + 1/5*C*a*x^5*e^2 + 1/5*A*c*x^5*e^2 + 1/2*C*
a*d*x^4*e + 1/2*A*c*d*x^4*e + 1/3*C*a*d^2*x^3 + 1/3*A*c*d^2*x^3 + 1/4*B*a*x
^4*e^2 + 2/3*B*a*d*x^3*e + 1/2*B*a*d^2*x^2 + 1/3*A*a*x^3*e^2 + A*a*d*x^2*e
+ A*a*d^2*x
```

Mupad [B]
time = 3.61, size = 143, normalized size = 0.82

$$x^3\left(\frac{A\,a\,e^2}{3}+\frac{A\,c\,d^2}{3}+\frac{C\,a\,d^2}{3}+\frac{2\,B\,a\,d\,e}{3}\right)+x^5\left(\frac{A\,c\,e^2}{5}+\frac{C\,a\,e^2}{5}+\frac{C\,c\,d^2}{5}+\frac{2\,B\,c\,d\,e}{5}\right)+x^4\left(\frac{B\,a\,e^2}{4}+\frac{B\,c\,d^2}{4}+\frac{A\,c\,d\,e}{2}+\frac{C\,a\,d\,e}{2}\right)+A\,a\,d^2\,x+\frac{a\,d\,x^2\,\left(2\,A\,e+B\,d\right)}{2}+\frac{c\,e\,x^6\,\left(B\,e+2\,C\,d\right)}{6}+\frac{C\,c\,e^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*(d + e*x)^2*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*a*e^2)/3 + (A*c*d^2)/3 + (C*a*d^2)/3 + (2*B*a*d*e)/3) + x^5*((A*c*e
^2)/5 + (C*a*e^2)/5 + (C*c*d^2)/5 + (2*B*c*d*e)/5) + x^4*((B*a*e^2)/4 + (B*
c*d^2)/4 + (A*c*d*e)/2 + (C*a*d*e)/2) + A*a*d^2*x + (a*d*x^2*(2*A*e + B*d))
/2 + (c*e*x^6*(B*e + 2*C*d))/6 + (C*c*e^2*x^7)/7
```

3.20 $\int (d+ex) (a+cx^2) (A+Bx+Cx^2) \, dx$

Optimal. Leaf size=86

$$aAdx+\frac{1}{2}a(Bd+Ae)x^2+\frac{1}{3}(Acd+aCd+aBe)x^3+\frac{1}{4}(Bcd+Ace+aCe)x^4+\frac{1}{5}c(Cd+Be)x^5+\frac{1}{6}cCex^6$$

```
[Out] a*A*d*x+1/2*a*(A*e+B*d)*x^2+1/3*(A*c*d+B*a*e+C*a*d)*x^3+1/4*(A*c*e+B*c*d+C*
a*e)*x^4+1/5*c*(B*e+C*d)*x^5+1/6*c*C*e*x^6
```

Rubi [A]
time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1642}

$$\frac{1}{4}x^4(aCe+Ace+Bcd)+\frac{1}{3}x^3(aBe+aCd+Acd)+\frac{1}{2}ax^2(Ae+Bd)+aAdx+\frac{1}{5}cx^5(Be+Cd)+\frac{1}{6}cCex^6$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d*x + (a*(B*d + A*e)*x^2)/2 + ((A*c*d + a*C*d + a*B*e)*x^3)/3 + ((B*c*d
 + A*c*e + a*C*e)*x^4)/4 + (c*(C*d + B*e)*x^5)/5 + (c*C*e*x^6)/6
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex) (a+cx^2) (A+Bx+Cx^2) \, dx = \int (aAd+a(Bd+Ae)x+(Acd+aCd+aBe)x^2+(Bcd+A$$
$$= aAdx+\frac{1}{2}a(Bd+Ae)x^2+\frac{1}{3}(Acd+aCd+aBe)x^3+\frac{1}{4}(Bcd$$

Mathematica [A]
time = 0.02, size = 86, normalized size = 1.00

$$aAdx+\frac{1}{2}a(Bd+Ae)x^2+\frac{1}{3}(Acd+aCd+aBe)x^3+\frac{1}{4}(Bcd+Ace+aCe)x^4+\frac{1}{5}c(Cd+Be)x^5+\frac{1}{6}cCex^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d*x + (a*(B*d + A*e)*x^2)/2 + ((A*c*d + a*C*d + a*B*e)*x^3)/3 + ((B*c*d
 + A*c*e + a*C*e)*x^4)/4 + (c*(C*d + B*e)*x^5)/5 + (c*C*e*x^6)/6
```

Maple [A]
time = 0.05, size = 79, normalized size = 0.92

method	result
default	$\frac{cCe\,x^6}{6}+\frac{(ceB+cdC)x^5}{5}+\frac{(Ace+Bcd+aCe)x^4}{4}+\frac{(Acd+aBe+Cad)x^3}{3}+\frac{(Aae+Bad)x^2}{2}+aAdx$
norman	$\frac{cCe\,x^6}{6}+\left(\frac{1}{5}ceB+\frac{1}{5}cdC\right)x^5+\left(\frac{1}{4}Ace+\frac{1}{4}Bcd+\frac{1}{4}aCe\right)x^4+\left(\frac{1}{3}Acd+\frac{1}{3}aBe+\frac{1}{3}Cad\right)x^3+\left(\frac{1}{2}Aae\right.$
gosper	$\frac{1}{6}cCe\,x^6+\frac{1}{5}x^5ceB+\frac{1}{5}x^5cdC+\frac{1}{4}x^4Ace+\frac{1}{4}x^4Bcd+\frac{1}{4}x^4aCe+\frac{1}{3}x^3Acd+\frac{1}{3}x^3aBe+\frac{1}{3}x^3Cad+\frac{1}{2}$
risch	$\frac{1}{6}cCe\,x^6+\frac{1}{5}x^5ceB+\frac{1}{5}x^5cdC+\frac{1}{4}x^4Ace+\frac{1}{4}x^4Bcd+\frac{1}{4}x^4aCe+\frac{1}{3}x^3Acd+\frac{1}{3}x^3aBe+\frac{1}{3}x^3Cad+\frac{1}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c*C*e*x^6+1/5*(B*c*e+C*c*d)*x^5+1/4*(A*c*e+B*c*d+C*a*e)*x^4+1/3*(A*c*d+
B*a*e+C*a*d)*x^3+1/2*(A*a*e+B*a*d)*x^2+a*A*d*x
```

Maxima [A]
time = 0.30, size = 85, normalized size = 0.99

$$\frac{1}{6}\,Ccx^6e+\frac{1}{5}\,(Ccd+Bce)x^5+\frac{1}{4}\,(Bcd+Cae+Ace)x^4+Aadx+\frac{1}{3}\,(Bae+(Ca+Ac)d)x^3+\frac{1}{2}\,(Bad+Aae)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/6*C*c*x^6*e + 1/5*(C*c*d + B*c*e)*x^5 + 1/4*(B*c*d + C*a*e + A*c*e)*x^4 +
 A*a*d*x + 1/3*(B*a*e + (C*a + A*c)*d)*x^3 + 1/2*(B*a*d + A*a*e)*x^2
```

Fricas [A]
time = 0.40, size = 88, normalized size = 1.02

$$\frac{1}{5}\,Ccdx^5+\frac{1}{4}\,Bcdx^4+\frac{1}{2}\,Badx^2+\frac{1}{3}\,(Ca+Ac)dx^3+Aadx+\frac{1}{60}\,\left(10\,Ccx^6+12\,Bcx^5+20\,Bax^3+15\,(Ca+Ac)x^4+30\,Aax^2\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/5*C*c*d*x^5 + 1/4*B*c*d*x^4 + 1/2*B*a*d*x^2 + 1/3*(C*a + A*c)*d*x^3 + A*a
*d*x + 1/60*(10*C*c*x^6 + 12*B*c*x^5 + 20*B*a*x^3 + 15*(C*a + A*c)*x^4 + 30
*A*a*x^2)*e
```

Sympy [A]
time = 0.01, size = 97, normalized size = 1.13

$$Aadx+\frac{Ccex^6}{6}+x^5\left(\frac{Bce}{5}+\frac{Ccd}{5}\right)+x^4\left(\frac{Ace}{4}+\frac{Bcd}{4}+\frac{Cae}{4}\right)+x^3\left(\frac{Acd}{3}+\frac{Bae}{3}+\frac{Cad}{3}\right)+x^2\left(\frac{Aae}{2}+\frac{Bad}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*d*x + C*c*e*x**6/6 + x**5*(B*c*e/5 + C*c*d/5) + x**4*(A*c*e/4 + B*c*d/4
 + C*a*e/4) + x**3*(A*c*d/3 + B*a*e/3 + C*a*d/3) + x**2*(A*a*e/2 + B*a*d/2)
```

Giac [A]
time = 3.14, size = 100, normalized size = 1.16

$$\frac{1}{6}\,Ccx^6e+\frac{1}{5}\,Ccdx^5+\frac{1}{5}\,Bcx^5e+\frac{1}{4}\,Bcdx^4+\frac{1}{4}\,Cax^4e+\frac{1}{4}\,Acx^4e+\frac{1}{3}\,Cadx^3+\frac{1}{3}\,Acdx^3+\frac{1}{3}\,Bax^3e+\frac{1}{2}\,Badx^2+\frac{1}{2}\,Aax^2e+Aadx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/6*C*c*x^6*e + 1/5*C*c*d*x^5 + 1/5*B*c*x^5*e + 1/4*B*c*d*x^4 + 1/4*C*a*x^4
*e + 1/4*A*c*x^4*e + 1/3*C*a*d*x^3 + 1/3*A*c*d*x^3 + 1/3*B*a*x^3*e + 1/2*B*
a*d*x^2 + 1/2*A*a*x^2*e + A*a*d*x
```

Mupad [B]
time = 3.56, size = 80, normalized size = 0.93

$$\frac{C\,c\,e\,x^6}{6}+\frac{c\,\left(B\,e+C\,d\right)\,x^5}{5}+\left(\frac{A\,c\,e}{4}+\frac{B\,c\,d}{4}+\frac{C\,a\,e}{4}\right)\,x^4+\left(\frac{A\,c\,d}{3}+\frac{B\,a\,e}{3}+\frac{C\,a\,d}{3}\right)\,x^3+\frac{a\,\left(A\,e+B\,d\right)\,x^2}{2}+A\,a\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*(d + e*x)*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*c*d)/3 + (B*a*e)/3 + (C*a*d)/3) + x^4*((A*c*e)/4 + (B*c*d)/4 + (C*a
*e)/4) + (a*x^2*(A*e + B*d))/2 + (c*x^5*(B*e + C*d))/5 + (C*c*e*x^6)/6 + A*
a*d*x
```

3.21 $\int (a+cx^2)(A+Bx+Cx^2)\,dx$

Optimal. Leaf size=46

$$aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

```
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*c+C*a)*x^3+1/4*B*c*x^4+1/5*c*C*x^5
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{1671\}$

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}aBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(A + B*x + C*x^2),x]
[Out] a*A*x + (a*B*x^2)/2 + ((A*c + a*C)*x^3)/3 + (B*c*x^4)/4 + (c*C*x^5)/5
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (a+cx^2)(A+Bx+Cx^2)\,dx &= \int (aA+aBx+(Ac+aC)x^2+Bcx^3+cCx^4)\,dx\\ &= aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.00

$$aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(A + B*x + C*x^2),x]
[Out] a*A*x + (a*B*x^2)/2 + ((A*c + a*C)*x^3)/3 + (B*c*x^4)/4 + (c*C*x^5)/5
```

Maple [A]
time = 0.05, size = 39, normalized size = 0.85

method	result	size
default	$aAx+\frac{aB\,x^{2}}{2}+\frac{(Ac+aC)x^{3}}{3}+\frac{Bc\,x^{4}}{4}+\frac{cC\,x^{5}}{5}$	39
norman	$\frac{cC\,x^{5}}{5}+\frac{Bc\,x^{4}}{4}+\left(\frac{Ac}{3}+\frac{aC}{3}\right)x^{3}+\frac{aB\,x^{2}}{2}+aAx$	40
gosper	$\frac{1}{5}cC\,x^{5}+\frac{1}{4}Bc\,x^{4}+\frac{1}{3}x^{3}Ac+\frac{1}{3}x^{3}aC+\frac{1}{2}aB\,x^{2}+aAx$	41
risch	$\frac{1}{5}cC\,x^{5}+\frac{1}{4}Bc\,x^{4}+\frac{1}{3}x^{3}Ac+\frac{1}{3}x^{3}aC+\frac{1}{2}aB\,x^{2}+aAx$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*c+C*a)*x^3+1/4*B*c*x^4+1/5*c*C*x^5
```

Maxima [A]
time = 0.32, size = 38, normalized size = 0.83

$$\frac{1}{5}Ccx^{5}+\frac{1}{4}Bcx^{4}+\frac{1}{2}Bax^{2}+\frac{1}{3}(Ca+Ac)x^{3}+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/2*B*a*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

Fricas [A]
time = 0.34, size = 38, normalized size = 0.83

$$\frac{1}{5}Ccx^{5}+\frac{1}{4}Bcx^{4}+\frac{1}{2}Bax^{2}+\frac{1}{3}(Ca+Ac)x^{3}+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/2*B*a*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

Sympy [A]
time = 0.01, size = 42, normalized size = 0.91

$$Aax+\frac{Bax^{2}}{2}+\frac{Bcx^{4}}{4}+\frac{Ccx^{5}}{5}+x^{3}\left(\frac{Ac}{3}+\frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*x + B*a*x**2/2 + B*c*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 + C*a/3)
```

Giac [A]
time = 3.00, size = 40, normalized size = 0.87

$$\frac{1}{5}Ccx^5 + \frac{1}{4}Bcx^4 + \frac{1}{3}Cax^3 + \frac{1}{3}Acx^3 + \frac{1}{2}Bax^2 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/3*C*a*x^3 + 1/3*A*c*x^3 + 1/2*B*a*x^2 + A*a*x
```

Mupad [B]
time = 0.03, size = 39, normalized size = 0.85

$$\frac{C\,c\,x^5}{5} + \frac{B\,c\,x^4}{4} + \left(\frac{A\,c}{3} + \frac{C\,a}{3}\right)\,x^3 + \frac{B\,a\,x^2}{2} + A\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*(A + B*x + C*x^2),x)
[Out] x^3*((A*c)/3 + (C*a)/3) + A*a*x + (B*a*x^2)/2 + (B*c*x^4)/4 + (C*c*x^5)/5
```

3.22 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx$

Optimal. Leaf size=145

$$-\frac{(ae^2(Cd-Be)+cd(Cd^2-e(Bd-Ae)))\,x}{e^4}+\frac{(aCe^2+c(Cd^2-e(Bd-Ae)))\,x^2}{2e^3}-\frac{c(Cd-Be)x^3}{3e^2}+\frac{cCx^4}{4e}-$$

```
[Out] -(a*e^2*(-B*e+C*d)+c*d*(C*d^2-e*(-A*e+B*d)))*x/e^4+1/2*(a*C*e^2+c*(C*d^2-e*
(-A*e+B*d)))*x^2/e^3-1/3*c*(-B*e+C*d)*x^3/e^2+1/4*c*C*x^4/e+(a*e^2+c*d^2)*(
A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e^5
```

Rubi [A]
time = 0.15, antiderivative size = 143, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1642}

$$-\frac{x(ae^2(Cd-Be)-cde(Bd-Ae)+cCd^3)}{e^4}+\frac{(ae^2+cd^2)\log(d+ex)\,(Ae^2-Bde+Cd^2)}{e^5}+\frac{x^2(aCe^2-ce(Bd-Ae)+cCd^2)}{2e^3}-\frac{cx^3(Cd-Be)}{3e^2}+\frac{cCx^4}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] -(((c*C*d^3 - c*d*e*(B*d - A*e) + a*e^2*(C*d - B*e))*x)/e^4) + ((c*C*d^2 +
a*C*e^2 - c*e*(B*d - A*e))*x^2)/(2*e^3) - (c*(C*d - B*e)*x^3)/(3*e^2) + (c*
C*x^4)/(4*e) + ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/e^5
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx &= \int \left(\frac{-ae^2(Cd-Be)-c(Cd^3-de(Bd-Ae))}{e^4}+\frac{(cCd^2+aCe^2-ce(B}{e^3}\right.\\ &= -\frac{(cCd^3-cde(Bd-Ae)+ae^2(Cd-Be))\,x}{e^4}+\frac{(cCd^2+aCe^2-ce(Bd}{2e^3}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 136, normalized size = 0.94

$$\frac{ex(6ae^2(-2Cd+2Be+Cex)+cC(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+2ce(3Ae(-2d+ex)+B(6d^2-3dex+2e^2x^2)))+12(cd^2+ae^2)\,(Cd^2+e(-Bd+Ae))\log(d+ex)}{12e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] (e*x*(6*a*e^2*(-2*C*d + 2*B*e + C*e*x) + c*C*(-12*d^3 + 6*d^2*e*x - 4*d*e^2
*x^2 + 3*e^3*x^3) + 2*c*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3*d*e*x + 2*e^2*
x^2))) + 12*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e))*Log[d + e*x])/(12*e^
5)
```

Maple [A]
time = 0.08, size = 178, normalized size = 1.23

method	result
norman	$\frac{(Ace^2-Bcde+aCe^2+Ccd^2)x^2}{2e^3}-\frac{(de^2cA-Bae^3-Bcd^2e+Cade^2+Ccd^3)x}{e^4}+\frac{cCx^4}{4e}+\frac{c(Be-Cd)x^3}{3e^2}+\frac{(Aae^4+Acd^2e^2-Ba$
default	$\frac{(Aae^4+Acd^2e^2-Bade^3-Bcd^3e+Cad^2e^2+Ccd^4)\ln(ex+d)}{e^5}-\frac{-\frac{1}{4}cCx^4e^3-\frac{1}{3}Bce^3x^3+\frac{1}{3}Ccde^2x^3-\frac{1}{2}Ace^3x^2+\frac{1}{2}Bcde^2x^2-\frac{1}{2}Ca}{e^4}$
risch	$\frac{cCx^4}{4e}+\frac{Bcx^3}{3e}-\frac{Ccdx^3}{3e^2}+\frac{Acx^2}{2e}-\frac{Bcdx^2}{2e^2}+\frac{Cax^2}{2e}+\frac{Ccd^2x^2}{2e^3}-\frac{Acdx}{e^2}+\frac{Bax}{e}+\frac{Bcd^2x}{e^3}-\frac{Cadx}{e^2}-\frac{Ccd^3x}{e^4}+\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] (A*a*e^4+A*c*d^2*e^2-B*a*d*e^3-B*c*d^3*e+C*a*d^2*e^2+C*c*d^4)/e^5*ln(e*x+d)
-1/e^4*(-1/4*c*C*x^4*e^3-1/3*B*c*e^3*x^3+1/3*C*c*d*e^2*x^3-1/2*A*c*e^3*x^2+
1/2*B*c*d*e^2*x^2-1/2*C*a*e^3*x^2-1/2*C*c*d^2*e*x^2+A*c*d*e^2*x-B*a*e^3*x-B
*c*d^2*e*x+C*a*d*e^2*x+C*c*d^3*x)
```

Maxima [A]
time = 0.28, size = 155, normalized size = 1.07

$$(Ccd^4 - Bcd^3e - Bade^3 + (Cae^2 + Ace^2)d^2 + Aae^4)e^{(-5)}\log(xe + d) + \frac{1}{12}(3Ccx^4e^3 - 4(Ccde^2 - Bce^3)x^3 + 6(Ccd^2e - Bcde^2 + Cae^3 + Ace^3)x^2 - 12(Ccd^3 - Bcd^2e - Bae^3 + (Cae^2 + Ace^2)d)x)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")
```

```
[Out] (C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + (C*a*e^2 + A*c*e^2)*d^2 + A*a*e^4)*e^(-5
)*log(x*e + d) + 1/12*(3*C*c*x^4*e^3 - 4*(C*c*d*e^2 - B*c*e^3)*x^3 + 6*(C*c
*d^2*e - B*c*d*e^2 + C*a*e^3 + A*c*e^3)*x^2 - 12*(C*c*d^3 - B*c*d^2*e - B*a
*e^3 + (C*a*e^2 + A*c*e^2)*d)*x)*e^(-4)
```

Fricas [A]
time = 0.34, size = 155, normalized size = 1.07

$$-\frac{1}{12}(12Ccd^3xe - (3Ccx^4 + 4Bcx^3 + 12Bax + 6(Ca + Ac)x^2)e^4 + 2(2Ccdx^3 + 3Bcdx^2 + 6(Ca + Ac)dx)e^3 - 6(Ccd^2x^2 + 2Bcd^2x)e^2 - 12(Ccd^4 - Bcd^3e - Bade^3 + (Ca + Ac)d^2e^2 + Aae^4)\log(xe + d))e^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/12*(12*C*c*d^3*x*e - (3*C*c*x^4 + 4*B*c*x^3 + 12*B*a*x + 6*(C*a + A*c)*x
^2)*e^4 + 2*(2*C*c*d*x^3 + 3*B*c*d*x^2 + 6*(C*a + A*c)*d*x)*e^3 - 6*(C*c*d^
2*x^2 + 2*B*c*d^2*x)*e^2 - 12*(C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + (C*a + A*c
)*d^2*e^2 + A*a*e^4)*log(x*e + d))*e^(-5)
```

Sympy [A]
time = 0.24, size = 148, normalized size = 1.02

$$\frac{Ccx^4}{4e} + x^3\left(\frac{Bc}{3e} - \frac{Ccd}{3e^2}\right) + x^2\left(\frac{Ac}{2e} - \frac{Bcd}{2e^2} + \frac{Ca}{2e} + \frac{Ccd^2}{2e^3}\right) + x\left(-\frac{Acd}{e^2} + \frac{Ba}{e} + \frac{Bcd^2}{e^3} - \frac{Cad}{e^2} - \frac{Ccd^3}{e^4}\right) + \frac{(ae^2+cd^2)\,(Ae^2-Bde+Cd^2)\log{(d+ex)}}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d),x)
```

```
[Out] C*c*x**4/(4*e) + x**3*(B*c/(3*e) - C*c*d/(3*e**2)) + x**2*(A*c/(2*e) - B*c*
d/(2*e**2) + C*a/(2*e) + C*c*d**2/(2*e**3)) + x*(-A*c*d/e**2 + B*a/e + B*c*
d**2/e**3 - C*a*d/e**2 - C*c*d**3/e**4) + (a*e**2 + c*d**2)*(A*e**2 - B*d*e
 + C*d**2)*log(d + e*x)/e**5
```

Giac [A]
time = 4.00, size = 170, normalized size = 1.17

$$\left(Ccd^4 - Bcd^3e + Cad^2e^2 + Acd^2e^2 - Bade^3 + Aae^4\right)e^{(-5)}\log\left(|xe+d|\right) + \frac{1}{12}\left(3\,Ccx^4e^3 - 4\,Ccdx^3e^2 + 6\,Ccd^2x^2e - 12\,Ccd^3x + 4\,Bcx^3e^3 - 6\,Bcdx^2e^2 + 12\,Bcd^2xe + 6\,Cax^2e^3 + 6\,Acx^2e^3 - 12\,Cadxe^2 - 12\,Acdxe^2 + 12\,Baxe^3\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

```
[Out] (C*c*d^4 - B*c*d^3*e + C*a*d^2*e^2 + A*c*d^2*e^2 - B*a*d*e^3 + A*a*e^4)*e^(
-5)*log(abs(x*e + d)) + 1/12*(3*C*c*x^4*e^3 - 4*C*c*d*x^3*e^2 + 6*C*c*d^2*x
^2*e - 12*C*c*d^3*x + 4*B*c*x^3*e^3 - 6*B*c*d*x^2*e^2 + 12*B*c*d^2*x*e + 6*
C*a*x^2*e^3 + 6*A*c*x^2*e^3 - 12*C*a*d*x*e^2 - 12*A*c*d*x*e^2 + 12*B*a*x*e^
3)*e^(-4)
```

Mupad [B]
time = 3.62, size = 175, normalized size = 1.21

$$x^3\left(\frac{B\,c}{3\,e} - \frac{C\,c\,d}{3\,e^2}\right) - x\left(\frac{d\left(\frac{A\,c+C\,a}{e} - \frac{d\left(\frac{B\,c}{e} - \frac{C\,c\,d}{e^2}\right)}{e}\right)}{e} - \frac{B\,a}{e}\right) + x^2\left(\frac{A\,c+C\,a}{2\,e} - \frac{d\left(\frac{B\,c}{e} - \frac{C\,c\,d}{e^2}\right)}{2\,e}\right) + \frac{\ln\left(d+e\,x\right)\,\left(A\,a\,e^4 + C\,c\,d^4 - B\,a\,d\,e^3 - B\,c\,d^3\,e + A\,c\,d^2\,e^2 + C\,a\,d^2\,e^2\right)}{e^5} + \frac{C\,c\,x^4}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x)
```

```
[Out] x^3*((B*c)/(3*e) - (C*c*d)/(3*e^2)) - x*((d*((A*c + C*a)/e - (d*((B*c)/e -
(C*c*d)/e^2))/e))/e - (B*a)/e) + x^2*((A*c + C*a)/(2*e) - (d*((B*c)/e - (C*
c*d)/e^2))/(2*e)) + (log(d + e*x)*(A*a*e^4 + C*c*d^4 - B*a*d*e^3 - B*c*d^3*
e + A*c*d^2*e^2 + C*a*d^2*e^2))/e^5 + (C*c*x^4)/(4*e)
```

3.23 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2} dx$

Optimal. Leaf size=153

$$\frac{(aCe^2+c(3Cd^2-e(2Bd-Ae)))\,x}{e^4}-\frac{c(2Cd-Be)x^2}{2e^3}+\frac{cCx^3}{3e^2}-\frac{(cd^2+ae^2)\,(Cd^2-Bde+Ae^2)}{e^5(d+ex)}-\frac{(ae^2(2Cd-$$

```
[Out] (a*C*e^2+c*(3*C*d^2-e*(-A*e+2*B*d)))*x/e^4-1/2*c*(-B*e+2*C*d)*x^2/e^3+1/3*c
*C*x^3/e^2-(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)/e^5/(e*x+d)-(a*e^2*(-B*e+2*C*d
)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*ln(e*x+d)/e^5
```

Rubi [A]
time = 0.13, antiderivative size = 151, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1642}

$$-\frac{\log(d+ex)\,(ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3)}{e^5}-\frac{(ae^2+cd^2)\,(Ae^2-Bde+Cd^2)}{e^5(d+ex)}+\frac{x(aCe^2-ce(2Bd-Ae)+3cCd^2)}{e^4}-\frac{cx^2(2Cd-Be)}{2e^3}+\frac{cCx^3}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] ((3*c*C*d^2 + a*C*e^2 - c*e*(2*B*d - A*e))*x)/e^4 - (c*(2*C*d - B*e)*x^2)/(
2*e^3) + (c*C*x^3)/(3*e^2) - ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2))/(e^5
*(d + e*x)) - ((4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))*Lo
g[d + e*x])/e^5
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^2)\,(A+Bx+Cx^2)}{(d+ex)^2}\,dx=\int \left(\frac{3cCd^2+aCe^2-ce(2Bd-Ae)}{e^4}+\frac{c(-2Cd+Be)x}{e^3}+\frac{cCx^2}{e^2}+\frac{(cd^2$$

$$=\frac{(3cCd^2+aCe^2-ce(2Bd-Ae))\,x}{e^4}-\frac{c(2Cd-Be)x^2}{2e^3}+\frac{cCx^3}{3e^2}-\frac{(cd^2+$$

Mathematica [A]
time = 0.11, size = 142, normalized size = 0.93

$$\frac{6e(3cCd^2+aCe^2+ce(-2Bd+Ae))\,x+3ce^2(-2Cd+Be)x^2+2cCe^3x^3-\frac{6(cd^2+ae^2)(Cd^2+e(-Bd+Ae))}{d+ex}+6(-4cCd^3+cde(3Bd-2Ae)+ae^2(-2Cd+Be))\log(d+ex)}{6e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] (6*e*(3*c*C*d^2 + a*C*e^2 + c*e*(-2*B*d + A*e))*x + 3*c*e^2*(-2*C*d + B*e)*
x^2 + 2*c*C*e^3*x^3 - (6*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d + e
*x) + 6*(-4*c*C*d^3 + c*d*e*(3*B*d - 2*A*e) + a*e^2*(-2*C*d + B*e))*Log[d +
 e*x])/(6*e^5)
```

Maple [A]
time = 0.08, size = 172, normalized size = 1.12

method	result
default	$\frac{\left(-2d\,e^{2}cA+Ba\,e^{3}+3Bc\,d^{2}e-2Cad\,e^{2}-4Cc\,d^{3}\right)\ln(ex+d)}{e^{5}}-\frac{Aa\,e^{4}+Ac\,d^{2}e^{2}-Bad\,e^{3}-Bc\,d^{3}e+Ca\,d^{2}e^{2}+Cc\,d^{4}}{e^{5}(ex+d)}+\frac{\frac{1}{3}cC\,x^{3}e^{2}+\frac{1}{2}B$
norman	$\frac{\frac{\left(Aa\,e^{4}+2Ac\,d^{2}e^{2}-Bad\,e^{3}-3Bc\,d^{3}e+2Ca\,d^{2}e^{2}+4Cc\,d^{4}\right)x}{e^{4}d}+\frac{\left(2Ac\,e^{2}-3Bcde+2aC\,e^{2}+4Cc\,d^{2}\right)x^{2}}{2e^{3}}+\frac{cC\,x^{4}}{3e}+\frac{c(3Be-4Cd)x^{3}}{6e^{2}}}{ex+d}-\frac{(2d\,e^{2}cA-Ba$
risch	$\frac{cC\,x^{3}}{3e^{2}}+\frac{Bc\,x^{2}}{2e^{2}}-\frac{Ccd\,x^{2}}{e^{3}}+\frac{Acx}{e^{2}}-\frac{2Bcdx}{e^{3}}+\frac{aCx}{e^{2}}+\frac{3Cc\,d^{2}x}{e^{4}}-\frac{Aa}{e(ex+d)}-\frac{Ac\,d^{2}}{e^{3}(ex+d)}+\frac{Bad}{e^{2}(ex+d)}+\frac{Bc\,d^{3}}{e^{4}(ex+d)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-2*A*c*d*e^2+B*a*e^3+3*B*c*d^2*e-2*C*a*d*e^2-4*C*c*d^3)/e^5*ln(e*x+d)-(A*a
*e^4+A*c*d^2*e^2-B*a*d*e^3-B*c*d^3*e+C*a*d^2*e^2+C*c*d^4)/e^5/(e*x+d)+1/e^4
*(1/3*c*C*x^3*e^2+1/2*B*c*e^2*x^2-C*c*d*e*x^2+A*c*e^2*x-2*B*c*d*e*x+a*C*e^2
*x+3*C*c*d^2*x)
```

Maxima [A]
time = 0.32, size = 166, normalized size = 1.08

$$-\left(4\,Ccd^{3}-3\,Bcd^{2}e-Bae^{3}+2\,\left(Cae^{2}+Ace^{2}\right)d\right)e^{(-5)}\log\left(xe+d\right)+\frac{1}{6}\,\left(2\,Ccx^{3}e^{2}-3\,\left(2\,Ccde-Bce^{2}\right)x^{2}+6\,\left(3\,Ccd^{2}-2\,Bcde+Cae^{2}+Ace^{2}\right)x\right)e^{(-4)}-\frac{Ccd^{4}-Bcd^{3}e-Bade^{3}+\left(Cae^{2}+Ace^{2}\right)d^{2}+Aae^{4}}{xe^{6}+de^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(4*C*c*d^3 - 3*B*c*d^2*e - B*a*e^3 + 2*(C*a*e^2 + A*c*e^2)*d)*e^(-5)*log(x
*e + d) + 1/6*(2*C*c*x^3*e^2 - 3*(2*C*c*d*e - B*c*e^2)*x^2 + 6*(3*C*c*d^2 -
 2*B*c*d*e + C*a*e^2 + A*c*e^2)*x)*e^(-4) - (C*c*d^4 - B*c*d^3*e - B*a*d*e^
3 + (C*a*e^2 + A*c*e^2)*d^2 + A*a*e^4)/(x*e^6 + d*e^5)
```

Fricas [A]
time = 0.34, size = 234, normalized size = 1.53

$$-\frac{6\,Ccd^{4}-\left(2\,Ccx^{4}+3\,Bcx^{3}+6\,\left(Ca+Ac\right)x^{2}-6\,Aa\right)e^{4}+\left(4\,Ccdx^{3}+9\,Bcdx^{2}-6\,Bad-6\,\left(Ca+Ac\right)dx\right)e^{3}-6\,\left(2\,Ccd^{2}x^{2}-2\,Bcd^{2}x-\left(Ca+Ac\right)d^{2}\right)e^{2}-6\,\left(3\,Ccd^{3}x+Bcd^{3}\right)e+6\,\left(4\,Ccd^{4}-Baxe^{4}-\left(Bad-2\,\left(Ca+Ac\right)dx\right)e^{3}-\left(3\,Bcd^{2}x-2\,\left(Ca+Ac\right)d^{2}\right)e^{2}+\left(4\,Ccd^{3}x-3\,Bcd^{3}\right)e\right)\log\left(xe+d\right)}{6\,\left(xe^{6}+de^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*C*c*d^4 - (2*C*c*x^4 + 3*B*c*x^3 + 6*(C*a + A*c)*x^2 - 6*A*a)*e^4 +
 (4*C*c*d*x^3 + 9*B*c*d*x^2 - 6*B*a*d - 6*(C*a + A*c)*d*x)*e^3 - 6*(2*C*c*d
^2*x^2 - 2*B*c*d^2*x - (C*a + A*c)*d^2)*e^2 - 6*(3*C*c*d^3*x + B*c*d^3)*e +
 6*(4*C*c*d^4 - B*a*x*e^4 - (B*a*d - 2*(C*a + A*c)*d*x)*e^3 - (3*B*c*d^2*x
- 2*(C*a + A*c)*d^2)*e^2 + (4*C*c*d^3*x - 3*B*c*d^3)*e)*log(x*e + d))/(x*e^
6 + d*e^5)
```

Sympy [A]
time = 0.57, size = 185, normalized size = 1.21

$$\frac{Ccx^3}{3e^2} + x^2\left(\frac{Bc}{2e^2} - \frac{Ccd}{e^3}\right) + x\left(\frac{Ac}{e^2} - \frac{2Bcd}{e^3} + \frac{Ca}{e^2} + \frac{3Ccd^2}{e^4}\right) + \frac{-Aae^4 - Acd^2e^2 + Bade^3 + Bcd^3e - Cad^2e^2 - Ccd^4}{de^5 + e^6x} - \frac{(2Acde^2 - Bae^3 - 3Bcd^2e + 2Cade^2 + 4Ccd^3)\log(d + ex)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

```
[Out] C*c*x**3/(3*e**2) + x**2*(B*c/(2*e**2) - C*c*d/e**3) + x*(A*c/e**2 - 2*B*c*
d/e**3 + C*a/e**2 + 3*C*c*d**2/e**4) + (-A*a*e**4 - A*c*d**2*e**2 + B*a*d*e
**3 + B*c*d**3*e - C*a*d**2*e**2 - C*c*d**4)/(d*e**5 + e**6*x) - (2*A*c*d*e
**2 - B*a*e**3 - 3*B*c*d**2*e + 2*C*a*d*e**2 + 4*C*c*d**3)*log(d + e*x)/e**
5
```

Giac [A]
time = 5.22, size = 240, normalized size = 1.57

$$\frac{1}{6}\left(2\,Cc - \frac{3\,(4\,Ccde - Bce^2)e^{(-1)}}{xe + d} + \frac{6\,(6\,Ccd^2e^2 - 3\,Bcde^3 + Cae^4 + Ace^4)e^{(-2)}}{(xe + d)^2}\right)(xe + d)^3e^{(-5)} + (4\,Ccd^3 - 3\,Bcd^2e + 2\,Cade^2 + 2\,Acde^2 - Bae^3)e^{(-5)}\log\left(\frac{|xe + d|e^{(-1)}}{(xe + d)^2}\right) - \left(\frac{Ccd^4e^3}{xe + d} - \frac{Bcd^3e^4}{xe + d} + \frac{Cad^2e^5}{xe + d} + \frac{Acd^2e^5}{xe + d} - \frac{Bade^6}{xe + d} + \frac{Aae^7}{xe + d}\right)e^{(-8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/6*(2*C*c - 3*(4*C*c*d*e - B*c*e^2)*e^(-1)/(x*e + d) + 6*(6*C*c*d^2*e^2 -
3*B*c*d*e^3 + C*a*e^4 + A*c*e^4)*e^(-2)/(x*e + d)^2)*(x*e + d)^3*e^(-5) + (
4*C*c*d^3 - 3*B*c*d^2*e + 2*C*a*d*e^2 + 2*A*c*d*e^2 - B*a*e^3)*e^(-5)*log(a
bs(x*e + d)*e^(-1)/(x*e + d)^2) - (C*c*d^4*e^3/(x*e + d) - B*c*d^3*e^4/(x*e
 + d) + C*a*d^2*e^5/(x*e + d) + A*c*d^2*e^5/(x*e + d) - B*a*d*e^6/(x*e + d)
 + A*a*e^7/(x*e + d))*e^(-8)
```

Mupad [B]
time = 0.09, size = 192, normalized size = 1.25

$$x^2\left(\frac{B\,c}{2\,e^2} - \frac{C\,c\,d}{e^3}\right) - x\left(\frac{2\,d\,\left(\frac{B\,c}{e^2} - \frac{2\,C\,c\,d}{e^3}\right)}{e} - \frac{A\,c + C\,a}{e^2} + \frac{C\,c\,d^2}{e^4}\right) - \frac{\ln(d + e\,x)\,(4\,C\,c\,d^3 - B\,a\,e^3 + 2\,A\,c\,d\,e^2 + 2\,C\,a\,d\,e^2 - 3\,B\,c\,d^2\,e)}{e^5} - \frac{A\,a\,e^4 + C\,c\,d^4 - B\,a\,d\,e^3 - B\,c\,d^3\,e + A\,c\,d^2\,e^2 + C\,a\,d^2\,e^2}{e\,(x\,e^5 + d\,e^4)} + \frac{C\,c\,x^3}{3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

```
[Out] x^2*((B*c)/(2*e^2) - (C*c*d)/e^3) - x*((2*d*((B*c)/e^2 - (2*C*c*d)/e^3))/e
- (A*c + C*a)/e^2 + (C*c*d^2)/e^4) - (log(d + e*x)*(4*C*c*d^3 - B*a*e^3 + 2
*A*c*d*e^2 + 2*C*a*d*e^2 - 3*B*c*d^2*e))/e^5 - (A*a*e^4 + C*c*d^4 - B*a*d*e
^3 - B*c*d^3*e + A*c*d^2*e^2 + C*a*d^2*e^2)/(e*(d*e^4 + e^5*x)) + (C*c*x^3)
/(3*e^2)
```

3.24 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3}\,dx$

Optimal. Leaf size=156

$$-\frac{c(3Cd-Be)x}{e^4}+\frac{cCx^2}{2e^3}-\frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)}{2e^5(d+ex)^2}+\frac{ae^2(2Cd-Be)+cd(4Cd^2-e(3Bd-2Ae))}{e^5(d+ex)}+\frac{(a$$

```
[Out] -c*(-B*e+3*C*d)*x/e^4+1/2*c*C*x^2/e^3-1/2*(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)
/e^5/(e*x+d)^2+(a*e^2*(-B*e+2*C*d)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))/e^5/(e*x
+d)+(a*C*e^2+c*(6*C*d^2-e*(-A*e+3*B*d)))*ln(e*x+d)/e^5
```

Rubi [A]
time = 0.12, antiderivative size = 154, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1642}

$$\frac{ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3}{e^5(d+ex)}-\frac{(ae^2+cd^2)(Ae^2-Bde+Cd^2)}{2e^5(d+ex)^2}+\frac{\log(d+ex)\left(aCe^2-ce(3Bd-Ae)+6cCd^2\right)}{e^5}-\frac{cx(3Cd-Be)}{e^4}+\frac{cCx^2}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] -((c*(3*C*d - B*e)*x)/e^4) + (c*C*x^2)/(2*e^3) - ((c*d^2 + a*e^2)*(C*d^2 -
B*d*e + A*e^2))/(2*e^5*(d + e*x)^2) + (4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) +
a*e^2*(2*C*d - B*e))/(e^5*(d + e*x)) + ((6*c*C*d^2 + a*C*e^2 - c*e*(3*B*d -
 A*e))*Log[d + e*x])/e^5
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3}\,dx = \int \left(\frac{c(-3Cd+Be)}{e^4}+\frac{cCx}{e^3}+\frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)}{e^4(d+ex)^3}+\frac{-4cCd}{}\right.$$

$$= -\frac{c(3Cd-Be)x}{e^4}+\frac{cCx^2}{2e^3}-\frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)}{2e^5(d+ex)^2}+\frac{4cCd^3-c}{}$$

Mathematica [A]
time = 0.07, size = 176, normalized size = 1.13

$$\frac{c(-3Cd+Be)x}{e^4}+\frac{cCx^2}{2e^3}+\frac{-cCd^4+Bcd^3e-Acd^2e^2-aCd^2e^2+aBde^3-aAe^4}{2e^5(d+ex)^2}+\frac{4cCd^3-3Bcd^2e+2Acde^2+2aCde^2-aBe^3}{e^5(d+ex)}+\frac{(6cCd^2-3Bcde+Ace^2+aCe^2)\log(d+ex)}{e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] (c*(-3*C*d + B*e)*x)/e^4 + (c*C*x^2)/(2*e^3) + (-(c*C*d^4) + B*c*d^3*e - A*
c*d^2*e^2 - a*C*d^2*e^2 + a*B*d*e^3 - a*A*e^4)/(2*e^5*(d + e*x)^2) + (4*c*C
*d^3 - 3*B*c*d^2*e + 2*A*c*d*e^2 + 2*a*C*d*e^2 - a*B*e^3)/(e^5*(d + e*x)) +
 ((6*c*C*d^2 - 3*B*c*d*e + A*c*e^2 + a*C*e^2)*Log[d + e*x])/e^5
```

Maple [A]
time = 0.09, size = 169, normalized size = 1.08

method	result
default	$\frac{\left(Ac\,e^{2}-3Bcde+aC\,e^{2}+6Cc\,d^{2}\right)\ln(ex+d)}{e^{5}}-\frac{Aa\,e^{4}+Ac\,d^{2}e^{2}-Bad\,e^{3}-Bc\,d^{3}e+Ca\,d^{2}e^{2}+Cc\,d^{4}}{2e^{5}(ex+d)^{2}}-\frac{-2d\,e^{2}cA+Ba\,e^{3}+3Bc\,d^{2}e-2C\cdots}{e^{5}(ex+d)}$
norman	$\frac{\frac{\left(2d\,e^{2}cA-Ba\,e^{3}-6Bc\,d^{2}e+2Cad\,e^{2}+12Cc\,d^{3}\right)x}{e^{4}}+\frac{c(Be-2Cd)x^{3}}{e^{2}}-\frac{Aa\,e^{4}-3Ac\,d^{2}e^{2}+Bad\,e^{3}+9Bc\,d^{3}e-3Ca\,d^{2}e^{2}-18Cc\,d^{4}}{2e^{5}}+\frac{cC\,x^{4}}{2e}}{(ex+d)^{2}}+\frac{(Ac\,e^{2}\cdots}{}$
risch	$\frac{cC\,x^{2}}{2e^{3}}+\frac{cBx}{e^{3}}-\frac{3cCdx}{e^{4}}+\frac{\left(2d\,e^{2}cA-Ba\,e^{3}-3Bc\,d^{2}e+2Cad\,e^{2}+4Cc\,d^{3}\right)x-\frac{Aa\,e^{4}-3Ac\,d^{2}e^{2}+Bad\,e^{3}+5Bc\,d^{3}e-3Ca\,d^{2}e^{2}-7Cc\,d^{4}}{2e}}{e^{4}(ex+d)^{2}}-\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^5*(A*c*e^2-3*B*c*d*e+C*a*e^2+6*C*c*d^2)*ln(e*x+d)-1/2*(A*a*e^4+A*c*d^2*
e^2-B*a*d*e^3-B*c*d^3*e+C*a*d^2*e^2+C*c*d^4)/e^5/(e*x+d)^2-(-2*A*c*d*e^2+B*
a*e^3+3*B*c*d^2*e-2*C*a*d*e^2-4*C*c*d^3)/e^5/(e*x+d)+c/e^4*(1/2*C*x^2*e+B*e
*x-3*C*d*x)
```

Maxima [A]
time = 0.29, size = 175, normalized size = 1.12

$$(6\,Ccd^2 - 3\,Bcde + Cae^2 + Ace^2)e^{(-5)}\log\left(xe + d\right) + \frac{1}{2}\,(Ccx^2e - 2\,(3\,Ccd - Bce)x)e^{(-4)} + \frac{7\,Ccd^4 - 5\,Bcd^3e - Bade^3 + 3\,(Cae^2 + Ace^2)d^2 - Aae^4 + 2\,(4\,Ccd^3e - 3\,Bcd^2e^2 - Bae^4 + 2\,(Cae^3 + Ace^3)d)x}{2\,(x^2e^7 + 2\,dxe^6 + d^2e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] (6*C*c*d^2 - 3*B*c*d*e + C*a*e^2 + A*c*e^2)*e^(-5)*log(x*e + d) + 1/2*(C*c*
x^2*e - 2*(3*C*c*d - B*c*e)*x)*e^(-4) + 1/2*(7*C*c*d^4 - 5*B*c*d^3*e - B*a*
d*e^3 + 3*(C*a*e^2 + A*c*e^2)*d^2 - A*a*e^4 + 2*(4*C*c*d^3*e - 3*B*c*d^2*e^
2 - B*a*e^4 + 2*(C*a*e^3 + A*c*e^3)*d)*x)/(x^2*e^7 + 2*d*x*e^6 + d^2*e^5)
```

Fricas [A]
time = 0.33, size = 254, normalized size = 1.63

$$\frac{7\,Ccd^4 + (Ccx^4 + 2\,Bcx^3 - 2\,Bax - Aa)e^4 - (4\,Ccdx^3 - 4\,Bcdx^2 + Bad - 4\,(Ca + Ac)dx)e^3 - (11\,Ccd^2x^2 + 4\,Bcd^2x - 3\,(Ca + Ac)d^2)e^2 + (2\,Ccd^3x - 5\,Bcd^3)e + 2\,(6\,Ccd^4 + (Ca + Ac)x^2e^4 - (3\,Bcdx^2 - 2\,(Ca + Ac)dx)e^3 + (6\,Ccd^2x^2 - 6\,Bcd^2x + (Ca + Ac)d^2)e^2 + 3\,(4\,Ccd^3x - Bcd^3)e)\log\left(xe + d\right)}{2\,(x^2e^7 + 2\,dxe^6 + d^2e^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(7*C*c*d^4 + (C*c*x^4 + 2*B*c*x^3 - 2*B*a*x - A*a)*e^4 - (4*C*c*d*x^3 -
 4*B*c*d*x^2 + B*a*d - 4*(C*a + A*c)*d*x)*e^3 - (11*C*c*d^2*x^2 + 4*B*c*d^2
*x - 3*(C*a + A*c)*d^2)*e^2 + (2*C*c*d^3*x - 5*B*c*d^3)*e + 2*(6*C*c*d^4 +
(C*a + A*c)*x^2*e^4 - (3*B*c*d*x^2 - 2*(C*a + A*c)*d*x)*e^3 + (6*C*c*d^2*x^
2 - 6*B*c*d^2*x + (C*a + A*c)*d^2)*e^2 + 3*(4*C*c*d^3*x - B*c*d^3)*e)*log(x
*e + d))/(x^2*e^7 + 2*d*x*e^6 + d^2*e^5)
```

Sympy [A]
time = 2.18, size = 206, normalized size = 1.32

$$\frac{Ccx^2}{2e^3} + x\left(\frac{Bc}{e^3} - \frac{3Ccd}{e^4}\right) + \frac{-Aae^4 + 3Acd^2e^2 - Bade^3 - 5Bcd^3e + 3Cad^2e^2 + 7Ccd^4 + x(4Acde^3 - 2Bae^4 - 6Bcd^2e^2 + 4Cade^3 + 8Ccd^3e)}{2d^2e^5 + 4de^6x + 2e^7x^2} + \frac{(Ace^2 - 3Bcde + Cae^2 + 6Ccd^2)\log(d + ex)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

```
[Out] C*c*x**2/(2*e**3) + x*(B*c/e**3 - 3*C*c*d/e**4) + (-A*a*e**4 + 3*A*c*d**2*e
**2 - B*a*d*e**3 - 5*B*c*d**3*e + 3*C*a*d**2*e**2 + 7*C*c*d**4 + x*(4*A*c*d
*e**3 - 2*B*a*e**4 - 6*B*c*d**2*e**2 + 4*C*a*d*e**3 + 8*C*c*d**3*e))/(2*d**
2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + (A*c*e**2 - 3*B*c*d*e + C*a*e**2 + 6*C
*c*d**2)*log(d + e*x)/e**5
```

Giac [A]
time = 4.10, size = 167, normalized size = 1.07

$$(6\,Ccd^2 - 3\,Bcde + Cae^2 + Ace^2)e^{(-5)}\log\left(|xe + d|\right) + \frac{1}{2}\left(Ccx^2e^3 - 6\,Ccdxe^2 + 2\,Bcxe^3\right)e^{(-6)} + \frac{(7\,Ccd^4 - 5\,Bcd^3e + 3\,Cad^2e^2 + 3\,Acd^2e^2 - Bade^3 - Aae^4 + 2\,(4\,Ccd^3e - 3\,Bcd^2e^2 + 2\,Cade^3 + 2\,Acde^3 - Bae^4)x)e^{(-5)}}{2\,(xe + d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] (6*C*c*d^2 - 3*B*c*d*e + C*a*e^2 + A*c*e^2)*e^(-5)*log(abs(x*e + d)) + 1/2*
(C*c*x^2*e^3 - 6*C*c*d*x*e^2 + 2*B*c*x*e^3)*e^(-6) + 1/2*(7*C*c*d^4 - 5*B*c
*d^3*e + 3*C*a*d^2*e^2 + 3*A*c*d^2*e^2 - B*a*d*e^3 - A*a*e^4 + 2*(4*C*c*d^3
*e - 3*B*c*d^2*e^2 + 2*C*a*d*e^3 + 2*A*c*d*e^3 - B*a*e^4)*x)*e^(-5)/(x*e +
d)^2
```

Mupad [B]
time = 0.09, size = 185, normalized size = 1.19

$$\frac{x\,(4\,C\,c\,d^3 - B\,a\,e^3 + 2\,A\,c\,d\,e^2 + 2\,C\,a\,d\,e^2 - 3\,B\,c\,d^2\,e) - \frac{A\,a\,e^4 - 7\,C\,c\,d^4 + B\,a\,d\,e^3 + 5\,B\,c\,d^3\,e - 3\,A\,c\,d^2\,e^2 - 3\,C\,a\,d^2\,e^2}{2\,e}}{d^2\,e^4 + 2\,d\,e^5\,x + e^6\,x^2} + x\left(\frac{B\,c}{e^3} - \frac{3\,C\,c\,d}{e^4}\right) + \frac{\ln(d + e\,x)\,(A\,c\,e^2 + C\,a\,e^2 + 6\,C\,c\,d^2 - 3\,B\,c\,d\,e)}{e^5} + \frac{C\,c\,x^2}{2\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

```
[Out] (x*(4*C*c*d^3 - B*a*e^3 + 2*A*c*d*e^2 + 2*C*a*d*e^2 - 3*B*c*d^2*e) - (A*a*e
^4 - 7*C*c*d^4 + B*a*d*e^3 + 5*B*c*d^3*e - 3*A*c*d^2*e^2 - 3*C*a*d^2*e^2)/(
2*e))/(d^2*e^4 + e^6*x^2 + 2*d*e^5*x) + x*((B*c)/e^3 - (3*C*c*d)/e^4) + (lo
g(d + e*x)*(A*c*e^2 + C*a*e^2 + 6*C*c*d^2 - 3*B*c*d*e))/e^5 + (C*c*x^2)/(2*
e^3)
```

3.25 $\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx$

Optimal. Leaf size=304

$$a^2Ad^3x+\frac{1}{3}ad\left(ad(Cd+3Be)+A\left(2cd^2+3ae^2\right)\right)x^3+\frac{1}{4}a^2e\left(3Cd^2+e(3Bd+Ae)\right)x^4+\frac{1}{5}\left(Acd\left(cd^2+6ae^2\right)+\right.$$

```
[Out] a^2*A*d^3*x+1/3*a*d*(a*d*(3*B*e+C*d)+A*(3*a*e^2+2*c*d^2))*x^3+1/4*a^2*e*(3*
C*d^2+e*(A*e+3*B*d))*x^4+1/5*(A*c*d*(6*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d)+2*
c*d^2*(3*B*e+C*d)))*x^5+1/6*a*e*(a*C*e^2+2*c*(3*C*d^2+e*(A*e+3*B*d)))*x^6+1
/7*c*(2*a*e^2*(B*e+3*C*d)+c*d*(C*d^2+3*e*(A*e+B*d)))*x^7+1/8*c*e*(2*a*C*e^2
+c*(3*C*d^2+e*(A*e+3*B*d)))*x^8+1/9*c^2*e^2*(B*e+3*C*d)*x^9+1/10*c^2*C*e^3*
x^10+1/6*d^2*(3*A*e+B*d)*(c*x^2+a)^3/c
```

Rubi [A]

time = 0.29, antiderivative size = 301, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1596, 1824}

$$\frac{1}{4}a^2ex^4\left(e(Ae+3Bd)+3Cd^2\right)+a^2Ad^3x+\frac{1}{7}cx^7\left(2ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3\right)+\frac{1}{8}cex^8\left(2aCe^2+ce(Ae+3Bd)+3cCd^2\right)+\frac{1}{6}aex^6\left(aCe^2+2ce(Ae+3Bd)+6cCd^2\right)+\frac{1}{5}x^5\left(Acd\left(6ae^2+cd^2\right)+a\left(ae^2(Be+3Cd)+2cd^2(3Be+Cd)\right)\right)+\frac{1}{3}adx^3\left(A\left(3ae^2+2cd^2\right)+ad(3Be+Cd)\right)+\frac{d^2\left(a+cx^2\right)^3(3Ae+Bd)}{6c}+\frac{1}{9}c^2e^2x^9(Be+3Cd)+\frac{1}{10}c^2Ce^3x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^3*x + (a*d*(a*d*(C*d + 3*B*e) + A*(2*c*d^2 + 3*a*e^2))*x^3)/3 + (a^
2*e*(3*C*d^2 + e*(3*B*d + A*e))*x^4)/4 + ((A*c*d*(c*d^2 + 6*a*e^2) + a*(a*e
^2*(3*C*d + B*e) + 2*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a*e*(6*c*C*d^2 + a*C*e
^2 + 2*c*e*(3*B*d + A*e))*x^6)/6 + (c*(c*C*d^3 + 3*c*d*e*(B*d + A*e) + 2*a*
e^2*(3*C*d + B*e))*x^7)/7 + (c*e*(3*c*C*d^2 + 2*a*C*e^2 + c*e*(3*B*d + A*e)
)*x^8)/8 + (c^2*e^2*(3*C*d + B*e)*x^9)/9 + (c^2*C*e^3*x^10)/10 + (d^2*(B*d
+ 3*A*e)*(a + c*x^2)^3)/(6*c)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right)\,dx = \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^3}{6c} + \int \left(a+cx^2\right)^2 \left(-\left(Bd^3+3Ad^2e\right)x\right.$$

$$= \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^3}{6c} + \int \left(a^2Ad^3 + ad\left(ad(Cd+3Be)+\right.\right.$$

$$= a^2Ad^3x + \frac{1}{3}ad\left(ad(Cd+3Be)+A\left(2cd^2+3ae^2\right)\right)x^3 + \frac{1}{4}a^2e(3$$

Mathematica [A]
time = 0.09, size = 335, normalized size = 1.10

$$a^2Ad^3x+\frac{1}{2}a^2d^2(Bd+3Ae)x^2+\frac{1}{3}ad\left(ad(Cd+3Be)+A\left(2cd^2+3ae^2\right)\right)x^3+\frac{1}{4}a\left(2Bcd^3+6Acd^2e+3aCd^2e+3aBde^2+aAe^3\right)x^4+\frac{1}{5}\left(Acd\left(cd^2+6ae^2\right)+a\left(ae^2(3Cd+Be)+2cd^2(Cd+3Be)\right)\right)x^5+\frac{1}{6}\left(aCe\left(6cd^2+ae^2\right)+Ace\left(3cd^2+2ae^2\right)+Bcd\left(cd^2+6ae^2\right)\right)x^6+\frac{1}{7}c\left(cCd^3+3cde(Bd+Ae)+2ae^2(3Cd+Be)\right)x^7+\frac{1}{8}ce\left(3cCd^2+2aCe^2+ce(3Bd+Ae)\right)x^8+\frac{1}{9}c^2e^2(3Cd+Be)x^9+\frac{1}{10}c^2Ce^3x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^3*x + (a^2*d^2*(B*d + 3*A*e)*x^2)/2 + (a*d*(a*d*(C*d + 3*B*e) + A*(
2*c*d^2 + 3*a*e^2))*x^3)/3 + (a*(2*B*c*d^3 + 6*A*c*d^2*e + 3*a*C*d^2*e + 3*
a*B*d*e^2 + a*A*e^3)*x^4)/4 + ((A*c*d*(c*d^2 + 6*a*e^2) + a*(a*e^2*(3*C*d +
 B*e) + 2*c*d^2*(C*d + 3*B*e)))*x^5)/5 + ((a*C*e*(6*c*d^2 + a*e^2) + A*c*e*
(3*c*d^2 + 2*a*e^2) + B*c*d*(c*d^2 + 6*a*e^2))*x^6)/6 + (c*(c*C*d^3 + 3*c*d
*e*(B*d + A*e) + 2*a*e^2*(3*C*d + B*e))*x^7)/7 + (c*e*(3*c*C*d^2 + 2*a*C*e^
2 + c*e*(3*B*d + A*e))*x^8)/8 + (c^2*e^2*(3*C*d + B*e)*x^9)/9 + (c^2*C*e^3*
x^10)/10
```

Maple [A]
time = 0.10, size = 385, normalized size = 1.27

method	result
norman	$\frac{c^2 C e^3 x^{10}}{10}+\left(\frac{1}{9}e^3c^2B+\frac{1}{3}c^2d\,e^2C\right)x^9+\left(\frac{1}{8}e^3c^2A+\frac{3}{8}c^2d\,e^2B+\frac{1}{4}Cac\,e^3+\frac{3}{8}C\,c^2d^2e\right)x^8+\left(\frac{3}{7}c^2d\,e^2A\right.$
default	$\frac{c^2 C e^3 x^{10}}{10}+\frac{(e^3c^2B+3c^2d\,e^2C)x^9}{9}+\frac{((2e^3ac+3d^2e\,c^2)C+3c^2d\,e^2B+e^3c^2A)x^8}{8}+\frac{((6d\,e^2ac+d^3c^2)C+(2e^3ac+3d^2e\,c^2)B+3c^2d}{7}$
gosper	$\frac{3}{2}x^2d^2e\,a^2A+\frac{3}{5}x^5C\,a^2d\,e^2+\frac{2}{5}x^5Cac\,d^3+\frac{3}{4}x^4B\,a^2d\,e^2+\frac{1}{2}x^4Bac\,d^3+\frac{3}{4}x^4d^2e\,a^2C+x^3A\,a^2d\,e^2+$
risch	$\frac{3}{2}x^2d^2e\,a^2A+\frac{3}{5}x^5C\,a^2d\,e^2+\frac{2}{5}x^5Cac\,d^3+\frac{3}{4}x^4B\,a^2d\,e^2+\frac{1}{2}x^4Bac\,d^3+\frac{3}{4}x^4d^2e\,a^2C+x^3A\,a^2d\,e^2+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*c^2*C*e^3*x^10+1/9*(B*c^2*e^3+3*C*c^2*d*e^2)*x^9+1/8*((2*a*c*e^3+3*c^2
*d^2*e)*C+3*c^2*d*e^2*B+e^3*c^2*A)*x^8+1/7*((6*a*c*d*e^2+c^2*d^3)*C+(2*a*c*
```

```
e^3+3*c^2*d^2*e)*B+3*c^2*d*e^2*A)*x^7+1/6*((a^2*e^3+6*a*c*d^2*e)*C+(6*a*c*d
*e^2+c^2*d^3)*B+(2*a*c*e^3+3*c^2*d^2*e)*A)*x^6+1/5*((3*a^2*d*e^2+2*a*c*d^3)
*C+(a^2*e^3+6*a*c*d^2*e)*B+(6*a*c*d*e^2+c^2*d^3)*A)*x^5+1/4*(3*d^2*e*a^2*C+
(3*a^2*d*e^2+2*a*c*d^3)*B+(a^2*e^3+6*a*c*d^2*e)*A)*x^4+1/3*(d^3*a^2*C+3*d^2
*e*a^2*B+(3*a^2*d*e^2+2*a*c*d^3)*A)*x^3+1/2*(3*A*a^2*d^2*e+B*a^2*d^3)*x^2+a
^2*A*d^3*x
```

Maxima [A]
time = 0.28, size = 361, normalized size = 1.19

$$\frac{1}{10}\,Cc^2x^{10}e^3+\frac{1}{9}\,(3\,Cc^2de^2+Bc^2e^3)x^9+\frac{1}{8}\,(3\,Cc^2d^2e+3\,Bc^2de^2+2\,Cace^3+Ac^2e^3)x^8+\frac{1}{7}\,(Cc^2d^3+3\,Bc^2d^2e+2\,Bace^3+3\,(2\,Cace^2+Ac^2e^2)d)x^7+Aa^2d^3x+\frac{1}{6}\,(Bc^2d^3+6\,Bacde^2+Ca^2e^3+2\,Aace^3+3\,(2\,Cace+Ac^2e)d^2)x^6+\frac{1}{5}\,(6\,Bacd^2e+(2\,Cac+Ac^2)d^3+Ba^2e^3+3\,(Ca^2e^2+2\,Aace^2)d)x^5+\frac{1}{4}\,(2\,Bacd^3+3\,Ba^2de^2+Aa^2e^3+3\,(Ca^2e+2\,Aace)d^2)x^4+\frac{1}{3}\,(3\,Ba^2d^2e+3\,Aa^2de^2+(Ca^2+2\,Aac)d^3)x^3+\frac{1}{2}\,(Ba^2d^3+3\,Aa^2d^2e)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*C*c^2*x^10*e^3 + 1/9*(3*C*c^2*d*e^2 + B*c^2*e^3)*x^9 + 1/8*(3*C*c^2*d^
2*e + 3*B*c^2*d*e^2 + 2*C*a*c*e^3 + A*c^2*e^3)*x^8 + 1/7*(C*c^2*d^3 + 3*B*c
^2*d^2*e + 2*B*a*c*e^3 + 3*(2*C*a*c*e^2 + A*c^2*e^2)*d)*x^7 + A*a^2*d^3*x +
 1/6*(B*c^2*d^3 + 6*B*a*c*d*e^2 + C*a^2*e^3 + 2*A*a*c*e^3 + 3*(2*C*a*c*e +
A*c^2*e)*d^2)*x^6 + 1/5*(6*B*a*c*d^2*e + (2*C*a*c + A*c^2)*d^3 + B*a^2*e^3
+ 3*(C*a^2*e^2 + 2*A*a*c*e^2)*d)*x^5 + 1/4*(2*B*a*c*d^3 + 3*B*a^2*d*e^2 + A
*a^2*e^3 + 3*(C*a^2*e + 2*A*a*c*e)*d^2)*x^4 + 1/3*(3*B*a^2*d^2*e + 3*A*a^2*
d*e^2 + (C*a^2 + 2*A*a*c)*d^3)*x^3 + 1/2*(B*a^2*d^3 + 3*A*a^2*d^2*e)*x^2
```

Fricas [A]
time = 0.35, size = 366, normalized size = 1.20

$$\frac{1}{7}\,Cc^2d^3x^7+\frac{1}{6}\,Bc^2d^3x^6+\frac{1}{2}\,Bacd^3x^4+\frac{1}{5}\,(2\,Cac+Ac^2)d^3x^5+\frac{1}{2}\,Ba^2d^3x^2+Aa^2d^3x+\frac{1}{3}\,(Ca^2+2\,Aac)d^3x^3+\frac{1}{2520}\,(252\,Cc^2x^{10}+280\,Bc^2x^9+720\,Bacx^7+315\,(2\,Cac+Ac^2)x^8+504\,Ba^2x^5+630\,Aa^2x^4+420\,(Ca^2+2\,Aac)x^6)e^3+\frac{1}{840}\,(280\,Cc^2dx^9+315\,Bc^2dx^8+840\,Bacdx^6+360\,(2\,Cac+Ac^2)dx^7+630\,Ba^2dx^4+840\,Aa^2dx^3+504\,(Ca^2+2\,Aac)dx^5)e^2+\frac{1}{280}\,(105\,Cc^2d^2x^8+120\,Bc^2d^2x^7+336\,Bacd^2x^5+140\,(2\,Cac+Ac^2)d^2x^6+280\,Ba^2d^2x^3+420\,Aa^2d^2x^2+210\,(Ca^2+2\,Aac)d^2x^4)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/7*C*c^2*d^3*x^7 + 1/6*B*c^2*d^3*x^6 + 1/2*B*a*c*d^3*x^4 + 1/5*(2*C*a*c +
A*c^2)*d^3*x^5 + 1/2*B*a^2*d^3*x^2 + A*a^2*d^3*x + 1/3*(C*a^2 + 2*A*a*c)*d^
3*x^3 + 1/2520*(252*C*c^2*x^10 + 280*B*c^2*x^9 + 720*B*a*c*x^7 + 315*(2*C*a
*c + A*c^2)*x^8 + 504*B*a^2*x^5 + 630*A*a^2*x^4 + 420*(C*a^2 + 2*A*a*c)*x^6
)*e^3 + 1/840*(280*C*c^2*d*x^9 + 315*B*c^2*d*x^8 + 840*B*a*c*d*x^6 + 360*(2
*C*a*c + A*c^2)*d*x^7 + 630*B*a^2*d*x^4 + 840*A*a^2*d*x^3 + 504*(C*a^2 + 2*
A*a*c)*d*x^5)*e^2 + 1/280*(105*C*c^2*d^2*x^8 + 120*B*c^2*d^2*x^7 + 336*B*a*
c*d^2*x^5 + 140*(2*C*a*c + A*c^2)*d^2*x^6 + 280*B*a^2*d^2*x^3 + 420*A*a^2*d
^2*x^2 + 210*(C*a^2 + 2*A*a*c)*d^2*x^4)*e
```

Sympy [A]
time = 0.03, size = 445, normalized size = 1.46

$$Aa^2d^3x+\frac{Cc^2e^3x^{10}}{10}+x^9\left(\frac{Bc^2e^3}{9}+\frac{Cc^2de^2}{3}\right)+x^8\left(\frac{Ac^2e^3}{8}+\frac{3Bc^2de^2}{8}+\frac{Cace^3}{4}+\frac{3Cc^2d^2e}{8}\right)+x^7\cdot\left(\frac{3Ac^2de^2}{7}+\frac{2Bace^3}{7}+\frac{3Bc^2d^2e}{7}+\frac{6Cacde^2}{7}+\frac{Cc^2d^3}{7}\right)+x^6\left(\frac{Aace^3}{3}+\frac{Ac^2d^2e}{2}+Bacde^2+\frac{Bc^2d^3}{6}+\frac{Ca^2e^3}{6}+Cacd^2e\right)+x^5\cdot\left(\frac{6Aacde^2}{5}+\frac{Ac^2d^3}{5}+\frac{Ba^2e^3}{5}+\frac{6Bacd^2e}{5}+\frac{3Ca^2de^2}{5}+\frac{2Cacd^3}{5}\right)+x^4\left(\frac{Aa^2e^3}{4}+\frac{3Aacd^2e}{2}+\frac{3Ba^2de^2}{4}+\frac{Bacd^3}{2}+\frac{3Ca^2d^2e}{4}\right)+x^3\left(Aa^2de^2+\frac{2Aacd^3}{3}+Ba^2d^2e+\frac{Ca^2d^3}{3}\right)+x^2\cdot\left(\frac{3Aa^2d^2e}{2}+\frac{Ba^2d^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*d**3*x + C*c**2*e**3*x**10/10 + x**9*(B*c**2*e**3/9 + C*c**2*d*e**2/
3) + x**8*(A*c**2*e**3/8 + 3*B*c**2*d*e**2/8 + C*a*c*e**3/4 + 3*C*c**2*d**2
*e/8) + x**7*(3*A*c**2*d*e**2/7 + 2*B*a*c*e**3/7 + 3*B*c**2*d**2*e/7 + 6*C*
a*c*d*e**2/7 + C*c**2*d**3/7) + x**6*(A*a*c*e**3/3 + A*c**2*d**2*e/2 + B*a*
c*d*e**2 + B*c**2*d**3/6 + C*a**2*e**3/6 + C*a*c*d**2*e) + x**5*(6*A*a*c*d*
e**2/5 + A*c**2*d**3/5 + B*a**2*e**3/5 + 6*B*a*c*d**2*e/5 + 3*C*a**2*d*e**2
/5 + 2*C*a*c*d**3/5) + x**4*(A*a**2*e**3/4 + 3*A*a*c*d**2*e/2 + 3*B*a**2*d*
e**2/4 + B*a*c*d**3/2 + 3*C*a**2*d**2*e/4) + x**3*(A*a**2*d*e**2 + 2*A*a*c*
d**3/3 + B*a**2*d**2*e + C*a**2*d**3/3) + x**2*(3*A*a**2*d**2*e/2 + B*a**2*
d**3/2)
```

Giac [A]
time = 4.22, size = 423, normalized size = 1.39

$$\frac{1}{10}Cc^2x^{10}e^3+\frac{1}{3}Cc^2dx^9e^2+\frac{3}{8}Cc^2d^2x^8e+\frac{1}{7}Cc^2d^3x^7+\frac{1}{9}Bc^2x^9e^3+\frac{3}{8}Bc^2dx^8e^2+\frac{3}{7}Bc^2d^2x^7e+\frac{1}{6}Bc^2d^3x^6+\frac{1}{4}Cacx^8e^3+\frac{1}{8}Ac^2x^8e^3+\frac{6}{7}Cacdx^7e^2+\frac{3}{7}Ac^2dx^7e^2+Cacd^2x^6e+\frac{1}{2}Ac^2d^2x^6e+\frac{2}{5}Cacd^3x^5+\frac{1}{5}Ac^2d^3x^5+\frac{2}{7}Bacx^7e^3+Bacdx^6e^2+\frac{6}{5}Bacd^2x^5e+\frac{1}{2}Bacd^3x^4+\frac{1}{6}Ca^2x^6e^3+\frac{1}{3}Aacx^6e^3+\frac{3}{5}Ca^2dx^5e^2+\frac{6}{5}Aacdx^5e^2+\frac{3}{4}Ca^2d^2x^4e+\frac{3}{2}Aacd^2x^4e+\frac{1}{3}Ca^2d^3x^3+\frac{2}{3}Aacd^3x^3+\frac{1}{5}Ba^2x^5e^3+\frac{3}{4}Ba^2dx^4e^2+Ba^2d^2x^3e+\frac{1}{2}Ba^2d^3x^2+\frac{1}{4}Aa^2x^4e^3+Aa^2dx^3e^2+\frac{3}{2}Aa^2d^2x^2e+Aa^2d^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*C*c^2*x^10*e^3 + 1/3*C*c^2*d*x^9*e^2 + 3/8*C*c^2*d^2*x^8*e + 1/7*C*c^2
*d^3*x^7 + 1/9*B*c^2*x^9*e^3 + 3/8*B*c^2*d*x^8*e^2 + 3/7*B*c^2*d^2*x^7*e +
1/6*B*c^2*d^3*x^6 + 1/4*C*a*c*x^8*e^3 + 1/8*A*c^2*x^8*e^3 + 6/7*C*a*c*d*x^7
*e^2 + 3/7*A*c^2*d*x^7*e^2 + C*a*c*d^2*x^6*e + 1/2*A*c^2*d^2*x^6*e + 2/5*C*
a*c*d^3*x^5 + 1/5*A*c^2*d^3*x^5 + 2/7*B*a*c*x^7*e^3 + B*a*c*d*x^6*e^2 + 6/5
*B*a*c*d^2*x^5*e + 1/2*B*a*c*d^3*x^4 + 1/6*C*a^2*x^6*e^3 + 1/3*A*a*c*x^6*e^
3 + 3/5*C*a^2*d*x^5*e^2 + 6/5*A*a*c*d*x^5*e^2 + 3/4*C*a^2*d^2*x^4*e + 3/2*A
*a*c*d^2*x^4*e + 1/3*C*a^2*d^3*x^3 + 2/3*A*a*c*d^3*x^3 + 1/5*B*a^2*x^5*e^3
+ 3/4*B*a^2*d*x^4*e^2 + B*a^2*d^2*x^3*e + 1/2*B*a^2*d^3*x^2 + 1/4*A*a^2*x^4
*e^3 + A*a^2*d*x^3*e^2 + 3/2*A*a^2*d^2*x^2*e + A*a^2*d^3*x
```

Mupad [B]
time = 0.14, size = 332, normalized size = 1.09

$$x^5\left(\frac{3Ca^2de^2}{5}+\frac{Ba^2e^3}{5}+\frac{2Cacd^3}{5}+\frac{6Bacd^2e}{5}+\frac{6Aacde^2}{5}+\frac{Ac^2d^3}{5}\right)+x^6\left(\frac{Ca^2e^3}{6}+Cacd^2e+Bacde^2+\frac{Aace^3}{3}+\frac{Bc^2d^3}{6}+\frac{Ac^2d^2e}{2}\right)+\frac{ax^4\left(Aae^3+2Bcd^3+3Bade^2+6Acd^2e+3Cad^2e\right)}{4}+\frac{cx^7\left(2Bae^3+Ccd^3+3Acde^2+6Cade^2+3Bcd^2e\right)}{7}+\frac{Cc^2e^3x^{10}}{10}+\frac{a^2d^2x^2\left(3Ae+Bd\right)}{2}+\frac{c^2e^2x^9\left(Be+3Cd\right)}{9}+\frac{adx^3\left(3Aae^2+2Acd^2+Cad^2+3Bade\right)}{3}+\frac{cex^8\left(Ace^2+2Cae^2+3Ccd^2+3Bcde\right)}{8}+Aa^2d^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^2*(d + e*x)^3*(A + B*x + C*x^2),x)
```

```
[Out] x^5*((A*c^2*d^3)/5 + (B*a^2*e^3)/5 + (2*C*a*c*d^3)/5 + (3*C*a^2*d*e^2)/5 +
(6*A*a*c*d*e^2)/5 + (6*B*a*c*d^2*e)/5) + x^6*((B*c^2*d^3)/6 + (C*a^2*e^3)/6
 + (A*a*c*e^3)/3 + (A*c^2*d^2*e)/2 + B*a*c*d*e^2 + C*a*c*d^2*e) + (a*x^4*(A
*a*e^3 + 2*B*c*d^3 + 3*B*a*d*e^2 + 6*A*c*d^2*e + 3*C*a*d^2*e))/4 + (c*x^7*(
```

```
2*B*a*e^3 + C*c*d^3 + 3*A*c*d*e^2 + 6*C*a*d*e^2 + 3*B*c*d^2*e))/7 + (C*c^2*
e^3*x^10)/10 + (a^2*d^2*x^2*(3*A*e + B*d))/2 + (c^2*e^2*x^9*(B*e + 3*C*d))/
9 + (a*d*x^3*(3*A*a*e^2 + 2*A*c*d^2 + C*a*d^2 + 3*B*a*d*e))/3 + (c*e*x^8*(A
*c*e^2 + 2*C*a*e^2 + 3*C*c*d^2 + 3*B*c*d*e))/8 + A*a^2*d^3*x
```

3.26 $\int (d+ex)^2 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx$

Optimal. Leaf size=217

$$a^2Ad^2x+\frac{1}{3}a\left(ad(Cd+2Be)+A\left(2cd^2+ae^2\right)\right)x^3+\frac{1}{4}a^2e(2Cd+Be)x^4+\frac{1}{5}\left(Ac\left(cd^2+2ae^2\right)+a\left(aCe^2+2cd(C\right.\right.$$

```
[Out] a^2*A*d^2*x+1/3*a*(a*d*(2*B*e+C*d)+A*(a*e^2+2*c*d^2))*x^3+1/4*a^2*e*(B*e+2*
C*d)*x^4+1/5*(A*c*(2*a*e^2+c*d^2)+a*(a*C*e^2+2*c*d*(2*B*e+C*d)))*x^5+1/3*a*
c*e*(B*e+2*C*d)*x^6+1/7*c*(2*a*C*e^2+c*(C*d^2+e*(A*e+2*B*d)))*x^7+1/8*c^2*e
*(B*e+2*C*d)*x^8+1/9*c^2*C*e^2*x^9+1/6*d*(2*A*e+B*d)*(c*x^2+a)^3/c
```

Rubi [A]
time = 0.19, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1596, 1824}

$$a^2Ad^2x+\frac{1}{4}a^2ex^4(Be+2Cd)+\frac{1}{7}cx^7\left(2aCe^2+ce(Ae+2Bd)+cCd^2\right)+\frac{1}{5}x^5\left(Ac\left(2ae^2+cd^2\right)+a\left(aCe^2+2cd(2Be+Cd)\right)\right)+\frac{1}{3}ax^3\left(A\left(ae^2+2cd^2\right)+ad(2Be+Cd)\right)+\frac{d\left(a+cx^2\right)^3(2Ae+Bd)}{6c}+\frac{1}{3}acex^6(Be+2Cd)+\frac{1}{8}c^2ex^8(Be+2Cd)+\frac{1}{9}c^2Ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^2*x + (a*(a*d*(C*d + 2*B*e) + A*(2*c*d^2 + a*e^2))*x^3)/3 + (a^2*e*
(2*C*d + B*e)*x^4)/4 + ((A*c*(c*d^2 + 2*a*e^2) + a*(a*C*e^2 + 2*c*d*(C*d +
2*B*e)))*x^5)/5 + (a*c*e*(2*C*d + B*e)*x^6)/3 + (c*(c*C*d^2 + 2*a*C*e^2 + c
*e*(2*B*d + A*e))*x^7)/7 + (c^2*e*(2*C*d + B*e)*x^8)/8 + (c^2*C*e^2*x^9)/9
+ (d*(B*d + 2*A*e)*(a + c*x^2)^3)/(6*c)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^2 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx = \frac{d(Bd+2Ae)\left(a+cx^2\right)^3}{6c} + \int \left(a+cx^2\right)^2 \left(-\left(Bd^2+2Ade\right) x\right.$$

$$= \frac{d(Bd+2Ae)\left(a+cx^2\right)^3}{6c} + \int \left(a^2Ad^2 + a\big(ad(Cd+2Be) + \right.$$

$$= a^2Ad^2x + \frac{1}{3}a\big(ad(Cd+2Be) + A\big(2cd^2+ae^2\big)\big)\, x^3 + \frac{1}{4}a^2e(2C$$

Mathematica [A]
time = 0.06, size = 241, normalized size = 1.11

$$a^2Ad^2x + \frac{1}{2}a^2d(Bd+2Ae)x^2 + \frac{1}{3}a\left(ad(Cd+2Be) + A\left(2cd^2+ae^2\right)\right)x^3 + \frac{1}{4}a\left(2Bcd^2+4Acde+2aCde+aBe^2\right)x^4 + \frac{1}{5}\left(Ac\left(cd^2+2ae^2\right) + a\left(aCe^2+2cd(Cd+2Be)\right)\right)x^5 + \frac{1}{6}c\left(Bcd^2+2Acde+4aCde+2aBe^2\right)x^6 + \frac{1}{7}c\left(cCd^2+2aCe^2+ce(2Bd+Ae)\right)x^7 + \frac{1}{8}c^2e(2Cd+Be)x^8 + \frac{1}{9}c^2Ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^2*x + (a^2*d*(B*d + 2*A*e)*x^2)/2 + (a*(a*d*(C*d + 2*B*e) + A*(2*c*
d^2 + a*e^2))*x^3)/3 + (a*(2*B*c*d^2 + 4*A*c*d*e + 2*a*C*d*e + a*B*e^2)*x^4
)/4 + ((A*c*(c*d^2 + 2*a*e^2) + a*(a*C*e^2 + 2*c*d*(C*d + 2*B*e)))*x^5)/5 +
 (c*(B*c*d^2 + 2*A*c*d*e + 4*a*C*d*e + 2*a*B*e^2)*x^6)/6 + (c*(c*C*d^2 + 2*
a*C*e^2 + c*e*(2*B*d + A*e))*x^7)/7 + (c^2*e*(2*C*d + B*e)*x^8)/8 + (c^2*C*
e^2*x^9)/9
```

Maple [A]
time = 0.11, size = 268, normalized size = 1.24

method	result
norman	$\frac{c^2Ce^2x^9}{9} + \left(\frac{1}{8}c^2e^2B + \frac{1}{4}c^2deC\right)x^8 + \left(\frac{1}{7}c^2e^2A + \frac{2}{7}c^2deB + \frac{2}{7}Cace^2 + \frac{1}{7}Cc^2d^2\right)x^7 + \left(\frac{1}{3}c^2deA + \frac{1}{3}\right.$
default	$\frac{c^2Ce^2x^9}{9} + \frac{(c^2e^2B+2c^2deC)x^8}{8} + \frac{((2ace^2+c^2d^2)C+2c^2deB+c^2e^2A)x^7}{7} + \frac{(4acdeC+(2ace^2+c^2d^2)B+2c^2deA)x^6}{6} + \frac{((e^2a}{}$
gosper	$\frac{2}{3}x^6acdeC + a^2Ad^2x + \frac{2}{7}x^7Cace^2 + \frac{1}{3}x^6c^2deA + \frac{1}{3}x^6Bace^2 + \frac{2}{5}x^5Aace^2 + \frac{2}{5}x^5Cacd^2 + \frac{1}{2}x^4Ba$
risch	$\frac{2}{3}x^6acdeC + a^2Ad^2x + \frac{2}{7}x^7Cace^2 + \frac{1}{3}x^6c^2deA + \frac{1}{3}x^6Bace^2 + \frac{2}{5}x^5Aace^2 + \frac{2}{5}x^5Cacd^2 + \frac{1}{2}x^4Ba$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*c^2*C*e^2*x^9+1/8*(B*c^2*e^2+2*C*c^2*d*e)*x^8+1/7*((2*a*c*e^2+c^2*d^2)*
C+2*c^2*d*e*B+c^2*e^2*A)*x^7+1/6*(4*a*c*d*e*C+(2*a*c*e^2+c^2*d^2)*B+2*c^2*d
*e*A)*x^6+1/5*((a^2*e^2+2*a*c*d^2)*C+4*a*c*d*e*B+(2*a*c*e^2+c^2*d^2)*A)*x^5
+1/4*(2*d*e*a^2*C+(a^2*e^2+2*a*c*d^2)*B+4*a*c*d*e*A)*x^4+1/3*(a^2*d^2*C+2*d
*e*a^2*B+(a^2*e^2+2*a*c*d^2)*A)*x^3+1/2*(2*A*a^2*d*e+B*a^2*d^2)*x^2+a^2*A*d
^2*x
```

Maxima [A]
time = 0.29, size = 261, normalized size = 1.20

$$\frac{1}{9}Cc^2x^9e^2+\frac{1}{8}\left(2Cc^2de+Bc^2e^2\right)x^8+\frac{1}{7}\left(Cc^2d^2+2Bc^2de+2Cace^2+Ac^2e^2\right)x^7+\frac{1}{6}\left(Bc^2d^2+2Bace^2+2\left(2Cace+Ac^2e\right)d\right)x^6+Aa^2d^2x+\frac{1}{5}\left(4Bacde+Ca^2e^2+2Aace^2+\left(2Cac+Ac^2\right)d^2\right)x^5+\frac{1}{4}\left(2Bacd^2+Ba^2e^2+2\left(Ca^2e+2Aace\right)d\right)x^4+\frac{1}{3}\left(2Ba^2de+Aa^2e^2+\left(Ca^2+2Aac\right)d^2\right)x^3+\frac{1}{2}\left(Ba^2d^2+2Aa^2de\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*C*c^2*x^9*e^2 + 1/8*(2*C*c^2*d*e + B*c^2*e^2)*x^8 + 1/7*(C*c^2*d^2 + 2*
B*c^2*d*e + 2*C*a*c*e^2 + A*c^2*e^2)*x^7 + 1/6*(B*c^2*d^2 + 2*B*a*c*e^2 + 2
*(2*C*a*c*e + A*c^2*e)*d)*x^6 + A*a^2*d^2*x + 1/5*(4*B*a*c*d*e + C*a^2*e^2
+ 2*A*a*c*e^2 + (2*C*a*c + A*c^2)*d^2)*x^5 + 1/4*(2*B*a*c*d^2 + B*a^2*e^2 +
 2*(C*a^2*e + 2*A*a*c*e)*d)*x^4 + 1/3*(2*B*a^2*d*e + A*a^2*e^2 + (C*a^2 + 2
*A*a*c)*d^2)*x^3 + 1/2*(B*a^2*d^2 + 2*A*a^2*d*e)*x^2
```

Fricas [A]
time = 0.36, size = 264, normalized size = 1.22

$$\frac{1}{7}Cc^2d^2x^7+\frac{1}{6}Bc^2d^2x^6+\frac{1}{2}Bacd^2x^4+\frac{1}{5}\left(2Cac+Ac^2\right)d^2x^5+\frac{1}{2}Ba^2d^2x^2+Aa^2d^2x+\frac{1}{3}\left(Ca^2+2Aac\right)d^2x^3+\frac{1}{2520}\left(280Cc^2x^9+315Bc^2x^8+840Bacx^6+360\left(2Cac+Ac^2\right)x^7+630Ba^2x^4+840Aa^2x^3+504\left(Ca^2+2Aac\right)x^5\right)e^2+\frac{1}{420}\left(105Cc^2dx^8+120Bc^2dx^7+336Bacdx^5+140\left(2Cac+Ac^2\right)dx^6+280Ba^2dx^3+420Aa^2dx^2+210\left(Ca^2+2Aac\right)dx^4\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/7*C*c^2*d^2*x^7 + 1/6*B*c^2*d^2*x^6 + 1/2*B*a*c*d^2*x^4 + 1/5*(2*C*a*c +
A*c^2)*d^2*x^5 + 1/2*B*a^2*d^2*x^2 + A*a^2*d^2*x + 1/3*(C*a^2 + 2*A*a*c)*d^
2*x^3 + 1/2520*(280*C*c^2*x^9 + 315*B*c^2*x^8 + 840*B*a*c*x^6 + 360*(2*C*a*
c + A*c^2)*x^7 + 630*B*a^2*x^4 + 840*A*a^2*x^3 + 504*(C*a^2 + 2*A*a*c)*x^5)
*e^2 + 1/420*(105*C*c^2*d*x^8 + 120*B*c^2*d*x^7 + 336*B*a*c*d*x^5 + 140*(2*
C*a*c + A*c^2)*d*x^6 + 280*B*a^2*d*x^3 + 420*A*a^2*d*x^2 + 210*(C*a^2 + 2*A
*a*c)*d*x^4)*e
```

Sympy [A]
time = 0.03, size = 311, normalized size = 1.43

$$Aa^2d^2x+\frac{Cc^2e^2x^9}{9}+x^8\left(\frac{Bc^2e^2}{8}+\frac{Cc^2de}{4}\right)+x^7\left(\frac{Ac^2e^2}{7}+\frac{2Bc^2de}{7}+\frac{2Cace^2}{7}+\frac{Cc^2d^2}{7}\right)+x^6\left(\frac{Ac^2de}{3}+\frac{Bace^2}{3}+\frac{Bc^2d^2}{6}+\frac{2Cacde}{3}\right)+x^5\cdot\left(\frac{2Aace^2}{5}+\frac{Ac^2d^2}{5}+\frac{4Bacde}{5}+\frac{Ca^2e^2}{5}+\frac{2Cacd^2}{5}\right)+x^4\left(Aacde+\frac{Ba^2e^2}{4}+\frac{Bacd^2}{2}+\frac{Ca^2de}{2}\right)+x^3\left(\frac{Aa^2e^2}{3}+\frac{2Aacd^2}{3}+\frac{2Ba^2de}{3}+\frac{Ca^2d^2}{3}\right)+x^2\left(Aa^2de+\frac{Ba^2d^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*d**2*x + C*c**2*e**2*x**9/9 + x**8*(B*c**2*e**2/8 + C*c**2*d*e/4) +
x**7*(A*c**2*e**2/7 + 2*B*c**2*d*e/7 + 2*C*a*c*e**2/7 + C*c**2*d**2/7) + x*
*6*(A*c**2*d*e/3 + B*a*c*e**2/3 + B*c**2*d**2/6 + 2*C*a*c*d*e/3) + x**5*(2*
A*a*c*e**2/5 + A*c**2*d**2/5 + 4*B*a*c*d*e/5 + C*a**2*e**2/5 + 2*C*a*c*d**2
/5) + x**4*(A*a*c*d*e + B*a**2*e**2/4 + B*a*c*d**2/2 + C*a**2*d*e/2) + x**3
*(A*a**2*e**2/3 + 2*A*a*c*d**2/3 + 2*B*a**2*d*e/3 + C*a**2*d**2/3) + x**2*(
A*a**2*d*e + B*a**2*d**2/2)
```

Giac [A]
time = 4.27, size = 302, normalized size = 1.39

$$\frac{1}{9} Cc^2x^9e^2 + \frac{1}{4} Cc^2dx^8e + \frac{1}{7} Cc^2d^2x^7 + \frac{1}{8} Bc^2x^8e^2 + \frac{2}{7} Bc^2dx^7e + \frac{1}{6} Bc^2d^2x^6 + \frac{2}{7} Cacx^7e^2 + \frac{1}{7} Ac^2x^7e^2 + \frac{2}{3} Cacdx^6e + \frac{1}{3} Ac^2dx^6e + \frac{2}{5} Cacd^2x^5 + \frac{1}{5} Ac^2d^2x^5 + \frac{1}{3} Bacx^6e^2 + \frac{4}{5} Bacdx^5e + \frac{1}{2} Bacd^2x^4 + \frac{1}{5} Ca^2x^5e^2 + \frac{2}{5} Aacx^5e^2 + \frac{1}{2} Ca^2dx^4e + Aacdx^4e + \frac{1}{3} Ca^2d^2x^3 + \frac{2}{3} Aacd^2x^3 + \frac{1}{4} Ba^2x^4e^2 + \frac{2}{3} Ba^2dx^3e + \frac{1}{2} Ba^2d^2x^2 + \frac{1}{3} Aa^2x^3e^2 + Aa^2dx^2e + Aa^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*C*c^2*x^9*e^2 + 1/4*C*c^2*d*x^8*e + 1/7*C*c^2*d^2*x^7 + 1/8*B*c^2*x^8*e
^2 + 2/7*B*c^2*d*x^7*e + 1/6*B*c^2*d^2*x^6 + 2/7*C*a*c*x^7*e^2 + 1/7*A*c^2*
x^7*e^2 + 2/3*C*a*c*d*x^6*e + 1/3*A*c^2*d*x^6*e + 2/5*C*a*c*d^2*x^5 + 1/5*A
*c^2*d^2*x^5 + 1/3*B*a*c*x^6*e^2 + 4/5*B*a*c*d*x^5*e + 1/2*B*a*c*d^2*x^4 +
1/5*C*a^2*x^5*e^2 + 2/5*A*a*c*x^5*e^2 + 1/2*C*a^2*d*x^4*e + A*a*c*d*x^4*e +
 1/3*C*a^2*d^2*x^3 + 2/3*A*a*c*d^2*x^3 + 1/4*B*a^2*x^4*e^2 + 2/3*B*a^2*d*x^
3*e + 1/2*B*a^2*d^2*x^2 + 1/3*A*a^2*x^3*e^2 + A*a^2*d*x^2*e + A*a^2*d^2*x
```

Mupad [B]
time = 3.72, size = 244, normalized size = 1.12

$$x^3\left(\frac{C\,a^2\,d^2}{3}+\frac{2\,B\,a^2\,d\,e}{3}+\frac{A\,a^2\,e^2}{3}+\frac{2\,A\,c\,a\,d^2}{3}\right)+x^7\left(\frac{C\,c^2\,d^2}{7}+\frac{2\,B\,c^2\,d\,e}{7}+\frac{A\,c^2\,e^2}{7}+\frac{2\,C\,a\,c\,e^2}{7}\right)+x^5\left(\frac{C\,a^2\,e^2}{5}+\frac{2\,C\,a\,c\,d^2}{5}+\frac{4\,B\,a\,c\,d\,e}{5}+\frac{2\,A\,a\,c\,e^2}{5}+\frac{A\,c^2\,d^2}{5}\right)+\frac{a\,x^4\,\left(B\,a\,e^2+2\,B\,c\,d^2+4\,A\,c\,d\,e+2\,C\,a\,d\,e\right)}{4}+\frac{c\,x^6\,\left(2\,B\,a\,e^2+B\,c\,d^2+2\,A\,c\,d\,e+4\,C\,a\,d\,e\right)}{6}+\frac{C\,c^2\,e^2\,x^9}{9}+A\,a^2\,d^2\,x+\frac{a^2\,d\,x^2\,\left(2\,A\,e+B\,d\right)}{2}+\frac{c^2\,e\,x^8\,\left(B\,e+2\,C\,d\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^2*(d + e*x)^2*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*a^2*e^2)/3 + (C*a^2*d^2)/3 + (2*A*a*c*d^2)/3 + (2*B*a^2*d*e)/3) + x
^7*((A*c^2*e^2)/7 + (C*c^2*d^2)/7 + (2*C*a*c*e^2)/7 + (2*B*c^2*d*e)/7) + x^
5*((A*c^2*d^2)/5 + (C*a^2*e^2)/5 + (2*A*a*c*e^2)/5 + (2*C*a*c*d^2)/5 + (4*B
*a*c*d*e)/5) + (a*x^4*(B*a*e^2 + 2*B*c*d^2 + 4*A*c*d*e + 2*C*a*d*e))/4 + (c
*x^6*(2*B*a*e^2 + B*c*d^2 + 2*A*c*d*e + 4*C*a*d*e))/6 + (C*c^2*e^2*x^9)/9 +
 A*a^2*d^2*x + (a^2*d*x^2*(2*A*e + B*d))/2 + (c^2*e*x^8*(B*e + 2*C*d))/8
```

3.27 $\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=128

$$a^2Adx+\frac{1}{3}a(2Acd+aCd+aBe)x^3+\frac{1}{4}a^2Cex^4+\frac{1}{5}c(Acd+2a(Cd+Be))x^5+\frac{1}{3}acCex^6+\frac{1}{7}c^2(Cd+Be)x^7+\frac{1}{8}c^2Cex$$

```
[Out] a^2*A*d*x+1/3*a*(2*A*c*d+B*a*e+C*a*d)*x^3+1/4*a^2*C*e*x^4+1/5*c*(A*c*d+2*a*
(B*e+C*d))*x^5+1/3*a*c*C*e*x^6+1/7*c^2*(B*e+C*d)*x^7+1/8*c^2*C*e*x^8+1/6*(A
*e+B*d)*(c*x^2+a)^3/c
```

Rubi [A]
time = 0.10, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1596, 1824}

$$a^2Adx+\frac{1}{4}a^2Cex^4+\frac{1}{5}cx^5(2a(Be+Cd)+Acd)+\frac{1}{3}ax^3(aBe+aCd+2Acd)+\frac{\left(a+cx^2\right)^3(Ae+Bd)}{6c}+\frac{1}{3}acCex^6+\frac{1}{7}c^2x^7(Be+Cd)+\frac{1}{8}c^2Cex^8$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d*x + (a*(2*A*c*d + a*C*d + a*B*e)*x^3)/3 + (a^2*C*e*x^4)/4 + (c*(A*c
*d + 2*a*(C*d + B*e))*x^5)/5 + (a*c*C*e*x^6)/3 + (c^2*(C*d + B*e)*x^7)/7 +
(c^2*C*e*x^8)/8 + ((B*d + A*e)*(a + c*x^2)^3)/(6*c)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx = \frac{(Bd+Ae)\left(a+cx^2\right)^3}{6c} + \int \left(a+cx^2\right)^2\left(-(Bd+Ae)x+(d+\right.$$

$$= \frac{(Bd+Ae)\left(a+cx^2\right)^3}{6c} + \int \left(a^2Ad+a(2Acd+aCd+aBe)x^2\right.$$

$$= a^2Adx+\frac{1}{3}a(2Acd+aCd+aBe)x^3+\frac{1}{4}a^2Cex^4+\frac{1}{5}c(Acd+$$

Mathematica [A]
time = 0.04, size = 144, normalized size = 1.12

$$a^2Adx+\frac{1}{2}a^2(Bd+Ae)x^2+\frac{1}{3}a(2Acd+aCd+aBe)x^3+\frac{1}{4}a(2Bcd+2Ace+aCe)x^4+\frac{1}{5}c(Acd+2aCd+2aBe)x^5+\frac{1}{6}c(Bcd+Ace+2aCe)x^6+\frac{1}{7}c^2(Cd+Be)x^7+\frac{1}{8}c^2Cex^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d*x + (a^2*(B*d + A*e)*x^2)/2 + (a*(2*A*c*d + a*C*d + a*B*e)*x^3)/3 +
 (a*(2*B*c*d + 2*A*c*e + a*C*e)*x^4)/4 + (c*(A*c*d + 2*a*C*d + 2*a*B*e)*x^5
)/5 + (c*(B*c*d + A*c*e + 2*a*C*e)*x^6)/6 + (c^2*(C*d + B*e)*x^7)/7 + (c^2*
C*e*x^8)/8
```

Maple [A]
time = 0.09, size = 151, normalized size = 1.18

method	result
default	$\frac{c^2Ce\,x^8}{8}+\frac{(c^2eB+c^2dC)x^7}{7}+\frac{(c^2eA+c^2dB+2aceC)x^6}{6}+\frac{(A\,c^2d+2Bace+2acdC)x^5}{5}+\frac{(2aceA+2acdB+e\,a^2C)x^4}{4}+\frac{(2ac$
norman	$\frac{c^2Ce\,x^8}{8}+\left(\frac{1}{7}c^2eB+\frac{1}{7}c^2dC\right)x^7+\left(\frac{1}{6}c^2eA+\frac{1}{6}c^2dB+\frac{1}{3}aceC\right)x^6+\left(\frac{1}{5}A\,c^2d+\frac{2}{5}Bace+\frac{2}{5}acdC\right)x^5$
gosper	$\frac{1}{8}c^2Ce\,x^8+\frac{1}{7}x^7c^2eB+\frac{1}{7}x^7c^2dC+\frac{1}{6}x^6c^2eA+\frac{1}{6}x^6c^2dB+\frac{1}{3}acCe\,x^6+\frac{1}{5}x^5A\,c^2d+\frac{2}{5}x^5Bace+\frac{2}{5}x$
risch	$\frac{1}{8}c^2Ce\,x^8+\frac{1}{7}x^7c^2eB+\frac{1}{7}x^7c^2dC+\frac{1}{6}x^6c^2eA+\frac{1}{6}x^6c^2dB+\frac{1}{3}acCe\,x^6+\frac{1}{5}x^5A\,c^2d+\frac{2}{5}x^5Bace+\frac{2}{5}x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c^2*C*e*x^8+1/7*(B*c^2*e+C*c^2*d)*x^7+1/6*(A*c^2*e+B*c^2*d+2*C*a*c*e)*x
^6+1/5*(A*c^2*d+2*B*a*c*e+2*C*a*c*d)*x^5+1/4*(2*A*a*c*e+2*B*a*c*d+C*a^2*e)*
x^4+1/3*(2*A*a*c*d+B*a^2*e+C*a^2*d)*x^3+1/2*(A*a^2*e+B*a^2*d)*x^2+a^2*A*d*x
```

Maxima [A]
time = 0.27, size = 161, normalized size = 1.26

$$\frac{1}{8}Cc^2x^8e+\frac{1}{7}\left(Cc^2d+Bc^2e\right)x^7+\frac{1}{6}\left(Bc^2d+2\,Cace+Ac^2e\right)x^6+\frac{1}{5}\left(2\,Bace+\left(2\,Cac+Ac^2\right)d\right)x^5+Aa^2dx+\frac{1}{4}\left(2\,Bacd+Ca^2e+2\,Aace\right)x^4+\frac{1}{3}\left(Ba^2e+\left(Ca^2+2\,Aac\right)d\right)x^3+\frac{1}{2}\left(Ba^2d+Aa^2e\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*C*c^2*x^8*e + 1/7*(C*c^2*d + B*c^2*e)*x^7 + 1/6*(B*c^2*d + 2*C*a*c*e +
A*c^2*e)*x^6 + 1/5*(2*B*a*c*e + (2*C*a*c + A*c^2)*d)*x^5 + A*a^2*d*x + 1/4*
(2*B*a*c*d + C*a^2*e + 2*A*a*c*e)*x^4 + 1/3*(B*a^2*e + (C*a^2 + 2*A*a*c)*d)
*x^3 + 1/2*(B*a^2*d + A*a^2*e)*x^2
```

Fricas [A]
time = 0.36, size = 162, normalized size = 1.27

$$\frac{1}{7}\,Cc^2dx^7+\frac{1}{6}\,Bc^2dx^6+\frac{1}{2}\,Bacdx^4+\frac{1}{5}\,(2\,Cac+Ac^2)dx^5+\frac{1}{2}\,Ba^2dx^2+Aa^2dx+\frac{1}{3}\,(Ca^2+2\,Aac)dx^3+\frac{1}{840}\,(105\,Cc^2x^8+120\,Bc^2x^7+336\,Bacx^5+140\,(2\,Cac+Ac^2)x^6+280\,Ba^2x^3+420\,Aa^2x^2+210\,(Ca^2+2\,Aac)x^4)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/7*C*c^2*d*x^7 + 1/6*B*c^2*d*x^6 + 1/2*B*a*c*d*x^4 + 1/5*(2*C*a*c + A*c^2)
*d*x^5 + 1/2*B*a^2*d*x^2 + A*a^2*d*x + 1/3*(C*a^2 + 2*A*a*c)*d*x^3 + 1/840*
(105*C*c^2*x^8 + 120*B*c^2*x^7 + 336*B*a*c*x^5 + 140*(2*C*a*c + A*c^2)*x^6
+ 280*B*a^2*x^3 + 420*A*a^2*x^2 + 210*(C*a^2 + 2*A*a*c)*x^4)*e
```

Sympy [A]
time = 0.02, size = 180, normalized size = 1.41

$$Aa^2dx+\frac{Cc^2ex^8}{8}+x^7\left(\frac{Bc^2e}{7}+\frac{Cc^2d}{7}\right)+x^6\left(\frac{Ac^2e}{6}+\frac{Bc^2d}{6}+\frac{Cace}{3}\right)+x^5\left(\frac{Ac^2d}{5}+\frac{2Bace}{5}+\frac{2Cacd}{5}\right)+x^4\left(\frac{Aace}{2}+\frac{Bacd}{2}+\frac{Ca^2e}{4}\right)+x^3\cdot\left(\frac{2Aacd}{3}+\frac{Ba^2e}{3}+\frac{Ca^2d}{3}\right)+x^2\left(\frac{Aa^2e}{2}+\frac{Ba^2d}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*d*x + C*c**2*e*x**8/8 + x**7*(B*c**2*e/7 + C*c**2*d/7) + x**6*(A*c**
2*e/6 + B*c**2*d/6 + C*a*c*e/3) + x**5*(A*c**2*d/5 + 2*B*a*c*e/5 + 2*C*a*c*
d/5) + x**4*(A*a*c*e/2 + B*a*c*d/2 + C*a**2*e/4) + x**3*(2*A*a*c*d/3 + B*a*
*2*e/3 + C*a**2*d/3) + x**2*(A*a**2*e/2 + B*a**2*d/2)
```

Giac [A]
time = 4.67, size = 181, normalized size = 1.41

$$\frac{1}{8}\,Cc^2x^8e+\frac{1}{7}\,Cc^2dx^7+\frac{1}{7}\,Bc^2x^7e+\frac{1}{6}\,Bc^2dx^6+\frac{1}{3}\,Cacx^6e+\frac{1}{6}\,Ac^2x^6e+\frac{2}{5}\,Cacdx^5+\frac{1}{5}\,Ac^2dx^5+\frac{2}{5}\,Bacx^5e+\frac{1}{2}\,Bacdx^4+\frac{1}{4}\,Ca^2x^4e+\frac{1}{2}\,Aacx^4e+\frac{1}{3}\,Ca^2dx^3+\frac{2}{3}\,Aacdx^3+\frac{1}{3}\,Ba^2x^3e+\frac{1}{2}\,Ba^2dx^2+\frac{1}{2}\,Aa^2x^2e+Aa^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*C*c^2*x^8*e + 1/7*C*c^2*d*x^7 + 1/7*B*c^2*x^7*e + 1/6*B*c^2*d*x^6 + 1/3
*C*a*c*x^6*e + 1/6*A*c^2*x^6*e + 2/5*C*a*c*d*x^5 + 1/5*A*c^2*d*x^5 + 2/5*B*
a*c*x^5*e + 1/2*B*a*c*d*x^4 + 1/4*C*a^2*x^4*e + 1/2*A*a*c*x^4*e + 1/3*C*a^2
```

```
*d*x^3 + 2/3*A*a*c*d*x^3 + 1/3*B*a^2*x^3*e + 1/2*B*a^2*d*x^2 + 1/2*A*a^2*x^
2*e + A*a^2*d*x
```

Mupad [B]
time = 3.69, size = 140, normalized size = 1.09

$$x^3\left(\frac{B\,a^2\,e}{3}+\frac{C\,a^2\,d}{3}+\frac{2\,A\,a\,c\,d}{3}\right)+x^6\left(\frac{A\,c^2\,e}{6}+\frac{B\,c^2\,d}{6}+\frac{C\,a\,c\,e}{3}\right)+\frac{c\,x^5\,\left(A\,c\,d+2\,B\,a\,e+2\,C\,a\,d\right)}{5}+\frac{a\,x^4\,\left(2\,A\,c\,e+2\,B\,c\,d+C\,a\,e\right)}{4}+\frac{a^2\,x^2\,\left(A\,e+B\,d\right)}{2}+\frac{c^2\,x^7\,\left(B\,e+C\,d\right)}{7}+A\,a^2\,d\,x+\frac{C\,c^2\,e\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^2*(d + e*x)*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((B*a^2*e)/3 + (C*a^2*d)/3 + (2*A*a*c*d)/3) + x^6*((A*c^2*e)/6 + (B*c^2
*d)/6 + (C*a*c*e)/3) + (c*x^5*(A*c*d + 2*B*a*e + 2*C*a*d))/5 + (a*x^4*(2*A*
c*e + 2*B*c*d + C*a*e))/4 + (a^2*x^2*(A*e + B*d))/2 + (c^2*x^7*(B*e + C*d))
/7 + A*a^2*d*x + (C*c^2*e*x^8)/8
```

3.28 $\int \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx$

Optimal. Leaf size=67

$$a^2Ax+\frac{1}{3}a(2Ac+aC)x^3+\frac{1}{5}c(Ac+2aC)x^5+\frac{1}{7}c^2Cx^7+\frac{B\left(a+cx^2\right)^3}{6c}$$

```
[Out] a^2*A*x+1/3*a*(2*A*c+C*a)*x^3+1/5*c*(A*c+2*C*a)*x^5+1/7*c^2*C*x^7+1/6*B*(c*
x^2+a)^3/c
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1596, 380}

$$a^2Ax+\frac{1}{5}cx^5(2aC+Ac)+\frac{1}{3}ax^3(aC+2Ac)+\frac{B\left(a+cx^2\right)^3}{6c}+\frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*x + (a*(2*A*c + a*C)*x^3)/3 + (c*(A*c + 2*a*C)*x^5)/5 + (c^2*C*x^7)/7
 + (B*(a + c*x^2)^3)/(6*c)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right)\, dx &= \frac{B(a+cx^2)^3}{6c} + \int \left(a+cx^2\right)^2 \left(A+Cx^2\right)\, dx \\
&= \frac{B(a+cx^2)^3}{6c} + \int \left(a^2A + a(2Ac+aC)x^2 + c(Ac+2aC)x^4 + c^2Cx^6\right) \\
&= a^2Ax + \frac{1}{3}a(2Ac+aC)x^3 + \frac{1}{5}c(Ac+2aC)x^5 + \frac{1}{7}c^2Cx^7 + \frac{B(a+cx^2)}{6c}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 69, normalized size = 1.03

$$\frac{1}{210}x\left(35a^2(6A+x(3B+2Cx))+7acx^2(20A+3x(5B+4Cx))+c^2x^4(42A+5x(7B+6Cx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] (x*(35*a^2*(6*A + x*(3*B + 2*C*x)) + 7*a*c*x^2*(20*A + 3*x*(5*B + 4*C*x)) +
 c^2*x^4*(42*A + 5*x*(7*B + 6*C*x))))/210
```

Maple [A]
time = 0.10, size = 75, normalized size = 1.12

method	result	si
default	$\frac{c^2 C x^7}{7}+\frac{c^2 B x^6}{6}+\frac{(A c^2+2acC)x^5}{5}+\frac{acB x^4}{2}+\frac{(2acA+a^2C)x^3}{3}+\frac{a^2 B x^2}{2}+a^2Ax$	75
norman	$\frac{c^2 C x^7}{7}+\frac{c^2 B x^6}{6}+\left(\frac{1}{5}A c^2+\frac{2}{5}acC\right)x^5+\frac{acB x^4}{2}+\left(\frac{2}{3}acA+\frac{1}{3}a^2C\right)x^3+\frac{a^2 B x^2}{2}+a^2Ax$	75
gosper	$\frac{1}{7}c^2C x^7+\frac{1}{6}c^2B x^6+\frac{1}{5}x^5A c^2+\frac{2}{5}x^5acC+\frac{1}{2}acB x^4+\frac{2}{3}x^3acA+\frac{1}{3}x^3a^2C+\frac{1}{2}a^2B x^2+a^2Ax$	77
risch	$\frac{1}{7}c^2C x^7+\frac{1}{6}c^2B x^6+\frac{1}{5}x^5A c^2+\frac{2}{5}x^5acC+\frac{1}{2}acB x^4+\frac{2}{3}x^3acA+\frac{1}{3}x^3a^2C+\frac{1}{2}a^2B x^2+a^2Ax$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*c^2*C*x^7+1/6*c^2*B*x^6+1/5*(A*c^2+2*C*a*c)*x^5+1/2*a*c*B*x^4+1/3*(2*A*
a*c+C*a^2)*x^3+1/2*a^2*B*x^2+a^2*A*x
```

Maxima [A]
time = 0.28, size = 74, normalized size = 1.10

$$\frac{1}{7}\,Cc^2x^7+\frac{1}{6}\,Bc^2x^6+\frac{1}{2}\,Bacx^4+\frac{1}{5}\left(2\,Cac+Ac^2\right)x^5+\frac{1}{2}\,Ba^2x^2+Aa^2x+\frac{1}{3}\left(Ca^2+2\,Aac\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 1/2*B*a*c*x^4 + 1/5*(2*C*a*c + A*c^2)*x^5 +
 1/2*B*a^2*x^2 + A*a^2*x + 1/3*(C*a^2 + 2*A*a*c)*x^3
```

Fricas [A]
time = 0.33, size = 74, normalized size = 1.10

$$\frac{1}{7}Cc^2x^7+\frac{1}{6}Bc^2x^6+\frac{1}{2}Bacx^4+\frac{1}{5}\left(2\,Cac+Ac^2\right)x^5+\frac{1}{2}Ba^2x^2+Aa^2x+\frac{1}{3}\left(Ca^2+2\,Aac\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 1/2*B*a*c*x^4 + 1/5*(2*C*a*c + A*c^2)*x^5 +
 1/2*B*a^2*x^2 + A*a^2*x + 1/3*(C*a^2 + 2*A*a*c)*x^3
```

Sympy [A]
time = 0.01, size = 83, normalized size = 1.24

$$Aa^2x+\frac{Ba^2x^2}{2}+\frac{Bacx^4}{2}+\frac{Bc^2x^6}{6}+\frac{Cc^2x^7}{7}+x^5\left(\frac{Ac^2}{5}+\frac{2Cac}{5}\right)+x^3\cdot\left(\frac{2Aac}{3}+\frac{Ca^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x + B*a**2*x**2/2 + B*a*c*x**4/2 + B*c**2*x**6/6 + C*c**2*x**7/7 + x
**5*(A*c**2/5 + 2*C*a*c/5) + x**3*(2*A*a*c/3 + C*a**2/3)
```

Giac [A]
time = 3.84, size = 76, normalized size = 1.13

$$\frac{1}{7}Cc^2x^7+\frac{1}{6}Bc^2x^6+\frac{2}{5}Cacx^5+\frac{1}{5}Ac^2x^5+\frac{1}{2}Bacx^4+\frac{1}{3}Ca^2x^3+\frac{2}{3}Aacx^3+\frac{1}{2}Ba^2x^2+Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 2/5*C*a*c*x^5 + 1/5*A*c^2*x^5 + 1/2*B*a*c*x
^4 + 1/3*C*a^2*x^3 + 2/3*A*a*c*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

Mupad [B]
time = 0.04, size = 74, normalized size = 1.10

$$x^3\left(\frac{C\,a^2}{3}+\frac{2\,A\,c\,a}{3}\right)+x^5\left(\frac{A\,c^2}{5}+\frac{2\,C\,a\,c}{5}\right)+\frac{B\,a^2\,x^2}{2}+\frac{B\,c^2\,x^6}{6}+\frac{C\,c^2\,x^7}{7}+A\,a^2\,x+\frac{B\,a\,c\,x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^2*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((C*a^2)/3 + (2*A*a*c)/3) + x^5*((A*c^2)/5 + (2*C*a*c)/5) + (B*a^2*x^2)
/2 + (B*c^2*x^6)/6 + (C*c^2*x^7)/7 + A*a^2*x + (B*a*c*x^4)/2
```

3.29 $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{d+ex}\,dx$

Optimal. Leaf size=297

$$-\frac{(a^2e^4(Cd-Be)+c^2d^3(Cd^2-e(Bd-Ae))+2acde^2(Cd^2-e(Bd-Ae)))\,x}{e^6}+\frac{(a^2Ce^4+c^2d^2(Cd^2-e(B$$

```
[Out] -(a^2*e^4*(-B*e+C*d)+c^2*d^3*(C*d^2-e*(-A*e+B*d))+2*a*c*d*e^2*(C*d^2-e*(-A*
e+B*d)))*x/e^6+1/2*(a^2*C*e^4+c^2*d^2*(C*d^2-e*(-A*e+B*d))+2*a*c*e^2*(C*d^2
-e*(-A*e+B*d)))*x^2/e^5-1/3*c*(2*a*e^2*(-B*e+C*d)+c*d*(C*d^2-e*(-A*e+B*d)))
*x^3/e^4+1/4*c*(2*a*C*e^2+c*(C*d^2-e*(-A*e+B*d)))*x^4/e^3-1/5*c^2*(-B*e+C*d
)*x^5/e^2+1/6*c^2*C*x^6/e+(a*e^2+c*d^2)^2*(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e^7
```

Rubi [A]
time = 0.40, antiderivative size = 295, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1642}

$$\frac{x^2(a^2Ce^4+2ace^2(Cd^2-e(Bd-Ae))+c^2(Cd^4-d^2e(Bd-Ae)))}{2e^5}-\frac{x(a^2e^4(Cd-Be)+2acde^2(Cd^2-e(Bd-Ae))+c^2(Cd^5-d^3e(Bd-Ae)))}{e^6}-\frac{cx^3(2ae^2(Cd-Be)-cde(Bd-Ae)+cCd^3)}{3e^4}+\frac{(ae^2+cd^2)^2\log(d+ex)\,(Ae^2-Bde+Cd^2)}{e^7}+\frac{cx^4(2aCe^2-ce(Bd-Ae)+cCd^2)}{4e^3}-\frac{c^2x^5(Cd-Be)}{5e^2}+\frac{c^2Cx^6}{6e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] -(((a^2*e^4*(C*d - B*e) + 2*a*c*d*e^2*(C*d^2 - e*(B*d - A*e)) + c^2*(C*d^5
- d^3*e*(B*d - A*e)))*x)/e^6) + ((a^2*C*e^4 + 2*a*c*e^2*(C*d^2 - e*(B*d - A
*e)) + c^2*(C*d^4 - d^2*e*(B*d - A*e)))*x^2)/(2*e^5) - (c*(c*C*d^3 - c*d*e*
(B*d - A*e) + 2*a*e^2*(C*d - B*e))*x^3)/(3*e^4) + (c*(c*C*d^2 + 2*a*C*e^2 -
 c*e*(B*d - A*e))*x^4)/(4*e^3) - (c^2*(C*d - B*e)*x^5)/(5*e^2) + (c^2*C*x^6
)/(6*e) + ((c*d^2 + a*e^2)^2*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/e^7
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{d+ex}\,dx = \int \Bigg(\frac{-a^2e^4(Cd-Be)-2acde^2(Cd^2-e(Bd-Ae))-c^2(Cd^5-d^3e(B}{e^6}$$

$$= -\frac{(a^2e^4(Cd-Be)+2acde^2(Cd^2-e(Bd-Ae))+c^2(Cd^5-d^3e(Bd-}{e^6}$$

Mathematica [A]
time = 0.11, size = 285, normalized size = 0.96

$$\frac{ex(30a^2e^4(-2Cd+2Be+Cex)+10ace^2(C(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+2e(3Ae(-2d+ex)+B(6d^2-3dex+2e^2x^2)))+c^2(C(-60d^5+30d^4ex-20d^3e^2x^2+15d^2e^3x^3-12de^4x^4+10e^5x^5)+e(5Ae(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+B(60d^4-30d^3ex+20d^2e^2x^2-15de^3x^3+12e^4x^4))))+60(cd^2+ae^2)^2(Cd^2+e(-Bd+Ae))\log(d+ex)}{60e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] (e*x*(30*a^2*e^4*(-2*C*d + 2*B*e + C*e*x) + 10*a*c*e^2*(C*(-12*d^3 + 6*d^2*
e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + 2*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3*d*e
*x + 2*e^2*x^2))) + c^2*(C*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 15*d^2*
e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + e*(5*A*e*(-12*d^3 + 6*d^2*e*x - 4*d*
e^2*x^2 + 3*e^3*x^3) + B*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x
^3 + 12*e^4*x^4)))) + 60*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e))*Log[d
 + e*x])/(60*e^7)
```

Maple [A]
time = 0.09, size = 441, normalized size = 1.48

method	result
norman	$\frac{(2Aac e^4+A c^2d^2e^2-2Bacd e^3-B c^2d^3e+a^2C e^4+2Cac d^2e^2+C c^2d^4)x^2}{2e^5} - \frac{(2Aacd e^4+A c^2d^3e^2-B a^2e^5-2Bac d^2e^3-B c^2d^4e+C}{e^6}$
default	$\frac{(A a^2e^6+2Aac d^2e^4+A c^2d^4e^2-B a^2d e^5-2Bac d^3e^3-B c^2d^5e+C a^2d^2e^4+2Cac d^4e^2+C c^2d^6)\ln(ex+d)}{e^7} - \frac{Bacd e^4x^2+\frac{1}{3}A c^2d e^4x}{}$
risch	$\frac{B c^2x^5}{5e}+\frac{A c^2x^4}{4e}+\frac{C a^2x^2}{2e}+\frac{B a^2x}{e}+\frac{\ln(ex+d)A a^2}{e}-\frac{2\ln(ex+d)Bac d^3}{e^4}+\frac{2\ln(ex+d)Cac d^4}{e^5}-\frac{2Cacd x^3}{3e^2}+\frac{Cac d^2x^2}{e^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] (A*a^2*e^6+2*A*a*c*d^2*e^4+A*c^2*d^4*e^2-B*a^2*d*e^5-2*B*a*c*d^3*e^3-B*c^2*
d^5*e+C*a^2*d^2*e^4+2*C*a*c*d^4*e^2+C*c^2*d^6)/e^7*ln(e*x+d)-1/e^6*(B*a*c*d
*e^4*x^2+1/3*A*c^2*d*e^4*x^3-2/3*B*a*c*e^5*x^3-1/3*B*c^2*d^2*e^3*x^3+1/3*C*
c^2*d^3*e^2*x^3-A*a*c*e^5*x^2-1/2*A*c^2*d^2*e^3*x^2+1/2*B*c^2*d^3*e^2*x^2-1
/2*C*c^2*d^4*e*x^2+1/4*B*c^2*d*e^4*x^4-1/2*C*a*c*e^5*x^4-1/4*C*c^2*d^2*e^3*
x^4+1/5*C*c^2*d*e^4*x^5-B*c^2*d^4*e*x+C*a^2*d*e^4*x+A*c^2*d^3*e^2*x-1/6*c^2
*C*x^6*e^5-1/5*B*c^2*e^5*x^5-1/4*A*c^2*e^5*x^4-1/2*C*a^2*e^5*x^2-B*a^2*e^5*
x+C*c^2*d^5*x+2/3*C*a*c*d*e^4*x^3-C*a*c*d^2*e^3*x^2+2*A*a*c*d*e^4*x-2*B*a*c
*d^2*e^3*x+2*C*a*c*d^3*e^2*x)
```

Maxima [A]
time = 0.33, size = 368, normalized size = 1.24

$$(Cc^2d^6-Bc^2d^5e-2\,Bacd^3e^3+(2\,Cace^2+Ac^2e^2)d^4-Ba^2de^5+Aa^2e^6+(Ca^2e^4+2\,Aace^4)d^2)e^{(-7)}\log(xe+d)+\frac{1}{60}(10\,Cc^2x^6e^5-12\,(Cc^2de^4-Bc^2e^5)x^5+15\,(Cc^2d^2e^3-Bc^2de^4+2\,Cace^5+Ac^2e^5)x^4-20\,(Cc^2d^3e^2-Bc^2d^2e^3-2\,Bace^5+(2\,Cace^4+Ac^2e^4)d)x^3+30\,(Cc^2d^4e-Bc^2d^3e^2-2\,Bacde^4+Ca^2e^5+2\,Aace^5+(2\,Cace^3+Ac^2e^3)d^2)x^2-60\,(Cc^2d^5-Bc^2d^4e-2\,Bacd^2e^3+(2\,Cace^2+Ac^2e^2)d^3-Ba^2e^5+(Ca^2e^4+2\,Aace^4)d)x)e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")
```

```
[Out] (C*c^2*d^6 - B*c^2*d^5*e - 2*B*a*c*d^3*e^3 + (2*C*a*c*e^2 + A*c^2*e^2)*d^4
- B*a^2*d*e^5 + A*a^2*e^6 + (C*a^2*e^4 + 2*A*a*c*e^4)*d^2)*e^(-7)*log(x*e +
 d) + 1/60*(10*C*c^2*x^6*e^5 - 12*(C*c^2*d*e^4 - B*c^2*e^5)*x^5 + 15*(C*c^2
*d^2*e^3 - B*c^2*d*e^4 + 2*C*a*c*e^5 + A*c^2*e^5)*x^4 - 20*(C*c^2*d^3*e^2 -
 B*c^2*d^2*e^3 - 2*B*a*c*e^5 + (2*C*a*c*e^4 + A*c^2*e^4)*d)*x^3 + 30*(C*c^2
*d^4*e - B*c^2*d^3*e^2 - 2*B*a*c*d*e^4 + C*a^2*e^5 + 2*A*a*c*e^5 + (2*C*a*c
*e^3 + A*c^2*e^3)*d^2)*x^2 - 60*(C*c^2*d^5 - B*c^2*d^4*e - 2*B*a*c*d^2*e^3
+ (2*C*a*c*e^2 + A*c^2*e^2)*d^3 - B*a^2*e^5 + (C*a^2*e^4 + 2*A*a*c*e^4)*d)*
x)*e^(-6)
```

Fricas [A]
time = 0.34, size = 370, normalized size = 1.25

$$-\frac{1}{60}\left(60\,Cc^{2}d^{5}xe-\left(10\,Cc^{2}x^{6}+12\,Bc^{2}x^{5}+40\,Bacx^{3}+15\left(2\,Cac+Ac^{2}\right)x^{4}+60\,Ba^{2}x+30\left(Ca^{2}+2\,Aac\right)x^{2}\right)e^{6}+\left(12\,Cc^{2}dx^{5}+15\,Bc^{2}dx^{4}+60\,Bacdx^{2}+20\left(2\,Cac+Ac^{2}\right)dx^{3}+60\left(Ca^{2}+2\,Aac\right)dx\right)e^{5}-5\left(3\,Cc^{2}d^{2}x^{4}+4\,Bc^{2}d^{2}x^{3}+24\,Bacd^{2}x+6\left(2\,Cac+Ac^{2}\right)d^{2}x^{2}\right)e^{4}+10\left(2\,Cc^{2}d^{3}x^{3}+3\,Bc^{2}d^{3}x^{2}+6\left(2\,Cac+Ac^{2}\right)d^{3}x\right)e^{3}-30\left(Cc^{2}d^{4}x^{2}+2\,Bc^{2}d^{4}x\right)e^{2}-60\left(Cc^{2}d^{6}-Bc^{2}d^{5}e-2\,Bacd^{3}e^{3}+\left(2\,Cac+Ac^{2}\right)d^{4}e^{2}-Ba^{2}de^{5}+Aa^{2}e^{6}+\left(Ca^{2}+2\,Aac\right)d^{2}e^{4}\right)\log\left(xe+d\right)\right)e^{\left(-7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/60*(60*C*c^2*d^5*x*e - (10*C*c^2*x^6 + 12*B*c^2*x^5 + 40*B*a*c*x^3 + 15*
(2*C*a*c + A*c^2)*x^4 + 60*B*a^2*x + 30*(C*a^2 + 2*A*a*c)*x^2)*e^6 + (12*C*
c^2*d*x^5 + 15*B*c^2*d*x^4 + 60*B*a*c*d*x^2 + 20*(2*C*a*c + A*c^2)*d*x^3 +
60*(C*a^2 + 2*A*a*c)*d*x)*e^5 - 5*(3*C*c^2*d^2*x^4 + 4*B*c^2*d^2*x^3 + 24*B
*a*c*d^2*x + 6*(2*C*a*c + A*c^2)*d^2*x^2)*e^4 + 10*(2*C*c^2*d^3*x^3 + 3*B*c
^2*d^3*x^2 + 6*(2*C*a*c + A*c^2)*d^3*x)*e^3 - 30*(C*c^2*d^4*x^2 + 2*B*c^2*d
^4*x)*e^2 - 60*(C*c^2*d^6 - B*c^2*d^5*e - 2*B*a*c*d^3*e^3 + (2*C*a*c + A*c^
2)*d^4*e^2 - B*a^2*d*e^5 + A*a^2*e^6 + (C*a^2 + 2*A*a*c)*d^2*e^4)*log(x*e +
 d))*e^(-7)
```

Sympy [A]
time = 0.47, size = 359, normalized size = 1.21

$$\frac{Cc^{2}x^{6}}{6e}+x^{5}\left(\frac{Bc^{2}}{5e}-\frac{Cc^{2}d}{5e^{2}}\right)+x^{4}\left(\frac{Ac^{2}}{4e}-\frac{Bc^{2}d}{4e^{2}}+\frac{Cac}{2e}+\frac{Cc^{2}d^{2}}{4e^{3}}\right)+x^{3}\left(-\frac{Ac^{2}d}{3e^{2}}+\frac{2Bac}{3e}+\frac{Bc^{2}d^{2}}{3e^{3}}-\frac{2Cacd}{3e^{2}}-\frac{Cc^{2}d^{3}}{3e^{4}}\right)+x^{2}\left(\frac{Aac}{e}+\frac{Ac^{2}d^{2}}{2e^{3}}-\frac{Bacd}{e^{2}}-\frac{Bc^{2}d^{3}}{2e^{4}}+\frac{Ca^{2}}{2e}+\frac{Cacd^{2}}{e^{3}}+\frac{Cc^{2}d^{4}}{2e^{5}}\right)+x\left(-\frac{2Aacd}{e^{2}}-\frac{Ac^{2}d^{3}}{e^{4}}+\frac{Ba^{2}}{e}+\frac{2Bacd^{2}}{e^{3}}+\frac{Bc^{2}d^{4}}{e^{5}}-\frac{Ca^{2}d}{e^{2}}-\frac{2Cacd^{3}}{e^{4}}-\frac{Cc^{2}d^{5}}{e^{6}}\right)+\frac{\left(ae^{2}+cd^{2}\right)^{2}\left(Ae^{2}-Bde+Cd^{2}\right)\log\left(d+ex\right)}{e^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d),x)
```

```
[Out] C*c**2*x**6/(6*e) + x**5*(B*c**2/(5*e) - C*c**2*d/(5*e**2)) + x**4*(A*c**2/
(4*e) - B*c**2*d/(4*e**2) + C*a*c/(2*e) + C*c**2*d**2/(4*e**3)) + x**3*(-A*
c**2*d/(3*e**2) + 2*B*a*c/(3*e) + B*c**2*d**2/(3*e**3) - 2*C*a*c*d/(3*e**2)
 - C*c**2*d**3/(3*e**4)) + x**2*(A*a*c/e + A*c**2*d**2/(2*e**3) - B*a*c*d/e
**2 - B*c**2*d**3/(2*e**4) + C*a**2/(2*e) + C*a*c*d**2/e**3 + C*c**2*d**4/(
2*e**5)) + x*(-2*A*a*c*d/e**2 - A*c**2*d**3/e**4 + B*a**2/e + 2*B*a*c*d**2/
e**3 + B*c**2*d**4/e**5 - C*a**2*d/e**2 - 2*C*a*c*d**3/e**4 - C*c**2*d**5/e
**6) + (a*e**2 + c*d**2)**2*(A*e**2 - B*d*e + C*d**2)*log(d + e*x)/e**7
```

Giac [A]
time = 4.54, size = 416, normalized size = 1.40

$$(Cc^2d^6 - Bc^2d^5e + 2\,Cacd^4e^2 + Ac^2d^4e^2 - 2\,Bacd^3e^3 + Ca^2d^2e^4 + 2\,Aacd^2e^4 - Ba^2de^5 + Aa^2e^6)e^{(-7)}\log(|xe+d|) + \frac{1}{60}(10\,Cc^2x^6e^5 - 12\,Cc^2dx^5e^4 + 15\,Cc^2d^2x^4e^3 - 20\,Cc^2d^3x^3e^2 + 30\,Cc^2d^4x^2e - 60\,Cc^2d^5x + 12\,Bc^2x^5e^5 - 15\,Bc^2dx^4e^4 + 20\,Bc^2d^2x^3e^3 - 30\,Bc^2d^3x^2e^2 + 60\,Bc^2d^4xe + 30\,Cacx^4e^5 + 15\,Ac^2x^4e^5 - 40\,Cacdx^3e^4 - 20\,Ac^2dx^3e^4 + 60\,Cacd^2x^2e^3 + 30\,Ac^2d^2x^2e^3 - 120\,Cacd^3xe^2 - 60\,Ac^2d^3xe^2 + 40\,Bacx^3e^5 - 60\,Bacdx^2e^4 + 120\,Bacd^2xe^3 + 30\,Ca^2x^2e^5 + 60\,Aacx^2e^5 - 60\,Ca^2dxe^4 - 120\,Aacdxe^4 + 60\,Ba^2xe^5)e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

```
[Out] (C*c^2*d^6 - B*c^2*d^5*e + 2*C*a*c*d^4*e^2 + A*c^2*d^4*e^2 - 2*B*a*c*d^3*e^
3 + C*a^2*d^2*e^4 + 2*A*a*c*d^2*e^4 - B*a^2*d*e^5 + A*a^2*e^6)*e^(-7)*log(a
bs(x*e + d)) + 1/60*(10*C*c^2*x^6*e^5 - 12*C*c^2*d*x^5*e^4 + 15*C*c^2*d^2*x
^4*e^3 - 20*C*c^2*d^3*x^3*e^2 + 30*C*c^2*d^4*x^2*e - 60*C*c^2*d^5*x + 12*B*
c^2*x^5*e^5 - 15*B*c^2*d*x^4*e^4 + 20*B*c^2*d^2*x^3*e^3 - 30*B*c^2*d^3*x^2*
e^2 + 60*B*c^2*d^4*x*e + 30*C*a*c*x^4*e^5 + 15*A*c^2*x^4*e^5 - 40*C*a*c*d*x
^3*e^4 - 20*A*c^2*d*x^3*e^4 + 60*C*a*c*d^2*x^2*e^3 + 30*A*c^2*d^2*x^2*e^3 -
 120*C*a*c*d^3*x*e^2 - 60*A*c^2*d^3*x*e^2 + 40*B*a*c*x^3*e^5 - 60*B*a*c*d*x
^2*e^4 + 120*B*a*c*d^2*x*e^3 + 30*C*a^2*x^2*e^5 + 60*A*a*c*x^2*e^5 - 60*C*a
^2*d*x*e^4 - 120*A*a*c*d*x*e^4 + 60*B*a^2*x*e^5)*e^(-6)
```

Mupad [B]
time = 3.68, size = 422, normalized size = 1.42

$$x^5\left(\frac{B\,c^2}{5\,e}-\frac{C\,c^2\,d}{5\,e^2}\right)-x\left(\frac{d\left(\frac{C\,a^2+2\,A\,a\,c}{e}+\frac{d\left(\frac{d\left(\frac{A\,c^2+2\,C\,a\,c}{e}-\frac{d\left(\frac{B\,c^2}{e}-\frac{C\,c^2\,d}{e^2}\right)}{e}\right)}{e}-\frac{2\,B\,a\,c}{e}\right)}{e}\right)}{e}-\frac{B\,a^2}{e}\right)+x^4\left(\frac{A\,c^2+2\,C\,a\,c}{4\,e}-\frac{d\left(\frac{B\,c^2}{e}-\frac{C\,c^2\,d}{e^2}\right)}{4\,e}\right)-x^3\left(\frac{d\left(\frac{A\,c^2+2\,C\,a\,c}{e}-\frac{d\left(\frac{B\,c^2}{e}-\frac{C\,c^2\,d}{e^2}\right)}{e}\right)}{3\,e}-\frac{2\,B\,a\,c}{3\,e}\right)+x^2\left(\frac{C\,a^2+2\,A\,a\,c}{2\,e}+\frac{d\left(\frac{d\left(\frac{A\,c^2+2\,C\,a\,c}{e}-\frac{d\left(\frac{B\,c^2}{e}-\frac{C\,c^2\,d}{e^2}\right)}{e}\right)}{e}-\frac{2\,B\,a\,c}{e}\right)}{2\,e}\right)+\frac{\ln\left(d+e\,x\right)\,\left(C\,a^2\,d^2\,e^4-B\,a^2\,d\,e^5+A\,a^2\,e^6+2\,C\,a\,c\,d^4\,e^2-2\,B\,a\,c\,d^3\,e^3+2\,A\,a\,c\,d^2\,e^4+C\,c^2\,d^6-B\,c^2\,d^5\,e+A\,c^2\,d^4\,e^2\right)}{e^7}+\frac{C\,c^2\,x^6}{6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x),x)
```

```
[Out] x^5*((B*c^2)/(5*e) - (C*c^2*d)/(5*e^2)) - x*((d*((C*a^2 + 2*A*a*c)/e + (d*(
(d*((A*c^2 + 2*C*a*c)/e - (d*((B*c^2)/e - (C*c^2*d)/e^2))/e))/e - (2*B*a*c)
/e))/e))/e - (B*a^2)/e) + x^4*((A*c^2 + 2*C*a*c)/(4*e) - (d*((B*c^2)/e - (C
*c^2*d)/e^2))/(4*e)) - x^3*((d*((A*c^2 + 2*C*a*c)/e - (d*((B*c^2)/e - (C*c^
2*d)/e^2))/e))/(3*e) - (2*B*a*c)/(3*e)) + x^2*((C*a^2 + 2*A*a*c)/(2*e) + (d
*((d*((A*c^2 + 2*C*a*c)/e - (d*((B*c^2)/e - (C*c^2*d)/e^2))/e))/e - (2*B*a*
c)/e))/(2*e)) + (log(d + e*x)*(A*a^2*e^6 + C*c^2*d^6 - B*a^2*d*e^5 - B*c^2*
d^5*e + A*c^2*d^4*e^2 + C*a^2*d^2*e^4 + 2*A*a*c*d^2*e^4 - 2*B*a*c*d^3*e^3 +
 2*C*a*c*d^4*e^2))/e^7 + (C*c^2*x^6)/(6*e)
```

3.30 $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^2}\,dx$

Optimal. Leaf size=292

$$\frac{(a^2Ce^4+c^2d^2(5Cd^2-e(4Bd-3Ae))+2ace^2(3Cd^2-e(2Bd-Ae)))\,x}{e^6}-\frac{c(2ae^2(2Cd-Be)+cd(4Cd^2-$$

```
[Out] (a^2*C*e^4+c^2*d^2*(5*C*d^2-e*(-3*A*e+4*B*d))+2*a*c*e^2*(3*C*d^2-e*(-A*e+2*
B*d)))*x/e^6-1/2*c*(2*a*e^2*(-B*e+2*C*d)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*x^
2/e^5+1/3*c*(2*a*C*e^2+c*(3*C*d^2-e*(-A*e+2*B*d)))*x^3/e^4-1/4*c^2*(-B*e+2*
C*d)*x^4/e^3+1/5*c^2*C*x^5/e^2-(a*e^2+c*d^2)^2*(A*e^2-B*d*e+C*d^2)/e^7/(e*x
+d)-(a*e^2+c*d^2)*(a*e^2*(-B*e+2*C*d)+c*d*(6*C*d^2-e*(-4*A*e+5*B*d)))*ln(e*
x+d)/e^7
```

Rubi [A]

time = 0.32, antiderivative size = 289, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = $\{1642\}$

$$\frac{x(a^2Ce^4+2ace^2(3Cd^2-e(2Bd-Ae))+c^2(5Cd^4-d^2e(4Bd-3Ae)))}{e^6}-\frac{cx^2(2ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3)}{2e^5}-\frac{(ae^2+cd^2)^2(Ae^2-Bde+Cd^2)}{e^7(d+ex)}+\frac{cx^3(2aCe^2-ce(2Bd-Ae)+3cCd^2)}{3e^4}-\frac{(ae^2+cd^2)\log(d+ex)(ae^2(2Cd-Be)-cde(5Bd-4Ae)+6cCd^3)}{e^7}-\frac{c^2x^4(2Cd-Be)}{4e^3}+\frac{c^2Cx^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] ((a^2*C*e^4 + c^2*(5*C*d^4 - d^2*e*(4*B*d - 3*A*e)) + 2*a*c*e^2*(3*C*d^2 -
e*(2*B*d - A*e)))*x)/e^6 - (c*(4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + 2*a*e^2*
(2*C*d - B*e))*x^2)/(2*e^5) + (c*(3*c*C*d^2 + 2*a*C*e^2 - c*e*(2*B*d - A*e)
)*x^3)/(3*e^4) - (c^2*(2*C*d - B*e)*x^4)/(4*e^3) + (c^2*C*x^5)/(5*e^2) - ((
c*d^2 + a*e^2)^2*(C*d^2 - B*d*e + A*e^2))/(e^7*(d + e*x)) - ((c*d^2 + a*e^2
)*(6*c*C*d^3 - c*d*e*(5*B*d - 4*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e*x])/e
^7
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^2}\,dx = \int \left(\frac{a^2Ce^4+c^2(5Cd^4-d^2e(4Bd-3Ae))+2ace^2(3Cd^2-e(2Bd-A}{e^6}\right.$$

$$=\frac{(a^2Ce^4+c^2(5Cd^4-d^2e(4Bd-3Ae))+2ace^2(3Cd^2-e(2Bd-Ae))}{e^6}$$

Mathematica [A]
time = 0.19, size = 272, normalized size = 0.93

$$\frac{60e(a^2Ce^4+2ace^2(3Cd^2+e(-2Bd+Ae))+c^2(5Cd^4+d^2e(-4Bd+3Ae)))x-30ce^2(4cCd^3+cde(-3Bd+2Ae)-2ae^2(-2Cd+Be))x^2+20ce^3(3cCd^2+2aCe^2+ce(-2Bd+Ae))x^3+15c^2e^4(-2Cd+Be)x^4+12c^2Ce^5x^5-\frac{60(cd^2+ae^2)^2(Cd^2+e(-Bd+Ae))}{d+ex}-60(cd^2+ae^2)(6cCd^3+cde(-5Bd+4Ae)+ae^2(2Cd-Be))\log(d+ex)}{60e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] (60*e*(a^2*C*e^4 + 2*a*c*e^2*(3*C*d^2 + e*(-2*B*d + A*e)) + c^2*(5*C*d^4 +
d^2*e*(-4*B*d + 3*A*e)))*x - 30*c*e^2*(4*c*C*d^3 + c*d*e*(-3*B*d + 2*A*e) -
 2*a*e^2*(-2*C*d + B*e))*x^2 + 20*c*e^3*(3*c*C*d^2 + 2*a*C*e^2 + c*e*(-2*B*
d + A*e))*x^3 + 15*c^2*e^4*(-2*C*d + B*e)*x^4 + 12*c^2*C*e^5*x^5 - (60*(c*d
^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) - 60*(c*d^2 + a*e^2)*(6
*c*C*d^3 + c*d*e*(-5*B*d + 4*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e*x])/(60*
e^7)
```

Maple [A]
time = 0.09, size = 426, normalized size = 1.46

method	result
norman	$\frac{\left(A\,a^2e^6+4Aac\,d^2e^4+4A\,c^2d^4e^2-B\,a^2d\,e^5-6Bac\,d^3e^3-5B\,c^2d^5e+2C\,a^2d^2e^4+8Cac\,d^4e^2+6C\,c^2d^6\right)x}{e^6d}+\frac{\left(4Aac\,e^4+4A\,c^2d^2e^2-6Bacd\,e^3-5B\,c^2d^3 \ldots\right.}{2e^5}$
default	$\frac{\left(-4Aacd\,e^4-4A\,c^2d^3e^2+B\,a^2e^5+6Bac\,d^2e^3+5B\,c^2d^4e-2C\,a^2d\,e^4-8Cac\,d^3e^2-6C\,c^2d^5\right)\ln(ex+d)}{e^7}-\frac{A\,a^2e^6+2Aac\,d^2e^4+A\,c^2d^4e^2 \ldots}{}$
risch	$-\frac{4\ln(ex+d)Aacd}{e^3}+\frac{6\ln(ex+d)Bac\,d^2}{e^4}-\frac{8\ln(ex+d)Cac\,d^3}{e^5}-\frac{2Cac\,d^4}{e^5(ex+d)}-\frac{2Cacd\,x^2}{e^3}-\frac{4Bacdx}{e^3}+\frac{6Cac\,d^2x}{e^4}-\frac{2Aac\,d^2}{e^3(ex+d)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-4*A*a*c*d*e^4-4*A*c^2*d^3*e^2+B*a^2*e^5+6*B*a*c*d^2*e^3+5*B*c^2*d^4*e-2*C
*a^2*d*e^4-8*C*a*c*d^3*e^2-6*C*c^2*d^5)/e^7*ln(e*x+d)-(A*a^2*e^6+2*A*a*c*d^
2*e^4+A*c^2*d^4*e^2-B*a^2*d*e^5-2*B*a*c*d^3*e^3-B*c^2*d^5*e+C*a^2*d^2*e^4+2
*C*a*c*d^4*e^2+C*c^2*d^6)/e^7/(e*x+d)+1/e^6*(1/5*c^2*C*x^5*e^4+1/4*B*c^2*e^
4*x^4-1/2*C*c^2*d*e^3*x^4+1/3*A*c^2*e^4*x^3-2/3*B*c^2*d*e^3*x^3+2/3*C*a*c*e
^4*x^3+C*c^2*d^2*e^2*x^3-A*c^2*d*e^3*x^2+B*a*c*e^4*x^2+3/2*B*c^2*d^2*e^2*x^
2-2*C*a*c*d*e^3*x^2-2*C*c^2*d^3*e*x^2+2*A*a*c*e^4*x+3*A*c^2*d^2*e^2*x-4*B*a
*c*d*e^3*x-4*B*c^2*d^3*e*x+a^2*C*e^4*x+6*C*a*c*d^2*e^2*x+5*C*c^2*d^4*x)
```

Maxima [A]
time = 0.28, size = 384, normalized size = 1.32

$$-\left(6\,Cc^2d^5-5\,Bc^2d^4e-6\,Bacd^2e^3+4\,\left(2\,Cace^2+Ac^2e^2\right)d^3-Ba^2e^5+2\,\left(Ca^2e^4+2\,Aace^4\right)d\right)e^{(-7)}\log\left(xe+d\right)+\frac{1}{60}\,\left(12\,Cc^2x^5e^4-15\,\left(2\,Cc^2de^3-Bc^2e^4\right)x^4+20\,\left(3\,Cc^2d^2e^2-2\,Bc^2de^3+2\,Cace^4+Ac^2e^4\right)x^3-30\,\left(4\,Cc^2d^3e-3\,Bc^2d^2e^2-2\,Bace^4+2\,\left(2\,Cace^3+Ac^2e^3\right)d\right)x^2+60\,\left(5\,Cc^2d^4-4\,Bc^2d^3e-4\,Bacde^3+Ca^2e^4+2\,Aace^4+3\,\left(2\,Cace^2+Ac^2e^2\right)d^2\right)x\right)e^{(-6)}-\frac{Cc^2d^6-Bc^2d^5e-2\,Bacd^3e^3+\left(2\,Cace^2+Ac^2e^2\right)d^4-Ba^2de^5+Aa^2e^6+\left(Ca^2e^4+2\,Aace^4\right)d^2}{xe^8+de^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(6*C*c^2*d^5 - 5*B*c^2*d^4*e - 6*B*a*c*d^2*e^3 + 4*(2*C*a*c*e^2 + A*c^2*e^
2)*d^3 - B*a^2*e^5 + 2*(C*a^2*e^4 + 2*A*a*c*e^4)*d)*e^(-7)*log(x*e + d) + 1
/60*(12*C*c^2*x^5*e^4 - 15*(2*C*c^2*d*e^3 - B*c^2*e^4)*x^4 + 20*(3*C*c^2*d^
2*e^2 - 2*B*c^2*d*e^3 + 2*C*a*c*e^4 + A*c^2*e^4)*x^3 - 30*(4*C*c^2*d^3*e -
3*B*c^2*d^2*e^2 - 2*B*a*c*e^4 + 2*(2*C*a*c*e^3 + A*c^2*e^3)*d)*x^2 + 60*(5*
C*c^2*d^4 - 4*B*c^2*d^3*e - 4*B*a*c*d*e^3 + C*a^2*e^4 + 2*A*a*c*e^4 + 3*(2*
C*a*c*e^2 + A*c^2*e^2)*d^2)*x)*e^(-6) - (C*c^2*d^6 - B*c^2*d^5*e - 2*B*a*c*
d^3*e^3 + (2*C*a*c*e^2 + A*c^2*e^2)*d^4 - B*a^2*d*e^5 + A*a^2*e^6 + (C*a^2*
e^4 + 2*A*a*c*e^4)*d^2)/(x*e^8 + d*e^7)
```

Fricas [A]
time = 0.37, size = 525, normalized size = 1.80

$$-\frac{60 \, C c^{2} d^{6} - {\left(12 \, C c^{2} x^{6} + 15 \, B c^{2} x^{5} + 60 \, B a c x^{3} + 20 \, {\left(2 \, C a c + A c^{2}\right)} x^{4} - 60 \, A a^{2} + 60 \, {\left(C a^{2} + 2 \, A a c\right)} x^{2}\right)} e^{6} + {\left(18 \, C c^{2} d x^{5} + 25 \, B c^{2} d x^{4} + 180 \, B a c d x^{2} + 40 \, {\left(2 \, C a c + A c^{2}\right)} d x^{3} - 60 \, B a^{2} d - 60 \, {\left(C a^{2} + 2 \, A a c\right)} d x\right)} e^{5} - 10 \, {\left(3 \, C c^{2} d^{2} x^{4} + 5 \, B c^{2} d^{2} x^{3} - 24 \, B a c d^{2} x + 12 \, {\left(2 \, C a c + A c^{2}\right)} d^{2} x^{2} - 6 \, {\left(C a^{2} + 2 \, A a c\right)} d^{2}\right)} e^{4} + 30 \, {\left(2 \, C c^{2} d^{3} x^{3} + 5 \, B c^{2} d^{3} x^{2} - 4 \, B a c d^{3} - 6 \, {\left(2 \, C a c + A c^{2}\right)} d^{3} x\right)} e^{3} - 60 \, {\left(3 \, C c^{2} d^{4} x^{2} - 4 \, B c^{2} d^{4} x - {\left(2 \, C a c + A c^{2}\right)} d^{4}\right)} e^{2} - 60 \, {\left(5 \, C c^{2} d^{5} x + B c^{2} d^{5}\right)} e + 60 \, {\left(6 \, C c^{2} d^{6} - B a^{2} x e^{6} - {\left(B a^{2} d - 2 \, {\left(C a^{2} + 2 \, A a c\right)} d x\right)} e^{5} - 2 \, {\left(3 \, B a c d^{2} x - {\left(C a^{2} + 2 \, A a c\right)} d^{2}\right)} e^{4} - 2 \, {\left(3 \, B a c d^{3} - 2 \, {\left(2 \, C a c + A c^{2}\right)} d^{3} x\right)} e^{3} - {\left(5 \, B c^{2} d^{4} x - 4 \, {\left(2 \, C a c + A c^{2}\right)} d^{4}\right)} e^{2} + {\left(6 \, C c^{2} d^{5} x - 5 \, B c^{2} d^{5}\right)} e\right)} \log\left(x e + d\right)}{60 \, {\left(x e^{8} + d e^{7}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/60*(60*C*c^2*d^6 - (12*C*c^2*x^6 + 15*B*c^2*x^5 + 60*B*a*c*x^3 + 20*(2*C
*a*c + A*c^2)*x^4 - 60*A*a^2 + 60*(C*a^2 + 2*A*a*c)*x^2)*e^6 + (18*C*c^2*d*
x^5 + 25*B*c^2*d*x^4 + 180*B*a*c*d*x^2 + 40*(2*C*a*c + A*c^2)*d*x^3 - 60*B*
a^2*d - 60*(C*a^2 + 2*A*a*c)*d*x)*e^5 - 10*(3*C*c^2*d^2*x^4 + 5*B*c^2*d^2*x
^3 - 24*B*a*c*d^2*x + 12*(2*C*a*c + A*c^2)*d^2*x^2 - 6*(C*a^2 + 2*A*a*c)*d^
2)*e^4 + 30*(2*C*c^2*d^3*x^3 + 5*B*c^2*d^3*x^2 - 4*B*a*c*d^3 - 6*(2*C*a*c +
 A*c^2)*d^3*x)*e^3 - 60*(3*C*c^2*d^4*x^2 - 4*B*c^2*d^4*x - (2*C*a*c + A*c^2
)*d^4)*e^2 - 60*(5*C*c^2*d^5*x + B*c^2*d^5)*e + 60*(6*C*c^2*d^6 - B*a^2*x*e
^6 - (B*a^2*d - 2*(C*a^2 + 2*A*a*c)*d*x)*e^5 - 2*(3*B*a*c*d^2*x - (C*a^2 +
2*A*a*c)*d^2)*e^4 - 2*(3*B*a*c*d^3 - 2*(2*C*a*c + A*c^2)*d^3*x)*e^3 - (5*B*
c^2*d^4*x - 4*(2*C*a*c + A*c^2)*d^4)*e^2 + (6*C*c^2*d^5*x - 5*B*c^2*d^5)*e)
*log(x*e + d))/(x*e^8 + d*e^7)
```

Sympy [A]
time = 1.24, size = 416, normalized size = 1.42

$$\frac{Cc^{2}x^{5}}{5e^{2}} + x^{4}\left(\frac{Bc^{2}}{4e^{2}} - \frac{Cc^{2}d}{2e^{3}}\right) + x^{3}\left(\frac{Ac^{2}}{3e^{2}} - \frac{2Bc^{2}d}{3e^{3}} + \frac{2Cac}{3e^{2}} + \frac{Cc^{2}d^{2}}{e^{4}}\right) + x^{2}\left(-\frac{Ac^{2}d}{e^{3}} + \frac{Bac}{e^{2}} + \frac{3Bc^{2}d^{2}}{2e^{4}} - \frac{2Cacd}{e^{3}} - \frac{2Cc^{2}d^{3}}{e^{5}}\right) + x\left(\frac{2Aac}{e^{2}} + \frac{3Ac^{2}d^{2}}{e^{4}} - \frac{4Bacd}{e^{3}} - \frac{4Bc^{2}d^{3}}{e^{5}} + \frac{Ca^{2}}{e^{2}} + \frac{6Cacd^{2}}{e^{4}} + \frac{5Cc^{2}d^{4}}{e^{6}}\right) + \frac{-Aa^{2}e^{6} - 2Aacd^{2}e^{4} - Ac^{2}d^{4}e^{2} + Ba^{2}de^{5} + 2Bacd^{3}e^{3} + Bc^{2}d^{5}e - Ca^{2}d^{2}e^{4} - 2Cacd^{4}e^{2} - Cc^{2}d^{6}}{de^{7} + e^{8}x} - \frac{(ae^{2} + cd^{2})(4Acde^{2} - Bae^{3} - 5Bcd^{2}e + 2Cade^{2} + 6Ccd^{3})\log(d + ex)}{e^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

```
[Out] C*c**2*x**5/(5*e**2) + x**4*(B*c**2/(4*e**2) - C*c**2*d/(2*e**3)) + x**3*(A
*c**2/(3*e**2) - 2*B*c**2*d/(3*e**3) + 2*C*a*c/(3*e**2) + C*c**2*d**2/e**4)
 + x**2*(-A*c**2*d/e**3 + B*a*c/e**2 + 3*B*c**2*d**2/(2*e**4) - 2*C*a*c*d/e
**3 - 2*C*c**2*d**3/e**5) + x*(2*A*a*c/e**2 + 3*A*c**2*d**2/e**4 - 4*B*a*c*
d/e**3 - 4*B*c**2*d**3/e**5 + C*a**2/e**2 + 6*C*a*c*d**2/e**4 + 5*C*c**2*d*
```

```
*4/e**6) + (-A*a**2*e**6 - 2*A*a*c*d**2*e**4 - A*c**2*d**4*e**2 + B*a**2*d*
e**5 + 2*B*a*c*d**3*e**3 + B*c**2*d**5*e - C*a**2*d**2*e**4 - 2*C*a*c*d**4*
e**2 - C*c**2*d**6)/(d*e**7 + e**8*x) - (a*e**2 + c*d**2)*(4*A*c*d*e**2 - B
*a*e**3 - 5*B*c*d**2*e + 2*C*a*d*e**2 + 6*C*c*d**3)*log(d + e*x)/e**7
```

Giac [A]
time = 4.65, size = 497, normalized size = 1.70

$$\frac{1}{60}\left(12\,Cc^2 - \frac{15\,(6\,Cc^2de - Bc^2e^2)e^{(-1)}}{xe+d} + \frac{20\,(15\,Cc^2d^2e^2 - 5\,Bc^2de^3 + 2\,Cace^4 + Ac^2e^4)e^{(-2)}}{(xe+d)^2} - \frac{60\,(10\,Cc^2d^3e^3 - 5\,Bc^2d^2e^4 + 4\,Cacde^5 + 2\,Ac^2de^5 - Bace^6)e^{(-3)}}{(xe+d)^3} + \frac{60\,(15\,Cc^2d^4e^4 - 10\,Bc^2d^3e^5 + 12\,Cacd^2e^6 + 6\,Ac^2d^2e^6 - 6\,Bacde^7 + Ca^2e^8 + 2\,Aace^8)e^{(-4)}}{(xe+d)^4}\right)(xe+d)^5e^{(-7)} + (6\,Cc^2d^5 - 5\,Bc^2d^4e + 8\,Cacd^3e^2 + 4\,Ac^2d^3e^2 - 6\,Bacd^2e^3 + 2\,Ca^2de^4 + 4\,Aacde^4 - Ba^2e^5)e^{(-7)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right) - \left(\frac{Cc^2d^6e^5}{xe+d} - \frac{Bc^2d^5e^6}{xe+d} + \frac{2\,Cacd^4e^7}{xe+d} + \frac{Ac^2d^4e^7}{xe+d} - \frac{2\,Bacd^3e^8}{xe+d} + \frac{Ca^2d^2e^9}{xe+d} + \frac{2\,Aacd^2e^9}{xe+d} - \frac{Ba^2de^{10}}{xe+d} + \frac{Aa^2e^{11}}{xe+d}\right)e^{(-12)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/60*(12*C*c^2 - 15*(6*C*c^2*d*e - B*c^2*e^2)*e^(-1)/(x*e + d) + 20*(15*C*c
^2*d^2*e^2 - 5*B*c^2*d*e^3 + 2*C*a*c*e^4 + A*c^2*e^4)*e^(-2)/(x*e + d)^2 -
60*(10*C*c^2*d^3*e^3 - 5*B*c^2*d^2*e^4 + 4*C*a*c*d*e^5 + 2*A*c^2*d*e^5 - B*
a*c*e^6)*e^(-3)/(x*e + d)^3 + 60*(15*C*c^2*d^4*e^4 - 10*B*c^2*d^3*e^5 + 12*
C*a*c*d^2*e^6 + 6*A*c^2*d^2*e^6 - 6*B*a*c*d*e^7 + C*a^2*e^8 + 2*A*a*c*e^8)*
e^(-4)/(x*e + d)^4)*(x*e + d)^5*e^(-7) + (6*C*c^2*d^5 - 5*B*c^2*d^4*e + 8*C
*a*c*d^3*e^2 + 4*A*c^2*d^3*e^2 - 6*B*a*c*d^2*e^3 + 2*C*a^2*d*e^4 + 4*A*a*c*
d*e^4 - B*a^2*e^5)*e^(-7)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) - (C*c^2*d^6
*e^5/(x*e + d) - B*c^2*d^5*e^6/(x*e + d) + 2*C*a*c*d^4*e^7/(x*e + d) + A*c^
2*d^4*e^7/(x*e + d) - 2*B*a*c*d^3*e^8/(x*e + d) + C*a^2*d^2*e^9/(x*e + d) +
 2*A*a*c*d^2*e^9/(x*e + d) - B*a^2*d*e^10/(x*e + d) + A*a^2*e^11/(x*e + d))
*e^(-12)
```

Mupad [B]
time = 0.12, size = 575, normalized size = 1.97

$$x^4\left(\frac{B\,c^2}{4\,e^2} - \frac{C\,c^2\,d}{2\,e^3}\right) + x\left(\frac{C\,a^2 + 2\,A\,c\,a}{e^2} + \frac{d^2\left(\frac{2\,d\left(\frac{B\,c^2}{e^2} - \frac{2\,C\,c^2\,d}{e^3}\right)}{e} - \frac{A\,c^2 + 2\,C\,a\,c}{e^2} + \frac{C\,c^2\,d^2}{e^4}\right)}{e^2} - \frac{2\,d\left(\frac{2\,d\left(\frac{2\,d\left(\frac{B\,c^2}{e^2} - \frac{2\,C\,c^2\,d}{e^3}\right)}{e} - \frac{A\,c^2 + 2\,C\,a\,c}{e^2} + \frac{C\,c^2\,d^2}{e^4}\right)}{e} - \frac{d^2\left(\frac{B\,c^2}{e^2} - \frac{2\,C\,c^2\,d}{e^3}\right)}{e^2} + \frac{2\,B\,a\,c}{e^2}\right)}{e}\right) - x^3\left(\frac{2\,d\left(\frac{B\,c^2}{e^2} - \frac{2\,C\,c^2\,d}{e^3}\right)}{3\,e} - \frac{A\,c^2 + 2\,C\,a\,c}{3\,e^2} + \frac{C\,c^2\,d^2}{3\,e^4}\right) + x^2\left(\frac{d\left(\frac{2\,d\left(\frac{B\,c^2}{e^2} - \frac{2\,C\,c^2\,d}{e^3}\right)}{e} - \frac{A\,c^2 + 2\,C\,a\,c}{e^2} + \frac{C\,c^2\,d^2}{e^4}\right)}{e} - \frac{d^2\left(\frac{B\,c^2}{e^2} - \frac{2\,C\,c^2\,d}{e^3}\right)}{2\,e^2} + \frac{B\,a\,c}{e^2}\right) - \frac{C\,a^2\,d^2\,e^4 - B\,a^2\,d\,e^5 + A\,a^2\,e^6 + 2\,C\,a\,c\,d^4\,e^2 - 2\,B\,a\,c\,d^3\,e^3 + 2\,A\,a\,c\,d^2\,e^4 + C\,c^2\,d^6 - B\,c^2\,d^5\,e + A\,c^2\,d^4\,e^2}{e\,\left(x\,e^7 + d\,e^6\right)} - \frac{\ln\left(d + e\,x\right)\,\left(2\,C\,a^2\,d\,e^4 - B\,a^2\,e^5 + 8\,C\,a\,c\,d^3\,e^2 - 6\,B\,a\,c\,d^2\,e^3 + 4\,A\,a\,c\,d\,e^4 + 6\,C\,c^2\,d^5 - 5\,B\,c^2\,d^4\,e + 4\,A\,c^2\,d^3\,e^2\right)}{e^7} + \frac{C\,c^2\,x^5}{5\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

```
[Out] x^4*((B*c^2)/(4*e^2) - (C*c^2*d)/(2*e^3)) + x*((C*a^2 + 2*A*a*c)/e^2 + (d^2
*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (A*c^2 + 2*C*a*c)/e^2 + (C*c^2*
d^2)/e^4))/e^2 - (2*d*((2*d*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (A*c
^2 + 2*C*a*c)/e^2 + (C*c^2*d^2)/e^4))/e - (d^2*((B*c^2)/e^2 - (2*C*c^2*d)/e
^3))/e^2 + (2*B*a*c)/e^2))/e) - x^3*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/
(3*e) - (A*c^2 + 2*C*a*c)/(3*e^2) + (C*c^2*d^2)/(3*e^4)) + x^2*((d*((2*d*((
B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (A*c^2 + 2*C*a*c)/e^2 + (C*c^2*d^2)/e^4)
)/e - (d^2*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/(2*e^2) + (B*a*c)/e^2) - (A*a^2
*e^6 + C*c^2*d^6 - B*a^2*d*e^5 - B*c^2*d^5*e + A*c^2*d^4*e^2 + C*a^2*d^2*e^
4 + 2*A*a*c*d^2*e^4 - 2*B*a*c*d^3*e^3 + 2*C*a*c*d^4*e^2)/(e*(d*e^6 + e^7*x)
) - (log(d + e*x)*(6*C*c^2*d^5 - B*a^2*e^5 + 2*C*a^2*d*e^4 - 5*B*c^2*d^4*e
+ 4*A*c^2*d^3*e^2 + 4*A*a*c*d*e^4 - 6*B*a*c*d^2*e^3 + 8*C*a*c*d^3*e^2))/e^7
 + (C*c^2*x^5)/(5*e^2)
```

3.31 $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^3}\,dx$

Optimal. Leaf size=295

$$-\frac{c(2ae^2(3Cd-Be)+cd(10Cd^2-3e(2Bd-Ae)))\,x}{e^6}+\frac{c(2aCe^2+c(6Cd^2-e(3Bd-Ae)))\,x^2}{2e^5}-\frac{c^2(3Cd-}{3e^4}$$

```
[Out] -c*(2*a*e^2*(-B*e+3*C*d)+c*d*(10*C*d^2-3*e*(-A*e+2*B*d)))*x/e^6+1/2*c*(2*a*
C*e^2+c*(6*C*d^2-e*(-A*e+3*B*d)))*x^2/e^5-1/3*c^2*(-B*e+3*C*d)*x^3/e^4+1/4*
c^2*C*x^4/e^3-1/2*(a*e^2+c*d^2)^2*(A*e^2-B*d*e+C*d^2)/e^7/(e*x+d)^2+(a*e^2+
c*d^2)*(a*e^2*(-B*e+2*C*d)+c*d*(6*C*d^2-e*(-4*A*e+5*B*d)))/e^7/(e*x+d)+(a^2
*C*e^4+c^2*d^2*(15*C*d^2-2*e*(-3*A*e+5*B*d))+2*a*c*e^2*(6*C*d^2-e*(-A*e+3*B
*d)))*ln(e*x+d)/e^7
```

Rubi [A]
time = 0.32, antiderivative size = 292, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1642}

$$\frac{\log(d+ex)\left(a^2Ce^4+2ace^2(6Cd^2-e(3Bd-Ae))+c^2(15Cd^4-2d^2e(5Bd-3Ae))\right)}{e^7}-\frac{cx(2ae^2(3Cd-Be)-3cde(2Bd-Ae)+10cCd^3)}{e^6}-\frac{(ae^2+cd^2)^2(Ae^2-Bde+Cd^2)}{2e^7(d+ex)^2}+\frac{cx^2(2aCe^2-ce(3Bd-Ae)+6cCd^2)}{2e^5}+\frac{(ae^2+cd^2)(ae^2(2Cd-Be)-cde(5Bd-4Ae)+6cCd^3)}{e^7(d+ex)}-\frac{c^2x^3(3Cd-Be)}{3e^4}+\frac{c^2Cx^4}{4e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] -((c*(10*c*C*d^3 - 3*c*d*e*(2*B*d - A*e) + 2*a*e^2*(3*C*d - B*e))*x)/e^6) +
 (c*(6*c*C*d^2 + 2*a*C*e^2 - c*e*(3*B*d - A*e))*x^2)/(2*e^5) - (c^2*(3*C*d
- B*e)*x^3)/(3*e^4) + (c^2*C*x^4)/(4*e^3) - ((c*d^2 + a*e^2)^2*(C*d^2 - B*d
*e + A*e^2))/(2*e^7*(d + e*x)^2) + ((c*d^2 + a*e^2)*(6*c*C*d^3 - c*d*e*(5*B
*d - 4*A*e) + a*e^2*(2*C*d - B*e)))/(e^7*(d + e*x)) + ((a^2*C*e^4 + c^2*(15
*C*d^4 - 2*d^2*e*(5*B*d - 3*A*e)) + 2*a*c*e^2*(6*C*d^2 - e*(3*B*d - A*e)))*
Log[d + e*x])/e^7
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^2)^2\,(A+Bx+Cx^2)}{(d+ex)^3}\,dx = \int \left(\frac{c(-10cCd^3+3cde(2Bd-Ae)-2ae^2(3Cd-Be))}{e^6}+\frac{c(6cCd^2+}{}\right.$$

$$= -\frac{c(10cCd^3-3cde(2Bd-Ae)+2ae^2(3Cd-Be))\,x}{e^6}+\frac{c(6cCd^2+2aC}{}$$

Mathematica [A]
time = 0.08, size = 274, normalized size = 0.93

$$\frac{-12ce(10cCd^3+3cde(-2Bd+Ae)-2ae^2(-3Cd+Be))x+6ce^2(6cCd^2+2aCe^2+ce(-3Bd+Ae))x^2+4c^2e^3(-3Cd+Be)x^3+3c^2Ce^4x^4-\frac{6(cd^2+ae^2)^2(Cd^2+e(-Bd+Ae))}{(d+ex)^2}+\frac{12(cd^2+ae^2)(6cCd^3+cde(-5Bd+4Ae)+ae^2(2Cd-Be))}{d+ex}+12(a^2Ce^4+2ace^2(6Cd^2+e(-3Bd+Ae))+c^2(15Cd^4+2d^2e(-5Bd+3Ae)))\log(d+ex)}{12e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] (-12*c*e*(10*c*C*d^3 + 3*c*d*e*(-2*B*d + A*e) - 2*a*e^2*(-3*C*d + B*e))*x +
 6*c*e^2*(6*c*C*d^2 + 2*a*C*e^2 + c*e*(-3*B*d + A*e))*x^2 + 4*c^2*e^3*(-3*C
*d + B*e)*x^3 + 3*c^2*C*e^4*x^4 - (6*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) +
 A*e)))/(d + e*x)^2 + (12*(c*d^2 + a*e^2)*(6*c*C*d^3 + c*d*e*(-5*B*d + 4*A*
e) + a*e^2*(2*C*d - B*e)))/(d + e*x) + 12*(a^2*C*e^4 + 2*a*c*e^2*(6*C*d^2 +
 e*(-3*B*d + A*e)) + c^2*(15*C*d^4 + 2*d^2*e*(-5*B*d + 3*A*e)))*Log[d + e*x
])/(12*e^7)
```

Maple [A]
time = 0.08, size = 399, normalized size = 1.35

method	result
norman	$\frac{\frac{\left(4Aacde^4+12Ac^2d^3e^2-Ba^2e^5-12Bacd^2e^3-20Bc^2d^4e+2Ca^2de^4+24Cacd^3e^2+30Cc^2d^5\right)x}{e^6}-\frac{Aa^2e^6-6Aacd^2e^4-18Ac^2d^4e^2+Ba^2de^5+18B\cdots}{\cdots}}{\cdots}$
default	$\frac{\left(2Aace^4+6Ac^2d^2e^2-6Bacde^3-10Bc^2d^3e+a^2Ce^4+12Cacd^2e^2+15Cc^2d^4\right)\ln(ex+d)}{e^7}-\frac{Aa^2e^6+2Aacd^2e^4+Ac^2d^4e^2-Ba^2de^5-\cdots}{2\cdots}$
risch	$\frac{c^2Cx^4}{4e^3}+\frac{c^2Bx^3}{3e^3}-\frac{c^2Cdx^3}{e^4}+\frac{c^2Ax^2}{2e^3}-\frac{3c^2Bdx^2}{2e^4}+\frac{cCax^2}{e^3}+\frac{3c^2Cd^2x^2}{e^5}-\frac{3c^2Adx}{e^4}+\frac{2cBax}{e^3}+\frac{6c^2Bd^2x}{e^5}-\frac{6cCadx}{e^4}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^7*(2*A*a*c*e^4+6*A*c^2*d^2*e^2-6*B*a*c*d*e^3-10*B*c^2*d^3*e+C*a^2*e^4+1
2*C*a*c*d^2*e^2+15*C*c^2*d^4)*ln(e*x+d)-1/2*(A*a^2*e^6+2*A*a*c*d^2*e^4+A*c^
2*d^4*e^2-B*a^2*d*e^5-2*B*a*c*d^3*e^3-B*c^2*d^5*e+C*a^2*d^2*e^4+2*C*a*c*d^4
*e^2+C*c^2*d^6)/e^7/(e*x+d)^2-(-4*A*a*c*d*e^4-4*A*c^2*d^3*e^2+B*a^2*e^5+6*B
*a*c*d^2*e^3+5*B*c^2*d^4*e-2*C*a^2*d*e^4-8*C*a*c*d^3*e^2-6*C*c^2*d^5)/e^7/(
e*x+d)-c/e^6*(-1/4*c*C*x^4*e^3-1/3*B*c*e^3*x^3+C*c*d*e^2*x^3-1/2*A*c*e^3*x^
2+3/2*B*c*d*e^2*x^2-C*a*e^3*x^2-3*C*c*d^2*e*x^2+3*A*c*d*e^2*x-2*B*a*e^3*x-6
*B*c*d^2*e*x+6*C*a*d*e^2*x+10*C*c*d^3*x)
```

Maxima [A]
time = 0.29, size = 395, normalized size = 1.34

$$\left(15\,Cc^2d^4-10\,Bc^2d^3e-6\,Bacde^3+Ca^2e^4+2\,Aace^4+6\,\left(2\,Cace^2+Ac^2e^2\right)d^2\right)e^{(-7)}\log\left(xe+d\right)+\frac{1}{12}\,\left(3\,Cc^2x^4e^3-4\,\left(3\,Cc^2de^2-Bc^2e^3\right)x^3+6\,\left(6\,Cc^2d^2e-3\,Bc^2de^2+2\,Cace^3+Ac^2e^3\right)x^2-12\,\left(10\,Cc^2d^3-6\,Bc^2d^2e-2\,Bace^3+3\,\left(2\,Cace^2+Ac^2e^2\right)d\right)x\right)e^{(-6)}+\frac{11\,Cc^2d^6-9\,Bc^2d^5e-10\,Bacd^3e^3+7\,\left(2\,Cace^2+Ac^2e^2\right)d^4-Ba^2de^5-Aa^2e^6+3\,\left(Ca^2e^4+2\,Aace^4\right)d^2+2\,\left(6\,Cc^2d^5e-5\,Bc^2d^4e^2-6\,Bacd^2e^4+4\,\left(2\,Cace^3+Ac^2e^3\right)d^3-Ba^2e^6+2\,\left(Ca^2e^5+2\,Aace^5\right)d\right)x}{2\,\left(x^2e^9+2\,dxe^8+d^2e^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] (15*C*c^2*d^4 - 10*B*c^2*d^3*e - 6*B*a*c*d*e^3 + C*a^2*e^4 + 2*A*a*c*e^4 +
6*(2*C*a*c*e^2 + A*c^2*e^2)*d^2)*e^(-7)*log(x*e + d) + 1/12*(3*C*c^2*x^4*e^
3 - 4*(3*C*c^2*d*e^2 - B*c^2*e^3)*x^3 + 6*(6*C*c^2*d^2*e - 3*B*c^2*d*e^2 +
2*C*a*c*e^3 + A*c^2*e^3)*x^2 - 12*(10*C*c^2*d^3 - 6*B*c^2*d^2*e - 2*B*a*c*e
^3 + 3*(2*C*a*c*e^2 + A*c^2*e^2)*d)*x)*e^(-6) + 1/2*(11*C*c^2*d^6 - 9*B*c^2
*d^5*e - 10*B*a*c*d^3*e^3 + 7*(2*C*a*c*e^2 + A*c^2*e^2)*d^4 - B*a^2*d*e^5 -
 A*a^2*e^6 + 3*(C*a^2*e^4 + 2*A*a*c*e^4)*d^2 + 2*(6*C*c^2*d^5*e - 5*B*c^2*d
^4*e^2 - 6*B*a*c*d^2*e^4 + 4*(2*C*a*c*e^3 + A*c^2*e^3)*d^3 - B*a^2*e^6 + 2*
(C*a^2*e^5 + 2*A*a*c*e^5)*d)*x)/(x^2*e^9 + 2*d*x*e^8 + d^2*e^7)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(289) = 578$.
time = 0.35, size = 580, normalized size = 1.97

$$\frac{66\,Cc^2d^6 + (3\,Cc^2x^6 + 4\,Bc^2x^5 + 24\,Bacx^3 + 6\,(2\,Cac + Ac^2)x^4 - 12\,Ba^2x - 6\,Aa^2)e^6 - 2\,(3\,Cc^2dx^5 + 5\,Bc^2dx^4 - 24\,Bacdx^2 + 12\,(2\,Cac + Ac^2)dx^3 + 3\,Ba^2d - 12\,(Ca^2 + 2\,Aac)dx)e^5 + (15\,Cc^2d^2x^4 + 40\,Bc^2d^2x^3 - 48\,Bacd^2x - 66\,(2\,Cac + Ac^2)d^2x^2 + 18\,(Ca^2 + 2\,Aac)d^2)e^4 - 6\,(10\,Cc^2d^3x^3 - 21\,Bc^2d^3x^2 + 10\,Bacd^3 - 2\,(2\,Cac + Ac^2)d^3x)e^3 - 6\,(34\,Cc^2d^4x^2 - 2\,Bc^2d^4x - 7\,(2\,Cac + Ac^2)d^4)e^2 - 6\,(8\,Cc^2d^5x + 9\,Bc^2d^5)e + 12\,(15\,Cc^2d^6 + (Ca^2 + 2\,Aac)x^2e^6 - 2\,(3\,Bacdx^2 - (Ca^2 + 2\,Aac)dx)e^5 - (12\,Bacd^2x - 6\,(2\,Cac + Ac^2)d^2x^2 - (Ca^2 + 2\,Aac)d^2)e^4 - 2\,(5\,Bc^2d^3x^2 + 3\,Bacd^3 - 6\,(2\,Cac + Ac^2)d^3x)e^3 + (15\,Cc^2d^4x^2 - 20\,Bc^2d^4x + 6\,(2\,Cac + Ac^2)d^4)e^2 + 10\,(3\,Cc^2d^5x - Bc^2d^5)e)\log\left(xe + d\right)}{12\,(x^2e^9 + 2\,dxe^8 + d^2e^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(66*C*c^2*d^6 + (3*C*c^2*x^6 + 4*B*c^2*x^5 + 24*B*a*c*x^3 + 6*(2*C*a*c
 + A*c^2)*x^4 - 12*B*a^2*x - 6*A*a^2)*e^6 - 2*(3*C*c^2*d*x^5 + 5*B*c^2*d*x^
4 - 24*B*a*c*d*x^2 + 12*(2*C*a*c + A*c^2)*d*x^3 + 3*B*a^2*d - 12*(C*a^2 + 2
*A*a*c)*d*x)*e^5 + (15*C*c^2*d^2*x^4 + 40*B*c^2*d^2*x^3 - 48*B*a*c*d^2*x -
66*(2*C*a*c + A*c^2)*d^2*x^2 + 18*(C*a^2 + 2*A*a*c)*d^2)*e^4 - 6*(10*C*c^2*
d^3*x^3 - 21*B*c^2*d^3*x^2 + 10*B*a*c*d^3 - 2*(2*C*a*c + A*c^2)*d^3*x)*e^3
- 6*(34*C*c^2*d^4*x^2 - 2*B*c^2*d^4*x - 7*(2*C*a*c + A*c^2)*d^4)*e^2 - 6*(8
*C*c^2*d^5*x + 9*B*c^2*d^5)*e + 12*(15*C*c^2*d^6 + (C*a^2 + 2*A*a*c)*x^2*e^
6 - 2*(3*B*a*c*d*x^2 - (C*a^2 + 2*A*a*c)*d*x)*e^5 - (12*B*a*c*d^2*x - 6*(2*
C*a*c + A*c^2)*d^2*x^2 - (C*a^2 + 2*A*a*c)*d^2)*e^4 - 2*(5*B*c^2*d^3*x^2 +
3*B*a*c*d^3 - 6*(2*C*a*c + A*c^2)*d^3*x)*e^3 + (15*C*c^2*d^4*x^2 - 20*B*c^2
*d^4*x + 6*(2*C*a*c + A*c^2)*d^4)*e^2 + 10*(3*C*c^2*d^5*x - B*c^2*d^5)*e)*l
og(x*e + d))/(x^2*e^9 + 2*d*x*e^8 + d^2*e^7)
```

Sympy [**A**]
time = 5.69, size = 474, normalized size = 1.61

$$\frac{Cc^2x^4}{4e^3} + x^3\left(\frac{Bc^2}{3e^3} - \frac{Cc^2d}{e^4}\right) + x^2\left(\frac{Ac^2}{2e^3} - \frac{3Bc^2d}{2e^4} + \frac{Cac}{e^3} + \frac{3Cc^2d^2}{e^5}\right) + x\left(-\frac{3Ac^2d}{e^4} + \frac{2Bac}{e^3} + \frac{6Bc^2d^2}{e^5} - \frac{6Cacd}{e^4} - \frac{10Cc^2d^3}{e^6}\right) + \frac{-Aa^2e^6 + 6Aacd^2e^4 + 7Ac^2d^4e^2 - Ba^2de^5 - 10Bacd^3e^3 - 9Bc^2d^5e + 3Ca^2d^2e^4 + 14Cacd^4e^2 + 11Cc^2d^6 + x(8Aacde^5 + 8Ac^2d^3e^3 - 2Ba^2e^6 - 12Bacd^2e^4 - 10Bc^2d^4e^2 + 4Ca^2de^5 + 16Cacd^3e^3 + 12Cc^2d^5e)}{2d^2e^7 + 4de^8x + 2e^9x^2} + \frac{(2Aace^4 + 6Ac^2d^2e^2 - 6Bacde^3 - 10Bc^2d^3e + Ca^2e^4 + 12Cacd^2e^2 + 15Cc^2d^4)\log{\left(d + ex \right)}}{e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

```
[Out] C*c**2*x**4/(4*e**3) + x**3*(B*c**2/(3*e**3) - C*c**2*d/e**4) + x**2*(A*c**
2/(2*e**3) - 3*B*c**2*d/(2*e**4) + C*a*c/e**3 + 3*C*c**2*d**2/e**5) + x*(-3
*A*c**2*d/e**4 + 2*B*a*c/e**3 + 6*B*c**2*d**2/e**5 - 6*C*a*c*d/e**4 - 10*C*
```

```
c**2*d**3/e**6) + (-A*a**2*e**6 + 6*A*a*c*d**2*e**4 + 7*A*c**2*d**4*e**2 -
B*a**2*d*e**5 - 10*B*a*c*d**3*e**3 - 9*B*c**2*d**5*e + 3*C*a**2*d**2*e**4 +
 14*C*a*c*d**4*e**2 + 11*C*c**2*d**6 + x*(8*A*a*c*d*e**5 + 8*A*c**2*d**3*e*
*3 - 2*B*a**2*e**6 - 12*B*a*c*d**2*e**4 - 10*B*c**2*d**4*e**2 + 4*C*a**2*d*
e**5 + 16*C*a*c*d**3*e**3 + 12*C*c**2*d**5*e))/(2*d**2*e**7 + 4*d*e**8*x +
2*e**9*x**2) + (2*A*a*c*e**4 + 6*A*c**2*d**2*e**2 - 6*B*a*c*d*e**3 - 10*B*c
**2*d**3*e + C*a**2*e**4 + 12*C*a*c*d**2*e**2 + 15*C*c**2*d**4)*log(d + e*x
)/e**7
```

Giac [**A**]
time = 4.49, size = 397, normalized size = 1.35

$$\left(15\,Cc^2d^4 - 10\,Bc^2d^3e + 12\,Cacd^2e^2 + 6\,Ac^2d^2e^2 - 6\,Bacde^3 + Ca^2e^4 + 2\,Aace^4\right)e^{(-7)}\log\left(|xe+d|\right) + \frac{1}{12}\left(3\,Cc^2x^4e^9 - 12\,Cc^2dx^3e^8 + 36\,Cc^2d^2x^2e^7 - 120\,Cc^2d^3xe^6 + 4\,Bc^2x^3e^9 - 18\,Bc^2dx^2e^8 + 72\,Bc^2d^2xe^7 + 12\,Cacx^2e^9 + 6\,Ac^2x^2e^9 - 72\,Cacdxe^8 - 36\,Ac^2dxe^8 + 24\,Bacxe^9\right)e^{(-12)} + \frac{\left(11\,Cc^2d^6 - 9\,Bc^2d^5e + 14\,Cacd^4e^2 + 7\,Ac^2d^4e^2 - 10\,Bacd^3e^3 + 3\,Ca^2d^2e^4 + 6\,Aacd^2e^4 - Ba^2de^5 - Aa^2e^6 + 2\left(6\,Cc^2d^5e - 5\,Bc^2d^4e^2 + 8\,Cacd^3e^3 + 4\,Ac^2d^3e^3 - 6\,Bacd^2e^4 + 2\,Ca^2de^5 + 4\,Aacde^5 - Ba^2e^6\right)x\right)e^{(-7)}}{2\left(xe+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] (15*C*c^2*d^4 - 10*B*c^2*d^3*e + 12*C*a*c*d^2*e^2 + 6*A*c^2*d^2*e^2 - 6*B*a
*c*d*e^3 + C*a^2*e^4 + 2*A*a*c*e^4)*e^(-7)*log(abs(x*e + d)) + 1/12*(3*C*c^
2*x^4*e^9 - 12*C*c^2*d*x^3*e^8 + 36*C*c^2*d^2*x^2*e^7 - 120*C*c^2*d^3*x*e^6
 + 4*B*c^2*x^3*e^9 - 18*B*c^2*d*x^2*e^8 + 72*B*c^2*d^2*x*e^7 + 12*C*a*c*x^2
*e^9 + 6*A*c^2*x^2*e^9 - 72*C*a*c*d*x*e^8 - 36*A*c^2*d*x*e^8 + 24*B*a*c*x*e
^9)*e^(-12) + 1/2*(11*C*c^2*d^6 - 9*B*c^2*d^5*e + 14*C*a*c*d^4*e^2 + 7*A*c^
2*d^4*e^2 - 10*B*a*c*d^3*e^3 + 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 - B*a^2*d*
e^5 - A*a^2*e^6 + 2*(6*C*c^2*d^5*e - 5*B*c^2*d^4*e^2 + 8*C*a*c*d^3*e^3 + 4*
A*c^2*d^3*e^3 - 6*B*a*c*d^2*e^4 + 2*C*a^2*d*e^5 + 4*A*a*c*d*e^5 - B*a^2*e^6
)*x)*e^(-7)/(x*e + d)^2
```

Mupad [**B**]
time = 3.82, size = 495, normalized size = 1.68

$$x\left(\frac{3\,d\left(\frac{3\,d\left(\frac{B\,c^2}{e^3}-\frac{3\,C\,c^2\,d}{e^4}\right)}{e}-\frac{A\,c^2+2\,C\,a\,c}{e^3}+\frac{3\,C\,c^2\,d^2}{e^5}\right)}{e}-\frac{3\,d^2\left(\frac{B\,c^2}{e^3}-\frac{3\,C\,c^2\,d}{e^4}\right)}{e^2}+\frac{2\,B\,a\,c}{e^3}-\frac{C\,c^2\,d^3}{e^6}\right)+x^3\left(\frac{B\,c^2}{3\,e^3}-\frac{C\,c^2\,d}{e^4}\right)-x^2\left(\frac{3\,d\left(\frac{B\,c^2}{e^3}-\frac{3\,C\,c^2\,d}{e^4}\right)}{2\,e}-\frac{A\,c^2+2\,C\,a\,c}{2\,e^3}+\frac{3\,C\,c^2\,d^2}{2\,e^5}\right)+\frac{\frac{3\,C\,a^2\,d^2\,e^4-B\,a^2\,d\,e^5-A\,a^2\,e^6+14\,C\,a\,c\,d^4\,e^2-10\,B\,a\,c\,d^3\,e^3+6\,A\,a\,c\,d^2\,e^4+11\,C\,c^2\,d^6-9\,B\,c^2\,d^5\,e+7\,A\,c^2\,d^4\,e^2}{2\,e}+x\left(2\,C\,a^2\,d\,e^4-B\,a^2\,e^5+8\,C\,a\,c\,d^3\,e^2-6\,B\,a\,c\,d^2\,e^3+4\,A\,a\,c\,d\,e^4+6\,C\,c^2\,d^5-5\,B\,c^2\,d^4\,e+4\,A\,c^2\,d^3\,e^2\right)}{d^2\,e^6+2\,d\,e^7\,x+e^8\,x^2}+\frac{\ln\left(d+e\,x\right)\left(C\,a^2\,e^4+12\,C\,a\,c\,d^2\,e^2-6\,B\,a\,c\,d\,e^3+2\,A\,a\,c\,e^4+15\,C\,c^2\,d^4-10\,B\,c^2\,d^3\,e+6\,A\,c^2\,d^2\,e^2\right)}{e^7}+\frac{C\,c^2\,x^4}{4\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

```
[Out] x*((3*d*((3*d*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/e - (A*c^2 + 2*C*a*c)/e^3 +
(3*C*c^2*d^2)/e^5))/e - (3*d^2*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/e^2 + (2*B*
a*c)/e^3 - (C*c^2*d^3)/e^6) + x^3*((B*c^2)/(3*e^3) - (C*c^2*d)/e^4) - x^2*(
(3*d*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/(2*e) - (A*c^2 + 2*C*a*c)/(2*e^3) + (
3*C*c^2*d^2)/(2*e^5)) + ((11*C*c^2*d^6 - A*a^2*e^6 - B*a^2*d*e^5 - 9*B*c^2*
d^5*e + 7*A*c^2*d^4*e^2 + 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 - 10*B*a*c*d^3*
e^3 + 14*C*a*c*d^4*e^2)/(2*e) + x*(6*C*c^2*d^5 - B*a^2*e^5 + 2*C*a^2*d*e^4
- 5*B*c^2*d^4*e + 4*A*c^2*d^3*e^2 + 4*A*a*c*d*e^4 - 6*B*a*c*d^2*e^3 + 8*C*a
*c*d^3*e^2))/(d^2*e^6 + e^8*x^2 + 2*d*e^7*x) + (log(d + e*x)*(C*a^2*e^4 + 1
5*C*c^2*d^4 + 2*A*a*c*e^4 - 10*B*c^2*d^3*e + 6*A*c^2*d^2*e^2 - 6*B*a*c*d*e^
3 + 12*C*a*c*d^2*e^2))/e^7 + (C*c^2*x^4)/(4*e^3)
```

3.32 $\int (d+ex)^3 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) \, dx$

Optimal. Leaf size=404

$$a^3Ad^3x+\frac{1}{3}a^2d\left(ad(Cd+3Be)+3A\left(cd^2+ae^2\right)\right)x^3+\frac{1}{4}a^3e\left(3Cd^2+e(3Bd+Ae)\right)x^4+\frac{1}{5}a\left(3Acd\left(cd^2+3ae^2\right)\right.$$

```
[Out] a^3*A*d^3*x+1/3*a^2*d*(a*d*(3*B*e+C*d)+3*A*(a*e^2+c*d^2))*x^3+1/4*a^3*e*(3*
C*d^2+e*(A*e+3*B*d))*x^4+1/5*a*(3*A*c*d*(3*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d
)+3*c*d^2*(3*B*e+C*d)))*x^5+1/6*a^2*e*(a*C*e^2+3*c*(3*C*d^2+e*(A*e+3*B*d)))
*x^6+1/7*c*(A*c*d*(9*a*e^2+c*d^2)+3*a*(a*e^2*(B*e+3*C*d)+c*d^2*(3*B*e+C*d))
)*x^7+3/8*a*c*e*(a*C*e^2+c*(3*C*d^2+e*(A*e+3*B*d)))*x^8+1/9*c^2*(3*a*e^2*(B
*e+3*C*d)+c*d*(C*d^2+3*e*(A*e+B*d)))*x^9+1/10*c^2*e*(3*a*C*e^2+c*(3*C*d^2+e
*(A*e+3*B*d)))*x^10+1/11*c^3*e^2*(B*e+3*C*d)*x^11+1/12*c^3*C*e^3*x^12+1/8*d
^2*(3*A*e+B*d)*(c*x^2+a)^4/c
```

Rubi [A]
time = 0.42, antiderivative size = 400, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1596, 1824}

$$\frac{1}{4}a^3ex^4\left(e(Ae+3Bd)+3Cd^2\right)+a^3Ad^3x+\frac{1}{6}a^2ex^6\left(aCe^2+3ce(Ae+3Bd)+9cCd^2\right)+\frac{1}{3}a^2dx^3\left(3A\left(ae^2+cd^2\right)+ad(3Be+Cd)\right)+\frac{1}{9}c^2x^9\left(3ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3\right)+\frac{1}{10}c^2ex^{10}\left(3aCe^2+ce(Ae+3Bd)+3cCd^2\right)+\frac{3}{8}acex^8\left(aCe^2+ce(Ae+3Bd)+3cCd^2\right)+\frac{1}{7}cx^7\left(Acd\left(9ae^2+cd^2\right)+3a\left(ae^2(Be+3Cd)+cd^2(3Be+Cd)\right)\right)+\frac{1}{5}ax^5\left(3Acd\left(3ae^2+cd^2\right)+a\left(ae^2(Be+3Cd)+3cd^2(3Be+Cd)\right)\right)+\frac{d^2\left(a+cx^2\right)^4(3Ae+Bd)}{8c}+\frac{1}{11}c^3e^2x^{11}(Be+3Cd)+\frac{1}{12}c^3Ce^3x^{12}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^3*x + (a^2*d*(a*d*(C*d + 3*B*e) + 3*A*(c*d^2 + a*e^2))*x^3)/3 + (a^
3*e*(3*C*d^2 + e*(3*B*d + A*e))*x^4)/4 + (a*(3*A*c*d*(c*d^2 + 3*a*e^2) + a*
(a*e^2*(3*C*d + B*e) + 3*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a^2*e*(9*c*C*d^2 +
 a*C*e^2 + 3*c*e*(3*B*d + A*e))*x^6)/6 + (c*(A*c*d*(c*d^2 + 9*a*e^2) + 3*a*
(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*x^7)/7 + (3*a*c*e*(3*c*C*d^2 +
 a*C*e^2 + c*e*(3*B*d + A*e))*x^8)/8 + (c^2*(c*C*d^3 + 3*c*d*e*(B*d + A*e)
+ 3*a*e^2*(3*C*d + B*e))*x^9)/9 + (c^2*e*(3*c*C*d^2 + 3*a*C*e^2 + c*e*(3*B*
d + A*e))*x^10)/10 + (c^3*e^2*(3*C*d + B*e)*x^11)/11 + (c^3*C*e^3*x^12)/12
+ (d^2*(B*d + 3*A*e)*(a + c*x^2)^4)/(8*c)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] && !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right)\,dx = \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^4}{8c} + \int \left(a+cx^2\right)^3 \left(-\left(Bd^3+3Ad^2e\right)x\right.$$

$$= \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^4}{8c} + \int \left(a^3Ad^3 + a^2d\left(ad(Cd+3Be)+\right.\right.$$

$$= a^3Ad^3x + \frac{1}{3}a^2d\left(ad(Cd+3Be)+3A\left(cd^2+ae^2\right)\right)x^3 + \frac{1}{4}a^3e(3$$

Mathematica [A]
time = 0.14, size = 459, normalized size = 1.14

$$a^3Ad^3x+\frac{1}{2}a^3d^2(Bd+3Ae)x^2+\frac{1}{3}a^2d\left(ad(Cd+3Be)+3A\left(cd^2+ae^2\right)\right)x^3+\frac{1}{4}a^2\left(3Bcd^3+9Acd^2e+3aCd^2e+3aBde^2+aAe^3\right)x^4+\frac{1}{5}a\left(3Acd\left(cd^2+3ae^2\right)+a\left(ae^2(3Cd+Be)+3cd^2(Cd+3Be)\right)\right)x^5+\frac{1}{6}a\left(3Ace\left(3cd^2+ae^2\right)+aCe\left(9cd^2+ae^2\right)+3Bcd\left(cd^2+3ae^2\right)\right)x^6+\frac{1}{7}c\left(Acd\left(cd^2+9ae^2\right)+3a\left(ae^2(3Cd+Be)+cd^2(Cd+3Be)\right)\right)x^7+\frac{1}{8}c\left(Bcd\left(cd^2+9ae^2\right)+3e\left(Ac\left(cd^2+ae^2\right)+aC\left(3cd^2+ae^2\right)\right)\right)x^8+\frac{1}{9}c^2\left(cCd^3+3cde(Bd+Ae)+3ae^2(3Cd+Be)\right)x^9+\frac{1}{10}c^2e\left(3cCd^2+3aCe^2+ce(3Bd+Ae)\right)x^{10}+\frac{1}{11}c^3e^2(3Cd+Be)x^{11}+\frac{1}{12}c^3Ce^3x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^3*x + (a^3*d^2*(B*d + 3*A*e)*x^2)/2 + (a^2*d*(a*d*(C*d + 3*B*e) + 3
*A*(c*d^2 + a*e^2))*x^3)/3 + (a^2*(3*B*c*d^3 + 9*A*c*d^2*e + 3*a*C*d^2*e +
3*a*B*d*e^2 + a*A*e^3)*x^4)/4 + (a*(3*A*c*d*(c*d^2 + 3*a*e^2) + a*(a*e^2*(3
*C*d + B*e) + 3*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a*(3*A*c*e*(3*c*d^2 + a*e^2
) + a*C*e*(9*c*d^2 + a*e^2) + 3*B*c*d*(c*d^2 + 3*a*e^2))*x^6)/6 + (c*(A*c*d
*(c*d^2 + 9*a*e^2) + 3*a*(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*x^7)/
7 + (c*(B*c*d*(c*d^2 + 9*a*e^2) + 3*e*(A*c*(c*d^2 + a*e^2) + a*C*(3*c*d^2 +
 a*e^2)))*x^8)/8 + (c^2*(c*C*d^3 + 3*c*d*e*(B*d + A*e) + 3*a*e^2*(3*C*d + B
*e))*x^9)/9 + (c^2*e*(3*c*C*d^2 + 3*a*C*e^2 + c*e*(3*B*d + A*e))*x^10)/10 +
 (c^3*e^2*(3*C*d + B*e)*x^11)/11 + (c^3*C*e^3*x^12)/12
```

Maple [A]
time = 0.10, size = 553, normalized size = 1.37 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*c^3*C*e^3*x^12+1/11*(B*c^3*e^3+3*C*c^3*d*e^2)*x^11+1/10*((3*a*c^2*e^3+
3*c^3*d^2*e)*C+3*d*e^2*c^3*B+e^3*c^3*A)*x^10+1/9*((9*a*c^2*d*e^2+c^3*d^3)*C
+(3*a*c^2*e^3+3*c^3*d^2*e)*B+3*d*e^2*c^3*A)*x^9+1/8*((3*a^2*c*e^3+9*a*c^2*d
^2*e)*C+(9*a*c^2*d*e^2+c^3*d^3)*B+(3*a*c^2*e^3+3*c^3*d^2*e)*A)*x^8+1/7*((9*
```

```
a^2*c*d*e^2+3*a*c^2*d^3)*C+(3*a^2*c*e^3+9*a*c^2*d^2*e)*B+(9*a*c^2*d*e^2+c^3
*d^3)*A)*x^7+1/6*((a^3*e^3+9*a^2*c*d^2*e)*C+(9*a^2*c*d*e^2+3*a*c^2*d^3)*B+(
3*a^2*c*e^3+9*a*c^2*d^2*e)*A)*x^6+1/5*((3*a^3*d*e^2+3*a^2*c*d^3)*C+(a^3*e^3
+9*a^2*c*d^2*e)*B+(9*a^2*c*d*e^2+3*a*c^2*d^3)*A)*x^5+1/4*(3*d^2*e*a^3*C+(3*
a^3*d*e^2+3*a^2*c*d^3)*B+(a^3*e^3+9*a^2*c*d^2*e)*A)*x^4+1/3*(d^3*a^3*C+3*d^
2*e*a^3*B+(3*a^3*d*e^2+3*a^2*c*d^3)*A)*x^3+1/2*(3*A*a^3*d^2*e+B*a^3*d^3)*x^
2+a^3*A*d^3*x
```

Maxima [A]
time = 0.32, size = 516, normalized size = 1.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/12*C*c^3*x^12*e^3 + 1/11*(3*C*c^3*d*e^2 + B*c^3*e^3)*x^11 + 1/10*(3*C*c^3
*d^2*e + 3*B*c^3*d*e^2 + 3*C*a*c^2*e^3 + A*c^3*e^3)*x^10 + 1/9*(C*c^3*d^3 +
 3*B*c^3*d^2*e + 3*B*a*c^2*e^3 + 3*(3*C*a*c^2*e^2 + A*c^3*e^2)*d)*x^9 + 1/8
*(B*c^3*d^3 + 9*B*a*c^2*d*e^2 + 3*C*a^2*c*e^3 + 3*A*a*c^2*e^3 + 3*(3*C*a*c^
2*e + A*c^3*e)*d^2)*x^8 + A*a^3*d^3*x + 1/7*(9*B*a*c^2*d^2*e + 3*B*a^2*c*e^
3 + (3*C*a*c^2 + A*c^3)*d^3 + 9*(C*a^2*c*e^2 + A*a*c^2*e^2)*d)*x^7 + 1/6*(3
*B*a*c^2*d^3 + 9*B*a^2*c*d*e^2 + C*a^3*e^3 + 3*A*a^2*c*e^3 + 9*(C*a^2*c*e +
 A*a*c^2*e)*d^2)*x^6 + 1/5*(9*B*a^2*c*d^2*e + B*a^3*e^3 + 3*(C*a^2*c + A*a*
c^2)*d^3 + 3*(C*a^3*e^2 + 3*A*a^2*c*e^2)*d)*x^5 + 1/4*(3*B*a^2*c*d^3 + 3*B*
a^3*d*e^2 + A*a^3*e^3 + 3*(C*a^3*e + 3*A*a^2*c*e)*d^2)*x^4 + 1/3*(3*B*a^3*d
^2*e + 3*A*a^3*d*e^2 + (C*a^3 + 3*A*a^2*c)*d^3)*x^3 + 1/2*(B*a^3*d^3 + 3*A*
a^3*d^2*e)*x^2
```

Fricas [A]
time = 0.37, size = 516, normalized size = 1.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/9*C*c^3*d^3*x^9 + 1/8*B*c^3*d^3*x^8 + 1/2*B*a*c^2*d^3*x^6 + 3/4*B*a^2*c*d
^3*x^4 + 1/7*(3*C*a*c^2 + A*c^3)*d^3*x^7 + 1/2*B*a^3*d^3*x^2 + 3/5*(C*a^2*c
 + A*a*c^2)*d^3*x^5 + A*a^3*d^3*x + 1/3*(C*a^3 + 3*A*a^2*c)*d^3*x^3 + 1/924
0*(770*C*c^3*x^12 + 840*B*c^3*x^11 + 3080*B*a*c^2*x^9 + 3960*B*a^2*c*x^7 +
924*(3*C*a*c^2 + A*c^3)*x^10 + 1848*B*a^3*x^5 + 3465*(C*a^2*c + A*a*c^2)*x^
8 + 2310*A*a^3*x^4 + 1540*(C*a^3 + 3*A*a^2*c)*x^6)*e^3 + 1/9240*(2520*C*c^3
*d*x^11 + 2772*B*c^3*d*x^10 + 10395*B*a*c^2*d*x^8 + 13860*B*a^2*c*d*x^6 + 3
080*(3*C*a*c^2 + A*c^3)*d*x^9 + 6930*B*a^3*d*x^4 + 11880*(C*a^2*c + A*a*c^2
)*d*x^7 + 9240*A*a^3*d*x^3 + 5544*(C*a^3 + 3*A*a^2*c)*d*x^5)*e^2 + 1/840*(2
```

```
52*C*c^3*d^2*x^10 + 280*B*c^3*d^2*x^9 + 1080*B*a*c^2*d^2*x^7 + 1512*B*a^2*c
*d^2*x^5 + 315*(3*C*a*c^2 + A*c^3)*d^2*x^8 + 840*B*a^3*d^2*x^3 + 1260*(C*a^
2*c + A*a*c^2)*d^2*x^6 + 1260*A*a^3*d^2*x^2 + 630*(C*a^3 + 3*A*a^2*c)*d^2*x
^4)*e
```

Sympy [A]
time = 0.04, size = 646, normalized size = 1.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*d**3*x + C*c**3*e**3*x**12/12 + x**11*(B*c**3*e**3/11 + 3*C*c**3*d*e
**2/11) + x**10*(A*c**3*e**3/10 + 3*B*c**3*d*e**2/10 + 3*C*a*c**2*e**3/10 +
 3*C*c**3*d**2*e/10) + x**9*(A*c**3*d*e**2/3 + B*a*c**2*e**3/3 + B*c**3*d**
2*e/3 + C*a*c**2*d*e**2 + C*c**3*d**3/9) + x**8*(3*A*a*c**2*e**3/8 + 3*A*c*
*3*d**2*e/8 + 9*B*a*c**2*d*e**2/8 + B*c**3*d**3/8 + 3*C*a**2*c*e**3/8 + 9*C
*a*c**2*d**2*e/8) + x**7*(9*A*a*c**2*d*e**2/7 + A*c**3*d**3/7 + 3*B*a**2*c*
e**3/7 + 9*B*a*c**2*d**2*e/7 + 9*C*a**2*c*d*e**2/7 + 3*C*a*c**2*d**3/7) + x
**6*(A*a**2*c*e**3/2 + 3*A*a*c**2*d**2*e/2 + 3*B*a**2*c*d*e**2/2 + B*a*c**2
*d**3/2 + C*a**3*e**3/6 + 3*C*a**2*c*d**2*e/2) + x**5*(9*A*a**2*c*d*e**2/5
+ 3*A*a*c**2*d**3/5 + B*a**3*e**3/5 + 9*B*a**2*c*d**2*e/5 + 3*C*a**3*d*e**2
/5 + 3*C*a**2*c*d**3/5) + x**4*(A*a**3*e**3/4 + 9*A*a**2*c*d**2*e/4 + 3*B*a
**3*d*e**2/4 + 3*B*a**2*c*d**3/4 + 3*C*a**3*d**2*e/4) + x**3*(A*a**3*d*e**2
 + A*a**2*c*d**3 + B*a**3*d**2*e + C*a**3*d**3/3) + x**2*(3*A*a**3*d**2*e/2
 + B*a**3*d**3/2)
```

Giac [A]
time = 4.37, size = 606, normalized size = 1.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/12*C*c^3*x^12*e^3 + 3/11*C*c^3*d*x^11*e^2 + 3/10*C*c^3*d^2*x^10*e + 1/9*C
*c^3*d^3*x^9 + 1/11*B*c^3*x^11*e^3 + 3/10*B*c^3*d*x^10*e^2 + 1/3*B*c^3*d^2*
x^9*e + 1/8*B*c^3*d^3*x^8 + 3/10*C*a*c^2*x^10*e^3 + 1/10*A*c^3*x^10*e^3 + C
*a*c^2*d*x^9*e^2 + 1/3*A*c^3*d*x^9*e^2 + 9/8*C*a*c^2*d^2*x^8*e + 3/8*A*c^3*
d^2*x^8*e + 3/7*C*a*c^2*d^3*x^7 + 1/7*A*c^3*d^3*x^7 + 1/3*B*a*c^2*x^9*e^3 +
 9/8*B*a*c^2*d*x^8*e^2 + 9/7*B*a*c^2*d^2*x^7*e + 1/2*B*a*c^2*d^3*x^6 + 3/8*
C*a^2*c*x^8*e^3 + 3/8*A*a*c^2*x^8*e^3 + 9/7*C*a^2*c*d*x^7*e^2 + 9/7*A*a*c^2
*d*x^7*e^2 + 3/2*C*a^2*c*d^2*x^6*e + 3/2*A*a*c^2*d^2*x^6*e + 3/5*C*a^2*c*d^
3*x^5 + 3/5*A*a*c^2*d^3*x^5 + 3/7*B*a^2*c*x^7*e^3 + 3/2*B*a^2*c*d*x^6*e^2 +
 9/5*B*a^2*c*d^2*x^5*e + 3/4*B*a^2*c*d^3*x^4 + 1/6*C*a^3*x^6*e^3 + 1/2*A*a^
```

```
2*c*x^6*e^3 + 3/5*C*a^3*d*x^5*e^2 + 9/5*A*a^2*c*d*x^5*e^2 + 3/4*C*a^3*d^2*x
^4*e + 9/4*A*a^2*c*d^2*x^4*e + 1/3*C*a^3*d^3*x^3 + A*a^2*c*d^3*x^3 + 1/5*B*
a^3*x^5*e^3 + 3/4*B*a^3*d*x^4*e^2 + B*a^3*d^2*x^3*e + 1/2*B*a^3*d^3*x^2 + 1
/4*A*a^3*x^4*e^3 + A*a^3*d*x^3*e^2 + 3/2*A*a^3*d^2*x^2*e + A*a^3*d^3*x
```

Mupad [B]
time = 4.05, size = 490, normalized size = 1.21

$$x^5\left(\frac{3Ca^3de^2}{5}+\frac{Ba^3e^3}{5}+\frac{3Ca^2cd^3}{5}+\frac{9Ba^2cd^2e}{5}+\frac{9Aa^2cde^2}{5}+\frac{3Aac^2d^3}{5}\right)+x^8\left(\frac{3Ca^2ce^3}{8}+\frac{9Cac^2d^2e}{8}+\frac{9Bac^2de^2}{8}+\frac{3Aac^2e^3}{8}+\frac{Bc^3d^3}{8}+\frac{3Ac^3d^2e}{8}\right)+x^6\left(\frac{Ca^3e^3}{6}+\frac{3Ca^2cd^2e}{2}+\frac{3Ba^2cde^2}{2}+\frac{Aa^2ce^3}{2}+\frac{Bac^2d^3}{2}+\frac{3Aac^2d^2e}{2}\right)+x^7\left(\frac{9Ca^2cde^2}{7}+\frac{3Ba^2ce^3}{7}+\frac{3Cac^2d^3}{7}+\frac{9Bac^2d^2e}{7}+\frac{9Aac^2de^2}{7}+\frac{Ac^3d^3}{7}\right)+\frac{a^2x^4\left(Aae^3+3Bcd^3+3Bade^2+9Acd^2e+3Cad^2e\right)}{4}+\frac{c^2x^9\left(3Bae^3+Ccd^3+3Acde^2+9Cade^2+3Bcd^2e\right)}{9}+\frac{Cc^3e^3x^{12}}{12}+\frac{a^3d^2x^2\left(3Ae+Bd\right)}{2}+\frac{c^3e^2x^{11}\left(Be+3Cd\right)}{11}+Aa^3d^3x+\frac{a^2dx^3\left(3Aae^2+3Acd^2+Cad^2+3Bade\right)}{3}+\frac{c^2ex^{10}\left(Ace^2+3Cae^2+3Ccd^2+3Bcde\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^3*(d + e*x)^3*(A + B*x + C*x^2),x)
```

```
[Out] x^5*((B*a^3*e^3)/5 + (3*A*a*c^2*d^3)/5 + (3*C*a^2*c*d^3)/5 + (3*C*a^3*d*e^2
)/5 + (9*A*a^2*c*d*e^2)/5 + (9*B*a^2*c*d^2*e)/5) + x^8*((B*c^3*d^3)/8 + (3*
A*a*c^2*e^3)/8 + (3*C*a^2*c*e^3)/8 + (3*A*c^3*d^2*e)/8 + (9*B*a*c^2*d*e^2)/
8 + (9*C*a*c^2*d^2*e)/8) + x^6*((C*a^3*e^3)/6 + (A*a^2*c*e^3)/2 + (B*a*c^2*
d^3)/2 + (3*A*a*c^2*d^2*e)/2 + (3*B*a^2*c*d*e^2)/2 + (3*C*a^2*c*d^2*e)/2) +
 x^7*((A*c^3*d^3)/7 + (3*B*a^2*c*e^3)/7 + (3*C*a*c^2*d^3)/7 + (9*A*a*c^2*d*
e^2)/7 + (9*B*a*c^2*d^2*e)/7 + (9*C*a^2*c*d*e^2)/7) + (a^2*x^4*(A*a*e^3 + 3
*B*c*d^3 + 3*B*a*d*e^2 + 9*A*c*d^2*e + 3*C*a*d^2*e))/4 + (c^2*x^9*(3*B*a*e^
3 + C*c*d^3 + 3*A*c*d*e^2 + 9*C*a*d*e^2 + 3*B*c*d^2*e))/9 + (C*c^3*e^3*x^12
)/12 + (a^3*d^2*x^2*(3*A*e + B*d))/2 + (c^3*e^2*x^11*(B*e + 3*C*d))/11 + A*
a^3*d^3*x + (a^2*d*x^3*(3*A*a*e^2 + 3*A*c*d^2 + C*a*d^2 + 3*B*a*d*e))/3 + (
c^2*e*x^10*(A*c*e^2 + 3*C*a*e^2 + 3*C*c*d^2 + 3*B*c*d*e))/10
```

3.33 $\int (d+ex)^2 (a+cx^2)^3 (A+Bx+Cx^2)\, dx$

Optimal. Leaf size=289

$$a^3Ad^2x+\frac{1}{3}a^2\left(ad(Cd+2Be)+A\left(3cd^2+ae^2\right)\right)x^3+\frac{1}{4}a^3e(2Cd+Be)x^4+\frac{1}{5}a\left(3Ac\left(cd^2+ae^2\right)+a\left(aCe^2+3cd(\right.\right.$$

```
[Out] a^3*A*d^2*x+1/3*a^2*(a*d*(2*B*e+C*d)+A*(a*e^2+3*c*d^2))*x^3+1/4*a^3*e*(B*e+
2*C*d)*x^4+1/5*a*(3*A*c*(a*e^2+c*d^2)+a*(a*C*e^2+3*c*d*(2*B*e+C*d)))*x^5+1/
2*a^2*c*e*(B*e+2*C*d)*x^6+1/7*c*(A*c*(3*a*e^2+c*d^2)+3*a*(a*C*e^2+c*d*(2*B*
e+C*d)))*x^7+3/8*a*c^2*e*(B*e+2*C*d)*x^8+1/9*c^2*(3*a*C*e^2+c*(C*d^2+e*(A*e
+2*B*d)))*x^9+1/10*c^3*e*(B*e+2*C*d)*x^10+1/11*c^3*C*e^2*x^11+1/8*d*(2*A*e+
B*d)*(c*x^2+a)^4/c
```

Rubi [A]
time = 0.27, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1596, 1824}

$$a^3Ad^2x+\frac{1}{4}a^3ex^4(Be+2Cd)+\frac{1}{3}a^2x^3\left(A\left(ae^2+3cd^2\right)+ad(2Be+Cd)\right)+\frac{1}{2}a^2cex^6(Be+2Cd)+\frac{1}{9}c^2x^9\left(3aCe^2+ce(Ae+2Bd)+cCd^2\right)+\frac{1}{7}cx^7\left(Ac\left(3ae^2+cd^2\right)+3a\left(aCe^2+cd(2Be+Cd)\right)\right)+\frac{1}{5}ax^5\left(3Ac\left(ae^2+cd^2\right)+a\left(aCe^2+3cd(2Be+Cd)\right)\right)+\frac{d\left(a+cx^2\right)^4(2Ae+Bd)}{8c}+\frac{3}{8}ac^2ex^8(Be+2Cd)+\frac{1}{10}c^3ex^{10}(Be+2Cd)+\frac{1}{11}c^3Ce^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^2*x + (a^2*(a*d*(C*d + 2*B*e) + A*(3*c*d^2 + a*e^2))*x^3)/3 + (a^3*
e*(2*C*d + B*e)*x^4)/4 + (a*(3*A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + 3*c*d*(C*
d + 2*B*e)))*x^5)/5 + (a^2*c*e*(2*C*d + B*e)*x^6)/2 + (c*(A*c*(c*d^2 + 3*a*
e^2) + 3*a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x^7)/7 + (3*a*c^2*e*(2*C*d + B*e)
*x^8)/8 + (c^2*(c*C*d^2 + 3*a*C*e^2 + c*e*(2*B*d + A*e))*x^9)/9 + (c^3*e*(2
*C*d + B*e)*x^10)/10 + (c^3*C*e^2*x^11)/11 + (d*(B*d + 2*A*e)*(a + c*x^2)^4
)/(8*c)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) \, dx &= \frac{d(Bd+2Ae)\left(a+cx^2\right)^4}{8c} + \int \left(a+cx^2\right)^3 \left(-\left(Bd^2+2Ade\right) x\right.\\ &= \frac{d(Bd+2Ae)\left(a+cx^2\right)^4}{8c} + \int \left(a^3Ad^2+a^2\left(ad(Cd+2Be)+\right.\right.\\ &= a^3Ad^2x+\frac{1}{3}a^2\left(ad(Cd+2Be)+A\left(3cd^2+ae^2\right)\right)x^3+\frac{1}{4}a^3e(2\end{aligned}$$

Mathematica [A]
time = 0.09, size = 329, normalized size = 1.14

$$a^3Ad^2x+\frac{1}{2}a^3d(Bd+2Ae)x^2+\frac{1}{3}a^2\left(ad(Cd+2Be)+A\left(3cd^2+ae^2\right)\right)x^3+\frac{1}{4}a^2\left(3Bcd^2+6Acde+2aCde+aBe^2\right)x^4+\frac{1}{5}a\left(3Ac\left(cd^2+ae^2\right)+a\left(aCe^2+3cd(Cd+2Be)\right)\right)x^5+\frac{1}{2}ac\left(2(Ac+aC)de+B\left(cd^2+ae^2\right)\right)x^6+\frac{1}{7}c\left(Ac\left(cd^2+3ae^2\right)+3a\left(aCe^2+cd(Cd+2Be)\right)\right)x^7+\frac{1}{8}c^2\left(Bcd^2+2Acde+6aCde+3aBe^2\right)x^8+\frac{1}{9}c^2\left(cCd^2+3aCe^2+ce(2Bd+Ae)\right)x^9+\frac{1}{10}c^3e(2Cd+Be)x^{10}+\frac{1}{11}c^3Ce^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^2*x + (a^3*d*(B*d + 2*A*e)*x^2)/2 + (a^2*(a*d*(C*d + 2*B*e) + A*(3*
c*d^2 + a*e^2))*x^3)/3 + (a^2*(3*B*c*d^2 + 6*A*c*d*e + 2*a*C*d*e + a*B*e^2)
*x^4)/4 + (a*(3*A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + 3*c*d*(C*d + 2*B*e)))*x^
5)/5 + (a*c*(2*(A*c + a*C)*d*e + B*(c*d^2 + a*e^2))*x^6)/2 + (c*(A*c*(c*d^2
 + 3*a*e^2) + 3*a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x^7)/7 + (c^2*(B*c*d^2 + 2
*A*c*d*e + 6*a*C*d*e + 3*a*B*e^2)*x^8)/8 + (c^2*(c*C*d^2 + 3*a*C*e^2 + c*e*
(2*B*d + A*e))*x^9)/9 + (c^3*e*(2*C*d + B*e)*x^10)/10 + (c^3*C*e^2*x^11)/11
```

Maple [A]
time = 0.10, size = 388, normalized size = 1.34

method	result
norman	$\frac{c^3Ce^2x^{11}}{11}+\left(\frac{1}{10}e^2c^3B+\frac{1}{5}c^3deC\right)x^{10}+\left(\frac{1}{9}e^2c^3A+\frac{2}{9}c^3deB+\frac{1}{3}Ca\,c^2e^2+\frac{1}{9}C\,c^3d^2\right)x^9+\left(\frac{1}{4}c^3deA+\right.$
default	$\frac{c^3Ce^2x^{11}}{11}+\frac{\left(e^2c^3B+2c^3deC\right)x^{10}}{10}+\frac{\left(\left(3e^2c^2a+c^3d^2\right)C+2c^3deB+e^2c^3A\right)x^9}{9}+\frac{\left(6de\,c^2aC+\left(3e^2c^2a+c^3d^2\right)B+2c^3deA\right)x^8}{8}+$
gosper	$\frac{2}{3}x^3de\,a^3B+x^2de\,a^3A+\frac{1}{2}x^6Ba\,c^2d^2+\frac{3}{5}x^5A\,a^2c\,e^2+\frac{3}{5}x^5Aa\,c^2d^2+\frac{3}{5}x^5C\,a^2c\,d^2+\frac{3}{4}x^4B\,a^2c\,d^2+$
risch	$\frac{2}{3}x^3de\,a^3B+x^2de\,a^3A+\frac{1}{2}x^6Ba\,c^2d^2+\frac{3}{5}x^5A\,a^2c\,e^2+\frac{3}{5}x^5Aa\,c^2d^2+\frac{3}{5}x^5C\,a^2c\,d^2+\frac{3}{4}x^4B\,a^2c\,d^2+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c^3*C*e^2*x^11+1/10*(B*c^3*e^2+2*C*c^3*d*e)*x^10+1/9*((3*a*c^2*e^2+c^3
*d^2)*C+2*c^3*d*e*B+e^2*c^3*A)*x^9+1/8*(6*d*e*c^2*a*C+(3*a*c^2*e^2+c^3*d^2)
*B+2*c^3*d*e*A)*x^8+1/7*((3*a^2*c*e^2+3*a*c^2*d^2)*C+6*d*e*c^2*a*B+(3*a*c^2
```

```
*e^2+c^3*d^2)*A)*x^7+1/6*(6*d*e*a^2*c*C+(3*a^2*c*e^2+3*a*c^2*d^2)*B+6*d*e*c
^2*a*A)*x^6+1/5*((a^3*e^2+3*a^2*c*d^2)*C+6*d*e*a^2*c*B+(3*a^2*c*e^2+3*a*c^2
*d^2)*A)*x^5+1/4*(2*d*e*a^3*C+(a^3*e^2+3*a^2*c*d^2)*B+6*d*e*a^2*c*A)*x^4+1/
3*(d^2*a^3*C+2*d*e*a^3*B+(a^3*e^2+3*a^2*c*d^2)*A)*x^3+1/2*(2*A*a^3*d*e+B*a^
3*d^2)*x^2+a^3*A*d^2*x
```

Maxima [A]
time = 0.28, size = 374, normalized size = 1.29

$$\frac{1}{11}\,Cc^3x^{11}e^2+\frac{1}{10}\left(2\,Cc^3de+Bc^3e^2\right)x^{10}+\frac{1}{9}\left(Cc^3d^2+2\,Bc^3de+3\,Cac^2e^2+Ac^3e^2\right)x^9+\frac{1}{8}\left(Bc^3d^2+3\,Bac^2e^2+2\left(3\,Cac^2e+Ac^3e\right)d\right)x^8+\frac{1}{7}\left(6\,Bac^2de+3\,Ca^2ce^2+3\,Aac^2e^2+\left(3\,Cac^2+Ac^3\right)d^2\right)x^7+Aa^3d^2x+\frac{1}{2}\left(Bac^2d^2+Ba^2ce^2+2\left(Ca^2ce+Aac^2e\right)d\right)x^6+\frac{1}{5}\left(6\,Ba^2cde+Ca^3e^2+3\,Aa^2ce^2+3\left(Ca^2c+Aac^2\right)d^2\right)x^5+\frac{1}{4}\left(3\,Ba^2cd^2+Ba^3e^2+2\left(Ca^3e+3\,Aa^2ce\right)d\right)x^4+\frac{1}{3}\left(2\,Ba^3de+Aa^3e^2+\left(Ca^3+3\,Aa^2c\right)d^2\right)x^3+\frac{1}{2}\left(Ba^3d^2+2\,Aa^3de\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/11*C*c^3*x^11*e^2 + 1/10*(2*C*c^3*d*e + B*c^3*e^2)*x^10 + 1/9*(C*c^3*d^2
+ 2*B*c^3*d*e + 3*C*a*c^2*e^2 + A*c^3*e^2)*x^9 + 1/8*(B*c^3*d^2 + 3*B*a*c^2
*e^2 + 2*(3*C*a*c^2*e + A*c^3*e)*d)*x^8 + 1/7*(6*B*a*c^2*d*e + 3*C*a^2*c*e^
2 + 3*A*a*c^2*e^2 + (3*C*a*c^2 + A*c^3)*d^2)*x^7 + A*a^3*d^2*x + 1/2*(B*a*c
^2*d^2 + B*a^2*c*e^2 + 2*(C*a^2*c*e + A*a*c^2*e)*d)*x^6 + 1/5*(6*B*a^2*c*d*
e + C*a^3*e^2 + 3*A*a^2*c*e^2 + 3*(C*a^2*c + A*a*c^2)*d^2)*x^5 + 1/4*(3*B*a
^2*c*d^2 + B*a^3*e^2 + 2*(C*a^3*e + 3*A*a^2*c*e)*d)*x^4 + 1/3*(2*B*a^3*d*e
+ A*a^3*e^2 + (C*a^3 + 3*A*a^2*c)*d^2)*x^3 + 1/2*(B*a^3*d^2 + 2*A*a^3*d*e)*
x^2
```

Fricas [A]
time = 0.33, size = 374, normalized size = 1.29

$$\frac{1}{9}\,Cc^3d^2x^9+\frac{1}{8}\,Bc^3d^2x^8+\frac{1}{2}\,Bac^2d^2x^6+\frac{3}{4}\,Ba^2cd^2x^4+\frac{1}{7}\left(3\,Cac^2+Ac^3\right)d^2x^7+\frac{1}{2}\,Ba^3d^2x^2+\frac{3}{5}\left(Ca^2c+Aac^2\right)d^2x^5+Aa^3d^2x+\frac{1}{3}\left(Ca^3+3\,Aa^2c\right)d^2x^3+\frac{1}{27720}\left(2520\,Cc^3x^{11}+2772\,Bc^3x^{10}+10395\,Bac^2x^8+13860\,Ba^2cx^6+3080\left(3\,Cac^2+Ac^3\right)x^9+6930\,Ba^3x^4+11880\left(Ca^2c+Aac^2\right)x^7+9240\,Aa^3x^3+5544\left(Ca^3+3\,Aa^2c\right)x^5\right)e^2+\frac{1}{1260}\left(252\,Cc^3dx^{10}+280\,Bc^3dx^9+1080\,Bac^2dx^7+1512\,Ba^2cdx^5+315\left(3\,Cac^2+Ac^3\right)dx^8+840\,Ba^3dx^3+1260\left(Ca^2c+Aac^2\right)dx^6+1260\,Aa^3dx^2+630\left(Ca^3+3\,Aa^2c\right)dx^4\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/9*C*c^3*d^2*x^9 + 1/8*B*c^3*d^2*x^8 + 1/2*B*a*c^2*d^2*x^6 + 3/4*B*a^2*c*d
^2*x^4 + 1/7*(3*C*a*c^2 + A*c^3)*d^2*x^7 + 1/2*B*a^3*d^2*x^2 + 3/5*(C*a^2*c
 + A*a*c^2)*d^2*x^5 + A*a^3*d^2*x + 1/3*(C*a^3 + 3*A*a^2*c)*d^2*x^3 + 1/277
20*(2520*C*c^3*x^11 + 2772*B*c^3*x^10 + 10395*B*a*c^2*x^8 + 13860*B*a^2*c*x
^6 + 3080*(3*C*a*c^2 + A*c^3)*x^9 + 6930*B*a^3*x^4 + 11880*(C*a^2*c + A*a*c
^2)*x^7 + 9240*A*a^3*x^3 + 5544*(C*a^3 + 3*A*a^2*c)*x^5)*e^2 + 1/1260*(252*
C*c^3*d*x^10 + 280*B*c^3*d*x^9 + 1080*B*a*c^2*d*x^7 + 1512*B*a^2*c*d*x^5 +
315*(3*C*a*c^2 + A*c^3)*d*x^8 + 840*B*a^3*d*x^3 + 1260*(C*a^2*c + A*a*c^2)*
d*x^6 + 1260*A*a^3*d*x^2 + 630*(C*a^3 + 3*A*a^2*c)*d*x^4)*e
```

Sympy [A]
time = 0.03, size = 447, normalized size = 1.55

$$Aa^3d^2x+\frac{Cc^3e^2x^{11}}{11}+x^{10}\left(\frac{Bc^3e^2}{10}+\frac{Cc^3de}{5}\right)+x^9\left(\frac{Ac^3e^2}{9}+\frac{2Bc^3de}{9}+\frac{Cac^2e^2}{3}+\frac{Cc^3d^2}{9}\right)+x^8\left(\frac{Ac^3de}{4}+\frac{3Bac^2e^2}{8}+\frac{Bc^3d^2}{8}+\frac{3Cac^2de}{4}\right)+x^7\cdot\left(\frac{3Aac^2e^2}{7}+\frac{Ac^3d^2}{7}+\frac{6Bac^2de}{7}+\frac{3Ca^2ce^2}{7}+\frac{3Cac^2d^2}{7}\right)+x^6\left(Aac^2de+\frac{Ba^2ce^2}{2}+\frac{Bac^2d^2}{2}+Ca^2cde\right)+x^5\cdot\left(\frac{3Aa^2ce^2}{5}+\frac{3Aac^2d^2}{5}+\frac{6Ba^2cde}{5}+\frac{Ca^3e^2}{5}+\frac{3Ca^2cd^2}{5}\right)+x^4\cdot\left(\frac{3Aa^2cde}{2}+\frac{Ba^3e^2}{4}+\frac{3Ba^2cd^2}{4}+\frac{Ca^3de}{2}\right)+x^3\left(\frac{Aa^3e^2}{3}+Aa^2cd^2+\frac{2Ba^3de}{3}+\frac{Ca^3d^2}{3}\right)+x^2\left(Aa^3de+\frac{Ba^3d^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*d**2*x + C*c**3*e**2*x**11/11 + x**10*(B*c**3*e**2/10 + C*c**3*d*e/5
) + x**9*(A*c**3*e**2/9 + 2*B*c**3*d*e/9 + C*a*c**2*e**2/3 + C*c**3*d**2/9)
 + x**8*(A*c**3*d*e/4 + 3*B*a*c**2*e**2/8 + B*c**3*d**2/8 + 3*C*a*c**2*d*e/
4) + x**7*(3*A*a*c**2*e**2/7 + A*c**3*d**2/7 + 6*B*a*c**2*d*e/7 + 3*C*a**2*
c*e**2/7 + 3*C*a*c**2*d**2/7) + x**6*(A*a*c**2*d*e + B*a**2*c*e**2/2 + B*a*
c**2*d**2/2 + C*a**2*c*d*e) + x**5*(3*A*a**2*c*e**2/5 + 3*A*a*c**2*d**2/5 +
 6*B*a**2*c*d*e/5 + C*a**3*e**2/5 + 3*C*a**2*c*d**2/5) + x**4*(3*A*a**2*c*d
*e/2 + B*a**3*e**2/4 + 3*B*a**2*c*d**2/4 + C*a**3*d*e/2) + x**3*(A*a**3*e**
2/3 + A*a**2*c*d**2 + 2*B*a**3*d*e/3 + C*a**3*d**2/3) + x**2*(A*a**3*d*e +
B*a**3*d**2/2)
```

Giac [A]
time = 5.22, size = 432, normalized size = 1.49

$$\frac{1}{11}Cc^3x^{11}e^2+\frac{1}{5}Cc^3dx^{10}e+\frac{1}{9}Cc^3d^2x^9+\frac{1}{10}Bc^3x^{10}e^2+\frac{2}{9}Bc^3dx^9e+\frac{1}{8}Bc^3d^2x^8+\frac{1}{3}Cac^2x^9e^2+\frac{1}{9}Ac^3x^9e^2+\frac{3}{4}Cac^2dx^8e+\frac{1}{4}Ac^3dx^8e+\frac{3}{7}Cac^2d^2x^7+\frac{1}{7}Ac^3d^2x^7+\frac{3}{8}Bac^2x^8e^2+\frac{6}{7}Bac^2dx^7e+\frac{1}{2}Bac^2d^2x^6+\frac{3}{7}Ca^2cx^7e^2+\frac{3}{7}Aac^2x^7e^2+Ca^2cdx^6e+Aac^2dx^6e+\frac{3}{5}Ca^2cd^2x^5+\frac{3}{5}Aac^2d^2x^5+\frac{1}{2}Ba^2cx^6e^2+\frac{6}{5}Ba^2cdx^5e+\frac{3}{4}Ba^2cd^2x^4+\frac{1}{5}Ca^3x^5e^2+\frac{3}{5}Aa^2cx^5e^2+\frac{1}{2}Ca^3dx^4e+\frac{3}{2}Aa^2cdx^4e+\frac{1}{3}Ca^3d^2x^3+Aa^2cd^2x^3+\frac{1}{4}Ba^3x^4e^2+\frac{2}{3}Ba^3dx^3e+\frac{1}{2}Ba^3d^2x^2+\frac{1}{3}Aa^3x^3e^2+Aa^3dx^2e+Aa^3d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/11*C*c^3*x^11*e^2 + 1/5*C*c^3*d*x^10*e + 1/9*C*c^3*d^2*x^9 + 1/10*B*c^3*x
^10*e^2 + 2/9*B*c^3*d*x^9*e + 1/8*B*c^3*d^2*x^8 + 1/3*C*a*c^2*x^9*e^2 + 1/9
*A*c^3*x^9*e^2 + 3/4*C*a*c^2*d*x^8*e + 1/4*A*c^3*d*x^8*e + 3/7*C*a*c^2*d^2*
x^7 + 1/7*A*c^3*d^2*x^7 + 3/8*B*a*c^2*x^8*e^2 + 6/7*B*a*c^2*d*x^7*e + 1/2*B
*a*c^2*d^2*x^6 + 3/7*C*a^2*c*x^7*e^2 + 3/7*A*a*c^2*x^7*e^2 + C*a^2*c*d*x^6*
e + A*a*c^2*d*x^6*e + 3/5*C*a^2*c*d^2*x^5 + 3/5*A*a*c^2*d^2*x^5 + 1/2*B*a^2
*c*x^6*e^2 + 6/5*B*a^2*c*d*x^5*e + 3/4*B*a^2*c*d^2*x^4 + 1/5*C*a^3*x^5*e^2
+ 3/5*A*a^2*c*x^5*e^2 + 1/2*C*a^3*d*x^4*e + 3/2*A*a^2*c*d*x^4*e + 1/3*C*a^3
*d^2*x^3 + A*a^2*c*d^2*x^3 + 1/4*B*a^3*x^4*e^2 + 2/3*B*a^3*d*x^3*e + 1/2*B*
a^3*d^2*x^2 + 1/3*A*a^3*x^3*e^2 + A*a^3*d*x^2*e + A*a^3*d^2*x
```

Mupad [B]
time = 3.94, size = 343, normalized size = 1.19

$$x^3\left(\frac{C\,a^3\,d^2}{3}+\frac{2\,B\,a^3\,d\,e}{3}+\frac{A\,a^3\,e^2}{3}+A\,c\,a^2\,d^2\right)+x^9\left(\frac{C\,c^3\,d^2}{9}+\frac{2\,B\,c^3\,d\,e}{9}+\frac{A\,c^3\,e^2}{9}+\frac{C\,a\,c^2\,e^2}{3}\right)+x^5\left(\frac{C\,a^3\,e^2}{5}+\frac{3\,C\,a^2\,c\,d^2}{5}+\frac{6\,B\,a^2\,c\,d\,e}{5}+\frac{3\,A\,a^2\,c\,e^2}{5}+\frac{3\,A\,a\,c^2\,d^2}{5}\right)+x^7\left(\frac{3\,C\,a^2\,c\,e^2}{7}+\frac{3\,C\,a\,c^2\,d^2}{7}+\frac{6\,B\,a\,c^2\,d\,e}{7}+\frac{3\,A\,a\,c^2\,e^2}{7}+\frac{A\,c^3\,d^2}{7}\right)+\frac{a^2\,x^4\,\left(B\,a\,e^2+3\,B\,c\,d^2+6\,A\,c\,d\,e+2\,C\,a\,d\,e\right)}{4}+\frac{c^2\,x^8\,\left(3\,B\,a\,e^2+B\,c\,d^2+2\,A\,c\,d\,e+6\,C\,a\,d\,e\right)}{8}+\frac{C\,c^3\,e^2\,x^{11}}{11}+\frac{a\,c\,x^6\,\left(B\,a\,e^2+B\,c\,d^2+2\,A\,c\,d\,e+2\,C\,a\,d\,e\right)}{2}+A\,a^3\,d^2\,x+\frac{a^3\,d\,x^2\,\left(2\,A\,e+B\,d\right)}{2}+\frac{c^3\,e\,x^{10}\,\left(B\,e+2\,C\,d\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^3*(d + e*x)^2*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*a^3*e^2)/3 + (C*a^3*d^2)/3 + (2*B*a^3*d*e)/3 + A*a^2*c*d^2) + x^9*(
(A*c^3*e^2)/9 + (C*c^3*d^2)/9 + (2*B*c^3*d*e)/9 + (C*a*c^2*e^2)/3) + x^5*((
C*a^3*e^2)/5 + (3*A*a*c^2*d^2)/5 + (3*A*a^2*c*e^2)/5 + (3*C*a^2*c*d^2)/5 +
(6*B*a^2*c*d*e)/5) + x^7*((A*c^3*d^2)/7 + (3*A*a*c^2*e^2)/7 + (3*C*a*c^2*d^
```

```
2)/7 + (3*C*a^2*c*e^2)/7 + (6*B*a*c^2*d*e)/7) + (a^2*x^4*(B*a*e^2 + 3*B*c*d
^2 + 6*A*c*d*e + 2*C*a*d*e))/4 + (c^2*x^8*(3*B*a*e^2 + B*c*d^2 + 2*A*c*d*e
+ 6*C*a*d*e))/8 + (C*c^3*e^2*x^11)/11 + (a*c*x^6*(B*a*e^2 + B*c*d^2 + 2*A*c
*d*e + 2*C*a*d*e))/2 + A*a^3*d^2*x + (a^3*d*x^2*(2*A*e + B*d))/2 + (c^3*e*x
^10*(B*e + 2*C*d))/10
```

3.34 $\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=169

$$a^3Adx+\frac{1}{3}a^2(3Acd+aCd+aBe)x^3+\frac{1}{4}a^3Cex^4+\frac{3}{5}ac(Acd+aCd+aBe)x^5+\frac{1}{2}a^2cCex^6+\frac{1}{7}c^2(Acd+3a(Cd+Be)$$

```
[Out] a^3*A*d*x+1/3*a^2*(3*A*c*d+B*a*e+C*a*d)*x^3+1/4*a^3*C*e*x^4+3/5*a*c*(A*c*d+
B*a*e+C*a*d)*x^5+1/2*a^2*c*C*e*x^6+1/7*c^2*(A*c*d+3*a*(B*e+C*d))*x^7+3/8*a*
c^2*C*e*x^8+1/9*c^3*(B*e+C*d)*x^9+1/10*c^3*C*e*x^10+1/8*(A*e+B*d)*(c*x^2+a)
^4/c
```

Rubi [A]
time = 0.12, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1596, 1824}

$$a^3Adx+\frac{1}{4}a^3Cex^4+\frac{1}{3}a^2x^3(aBe+aCd+3Acd)+\frac{1}{2}a^2cCex^6+\frac{1}{7}c^2x^7(3a(Be+Cd)+Acd)+\frac{3}{5}acx^5(aBe+aCd+Acd)+\frac{\left(a+cx^2\right)^4(Ae+Bd)}{8c}+\frac{3}{8}ac^2Cex^8+\frac{1}{9}c^3x^9(Be+Cd)+\frac{1}{10}c^3Cex^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d*x + (a^2*(3*A*c*d + a*C*d + a*B*e)*x^3)/3 + (a^3*C*e*x^4)/4 + (3*a*
c*(A*c*d + a*C*d + a*B*e)*x^5)/5 + (a^2*c*C*e*x^6)/2 + (c^2*(A*c*d + 3*a*(C
*d + B*e))*x^7)/7 + (3*a*c^2*C*e*x^8)/8 + (c^3*(C*d + B*e)*x^9)/9 + (c^3*C*
e*x^10)/10 + ((B*d + A*e)*(a + c*x^2)^4)/(8*c)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex) \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) \, dx = \frac{(Bd+Ae)\left(a+cx^2\right)^4}{8c} + \int \left(a+cx^2\right)^3 \left(-(Bd+Ae)x+(d+e\right.$$

$$= \frac{(Bd+Ae)\left(a+cx^2\right)^4}{8c} + \int \left(a^3Ad+a^2(3Acd+aCd+aBe)x^2\right.$$

$$= a^3Adx + \frac{1}{3}a^2(3Acd+aCd+aBe)x^3 + \frac{1}{4}a^3Cex^4 + \frac{3}{5}ac(Acd+$$

Mathematica [A]
time = 0.05, size = 196, normalized size = 1.16

$$a^3Adx+\frac{1}{2}a^3(Bd+Ae)x^2+\frac{1}{3}a^2(3Acd+aCd+aBe)x^3+\frac{1}{4}a^2(3Bcd+3Ace+aCe)x^4+\frac{3}{5}ac(Acd+aCd+aBe)x^5+\frac{1}{2}ac(Bcd+Ace+aCe)x^6+\frac{1}{7}c^2(Acd+3aCd+3aBe)x^7+\frac{1}{8}c^2(Bcd+Ace+3aCe)x^8+\frac{1}{9}c^3(Cd+Be)x^9+\frac{1}{10}c^3Cex^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d*x + (a^3*(B*d + A*e)*x^2)/2 + (a^2*(3*A*c*d + a*C*d + a*B*e)*x^3)/3
 + (a^2*(3*B*c*d + 3*A*c*e + a*C*e)*x^4)/4 + (3*a*c*(A*c*d + a*C*d + a*B*e)
*x^5)/5 + (a*c*(B*c*d + A*c*e + a*C*e)*x^6)/2 + (c^2*(A*c*d + 3*a*C*d + 3*a
*B*e)*x^7)/7 + (c^2*(B*c*d + A*c*e + 3*a*C*e)*x^8)/8 + (c^3*(C*d + B*e)*x^9
)/9 + (c^3*C*e*x^10)/10
```

Maple [A]
time = 0.09, size = 223, normalized size = 1.32

method	result
default	$\frac{c^3Ce\,x^{10}}{10}+\frac{(e\,c^3B+c^3dC)x^9}{9}+\frac{(e\,c^3A+c^3dB+3a\,c^2eC)x^8}{8}+\frac{(c^3dA+3a\,c^2eB+3d\,c^2aC)x^7}{7}+\frac{(3Aa\,c^2e+3Ba\,c^2d+3a^2ceC)x^6}{6}$
norman	$\frac{c^3Ce\,x^{10}}{10}+\left(\frac{1}{9}e\,c^3B+\frac{1}{9}c^3dC\right)x^9+\left(\frac{1}{8}e\,c^3A+\frac{1}{8}c^3dB+\frac{3}{8}a\,c^2eC\right)x^8+\left(\frac{1}{7}c^3dA+\frac{3}{7}a\,c^2eB+\frac{3}{7}d\,c^2aC\right.$
gosper	$\frac{1}{10}c^3Ce\,x^{10}+\frac{1}{9}x^9e\,c^3B+\frac{1}{9}x^9c^3dC+\frac{1}{8}x^8e\,c^3A+\frac{1}{8}x^8c^3dB+\frac{3}{8}a\,c^2Ce\,x^8+\frac{1}{7}x^7c^3dA+\frac{3}{7}x^7a\,c^2eB+$
risch	$\frac{1}{10}c^3Ce\,x^{10}+\frac{1}{9}x^9e\,c^3B+\frac{1}{9}x^9c^3dC+\frac{1}{8}x^8e\,c^3A+\frac{1}{8}x^8c^3dB+\frac{3}{8}a\,c^2Ce\,x^8+\frac{1}{7}x^7c^3dA+\frac{3}{7}x^7a\,c^2eB+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*c^3*C*e*x^10+1/9*(B*c^3*e+C*c^3*d)*x^9+1/8*(A*c^3*e+B*c^3*d+3*C*a*c^2*
e)*x^8+1/7*(A*c^3*d+3*B*a*c^2*e+3*C*a*c^2*d)*x^7+1/6*(3*A*a*c^2*e+3*B*a*c^2
*d+3*C*a^2*c*e)*x^6+1/5*(3*A*a*c^2*d+3*B*a^2*c*e+3*C*a^2*c*d)*x^5+1/4*(3*A*
a^2*c*e+3*B*a^2*c*d+C*a^3*e)*x^4+1/3*(3*A*a^2*c*d+B*a^3*e+C*a^3*d)*x^3+1/2*
(A*a^3*e+B*a^3*d)*x^2+a^3*A*d*x
```

Maxima [A]
time = 0.28, size = 231, normalized size = 1.37

$$\frac{1}{10}Cc^3x^{10}e+\frac{1}{9}\left(Cc^3d+Bc^3e\right)x^9+\frac{1}{8}\left(Bc^3d+3Cac^2e+Ac^3e\right)x^8+\frac{1}{7}\left(3Bac^2e+\left(3Cac^2+Ac^3\right)d\right)x^7+\frac{1}{2}\left(Bac^2d+Ca^2ce+Aac^2e\right)x^6+Aa^3dx+\frac{3}{5}\left(Ba^2ce+\left(Ca^2c+Aac^2\right)d\right)x^5+\frac{1}{4}\left(3Ba^2cd+Ca^3e+3Aa^2ce\right)x^4+\frac{1}{3}\left(Ba^3e+\left(Ca^3+3Aa^2c\right)d\right)x^3+\frac{1}{2}\left(Ba^3d+Aa^3e\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*C*c^3*x^10*e + 1/9*(C*c^3*d + B*c^3*e)*x^9 + 1/8*(B*c^3*d + 3*C*a*c^2*
e + A*c^3*e)*x^8 + 1/7*(3*B*a*c^2*e + (3*C*a*c^2 + A*c^3)*d)*x^7 + 1/2*(B*a
*c^2*d + C*a^2*c*e + A*a*c^2*e)*x^6 + A*a^3*d*x + 3/5*(B*a^2*c*e + (C*a^2*c
 + A*a*c^2)*d)*x^5 + 1/4*(3*B*a^2*c*d + C*a^3*e + 3*A*a^2*c*e)*x^4 + 1/3*(B
*a^3*e + (C*a^3 + 3*A*a^2*c)*d)*x^3 + 1/2*(B*a^3*d + A*a^3*e)*x^2
```

Fricas [A]
time = 0.36, size = 232, normalized size = 1.37

$$\frac{1}{9}Cc^3dx^9+\frac{1}{8}Bc^3dx^8+\frac{1}{2}Bac^2dx^6+\frac{3}{4}Ba^2cdx^4+\frac{1}{7}\left(3Cac^2+Ac^3\right)dx^7+\frac{1}{2}Ba^3dx^2+\frac{3}{5}\left(Ca^2c+Aac^2\right)dx^5+Aa^3dx+\frac{1}{3}\left(Ca^3+3Aa^2c\right)dx^3+\frac{1}{2520}\left(252Cc^3x^{10}+280Bc^3x^9+1080Bac^2x^7+1512Ba^2cx^5+315\left(3Cac^2+Ac^3\right)x^8+840Ba^3x^3+1260\left(Ca^2c+Aac^2\right)x^6+1260Aa^3x^2+630\left(Ca^3+3Aa^2c\right)x^4\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/9*C*c^3*d*x^9 + 1/8*B*c^3*d*x^8 + 1/2*B*a*c^2*d*x^6 + 3/4*B*a^2*c*d*x^4 +
 1/7*(3*C*a*c^2 + A*c^3)*d*x^7 + 1/2*B*a^3*d*x^2 + 3/5*(C*a^2*c + A*a*c^2)*
d*x^5 + A*a^3*d*x + 1/3*(C*a^3 + 3*A*a^2*c)*d*x^3 + 1/2520*(252*C*c^3*x^10
+ 280*B*c^3*x^9 + 1080*B*a*c^2*x^7 + 1512*B*a^2*c*x^5 + 315*(3*C*a*c^2 + A*
c^3)*x^8 + 840*B*a^3*x^3 + 1260*(C*a^2*c + A*a*c^2)*x^6 + 1260*A*a^3*x^2 +
630*(C*a^3 + 3*A*a^2*c)*x^4)*e
```

Sympy [A]
time = 0.02, size = 265, normalized size = 1.57

$$Aa^3dx+\frac{Cc^3ex^{10}}{10}+x^9\left(\frac{Bc^3e}{9}+\frac{Cc^3d}{9}\right)+x^8\left(\frac{Ac^3e}{8}+\frac{Bc^3d}{8}+\frac{3Cac^2e}{8}\right)+x^7\left(\frac{Ac^3d}{7}+\frac{3Bac^2e}{7}+\frac{3Cac^2d}{7}\right)+x^6\left(\frac{Aac^2e}{2}+\frac{Bac^2d}{2}+\frac{Ca^2ce}{2}\right)+x^5\cdot\left(\frac{3Aac^2d}{5}+\frac{3Ba^2ce}{5}+\frac{3Ca^2cd}{5}\right)+x^4\cdot\left(\frac{3Aa^2ce}{4}+\frac{3Ba^2cd}{4}+\frac{Ca^3e}{4}\right)+x^3\left(Aa^2cd+\frac{Ba^3e}{3}+\frac{Ca^3d}{3}\right)+x^2\left(\frac{Aa^3e}{2}+\frac{Ba^3d}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*d*x + C*c**3*e*x**10/10 + x**9*(B*c**3*e/9 + C*c**3*d/9) + x**8*(A*c
**3*e/8 + B*c**3*d/8 + 3*C*a*c**2*e/8) + x**7*(A*c**3*d/7 + 3*B*a*c**2*e/7
+ 3*C*a*c**2*d/7) + x**6*(A*a*c**2*e/2 + B*a*c**2*d/2 + C*a**2*c*e/2) + x**
5*(3*A*a*c**2*d/5 + 3*B*a**2*c*e/5 + 3*C*a**2*c*d/5) + x**4*(3*A*a**2*c*e/4
 + 3*B*a**2*c*d/4 + C*a**3*e/4) + x**3*(A*a**2*c*d + B*a**3*e/3 + C*a**3*d/
3) + x**2*(A*a**3*e/2 + B*a**3*d/2)
```

Giac [A]
time = 4.04, size = 261, normalized size = 1.54

$$\frac{1}{10}Cc^3x^{10}e+\frac{1}{9}Cc^3dx^9+\frac{1}{9}Bc^3x^9e+\frac{1}{8}Bc^3dx^8+\frac{3}{8}Cac^2x^8e+\frac{1}{8}Ac^3x^8e+\frac{3}{7}Cac^2dx^7+\frac{1}{7}Ac^3dx^7+\frac{3}{7}Bac^2x^7e+\frac{1}{2}Bac^2dx^6+\frac{1}{2}Ca^2cx^6e+\frac{1}{2}Aac^2x^6e+\frac{3}{5}Ca^2cdx^5+\frac{3}{5}Aac^2dx^5+\frac{3}{5}Ba^2cx^5e+\frac{3}{4}Ba^2cdx^4+\frac{1}{4}Ca^3x^4e+\frac{3}{4}Aa^2cx^4e+\frac{1}{3}Ca^3dx^3+Aa^2cdx^3+\frac{1}{3}Ba^3x^3e+\frac{1}{2}Ba^3dx^2+\frac{1}{2}Aa^3x^2e+Aa^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*C*c^3*x^10*e + 1/9*C*c^3*d*x^9 + 1/9*B*c^3*x^9*e + 1/8*B*c^3*d*x^8 + 3
/8*C*a*c^2*x^8*e + 1/8*A*c^3*x^8*e + 3/7*C*a*c^2*d*x^7 + 1/7*A*c^3*d*x^7 +
3/7*B*a*c^2*x^7*e + 1/2*B*a*c^2*d*x^6 + 1/2*C*a^2*c*x^6*e + 1/2*A*a*c^2*x^6
*e + 3/5*C*a^2*c*d*x^5 + 3/5*A*a*c^2*d*x^5 + 3/5*B*a^2*c*x^5*e + 3/4*B*a^2*
c*d*x^4 + 1/4*C*a^3*x^4*e + 3/4*A*a^2*c*x^4*e + 1/3*C*a^3*d*x^3 + A*a^2*c*d
*x^3 + 1/3*B*a^3*x^3*e + 1/2*B*a^3*d*x^2 + 1/2*A*a^3*x^2*e + A*a^3*d*x
```

Mupad [B]
time = 0.10, size = 187, normalized size = 1.11

$$x^3\left(\frac{B\,a^3\,e}{3}+\frac{C\,a^3\,d}{3}+A\,a^2\,c\,d\right)+x^8\left(\frac{A\,c^3\,e}{8}+\frac{B\,c^3\,d}{8}+\frac{3\,C\,a\,c^2\,e}{8}\right)+\frac{a^3\,x^2\,\left(A\,e+B\,d\right)}{2}+\frac{c^3\,x^9\,\left(B\,e+C\,d\right)}{9}+\frac{c^2\,x^7\,\left(A\,c\,d+3\,B\,a\,e+3\,C\,a\,d\right)}{7}+\frac{a^2\,x^4\,\left(3\,A\,c\,e+3\,B\,c\,d+C\,a\,e\right)}{4}+A\,a^3\,d\,x+\frac{3\,a\,c\,x^5\,\left(A\,c\,d+B\,a\,e+C\,a\,d\right)}{5}+\frac{a\,c\,x^6\,\left(A\,c\,e+B\,c\,d+C\,a\,e\right)}{2}+\frac{C\,c^3\,e\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^3*(d + e*x)*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((B*a^3*e)/3 + (C*a^3*d)/3 + A*a^2*c*d) + x^8*((A*c^3*e)/8 + (B*c^3*d)/
8 + (3*C*a*c^2*e)/8) + (a^3*x^2*(A*e + B*d))/2 + (c^3*x^9*(B*e + C*d))/9 +
(c^2*x^7*(A*c*d + 3*B*a*e + 3*C*a*d))/7 + (a^2*x^4*(3*A*c*e + 3*B*c*d + C*a
*e))/4 + A*a^3*d*x + (3*a*c*x^5*(A*c*d + B*a*e + C*a*d))/5 + (a*c*x^6*(A*c*
e + B*c*d + C*a*e))/2 + (C*c^3*e*x^10)/10
```

3.35 $\int (a+cx^2)^3 (A+Bx+Cx^2) \, dx$

Optimal. Leaf size=87

$$a^3Ax+\frac{1}{3}a^2(3Ac+aC)x^3+\frac{3}{5}ac(Ac+aC)x^5+\frac{1}{7}c^2(Ac+3aC)x^7+\frac{1}{9}c^3Cx^9+\frac{B(a+cx^2)^4}{8c}$$

```
[Out] a^3*A*x+1/3*a^2*(3*A*c+C*a)*x^3+3/5*a*c*(A*c+C*a)*x^5+1/7*c^2*(A*c+3*C*a)*x
^7+1/9*c^3*C*x^9+1/8*B*(c*x^2+a)^4/c
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1596, 380}

$$a^3Ax+\frac{1}{3}a^2x^3(aC+3Ac)+\frac{1}{7}c^2x^7(3aC+Ac)+\frac{3}{5}acx^5(aC+Ac)+\frac{B(a+cx^2)^4}{8c}+\frac{1}{9}c^3Cx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*x + (a^2*(3*A*c + a*C)*x^3)/3 + (3*a*c*(A*c + a*C)*x^5)/5 + (c^2*(A*c
 + 3*a*C)*x^7)/7 + (c^3*C*x^9)/9 + (B*(a + c*x^2)^4)/(8*c)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\int (a+cx^2)^3 (A+Bx+Cx^2) \, dx = \frac{B(a+cx^2)^4}{8c} + \int (a+cx^2)^3 (A+Cx^2) \, dx$$
$$= \frac{B(a+cx^2)^4}{8c} + \int (a^3A + a^2(3Ac+aC)x^2 + 3ac(Ac+aC)x^4 + c^2(Ac$$
$$= a^3Ax + \frac{1}{3}a^2(3Ac+aC)x^3 + \frac{3}{5}ac(Ac+aC)x^5 + \frac{1}{7}c^2(Ac+3aC)x^7 + \frac{1}{9}$$

Mathematica [A]
time = 0.02, size = 100, normalized size = 1.15

$$\frac{1}{6}a^3x(6A+x(3B+2Cx)) + \frac{1}{20}a^2cx^3(20A+3x(5B+4Cx)) + \frac{1}{70}ac^2x^5(42A+5x(7B+6Cx)) + \frac{1}{504}c^3x^7(72A+7x(9B+8Cx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] (a^3*x*(6*A + x*(3*B + 2*C*x)))/6 + (a^2*c*x^3*(20*A + 3*x*(5*B + 4*C*x)))/
20 + (a*c^2*x^5*(42*A + 5*x*(7*B + 6*C*x)))/70 + (c^3*x^7*(72*A + 7*x*(9*B
+ 8*C*x)))/504
```

Maple [A]
time = 0.09, size = 111, normalized size = 1.28

method	result
norman	$\frac{c^3 C x^9}{9} + \frac{c^3 B x^8}{8} + \left(\frac{1}{7}c^3A + \frac{3}{7}c^2aC\right)x^7 + \frac{c^2aB x^6}{2} + \left(\frac{3}{5}c^2aA + \frac{3}{5}a^2cC\right)x^5 + \frac{3a^2cB x^4}{4} + \left(a^2cA + \frac{1}{3}a^3C\right)$
default	$\frac{c^3 C x^9}{9} + \frac{c^3 B x^8}{8} + \frac{(c^3A+3c^2aC)x^7}{7} + \frac{c^2aB x^6}{2} + \frac{(3c^2aA+3a^2cC)x^5}{5} + \frac{3a^2cB x^4}{4} + \frac{(3a^2cA+a^3C)x^3}{3} + \frac{a^3B x^2}{2} + a^3A$
gosper	$\frac{1}{9}c^3C x^9 + \frac{1}{8}c^3B x^8 + \frac{1}{7}x^7c^3A + \frac{3}{7}x^7c^2aC + \frac{1}{2}c^2aB x^6 + \frac{3}{5}x^5c^2aA + \frac{3}{5}x^5a^2cC + \frac{3}{4}a^2cB x^4 + x^3a^2c$
risch	$\frac{1}{9}c^3C x^9 + \frac{1}{8}c^3B x^8 + \frac{1}{7}x^7c^3A + \frac{3}{7}x^7c^2aC + \frac{1}{2}c^2aB x^6 + \frac{3}{5}x^5c^2aA + \frac{3}{5}x^5a^2cC + \frac{3}{4}a^2cB x^4 + x^3a^2c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*c^3*C*x^9+1/8*c^3*B*x^8+1/7*(A*c^3+3*C*a*c^2)*x^7+1/2*c^2*a*B*x^6+1/5*(
3*A*a*c^2+3*C*a^2*c)*x^5+3/4*a^2*c*B*x^4+1/3*(3*A*a^2*c+C*a^3)*x^3+1/2*a^3*
B*x^2+a^3*A*x
```

Maxima [A]
time = 0.28, size = 108, normalized size = 1.24

$$\frac{1}{9}Cc^3x^9 + \frac{1}{8}Bc^3x^8 + \frac{1}{2}Bac^2x^6 + \frac{3}{4}Ba^2cx^4 + \frac{1}{7}\left(3Cac^2 + Ac^3\right)x^7 + \frac{1}{2}Ba^3x^2 + \frac{3}{5}\left(Ca^2c + Aac^2\right)x^5 + Aa^3x + \frac{1}{3}\left(Ca^3 + 3Aa^2c\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*C*c^3*x^9 + 1/8*B*c^3*x^8 + 1/2*B*a*c^2*x^6 + 3/4*B*a^2*c*x^4 + 1/7*(3*
C*a*c^2 + A*c^3)*x^7 + 1/2*B*a^3*x^2 + 3/5*(C*a^2*c + A*a*c^2)*x^5 + A*a^3*
x + 1/3*(C*a^3 + 3*A*a^2*c)*x^3
```

Fricas [A]
time = 0.33, size = 108, normalized size = 1.24

$$\frac{1}{9}Cc^3x^9 + \frac{1}{8}Bc^3x^8 + \frac{1}{2}Bac^2x^6 + \frac{3}{4}Ba^2cx^4 + \frac{1}{7}(3Cac^2 + Ac^3)x^7 + \frac{1}{2}Ba^3x^2 + \frac{3}{5}(Ca^2c + Aac^2)x^5 + Aa^3x + \frac{1}{3}(Ca^3 + 3Aa^2c)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/9*C*c^3*x^9 + 1/8*B*c^3*x^8 + 1/2*B*a*c^2*x^6 + 3/4*B*a^2*c*x^4 + 1/7*(3*
C*a*c^2 + A*c^3)*x^7 + 1/2*B*a^3*x^2 + 3/5*(C*a^2*c + A*a*c^2)*x^5 + A*a^3*
x + 1/3*(C*a^3 + 3*A*a^2*c)*x^3
```

Sympy [A]
time = 0.01, size = 122, normalized size = 1.40

$$Aa^3x + \frac{Ba^3x^2}{2} + \frac{3Ba^2cx^4}{4} + \frac{Bac^2x^6}{2} + \frac{Bc^3x^8}{8} + \frac{Cc^3x^9}{9} + x^7\left(\frac{Ac^3}{7} + \frac{3Cac^2}{7}\right) + x^5\cdot\left(\frac{3Aac^2}{5} + \frac{3Ca^2c}{5}\right) + x^3\left(Aa^2c + \frac{Ca^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x + B*a**3*x**2/2 + 3*B*a**2*c*x**4/4 + B*a*c**2*x**6/2 + B*c**3*x**
8/8 + C*c**3*x**9/9 + x**7*(A*c**3/7 + 3*C*a*c**2/7) + x**5*(3*A*a*c**2/5 +
 3*C*a**2*c/5) + x**3*(A*a**2*c + C*a**3/3)
```

Giac [A]
time = 3.71, size = 111, normalized size = 1.28

$$\frac{1}{9}Cc^3x^9 + \frac{1}{8}Bc^3x^8 + \frac{3}{7}Cac^2x^7 + \frac{1}{7}Ac^3x^7 + \frac{1}{2}Bac^2x^6 + \frac{3}{5}Ca^2cx^5 + \frac{3}{5}Aac^2x^5 + \frac{3}{4}Ba^2cx^4 + \frac{1}{3}Ca^3x^3 + Aa^2cx^3 + \frac{1}{2}Ba^3x^2 + Aa^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*C*c^3*x^9 + 1/8*B*c^3*x^8 + 3/7*C*a*c^2*x^7 + 1/7*A*c^3*x^7 + 1/2*B*a*c
^2*x^6 + 3/5*C*a^2*c*x^5 + 3/5*A*a*c^2*x^5 + 3/4*B*a^2*c*x^4 + 1/3*C*a^3*x^
3 + A*a^2*c*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

Mupad [B]
time = 0.06, size = 103, normalized size = 1.18

$$x^3\left(\frac{C\,a^3}{3} + A\,c\,a^2\right) + x^7\left(\frac{A\,c^3}{7} + \frac{3\,C\,a\,c^2}{7}\right) + \frac{B\,a^3\,x^2}{2} + \frac{B\,c^3\,x^8}{8} + \frac{C\,c^3\,x^9}{9} + A\,a^3\,x + \frac{3\,a\,c\,x^5\,(A\,c + C\,a)}{5} + \frac{3\,B\,a^2\,c\,x^4}{4} + \frac{B\,a\,c^2\,x^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^3*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((C*a^3)/3 + A*a^2*c) + x^7*((A*c^3)/7 + (3*C*a*c^2)/7) + (B*a^3*x^2)/2
 + (B*c^3*x^8)/8 + (C*c^3*x^9)/9 + A*a^3*x + (3*a*c*x^5*(A*c + C*a))/5 + (3
*B*a^2*c*x^4)/4 + (B*a*c^2*x^6)/2
```

3.36 $\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{d+ex} \, dx$

Optimal. Leaf size=490

$$-\frac{(cd^2+ae^2)^2 (ae^2(2Cd-Be)+cd(8Cd^2-e(7Bd-6Ae)))\,x}{e^8}+\frac{(cd^2+ae^2)\left(a^2Ce^4+c^2d^2(28Cd^2-3e(7B\right.}{}$$

```
[Out] -(a*e^2+c*d^2)^2*(a*e^2*(-B*e+2*C*d)+c*d*(8*C*d^2-e*(-6*A*e+7*B*d)))*x/e^8+
1/2*(a*e^2+c*d^2)*(a^2*C*e^4+c^2*d^2*(28*C*d^2-3*e*(-5*A*e+7*B*d))+a*c*e^2*
(17*C*d^2-3*e*(-A*e+3*B*d)))*(e*x+d)^2/e^9-1/3*c*(3*a^2*e^4*(-B*e+4*C*d)+c^
2*d^3*(56*C*d^2-5*e*(-4*A*e+7*B*d))+6*a*c*d*e^2*(10*C*d^2-e*(-2*A*e+5*B*d))
)*(e*x+d)^3/e^9+1/4*c*(3*a^2*C*e^4+5*c^2*d^2*(14*C*d^2-e*(-3*A*e+7*B*d))+3*
a*c*e^2*(15*C*d^2-e*(-A*e+5*B*d)))*(e*x+d)^4/e^9-1/5*c^2*(3*a*e^2*(-B*e+6*C
*d)+c*d*(56*C*d^2-3*e*(-2*A*e+7*B*d)))*(e*x+d)^5/e^9+1/6*c^2*(3*a*C*e^2+c*(
28*C*d^2-e*(-A*e+7*B*d)))*(e*x+d)^6/e^9-1/7*c^3*(-B*e+8*C*d)*(e*x+d)^7/e^9+
1/8*c^3*C*(e*x+d)^8/e^9+(a*e^2+c*d^2)^3*(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e^9
```

Rubi [A]
time = 0.66, antiderivative size = 487, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = $\{1642\}$

$$\frac{c(d+ex)^4 (3a^2Ce^4+3ace^2(15Cd^2-e(5Bd-Ae))+5c^2(14Cd^4-d^2e(7Bd-3Ae)))}{4e^9}+\frac{(d+ex)^2 (ae^2+cd^2) (a^2Ce^4+ace^2(17Cd^2-3e(3Bd-Ae))+c^2(28Cd^4-3d^2e(7Bd-5Ae)))}{2e^9}-\frac{c(d+ex)^3 (3a^2e^4(4Cd-Be)+6acde^2(10Cd^2-e(5Bd-2Ae))+c^2(56Cd^5-5d^3e(7Bd-4Ae)))}{3e^9}-\frac{c^2(d+ex)^5 (3ae^2(6Cd-Be)-3cde(7Bd-2Ae)+56cCd^3)}{5e^9}+\frac{c^2(d+ex)^6 (3aCe^2-ce(7Bd-Ae)+28cCd^2)}{6e^9}+\frac{(ae^2+cd^2)^3 \log(d+ex) (Ae^2-Bde+Cd^2)}{e^9}-\frac{x(ae^2+cd^2)^2 (ae^2(2Cd-Be)-cde(7Bd-6Ae)+8cCd^3)}{e^8}-\frac{c^3(d+ex)^7(8Cd-Be)}{7e^9}+\frac{c^3C(d+ex)^8}{8e^9}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] -(((c*d^2 + a*e^2)^2*(8*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B*
e))*x)/e^8) + ((c*d^2 + a*e^2)*(a^2*C*e^4 + c^2*(28*C*d^4 - 3*d^2*e*(7*B*d
- 5*A*e)) + a*c*e^2*(17*C*d^2 - 3*e*(3*B*d - A*e)))*(d + e*x)^2)/(2*e^9) -
(c*(3*a^2*e^4*(4*C*d - B*e) + c^2*(56*C*d^5 - 5*d^3*e*(7*B*d - 4*A*e)) + 6*
a*c*d*e^2*(10*C*d^2 - e*(5*B*d - 2*A*e)))*(d + e*x)^3)/(3*e^9) + (c*(3*a^2*
C*e^4 + 5*c^2*(14*C*d^4 - d^2*e*(7*B*d - 3*A*e)) + 3*a*c*e^2*(15*C*d^2 - e*
(5*B*d - A*e)))*(d + e*x)^4)/(4*e^9) - (c^2*(56*c*C*d^3 - 3*c*d*e*(7*B*d -
2*A*e) + 3*a*e^2*(6*C*d - B*e))*(d + e*x)^5)/(5*e^9) + (c^2*(28*c*C*d^2 + 3
*a*C*e^2 - c*e*(7*B*d - A*e))*(d + e*x)^6)/(6*e^9) - (c^3*(8*C*d - B*e)*(d
+ e*x)^7)/(7*e^9) + (c^3*C*(d + e*x)^8)/(8*e^9) + ((c*d^2 + a*e^2)^3*(C*d^2
 - B*d*e + A*e^2)*Log[d + e*x])/e^9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{d+ex}\,dx = \int \left(\frac{(cd^2+ae^2)^2 (-8cCd^3+cde(7Bd-6Ae)-ae^2(2Cd-Be))}{e^8} + \frac{(cd^2}{}\right.$$

$$= -\frac{(cd^2+ae^2)^2 (8cCd^3-cde(7Bd-6Ae)+ae^2(2Cd-Be))\,x}{e^8} + \frac{(cd^2+}{}$$

Mathematica [A]
time = 0.34, size = 498, normalized size = 1.02

$$\frac{x(420a^3e^6(-2Cd+2Be+Cex)+210a^2ce^4(C(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+2e(3Ae(-2d+ex)+B(6d^2-3dex+2e^2x^2)))+42ac^2e^2(C(-60d^5+30d^4ex-20d^3e^2x^2+15d^2e^3x^3-12de^4x^4+10e^5x^5)+e(5Ae(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+B(60d^4-30d^3ex+20d^2e^2x^2-15de^3x^3+12e^4x^4)))+c^3(C(-840d^7+420d^6ex-280d^5e^2x^2+210d^4e^3x^3-168d^3e^4x^4+140d^2e^5x^5-120de^6x^6+105e^7x^7)+2e(7Ae(-60d^5+30d^4ex-20d^3e^2x^2+15d^2e^3x^3-12de^4x^4+10e^5x^5)+B(420d^6-210d^5ex+140d^4e^2x^2-105d^3e^3x^3+84d^2e^4x^4-70de^5x^5+60e^6x^6))))}{840e^8}+\frac{(cd^2+ae^2)^3(Cd^2+e(-Bd+Ae))\log(d+ex)}{e^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] (x*(420*a^3*e^6*(-2*C*d + 2*B*e + C*e*x) + 210*a^2*c*e^4*(C*(-12*d^3 + 6*d^
2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + 2*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3*d
*e*x + 2*e^2*x^2))) + 42*a*c^2*e^2*(C*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^
2 + 15*d^2*e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + e*(5*A*e*(-12*d^3 + 6*d^2
*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + B*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 -
 15*d*e^3*x^3 + 12*e^4*x^4))) + c^3*(C*(-840*d^7 + 420*d^6*e*x - 280*d^5*e^
2*x^2 + 210*d^4*e^3*x^3 - 168*d^3*e^4*x^4 + 140*d^2*e^5*x^5 - 120*d*e^6*x^6
 + 105*e^7*x^7) + 2*e*(7*A*e*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 15*d^
2*e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + B*(420*d^6 - 210*d^5*e*x + 140*d^4
*e^2*x^2 - 105*d^3*e^3*x^3 + 84*d^2*e^4*x^4 - 70*d*e^5*x^5 + 60*e^6*x^6))))
)/(840*e^8) + ((c*d^2 + a*e^2)^3*(C*d^2 + e*(-(B*d) + A*e))*Log[d + e*x])/e
^9
```

Maple [A]
time = 0.09, size = 811, normalized size = 1.66 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] (A*a^3*e^8+3*A*a^2*c*d^2*e^6+3*A*a*c^2*d^4*e^4+A*c^3*d^6*e^2-B*a^3*d*e^7-3*
B*a^2*c*d^3*e^5-3*B*a*c^2*d^5*e^3-B*c^3*d^7*e+C*a^3*d^2*e^6+3*C*a^2*c*d^4*e
^4+3*C*a*c^2*d^6*e^2+C*c^3*d^8)/e^9*ln(e*x+d)-1/e^8*(C*a^2*c*d*e^6*x^3+C*a*
c^2*d^3*e^4*x^3+A*a*c^2*d*e^6*x^3+A*c^3*d^5*e^2*x-B*c^3*d^6*e*x+C*a^3*d*e^6
*x-1/2*C*a^3*e^7*x^2-1/8*c^3*C*x^8*e^7-1/7*B*c^3*e^7*x^7-1/6*A*c^3*e^7*x^6-
B*a^3*e^7*x+C*c^3*d^7*x-3/2*C*a*c^2*d^4*e^3*x^2+3/2*B*a^2*c*d*e^6*x^2+3/2*B
*a*c^2*d^3*e^4*x^2-3/2*C*a^2*c*d^2*e^5*x^2-B*a*c^2*d^2*e^5*x^3-3/2*A*a*c^2*
d^2*e^5*x^2+3/5*C*a*c^2*d*e^6*x^5+3/4*B*a*c^2*d*e^6*x^4-3/4*C*a*c^2*d^2*e^5
```

```
*x^4+3*C*a*c^2*d^5*e^2*x+3*A*a^2*c*d*e^6*x+3*A*a*c^2*d^3*e^4*x-3*B*a^2*c*d^
2*e^5*x-3*B*a*c^2*d^4*e^3*x+3*C*a^2*c*d^3*e^4*x-1/2*C*c^3*d^6*e*x^2+1/2*B*c
^3*d^5*e^2*x^2-1/2*A*c^3*d^4*e^3*x^2+1/3*C*c^3*d^5*e^2*x^3-3/2*A*a^2*c*e^7*
x^2-1/3*B*c^3*d^4*e^3*x^3-1/2*C*a*c^2*e^7*x^6-1/6*C*c^3*d^2*e^5*x^6+1/5*A*c
^3*d*e^6*x^5-3/5*B*a*c^2*e^7*x^5-1/5*B*c^3*d^2*e^5*x^5+1/5*C*c^3*d^3*e^4*x^
5-3/4*A*a*c^2*e^7*x^4-1/4*A*c^3*d^2*e^5*x^4+1/4*B*c^3*d^3*e^4*x^4-3/4*C*a^2
*c*e^7*x^4-1/4*C*c^3*d^4*e^3*x^4+1/3*A*c^3*d^3*e^4*x^3-B*a^2*c*e^7*x^3+1/7*
C*c^3*d*e^6*x^7+1/6*B*c^3*d*e^6*x^6)
```

Maxima [A]
time = 0.29, size = 659, normalized size = 1.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")
```

```
[Out] (C*c^3*d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 + (3*C*a*c
^2*e^2 + A*c^3*e^2)*d^6 - B*a^3*d*e^7 + 3*(C*a^2*c*e^4 + A*a*c^2*e^4)*d^4 +
 A*a^3*e^8 + (C*a^3*e^6 + 3*A*a^2*c*e^6)*d^2)*e^(-9)*log(x*e + d) + 1/840*(
105*C*c^3*x^8*e^7 - 120*(C*c^3*d*e^6 - B*c^3*e^7)*x^7 + 140*(C*c^3*d^2*e^5
- B*c^3*d*e^6 + 3*C*a*c^2*e^7 + A*c^3*e^7)*x^6 - 168*(C*c^3*d^3*e^4 - B*c^3
*d^2*e^5 - 3*B*a*c^2*e^7 + (3*C*a*c^2*e^6 + A*c^3*e^6)*d)*x^5 + 210*(C*c^3*
d^4*e^3 - B*c^3*d^3*e^4 - 3*B*a*c^2*d*e^6 + 3*C*a^2*c*e^7 + 3*A*a*c^2*e^7 +
 (3*C*a*c^2*e^5 + A*c^3*e^5)*d^2)*x^4 - 280*(C*c^3*d^5*e^2 - B*c^3*d^4*e^3
- 3*B*a*c^2*d^2*e^5 - 3*B*a^2*c*e^7 + (3*C*a*c^2*e^4 + A*c^3*e^4)*d^3 + 3*(
C*a^2*c*e^6 + A*a*c^2*e^6)*d)*x^3 + 420*(C*c^3*d^6*e - B*c^3*d^5*e^2 - 3*B*
a*c^2*d^3*e^4 - 3*B*a^2*c*d*e^6 + (3*C*a*c^2*e^3 + A*c^3*e^3)*d^4 + C*a^3*e
^7 + 3*A*a^2*c*e^7 + 3*(C*a^2*c*e^5 + A*a*c^2*e^5)*d^2)*x^2 - 840*(C*c^3*d^
7 - B*c^3*d^6*e - 3*B*a*c^2*d^4*e^3 - 3*B*a^2*c*d^2*e^5 + (3*C*a*c^2*e^2 +
A*c^3*e^2)*d^5 - B*a^3*e^7 + 3*(C*a^2*c*e^4 + A*a*c^2*e^4)*d^3 + (C*a^3*e^6
 + 3*A*a^2*c*e^6)*d)*x)*e^(-8)
```

Fricas [A]
time = 0.37, size = 662, normalized size = 1.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/840*(840*C*c^3*d^7*x*e - (105*C*c^3*x^8 + 120*B*c^3*x^7 + 504*B*a*c^2*x^
5 + 840*B*a^2*c*x^3 + 140*(3*C*a*c^2 + A*c^3)*x^6 + 840*B*a^3*x + 630*(C*a^
2*c + A*a*c^2)*x^4 + 420*(C*a^3 + 3*A*a^2*c)*x^2)*e^8 + 2*(60*C*c^3*d*x^7 +
 70*B*c^3*d*x^6 + 315*B*a*c^2*d*x^4 + 630*B*a^2*c*d*x^2 + 84*(3*C*a*c^2 + A
*c^3)*d*x^5 + 420*(C*a^2*c + A*a*c^2)*d*x^3 + 420*(C*a^3 + 3*A*a^2*c)*d*x)*
```

```
e^7 - 14*(10*C*c^3*d^2*x^6 + 12*B*c^3*d^2*x^5 + 60*B*a*c^2*d^2*x^3 + 180*B*
a^2*c*d^2*x + 15*(3*C*a*c^2 + A*c^3)*d^2*x^4 + 90*(C*a^2*c + A*a*c^2)*d^2*x
^2)*e^6 + 14*(12*C*c^3*d^3*x^5 + 15*B*c^3*d^3*x^4 + 90*B*a*c^2*d^3*x^2 + 20
*(3*C*a*c^2 + A*c^3)*d^3*x^3 + 180*(C*a^2*c + A*a*c^2)*d^3*x)*e^5 - 70*(3*C
*c^3*d^4*x^4 + 4*B*c^3*d^4*x^3 + 36*B*a*c^2*d^4*x + 6*(3*C*a*c^2 + A*c^3)*d
^4*x^2)*e^4 + 140*(2*C*c^3*d^5*x^3 + 3*B*c^3*d^5*x^2 + 6*(3*C*a*c^2 + A*c^3
)*d^5*x)*e^3 - 420*(C*c^3*d^6*x^2 + 2*B*c^3*d^6*x)*e^2 - 840*(C*c^3*d^8 - B
*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 + (3*C*a*c^2 + A*c^3)*d^
6*e^2 - B*a^3*d*e^7 + 3*(C*a^2*c + A*a*c^2)*d^4*e^4 + A*a^3*e^8 + (C*a^3 +
3*A*a^2*c)*d^2*e^6)*log(x*e + d))*e^(-9)
```

Sympy [A]
time = 0.78, size = 685, normalized size = 1.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d),x)
```

```
[Out] C*c**3*x**8/(8*e) + x**7*(B*c**3/(7*e) - C*c**3*d/(7*e**2)) + x**6*(A*c**3/
(6*e) - B*c**3*d/(6*e**2) + C*a*c**2/(2*e) + C*c**3*d**2/(6*e**3)) + x**5*(
-A*c**3*d/(5*e**2) + 3*B*a*c**2/(5*e) + B*c**3*d**2/(5*e**3) - 3*C*a*c**2*d
/(5*e**2) - C*c**3*d**3/(5*e**4)) + x**4*(3*A*a*c**2/(4*e) + A*c**3*d**2/(4
*e**3) - 3*B*a*c**2*d/(4*e**2) - B*c**3*d**3/(4*e**4) + 3*C*a**2*c/(4*e) +
3*C*a*c**2*d**2/(4*e**3) + C*c**3*d**4/(4*e**5)) + x**3*(-A*a*c**2*d/e**2 -
 A*c**3*d**3/(3*e**4) + B*a**2*c/e + B*a*c**2*d**2/e**3 + B*c**3*d**4/(3*e*
*5) - C*a**2*c*d/e**2 - C*a*c**2*d**3/e**4 - C*c**3*d**5/(3*e**6)) + x**2*(
3*A*a**2*c/(2*e) + 3*A*a*c**2*d**2/(2*e**3) + A*c**3*d**4/(2*e**5) - 3*B*a*
*2*c*d/(2*e**2) - 3*B*a*c**2*d**3/(2*e**4) - B*c**3*d**5/(2*e**6) + C*a**3/
(2*e) + 3*C*a**2*c*d**2/(2*e**3) + 3*C*a*c**2*d**4/(2*e**5) + C*c**3*d**6/(
2*e**7)) + x*(-3*A*a**2*c*d/e**2 - 3*A*a*c**2*d**3/e**4 - A*c**3*d**5/e**6
+ B*a**3/e + 3*B*a**2*c*d**2/e**3 + 3*B*a*c**2*d**4/e**5 + B*c**3*d**6/e**7
 - C*a**3*d/e**2 - 3*C*a**2*c*d**3/e**4 - 3*C*a*c**2*d**5/e**6 - C*c**3*d**
7/e**8) + (a*e**2 + c*d**2)**3*(A*e**2 - B*d*e + C*d**2)*log(d + e*x)/e**9
```

Giac [A]
time = 3.89, size = 764, normalized size = 1.56

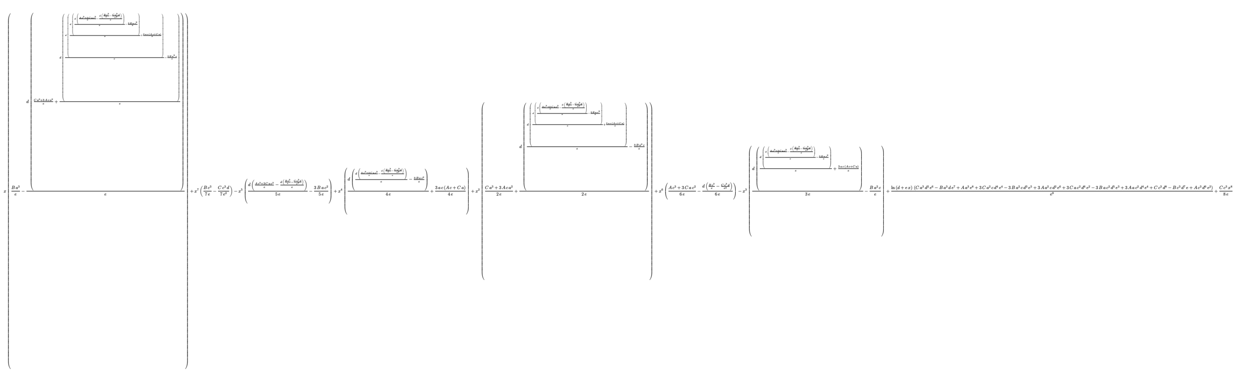
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

```
[Out] (C*c^3*d^8 - B*c^3*d^7*e + 3*C*a*c^2*d^6*e^2 + A*c^3*d^6*e^2 - 3*B*a*c^2*d^
5*e^3 + 3*C*a^2*c*d^4*e^4 + 3*A*a*c^2*d^4*e^4 - 3*B*a^2*c*d^3*e^5 + C*a^3*d
^2*e^6 + 3*A*a^2*c*d^2*e^6 - B*a^3*d*e^7 + A*a^3*e^8)*e^(-9)*log(abs(x*e +
```

```
d)) + 1/840*(105*C*c^3*x^8*e^7 - 120*C*c^3*d*x^7*e^6 + 140*C*c^3*d^2*x^6*e^
5 - 168*C*c^3*d^3*x^5*e^4 + 210*C*c^3*d^4*x^4*e^3 - 280*C*c^3*d^5*x^3*e^2 +
 420*C*c^3*d^6*x^2*e - 840*C*c^3*d^7*x + 120*B*c^3*x^7*e^7 - 140*B*c^3*d*x^
6*e^6 + 168*B*c^3*d^2*x^5*e^5 - 210*B*c^3*d^3*x^4*e^4 + 280*B*c^3*d^4*x^3*e
^3 - 420*B*c^3*d^5*x^2*e^2 + 840*B*c^3*d^6*x*e + 420*C*a*c^2*x^6*e^7 + 140*
A*c^3*x^6*e^7 - 504*C*a*c^2*d*x^5*e^6 - 168*A*c^3*d*x^5*e^6 + 630*C*a*c^2*d
^2*x^4*e^5 + 210*A*c^3*d^2*x^4*e^5 - 840*C*a*c^2*d^3*x^3*e^4 - 280*A*c^3*d^
3*x^3*e^4 + 1260*C*a*c^2*d^4*x^2*e^3 + 420*A*c^3*d^4*x^2*e^3 - 2520*C*a*c^2
*d^5*x*e^2 - 840*A*c^3*d^5*x*e^2 + 504*B*a*c^2*x^5*e^7 - 630*B*a*c^2*d*x^4*
e^6 + 840*B*a*c^2*d^2*x^3*e^5 - 1260*B*a*c^2*d^3*x^2*e^4 + 2520*B*a*c^2*d^4
*x*e^3 + 630*C*a^2*c*x^4*e^7 + 630*A*a*c^2*x^4*e^7 - 840*C*a^2*c*d*x^3*e^6
- 840*A*a*c^2*d*x^3*e^6 + 1260*C*a^2*c*d^2*x^2*e^5 + 1260*A*a*c^2*d^2*x^2*e
^5 - 2520*C*a^2*c*d^3*x*e^4 - 2520*A*a*c^2*d^3*x*e^4 + 840*B*a^2*c*x^3*e^7
- 1260*B*a^2*c*d*x^2*e^6 + 2520*B*a^2*c*d^2*x*e^5 + 420*C*a^3*x^2*e^7 + 126
0*A*a^2*c*x^2*e^7 - 840*C*a^3*d*x*e^6 - 2520*A*a^2*c*d*x*e^6 + 840*B*a^3*x*
e^7)*e^(-8)
```

Mupad [B]
time = 3.88, size = 741, normalized size = 1.51



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x),x)
```

```
[Out] x*((B*a^3)/e - (d*((C*a^3 + 3*A*a^2*c)/e + (d*((d*((d*((d*((A*c^3 + 3*C*a*c
^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/e + (3*a*c*
(A*c + C*a))/e))/e - (3*B*a^2*c)/e))/e))/e) + x^7*((B*c^3)/(7*e) - (C*c^3*d
)/(7*e^2)) - x^5*((d*((A*c^3 + 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2
))/e))/(5*e) - (3*B*a*c^2)/(5*e)) + x^4*((d*((d*((A*c^3 + 3*C*a*c^2)/e - (d
*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/(4*e) + (3*a*c*(A*c +
 C*a))/(4*e)) + x^2*((C*a^3 + 3*A*a^2*c)/(2*e) + (d*((d*((d*((d*((A*c^3 + 3
*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/e + (
3*a*c*(A*c + C*a))/e))/e - (3*B*a^2*c)/e))/(2*e)) + x^6*((A*c^3 + 3*C*a*c^2
)/(6*e) - (d*((B*c^3)/e - (C*c^3*d)/e^2))/(6*e)) - x^3*((d*((d*((d*((A*c^3
+ 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/e
+ (3*a*c*(A*c + C*a))/e))/(3*e) - (B*a^2*c)/e) + (log(d + e*x)*(A*a^3*e^8 +
```

```
C*c^3*d^8 - B*a^3*d*e^7 - B*c^3*d^7*e + A*c^3*d^6*e^2 + C*a^3*d^2*e^6 + 3*
A*a*c^2*d^4*e^4 + 3*A*a^2*c*d^2*e^6 - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5
 + 3*C*a*c^2*d^6*e^2 + 3*C*a^2*c*d^4*e^4))/e^9 + (C*c^3*x^8)/(8*e)
```

3.37 $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^2}\,dx$

Optimal. Leaf size=486

$$\frac{(a^3Ce^6 + c^3d^4(7Cd^2 - e(6Bd - 5Ae)) + 3ac^2d^2e^2(5Cd^2 - e(4Bd - 3Ae)) + 3a^2ce^4(3Cd^2 - e(2Bd - Ae)))}{e^8}$$

```
[Out] (a^3*C*e^6+c^3*d^4*(7*C*d^2-e*(-5*A*e+6*B*d))+3*a*c^2*d^2*e^2*(5*C*d^2-e*(-
3*A*e+4*B*d))+3*a^2*c*e^4*(3*C*d^2-e*(-A*e+2*B*d)))*x/e^8-1/2*c*(3*a^2*e^4*
(-B*e+2*C*d)+c^2*d^3*(6*C*d^2-e*(-4*A*e+5*B*d))+3*a*c*d*e^2*(4*C*d^2-e*(-2*
A*e+3*B*d)))*x^2/e^7+1/3*c*(3*a^2*C*e^4+c^2*d^2*(5*C*d^2-e*(-3*A*e+4*B*d))+
3*a*c*e^2*(3*C*d^2-e*(-A*e+2*B*d)))*x^3/e^6-1/4*c^2*(3*a*e^2*(-B*e+2*C*d)+c
*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*x^4/e^5+1/5*c^2*(3*a*C*e^2+c*(3*C*d^2-e*(-A*
e+2*B*d)))*x^5/e^4-1/6*c^3*(-B*e+2*C*d)*x^6/e^3+1/7*c^3*C*x^7/e^2-(a*e^2+c*
d^2)^3*(A*e^2-B*d*e+C*d^2)/e^9/(e*x+d)-(a*e^2+c*d^2)^2*(a*e^2*(-B*e+2*C*d)+
c*d*(8*C*d^2-e*(-6*A*e+7*B*d)))*ln(e*x+d)/e^9
```

Rubi [A]

time = 0.59, antiderivative size = 483, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = $\{1642\}$

$$\frac{cx^3(3a^2Ce^4 + 3ace^2(3Cd^2 - e(2Bd - Ae)) + c^2(5Cd^4 - d^2e(4Bd - 3Ae)))}{3e^6} - \frac{cx^2(3a^2e^4(2Cd - Be) + 3acde^2(4Cd^2 - e(3Bd - 2Ae)) + c^2(6Cd^5 - d^3e(5Bd - 4Ae)))}{2e^7} + \frac{x(a^3Ce^6 + 3a^2ce^4(3Cd^2 - e(2Bd - Ae)) + 3ac^2d^2e^2(5Cd^2 - e(4Bd - 3Ae)) + c^3(7Cd^6 - d^4e(6Bd - 5Ae)))}{e^8} - \frac{c^2x^4(3ae^2(2Cd - Be) - cde(3Bd - 2Ae) + 4cCd^3)}{4e^5} + \frac{c^2x^5(3aCe^2 - ce(2Bd - Ae) + 3cCd^2)}{5e^4} - \frac{(ae^2 + cd^2)^3(Ae^2 - Bde + Cd^2)}{e^9(d + ex)} - \frac{(ae^2 + cd^2)^2\log(d + ex)(ae^2(2Cd - Be) - cde(7Bd - 6Ae) + 8cCd^3)}{e^9} - \frac{c^3x^6(2Cd - Be)}{6e^3} + \frac{c^3Cx^7}{7e^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] ((a^3*C*e^6 + c^3*(7*C*d^6 - d^4*e*(6*B*d - 5*A*e)) + 3*a*c^2*d^2*e^2*(5*C*
d^2 - e*(4*B*d - 3*A*e)) + 3*a^2*c*e^4*(3*C*d^2 - e*(2*B*d - A*e)))*x)/e^8
- (c*(3*a^2*e^4*(2*C*d - B*e) + c^2*(6*C*d^5 - d^3*e*(5*B*d - 4*A*e)) + 3*a
*c*d*e^2*(4*C*d^2 - e*(3*B*d - 2*A*e)))*x^2)/(2*e^7) + (c*(3*a^2*C*e^4 + c^
2*(5*C*d^4 - d^2*e*(4*B*d - 3*A*e)) + 3*a*c*e^2*(3*C*d^2 - e*(2*B*d - A*e))
)*x^3)/(3*e^6) - (c^2*(4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + 3*a*e^2*(2*C*d -
 B*e))*x^4)/(4*e^5) + (c^2*(3*c*C*d^2 + 3*a*C*e^2 - c*e*(2*B*d - A*e))*x^5)
/(5*e^4) - (c^3*(2*C*d - B*e)*x^6)/(6*e^3) + (c^3*C*x^7)/(7*e^2) - ((c*d^2
+ a*e^2)^3*(C*d^2 - B*d*e + A*e^2))/(e^9*(d + e*x)) - ((c*d^2 + a*e^2)^2*(8
*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e*x])/e^9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^2} dx = \int \left(\frac{a^3Ce^6 + c^3(7Cd^6 - d^4e(6Bd - 5Ae)) + 3ac^2d^2e^2(5Cd^2 - e(4Bd -}{e^8} \right.$$

$$= \frac{(a^3Ce^6 + c^3(7Cd^6 - d^4e(6Bd - 5Ae)) + 3ac^2d^2e^2(5Cd^2 - e(4Bd - 3Ae}{e^8}$$

Mathematica [A]
time = 0.25, size = 641, normalized size = 1.32

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] (420*a^3*e^6*(e*(B*d - A*e) + C*(-d^2 + d*e*x + e^2*x^2)) + 210*a^2*c*e^4*(
2*C*(-3*d^4 + 9*d^3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + 3*e*(2*A
*e*(-d^2 + d*e*x + e^2*x^2) + B*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*x^3)
)) + 21*a*c^2*e^2*(-6*C*(10*d^6 - 50*d^5*e*x - 30*d^4*e^2*x^2 + 10*d^3*e^3*
x^3 - 5*d^2*e^4*x^4 + 3*d*e^5*x^5 - 2*e^6*x^6) + 5*e*(4*A*e*(-3*d^4 + 9*d^3
*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + B*(12*d^5 - 48*d^4*e*x - 30
*d^3*e^2*x^2 + 10*d^2*e^3*x^3 - 5*d*e^4*x^4 + 3*e^5*x^5))) + c^3*(-4*C*(105
*d^8 - 735*d^7*e*x - 420*d^6*e^2*x^2 + 140*d^5*e^3*x^3 - 70*d^4*e^4*x^4 + 4
2*d^3*e^5*x^5 - 28*d^2*e^6*x^6 + 20*d*e^7*x^7 - 15*e^8*x^8) + 7*e*(6*A*e*(-
10*d^6 + 50*d^5*e*x + 30*d^4*e^2*x^2 - 10*d^3*e^3*x^3 + 5*d^2*e^4*x^4 - 3*d
*e^5*x^5 + 2*e^6*x^6) + B*(60*d^7 - 360*d^6*e*x - 210*d^5*e^2*x^2 + 70*d^4*
e^3*x^3 - 35*d^3*e^4*x^4 + 21*d^2*e^5*x^5 - 14*d*e^6*x^6 + 10*e^7*x^7))) -
420*(c*d^2 + a*e^2)^2*(8*c*C*d^3 + c*d*e*(-7*B*d + 6*A*e) + a*e^2*(2*C*d -
B*e))*(d + e*x)*Log[d + e*x])/(420*e^9*(d + e*x))
```

Maple [A]
time = 0.09, size = 788, normalized size = 1.62 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-6*A*a^2*c*d*e^6-12*A*a*c^2*d^3*e^4-6*A*c^3*d^5*e^2+B*a^3*e^7+9*B*a^2*c*d^
2*e^5+15*B*a*c^2*d^4*e^3+7*B*c^3*d^6*e-2*C*a^3*d*e^6-12*C*a^2*c*d^3*e^4-18*
C*a*c^2*d^5*e^2-8*C*c^3*d^7)/e^9*ln(e*x+d)-(A*a^3*e^8+3*A*a^2*c*d^2*e^6+3*A
*a*c^2*d^4*e^4+A*c^3*d^6*e^2-B*a^3*d*e^7-3*B*a^2*c*d^3*e^5-3*B*a*c^2*d^5*e^
3-B*c^3*d^7*e+C*a^3*d^2*e^6+3*C*a^2*c*d^4*e^4+3*C*a*c^2*d^6*e^2+C*c^3*d^8)/
e^9/(e*x+d)+1/e^8*(9/2*B*a*c^2*d^2*e^4*x^2-3*C*a^2*c*d*e^5*x^2-6*C*a*c^2*d^
3*e^3*x^2+9*A*a*c^2*d^2*e^4*x-6*B*a^2*c*d*e^5*x-12*B*a*c^2*d^3*e^3*x+9*C*a^
2*c*d^2*e^4*x+15*C*a*c^2*d^4*e^2*x-3*A*a*c^2*d*e^5*x^2+3*C*a*c^2*d^2*e^4*x^
```

```
3-3/2*C*a*c^2*d*e^5*x^4-2*B*a*c^2*d*e^5*x^3+C*a^2*c*e^6*x^3+A*a*c^2*e^6*x^3
+A*c^3*d^2*e^4*x^3+a^3*C*e^6*x+7*C*c^3*d^6*x+1/7*c^3*C*x^7*e^6+1/6*B*c^3*e^
6*x^6+1/5*A*c^3*e^6*x^5+5/2*B*c^3*d^4*e^2*x^2-3*C*c^3*d^5*e*x^2+3*A*a^2*c*e
^6*x+5*A*c^3*d^4*e^2*x-6*B*c^3*d^5*e*x-1/3*C*c^3*d*e^5*x^6-2/5*B*c^3*d*e^5*
x^5+3/5*C*a*c^2*e^6*x^5+3/5*C*c^3*d^2*e^4*x^5-1/2*A*c^3*d*e^5*x^4+3/4*B*a*c
^2*e^6*x^4+3/4*B*c^3*d^2*e^4*x^4-C*c^3*d^3*e^3*x^4-4/3*B*c^3*d^3*e^3*x^3+5/
3*C*c^3*d^4*e^2*x^3-2*A*c^3*d^3*e^3*x^2+3/2*B*a^2*c*e^6*x^2)
```

Maxima [A]
time = 0.30, size = 679, normalized size = 1.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(8*C*c^3*d^7 - 7*B*c^3*d^6*e - 15*B*a*c^2*d^4*e^3 - 9*B*a^2*c*d^2*e^5 + 6*
(3*C*a*c^2*e^2 + A*c^3*e^2)*d^5 - B*a^3*e^7 + 12*(C*a^2*c*e^4 + A*a*c^2*e^4
)*d^3 + 2*(C*a^3*e^6 + 3*A*a^2*c*e^6)*d)*e^(-9)*log(x*e + d) + 1/420*(60*C*
c^3*x^7*e^6 - 70*(2*C*c^3*d*e^5 - B*c^3*e^6)*x^6 + 84*(3*C*c^3*d^2*e^4 - 2*
B*c^3*d*e^5 + 3*C*a*c^2*e^6 + A*c^3*e^6)*x^5 - 105*(4*C*c^3*d^3*e^3 - 3*B*c
^3*d^2*e^4 - 3*B*a*c^2*e^6 + 2*(3*C*a*c^2*e^5 + A*c^3*e^5)*d)*x^4 + 140*(5*
C*c^3*d^4*e^2 - 4*B*c^3*d^3*e^3 - 6*B*a*c^2*d*e^5 + 3*C*a^2*c*e^6 + 3*A*a*c
^2*e^6 + 3*(3*C*a*c^2*e^4 + A*c^3*e^4)*d^2)*x^3 - 210*(6*C*c^3*d^5*e - 5*B*
c^3*d^4*e^2 - 9*B*a*c^2*d^2*e^4 - 3*B*a^2*c*e^6 + 4*(3*C*a*c^2*e^3 + A*c^3*
e^3)*d^3 + 6*(C*a^2*c*e^5 + A*a*c^2*e^5)*d)*x^2 + 420*(7*C*c^3*d^6 - 6*B*c^
3*d^5*e - 12*B*a*c^2*d^3*e^3 - 6*B*a^2*c*d*e^5 + 5*(3*C*a*c^2*e^2 + A*c^3*e
^2)*d^4 + C*a^3*e^6 + 3*A*a^2*c*e^6 + 9*(C*a^2*c*e^4 + A*a*c^2*e^4)*d^2)*x)
*e^(-8) - (C*c^3*d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5
+ (3*C*a*c^2*e^2 + A*c^3*e^2)*d^6 - B*a^3*d*e^7 + 3*(C*a^2*c*e^4 + A*a*c^2*
e^4)*d^4 + A*a^3*e^8 + (C*a^3*e^6 + 3*A*a^2*c*e^6)*d^2)/(x*e^10 + d*e^9)
```

Fricas [A]
time = 0.34, size = 892, normalized size = 1.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/420*(420*C*c^3*d^8 - (60*C*c^3*x^8 + 70*B*c^3*x^7 + 315*B*a*c^2*x^5 + 63
0*B*a^2*c*x^3 + 84*(3*C*a*c^2 + A*c^3)*x^6 + 420*(C*a^2*c + A*a*c^2)*x^4 -
420*A*a^3 + 420*(C*a^3 + 3*A*a^2*c)*x^2)*e^8 + (80*C*c^3*d*x^7 + 98*B*c^3*d
*x^6 + 525*B*a*c^2*d*x^4 + 1890*B*a^2*c*d*x^2 + 126*(3*C*a*c^2 + A*c^3)*d*x
^5 - 420*B*a^3*d + 840*(C*a^2*c + A*a*c^2)*d*x^3 - 420*(C*a^3 + 3*A*a^2*c)*
d*x)*e^7 - 7*(16*C*c^3*d^2*x^6 + 21*B*c^3*d^2*x^5 + 150*B*a*c^2*d^2*x^3 - 3
```

```
60*B*a^2*c*d^2*x + 30*(3*C*a*c^2 + A*c^3)*d^2*x^4 + 360*(C*a^2*c + A*a*c^2)
*d^2*x^2 - 60*(C*a^3 + 3*A*a^2*c)*d^2)*e^6 + 7*(24*C*c^3*d^3*x^5 + 35*B*c^3
*d^3*x^4 + 450*B*a*c^2*d^3*x^2 - 180*B*a^2*c*d^3 + 60*(3*C*a*c^2 + A*c^3)*d
^3*x^3 - 540*(C*a^2*c + A*a*c^2)*d^3*x)*e^5 - 70*(4*C*c^3*d^4*x^4 + 7*B*c^3
*d^4*x^3 - 72*B*a*c^2*d^4*x + 18*(3*C*a*c^2 + A*c^3)*d^4*x^2 - 18*(C*a^2*c
+ A*a*c^2)*d^4)*e^4 + 70*(8*C*c^3*d^5*x^3 + 21*B*c^3*d^5*x^2 - 18*B*a*c^2*d
^5 - 30*(3*C*a*c^2 + A*c^3)*d^5*x)*e^3 - 420*(4*C*c^3*d^6*x^2 - 6*B*c^3*d^6
*x - (3*C*a*c^2 + A*c^3)*d^6)*e^2 - 420*(7*C*c^3*d^7*x + B*c^3*d^7)*e + 420
*(8*C*c^3*d^8 - B*a^3*x*e^8 - (B*a^3*d - 2*(C*a^3 + 3*A*a^2*c)*d*x)*e^7 - (
9*B*a^2*c*d^2*x - 2*(C*a^3 + 3*A*a^2*c)*d^2)*e^6 - 3*(3*B*a^2*c*d^3 - 4*(C*
a^2*c + A*a*c^2)*d^3*x)*e^5 - 3*(5*B*a*c^2*d^4*x - 4*(C*a^2*c + A*a*c^2)*d^
4)*e^4 - 3*(5*B*a*c^2*d^5 - 2*(3*C*a*c^2 + A*c^3)*d^5*x)*e^3 - (7*B*c^3*d^6
*x - 6*(3*C*a*c^2 + A*c^3)*d^6)*e^2 + (8*C*c^3*d^7*x - 7*B*c^3*d^7)*e)*log(
x*e + d))/(x*e^10 + d*e^9)
```

Sympy [**A**]
time = 2.15, size = 748, normalized size = 1.54

$$\frac{Cc^3x^7}{7e^2} + x^6\left(\frac{Bc^3}{6e^2} - \frac{Cc^3d}{3e^3}\right) + x^5\left(\frac{Ac^3}{5e^2} - \frac{2Bc^3d}{5e^3} + \frac{3Cac^2}{5e^2} + \frac{3Cc^3d^2}{5e^4}\right) + x^4\left(-\frac{Ac^3d}{2e^3} + \frac{3Bac^2}{4e^2} + \frac{3Bc^3d^2}{4e^4} - \frac{3Cac^2d}{2e^3} - \frac{Cc^3d^3}{e^5}\right) + x^3\left(\frac{Aac^2}{e^2} + \frac{Ac^3d^2}{e^4} - \frac{2Bac^2d}{e^3} - \frac{4Bc^3d^3}{3e^5} + \frac{Ca^2c}{e^2} + \frac{3Cac^2d^2}{e^4} + \frac{5Cc^3d^4}{3e^6}\right) + x^2\left(-\frac{3Aac^2d}{e^3} - \frac{2Ac^3d^3}{e^5} + \frac{3Ba^2c}{2e^2} + \frac{9Bac^2d^2}{2e^4} + \frac{5Bc^3d^4}{2e^6} - \frac{3Ca^2cd}{e^3} - \frac{6Cac^2d^3}{e^5} - \frac{3Cc^3d^5}{e^7}\right) + x\left(\frac{3Aa^2c}{e^2} + \frac{9Aac^2d^2}{e^4} + \frac{5Ac^3d^4}{e^6} - \frac{6Ba^2cd}{e^3} - \frac{12Bac^2d^3}{e^5} - \frac{6Bc^3d^5}{e^7} + \frac{Ca^3}{e^2} + \frac{9Ca^2cd^2}{e^4} + \frac{15Cac^2d^4}{e^6} + \frac{7Cc^3d^6}{e^8}\right) + \frac{-Aa^3e^8 - 3Aa^2cd^2e^6 - 3Aac^2d^4e^4 - Ac^3d^6e^2 + Ba^3de^7 + 3Ba^2cd^3e^5 + 3Bac^2d^5e^3 + Bc^3d^7e - Ca^3d^2e^6 - 3Ca^2cd^4e^4 - 3Cac^2d^6e^2 - Cc^3d^8}{de^9 + e^{10}x} - \frac{(ae^2 + cd^2)^2 \cdot (6Acde^2 - Bae^3 - 7Bcd^2e + 2Cade^2 + 8Ccd^3)\log(d + ex)}{e^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

```
[Out] C*c**3*x**7/(7*e**2) + x**6*(B*c**3/(6*e**2) - C*c**3*d/(3*e**3)) + x**5*(A
*c**3/(5*e**2) - 2*B*c**3*d/(5*e**3) + 3*C*a*c**2/(5*e**2) + 3*C*c**3*d**2/
(5*e**4)) + x**4*(-A*c**3*d/(2*e**3) + 3*B*a*c**2/(4*e**2) + 3*B*c**3*d**2/
(4*e**4) - 3*C*a*c**2*d/(2*e**3) - C*c**3*d**3/e**5) + x**3*(A*a*c**2/e**2
+ A*c**3*d**2/e**4 - 2*B*a*c**2*d/e**3 - 4*B*c**3*d**3/(3*e**5) + C*a**2*c/
e**2 + 3*C*a*c**2*d**2/e**4 + 5*C*c**3*d**4/(3*e**6)) + x**2*(-3*A*a*c**2*d
/e**3 - 2*A*c**3*d**3/e**5 + 3*B*a**2*c/(2*e**2) + 9*B*a*c**2*d**2/(2*e**4)
 + 5*B*c**3*d**4/(2*e**6) - 3*C*a**2*c*d/e**3 - 6*C*a*c**2*d**3/e**5 - 3*C*
c**3*d**5/e**7) + x*(3*A*a**2*c/e**2 + 9*A*a*c**2*d**2/e**4 + 5*A*c**3*d**4
/e**6 - 6*B*a**2*c*d/e**3 - 12*B*a*c**2*d**3/e**5 - 6*B*c**3*d**5/e**7 + C*
a**3/e**2 + 9*C*a**2*c*d**2/e**4 + 15*C*a*c**2*d**4/e**6 + 7*C*c**3*d**6/e*
*8) + (-A*a**3*e**8 - 3*A*a**2*c*d**2*e**6 - 3*A*a*c**2*d**4*e**4 - A*c**3*
d**6*e**2 + B*a**3*d*e**7 + 3*B*a**2*c*d**3*e**5 + 3*B*a*c**2*d**5*e**3 + B
*c**3*d**7*e - C*a**3*d**2*e**6 - 3*C*a**2*c*d**4*e**4 - 3*C*a*c**2*d**6*e*
*2 - C*c**3*d**8)/(d*e**9 + e**10*x) - (a*e**2 + c*d**2)**2*(6*A*c*d*e**2 -
 B*a*e**3 - 7*B*c*d**2*e + 2*C*a*d*e**2 + 8*C*c*d**3)*log(d + e*x)/e**9
```

Giac [**A**]
time = 3.85, size = 838, normalized size = 1.72

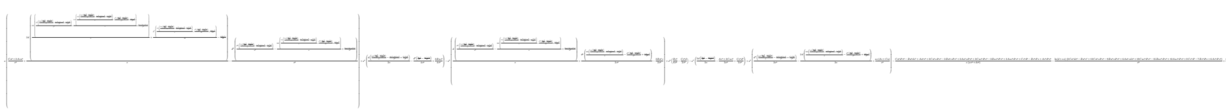
$$\frac{1}{420}\left(60\,Cc^3 - \frac{70\,(8\,Cc^3de - Bc^3e^2)e^{(-1)}}{xe + d} + \frac{84\,(28\,Cc^3d^2e^2 - 7\,Bc^3de^3 + 3\,Cac^2e^4 + Ac^3e^4)e^{(-2)}}{(xe + d)^2} - \frac{105\,(56\,Cc^3d^3e^3 - 21\,Bc^3d^2e^4 + 18\,Cac^2de^5 + 6\,Ac^3de^5 - 3\,Bac^2e^6)e^{(-3)}}{(xe + d)^3} + \frac{140\,(70\,Cc^3d^4e^4 - 35\,Bc^3d^3e^5 + 45\,Cac^2d^2e^6 + 15\,Ac^3d^2e^6 - 15\,Bac^2de^7 + 3\,Ca^2ce^8 + 3\,Aac^2e^8)e^{(-4)}}{(xe + d)^4} - \frac{210\,(56\,Cc^3d^5e^5 - 35\,Bc^3d^4e^6 + 60\,Cac^2d^3e^7 + 20\,Ac^3d^3e^7 - 30\,Bac^2d^2e^8 + 12\,Ca^2cde^9 + 12\,Aac^2de^9 - 3\,Ba^2ce^{10})e^{(-5)}}{(xe + d)^5} + \frac{420\,(28\,Cc^3d^6e^6 - 21\,Bc^3d^5e^7 + 45\,Cac^2d^4e^8 + 15\,Ac^3d^4e^8 - 30\,Bac^2d^3e^9 + 18\,Ca^2cd^2e^{10} + 18\,Aac^2d^2e^{10} - 9\,Ba^2cde^{11} + Ca^3e^{12} + 3\,Aa^2ce^{12})e^{(-6)}}{(xe + d)^6}\right)(xe + d)^7e^{(-9)} + (8\,Cc^3d^7 - 7\,Bc^3d^6e + 18\,Cac^2d^5e^2 + 6\,Ac^3d^5e^2 - 15\,Bac^2d^4e^3 + 12\,Ca^2cd^3e^4 + 12\,Aac^2d^3e^4 - 9\,Ba^2cd^2e^5 + 2\,Ca^3de^6 + 6\,Aa^2cde^6 - Ba^3e^7)e^{(-9)}\log\left(\frac{|xe + d|e^{(-1)}}{(xe + d)^2}\right) - \left(\frac{Cc^3d^8e^7}{xe + d} - \frac{Bc^3d^7e^8}{xe + d} + \frac{3\,Cac^2d^6e^9}{xe + d} + \frac{Ac^3d^6e^9}{xe + d} - \frac{3\,Bac^2d^5e^{10}}{xe + d} + \frac{3\,Ca^2cd^4e^{11}}{xe + d} + \frac{3\,Aac^2d^4e^{11}}{xe + d} - \frac{3\,Ba^2cd^3e^{12}}{xe + d} + \frac{Ca^3d^2e^{13}}{xe + d} + \frac{3\,Aa^2cd^2e^{13}}{xe + d} - \frac{Ba^3de^{14}}{xe + d} + \frac{Aa^3e^{15}}{xe + d}\right)e^{(-16)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/420*(60*C*c^3 - 70*(8*C*c^3*d*e - B*c^3*e^2)*e^(-1)/(x*e + d) + 84*(28*C*
c^3*d^2*e^2 - 7*B*c^3*d*e^3 + 3*C*a*c^2*e^4 + A*c^3*e^4)*e^(-2)/(x*e + d)^2
 - 105*(56*C*c^3*d^3*e^3 - 21*B*c^3*d^2*e^4 + 18*C*a*c^2*d*e^5 + 6*A*c^3*d*
e^5 - 3*B*a*c^2*e^6)*e^(-3)/(x*e + d)^3 + 140*(70*C*c^3*d^4*e^4 - 35*B*c^3*
d^3*e^5 + 45*C*a*c^2*d^2*e^6 + 15*A*c^3*d^2*e^6 - 15*B*a*c^2*d*e^7 + 3*C*a^
2*c*e^8 + 3*A*a*c^2*e^8)*e^(-4)/(x*e + d)^4 - 210*(56*C*c^3*d^5*e^5 - 35*B*
c^3*d^4*e^6 + 60*C*a*c^2*d^3*e^7 + 20*A*c^3*d^3*e^7 - 30*B*a*c^2*d^2*e^8 +
12*C*a^2*c*d*e^9 + 12*A*a*c^2*d*e^9 - 3*B*a^2*c*e^10)*e^(-5)/(x*e + d)^5 +
420*(28*C*c^3*d^6*e^6 - 21*B*c^3*d^5*e^7 + 45*C*a*c^2*d^4*e^8 + 15*A*c^3*d^
4*e^8 - 30*B*a*c^2*d^3*e^9 + 18*C*a^2*c*d^2*e^10 + 18*A*a*c^2*d^2*e^10 - 9*
B*a^2*c*d*e^11 + C*a^3*e^12 + 3*A*a^2*c*e^12)*e^(-6)/(x*e + d)^6)*(x*e + d)
^7*e^(-9) + (8*C*c^3*d^7 - 7*B*c^3*d^6*e + 18*C*a*c^2*d^5*e^2 + 6*A*c^3*d^5
*e^2 - 15*B*a*c^2*d^4*e^3 + 12*C*a^2*c*d^3*e^4 + 12*A*a*c^2*d^3*e^4 - 9*B*a
^2*c*d^2*e^5 + 2*C*a^3*d*e^6 + 6*A*a^2*c*d*e^6 - B*a^3*e^7)*e^(-9)*log(abs(
x*e + d)*e^(-1)/(x*e + d)^2) - (C*c^3*d^8*e^7/(x*e + d) - B*c^3*d^7*e^8/(x*
e + d) + 3*C*a*c^2*d^6*e^9/(x*e + d) + A*c^3*d^6*e^9/(x*e + d) - 3*B*a*c^2*
d^5*e^10/(x*e + d) + 3*C*a^2*c*d^4*e^11/(x*e + d) + 3*A*a*c^2*d^4*e^11/(x*e
 + d) - 3*B*a^2*c*d^3*e^12/(x*e + d) + C*a^3*d^2*e^13/(x*e + d) + 3*A*a^2*c
*d^2*e^13/(x*e + d) - B*a^3*d*e^14/(x*e + d) + A*a^3*e^15/(x*e + d))*e^(-16
)
```

Mupad [B]
time = 3.99, size = 1511, normalized size = 3.11



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

```
[Out] x*((C*a^3 + 3*A*a^2*c)/e^2 + (2*d*((2*d*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^3
*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e^2 - (2*d*((2*d*
((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3
*d^2)/e^4))/e - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2
))/e + (3*a*c*(A*c + C*a))/e^2))/e + (d^2*((2*d*((2*d*((B*c^3)/e^2 - (2*C*c
^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c^
3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/e^2 - (3*B*a^2*c)/e^2))/
e - (d^2*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^
2)/e^2 + (C*c^3*d^2)/e^4))/e^2 - (2*d*((2*d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d
)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c^3)/e
^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/e + (3*a*c*(A*c + C*a))/e^2)
)/e^2) + x^4*((d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*
c^2)/e^2 + (C*c^3*d^2)/e^4))/(2*e) - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/
(4*e^2) + (3*B*a*c^2)/(4*e^2)) - x^2*((d*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^
```

```
3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e^2 - (2*d*((2*d
*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^
3*d^2)/e^4))/e - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^
2))/e + (3*a*c*(A*c + C*a))/e^2))/e + (d^2*((2*d*((2*d*((B*c^3)/e^2 - (2*C*
c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c
^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/(2*e^2) - (3*B*a^2*c)/(
2*e^2)) + x^6*((B*c^3)/(6*e^2) - (C*c^3*d)/(3*e^3)) - x^5*((2*d*((B*c^3)/e^
2 - (2*C*c^3*d)/e^3))/(5*e) - (A*c^3 + 3*C*a*c^2)/(5*e^2) + (C*c^3*d^2)/(5*
e^4)) + x^3*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a
*c^2)/e^2 + (C*c^3*d^2)/e^4))/(3*e^2) - (2*d*((2*d*((2*d*((B*c^3)/e^2 - (2*
C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B
*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/(3*e) + (a*c*(A*c + C
*a))/e^2) - (A*a^3*e^8 + C*c^3*d^8 - B*a^3*d*e^7 - B*c^3*d^7*e + A*c^3*d^6*
e^2 + C*a^3*d^2*e^6 + 3*A*a*c^2*d^4*e^4 + 3*A*a^2*c*d^2*e^6 - 3*B*a*c^2*d^5
*e^3 - 3*B*a^2*c*d^3*e^5 + 3*C*a*c^2*d^6*e^2 + 3*C*a^2*c*d^4*e^4)/(e*(d*e^8
 + e^9*x)) - (log(d + e*x)*(8*C*c^3*d^7 - B*a^3*e^7 + 2*C*a^3*d*e^6 - 7*B*c
^3*d^6*e + 6*A*c^3*d^5*e^2 + 12*A*a*c^2*d^3*e^4 - 15*B*a*c^2*d^4*e^3 - 9*B*
a^2*c*d^2*e^5 + 18*C*a*c^2*d^5*e^2 + 12*C*a^2*c*d^3*e^4 + 6*A*a^2*c*d*e^6))
/e^9 + (C*c^3*x^7)/(7*e^2)
```

3.38 $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^3}\,dx$

Optimal. Leaf size=466

$$-\frac{c(3a^2e^4(3Cd-Be)+c^2d^3(21Cd^2-5e(3Bd-2Ae))+3acde^2(10Cd^2-3e(2Bd-Ae)))\,x}{e^8}+\frac{c(3a^2Ce^4+$$

```
[Out] -c*(3*a^2*e^4*(-B*e+3*C*d)+c^2*d^3*(21*C*d^2-5*e*(-2*A*e+3*B*d))+3*a*c*d*e^
2*(10*C*d^2-3*e*(-A*e+2*B*d)))*x/e^8+1/2*c*(3*a^2*C*e^4+c^2*d^2*(15*C*d^2-2
*e*(-3*A*e+5*B*d))+3*a*c*e^2*(6*C*d^2-e*(-A*e+3*B*d)))*x^2/e^7-1/3*c^2*(3*a
*e^2*(-B*e+3*C*d)+c*d*(10*C*d^2-3*e*(-A*e+2*B*d)))*x^3/e^6+1/4*c^2*(3*a*C*e
^2+c*(6*C*d^2-e*(-A*e+3*B*d)))*x^4/e^5-1/5*c^3*(-B*e+3*C*d)*x^5/e^4+1/6*c^3
*C*x^6/e^3-1/2*(a*e^2+c*d^2)^3*(A*e^2-B*d*e+C*d^2)/e^9/(e*x+d)^2+(a*e^2+c*d
^2)^2*(a*e^2*(-B*e+2*C*d)+c*d*(8*C*d^2-e*(-6*A*e+7*B*d)))/e^9/(e*x+d)+(a*e^
2+c*d^2)*(a^2*C*e^4+c^2*d^2*(28*C*d^2-3*e*(-5*A*e+7*B*d))+a*c*e^2*(17*C*d^2
-3*e*(-A*e+3*B*d)))*ln(e*x+d)/e^9
```

Rubi [A]
time = 0.58, antiderivative size = 463, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1642}

$$\frac{(ae^2+cd^2)\log(d+ex)\left(a^2Ce^4+ace^2(17Cd^2-3e(3Bd-Ae))+c^2(28Cd^4-3d^2e(7Bd-5Ae))\right)}{e^9}+\frac{cx^2(3a^2Ce^4+3ace^2(6Cd^2-e(3Bd-Ae))+c^2(15Cd^4-2d^2e(5Bd-3Ae)))}{2e^7}-\frac{cx(3a^2e^4(3Cd-Be)+3acde^2(10Cd^2-3e(2Bd-Ae))+c^2(21Cd^5-5d^3e(3Bd-2Ae)))}{e^8}-\frac{c^2x^3(3ae^2(3Cd-Be)-3cde(2Bd-Ae)+10cCd^3)}{3e^6}+\frac{c^2x^4(3aCe^2-ce(3Bd-Ae)+6cCd^2)}{4e^5}-\frac{(ae^2+cd^2)^3(Ae^2-Bde+Cd^2)}{2e^9(d+ex)^2}+\frac{(ae^2+cd^2)^2(ae^2(2Cd-Be)-cde(7Bd-6Ae)+8cCd^3)}{e^9(d+ex)}-\frac{c^3x^5(3Cd-Be)}{5e^4}+\frac{c^3Cx^6}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] -((c*(3*a^2*e^4*(3*C*d - B*e) + c^2*(21*C*d^5 - 5*d^3*e*(3*B*d - 2*A*e)) +
3*a*c*d*e^2*(10*C*d^2 - 3*e*(2*B*d - A*e)))*x)/e^8) + (c*(3*a^2*C*e^4 + c^2
*(15*C*d^4 - 2*d^2*e*(5*B*d - 3*A*e)) + 3*a*c*e^2*(6*C*d^2 - e*(3*B*d - A*e
)))*x^2)/(2*e^7) - (c^2*(10*c*C*d^3 - 3*c*d*e*(2*B*d - A*e) + 3*a*e^2*(3*C*
d - B*e))*x^3)/(3*e^6) + (c^2*(6*c*C*d^2 + 3*a*C*e^2 - c*e*(3*B*d - A*e))*x
^4)/(4*e^5) - (c^3*(3*C*d - B*e)*x^5)/(5*e^4) + (c^3*C*x^6)/(6*e^3) - ((c*d
^2 + a*e^2)^3*(C*d^2 - B*d*e + A*e^2))/(2*e^9*(d + e*x)^2) + ((c*d^2 + a*e^
2)^2*(8*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B*e)))/(e^9*(d + e
*x)) + ((c*d^2 + a*e^2)*(a^2*C*e^4 + c^2*(28*C*d^4 - 3*d^2*e*(7*B*d - 5*A*e
)) + a*c*e^2*(17*C*d^2 - 3*e*(3*B*d - A*e)))*Log[d + e*x])/e^9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^3} dx = \int \left(\frac{c(-3a^2e^4(3Cd-Be)-c^2(21Cd^5-5d^3e(3Bd-2Ae))-3acde^2(1\ldots}{e^8} \right.$$

$$= -\frac{c(3a^2e^4(3Cd-Be)+c^2(21Cd^5-5d^3e(3Bd-2Ae))+3acde^2(10Cd^2\ldots}{e^8}$$

Mathematica [A]
time = 0.14, size = 438, normalized size = 0.94

$$\frac{-60ce(-3a^2e^4(-3Cd+Be)+3acde^2(10Cd^2+3e(-2Bd+Ae))+c^2(21Cd^5+5d^3e(-3Bd+2Ae)))x+30ce^2(3a^2Ce^4+3ace^2(6Cd^2+e(-3Bd+Ae))+c^2(15Cd^4+2d^2e(-5Bd+3Ae)))x^2-20c^2e^3(10cCd^3+3cde(-2Bd+Ae)-3ae^2(-3Cd+Be))x^3+15c^2e^4(6cCd^2+3aCe^2+ce(-3Bd+Ae))x^4+12c^3e^5(-3Cd+Be)x^5+10c^3Ce^6x^6-\frac{30(cd^2+ae^2)^3(Cd^2+e(-Bd+Ae))}{(d+ex)^2}+\frac{60(cd^2+ae^2)^2(8cCd^3+cde(-7Bd+6Ae)+ae^2(2Cd-Be))}{d+ex}+60(cd^2+ae^2)(a^2Ce^4+ace^2(17Cd^2+3e(-3Bd+Ae))+c^2(28Cd^4+3d^2e(-7Bd+5Ae)))\log(d+ex)}{60e^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] (-60*c*e*(-3*a^2*e^4*(-3*C*d + B*e) + 3*a*c*d*e^2*(10*C*d^2 + 3*e*(-2*B*d +
 A*e)) + c^2*(21*C*d^5 + 5*d^3*e*(-3*B*d + 2*A*e)))*x + 30*c*e^2*(3*a^2*C*e
^4 + 3*a*c*e^2*(6*C*d^2 + e*(-3*B*d + A*e)) + c^2*(15*C*d^4 + 2*d^2*e*(-5*B
*d + 3*A*e)))*x^2 - 20*c^2*e^3*(10*c*C*d^3 + 3*c*d*e*(-2*B*d + A*e) - 3*a*e
^2*(-3*C*d + B*e))*x^3 + 15*c^2*e^4*(6*c*C*d^2 + 3*a*C*e^2 + c*e*(-3*B*d +
A*e))*x^4 + 12*c^3*e^5*(-3*C*d + B*e)*x^5 + 10*c^3*C*e^6*x^6 - (30*(c*d^2 +
 a*e^2)^3*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2 + (60*(c*d^2 + a*e^2)^2*(
8*c*C*d^3 + c*d*e*(-7*B*d + 6*A*e) + a*e^2*(2*C*d - B*e)))/(d + e*x) + 60*(
c*d^2 + a*e^2)*(a^2*C*e^4 + a*c*e^2*(17*C*d^2 + 3*e*(-3*B*d + A*e)) + c^2*(
28*C*d^4 + 3*d^2*e*(-7*B*d + 5*A*e)))*Log[d + e*x])/(60*e^9)
```

Maple [A]
time = 0.09, size = 753, normalized size = 1.62 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^9*(3*A*a^2*c*e^6+18*A*a*c^2*d^2*e^4+15*A*c^3*d^4*e^2-9*B*a^2*c*d*e^5-30
*B*a*c^2*d^3*e^3-21*B*c^3*d^5*e+C*a^3*e^6+18*C*a^2*c*d^2*e^4+45*C*a*c^2*d^4
*e^2+28*C*c^3*d^6)*ln(e*x+d)-1/2*(A*a^3*e^8+3*A*a^2*c*d^2*e^6+3*A*a*c^2*d^4
*e^4+A*c^3*d^6*e^2-B*a^3*d*e^7-3*B*a^2*c*d^3*e^5-3*B*a*c^2*d^5*e^3-B*c^3*d^
7*e+C*a^3*d^2*e^6+3*C*a^2*c*d^4*e^4+3*C*a*c^2*d^6*e^2+C*c^3*d^8)/e^9/(e*x+d
)^2-(-6*A*a^2*c*d*e^6-12*A*a*c^2*d^3*e^4-6*A*c^3*d^5*e^2+B*a^3*e^7+9*B*a^2*
c*d^2*e^5+15*B*a*c^2*d^4*e^3+7*B*c^3*d^6*e-2*C*a^3*d*e^6-12*C*a^2*c*d^3*e^4
-18*C*a*c^2*d^5*e^2-8*C*c^3*d^7)/e^9/(e*x+d)-c/e^8*(9/2*B*a*c*d*e^4*x^2+A*c
^2*d*e^4*x^3-B*a*c*e^5*x^3-2*B*c^2*d^2*e^3*x^3+10/3*C*c^2*d^3*e^2*x^3-3/2*A
*a*c*e^5*x^2-3*A*c^2*d^2*e^3*x^2+5*B*c^2*d^3*e^2*x^2-15/2*C*c^2*d^4*e*x^2+3
/4*B*c^2*d*e^4*x^4-3/4*C*a*c*e^5*x^4-3/2*C*c^2*d^2*e^3*x^4+3/5*C*c^2*d*e^4*
x^5-15*B*c^2*d^4*e*x+9*C*a^2*d*e^4*x+10*A*c^2*d^3*e^2*x-1/6*c^2*C*x^6*e^5-1
```

```
/5*B*c^2*e^5*x^5-1/4*A*c^2*e^5*x^4-3/2*C*a^2*e^5*x^2-3*B*a^2*e^5*x+21*C*c^2
*d^5*x+3*C*a*c*d*e^4*x^3-9*C*a*c*d^2*e^3*x^2+9*A*a*c*d*e^4*x-18*B*a*c*d^2*e
^3*x+30*C*a*c*d^3*e^2*x)
```

Maxima [A]
time = 0.31, size = 690, normalized size = 1.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] (28*C*c^3*d^6 - 21*B*c^3*d^5*e - 30*B*a*c^2*d^3*e^3 - 9*B*a^2*c*d*e^5 + 15*
(3*C*a*c^2*e^2 + A*c^3*e^2)*d^4 + C*a^3*e^6 + 3*A*a^2*c*e^6 + 18*(C*a^2*c*e
^4 + A*a*c^2*e^4)*d^2)*e^(-9)*log(x*e + d) + 1/60*(10*C*c^3*x^6*e^5 - 12*(3
*C*c^3*d*e^4 - B*c^3*e^5)*x^5 + 15*(6*C*c^3*d^2*e^3 - 3*B*c^3*d*e^4 + 3*C*a
*c^2*e^5 + A*c^3*e^5)*x^4 - 20*(10*C*c^3*d^3*e^2 - 6*B*c^3*d^2*e^3 - 3*B*a*
c^2*e^5 + 3*(3*C*a*c^2*e^4 + A*c^3*e^4)*d)*x^3 + 30*(15*C*c^3*d^4*e - 10*B*
c^3*d^3*e^2 - 9*B*a*c^2*d*e^4 + 3*C*a^2*c*e^5 + 3*A*a*c^2*e^5 + 6*(3*C*a*c^
2*e^3 + A*c^3*e^3)*d^2)*x^2 - 60*(21*C*c^3*d^5 - 15*B*c^3*d^4*e - 18*B*a*c^
2*d^2*e^3 - 3*B*a^2*c*e^5 + 10*(3*C*a*c^2*e^2 + A*c^3*e^2)*d^3 + 9*(C*a^2*c
*e^4 + A*a*c^2*e^4)*d)*x)*e^(-8) + 1/2*(15*C*c^3*d^8 - 13*B*c^3*d^7*e - 27*
B*a*c^2*d^5*e^3 - 15*B*a^2*c*d^3*e^5 + 11*(3*C*a*c^2*e^2 + A*c^3*e^2)*d^6 -
 B*a^3*d*e^7 + 21*(C*a^2*c*e^4 + A*a*c^2*e^4)*d^4 - A*a^3*e^8 + 3*(C*a^3*e^
6 + 3*A*a^2*c*e^6)*d^2 + 2*(8*C*c^3*d^7*e - 7*B*c^3*d^6*e^2 - 15*B*a*c^2*d^
4*e^4 - 9*B*a^2*c*d^2*e^6 + 6*(3*C*a*c^2*e^3 + A*c^3*e^3)*d^5 - B*a^3*e^8 +
 12*(C*a^2*c*e^5 + A*a*c^2*e^5)*d^3 + 2*(C*a^3*e^7 + 3*A*a^2*c*e^7)*d)*x)/(
x^2*e^11 + 2*d*x*e^10 + d^2*e^9)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 984 vs. $2(458) = 916$.
time = 0.37, size = 984, normalized size = 2.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/60*(450*C*c^3*d^8 + (10*C*c^3*x^8 + 12*B*c^3*x^7 + 60*B*a*c^2*x^5 + 180*B
*a^2*c*x^3 + 15*(3*C*a*c^2 + A*c^3)*x^6 - 60*B*a^3*x + 90*(C*a^2*c + A*a*c^
2)*x^4 - 30*A*a^3)*e^8 - (16*C*c^3*d*x^7 + 21*B*c^3*d*x^6 + 150*B*a*c^2*d*x
^4 - 360*B*a^2*c*d*x^2 + 30*(3*C*a*c^2 + A*c^3)*d*x^5 + 30*B*a^3*d + 360*(C
*a^2*c + A*a*c^2)*d*x^3 - 120*(C*a^3 + 3*A*a^2*c)*d*x)*e^7 + (28*C*c^3*d^2*
x^6 + 42*B*c^3*d^2*x^5 + 600*B*a*c^2*d^2*x^3 - 360*B*a^2*c*d^2*x + 75*(3*C*
a*c^2 + A*c^3)*d^2*x^4 - 990*(C*a^2*c + A*a*c^2)*d^2*x^2 + 90*(C*a^3 + 3*A*
a^2*c)*d^2)*e^6 - (56*C*c^3*d^3*x^5 + 105*B*c^3*d^3*x^4 - 1890*B*a*c^2*d^3*
```

```
x^2 + 450*B*a^2*c*d^3 + 300*(3*C*a*c^2 + A*c^3)*d^3*x^3 - 180*(C*a^2*c + A*
a*c^2)*d^3*x)*e^5 + 10*(14*C*c^3*d^4*x^4 + 42*B*c^3*d^4*x^3 + 18*B*a*c^2*d^
4*x - 102*(3*C*a*c^2 + A*c^3)*d^4*x^2 + 63*(C*a^2*c + A*a*c^2)*d^4)*e^4 - 1
0*(56*C*c^3*d^5*x^3 - 150*B*c^3*d^5*x^2 + 81*B*a*c^2*d^5 + 24*(3*C*a*c^2 +
A*c^3)*d^5*x)*e^3 - 30*(69*C*c^3*d^6*x^2 - 16*B*c^3*d^6*x - 11*(3*C*a*c^2 +
 A*c^3)*d^6)*e^2 - 390*(2*C*c^3*d^7*x + B*c^3*d^7)*e + 60*(28*C*c^3*d^8 + (
C*a^3 + 3*A*a^2*c)*x^2*e^8 - (9*B*a^2*c*d*x^2 - 2*(C*a^3 + 3*A*a^2*c)*d*x)*
e^7 - (18*B*a^2*c*d^2*x - 18*(C*a^2*c + A*a*c^2)*d^2*x^2 - (C*a^3 + 3*A*a^2
*c)*d^2)*e^6 - 3*(10*B*a*c^2*d^3*x^2 + 3*B*a^2*c*d^3 - 12*(C*a^2*c + A*a*c^
2)*d^3*x)*e^5 - 3*(20*B*a*c^2*d^4*x - 5*(3*C*a*c^2 + A*c^3)*d^4*x^2 - 6*(C*
a^2*c + A*a*c^2)*d^4)*e^4 - 3*(7*B*c^3*d^5*x^2 + 10*B*a*c^2*d^5 - 10*(3*C*a
*c^2 + A*c^3)*d^5*x)*e^3 + (28*C*c^3*d^6*x^2 - 42*B*c^3*d^6*x + 15*(3*C*a*c
^2 + A*c^3)*d^6)*e^2 + 7*(8*C*c^3*d^7*x - 3*B*c^3*d^7)*e)*log(x*e + d))/(x^
2*e^11 + 2*d*x*e^10 + d^2*e^9)
```

Sympy [A]
time = 11.17, size = 816, normalized size = 1.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

```
[Out] C*c**3*x**6/(6*e**3) + x**5*(B*c**3/(5*e**3) - 3*C*c**3*d/(5*e**4)) + x**4*
(A*c**3/(4*e**3) - 3*B*c**3*d/(4*e**4) + 3*C*a*c**2/(4*e**3) + 3*C*c**3*d**
2/(2*e**5)) + x**3*(-A*c**3*d/e**4 + B*a*c**2/e**3 + 2*B*c**3*d**2/e**5 - 3
*C*a*c**2*d/e**4 - 10*C*c**3*d**3/(3*e**6)) + x**2*(3*A*a*c**2/(2*e**3) + 3
*A*c**3*d**2/e**5 - 9*B*a*c**2*d/(2*e**4) - 5*B*c**3*d**3/e**6 + 3*C*a**2*c
/(2*e**3) + 9*C*a*c**2*d**2/e**5 + 15*C*c**3*d**4/(2*e**7)) + x*(-9*A*a*c**
2*d/e**4 - 10*A*c**3*d**3/e**6 + 3*B*a**2*c/e**3 + 18*B*a*c**2*d**2/e**5 +
15*B*c**3*d**4/e**7 - 9*C*a**2*c*d/e**4 - 30*C*a*c**2*d**3/e**6 - 21*C*c**3
*d**5/e**8) + (-A*a**3*e**8 + 9*A*a**2*c*d**2*e**6 + 21*A*a*c**2*d**4*e**4
+ 11*A*c**3*d**6*e**2 - B*a**3*d*e**7 - 15*B*a**2*c*d**3*e**5 - 27*B*a*c**2
*d**5*e**3 - 13*B*c**3*d**7*e + 3*C*a**3*d**2*e**6 + 21*C*a**2*c*d**4*e**4
+ 33*C*a*c**2*d**6*e**2 + 15*C*c**3*d**8 + x*(12*A*a**2*c*d*e**7 + 24*A*a*c
**2*d**3*e**5 + 12*A*c**3*d**5*e**3 - 2*B*a**3*e**8 - 18*B*a**2*c*d**2*e**6
 - 30*B*a*c**2*d**4*e**4 - 14*B*c**3*d**6*e**2 + 4*C*a**3*d*e**7 + 24*C*a**
2*c*d**3*e**5 + 36*C*a*c**2*d**5*e**3 + 16*C*c**3*d**7*e))/(2*d**2*e**9 + 4
*d*e**10*x + 2*e**11*x**2) + (a*e**2 + c*d**2)*(3*A*a*c*e**4 + 15*A*c**2*d*
*2*e**2 - 9*B*a*c*d*e**3 - 21*B*c**2*d**3*e + C*a**2*e**4 + 17*C*a*c*d**2*e
**2 + 28*C*c**2*d**4)*log(d + e*x)/e**9
```

Giac [A]
time = 4.08, size = 727, normalized size = 1.56

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")

[Out] (28*C*c^3*d^6 - 21*B*c^3*d^5*e + 45*C*a*c^2*d^4*e^2 + 15*A*c^3*d^4*e^2 - 30*B*a*c^2*d^3*e^3 + 18*C*a^2*c*d^2*e^4 + 18*A*a*c^2*d^2*e^4 - 9*B*a^2*c*d*e^5 + C*a^3*e^6 + 3*A*a^2*c*e^6)*e^(-9)*log(abs(x*e + d)) + 1/60*(10*C*c^3*x^6*e^15 - 36*C*c^3*d*x^5*e^14 + 90*C*c^3*d^2*x^4*e^13 - 200*C*c^3*d^3*x^3*e^12 + 450*C*c^3*d^4*x^2*e^11 - 1260*C*c^3*d^5*x*e^10 + 12*B*c^3*x^5*e^15 - 45*B*c^3*d*x^4*e^14 + 120*B*c^3*d^2*x^3*e^13 - 300*B*c^3*d^3*x^2*e^12 + 900*B*c^3*d^4*x*e^11 + 45*C*a*c^2*x^4*e^15 + 15*A*c^3*x^4*e^15 - 180*C*a*c^2*d*x^3*e^14 - 60*A*c^3*d*x^3*e^14 + 540*C*a*c^2*d^2*x^2*e^13 + 180*A*c^3*d^2*x^2*e^13 - 1800*C*a*c^2*d^3*x*e^12 - 600*A*c^3*d^3*x*e^12 + 60*B*a*c^2*x^3*e^15 - 270*B*a*c^2*d*x^2*e^14 + 1080*B*a*c^2*d^2*x*e^13 + 90*C*a^2*c*x^2*e^15 + 90*A*a*c^2*x^2*e^15 - 540*C*a^2*c*d*x*e^14 - 540*A*a*c^2*d*x*e^14 + 180*B*a^2*c*x*e^15)*e^(-18) + 1/2*(15*C*c^3*d^8 - 13*B*c^3*d^7*e + 33*C*a*c^2*d^6*e^2 + 11*A*c^3*d^6*e^2 - 27*B*a*c^2*d^5*e^3 + 21*C*a^2*c*d^4*e^4 + 21*A*a*c^2*d^4*e^4 - 15*B*a^2*c*d^3*e^5 + 3*C*a^3*d^2*e^6 + 9*A*a^2*c*d^2*e^6 - B*a^3*d*e^7 - A*a^3*e^8 + 2*(8*C*c^3*d^7*e - 7*B*c^3*d^6*e^2 + 18*C*a*c^2*d^5*e^3 + 6*A*c^3*d^5*e^3 - 15*B*a*c^2*d^4*e^4 + 12*C*a^2*c*d^3*e^5 + 12*A*a*c^2*d^3*e^5 - 9*B*a^2*c*d^2*e^6 + 2*C*a^3*d*e^7 + 6*A*a^2*c*d*e^7 - B*a^3*e^8)*x)*e^(-9)/(x*e + d)^2

Mupad [B]
time = 3.94, size = 1290, normalized size = 2.77

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^3,x)

[Out] x^3*((d*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e - (d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (B*a*c^2)/e^3 - (C*c^3*d^3)/(3*e^6)) + x*((3*d*((3*d*((3*d*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e - (3*d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*B*a*c^2)/e^3 - (C*c^3*d^3)/e^6))/e - (3*d^2*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e^2 + (d^3*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^3 - (3*a*c*(A*c + C*a))/e^3))/e + (d^3*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e^3 - (3*d^2*((3*d*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e - (3*d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*B*a*c^2)/e^3 - (C*c^3*d^3)/e^6))/e^2 + (3*B*a^2*c)/e^3) + x^5*((B*c^3)/(5*e^3) - (3*C*c^3*d)/(5*e^4)) - x^4*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/(4*e) - (A*c^3 + 3*C*a*c^2)/(4*e^3) + (3*C*c^3*d^2)/(4*e^5)) - x^2*((3*d*((3*d*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^

```
5))/e - (3*d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*B*a*c^2)/e^3 - (C*
c^3*d^3)/e^6))/(2*e) - (3*d^2*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A
*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/(2*e^2) + (d^3*((B*c^3)/e^3 - (
3*C*c^3*d)/e^4))/(2*e^3) - (3*a*c*(A*c + C*a))/(2*e^3)) + ((15*C*c^3*d^8 -
A*a^3*e^8 - B*a^3*d*e^7 - 13*B*c^3*d^7*e + 11*A*c^3*d^6*e^2 + 3*C*a^3*d^2*e
^6 + 21*A*a*c^2*d^4*e^4 + 9*A*a^2*c*d^2*e^6 - 27*B*a*c^2*d^5*e^3 - 15*B*a^2
*c*d^3*e^5 + 33*C*a*c^2*d^6*e^2 + 21*C*a^2*c*d^4*e^4)/(2*e) + x*(8*C*c^3*d^
7 - B*a^3*e^7 + 2*C*a^3*d*e^6 - 7*B*c^3*d^6*e + 6*A*c^3*d^5*e^2 + 12*A*a*c^
2*d^3*e^4 - 15*B*a*c^2*d^4*e^3 - 9*B*a^2*c*d^2*e^5 + 18*C*a*c^2*d^5*e^2 + 1
2*C*a^2*c*d^3*e^4 + 6*A*a^2*c*d*e^6))/(d^2*e^8 + e^10*x^2 + 2*d*e^9*x) + (l
og(d + e*x)*(C*a^3*e^6 + 28*C*c^3*d^6 + 3*A*a^2*c*e^6 - 21*B*c^3*d^5*e + 15
*A*c^3*d^4*e^2 + 18*A*a*c^2*d^2*e^4 - 30*B*a*c^2*d^3*e^3 + 45*C*a*c^2*d^4*e
^2 + 18*C*a^2*c*d^2*e^4 - 9*B*a^2*c*d*e^5))/e^9 + (C*c^3*x^6)/(6*e^3)
```

3.39 $\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2} dx$

Optimal. Leaf size=17

$$\frac{(a+bx^2)^2}{c+dx}$$

```
[Out] (b*x^2+a)^2/(d*x+c)
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = $\{1604\}$

$$\frac{(a+bx^2)^2}{c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(-(a*d) + 4*b*c*x + 3*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^2/(c + d*x)
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx = \frac{(a+bx^2)^2}{c+dx}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 62 vs. 2(17) = 34.
time = 0.02, size = 62, normalized size = 3.65

$$\frac{a^2d^4+2abd^2(c^2+cdx+d^2x^2)+b^2(c^4+c^3dx+d^4x^4)}{d^4(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(-(a*d) + 4*b*c*x + 3*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a^2*d^4 + 2*a*b*d^2*(c^2 + c*d*x + d^2*x^2) + b^2*(c^4 + c^3*d*x + d^4*x^4
))/(d^4*(c + d*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(17) = 34$.
time = 0.08, size = 76, normalized size = 4.47

method	result	size
gosper	$\frac{b^2x^4+2ab\,x^2+a^2}{dx+c}$	27
norman	$\frac{b^2x^4+2ab\,x^2-\frac{d\,a^2x}{c}}{dx+c}$	34
default	$\frac{b(b\,x^3d^2-bcd\,x^2+2a\,d^2x+b\,c^2x)}{d^3}-\frac{-a^2d^4-2ab\,c^2d^2-b^2c^4}{d^4(dx+c)}$	76
risch	$\frac{b^2x^3}{d}-\frac{b^2c\,x^2}{d^2}+\frac{2bax}{d}+\frac{b^2c^2x}{d^3}+\frac{a^2}{dx+c}+\frac{2ab\,c^2}{d^2(dx+c)}+\frac{b^2c^4}{d^4(dx+c)}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b/d^3*(b*d^2*x^3-b*c*d*x^2+2*a*d^2*x+b*c^2*x)-(-a^2*d^4-2*a*b*c^2*d^2-b^2*c
^4)/d^4/(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(17) = 34$.
time = 0.28, size = 82, normalized size = 4.82

$$\frac{b^2c^4+2\,abc^2d^2+a^2d^4}{d^5x+cd^4}+\frac{b^2d^2x^3-b^2cdx^2+(b^2c^2+2\,abd^2)x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(d^5*x + c*d^4) + (b^2*d^2*x^3 - b^2*c*
d*x^2 + (b^2*c^2 + 2*a*b*d^2)*x)/d^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(17) = 34$.
time = 0.33, size = 78, normalized size = 4.59

$$\frac{b^2d^4x^4+2\,abd^4x^2+b^2c^4+2\,abc^2d^2+a^2d^4+(b^2c^3d+2\,abcd^3)x}{d^5x+cd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] (b^2*d^4*x^4 + 2*a*b*d^4*x^2 + b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4 + (b^2*c^3
*d + 2*a*b*c*d^3)*x)/(d^5*x + c*d^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(12) = 24$.
time = 0.16, size = 73, normalized size = 4.29

$$-\frac{b^2cx^2}{d^2}+\frac{b^2x^3}{d}+x\left(\frac{2ab}{d}+\frac{b^2c^2}{d^3}\right)+\frac{a^2d^4+2abc^2d^2+b^2c^4}{cd^4+d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(3*b*d*x**2+4*b*c*x-a*d)/(d*x+c)**2,x)
```

```
[Out] -b**2*c*x**2/d**2 + b**2*x**3/d + x*(2*a*b/d + b**2*c**2/d**3) + (a**2*d**4
 + 2*a*b*c**2*d**2 + b**2*c**4)/(c*d**4 + d**5*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(17) = 34$.
time = 4.69, size = 111, normalized size = 6.53

$$\frac{\left(b^2-\frac{4b^2c}{dx+c}+\frac{6b^2c^2}{(dx+c)^2}+\frac{2abd^2}{(dx+c)^2}\right)(dx+c)^3}{d^4}+\frac{\frac{b^2c^4d^3}{dx+c}+\frac{2abc^2d^5}{dx+c}+\frac{a^2d^7}{dx+c}}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (b^2 - 4*b^2*c/(d*x + c) + 6*b^2*c^2/(d*x + c)^2 + 2*a*b*d^2/(d*x + c)^2)*(
d*x + c)^3/d^4 + (b^2*c^4*d^3/(d*x + c) + 2*a*b*c^2*d^5/(d*x + c) + a^2*d^7
/(d*x + c))/d^7
```

Mupad [B]
time = 0.08, size = 85, normalized size = 5.00

$$x\left(\frac{b^2\,c^2}{d^3}+\frac{2\,a\,b}{d}\right)+\frac{b^2\,x^3}{d}+\frac{a^2\,d^4+2\,a\,b\,c^2\,d^2+b^2\,c^4}{d\,\left(x\,d^4+c\,d^3\right)}-\frac{b^2\,c\,x^2}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(4*b*c*x - a*d + 3*b*d*x^2))/(c + d*x)^2,x)
```

```
[Out] x*((b^2*c^2)/d^3 + (2*a*b)/d) + (b^2*x^3)/d + (a^2*d^4 + b^2*c^4 + 2*a*b*c^
2*d^2)/(d*(c*d^3 + d^4*x)) - (b^2*c*x^2)/d^2
```

3.40 $\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2} dx$

Optimal. Leaf size=17

$$\frac{(a+bx^2)^2}{c+dx}$$

```
[Out] (b*x^2+a)^2/(d*x+c)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {1604}

$$\frac{(a+bx^2)^2}{c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(-(a*d) + b*x*(4*c + 3*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^2/(c + d*x)
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2} dx = \frac{(a+bx^2)^2}{c+dx}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 62 vs. 2(17) = 34.
time = 0.01, size = 62, normalized size = 3.65

$$\frac{a^2d^4 + 2abd^2(c^2 + cdx + d^2x^2) + b^2(c^4 + c^3dx + d^4x^4)}{d^4(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(-(a*d) + b*x*(4*c + 3*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a^2*d^4 + 2*a*b*d^2*(c^2 + c*d*x + d^2*x^2) + b^2*(c^4 + c^3*d*x + d^4*x^4
))/(d^4*(c + d*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(17) = 34$.
time = 0.07, size = 76, normalized size = 4.47

method	result	size
gosper	$\frac{b^{2} x^{4}+2 a b \,x^{2}+a^{2}}{d x +c}$	27
norman	$\frac{b^{2} x^{4}+2 a b \,x^{2}-\frac{d \,a^{2} x}{c}}{d x +c}$	34
default	$\frac{b \left(b \,x^{3} d^{2}-b c d \,x^{2}+2 a \,d^{2} x +b \,c^{2} x \right)}{d^{3}}-\frac{-a^{2} d^{4}-2 a b \,c^{2} d^{2}-b^{2} c^{4}}{d^{4} \left(d x +c \right)}$	76
risch	$\frac{b^{2} x^{3}}{d}-\frac{b^{2} c \,x^{2}}{d^{2}}+\frac{2 b a x}{d}+\frac{b^{2} c^{2} x}{d^{3}}+\frac{a^{2}}{d x +c}+\frac{2 a b \,c^{2}}{d^{2} \left(d x +c \right)}+\frac{b^{2} c^{4}}{d^{4} \left(d x +c \right)}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b/d^3*(b*d^2*x^3-b*c*d*x^2+2*a*d^2*x+b*c^2*x)-(-a^2*d^4-2*a*b*c^2*d^2-b^2*c
^4)/d^4/(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(17) = 34$.
time = 0.27, size = 82, normalized size = 4.82

$$\frac{b^2c^4 + 2\,abc^2d^2 + a^2d^4}{d^5x + cd^4} + \frac{b^2d^2x^3 - b^2cdx^2 + (b^2c^2 + 2\,abd^2)x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(d^5*x + c*d^4) + (b^2*d^2*x^3 - b^2*c*
d*x^2 + (b^2*c^2 + 2*a*b*d^2)*x)/d^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(17) = 34$.
time = 0.36, size = 78, normalized size = 4.59

$$\frac{b^2d^4x^4 + 2\,abd^4x^2 + b^2c^4 + 2\,abc^2d^2 + a^2d^4 + (b^2c^3d + 2\,abcd^3)x}{d^5x + cd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^4*x^4 + 2*a*b*d^4*x^2 + b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4 + (b^2*c^3
*d + 2*a*b*c*d^3)*x)/(d^5*x + c*d^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(12) = 24$.
time = 0.16, size = 73, normalized size = 4.29

$$-\frac{b^2cx^2}{d^2} + \frac{b^2x^3}{d} + x\left(\frac{2ab}{d} + \frac{b^2c^2}{d^3}\right) + \frac{a^2d^4 + 2abc^2d^2 + b^2c^4}{cd^4 + d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)**2,x)
```

```
[Out] -b**2*c*x**2/d**2 + b**2*x**3/d + x*(2*a*b/d + b**2*c**2/d**3) + (a**2*d**4
 + 2*a*b*c**2*d**2 + b**2*c**4)/(c*d**4 + d**5*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(17) = 34$.
time = 2.77, size = 111, normalized size = 6.53

$$\frac{\left(b^2 - \frac{4\,b^2c}{dx+c} + \frac{6\,b^2c^2}{(dx+c)^2} + \frac{2\,abd^2}{(dx+c)^2}\right)(dx+c)^3}{d^4} + \frac{\frac{b^2c^4d^3}{dx+c} + \frac{2\,abc^2d^5}{dx+c} + \frac{a^2d^7}{dx+c}}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (b^2 - 4*b^2*c/(d*x + c) + 6*b^2*c^2/(d*x + c)^2 + 2*a*b*d^2/(d*x + c)^2)*(
d*x + c)^3/d^4 + (b^2*c^4*d^3/(d*x + c) + 2*a*b*c^2*d^5/(d*x + c) + a^2*d^7
/(d*x + c))/d^7
```

Mupad [B]
time = 3.84, size = 85, normalized size = 5.00

$$x\left(\frac{b^2\,c^2}{d^3} + \frac{2\,a\,b}{d}\right) + \frac{b^2\,x^3}{d} + \frac{a^2\,d^4 + 2\,a\,b\,c^2\,d^2 + b^2\,c^4}{d\,\left(x\,d^4 + c\,d^3\right)} - \frac{b^2\,c\,x^2}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*d - b*x*(4*c + 3*d*x))*(a + b*x^2))/(c + d*x)^2,x)
```

```
[Out] x*((b^2*c^2)/d^3 + (2*a*b)/d) + (b^2*x^3)/d + (a^2*d^4 + b^2*c^4 + 2*a*b*c^
2*d^2)/(d*(c*d^3 + d^4*x)) - (b^2*c*x^2)/d^2
```

3.41 $\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2} dx$

Optimal. Leaf size=17

$$\frac{(a+bx^2)^3}{c+dx}$$

[Out] (b*x^2+a)^3/(d*x+c)

Rubi [A]
time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1604}

$$\frac{(a+bx^2)^3}{c+dx}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^2)^2*(-(a*d) + 6*b*c*x + 5*b*d*x^2))/(c + d*x)^2,x]

[Out] (a + b*x^2)^3/(c + d*x)

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2} dx = \frac{(a+bx^2)^3}{c+dx}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(17) = 34.
time = 0.03, size = 90, normalized size = 5.29

$$\frac{a^3d^6+3a^2bd^4(c^2+cdx+d^2x^2)+3ab^2d^2(c^4+c^3dx+d^4x^4)+b^3(c^6+c^5dx+d^6x^6)}{d^6(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(-(a*d) + 6*b*c*x + 5*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a^3*d^6 + 3*a^2*b*d^4*(c^2 + c*d*x + d^2*x^2) + 3*a*b^2*d^2*(c^4 + c^3*d*x
 + d^4*x^4) + b^3*(c^6 + c^5*d*x + d^6*x^6))/(d^6*(c + d*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(17) = 34$.
time = 0.10, size = 157, normalized size = 9.24

method	result
gosper	$\frac{b^{3} x^{6}+3 a \,b^{2} x^{4}+3 a^{2} b \,x^{2}+a^{3}}{d x +c}$
norman	$\frac{b^{3} x^{6}+3 a \,b^{2} x^{4}+3 a^{2} b \,x^{2}-\frac{d \,a^{3} x}{c}}{d x +c}$
default	$\frac{b \left(b^{2} x^{5} d^{4}-b^{2} c \,x^{4} d^{3}+3 a b \,d^{4} x^{3}+b^{2} c^{2} d^{2} x^{3}-3 a b c \,d^{3} x^{2}-b^{2} c^{3} d \,x^{2}+3 a^{2} d^{4} x +3 a b \,c^{2} d^{2} x +b^{2} c^{4} x \right)}{d^{5}}-\frac{-a^{3} d^{6}-3 a^{2} b \,c^{2} d^{4}-3 a \,b^{2} c^{4} d^{2}-b^{3} c^{6}}{d^{6} \left(d x +c \right)}$
risch	$\frac{b^{3} x^{5}}{d}-\frac{b^{3} c \,x^{4}}{d^{2}}+\frac{3 b^{2} a \,x^{3}}{d}+\frac{b^{3} c^{2} x^{3}}{d^{3}}-\frac{3 b^{2} a c \,x^{2}}{d^{2}}-\frac{b^{3} c^{3} x^{2}}{d^{4}}+\frac{3 b \,a^{2} x}{d}+\frac{3 b^{2} a \,c^{2} x}{d^{3}}+\frac{b^{3} c^{4} x}{d^{5}}+\frac{a^{3}}{d x +c}+\frac{3 a^{2} b \,c^{2}}{d^{2} \left(d x +c \right)}+\frac{3 a \,b^{2}}{d^{4} (dx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b/d^5*(b^2*d^4*x^5-b^2*c*d^3*x^4+3*a*b*d^4*x^3+b^2*c^2*d^2*x^3-3*a*b*c*d^3*
x^2-b^2*c^3*d*x^2+3*a^2*d^4*x+3*a*b*c^2*d^2*x+b^2*c^4*x)-(-a^3*d^6-3*a^2*b*
c^2*d^4-3*a*b^2*c^4*d^2-b^3*c^6)/d^6/(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(17) = 34$.
time = 0.28, size = 160, normalized size = 9.41

$$\frac{b^3c^6 + 3\,ab^2c^4d^2 + 3\,a^2bc^2d^4 + a^3d^6}{d^7x + cd^6} + \frac{b^3d^4x^5 - b^3cd^3x^4 + (b^3c^2d^2 + 3\,ab^2d^4)x^3 - (b^3c^3d + 3\,ab^2cd^3)x^2 + (b^3c^4 + 3\,ab^2c^2d^2 + 3\,a^2bd^4)x}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="maxim
a")
```

```
[Out] (b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)/(d^7*x + c*d^6) + (
b^3*d^4*x^5 - b^3*c*d^3*x^4 + (b^3*c^2*d^2 + 3*a*b^2*d^4)*x^3 - (b^3*c^3*d
+ 3*a*b^2*c*d^3)*x^2 + (b^3*c^4 + 3*a*b^2*c^2*d^2 + 3*a^2*b*d^4)*x)/d^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(17) = 34$.
time = 0.35, size = 120, normalized size = 7.06

$$\frac{b^3d^6x^6 + 3\,ab^2d^6x^4 + 3\,a^2bd^6x^2 + b^3c^6 + 3\,ab^2c^4d^2 + 3\,a^2bc^2d^4 + a^3d^6 + (b^3c^5d + 3\,ab^2c^3d^3 + 3\,a^2bcd^5)x}{d^7x + cd^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="frica
s")
```

```
[Out] (b^3*d^6*x^6 + 3*a*b^2*d^6*x^4 + 3*a^2*b*d^6*x^2 + b^3*c^6 + 3*a*b^2*c^4*d^
2 + 3*a^2*b*c^2*d^4 + a^3*d^6 + (b^3*c^5*d + 3*a*b^2*c^3*d^3 + 3*a^2*b*c*d^
5)*x)/(d^7*x + c*d^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(12) = 24$.
time = 0.27, size = 153, normalized size = 9.00

$$-\frac{b^3cx^4}{d^2}+\frac{b^3x^5}{d}+x^3\cdot\left(\frac{3ab^2}{d}+\frac{b^3c^2}{d^3}\right)+x^2\left(-\frac{3ab^2c}{d^2}-\frac{b^3c^3}{d^4}\right)+x\left(\frac{3a^2b}{d}+\frac{3ab^2c^2}{d^3}+\frac{b^3c^4}{d^5}\right)+\frac{a^3d^6+3a^2bc^2d^4+3ab^2c^4d^2+b^3c^6}{cd^6+d^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(5*b*d*x**2+6*b*c*x-a*d)/(d*x+c)**2,x)
```

```
[Out] -b**3*c*x**4/d**2 + b**3*x**5/d + x**3*(3*a*b**2/d + b**3*c**2/d**3) + x**2
*(-3*a*b**2*c/d**2 - b**3*c**3/d**4) + x*(3*a**2*b/d + 3*a*b**2*c**2/d**3 +
 b**3*c**4/d**5) + (a**3*d**6 + 3*a**2*b*c**2*d**4 + 3*a*b**2*c**4*d**2 + b
**3*c**6)/(c*d**6 + d**7*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(17) = 34$.
time = 4.54, size = 216, normalized size = 12.71

$$\frac{\left(b^3-\frac{6b^3c}{dx+c}+\frac{15b^3c^2}{(dx+c)^2}-\frac{20b^3c^3}{(dx+c)^3}+\frac{15b^3c^4}{(dx+c)^4}+\frac{3ab^2d^2}{(dx+c)^2}-\frac{12ab^2cd^2}{(dx+c)^3}+\frac{18ab^2c^2d^2}{(dx+c)^4}+\frac{3a^2bd^4}{(dx+c)^4}\right)(dx+c)^5}{d^6}+\frac{\frac{b^3c^6d^5}{dx+c}+\frac{3ab^2c^4d^7}{dx+c}+\frac{3a^2bc^2d^9}{dx+c}+\frac{a^3d^{11}}{dx+c}}{d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="giac"
)
```

```
[Out] (b^3 - 6*b^3*c/(d*x + c) + 15*b^3*c^2/(d*x + c)^2 - 20*b^3*c^3/(d*x + c)^3
+ 15*b^3*c^4/(d*x + c)^4 + 3*a*b^2*d^2/(d*x + c)^2 - 12*a*b^2*c*d^2/(d*x +
c)^3 + 18*a*b^2*c^2*d^2/(d*x + c)^4 + 3*a^2*b*d^4/(d*x + c)^4)*(d*x + c)^5/
d^6 + (b^3*c^6*d^5/(d*x + c) + 3*a*b^2*c^4*d^7/(d*x + c) + 3*a^2*b*c^2*d^9/
(d*x + c) + a^3*d^11/(d*x + c))/d^11
```

Mupad [B]
time = 3.78, size = 252, normalized size = 14.82

$$x^3\left(\frac{3\,a\,b^2}{d}+\frac{b^3\,c^2}{d^3}\right)-x\left(\frac{2\,c\left(\frac{4\,b^3\,c^3}{d^4}-\frac{2\,c\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d}+\frac{12\,a\,b^2\,c}{d^2}\right)}{d}+\frac{c^2\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d^2}-\frac{3\,a^2\,b}{d}\right)+x^2\left(\frac{2\,b^3\,c^3}{d^4}-\frac{c\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d}+\frac{6\,a\,b^2\,c}{d^2}\right)+\frac{a^3\,d^6+3\,a^2\,b\,c^2\,d^4+3\,a\,b^2\,c^4\,d^2+b^3\,c^6}{d\,\left(x\,d^6+c\,d^5\right)}+\frac{b^3\,x^5}{d}-\frac{b^3\,c\,x^4}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(6*b*c*x - a*d + 5*b*d*x^2))/(c + d*x)^2,x)
```

```
[Out] x^3*((3*a*b^2)/d + (b^3*c^2)/d^3) - x*((2*c*((4*b^3*c^3)/d^4 - (2*c*((9*a*b
^2)/d + (3*b^3*c^2)/d^3))/d + (12*a*b^2*c)/d^2))/d + (c^2*((9*a*b^2)/d + (3
*b^3*c^2)/d^3))/d^2 - (3*a^2*b)/d) + x^2*((2*b^3*c^3)/d^4 - (c*((9*a*b^2)/d
 + (3*b^3*c^2)/d^3))/d + (6*a*b^2*c)/d^2) + (a^3*d^6 + b^3*c^6 + 3*a*b^2*c^
4*d^2 + 3*a^2*b*c^2*d^4)/(d*(c*d^5 + d^6*x)) + (b^3*x^5)/d - (b^3*c*x^4)/d^
2
```

3.42 $\int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2} dx$

Optimal. Leaf size=17

$$\frac{(a+bx^2)^3}{c+dx}$$

```
[Out] (b*x^2+a)^3/(d*x+c)
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {1604}

$$\frac{(a+bx^2)^3}{c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(-(a*d) + b*x*(6*c + 5*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^3/(c + d*x)
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2} dx = \frac{(a+bx^2)^3}{c+dx}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(17) = 34.
time = 0.02, size = 90, normalized size = 5.29

$$\frac{a^3d^6+3a^2bd^4(c^2+cdx+d^2x^2)+3ab^2d^2(c^4+c^3dx+d^4x^4)+b^3(c^6+c^5dx+d^6x^6)}{d^6(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(-(a*d) + b*x*(6*c + 5*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a^3*d^6 + 3*a^2*b*d^4*(c^2 + c*d*x + d^2*x^2) + 3*a*b^2*d^2*(c^4 + c^3*d*x
 + d^4*x^4) + b^3*(c^6 + c^5*d*x + d^6*x^6))/(d^6*(c + d*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(17) = 34$.
time = 0.07, size = 157, normalized size = 9.24

method	result
gosper	$\frac{b^{3} x^{6}+3 a \,b^{2} x^{4}+3 a^{2} b \,x^{2}+a^{3}}{d x +c}$
norman	$\frac{b^{3} x^{6}+3 a \,b^{2} x^{4}+3 a^{2} b \,x^{2}-\frac{d \,a^{3} x}{c}}{d x +c}$
default	$\frac{b \left(b^{2} x^{5} d^{4}-b^{2} c \,x^{4} d^{3}+3 a b \,d^{4} x^{3}+b^{2} c^{2} d^{2} x^{3}-3 a b c \,d^{3} x^{2}-b^{2} c^{3} d \,x^{2}+3 a^{2} d^{4} x +3 a b \,c^{2} d^{2} x +b^{2} c^{4} x \right)}{d^{5}}-\frac{-a^{3} d^{6}-3 a^{2} b \,c^{2} d^{4}-3 a \,b^{2} c^{4} d^{2}-b^{3} c^{6}}{d^{6} \left(d x +c \right)}$
risch	$\frac{b^{3} x^{5}}{d}-\frac{b^{3} c \,x^{4}}{d^{2}}+\frac{3 b^{2} a \,x^{3}}{d}+\frac{b^{3} c^{2} x^{3}}{d^{3}}-\frac{3 b^{2} a c \,x^{2}}{d^{2}}-\frac{b^{3} c^{3} x^{2}}{d^{4}}+\frac{3 b \,a^{2} x}{d}+\frac{3 b^{2} a \,c^{2} x}{d^{3}}+\frac{b^{3} c^{4} x}{d^{5}}+\frac{a^{3}}{d x +c}+\frac{3 a^{2} b \,c^{2}}{d^{2} \left(d x +c \right)}+\frac{3 a \,b^{2} \cdots}{d^{4} (d x \cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b/d^5*(b^2*d^4*x^5-b^2*c*d^3*x^4+3*a*b*d^4*x^3+b^2*c^2*d^2*x^3-3*a*b*c*d^3*
x^2-b^2*c^3*d*x^2+3*a^2*d^4*x+3*a*b*c^2*d^2*x+b^2*c^4*x)-(-a^3*d^6-3*a^2*b*
c^2*d^4-3*a*b^2*c^4*d^2-b^3*c^6)/d^6/(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(17) = 34$.
time = 0.28, size = 160, normalized size = 9.41

$$\frac{b^3c^6 + 3\,ab^2c^4d^2 + 3\,a^2bc^2d^4 + a^3d^6}{d^7x + cd^6} + \frac{b^3d^4x^5 - b^3cd^3x^4 + (b^3c^2d^2 + 3\,ab^2d^4)x^3 - (b^3c^3d + 3\,ab^2cd^3)x^2 + (b^3c^4 + 3\,ab^2c^2d^2 + 3\,a^2bd^4)x}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="maxima
")
```

```
[Out] (b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)/(d^7*x + c*d^6) + (
b^3*d^4*x^5 - b^3*c*d^3*x^4 + (b^3*c^2*d^2 + 3*a*b^2*d^4)*x^3 - (b^3*c^3*d
+ 3*a*b^2*c*d^3)*x^2 + (b^3*c^4 + 3*a*b^2*c^2*d^2 + 3*a^2*b*d^4)*x)/d^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(17) = 34$.
time = 0.33, size = 120, normalized size = 7.06

$$\frac{b^3d^6x^6 + 3\,ab^2d^6x^4 + 3\,a^2bd^6x^2 + b^3c^6 + 3\,ab^2c^4d^2 + 3\,a^2bc^2d^4 + a^3d^6 + (b^3c^5d + 3\,ab^2c^3d^3 + 3\,a^2bcd^5)x}{d^7x + cd^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="fricas
")
```

```
[Out] (b^3*d^6*x^6 + 3*a*b^2*d^6*x^4 + 3*a^2*b*d^6*x^2 + b^3*c^6 + 3*a*b^2*c^4*d^
2 + 3*a^2*b*c^2*d^4 + a^3*d^6 + (b^3*c^5*d + 3*a*b^2*c^3*d^3 + 3*a^2*b*c*d^
5)*x)/(d^7*x + c*d^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(12) = 24$.
time = 0.28, size = 153, normalized size = 9.00

$$-\frac{b^3cx^4}{d^2}+\frac{b^3x^5}{d}+x^3\cdot\left(\frac{3ab^2}{d}+\frac{b^3c^2}{d^3}\right)+x^2\left(-\frac{3ab^2c}{d^2}-\frac{b^3c^3}{d^4}\right)+x\left(\frac{3a^2b}{d}+\frac{3ab^2c^2}{d^3}+\frac{b^3c^4}{d^5}\right)+\frac{a^3d^6+3a^2bc^2d^4+3ab^2c^4d^2+b^3c^6}{cd^6+d^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)**2,x)
```

```
[Out] -b**3*c*x**4/d**2 + b**3*x**5/d + x**3*(3*a*b**2/d + b**3*c**2/d**3) + x**2
*(-3*a*b**2*c/d**2 - b**3*c**3/d**4) + x*(3*a**2*b/d + 3*a*b**2*c**2/d**3 +
 b**3*c**4/d**5) + (a**3*d**6 + 3*a**2*b*c**2*d**4 + 3*a*b**2*c**4*d**2 + b
**3*c**6)/(c*d**6 + d**7*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(17) = 34$.
time = 3.62, size = 216, normalized size = 12.71

$$\frac{\left(b^3-\frac{6b^3c}{dx+c}+\frac{15b^3c^2}{(dx+c)^2}-\frac{20b^3c^3}{(dx+c)^3}+\frac{15b^3c^4}{(dx+c)^4}+\frac{3ab^2d^2}{(dx+c)^2}-\frac{12ab^2cd^2}{(dx+c)^3}+\frac{18ab^2c^2d^2}{(dx+c)^4}+\frac{3a^2bd^4}{(dx+c)^4}\right)(dx+c)^5}{d^6}+\frac{\frac{b^3c^6d^5}{dx+c}+\frac{3ab^2c^4d^7}{dx+c}+\frac{3a^2bc^2d^9}{dx+c}+\frac{a^3d^{11}}{dx+c}}{d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (b^3 - 6*b^3*c/(d*x + c) + 15*b^3*c^2/(d*x + c)^2 - 20*b^3*c^3/(d*x + c)^3
+ 15*b^3*c^4/(d*x + c)^4 + 3*a*b^2*d^2/(d*x + c)^2 - 12*a*b^2*c*d^2/(d*x +
c)^3 + 18*a*b^2*c^2*d^2/(d*x + c)^4 + 3*a^2*b*d^4/(d*x + c)^4)*(d*x + c)^5/
d^6 + (b^3*c^6*d^5/(d*x + c) + 3*a*b^2*c^4*d^7/(d*x + c) + 3*a^2*b*c^2*d^9/
(d*x + c) + a^3*d^11/(d*x + c))/d^11
```

Mupad [B]
time = 0.05, size = 252, normalized size = 14.82

$$x^3\left(\frac{3\,a\,b^2}{d}+\frac{b^3\,c^2}{d^3}\right)-x\left(\frac{2\,c\left(\frac{4\,b^3\,c^3}{d^4}-\frac{2\,c\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d}+\frac{12\,a\,b^2\,c}{d^2}\right)}{d}+\frac{c^2\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d^2}-\frac{3\,a^2\,b}{d}\right)+x^2\left(\frac{2\,b^3\,c^3}{d^4}-\frac{c\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d}+\frac{6\,a\,b^2\,c}{d^2}\right)+\frac{a^3\,d^6+3\,a^2\,b\,c^2\,d^4+3\,a\,b^2\,c^4\,d^2+b^3\,c^6}{d\,\left(x\,d^6+c\,d^5\right)}+\frac{b^3\,x^5}{d}-\frac{b^3\,c\,x^4}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*d - b*x*(6*c + 5*d*x))*(a + b*x^2)^2)/(c + d*x)^2,x)
```

```
[Out] x^3*((3*a*b^2)/d + (b^3*c^2)/d^3) - x*((2*c*((4*b^3*c^3)/d^4 - (2*c*((9*a*b
^2)/d + (3*b^3*c^2)/d^3))/d + (12*a*b^2*c)/d^2))/d + (c^2*((9*a*b^2)/d + (3
*b^3*c^2)/d^3))/d^2 - (3*a^2*b)/d) + x^2*((2*b^3*c^3)/d^4 - (c*((9*a*b^2)/d
 + (3*b^3*c^2)/d^3))/d + (6*a*b^2*c)/d^2) + (a^3*d^6 + b^3*c^6 + 3*a*b^2*c^
4*d^2 + 3*a^2*b*c^2*d^4)/(d*(c*d^5 + d^6*x)) + (b^3*x^5)/d - (b^3*c*x^4)/d^
2
```

3.43 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{a+cx^2}\,dx$

Optimal. Leaf size=240

$$-\frac{(ae^2(3Cd+Be)-cd(Cd^2+3e(Bd+Ae)))\,x}{c^2}-\frac{e(aCe^2-c(3Cd^2+e(3Bd+Ae)))\,x^2}{2c^2}+\frac{e^2(3Cd+Be)x^3}{3c}$$

```
[Out] -(a*e^2*(B*e+3*C*d)-c*d*(C*d^2+3*e*(A*e+B*d)))*x/c^2-1/2*e*(a*C*e^2-c*(3*C*
d^2+e*(A*e+3*B*d)))*x^2/c^2+1/3*e^2*(B*e+3*C*d)*x^3/c+1/4*C*e^3*x^4/c+1/2*(
B*c*d*(-3*a*e^2+c*d^2)+(A*c-C*a)*e*(-a*e^2+3*c*d^2))*ln(c*x^2+a)/c^3+(A*c*d
*(-3*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d)-c*d^2*(3*B*e+C*d)))*arctan(x*c^(1/2)
/a^(1/2))/c^(5/2)/a^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 237, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1643, 649, 211, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(Acd(cd^2-3ae^2)+a(ae^2(Be+3Cd)-cd^2(3Be+Cd)))}{\sqrt{a}\,c^{5/2}}+\frac{\log(a+cx^2)\,(e(Ac-aC)\,(3cd^2-ae^2)+Bcd(cd^2-3ae^2))}{2c^3}+\frac{x(-ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3)}{c^2}+\frac{ex^2(-aCe^2+ce(Ae+3Bd)+3cCd^2)}{2c^2}+\frac{e^2x^3(Be+3Cd)}{3c}+\frac{Ce^3x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((c*C*d^3 + 3*c*d*e*(B*d + A*e) - a*e^2*(3*C*d + B*e))*x)/c^2 + (e*(3*c*C*d
^2 - a*C*e^2 + c*e*(3*B*d + A*e))*x^2)/(2*c^2) + (e^2*(3*C*d + B*e)*x^3)/(3
*c) + (C*e^3*x^4)/(4*c) + ((A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d + B*e
) - c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) +
((B*c*d*(c*d^2 - 3*a*e^2) + (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2]
)/(2*c^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx = \int \left(\frac{cCd^3+3cde(Bd+Ae)-ae^2(3Cd+Be)}{c^2}+\frac{e(3cCd^2-aCe^2+ce(3\right.$$

$$= \frac{\left(cCd^3+3cde(Bd+Ae)-ae^2(3Cd+Be)\right)x}{c^2}+\frac{e(3cCd^2-aCe^2+ce(3I}{2c^2}$$

$$= \frac{\left(cCd^3+3cde(Bd+Ae)-ae^2(3Cd+Be)\right)x}{c^2}+\frac{e(3cCd^2-aCe^2+ce(3I}{2c^2}$$

$$= \frac{\left(cCd^3+3cde(Bd+Ae)-ae^2(3Cd+Be)\right)x}{c^2}+\frac{e(3cCd^2-aCe^2+ce(3I}{2c^2}$$

Mathematica [A]
time = 0.15, size = 223, normalized size = 0.93

$$\frac{\left(Acd(cd^2-3ae^2)+a(ae^2(3Cd+Be)-cd^2(Cd+3Be))\right)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,c^{5/2}}+\frac{cx(-6ae^2(6Cd+2Be+Cex)+3cC(4d^3+6d^2ex+4de^2x^2+e^3x^3)+2ce(3Ae(6d+ex)+B(18d^2+9dex+2e^2x^2)))+6(Bcd(cd^2-3ae^2)+(Ac-aC)e(3cd^2-ae^2))\log\left(a+cx^2\right)}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d + B*e) - c*d^2*(C*d + 3*B*e)))*
ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + (c*x*(-6*a*e^2*(6*C*d + 2*
B*e + C*e*x) + 3*c*C*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + 2*c*e*(3
*A*e*(6*d + e*x) + B*(18*d^2 + 9*d*e*x + 2*e^2*x^2))) + 6*(B*c*d*(c*d^2 - 3
*a*e^2) + (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2])/(12*c^3)
```

Maple [A]
time = 0.13, size = 260, normalized size = 1.08

method	result
default	$\frac{\frac{1}{4}cC\,x^4e^3+\frac{1}{3}Bc\,e^3x^3+Ccd\,e^2x^3+\frac{1}{2}Ac\,e^3x^2+\frac{3}{2}Bcd\,e^2x^2-\frac{1}{2}Ca\,e^3x^2+\frac{3}{2}Cc\,d^2e\,x^2+3Acd\,e^2x-Ba\,e^3x+3Bc\,d^2ex-3Cad\,e^2x+Cc\,d^3x}{c^2}$

risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/4*c*C*x^4*e^3+1/3*B*c*e^3*x^3+C*c*d*e^2*x^3+1/2*A*c*e^3*x^2+3/2*B*
c*d*e^2*x^2-1/2*C*a*e^3*x^2+3/2*C*c*d^2*e*x^2+3*A*c*d*e^2*x-B*a*e^3*x+3*B*c
*d^2*e*x-3*C*a*d*e^2*x+C*c*d^3*x)+1/c^2*(1/2*(-A*a*c*e^3+3*A*c^2*d^2*e-3*B*
a*c*d*e^2+B*c^2*d^3+C*a^2*e^3-3*C*a*c*d^2*e)/c*ln(c*x^2+a)+(-3*A*a*c*d*e^2+
A*c^2*d^3+B*a^2*e^3-3*B*a*c*d^2*e+3*C*a^2*d*e^2-C*a*c*d^3)/(a*c)^(1/2)*arct
an(c*x/(a*c)^(1/2)))
```

Maxima [A]
time = 0.50, size = 242, normalized size = 1.01

$$-\frac{\left(3\,Bacd^{2}e+\left(Cac-Ac^{2}\right)d^{3}-Ba^{2}e^{3}-3\,\left(Ca^{2}e^{2}-Aace^{2}\right)d\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c^{2}}+\frac{3\,Ccx^{4}e^{3}+4\,\left(3\,Ccde^{2}+Bce^{3}\right)x^{3}+6\,\left(3\,Ccd^{2}e+3\,Bcde^{2}-Cae^{3}+Ace^{3}\right)x^{2}+12\,\left(Ccd^{3}+3\,Bcd^{2}e-Bae^{3}-3\,\left(Cae^{2}-Ace^{2}\right)d\right)x}{12\,c^{2}}+\frac{\left(Bc^{2}d^{3}-3\,Bacde^{2}+Ca^{2}e^{3}-Aace^{3}-3\,\left(Cace-Ac^{2}e\right)d^{2}\right)\log\left(cx^{2}+a\right)}{2\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -(3*B*a*c*d^2*e + (C*a*c - A*c^2)*d^3 - B*a^2*e^3 - 3*(C*a^2*e^2 - A*a*c*e^
2)*d)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/12*(3*C*c*x^4*e^3 + 4*(3*C*
c*d*e^2 + B*c*e^3)*x^3 + 6*(3*C*c*d^2*e + 3*B*c*d*e^2 - C*a*e^3 + A*c*e^3)*
x^2 + 12*(C*c*d^3 + 3*B*c*d^2*e - B*a*e^3 - 3*(C*a*e^2 - A*c*e^2)*d)*x)/c^2
 + 1/2*(B*c^2*d^3 - 3*B*a*c*d*e^2 + C*a^2*e^3 - A*a*c*e^3 - 3*(C*a*c*e - A*
c^2*e)*d^2)*log(c*x^2 + a)/c^3
```

Fricas [A]
time = 0.38, size = 578, normalized size = 2.41

$$\left[\frac{12\,Cac^{2}d^{3}x+6\,\left(3\,Bacd^{2}e+\left(Cac-Ac^{2}\right)d^{3}-Ba^{2}e^{3}-3\,\left(Ca^{2}-Aac\right)de^{2}\right)\sqrt{-ac}\,\log\left(\frac{cx^{2}-2\,\sqrt{-ac}\,x-a}{cx^{2}+a}\right)+\left(3\,Cac^{2}x^{4}+4\,Bac^{2}x^{3}-12\,Ba^{2}cx-6\,\left(Ca^{2}c-Aac^{2}\right)x^{2}\right)e^{3}+6\,\left(2\,Cac^{2}dx^{3}+3\,Bac^{2}dx^{2}-6\,\left(Ca^{2}c-Aac^{2}\right)dx\right)e^{2}+18\,\left(Cac^{2}d^{2}x^{2}+2\,Bac^{2}d^{2}x\right)e+6\,\left(Bac^{2}d^{3}-3\,Ba^{2}cde^{2}-3\,\left(Ca^{2}c-Aac^{2}\right)d^{2}e+\left(Ca^{3}-Aa^{2}c\right)e^{3}\right)\log\left(cx^{2}+a\right)}{12\,ac^{3}},\ \frac{12\,Cac^{2}d^{3}x-12\,\left(3\,Bacd^{2}e+\left(Cac-Ac^{2}\right)d^{3}-Ba^{2}e^{3}-3\,\left(Ca^{2}-Aac\right)de^{2}\right)\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,x}{a}\right)+\left(3\,Cac^{2}x^{4}+4\,Bac^{2}x^{3}-12\,Ba^{2}cx-6\,\left(Ca^{2}c-Aac^{2}\right)x^{2}\right)e^{3}+6\,\left(2\,Cac^{2}dx^{3}+3\,Bac^{2}dx^{2}-6\,\left(Ca^{2}c-Aac^{2}\right)dx\right)e^{2}+18\,\left(Cac^{2}d^{2}x^{2}+2\,Bac^{2}d^{2}x\right)e+6\,\left(Bac^{2}d^{3}-3\,Ba^{2}cde^{2}-3\,\left(Ca^{2}c-Aac^{2}\right)d^{2}e+\left(Ca^{3}-Aa^{2}c\right)e^{3}\right)\log\left(cx^{2}+a\right)}{12\,ac^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(12*C*a*c^2*d^3*x + 6*(3*B*a*c*d^2*e + (C*a*c - A*c^2)*d^3 - B*a^2*e^
3 - 3*(C*a^2 - A*a*c)*d*e^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c
*x^2 + a)) + (3*C*a*c^2*x^4 + 4*B*a*c^2*x^3 - 12*B*a^2*c*x - 6*(C*a^2*c - A
*a*c^2)*x^2)*e^3 + 6*(2*C*a*c^2*d*x^3 + 3*B*a*c^2*d*x^2 - 6*(C*a^2*c - A*a*
c^2)*d*x)*e^2 + 18*(C*a*c^2*d^2*x^2 + 2*B*a*c^2*d^2*x)*e + 6*(B*a*c^2*d^3 -
 3*B*a^2*c*d*e^2 - 3*(C*a^2*c - A*a*c^2)*d^2*e + (C*a^3 - A*a^2*c)*e^3)*log
(c*x^2 + a))/(a*c^3), 1/12*(12*C*a*c^2*d^3*x - 12*(3*B*a*c*d^2*e + (C*a*c -
 A*c^2)*d^3 - B*a^2*e^3 - 3*(C*a^2 - A*a*c)*d*e^2)*sqrt(a*c)*arctan(sqrt(a*
c)*x/a) + (3*C*a*c^2*x^4 + 4*B*a*c^2*x^3 - 12*B*a^2*c*x - 6*(C*a^2*c - A*a*
c^2)*x^2)*e^3 + 6*(2*C*a*c^2*d*x^3 + 3*B*a*c^2*d*x^2 - 6*(C*a^2*c - A*a*c^2
```

```
)*d*x)*e^2 + 18*(C*a*c^2*d^2*x^2 + 2*B*a*c^2*d^2*x)*e + 6*(B*a*c^2*d^3 - 3*
B*a^2*c*d*e^2 - 3*(C*a^2*c - A*a*c^2)*d^2*e + (C*a^3 - A*a^2*c)*e^3)*log(c*
x^2 + a))/(a*c^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. $2(224) = 448$.
time = 5.81, size = 1008, normalized size = 4.20

$$\frac{Ce^3x^4}{4c} + x^3\left(\frac{Be^3}{3c} + \frac{Cde^2}{c}\right) + x^2\left(\frac{Ae^3}{2c} + \frac{3Bde^2}{2c} - \frac{Cae^3}{2c^2} + \frac{3Cd^2e}{2c}\right) + x\left(\frac{3Ade^2}{c} - \frac{Bae^3}{c^2} + \frac{3Bd^2e}{c} - \frac{3Cade^2}{c^2} + \frac{Cd^3}{c}\right) + \left(\frac{-Aace^3 + 3Ac^2d^2e - 3Bacde^2 + Bc^2d^3 + Ca^2e^3 - 3Cacd^2e}{2c^3} - \frac{\sqrt{-ac^7}\left(-3Aacde^2 + Ac^2d^3 + Ba^2e^3 - 3Bacd^2e + 3Ca^2de^2 - Cacd^3\right)}{2ac^6}\right)\log\left(x + \frac{Aa^2ce^3 - 3Aac^2d^2e + 3Ba^2cde^2 - Bac^2d^3 - Ca^3e^3 + 3Ca^2cd^2e + 2ac^3\left(\frac{-Aace^3 + 3Ac^2d^2e - 3Bacde^2 + Bc^2d^3 + Ca^2e^3 - 3Cacd^2e}{2c^3} - \frac{\sqrt{-ac^7}\left(-3Aacde^2 + Ac^2d^3 + Ba^2e^3 - 3Bacd^2e + 3Ca^2de^2 - Cacd^3\right)}{2ac^6}\right)}{-3Aac^2de^2 + Ac^3d^3 + Ba^2ce^3 - 3Bac^2d^2e + 3Ca^2cde^2 - Cac^2d^3}\right) + \left(\frac{-Aace^3 + 3Ac^2d^2e - 3Bacde^2 + Bc^2d^3 + Ca^2e^3 - 3Cacd^2e}{2c^3} + \frac{\sqrt{-ac^7}\left(-3Aacde^2 + Ac^2d^3 + Ba^2e^3 - 3Bacd^2e + 3Ca^2de^2 - Cacd^3\right)}{2ac^6}\right)\log\left(x + \frac{Aa^2ce^3 - 3Aac^2d^2e + 3Ba^2cde^2 - Bac^2d^3 - Ca^3e^3 + 3Ca^2cd^2e + 2ac^3\left(\frac{-Aace^3 + 3Ac^2d^2e - 3Bacde^2 + Bc^2d^3 + Ca^2e^3 - 3Cacd^2e}{2c^3} + \frac{\sqrt{-ac^7}\left(-3Aacde^2 + Ac^2d^3 + Ba^2e^3 - 3Bacd^2e + 3Ca^2de^2 - Cacd^3\right)}{2ac^6}\right)}{-3Aac^2de^2 + Ac^3d^3 + Ba^2ce^3 - 3Bac^2d^2e + 3Ca^2cde^2 - Cac^2d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*e**3*x**4/(4*c) + x**3*(B*e**3/(3*c) + C*d*e**2/c) + x**2*(A*e**3/(2*c) +
 3*B*d*e**2/(2*c) - C*a*e**3/(2*c**2) + 3*C*d**2*e/(2*c)) + x*(3*A*d*e**2/c
 - B*a*e**3/c**2 + 3*B*d**2*e/c - 3*C*a*d*e**2/c**2 + C*d**3/c) + ((-A*a*c*
e**3 + 3*A*c**2*d**2*e - 3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*e**3 - 3*C*a
*c*d**2*e)/(2*c**3) - sqrt(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d**3 + B*a**2
*e**3 - 3*B*a*c*d**2*e + 3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**6))*log(x +
(A*a**2*c*e**3 - 3*A*a*c**2*d**2*e + 3*B*a**2*c*d*e**2 - B*a*c**2*d**3 - C*
a**3*e**3 + 3*C*a**2*c*d**2*e + 2*a*c**3*((-A*a*c*e**3 + 3*A*c**2*d**2*e -
3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*e**3 - 3*C*a*c*d**2*e)/(2*c**3) - sqr
t(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d**3 + B*a**2*e**3 - 3*B*a*c*d**2*e +
3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**6)))/(-3*A*a*c**2*d*e**2 + A*c**3*d**
3 + B*a**2*c*e**3 - 3*B*a*c**2*d**2*e + 3*C*a**2*c*d*e**2 - C*a*c**2*d**3))
 + ((-A*a*c*e**3 + 3*A*c**2*d**2*e - 3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*
e**3 - 3*C*a*c*d**2*e)/(2*c**3) + sqrt(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d
**3 + B*a**2*e**3 - 3*B*a*c*d**2*e + 3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**
6))*log(x + (A*a**2*c*e**3 - 3*A*a*c**2*d**2*e + 3*B*a**2*c*d*e**2 - B*a*c*
*2*d**3 - C*a**3*e**3 + 3*C*a**2*c*d**2*e + 2*a*c**3*((-A*a*c*e**3 + 3*A*c*
*2*d**2*e - 3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*e**3 - 3*C*a*c*d**2*e)/(2
*c**3) + sqrt(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d**3 + B*a**2*e**3 - 3*B*a
*c*d**2*e + 3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**6)))/(-3*A*a*c**2*d*e**2
+ A*c**3*d**3 + B*a**2*c*e**3 - 3*B*a*c**2*d**2*e + 3*C*a**2*c*d*e**2 - C*a
*c**2*d**3))
```

Giac [A]
time = 3.43, size = 279, normalized size = 1.16

$$-\frac{\left(Cacd^3 - Ac^2d^3 + 3\,Bacd^2e - 3\,Ca^2de^2 + 3\,Aacde^2 - Ba^2e^3\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c^2} + \frac{\left(Bc^2d^3 - 3\,Cacd^2e + 3\,Ac^2d^2e - 3\,Bacde^2 + Ca^2e^3 - Aace^3\right)\log\left(cx^2 + a\right)}{2\,c^3} + \frac{3\,Cc^3x^4e^3 + 12\,Cc^3dx^3e^2 + 18\,Cc^3d^2x^2e + 12\,Cc^3d^3x + 4\,Bc^3x^3e^3 + 18\,Bc^3dx^2e^2 + 36\,Bc^3d^2xe - 6\,Cac^2x^2e^3 + 6\,Ac^3x^2e^3 - 36\,Cac^2dxe^2 + 36\,Ac^3dxe^2 - 12\,Bac^2xe^3}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a*c*d^3 - A*c^2*d^3 + 3*B*a*c*d^2*e - 3*C*a^2*d*e^2 + 3*A*a*c*d*e^2 - B
*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/2*(B*c^2*d^3 - 3*C*a*c*
```

```
d^2*e + 3*A*c^2*d^2*e - 3*B*a*c*d*e^2 + C*a^2*e^3 - A*a*c*e^3)*log(c*x^2 +
a)/c^3 + 1/12*(3*C*c^3*x^4*e^3 + 12*C*c^3*d*x^3*e^2 + 18*C*c^3*d^2*x^2*e +
12*C*c^3*d^3*x + 4*B*c^3*x^3*e^3 + 18*B*c^3*d*x^2*e^2 + 36*B*c^3*d^2*x*e -
6*C*a*c^2*x^2*e^3 + 6*A*c^3*x^2*e^3 - 36*C*a*c^2*d*x*e^2 + 36*A*c^3*d*x*e^2
 - 12*B*a*c^2*x*e^3)/c^4
```

Mupad [B]
time = 3.99, size = 277, normalized size = 1.15

$$x^2\left(\frac{3\,C\,d^2\,e+3\,B\,d\,e^2+A\,e^3}{2\,c}-\frac{C\,a\,e^3}{2\,c^2}\right)+x\left(\frac{C\,d^3+3\,B\,d^2\,e+3\,A\,d\,e^2}{c}-\frac{a\,\left(B\,e^3+3\,C\,d\,e^2\right)}{c^2}\right)+\frac{x^3\,\left(B\,e^3+3\,C\,d\,e^2\right)}{3\,c}+\frac{C\,e^3\,x^4}{4\,c}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(3\,C\,a^2\,d\,e^2+B\,a^2\,e^3-C\,a\,c\,d^3-3\,B\,a\,c\,d^2\,e-3\,A\,a\,c\,d\,e^2+A\,c^2\,d^3\right)}{\sqrt{a}\,c^{5/2}}+\frac{\ln\left(c\,x^2+a\right)\,\left(4\,C\,a^3\,c^3\,e^3-12\,C\,a^2\,c^4\,d^2\,e-12\,B\,a^2\,c^4\,d\,e^2-4\,A\,a^2\,c^4\,e^3+4\,B\,a\,c^5\,d^3+12\,A\,a\,c^5\,d^2\,e\right)}{8\,a\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2),x)
```

```
[Out] x^2*((A*e^3 + 3*B*d*e^2 + 3*C*d^2*e)/(2*c) - (C*a*e^3)/(2*c^2)) + x*((C*d^3
 + 3*A*d*e^2 + 3*B*d^2*e)/c - (a*(B*e^3 + 3*C*d*e^2))/c^2) + (x^3*(B*e^3 +
3*C*d*e^2))/(3*c) + (C*e^3*x^4)/(4*c) + (atan((c^(1/2)*x)/a^(1/2))*(A*c^2*d
^3 + B*a^2*e^3 - C*a*c*d^3 + 3*C*a^2*d*e^2 - 3*A*a*c*d*e^2 - 3*B*a*c*d^2*e)
)/(a^(1/2)*c^(5/2)) + (log(a + c*x^2)*(4*B*a*c^5*d^3 - 4*A*a^2*c^4*e^3 + 4*
C*a^3*c^3*e^3 - 12*B*a^2*c^4*d*e^2 - 12*C*a^2*c^4*d^2*e + 12*A*a*c^5*d^2*e)
)/(8*a*c^6)
```

3.44 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{a+cx^2}\,dx$

Optimal. Leaf size=168

$$-\frac{(aCe^2-c(Cd^2+e(2Bd+Ae)))\,x}{c^2}+\frac{e(2Cd+Be)x^2}{2c}+\frac{Ce^2x^3}{3c}+\frac{(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd+2Be))}{\sqrt{a}\,c^{5/2}}$$

```
[Out] -(a*C*e^2-c*(C*d^2+e*(A*e+2*B*d)))*x/c^2+1/2*e*(B*e+2*C*d)*x^2/c+1/3*C*e^2*
x^3/c+1/2*(2*A*c*d*e-B*a*e^2+B*c*d^2-2*C*a*d*e)*ln(c*x^2+a)/c^2+(A*c*(-a*e^
2+c*d^2)+a*(a*C*e^2-c*d*(2*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/c^(5/2)/a^(
1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 166, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1643, 649, 211, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(Ac(cd^2-ae^2)+a(aCe^2-cd(2Be+Cd)))}{\sqrt{a}\,c^{5/2}}+\frac{\log(a+cx^2)(-aBe^2-2aCde+2Acde+Bcd^2)}{2c^2}+\frac{x(-aCe^2+ce(Ae+2Bd)+cCd^2)}{c^2}+\frac{ex^2(Be+2Cd)}{2c}+\frac{Ce^2x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((c*C*d^2 - a*C*e^2 + c*e*(2*B*d + A*e))*x)/c^2 + (e*(2*C*d + B*e)*x^2)/(2*
c) + (C*e^2*x^3)/(3*c) + ((A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*d*(C*d + 2*
B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + ((B*c*d^2 + 2*A*c*d
*e - 2*a*C*d*e - a*B*e^2)*Log[a + c*x^2])/(2*c^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{a+cx^2}\,dx = \int \left(\frac{cCd^2-aCe^2+ce(2Bd+Ae)}{c^2} + \frac{e(2Cd+Be)x}{c} + \frac{Ce^2x^2}{c} + \frac{Ac(cd\right.$$

$$= \frac{(cCd^2-aCe^2+ce(2Bd+Ae))\,x}{c^2} + \frac{e(2Cd+Be)x^2}{2c} + \frac{Ce^2x^3}{3c} + \frac{\int \frac{Ac(c}{}}{}$$

$$= \frac{(cCd^2-aCe^2+ce(2Bd+Ae))\,x}{c^2} + \frac{e(2Cd+Be)x^2}{2c} + \frac{Ce^2x^3}{3c} + \frac{(Bcd^2}{}$$

$$= \frac{(cCd^2-aCe^2+ce(2Bd+Ae))\,x}{c^2} + \frac{e(2Cd+Be)x^2}{2c} + \frac{Ce^2x^3}{3c} + \frac{(Ac(c}{}$$

Mathematica [A]
time = 0.11, size = 155, normalized size = 0.92

$$\frac{(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd+2Be)))\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,c^{5/2}} + \frac{x(-6aCe^2+3ce(4Bd+2Ae+Bex)+2cC(3d^2+3dex+e^2x^2))+3(Bcd^2+2Acde-2aCde-aBe^2)\log(a+cx^2)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*d*(C*d + 2*B*e)))*ArcTan[(Sqrt[c]*x)
/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + (x*(-6*a*C*e^2 + 3*c*e*(4*B*d + 2*A*e + B*e*
x) + 2*c*C*(3*d^2 + 3*d*e*x + e^2*x^2)) + 3*(B*c*d^2 + 2*A*c*d*e - 2*a*C*d*
e - a*B*e^2)*Log[a + c*x^2])/(6*c^2)
```

Maple [A]
time = 0.13, size = 169, normalized size = 1.01

method	result
default	$\frac{\frac{1}{3}cC\,x^3e^2+\frac{1}{2}Bc\,e^2x^2+Cde\,x^2+Ac\,e^2x+2Bcdex-aC\,e^2x+Cc\,d^2x}{c^2} + \frac{\frac{\left(2c^2deA-Bac\,e^2+B\,c^2d^2-2acdeC\right)\ln\left(c\,x^2+a\right)}{2c}+\frac{\left(-Aac\,e^2+A\,c^2\right.}{}}{c^2}$
risch	$\frac{C\,e^2x^3}{3c}+\frac{B\,e^2x^2}{2c}+\frac{Cde\,x^2}{c}+\frac{A\,e^2x}{c}+\frac{2Bdex}{c}-\frac{aC\,e^2x}{c^2}+\frac{C\,d^2x}{c}+\frac{\ln\left(-A\,a^2c\,e^2+Aa\,c^2d^2-2de\,a^2cB+C\,a^3e^2-C\,a^2c\,d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/3*c*C*x^3*e^2+1/2*B*c*e^2*x^2+C*c*d*e*x^2+A*c*e^2*x+2*B*c*d*e*x-a*
C*e^2*x+C*c*d^2*x)+1/c^2*(1/2*(2*A*c^2*d*e-B*a*c*e^2+B*c^2*d^2-2*C*a*c*d*e)
/c*ln(c*x^2+a)+(-A*a*c*e^2+A*c^2*d^2-2*B*a*c*d*e+C*a^2*e^2-C*a*c*d^2)/(a*c)
^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Maxima [A]
time = 0.49, size = 160, normalized size = 0.95

$$\frac{(Bcd^2 - Bae^2 - 2(Cae - Ace)d)\log(cx^2 + a)}{2c^2} - \frac{(2Bacde - Ca^2e^2 + Aace^2 + (Cac - Ac^2)d^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c^2} + \frac{2Ccx^3e^2 + 3(2Ccde + Bce^2)x^2 + 6(Ccd^2 + 2Bcde - Cae^2 + Ace^2)x}{6c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*c*d^2 - B*a*e^2 - 2*(C*a*e - A*c*e)*d)*log(c*x^2 + a)/c^2 - (2*B*a*c
*d*e - C*a^2*e^2 + A*a*c*e^2 + (C*a*c - A*c^2)*d^2)*arctan(c*x/sqrt(a*c))/(
sqrt(a*c)*c^2) + 1/6*(2*C*c*x^3*e^2 + 3*(2*C*c*d*e + B*c*e^2)*x^2 + 6*(C*c*
d^2 + 2*B*c*d*e - C*a*e^2 + A*c*e^2)*x)/c^2
```

Fricas [A]
time = 0.39, size = 400, normalized size = 2.38

$$\left[\frac{6Cac^2d^2x - 3(2Bacde + (Cac - Ac^2)d^2 - (Ca^2 - Aac)e^2)\sqrt{-ac}\log\left(\frac{cx^2 + 2\sqrt{-ac}\,x - a}{cx^2 + a}\right) + (2Cac^2x^3 + 3Bac^2x^2 - 6(Ca^2c - Aac^2)x)e^2 + 6(Cac^2dx^2 + 2Bac^2dx)e + 3(Bac^2d^2 - Ba^2ce^2 - 2(Ca^2c - Aac^2)de)\log(cx^2 + a)}{6ac^3}, \frac{6Cac^2d^2x - 6(2Bacde + (Cac - Ac^2)d^2 - (Ca^2 - Aac)e^2)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\,x}{a}\right) + (2Cac^2x^3 + 3Bac^2x^2 - 6(Ca^2c - Aac^2)x)e^2 + 6(Cac^2dx^2 + 2Bac^2dx)e + 3(Bac^2d^2 - Ba^2ce^2 - 2(Ca^2c - Aac^2)de)\log(cx^2 + a)}{6ac^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(6*C*a*c^2*d^2*x - 3*(2*B*a*c*d*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a
*c)*e^2)*sqrt(-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + (2*C*a*
c^2*x^3 + 3*B*a*c^2*x^2 - 6*(C*a^2*c - A*a*c^2)*x)*e^2 + 6*(C*a*c^2*d*x^2 +
 2*B*a*c^2*d*x)*e + 3*(B*a*c^2*d^2 - B*a^2*c*e^2 - 2*(C*a^2*c - A*a*c^2)*d*
e)*log(c*x^2 + a))/(a*c^3), 1/6*(6*C*a*c^2*d^2*x - 6*(2*B*a*c*d*e + (C*a*c
- A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (2*C*
a*c^2*x^3 + 3*B*a*c^2*x^2 - 6*(C*a^2*c - A*a*c^2)*x)*e^2 + 6*(C*a*c^2*d*x^2
 + 2*B*a*c^2*d*x)*e + 3*(B*a*c^2*d^2 - B*a^2*c*e^2 - 2*(C*a^2*c - A*a*c^2)*
d*e)*log(c*x^2 + a))/(a*c^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(156) = 312$.
time = 1.59, size = 638, normalized size = 3.80

$$\frac{Ce^2x^3}{3c} + x^2\left(\frac{Be^2}{2c} + \frac{Cde}{c}\right) + x\left(\frac{Ae^2}{c} + \frac{2Bde}{c} - \frac{Cae^2}{c^2} + \frac{Cd^2}{c}\right) + \left(-\frac{-2Acde + Bae^2 - Bcd^2 + 2Cade}{2c^2} - \frac{\sqrt{-ac^5}\,(-Aace^2 + Ac^2d^2 - 2Bacde + Ca^2e^2 - Cacd^2)}{2ac^5}\right)\log\left(x + \frac{-2Aacde + Ba^2e^2 - Bacd^2 + 2Ca^2de + 2ac^2\left(-\frac{-2Acde + Bae^2 - Bcd^2 + 2Cade}{2c^2} - \frac{\sqrt{-ac^5}\,(-Aace^2 + Ac^2d^2 - 2Bacde + Ca^2e^2 - Cacd^2)}{2ac^5}\right)}{-Aace^2 + Ac^2d^2 - 2Bacde + Ca^2e^2 - Cacd^2}\right) + \left(-\frac{-2Acde + Bae^2 - Bcd^2 + 2Cade}{2c^2} + \frac{\sqrt{-ac^5}\,(-Aace^2 + Ac^2d^2 - 2Bacde + Ca^2e^2 - Cacd^2)}{2ac^5}\right)\log\left(x + \frac{-2Aacde + Ba^2e^2 - Bacd^2 + 2Ca^2de + 2ac^2\left(-\frac{-2Acde + Bae^2 - Bcd^2 + 2Cade}{2c^2} + \frac{\sqrt{-ac^5}\,(-Aace^2 + Ac^2d^2 - 2Bacde + Ca^2e^2 - Cacd^2)}{2ac^5}\right)}{-Aace^2 + Ac^2d^2 - 2Bacde + Ca^2e^2 - Cacd^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*e**2*x**3/(3*c) + x**2*(B*e**2/(2*c) + C*d*e/c) + x*(A*e**2/c + 2*B*d*e/c
 - C*a*e**2/c**2 + C*d**2/c) + (-(-2*A*c*d*e + B*a*e**2 - B*c*d**2 + 2*C*a*
d*e)/(2*c**2) - sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*
a**2*e**2 - C*a*c*d**2)/(2*a*c**5))*log(x + (-2*A*a*c*d*e + B*a**2*e**2 - B
*a*c*d**2 + 2*C*a**2*d*e + 2*a*c**2*(-(-2*A*c*d*e + B*a*e**2 - B*c*d**2 + 2
*C*a*d*e)/(2*c**2) - sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e
 + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5)))/(-A*a*c*e**2 + A*c**2*d**2 - 2*B*
a*c*d*e + C*a**2*e**2 - C*a*c*d**2)) + (-(-2*A*c*d*e + B*a*e**2 - B*c*d**2
+ 2*C*a*d*e)/(2*c**2) + sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*
d*e + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5))*log(x + (-2*A*a*c*d*e + B*a**2*
e**2 - B*a*c*d**2 + 2*C*a**2*d*e + 2*a*c**2*(-(-2*A*c*d*e + B*a*e**2 - B*c*
d**2 + 2*C*a*d*e)/(2*c**2) + sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B
*a*c*d*e + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5)))/(-A*a*c*e**2 + A*c**2*d**
2 - 2*B*a*c*d*e + C*a**2*e**2 - C*a*c*d**2))
```

Giac [A]
time = 4.47, size = 176, normalized size = 1.05

$$\frac{(Bcd^2 - 2\,Cade + 2\,Acde - Bae^2)\log\left(cx^2 + a\right)}{2\,c^2} - \frac{(Cacd^2 - Ac^2d^2 + 2\,Bacde - Ca^2e^2 + Aace^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c^2} + \frac{2\,Cc^2x^3e^2 + 6\,Cc^2dx^2e + 6\,Cc^2d^2x + 3\,Bc^2x^2e^2 + 12\,Bc^2dxe - 6\,Cacxe^2 + 6\,Ac^2xe^2}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d^2 - 2*C*a*d*e + 2*A*c*d*e - B*a*e^2)*log(c*x^2 + a)/c^2 - (C*a*c
*d^2 - A*c^2*d^2 + 2*B*a*c*d*e - C*a^2*e^2 + A*a*c*e^2)*arctan(c*x/sqrt(a*c
))/(sqrt(a*c)*c^2) + 1/6*(2*C*c^2*x^3*e^2 + 6*C*c^2*d*x^2*e + 6*C*c^2*d^2*x
 + 3*B*c^2*x^2*e^2 + 12*B*c^2*d*x*e - 6*C*a*c*x*e^2 + 6*A*c^2*x*e^2)/c^3
```

Mupad [B]
time = 3.90, size = 181, normalized size = 1.08

$$x\left(\frac{C\,d^2 + 2\,B\,d\,e + A\,e^2}{c} - \frac{C\,a\,e^2}{c^2}\right) + \frac{x^2\,(B\,e^2 + 2\,C\,d\,e)}{2\,c} + \frac{C\,e^2\,x^3}{3\,c} - \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(-C\,a^2\,e^2 + C\,a\,c\,d^2 + 2\,B\,a\,c\,d\,e + A\,a\,c\,e^2 - A\,c^2\,d^2)}{\sqrt{a}\,c^{5/2}} + \frac{\ln\left(c\,x^2 + a\right)\,(-8\,C\,a^2\,c^3\,d\,e - 4\,B\,a^2\,c^3\,e^2 + 4\,B\,a\,c^4\,d^2 + 8\,A\,a\,c^4\,d\,e)}{8\,a\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2),x)
```

```
[Out] x*((A*e^2 + C*d^2 + 2*B*d*e)/c - (C*a*e^2)/c^2) + (x^2*(B*e^2 + 2*C*d*e))/(
2*c) + (C*e^2*x^3)/(3*c) - (atan((c^(1/2)*x)/a^(1/2))*(A*a*c*e^2 - C*a^2*e^
2 - A*c^2*d^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(a^(1/2)*c^(5/2)) + (log(a + c*x^
2)*(4*B*a*c^4*d^2 - 4*B*a^2*c^3*e^2 + 8*A*a*c^4*d*e - 8*C*a^2*c^3*d*e))/(8*
a*c^5)
```

3.45 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2} \, dx$

Optimal. Leaf size=93

$$\frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{(Acd-a(Cd+Be))\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,c^{3/2}}+\frac{(Bcd+Ace-aCe)\log\left(a+cx^2\right)}{2c^2}$$

```
[Out] (B*e+C*d)*x/c+1/2*C*e*x^2/c+1/2*(A*c*e+B*c*d-C*a*e)*ln(c*x^2+a)/c^2+(A*c*d-
a*(B*e+C*d))*arctan(x*c^(1/2)/a^(1/2))/c^(3/2)/a^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1643, 649, 211, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(Acd-a(Be+Cd))}{\sqrt{a}\,c^{3/2}}+\frac{\log\left(a+cx^2\right)(-aCe+Ace+Bcd)}{2c^2}+\frac{x(Be+Cd)}{c}+\frac{Cex^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((C*d + B*e)*x)/c + (C*e*x^2)/(2*c) + ((A*c*d - a*(C*d + B*e))*ArcTan[(Sqrt
[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + ((B*c*d + A*c*e - a*C*e)*Log[a + c*x^2
])/(2*c^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

```
d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx = \int \left(\frac{Cd+Be}{c}+\frac{Cex}{c}+\frac{Acd-a(Cd+Be)+(Bcd+Ace-aCe)x}{c\left(a+cx^2\right)}\right) dx$$
$$= \frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{\int \frac{Acd-a(Cd+Be)+(Bcd+Ace-aCe)x}{a+cx^2}\,dx}{c}$$
$$= \frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{(Bcd+Ace-aCe)\int \frac{x}{a+cx^2}\,dx}{c}+\frac{(Acd-a(Cd+$$
$$= \frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{(Acd-a(Cd+Be))\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,c^{3/2}}+\frac{(Bcd+$$

Mathematica [A]
time = 0.06, size = 86, normalized size = 0.92

$$\frac{cx(2Cd+2Be+Cex)-\frac{2\sqrt{c}\,(-Acd+aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}}+(Bcd+Ace-aCe)\log\left(a+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] (c*x*(2*C*d + 2*B*e + C*e*x) - (2*Sqrt[c]*(-(A*c*d) + a*C*d + a*B*e)*ArcTan
[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] + (B*c*d + A*c*e - a*C*e)*Log[a + c*x^2])/(2
*c^2)
```

Maple [A]
time = 0.11, size = 84, normalized size = 0.90

method	result
default	$\frac{\frac{1}{2}C\,x^2e+Bex+Cdx}{c}+\frac{\frac{(Ace+Bcd-aCe)\ln\left(c\,x^2+a\right)}{2c}+\frac{(Acd-aBe-Cad)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{c}$
risch	$\frac{Ce\,x^2}{2c}+\frac{Bex}{c}+\frac{Cdx}{c}+\frac{\ln\left(acdA-e\,a^2B-d\,a^2C-\sqrt{-ac\left(Acd-aBe-Cad\right)^2}\,x\right)eA}{2c}+\frac{\ln\left(acdA-e\,a^2B-d\,a^2C-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*C*x^2*e+B*e*x+C*d*x)+1/c*(1/2*(A*c*e+B*c*d-C*a*e)/c*ln(c*x^2+a)+(A
*c*d-B*a*e-C*a*d)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Maxima [A]
time = 0.48, size = 89, normalized size = 0.96

$$-\frac{(Bae + (Ca - Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c} + \frac{Cx^2e + 2\,(Cd + Be)x}{2\,c} + \frac{(Bcd - Cae + Ace)\log\left(cx^2 + a\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -(B*a*e + (C*a - A*c)*d)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + 1/2*(C*x^2*e
 + 2*(C*d + B*e)*x)/c + 1/2*(B*c*d - C*a*e + A*c*e)*log(c*x^2 + a)/c^2
```

Fricas [A]
time = 0.36, size = 214, normalized size = 2.30

$$\left[\frac{2\,Cacdx - (Bae + (Ca - Ac)d)\sqrt{-ac}\,\log\left(\frac{cx^2 + 2\,\sqrt{-ac}\,x - a}{cx^2 + a}\right) + (Cacx^2 + 2\,Bacx)e + (Bacd - (Ca^2 - Aac)e)\log\left(cx^2 + a\right)}{2\,ac^2}, \frac{2\,Cacdx - 2\,(Bae + (Ca - Ac)d)\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,x}{a}\right) + (Cacx^2 + 2\,Bacx)e + (Bacd - (Ca^2 - Aac)e)\log\left(cx^2 + a\right)}{2\,ac^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*C*a*c*d*x - (B*a*e + (C*a - A*c)*d)*sqrt(-a*c)*log((c*x^2 + 2*sqrt(
-a*c)*x - a)/(c*x^2 + a)) + (C*a*c*x^2 + 2*B*a*c*x)*e + (B*a*c*d - (C*a^2 -
 A*a*c)*e)*log(c*x^2 + a))/(a*c^2), 1/2*(2*C*a*c*d*x - 2*(B*a*e + (C*a - A*
c)*d)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (C*a*c*x^2 + 2*B*a*c*x)*e + (B*a*c*
d - (C*a^2 - A*a*c)*e)*log(c*x^2 + a))/(a*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(88) = 176$.
time = 0.79, size = 337, normalized size = 3.62

$$\frac{Cex^2}{2c} + x\left(\frac{Be}{c} + \frac{Cd}{c}\right) + \left(-\frac{-Ace - Bcd + Cae}{2c^2} - \frac{\sqrt{-ac^5}\,(-Acd + Bae + Cad)}{2ac^4}\right)\log\left(x + \frac{Aace + Bacd - Ca^2e - 2ac^2\left(-\frac{-Ace - Bcd + Cae}{2c^2} - \frac{\sqrt{-ac^5}\,(-Acd + Bae + Cad)}{2ac^4}\right)}{-Ac^2d + Bace + Cacd}\right) + \left(-\frac{-Ace - Bcd + Cae}{2c^2} + \frac{\sqrt{-ac^5}\,(-Acd + Bae + Cad)}{2ac^4}\right)\log\left(x + \frac{Aace + Bacd - Ca^2e - 2ac^2\left(-\frac{-Ace - Bcd + Cae}{2c^2} + \frac{\sqrt{-ac^5}\,(-Acd + Bae + Cad)}{2ac^4}\right)}{-Ac^2d + Bace + Cacd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*e*x**2/(2*c) + x*(B*e/c + C*d/c) + (-(-A*c*e - B*c*d + C*a*e)/(2*c**2) -
sqrt(-a*c**5)*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4))*log(x + (A*a*c*e + B*a*c
*d - C*a**2*e - 2*a*c**2*(-(-A*c*e - B*c*d + C*a*e)/(2*c**2) - sqrt(-a*c**5
)*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4)))/(-A*c**2*d + B*a*c*e + C*a*c*d)) +
(-(-A*c*e - B*c*d + C*a*e)/(2*c**2) + sqrt(-a*c**5)*(-A*c*d + B*a*e + C*a*d
```

```
)/(2*a*c**4))*log(x + (A*a*c*e + B*a*c*d - C*a**2*e - 2*a*c**2*(-(-A*c*e -
B*c*d + C*a*e)/(2*c**2) + sqrt(-a*c**5)*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4)
))/(-A*c**2*d + B*a*c*e + C*a*c*d))
```

Giac [A]
time = 2.88, size = 91, normalized size = 0.98

$$-\frac{(Cad - Acd + Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c} + \frac{(Bcd - Cae + Ace)\log\left(cx^2 + a\right)}{2\,c^2} + \frac{Ccx^2e + 2\,Ccdx + 2\,Bcxe}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a*d - A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + 1/2*(B*c*d -
 C*a*e + A*c*e)*log(c*x^2 + a)/c^2 + 1/2*(C*c*x^2*e + 2*C*c*d*x + 2*B*c*x*e
)/c^2
```

Mupad [B]
time = 3.78, size = 97, normalized size = 1.04

$$\frac{x\,(B\,e + C\,d)}{c} - \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(B\,a\,e - A\,c\,d + C\,a\,d)}{\sqrt{a}\,c^{3/2}} + \frac{C\,e\,x^2}{2\,c} + \frac{\ln\left(c\,x^2 + a\right)\,\left(4\,A\,a\,c^3\,e + 4\,B\,a\,c^3\,d - 4\,C\,a^2\,c^2\,e\right)}{8\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2),x)
```

```
[Out] (x*(B*e + C*d))/c - (atan((c^(1/2)*x)/a^(1/2))*(B*a*e - A*c*d + C*a*d))/(a^
(1/2)*c^(3/2)) + (C*e*x^2)/(2*c) + (log(a + c*x^2)*(4*A*a*c^3*e + 4*B*a*c^3
*d - 4*C*a^2*c^2*e))/(8*a*c^4)
```

3.46 $\int \frac{A+Bx+Cx^2}{a+cx^2} dx$

Optimal. Leaf size=55

$$\frac{Cx}{c} + \frac{(Ac-aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,c^{3/2}} + \frac{B\log\left(a+cx^2\right)}{2c}$$

[Out] C*x/c+1/2*B*ln(c*x^2+a)/c+(A*c-C*a)*arctan(x*c^(1/2)/a^(1/2))/c^(3/2)/a^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1824, 649, 211, 266}

$$\frac{(Ac-aC)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,c^{3/2}} + \frac{B\log\left(a+cx^2\right)}{2c} + \frac{Cx}{c}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x + C*x^2)/(a + c*x^2),x]

[Out] (C*x)/c + ((A*c - a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + (B*Log[a + c*x^2])/(2*c)

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx &= \int \left(\frac{C}{c}+\frac{Ac-aC+Bcx}{c\,(a+cx^2)}\right)dx\\ &= \frac{Cx}{c}+\frac{\int \frac{Ac-aC+Bcx}{a+cx^2}\,dx}{c}\\ &= \frac{Cx}{c}+B\int \frac{x}{a+cx^2}\,dx+\frac{(Ac-aC)\int \frac{1}{a+cx^2}\,dx}{c}\\ &= \frac{Cx}{c}+\frac{(Ac-aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 56, normalized size = 1.02

$$\frac{Cx}{c}-\frac{(-Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2),x]
```

```
[Out] (C*x)/c - ((-(A*c) + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) +
(B*Log[a + c*x^2])/(2*c)
```

Maple [**A**]
time = 0.10, size = 47, normalized size = 0.85

method	result
default	$\frac{Cx}{c}+\frac{\frac{B\ln\left(c\,x^2+a\right)}{2}+\frac{(Ac-aC)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{c}$
risch	$\frac{Cx}{c}+\frac{\ln\left(acA-a^2C-\sqrt{-ac\,(Ac-aC)^2}\;x\right)B}{2c}+\frac{\ln\left(acA-a^2C-\sqrt{-ac\,(Ac-aC)^2}\;x\right)\sqrt{-ac\,(Ac-aC)}}{2c^2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*x/c+1/c*(1/2*B*ln(c*x^2+a)+(A*c-C*a)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Maxima [A]
time = 0.50, size = 48, normalized size = 0.87

$$\frac{Cx}{c} + \frac{B\log\left(cx^2+a\right)}{2\,c} - \frac{(Ca-Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] C*x/c + 1/2*B*log(c*x^2 + a)/c - (C*a - A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*
c)*c)
```

Fricas [A]
time = 0.36, size = 125, normalized size = 2.27

$$\left[\frac{2\,Cacx + Bac\log\left(cx^2+a\right) + (Ca-Ac)\sqrt{-ac}\,\log\left(\frac{cx^2-2\,\sqrt{-ac}\,x-a}{cx^2+a}\right)}{2\,ac^2}, \frac{2\,Cacx + Bac\log\left(cx^2+a\right) - 2\,(Ca-Ac)\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,x}{a}\right)}{2\,ac^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*C*a*c*x + B*a*c*log(c*x^2 + a) + (C*a - A*c)*sqrt(-a*c)*log((c*x^2
- 2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a*c^2), 1/2*(2*C*a*c*x + B*a*c*log(c*x
^2 + a) - 2*(C*a - A*c)*sqrt(a*c)*arctan(sqrt(a*c)*x/a))/(a*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(48) = 96$.
time = 0.24, size = 156, normalized size = 2.84

$$\frac{Cx}{c} + \left(\frac{B}{2c} - \frac{\sqrt{-ac^3}\,(-Ac+Ca)}{2ac^3}\right)\log\left(x + \frac{Ba - 2ac\left(\frac{B}{2c} - \frac{\sqrt{-ac^3}\,(-Ac+Ca)}{2ac^3}\right)}{-Ac+Ca}\right) + \left(\frac{B}{2c} + \frac{\sqrt{-ac^3}\,(-Ac+Ca)}{2ac^3}\right)\log\left(x + \frac{Ba - 2ac\left(\frac{B}{2c} + \frac{\sqrt{-ac^3}\,(-Ac+Ca)}{2ac^3}\right)}{-Ac+Ca}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*x/c + (B/(2*c) - sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3))*log(x + (B*a - 2*
a*c*(B/(2*c) - sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3)))/(-A*c + C*a)) + (B/(
2*c) + sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3))*log(x + (B*a - 2*a*c*(B/(2*c)
 + sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3)))/(-A*c + C*a))
```

Giac [A]
time = 3.94, size = 48, normalized size = 0.87

$$\frac{Cx}{c} + \frac{B\log\left(cx^2+a\right)}{2\,c} - \frac{(Ca-Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] C*x/c + 1/2*B*log(c*x^2 + a)/c - (C*a - A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*
c)*c)
```

Mupad [B]
time = 3.73, size = 56, normalized size = 1.02

$$\frac{B\,\ln\left(c\,x^2+a\right)}{2\,c}+\frac{C\,x}{c}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}-\frac{C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2),x)
```

```
[Out] (B*log(a + c*x^2))/(2*c) + (C*x)/c + (A*atan((c^(1/2)*x)/a^(1/2)))/(a^(1/2)
*c^(1/2)) - (C*a^(1/2)*atan((c^(1/2)*x)/a^(1/2)))/c^(3/2)
```

3.47 $\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)}\,dx$

Optimal. Leaf size=133

$$\frac{(Acd-aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}\,(cd^2+ae^2)}+\frac{(Cd^2-Bde+Ae^2)\log(d+ex)}{e\,(cd^2+ae^2)}+\frac{(Bcd-Ace+aCe)\log\left(a+cx^2\right)}{2c\,(cd^2+ae^2)}$$

```
[Out] (A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e/(a*e^2+c*d^2)+1/2*(-A*c*e+B*c*d+C*a*e)*ln(c
*x^2+a)/c/(a*e^2+c*d^2)+(A*c*d+B*a*e-C*a*d)*arctan(x*c^(1/2)/a^(1/2))/(a*e^
2+c*d^2)/a^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1643, 649, 211, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe-aCd+Acd)}{\sqrt{a}\,\sqrt{c}\,(ae^2+cd^2)}+\frac{\log\left(a+cx^2\right)(aCe-Ace+Bcd)}{2c\,(ae^2+cd^2)}+\frac{\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{e\,(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)),x]
```

```
[Out] ((A*c*d - a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]*(c*d
^2 + a*e^2)) + ((C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/(e*(c*d^2 + a*e^2)) +
 ((B*c*d - A*c*e + a*C*e)*Log[a + c*x^2])/(2*c*(c*d^2 + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)}\,dx = \int \left(\frac{Cd^2-Bde+Ae^2}{(cd^2+ae^2)(d+ex)} + \frac{Acd-aCd+aBe+(Bcd-Ace+aCe)x}{(cd^2+ae^2)(a+cx^2)}\right)dx$$
$$= \frac{(Cd^2-Bde+Ae^2)\log(d+ex)}{e(cd^2+ae^2)} + \frac{\int \frac{Acd-aCd+aBe+(Bcd-Ace+aCe)x}{a+cx^2}\,dx}{cd^2+ae^2}$$
$$= \frac{(Cd^2-Bde+Ae^2)\log(d+ex)}{e(cd^2+ae^2)} + \frac{(Acd-aCd+aBe)\int \frac{1}{a+cx^2}\,dx}{cd^2+ae^2} + \frac{(Bcd-Ac$$
$$= \frac{(Acd-aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}\,(cd^2+ae^2)} + \frac{(Cd^2-Bde+Ae^2)\log(d+ex)}{e(cd^2+ae^2)} + \frac{(Bcd$$

Mathematica [A]
time = 0.07, size = 120, normalized size = 0.90

$$\frac{2\sqrt{c}\,e(Acd-aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)+\sqrt{a}\,(2c(Cd^2-Bde+Ae^2)\log(d+ex)+e(Bcd-Ace+aCe)\log(a+cx^2))}{2\sqrt{a}\,ce(cd^2+ae^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)),x]
```

```
[Out] (2*Sqrt[c]*e*(A*c*d - a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]] + Sqrt[a]*
(2*c*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x] + e*(B*c*d - A*c*e + a*C*e)*Log[a
 + c*x^2]))/(2*Sqrt[a]*c*e*(c*d^2 + a*e^2))
```

Maple [A]
time = 0.13, size = 112, normalized size = 0.84

method	result	size
default	$\frac{(Ae^2-Bde+Cd^2)\ln(ex+d)}{e(ae^2+cd^2)}+\frac{\frac{(-Ace+Bcd+aCe)\ln(cx^2+a)}{2c}+\frac{(Acd+aBe-Cad)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{ae^2+cd^2}$	112
risch	Expression too large to display	6674

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] (A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e/(a*e^2+c*d^2)+1/(a*e^2+c*d^2)*(1/2*(-A*c*e+
B*c*d+C*a*e)/c*ln(c*x^2+a)+(A*c*d+B*a*e-C*a*d)/(a*c)^(1/2)*arctan(c*x/(a*c)
^(1/2)))
```

Maxima [A]
time = 0.49, size = 124, normalized size = 0.93

$$\frac{(Bcd+Cae-Ace)\log\left(cx^2+a\right)}{2\left(c^2d^2+ace^2\right)}+\frac{\left(Cd^2-Bde+Ae^2\right)\log\left(xe+d\right)}{cd^2e+ae^3}+\frac{(Bae-(Ca-Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(cd^2+ae^2\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*c*d + C*a*e - A*c*e)*log(c*x^2 + a)/(c^2*d^2 + a*c*e^2) + (C*d^2 - B
*d*e + A*e^2)*log(x*e + d)/(c*d^2*e + a*e^3) + (B*a*e - (C*a - A*c)*d)*arct
an(c*x/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))
```

Fricas [A]
time = 4.38, size = 264, normalized size = 1.98

$$\left[-\frac{\left(Bae^2-(Ca-Ac)de\right)\sqrt{-ac}\log\left(\frac{cx^2-2\sqrt{-ac}\,x-a}{cx^2+a}\right)-\left(Bacde+\left(Ca^2-Aac\right)e^2\right)\log\left(cx^2+a\right)-2\left(Cacd^2-Bacde+Aace^2\right)\log\left(xe+d\right)}{2\left(ac^2d^2e+a^2ce^3\right)},\ \frac{2\left(Bae^2-(Ca-Ac)de\right)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\,x}{a}\right)+\left(Bacde+\left(Ca^2-Aac\right)e^2\right)\log\left(cx^2+a\right)+2\left(Cacd^2-Bacde+Aace^2\right)\log\left(xe+d\right)}{2\left(ac^2d^2e+a^2ce^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*((B*a*e^2 - (C*a - A*c)*d*e)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x -
 a)/(c*x^2 + a)) - (B*a*c*d*e + (C*a^2 - A*a*c)*e^2)*log(c*x^2 + a) - 2*(C*
a*c*d^2 - B*a*c*d*e + A*a*c*e^2)*log(x*e + d))/(a*c^2*d^2*e + a^2*c*e^3), 1
/2*(2*(B*a*e^2 - (C*a - A*c)*d*e)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (B*a*c*
d*e + (C*a^2 - A*a*c)*e^2)*log(c*x^2 + a) + 2*(C*a*c*d^2 - B*a*c*d*e + A*a*
c*e^2)*log(x*e + d))/(a*c^2*d^2*e + a^2*c*e^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.72, size = 125, normalized size = 0.94

$$\frac{(Bcd+Cae-Ace)\log\left(cx^2+a\right)}{2\left(c^2d^2+ace^2\right)}+\frac{\left(Cd^2-Bde+Ae^2\right)\log\left(|xe+d|\right)}{cd^2e+ae^3}-\frac{(Cad-Acd-Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(cd^2+ae^2\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d + C*a*e - A*c*e)*log(c*x^2 + a)/(c^2*d^2 + a*c*e^2) + (C*d^2 - B
*d*e + A*e^2)*log(abs(x*e + d))/(c*d^2*e + a*e^3) - (C*a*d - A*c*d - B*a*e)
*arctan(c*x/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))
```

Mupad [B]
time = 6.49, size = 840, normalized size = 6.32

$$\frac{\ln(d+ex)\,(Cd^2-Bde+Ae^2)}{cd^2e+ae^3}-\frac{\ln\left(x\,(ceB^2-cdBC+aeC^2-AceC)+C^2ad+\frac{\left(c^2\left(\frac{Aae}{2}-\frac{Bad}{2}\right)-c\left(\frac{Ca^2e}{2}-\frac{Ad\sqrt{-ac^3}}{2}\right)+\frac{Bae\sqrt{-ac^3}}{2}-\frac{Cad\sqrt{-ac^3}}{2}\right)\left(\frac{(x\,(6ac^2e^3-2c^3d^2e)+8ac^2de^2)\left(c^2\left(\frac{Aae}{2}-\frac{Bad}{2}\right)-c\left(\frac{Ca^2e}{2}-\frac{Ad\sqrt{-ac^3}}{2}\right)+\frac{Bae\sqrt{-ac^3}}{2}-\frac{Cad\sqrt{-ac^3}}{2}\right)}{a^2c^2e^2+ac^3d^2}-x\,(2Cc^2d^2-Bc^2de+3Ac^2e^2-5Cace^2)+Bace^2-Ac^2de+5Cacde\right)}{a^2c^2e^2+ac^3d^2}+ABce-ACcd\right)\left(c^2\left(\frac{Aae}{2}-\frac{Bad}{2}\right)-c\left(\frac{Ca^2e}{2}-\frac{Ad\sqrt{-ac^3}}{2}\right)+\frac{Bae\sqrt{-ac^3}}{2}-\frac{Cad\sqrt{-ac^3}}{2}\right)}{a^2c^2e^2+ac^3d^2}-\frac{\ln\left(x\,(ceB^2-cdBC+aeC^2-AceC)+C^2ad+\frac{\left(c^2\left(\frac{Aae}{2}-\frac{Bad}{2}\right)-c\left(\frac{Ca^2e}{2}+\frac{Ad\sqrt{-ac^3}}{2}\right)-\frac{Bae\sqrt{-ac^3}}{2}+\frac{Cad\sqrt{-ac^3}}{2}\right)\left(\frac{(x\,(6ac^2e^3-2c^3d^2e)+8ac^2de^2)\left(c^2\left(\frac{Aae}{2}-\frac{Bad}{2}\right)-c\left(\frac{Ca^2e}{2}+\frac{Ad\sqrt{-ac^3}}{2}\right)-\frac{Bae\sqrt{-ac^3}}{2}+\frac{Cad\sqrt{-ac^3}}{2}\right)}{a^2c^2e^2+ac^3d^2}-x\,(2Cc^2d^2-Bc^2de+3Ac^2e^2-5Cace^2)+Bace^2-Ac^2de+5Cacde\right)}{a^2c^2e^2+ac^3d^2}+ABce-ACcd\right)\left(c^2\left(\frac{Aae}{2}-\frac{Bad}{2}\right)-c\left(\frac{Ca^2e}{2}+\frac{Ad\sqrt{-ac^3}}{2}\right)-\frac{Bae\sqrt{-ac^3}}{2}+\frac{Cad\sqrt{-ac^3}}{2}\right)}{a^2c^2e^2+ac^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)*(d + e*x)),x)
```

```
[Out] (log(d + e*x)*(A*e^2 + C*d^2 - B*d*e))/(a*e^3 + c*d^2*e) - (log(x*(C^2*a*e
+ B^2*c*e - A*C*c*e - B*C*c*d) + C^2*a*d + ((c^2*((A*a*e)/2 - (B*a*d)/2) -
c*((C*a^2*e)/2 - (A*d*(-a*c^3)^(1/2))/2) + (B*a*e*(-a*c^3)^(1/2))/2 - (C*a*
d*(-a*c^3)^(1/2))/2)*(((x*(6*a*c^2*e^3 - 2*c^3*d^2*e) + 8*a*c^2*d*e^2)*(c^2
*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 - (A*d*(-a*c^3)^(1/2))/2) + (B*a*
e*(-a*c^3)^(1/2))/2 - (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^2 + a^2*c^2*e^2)
- x*(3*A*c^2*e^2 + 2*C*c^2*d^2 - 5*C*a*c*e^2 - B*c^2*d*e) + B*a*c*e^2 - A*c
^2*d*e + 5*C*a*c*d*e))/(a*c^3*d^2 + a^2*c^2*e^2) + A*B*c*e - A*C*c*d)*(c^2*
((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 - (A*d*(-a*c^3)^(1/2))/2) + (B*a*e
*(-a*c^3)^(1/2))/2 - (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^2 + a^2*c^2*e^2) -
 (log(x*(C^2*a*e + B^2*c*e - A*C*c*e - B*C*c*d) + C^2*a*d + ((c^2*((A*a*e)/
2 - (B*a*d)/2) - c*((C*a^2*e)/2 + (A*d*(-a*c^3)^(1/2))/2) - (B*a*e*(-a*c^3)
^(1/2))/2 + (C*a*d*(-a*c^3)^(1/2))/2)*(((x*(6*a*c^2*e^3 - 2*c^3*d^2*e) + 8*
a*c^2*d*e^2)*(c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 + (A*d*(-a*c^3)^
(1/2))/2) - (B*a*e*(-a*c^3)^(1/2))/2 + (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^
2 + a^2*c^2*e^2) - x*(3*A*c^2*e^2 + 2*C*c^2*d^2 - 5*C*a*c*e^2 - B*c^2*d*e)
+ B*a*c*e^2 - A*c^2*d*e + 5*C*a*c*d*e))/(a*c^3*d^2 + a^2*c^2*e^2) + A*B*c*e
 - A*C*c*d)*(c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 + (A*d*(-a*c^3)^(
1/2))/2) - (B*a*e*(-a*c^3)^(1/2))/2 + (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^2
 + a^2*c^2*e^2)
```

3.48 $\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)}\,dx$

Optimal. Leaf size=214

$$-\frac{Cd^2-Bde+Ae^2}{e\,(cd^2+ae^2)\,(d+ex)}+\frac{(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}\,\left(cd^2+ae^2\right)^2}-\frac{(Bcd^2-2Acde+2a\cdots}{(cd^2\cdots}$$

```
[Out] (-A*e^2+B*d*e-C*d^2)/e/(a*e^2+c*d^2)/(e*x+d)-(-2*A*c*d*e-B*a*e^2+B*c*d^2+2*
C*a*d*e)*ln(e*x+d)/(a*e^2+c*d^2)^2+1/2*(-2*A*c*d*e-B*a*e^2+B*c*d^2+2*C*a*d*
e)*ln(c*x^2+a)/(a*e^2+c*d^2)^2+(A*c*(-a*e^2+c*d^2)+a*(a*C*e^2-c*d*(-2*B*e+C
*d)))*arctan(x*c^(1/2)/a^(1/2))/(a*e^2+c*d^2)^2/a^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1643, 649, 211, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))}{\sqrt{a}\,\sqrt{c}\,(ae^2+cd^2)^2}+\frac{\log(a+cx^2)(-aBe^2+2aCde-2Acde+Bcd^2)}{2(ae^2+cd^2)^2}-\frac{Ae^2-Bde+Cd^2}{e(d+ex)(ae^2+cd^2)}-\frac{\log(d+ex)(-aBe^2+2aCde-2Acde+Bcd^2)}{(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)),x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)/(e*(c*d^2 + a*e^2)*(d + e*x))) + ((A*c*(c*d^2 - a
*e^2) + a*(a*C*e^2 - c*d*(C*d - 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt
[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) - ((B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^
2)*Log[d + e*x])/(c*d^2 + a*e^2)^2 + ((B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*
B*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)}\,dx = \int \left(\frac{Cd^2-Bde+Ae^2}{(cd^2+ae^2)\,(d+ex)^2} + \frac{e(-Bcd^2+2Acde-2aCde+aBe^2)}{(cd^2+ae^2)^2\,(d+ex)} + \frac{Ac(cd^2-\cdots}{\cdots}\right.$$

$$= -\frac{Cd^2-Bde+Ae^2}{e\,(cd^2+ae^2)\,(d+ex)} - \frac{(Bcd^2-2Acde+2aCde-aBe^2)\log(d+ex)}{(cd^2+ae^2)^2} + \frac{\int \cdots}{\cdots}$$

$$= -\frac{Cd^2-Bde+Ae^2}{e\,(cd^2+ae^2)\,(d+ex)} - \frac{(Bcd^2-2Acde+2aCde-aBe^2)\log(d+ex)}{(cd^2+ae^2)^2} + \frac{(c(\cdots}{\cdots}$$

$$= -\frac{Cd^2-Bde+Ae^2}{e\,(cd^2+ae^2)\,(d+ex)} + \frac{(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\tan^{-1}\left(\cdots\right.}{\sqrt{a}\,\sqrt{c}\,(cd^2+ae^2)^2}$$

Mathematica [A]
time = 0.22, size = 188, normalized size = 0.88

$$\frac{-\frac{2(cd^2+ae^2)(Cd^2+e(-Bd+Ae))}{e(d+ex)} + \frac{2(Ac(cd^2-ae^2)+a(aCe^2+cd(-Cd+2Be)))\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}} + (-2Bcd^2+4Acde-4aCde+2aBe^2)\log(d+ex) + (Bcd^2-2Acde+2aCde-aBe^2)\log(a+cx^2)}{2\,(cd^2+ae^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)),x]
```

```
[Out] ((-2*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(e*(d + e*x)) + (2*(A*c*(c
*d^2 - a*e^2) + a*(a*C*e^2 + c*d*(-(C*d) + 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt
[a]])/(Sqrt[a]*Sqrt[c]) + (-2*B*c*d^2 + 4*A*c*d*e - 4*a*C*d*e + 2*a*B*e^2)*
Log[d + e*x] + (B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2)*Log[a + c*x^2])/
(2*(c*d^2 + a*e^2)^2)
```

Maple [A]
time = 0.16, size = 204, normalized size = 0.95

method	result
default	$-\frac{A e^2-Bde+C d^2}{(a e^2+c d^2)e(ex+d)} + \frac{(2cdeA+Ba e^2-Bc d^2-2adeC)\ln(ex+d)}{(a e^2+c d^2)^2} + \frac{\frac{\left(-2c^2 deA-Bac e^2+B c^2 d^2+2acdeC\right)\ln\left(c x^2+a\right)}{2c}+\frac{\left(-Aac e^2+\cdots\right.}{\cdots}}{(a e^2+c d^2)\cdots}$

risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)/e/(e*x+d)+(2*A*c*d*e+B*a*e^2-B*c*d^2-2*C
*a*d*e)/(a*e^2+c*d^2)^2*ln(e*x+d)+1/(a*e^2+c*d^2)^2*(1/2*(-2*A*c^2*d*e-B*a*
c*e^2+B*c^2*d^2+2*C*a*c*d*e)/c*ln(c*x^2+a)+(-A*a*c*e^2+A*c^2*d^2+2*B*a*c*d*
e+C*a^2*e^2-C*a*c*d^2)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Maxima [A]
time = 0.52, size = 252, normalized size = 1.18

$$\frac{(Bcd^2 - Bae^2 + 2(Cae - Ace)d)\log(cx^2+a)}{2(c^2d^4+2acd^2e^2+a^2e^4)} - \frac{(Bcd^2 - Bae^2 + 2(Cae-Ace)d)\log(xe+d)}{c^2d^4+2acd^2e^2+a^2e^4} + \frac{(2Bacde + Ca^2e^2 - Aace^2 - (Cac - Ac^2)d^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(c^2d^4+2acd^2e^2+a^2e^4)\sqrt{ac}} - \frac{Cd^2 - Bde + Ae^2}{cd^3e + ade^3 + (cd^2e^2+ae^4)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*c*d^2 - B*a*e^2 + 2*(C*a*e - A*c*e)*d)*log(c*x^2 + a)/(c^2*d^4 + 2*a
*c*d^2*e^2 + a^2*e^4) - (B*c*d^2 - B*a*e^2 + 2*(C*a*e - A*c*e)*d)*log(x*e +
 d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + (2*B*a*c*d*e + C*a^2*e^2 - A*a*c*
e^2 - (C*a*c - A*c^2)*d^2)*arctan(c*x/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2
+ a^2*e^4)*sqrt(a*c)) - (C*d^2 - B*d*e + A*e^2)/(c*d^3*e + a*d*e^3 + (c*d^2
*e^2 + a*e^4)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(203) = 406.
time = 20.07, size = 873, normalized size = 4.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*C*a*c^2*d^4 - 2*B*a*c^2*d^3*e - 2*B*a^2*c*d*e^3 + 2*A*a^2*c*e^4 +
2*(C*a^2*c + A*a*c^2)*d^2*e^2 + ((C*a*c - A*c^2)*d^3*e - (C*a^2 - A*a*c)*x*
e^4 - (2*B*a*c*d*x + (C*a^2 - A*a*c)*d)*e^3 - (2*B*a*c*d^2 - (C*a*c - A*c^2
)*d^2*x)*e^2)*sqrt(-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - (B
*a*c^2*d^3*e - B*a^2*c*x*e^4 - (B*a^2*c*d - 2*(C*a^2*c - A*a*c^2)*d*x)*e^3
+ (B*a*c^2*d^2*x + 2*(C*a^2*c - A*a*c^2)*d^2)*e^2)*log(c*x^2 + a) + 2*(B*a*
c^2*d^3*e - B*a^2*c*x*e^4 - (B*a^2*c*d - 2*(C*a^2*c - A*a*c^2)*d*x)*e^3 + (
B*a*c^2*d^2*x + 2*(C*a^2*c - A*a*c^2)*d^2)*e^2)*log(x*e + d))/(a*c^3*d^4*x*
e^2 + a*c^3*d^5*e + 2*a^2*c^2*d^2*x*e^4 + 2*a^2*c^2*d^3*e^3 + a^3*c*x*e^6 +
 a^3*c*d*e^5), -1/2*(2*C*a*c^2*d^4 - 2*B*a*c^2*d^3*e - 2*B*a^2*c*d*e^3 + 2*
```

```
A*a^2*c*e^4 + 2*(C*a^2*c + A*a*c^2)*d^2*e^2 + 2*((C*a*c - A*c^2)*d^3*e - (C
*a^2 - A*a*c)*x*e^4 - (2*B*a*c*d*x + (C*a^2 - A*a*c)*d)*e^3 - (2*B*a*c*d^2
- (C*a*c - A*c^2)*d^2*x)*e^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (B*a*c^2*d^
3*e - B*a^2*c*x*e^4 - (B*a^2*c*d - 2*(C*a^2*c - A*a*c^2)*d*x)*e^3 + (B*a*c^
2*d^2*x + 2*(C*a^2*c - A*a*c^2)*d^2)*e^2)*log(c*x^2 + a) + 2*(B*a*c^2*d^3*e
 - B*a^2*c*x*e^4 - (B*a^2*c*d - 2*(C*a^2*c - A*a*c^2)*d*x)*e^3 + (B*a*c^2*d
^2*x + 2*(C*a^2*c - A*a*c^2)*d^2)*e^2)*log(x*e + d))/(a*c^3*d^4*x*e^2 + a*c
^3*d^5*e + 2*a^2*c^2*d^2*x*e^4 + 2*a^2*c^2*d^3*e^3 + a^3*c*x*e^6 + a^3*c*d*
e^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.27, size = 270, normalized size = 1.26

$$-\frac{\left(Cacd^2e^2 - Ac^2d^2e^2 - 2\,Bacde^3 - Ca^2e^4 + Aace^4\right)\arctan\left(\frac{\left(cd - \frac{cd^2}{xe+d} - \frac{ae^2}{xe+d}\right)e^{(-1)}}{\sqrt{ac}}\right)e^{(-2)}}{\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)\sqrt{ac}} + \frac{\left(Bcd^2 + 2\,Cade - 2\,Acde - Bae^2\right)\log\left(c - \frac{2\,cd}{xe+d} + \frac{cd^2}{(xe+d)^2} + \frac{ae^2}{(xe+d)^2}\right)}{2\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)} - \frac{\frac{Cd^2e}{xe+d} - \frac{Bde^2}{xe+d} + \frac{Ae^3}{xe+d}}{cd^2e^2 + ae^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a*c*d^2*e^2 - A*c^2*d^2*e^2 - 2*B*a*c*d*e^3 - C*a^2*e^4 + A*a*c*e^4)*ar
ctan((c*d - c*d^2/(x*e + d) - a*e^2/(x*e + d))*e^(-1)/sqrt(a*c))*e^(-2)/((c
^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) + 1/2*(B*c*d^2 + 2*C*a*d*e - 2
*A*c*d*e - B*a*e^2)*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*
e + d)^2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - (C*d^2*e/(x*e + d) - B*d*e^
2/(x*e + d) + A*e^3/(x*e + d))/(c*d^2*e^2 + a*e^4)
```

Mupad [**B**]
time = 6.77, size = 1199, normalized size = 5.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)*(d + e*x)^2),x)
```

```
[Out] (log(C*c*d^4*(-a*c)^(3/2) - A*a*e^4*(-a*c)^(3/2) + 3*B*a*c^3*d^4 + 3*B*a^3*
c*e^4 + A*c^4*d^4*x + A*c^3*d^4*(-a*c)^(1/2) - C*a^3*e^4*(-a*c)^(1/2) - C*a
```

```
*c^3*d^4*x - C*a^3*c*e^4*x + 14*A*c*d^2*e^2*(-a*c)^(3/2) - 14*C*a*d^2*e^2*(
-a*c)^(3/2) - 3*B*c^3*d^4*x*(-a*c)^(1/2) + 8*A*a^2*c^2*d*e^3 + 8*C*a^2*c^2*
d^3*e + A*a^2*c^2*e^4*x - 10*B*a^2*c^2*d^2*e^2 + 8*B*a*d*e^3*(-a*c)^(3/2) -
 8*B*c*d^3*e*(-a*c)^(3/2) + 3*B*a*e^4*x*(-a*c)^(3/2) - 8*A*a*c^3*d^3*e - 8*
C*a^3*c*d*e^3 + 14*C*a^2*c^2*d^2*e^2*x + 8*A*c*d*e^3*x*(-a*c)^(3/2) - 8*C*a
*d*e^3*x*(-a*c)^(3/2) + 8*C*c*d^3*e*x*(-a*c)^(3/2) + 8*B*a*c^3*d^3*e*x + 8*
A*c^3*d^3*e*x*(-a*c)^(1/2) - 10*B*c*d^2*e^2*x*(-a*c)^(3/2) - 14*A*a*c^3*d^2
*e^2*x - 8*B*a^2*c^2*d*e^3*x)*(c^2*(a*((B*d^2)/2 - A*d*e) + (A*d^2*(-a*c)^(
1/2))/2) - c*(a^2*((B*e^2)/2 - C*d*e) + a*((A*e^2*(-a*c)^(1/2))/2 + (C*d^2*
(-a*c)^(1/2))/2 - B*d*e*(-a*c)^(1/2))) + (C*a^2*e^2*(-a*c)^(1/2))/2))/(a*c^
3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - (log(d + e*x)*(c*(B*d^2 - 2*A*d*e)
 - a*(B*e^2 - 2*C*d*e)))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) - (log(A*a*e^4
*(-a*c)^(3/2) - C*c*d^4*(-a*c)^(3/2) + 3*B*a*c^3*d^4 + 3*B*a^3*c*e^4 + A*c^
4*d^4*x - A*c^3*d^4*(-a*c)^(1/2) + C*a^3*e^4*(-a*c)^(1/2) - C*a*c^3*d^4*x -
 C*a^3*c*e^4*x - 14*A*c*d^2*e^2*(-a*c)^(3/2) + 14*C*a*d^2*e^2*(-a*c)^(3/2)
+ 3*B*c^3*d^4*x*(-a*c)^(1/2) + 8*A*a^2*c^2*d*e^3 + 8*C*a^2*c^2*d^3*e + A*a^
2*c^2*e^4*x - 10*B*a^2*c^2*d^2*e^2 - 8*B*a*d*e^3*(-a*c)^(3/2) + 8*B*c*d^3*e
*(-a*c)^(3/2) - 3*B*a*e^4*x*(-a*c)^(3/2) - 8*A*a*c^3*d^3*e - 8*C*a^3*c*d*e^
3 + 14*C*a^2*c^2*d^2*e^2*x - 8*A*c*d*e^3*x*(-a*c)^(3/2) + 8*C*a*d*e^3*x*(-a
*c)^(3/2) - 8*C*c*d^3*e*x*(-a*c)^(3/2) + 8*B*a*c^3*d^3*e*x - 8*A*c^3*d^3*e*
x*(-a*c)^(1/2) + 10*B*c*d^2*e^2*x*(-a*c)^(3/2) - 14*A*a*c^3*d^2*e^2*x - 8*B
*a^2*c^2*d*e^3*x)*(c*(a^2*((B*e^2)/2 - C*d*e) - a*((A*e^2*(-a*c)^(1/2))/2 +
 (C*d^2*(-a*c)^(1/2))/2 - B*d*e*(-a*c)^(1/2))) - c^2*(a*((B*d^2)/2 - A*d*e)
 - (A*d^2*(-a*c)^(1/2))/2) + (C*a^2*e^2*(-a*c)^(1/2))/2))/(a*c^3*d^4 + a^3*
c*e^4 + 2*a^2*c^2*d^2*e^2) - (A*e^2 + C*d^2 - B*d*e)/(e*(a*e^2 + c*d^2)*(d
+ e*x))
```

3.49 $\int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)}\,dx$

Optimal. Leaf size=305

$$-\frac{Cd^2-Bde+Ae^2}{2e\left(cd^2+ae^2\right)(d+ex)^2}+\frac{Bcd^2-2Acde+2aCde-aBe^2}{\left(cd^2+ae^2\right)^2(d+ex)}+\frac{\sqrt{c}\,\left(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-\ldots\right.}{\sqrt{a}\,\left(cd^2+ae^2\ldots\right.}$$

```
[Out] 1/2*(-A*e^2+B*d*e-C*d^2)/e/(a*e^2+c*d^2)/(e*x+d)^2+(-2*A*c*d*e-B*a*e^2+B*c*
d^2+2*C*a*d*e)/(a*e^2+c*d^2)^2/(e*x+d)-(B*c*d*(-3*a*e^2+c*d^2)-(A*c-C*a)*e*
(-a*e^2+3*c*d^2))*ln(e*x+d)/(a*e^2+c*d^2)^3+1/2*(B*c*d*(-3*a*e^2+c*d^2)-(A*
c-C*a)*e*(-a*e^2+3*c*d^2))*ln(c*x^2+a)/(a*e^2+c*d^2)^3+(A*c*d*(-3*a*e^2+c*d
^2)-a*(c*d^2*(-3*B*e+C*d)-a*e^2*(-B*e+3*C*d)))*arctan(x*c^(1/2)/a^(1/2))*c^
(1/2)/(a*e^2+c*d^2)^3/a^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1643, 649, 211, 266}

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3Cd-Be))\right)}{\sqrt{a}\,\left(ae^2+cd^2\right)^3}+\frac{\log\left(a+cx^2\right)\left(Bcd(cd^2-3ae^2)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)}{2\left(ae^2+cd^2\right)^3}-\frac{Ae^2-Bde+Cd^2}{2e(d+ex)^2\left(ae^2+cd^2\right)}+\frac{-aBe^2+2aCde-2Acde+Bcd^2}{(d+ex)\left(ae^2+cd^2\right)^2}-\frac{\log(d+ex)\left(Bcd(cd^2-3ae^2)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)}{\left(ae^2+cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)),x]
```

```
[Out] -1/2*(C*d^2 - B*d*e + A*e^2)/(e*(c*d^2 + a*e^2)*(d + e*x)^2) + (B*c*d^2 - 2
*A*c*d*e + 2*a*C*d*e - a*B*e^2)/((c*d^2 + a*e^2)^2*(d + e*x)) + (Sqrt[c]*(A
*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e)))*Arc
Tan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)^3) - ((B*c*d*(c*d^2 - 3*
a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[d + e*x])/(c*d^2 + a*e^2)^3 +
 ((B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2
])/(2*(c*d^2 + a*e^2)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)}\,dx = \int \left(\frac{Cd^2-Bde+Ae^2}{(cd^2+ae^2)(d+ex)^3}+\frac{e(-Bcd^2+2Acde-2aCde+aBe^2)}{(cd^2+ae^2)^2(d+ex)^2}+\frac{e(-Bcd(cd\ldots}{\ldots}\right.$$

$$= -\frac{Cd^2-Bde+Ae^2}{2e(cd^2+ae^2)(d+ex)^2}+\frac{Bcd^2-2Acde+2aCde-aBe^2}{(cd^2+ae^2)^2(d+ex)}-\frac{(Bcd(cd^2-3ae\ldots}{\ldots}$$

$$= -\frac{Cd^2-Bde+Ae^2}{2e(cd^2+ae^2)(d+ex)^2}+\frac{Bcd^2-2Acde+2aCde-aBe^2}{(cd^2+ae^2)^2(d+ex)}-\frac{(Bcd(cd^2-3ae\ldots}{\ldots}$$

$$= -\frac{Cd^2-Bde+Ae^2}{2e(cd^2+ae^2)(d+ex)^2}+\frac{Bcd^2-2Acde+2aCde-aBe^2}{(cd^2+ae^2)^2(d+ex)}+\frac{\sqrt{c}\,(Acd(cd^2-\ldots}{\ldots}$$

Mathematica [A]
time = 0.21, size = 277, normalized size = 0.91

$$\frac{-\frac{(cd^2+ae^2)^2(Cd^2+e(-Bd+Ae))}{e(d+ex)^2}+\frac{2(cd^2+ae^2)(Bcd^2-2Acde+2aCde-aBe^2)}{d+ex}+\frac{2\sqrt{c}\,(Acd(cd^2-3ae^2)+a(ae^2(3Cd-Be)+cd^2(-Cd+3Be)))\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}}-2(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))\log(d+ex)+(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))\log(a+cx^2)}{2(cd^2+ae^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)),x]
```

```
[Out] (-(((c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(e*(d + e*x)^2)) + (2*(c*
d^2 + a*e^2)*(B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2))/(d + e*x) + (2*Sq
rt[c]*(A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d - B*e) + c*d^2*(-(C*d) + 3
*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] - 2*(B*c*d*(c*d^2 - 3*a*e^2) -
 (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[d + e*x] + (B*c*d*(c*d^2 - 3*a*e^2) -
 (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

Maple [A]
time = 0.18, size = 317, normalized size = 1.04

method	result
default	$-\frac{A\,e^{2}-Bde+C\,d^{2}}{2(a\,e^{2}+c\,d^{2})e(ex+d)^{2}}-\frac{2cdeA+Ba\,e^{2}-Bc\,d^{2}-2adeC}{(a\,e^{2}+c\,d^{2})^{2}(ex+d)}-\frac{(Aac\,e^{3}-3A\,c^{2}d^{2}e-3Bacd\,e^{2}+B\,c^{2}d^{3}-C\,a^{2}e^{3}+3Cac\,d^{2}e)\ln(ex+d}{(a\,e^{2}+c\,d^{2})^{3}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)/e/(e*x+d)^2-(2*A*c*d*e+B*a*e^2-B*c*d
^2-2*C*a*d*e)/(a*e^2+c*d^2)^2/(e*x+d)-(A*a*c*e^3-3*A*c^2*d^2*e-3*B*a*c*d*e^
2+B*c^2*d^3-C*a^2*e^3+3*C*a*c*d^2*e)/(a*e^2+c*d^2)^3*ln(e*x+d)-c/(a*e^2+c*d
^2)^3*(1/2*(-A*a*c*e^3+3*A*c^2*d^2*e+3*B*a*c*d*e^2-B*c^2*d^3+C*a^2*e^3-3*C*
a*c*d^2*e)/c*ln(c*x^2+a)+(3*A*a*c*d*e^2-A*c^2*d^3+B*a^2*e^3-3*B*a*c*d^2*e-3
*C*a^2*d*e^2+C*a*c*d^3)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

Maxima [A]
time = 0.54, size = 480, normalized size = 1.57

$$\frac{(Bc^{2}d^{3}-3\,Bacde^{2}-Ca^{2}e^{3}+Aace^{3}+3\,(Cace-Ac^{2}e)d^{2})\log(cx^{2}+a)}{2\,(c^{3}d^{6}+3\,ac^{2}d^{4}e^{2}+3\,a^{2}cd^{2}e^{4}+a^{3}e^{6})}-\frac{(Bc^{2}d^{3}-3\,Bacde^{2}-Ca^{2}e^{3}+Aace^{3}+3\,(Cace-Ac^{2}e)d^{2})\log(xe+d)}{c^{3}d^{6}+3\,ac^{2}d^{4}e^{2}+3\,a^{2}cd^{2}e^{4}+a^{3}e^{6}}+\frac{(3\,Bac^{2}d^{2}e-Ba^{2}ce^{3}-(Cac^{2}-Ac^{3})d^{3}+3\,(Ca^{2}ce^{2}-Aac^{2}e^{2})d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(c^{3}d^{6}+3\,ac^{2}d^{4}e^{2}+3\,a^{2}cd^{2}e^{4}+a^{3}e^{6})\sqrt{ac}}-\frac{Ccd^{4}-3\,Bcd^{3}e+Bade^{3}-(3\,Cae^{2}-5\,Ace^{2})d^{2}+Aae^{4}-2\,(Bcd^{2}e^{2}-Bae^{4}+2\,(Cae^{3}-Ace^{3})d)x}{2\,(c^{2}d^{6}e+2\,acd^{4}e^{3}+a^{2}d^{2}e^{5}+(c^{2}d^{4}e^{3}+2\,acd^{2}e^{5}+a^{2}e^{7})x^{2}+2\,(c^{2}d^{5}e^{2}+2\,acd^{3}e^{4}+a^{2}de^{6})x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*c^2*d^3 - 3*B*a*c*d*e^2 - C*a^2*e^3 + A*a*c*e^3 + 3*(C*a*c*e - A*c^2
*e)*d^2)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*
e^6) - (B*c^2*d^3 - 3*B*a*c*d*e^2 - C*a^2*e^3 + A*a*c*e^3 + 3*(C*a*c*e - A*
c^2*e)*d^2)*log(x*e + d)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3
*e^6) + (3*B*a*c^2*d^2*e - B*a^2*c*e^3 - (C*a*c^2 - A*c^3)*d^3 + 3*(C*a^2*c
*e^2 - A*a*c^2*e^2)*d)*arctan(c*x/sqrt(a*c))/((c^3*d^6 + 3*a*c^2*d^4*e^2 +
3*a^2*c*d^2*e^4 + a^3*e^6)*sqrt(a*c)) - 1/2*(C*c*d^4 - 3*B*c*d^3*e + B*a*d*
e^3 - (3*C*a*e^2 - 5*A*c*e^2)*d^2 + A*a*e^4 - 2*(B*c*d^2*e^2 - B*a*e^4 + 2*
(C*a*e^3 - A*c*e^3)*d)*x)/(c^2*d^6*e + 2*a*c*d^4*e^3 + a^2*d^2*e^5 + (c^2*d
^4*e^3 + 2*a*c*d^2*e^5 + a^2*e^7)*x^2 + 2*(c^2*d^5*e^2 + 2*a*c*d^3*e^4 + a^
2*d*e^6)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 844 vs. $2(287) = 574$.
time = 75.98, size = 1712, normalized size = 5.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(C*c^2*d^6 - 3*B*c^2*d^5*e - 3*(C*a^2 - 2*A*a*c)*d^2*e^4 - ((C*a*c -
A*c^2)*d^5*e + B*a^2*x^2*e^6 + (2*B*a^2*d*x - 3*(C*a^2 - A*a*c)*d*x^2)*e^5
- (3*B*a*c*d^2*x^2 - B*a^2*d^2 + 6*(C*a^2 - A*a*c)*d^2*x)*e^4 - (6*B*a*c*d^
3*x - (C*a*c - A*c^2)*d^3*x^2 + 3*(C*a^2 - A*a*c)*d^3)*e^3 - (3*B*a*c*d^4 -
 2*(C*a*c - A*c^2)*d^4*x)*e^2)*sqrt(-c/a)*log((c*x^2 - 2*a*x*sqrt(-c/a) - a
)/(c*x^2 + a)) + (2*B*a^2*x + A*a^2)*e^6 + (B*a^2*d - 4*(C*a^2 - A*a*c)*d*x
)*e^5 - 2*(B*a*c*d^3 + 2*(C*a*c - A*c^2)*d^3*x)*e^3 - (2*B*c^2*d^4*x + (2*C
*a*c - 5*A*c^2)*d^4)*e^2 - (B*c^2*d^5*e - (C*a^2 - A*a*c)*x^2*e^6 - (3*B*a*
c*d*x^2 + 2*(C*a^2 - A*a*c)*d*x)*e^5 - (6*B*a*c*d^2*x - 3*(C*a*c - A*c^2)*d
^2*x^2 + (C*a^2 - A*a*c)*d^2)*e^4 + (B*c^2*d^3*x^2 - 3*B*a*c*d^3 + 6*(C*a*c
 - A*c^2)*d^3*x)*e^3 + (2*B*c^2*d^4*x + 3*(C*a*c - A*c^2)*d^4)*e^2)*log(c*x
^2 + a) + 2*(B*c^2*d^5*e - (C*a^2 - A*a*c)*x^2*e^6 - (3*B*a*c*d*x^2 + 2*(C*
a^2 - A*a*c)*d*x)*e^5 - (6*B*a*c*d^2*x - 3*(C*a*c - A*c^2)*d^2*x^2 + (C*a^2
 - A*a*c)*d^2)*e^4 + (B*c^2*d^3*x^2 - 3*B*a*c*d^3 + 6*(C*a*c - A*c^2)*d^3*x
)*e^3 + (2*B*c^2*d^4*x + 3*(C*a*c - A*c^2)*d^4)*e^2)*log(x*e + d))/(2*c^3*d
^7*x*e^2 + c^3*d^8*e + 6*a*c^2*d^5*x*e^4 + 6*a^2*c*d^3*x*e^6 + a^3*x^2*e^9
+ 2*a^3*d*x*e^8 + (3*a^2*c*d^2*x^2 + a^3*d^2)*e^7 + 3*(a*c^2*d^4*x^2 + a^2*
c*d^4)*e^5 + (c^3*d^6*x^2 + 3*a*c^2*d^6)*e^3), -1/2*(C*c^2*d^6 - 3*B*c^2*d^
5*e - 3*(C*a^2 - 2*A*a*c)*d^2*e^4 + 2*((C*a*c - A*c^2)*d^5*e + B*a^2*x^2*e^
6 + (2*B*a^2*d*x - 3*(C*a^2 - A*a*c)*d*x^2)*e^5 - (3*B*a*c*d^2*x^2 - B*a^2*
d^2 + 6*(C*a^2 - A*a*c)*d^2*x)*e^4 - (6*B*a*c*d^3*x - (C*a*c - A*c^2)*d^3*x
^2 + 3*(C*a^2 - A*a*c)*d^3)*e^3 - (3*B*a*c*d^4 - 2*(C*a*c - A*c^2)*d^4*x)*e
^2)*sqrt(c/a)*arctan(x*sqrt(c/a)) + (2*B*a^2*x + A*a^2)*e^6 + (B*a^2*d - 4*
(C*a^2 - A*a*c)*d*x)*e^5 - 2*(B*a*c*d^3 + 2*(C*a*c - A*c^2)*d^3*x)*e^3 - (2
*B*c^2*d^4*x + (2*C*a*c - 5*A*c^2)*d^4)*e^2 - (B*c^2*d^5*e - (C*a^2 - A*a*c
)*x^2*e^6 - (3*B*a*c*d*x^2 + 2*(C*a^2 - A*a*c)*d*x)*e^5 - (6*B*a*c*d^2*x -
3*(C*a*c - A*c^2)*d^2*x^2 + (C*a^2 - A*a*c)*d^2)*e^4 + (B*c^2*d^3*x^2 - 3*B
*a*c*d^3 + 6*(C*a*c - A*c^2)*d^3*x)*e^3 + (2*B*c^2*d^4*x + 3*(C*a*c - A*c^2
)*d^4)*e^2)*log(c*x^2 + a) + 2*(B*c^2*d^5*e - (C*a^2 - A*a*c)*x^2*e^6 - (3*
B*a*c*d*x^2 + 2*(C*a^2 - A*a*c)*d*x)*e^5 - (6*B*a*c*d^2*x - 3*(C*a*c - A*c^
2)*d^2*x^2 + (C*a^2 - A*a*c)*d^2)*e^4 + (B*c^2*d^3*x^2 - 3*B*a*c*d^3 + 6*(C
*a*c - A*c^2)*d^3*x)*e^3 + (2*B*c^2*d^4*x + 3*(C*a*c - A*c^2)*d^4)*e^2)*log
(x*e + d))/(2*c^3*d^7*x*e^2 + c^3*d^8*e + 6*a*c^2*d^5*x*e^4 + 6*a^2*c*d^3*x
*e^6 + a^3*x^2*e^9 + 2*a^3*d*x*e^8 + (3*a^2*c*d^2*x^2 + a^3*d^2)*e^7 + 3*(a
*c^2*d^4*x^2 + a^2*c*d^4)*e^5 + (c^3*d^6*x^2 + 3*a*c^2*d^6)*e^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.41, size = 489, normalized size = 1.60

$$\frac{(Bc^2d^3 + 3\,Cacd^2e - 3\,Ac^2d^2e - 3\,Bacde^2 - Ca^2e^3 + Aace^3)\log(cx^2 + a)}{2\,(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6)} - \frac{(Bc^2d^3e + 3\,Cacd^2e^2 - 3\,Ac^2d^2e^2 - 3\,Bacde^3 - Ca^2e^4 + Aace^4)\log(|xe + d|)}{c^3d^6e + 3\,ac^2d^4e^3 + 3\,a^2cd^2e^5 + a^3e^7} - \frac{(Cac^2d^3 - Ac^3d^3 - 3\,Bac^2d^2e - 3\,Ca^2cde^2 + 3\,Aac^2de^2 + Ba^2ce^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6)\sqrt{ac}} - \frac{(Cc^2d^6 - 3\,Bc^2d^5e - 2\,Cacd^4e^2 + 5\,Ac^2d^4e^2 - 2\,Bacd^3e^3 - 3\,Ca^2d^2e^4 + 6\,Aacd^2e^4 + Ba^2de^5 + Aa^2e^6 - 2\,(Bc^2d^4e^2 + 2\,Cacd^3e^3 - 2\,Ac^2d^3e^3 + 2\,Ca^2de^5 - 2\,Aacde^5 - Ba^2e^6)x)e^{(-1)}}{2\,(cd^2 + ae^2)^3(xe + d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*c^2*d^3 + 3*C*a*c*d^2*e - 3*A*c^2*d^2*e - 3*B*a*c*d*e^2 - C*a^2*e^3
+ A*a*c*e^3)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 +
a^3*e^6) - (B*c^2*d^3*e + 3*C*a*c*d^2*e^2 - 3*A*c^2*d^2*e^2 - 3*B*a*c*d*e^3
 - C*a^2*e^4 + A*a*c*e^4)*log(abs(x*e + d))/(c^3*d^6*e + 3*a*c^2*d^4*e^3 +
3*a^2*c*d^2*e^5 + a^3*e^7) - (C*a*c^2*d^3 - A*c^3*d^3 - 3*B*a*c^2*d^2*e - 3
*C*a^2*c*d*e^2 + 3*A*a*c^2*d*e^2 + B*a^2*c*e^3)*arctan(c*x/sqrt(a*c))/((c^3
*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*sqrt(a*c)) - 1/2*(C*c^2
*d^6 - 3*B*c^2*d^5*e - 2*C*a*c*d^4*e^2 + 5*A*c^2*d^4*e^2 - 2*B*a*c*d^3*e^3
- 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 + B*a^2*d*e^5 + A*a^2*e^6 - 2*(B*c^2*d^
4*e^2 + 2*C*a*c*d^3*e^3 - 2*A*c^2*d^3*e^3 + 2*C*a^2*d*e^5 - 2*A*a*c*d*e^5 -
 B*a^2*e^6)*x)*e^(-1)/((c*d^2 + a*e^2)^3*(x*e + d)^2)
```

Mupad [B]
time = 9.19, size = 2500, normalized size = 8.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)*(d + e*x)^3),x)
```

```
[Out] (log(d + e*x)*(e^3*(C*a^2 - A*a*c) - B*c^2*d^3 + d^2*e*(3*A*c^2 - 3*C*a*c)
+ 3*B*a*c*d*e^2))/(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4) -
 (log(9*A^2*a^5*e^10*(-a*c)^(5/2) + A^2*c^5*d^10*(-a*c)^(5/2) - B^2*a^7*e^1
0*(-a*c)^(3/2) - 9*B^2*c^3*d^10*(-a*c)^(7/2) + 9*C^2*a^9*e^10*(-a*c)^(1/2)
+ C^2*c*d^10*(-a*c)^(9/2) + 9*C^2*a^9*c*e^10*x - 6*A^2*a*d^4*e^6*(-a*c)^(9/
2) - 6*B^2*a*d^6*e^4*(-a*c)^(9/2) + 106*A^2*c*d^6*e^4*(-a*c)^(9/2) + 77*C^2
*a*d^8*e^2*(-a*c)^(9/2) - 27*B^2*c*d^8*e^2*(-a*c)^(9/2) + A^2*a^2*c^8*d^10*
x + 9*A^2*a^7*c^3*e^10*x + 9*B^2*a^3*c^7*d^10*x + B^2*a^8*c^2*e^10*x + C^2*
a^4*c^6*d^10*x + 27*A^2*a^3*d^2*e^8*(-a*c)^(7/2) - 106*B^2*a^3*d^4*e^6*(-a*
c)^(7/2) + 77*B^2*a^5*d^2*e^8*(-a*c)^(5/2) - 77*A^2*c^3*d^8*e^2*(-a*c)^(7/2
) - 106*C^2*a^3*d^6*e^4*(-a*c)^(7/2) - 6*C^2*a^5*d^4*e^6*(-a*c)^(5/2) + 27*
C^2*a^7*d^2*e^8*(-a*c)^(3/2) + 18*A*C*a^7*e^10*(-a*c)^(3/2) + 2*A*C*c^3*d^1
0*(-a*c)^(7/2) + 224*A*B*a*d^5*e^5*(-a*c)^(9/2) - 48*A*B*a^5*d*e^9*(-a*c)^(
5/2) - 212*A*C*a*d^6*e^4*(-a*c)^(9/2) + 64*A*B*c*d^7*e^3*(-a*c)^(9/2) + 48*
A*B*c^3*d^9*e*(-a*c)^(7/2) - 64*B*C*a*d^7*e^3*(-a*c)^(9/2) - 48*B*C*a^7*d*e
^9*(-a*c)^(3/2) - 154*A*C*c*d^8*e^2*(-a*c)^(9/2) + 77*A^2*a^3*c^7*d^8*e^2*x
```

+ 106*A^2*a^4*c^6*d^6*e^4*x - 6*A^2*a^5*c^5*d^4*e^6*x - 27*A^2*a^6*c^4*d^2
*e^8*x - 27*B^2*a^4*c^6*d^8*e^2*x - 6*B^2*a^5*c^5*d^6*e^4*x + 106*B^2*a^6*c
^4*d^4*e^6*x + 77*B^2*a^7*c^3*d^2*e^8*x + 77*C^2*a^5*c^5*d^8*e^2*x + 106*C^
2*a^6*c^4*d^6*e^4*x - 6*C^2*a^7*c^3*d^4*e^6*x - 27*C^2*a^8*c^2*d^2*e^8*x -
2*A*C*a^3*c^7*d^10*x - 18*A*C*a^8*c^2*e^10*x - 64*A*B*a^3*d^3*e^7*(-a*c)^(7
/2) - 12*A*C*a^3*d^4*e^6*(-a*c)^(7/2) + 54*A*C*a^5*d^2*e^8*(-a*c)^(5/2) + 2
24*B*C*a^3*d^5*e^5*(-a*c)^(7/2) - 64*B*C*a^5*d^3*e^7*(-a*c)^(5/2) + 48*B*C*
c*d^9*e*(-a*c)^(9/2) - 48*A*B*a^3*c^7*d^9*e*x - 48*A*B*a^7*c^3*d*e^9*x + 48
*B*C*a^4*c^6*d^9*e*x + 48*B*C*a^8*c^2*d*e^9*x + 64*A*B*a^4*c^6*d^7*e^3*x +
224*A*B*a^5*c^5*d^5*e^5*x + 64*A*B*a^6*c^4*d^3*e^7*x - 154*A*C*a^4*c^6*d^8*
e^2*x - 212*A*C*a^5*c^5*d^6*e^4*x + 12*A*C*a^6*c^4*d^4*e^6*x + 54*A*C*a^7*c
^3*d^2*e^8*x - 64*B*C*a^5*c^5*d^7*e^3*x - 224*B*C*a^6*c^4*d^5*e^5*x - 64*B*
C*a^7*c^3*d^3*e^7*x)*(e^2*((3*B*a^2*c*d)/2 - (3*C*a^2*d*(-a*c)^(1/2))/2 + (
3*A*a*c*d*(-a*c)^(1/2))/2) + e^3*((C*a^3)/2 - (A*a^2*c)/2 + (B*a^2*(-a*c)^(
1/2))/2) - e*((3*C*a^2*c*d^2)/2 - (3*A*a*c^2*d^2)/2 + (3*B*a*c*d^2*(-a*c)^(
1/2))/2) - (B*a*c^2*d^3)/2 - (A*c^2*d^3*(-a*c)^(1/2))/2 + (C*a*c*d^3*(-a*c)
^(1/2))/2))/(a^4*e^6 + a*c^3*d^6 + 3*a^3*c*d^2*e^4 + 3*a^2*c^2*d^4*e^2) - (
log(B^2*a^7*e^10*(-a*c)^(3/2) - A^2*c^5*d^10*(-a*c)^(5/2) - 9*A^2*a^5*e^10*
(-a*c)^(5/2) + 9*B^2*c^3*d^10*(-a*c)^(7/2) - 9*C^2*a^9*e^10*(-a*c)^(1/2) -
C^2*c*d^10*(-a*c)^(9/2) + 9*C^2*a^9*c*e^10*x + 6*A^2*a*d^4*e^6*(-a*c)^(9/2)
+ 6*B^2*a*d^6*e^4*(-a*c)^(9/2) - 106*A^2*c*d^6*e^4*(-a*c)^(9/2) - 77*C^2*a
*d^8*e^2*(-a*c)^(9/2) + 27*B^2*c*d^8*e^2*(-a*c)^(9/2) + A^2*a^2*c^8*d^10*x
+ 9*A^2*a^7*c^3*e^10*x + 9*B^2*a^3*c^7*d^10*x + B^2*a^8*c^2*e^10*x + C^2*a^
4*c^6*d^10*x - 27*A^2*a^3*d^2*e^8*(-a*c)^(7/2) + 106*B^2*a^3*d^4*e^6*(-a*c)
^(7/2) - 77*B^2*a^5*d^2*e^8*(-a*c)^(5/2) + 77*A^2*c^3*d^8*e^2*(-a*c)^(7/2)
+ 106*C^2*a^3*d^6*e^4*(-a*c)^(7/2) + 6*C^2*a^5*d^4*e^6*(-a*c)^(5/2) - 27*C^
2*a^7*d^2*e^8*(-a*c)^(3/2) - 18*A*C*a^7*e^10*(-a*c)^(3/2) - 2*A*C*c^3*d^10*
(-a*c)^(7/2) - 224*A*B*a*d^5*e^5*(-a*c)^(9/2) + 48*A*B*a^5*d*e^9*(-a*c)^(5/
2) + 212*A*C*a*d^6*e^4*(-a*c)^(9/2) - 64*A*B*c*d^7*e^3*(-a*c)^(9/2) - 48*A*
B*c^3*d^9*e*(-a*c)^(7/2) + 64*B*C*a*d^7*e^3*(-a*c)^(9/2) + 48*B*C*a^7*d*e^9
*(-a*c)^(3/2) + 154*A*C*c*d^8*e^2*(-a*c)^(9/2) + 77*A^2*a^3*c^7*d^8*e^2*x +
106*A^2*a^4*c^6*d^6*e^4*x - 6*A^2*a^5*c^5*d^4*e^6*x - 27*A^2*a^6*c^4*d^2*e
^8*x - 27*B^2*a^4*c^6*d^8*e^2*x - 6*B^2*a^5*c^5*d^6*e^4*x + 106*B^2*a^6*c^4
*d^4*e^6*x + 77*B^2*a^7*c^3*d^2*e^8*x + 77*C^2*a^5*c^5*d^8*e^2*x + 106*C^2*
a^6*c^4*d^6*e^4*x - 6*C^2*a^7*c^3*d^4*e^6*x - 27*C^2*a^8*c^2*d^2*e^8*x - 2*
A*C*a^3*c^7*d^10*x - 18*A*C*a^8*c^2*e^10*x + 64*A*B*a^3*d^3*e^7*(-a*c)^(7/2
) + 12*A*C*a^3*d^4*e^6*(-a*c)^(7/2) - 54*A*C*a^5*d^2*e^8*(-a*c)^(5/2) - 224
*B*C*a^3*d^5*e^5*(-a*c)^(7/2) + 64*B*C*a^5*d^3*e^7*(-a*c)^(5/2) - 48*B*C*c*
d^9*e*(-a*c)^(9/2) - 48*A*B*a^3*c^7*d^9*e*x - 48*A*B*a^7*c^3*d*e^9*x + 48*B
*C*a^4*c^6*d^9*e*x + 48*B*C*a^8*c^2*d*e^9*x + 64*A*B*a^4*c^6*d^7*e^3*x + 22
4*A*B*a^5*c^5*d^5*e^5*x + 64*A*B*a^6*c^4*d^3*e^7*x - 154*A*C*a^4*c^6*d^8*e^
2*x - 212*A*C*a^5*c^5*d^6*e^4*x + 12*A*C*a^6*c^4*d^4*e^6*x + 54*A*C*a^7*c^3
*d^2*e^8*x - 64*B*C*a^5*c^5*d^7*e^3*x - 224*B*C*a^6*c^4*d^5*e^5*x - 64*B*C*
a^7*c^3*d^3*e^7*x)*(e^2*((3*B*a^2*c*d)/2 + (3*C*a^2*d*(-a*c)^(1/2))/2 - (3*
A*a*c*d*(-a*c)^(1/2))/2) - e^3*((A*a^2*c)/2 - (C*a^3)/2 + (B*a^2*(-a*c)^(1/

```
2))/2) + e*((3*A*a*c^2*d^2)/2 - (3*C*a^2*c*d^2)/2 + (3*B*a*c*d^2*(-a*c)^(1/
2))/2) - (B*a*c^2*d^3)/2 + (A*c^2*d^3*(-a*c)^(1/2))/2 - (C*a*c*d^3*(-a*c)^(
1/2))/2))/(a^4*e^6 + a*c^3*d^6 + 3*a^3*c*d^2*e^4 + 3*a^2*c^2*d^4*e^2) - ((A
*a*e^4 + C*c*d^4 + B*a*d*e^3 - 3*B*c*d^3*e + 5*A*c*d^2*e^2 - 3*C*a*d^2*e^2)
/(2*e*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(B*a*e^3 + 2*A*c*d*e^2 - 2*
C*a*d*e^2 - B*c*d^2*e))/(a^2*e^4 + c^2*d^4 + 2*...
```

3.50 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx$

Optimal. Leaf size=216

$$-\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2}-\frac{(Ac-2aC)e^3x^2}{2ac^2}-\frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\,(a+cx^2)}+\frac{(Acd(cd^2+3ae^2)-a(3ae^2(}{}$$

```
[Out] -3/2*e^2*(A*c*d-a*(B*e+3*C*d))*x/a/c^2-1/2*(A*c-2*C*a)*e^3*x^2/a/c^2-1/2*(a
*B-(A*c-C*a)*x)*(e*x+d)^3/a/c/(c*x^2+a)+1/2*(A*c*d*(3*a*e^2+c*d^2)-a*(3*a*e
^2*(B*e+3*C*d)-c*d^2*(3*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/a^(3/2)/c^(5/2
)-1/2*e*(2*a*C*e^2-c*(3*C*d^2+e*(A*e+3*B*d)))*ln(c*x^2+a)/c^3
```

Rubi [A]
time = 0.29, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1659, 815, 649, 211, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(Acd(3ae^2+cd^2)-a(3ae^2(Be+3Cd)-cd^2(3Be+Cd)))}{2a^{3/2}c^{5/2}}-\frac{e\log(a+cx^2)(2aCe^2-c(e(Ae+3Bd)+3Cd^2))}{2c^3}-\frac{3e^2x(Acd-a(Be+3Cd))}{2ac^2}-\frac{(d+ex)^3(aB-x(Ac-aC))}{2ac\,(a+cx^2)}-\frac{e^3x^2(Ac-2aC)}{2ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] (-3*e^2*(A*c*d - a*(3*C*d + B*e))*x)/(2*a*c^2) - ((A*c - 2*a*C)*e^3*x^2)/(2
*a*c^2) - ((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(2*a*c*(a + c*x^2)) + ((A*c*d
*(c*d^2 + 3*a*e^2) - a*(3*a*e^2*(3*C*d + B*e) - c*d^2*(C*d + 3*B*e)))*ArcTa
n[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(5/2)) - (e*(2*a*C*e^2 - c*(3*C*d^2 +
e*(3*B*d + A*e)))*Log[a + c*x^2])/(2*c^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^2} dx = -\frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac(a+cx^2)} - \frac{\int \frac{(d+ex)^2(-Acd-aCd-3aBe+2(Ac-2aC)ex)}{a+cx^2}}{2ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac(a+cx^2)} - \frac{\int \left(\frac{3e^2(Acd-3aCd-aBe)}{c} + \frac{2(Ac-2aC)e^3x}{c} - \right.}{}$$

$$= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)}{2ac(a+cx^2}$$

$$= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)}{2ac(a+cx^2}$$

$$= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)}{2ac(a+cx^2}$$

Mathematica [A]
time = 0.15, size = 233, normalized size = 1.08

$$\frac{2ce^2(3Cd+Be)x + cCe^3x^2 + \frac{-a^3Ce^3+Ac^3d^3x-ac^2d(Cd^2x+3Ae(d+ex)+Bd(d+3ex))+a^2ce(3Cd(d+ex)+e(3Bd+Ae+Bex))}{a(a+cx^2)} + \frac{\sqrt{c}\,(Acd(cd^2+3ae^2)+a(-3ae^2(3Cd+Be)+cd^2(Cd+3Be)))\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{a^{3/2}} + e(3cCd^2-2aCe^2+ce(3Bd+Ae))\log(a+cx^2)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] (2*c*e^2*(3*C*d + B*e)*x + c*C*e^3*x^2 + (-(a^3*C*e^3) + A*c^3*d^3*x - a*c^
2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d*(d + 3*e*x)) + a^2*c*e*(3*C*d*(d + e*x
) + e*(3*B*d + A*e + B*e*x)))/(a*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c*d^2 + 3*
a*e^2) + a*(-3*a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*
x)/Sqrt[a]])/a^(3/2) + e*(3*c*C*d^2 - 2*a*C*e^2 + c*e*(3*B*d + A*e))*Log[a
+ c*x^2])/(2*c^3)
```

Maple [A]
time = 0.14, size = 284, normalized size = 1.31

method	result
default	$\frac{e^2\left(\frac{1}{2}C\,x^2e+Bex+3Cdx\right)}{c^2}+\frac{-\frac{\left(3Aacd\,e^2-A\,c^2d^3-B\,a^2e^3+3Bac\,d^2e-3C\,a^2d\,e^2+Cac\,d^3\right)x}{2a}+\frac{Aac\,e^3-3A\,c^2d^2e+3Bacd\,e^2-B\,c^2d^3-C\,a^2e^3+3}{2c}}{c\,x^2+a}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e^2/c^2*(1/2*C*x^2*e+B*e*x+3*C*d*x)+1/c^2*((-1/2*(3*A*a*c*d*e^2-A*c^2*d^3-B
*a^2*e^3+3*B*a*c*d^2*e-3*C*a^2*d*e^2+C*a*c*d^3)/a*x+1/2*(A*a*c*e^3-3*A*c^2*
d^2*e+3*B*a*c*d*e^2-B*c^2*d^3-C*a^2*e^3+3*C*a*c*d^2*e)/c)/(c*x^2+a)+1/2/a*(
1/2*(2*A*a*c*e^3+6*B*a*c*d*e^2-4*C*a^2*e^3+6*C*a*c*d^2*e)/c*ln(c*x^2+a)+(3*
A*a*c*d*e^2+A*c^2*d^3-3*B*a^2*e^3+3*B*a*c*d^2*e-9*C*a^2*d*e^2+C*a*c*d^3)/(a
*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Maxima [A]
time = 0.51, size = 284, normalized size = 1.31

$$-\frac{Bac^2d^3-3\,Ba^2cde^2+Ca^3e^3-Aa^2ce^3-3\,(Ca^2ce-Aac^2e)d^2+(3\,Bac^2d^2e-Ba^2ce^3+(Cac^2-Ac^3)d^3-3\,(Ca^2ce^2-Aac^2e^2)d)x}{2\,(ac^4x^2+a^2c^3)}+\frac{Cx^2e^3+2\,(3\,Cde^2+Be^3)x}{2\,c^2}+\frac{(3\,Ccd^2e+3\,Bcde^2-2\,Cae^3+Ace^3)\log(cx^2+a)}{2\,c^3}+\frac{(3\,Bacd^2e+(Cac+Ac^2)d^3-3\,Ba^2e^3-3\,(3\,Ca^2e^2-Aace^2)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a*c^2*d^3 - 3*B*a^2*c*d*e^2 + C*a^3*e^3 - A*a^2*c*e^3 - 3*(C*a^2*c*
e - A*a*c^2*e)*d^2 + (3*B*a*c^2*d^2*e - B*a^2*c*e^3 + (C*a*c^2 - A*c^3)*d^3
 - 3*(C*a^2*c*e^2 - A*a*c^2*e^2)*d)*x)/(a*c^4*x^2 + a^2*c^3) + 1/2*(C*x^2*e
^3 + 2*(3*C*d*e^2 + B*e^3)*x)/c^2 + 1/2*(3*C*c*d^2*e + 3*B*c*d*e^2 - 2*C*a*
e^3 + A*c*e^3)*log(c*x^2 + a)/c^3 + 1/2*(3*B*a*c*d^2*e + (C*a*c + A*c^2)*d^
3 - 3*B*a^2*e^3 - 3*(3*C*a^2*e^2 - A*a*c*e^2)*d)*arctan(c*x/sqrt(a*c))/(sqr
t(a*c)*a*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 436 vs. 2(198) = 396.
time = 0.37, size = 895, normalized size = 4.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c^2*d^3 + 2*(C*a^2*c^2 - A*a*c^3)*d^3*x - ((C*a*c^2 + A*c^3)
*d^3*x^2 + (C*a^2*c + A*a*c^2)*d^3 - 3*(B*a^2*c*x^2 + B*a^3)*e^3 - 3*((3*C*
a^2*c - A*a*c^2)*d*x^2 + (3*C*a^3 - A*a^2*c)*d)*e^2 + 3*(B*a*c^2*d^2*x^2 +
B*a^2*c*d^2)*e)*sqrt(-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) -
2*(C*a^2*c^2*x^4 + 2*B*a^2*c^2*x^3 + C*a^3*c*x^2 + 3*B*a^3*c*x - C*a^4 + A*
a^3*c)*e^3 - 6*(2*C*a^2*c^2*d*x^3 + B*a^3*c*d + (3*C*a^3*c - A*a^2*c^2)*d*x
)*e^2 + 6*(B*a^2*c^2*d^2*x - (C*a^3*c - A*a^2*c^2)*d^2)*e + 2*((2*C*a^4 - A
*a^3*c + (2*C*a^3*c - A*a^2*c^2)*x^2)*e^3 - 3*(B*a^2*c^2*d*x^2 + B*a^3*c*d)
*e^2 - 3*(C*a^2*c^2*d^2*x^2 + C*a^3*c*d^2)*e)*log(c*x^2 + a))/(a^2*c^4*x^2
+ a^3*c^3), -1/2*(B*a^2*c^2*d^3 + (C*a^2*c^2 - A*a*c^3)*d^3*x - ((C*a*c^2 +
 A*c^3)*d^3*x^2 + (C*a^2*c + A*a*c^2)*d^3 - 3*(B*a^2*c*x^2 + B*a^3)*e^3 - 3
*((3*C*a^2*c - A*a*c^2)*d*x^2 + (3*C*a^3 - A*a^2*c)*d)*e^2 + 3*(B*a*c^2*d^2
*x^2 + B*a^2*c*d^2)*e)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (C*a^2*c^2*x^4 + 2
*B*a^2*c^2*x^3 + C*a^3*c*x^2 + 3*B*a^3*c*x - C*a^4 + A*a^3*c)*e^3 - 3*(2*C*
a^2*c^2*d*x^3 + B*a^3*c*d + (3*C*a^3*c - A*a^2*c^2)*d*x)*e^2 + 3*(B*a^2*c^2
*d^2*x - (C*a^3*c - A*a^2*c^2)*d^2)*e + ((2*C*a^4 - A*a^3*c + (2*C*a^3*c -
A*a^2*c^2)*x^2)*e^3 - 3*(B*a^2*c^2*d*x^2 + B*a^3*c*d)*e^2 - 3*(C*a^2*c^2*d^
2*x^2 + C*a^3*c*d^2)*e)*log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(197) = 394$.
time = 28.66, size = 952, normalized size = 4.41

$$\frac{Ce^3x^2}{2c^2}+x\left(\frac{Be^3}{c^2}+\frac{3Cde^2}{c^2}\right)+\left(-\frac{e(-Ace^2-3Bcde+2Cae^2-3Ccd^2)}{2c^3}-\frac{\sqrt{-a^3c^7}\,(-3Aacde^2-Ac^2d^3+3Ba^2e^3-3Bacd^2e+9Ca^2de^2-Cacd^3)}{4a^3c^6}\right)\log\left(x+\frac{2Aa^2ce^3+6Ba^2cde^2-4Ca^3e^3+6Ca^2cd^2e-4a^2c^3\left(-\frac{e(-Ace^2-3Bcde+2Cae^2-3Ccd^2)}{2c^3}-\frac{\sqrt{-a^3c^7}\,(-3Aacde^2-Ac^2d^3+3Ba^2e^3-3Bacd^2e+9Ca^2de^2-Cacd^3)}{4a^3c^6}\right)}{-3Aac^2de^2-Ac^3d^3+3Ba^2ce^3-3Bac^2d^2e+9Ca^2cde^2-Cac^2d^3}\right)+\left(-\frac{e(-Ace^2-3Bcde+2Cae^2-3Ccd^2)}{2c^3}+\frac{\sqrt{-a^3c^7}\,(-3Aacde^2-Ac^2d^3+3Ba^2e^3-3Bacd^2e+9Ca^2de^2-Cacd^3)}{4a^3c^6}\right)\log\left(x+\frac{2Aa^2ce^3+6Ba^2cde^2-4Ca^3e^3+6Ca^2cd^2e-4a^2c^3\left(-\frac{e(-Ace^2-3Bcde+2Cae^2-3Ccd^2)}{2c^3}+\frac{\sqrt{-a^3c^7}\,(-3Aacde^2-Ac^2d^3+3Ba^2e^3-3Bacd^2e+9Ca^2de^2-Cacd^3)}{4a^3c^6}\right)}{-3Aac^2de^2-Ac^3d^3+3Ba^2ce^3-3Bac^2d^2e+9Ca^2cde^2-Cac^2d^3}\right)+\frac{Aa^2ce^3-3Aac^2d^2e+3Ba^2cde^2-Bac^2d^3-Ca^3e^3+3Ca^2cd^2e+x(-3Aac^2de^2+Ac^3d^3+Ba^2ce^3-3Bac^2d^2e+3Ca^2cde^2-Cac^2d^3)}{2a^2c^3+2ac^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] C*e**3*x**2/(2*c**2) + x*(B*e**3/c**2 + 3*C*d*e**2/c**2) + (-e*(-A*c*e**2 -
 3*B*c*d*e + 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) - sqrt(-a**3*c**7)*(-3*A*a*c
*d*e**2 - A*c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 -
C*a*c*d**3)/(4*a**3*c**6))*log(x + (2*A*a**2*c*e**3 + 6*B*a**2*c*d*e**2 - 4
*C*a**3*e**3 + 6*C*a**2*c*d**2*e - 4*a**2*c**3*(-e*(-A*c*e**2 - 3*B*c*d*e +
 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) - sqrt(-a**3*c**7)*(-3*A*a*c*d*e**2 - A*
c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 - C*a*c*d**3)/
(4*a**3*c**6)))/(-3*A*a*c**2*d*e**2 - A*c**3*d**3 + 3*B*a**2*c*e**3 - 3*B*a
*c**2*d**2*e + 9*C*a**2*c*d*e**2 - C*a*c**2*d**3)) + (-e*(-A*c*e**2 - 3*B*c
*d*e + 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) + sqrt(-a**3*c**7)*(-3*A*a*c*d*e**
2 - A*c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 - C*a*c*
d**3)/(4*a**3*c**6))*log(x + (2*A*a**2*c*e**3 + 6*B*a**2*c*d*e**2 - 4*C*a**
3*e**3 + 6*C*a**2*c*d**2*e - 4*a**2*c**3*(-e*(-A*c*e**2 - 3*B*c*d*e + 2*C*a
*e**2 - 3*C*c*d**2)/(2*c**3) + sqrt(-a**3*c**7)*(-3*A*a*c*d*e**2 - A*c**2*d
```

```
**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 - C*a*c*d**3)/(4*a**
3*c**6)))/(-3*A*a*c**2*d*e**2 - A*c**3*d**3 + 3*B*a**2*c*e**3 - 3*B*a*c**2*
d**2*e + 9*C*a**2*c*d*e**2 - C*a*c**2*d**3)) + (A*a**2*c*e**3 - 3*A*a*c**2*
d**2*e + 3*B*a**2*c*d*e**2 - B*a*c**2*d**3 - C*a**3*e**3 + 3*C*a**2*c*d**2*
e + x*(-3*A*a*c**2*d*e**2 + A*c**3*d**3 + B*a**2*c*e**3 - 3*B*a*c**2*d**2*e
 + 3*C*a**2*c*d*e**2 - C*a*c**2*d**3))/(2*a**2*c**3 + 2*a*c**4*x**2)
```

Giac [A]
time = 6.28, size = 289, normalized size = 1.34

$$\frac{(3\,Ccd^2e + 3\,Bcde^2 - 2\,Cae^3 + Ace^3)\log\left(cx^2 + a\right)}{2\,c^3} + \frac{(Cacd^3 + Ac^2d^3 + 3\,Bacd^2e - 9\,Ca^2de^2 + 3\,Aacde^2 - 3\,Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,ac^2} + \frac{Cc^2x^2e^3 + 6\,Cc^2dxe^2 + 2\,Bc^2xe^3}{2\,c^4} - \frac{Bac^2d^3 - 3\,Ca^2cd^2e + 3\,Aac^2d^2e - 3\,Ba^2cde^2 + Ca^3e^3 - Aa^2ce^3 + (Cac^2d^3 - Ac^3d^3 + 3\,Bac^2d^2e - 3\,Ca^2cde^2 + 3\,Aac^2de^2 - Ba^2ce^3)x}{2\,(cx^2 + a)ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*C*c*d^2*e + 3*B*c*d*e^2 - 2*C*a*e^3 + A*c*e^3)*log(c*x^2 + a)/c^3 +
1/2*(C*a*c*d^3 + A*c^2*d^3 + 3*B*a*c*d^2*e - 9*C*a^2*d*e^2 + 3*A*a*c*d*e^2
- 3*B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2) + 1/2*(C*c^2*x^2*e^3
 + 6*C*c^2*d*x*e^2 + 2*B*c^2*x*e^3)/c^4 - 1/2*(B*a*c^2*d^3 - 3*C*a^2*c*d^2*
e + 3*A*a*c^2*d^2*e - 3*B*a^2*c*d*e^2 + C*a^3*e^3 - A*a^2*c*e^3 + (C*a*c^2*
d^3 - A*c^3*d^3 + 3*B*a*c^2*d^2*e - 3*C*a^2*c*d*e^2 + 3*A*a*c^2*d*e^2 - B*a
^2*c*e^3)*x)/((c*x^2 + a)*a*c^3)
```

Mupad [B]
time = 4.01, size = 303, normalized size = 1.40

$$\frac{x\,\left(B\,e^3+3\,C\,d\,e^2\right)}{c^2}-\frac{\frac{C\,a^2\,e^3-3\,C\,a\,c\,d^2\,e-3\,B\,a\,c\,d\,e^2-A\,a\,c\,e^3+B\,c^2\,d^3+3\,A\,c^2\,d^2\,e}{2\,c}-\frac{x\,\left(3\,C\,a^2\,d\,e^2+B\,a^2\,e^3-C\,a\,c\,d^3-3\,B\,a\,c\,d^2\,e-3\,A\,a\,c\,d\,e^2+A\,c^2\,d^3\right)}{2\,a}}{c^3\,x^2+a\,c^2}+\frac{C\,e^3\,x^2}{2\,c^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(-9\,C\,a^2\,d\,e^2-3\,B\,a^2\,e^3+C\,a\,c\,d^3+3\,B\,a\,c\,d^2\,e+3\,A\,a\,c\,d\,e^2+A\,c^2\,d^3\right)}{2\,a^{3/2}\,c^{5/2}}+\frac{\ln\left(c\,x^2+a\right)\,\left(-32\,C\,a^4\,c^3\,e^3+48\,C\,a^3\,c^4\,d^2\,e+48\,B\,a^3\,c^4\,d\,e^2+16\,A\,a^3\,c^4\,e^3\right)}{32\,a^3\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2,x)
```

```
[Out] (x*(B*e^3 + 3*C*d*e^2))/c^2 - ((B*c^2*d^3 + C*a^2*e^3 - A*a*c*e^3 + 3*A*c^2
*d^2*e - 3*B*a*c*d*e^2 - 3*C*a*c*d^2*e)/(2*c) - (x*(A*c^2*d^3 + B*a^2*e^3 -
 C*a*c*d^3 + 3*C*a^2*d*e^2 - 3*A*a*c*d*e^2 - 3*B*a*c*d^2*e))/(2*a))/(a*c^2
+ c^3*x^2) + (C*e^3*x^2)/(2*c^2) + (atan((c^(1/2)*x)/a^(1/2))*(A*c^2*d^3 -
3*B*a^2*e^3 + C*a*c*d^3 - 9*C*a^2*d*e^2 + 3*A*a*c*d*e^2 + 3*B*a*c*d^2*e))/(
2*a^(3/2)*c^(5/2)) + (log(a + c*x^2)*(16*A*a^3*c^4*e^3 - 32*C*a^4*c^3*e^3 +
 48*B*a^3*c^4*d*e^2 + 48*C*a^3*c^4*d^2*e))/(32*a^3*c^6)
```

3.51 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx$

Optimal. Leaf size=146

$$-\frac{(Ac-3aC)e^2x}{2ac^2}-\frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\,(a+cx^2)}+\frac{(a(Ac-3aC)e^2+cd(Acd+aCd+2aBe))\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{a}}\right.}{2a^{3/2}c^{5/2}}$$

```
[Out] -1/2*(A*c-3*C*a)*e^2*x/a/c^2-1/2*(a*B-(A*c-C*a)*x)*(e*x+d)^2/a/c/(c*x^2+a)+
1/2*(a*(A*c-3*C*a)*e^2+c*d*(A*c*d+2*B*a*e+C*a*d))*arctan(x*c^(1/2)/a^(1/2))
/a^(3/2)/c^(5/2)+1/2*e*(B*e+2*C*d)*ln(c*x^2+a)/c^2
```

Rubi [A]
time = 0.15, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1659, 788, 649, 211, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(cd(2aBe+aCd+Acd)+ae^2(Ac-3aC))}{2a^{3/2}c^{5/2}}-\frac{(d+ex)^2(aB-x(Ac-aC))}{2ac\,(a+cx^2)}-\frac{e^2x(Ac-3aC)}{2ac^2}+\frac{e\log\left(a+cx^2\right)(Be+2Cd)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] -1/2*((A*c - 3*a*C)*e^2*x)/(a*c^2) - ((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(2
*a*c*(a + c*x^2)) + ((a*(A*c - 3*a*C)*e^2 + c*d*(A*c*d + a*C*d + 2*a*B*e))*
ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(5/2)) + (e*(2*C*d + B*e)*Log[a +
 c*x^2])/(2*c^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)(-Acd-aCd-2aBe+(Ac-3aC)ex)}{a+cx^2}\,dx}{2ac}$$

$$= -\frac{(Ac-3aC)e^2x}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)} - \int \frac{-a(Ac-3aC)e^2+cd(-\ldots}{\ldots}$$

$$= -\frac{(Ac-3aC)e^2x}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)} + \frac{(e(2Cd+Be))\int \frac{\ldots}{a+\ldots}}{c}$$

$$= -\frac{(Ac-3aC)e^2x}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)} + \frac{(a(Ac-3aC)e^2+\ldots}{\ldots}$$

Mathematica [A]
time = 0.10, size = 175, normalized size = 1.20

$$\frac{2\sqrt{c}\,Ce^2x+\frac{\sqrt{c}\left(Ac^2d^2x+a^2e(2Cd+Be+Cex)-ac\left(Cd^2x+Ae(2d+ex)+Bd(d+2ex)\right)\right)}{a\left(a+cx^2\right)}+\frac{\left(Ac\left(cd^2+ae^2\right)+a\left(-3aCe^2+cd(Cd+2Be)\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{a^{3/2}}+\sqrt{c}\,e(2Cd+Be)\log\left(a+cx^2\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] (2*Sqrt[c]*C*e^2*x + (Sqrt[c]*(A*c^2*d^2*x + a^2*e*(2*C*d + B*e + C*e*x) -
a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x))))/(a*(a + c*x^2)) + ((A*c
*(c*d^2 + a*e^2) + a*(-3*a*C*e^2 + c*d*(C*d + 2*B*e)))*ArcTan[(Sqrt[c]*x)/S
qrt[a]])/a^(3/2) + Sqrt[c]*e*(2*C*d + B*e)*Log[a + c*x^2])/(2*c^(5/2))
```

Maple [A]
time = 0.14, size = 187, normalized size = 1.28

method	result
default	$\frac{C e^{2} x}{c^{2}}+\frac{-\frac{\left(A a c \,e^{2}-A \,c^{2} d^{2}+2 a c d e B-a^{2} C \,e^{2}+C a c \,d^{2}\right) x}{2 a}-c d e A+\frac{B a \,e^{2}}{2}-\frac{B c \,d^{2}}{2}+a d e C}{c \,x^{2}+a}+\frac{\frac{\left(2 B a c \,e^{2}+4 a c d e C\right) \ln \left(c \,x^{2}+a\right)}{2 c}+\frac{\left(A a c \,e^{2}+A \,c^{2} d^{2}+\cdots\right.}{\cdots}}{2 a}$ ... c^{2}
risch	$\frac{C e^{2} x}{c^{2}}+\frac{-\frac{\left(A a c \,e^{2}-A \,c^{2} d^{2}+2 a c d e B-a^{2} C \,e^{2}+C a c \,d^{2}\right) x}{2 a}-c d e A+\frac{B a \,e^{2}}{2}-\frac{B c \,d^{2}}{2}+a d e C}{c^{2} \left(c \,x^{2}+a \right)}+\frac{\ln \left(A \,a^{2} c \,e^{2}+A a \,c^{2} d^{2}+2 d e \,a^{2} c B-3 C \,a^{3} e^{2}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] C*e^2/c^2*x+1/c^2*((-1/2*(A*a*c*e^2-A*c^2*d^2+2*B*a*c*d*e-C*a^2*e^2+C*a*c*d
^2)/a*x-c*d*e*A+1/2*B*a*e^2-1/2*B*c*d^2+a*d*e*C)/(c*x^2+a)+1/2/a*(1/2*(2*B*
a*c*e^2+4*C*a*c*d*e)/c*ln(c*x^2+a)+(A*a*c*e^2+A*c^2*d^2+2*B*a*c*d*e-3*C*a^2
*e^2+C*a*c*d^2)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Maxima [A]
time = 0.50, size = 186, normalized size = 1.27

$$\frac{Cxe^2}{c^2}-\frac{Bacd^2-Ba^2e^2-2\left(Ca^2e-Aace\right)d+\left(2\,Bacde-Ca^2e^2+Aace^2+\left(Cac-Ac^2\right)d^2\right)x}{2\left(ac^3x^2+a^2c^2\right)}+\frac{\left(2\,Cde+Be^2\right)\log\left(cx^2+a\right)}{2\,c^2}+\frac{\left(2\,Bacde-3\,Ca^2e^2+Aace^2+\left(Cac+Ac^2\right)d^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] C*x*e^2/c^2 - 1/2*(B*a*c*d^2 - B*a^2*e^2 - 2*(C*a^2*e - A*a*c*e)*d + (2*B*a
*c*d*e - C*a^2*e^2 + A*a*c*e^2 + (C*a*c - A*c^2)*d^2)*x)/(a*c^3*x^2 + a^2*c
^2) + 1/2*(2*C*d*e + B*e^2)*log(c*x^2 + a)/c^2 + 1/2*(2*B*a*c*d*e - 3*C*a^2
*e^2 + A*a*c*e^2 + (C*a*c + A*c^2)*d^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*
c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(134) = 268$.
time = 0.41, size = 623, normalized size = 4.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c^2*d^2 + 2*(C*a^2*c^2 - A*a*c^3)*d^2*x + ((C*a*c^2 + A*c^3)
*d^2*x^2 + (C*a^2*c + A*a*c^2)*d^2 - (3*C*a^3 - A*a^2*c + (3*C*a^2*c - A*a*
```

```
c^2)*x^2)*e^2 + 2*(B*a*c^2*d*x^2 + B*a^2*c*d)*e)*sqrt(-a*c)*log((c*x^2 - 2*
sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(2*C*a^2*c^2*x^3 + B*a^3*c + (3*C*a^3*c
- A*a^2*c^2)*x)*e^2 + 4*(B*a^2*c^2*d*x - (C*a^3*c - A*a^2*c^2)*d)*e - 2*((B
*a^2*c^2*x^2 + B*a^3*c)*e^2 + 2*(C*a^2*c^2*d*x^2 + C*a^3*c*d)*e)*log(c*x^2
+ a))/(a^2*c^4*x^2 + a^3*c^3), -1/2*(B*a^2*c^2*d^2 + (C*a^2*c^2 - A*a*c^3)*
d^2*x - ((C*a*c^2 + A*c^3)*d^2*x^2 + (C*a^2*c + A*a*c^2)*d^2 - (3*C*a^3 - A
*a^2*c + (3*C*a^2*c - A*a*c^2)*x^2)*e^2 + 2*(B*a*c^2*d*x^2 + B*a^2*c*d)*e)*
sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (2*C*a^2*c^2*x^3 + B*a^3*c + (3*C*a^3*c -
 A*a^2*c^2)*x)*e^2 + 2*(B*a^2*c^2*d*x - (C*a^3*c - A*a^2*c^2)*d)*e - ((B*a^
2*c^2*x^2 + B*a^3*c)*e^2 + 2*(C*a^2*c^2*d*x^2 + C*a^3*c*d)*e)*log(c*x^2 + a
))/(a^2*c^4*x^2 + a^3*c^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(138) = 276$.
time = 7.85, size = 593, normalized size = 4.06

$$\frac{Ce^2x}{c^2} + \left(\frac{e(Be+2Cd)}{2c^2} - \frac{\sqrt{-a^3c^5}\,(-Aace^2 - Ac^2d^2 - 2Bacde + 3Ca^2e^2 - Cacd^2)}{4a^3c^5}\right)\log\left(x + \frac{2Ba^2e^2 + 4Ca^2de - 4a^2c^2\left(\frac{e(Be+2Cd)}{2c^2} - \frac{\sqrt{-a^3c^5}\,(-Aace^2 - Ac^2d^2 - 2Bacde + 3Ca^2e^2 - Cacd^2)}{4a^3c^5}\right)}{-Aace^2 - Ac^2d^2 - 2Bacde + 3Ca^2e^2 - Cacd^2}\right) + \left(\frac{e(Be+2Cd)}{2c^2} + \frac{\sqrt{-a^3c^5}\,(-Aace^2 - Ac^2d^2 - 2Bacde + 3Ca^2e^2 - Cacd^2)}{4a^3c^5}\right)\log\left(x + \frac{2Ba^2e^2 + 4Ca^2de - 4a^2c^2\left(\frac{e(Be+2Cd)}{2c^2} + \frac{\sqrt{-a^3c^5}\,(-Aace^2 - Ac^2d^2 - 2Bacde + 3Ca^2e^2 - Cacd^2)}{4a^3c^5}\right)}{-Aace^2 - Ac^2d^2 - 2Bacde + 3Ca^2e^2 - Cacd^2}\right) + \frac{-2Aacde + Ba^2e^2 - Bacd^2 + 2Ca^2de + x(-Aace^2 + Ac^2d^2 - 2Bacde + Ca^2e^2 - Cacd^2)}{2a^2c^2 + 2ac^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] C*e**2*x/c**2 + (e*(B*e + 2*C*d)/(2*c**2) - sqrt(-a**3*c**5)*(-A*a*c*e**2 -
 A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)/(4*a**3*c**5))*log
(x + (2*B*a**2*e**2 + 4*C*a**2*d*e - 4*a**2*c**2*(e*(B*e + 2*C*d)/(2*c**2)
- sqrt(-a**3*c**5)*(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2
 - C*a*c*d**2)/(4*a**3*c**5)))/(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3
*C*a**2*e**2 - C*a*c*d**2)) + (e*(B*e + 2*C*d)/(2*c**2) + sqrt(-a**3*c**5)*
(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)/(4*a
**3*c**5))*log(x + (2*B*a**2*e**2 + 4*C*a**2*d*e - 4*a**2*c**2*(e*(B*e + 2*
C*d)/(2*c**2) + sqrt(-a**3*c**5)*(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e +
 3*C*a**2*e**2 - C*a*c*d**2)/(4*a**3*c**5)))/(-A*a*c*e**2 - A*c**2*d**2 - 2
*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)) + (-2*A*a*c*d*e + B*a**2*e**2 - B
*a*c*d**2 + 2*C*a**2*d*e + x*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*a
**2*e**2 - C*a*c*d**2))/(2*a**2*c**2 + 2*a*c**3*x**2)
```

Giac [A]
time = 3.63, size = 184, normalized size = 1.26

$$\frac{Cxe^2}{c^2} + \frac{(2\,Cde + Be^2)\log\left(cx^2 + a\right)}{2\,c^2} + \frac{(Cacd^2 + Ac^2d^2 + 2\,Bacde - 3\,Ca^2e^2 + Aace^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,ac^2} - \frac{Bacd^2 - 2\,Ca^2de + 2\,Aacde - Ba^2e^2 + (Cacd^2 - Ac^2d^2 + 2\,Bacde - Ca^2e^2 + Aace^2)x}{2\,(cx^2 + a)ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] C*x*e^2/c^2 + 1/2*(2*C*d*e + B*e^2)*log(c*x^2 + a)/c^2 + 1/2*(C*a*c*d^2 + A
*c^2*d^2 + 2*B*a*c*d*e - 3*C*a^2*e^2 + A*a*c*e^2)*arctan(c*x/sqrt(a*c))/(sq
```

```
rt(a*c)*a*c^2) - 1/2*(B*a*c*d^2 - 2*C*a^2*d*e + 2*A*a*c*d*e - B*a^2*e^2 + (
C*a*c*d^2 - A*c^2*d^2 + 2*B*a*c*d*e - C*a^2*e^2 + A*a*c*e^2)*x)/((c*x^2 + a
)*a*c^2)
```

Mupad [B]
time = 0.23, size = 195, normalized size = 1.34

$$\frac{C\,e^2\,x}{c^2}-\frac{\frac{x\,\left(-C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2-A\,c^2\,d^2\right)}{2\,a}-\frac{B\,a\,e^2}{2}+\frac{B\,c\,d^2}{2}+A\,c\,d\,e-C\,a\,d\,e}{c^3\,x^2+a\,c^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(-3\,C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2+A\,c^2\,d^2\right)}{2\,a^{3/2}\,c^{5/2}}+\frac{\ln\left(c\,x^2+a\right)\,\left(16\,B\,a^3\,c^3\,e^2+32\,C\,d\,a^3\,c^3\,e\right)}{32\,a^3\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^2,x)
```

```
[Out] (C*e^2*x)/c^2 - ((x*(A*a*c*e^2 - C*a^2*e^2 - A*c^2*d^2 + C*a*c*d^2 + 2*B*a*
c*d*e))/(2*a) - (B*a*e^2)/2 + (B*c*d^2)/2 + A*c*d*e - C*a*d*e)/(a*c^2 + c^3
*x^2) + (atan((c^(1/2)*x)/a^(1/2))*(A*c^2*d^2 - 3*C*a^2*e^2 + A*a*c*e^2 + C
*a*c*d^2 + 2*B*a*c*d*e))/(2*a^(3/2)*c^(5/2)) + (log(a + c*x^2)*(16*B*a^3*c^
3*e^2 + 32*C*a^3*c^3*d*e))/(32*a^3*c^5)
```

3.52 $\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{(aB-(Ac-aC)x)(d+ex)}{2ac\,(a+cx^2)}+\frac{(Acd+aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}+\frac{Ce\log\left(a+cx^2\right)}{2c^2}$$

[Out] -1/2*(a*B-(A*c-C*a)*x)*(e*x+d)/a/c/(c*x^2+a)+1/2*(A*c*d+B*a*e+C*a*d)*arctan(x*c^(1/2)/a^(1/2))/a^(3/2)/c^(3/2)+1/2*C*e*ln(c*x^2+a)/c^2

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1659, 649, 211, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd+Acd)}{2a^{3/2}c^{3/2}}-\frac{(d+ex)(aB-x(Ac-aC))}{2ac\,(a+cx^2)}+\frac{Ce\log\left(a+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^2,x]

[Out] -1/2*((a*B - (A*c - a*C)*x)*(d + e*x))/(a*c*(a + c*x^2)) + ((A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2)) + (C*e*Log[a + c*x^2])/(2*c^2)

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx &= -\frac{(aB-(Ac-aC)x)(d+ex)}{2ac\left(a+cx^2\right)}-\frac{\int \frac{-Acd-a(Cd+Be)-2aCex}{a+cx^2}\,dx}{2ac}\\ &= -\frac{(aB-(Ac-aC)x)(d+ex)}{2ac\left(a+cx^2\right)}+\frac{(Ce)\int \frac{x}{a+cx^2}\,dx}{c}+\frac{(Acd+aCd+aBe)}{2ac}\\ &= -\frac{(aB-(Ac-aC)x)(d+ex)}{2ac\left(a+cx^2\right)}+\frac{(Acd+aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}+\end{aligned}$$

Mathematica [A]
time = 0.07, size = 102, normalized size = 1.05

$$\frac{\frac{a^2Ce+Ac^2dx-ac(Ae+Cdx+B(d+ex))}{a(a+cx^2)}+\frac{\sqrt{c}\,(Acd+aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{a^{3/2}}+Ce\log\left(a+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] ((a^2*C*e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x)))/(a*(a + c*x^2)) +
(Sqrt[c]*(A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(3/2) + C*e
*Log[a + c*x^2])/(2*c^2)
```

Maple [A]
time = 0.10, size = 108, normalized size = 1.11

method	result

default	$\frac{\frac{(Acd-aBe-Cad)x}{2ac}-\frac{Ace+Bcd-aCe}{2c^2}}{c\,x^2+a}+\frac{\frac{aCe\ln\left(c\,x^2+a\right)}{c}+\frac{(Acd+aBe+Cad)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{2ac}$
risch	$\frac{\frac{(Acd-aBe-Cad)x}{2ac}-\frac{Ace+Bcd-aCe}{2c^2}}{c\,x^2+a}+\frac{\ln\left(acdA+e\,a^2B+d\,a^2C-\sqrt{-ac\,(Acd+aBe+Cad)^2}\;x\right)eC}{2c^2}+\frac{\ln\left(acdA+e\,a^2B\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*(A*c*d-B*a*e-C*a*d)/a/c*x-1/2*(A*c*e+B*c*d-C*a*e)/c^2)/(c*x^2+a)+1/2/a
/c*(a*C*e/c*ln(c*x^2+a)+(A*c*d+B*a*e+C*a*d)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1
/2)))
```

Maxima [A]
time = 0.50, size = 116, normalized size = 1.20

$$\frac{Ce\log\left(cx^2+a\right)}{2\,c^2}-\frac{Bacd-Ca^2e+Aace+(Bace+(Cac-Ac^2)d)x}{2\,(ac^3x^2+a^2c^2)}+\frac{(Bae+(Ca+Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*C*e*log(c*x^2 + a)/c^2 - 1/2*(B*a*c*d - C*a^2*e + A*a*c*e + (B*a*c*e +
(C*a*c - A*c^2)*d)*x)/(a*c^3*x^2 + a^2*c^2) + 1/2*(B*a*e + (C*a + A*c)*d)*a
rctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c)
```

Fricas [A]
time = 0.39, size = 340, normalized size = 3.51

$$\left[-\frac{2\,Ba^2cd+2\,(Ca^2c-Aac^2)dx-2\,(Ca^2cx^2+Ca^3)e\log\left(cx^2+a\right)+((Cac+Ac^2)dx^2+(Ca^2+Aac)d+(Bacx^2+Ba^2)e)\sqrt{-ac}\,\log\left(\frac{cx^2-2\sqrt{-ac}\,x-a}{cx^2+a}\right)+2\,(Ba^2cx-Ca^3+Aa^2c)e}{4\,(a^2c^3x^2+a^3c^2)},\,-\frac{Ba^2cd+(Ca^2c-Aac^2)dx-(Ca^2cx^2+Ca^3)e\log\left(cx^2+a\right)-((Cac+Ac^2)dx^2+(Ca^2+Aac)d+(Bacx^2+Ba^2)e)\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,x}{a}\right)+(Ba^2cx-Ca^3+Aa^2c)e}{2\,(a^2c^3x^2+a^3c^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c*d + 2*(C*a^2*c - A*a*c^2)*d*x - 2*(C*a^2*c*x^2 + C*a^3)*e*
log(c*x^2 + a) + ((C*a*c + A*c^2)*d*x^2 + (C*a^2 + A*a*c)*d + (B*a*c*x^2 +
B*a^2)*e)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(B*a
^2*c*x - C*a^3 + A*a^2*c)*e)/(a^2*c^3*x^2 + a^3*c^2), -1/2*(B*a^2*c*d + (C*
a^2*c - A*a*c^2)*d*x - (C*a^2*c*x^2 + C*a^3)*e*log(c*x^2 + a) - ((C*a*c + A
*c^2)*d*x^2 + (C*a^2 + A*a*c)*d + (B*a*c*x^2 + B*a^2)*e)*sqrt(a*c)*arctan(s
qrt(a*c)*x/a) + (B*a^2*c*x - C*a^3 + A*a^2*c)*e)/(a^2*c^3*x^2 + a^3*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(90) = 180$.
time = 2.82, size = 318, normalized size = 3.28

$$\left(\frac{Ce}{2c^2} - \frac{\sqrt{-a^3c^5}\,(Acd + Bae + Cad)}{4a^3c^4}\right)\log\left(x + \frac{-2Ca^2e + 4a^2c^2\left(\frac{Ce}{2c^2} - \frac{\sqrt{-a^3c^5}\,(Acd+Bae+Cad)}{4a^3c^4}\right)}{Ac^2d + Bace + Cacd}\right) + \left(\frac{Ce}{2c^2} + \frac{\sqrt{-a^3c^5}\,(Acd + Bae + Cad)}{4a^3c^4}\right)\log\left(x + \frac{-2Ca^2e + 4a^2c^2\left(\frac{Ce}{2c^2} + \frac{\sqrt{-a^3c^5}\,(Acd+Bae+Cad)}{4a^3c^4}\right)}{Ac^2d + Bace + Cacd}\right) + \frac{-Aace - Bacd + Ca^2e + x(Ac^2d - Bace - Cacd)}{2a^2c^2 + 2ac^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] (C*e/(2*c**2) - sqrt(-a**3*c**5)*(A*c*d + B*a*e + C*a*d)/(4*a**3*c**4))*log
(x + (-2*C*a**2*e + 4*a**2*c**2*(C*e/(2*c**2) - sqrt(-a**3*c**5)*(A*c*d + B
*a*e + C*a*d)/(4*a**3*c**4)))/(A*c**2*d + B*a*c*e + C*a*c*d)) + (C*e/(2*c**
2) + sqrt(-a**3*c**5)*(A*c*d + B*a*e + C*a*d)/(4*a**3*c**4))*log(x + (-2*C*
a**2*e + 4*a**2*c**2*(C*e/(2*c**2) + sqrt(-a**3*c**5)*(A*c*d + B*a*e + C*a*
d)/(4*a**3*c**4)))/(A*c**2*d + B*a*c*e + C*a*c*d)) + (-A*a*c*e - B*a*c*d +
C*a**2*e + x*(A*c**2*d - B*a*c*e - C*a*c*d))/(2*a**2*c**2 + 2*a*c**3*x**2)
```

Giac [A]
time = 3.75, size = 112, normalized size = 1.15

$$\frac{Ce\log\left(cx^2 + a\right)}{2\,c^2} + \frac{(Cad + Acd + Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,ac} - \frac{(Cad - Acd + Bae)x + \frac{Bacd - Ca^2e + Aace}{c}}{2\,(cx^2 + a)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*C*e*log(c*x^2 + a)/c^2 + 1/2*(C*a*d + A*c*d + B*a*e)*arctan(c*x/sqrt(a*
c))/(sqrt(a*c)*a*c) - 1/2*((C*a*d - A*c*d + B*a*e)*x + (B*a*c*d - C*a^2*e +
 A*a*c*e)/c)/((c*x^2 + a)*a*c)
```

Mupad [B]
time = 0.14, size = 191, normalized size = 1.97

$$\frac{C\,e\,\ln\left(c\,x^2 + a\right)}{2\,c^2} - \frac{B\,d}{2\,\left(c^2\,x^2 + a\,c\right)} - \frac{B\,e\,x}{2\,\left(c^2\,x^2 + a\,c\right)} - \frac{C\,d\,x}{2\,\left(c^2\,x^2 + a\,c\right)} - \frac{A\,e}{2\,\left(c^2\,x^2 + a\,c\right)} + \frac{C\,a\,e}{2\,\left(c^3\,x^2 + a\,c^2\right)} + \frac{A\,d\,x}{2\,\left(a^2 + c\,a\,x^2\right)} + \frac{A\,d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{c}} + \frac{B\,e\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,c^{3/2}} + \frac{C\,d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^2,x)
```

```
[Out] (C*e*log(a + c*x^2))/(2*c^2) - (B*d)/(2*(a*c + c^2*x^2)) - (B*e*x)/(2*(a*c
+ c^2*x^2)) - (C*d*x)/(2*(a*c + c^2*x^2)) - (A*e)/(2*(a*c + c^2*x^2)) + (C*
a*e)/(2*(a*c^2 + c^3*x^2)) + (A*d*x)/(2*(a^2 + a*c*x^2)) + (A*d*atan((c^(1/
2)*x)/a^(1/2)))/(2*a^(3/2)*c^(1/2)) + (B*e*atan((c^(1/2)*x)/a^(1/2)))/(2*a^
(1/2)*c^(3/2)) + (C*d*atan((c^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*c^(3/2))
```

3.53 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^2} dx$

Optimal. Leaf size=69

$$-\frac{aB-(Ac-aC)x}{2ac\left(a+cx^2\right)}+\frac{(Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}$$

```
[Out] 1/2*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)+1/2*(A*c+C*a)*arctan(x*c^(1/2)/a^(1/2)
)/a^(3/2)/c^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1828, 12, 211}

$$\frac{(aC+Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}-\frac{aB-x(Ac-aC)}{2ac\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^2,x]
```

```
[Out] -1/2*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)) + ((A*c + a*C)*ArcTan[(Sqrt[c]
*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^2}\,dx = -\frac{aB-(Ac-aC)x}{2ac\,(a+cx^2)} - \frac{\int \frac{-A-\frac{aC}{c}}{a+cx^2}\,dx}{2a}$$
$$= -\frac{aB-(Ac-aC)x}{2ac\,(a+cx^2)} + \frac{(Ac+aC)\int \frac{1}{a+cx^2}\,dx}{2ac}$$
$$= -\frac{aB-(Ac-aC)x}{2ac\,(a+cx^2)} + \frac{(Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}$$

Mathematica [A]
time = 0.04, size = 68, normalized size = 0.99

$$\frac{-aB+Acx-aCx}{2ac\,(a+cx^2)} + \frac{(Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^2,x]
[Out] (-(a*B) + A*c*x - a*C*x)/(2*a*c*(a + c*x^2)) + ((A*c + a*C)*ArcTan[(Sqrt[c]
*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

Maple [A]
time = 0.10, size = 65, normalized size = 0.94

method	result	size
default	$\frac{\frac{(Ac-aC)x}{2ac}-\frac{B}{2c}}{c\,x^2+a}+\frac{(Ac+aC)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2ac\sqrt{ac}}$	65
risch	$\frac{\frac{(Ac-aC)x}{2ac}-\frac{B}{2c}}{c\,x^2+a}-\frac{\ln\left(cx+\sqrt{-ac}\right)A}{4\sqrt{-ac}\,a}-\frac{\ln\left(cx+\sqrt{-ac}\right)C}{4\sqrt{-ac}\,c}+\frac{\ln\left(-cx+\sqrt{-ac}\right)A}{4\sqrt{-ac}\,a}+\frac{\ln\left(-cx+\sqrt{-ac}\right)C}{4\sqrt{-ac}\,c}$	130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] (1/2*(A*c-C*a)/a/c*x-1/2*B/c)/(c*x^2+a)+1/2*(A*c+C*a)/a/c/(a*c)^(1/2)*arcta
n(c*x/(a*c)^(1/2))
```

Maxima [A]
time = 0.49, size = 62, normalized size = 0.90

$$-\frac{Ba+(Ca-Ac)x}{2\,(ac^2x^2+a^2c)}+\frac{(Ca+Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a + (C*a - A*c)*x)/(a*c^2*x^2 + a^2*c) + 1/2*(C*a + A*c)*arctan(c*x
/sqrt(a*c))/(sqrt(a*c)*a*c)
```

Fricas [A]
time = 0.35, size = 195, normalized size = 2.83

$$\left[-\frac{2\,Ba^2c+(Ca^2+Aac+(Cac+Ac^2)x^2)\sqrt{-ac}\,\log\left(\frac{cx^2-2\,\sqrt{-ac}\,x-a}{cx^2+a}\right)+2\,(Ca^2c-Aac^2)x}{4\,(a^2c^3x^2+a^3c^2)},-\frac{Ba^2c-(Ca^2+Aac+(Cac+Ac^2)x^2)\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,x}{a}\right)+(Ca^2c-Aac^2)x}{2\,(a^2c^3x^2+a^3c^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c + (C*a^2 + A*a*c + (C*a*c + A*c^2)*x^2)*sqrt(-a*c)*log((c*
x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(C*a^2*c - A*a*c^2)*x)/(a^2*c^3*
x^2 + a^3*c^2), -1/2*(B*a^2*c - (C*a^2 + A*a*c + (C*a*c + A*c^2)*x^2)*sqrt(
a*c)*arctan(sqrt(a*c)*x/a) + (C*a^2*c - A*a*c^2)*x)/(a^2*c^3*x^2 + a^3*c^2)
]
```

Sympy [A]
time = 0.34, size = 116, normalized size = 1.68

$$-\frac{\sqrt{-\frac{1}{a^3c^3}}\,(Ac+Ca)\log\left(-a^2c\sqrt{-\frac{1}{a^3c^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3c^3}}\,(Ac+Ca)\log\left(a^2c\sqrt{-\frac{1}{a^3c^3}}+x\right)}{4}+\frac{-Ba+x(Ac-Ca)}{2a^2c+2ac^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**3*c**3))*(A*c + C*a)*log(-a**2*c*sqrt(-1/(a**3*c**3)) + x)/4 +
 sqrt(-1/(a**3*c**3))*(A*c + C*a)*log(a**2*c*sqrt(-1/(a**3*c**3)) + x)/4 +
(-B*a + x*(A*c - C*a))/(2*a**2*c + 2*a*c**2*x**2)
```

Giac [A]
time = 2.75, size = 60, normalized size = 0.87

$$\frac{(Ca+Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,ac}-\frac{Cax-Acx+Ba}{2\,(cx^2+a)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*a + A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c) - 1/2*(C*a*x - A*c*x
+ B*a)/((c*x^2 + a)*a*c)
```

Mupad [B]
time = 0.10, size = 60, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(A\,c+C\,a)}{2\,a^{3/2}\,c^{3/2}}-\frac{\frac{B}{2\,c}-\frac{x\,(A\,c-C\,a)}{2\,a\,c}}{c\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^2,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(A*c + C*a))/(2*a^(3/2)*c^(3/2)) - (B/(2*c) - (x
*(A*c - C*a))/(2*a*c))/(a + c*x^2)
```

3.54 $\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2} \, dx$

Optimal. Leaf size=226

$$-\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)}+\frac{\left(a(Cd-Be)\left(cd^2-ae^2\right)+Acd\left(cd^2+3ae^2\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}\,\left(cd^2+ae^2\right)^2}$$

```
[Out] 1/2*(-a*(-A*c*e+B*c*d+C*a*e)+c*(A*c*d+B*a*e-C*a*d)*x)/a/c/(a*e^2+c*d^2)/(c*
x^2+a)+e*(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/(a*e^2+c*d^2)^2-1/2*e*(A*e^2-B*d*e+C
*d^2)*ln(c*x^2+a)/(a*e^2+c*d^2)^2+1/2*(a*(-B*e+C*d)*(-a*e^2+c*d^2)+A*c*d*(3
*a*e^2+c*d^2))*arctan(x*c^(1/2)/a^(1/2))/a^(3/2)/(a*e^2+c*d^2)^2/c^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 815, 649, 211, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)+a\left(cd^2-ae^2\right)(Cd-Be)\right)}{2a^{3/2}\sqrt{c}\,\left(ae^2+cd^2\right)^2}-\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{2ac\left(a+cx^2\right)\left(ae^2+cd^2\right)}-\frac{e\log\left(a+cx^2\right)\left(Ae^2-Bde+Cd^2\right)}{2\left(ae^2+cd^2\right)^2}+\frac{e\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^2),x]
```

```
[Out] -1/2*(a*(B*c*d - A*c*e + a*C*e) - c*(A*c*d - a*C*d + a*B*e)*x)/(a*c*(c*d^2
+ a*e^2)*(a + c*x^2)) + ((a*(C*d - B*e)*(c*d^2 - a*e^2) + A*c*d*(c*d^2 + 3*
a*e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[c]*(c*d^2 + a*e^2)^2)
+ (e*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^2 - (e*(C*d^2 -
B*d*e + A*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx = -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{-\frac{c\left(ad(Cd-Be)+A\left(cd^2+2ae^2\right)\right)}{cd^2+ae^2} - \cdots}{(d+ex)(a+cx^2)\cdots}}{2ac}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \left(-\frac{2ace^2\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)} + \cdots\right)}{\cdots}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e\left(Cd^2-Bde+Ae^2\right)\log(d \cdots}{\left(cd^2+ae^2\right)^2}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e\left(Cd^2-Bde+Ae^2\right)\log(d \cdots}{\left(cd^2+ae^2\right)^2}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(a(Cd-Be)\left(cd^2-ae^2\right)+\cdots\right.}{2a^{3/2}\sqrt{\cdots}}$$

Mathematica [A]
time = 0.15, size = 195, normalized size = 0.86

$$\frac{\frac{\left(cd^2+ae^2\right)\left(-a^2Ce+Ac^2dx+ac(-Bd+Ae-Cdx+Bex)\right)}{ac(a+cx^2)}+\frac{\left(a(Cd-Be)\left(cd^2-ae^2\right)+Acd\left(cd^2+3ae^2\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{c}}+2e\left(Cd^2+e(-Bd+Ae)\right)\log(d+ex)-e\left(Cd^2+e(-Bd+Ae)\right)\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^2),x]
```

```
[Out] (((c*d^2 + a*e^2)*(-(a^2*C*e) + A*c^2*d*x + a*c*(-(B*d) + A*e - C*d*x + B*e
*x)))/(a*c*(a + c*x^2)) + ((a*(C*d - B*e)*(c*d^2 - a*e^2) + A*c*d*(c*d^2 +
3*a*e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[c]) + 2*e*(C*d^2 + e*(
-(B*d) + A*e))*Log[d + e*x] - e*(C*d^2 + e*(-(B*d) + A*e))*Log[a + c*x^2])/
(2*(c*d^2 + a*e^2)^2)
```

Maple [A]
time = 0.12, size = 293, normalized size = 1.30

method	result
default	$\frac{e\left(A\,e^{2}-Bde+C\,d^{2}\right)\ln\left(ex+d\right)}{\left(a\,e^{2}+c\,d^{2}\right)^{2}}+\frac{\frac{\left(Aacd\,e^{2}+A\,c^{2}d^{3}+B\,a^{2}e^{3}+Bac\,d^{2}e-C\,a^{2}d\,e^{2}-Cac\,d^{3}\right)x}{2a}+\frac{Aac\,e^{3}+A\,c^{2}d^{2}e-Bacd\,e^{2}-B\,c^{2}d^{3}-C\,a^{2}e^{3}-Cac}{2c}}{c\,x^{2}+a}$
risch	$\frac{\frac{\left(Acd+aBe-Cad\right)x}{2a\left(a\,e^{2}+c\,d^{2}\right)}+\frac{Ace-Bcd-aCe}{2\left(a\,e^{2}+c\,d^{2}\right)c}}{c\,x^{2}+a}+\frac{e^{3}\ln\left(ex+d\right)A}{a^{2}e^{4}+2ac\,d^{2}e^{2}+c^{2}d^{4}}-\frac{e^{2}\ln\left(ex+d\right)Bd}{a^{2}e^{4}+2ac\,d^{2}e^{2}+c^{2}d^{4}}+\frac{e\ln\left(ex+d\right)C\,d^{2}}{a^{2}e^{4}+2ac\,d^{2}e^{2}+c^{2}d^{4}}+\Bigg(\_R=\mathrm{RootOf}\Big(\Big(a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e*(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/(a*e^2+c*d^2)^2+1/(a*e^2+c*d^2)^2*((1/2*(A*
a*c*d*e^2+A*c^2*d^3+B*a^2*e^3+B*a*c*d^2*e-C*a^2*d*e^2-C*a*c*d^3)/a*x+1/2*(A
*a*c*e^3+A*c^2*d^2*e-B*a*c*d*e^2-B*c^2*d^3-C*a^2*e^3-C*a*c*d^2*e)/c)/(c*x^2
+a)+1/2/a*(1/2*(-2*A*a*c*e^3+2*B*a*c*d*e^2-2*C*a*c*d^2*e)/c*ln(c*x^2+a)+(3*
A*a*c*d*e^2+A*c^2*d^3+B*a^2*e^3-B*a*c*d^2*e-C*a^2*d*e^2+C*a*c*d^3)/(a*c)^(1
/2)*arctan(c*x/(a*c)^(1/2))))
```

Maxima [A]
time = 0.51, size = 287, normalized size = 1.27

$$-\frac{\left(Cd^{2}e-Bde^{2}+Ae^{3}\right)\log\left(cx^{2}+a\right)}{2\left(c^{2}d^{4}+2\,acd^{2}e^{2}+a^{2}e^{4}\right)}+\frac{\left(Cd^{2}e-Bde^{2}+Ae^{3}\right)\log\left(xe+d\right)}{c^{2}d^{4}+2\,acd^{2}e^{2}+a^{2}e^{4}}-\frac{\left(Bacd^{2}e-\left(Cac+Ac^{2}\right)d^{3}-Ba^{2}e^{3}+\left(Ca^{2}e^{2}-3\,Aace^{2}\right)d\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\left(ac^{2}d^{4}+2\,a^{2}cd^{2}e^{2}+a^{3}e^{4}\right)\sqrt{ac}}-\frac{Bacd+Ca^{2}e-Aace-\left(Bace-\left(Cac-Ac^{2}\right)d\right)x}{2\left(a^{2}c^{2}d^{2}+a^{3}ce^{2}+\left(ac^{3}d^{2}+a^{2}c^{2}e^{2}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(C*d^2*e - B*d*e^2 + A*e^3)*log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4) + (C*d^2*e - B*d*e^2 + A*e^3)*log(x*e + d)/(c^2*d^4 + 2*a*c*d^2*e^
2 + a^2*e^4) - 1/2*(B*a*c*d^2*e - (C*a*c + A*c^2)*d^3 - B*a^2*e^3 + (C*a^2*
e^2 - 3*A*a*c*e^2)*d)*arctan(c*x/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 +
 a^3*e^4)*sqrt(a*c)) - 1/2*(B*a*c*d + C*a^2*e - A*a*c*e - (B*a*c*e - (C*a*c
 - A*c^2)*d)*x)/(a^2*c^2*d^2 + a^3*c*e^2 + (a*c^3*d^2 + a^2*c^2*e^2)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(213) = 426$.

time = 20.73, size = 1027, normalized size = 4.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c^2*d^3 + 2*(C*a^2*c^2 - A*a*c^3)*d^3*x + ((C*a*c^2 + A*c^3)
*d^3*x^2 + (C*a^2*c + A*a*c^2)*d^3 + (B*a^2*c*x^2 + B*a^3)*e^3 - ((C*a^2*c
- 3*A*a*c^2)*d*x^2 + (C*a^3 - 3*A*a^2*c)*d)*e^2 - (B*a*c^2*d^2*x^2 + B*a^2*
c*d^2)*e)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(B*a
^3*c*x - C*a^4 + A*a^3*c)*e^3 + 2*(B*a^3*c*d + (C*a^3*c - A*a^2*c^2)*d*x)*e
^2 - 2*(B*a^2*c^2*d^2*x - (C*a^3*c - A*a^2*c^2)*d^2)*e + 2*((A*a^2*c^2*x^2
+ A*a^3*c)*e^3 - (B*a^2*c^2*d*x^2 + B*a^3*c*d)*e^2 + (C*a^2*c^2*d^2*x^2 + C
*a^3*c*d^2)*e)*log(c*x^2 + a) - 4*((A*a^2*c^2*x^2 + A*a^3*c)*e^3 - (B*a^2*c
^2*d*x^2 + B*a^3*c*d)*e^2 + (C*a^2*c^2*d^2*x^2 + C*a^3*c*d^2)*e)*log(x*e +
d))/(a^2*c^4*d^4*x^2 + a^3*c^3*d^4 + (a^4*c^2*x^2 + a^5*c)*e^4 + 2*(a^3*c^3
*d^2*x^2 + a^4*c^2*d^2)*e^2), -1/2*(B*a^2*c^2*d^3 + (C*a^2*c^2 - A*a*c^3)*d
^3*x - ((C*a*c^2 + A*c^3)*d^3*x^2 + (C*a^2*c + A*a*c^2)*d^3 + (B*a^2*c*x^2
+ B*a^3)*e^3 - ((C*a^2*c - 3*A*a*c^2)*d*x^2 + (C*a^3 - 3*A*a^2*c)*d)*e^2 -
(B*a*c^2*d^2*x^2 + B*a^2*c*d^2)*e)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (B*a^3
*c*x - C*a^4 + A*a^3*c)*e^3 + (B*a^3*c*d + (C*a^3*c - A*a^2*c^2)*d*x)*e^2 -
 (B*a^2*c^2*d^2*x - (C*a^3*c - A*a^2*c^2)*d^2)*e + ((A*a^2*c^2*x^2 + A*a^3*
c)*e^3 - (B*a^2*c^2*d*x^2 + B*a^3*c*d)*e^2 + (C*a^2*c^2*d^2*x^2 + C*a^3*c*d
^2)*e)*log(c*x^2 + a) - 2*((A*a^2*c^2*x^2 + A*a^3*c)*e^3 - (B*a^2*c^2*d*x^2
 + B*a^3*c*d)*e^2 + (C*a^2*c^2*d^2*x^2 + C*a^3*c*d^2)*e)*log(x*e + d))/(a^2
*c^4*d^4*x^2 + a^3*c^3*d^4 + (a^4*c^2*x^2 + a^5*c)*e^4 + 2*(a^3*c^3*d^2*x^2
 + a^4*c^2*d^2)*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.91, size = 350, normalized size = 1.55

$$-\frac{(Cd^2e - Bde^2 + Ae^3)\log(cx^2 + a)}{2\,(c^2d^4 + 2\,acd^2e^2 + a^2e^4)} + \frac{(Cd^2e^2 - Bde^3 + Ae^4)\log(|xe + d|)}{c^2d^4e + 2\,acd^2e^3 + a^2e^5} + \frac{(Cacd^3 + Ac^2d^3 - Bacd^2e - Ca^2de^2 + 3\,Aacde^2 + Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,(ac^2d^4 + 2\,a^2cd^2e^2 + a^3e^4)\sqrt{ac}} - \frac{Bac^2d^3 + Ca^2cd^2e - Aac^2d^2e + Ba^2cde^2 + Ca^3e^3 - Aa^2ce^3 + (Cac^2d^3 - Ac^3d^3 - Bac^2d^2e + Ca^2cde^2 - Aac^2de^2 - Ba^2ce^3)x}{2\,(cd^2 + ae^2)^2(cx^2 + a)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(C*d^2*e - B*d*e^2 + A*e^3)*log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4) + (C*d^2*e^2 - B*d*e^3 + A*e^4)*log(abs(x*e + d))/(c^2*d^4*e + 2*a
*c*d^2*e^3 + a^2*e^5) + 1/2*(C*a*c*d^3 + A*c^2*d^3 - B*a*c*d^2*e - C*a^2*d*
e^2 + 3*A*a*c*d*e^2 + B*a^2*e^3)*arctan(c*x/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*
c*d^2*e^2 + a^3*e^4)*sqrt(a*c)) - 1/2*(B*a*c^2*d^3 + C*a^2*c*d^2*e - A*a*c^
2*d^2*e + B*a^2*c*d*e^2 + C*a^3*e^3 - A*a^2*c*e^3 + (C*a*c^2*d^3 - A*c^3*d^
3 - B*a*c^2*d^2*e + C*a^2*c*d*e^2 - A*a*c^2*d*e^2 - B*a^2*c*e^3)*x)/((c*d^2
 + a*e^2)^2*(c*x^2 + a)*a*c)
```

Mupad [B]
time = 7.68, size = 1493, normalized size = 6.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^2*(d + e*x)),x)
```

```
[Out] (log(A*c^3*d^5*(-a^3*c)^(1/2) - B*a^3*e^5*(-a^3*c)^(1/2) + 6*A*a^4*c*e^5 -
B*a^4*c*e^5*x - 2*A*a^2*c^3*d^4*e - 8*C*a^3*c^2*d^4*e + 8*C*a^4*c*d^2*e^3 +
 C*a^2*c^3*d^5*x + C*a*c^2*d^5*(-a^3*c)^(1/2) + C*a^3*d*e^4*(-a^3*c)^(1/2)
- 12*A*a^3*c^2*d^2*e^3 + 8*B*a^3*c^2*d^3*e^2 - 8*B*a^4*c*d*e^4 + A*a*c^4*d^
5*x + 2*A*a^2*c^3*d^3*e^2*x + 14*B*a^3*c^2*d^2*e^3*x - 14*C*a^3*c^2*d^3*e^2
*x + 2*A*a*c^2*d^3*e^2*(-a^3*c)^(1/2) + 14*B*a^2*c*d^2*e^3*(-a^3*c)^(1/2) -
 14*C*a^2*c*d^3*e^2*(-a^3*c)^(1/2) + C*a^4*c*d*e^4*x - 15*A*a^3*c^2*d*e^4*x
 - B*a^2*c^3*d^4*e*x - 15*A*a^2*c*d*e^4*(-a^3*c)^(1/2) - B*a*c^2*d^4*e*(-a^
3*c)^(1/2) - 6*A*a^2*c*e^5*x*(-a^3*c)^(1/2) + 2*A*c^3*d^4*e*x*(-a^3*c)^(1/2
) + 8*B*a^2*c*d*e^4*x*(-a^3*c)^(1/2) + 8*C*a*c^2*d^4*e*x*(-a^3*c)^(1/2) + 1
2*A*a*c^2*d^2*e^3*x*(-a^3*c)^(1/2) - 8*B*a*c^2*d^3*e^2*x*(-a^3*c)^(1/2) - 8
*C*a^2*c*d^2*e^3*x*(-a^3*c)^(1/2))*(a^2*((B*e^3*(-a^3*c)^(1/2))/4 - (C*d*e^
2*(-a^3*c)^(1/2))/4) - c*(a^3*((A*e^3)/2 - (B*d*e^2)/2 + (C*d^2*e)/2) - a*(
(C*d^3*(-a^3*c)^(1/2))/4 + (3*A*d*e^2*(-a^3*c)^(1/2))/4 - (B*d^2*e*(-a^3*c)
^(1/2))/4)) + (A*c^2*d^3*(-a^3*c)^(1/2))/4))/(a^5*c*e^4 + a^3*c^3*d^4 + 2*a
^4*c^2*d^2*e^2) - (log(A*c^3*d^5*(-a^3*c)^(1/2) - B*a^3*e^5*(-a^3*c)^(1/2)
- 6*A*a^4*c*e^5 + B*a^4*c*e^5*x + 2*A*a^2*c^3*d^4*e + 8*C*a^3*c^2*d^4*e - 8
*C*a^4*c*d^2*e^3 - C*a^2*c^3*d^5*x + C*a*c^2*d^5*(-a^3*c)^(1/2) + C*a^3*d*e
^4*(-a^3*c)^(1/2) + 12*A*a^3*c^2*d^2*e^3 - 8*B*a^3*c^2*d^3*e^2 + 8*B*a^4*c*
d*e^4 - A*a*c^4*d^5*x - 2*A*a^2*c^3*d^3*e^2*x - 14*B*a^3*c^2*d^2*e^3*x + 14
*C*a^3*c^2*d^3*e^2*x + 2*A*a*c^2*d^3*e^2*(-a^3*c)^(1/2) + 14*B*a^2*c*d^2*e^
3*(-a^3*c)^(1/2) - 14*C*a^2*c*d^3*e^2*(-a^3*c)^(1/2) - C*a^4*c*d*e^4*x + 15
*A*a^3*c^2*d*e^4*x + B*a^2*c^3*d^4*e*x - 15*A*a^2*c*d*e^4*(-a^3*c)^(1/2) -
B*a*c^2*d^4*e*(-a^3*c)^(1/2) - 6*A*a^2*c*e^5*x*(-a^3*c)^(1/2) + 2*A*c^3*d^4
*e*x*(-a^3*c)^(1/2) + 8*B*a^2*c*d*e^4*x*(-a^3*c)^(1/2) + 8*C*a*c^2*d^4*e*x*
(-a^3*c)^(1/2) + 12*A*a*c^2*d^2*e^3*x*(-a^3*c)^(1/2) - 8*B*a*c^2*d^3*e^2*x*
(-a^3*c)^(1/2) - 8*C*a^2*c*d^2*e^3*x*(-a^3*c)^(1/2))*(c*(a^3*((A*e^3)/2 - (
```

```
B*d*e^2)/2 + (C*d^2*e)/2) + a*((C*d^3*(-a^3*c)^(1/2))/4 + (3*A*d*e^2*(-a^3*
c)^(1/2))/4 - (B*d^2*e*(-a^3*c)^(1/2))/4)) + a^2*((B*e^3*(-a^3*c)^(1/2))/4
- (C*d*e^2*(-a^3*c)^(1/2))/4) + (A*c^2*d^3*(-a^3*c)^(1/2))/4))/(a^5*c*e^4 +
 a^3*c^3*d^4 + 2*a^4*c^2*d^2*e^2) - ((B*c*d - A*c*e + C*a*e)/(2*c*(a*e^2 +
c*d^2)) - (x*(A*c*d + B*a*e - C*a*d))/(2*a*(a*e^2 + c*d^2)))/(a + c*x^2) +
(e*log(d + e*x)*(A*e^2 + C*d^2 - B*d*e))/(a*e^2 + c*d^2)^2
```

3.55 $\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx$

Optimal. Leaf size=374

$$-\frac{e(Cd^2-Bde+Ae^2)}{\left(cd^2+ae^2\right)^2(d+ex)}-\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\,x}{2a\left(cd^2+ae^2\right)^2(a+cx^2)}$$

```
[Out] -e*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^2/(e*x+d)+1/2*(-a*(-2*A*c*d*e-B*a*e^2+
B*c*d^2+2*C*a*d*e)+(A*c*(-a*e^2+c*d^2)+a*(a*C*e^2-c*d*(-2*B*e+C*d)))*x)/a/(
a*e^2+c*d^2)^2/(c*x^2+a)-e*(a*e^2*(-B*e+2*C*d)-c*d*(2*C*d^2-e*(-4*A*e+3*B*d
)))*ln(e*x+d)/(a*e^2+c*d^2)^3+1/2*e*(a*e^2*(-B*e+2*C*d)-c*d*(2*C*d^2-e*(-4*
A*e+3*B*d)))*ln(c*x^2+a)/(a*e^2+c*d^2)^3+1/2*(A*c*(-3*a^2*e^4+6*a*c*d^2*e^2
+c^2*d^4)+a*(a^2*C*e^4+c^2*d^3*(-2*B*e+C*d)-6*a*c*d*e^2*(-B*e+C*d)))*arctan
(x*c^(1/2)/a^(1/2))/a^(3/2)/(a*e^2+c*d^2)^3/c^(1/2)
```

Rubi [A]

time = 0.59, antiderivative size = 371, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 1643, 649, 211, 266}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Ac(-3a^2e^4+6acd^2e^2+c^2d^4)+a(a^2Ce^4-6acde^2(Cd-Be)+c^2d^3(Cd-2Be))\right)}{2a^{3/2}\sqrt{c}\left(ae^2+cd^2\right)^3}-\frac{a(-aBe^2+2aCde-2Acde+Bcd^2)-x(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)^2}-\frac{e(Ae^2-Bde+Cd^2)}{(d+ex)\left(ae^2+cd^2\right)^2}-\frac{e\log\left(a+cx^2\right)\left(-ae^2(2Cd-Be)-cde(3Bd-4Ae)+2cCd^3\right)}{2\left(ae^2+cd^2\right)^3}+\frac{e\log(d+ex)\left(-ae^2(2Cd-Be)-cde(3Bd-4Ae)+2cCd^3\right)}{\left(ae^2+cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^2),x]
```

```
[Out] -((e*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^2*(d + e*x))) - (a*(B*c*d^2
 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2) - (A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*
d*(C*d - 2*B*e)))*x)/(2*a*(c*d^2 + a*e^2)^2*(a + c*x^2)) + ((A*c*(c^2*d^4 +
 6*a*c*d^2*e^2 - 3*a^2*e^4) + a*(a^2*C*e^4 + c^2*d^3*(C*d - 2*B*e) - 6*a*c*
d*e^2*(C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[c]*(c*d^2
 + a*e^2)^3) + (e*(2*c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e))*
Log[d + e*x])/(c*d^2 + a*e^2)^3 - (e*(2*c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a
*e^2*(2*C*d - B*e))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx = -\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2}{2a\left(cd^2+ae^2\right)^2(a+cx^2)}$$

$$= -\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2}{2a\left(cd^2+ae^2\right)^2(a+cx^2)}$$

$$= -\frac{e(Cd^2-Bde+Ae^2)}{\left(cd^2+ae^2\right)^2(d+ex)}-\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae}{2a\left(cd^2+ae^2\right)^2(a+c}$$

$$= -\frac{e(Cd^2-Bde+Ae^2)}{\left(cd^2+ae^2\right)^2(d+ex)}-\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae}{2a\left(cd^2+ae^2\right)^2(a+c}$$

$$= -\frac{e(Cd^2-Bde+Ae^2)}{\left(cd^2+ae^2\right)^2(d+ex)}-\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae}{2a\left(cd^2+ae^2\right)^2(a+c}$$

Mathematica [A]
time = 0.28, size = 320, normalized size = 0.86

$$\frac{-\frac{2e(cd^2+ae^2)(Cd^2+e(-Bd+Ae))}{d+ex}+\frac{(cd^2+ae^2)(Ac^2d^2x+a^2e(-2Cd+Be+Cex)-ac(Cd^2x+Bd(d-2ex)+Ae(-2d+ex)))}{a(a+cx^2)}+\frac{(Ac(c^2d^4+6acd^2e^2-3a^2e^4)+a(a^2Ce^4+c^2d^3(Cd-2Be)+6acde^2(-Cd+Be)))\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{c}}+2e(2cCd^3+cde(-3Bd+4Ae)+ae^2(-2Cd+Be))\log(d+ex)-e(2cCd^3+cde(-3Bd+4Ae)+ae^2(-2Cd+Be))\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^2),x]
```

```
[Out] ((-2*e*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) + ((c*d^2 + a*
e^2)*(A*c^2*d^2*x + a^2*e*(-2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + B*d*(d -
2*e*x) + A*e*(-2*d + e*x))))/(a*(a + c*x^2)) + ((A*c*(c^2*d^4 + 6*a*c*d^2*e
^2 - 3*a^2*e^4) + a*(a^2*C*e^4 + c^2*d^3*(C*d - 2*B*e) + 6*a*c*d*e^2*(-(C*d
) + B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[c]) + 2*e*(2*c*C*d^3
+ c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e))*Log[d + e*x] - e*(2*c*C*d^
3 + c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e))*Log[a + c*x^2])/(2*(c*d^
2 + a*e^2)^3)
```

Maple [A]
time = 0.14, size = 424, normalized size = 1.13

method	result
default	$\frac{e\left(4d\,e^{2}cA+Ba\,e^{3}-3Bc\,d^{2}e-2Cad\,e^{2}+2Cc\,d^{3}\right)\ln\left(ex+d\right)}{\left(a\,e^{2}+c\,d^{2}\right)^{3}}-\frac{e\left(A\,e^{2}-Bde+C\,d^{2}\right)}{\left(a\,e^{2}+c\,d^{2}\right)^{2}\left(ex+d\right)}-\frac{\frac{\left(A\,a^{2}c\,e^{4}-A\,c^{3}d^{4}-2B\,a^{2}cd\,e^{3}-2Ba\,c^{2}d^{3}e-C\,a^{3}e^{4}}{2a}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e*(4*A*c*d*e^2+B*a*e^3-3*B*c*d^2*e-2*C*a*d*e^2+2*C*c*d^3)/(a*e^2+c*d^2)^3*l
n(e*x+d)-e*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^2/(e*x+d)-1/(a*e^2+c*d^2)^3*((
1/2*(A*a^2*c*e^4-A*c^3*d^4-2*B*a^2*c*d*e^3-2*B*a*c^2*d^3*e-C*a^3*e^4+C*a*c^
2*d^4)/a*x-A*a*c*d*e^3-A*c^2*d^3*e-1/2*B*a^2*e^4+1/2*B*c^2*d^4+C*a^2*d*e^3+
C*a*c*d^3*e)/(c*x^2+a)+1/2/a*(1/2*(8*A*a*c^2*d*e^3+2*B*a^2*c*e^4-6*B*a*c^2*
d^2*e^2-4*C*a^2*c*d*e^3+4*C*a*c^2*d^3*e)/c*ln(c*x^2+a)+(3*A*a^2*c*e^4-6*A*a
*c^2*d^2*e^2-A*c^3*d^4-6*B*a^2*c*d*e^3+2*B*a*c^2*d^3*e-C*a^3*e^4+6*C*a^2*c*
d^2*e^2-C*a*c^2*d^4)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Maxima [A]
time = 0.51, size = 593, normalized size = 1.59

$$-\frac{\left(2\,Ccd^{3}e-3\,Bcd^{2}e^{2}+Bae^{4}-2\,\left(Cae^{3}-2\,Ace^{3}\right)d\right)\log\left(cx^{2}+a\right)}{2\,\left(c^{3}d^{6}+3\,ac^{2}d^{4}e^{2}+3\,a^{2}cd^{2}e^{4}+a^{3}e^{6}\right)}+\frac{\left(2\,Ccd^{3}e-3\,Bcd^{2}e^{2}+Bae^{4}-2\,\left(Cae^{3}-2\,Ace^{3}\right)d\right)\log\left(xe+d\right)}{c^{3}d^{6}+3\,ac^{2}d^{4}e^{2}+3\,a^{2}cd^{2}e^{4}+a^{3}e^{6}}-\frac{\left(2\,Bac^{2}d^{3}e-6\,Ba^{2}cde^{3}-\left(Cac^{2}+Ac^{3}\right)d^{4}-Ca^{3}e^{4}+3\,Aa^{2}ce^{4}+6\,\left(Ca^{2}ce^{2}-Aac^{2}e^{2}\right)d^{2}\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\left(ac^{3}d^{6}+3\,a^{2}c^{2}d^{4}e^{2}+3\,a^{3}cd^{2}e^{4}+a^{4}e^{6}\right)\sqrt{ac}}-\frac{Bacd^{3}-3\,Ba^{2}de^{2}+2\,Aa^{2}e^{3}+2\,\left(2\,Ca^{2}e-Aace\right)d^{2}-\left(4\,Bacde^{2}+Ca^{2}e^{3}-3\,Aace^{3}-\left(3\,Cace-Ac^{2}e\right)d^{2}\right)x^{2}-\left(Bacd^{2}e-\left(Cac-Ac^{2}\right)d^{3}+Ba^{2}e^{3}-\left(Ca^{2}e^{2}-Aace^{2}\right)d\right)x}{2\,\left(a^{2}c^{2}d^{5}+2\,a^{3}cd^{3}e^{2}+a^{4}de^{4}+\left(ac^{3}d^{4}e+2\,a^{2}c^{2}d^{2}e^{3}+a^{3}ce^{5}\right)x^{3}+\left(ac^{3}d^{5}+2\,a^{2}c^{2}d^{3}e^{2}+a^{3}cde^{4}\right)x^{2}+\left(a^{2}c^{2}d^{4}e+2\,a^{3}cd^{2}e^{3}+a^{4}e^{5}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*C*c*d^3*e - 3*B*c*d^2*e^2 + B*a*e^4 - 2*(C*a*e^3 - 2*A*c*e^3)*d)*lo
g(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (2*C
*c*d^3*e - 3*B*c*d^2*e^2 + B*a*e^4 - 2*(C*a*e^3 - 2*A*c*e^3)*d)*log(x*e + d
```

```
)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - 1/2*(2*B*a*c^2*
d^3*e - 6*B*a^2*c*d*e^3 - (C*a*c^2 + A*c^3)*d^4 - C*a^3*e^4 + 3*A*a^2*c*e^4
 + 6*(C*a^2*c*e^2 - A*a*c^2*e^2)*d^2)*arctan(c*x/sqrt(a*c))/((a*c^3*d^6 + 3
*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(a*c)) - 1/2*(B*a*c*d^3 -
 3*B*a^2*d*e^2 + 2*A*a^2*e^3 + 2*(2*C*a^2*e - A*a*c*e)*d^2 - (4*B*a*c*d*e^2
 + C*a^2*e^3 - 3*A*a*c*e^3 - (3*C*a*c*e - A*c^2*e)*d^2)*x^2 - (B*a*c*d^2*e
- (C*a*c - A*c^2)*d^3 + B*a^2*e^3 - (C*a^2*e^2 - A*a*c*e^2)*d)*x)/(a^2*c^2*
d^5 + 2*a^3*c*d^3*e^2 + a^4*d*e^4 + (a*c^3*d^4*e + 2*a^2*c^2*d^2*e^3 + a^3*
c*e^5)*x^3 + (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*x^2 + (a^2*c^2*d
^4*e + 2*a^3*c*d^2*e^3 + a^4*e^5)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1415 vs. $2(362) = 724$.
time = 126.70, size = 2853, normalized size = 7.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c^3*d^5 + 2*(C*a^2*c^3 - A*a*c^4)*d^5*x - ((C*a*c^3 + A*c^4)
*d^5*x^2 + (C*a^2*c^2 + A*a*c^3)*d^5 + ((C*a^3*c - 3*A*a^2*c^2)*x^3 + (C*a^
4 - 3*A*a^3*c)*x)*e^5 + (6*B*a^2*c^2*d*x^3 + 6*B*a^3*c*d*x + (C*a^3*c - 3*A
*a^2*c^2)*d*x^2 + (C*a^4 - 3*A*a^3*c)*d)*e^4 + 6*(B*a^2*c^2*d^2*x^2 + B*a^3
*c*d^2 - (C*a^2*c^2 - A*a*c^3)*d^2*x^3 - (C*a^3*c - A*a^2*c^2)*d^2*x)*e^3 -
 2*(B*a*c^3*d^3*x^3 + B*a^2*c^2*d^3*x + 3*(C*a^2*c^2 - A*a*c^3)*d^3*x^2 + 3
*(C*a^3*c - A*a^2*c^2)*d^3)*e^2 - (2*B*a*c^3*d^4*x^2 + 2*B*a^2*c^2*d^4 - (C
*a*c^3 + A*c^4)*d^4*x^3 - (C*a^2*c^2 + A*a*c^3)*d^4*x)*e)*sqrt(-a*c)*log((c
*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(B*a^4*c*x - 2*A*a^4*c + (C*a^4
*c - 3*A*a^3*c^2)*x^2)*e^5 - 2*(4*B*a^3*c^2*d*x^2 + 3*B*a^4*c*d - (C*a^4*c
- A*a^3*c^2)*d*x)*e^4 - 4*(B*a^3*c^2*d^2*x - 2*C*a^4*c*d^2 - (C*a^3*c^2 + A
*a^2*c^3)*d^2*x^2)*e^3 - 4*(2*B*a^2*c^3*d^3*x^2 + B*a^3*c^2*d^3 - (C*a^3*c^
2 - A*a^2*c^3)*d^3*x)*e^2 - 2*(B*a^2*c^3*d^4*x - (3*C*a^2*c^3 - A*a*c^4)*d^
4*x^2 - 2*(2*C*a^3*c^2 - A*a^2*c^3)*d^4)*e + 2*((B*a^3*c^2*x^3 + B*a^4*c*x)
*e^5 + (B*a^3*c^2*d*x^2 + B*a^4*c*d - 2*(C*a^3*c^2 - 2*A*a^2*c^3)*d*x^3 - 2
*(C*a^4*c - 2*A*a^3*c^2)*d*x)*e^4 - (3*B*a^2*c^3*d^2*x^3 + 3*B*a^3*c^2*d^2*
x + 2*(C*a^3*c^2 - 2*A*a^2*c^3)*d^2*x^2 + 2*(C*a^4*c - 2*A*a^3*c^2)*d^2)*e^
3 + (2*C*a^2*c^3*d^3*x^3 - 3*B*a^2*c^3*d^3*x^2 + 2*C*a^3*c^2*d^3*x - 3*B*a^
3*c^2*d^3)*e^2 + 2*(C*a^2*c^3*d^4*x^2 + C*a^3*c^2*d^4)*e)*log(c*x^2 + a) -
4*((B*a^3*c^2*x^3 + B*a^4*c*x)*e^5 + (B*a^3*c^2*d*x^2 + B*a^4*c*d - 2*(C*a^
3*c^2 - 2*A*a^2*c^3)*d*x^3 - 2*(C*a^4*c - 2*A*a^3*c^2)*d*x)*e^4 - (3*B*a^2*
c^3*d^2*x^3 + 3*B*a^3*c^2*d^2*x + 2*(C*a^3*c^2 - 2*A*a^2*c^3)*d^2*x^2 + 2*(
C*a^4*c - 2*A*a^3*c^2)*d^2)*e^3 + (2*C*a^2*c^3*d^3*x^3 - 3*B*a^2*c^3*d^3*x^
2 + 2*C*a^3*c^2*d^3*x - 3*B*a^3*c^2*d^3)*e^2 + 2*(C*a^2*c^3*d^4*x^2 + C*a^3
*c^2*d^4)*e)*log(x*e + d))/(a^2*c^5*d^7*x^2 + a^3*c^4*d^7 + (a^5*c^2*x^3 +
```

```
a^6*c*x)*e^7 + (a^5*c^2*d*x^2 + a^6*c*d)*e^6 + 3*(a^4*c^3*d^2*x^3 + a^5*c^2
*d^2*x)*e^5 + 3*(a^4*c^3*d^3*x^2 + a^5*c^2*d^3)*e^4 + 3*(a^3*c^4*d^4*x^3 +
a^4*c^3*d^4*x)*e^3 + 3*(a^3*c^4*d^5*x^2 + a^4*c^3*d^5)*e^2 + (a^2*c^5*d^6*x
^3 + a^3*c^4*d^6*x)*e), -1/2*(B*a^2*c^3*d^5 + (C*a^2*c^3 - A*a*c^4)*d^5*x -
 ((C*a*c^3 + A*c^4)*d^5*x^2 + (C*a^2*c^2 + A*a*c^3)*d^5 + ((C*a^3*c - 3*A*a
^2*c^2)*x^3 + (C*a^4 - 3*A*a^3*c)*x)*e^5 + (6*B*a^2*c^2*d*x^3 + 6*B*a^3*c*d
*x + (C*a^3*c - 3*A*a^2*c^2)*d*x^2 + (C*a^4 - 3*A*a^3*c)*d)*e^4 + 6*(B*a^2*
c^2*d^2*x^2 + B*a^3*c*d^2 - (C*a^2*c^2 - A*a*c^3)*d^2*x^3 - (C*a^3*c - A*a^
2*c^2)*d^2*x)*e^3 - 2*(B*a*c^3*d^3*x^3 + B*a^2*c^2*d^3*x + 3*(C*a^2*c^2 - A
*a*c^3)*d^3*x^2 + 3*(C*a^3*c - A*a^2*c^2)*d^3)*e^2 - (2*B*a*c^3*d^4*x^2 + 2
*B*a^2*c^2*d^4 - (C*a*c^3 + A*c^4)*d^4*x^3 - (C*a^2*c^2 + A*a*c^3)*d^4*x)*e
)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (B*a^4*c*x - 2*A*a^4*c + (C*a^4*c - 3*A
*a^3*c^2)*x^2)*e^5 - (4*B*a^3*c^2*d*x^2 + 3*B*a^4*c*d - (C*a^4*c - A*a^3*c^
2)*d*x)*e^4 - 2*(B*a^3*c^2*d^2*x - 2*C*a^4*c*d^2 - (C*a^3*c^2 + A*a^2*c^3)*
d^2*x^2)*e^3 - 2*(2*B*a^2*c^3*d^3*x^2 + B*a^3*c^2*d^3 - (C*a^3*c^2 - A*a^2*
c^3)*d^3*x)*e^2 - (B*a^2*c^3*d^4*x - (3*C*a^2*c^3 - A*a*c^4)*d^4*x^2 - 2*(2
*C*a^3*c^2 - A*a^2*c^3)*d^4)*e + ((B*a^3*c^2*x^3 + B*a^4*c*x)*e^5 + (B*a^3*
c^2*d*x^2 + B*a^4*c*d - 2*(C*a^3*c^2 - 2*A*a^2*c^3)*d*x^3 - 2*(C*a^4*c - 2*
A*a^3*c^2)*d*x)*e^4 - (3*B*a^2*c^3*d^2*x^3 + 3*B*a^3*c^2*d^2*x + 2*(C*a^3*c
^2 - 2*A*a^2*c^3)*d^2*x^2 + 2*(C*a^4*c - 2*A*a^3*c^2)*d^2)*e^3 + (2*C*a^2*c
^3*d^3*x^3 - 3*B*a^2*c^3*d^3*x^2 + 2*C*a^3*c^2*d^3*x - 3*B*a^3*c^2*d^3)*e^2
 + 2*(C*a^2*c^3*d^4*x^2 + C*a^3*c^2*d^4)*e)*log(c*x^2 + a) - 2*((B*a^3*c^2*
x^3 + B*a^4*c*x)*e^5 + (B*a^3*c^2*d*x^2 + B*a^4*c*d - 2*(C*a^3*c^2 - 2*A*a^
2*c^3)*d*x^3 - 2*(C*a^4*c - 2*A*a^3*c^2)*d*x)*e^4 - (3*B*a^2*c^3*d^2*x^3 +
3*B*a^3*c^2*d^2*x + 2*(C*a^3*c^2 - 2*A*a^2*c^3)*d^2*x^2 + 2*(C*a^4*c - 2*A*
a^3*c^2)*d^2)*e^3 + (2*C*a^2*c^3*d^3*x^3 - 3*B*a^2*c^3*d^3*x^2 + 2*C*a^3*c^
2*d^3*x - 3*B*a^3*c^2*d^3)*e^2 + 2*(C*a^2*c^3*d^4*x^2 + C*a^3*c^2*d^4)*e)*l
og(x*e + d))/(a^2*c^5*d^7*x^2 + a^3*c^4*d^7 + (a^5*c^2*x^3 + a^6*c*x)*e^7 +
 (a^5*c^2*d*x^2 + a^6*c*d)*e^6 + 3*(a^4*c^3*d^2*x^3 + a^5*c^2*d^2*x)*e^5 +
3*(a^4*c^3*d^3*x^2 + a^5*c^2*d^3)*e^4 + 3*(a^3*c^4*d^4*x^3 + a^4*c^3*d^4*x)
*e^3 + 3*(a^3*c^4*d^5*x^2 + a^4*c^3*d^5)*e^2 + (a^2*c^5*d^6*x^3 + a^3*c^4*d
^6*x)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.64, size = 608, normalized size = 1.63

$$\frac{\left(Cac^2d^4e^2 + Ac^3d^4e^2 - 2\,Bac^2d^3e^3 - 6\,Ca^2cd^2e^4 + 6\,Aac^2d^2e^4 + 6\,Ba^2cde^5 + Ca^3e^6 - 3\,Aa^2ce^6\right)\arctan\left(\frac{\left(cd - \frac{cd^2}{xe+d} - \frac{ae^2}{xe+d}\right)e^{(-1)}}{\sqrt{ac}}\right)e^{(-2)}}{2\left(ac^3d^6 + 3\,a^2c^2d^4e^2 + 3\,a^3cd^2e^4 + a^4e^6\right)\sqrt{ac}} - \frac{\left(2\,Ccd^3e - 3\,Bcd^2e^2 - 2\,Cade^3 + 4\,Acde^3 + Bae^4\right)\log\left(c - \frac{2\,cd}{xe+d} + \frac{cd^2}{(xe+d)^2} + \frac{ae^2}{(xe+d)^2}\right)}{2\left(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6\right)} - \frac{\frac{Cd^2e^5}{xe+d} - \frac{Bde^6}{xe+d} + \frac{Ae^7}{xe+d}}{c^2d^4e^4 + 2\,acd^2e^6 + a^2e^8} - \frac{\frac{Cac^2d^3e - Ac^3d^3e - 3\,Bac^2d^2e^2 - 3\,Ca^2cde^3 + 3\,Aac^2de^3 + Ba^2ce^4}{cd^2+ae^2} - \frac{\left(Cac^2d^4e^2 - Ac^3d^4e^2 - 4\,Bac^2d^3e^3 - 6\,Ca^2cd^2e^4 + 6\,Aac^2d^2e^4 + 4\,Ba^2cde^5 + Ca^3e^6 - Aa^2ce^6\right)e^{(-1)}}{(cd^2+ae^2)(xe+d)}}{2\left(cd^2+ae^2\right)^2a\left(c - \frac{2\,cd}{xe+d} + \frac{cd^2}{(xe+d)^2} + \frac{ae^2}{(xe+d)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*a*c^2*d^4*e^2 + A*c^3*d^4*e^2 - 2*B*a*c^2*d^3*e^3 - 6*C*a^2*c*d^2*e^
4 + 6*A*a*c^2*d^2*e^4 + 6*B*a^2*c*d*e^5 + C*a^3*e^6 - 3*A*a^2*c*e^6)*arctan
((c*d - c*d^2/(x*e + d) - a*e^2/(x*e + d))*e^(-1)/sqrt(a*c))*e^(-2)/((a*c^3
*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(a*c)) - 1/2*(2*C
*c*d^3*e - 3*B*c*d^2*e^2 - 2*C*a*d*e^3 + 4*A*c*d*e^3 + B*a*e^4)*log(c - 2*c
*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2)/(c^3*d^6 + 3*a*c^2*d^
4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - (C*d^2*e^5/(x*e + d) - B*d*e^6/(x*e +
d) + A*e^7/(x*e + d))/(c^2*d^4*e^4 + 2*a*c*d^2*e^6 + a^2*e^8) - 1/2*((C*a*c
^2*d^3*e - A*c^3*d^3*e - 3*B*a*c^2*d^2*e^2 - 3*C*a^2*c*d*e^3 + 3*A*a*c^2*d*
e^3 + B*a^2*c*e^4)/(c*d^2 + a*e^2) - (C*a*c^2*d^4*e^2 - A*c^3*d^4*e^2 - 4*B
*a*c^2*d^3*e^3 - 6*C*a^2*c*d^2*e^4 + 6*A*a*c^2*d^2*e^4 + 4*B*a^2*c*d*e^5 +
C*a^3*e^6 - A*a^2*c*e^6)*e^(-1)/((c*d^2 + a*e^2)*(x*e + d)))/((c*d^2 + a*e^
2)^2*a*(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2))
```

Mupad [B]
time = 9.91, size = 2094, normalized size = 5.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^2*(d + e*x)^2),x)
```

```
[Out] ((x^2*(C*a^2*e^3 - 3*A*a*c*e^3 + A*c^2*d^2*e + 4*B*a*c*d*e^2 - 3*C*a*c*d^2*
e))/(2*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (2*A*a*e^3 + B*c*d^3 - 3*B*
a*d*e^2 - 2*A*c*d^2*e + 4*C*a*d^2*e)/(2*(a*e^2 + c*d^2)^2) + (x*(A*c*d + B*
a*e - C*a*d))/(2*a*(a*e^2 + c*d^2)))/(a*d + a*e*x + c*d*x^2 + c*e*x^3) - (l
og(3*A*e^6*(-a^3*c)^(3/2) - A*c^4*d^6*(-a^3*c)^(1/2) + C*a^4*e^6*(-a^3*c)^(
1/2) + 31*C*d^2*e^4*(-a^3*c)^(3/2) + 6*B*a^5*c*e^6 - 18*B*d*e^5*(-a^3*c)^(3
/2) - 6*B*e^6*x*(-a^3*c)^(3/2) - C*a^5*c*e^6*x + 14*C*d*e^5*x*(-a^3*c)^(3/2
) - 2*A*a^2*c^4*d^5*e + 30*A*a^4*c^2*d*e^5 - 14*C*a^3*c^3*d^5*e + 3*A*a^4*c
^2*e^6*x + C*a^2*c^4*d^6*x - C*a*c^3*d^6*(-a^3*c)^(1/2) - 36*A*a^3*c^3*d^3*
e^3 + 22*B*a^3*c^3*d^4*e^2 - 36*B*a^4*c^2*d^2*e^4 + 36*C*a^4*c^2*d^3*e^3 -
14*C*a^5*c*d*e^5 + A*a*c^5*d^6*x + 5*A*a^2*c^4*d^4*e^2*x - 57*A*a^3*c^3*d^2
*e^4*x + 44*B*a^3*c^3*d^3*e^3*x - 31*C*a^3*c^3*d^4*e^2*x + 31*C*a^4*c^2*d^2
*e^4*x - 5*A*a*c^3*d^4*e^2*(-a^3*c)^(1/2) + 57*A*a^2*c^2*d^2*e^4*(-a^3*c)^(
1/2) - 44*B*a^2*c^2*d^3*e^3*(-a^3*c)^(1/2) + 31*C*a^2*c^2*d^4*e^2*(-a^3*c)^
(1/2) - 2*B*a^2*c^4*d^5*e*x - 18*B*a^4*c^2*d*e^5*x + 2*B*a*c^3*d^5*e*(-a^3*
c)^(1/2) - 2*A*c^4*d^5*e*x*(-a^3*c)^(1/2) - 36*B*a^2*c^2*d^2*e^4*x*(-a^3*c)
^(1/2) + 36*C*a^2*c^2*d^3*e^3*x*(-a^3*c)^(1/2) - 14*C*a*c^3*d^5*e*x*(-a^3*c
)^(1/2) - 36*A*a*c^3*d^3*e^3*x*(-a^3*c)^(1/2) + 30*A*a^2*c^2*d*e^5*x*(-a^3*
c)^(1/2) + 22*B*a*c^3*d^4*e^2*x*(-a^3*c)^(1/2))*(c^2*(a*((C*d^4*(-a^3*c)^(1
/2))/4 + (3*A*d^2*e^2*(-a^3*c)^(1/2))/2 - (B*d^3*e*(-a^3*c)^(1/2))/2) + a^3
```

```
*(2*A*d*e^3 - (3*B*d^2*e^2)/2 + C*d^3*e)) - c*(a^2*((3*A*e^4*(-a^3*c)^(1/2)
)/4 + (3*C*d^2*e^2*(-a^3*c)^(1/2))/2 - (3*B*d*e^3*(-a^3*c)^(1/2))/2) - a^4*
((B*e^4)/2 - C*d*e^3)) + (A*c^3*d^4*(-a^3*c)^(1/2))/4 + (C*a^3*e^4*(-a^3*c)
^(1/2))/4))/(a^6*c*e^6 + a^3*c^4*d^6 + 3*a^4*c^3*d^4*e^2 + 3*a^5*c^2*d^2*e^
4) + (log(3*A*e^6*(-a^3*c)^(3/2) - A*c^4*d^6*(-a^3*c)^(1/2) + C*a^4*e^6*(-a
^3*c)^(1/2) + 31*C*d^2*e^4*(-a^3*c)^(3/2) - 6*B*a^5*c*e^6 - 18*B*d*e^5*(-a^
3*c)^(3/2) - 6*B*e^6*x*(-a^3*c)^(3/2) + C*a^5*c*e^6*x + 14*C*d*e^5*x*(-a^3*
c)^(3/2) + 2*A*a^2*c^4*d^5*e - 30*A*a^4*c^2*d*e^5 + 14*C*a^3*c^3*d^5*e - 3*
A*a^4*c^2*e^6*x - C*a^2*c^4*d^6*x - C*a*c^3*d^6*(-a^3*c)^(1/2) + 36*A*a^3*c
^3*d^3*e^3 - 22*B*a^3*c^3*d^4*e^2 + 36*B*a^4*c^2*d^2*e^4 - 36*C*a^4*c^2*d^3
*e^3 + 14*C*a^5*c*d*e^5 - A*a*c^5*d^6*x - 5*A*a^2*c^4*d^4*e^2*x + 57*A*a^3*
c^3*d^2*e^4*x - 44*B*a^3*c^3*d^3*e^3*x + 31*C*a^3*c^3*d^4*e^2*x - 31*C*a^4*
c^2*d^2*e^4*x - 5*A*a*c^3*d^4*e^2*(-a^3*c)^(1/2) + 57*A*a^2*c^2*d^2*e^4*(-a
^3*c)^(1/2) - 44*B*a^2*c^2*d^3*e^3*(-a^3*c)^(1/2) + 31*C*a^2*c^2*d^4*e^2*(-
a^3*c)^(1/2) + 2*B*a^2*c^4*d^5*e*x + 18*B*a^4*c^2*d*e^5*x + 2*B*a*c^3*d^5*e
*(-a^3*c)^(1/2) - 2*A*c^4*d^5*e*x*(-a^3*c)^(1/2) - 36*B*a^2*c^2*d^2*e^4*x*(
-a^3*c)^(1/2) + 36*C*a^2*c^2*d^3*e^3*x*(-a^3*c)^(1/2) - 14*C*a*c^3*d^5*e*x*
(-a^3*c)^(1/2) - 36*A*a*c^3*d^3*e^3*x*(-a^3*c)^(1/2) + 30*A*a^2*c^2*d*e^5*x
*(-a^3*c)^(1/2) + 22*B*a*c^3*d^4*e^2*x*(-a^3*c)^(1/2))*(c^2*(a*((C*d^4*(-a^
3*c)^(1/2))/4 + (3*A*d^2*e^2*(-a^3*c)^(1/2))/2 - (B*d^3*e*(-a^3*c)^(1/2))/2
) - a^3*(2*A*d*e^3 - (3*B*d^2*e^2)/2 + C*d^3*e)) - c*(a^2*((3*A*e^4*(-a^3*c
)^(1/2))/4 + (3*C*d^2*e^2*(-a^3*c)^(1/2))/2 - (3*B*d*e^3*(-a^3*c)^(1/2))/2)
 + a^4*((B*e^4)/2 - C*d*e^3)) + (A*c^3*d^4*(-a^3*c)^(1/2))/4 + (C*a^3*e^4*(
-a^3*c)^(1/2))/4))/(a^6*c*e^6 + a^3*c^4*d^6 + 3*a^4*c^3*d^4*e^2 + 3*a^5*c^2
*d^2*e^4) + (log(d + e*x)*(a*(B*e^4 - 2*C*d*e^3) + c*(4*A*d*e^3 - 3*B*d^2*e
^2 + 2*C*d^3*e)))/(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)
```

3.56 $\int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)^2}\,dx$

Optimal. Leaf size=524

$$-\frac{e(Cd^2-Bde+Ae^2)}{2\left(cd^2+ae^2\right)^2(d+ex)^2}+\frac{e(ae^2(2Cd-Be)-cd(2Cd^2-e(3Bd-4Ae)))}{\left(cd^2+ae^2\right)^3(d+ex)}-\frac{a(Bcd(cd^2-3ae^2)-(Ac-a$$

```
[Out] -1/2*e*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^2/(e*x+d)^2+e*(a*e^2*(-B*e+2*C*d)-
c*d*(2*C*d^2-e*(-4*A*e+3*B*d)))/(a*e^2+c*d^2)^3/(e*x+d)+1/2*(-a*(B*c*d*(-3*
a*e^2+c*d^2)-(A*c-C*a)*e*(-a*e^2+3*c*d^2))+c*(A*c*d*(-3*a*e^2+c*d^2)-a*(c*d
^2*(-3*B*e+C*d)-a*e^2*(-B*e+3*C*d)))*x)/a/(a*e^2+c*d^2)^3/(c*x^2+a)+e*(a^2*
C*e^4+c^2*d^2*(3*C*d^2-2*e*(-5*A*e+3*B*d))-2*a*c*e^2*(4*C*d^2-e*(-A*e+3*B*d
)))*ln(e*x+d)/(a*e^2+c*d^2)^4-1/2*e*(a^2*C*e^4+c^2*d^2*(3*C*d^2-2*e*(-5*A*e
+3*B*d))-2*a*c*e^2*(4*C*d^2-e*(-A*e+3*B*d)))*ln(c*x^2+a)/(a*e^2+c*d^2)^4+1/
2*(A*c*d*(-15*a^2*e^4+10*a*c*d^2*e^2+c^2*d^4)-a*(2*a*c*d^2*e^2*(-9*B*e+7*C*
d)-c^2*d^4*(-3*B*e+C*d)-3*a^2*e^4*(-B*e+3*C*d)))*arctan(x*c^(1/2)/a^(1/2))*
c^(1/2)/a^(3/2)/(a*e^2+c*d^2)^4
```

Rubi [A]
time = 1.05, antiderivative size = 524, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 1643, 649, 211, 266}

$$-\frac{e\log\left(a+cx^2\right)\left(a^2Ce^4-2ace^2(4Cd^2-e(3Bd-Ae))+c^2(3Cd^4-2d^2e(3Bd-5Ae))\right)}{2\left(ae^2+cd^2\right)^4}+\frac{e\log(d+ex)\left(a^2Ce^4-2ace^2(4Cd^2-e(3Bd-Ae))+c^2(3Cd^4-2d^2e(3Bd-5Ae))\right)}{\left(ae^2+cd^2\right)^4}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd(-15a^2e^4+10acd^2e^2+c^2d^4)-a(-3a^2e^4(3Cd-Be)+2acd^2e^2(7Cd-9Be)-c^2d^4(Cd-3Be))\right)}{2a^{3/2}\left(ae^2+cd^2\right)^4}-\frac{a(Bcd(cd^2-3ae^2)-e(Ac-aC)(3cd^2-ae^2))-cx(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3Cd-Be)))}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)^3}-\frac{e(Ae^2-Bde+Cd^2)}{2(d+ex)^2\left(ae^2+cd^2\right)^2}-\frac{e(-ae^2(2Cd-Be)-cde(3Bd-4Ae)+2cCd^3)}{(d+ex)\left(ae^2+cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^2),x]
```

```
[Out] -1/2*(e*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^2*(d + e*x)^2) - (e*(2*c*
C*d^3 - c*d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e)))/((c*d^2 + a*e^2)^3*(d
 + e*x)) - (a*(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2)) -
 c*(A*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e))
)*x)/(2*a*(c*d^2 + a*e^2)^3*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c^2*d^4 + 10*a*
c*d^2*e^2 - 15*a^2*e^4) - a*(2*a*c*d^2*e^2*(7*C*d - 9*B*e) - c^2*d^4*(C*d -
 3*B*e) - 3*a^2*e^4*(3*C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)
*(c*d^2 + a*e^2)^4) + (e*(a^2*C*e^4 + c^2*(3*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e
)) - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e)))*Log[d + e*x])/(c*d^2 + a*e^2)^4
 - (e*(a^2*C*e^4 + c^2*(3*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e)) - 2*a*c*e^2*(4*C
*d^2 - e*(3*B*d - A*e)))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^2}\,dx = -\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd\cdots}{2a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)}$$

$$= -\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd\cdots}{2a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)}$$

$$= -\frac{e(Cd^2-Bde+Ae^2)}{2\left(cd^2+ae^2\right)^2(d+ex)^2} - \frac{e(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be))}{\left(cd^2+ae^2\right)^3(d+ex)} - \cdots$$

$$= -\frac{e(Cd^2-Bde+Ae^2)}{2\left(cd^2+ae^2\right)^2(d+ex)^2} - \frac{e(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be))}{\left(cd^2+ae^2\right)^3(d+ex)} - \cdots$$

$$= -\frac{e(Cd^2-Bde+Ae^2)}{2\left(cd^2+ae^2\right)^2(d+ex)^2} - \frac{e(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be))}{\left(cd^2+ae^2\right)^3(d+ex)} - \cdots$$

Mathematica [**A**]
time = 0.42, size = 466, normalized size = 0.89

$$\frac{-\frac{e(cd^2+ae^2)^2(Cd^2+e(-Bd+Ae))}{(d+ex)^2}-\frac{2e(cd^2+ae^2)(2cCd^3+cde(-3Bd+4Ae)+ae^2(-2Cd+Be))}{d+ex}+\frac{(cd^2+ae^2)(a^3Ce^3+Ac^3d^3x-ac^2d(Cd^2x+Bd(d-3ex)+3Ae(-d+ex))-a^2ce(3Cd(d-ex)+e(-3Bd+Ae+Bex)))}{a(a+cx^2)}+\frac{\sqrt{c}\,(Acd(c^2d^4+10acd^2e^2-15a^2e^4)+a(-2acd^2e^2(7Cd-9Be)+c^2d^4(Cd-3Be)-3a^2e^4(-3Cd+Be)))\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{a^{3/2}}+2(a^2Ce^5-2ace^3(4Cd^2+e(-3Bd+Ae))+c^2d^2e(3Cd^2+2e(-3Bd+5Ae)))\log(d+ex)-(a^2Ce^5-2ace^3(4Cd^2+e(-3Bd+Ae))+c^2d^2e(3Cd^2+2e(-3Bd+5Ae)))\log(a+cx^2)}{2\left(cd^2+ae^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^2),x]
```

```
[Out] (-((e*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2) - (2*e*(c*
d^2 + a*e^2)*(2*c*C*d^3 + c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e)))/(
d + e*x) + ((c*d^2 + a*e^2)*(a^3*C*e^3 + A*c^3*d^3*x - a*c^2*d*(C*d^2*x + B
*d*(d - 3*e*x) + 3*A*e*(-d + e*x)) - a^2*c*e*(3*C*d*(d - e*x) + e*(-3*B*d +
 A*e + B*e*x))))/(a*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c^2*d^4 + 10*a*c*d^2*e^
2 - 15*a^2*e^4) + a*(-2*a*c*d^2*e^2*(7*C*d - 9*B*e) + c^2*d^4*(C*d - 3*B*e)
 - 3*a^2*e^4*(-3*C*d + B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(3/2) + 2*(a^2
*C*e^5 - 2*a*c*e^3*(4*C*d^2 + e*(-3*B*d + A*e)) + c^2*d^2*e*(3*C*d^2 + 2*e*
(-3*B*d + 5*A*e)))*Log[d + e*x] - (a^2*C*e^5 - 2*a*c*e^3*(4*C*d^2 + e*(-3*B
*d + A*e)) + c^2*d^2*e*(3*C*d^2 + 2*e*(-3*B*d + 5*A*e)))*Log[a + c*x^2])/(2
*(c*d^2 + a*e^2)^4)
```

Maple [**A**]
time = 0.23, size = 643, normalized size = 1.23

method	result

default	$-\frac{e(4d\,e^{2}cA+Ba\,e^{3}-3Bc\,d^{2}e-2Cad\,e^{2}+2Cc\,d^{3})}{(a\,e^{2}+c\,d^{2})^{3}(ex+d)}-\frac{e(A\,e^{2}-Bde+C\,d^{2})}{2(a\,e^{2}+c\,d^{2})^{2}(ex+d)^{2}}-\frac{e(2Aac\,e^{4}-10A\,c^{2}d^{2}e^{2}-6Bacd\,e^{3}+6B\,c^{2}d^{3}e-a^{2}C\,e}{(a\,e^{2}+c\,d^{2})^{4}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -e*(4*A*c*d*e^2+B*a*e^3-3*B*c*d^2*e-2*C*a*d*e^2+2*C*c*d^3)/(a*e^2+c*d^2)^3/
(e*x+d)-1/2*e*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^2/(e*x+d)^2-e*(2*A*a*c*e^4-
10*A*c^2*d^2*e^2-6*B*a*c*d*e^3+6*B*c^2*d^3*e-C*a^2*e^4+8*C*a*c*d^2*e^2-3*C*
c^2*d^4)/(a*e^2+c*d^2)^4*ln(e*x+d)-c/(a*e^2+c*d^2)^4*((1/2*(3*A*a^2*c*d*e^4
+2*A*a*c^2*d^3*e^2-A*c^3*d^5+B*a^3*e^5-2*B*a^2*c*d^2*e^3-3*B*a*c^2*d^4*e-3*
C*a^3*d*e^4-2*C*a^2*c*d^3*e^2+C*a*c^2*d^5)/a*x+1/2*(A*a^2*c*e^5-2*A*a*c^2*d
^2*e^3-3*A*c^3*d^4*e-3*B*a^2*c*d*e^4-2*B*a*c^2*d^3*e^2+B*c^3*d^5-C*a^3*e^5+
2*C*a^2*c*d^2*e^3+3*C*a*c^2*d^4*e)/c)/(c*x^2+a)+1/2/a*(1/2*(-4*A*a^2*c*e^5+
20*A*a*c^2*d^2*e^3+12*B*a^2*c*d*e^4-12*B*a*c^2*d^3*e^2+2*C*a^3*e^5-16*C*a^2
*c*d^2*e^3+6*C*a*c^2*d^4*e)/c*ln(c*x^2+a)+(15*A*a^2*c*d*e^4-10*A*a*c^2*d^3*
e^2-A*c^3*d^5+3*B*a^3*e^5-18*B*a^2*c*d^2*e^3+3*B*a*c^2*d^4*e-9*C*a^3*d*e^4+
14*C*a^2*c*d^3*e^2-C*a*c^2*d^5)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Maxima [A]
time = 0.58, size = 1007, normalized size = 1.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*C*c^2*d^4*e - 6*B*c^2*d^3*e^2 + 6*B*a*c*d*e^4 + C*a^2*e^5 - 2*A*a*c
*e^5 - 2*(4*C*a*c*e^3 - 5*A*c^2*e^3)*d^2)*log(c*x^2 + a)/(c^4*d^8 + 4*a*c^3
*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) + (3*C*c^2*d^4*e
- 6*B*c^2*d^3*e^2 + 6*B*a*c*d*e^4 + C*a^2*e^5 - 2*A*a*c*e^5 - 2*(4*C*a*c*e^
3 - 5*A*c^2*e^3)*d^2)*log(x*e + d)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d
^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) - 1/2*(3*B*a*c^3*d^4*e - 18*B*a^2*c^2*d
^2*e^3 - (C*a*c^3 + A*c^4)*d^5 + 3*B*a^3*c*e^5 + 2*(7*C*a^2*c^2*e^2 - 5*A*a
*c^3*e^2)*d^3 - 3*(3*C*a^3*c*e^4 - 5*A*a^2*c^2*e^4)*d)*arctan(c*x/sqrt(a*c)
)/((a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a
^5*e^8)*sqrt(a*c)) - 1/2*(B*a*c^2*d^5 - 10*B*a^2*c*d^3*e^2 + B*a^3*d*e^4 +
(8*C*a^2*c*e - 3*A*a*c^2*e)*d^4 + A*a^3*e^5 - (9*B*a*c^2*d^2*e^3 - 3*B*a^2*
```

```
c*e^5 - (5*C*a*c^2*e^2 - A*c^3*e^2)*d^3 + (7*C*a^2*c*e^4 - 11*A*a*c^2*e^4)*
d)*x^3 - 2*(2*C*a^3*e^3 - 5*A*a^2*c*e^3)*d^2 - (12*B*a*c^2*d^3*e^2 - (7*C*a
*c^2*e - 2*A*c^3*e)*d^4 + C*a^3*e^5 - 2*A*a^2*c*e^5 + 6*(C*a^2*c*e^3 - 2*A*
a*c^2*e^3)*d^2)*x^2 - (B*a*c^2*d^4*e + 11*B*a^2*c*d^2*e^3 - (C*a*c^2 - A*c^
3)*d^5 - 2*B*a^3*e^5 - (7*C*a^2*c*e^2 - 3*A*a*c^2*e^2)*d^3 + 2*(3*C*a^3*e^4
 - 5*A*a^2*c*e^4)*d)*x)/(a^2*c^3*d^8 + 3*a^3*c^2*d^6*e^2 + 3*a^4*c*d^4*e^4
+ a^5*d^2*e^6 + (a*c^4*d^6*e^2 + 3*a^2*c^3*d^4*e^4 + 3*a^3*c^2*d^2*e^6 + a^
4*c*e^8)*x^4 + 2*(a*c^4*d^7*e + 3*a^2*c^3*d^5*e^3 + 3*a^3*c^2*d^3*e^5 + a^4
*c*d*e^7)*x^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*
c*d^2*e^6 + a^5*e^8)*x^2 + 2*(a^2*c^3*d^7*e + 3*a^3*c^2*d^5*e^3 + 3*a^4*c*d
^3*e^5 + a^5*d*e^7)*x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.29, size = 957, normalized size = 1.83

$$-\frac{(3Cc^2d^4e - 6Bc^2d^3e^2 - 8Cacd^2e^3 + 10Ac^2d^2e^3 + 6Bacde^4 + Ca^2e^5 - 2Aace^5)\log(cx^2+a)}{2(c^4d^8 + 4ac^3d^6e^2 + 6a^2c^2d^4e^4 + 4a^3cd^2e^6 + a^4e^8)} + \frac{(3Cc^2d^4e^2 - 6Bc^2d^3e^3 - 8Cacd^2e^4 + 10Ac^2d^2e^4 + 6Bacde^5 + Ca^2e^6 - 2Aace^6)\log(|xe+d|)}{c^4d^8e + 4ac^3d^6e^3 + 6a^2c^2d^4e^5 + 4a^3cd^2e^7 + a^4e^9} + \frac{(Cac^3d^5 + Ac^4d^5 - 3Bac^3d^4e - 14Ca^2c^2d^3e^2 + 10Aac^3d^3e^2 + 18Ba^2c^2d^2e^3 + 9Ca^3cde^4 - 15Aa^2c^2de^4 - 3Ba^3ce^5)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2(ac^4d^8 + 4a^2c^3d^6e^2 + 6a^3c^2d^4e^4 + 4a^4cd^2e^6 + a^5e^8)\sqrt{ac}} - \frac{Bac^3d^7 + 8Ca^2c^2d^6e - 3Aac^3d^6e - 9Ba^2c^2d^5e^2 + 4Ca^3cd^4e^3 + 7Aa^2c^2d^4e^3 - 9Ba^3cd^3e^4 - 4Ca^4d^2e^5 + 11Aa^3cd^2e^5 + Ba^4de^6 + Aa^4e^7 + (5Cac^3d^5e^2 - Ac^4d^5e^2 - 9Bac^3d^4e^3 - 2Ca^2c^2d^3e^4 + 10Aac^3d^3e^4 - 6Ba^2c^2d^2e^5 - 7Ca^3cde^6 + 11Aa^2c^2de^6 + 3Ba^3ce^7)x^3 + (7Cac^3d^6e - 2Ac^4d^6e - 12Bac^3d^5e^2 + Ca^2c^2d^4e^3 + 10Aac^3d^4e^3 - 12Ba^2c^2d^3e^4 - 7Ca^3cd^2e^5 + 14Aa^2c^2d^2e^5 - Ca^4e^7 + 2Aa^3ce^7)x^2 + (Cac^3d^7 - Ac^4d^7 - Bac^3d^6e + 8Ca^2c^2d^5e^2 - 4Aac^3d^5e^2 - 12Ba^2c^2d^4e^3 + Ca^3cd^3e^4 + 7Aa^2c^2d^3e^4 - 9Ba^3cd^2e^5 - 6Ca^4de^6 + 10Aa^3cde^6 + 2Ba^4e^7)x}{2(cd^2+ae^2)^4(cx^2+a)(xe+d)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*C*c^2*d^4*e - 6*B*c^2*d^3*e^2 - 8*C*a*c*d^2*e^3 + 10*A*c^2*d^2*e^3
+ 6*B*a*c*d*e^4 + C*a^2*e^5 - 2*A*a*c*e^5)*log(c*x^2 + a)/(c^4*d^8 + 4*a*c^
3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) + (3*C*c^2*d^4*e
^2 - 6*B*c^2*d^3*e^3 - 8*C*a*c*d^2*e^4 + 10*A*c^2*d^2*e^4 + 6*B*a*c*d*e^5 +
 C*a^2*e^6 - 2*A*a*c*e^6)*log(abs(x*e + d))/(c^4*d^8*e + 4*a*c^3*d^6*e^3 +
6*a^2*c^2*d^4*e^5 + 4*a^3*c*d^2*e^7 + a^4*e^9) + 1/2*(C*a*c^3*d^5 + A*c^4*d
```

```
^5 - 3*B*a*c^3*d^4*e - 14*C*a^2*c^2*d^3*e^2 + 10*A*a*c^3*d^3*e^2 + 18*B*a^2
*c^2*d^2*e^3 + 9*C*a^3*c*d*e^4 - 15*A*a^2*c^2*d*e^4 - 3*B*a^3*c*e^5)*arctan
(c*x/sqrt(a*c))/((a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4
*c*d^2*e^6 + a^5*e^8)*sqrt(a*c)) - 1/2*(B*a*c^3*d^7 + 8*C*a^2*c^2*d^6*e - 3
*A*a*c^3*d^6*e - 9*B*a^2*c^2*d^5*e^2 + 4*C*a^3*c*d^4*e^3 + 7*A*a^2*c^2*d^4*
e^3 - 9*B*a^3*c*d^3*e^4 - 4*C*a^4*d^2*e^5 + 11*A*a^3*c*d^2*e^5 + B*a^4*d*e^
6 + A*a^4*e^7 + (5*C*a*c^3*d^5*e^2 - A*c^4*d^5*e^2 - 9*B*a*c^3*d^4*e^3 - 2*
C*a^2*c^2*d^3*e^4 + 10*A*a*c^3*d^3*e^4 - 6*B*a^2*c^2*d^2*e^5 - 7*C*a^3*c*d*
e^6 + 11*A*a^2*c^2*d*e^6 + 3*B*a^3*c*e^7)*x^3 + (7*C*a*c^3*d^6*e - 2*A*c^4*
d^6*e - 12*B*a*c^3*d^5*e^2 + C*a^2*c^2*d^4*e^3 + 10*A*a*c^3*d^4*e^3 - 12*B*
a^2*c^2*d^3*e^4 - 7*C*a^3*c*d^2*e^5 + 14*A*a^2*c^2*d^2*e^5 - C*a^4*e^7 + 2*
A*a^3*c*e^7)*x^2 + (C*a*c^3*d^7 - A*c^4*d^7 - B*a*c^3*d^6*e + 8*C*a^2*c^2*d
^5*e^2 - 4*A*a*c^3*d^5*e^2 - 12*B*a^2*c^2*d^4*e^3 + C*a^3*c*d^3*e^4 + 7*A*a
^2*c^2*d^3*e^4 - 9*B*a^3*c*d^2*e^5 - 6*C*a^4*d*e^6 + 10*A*a^3*c*d*e^6 + 2*B
*a^4*e^7)*x)/((c*d^2 + a*e^2)^4*(c*x^2 + a)*(x*e + d)^2*a)
```

Mupad [B]
time = 14.48, size = 2828, normalized size = 5.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^2*(d + e*x)^3),x)
```

```
[Out] (log(C*c^2*d^7*(-a^3*c)^(3/2) - 3*B*a^6*e^7*(-a^3*c)^(1/2) - 6*C*a^8*e^7 +
12*A*a^7*c*e^7 - 3*B*a^7*c*e^7*x + 2*A*a^4*c^4*d^6*e + 20*C*a^5*c^3*d^6*e +
 72*C*a^7*c*d^2*e^5 - A*a^3*c^5*d^7*x - C*a^4*c^4*d^7*x + 39*A*a^2*d*e^6*(-
a^3*c)^(3/2) + 21*C*a^6*d*e^6*(-a^3*c)^(1/2) - 3*B*c^2*d^6*e*(-a^3*c)^(3/2)
 + 12*A*a^2*e^7*x*(-a^3*c)^(3/2) + 6*C*a^6*e^7*x*(-a^3*c)^(1/2) + 80*A*a^5*
c^3*d^4*e^3 - 102*A*a^6*c^2*d^2*e^5 - 42*B*a^5*c^3*d^5*e^2 + 108*B*a^6*c^2*
d^3*e^4 - 94*C*a^6*c^2*d^4*e^3 - A*a^2*c^4*d^7*(-a^3*c)^(1/2) - 93*B*a^2*d^
2*e^5*(-a^3*c)^(3/2) + 9*A*c^2*d^5*e^2*(-a^3*c)^(3/2) + 119*C*a^2*d^3*e^4*(
-a^3*c)^(3/2) - 42*B*a^7*c*d*e^6 - 9*A*a^4*c^4*d^5*e^2*x + 145*A*a^5*c^3*d^
3*e^4*x - 93*B*a^5*c^3*d^4*e^3*x + 93*B*a^6*c^2*d^2*e^5*x + 51*C*a^5*c^3*d^
5*e^2*x - 119*C*a^6*c^2*d^3*e^4*x + 80*A*c^2*d^4*e^3*x*(-a^3*c)^(3/2) + 72*
C*a^2*d^2*e^5*x*(-a^3*c)^(3/2) - 42*B*c^2*d^5*e^2*x*(-a^3*c)^(3/2) + 21*C*a
^7*c*d*e^6*x - 39*A*a^6*c^2*d*e^6*x + 3*B*a^4*c^4*d^6*e*x - 145*A*a*c*d^3*e
^4*(-a^3*c)^(3/2) + 93*B*a*c*d^4*e^3*(-a^3*c)^(3/2) - 51*C*a*c*d^5*e^2*(-a^
3*c)^(3/2) - 42*B*a^2*d*e^6*x*(-a^3*c)^(3/2) + 20*C*c^2*d^6*e*x*(-a^3*c)^(3
/2) - 102*A*a*c*d^2*e^5*x*(-a^3*c)^(3/2) + 108*B*a*c*d^3*e^4*x*(-a^3*c)^(3/
2) - 94*C*a*c*d^4*e^3*x*(-a^3*c)^(3/2) - 2*A*a^2*c^4*d^6*e*x*(-a^3*c)^(1/2)
)*(e^2*(3*B*a^3*c^2*d^3 + (5*A*a*c^2*d^3*(-a^3*c)^(1/2))/2 - (7*C*a^2*c*d^3
*(-a^3*c)^(1/2))/2) + e^3*(4*C*a^4*c*d^2 - 5*A*a^3*c^2*d^2 + (9*B*a^2*c*d^2
*(-a^3*c)^(1/2))/2) - e^4*(3*B*a^4*c*d - (9*C*a^3*d*(-a^3*c)^(1/2))/4 + (15
*A*a^2*c*d*(-a^3*c)^(1/2))/4) - e*((3*C*a^3*c^2*d^4)/2 + (3*B*a*c^2*d^4*(-a
```

```
^3*c)^(1/2))/4) - e^5*((C*a^5)/2 + (3*B*a^3*(-a^3*c)^(1/2))/4 - A*a^4*c) +
(A*c^3*d^5*(-a^3*c)^(1/2))/4 + (C*a*c^2*d^5*(-a^3*c)^(1/2))/4))/(a^7*e^8 +
a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4) - (l
og(3*B*a^6*e^7*(-a^3*c)^(1/2) - 6*C*a^8*e^7 - C*c^2*d^7*(-a^3*c)^(3/2) + 12
*A*a^7*c*e^7 - 3*B*a^7*c*e^7*x + 2*A*a^4*c^4*d^6*e + 20*C*a^5*c^3*d^6*e + 7
2*C*a^7*c*d^2*e^5 - A*a^3*c^5*d^7*x - C*a^4*c^4*d^7*x - 39*A*a^2*d*e^6*(-a^
3*c)^(3/2) - 21*C*a^6*d*e^6*(-a^3*c)^(1/2) + 3*B*c^2*d^6*e*(-a^3*c)^(3/2) -
 12*A*a^2*e^7*x*(-a^3*c)^(3/2) - 6*C*a^6*e^7*x*(-a^3*c)^(1/2) + 80*A*a^5*c^
3*d^4*e^3 - 102*A*a^6*c^2*d^2*e^5 - 42*B*a^5*c^3*d^5*e^2 + 108*B*a^6*c^2*d^
3*e^4 - 94*C*a^6*c^2*d^4*e^3 + A*a^2*c^4*d^7*(-a^3*c)^(1/2) + 93*B*a^2*d^2*
e^5*(-a^3*c)^(3/2) - 9*A*c^2*d^5*e^2*(-a^3*c)^(3/2) - 119*C*a^2*d^3*e^4*(-a
^3*c)^(3/2) - 42*B*a^7*c*d*e^6 - 9*A*a^4*c^4*d^5*e^2*x + 145*A*a^5*c^3*d^3*
e^4*x - 93*B*a^5*c^3*d^4*e^3*x + 93*B*a^6*c^2*d^2*e^5*x + 51*C*a^5*c^3*d^5*
e^2*x - 119*C*a^6*c^2*d^3*e^4*x - 80*A*c^2*d^4*e^3*x*(-a^3*c)^(3/2) - 72*C*
a^2*d^2*e^5*x*(-a^3*c)^(3/2) + 42*B*c^2*d^5*e^2*x*(-a^3*c)^(3/2) + 21*C*a^7
*c*d*e^6*x - 39*A*a^6*c^2*d*e^6*x + 3*B*a^4*c^4*d^6*e*x + 145*A*a*c*d^3*e^4
*(-a^3*c)^(3/2) - 93*B*a*c*d^4*e^3*(-a^3*c)^(3/2) + 51*C*a*c*d^5*e^2*(-a^3*
c)^(3/2) + 42*B*a^2*d*e^6*x*(-a^3*c)^(3/2) - 20*C*c^2*d^6*e*x*(-a^3*c)^(3/2
) + 102*A*a*c*d^2*e^5*x*(-a^3*c)^(3/2) - 108*B*a*c*d^3*e^4*x*(-a^3*c)^(3/2)
 + 94*C*a*c*d^4*e^3*x*(-a^3*c)^(3/2) + 2*A*a^2*c^4*d^6*e*x*(-a^3*c)^(1/2))*
(e^3*(5*A*a^3*c^2*d^2 - 4*C*a^4*c*d^2 + (9*B*a^2*c*d^2*(-a^3*c)^(1/2))/2) -
 e^2*(3*B*a^3*c^2*d^3 - (5*A*a*c^2*d^3*(-a^3*c)^(1/2))/2 + (7*C*a^2*c*d^3*(
-a^3*c)^(1/2))/2) + e^4*(3*B*a^4*c*d + (9*C*a^3*d*(-a^3*c)^(1/2))/4 - (15*A
*a^2*c*d*(-a^3*c)^(1/2))/4) + e*((3*C*a^3*c^2*d^4)/2 - (3*B*a*c^2*d^4*(-a^3
*c)^(1/2))/4) - e^5*((3*B*a^3*(-a^3*c)^(1/2))/4 - (C*a^5)/2 + A*a^4*c) + (A
*c^3*d^5*(-a^3*c)^(1/2))/4 + (C*a*c^2*d^5*(-a^3*c)^(1/2))/4))/(a^7*e^8 + a^
3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4) - ((A*
a^2*e^5 + B*c^2*d^5 + B*a^2*d*e^4 - 3*A*c^2*d^4*e - 4*C*a^2*d^2*e^3 + 8*C*a
*c*d^4*e + 10*A*a*c*d^2*e^3 - 10*B*a*c*d^3*e^2)/(2*(a*e^2 + c*d^2)*(a^2*e^4
 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x^3*(3*B*a^2*c*e^5 - A*c^3*d^3*e^2 - 9*B*a*
c^2*d^2*e^3 + 5*C*a*c^2*d^3*e^2 + 11*A*a*c^2*d*e^4 - 7*C*a^2*c*d*e^4))/(2*a
*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*(A*c^3*d^5 -
 2*B*a^3*e^5 - C*a*c^2*d^5 + 6*C*a^3*d*e^4 + 3*A*a*c^2*d^3*e^2 + 11*B*a^2*c
*d^2*e^3 - 7*C*a^2*c*d^3*e^2 - 10*A*a^2*c*d*e^4 + B*a*c^2*d^4*e))/(2*a*(a*e
^2 + c*d^2)*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x^2*(C*a^3*e^5 - 2*A*a^
2*c*e^5 + 2*A*c^3*d^4*e - 12*A*a*c^2*d^2*e^3 + 12*B*a*c^2*d^3*e^2 + 6*C*a^2
*c*d^2*e^3 - 7*C*a*c^2*d^4*e))/(2*a*(a*e^2 + c*d^2)*(a^2*e^4 + c^2*d^4 + 2*
a*c*d^2*e^2)))/(a*d^2 + x^2*(a*e^2 + c*d^2) + c*e^2*x^4 + 2*a*d*e*x + 2*c*d
*e*x^3) + (log(d + e*x)*(c^2*(10*A*d^2*e^3 - 6*B*d^3*e^2 + 3*C*d^4*e) - c*(
2*A*a*e^5 - 6*B*a*d*e^4 + 8*C*a*d^2*e^3) + C*a^2*e^5))/(a^4*e^8 + c^4*d^8 +
 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)
```

3.57 $\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^3} \, dx$

Optimal. Leaf size=209

$$-\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\,(a+cx^2)^2}-\frac{(d+ex)\,(ae(3Acd+5aCd+3aBe)-(3Ac^2d^2-a(4aCe^2-cd(Cd+3Be)$$

```
[Out] -1/4*(a*B-(A*c-C*a)*x)*(e*x+d)^3/a/c/(c*x^2+a)^2-1/8*(e*x+d)*(a*e*(3*A*c*d+
3*B*a*e+5*C*a*d)-(3*A*c^2*d^2-a*(4*a*C*e^2-c*d*(3*B*e+C*d)))*x)/a^2/c^2/(c*
x^2+a)+1/8*(3*A*c*d*(a*e^2+c*d^2)+a*(3*a*e^2*(B*e+3*C*d)+c*d^2*(3*B*e+C*d))
)*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/c^(5/2)+1/2*C*e^3*ln(c*x^2+a)/c^3
```

Rubi [A]
time = 0.18, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1659, 833, 649, 211, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(cd^2(3aBe+aCd+3Acd)+3ae^2(aBe+3aCd+Acd))}{8a^{5/2}c^{5/2}}-\frac{(d+ex)\,(ae(3aBe+5aCd+3Acd)-x(3Ac^2d^2-a(4aCe^2-cd(3Be+Cd))))}{8a^2c^2\,(a+cx^2)}-\frac{(d+ex)^3(aB-x(Ac-aC))}{4ac\,(a+cx^2)^2}+\frac{Ce^3\log(a+cx^2)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] -1/4*((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(a*c*(a + c*x^2)^2) - ((d + e*x)*(
a*e*(3*A*c*d + 5*a*C*d + 3*a*B*e) - (3*A*c^2*d^2 - a*(4*a*C*e^2 - c*d*(C*d
+ 3*B*e)))*x))/(8*a^2*c^2*(a + c*x^2)) + ((3*a*e^2*(A*c*d + 3*a*C*d + a*B*e
) + c*d^2*(3*A*c*d + a*C*d + 3*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5
/2)*c^(5/2)) + (C*e^3*Log[a + c*x^2])/(2*c^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} - \frac{\int \frac{(d+ex)^2(-3Acd-aCd-3aBe-4aCex)}{\left(a+cx^2\right)^2}\,dx}{4ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} - \frac{(d+ex)\left(ae(3Acd+5aCd+3aBe)\right.}{8a^2c}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} - \frac{(d+ex)\left(ae(3Acd+5aCd+3aBe)\right.}{8a^2c}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} - \frac{(d+ex)\left(ae(3Acd+5aCd+3aBe)\right.}{8a^2c}$$

Mathematica [A]
time = 0.17, size = 281, normalized size = 1.34

$$\frac{\frac{-2a^3Ce^3+2Ac^3d^3x-2ac^2d\left(Cd^2x+3Ae(d+ex)+Bd(d+3ex)\right)+2a^2ce(3Cd(d+ex)+e(3Bd+Ae+Bex))}{a\left(a+cx^2\right)^2}+\frac{8a^3Ce^3+3Ac^3d^3x+ac^2d\left(Cd^2+3e(Bd+Ae)\right)x-a^2ce(3Cd(4d+5ex)+e(12Bd+4Ae+5Bex))}{a^2\left(a+cx^2\right)}+\frac{\sqrt{c}\left(3Acd\left(cd^2+ae^2\right)+a\left(3ae^2(3Cd+Be)+cd^2(Cd+3Be)\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{a^{5/2}}+4Ce^3\log\left(a+cx^2\right)}{8c^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^3,x]

[Out] ((-2*a^3*C*e^3 + 2*A*c^3*d^3*x - 2*a*c^2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d *(d + 3*e*x)) + 2*a^2*c*e*(3*C*d*(d + e*x) + e*(3*B*d + A*e + B*e*x)))/(a*(a + c*x^2)^2) + (8*a^3*C*e^3 + 3*A*c^3*d^3*x + a*c^2*d*(C*d^2 + 3*e*(B*d + A*e))*x - a^2*c*e*(3*C*d*(4*d + 5*e*x) + e*(12*B*d + 4*A*e + 5*B*e*x)))/(a^ 2*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c*d^2 + a*e^2) + a*(3*a*e^2*(3*C*d + B* e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(5/2) + 4*C*e^3*L og[a + c*x^2])/(8*c^3)

Maple [A]
time = 0.15, size = 333, normalized size = 1.59

method	result
default	$\frac{\frac{\left(3Aacd\,e^{2}+3A\,c^{2}d^{3}-5B\,a^{2}e^{3}+3Bac\,d^{2}e-15C\,a^{2}d\,e^{2}+Cac\,d^{3}\right)x^{3}}{8a^{2}c}-\frac{e\left(Ac\,e^{2}+3Bcde-2aC\,e^{2}+3Cc\,d^{2}\right)x^{2}}{2c^{2}}-\frac{\left(3Aacd\,e^{2}-5A\,c^{2}d^{3}+3B\,a^{2}e^{3}+3Bac\,d^{2}\right.}{8c^{2}a}}{\left(c\,x^{2}+a\right)^{2}}$
risch	$\frac{\frac{\left(3Aacd\,e^{2}+3A\,c^{2}d^{3}-5B\,a^{2}e^{3}+3Bac\,d^{2}e-15C\,a^{2}d\,e^{2}+Cac\,d^{3}\right)x^{3}}{8a^{2}c}-\frac{e\left(Ac\,e^{2}+3Bcde-2aC\,e^{2}+3Cc\,d^{2}\right)x^{2}}{2c^{2}}-\frac{\left(3Aacd\,e^{2}-5A\,c^{2}d^{3}+3B\,a^{2}e^{3}+3Bac\,d^{2}\right.}{8c^{2}a}}{\left(c\,x^{2}+a\right)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] (1/8*(3*A*a*c*d*e^2+3*A*c^2*d^3-5*B*a^2*e^3+3*B*a*c*d^2*e-15*C*a^2*d*e^2+C* a*c*d^3)/a^2/c*x^3-1/2*e*(A*c*e^2+3*B*c*d*e-2*C*a*e^2+3*C*c*d^2)/c^2*x^2-1/ 8*(3*A*a*c*d*e^2-5*A*c^2*d^3+3*B*a^2*e^3+3*B*a*c*d^2*e+9*C*a^2*d*e^2+C*a*c* d^3)/c^2/a*x-1/4*(A*a*c*e^3+3*A*c^2*d^2*e+3*B*a*c*d*e^2+B*c^2*d^3-3*C*a^2*e ^3+3*C*a*c*d^2*e)/c^3)/(c*x^2+a)^2+1/8/a^2/c^2*(4*C*a^2*e^3/c*ln(c*x^2+a)+(3*A*a*c*d*e^2+3*A*c^2*d^3+3*B*a^2*e^3+3*B*a*c*d^2*e+9*C*a^2*d*e^2+C*a*c*d^3)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))

Maxima [A]
time = 0.52, size = 378, normalized size = 1.81

$$-\frac{2\,Ba^{2}c^{2}d^{3}+6\,Ba^{3}cde^{2}-6\,Ca^{4}e^{3}+2\,Aa^{3}ce^{3}-(3\,Bac^{3}d^{2}e-5\,Ba^{2}c^{2}e^{3}+(Cac^{3}+3\,Ac^{4})d^{3}-3\,(5\,Ca^{2}c^{2}e^{2}-Aac^{3}e^{2})d)x^{3}+6\,(Ca^{3}ce+Aa^{2}c^{2}e)d^{2}+4\,(3\,Ca^{2}c^{2}d^{2}e+3\,Ba^{2}c^{2}de^{2}-2\,Ca^{3}ce^{3}+Aa^{2}c^{2}e^{3})x^{2}+(3\,Ba^{2}c^{2}d^{2}e+3\,Ba^{3}ce^{3}+(Ca^{2}c^{2}-5\,Aac^{3})d^{3}+3\,(3\,Ca^{3}ce^{2}+Aa^{2}c^{2}e^{2})d)x}{8\,(a^{2}c^{5}x^{4}+2\,a^{3}c^{4}x^{2}+a^{4}c^{3})}+\frac{Ce^{3}\log\left(cx^{2}+a\right)}{2\,c^{3}}+\frac{(3\,Bacd^{2}e+(Cac+3\,Ac^{2})d^{3}+3\,Ba^{2}e^{3}+3\,(3\,Ca^{2}e^{2}+Aace^{2})d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")

[Out] -1/8*(2*B*a^2*c^2*d^3 + 6*B*a^3*c*d*e^2 - 6*C*a^4*e^3 + 2*A*a^3*c*e^3 - (3* B*a*c^3*d^2*e - 5*B*a^2*c^2*e^3 + (C*a*c^3 + 3*A*c^4)*d^3 - 3*(5*C*a^2*c^2* e^2 - A*a*c^3*e^2)*d)*x^3 + 6*(C*a^3*c*e + A*a^2*c^2*e)*d^2 + 4*(3*C*a^2*c^ 2*d^2*e + 3*B*a^2*c^2*d*e^2 - 2*C*a^3*c*e^3 + A*a^2*c^2*e^3)*x^2 + (3*B*a^2 *c^2*d^2*e + 3*B*a^3*c*e^3 + (C*a^2*c^2 - 5*A*a*c^3)*d^3 + 3*(3*C*a^3*c*e^2

```
 + A*a^2*c^2*e^2)*d)*x)/(a^2*c^5*x^4 + 2*a^3*c^4*x^2 + a^4*c^3) + 1/2*C*e^3
*log(c*x^2 + a)/c^3 + 1/8*(3*B*a*c*d^2*e + (C*a*c + 3*A*c^2)*d^3 + 3*B*a^2*
e^3 + 3*(3*C*a^2*e^2 + A*a*c*e^2)*d)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c
^2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(199) = 398$.
time = 0.36, size = 1111, normalized size = 5.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c^2*d^3 - 2*(C*a^2*c^3 + 3*A*a*c^4)*d^3*x^3 + 2*(C*a^3*c^2
- 5*A*a^2*c^3)*d^3*x - 8*(C*a^3*c^2*x^4 + 2*C*a^4*c*x^2 + C*a^5)*e^3*log(c*
x^2 + a) + ((C*a*c^3 + 3*A*c^4)*d^3*x^4 + 2*(C*a^2*c^2 + 3*A*a*c^3)*d^3*x^2
 + (C*a^3*c + 3*A*a^2*c^2)*d^3 + 3*(B*a^2*c^2*x^4 + 2*B*a^3*c*x^2 + B*a^4)*
e^3 + 3*((3*C*a^2*c^2 + A*a*c^3)*d*x^4 + 2*(3*C*a^3*c + A*a^2*c^2)*d*x^2 +
(3*C*a^4 + A*a^3*c)*d)*e^2 + 3*(B*a*c^3*d^2*x^4 + 2*B*a^2*c^2*d^2*x^2 + B*a
^3*c*d^2)*e)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(
5*B*a^3*c^2*x^3 + 3*B*a^4*c*x - 6*C*a^5 + 2*A*a^4*c - 4*(2*C*a^4*c - A*a^3*
c^2)*x^2)*e^3 + 6*(4*B*a^3*c^2*d*x^2 + 2*B*a^4*c*d + (5*C*a^3*c^2 - A*a^2*c
^3)*d*x^3 + (3*C*a^4*c + A*a^3*c^2)*d*x)*e^2 - 6*(B*a^2*c^3*d^2*x^3 - 4*C*a
^3*c^2*d^2*x^2 - B*a^3*c^2*d^2*x - 2*(C*a^4*c + A*a^3*c^2)*d^2)*e)/(a^3*c^5
*x^4 + 2*a^4*c^4*x^2 + a^5*c^3), -1/8*(2*B*a^3*c^2*d^3 - (C*a^2*c^3 + 3*A*a
*c^4)*d^3*x^3 + (C*a^3*c^2 - 5*A*a^2*c^3)*d^3*x - 4*(C*a^3*c^2*x^4 + 2*C*a^
4*c*x^2 + C*a^5)*e^3*log(c*x^2 + a) - ((C*a*c^3 + 3*A*c^4)*d^3*x^4 + 2*(C*a
^2*c^2 + 3*A*a*c^3)*d^3*x^2 + (C*a^3*c + 3*A*a^2*c^2)*d^3 + 3*(B*a^2*c^2*x^
4 + 2*B*a^3*c*x^2 + B*a^4)*e^3 + 3*((3*C*a^2*c^2 + A*a*c^3)*d*x^4 + 2*(3*C*
a^3*c + A*a^2*c^2)*d*x^2 + (3*C*a^4 + A*a^3*c)*d)*e^2 + 3*(B*a*c^3*d^2*x^4
+ 2*B*a^2*c^2*d^2*x^2 + B*a^3*c*d^2)*e)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (
5*B*a^3*c^2*x^3 + 3*B*a^4*c*x - 6*C*a^5 + 2*A*a^4*c - 4*(2*C*a^4*c - A*a^3*
c^2)*x^2)*e^3 + 3*(4*B*a^3*c^2*d*x^2 + 2*B*a^4*c*d + (5*C*a^3*c^2 - A*a^2*c
^3)*d*x^3 + (3*C*a^4*c + A*a^3*c^2)*d*x)*e^2 - 3*(B*a^2*c^3*d^2*x^3 - 4*C*a
^3*c^2*d^2*x^2 - B*a^3*c^2*d^2*x - 2*(C*a^4*c + A*a^3*c^2)*d^2)*e)/(a^3*c^5
*x^4 + 2*a^4*c^4*x^2 + a^5*c^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

[Out] Timed out

Giac [A]

time = 4.42, size = 348, normalized size = 1.67

$$\frac{Ce^3\log\left(cx^2+a\right)}{2\,c^3}+\frac{\left(Cacd^3+3\,Ac^2d^3+3\,Bacd^2e+9\,Ca^2de^2+3\,Aacde^2+3\,Ba^2e^3\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^2c^2}+\frac{\left(Cac^2d^3+3\,Ac^3d^3+3\,Bac^2d^2e-15\,Ca^2cde^2+3\,Aac^2de^2-5\,Ba^2ce^3\right)x^3-4\left(3\,Ca^2cd^2e+3\,Ba^2cde^2-2\,Ca^3e^3+Aa^2ce^3\right)x^2-\left(Ca^2cd^3-5\,Aac^2d^3+3\,Ba^2cd^2e+9\,Ca^3de^2+3\,Aa^2cde^2+3\,Ba^3e^3\right)x-\frac{2\left(Ba^2c^2d^3+3\,Ca^3cd^2e+3\,Aa^2c^2d^2e+3\,Ba^3cde^2-3\,Ca^4e^3+Aa^3ce^3\right)}{c}}{8\left(cx^2+a\right)^2a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*C*e^3*log(c*x^2 + a)/c^3 + 1/8*(C*a*c*d^3 + 3*A*c^2*d^3 + 3*B*a*c*d^2*e
 + 9*C*a^2*d*e^2 + 3*A*a*c*d*e^2 + 3*B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt
(a*c)*a^2*c^2) + 1/8*((C*a*c^2*d^3 + 3*A*c^3*d^3 + 3*B*a*c^2*d^2*e - 15*C*a
^2*c*d*e^2 + 3*A*a*c^2*d*e^2 - 5*B*a^2*c*e^3)*x^3 - 4*(3*C*a^2*c*d^2*e + 3*
B*a^2*c*d*e^2 - 2*C*a^3*e^3 + A*a^2*c*e^3)*x^2 - (C*a^2*c*d^3 - 5*A*a*c^2*d
^3 + 3*B*a^2*c*d^2*e + 9*C*a^3*d*e^2 + 3*A*a^2*c*d*e^2 + 3*B*a^3*e^3)*x - 2
*(B*a^2*c^2*d^3 + 3*C*a^3*c*d^2*e + 3*A*a^2*c^2*d^2*e + 3*B*a^3*c*d*e^2 - 3
*C*a^4*e^3 + A*a^3*c*e^3)/c)/((c*x^2 + a)^2*a^2*c^2)
```

Mupad [B]

time = 1.77, size = 920, normalized size = 4.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^3,x)
```

```
[Out] (5*A*d^3*x)/(8*(a^3 + 2*a^2*c*x^2 + a*c^2*x^4)) - (B*d^3)/(4*(a^2*c + c^3*x
^4 + 2*a*c^2*x^2)) + (3*C*a^2*e^3)/(4*(a^2*c^3 + c^5*x^4 + 2*a*c^4*x^2)) -
(3*A*d^2*e)/(4*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) + (C*d^3*x^3)/(8*(a^3 + 2*a
^2*c*x^2 + a*c^2*x^4)) - (C*d^3*x)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) - (A
*a*e^3)/(4*(a^2*c^2 + c^4*x^4 + 2*a*c^3*x^2)) - (A*e^3*x^2)/(2*(a^2*c + c^3
*x^4 + 2*a*c^2*x^2)) - (5*B*e^3*x^3)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) +
(C*e^3*log(a + c*x^2))/(2*c^3) - (3*B*a*d*e^2)/(4*(a^2*c^2 + c^4*x^4 + 2*a*
c^3*x^2)) - (3*C*a*d^2*e)/(4*(a^2*c^2 + c^4*x^4 + 2*a*c^3*x^2)) + (3*A*c*d^
3*x^3)/(8*(a^4 + 2*a^3*c*x^2 + a^2*c^2*x^4)) - (3*B*a*e^3*x)/(8*(a^2*c^2 +
c^4*x^4 + 2*a*c^3*x^2)) - (3*B*d*e^2*x^2)/(2*(a^2*c + c^3*x^4 + 2*a*c^2*x^2
)) - (3*C*d^2*e*x^2)/(2*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) - (15*C*d*e^2*x^3)
/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) + (C*a*e^3*x^2)/(a^2*c^2 + c^4*x^4 + 2
*a*c^3*x^2) + (3*A*d^3*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(5/2)*c^(1/2)) + (3*
B*e^3*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*c^(5/2)) + (C*d^3*atan((c^(1/2)
*x)/a^(1/2)))/(8*a^(3/2)*c^(3/2)) + (3*A*d*e^2*x^3)/(8*(a^3 + 2*a^2*c*x^2 +
 a*c^2*x^4)) + (3*B*d^2*e*x^3)/(8*(a^3 + 2*a^2*c*x^2 + a*c^2*x^4)) - (3*A*d
*e^2*x)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) - (3*B*d^2*e*x)/(8*(a^2*c + c^3
*x^4 + 2*a*c^2*x^2)) + (3*A*d*e^2*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(3/2)*c^(
3/2)) + (3*B*d^2*e*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(3/2)*c^(3/2)) + (9*C*d*
e^2*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*c^(5/2)) - (9*C*a*d*e^2*x)/(8*(a^
2*c^2 + c^4*x^4 + 2*a*c^3*x^2))
```

3.58 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx$

Optimal. Leaf size=156

$$-\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac\,(a+cx^2)^2}-\frac{(d+ex)(a(Ac+3aC)e-c(3Acd+aCd+2aBe)x)}{8a^2c^2\,(a+cx^2)}+\frac{(a(Ac+3aC)e^2+}{}$$

```
[Out] -1/4*(a*B-(A*c-C*a)*x)*(e*x+d)^2/a/c/(c*x^2+a)^2-1/8*(e*x+d)*(a*(A*c+3*C*a)
*e-c*(3*A*c*d+2*B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)+1/8*(a*(A*c+3*C*a)*e^2+c*
d*(3*A*c*d+2*B*a*e+C*a*d))*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/c^(5/2)
```

Rubi [A]
time = 0.14, antiderivative size = 175, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1659, 792, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(cd(2aBe+aCd+3Acd)+ae^2(3aC+Ac))}{8a^{5/2}c^{5/2}}-\frac{x(ae^2(3aC+Ac)-cd(2aBe+aCd+3Acd))+2ae(aBe+2aCd+2Acd)}{8a^2c^2\,(a+cx^2)}-\frac{(d+ex)^2(aB-x(Ac-aC))}{4ac\,(a+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] -1/4*((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(a*c*(a + c*x^2)^2) - (2*a*e*(2*A*
c*d + 2*a*C*d + a*B*e) + (a*(A*c + 3*a*C)*e^2 - c*d*(3*A*c*d + a*C*d + 2*a*
B*e))*x)/(8*a^2*c^2*(a + c*x^2)) + ((a*(A*c + 3*a*C)*e^2 + c*d*(3*A*c*d + a
*C*d + 2*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
```

```
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac\left(a+cx^2\right)^2} - \frac{\int \frac{(d+ex)(-3Acd-aCd-2aBe-(Ac+3aC)ex)}{\left(a+cx^2\right)^2}\,dx}{4ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac\left(a+cx^2\right)^2} - \frac{2ae(2Acd+2aCd+aBe)+(a(Ac+3 \ldots}{8a^2c^2\,(a \ldots}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac\left(a+cx^2\right)^2} - \frac{2ae(2Acd+2aCd+aBe)+(a(Ac+3 \ldots}{8a^2c^2\,(a \ldots}$$

Mathematica [A]
time = 0.10, size = 211, normalized size = 1.35

$$\frac{3Ac^2d^2x+ac(Cd^2+e(2Bd+Ae))\,x-a^2e(8Cd+4Be+5Cex)}{8a^2c^2\,(a+cx^2)}+\frac{Ac^2d^2x+a^2e(2Cd+Be+Cex)-ac(Cd^2x+Ae(2d+ex)+Bd(d+2ex))}{4ac^2\,(a+cx^2)^2}+\frac{(Ac(3cd^2+ae^2)+a(3aCe^2+cd(Cd+2Be)))\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] (3*A*c^2*d^2*x + a*c*(C*d^2 + e*(2*B*d + A*e))*x - a^2*e*(8*C*d + 4*B*e + 5
*C*e*x))/(8*a^2*c^2*(a + c*x^2)) + (A*c^2*d^2*x + a^2*e*(2*C*d + B*e + C*e*
x) - a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x)))/(4*a*c^2*(a + c*x^2
)^2) + ((A*c*(3*c*d^2 + a*e^2) + a*(3*a*C*e^2 + c*d*(C*d + 2*B*e)))*ArcTan[
(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(5/2))
```

Maple [A]
time = 0.10, size = 222, normalized size = 1.42

method	result
default	$\frac{\frac{\left(Aac\,e^2+3A\,c^2d^2+2acdeB-5a^2C\,e^2+Cac\,d^2\right)x^3}{8a^2c}-\frac{e(Be+2Cd)x^2}{2c}-\frac{\left(Aac\,e^2-5A\,c^2d^2+2acdeB+3a^2C\,e^2+Cac\,d^2\right)x}{8c^2a}-\frac{2cdeA+Ba\,e^2+Bc\,d^2+2adeC \ldots}{4c^2}}{\left(c\,x^2+a\right)^2}$
risch	$\frac{\frac{\left(Aac\,e^2+3A\,c^2d^2+2acdeB-5a^2C\,e^2+Cac\,d^2\right)x^3}{8a^2c}-\frac{e(Be+2Cd)x^2}{2c}-\frac{\left(Aac\,e^2-5A\,c^2d^2+2acdeB+3a^2C\,e^2+Cac\,d^2\right)x}{8c^2a}-\frac{2cdeA+Ba\,e^2+Bc\,d^2+2adeC \ldots}{4c^2}}{\left(c\,x^2+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(A*a*c*e^2+3*A*c^2*d^2+2*B*a*c*d*e-5*C*a^2*e^2+C*a*c*d^2)/a^2/c*x^3-1/
2*e*(B*e+2*C*d)*x^2/c-1/8*(A*a*c*e^2-5*A*c^2*d^2+2*B*a*c*d*e+3*C*a^2*e^2+C*
a*c*d^2)/c^2/a*x-1/4*(2*A*c*d*e+B*a*e^2+B*c*d^2+2*C*a*d*e)/c^2)/(c*x^2+a)^2
+1/8*(A*a*c*e^2+3*A*c^2*d^2+2*B*a*c*d*e+3*C*a^2*e^2+C*a*c*d^2)/a^2/c^2/(a*c
)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Maxima [A]
time = 0.50, size = 253, normalized size = 1.62

$$-\frac{2\,Ba^2cd^2+2\,Ba^3e^2-(2\,Bac^2de-5\,Ca^2ce^2+Aac^2e^2+(Cac^2+3\,Ac^3)d^2)x^3+4\,(2\,Ca^2cde+Ba^2ce^2)x^2+4\,(Ca^3e+Aa^2ce)d+(2\,Ba^2cde+3\,Ca^3e^2+Aa^2ce^2+(Ca^2c-5\,Aac^2)d^2)x}{8\,(a^2c^4x^4+2\,a^3c^3x^2+a^4c^2)}+\frac{(2\,Bacde+3\,Ca^2e^2+Aace^2+(Cac+3\,Ac^2)d^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(2*B*a^2*c*d^2 + 2*B*a^3*e^2 - (2*B*a*c^2*d*e - 5*C*a^2*c*e^2 + A*a*c^
2*e^2 + (C*a*c^2 + 3*A*c^3)*d^2)*x^3 + 4*(2*C*a^2*c*d*e + B*a^2*c*e^2)*x^2
+ 4*(C*a^3*e + A*a^2*c*e)*d + (2*B*a^2*c*d*e + 3*C*a^3*e^2 + A*a^2*c*e^2 +
(C*a^2*c - 5*A*a*c^2)*d^2)*x)/(a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2) + 1/8
*(2*B*a*c*d*e + 3*C*a^2*e^2 + A*a*c*e^2 + (C*a*c + 3*A*c^2)*d^2)*arctan(c*x
/sqrt(a*c))/(sqrt(a*c)*a^2*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(145) = 290$.
time = 0.37, size = 803, normalized size = 5.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c^2*d^2 - 2*(C*a^2*c^3 + 3*A*a*c^4)*d^2*x^3 + 2*(C*a^3*c^2
- 5*A*a^2*c^3)*d^2*x + ((C*a*c^3 + 3*A*c^4)*d^2*x^4 + 2*(C*a^2*c^2 + 3*A*a*
c^3)*d^2*x^2 + (C*a^3*c + 3*A*a^2*c^2)*d^2 + (3*C*a^4 + A*a^3*c + (3*C*a^2*
c^2 + A*a*c^3)*x^4 + 2*(3*C*a^3*c + A*a^2*c^2)*x^2)*e^2 + 2*(B*a*c^3*d*x^4
+ 2*B*a^2*c^2*d*x^2 + B*a^3*c*d)*e)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x
- a)/(c*x^2 + a)) + 2*(4*B*a^3*c^2*x^2 + 2*B*a^4*c + (5*C*a^3*c^2 - A*a^2*c
^3)*x^3 + (3*C*a^4*c + A*a^3*c^2)*x)*e^2 - 4*(B*a^2*c^3*d*x^3 - 4*C*a^3*c^2
*d*x^2 - B*a^3*c^2*d*x - 2*(C*a^4*c + A*a^3*c^2)*d)*e)/(a^3*c^5*x^4 + 2*a^4
*c^4*x^2 + a^5*c^3), -1/8*(2*B*a^3*c^2*d^2 - (C*a^2*c^3 + 3*A*a*c^4)*d^2*x^
3 + (C*a^3*c^2 - 5*A*a^2*c^3)*d^2*x - ((C*a*c^3 + 3*A*c^4)*d^2*x^4 + 2*(C*a
```

```
^2*c^2 + 3*A*a*c^3)*d^2*x^2 + (C*a^3*c + 3*A*a^2*c^2)*d^2 + (3*C*a^4 + A*a^
3*c + (3*C*a^2*c^2 + A*a*c^3)*x^4 + 2*(3*C*a^3*c + A*a^2*c^2)*x^2)*e^2 + 2*
(B*a*c^3*d*x^4 + 2*B*a^2*c^2*d*x^2 + B*a^3*c*d)*e)*sqrt(a*c)*arctan(sqrt(a*
c)*x/a) + (4*B*a^3*c^2*x^2 + 2*B*a^4*c + (5*C*a^3*c^2 - A*a^2*c^3)*x^3 + (3
*C*a^4*c + A*a^3*c^2)*x)*e^2 - 2*(B*a^2*c^3*d*x^3 - 4*C*a^3*c^2*d*x^2 - B*a
^3*c^2*d*x - 2*(C*a^4*c + A*a^3*c^2)*d)*e)/(a^3*c^5*x^4 + 2*a^4*c^4*x^2 + a
^5*c^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.20, size = 254, normalized size = 1.63

$$\frac{(Cacd^2+3\,Ac^2d^2+2\,Bacde+3\,Ca^2e^2+Aace^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^2c^2}+\frac{Cac^2d^2x^3+3\,Ac^3d^2x^3+2\,Bac^2dx^3e-5\,Ca^2cx^3e^2+Aac^2x^3e^2-8\,Ca^2cdx^2e-Ca^2cd^2x+5\,Aac^2d^2x-4\,Ba^2cx^2e^2-2\,Ba^2cdxe-2\,Ba^2cd^2-3\,Ca^3xe^2-Aa^2cxe^2-4\,Ca^3de-4\,Aa^2cde-2\,Ba^3e^2}{8\,(cx^2+a)^2a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a*c*d^2 + 3*A*c^2*d^2 + 2*B*a*c*d*e + 3*C*a^2*e^2 + A*a*c*e^2)*arcta
n(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c^2) + 1/8*(C*a*c^2*d^2*x^3 + 3*A*c^3*d^2*x
^3 + 2*B*a*c^2*d*x^3*e - 5*C*a^2*c*x^3*e^2 + A*a*c^2*x^3*e^2 - 8*C*a^2*c*d*
x^2*e - C*a^2*c*d^2*x + 5*A*a*c^2*d^2*x - 4*B*a^2*c*x^2*e^2 - 2*B*a^2*c*d*x
*e - 2*B*a^2*c*d^2 - 3*C*a^3*x*e^2 - A*a^2*c*x*e^2 - 4*C*a^3*d*e - 4*A*a^2*
c*d*e - 2*B*a^3*e^2)/((c*x^2 + a)^2*a^2*c^2)
```

Mupad [B]
time = 3.96, size = 230, normalized size = 1.47

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(3\,C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2+3\,A\,c^2\,d^2\right)}{8\,a^{5/2}\,c^{5/2}}-\frac{\frac{B\,a\,e^2+B\,c\,d^2+2\,A\,c\,d\,e+2\,C\,a\,d\,e}{4\,c^2}+\frac{x^2\,\left(B\,e^2+2\,C\,d\,e\right)}{2\,c}+\frac{x\,\left(3\,C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2-5\,A\,c^2\,d^2\right)}{8\,a\,c^2}-\frac{x^3\,\left(-5\,C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2+3\,A\,c^2\,d^2\right)}{8\,a^2\,c}}{a^2+2\,a\,c\,x^2+c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(3*A*c^2*d^2 + 3*C*a^2*e^2 + A*a*c*e^2 + C*a*c*d
^2 + 2*B*a*c*d*e))/(8*a^(5/2)*c^(5/2)) - ((B*a*e^2 + B*c*d^2 + 2*A*c*d*e +
2*C*a*d*e)/(4*c^2) + (x^2*(B*e^2 + 2*C*d*e))/(2*c) + (x*(3*C*a^2*e^2 - 5*A*
c^2*d^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(8*a*c^2) - (x^3*(3*A*c^2*d
^2 - 5*C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(8*a^2*c))/(a^2 +
c^2*x^4 + 2*a*c*x^2)
```

3.59 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx$

Optimal. Leaf size=130

$$-\frac{(aB-(Ac-aC)x)(d+ex)}{4ac\left(a+cx^2\right)^2}-\frac{2a(Ac+aC)e-c(3Acd+aCd+aBe)x}{8a^2c^2\left(a+cx^2\right)}+\frac{(3Acd+aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

```
[Out] -1/4*(a*B-(A*c-C*a)*x)*(e*x+d)/a/c/(c*x^2+a)^2+1/8*(-2*a*(A*c+C*a)*e+c*(3*A
*c*d+B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)+1/8*(3*A*c*d+B*a*e+C*a*d)*arctan(x*c
^(1/2)/a^(1/2))/a^(5/2)/c^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1659, 653, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd+3Acd)}{8a^{5/2}c^{3/2}}-\frac{2ae(aC+Ac)-cx(aBe+aCd+3Acd)}{8a^2c^2\left(a+cx^2\right)}-\frac{(d+ex)(aB-x(Ac-aC))}{4ac\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] -1/4*((a*B - (A*c - a*C)*x)*(d + e*x))/(a*c*(a + c*x^2)^2) - (2*a*(A*c + a*
C)*e - c*(3*A*c*d + a*C*d + a*B*e)*x)/(8*a^2*c^2*(a + c*x^2)) + ((3*A*c*d +
 a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
```

```
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)}{4ac\left(a+cx^2\right)^2}-\frac{\int \frac{-3Acd-a(Cd+Be)-2(Ac+aC)ex}{\left(a+cx^2\right)^2}\,dx}{4ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{4ac\left(a+cx^2\right)^2}-\frac{2a(Ac+aC)e-c(3Acd+aCd+aBe)x}{8a^2c^2\left(a+cx^2\right)}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{4ac\left(a+cx^2\right)^2}-\frac{2a(Ac+aC)e-c(3Acd+aCd+aBe)x}{8a^2c^2\left(a+cx^2\right)}$$

Mathematica [A]
time = 0.07, size = 137, normalized size = 1.05

$$\frac{\frac{\sqrt{a}\left(-4a^2Ce+3Ac^2dx+ac(Cd+Be)x\right)}{a+cx^2}+\frac{2a^{3/2}\left(a^2Ce+Ac^2dx-ac(Ae+Cdx+B(d+ex))\right)}{\left(a+cx^2\right)^2}+\sqrt{c}\,(3Acd+aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*(-4*a^2*C*e + 3*A*c^2*d*x + a*c*(C*d + B*e)*x))/(a + c*x^2) + (2*
a^(3/2)*(a^2*C*e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x))))/(a + c*x^2
)^2 + Sqrt[c]*(3*A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(
5/2)*c^2)
```

Maple [A]
time = 0.10, size = 124, normalized size = 0.95

method	result
default	$\frac{\frac{(3Acd+aBe+Cad)x^3}{8a^2}-\frac{Ce\,x^2}{2c}+\frac{(5Acd-aBe-Cad)x}{8ac}-\frac{Ace+Bcd+aCe}{4c^2}}{\left(c\,x^2+a\right)^2}+\frac{(3Acd+aBe+Cad)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8a^2c\sqrt{ac}}$
risch	$\frac{\frac{(3Acd+aBe+Cad)x^3}{8a^2}-\frac{Ce\,x^2}{2c}+\frac{(5Acd-aBe-Cad)x}{8ac}-\frac{Ace+Bcd+aCe}{4c^2}}{\left(c\,x^2+a\right)^2}-\frac{3\ln\left(cx+\sqrt{-ac}\right)Ad}{16\sqrt{-ac}\,a^2}-\frac{\ln\left(cx+\sqrt{-ac}\right)Be}{16\sqrt{-ac}\,ca}-\frac{\ln\left(cx+\sqrt{\right.}}{16\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*A*c*d+B*a*e+C*a*d)/a^2*x^3-1/2*C*e*x^2/c+1/8*(5*A*c*d-B*a*e-C*a*d)/
a/c*x-1/4*(A*c*e+B*c*d+C*a*e)/c^2)/(c*x^2+a)^2+1/8*(3*A*c*d+B*a*e+C*a*d)/a^
2/c/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Maxima [A]
time = 0.51, size = 166, normalized size = 1.28

$$-\frac{4\,Ca^2cx^2e+2\,Ba^2cd+2\,Ca^3e+2\,Aa^2ce-(Bac^2e+(Cac^2+3\,Ac^3)d)x^3+(Ba^2ce+(Ca^2c-5\,Aac^2)d)x}{8\,(a^2c^4x^4+2\,a^3c^3x^2+a^4c^2)}+\frac{(Bae+(Ca+3\,Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*C*a^2*c*x^2*e + 2*B*a^2*c*d + 2*C*a^3*e + 2*A*a^2*c*e - (B*a*c^2*e
+ (C*a*c^2 + 3*A*c^3)*d)*x^3 + (B*a^2*c*e + (C*a^2*c - 5*A*a*c^2)*d)*x)/(a^
2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2) + 1/8*(B*a*e + (C*a + 3*A*c)*d)*arctan
(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c)
```

Fricas [A]
time = 0.35, size = 480, normalized size = 3.69

$$\left[-\frac{4\,Ba^3cd-2\,(Ca^2c^2+3\,Aac^3)dx^3+2\,(Ca^3c-5\,Aa^2c^2)dx+((Cac^2+3\,Ac^3)dx^4+2\,(Ca^2c+3\,Aac^2)dx^2+(Ca^3+3\,Aa^2c)d+(Bac^2x^4+2\,Ba^2cx^2+Ba^3)e)\sqrt{-ac}\,\log\left(\frac{cx^2-2\,\sqrt{-ac}\,x-a}{cx^2+a}\right)-2\,(Ba^2c^2x^3-4\,Ca^3cx^2-Ba^3cx-2\,Ca^4-2\,Aa^3c)e}{16\,(a^3c^4x^4+2\,a^4c^3x^2+a^5c^2)},\,-\frac{2\,Ba^3cd-(Ca^2c^2+3\,Aac^3)dx^3+(Ca^3c-5\,Aa^2c^2)dx-((Cac^2+3\,Ac^3)dx^4+2\,(Ca^2c+3\,Aac^2)dx^2+(Ca^3+3\,Aa^2c)d+(Bac^2x^4+2\,Ba^2cx^2+Ba^3)e)\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,x}{a}\right)-(Ba^2c^2x^3-4\,Ca^3cx^2-Ba^3cx-2\,Ca^4-2\,Aa^3c)e}{8\,(a^3c^4x^4+2\,a^4c^3x^2+a^5c^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c*d - 2*(C*a^2*c^2 + 3*A*a*c^3)*d*x^3 + 2*(C*a^3*c - 5*A*a^
2*c^2)*d*x + ((C*a*c^2 + 3*A*c^3)*d*x^4 + 2*(C*a^2*c + 3*A*a*c^2)*d*x^2 + (
C*a^3 + 3*A*a^2*c)*d + (B*a*c^2*x^4 + 2*B*a^2*c*x^2 + B*a^3)*e)*sqrt(-a*c)*
log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(B*a^2*c^2*x^3 - 4*C*a^3*
c*x^2 - B*a^3*c*x - 2*C*a^4 - 2*A*a^3*c)*e)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 +
a^5*c^2), -1/8*(2*B*a^3*c*d - (C*a^2*c^2 + 3*A*a*c^3)*d*x^3 + (C*a^3*c - 5*
A*a^2*c^2)*d*x - ((C*a*c^2 + 3*A*c^3)*d*x^4 + 2*(C*a^2*c + 3*A*a*c^2)*d*x^2
 + (C*a^3 + 3*A*a^2*c)*d + (B*a*c^2*x^4 + 2*B*a^2*c*x^2 + B*a^3)*e)*sqrt(a*
c)*arctan(sqrt(a*c)*x/a) - (B*a^2*c^2*x^3 - 4*C*a^3*c*x^2 - B*a^3*c*x - 2*C
*a^4 - 2*A*a^3*c)*e)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*c^2)]
```

Sympy [A]
time = 14.85, size = 240, normalized size = 1.85

$$-\frac{\sqrt{-\frac{1}{a^5c^3}}\cdot(3Acd+Bae+Cad)\log\left(-a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5c^3}}\cdot(3Acd+Bae+Cad)\log\left(a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}+\frac{-2Aa^2ce-2Ba^2cd-2Ca^3e-4Ca^2cex^2+x^3\cdot(3Ac^3d+Bac^2e+Cac^2d)+x(5Aac^2d-Ba^2ce-Ca^2cd)}{8a^4c^2+16a^3c^3x^2+8a^2c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*c**3))*(3*A*c*d + B*a*e + C*a*d)*log(-a**3*c*sqrt(-1/(a**5*c
**3)) + x)/16 + sqrt(-1/(a**5*c**3))*(3*A*c*d + B*a*e + C*a*d)*log(a**3*c*s
qrt(-1/(a**5*c**3)) + x)/16 + (-2*A*a**2*c*e - 2*B*a**2*c*d - 2*C*a**3*e -
4*C*a**2*c*e*x**2 + x**3*(3*A*c**3*d + B*a*c**2*e + C*a*c**2*d) + x*(5*A*a*
c**2*d - B*a**2*c*e - C*a**2*c*d))/(8*a**4*c**2 + 16*a**3*c**3*x**2 + 8*a**
2*c**4*x**4)
```

Giac [A]
time = 3.87, size = 152, normalized size = 1.17

$$\frac{(Cad+3\,Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^2c}+\frac{Cac^2dx^3+3\,Ac^3dx^3+Bac^2x^3e-4\,Ca^2cx^2e-Ca^2cdx+5\,Aac^2dx-Ba^2cxe-2\,Ba^2cd-2\,Ca^3e-2\,Aa^2ce}{8\,(cx^2+a)^2a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a*d + 3*A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c) + 1/8
*(C*a*c^2*d*x^3 + 3*A*c^3*d*x^3 + B*a*c^2*x^3*e - 4*C*a^2*c*x^2*e - C*a^2*c
*d*x + 5*A*a*c^2*d*x - B*a^2*c*x*e - 2*B*a^2*c*d - 2*C*a^3*e - 2*A*a^2*c*e)
/((c*x^2 + a)^2*a^2*c^2)
```

Mupad [B]
time = 0.15, size = 128, normalized size = 0.98

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(3\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{8\,a^{5/2}\,c^{3/2}}-\frac{\frac{A\,c\,e+B\,c\,d+C\,a\,e}{4\,c^2}-\frac{x^3\,\left(3\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{8\,a^2}+\frac{C\,e\,x^2}{2\,c}+\frac{x\,\left(B\,a\,e-5\,A\,c\,d+C\,a\,d\right)}{8\,a\,c}}{a^2+2\,a\,c\,x^2+c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^3,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(3*A*c*d + B*a*e + C*a*d))/(8*a^(5/2)*c^(3/2)) -
 ((A*c*e + B*c*d + C*a*e)/(4*c^2) - (x^3*(3*A*c*d + B*a*e + C*a*d))/(8*a^2)
 + (C*e*x^2)/(2*c) + (x*(B*a*e - 5*A*c*d + C*a*d))/(8*a*c))/(a^2 + c^2*x^4
+ 2*a*c*x^2)
```

3.60 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^3}\,dx$

Optimal. Leaf size=98

$$-\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2}+\frac{(3Ac+aC)x}{8a^2c\left(a+cx^2\right)}+\frac{(3Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

```
[Out] 1/4*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^2+1/8*(3*A*c+C*a)*x/a^2/c/(c*x^2+a)+1/
8*(3*A*c+C*a)*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/c^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1828, 12, 205, 211}

$$\frac{(aC+3Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}+\frac{x(aC+3Ac)}{8a^2c\left(a+cx^2\right)}-\frac{aB-x(Ac-aC)}{4ac\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^3,x]
```

```
[Out] -1/4*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)^2) + ((3*A*c + a*C)*x)/(8*a^2*c
*(a + c*x^2)) + ((3*A*c + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3
/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(a+cx^2)^3}\,dx &= -\frac{aB-(Ac-aC)x}{4ac\,(a+cx^2)^2}-\frac{\int \frac{-3A-\frac{aC}{c}}{(a+cx^2)^2}\,dx}{4a}\\ &= -\frac{aB-(Ac-aC)x}{4ac\,(a+cx^2)^2}+\frac{(3Ac+aC)\int \frac{1}{(a+cx^2)^2}\,dx}{4ac}\\ &= -\frac{aB-(Ac-aC)x}{4ac\,(a+cx^2)^2}+\frac{(3Ac+aC)x}{8a^2c\,(a+cx^2)}+\frac{(3Ac+aC)\int \frac{1}{a+cx^2}\,dx}{8a^2c}\\ &= -\frac{aB-(Ac-aC)x}{4ac\,(a+cx^2)^2}+\frac{(3Ac+aC)x}{8a^2c\,(a+cx^2)}+\frac{(3Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 90, normalized size = 0.92

$$\frac{3Ac^2x^3-a^2(2B+Cx)+acx(5A+Cx^2)}{8a^2c\,(a+cx^2)^2}+\frac{(3Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^3,x]
```

```
[Out] (3*A*c^2*x^3 - a^2*(2*B + C*x) + a*c*x*(5*A + C*x^2))/(8*a^2*c*(a + c*x^2)^
2) + ((3*A*c + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3/2))
```

Maple [A]
time = 0.10, size = 83, normalized size = 0.85

method	result

default	$\frac{\frac{(3Ac+aC)x^3}{8a^2}+\frac{(5Ac-aC)x}{8ac}-\frac{B}{4c}}{(c\,x^2+a)^2}+\frac{(3Ac+aC)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8a^2c\sqrt{ac}}$
risch	$\frac{\frac{(3Ac+aC)x^3}{8a^2}+\frac{(5Ac-aC)x}{8ac}-\frac{B}{4c}}{(c\,x^2+a)^2}-\frac{3\ln\left(cx+\sqrt{-ac}\right)A}{16\sqrt{-ac}\;a^2}-\frac{\ln\left(cx+\sqrt{-ac}\right)C}{16\sqrt{-ac}\;ca}+\frac{3\ln\left(-cx+\sqrt{-ac}\right)A}{16\sqrt{-ac}\;a^2}+\frac{\ln\left(-cx+\sqrt{-\right.}}{16\sqrt{-ac}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*A*c+C*a)/a^2*x^3+1/8*(5*A*c-C*a)/a/c*x-1/4*B/c)/(c*x^2+a)^2+1/8*(3*
A*c+C*a)/a^2/c/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Maxima [A]
time = 0.51, size = 98, normalized size = 1.00

$$\frac{(Cac+3\,Ac^2)x^3-2\,Ba^2-(Ca^2-5\,Aac)x}{8\,(a^2c^3x^4+2\,a^3c^2x^2+a^4c)}+\frac{(Ca+3\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((C*a*c + 3*A*c^2)*x^3 - 2*B*a^2 - (C*a^2 - 5*A*a*c)*x)/(a^2*c^3*x^4 +
2*a^3*c^2*x^2 + a^4*c) + 1/8*(C*a + 3*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)
*a^2*c)
```

Fricas [A]
time = 0.34, size = 314, normalized size = 3.20

$$\left[-\frac{4\,Ba^3c-2\,(Ca^2c^2+3\,Aac^3)x^3+((Cac^2+3\,Ac^3)x^4+Ca^3+3\,Aa^2c+2\,(Ca^2c+3\,Aac^2)x^2)\sqrt{-ac}\,\log\left(\frac{cx^2-2\,\sqrt{-ac}\,x-a}{cx^2+a}\right)+2\,(Ca^3c-5\,Aa^2c^2)x}{16\,(a^3c^4x^4+2\,a^4c^3x^2+a^5c^2)},-\frac{2\,Ba^3c-(Ca^2c^2+3\,Aac^3)x^3-((Cac^2+3\,Ac^3)x^4+Ca^3+3\,Aa^2c+2\,(Ca^2c+3\,Aac^2)x^2)\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,x}{a}\right)+(Ca^3c-5\,Aa^2c^2)x}{8\,(a^3c^4x^4+2\,a^4c^3x^2+a^5c^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c - 2*(C*a^2*c^2 + 3*A*a*c^3)*x^3 + ((C*a*c^2 + 3*A*c^3)*x^
4 + C*a^3 + 3*A*a^2*c + 2*(C*a^2*c + 3*A*a*c^2)*x^2)*sqrt(-a*c)*log((c*x^2
- 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(C*a^3*c - 5*A*a^2*c^2)*x)/(a^3*c^4*
x^4 + 2*a^4*c^3*x^2 + a^5*c^2), -1/8*(2*B*a^3*c - (C*a^2*c^2 + 3*A*a*c^3)*x
^3 - ((C*a*c^2 + 3*A*c^3)*x^4 + C*a^3 + 3*A*a^2*c + 2*(C*a^2*c + 3*A*a*c^2)
*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (C*a^3*c - 5*A*a^2*c^2)*x)/(a^3*c^4
*x^4 + 2*a^4*c^3*x^2 + a^5*c^2)]
```

Sympy [A]
time = 0.62, size = 156, normalized size = 1.59

$$-\frac{\sqrt{-\frac{1}{a^5c^3}}\cdot(3Ac+Ca)\log\left(-a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5c^3}}\cdot(3Ac+Ca)\log\left(a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}+\frac{-2Ba^2+x^3\cdot(3Ac^2+Cac)+x(5Aac-Ca^2)}{8a^4c+16a^3c^2x^2+8a^2c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*c**3))*(3*A*c + C*a)*log(-a**3*c*sqrt(-1/(a**5*c**3)) + x)/1
6 + sqrt(-1/(a**5*c**3))*(3*A*c + C*a)*log(a**3*c*sqrt(-1/(a**5*c**3)) + x)
/16 + (-2*B*a**2 + x**3*(3*A*c**2 + C*a*c) + x*(5*A*a*c - C*a**2))/(8*a**4*
c + 16*a**3*c**2*x**2 + 8*a**2*c**3*x**4)
```

Giac [A]
time = 4.07, size = 84, normalized size = 0.86

$$\frac{(Ca+3\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^2c}+\frac{Cacx^3+3\,Ac^2x^3-Ca^2x+5\,Aacx-2\,Ba^2}{8\,(cx^2+a)^2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a + 3*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c) + 1/8*(C*a*c*x^3
+ 3*A*c^2*x^3 - C*a^2*x + 5*A*a*c*x - 2*B*a^2)/((c*x^2 + a)^2*a^2*c)
```

Mupad [B]
time = 3.84, size = 88, normalized size = 0.90

$$\frac{\frac{x^3\,(3\,A\,c+C\,a)}{8\,a^2}-\frac{B}{4\,c}+\frac{x\,(5\,A\,c-C\,a)}{8\,a\,c}}{a^2+2\,a\,c\,x^2+c^2\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(3\,A\,c+C\,a)}{8\,a^{5/2}\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^3,x)
```

```
[Out] ((x^3*(3*A*c + C*a))/(8*a^2) - B/(4*c) + (x*(5*A*c - C*a))/(8*a*c))/(a^2 +
c^2*x^4 + 2*a*c*x^2) + (atan((c^(1/2)*x)/a^(1/2))*(3*A*c + C*a))/(8*a^(5/2)
*c^(3/2))
```

3.61 $\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx$

Optimal. Leaf size=353

$$-\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2}+\frac{4a^2e\left(Cd^2-Bde+Ae^2\right)+\left(a(Cd-Be)\left(cd^2-3ae^2\right)+\right.}{8a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

```
[Out] 1/4*(-a*(-A*c*e+B*c*d+C*a*e)+c*(A*c*d+B*a*e-C*a*d)*x)/a/c/(a*e^2+c*d^2)/(c*
x^2+a)^2+1/8*(4*a^2*e*(A*e^2-B*d*e+C*d^2)+(a*(-B*e+C*d)*(-3*a*e^2+c*d^2)+A*
c*d*(7*a*e^2+3*c*d^2))*x)/a^2/(a*e^2+c*d^2)^2/(c*x^2+a)+e^3*(A*e^2-B*d*e+C*
d^2)*ln(e*x+d)/(a*e^2+c*d^2)^3-1/2*e^3*(A*e^2-B*d*e+C*d^2)*ln(c*x^2+a)/(a*e
^2+c*d^2)^3+1/8*(a*(-B*e+C*d)*(-3*a^2*e^4+6*a*c*d^2*e^2+c^2*d^4)+A*c*d*(15*
a^2*e^4+10*a*c*d^2*e^2+3*c^2*d^4))*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2
+c*d^2)^3/c^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1661, 837, 815, 649, 211, 266}

$$\frac{4a^2e\left(Ae^2-Bde+Cd^2\right)+x\left(Acd\left(7ae^2+3cd^2\right)+a\left(cd^2-3ae^2\right)(Cd-Be)\right)}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^2}+\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(15a^2e^4+10acd^2e^2+3c^2d^4\right)+a\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)(Cd-Be)\right)}{8a^{5/2}\sqrt{c}\left(ae^2+cd^2\right)^3}-\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{4ac\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}-\frac{e^3\log\left(a+cx^2\right)\left(Ae^2-Bde+Cd^2\right)}{2\left(ae^2+cd^2\right)^3}+\frac{e^3\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{\left(ae^2+cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^3),x]
```

```
[Out] -1/4*(a*(B*c*d - A*c*e + a*C*e) - c*(A*c*d - a*C*d + a*B*e)*x)/(a*c*(c*d^2
+ a*e^2)*(a + c*x^2)^2) + (4*a^2*e*(C*d^2 - B*d*e + A*e^2) + (a*(C*d - B*e)
*(c*d^2 - 3*a*e^2) + A*c*d*(3*c*d^2 + 7*a*e^2))*x)/(8*a^2*(c*d^2 + a*e^2)^2
*(a + c*x^2)) + ((a*(C*d - B*e)*(c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4) + A*c
*d*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/
(8*a^(5/2)*Sqrt[c]*(c*d^2 + a*e^2)^3) + (e^3*(C*d^2 - B*d*e + A*e^2)*Log[d
+ e*x])/(c*d^2 + a*e^2)^3 - (e^3*(C*d^2 - B*d*e + A*e^2)*Log[a + c*x^2])/(2
*(c*d^2 + a*e^2)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx = -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} - \frac{\int \frac{-\frac{c\left(ad(Cd-Be)+A\left(3cd^2+4ae^2\right)\right)}{cd^2+ae^2} - \cdots}{(d+ex)(a+cx^2)\cdots}}{4ac}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e(Cd^2-Bde+Ae^2)+\cdots}{8\cdots}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e(Cd^2-Bde+Ae^2)+\cdots}{8\cdots}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e(Cd^2-Bde+Ae^2)+\cdots}{8\cdots}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e(Cd^2-Bde+Ae^2)+\cdots}{8\cdots}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e(Cd^2-Bde+Ae^2)+\cdots}{8\cdots}$$

Mathematica [A]
time = 0.28, size = 321, normalized size = 0.91

$$\frac{\frac{2\left(cd^2+ae^2\right)^2\left(-a^2Ce+Ac^2dx+ac(-Bd+Ae-Cdx+Bex)\right)}{ac\left(a+cx^2\right)^2}+\frac{\left(cd^2+ae^2\right)\left(3Ac^2d^3x+acd(Cd^2+e(-Bd+7Ae))x+a^2e(Cd(4d-3ex)+e(-4Bd+4Ae+3Bex))\right)}{a^2\left(a+cx^2\right)}+\frac{\left(a(Cd-Be)\left(c^2d^4+6acd^2e^2-3a^2e^4\right)+Acd\left(3c^2d^4+10acd^2e^2+15a^2e^4\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{c}}+8e^3\left(Cd^2+e(-Bd+Ae)\right)\log(d+ex)-4e^3\left(Cd^2+e(-Bd+Ae)\right)\log\left(a+cx^2\right)}{8\left(cd^2+ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^3),x]
```

```
[Out] ((2*(c*d^2 + a*e^2)^2*(-(a^2*C*e) + A*c^2*d*x + a*c*(-(B*d) + A*e - C*d*x +
 B*e*x)))/(a*c*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(3*A*c^2*d^3*x + a*c*d*(C*
d^2 + e*(-(B*d) + 7*A*e))*x + a^2*e*(C*d*(4*d - 3*e*x) + e*(-4*B*d + 4*A*e
+ 3*B*e*x))))/(a^2*(a + c*x^2)) + ((a*(C*d - B*e)*(c^2*d^4 + 6*a*c*d^2*e^2
- 3*a^2*e^4) + A*c*d*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4))*ArcTan[(Sqr
t[c]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[c]) + 8*e^3*(C*d^2 + e*(-(B*d) + A*e))*Log[
d + e*x] - 4*e^3*(C*d^2 + e*(-(B*d) + A*e))*Log[a + c*x^2])/(8*(c*d^2 + a*e
^2)^3)
```

Maple [A]
time = 0.20, size = 605, normalized size = 1.71

method	result

default	$\frac{e^3(A e^2-Bde+C d^2)\ln(ex+d)}{(a e^2+c d^2)^3}+\frac{\frac{c\left(7A a^2cd e^4+10Aa c^2d^3e^2+3A c^3d^5+3B a^3e^5+2B a^2c d^2e^3-B c^2d^4ea-3C a^3d e^4-2C a^2c d^3e^2+c^2C d^5a\right)}{8a^2}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/(a*e^2+c*d^2)^3+1/(a*e^2+c*d^2)^3*((1/8*c
*(7*A*a^2*c*d*e^4+10*A*a*c^2*d^3*e^2+3*A*c^3*d^5+3*B*a^3*e^5+2*B*a^2*c*d^2*
e^3-B*a*c^2*d^4*e-3*C*a^3*d*e^4-2*C*a^2*c*d^3*e^2+C*a*c^2*d^5)/a^2*x^3+(1/2
*A*a*c*e^5+1/2*e^3*c^2*d^2*A-1/2*B*a*c*d*e^4-1/2*c^2*e^2*B*d^3+1/2*C*a*c*d^
2*e^3+1/2*C*c^2*d^4*e)*x^2+1/8*(9*A*a^2*c*d*e^4+14*A*a*c^2*d^3*e^2+5*A*c^3*
d^5+5*B*a^3*e^5+6*B*a^2*c*d^2*e^3+B*a*c^2*d^4*e-5*C*a^3*d*e^4-6*C*a^2*c*d^3
*e^2-C*a*c^2*d^5)/a*x+1/4*(3*A*a^2*c*e^5+4*A*a*c^2*d^2*e^3+A*c^3*d^4*e-3*B*
a^2*c*d*e^4-4*B*a*c^2*d^3*e^2-B*c^3*d^5-C*a^3*e^5+C*a*c^2*d^4*e)/c)/(c*x^2+
a)^2+1/8/a^2*(1/2*(-8*A*a^2*c*e^5+8*B*a^2*c*d*e^4-8*C*a^2*c*d^2*e^3)/c*ln(c
*x^2+a)+(15*A*a^2*c*d*e^4+10*A*a*c^2*d^3*e^2+3*A*c^3*d^5+3*B*a^3*e^5-6*B*a^
2*c*d^2*e^3-B*a*c^2*d^4*e-3*C*a^3*d*e^4+6*C*a^2*c*d^3*e^2+C*a*c^2*d^5)/(a*c
)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Maxima [A]
time = 0.53, size = 638, normalized size = 1.81

$$-\frac{(Cd^2e^3-Bde^4+Ae^5)\log(cx^2+a)}{2(c^3d^6+3ac^2d^4e^2+3a^2cd^2e^4+a^3e^6)}+\frac{(Cd^2e^3-Bde^4+Ae^5)\log(xe+d)}{c^3d^6+3ac^2d^4e^2+3a^2cd^2e^4+a^3e^6}-\frac{(Bac^2d^4e+6Ba^2cd^2e^3-(Cac^2+3Ac^3)d^5-3Ba^3e^5-2(3Ca^2ce^2+5Aac^2e^2)d^3+3(Ca^3e^4-5Aa^2ce^4)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8(a^2c^3d^6+3a^3c^2d^4e^2+3a^4cd^2e^4+a^5e^6)\sqrt{ac}}-\frac{2Ba^2c^2d^3+6Ba^3cde^2+2Ca^4e^3-6Aa^3ce^3+(Bac^3d^2e-3Ba^2c^2e^3-(Cac^3+3Ac^4)d^3+(3Ca^2c^2e^2-7Aac^3e^2)d)x^3-2(Ca^3ce+Aa^2c^2e)d^2-4(Ca^2c^2d^2e-Ba^2c^2de^2+Aa^2c^2e^3)x^2-(Ba^2c^2d^2e+5Ba^3ce^3-(Ca^2c^2-5Aac^3)d^3-(5Ca^3ce^2-9Aa^2c^2e^2)d)x}{8(a^4c^3d^4+2a^5c^2d^2e^2+a^6ce^4+(a^2c^5d^4+2a^3c^4d^2e^2+a^4c^3e^4)x^4+2(a^3c^4d^4+2a^4c^3d^2e^2+a^5c^2e^4)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(C*d^2*e^3 - B*d*e^4 + A*e^5)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (C*d^2*e^3 - B*d*e^4 + A*e^5)*log(x*e + d)
/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - 1/8*(B*a*c^2*d^4
*e + 6*B*a^2*c*d^2*e^3 - (C*a*c^2 + 3*A*c^3)*d^5 - 3*B*a^3*e^5 - 2*(3*C*a^2
*c*e^2 + 5*A*a*c^2*e^2)*d^3 + 3*(C*a^3*e^4 - 5*A*a^2*c*e^4)*d)*arctan(c*x/s
qrt(a*c))/((a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*sq
rt(a*c)) - 1/8*(2*B*a^2*c^2*d^3 + 6*B*a^3*c*d*e^2 + 2*C*a^4*e^3 - 6*A*a^3*c
*e^3 + (B*a*c^3*d^2*e - 3*B*a^2*c^2*e^3 - (C*a*c^3 + 3*A*c^4)*d^3 + (3*C*a^
2*c^2*e^2 - 7*A*a*c^3*e^2)*d)*x^3 - 2*(C*a^3*c*e + A*a^2*c^2*e)*d^2 - 4*(C*
a^2*c^2*d^2*e - B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2 - (B*a^2*c^2*d^2*e + 5
*B*a^3*c*e^3 - (C*a^2*c^2 - 5*A*a*c^3)*d^3 - (5*C*a^3*c*e^2 - 9*A*a^2*c^2*e
^2)*d)*x)/(a^4*c^3*d^4 + 2*a^5*c^2*d^2*e^2 + a^6*c*e^4 + (a^2*c^5*d^4 + 2*a
^3*c^4*d^2*e^2 + a^4*c^3*e^4)*x^4 + 2*(a^3*c^4*d^4 + 2*a^4*c^3*d^2*e^2 + a^
5*c^2*e^4)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1142 vs. $2(331) = 662$.
time = 169.84, size = 2306, normalized size = 6.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c^3*d^5 - 2*(C*a^2*c^4 + 3*A*a*c^5)*d^5*x^3 + 2*(C*a^3*c^3
- 5*A*a^2*c^4)*d^5*x + ((C*a*c^4 + 3*A*c^5)*d^5*x^4 + 2*(C*a^2*c^3 + 3*A*a*
c^4)*d^5*x^2 + (C*a^3*c^2 + 3*A*a^2*c^3)*d^5 + 3*(B*a^3*c^2*x^4 + 2*B*a^4*c
*x^2 + B*a^5)*e^5 - 3*((C*a^3*c^2 - 5*A*a^2*c^3)*d*x^4 + 2*(C*a^4*c - 5*A*a
^3*c^2)*d*x^2 + (C*a^5 - 5*A*a^4*c)*d)*e^4 - 6*(B*a^2*c^3*d^2*x^4 + 2*B*a^3
*c^2*d^2*x^2 + B*a^4*c*d^2)*e^3 + 2*((3*C*a^2*c^3 + 5*A*a*c^4)*d^3*x^4 + 2*
(3*C*a^3*c^2 + 5*A*a^2*c^3)*d^3*x^2 + (3*C*a^4*c + 5*A*a^3*c^2)*d^3)*e^2 -
(B*a*c^4*d^4*x^4 + 2*B*a^2*c^3*d^4*x^2 + B*a^3*c^2*d^4)*e)*sqrt(-a*c)*log((
c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(3*B*a^4*c^2*x^3 + 4*A*a^4*c^2
*x^2 + 5*B*a^5*c*x - 2*C*a^6 + 6*A*a^5*c)*e^5 + 2*(4*B*a^4*c^2*d*x^2 + 6*B*
a^5*c*d + (3*C*a^4*c^2 - 7*A*a^3*c^3)*d*x^3 + (5*C*a^5*c - 9*A*a^4*c^2)*d*x
)*e^4 - 4*(B*a^3*c^3*d^2*x^3 + 3*B*a^4*c^2*d^2*x + 4*A*a^4*c^2*d^2 + 2*(C*a
^4*c^2 + A*a^3*c^3)*d^2*x^2)*e^3 + 4*(2*B*a^3*c^3*d^3*x^2 + 4*B*a^4*c^2*d^3
 + (C*a^3*c^3 - 5*A*a^2*c^4)*d^3*x^3 + (3*C*a^4*c^2 - 7*A*a^3*c^3)*d^3*x)*e
^2 + 2*(B*a^2*c^4*d^4*x^3 - 4*C*a^3*c^3*d^4*x^2 - B*a^3*c^3*d^4*x - 2*(C*a^
4*c^2 + A*a^3*c^3)*d^4)*e + 8*((A*a^3*c^3*x^4 + 2*A*a^4*c^2*x^2 + A*a^5*c)*
e^5 - (B*a^3*c^3*d*x^4 + 2*B*a^4*c^2*d*x^2 + B*a^5*c*d)*e^4 + (C*a^3*c^3*d^
2*x^4 + 2*C*a^4*c^2*d^2*x^2 + C*a^5*c*d^2)*e^3)*log(c*x^2 + a) - 16*((A*a^3
*c^3*x^4 + 2*A*a^4*c^2*x^2 + A*a^5*c)*e^5 - (B*a^3*c^3*d*x^4 + 2*B*a^4*c^2*
d*x^2 + B*a^5*c*d)*e^4 + (C*a^3*c^3*d^2*x^4 + 2*C*a^4*c^2*d^2*x^2 + C*a^5*c
*d^2)*e^3)*log(x*e + d))/(a^3*c^6*d^6*x^4 + 2*a^4*c^5*d^6*x^2 + a^5*c^4*d^6
 + (a^6*c^3*x^4 + 2*a^7*c^2*x^2 + a^8*c)*e^6 + 3*(a^5*c^4*d^2*x^4 + 2*a^6*c
^3*d^2*x^2 + a^7*c^2*d^2)*e^4 + 3*(a^4*c^5*d^4*x^4 + 2*a^5*c^4*d^4*x^2 + a^
6*c^3*d^4)*e^2), -1/8*(2*B*a^3*c^3*d^5 - (C*a^2*c^4 + 3*A*a*c^5)*d^5*x^3 +
(C*a^3*c^3 - 5*A*a^2*c^4)*d^5*x - ((C*a*c^4 + 3*A*c^5)*d^5*x^4 + 2*(C*a^2*c
^3 + 3*A*a*c^4)*d^5*x^2 + (C*a^3*c^2 + 3*A*a^2*c^3)*d^5 + 3*(B*a^3*c^2*x^4
+ 2*B*a^4*c*x^2 + B*a^5)*e^5 - 3*((C*a^3*c^2 - 5*A*a^2*c^3)*d*x^4 + 2*(C*a^
4*c - 5*A*a^3*c^2)*d*x^2 + (C*a^5 - 5*A*a^4*c)*d)*e^4 - 6*(B*a^2*c^3*d^2*x^
4 + 2*B*a^3*c^2*d^2*x^2 + B*a^4*c*d^2)*e^3 + 2*((3*C*a^2*c^3 + 5*A*a*c^4)*d
^3*x^4 + 2*(3*C*a^3*c^2 + 5*A*a^2*c^3)*d^3*x^2 + (3*C*a^4*c + 5*A*a^3*c^2)*
d^3)*e^2 - (B*a*c^4*d^4*x^4 + 2*B*a^2*c^3*d^4*x^2 + B*a^3*c^2*d^4)*e)*sqrt(
a*c)*arctan(sqrt(a*c)*x/a) - (3*B*a^4*c^2*x^3 + 4*A*a^4*c^2*x^2 + 5*B*a^5*c
*x - 2*C*a^6 + 6*A*a^5*c)*e^5 + (4*B*a^4*c^2*d*x^2 + 6*B*a^5*c*d + (3*C*a^4
*c^2 - 7*A*a^3*c^3)*d*x^3 + (5*C*a^5*c - 9*A*a^4*c^2)*d*x)*e^4 - 2*(B*a^3*c
^3*d^2*x^3 + 3*B*a^4*c^2*d^2*x + 4*A*a^4*c^2*d^2 + 2*(C*a^4*c^2 + A*a^3*c^3
)*d^2*x^2)*e^3 + 2*(2*B*a^3*c^3*d^3*x^2 + 4*B*a^4*c^2*d^3 + (C*a^3*c^3 - 5*
```

```
A*a^2*c^4)*d^3*x^3 + (3*C*a^4*c^2 - 7*A*a^3*c^3)*d^3*x)*e^2 + (B*a^2*c^4*d^
4*x^3 - 4*C*a^3*c^3*d^4*x^2 - B*a^3*c^3*d^4*x - 2*(C*a^4*c^2 + A*a^3*c^3)*d
^4)*e + 4*((A*a^3*c^3*x^4 + 2*A*a^4*c^2*x^2 + A*a^5*c)*e^5 - (B*a^3*c^3*d*x
^4 + 2*B*a^4*c^2*d*x^2 + B*a^5*c*d)*e^4 + (C*a^3*c^3*d^2*x^4 + 2*C*a^4*c^2*
d^2*x^2 + C*a^5*c*d^2)*e^3)*log(c*x^2 + a) - 8*((A*a^3*c^3*x^4 + 2*A*a^4*c^
2*x^2 + A*a^5*c)*e^5 - (B*a^3*c^3*d*x^4 + 2*B*a^4*c^2*d*x^2 + B*a^5*c*d)*e^
4 + (C*a^3*c^3*d^2*x^4 + 2*C*a^4*c^2*d^2*x^2 + C*a^5*c*d^2)*e^3)*log(x*e +
d))/(a^3*c^6*d^6*x^4 + 2*a^4*c^5*d^6*x^2 + a^5*c^4*d^6 + (a^6*c^3*x^4 + 2*a
^7*c^2*x^2 + a^8*c)*e^6 + 3*(a^5*c^4*d^2*x^4 + 2*a^6*c^3*d^2*x^2 + a^7*c^2*
d^2)*e^4 + 3*(a^4*c^5*d^4*x^4 + 2*a^5*c^4*d^4*x^2 + a^6*c^3*d^4)*e^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 715 vs. 2(331) = 662.
time = 3.38, size = 715, normalized size = 2.03

$$-\frac{(Cd^2e^3 - Bde^4 + Ae^5)\log(cx^2+a)}{2(c^3d^6 + 3ac^2d^4e^2 + 3a^2cd^2e^4 + a^3e^6)} + \frac{(Cd^2e^4 - Bde^5 + Ae^6)\log(|xe+d|)}{c^3d^6e + 3ac^2d^4e^3 + 3a^2cd^2e^5 + a^3e^7} + \frac{(Cac^2d^5 + 3Ac^3d^5 - Bac^2d^4e + 6Ca^2cd^3e^2 + 10Aac^2d^3e^2 - 6Ba^2cd^2e^3 - 3Ca^3de^4 + 15Aa^2cde^4 + 3Ba^3e^5)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8(a^2c^3d^6 + 3a^3c^2d^4e^2 + 3a^4cd^2e^4 + a^5e^6)\sqrt{ac}} - \frac{2Ba^2c^3d^5 - 2Ca^3c^2d^4e - 2Aa^2c^3d^4e + 8Ba^3c^2d^3e^2 - 8Aa^3c^2d^2e^3 + 6Ba^4cde^4 + 2Ca^5e^5 - 6Aa^4ce^5 - (Cac^4d^5 + 3Ac^5d^5 - Bac^4d^4e - 2Ca^2c^3d^3e^2 + 10Aac^4d^3e^2 + 2Ba^2c^3d^2e^3 - 3Ca^3c^2de^4 + 7Aa^2c^3de^4 + 3Ba^3c^2e^5)x^3 - 4(Ca^2c^3d^4e - Ba^2c^3d^3e^2 + Ca^3c^2d^2e^3 + Aa^2c^3d^2e^3 - Ba^3c^2de^4 + Aa^3c^2e^5)x^2 + (Ca^2c^3d^5 - 5Aac^4d^5 - Ba^2c^3d^4e + 6Ca^3c^2d^3e^2 - 14Aa^2c^3d^3e^2 - 6Ba^3c^2d^2e^3 + 5Ca^4cde^4 - 9Aa^3c^2de^4 - 5Ba^4ce^5)x}{8(cd^2+ae^2)^3(cx^2+a)^2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(C*d^2*e^3 - B*d*e^4 + A*e^5)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (C*d^2*e^4 - B*d*e^5 + A*e^6)*log(abs(x*e
+ d))/(c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7) + 1/8*(C*a*
c^2*d^5 + 3*A*c^3*d^5 - B*a*c^2*d^4*e + 6*C*a^2*c*d^3*e^2 + 10*A*a*c^2*d^3*
e^2 - 6*B*a^2*c*d^2*e^3 - 3*C*a^3*d*e^4 + 15*A*a^2*c*d*e^4 + 3*B*a^3*e^5)*a
rctan(c*x/sqrt(a*c))/((a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 +
a^5*e^6)*sqrt(a*c)) - 1/8*(2*B*a^2*c^3*d^5 - 2*C*a^3*c^2*d^4*e - 2*A*a^2*c^
3*d^4*e + 8*B*a^3*c^2*d^3*e^2 - 8*A*a^3*c^2*d^2*e^3 + 6*B*a^4*c*d*e^4 + 2*C
*a^5*e^5 - 6*A*a^4*c*e^5 - (C*a*c^4*d^5 + 3*A*c^5*d^5 - B*a*c^4*d^4*e - 2*C
*a^2*c^3*d^3*e^2 + 10*A*a*c^4*d^3*e^2 + 2*B*a^2*c^3*d^2*e^3 - 3*C*a^3*c^2*d
*e^4 + 7*A*a^2*c^3*d*e^4 + 3*B*a^3*c^2*e^5)*x^3 - 4*(C*a^2*c^3*d^4*e - B*a^
2*c^3*d^3*e^2 + C*a^3*c^2*d^2*e^3 + A*a^2*c^3*d^2*e^3 - B*a^3*c^2*d*e^4 + A
*a^3*c^2*e^5)*x^2 + (C*a^2*c^3*d^5 - 5*A*a*c^4*d^5 - B*a^2*c^3*d^4*e + 6*C*
a^3*c^2*d^3*e^2 - 14*A*a^2*c^3*d^3*e^2 - 6*B*a^3*c^2*d^2*e^3 + 5*C*a^4*c*d*
e^4 - 9*A*a^3*c^2*d*e^4 - 5*B*a^4*c*e^5)*x)/((c*d^2 + a*e^2)^3*(c*x^2 + a)^
2*a^2*c)
```

Mupad [B]
time = 9.90, size = 2392, normalized size = 6.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^3*(d + e*x)),x)
[Out] ((x^2*(A*c*e^3 - B*c*d*e^2 + C*c*d^2*e))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*
e^2)) - (B*c^2*d^3 + C*a^2*e^3 - 3*A*a*c*e^3 - A*c^2*d^2*e + 3*B*a*c*d*e^2
- C*a*c*d^2*e)/(4*c*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(5*A*c^2*d^3
+ 5*B*a^2*e^3 - C*a*c*d^3 - 5*C*a^2*d*e^2 + 9*A*a*c*d*e^2 + B*a*c*d^2*e))/(
8*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x^3*(3*A*c^3*d^3 + 3*B*a^2*c*e^
3 + C*a*c^2*d^3 + 7*A*a*c^2*d*e^2 - B*a*c^2*d^2*e - 3*C*a^2*c*d*e^2))/(8*a^
2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a^2 + c^2*x^4 + 2*a*c*x^2) - (log(
3*A*c^4*d^7*(-a^5*c)^(1/2) - 3*B*a^4*e^7*(-a^5*c)^(1/2) - 24*A*a^6*c*e^7 +
3*B*a^6*c*e^7*x + 6*A*a^3*c^4*d^6*e + 2*C*a^4*c^3*d^6*e - 30*C*a^6*c*d^2*e^
5 - 3*A*a^2*c^5*d^7*x - C*a^3*c^4*d^7*x + C*a*c^3*d^7*(-a^5*c)^(1/2) + 3*C*
a^4*d*e^6*(-a^5*c)^(1/2) + 20*A*a^4*c^3*d^4*e^3 + 54*A*a^5*c^2*d^2*e^5 - 2*
B*a^4*c^3*d^5*e^2 - 36*B*a^5*c^2*d^3*e^4 + 36*C*a^5*c^2*d^4*e^3 + 30*B*a^6*
c*d*e^6 - 7*A*a^3*c^4*d^5*e^2*x - 5*A*a^4*c^3*d^3*e^4*x + 5*B*a^4*c^3*d^4*e
^3*x - 57*B*a^5*c^2*d^2*e^5*x - 5*C*a^4*c^3*d^5*e^2*x + 57*C*a^5*c^2*d^3*e^
4*x + 7*A*a*c^3*d^5*e^2*(-a^5*c)^(1/2) + 57*B*a^3*c*d^2*e^5*(-a^5*c)^(1/2)
- 57*C*a^3*c*d^3*e^4*(-a^5*c)^(1/2) - 3*C*a^6*c*d*e^6*x + 5*A*a^2*c^2*d^3*e
^4*(-a^5*c)^(1/2) - 5*B*a^2*c^2*d^4*e^3*(-a^5*c)^(1/2) + 5*C*a^2*c^2*d^5*e^
2*(-a^5*c)^(1/2) + 63*A*a^5*c^2*d*e^6*x + B*a^3*c^4*d^6*e*x - 63*A*a^3*c*d*
e^6*(-a^5*c)^(1/2) - B*a*c^3*d^6*e*(-a^5*c)^(1/2) - 24*A*a^3*c*e^7*x*(-a^5*
c)^(1/2) + 6*A*c^4*d^6*e*x*(-a^5*c)^(1/2) + 54*A*a^2*c^2*d^2*e^5*x*(-a^5*c)
^(1/2) - 36*B*a^2*c^2*d^3*e^4*x*(-a^5*c)^(1/2) + 36*C*a^2*c^2*d^4*e^3*x*(-a
^5*c)^(1/2) + 30*B*a^3*c*d*e^6*x*(-a^5*c)^(1/2) + 2*C*a*c^3*d^6*e*x*(-a^5*c
)^(1/2) + 20*A*a*c^3*d^4*e^3*x*(-a^5*c)^(1/2) - 2*B*a*c^3*d^5*e^2*x*(-a^5*c
)^(1/2) - 30*C*a^3*c*d^2*e^5*x*(-a^5*c)^(1/2))*(c*(a^2*((3*C*d^3*e^2*(-a^5*
c)^(1/2))/8 - (3*B*d^2*e^3*(-a^5*c)^(1/2))/8 + (15*A*d*e^4*(-a^5*c)^(1/2))/
16) + a^5*((A*e^5)/2 + (C*d^2*e^3)/2 - (B*d*e^4)/2)) + a^3*((3*B*e^5*(-a^5*
c)^(1/2))/16 - (3*C*d*e^4*(-a^5*c)^(1/2))/16) + a*c^2*((C*d^5*(-a^5*c)^(1/2
))/16 + (5*A*d^3*e^2*(-a^5*c)^(1/2))/8 - (B*d^4*e*(-a^5*c)^(1/2))/16) + (3*
A*c^3*d^5*(-a^5*c)^(1/2))/16))/(a^8*c*e^6 + a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2
 + 3*a^7*c^2*d^2*e^4) + (log(3*A*c^4*d^7*(-a^5*c)^(1/2) - 3*B*a^4*e^7*(-a^5
*c)^(1/2) + 24*A*a^6*c*e^7 - 3*B*a^6*c*e^7*x - 6*A*a^3*c^4*d^6*e - 2*C*a^4*
c^3*d^6*e + 30*C*a^6*c*d^2*e^5 + 3*A*a^2*c^5*d^7*x + C*a^3*c^4*d^7*x + C*a*
c^3*d^7*(-a^5*c)^(1/2) + 3*C*a^4*d*e^6*(-a^5*c)^(1/2) - 20*A*a^4*c^3*d^4*e^
3 - 54*A*a^5*c^2*d^2*e^5 + 2*B*a^4*c^3*d^5*e^2 + 36*B*a^5*c^2*d^3*e^4 - 36*
C*a^5*c^2*d^4*e^3 - 30*B*a^6*c*d*e^6 + 7*A*a^3*c^4*d^5*e^2*x + 5*A*a^4*c^3*
d^3*e^4*x - 5*B*a^4*c^3*d^4*e^3*x + 57*B*a^5*c^2*d^2*e^5*x + 5*C*a^4*c^3*d^
5*e^2*x - 57*C*a^5*c^2*d^3*e^4*x + 7*A*a*c^3*d^5*e^2*(-a^5*c)^(1/2) + 57*B*
```

a^3*c*d^2*e^5*(-a^5*c)^(1/2) - 57*C*a^3*c*d^3*e^4*(-a^5*c)^(1/2) + 3*C*a^6*c*d*e^6*x + 5*A*a^2*c^2*d^3*e^4*(-a^5*c)^(1/2) - 5*B*a^2*c^2*d^4*e^3*(-a^5*c)^(1/2) + 5*C*a^2*c^2*d^5*e^2*(-a^5*c)^(1/2) - 63*A*a^5*c^2*d*e^6*x - B*a^3*c^4*d^6*e*x - 63*A*a^3*c*d*e^6*(-a^5*c)^(1/2) - B*a*c^3*d^6*e*(-a^5*c)^(1/2) - 24*A*a^3*c*e^7*x*(-a^5*c)^(1/2) + 6*A*c^4*d^6*e*x*(-a^5*c)^(1/2) + 54*A*a^2*c^2*d^2*e^5*x*(-a^5*c)^(1/2) - 36*B*a^2*c^2*d^3*e^4*x*(-a^5*c)^(1/2) + 36*C*a^2*c^2*d^4*e^3*x*(-a^5*c)^(1/2) + 30*B*a^3*c*d*e^6*x*(-a^5*c)^(1/2) + 2*C*a*c^3*d^6*e*x*(-a^5*c)^(1/2) + 20*A*a*c^3*d^4*e^3*x*(-a^5*c)^(1/2) - 2*B*a*c^3*d^5*e^2*x*(-a^5*c)^(1/2) - 30*C*a^3*c*d^2*e^5*x*(-a^5*c)^(1/2))*(c*(a^2*((3*C*d^3*e^2*(-a^5*c)^(1/2))/8 - (3*B*d^2*e^3*(-a^5*c)^(1/2))/8 + (15*A*d*e^4*(-a^5*c)^(1/2))/16) - a^5*((A*e^5)/2 + (C*d^2*e^3)/2 - (B*d*e^4)/2)) + a^3*((3*B*e^5*(-a^5*c)^(1/2))/16 - (3*C*d*e^4*(-a^5*c)^(1/2))/16) + a*c^2*((C*d^5*(-a^5*c)^(1/2))/16 + (5*A*d^3*e^2*(-a^5*c)^(1/2))/8 - (B*d^4*e*(-a^5*c)^(1/2))/16) + (3*A*c^3*d^5*(-a^5*c)^(1/2))/16))/(a^8*c*e^6 + a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4) + (e^3*log(d + e*x)*(A*e^2 + C*d^2 - B*d*e))/(a*e^2 + c*d^2)^3

3.62 $\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)^3} \, dx$

Optimal. Leaf size=571

$$-\frac{e^3(Cd^2-Bde+Ae^2)}{(cd^2+ae^2)^3(d+ex)}-\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be))}{4a(cd^2+ae^2)^2(a+cx^2)^2}$$

```
[Out] -e^3*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^3/(e*x+d)+1/4*(-a*(-2*A*c*d*e-B*a*e^
2+B*c*d^2+2*C*a*d*e)+(A*c*(-a*e^2+c*d^2)+a*(a*C*e^2-c*d*(-2*B*e+C*d)))*x)/a
/(a*e^2+c*d^2)^2/(c*x^2+a)^2+1/8*(-4*a^2*e*(a*e^2*(-B*e+2*C*d)-c*d*(2*C*d^2
-e*(-4*A*e+3*B*d)))+(A*c*(-7*a^2*e^4+12*a*c*d^2*e^2+3*c^2*d^4)+a*(3*a^2*C*e
^4-2*a*c*d*e^2*(-7*B*e+6*C*d)+c^2*d^3*(-2*B*e+C*d)))*x)/a^2/(a*e^2+c*d^2)^3
/(c*x^2+a)-e^3*(a*e^2*(-B*e+2*C*d)-c*d*(4*C*d^2-e*(-6*A*e+5*B*d)))*ln(e*x+d
)/(a*e^2+c*d^2)^4+1/2*e^3*(a*e^2*(-B*e+2*C*d)-c*d*(4*C*d^2-e*(-6*A*e+5*B*d)
))*ln(c*x^2+a)/(a*e^2+c*d^2)^4+1/8*(3*A*c*(-5*a^3*e^6+15*a^2*c*d^2*e^4+5*a*
c^2*d^4*e^2+c^3*d^6)+a*(3*a^3*C*e^6+a*c^2*d^3*e^2*(-20*B*e+13*C*d)-3*a^2*c*
d*e^4*(-10*B*e+11*C*d)+c^3*d^5*(-2*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/a^(
5/2)/(a*e^2+c*d^2)^4/c^(1/2)
```

Rubi [A]
time = 1.30, antiderivative size = 566, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 1643, 649, 211, 266}

$$\frac{x(Ac(-7a^2e^4+12acd^2e^2+3c^2d^4)+a(3a^2Ce^4-2acde^2(6Cd-7Be)+c^2d^3(Cd-2Be)))+4a^2e(-ae^2(2Cd-Be)-cde(3Bd-4Ae)+2cCd^3)}{8a^2(a+cx^2)(ae^2+cd^2)^3}+\frac{\text{ArcTan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(3Ac(-5a^3e^6+15a^2cd^2e^4+5ac^2d^4e^2+c^3d^6)+a(3a^3Ce^6-3a^2cde^4(11Cd-10Be)+ac^2d^3e^2(13Cd-20Be)+c^3d^5(Cd-2Be)))}{8a^{5/2}\sqrt{c}(ae^2+cd^2)^4}-\frac{a(-aBe^2+2aCde-2Acde+Bcd^2)-x(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))}{4a(a+cx^2)^2(ae^2+cd^2)^2}-\frac{e^3(Ae^2-Bde+Cd^2)}{(d+ex)(ae^2+cd^2)^3}-\frac{e^3\log(a+cx^2)(-ae^2(2Cd-Be)-cde(5Bd-6Ae)+4cCd^3)}{2(ae^2+cd^2)^4}+\frac{e^3\log(d+ex)(-ae^2(2Cd-Be)-cde(5Bd-6Ae)+4cCd^3)}{(ae^2+cd^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^3),x]
```

```
[Out] -((e^3*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^3*(d + e*x))) - (a*(B*c*d^
2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2) - (A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 -
c*d*(C*d - 2*B*e)))*x)/(4*a*(c*d^2 + a*e^2)^2*(a + c*x^2)^2) + (4*a^2*e*(2*
c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e)) + (A*c*(3*c^2*d^4 +
12*a*c*d^2*e^2 - 7*a^2*e^4) + a*(3*a^2*C*e^4 - 2*a*c*d*e^2*(6*C*d - 7*B*e)
+ c^2*d^3*(C*d - 2*B*e)))*x)/(8*a^2*(c*d^2 + a*e^2)^3*(a + c*x^2)) + ((3*A*
c*(c^3*d^6 + 5*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - 5*a^3*e^6) + a*(3*a^3*C*e
^6 + a*c^2*d^3*e^2*(13*C*d - 20*B*e) - 3*a^2*c*d*e^4*(11*C*d - 10*B*e) + c^
3*d^5*(C*d - 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[c]*(c*d^
2 + a*e^2)^4) + (e^3*(4*c*C*d^3 - c*d*e*(5*B*d - 6*A*e) - a*e^2*(2*C*d - B*
e))*Log[d + e*x])/(c*d^2 + a*e^2)^4 - (e^3*(4*c*C*d^3 - c*d*e*(5*B*d - 6*A*
e) - a*e^2*(2*C*d - B*e))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)^3}\,dx = -\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2}{4a\,(cd^2+ae^2)^2\,(a+cx^2)^2}$$

$$= -\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2}{4a\,(cd^2+ae^2)^2\,(a+cx^2)^2}$$

$$= -\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2}{4a\,(cd^2+ae^2)^2\,(a+cx^2)^2}$$

$$= -\frac{e^3(Cd^2-Bde+Ae^2)}{(cd^2+ae^2)^3\,(d+ex)} - \frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae}{4a\,(cd^2+ae^2)^2\,(a+c}$$

$$= -\frac{e^3(Cd^2-Bde+Ae^2)}{(cd^2+ae^2)^3\,(d+ex)} - \frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae}{4a\,(cd^2+ae^2)^2\,(a+c}$$

$$= -\frac{e^3(Cd^2-Bde+Ae^2)}{(cd^2+ae^2)^3\,(d+ex)} - \frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae}{4a\,(cd^2+ae^2)^2\,(a+c}$$

Mathematica [A]
time = 0.48, size = 498, normalized size = 0.87

$$\frac{-\frac{8e^3(cd^2+ae^2)(Cd^2+e(-Bd+Ae))}{d+ex}+\frac{2(cd^2+ae^2)^2(Ac^2d^2x+a^2e(-2Cd+Be+Cex)-ac(Cd^2x+Bd(d-2ex)+Ae(-2d+ex)))}{a(a+cx^2)^2}+\frac{(cd^2+ae^2)(3Ac^3d^4x+ac^2d^2(Cd^2+2e(-Bd+6Ae))x+a^3e^3(-8Cd+4Be+3Cex)+a^2ce(4Cd^2(2d-3ex)+e(-2Bd(6d-7ex)+Ae(16d-7ex))))}{a^2(a+cx^2)}+\frac{(3Ac(c^3d^6+5ac^2d^4e^2+15a^2cd^2e^4-5a^3e^6)+a(3a^3Ce^6+ac^2d^3e^2(13Cd-20Be)+c^3d^5(Cd-2Be)+3a^2cde^4(-11Cd+10Be)))\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{c}}+8e^3(4cCd^3+cde(-5Bd+6Ae)+ae^2(-2Cd+Be))\log(d+ex)-4e^3(4cCd^3+cde(-5Bd+6Ae)+ae^2(-2Cd+Be))\log(a+cx^2)}{8(cd^2+ae^2)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^3),x]
```

```
[Out] ((-8*e^3*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) + (2*(c*d^2
+ a*e^2)^2*(A*c^2*d^2*x + a^2*e*(-2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + B*d
*(d - 2*e*x) + A*e*(-2*d + e*x))))/(a*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(3*
A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 2*e*(-(B*d) + 6*A*e))*x + a^3*e^3*(-8*C*d
+ 4*B*e + 3*C*e*x) + a^2*c*e*(4*C*d^2*(2*d - 3*e*x) + e*(-2*B*d*(6*d - 7*e*
x) + A*e*(16*d - 7*e*x)))))/(a^2*(a + c*x^2)) + ((3*A*c*(c^3*d^6 + 5*a*c^2*
d^4*e^2 + 15*a^2*c*d^2*e^4 - 5*a^3*e^6) + a*(3*a^3*C*e^6 + a*c^2*d^3*e^2*(1
3*C*d - 20*B*e) + c^3*d^5*(C*d - 2*B*e) + 3*a^2*c*d*e^4*(-11*C*d + 10*B*e))
)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[c]) + 8*e^3*(4*c*C*d^3 + c*d*e
*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e))*Log[d + e*x] - 4*e^3*(4*c*C*d^3 +
 c*d*e*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e))*Log[a + c*x^2])/(8*(c*d^2 +
 a*e^2)^4)
```

Maple [A]
time = 0.18, size = 833, normalized size = 1.46

method	result
default	$\frac{e^{3}\left(6d\,e^{2}cA+Ba\,e^{3}-5Bc\,d^{2}e-2Cad\,e^{2}+4Cc\,d^{3}\right)\ln\left(ex+d\right)}{\left(a\,e^{2}+c\,d^{2}\right)^{4}}-\frac{e^{3}\left(A\,e^{2}-Bde+C\,d^{2}\right)}{\left(a\,e^{2}+c\,d^{2}\right)^{3}\left(ex+d\right)}-\frac{c\left(7A\,a^{3}c\,e^{6}-5A\,a^{2}c^{2}d^{2}e^{4}-15Aa\,c^{3}d^{4}e^{2}-3A\,c^{4}d\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(6*A*c*d*e^2+B*a*e^3-5*B*c*d^2*e-2*C*a*d*e^2+4*C*c*d^3)/(a*e^2+c*d^2)^4
*ln(e*x+d)-e^3*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^3/(e*x+d)-1/(a*e^2+c*d^2)^
4*((1/8*c*(7*A*a^3*c*e^6-5*A*a^2*c^2*d^2*e^4-15*A*a*c^3*d^4*e^2-3*A*c^4*d^6
-14*B*a^3*c*d*e^5-12*B*a^2*c^2*d^3*e^3+2*B*a*c^3*d^5*e-3*C*a^4*e^6+9*C*a^3*
c*d^2*e^4+11*C*a^2*c^2*d^4*e^2-C*a*c^3*d^6)/a^2*x^3+(-2*A*a*c^2*d*e^5-2*e^3
*c^3*d^3*A-1/2*B*a^2*c*e^6+B*a*c^2*d^2*e^4+3/2*e^2*c^3*B*d^4+C*a^2*c*d*e^5-
C*c^3*d^5*e)*x^2+1/8*(9*A*a^3*c*e^6-3*A*a^2*c^2*d^2*e^4-17*A*a*c^3*d^4*e^2-
5*A*c^4*d^6-18*B*a^3*c*d*e^5-20*B*a^2*c^2*d^3*e^3-2*B*a*c^3*d^5*e-5*C*a^4*e
^6+7*C*a^3*c*d^2*e^4+13*C*a^2*c^2*d^4*e^2+C*a*c^3*d^6)/a*x-5/2*A*a^2*c*d*e^
5-3*A*a*c^2*d^3*e^3-1/2*A*c^3*d^5*e-3/4*B*a^3*e^6+3/4*B*a^2*c*d^2*e^4+7/4*B
*a*c^2*d^4*e^2+1/4*B*c^3*d^6+3/2*C*a^3*d*e^5+C*a^2*c*d^3*e^3-1/2*C*a*c^2*d^
5*e)/(c*x^2+a)^2+1/8/a^2*(1/2*(48*A*a^2*c^2*d*e^5+8*B*a^3*c*e^6-40*B*a^2*c^
2*d^2*e^4-16*C*a^3*c*d*e^5+32*C*a^2*c^2*d^3*e^3)/c*ln(c*x^2+a)+(15*A*a^3*c*
e^6-45*A*a^2*c^2*d^2*e^4-15*A*a*c^3*d^4*e^2-3*A*c^4*d^6-30*B*a^3*c*d*e^5+20
*B*a^2*c^2*d^3*e^3+2*B*a*c^3*d^5*e-3*C*a^4*e^6+33*C*a^3*c*d^2*e^4-13*C*a^2*
c^2*d^4*e^2-C*a*c^3*d^6)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1170 vs. $2(549) = 1098$.
time = 0.55, size = 1170, normalized size = 2.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 + B*a*e^6 - 2*(C*a*e^5 - 3*A*c*e^5)*d)*
log(c*x^2 + a)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2
*e^6 + a^4*e^8) + (4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 + B*a*e^6 - 2*(C*a*e^5 - 3
*A*c*e^5)*d)*log(x*e + d)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 +
4*a^3*c*d^2*e^6 + a^4*e^8) - 1/8*(2*B*a*c^3*d^5*e + 20*B*a^2*c^2*d^3*e^3 -
(C*a*c^3 + 3*A*c^4)*d^6 - 30*B*a^3*c*d*e^5 - 3*C*a^4*e^6 + 15*A*a^3*c*e^6 -
 (13*C*a^2*c^2*e^2 + 15*A*a*c^3*e^2)*d^4 + 3*(11*C*a^3*c*e^4 - 15*A*a^2*c^2
```

```
*e^4)*d^2)*arctan(c*x/sqrt(a*c))/((a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*
c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(a*c)) - 1/8*(2*B*a^2*c^2*d^5
+ 12*B*a^3*c*d^3*e^2 - 14*B*a^4*d*e^4 + 8*A*a^4*e^5 - 4*(C*a^3*c*e + A*a^2*
c^2*e)*d^4 + (2*B*a*c^3*d^3*e^2 - 22*B*a^2*c^2*d*e^4 - 3*C*a^3*c*e^5 + 15*A
*a^2*c^2*e^5 - (C*a*c^3*e + 3*A*c^4*e)*d^4 + 4*(5*C*a^2*c^2*e^3 - 3*A*a*c^3
*e^3)*d^2)*x^4 + (2*B*a*c^3*d^4*e - 2*B*a^2*c^2*d^2*e^3 - (C*a*c^3 + 3*A*c^
4)*d^5 - 4*B*a^3*c*e^5 + 4*(C*a^2*c^2*e^2 - 3*A*a*c^3*e^2)*d^3 + (5*C*a^3*c
*e^4 - 9*A*a^2*c^2*e^4)*d)*x^3 + 20*(C*a^4*e^3 - A*a^3*c*e^3)*d^2 + (10*B*a
^2*c^2*d^3*e^2 - 38*B*a^3*c*d*e^4 - 5*C*a^4*e^5 + 25*A*a^3*c*e^5 - (7*C*a^2
*c^2*e + 5*A*a*c^3*e)*d^4 + 4*(9*C*a^3*c*e^3 - 7*A*a^2*c^2*e^3)*d^2)*x^2 -
(6*B*a^3*c*d^2*e^3 - (C*a^2*c^2 - 5*A*a*c^3)*d^5 + 6*B*a^4*e^5 - 8*(C*a^3*c
*e^2 - 2*A*a^2*c^2*e^2)*d^3 - (7*C*a^4*e^4 - 11*A*a^3*c*e^4)*d)*x)/(a^4*c^3
*d^7 + 3*a^5*c^2*d^5*e^2 + 3*a^6*c*d^3*e^4 + a^7*d*e^6 + (a^2*c^5*d^6*e + 3
*a^3*c^4*d^4*e^3 + 3*a^4*c^3*d^2*e^5 + a^5*c^2*e^7)*x^5 + (a^2*c^5*d^7 + 3*
a^3*c^4*d^5*e^2 + 3*a^4*c^3*d^3*e^4 + a^5*c^2*d*e^6)*x^4 + 2*(a^3*c^4*d^6*e
 + 3*a^4*c^3*d^4*e^3 + 3*a^5*c^2*d^2*e^5 + a^6*c*e^7)*x^3 + 2*(a^3*c^4*d^7
+ 3*a^4*c^3*d^5*e^2 + 3*a^5*c^2*d^3*e^4 + a^6*c*d*e^6)*x^2 + (a^4*c^3*d^6*e
 + 3*a^5*c^2*d^4*e^3 + 3*a^6*c*d^2*e^5 + a^7*e^7)*x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a)**3,x)
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. 2(549) = 1098.
time = 4.66, size = 1107, normalized size = 1.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a*c^3*d^6*e^2 + 3*A*c^4*d^6*e^2 - 2*B*a*c^3*d^5*e^3 + 13*C*a^2*c^2*d
^4*e^4 + 15*A*a*c^3*d^4*e^4 - 20*B*a^2*c^2*d^3*e^5 - 33*C*a^3*c*d^2*e^6 + 4
5*A*a^2*c^2*d^2*e^6 + 30*B*a^3*c*d*e^7 + 3*C*a^4*e^8 - 15*A*a^3*c*e^8)*arct
an((c*d - c*d^2/(x*e + d) - a*e^2/(x*e + d))*e^(-1)/sqrt(a*c))*e^(-2)/((a^2
*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^
8)*sqrt(a*c)) - 1/2*(4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 - 2*C*a*d*e^5 + 6*A*c*d*
e^5 + B*a*e^6)*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d
)^2)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4
*e^8) - (C*d^2*e^9/(x*e + d) - B*d*e^10/(x*e + d) + A*e^11/(x*e + d))/(c^3*
d^6*e^6 + 3*a*c^2*d^4*e^8 + 3*a^2*c*d^2*e^10 + a^3*e^12) + 1/8*(C*a*c^4*d^5
*e + 3*A*c^5*d^5*e - 2*B*a*c^4*d^4*e^2 - 22*C*a^2*c^3*d^3*e^3 + 14*A*a*c^4*
d^3*e^3 + 32*B*a^2*c^3*d^2*e^4 + 17*C*a^3*c^2*d*e^5 - 29*A*a^2*c^3*d*e^5 -
6*B*a^3*c^2*e^6 - (3*C*a*c^4*d^6*e^2 + 9*A*c^5*d^6*e^2 - 6*B*a*c^4*d^5*e^3
- 77*C*a^2*c^3*d^4*e^4 + 41*A*a*c^4*d^4*e^4 + 116*B*a^2*c^3*d^3*e^5 + 77*C*
a^3*c^2*d^2*e^6 - 121*A*a^2*c^3*d^2*e^6 - 38*B*a^3*c^2*d*e^7 - 3*C*a^4*c*e^
8 + 7*A*a^3*c^2*e^8)*e^(-1)/(x*e + d) + (3*C*a*c^4*d^7*e^3 + 9*A*c^5*d^7*e^
3 - 6*B*a*c^4*d^6*e^4 - 89*C*a^2*c^3*d^5*e^5 + 45*A*a*c^4*d^5*e^5 + 140*B*a
^2*c^3*d^4*e^6 + 85*C*a^3*c^2*d^3*e^7 - 145*A*a^2*c^3*d^3*e^7 - 22*B*a^3*c^
2*d^2*e^8 + 17*C*a^4*c*d*e^9 - 21*A*a^3*c^2*d*e^9 - 8*B*a^4*c*e^10)*e^(-2)/
(x*e + d)^2 - (C*a*c^4*d^8*e^4 + 3*A*c^5*d^8*e^4 - 2*B*a*c^4*d^7*e^5 - 34*C
*a^2*c^3*d^6*e^6 + 18*A*a*c^4*d^6*e^6 + 58*B*a^2*c^3*d^5*e^7 + 20*C*a^3*c^2
*d^4*e^8 - 60*A*a^2*c^3*d^4*e^8 + 26*B*a^3*c^2*d^3*e^9 + 50*C*a^4*c*d^2*e^1
0 - 66*A*a^3*c^2*d^2*e^10 - 34*B*a^4*c*d*e^11 - 5*C*a^5*e^12 + 9*A*a^4*c*e^
12)*e^(-3)/(x*e + d)^3)/((c*d^2 + a*e^2)^4*a^2*(c - 2*c*d/(x*e + d) + c*d^2
/(x*e + d)^2 + a*e^2/(x*e + d)^2)^2)
```

Mupad [B]
time = 6.66, size = 2500, normalized size = 4.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^3*(d + e*x)^2),x)
```

```
[Out] symsum(log(root(17920*a^9*c^5*d^8*e^8*z^3 + 14336*a^10*c^4*d^6*e^10*z^3 + 1
4336*a^8*c^6*d^10*e^6*z^3 + 7168*a^11*c^3*d^4*e^12*z^3 + 7168*a^7*c^7*d^12*
e^4*z^3 + 2048*a^12*c^2*d^2*e^14*z^3 + 2048*a^6*c^8*d^14*e^2*z^3 + 256*a^5*
c^9*d^16*z^3 + 256*a^13*c*e^16*z^3 + 948*B*C*a^7*c*d*e^11*z - 12*A*B*a*c^7*
d^11*e*z + 9768*B*C*a^5*c^3*d^5*e^7*z - 7476*B*C*a^6*c^2*d^3*e^9*z - 328*B*
C*a^4*c^4*d^7*e^5*z - 92*B*C*a^3*c^5*d^9*e^3*z - 12486*A*C*a^5*c^3*d^4*e^8*
z + 5868*A*C*a^6*c^2*d^2*e^10*z + 282*A*C*a^3*c^5*d^8*e^4*z + 168*A*C*a^4*c
^4*d^6*e^6*z + 108*A*C*a^2*c^6*d^10*e^2*z + 14820*A*B*a^5*c^3*d^3*e^9*z - 8
40*A*B*a^4*c^4*d^5*e^7*z - 600*A*B*a^3*c^5*d^7*e^5*z - 180*A*B*a^2*c^6*d^9*
e^3*z - 4*B*C*a^2*c^6*d^11*e*z - 3204*A*B*a^6*c^2*d*e^11*z + 4239*C^2*a^6*c
```

^2*d^4*e^8*z - 3924*C^2*a^5*c^3*d^6*e^6*z + 103*C^2*a^4*c^4*d^8*e^4*z + 26*
C^2*a^3*c^5*d^10*e^2*z - 6000*B^2*a^5*c^3*d^4*e^8*z + 2820*B^2*a^6*c^2*d^2*
e^10*z + 280*B^2*a^4*c^4*d^6*e^6*z + 80*B^2*a^3*c^5*d^8*e^4*z + 4*B^2*a^2*c
^6*d^10*e^2*z - 8262*A^2*a^5*c^3*d^2*e^10*z + 1575*A^2*a^4*c^4*d^4*e^8*z +
1260*A^2*a^3*c^5*d^6*e^6*z + 495*A^2*a^2*c^6*d^8*e^4*z - 90*A*C*a^7*c*e^12*
z + 6*A*C*a*c^7*d^12*z - 966*C^2*a^7*c*d^2*e^10*z + 90*A^2*a*c^7*d^10*e^2*z
 + C^2*a^2*c^6*d^12*z + 225*A^2*a^6*c^2*e^12*z - 192*B^2*a^7*c*e^12*z + 9*A
^2*c^8*d^12*z + 9*C^2*a^8*e^12*z + 78*A*B*C*a*c^4*d^6*e^4 + 942*A*B*C*a^2*c
^3*d^4*e^6 - 342*A*B*C*a^3*c^2*d^2*e^8 - 129*B*C^2*a^4*c*d^2*e^8 + 990*A^2*
C*a^3*c^2*d*e^9 - 234*A^2*C*a*c^4*d^5*e^5 - 24*A*C^2*a*c^4*d^7*e^3 + 333*A^
2*B*a*c^4*d^4*e^6 - 252*A*B^2*a^3*c^2*d*e^9 - 60*A*B^2*a*c^4*d^5*e^5 + 204*
B^2*C*a^4*c*d*e^9 - 234*A*C^2*a^4*c*d*e^9 - 624*B^2*C*a^3*c^2*d^3*e^7 + 405
*B*C^2*a^3*c^2*d^4*e^6 - 36*B^2*C*a^2*c^3*d^5*e^5 + 21*B*C^2*a^2*c^3*d^6*e^
4 - 1296*A^2*C*a^2*c^3*d^3*e^7 + 396*A*C^2*a^3*c^2*d^3*e^7 - 330*A*C^2*a^2*
c^3*d^5*e^5 + 1863*A^2*B*a^2*c^3*d^2*e^8 - 672*A*B^2*a^2*c^3*d^3*e^7 + 90*A
*B*C*a^4*c*e^10 + 8*C^3*a^4*c*d^3*e^7 - 1350*A^3*a^2*c^3*d*e^9 - 324*A^3*a*
c^4*d^3*e^7 - 36*A^2*C*c^5*d^7*e^3 + 45*A^2*B*c^5*d^6*e^4 - 225*A^2*B*a^3*c
^2*e^10 - 86*C^3*a^3*c^2*d^5*e^5 - 4*C^3*a^2*c^3*d^7*e^3 + 316*B^3*a^3*c^2*
d^2*e^8 + 20*B^3*a^2*c^3*d^4*e^6 + 18*C^3*a^5*d*e^9 - 64*B^3*a^4*c*e^10 - 9
*B*C^2*a^5*e^10 - 54*A^3*c^5*d^5*e^5, z, k)*((120*A*a^8*c^2*e^13 - 24*C*a^9
*c*e^13 + 24*A*a^2*c^8*d^12*e - 112*B*a^8*c^2*d*e^12 + 8*C*a^3*c^7*d^12*e +
 144*A*a^3*c^7*d^10*e^3 + 456*A*a^4*c^6*d^8*e^5 + 864*A*a^5*c^5*d^6*e^7 + 9
36*A*a^6*c^4*d^4*e^9 + 528*A*a^7*c^3*d^2*e^11 - 16*B*a^3*c^7*d^11*e^2 - 176
*B*a^4*c^6*d^9*e^4 - 544*B*a^5*c^5*d^7*e^6 - 736*B*a^6*c^4*d^5*e^8 - 464*B*
a^7*c^3*d^3*e^10 + 112*C*a^4*c^6*d^10*e^3 + 344*C*a^5*c^5*d^8*e^5 + 416*C*a
^6*c^4*d^6*e^7 + 184*C*a^7*c^3*d^4*e^9 - 16*C*a^8*c^2*d^2*e^11)/(64*(a^10*e
^12 + a^4*c^6*d^12 + 6*a^9*c*d^2*e^10 + 6*a^5*c^5*d^10*e^2 + 15*a^6*c^4*d^8
*e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2*d^4*e^8)) + root(17920*a^9*c^5*d^8*e
^8*z^3 + 14336*a^10*c^4*d^6*e^10*z^3 + 14336*a^8*c^6*d^10*e^6*z^3 + 7168*a^
11*c^3*d^4*e^12*z^3 + 7168*a^7*c^7*d^12*e^4*z^3 + 2048*a^12*c^2*d^2*e^14*z^
3 + 2048*a^6*c^8*d^14*e^2*z^3 + 256*a^5*c^9*d^16*z^3 + 256*a^13*c*e^16*z^3
+ 948*B*C*a^7*c*d*e^11*z - 12*A*B*a*c^7*d^11*e*z + 9768*B*C*a^5*c^3*d^5*e^7
*z - 7476*B*C*a^6*c^2*d^3*e^9*z - 328*B*C*a^4*c^4*d^7*e^5*z - 92*B*C*a^3*c^
5*d^9*e^3*z - 12486*A*C*a^5*c^3*d^4*e^8*z + 5868*A*C*a^6*c^2*d^2*e^10*z + 2
82*A*C*a^3*c^5*d^8*e^4*z + 168*A*C*a^4*c^4*d^6*e^6*z + 108*A*C*a^2*c^6*d^10
*e^2*z + 14820*A*B*a^5*c^3*d^3*e^9*z - 840*A*B*a^4*c^4*d^5*e^7*z - 600*A*B*
a^3*c^5*d^7*e^5*z - 180*A*B*a^2*c^6*d^9*e^3*z - 4*B*C*a^2*c^6*d^11*e*z - 32
04*A*B*a^6*c^2*d*e^11*z + 4239*C^2*a^6*c^2*d^4*e^8*z - 3924*C^2*a^5*c^3*d^6
*e^6*z + 103*C^2*a^4*c^4*d^8*e^4*z + 26*C^2*a^3*c^5*d^10*e^2*z - 6000*B^2*a
^5*c^3*d^4*e^8*z + 2820*B^2*a^6*c^2*d^2*e^10*z + 280*B^2*a^4*c^4*d^6*e^6*z
+ 80*B^2*a^3*c^5*d^8*e^4*z + 4*B^2*a^2*c^6*d^10*e^2*z - 8262*A^2*a^5*c^3*d^
2*e^10*z + 1575*A^2*a^4*c^4*d^4*e^8*z + 1260*A^2*a^3*c^5*d^6*e^6*z + 495*A^
2*a^2*c^6*d^8*e^4*z - 90*A*C*a^7*c*e^12*z + 6*A*C*a*c^7*d^12*z - 966*C^2*a^
7*c*d^2*e^10*z + 90*A^2*a*c^7*d^10*e^2*z + C^2*a^2*c^6*d^12*z + 225*A^2*a^6
*c^2*e^12*z - 192*B^2*a^7*c*e^12*z + 9*A^2*c^8*d^12*z + 9*C^2*a^8*e^12*z +

78*A*B*C*a*c^4*d^6*e^4 + 942*A*B*C*a^2*c^3*d^4*e^6 - 342*A*B*C*a^3*c^2*d^2*
e^8 - 129*B*C^2*a^4*c*d^2*e^8 + 990*A^2*C*a^3*c^2*d*e^9 - 234*A^2*C*a*c^4*d
^5*e^5 - 24*A*C^2*a*c^4*d^7*e^3 + 333*A^2*B*a*c^4*d^4*e^6 - 252*A*B^2*a^3*c
^2*d*e^9 - 60*A*B^2*a*c^4*d^5*e^5 + 204*B^2*C*a^4*c*d*e^9 - 234*A*C^2*a^4*c
*d*e^9 - 624*B^2*C*a^3*c^2*d^3*e^7 + 405*B*C^2*a^3*c^2*d^4*e^6 - 36*B^2*C*a
^2*c^3*d^5*e^5 + 21*B*C^2*a^2*c^3*d^6*e^4 - 1296*A^2*C*a^2*c^3*d^3*e^7 + 39
6*A*C^2*a^3*c^2*d^3*e^7 - 330*A*C^2*a^2*c^3*d^5*e^5 + 1863*A^2*B*a^2*c^3*d^
2*e^8 - 672*A*B^2*a^2*c^3*d^3*e^7 + 90*A*B*C*a^4*c*e^10 + 8*C^3*a^4*c*d^3*e
^7 - 1350*A^3*a^2*c^3*d*e^9 - 324*A^3*a*c^4*d^3*e^7 - 36*A^2*C*c^5*d^7*e^3
+ 45*A^2*B*c^5*d^6*e^4 - 225*A^2*B*a^3*c^2*e^10 - 86*C^3*a^3*c^2*d^5*e^5 -
4*C^3*a^2*c^3*d^7*e^3 + 316*B^3*a^3*c^2*d^2*e^8 + 20*B^3*a^2*c^3*d^4*e^6 +
18*C^3*a^5*d*e^9 - 64*B^3*a^4*c*e^10 - 9*B*C^2*...

3.63 $\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^3} dx$

Optimal. Leaf size=753

$$-\frac{e^3(Cd^2-Bde+Ae^2)}{2\left(cd^2+ae^2\right)^3(d+ex)^2}+\frac{e^3(ae^2(2Cd-Be)-cd(4Cd^2-e(5Bd-6Ae)))}{\left(cd^2+ae^2\right)^4(d+ex)}-\frac{a(Bcd(cd^2-3ae^2)-(Ac-$$

```
[Out] -1/2*e^3*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^3/(e*x+d)^2+e^3*(a*e^2*(-B*e+2*C
*d)-c*d*(4*C*d^2-e*(-6*A*e+5*B*d)))/(a*e^2+c*d^2)^4/(e*x+d)+1/4*(-a*(B*c*d*
(-3*a*e^2+c*d^2)-(A*c-C*a)*e*(-a*e^2+3*c*d^2))+c*(A*c*d*(-3*a*e^2+c*d^2)-a*
(c*d^2*(-3*B*e+C*d)-a*e^2*(-B*e+3*C*d)))*x)/a/(a*e^2+c*d^2)^3/(c*x^2+a)^2+1
/8*(4*a^2*e*(a^2*C*e^4+c^2*d^2*(3*C*d^2-2*e*(-5*A*e+3*B*d))-2*a*c*e^2*(4*C*
d^2-e*(-A*e+3*B*d)))+c*(3*A*c*d*(-11*a^2*e^4+6*a*c*d^2*e^2+c^2*d^4)-a*(2*a*
c*d^2*e^2*(-19*B*e+13*C*d)-c^2*d^4*(-3*B*e+C*d)-7*a^2*e^4*(-B*e+3*C*d)))*x)
/a^2/(a*e^2+c*d^2)^4/(c*x^2+a)+e^3*(a^2*C*e^4-a*c*e^2*(3*A*e^2-9*B*d*e+13*C
*d^2)+c^2*d^2*(10*C*d^2-3*e*(-7*A*e+5*B*d)))*ln(e*x+d)/(a*e^2+c*d^2)^5-1/2*
e^3*(a^2*C*e^4-a*c*e^2*(3*A*e^2-9*B*d*e+13*C*d^2)+c^2*d^2*(10*C*d^2-3*e*(-7
*A*e+5*B*d)))*ln(c*x^2+a)/(a*e^2+c*d^2)^5+1/8*(3*A*c*d*(-35*a^3*e^6+35*a^2*
c*d^2*e^4+7*a*c^2*d^4*e^2+c^3*d^6)+a*(a*c^2*d^4*e^2*(-45*B*e+23*C*d)-5*a^2*
c*d^2*e^4*(-27*B*e+25*C*d)+c^3*d^6*(-3*B*e+C*d)+15*a^3*e^6*(-B*e+3*C*d)))*a
rctan(x*c^(1/2)/a^(1/2))*c^(1/2)/a^(5/2)/(a*e^2+c*d^2)^5
```

Rubi [A]
time = 2.33, antiderivative size = 753, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1661, 1643, 649, 211, 266}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^3),x]
```

```
[Out] -1/2*(e^3*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^3*(d + e*x)^2) - (e^3*(
4*c*C*d^3 - c*d*e*(5*B*d - 6*A*e) - a*e^2*(2*C*d - B*e)))/((c*d^2 + a*e^2)^
4*(d + e*x)) - (a*(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2
)) - c*(A*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B
*e)))*x)/(4*a*(c*d^2 + a*e^2)^3*(a + c*x^2)^2) + (4*a^2*e*(a^2*C*e^4 + c^2*
(3*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e)) - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e))
) + c*(3*A*c*d*(c^2*d^4 + 6*a*c*d^2*e^2 - 11*a^2*e^4) - a*(2*a*c*d^2*e^2*(1
3*C*d - 19*B*e) - c^2*d^4*(C*d - 3*B*e) - 7*a^2*e^4*(3*C*d - B*e)))*x)/(8*a
^2*(c*d^2 + a*e^2)^4*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c^3*d^6 + 7*a*c^2*d^
4*e^2 + 35*a^2*c*d^2*e^4 - 35*a^3*e^6) + a*(a*c^2*d^4*e^2*(23*C*d - 45*B*e)
 - 5*a^2*c*d^2*e^4*(25*C*d - 27*B*e) + c^3*d^6*(C*d - 3*B*e) + 15*a^3*e^6*(
```

```
3*C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*(c*d^2 + a*e^2)^5) +
 (e^3*(a^2*C*e^4 - a*c*e^2*(13*C*d^2 - 9*B*d*e + 3*A*e^2) + c^2*(10*C*d^4 -
 3*d^2*e*(5*B*d - 7*A*e)))*Log[d + e*x])/(c*d^2 + a*e^2)^5 - (e^3*(a^2*C*e^
4 - a*c*e^2*(13*C*d^2 - 9*B*d*e + 3*A*e^2) + c^2*(10*C*d^4 - 3*d^2*e*(5*B*d
 - 7*A*e)))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^5)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^3}\,dx = -\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd^2}{4a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)^2}$$

$$= -\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd^2}{4a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)^2}$$

$$= -\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd^2}{4a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)^2}$$

$$= -\frac{e^3(Cd^2-Bde+Ae^2)}{2\left(cd^2+ae^2\right)^3(d+ex)^2} - \frac{e^3(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be))}{\left(cd^2+ae^2\right)^4(d+ex)}$$

$$= -\frac{e^3(Cd^2-Bde+Ae^2)}{2\left(cd^2+ae^2\right)^3(d+ex)^2} - \frac{e^3(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be))}{\left(cd^2+ae^2\right)^4(d+ex)}$$

$$= -\frac{e^3(Cd^2-Bde+Ae^2)}{2\left(cd^2+ae^2\right)^3(d+ex)^2} - \frac{e^3(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be))}{\left(cd^2+ae^2\right)^4(d+ex)}$$

<u>**Mathematica**</u> [**A**]
time = 0.68, size = 672, normalized size = 0.89

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^3),x]
```

```
[Out] ((-4*e^3*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2 - (8*e^3
*(c*d^2 + a*e^2)*(4*c*C*d^3 + c*d*e*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e)
))/(d + e*x) + (2*(c*d^2 + a*e^2)^2*(a^3*C*e^3 + A*c^3*d^3*x - a*c^2*d*(C*d
^2*x + B*d*(d - 3*e*x) + 3*A*e*(-d + e*x)) - a^2*c*e*(3*C*d*(d - e*x) + e*(
-3*B*d + A*e + B*e*x))))/(a*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(4*a^4*C*e^5
+ 3*A*c^4*d^5*x + a*c^3*d^3*(C*d^2 + 3*e*(-(B*d) + 6*A*e))*x + a^3*c*e^3*(C
*d*(-32*d + 21*e*x) + e*(24*B*d - 8*A*e - 7*B*e*x)) + a^2*c^2*d*e*(2*C*d^2*
(6*d - 13*e*x) + e*(-24*B*d^2 + 40*A*d*e + 38*B*d*e*x - 33*A*e^2*x))))/(a^2
*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c^3*d^6 + 7*a*c^2*d^4*e^2 + 35*a^2*c*d^2
*e^4 - 35*a^3*e^6) + a*(a*c^2*d^4*e^2*(23*C*d - 45*B*e) - 5*a^2*c*d^2*e^4*(
25*C*d - 27*B*e) + c^3*d^6*(C*d - 3*B*e) - 15*a^3*e^6*(-3*C*d + B*e)))*ArcT
an[(Sqrt[c]*x)/Sqrt[a]])/a^(5/2) + 8*(a^2*C*e^7 + a*c*e^5*(-13*C*d^2 + 9*B*
d*e - 3*A*e^2) + c^2*d^2*e^3*(10*C*d^2 + 3*e*(-5*B*d + 7*A*e)))*Log[d + e*x
```

```
] - 4*(a^2*C*e^7 + a*c*e^5*(-13*C*d^2 + 9*B*d*e - 3*A*e^2) + c^2*d^2*e^3*(1
0*C*d^2 + 3*e*(-5*B*d + 7*A*e)))*Log[a + c*x^2])/(8*(c*d^2 + a*e^2)^5)
```

Maple [A]
time = 0.26, size = 1055, normalized size = 1.40

method	result
default	$-\frac{e^3(6d e^2 cA+Ba e^3-5Bc d^2 e-2Cad e^2+4Cc d^3)}{(a e^2+c d^2)^4(ex+d)}-\frac{e^3(A e^2-Bde+C d^2)}{2(a e^2+c d^2)^3(ex+d)^2}-\frac{e^3(3Aac e^4-21A c^2 d^2 e^2-9Bacd e^3+15B c^2 d^3 e-a^2}{(a e^2+c d^2)^5}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -e^3*(6*A*c*d*e^2+B*a*e^3-5*B*c*d^2*e-2*C*a*d*e^2+4*C*c*d^3)/(a*e^2+c*d^2)^
4/(e*x+d)-1/2*e^3*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^3/(e*x+d)^2-e^3*(3*A*a*
c*e^4-21*A*c^2*d^2*e^2-9*B*a*c*d*e^3+15*B*c^2*d^3*e-C*a^2*e^4+13*C*a*c*d^2*
e^2-10*C*c^2*d^4)/(a*e^2+c*d^2)^5*ln(e*x+d)-c/(a*e^2+c*d^2)^5*((1/8*c*(33*A
*a^3*c*d*e^6+15*A*a^2*c^2*d^3*e^4-21*A*a*c^3*d^5*e^2-3*A*c^4*d^7+7*B*a^4*e^
7-31*B*a^3*c*d^2*e^5-35*B*a^2*c^2*d^4*e^3+3*B*a*c^3*d^6*e-21*C*a^4*d*e^6+5*
C*a^3*c*d^3*e^4+25*C*a^2*c^2*d^5*e^2-C*a*c^3*d^7)/a^2*x^3+(A*a^2*c*e^7-4*A*
a*c^2*d^2*e^5-5*e^3*c^3*d^4*A-3*B*a^2*c*d*e^6+3*e^2*c^3*B*d^5-1/2*C*a^3*e^7
+7/2*C*a^2*c*d^2*e^5+5/2*C*a*c^2*d^4*e^3-3/2*C*c^3*d^6*e)*x^2+1/8*(39*A*a^3
*c*d*e^6+25*A*a^2*c^2*d^3*e^4-19*A*a*c^3*d^5*e^2-5*A*c^4*d^7+9*B*a^4*e^7-33
*B*a^3*c*d^2*e^5-45*B*a^2*c^2*d^4*e^3-3*B*a*c^3*d^6*e-27*C*a^4*d*e^6-5*C*a^
3*c*d^3*e^4+23*C*a^2*c^2*d^5*e^2+C*a*c^3*d^7)/a*x+1/4*(5*A*a^3*c*e^7-17*A*a
^2*c^2*d^2*e^5-25*A*a*c^3*d^4*e^3-3*A*c^4*d^6*e-15*B*a^3*c*d*e^6-5*B*a^2*c^
2*d^3*e^4+11*B*a*c^3*d^5*e^2+B*c^4*d^7-3*C*a^4*e^7+15*C*a^3*c*d^2*e^5+15*C*
a^2*c^2*d^4*e^3-3*C*a*c^3*d^6*e)/c)/(c*x^2+a)^2+1/8/a^2*(1/2*(-24*A*a^3*c*e
^7+168*A*a^2*c^2*d^2*e^5+72*B*a^3*c*d*e^6-120*B*a^2*c^2*d^3*e^4+8*C*a^4*e^7
-104*C*a^3*c*d^2*e^5+80*C*a^2*c^2*d^4*e^3)/c*ln(c*x^2+a)+(105*A*a^3*c*d*e^6
-105*A*a^2*c^2*d^3*e^4-21*A*a*c^3*d^5*e^2-3*A*c^4*d^7+15*B*a^4*e^7-135*B*a^
3*c*d^2*e^5+45*B*a^2*c^2*d^4*e^3+3*B*a*c^3*d^6*e-45*C*a^4*d*e^6+125*C*a^3*c
*d^3*e^4-23*C*a^2*c^2*d^5*e^2-C*a*c^3*d^7)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/
2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1793 vs. 2(726) = 1452.
time = 0.57, size = 1793, normalized size = 2.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 + 9*B*a*c*d*e^6 + C*a^2*e^7 - 3*A
*a*c*e^7 - (13*C*a*c*e^5 - 21*A*c^2*e^5)*d^2)*log(c*x^2 + a)/(c^5*d^10 + 5*
a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 +
 a^5*e^10) + (10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 + 9*B*a*c*d*e^6 + C*a^2*e
^7 - 3*A*a*c*e^7 - (13*C*a*c*e^5 - 21*A*c^2*e^5)*d^2)*log(x*e + d)/(c^5*d^1
0 + 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2
*e^8 + a^5*e^10) - 1/8*(3*B*a*c^4*d^6*e + 45*B*a^2*c^3*d^4*e^3 - 135*B*a^3*
c^2*d^2*e^5 - (C*a*c^4 + 3*A*c^5)*d^7 + 15*B*a^4*c*e^7 - (23*C*a^2*c^3*e^2
+ 21*A*a*c^4*e^2)*d^5 + 5*(25*C*a^3*c^2*e^4 - 21*A*a^2*c^3*e^4)*d^3 - 15*(3
*C*a^4*c*e^6 - 7*A*a^3*c^2*e^6)*d)*arctan(c*x/sqrt(a*c))/((a^2*c^5*d^10 + 5
*a^3*c^4*d^8*e^2 + 10*a^4*c^3*d^6*e^4 + 10*a^5*c^2*d^4*e^6 + 5*a^6*c*d^2*e^
8 + a^7*e^10)*sqrt(a*c)) - 1/8*(2*B*a^2*c^3*d^7 + 20*B*a^3*c^2*d^5*e^2 - 74
*B*a^4*c*d^3*e^4 + 4*B*a^5*d*e^6 - 6*(C*a^3*c^2*e + A*a^2*c^3*e)*d^6 + 4*A*
a^5*e^7 + (3*B*a*c^4*d^4*e^3 - 78*B*a^2*c^3*d^2*e^5 + 15*B*a^3*c^2*e^7 - (C
*a*c^4*e^2 + 3*A*c^5*e^2)*d^5 + 2*(29*C*a^2*c^3*e^4 - 9*A*a*c^4*e^4)*d^3 -
(37*C*a^3*c^2*e^6 - 81*A*a^2*c^3*e^6)*d)*x^5 + 4*(18*C*a^4*c*e^3 - 11*A*a^3
*c^2*e^3)*d^4 + 2*(3*B*a*c^4*d^5*e^2 - 48*B*a^2*c^3*d^3*e^4 - 3*B*a^3*c^2*d
*e^6 - (C*a*c^4*e + 3*A*c^5*e)*d^6 - 2*C*a^4*c*e^7 + 6*A*a^3*c^2*e^7 + 2*(1
9*C*a^2*c^3*e^3 - 9*A*a*c^4*e^3)*d^4 - (11*C*a^3*c^2*e^5 - 39*A*a^2*c^3*e^5
)*d^2)*x^4 + (3*B*a*c^4*d^6*e + 7*B*a^2*c^3*d^4*e^3 - 163*B*a^3*c^2*d^2*e^5
 - (C*a*c^4 + 3*A*c^5)*d^7 + 25*B*a^4*c*e^7 + (3*C*a^2*c^3*e^2 - 23*A*a*c^4
*e^2)*d^5 + (129*C*a^3*c^2*e^4 - 61*A*a^2*c^3*e^4)*d^3 - (67*C*a^4*c*e^6 -
151*A*a^3*c^2*e^6)*d)*x^3 - 2*(9*C*a^5*e^5 - 31*A*a^4*c*e^5)*d^2 + 2*(10*B*
a^2*c^3*d^5*e^2 - 88*B*a^3*c^2*d^3*e^4 - 2*B*a^4*c*d*e^6 - 5*(C*a^2*c^3*e +
 A*a*c^4*e)*d^6 - 3*C*a^5*e^7 + 9*A*a^4*c*e^7 + (71*C*a^3*c^2*e^3 - 37*A*a^
2*c^3*e^3)*d^4 - (23*C*a^4*c*e^5 - 73*A*a^3*c^2*e^5)*d^2)*x^2 + (B*a^2*c^3*
d^6*e - 2*B*a^3*c^2*d^4*e^3 - 91*B*a^4*c*d^2*e^5 + (C*a^2*c^3 - 5*A*a*c^4)*
d^7 + 8*B*a^5*e^7 + 2*(5*C*a^3*c^2*e^2 - 13*A*a^2*c^3*e^2)*d^5 + 7*(11*C*a^
4*c*e^4 - 7*A*a^3*c^2*e^4)*d^3 - 4*(7*C*a^5*e^6 - 17*A*a^4*c*e^6)*d)*x)/(a^
4*c^4*d^10 + 4*a^5*c^3*d^8*e^2 + 6*a^6*c^2*d^6*e^4 + 4*a^7*c*d^4*e^6 + a^8*
d^2*e^8 + (a^2*c^6*d^8*e^2 + 4*a^3*c^5*d^6*e^4 + 6*a^4*c^4*d^4*e^6 + 4*a^5*
c^3*d^2*e^8 + a^6*c^2*e^10)*x^6 + 2*(a^2*c^6*d^9*e + 4*a^3*c^5*d^7*e^3 + 6*
a^4*c^4*d^5*e^5 + 4*a^5*c^3*d^3*e^7 + a^6*c^2*d*e^9)*x^5 + (a^2*c^6*d^10 +
6*a^3*c^5*d^8*e^2 + 14*a^4*c^4*d^6*e^4 + 16*a^5*c^3*d^4*e^6 + 9*a^6*c^2*d^2
*e^8 + 2*a^7*c*e^10)*x^4 + 4*(a^3*c^5*d^9*e + 4*a^4*c^4*d^7*e^3 + 6*a^5*c^3
*d^5*e^5 + 4*a^6*c^2*d^3*e^7 + a^7*c*d*e^9)*x^3 + (2*a^3*c^5*d^10 + 9*a^4*c
^4*d^8*e^2 + 16*a^5*c^3*d^6*e^4 + 14*a^6*c^2*d^4*e^6 + 6*a^7*c*d^2*e^8 + a^
8*e^10)*x^2 + 2*(a^4*c^4*d^9*e + 4*a^5*c^3*d^7*e^3 + 6*a^6*c^2*d^5*e^5 + 4*
a^7*c*d^3*e^7 + a^8*d*e^9)*x)
```

Fricas [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1532 vs. 2(726) = 1452.
time = 3.67, size = 1532, normalized size = 2.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 - 13*C*a*c*d^2*e^5 + 21*A*c^2*d^2
*e^5 + 9*B*a*c*d*e^6 + C*a^2*e^7 - 3*A*a*c*e^7)*log(c*x^2 + a)/(c^5*d^10 +
5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8
 + a^5*e^10) + (10*C*c^2*d^4*e^4 - 15*B*c^2*d^3*e^5 - 13*C*a*c*d^2*e^6 + 21
*A*c^2*d^2*e^6 + 9*B*a*c*d*e^7 + C*a^2*e^8 - 3*A*a*c*e^8)*log(abs(x*e + d))
/(c^5*d^10*e + 5*a*c^4*d^8*e^3 + 10*a^2*c^3*d^6*e^5 + 10*a^3*c^2*d^4*e^7 +
5*a^4*c*d^2*e^9 + a^5*e^11) + 1/8*(C*a*c^4*d^7 + 3*A*c^5*d^7 - 3*B*a*c^4*d^
6*e + 23*C*a^2*c^3*d^5*e^2 + 21*A*a*c^4*d^5*e^2 - 45*B*a^2*c^3*d^4*e^3 - 12
5*C*a^3*c^2*d^3*e^4 + 105*A*a^2*c^3*d^3*e^4 + 135*B*a^3*c^2*d^2*e^5 + 45*C*
a^4*c*d*e^6 - 105*A*a^3*c^2*d*e^6 - 15*B*a^4*c*e^7)*arctan(c*x/sqrt(a*c))/(
(a^2*c^5*d^10 + 5*a^3*c^4*d^8*e^2 + 10*a^4*c^3*d^6*e^4 + 10*a^5*c^2*d^4*e^6
 + 5*a^6*c*d^2*e^8 + a^7*e^10)*sqrt(a*c)) + 1/8*(C*a*c^4*d^5*x^5*e^2 + 3*A*
c^5*d^5*x^5*e^2 + 2*C*a*c^4*d^6*x^4*e + 6*A*c^5*d^6*x^4*e + C*a*c^4*d^7*x^3
 + 3*A*c^5*d^7*x^3 - 3*B*a*c^4*d^4*x^5*e^3 - 6*B*a*c^4*d^5*x^4*e^2 - 3*B*a*
c^4*d^6*x^3*e - 58*C*a^2*c^3*d^3*x^5*e^4 + 18*A*a*c^4*d^3*x^5*e^4 - 76*C*a^
2*c^3*d^4*x^4*e^3 + 36*A*a*c^4*d^4*x^4*e^3 - 3*C*a^2*c^3*d^5*x^3*e^2 + 23*A
*a*c^4*d^5*x^3*e^2 + 10*C*a^2*c^3*d^6*x^2*e + 10*A*a*c^4*d^6*x^2*e - C*a^2*
```

```
c^3*d^7*x + 5*A*a*c^4*d^7*x + 78*B*a^2*c^3*d^2*x^5*e^5 + 96*B*a^2*c^3*d^3*x
^4*e^4 - 7*B*a^2*c^3*d^4*x^3*e^3 - 20*B*a^2*c^3*d^5*x^2*e^2 - B*a^2*c^3*d^6
*x*e - 2*B*a^2*c^3*d^7 + 37*C*a^3*c^2*d*x^5*e^6 - 81*A*a^2*c^3*d*x^5*e^6 +
22*C*a^3*c^2*d^2*x^4*e^5 - 78*A*a^2*c^3*d^2*x^4*e^5 - 129*C*a^3*c^2*d^3*x^3
*e^4 + 61*A*a^2*c^3*d^3*x^3*e^4 - 142*C*a^3*c^2*d^4*x^2*e^3 + 74*A*a^2*c^3*
d^4*x^2*e^3 - 10*C*a^3*c^2*d^5*x*e^2 + 26*A*a^2*c^3*d^5*x*e^2 + 6*C*a^3*c^2
*d^6*e + 6*A*a^2*c^3*d^6*e - 15*B*a^3*c^2*x^5*e^7 + 6*B*a^3*c^2*d*x^4*e^6 +
 163*B*a^3*c^2*d^2*x^3*e^5 + 176*B*a^3*c^2*d^3*x^2*e^4 + 2*B*a^3*c^2*d^4*x*
e^3 - 20*B*a^3*c^2*d^5*e^2 + 4*C*a^4*c*x^4*e^7 - 12*A*a^3*c^2*x^4*e^7 + 67*
C*a^4*c*d*x^3*e^6 - 151*A*a^3*c^2*d*x^3*e^6 + 46*C*a^4*c*d^2*x^2*e^5 - 146*
A*a^3*c^2*d^2*x^2*e^5 - 77*C*a^4*c*d^3*x*e^4 + 49*A*a^3*c^2*d^3*x*e^4 - 72*
C*a^4*c*d^4*e^3 + 44*A*a^3*c^2*d^4*e^3 - 25*B*a^4*c*x^3*e^7 + 4*B*a^4*c*d*x
^2*e^6 + 91*B*a^4*c*d^2*x*e^5 + 74*B*a^4*c*d^3*e^4 + 6*C*a^5*x^2*e^7 - 18*A
*a^4*c*x^2*e^7 + 28*C*a^5*d*x*e^6 - 68*A*a^4*c*d*x*e^6 + 18*C*a^5*d^2*e^5 -
 62*A*a^4*c*d^2*e^5 - 8*B*a^5*x*e^7 - 4*B*a^5*d*e^6 - 4*A*a^5*e^7)/((a^2*c^
4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*
(c*x^3*e + c*d*x^2 + a*x*e + a*d)^2)
```

Mupad [B]
time = 7.24, size = 2500, normalized size = 3.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^3*(d + e*x)^3),x)
```

```
[Out] ((x^5*(3*A*c^5*d^5*e^2 - 15*B*a^3*c^2*e^7 + 18*A*a*c^4*d^3*e^4 - 81*A*a^2*c
^3*d*e^6 - 3*B*a*c^4*d^4*e^3 + C*a*c^4*d^5*e^2 + 37*C*a^3*c^2*d*e^6 + 78*B*
a^2*c^3*d^2*e^5 - 58*C*a^2*c^3*d^3*e^4))/(8*a^2*(a^4*e^8 + c^4*d^8 + 4*a*c^
3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) - (2*A*a^3*e^7 + B*c^3*d^
7 + 2*B*a^3*d*e^6 - 3*A*c^3*d^6*e - 9*C*a^3*d^2*e^5 - 22*A*a*c^2*d^4*e^3 +
31*A*a^2*c*d^2*e^5 + 10*B*a*c^2*d^5*e^2 - 37*B*a^2*c*d^3*e^4 + 36*C*a^2*c*d
^4*e^3 - 3*C*a*c^2*d^6*e)/(4*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c
*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (x*(5*A*c^4*d^7 - 8*B*a^4*e^7 - C*a*c^3*d^
7 + 28*C*a^4*d*e^6 + 26*A*a*c^3*d^5*e^2 + 91*B*a^3*c*d^2*e^5 - 77*C*a^3*c*d
^3*e^4 + 49*A*a^2*c^2*d^3*e^4 + 2*B*a^2*c^2*d^4*e^3 - 10*C*a^2*c^2*d^5*e^2
- 68*A*a^3*c*d*e^6 - B*a*c^3*d^6*e))/(8*a*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*
e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (x^2*(3*C*a^4*e^7 - 9*A*a^3*c
*e^7 + 5*A*c^4*d^6*e + 37*A*a*c^3*d^4*e^3 - 10*B*a*c^3*d^5*e^2 + 23*C*a^3*c
*d^2*e^5 - 73*A*a^2*c^2*d^2*e^5 + 88*B*a^2*c^2*d^3*e^4 - 71*C*a^2*c^2*d^4*e
^3 + 2*B*a^3*c*d*e^6 + 5*C*a*c^3*d^6*e))/(4*a*(a^4*e^8 + c^4*d^8 + 4*a*c^3*
d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (x^3*(3*A*c^5*d^7 - 25*B*
a^4*c*e^7 + C*a*c^4*d^7 + 23*A*a*c^4*d^5*e^2 - 151*A*a^3*c^2*d*e^6 + 61*A*a
^2*c^3*d^3*e^4 - 7*B*a^2*c^3*d^4*e^3 + 163*B*a^3*c^2*d^2*e^5 - 3*C*a^2*c^3*
d^5*e^2 - 129*C*a^3*c^2*d^3*e^4 - 3*B*a*c^4*d^6*e + 67*C*a^4*c*d*e^6))/(8*a
```

^2*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e
^4)) + (x^4*(2*C*a^4*c*e^7 + 3*A*c^5*d^6*e - 6*A*a^3*c^2*e^7 + 18*A*a*c^4*d
^4*e^3 - 3*B*a*c^4*d^5*e^2 + 3*B*a^3*c^2*d*e^6 - 39*A*a^2*c^3*d^2*e^5 + 48*
B*a^2*c^3*d^3*e^4 - 38*C*a^2*c^3*d^4*e^3 + 11*C*a^3*c^2*d^2*e^5 + C*a*c^4*d
^6*e))/(4*a^2*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^
2*c^2*d^4*e^4)))/(x^2*(a^2*e^2 + 2*a*c*d^2) + x^4*(c^2*d^2 + 2*a*c*e^2) + a
^2*d^2 + c^2*e^2*x^6 + 2*a^2*d*e*x + 2*c^2*d*e*x^5 + 4*a*c*d*e*x^3) + symsu
m(log(root(2560*a^14*c*d^2*e^18*z^3 + 64512*a^10*c^5*d^10*e^10*z^3 + 53760*
a^11*c^4*d^8*e^12*z^3 + 53760*a^9*c^6*d^12*e^8*z^3 + 30720*a^12*c^3*d^6*e^1
4*z^3 + 30720*a^8*c^7*d^14*e^6*z^3 + 11520*a^13*c^2*d^4*e^16*z^3 + 11520*a^
7*c^8*d^16*e^4*z^3 + 2560*a^6*c^9*d^18*e^2*z^3 + 256*a^5*c^10*d^20*z^3 + 25
6*a^15*e^20*z^3 - 4806*B*C*a^8*c*d*e^13*z - 18*A*B*a*c^8*d^13*e*z - 147930*
B*C*a^6*c^3*d^5*e^9*z + 74760*B*C*a^5*c^4*d^7*e^7*z + 66588*B*C*a^7*c^2*d^3
*e^11*z - 1050*B*C*a^4*c^5*d^9*e^5*z - 228*B*C*a^3*c^6*d^11*e^3*z + 152052*
A*C*a^6*c^3*d^4*e^10*z - 109830*A*C*a^5*c^4*d^6*e^8*z - 32490*A*C*a^7*c^2*d
^2*e^12*z + 426*A*C*a^3*c^6*d^10*e^4*z - 360*A*C*a^4*c^5*d^8*e^6*z + 180*A*
C*a^2*c^7*d^12*e^2*z + 158130*A*B*a^5*c^4*d^5*e^9*z - 121356*A*B*a^6*c^3*d^
3*e^11*z - 3240*A*B*a^4*c^5*d^7*e^7*z - 1710*A*B*a^3*c^6*d^9*e^5*z - 396*A*
B*a^2*c^7*d^11*e^3*z - 6*B*C*a^2*c^7*d^13*e*z + 13518*A*B*a^7*c^2*d*e^13*z
+ 67615*C^2*a^6*c^3*d^6*e^8*z - 47538*C^2*a^7*c^2*d^4*e^10*z - 24860*C^2*a^
5*c^4*d^8*e^6*z + 279*C^2*a^4*c^5*d^10*e^4*z + 46*C^2*a^3*c^6*d^12*e^2*z +
71415*B^2*a^6*c^3*d^4*e^10*z - 55260*B^2*a^5*c^4*d^6*e^8*z - 19602*B^2*a^7*
c^2*d^2*e^12*z + 1215*B^2*a^4*c^5*d^8*e^6*z + 270*B^2*a^3*c^6*d^10*e^4*z +
9*B^2*a^2*c^7*d^12*e^2*z - 106722*A^2*a^5*c^4*d^4*e^10*z + 35217*A^2*a^6*c^
3*d^2*e^12*z + 6615*A^2*a^4*c^5*d^6*e^8*z + 3780*A^2*a^3*c^6*d^8*e^6*z + 10
71*A^2*a^2*c^7*d^10*e^4*z + 1152*A*C*a^8*c*e^14*z + 6*A*C*a*c^8*d^14*z + 70
17*C^2*a^8*c*d^2*e^12*z + 126*A^2*a*c^8*d^12*e^2*z + C^2*a^2*c^7*d^14*z - 1
728*A^2*a^7*c^2*e^14*z + 225*B^2*a^8*c*e^14*z + 9*A^2*c^9*d^14*z - 192*C^2*
a^9*e^14*z + 3168*A*B*C*a^4*c^2*d*e^10 + 270*A*B*C*a*c^5*d^7*e^4 - 6930*A*B
*C*a^3*c^3*d^3*e^8 + 5148*A*B*C*a^2*c^4*d^5*e^6 - 819*A^2*C*a*c^5*d^6*e^5 -
60*A*C^2*a*c^5*d^8*e^3 - 6102*A^2*B*a^3*c^3*d*e^10 + 1512*A^2*B*a*c^5*d^5*
e^6 - 270*A*B^2*a*c^5*d^6*e^5 - 378*B*C^2*a^5*c*d*e^10 - 5049*B^2*C*a^3*c^3
*d^4*e^7 + 4698*B^2*C*a^4*c^2*d^2*e^9 + 2508*B*C^2*a^3*c^3*d^5*e^6 - 1977*B
*C^2*a^4*c^2*d^3*e^8 - 180*B^2*C*a^2*c^4*d^6*e^5 + 75*B*C^2*a^2*c^4*d^7*e^4
+ 15921*A^2*C*a^3*c^3*d^2*e^9 - 7848*A^2*C*a^2*c^4*d^4*e^7 - 6363*A*C^2*a^
4*c^2*d^2*e^9 + 4926*A*C^2*a^3*c^3*d^4*e^7 - 1443*A*C^2*a^2*c^4*d^6*e^5 + 1
4283*A^2*B*a^2*c^4*d^3*e^8 - 4617*A*B^2*a^2*c^4*d^4*e^7 - 1944*A*B^2*a^3*c^
3*d^2*e^9 + 791*C^3*a^5*c*d^2*e^9 - 2025*B^3*a^4*c^2*d*e^10 - 1674*A^3*a*c^
5*d^4*e^7 - 90*A^2*C*c^6*d^8*e^3 + 135*A^2*B*c^6*d^7*e^4 - 1728*A^2*C*a^4*c
^2*e^11 + 675*A*B^2*a^4*c^2*e^11 - 225*B^2*C*a^5*c*e^11 + 576*A*C^2*a^5*c*e
^11 - 397*C^3*a^3*c^3*d^6*e^5 - 108*C^3*a^4*c^2*d^4*e^7 - 10*C^3*a^2*c^4*d^
8*e^3 + 3294*B^3*a^3*c^3*d^3*e^8 + 135*B^3*a^2*c^4*d^5*e^6 - 11853*A^3*a^2*
c^4*d^2*e^9 - 189*A^3*c^6*d^6*e^5 + 1728*A^3*a^3*c^3*e^11 - 64*C^3*a^6*e^11
, z, k)*(root(2560*a^14*c*d^2*e^18*z^3 + 64512*a^10*c^5*d^10*e^10*z^3 + 537
60*a^11*c^4*d^8*e^12*z^3 + 53760*a^9*c^6*d^12*e^8*z^3 + 30720*a^12*c^3*d^6*

```
e^14*z^3 + 30720*a^8*c^7*d^14*e^6*z^3 + 11520*a^13*c^2*d^4*e^16*z^3 + 11520
*a^7*c^8*d^16*e^4*z^3 + 2560*a^6*c^9*d^18*e^2*z^3 + 256*a^5*c^10*d^20*z^3 +
 256*a^15*e^20*z^3 - 4806*B*C*a^8*c*d*e^13*z - ...
```

3.64 $\int \frac{(d+ex)^4(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$

Optimal. Leaf size=234

$$-\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\,(a+cx^2)^3}-\frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+aCd+4aBe)x)}{24a^2c^2\,(a+cx^2)^2}-\frac{(a(Ac+5aC)e^2+c$$

```
[Out] -1/6*(a*B-(A*c-C*a)*x)*(e*x+d)^4/a/c/(c*x^2+a)^3-1/24*(e*x+d)^3*(a*(A*c+5*C
*a)*e-c*(5*A*c*d+4*B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)^2-1/16*(a*(A*c+5*C*a)*
e^2+c*d*(5*A*c*d+4*B*a*e+C*a*d))*(-c*d*x+a*e)*(e*x+d)/a^3/c^3/(c*x^2+a)+1/1
6*(a*e^2+c*d^2)*(a*(A*c+5*C*a)*e^2+c*d*(5*A*c*d+4*B*a*e+C*a*d))*arctan(x*c^
(1/2)/a^(1/2))/a^(7/2)/c^(7/2)
```

Rubi [A]
time = 0.18, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1659, 819, 737, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(ae^2+cd^2)\,(cd(4aBe+aCd+5Acd)+ae^2(5aC+Ac))}{16a^{7/2}c^{7/2}}-\frac{(d+ex)(ae-cdx)\,(cd(4aBe+aCd+5Acd)+ae^2(5aC+Ac))}{16a^3c^3\,(a+cx^2)}-\frac{(d+ex)^3(ae(5aC+Ac)-cx(4aBe+aCd+5Acd))}{24a^2c^2\,(a+cx^2)^2}-\frac{(d+ex)^4(aB-x(Ac-aC))}{6ac\,(a+cx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -1/6*((a*B - (A*c - a*C)*x)*(d + e*x)^4)/(a*c*(a + c*x^2)^3) - ((d + e*x)^3
*(a*(A*c + 5*a*C)*e - c*(5*A*c*d + a*C*d + 4*a*B*e)*x))/(24*a^2*c^2*(a + c*
x^2)^2) - ((a*(A*c + 5*a*C)*e^2 + c*d*(5*A*c*d + a*C*d + 4*a*B*e))*(a*e - c
*d*x)*(d + e*x))/(16*a^3*c^3*(a + c*x^2)) + ((c*d^2 + a*e^2)*(a*(A*c + 5*a*
C)*e^2 + c*d*(5*A*c*d + a*C*d + 4*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*
a^(7/2)*c^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[m*((c*d*f + a*e*g)/(2*a*c*(p + 1))), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^4\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3} - \frac{\int \frac{(d+ex)^3(-5Acd-aCd-4aBe-(Ac+5aC)ex)}{(a+cx^2)^3}\,d}{6ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+}{24a^2c^2\left(a+cx^2\right)^2}$$

Mathematica [A]
time = 0.20, size = 437, normalized size = 1.87

$$\frac{5Ac^3d^4x+ac^2d^2\left(Cd^2+4Bde+6Ae^2\right)x+a^2ce^2\left(6Cd^2+e(4Bd+Ae)\right)x-a^3e^3(32Cd+8Be+11Cex)}{16a^3c^3\left(a+cx^2\right)}+\frac{Ac^3d^4x-a^3e^3(4Cd+Be+Cex)-ac^2d^2\left(4Ade+Cd^2x+6Ae^2x+Bd(d+4ex)\right)+a^2ce\left(2Cd^2(2d+3ex)+e(Ae(4d+ex)+2Bd(3d+2ex))\right)}{6ac^3\left(a+cx^2\right)^3}+\frac{5Ac^3d^4x+ac^2d^2\left(Cd^2+4Bde+6Ae^2\right)x+a^3e^3(48Cd+12Be+13Cex)-a^2ce\left(6Cd^2(4d+7ex)+e(4Bd(9d+7ex)+Ae(24d+7ex))\right)}{24a^2c^3\left(a+cx^2\right)^2}+\frac{\left(cd^2+ae^2\right)\left(Ac\left(5cd^2+ae^2\right)+a\left(5aCe^2+cd(Cd+4Be)\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16a^{7/2}c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] (5*A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 4*B*d*e + 6*A*e^2)*x + a^2*c*e^2*(6*C*d
^2 + e*(4*B*d + A*e))*x - a^3*e^3*(32*C*d + 8*B*e + 11*C*e*x))/(16*a^3*c^3*
```

```
(a + c*x^2)) + (A*c^3*d^4*x - a^3*e^3*(4*C*d + B*e + C*e*x) - a*c^2*d^2*(4*
A*d*e + C*d^2*x + 6*A*e^2*x + B*d*(d + 4*e*x)) + a^2*c*e*(2*C*d^2*(2*d + 3*
e*x) + e*(A*e*(4*d + e*x) + 2*B*d*(3*d + 2*e*x))))/(6*a*c^3*(a + c*x^2)^3)
+ (5*A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 4*B*d*e + 6*A*e^2)*x + a^3*e^3*(48*C*
d + 12*B*e + 13*C*e*x) - a^2*c*e*(6*C*d^2*(4*d + 7*e*x) + e*(4*B*d*(9*d + 7
*e*x) + A*e*(24*d + 7*e*x))))/(24*a^2*c^3*(a + c*x^2)^2) + ((c*d^2 + a*e^2)
*(A*c*(5*c*d^2 + a*e^2) + a*(5*a*C*e^2 + c*d*(C*d + 4*B*e)))*ArcTan[(Sqrt[c
]*x)/Sqrt[a]])/(16*a^(7/2)*c^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(218) = 436$.
time = 0.13, size = 544, normalized size = 2.32

method	result
default	$\frac{\frac{\left(A\,a^{2} c\,e^{4}+6 A a\,c^{2} d^{2} e^{2}+5 A\,c^{3} d^{4}+4 B\,a^{2} c d\,e^{3}+4 B a\,c^{2} d^{3} e-11 C\,a^{3} e^{4}+6 C\,a^{2} c\,d^{2} e^{2}+C a\,c^{2} d^{4}\right) x^{5}}{16 a^{3} c}-\frac{e^{3} \left(B e+4 C d\right) x^{4}}{2 c}-\frac{\left(A\,a^{2} c\,e^{4}-6 A a\,c^{2} d^{2} e^{2}-5 A\,c^{3}}{}}{}$
risch	$\frac{\frac{\left(A\,a^{2} c\,e^{4}+6 A a\,c^{2} d^{2} e^{2}+5 A\,c^{3} d^{4}+4 B\,a^{2} c d\,e^{3}+4 B a\,c^{2} d^{3} e-11 C\,a^{3} e^{4}+6 C\,a^{2} c\,d^{2} e^{2}+C a\,c^{2} d^{4}\right) x^{5}}{16 a^{3} c}-\frac{e^{3} \left(B e+4 C d\right) x^{4}}{2 c}-\frac{\left(A\,a^{2} c\,e^{4}-6 A a\,c^{2} d^{2} e^{2}-5 A\,c^{3}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*(A*a^2*c*e^4+6*A*a*c^2*d^2*e^2+5*A*c^3*d^4+4*B*a^2*c*d*e^3+4*B*a*c^2*
d^3*e-11*C*a^3*e^4+6*C*a^2*c*d^2*e^2+C*a*c^2*d^4)/a^3/c*x^5-1/2*e^3*(B*e+4*
C*d)/c*x^4-1/6*(A*a^2*c*e^4-6*A*a*c^2*d^2*e^2-5*A*c^3*d^4+4*B*a^2*c*d*e^3-4
*B*a*c^2*d^3*e+5*C*a^3*e^4+6*C*a^2*c*d^2*e^2-C*a*c^2*d^4)/a^2/c^2*x^3-1/2*e
*(2*A*c*d*e^2+B*a*e^3+3*B*c*d^2*e+4*C*a*d*e^2+2*C*c*d^3)/c^2*x^2-1/16*(A*a^
2*c*e^4+6*A*a*c^2*d^2*e^2-11*A*c^3*d^4+4*B*a^2*c*d*e^3+4*B*a*c^2*d^3*e+5*C*
a^3*e^4+6*C*a^2*c*d^2*e^2+C*a*c^2*d^4)/a/c^3*x-1/6*(2*A*a*c*d*e^3+4*A*c^2*d
^3*e+B*a^2*e^4+3*B*a*c*d^2*e^2+B*c^2*d^4+4*C*a^2*d*e^3+2*C*a*c*d^3*e)/c^3)/
(c*x^2+a)^3+1/16*(A*a^2*c*e^4+6*A*a*c^2*d^2*e^2+5*A*c^3*d^4+4*B*a^2*c*d*e^3
+4*B*a*c^2*d^3*e+5*C*a^3*e^4+6*C*a^2*c*d^2*e^2+C*a*c^2*d^4)/a^3/c^3/(a*c)^(
1/2)*arctan(c*x/(a*c)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(222) = 444$.
time = 0.51, size = 597, normalized size = 2.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(8*B*a^3*c^2*d^4 + 24*B*a^4*c*d^2*e^2 + 8*B*a^5*e^4 - 3*(4*B*a*c^4*d^
3*e + 4*B*a^2*c^3*d*e^3 - 11*C*a^3*c^2*e^4 + A*a^2*c^3*e^4 + (C*a*c^4 + 5*A
*c^5)*d^4 + 6*(C*a^2*c^3*e^2 + A*a*c^4*e^2)*d^2)*x^5 + 24*(4*C*a^3*c^2*d*e^
3 + B*a^3*c^2*e^4)*x^4 + 16*(C*a^4*c*e + 2*A*a^3*c^2*e)*d^3 - 8*(4*B*a^2*c^
3*d^3*e - 4*B*a^3*c^2*d*e^3 - 5*C*a^4*c*e^4 - A*a^3*c^2*e^4 + (C*a^2*c^3 +
5*A*a*c^4)*d^4 - 6*(C*a^3*c^2*e^2 - A*a^2*c^3*e^2)*d^2)*x^3 + 24*(2*C*a^3*c
^2*d^3*e + 3*B*a^3*c^2*d^2*e^2 + B*a^4*c*e^4 + 2*(2*C*a^4*c*e^3 + A*a^3*c^2
*e^3)*d)*x^2 + 16*(2*C*a^5*e^3 + A*a^4*c*e^3)*d + 3*(4*B*a^3*c^2*d^3*e + 4*
B*a^4*c*d*e^3 + 5*C*a^5*e^4 + A*a^4*c*e^4 + (C*a^3*c^2 - 11*A*a^2*c^3)*d^4
+ 6*(C*a^4*c*e^2 + A*a^3*c^2*e^2)*d^2)*x)/(a^3*c^6*x^6 + 3*a^4*c^5*x^4 + 3*
a^5*c^4*x^2 + a^6*c^3) + 1/16*(4*B*a*c^2*d^3*e + 4*B*a^2*c*d*e^3 + (C*a*c^2
 + 5*A*c^3)*d^4 + 5*C*a^3*e^4 + A*a^2*c*e^4 + 6*(C*a^2*c*e^2 + A*a*c^2*e^2)
*d^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 919 vs. $2(222) = 444$.
time = 0.48, size = 1859, normalized size = 7.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*B*a^4*c^3*d^4 - 6*(C*a^2*c^5 + 5*A*a*c^6)*d^4*x^5 - 16*(C*a^3*c^
4 + 5*A*a^2*c^5)*d^4*x^3 + 6*(C*a^4*c^3 - 11*A*a^3*c^4)*d^4*x + 3*((C*a*c^5
 + 5*A*c^6)*d^4*x^6 + 3*(C*a^2*c^4 + 5*A*a*c^5)*d^4*x^4 + 3*(C*a^3*c^3 + 5*
A*a^2*c^4)*d^4*x^2 + (C*a^4*c^2 + 5*A*a^3*c^3)*d^4 + (5*C*a^6 + A*a^5*c + (
5*C*a^3*c^3 + A*a^2*c^4)*x^6 + 3*(5*C*a^4*c^2 + A*a^3*c^3)*x^4 + 3*(5*C*a^5
*c + A*a^4*c^2)*x^2)*e^4 + 4*(B*a^2*c^4*d*x^6 + 3*B*a^3*c^3*d*x^4 + 3*B*a^4
*c^2*d*x^2 + B*a^5*c*d)*e^3 + 6*((C*a^2*c^4 + A*a*c^5)*d^2*x^6 + 3*(C*a^3*c
^3 + A*a^2*c^4)*d^2*x^4 + 3*(C*a^4*c^2 + A*a^3*c^3)*d^2*x^2 + (C*a^5*c + A*
a^4*c^2)*d^2)*e^2 + 4*(B*a*c^5*d^3*x^6 + 3*B*a^2*c^4*d^3*x^4 + 3*B*a^3*c^3*
d^3*x^2 + B*a^4*c^2*d^3)*e)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*
x^2 + a)) + 2*(24*B*a^4*c^3*x^4 + 24*B*a^5*c^2*x^2 + 8*B*a^6*c + 3*(11*C*a^
4*c^3 - A*a^3*c^4)*x^5 + 8*(5*C*a^5*c^2 + A*a^4*c^3)*x^3 + 3*(5*C*a^6*c + A
*a^5*c^2)*x)*e^4 - 8*(3*B*a^3*c^4*d*x^5 - 24*C*a^4*c^3*d*x^4 - 8*B*a^4*c^3*
d*x^3 - 3*B*a^5*c^2*d*x - 12*(2*C*a^5*c^2 + A*a^4*c^3)*d*x^2 - 4*(2*C*a^6*c
 + A*a^5*c^2)*d)*e^3 + 12*(12*B*a^4*c^3*d^2*x^2 + 4*B*a^5*c^2*d^2 - 3*(C*a^
3*c^4 + A*a^2*c^5)*d^2*x^5 + 8*(C*a^4*c^3 - A*a^3*c^4)*d^2*x^3 + 3*(C*a^5*c
^2 + A*a^4*c^3)*d^2*x)*e^2 - 8*(3*B*a^2*c^5*d^3*x^5 + 8*B*a^3*c^4*d^3*x^3 -
 12*C*a^4*c^3*d^3*x^2 - 3*B*a^4*c^3*d^3*x - 4*(C*a^5*c^2 + 2*A*a^4*c^3)*d^3
)*e)/(a^4*c^7*x^6 + 3*a^5*c^6*x^4 + 3*a^6*c^5*x^2 + a^7*c^4), -1/48*(8*B*a^
4*c^3*d^4 - 3*(C*a^2*c^5 + 5*A*a*c^6)*d^4*x^5 - 8*(C*a^3*c^4 + 5*A*a^2*c^5)
*d^4*x^3 + 3*(C*a^4*c^3 - 11*A*a^3*c^4)*d^4*x - 3*((C*a*c^5 + 5*A*c^6)*d^4*
x^6 + 3*(C*a^2*c^4 + 5*A*a*c^5)*d^4*x^4 + 3*(C*a^3*c^3 + 5*A*a^2*c^4)*d^4*x
```

```
^2 + (C*a^4*c^2 + 5*A*a^3*c^3)*d^4 + (5*C*a^6 + A*a^5*c + (5*C*a^3*c^3 + A*
a^2*c^4)*x^6 + 3*(5*C*a^4*c^2 + A*a^3*c^3)*x^4 + 3*(5*C*a^5*c + A*a^4*c^2)*
x^2)*e^4 + 4*(B*a^2*c^4*d*x^6 + 3*B*a^3*c^3*d*x^4 + 3*B*a^4*c^2*d*x^2 + B*a
^5*c*d)*e^3 + 6*((C*a^2*c^4 + A*a*c^5)*d^2*x^6 + 3*(C*a^3*c^3 + A*a^2*c^4)*
d^2*x^4 + 3*(C*a^4*c^2 + A*a^3*c^3)*d^2*x^2 + (C*a^5*c + A*a^4*c^2)*d^2)*e^
2 + 4*(B*a*c^5*d^3*x^6 + 3*B*a^2*c^4*d^3*x^4 + 3*B*a^3*c^3*d^3*x^2 + B*a^4*
c^2*d^3)*e)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (24*B*a^4*c^3*x^4 + 24*B*a^5*
c^2*x^2 + 8*B*a^6*c + 3*(11*C*a^4*c^3 - A*a^3*c^4)*x^5 + 8*(5*C*a^5*c^2 + A
*a^4*c^3)*x^3 + 3*(5*C*a^6*c + A*a^5*c^2)*x)*e^4 - 4*(3*B*a^3*c^4*d*x^5 - 2
4*C*a^4*c^3*d*x^4 - 8*B*a^4*c^3*d*x^3 - 3*B*a^5*c^2*d*x - 12*(2*C*a^5*c^2 +
 A*a^4*c^3)*d*x^2 - 4*(2*C*a^6*c + A*a^5*c^2)*d)*e^3 + 6*(12*B*a^4*c^3*d^2*
x^2 + 4*B*a^5*c^2*d^2 - 3*(C*a^3*c^4 + A*a^2*c^5)*d^2*x^5 + 8*(C*a^4*c^3 -
A*a^3*c^4)*d^2*x^3 + 3*(C*a^5*c^2 + A*a^4*c^3)*d^2*x)*e^2 - 4*(3*B*a^2*c^5*
d^3*x^5 + 8*B*a^3*c^4*d^3*x^3 - 12*C*a^4*c^3*d^3*x^2 - 3*B*a^4*c^3*d^3*x -
4*(C*a^5*c^2 + 2*A*a^4*c^3)*d^3)*e)/(a^4*c^7*x^6 + 3*a^5*c^6*x^4 + 3*a^6*c^
5*x^2 + a^7*c^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 636 vs. 2(222) = 444.
time = 3.91, size = 636, normalized size = 2.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
[Out] 1/16*(C*a*c^2*d^4 + 5*A*c^3*d^4 + 4*B*a*c^2*d^3*e + 6*C*a^2*c*d^2*e^2 + 6*A
*a*c^2*d^2*e^2 + 4*B*a^2*c*d*e^3 + 5*C*a^3*e^4 + A*a^2*c*e^4)*arctan(c*x/sq
rt(a*c))/(sqrt(a*c)*a^3*c^3) + 1/48*(3*C*a*c^4*d^4*x^5 + 15*A*c^5*d^4*x^5 +
 12*B*a*c^4*d^3*x^5*e + 18*C*a^2*c^3*d^2*x^5*e^2 + 18*A*a*c^4*d^2*x^5*e^2 +
 8*C*a^2*c^3*d^4*x^3 + 40*A*a*c^4*d^4*x^3 + 12*B*a^2*c^3*d*x^5*e^3 + 32*B*a
^2*c^3*d^3*x^3*e - 33*C*a^3*c^2*x^5*e^4 + 3*A*a^2*c^3*x^5*e^4 - 96*C*a^3*c^
2*d*x^4*e^3 - 48*C*a^3*c^2*d^2*x^3*e^2 + 48*A*a^2*c^3*d^2*x^3*e^2 - 48*C*a^
3*c^2*d^3*x^2*e - 3*C*a^3*c^2*d^4*x + 33*A*a^2*c^3*d^4*x - 24*B*a^3*c^2*x^4
*e^4 - 32*B*a^3*c^2*d*x^3*e^3 - 72*B*a^3*c^2*d^2*x^2*e^2 - 12*B*a^3*c^2*d^3
*x*e - 8*B*a^3*c^2*d^4 - 40*C*a^4*c*x^3*e^4 - 8*A*a^3*c^2*x^3*e^4 - 96*C*a^
```

```
4*c*d*x^2*e^3 - 48*A*a^3*c^2*d*x^2*e^3 - 18*C*a^4*c*d^2*x*e^2 - 18*A*a^3*c^
2*d^2*x*e^2 - 16*C*a^4*c*d^3*e - 32*A*a^3*c^2*d^3*e - 24*B*a^4*c*x^2*e^4 -
12*B*a^4*c*d*x*e^3 - 24*B*a^4*c*d^2*e^2 - 15*C*a^5*x*e^4 - 3*A*a^4*c*x*e^4
- 32*C*a^5*d*e^3 - 16*A*a^4*c*d*e^3 - 8*B*a^5*e^4)/((c*x^2 + a)^3*a^3*c^3)
```

Mupad [B]
time = 4.38, size = 669, normalized size = 2.86

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x\,\left(c\,d^2+a\,e^2\right)\,\left(5\,C\,a^2\,e^2+C\,a\,c\,d^2+4\,B\,a\,c\,d\,e+A\,a\,c\,e^2+5\,A\,c^2\,d^2\right)}{\sqrt{a}\,\left(5\,C\,a^3\,e^4+6\,C\,a^2\,c\,d^2\,e^2+4\,B\,a^2\,c\,d\,e^3+A\,a^2\,c\,e^4+C\,a\,c^2\,d^4+4\,B\,a\,c^2\,d^3\,e+6\,A\,a\,c^2\,d^2\,e^2+5\,A\,c^3\,d^4\right)}\right)\,\left(c\,d^2+a\,e^2\right)\,\left(5\,C\,a^2\,e^2+C\,a\,c\,d^2+4\,B\,a\,c\,d\,e+A\,a\,c\,e^2+5\,A\,c^2\,d^2\right)}{16\,a^{7/2}\,c^{7/2}}-\frac{\frac{4\,C\,a^2\,d\,e^3+B\,a^2\,e^4+2\,C\,a\,c\,d^3\,e+3\,B\,a\,c\,d^2\,e^2+2\,A\,a\,c\,d\,e^3+B\,c^2\,d^4+4\,A\,c^2\,d^3\,e}{6\,c^3}+\frac{x^2\,\left(B\,a\,e^4+2\,A\,c\,d\,e^3+4\,C\,a\,d\,e^3+2\,C\,c\,d^3\,e+3\,B\,c\,d^2\,e^2\right)}{2\,c^2}+\frac{x^4\,\left(B\,e^4+4\,C\,d\,e^3\right)}{2\,c}+\frac{x\,\left(5\,C\,a^3\,e^4+6\,C\,a^2\,c\,d^2\,e^2+4\,B\,a^2\,c\,d\,e^3+A\,a^2\,c\,e^4+C\,a\,c^2\,d^4+4\,B\,a\,c^2\,d^3\,e+6\,A\,a\,c^2\,d^2\,e^2-11\,A\,c^3\,d^4\right)}{16\,a\,c^3}-\frac{x^3\,\left(-5\,C\,a^3\,e^4-6\,C\,a^2\,c\,d^2\,e^2-4\,B\,a^2\,c\,d\,e^3-A\,a^2\,c\,e^4+C\,a\,c^2\,d^4+4\,B\,a\,c^2\,d^3\,e+6\,A\,a\,c^2\,d^2\,e^2+5\,A\,c^3\,d^4\right)}{6\,a^2\,c^2}-\frac{x^5\,\left(-11\,C\,a^3\,e^4+6\,C\,a^2\,c\,d^2\,e^2+4\,B\,a^2\,c\,d\,e^3+A\,a^2\,c\,e^4+C\,a\,c^2\,d^4+4\,B\,a\,c^2\,d^3\,e+6\,A\,a\,c^2\,d^2\,e^2+5\,A\,c^3\,d^4\right)}{16\,a^3\,c}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

```
[Out] (atan((c^(1/2)*x*(a*e^2 + c*d^2)*(5*A*c^2*d^2 + 5*C*a^2*e^2 + A*a*c*e^2 + C
*a*c*d^2 + 4*B*a*c*d*e))/(a^(1/2)*(5*A*c^3*d^4 + 5*C*a^3*e^4 + A*a^2*c*e^4
+ C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 + 6*C*a^2*c*d^2*e^2 + 4*B*a*c^2*d^3*e + 4
*B*a^2*c*d*e^3)))*(a*e^2 + c*d^2)*(5*A*c^2*d^2 + 5*C*a^2*e^2 + A*a*c*e^2 +
C*a*c*d^2 + 4*B*a*c*d*e))/(16*a^(7/2)*c^(7/2)) - ((B*a^2*e^4 + B*c^2*d^4 +
4*A*c^2*d^3*e + 4*C*a^2*d*e^3 + 2*A*a*c*d*e^3 + 2*C*a*c*d^3*e + 3*B*a*c*d^2
*e^2)/(6*c^3) + (x^2*(B*a*e^4 + 2*A*c*d*e^3 + 4*C*a*d*e^3 + 2*C*c*d^3*e + 3
*B*c*d^2*e^2))/(2*c^2) + (x^4*(B*e^4 + 4*C*d*e^3))/(2*c) + (x*(5*C*a^3*e^4
- 11*A*c^3*d^4 + A*a^2*c*e^4 + C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 + 6*C*a^2*c*
d^2*e^2 + 4*B*a*c^2*d^3*e + 4*B*a^2*c*d*e^3))/(16*a*c^3) - (x^3*(5*A*c^3*d^
4 - 5*C*a^3*e^4 - A*a^2*c*e^4 + C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 - 6*C*a^2*c
*d^2*e^2 + 4*B*a*c^2*d^3*e - 4*B*a^2*c*d*e^3))/(6*a^2*c^2) - (x^5*(5*A*c^3*
d^4 - 11*C*a^3*e^4 + A*a^2*c*e^4 + C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 + 6*C*a^
2*c*d^2*e^2 + 4*B*a*c^2*d^3*e + 4*B*a^2*c*d*e^3))/(16*a^3*c))/(a^3 + c^3*x^
6 + 3*a^2*c*x^2 + 3*a*c^2*x^4)
```

3.65 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$

Optimal. Leaf size=254

$$-\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3}-\frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd+aCd+3aBe)x)}{24a^2c^2\left(a+cx^2\right)^2}-\frac{(d+ex)\left(ae(5Acd\right.}{}$$

```
[Out] -1/6*(a*B-(A*c-C*a)*x)*(e*x+d)^3/a/c/(c*x^2+a)^3-1/24*(e*x+d)^2*(2*a*(A*c+2
*C*a)*e-c*(5*A*c*d+3*B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)^2-1/48*(e*x+d)*(a*e*
(5*A*c*d+3*B*a*e+C*a*d)-(4*a*(A*c+2*C*a)*e^2+3*c*d*(5*A*c*d+3*B*a*e+C*a*d))
*x)/a^3/c^2/(c*x^2+a)+1/16*(A*c*d*(3*a*e^2+5*c*d^2)+a*(a*e^2*(B*e+3*C*d)+c*
d^2*(3*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/a^(7/2)/c^(5/2)
```

Rubi [A]
time = 0.33, antiderivative size = 288, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1659, 835, 792, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+5cd^2\right)+a\left(ae^2(Be+3Cd)+cd^2(3Be+Cd)\right)\right)}{16a^{7/2}c^{5/2}}-\frac{4ae\left(Ac\left(ae^2+5cd^2\right)+a\left(2aCe^2+cd(3Be+Cd)\right)\right)-cx\left(Acd\left(15cd^2-ae^2\right)+a\left(ae^2(7Cd-3Be)+3cd^2(3Be+Cd)\right)\right)}{48a^3c^3\left(a+cx^2\right)}-\frac{(d+ex)^2(2ae(2aC+Ac)-cx(3aBe+aCd+5Acd))}{24a^2c^2\left(a+cx^2\right)^2}-\frac{(d+ex)^3(aB-x(Ac-aC))}{6ac\left(a+cx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -1/6*((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(a*c*(a + c*x^2)^3) - ((d + e*x)^2
*(2*a*(A*c + 2*a*C)*e - c*(5*A*c*d + a*C*d + 3*a*B*e)*x))/(24*a^2*c^2*(a +
c*x^2)^2) - (4*a*e*(A*c*(5*c*d^2 + a*e^2) + a*(2*a*C*e^2 + c*d*(C*d + 3*B*e
))) - c*(A*c*d*(15*c*d^2 - a*e^2) + a*(a*e^2*(7*C*d - 3*B*e) + 3*c*d^2*(C*d
 + 3*B*e)))*x)/(48*a^3*c^3*(a + c*x^2)) + ((A*c*d*(5*c*d^2 + 3*a*e^2) + a*(
a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(1
6*a^(7/2)*c^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3} - \frac{\int \frac{(d+ex)^2(-5Acd-aCd-3aBe-2(Ac+2aC)ex)}{(a+cx^2)^3}}{6ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd}{24a^2c^2\left(a+cx^2\right)^2}$$

Mathematica [A]
time = 0.20, size = 350, normalized size = 1.38

$$\frac{-\frac{3\sqrt{a}\left(8a^3Ce^3-5Ac^3d^3x-a^2ce^2(3Cd+Be)x-ac^2d\left(Cd^2+3e(Bd+Ae)\right)x\right)}{a+cx^2}-\frac{8a^{5/2}\left(a^3Ce^3-Ac^3d^3x+ac^2d\left(Cd^2x+3Ae(d+ex)+Bd(d+3ex)\right)-a^2ce(3Cd(d+ex)+e(3Bd+Ae+Bex))\right)}{\left(a+cx^2\right)^3}+\frac{2a^{3/2}\left(12a^3Ce^3+5Ac^3d^3x+ac^2d\left(Cd^2+3e(Bd+Ae)\right)x-a^2ce(3Cd(6d+7ex)+e(18Bd+6Ae+7Bex))\right)}{\left(a+cx^2\right)^2}+3\sqrt{c}\left(Acd\left(5cd^2+3ae^2\right)+a\left(ae^2(3Cd+Be)+cd^2(Cd+3Be)\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{48a^{7/2}c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] ((-3*Sqrt[a]*(8*a^3*C*e^3 - 5*A*c^3*d^3*x - a^2*c*e^2*(3*C*d + B*e)*x - a*c
^2*d*(C*d^2 + 3*e*(B*d + A*e))*x))/(a + c*x^2) - (8*a^(5/2)*(a^3*C*e^3 - A*
```

c^3*d^3*x + a*c^2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d*(d + 3*e*x)) - a^2*c*e
*(3*C*d*(d + e*x) + e*(3*B*d + A*e + B*e*x))))/(a + c*x^2)^3 + (2*a^(3/2)*(
12*a^3*C*e^3 + 5*A*c^3*d^3*x + a*c^2*d*(C*d^2 + 3*e*(B*d + A*e))*x - a^2*c*
e*(3*C*d*(6*d + 7*e*x) + e*(18*B*d + 6*A*e + 7*B*e*x))))/(a + c*x^2)^2 + 3*
Sqrt[c]*(A*c*d*(5*c*d^2 + 3*a*e^2) + a*(a*e^2*(3*C*d + B*e) + c*d^2*(C*d +
3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(48*a^(7/2)*c^3)

Maple [A]
time = 0.12, size = 383, normalized size = 1.51

method	result
default	$\frac{\left(3Aacd\,e^2+5A\,c^2d^3+B\,a^2e^3+3Bac\,d^2e+3C\,a^2d\,e^2+Cac\,d^3\right)x^5}{16a^3}-\frac{C\,e^3x^4}{2c}+\frac{\left(3Aacd\,e^2+5A\,c^2d^3-B\,a^2e^3+3Bac\,d^2e-3C\,a^2d\,e^2+Cac\,d^3\right)x^3}{6a^2c}-\frac{e\left(Ac\right.}{}$
risch	$\frac{\left(3Aacd\,e^2+5A\,c^2d^3+B\,a^2e^3+3Bac\,d^2e+3C\,a^2d\,e^2+Cac\,d^3\right)x^5}{16a^3}-\frac{C\,e^3x^4}{2c}+\frac{\left(3Aacd\,e^2+5A\,c^2d^3-B\,a^2e^3+3Bac\,d^2e-3C\,a^2d\,e^2+Cac\,d^3\right)x^3}{6a^2c}-\frac{e\left(Ac\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)

[Out] (1/16*(3*A*a*c*d*e^2+5*A*c^2*d^3+B*a^2*e^3+3*B*a*c*d^2*e+3*C*a^2*d*e^2+C*a*
c*d^3)/a^3*x^5-1/2*C*e^3*x^4/c+1/6*(3*A*a*c*d*e^2+5*A*c^2*d^3-B*a^2*e^3+3*B
*a*c*d^2*e-3*C*a^2*d*e^2+C*a*c*d^3)/a^2/c*x^3-1/4*e*(A*c*e^2+3*B*c*d*e+2*C*
a*e^2+3*C*c*d^2)/c^2*x^2-1/16*(3*A*a*c*d*e^2-11*A*c^2*d^3+B*a^2*e^3+3*B*a*c
*d^2*e+3*C*a^2*d*e^2+C*a*c*d^3)/c^2/a*x-1/12*(A*a*c*e^3+6*A*c^2*d^2*e+3*B*a
*c*d*e^2+2*B*c^2*d^3+2*C*a^2*e^3+3*C*a*c*d^2*e)/c^3)/(c*x^2+a)^3+1/16*(3*A*
a*c*d*e^2+5*A*c^2*d^3+B*a^2*e^3+3*B*a*c*d^2*e+3*C*a^2*d*e^2+C*a*c*d^3)/a^3/
c^2/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))

Maxima [A]
time = 0.51, size = 460, normalized size = 1.81

$$-\frac{24\,Ca^3c^2x^4e^3+8\,Ba^3c^2d^3+12\,Ba^4cde^2+8\,Ca^5e^3+4\,Aa^4ce^3-3\,(3\,Bac^4d^2e+Ba^2c^3e^3+(Cac^4+5\,Ac^5)d^3+3\,(Ca^2c^3e^2+Aac^4e^2)d)x^5-8\,(3\,Ba^2c^3d^2e-Ba^3c^2e^3+(Ca^2c^3+5\,Aac^4)d^3-3\,(Ca^3c^2e^2-Aa^2c^3e^2)d)x^3+12\,(Ca^4ce+2\,Aa^3c^2e)d^2+12\,(3\,Ca^3c^2d^2e+3\,Ba^3c^2de^2+2\,Ca^4ce^3+Aa^3c^2e^3)x^2+3\,(3\,Ba^3c^2d^2e+Ba^4ce^3+(Ca^3c^2-11\,Aa^2c^3)d^3+3\,(Ca^4ce^2+Aa^3c^2e^2)d)x}{48\,(a^3c^6x^6+3\,a^4c^5x^4+3\,a^5c^4x^2+a^6c^3)}+\frac{(3\,Bacd^2e+(Cac+5\,Ac^2)d^3+Ba^2e^3+3\,(Ca^2e^2+Aace^2)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")

[Out] -1/48*(24*C*a^3*c^2*x^4*e^3 + 8*B*a^3*c^2*d^3 + 12*B*a^4*c*d*e^2 + 8*C*a^5*
e^3 + 4*A*a^4*c*e^3 - 3*(3*B*a*c^4*d^2*e + B*a^2*c^3*e^3 + (C*a*c^4 + 5*A*c
^5)*d^3 + 3*(C*a^2*c^3*e^2 + A*a*c^4*e^2)*d)*x^5 - 8*(3*B*a^2*c^3*d^2*e - B
*a^3*c^2*e^3 + (C*a^2*c^3 + 5*A*a*c^4)*d^3 - 3*(C*a^3*c^2*e^2 - A*a^2*c^3*e
^2)*d)*x^3 + 12*(C*a^4*c*e + 2*A*a^3*c^2*e)*d^2 + 12*(3*C*a^3*c^2*d^2*e + 3
*B*a^3*c^2*d*e^2 + 2*C*a^4*c*e^3 + A*a^3*c^2*e^3)*x^2 + 3*(3*B*a^3*c^2*d^2*
e + B*a^4*c*e^3 + (C*a^3*c^2 - 11*A*a^2*c^3)*d^3 + 3*(C*a^4*c*e^2 + A*a^3*c
^2*e^2)*d)*x)/(a^3*c^6*x^6 + 3*a^4*c^5*x^4 + 3*a^5*c^4*x^2 + a^6*c^3) + 1/1

```
6*(3*B*a*c*d^2*e + (C*a*c + 5*A*c^2)*d^3 + B*a^2*e^3 + 3*(C*a^2*e^2 + A*a*c
*e^2)*d)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(243) = 486$.
time = 0.43, size = 1380, normalized size = 5.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*B*a^4*c^2*d^3 - 6*(C*a^2*c^4 + 5*A*a*c^5)*d^3*x^5 - 16*(C*a^3*c^
3 + 5*A*a^2*c^4)*d^3*x^3 + 6*(C*a^4*c^2 - 11*A*a^3*c^3)*d^3*x + 3*((C*a*c^4
 + 5*A*c^5)*d^3*x^6 + 3*(C*a^2*c^3 + 5*A*a*c^4)*d^3*x^4 + 3*(C*a^3*c^2 + 5*
A*a^2*c^3)*d^3*x^2 + (C*a^4*c + 5*A*a^3*c^2)*d^3 + (B*a^2*c^3*x^6 + 3*B*a^3
*c^2*x^4 + 3*B*a^4*c*x^2 + B*a^5)*e^3 + 3*((C*a^2*c^3 + A*a*c^4)*d*x^6 + 3*
(C*a^3*c^2 + A*a^2*c^3)*d*x^4 + 3*(C*a^4*c + A*a^3*c^2)*d*x^2 + (C*a^5 + A*
a^4*c)*d)*e^2 + 3*(B*a*c^4*d^2*x^6 + 3*B*a^2*c^3*d^2*x^4 + 3*B*a^3*c^2*d^2*
x^2 + B*a^4*c*d^2)*e)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 +
a)) - 2*(3*B*a^3*c^3*x^5 - 24*C*a^4*c^2*x^4 - 8*B*a^4*c^2*x^3 - 3*B*a^5*c*x
 - 8*C*a^6 - 4*A*a^5*c - 12*(2*C*a^5*c + A*a^4*c^2)*x^2)*e^3 + 6*(12*B*a^4*
c^2*d*x^2 + 4*B*a^5*c*d - 3*(C*a^3*c^3 + A*a^2*c^4)*d*x^5 + 8*(C*a^4*c^2 -
A*a^3*c^3)*d*x^3 + 3*(C*a^5*c + A*a^4*c^2)*d*x)*e^2 - 6*(3*B*a^2*c^4*d^2*x^
5 + 8*B*a^3*c^3*d^2*x^3 - 12*C*a^4*c^2*d^2*x^2 - 3*B*a^4*c^2*d^2*x - 4*(C*a
^5*c + 2*A*a^4*c^2)*d^2)*e)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 +
a^7*c^3), -1/48*(8*B*a^4*c^2*d^3 - 3*(C*a^2*c^4 + 5*A*a*c^5)*d^3*x^5 - 8*(C
*a^3*c^3 + 5*A*a^2*c^4)*d^3*x^3 + 3*(C*a^4*c^2 - 11*A*a^3*c^3)*d^3*x - 3*((
C*a*c^4 + 5*A*c^5)*d^3*x^6 + 3*(C*a^2*c^3 + 5*A*a*c^4)*d^3*x^4 + 3*(C*a^3*c
^2 + 5*A*a^2*c^3)*d^3*x^2 + (C*a^4*c + 5*A*a^3*c^2)*d^3 + (B*a^2*c^3*x^6 +
3*B*a^3*c^2*x^4 + 3*B*a^4*c*x^2 + B*a^5)*e^3 + 3*((C*a^2*c^3 + A*a*c^4)*d*x
^6 + 3*(C*a^3*c^2 + A*a^2*c^3)*d*x^4 + 3*(C*a^4*c + A*a^3*c^2)*d*x^2 + (C*a
^5 + A*a^4*c)*d)*e^2 + 3*(B*a*c^4*d^2*x^6 + 3*B*a^2*c^3*d^2*x^4 + 3*B*a^3*c
^2*d^2*x^2 + B*a^4*c*d^2)*e)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (3*B*a^3*c^3
*x^5 - 24*C*a^4*c^2*x^4 - 8*B*a^4*c^2*x^3 - 3*B*a^5*c*x - 8*C*a^6 - 4*A*a^5
*c - 12*(2*C*a^5*c + A*a^4*c^2)*x^2)*e^3 + 3*(12*B*a^4*c^2*d*x^2 + 4*B*a^5*
c*d - 3*(C*a^3*c^3 + A*a^2*c^4)*d*x^5 + 8*(C*a^4*c^2 - A*a^3*c^3)*d*x^3 + 3
*(C*a^5*c + A*a^4*c^2)*d*x)*e^2 - 3*(3*B*a^2*c^4*d^2*x^5 + 8*B*a^3*c^3*d^2*
x^3 - 12*C*a^4*c^2*d^2*x^2 - 3*B*a^4*c^2*d^2*x - 4*(C*a^5*c + 2*A*a^4*c^2)*
d^2)*e)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.46, size = 475, normalized size = 1.87

$$\frac{\left(Cacd^3+5\,Ac^2d^3+3\,Bacd^2e+3\,Ca^2de^2+3\,Aacde^2+Ba^2e^3\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3c^2}+\frac{3\,Cac^4d^3x^5+15\,Ac^5d^3x^5+9\,Bac^4d^2x^5e+9\,Ca^2c^3dx^5e^2+9\,Aac^4dx^5e^2+8\,Ca^2c^3d^3x^3+40\,Aac^4d^3x^3+3\,Ba^2c^3x^5e^3+24\,Ba^2c^3d^2x^3e-24\,Ca^3c^2x^4e^3-24\,Ca^3c^2dx^3e^2+24\,Aa^2c^3dx^3e^2-36\,Ca^3c^2d^2x^2e-3\,Ca^3c^2d^3x+33\,Aa^2c^3d^3x-8\,Ba^3c^2x^3e^3-36\,Ba^3c^2dx^2e^2-9\,Ba^3c^2d^2xe-8\,Ba^3c^2d^3-24\,Ca^4cx^2e^3-12\,Aa^3c^2x^2e^3-9\,Ca^4cdxe^2-9\,Aa^3c^2dxe^2-12\,Ca^4cd^2e-24\,Aa^3c^2d^2e-3\,Ba^4cxe^3-12\,Ba^4cde^2-8\,Ca^5e^3-4\,Aa^4ce^3}{48\,(cx^2+a)^3a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*c*d^3 + 5*A*c^2*d^3 + 3*B*a*c*d^2*e + 3*C*a^2*d*e^2 + 3*A*a*c*d*e
^2 + B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2) + 1/48*(3*C*a*c^4
*d^3*x^5 + 15*A*c^5*d^3*x^5 + 9*B*a*c^4*d^2*x^5*e + 9*C*a^2*c^3*d*x^5*e^2 +
 9*A*a*c^4*d*x^5*e^2 + 8*C*a^2*c^3*d^3*x^3 + 40*A*a*c^4*d^3*x^3 + 3*B*a^2*c
^3*x^5*e^3 + 24*B*a^2*c^3*d^2*x^3*e - 24*C*a^3*c^2*x^4*e^3 - 24*C*a^3*c^2*d
*x^3*e^2 + 24*A*a^2*c^3*d*x^3*e^2 - 36*C*a^3*c^2*d^2*x^2*e - 3*C*a^3*c^2*d^
3*x + 33*A*a^2*c^3*d^3*x - 8*B*a^3*c^2*x^3*e^3 - 36*B*a^3*c^2*d*x^2*e^2 - 9
*B*a^3*c^2*d^2*x*e - 8*B*a^3*c^2*d^3 - 24*C*a^4*c*x^2*e^3 - 12*A*a^3*c^2*x^
2*e^3 - 9*C*a^4*c*d*x*e^2 - 9*A*a^3*c^2*d*x*e^2 - 12*C*a^4*c*d^2*e - 24*A*a
^3*c^2*d^2*e - 3*B*a^4*c*x*e^3 - 12*B*a^4*c*d*e^2 - 8*C*a^5*e^3 - 4*A*a^4*c
*e^3)/((c*x^2 + a)^3*a^3*c^3)
```

Mupad [B]
time = 4.07, size = 402, normalized size = 1.58

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(3\,C\,a^2\,d\,e^2+B\,a^2\,e^3+C\,a\,c\,d^3+3\,B\,a\,c\,d^2\,e+3\,A\,a\,c\,d\,e^2+5\,A\,c^2\,d^3\right)}{16\,a^{7/2}\,c^{5/2}}-\frac{\frac{2\,C\,a^2\,e^3+3\,C\,a\,c\,d^2\,e+3\,B\,a\,c\,d\,e^2+A\,a\,c\,e^3+2\,B\,c^2\,d^3+6\,A\,c^2\,d^2\,e}{12\,c^3}+\frac{x^2\,\left(A\,c\,e^3+2\,C\,a\,e^3+3\,B\,c\,d\,e^2+3\,C\,c\,d^2\,e\right)}{4\,c^2}-\frac{x^5\,\left(3\,C\,a^2\,d\,e^2+B\,a^2\,e^3+C\,a\,c\,d^3+3\,B\,a\,c\,d^2\,e+3\,A\,a\,c\,d\,e^2+5\,A\,c^2\,d^3\right)}{16\,a^3}+\frac{C\,e^3\,x^4}{2\,c}-\frac{x^3\,\left(-3\,C\,a^2\,d\,e^2-B\,a^2\,e^3+C\,a\,c\,d^3+3\,B\,a\,c\,d^2\,e+3\,A\,a\,c\,d\,e^2+5\,A\,c^2\,d^3\right)}{6\,a^2\,c}+\frac{x\,\left(3\,C\,a^2\,d\,e^2+B\,a^2\,e^3+C\,a\,c\,d^3+3\,B\,a\,c\,d^2\,e+3\,A\,a\,c\,d\,e^2-11\,A\,c^2\,d^3\right)}{16\,a\,c^2}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(5*A*c^2*d^3 + B*a^2*e^3 + C*a*c*d^3 + 3*C*a^2*d
*e^2 + 3*A*a*c*d*e^2 + 3*B*a*c*d^2*e))/(16*a^(7/2)*c^(5/2)) - ((2*B*c^2*d^3
 + 2*C*a^2*e^3 + A*a*c*e^3 + 6*A*c^2*d^2*e + 3*B*a*c*d*e^2 + 3*C*a*c*d^2*e)
/(12*c^3) + (x^2*(A*c*e^3 + 2*C*a*e^3 + 3*B*c*d*e^2 + 3*C*c*d^2*e))/(4*c^2)
 - (x^5*(5*A*c^2*d^3 + B*a^2*e^3 + C*a*c*d^3 + 3*C*a^2*d*e^2 + 3*A*a*c*d*e^
2 + 3*B*a*c*d^2*e))/(16*a^3) + (C*e^3*x^4)/(2*c) - (x^3*(5*A*c^2*d^3 - B*a^
2*e^3 + C*a*c*d^3 - 3*C*a^2*d*e^2 + 3*A*a*c*d*e^2 + 3*B*a*c*d^2*e))/(6*a^2*
c) + (x*(B*a^2*e^3 - 11*A*c^2*d^3 + C*a*c*d^3 + 3*C*a^2*d*e^2 + 3*A*a*c*d*e
^2 + 3*B*a*c*d^2*e))/(16*a*c^2))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4
)
```

3.66 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$

Optimal. Leaf size=225

$$-\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\,(a+cx^2)^3}-\frac{2ae(4Acd+2aCd+aBe)+(3a(Ac+aC)e^2-cd(5Acd+aCd+2aBe))}{24a^2c^2\,(a+cx^2)^2}$$

```
[Out] -1/6*(a*B-(A*c-C*a)*x)*(e*x+d)^2/a/c/(c*x^2+a)^3+1/24*(-2*a*e*(4*A*c*d+B*a*
e+2*C*a*d)-(3*a*(A*c+C*a)*e^2-c*d*(5*A*c*d+2*B*a*e+C*a*d))*x)/a^2/c^2/(c*x^
2+a)^2+1/16*(a*(A*c+C*a)*e^2+c*d*(5*A*c*d+2*B*a*e+C*a*d))*x/a^3/c^2/(c*x^2+
a)+1/16*(a*(A*c+C*a)*e^2+c*d*(5*A*c*d+2*B*a*e+C*a*d))*arctan(x*c^(1/2)/a^(1
/2))/a^(7/2)/c^(5/2)
```

Rubi [A]
time = 0.20, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1659, 792, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(cd(2aBe+aCd+5Acd)+ae^2(aC+Ac))}{16a^{7/2}c^{5/2}}+\frac{x(cd(2aBe+aCd+5Acd)+ae^2(aC+Ac))}{16a^3c^2\,(a+cx^2)}-\frac{x(3ae^2(aC+Ac)-cd(2aBe+aCd+5Acd))+2ae(aBe+2aCd+4Acd)}{24a^2c^2\,(a+cx^2)^2}-\frac{(d+ex)^2(aB-x(Ac-aC))}{6ac\,(a+cx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -1/6*((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(a*c*(a + c*x^2)^3) - (2*a*e*(4*A*
c*d + 2*a*C*d + a*B*e) + (3*a*(A*c + a*C)*e^2 - c*d*(5*A*c*d + a*C*d + 2*a*
B*e))*x)/(24*a^2*c^2*(a + c*x^2)^2) + ((a*(A*c + a*C)*e^2 + c*d*(5*A*c*d +
a*C*d + 2*a*B*e))*x)/(16*a^3*c^2*(a + c*x^2)) + ((a*(A*c + a*C)*e^2 + c*d*(
5*A*c*d + a*C*d + 2*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(5/2
))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3} - \frac{\int \frac{(d+ex)(-5Acd-aCd-2aBe-3(Ac+aC)ex)}{\left(a+cx^2\right)^3}\,dx}{6ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3} - \frac{2ae(4Acd+2aCd+aBe)+(3a(Ac+}{24a^2c^2\,(a}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3} - \frac{2ae(4Acd+2aCd+aBe)+(3a(Ac+}{24a^2c^2\,(a}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3} - \frac{2ae(4Acd+2aCd+aBe)+(3a(Ac+}{24a^2c^2\,(a}$$

Mathematica [A]
time = 0.11, size = 266, normalized size = 1.18

$$\frac{\left(Ac\left(5cd^2+ae^2\right)+a\left(aCe^2+cd(Cd+2Be)\right)\right)x}{16a^3c^2\left(a+cx^2\right)}+\frac{5Ac^2d^2x+ac\left(Cd^2+e(2Bd+Ae)\right)x-a^2e(12Cd+6Be+7Cex)}{24a^2c^2\left(a+cx^2\right)^2}+\frac{Ac^2d^2x+a^2e(2Cd+Be+Cex)-ac\left(Cd^2x+Ae(2d+ex)+Bd(d+2ex)\right)}{6ac^2\left(a+cx^2\right)^3}+\frac{\left(Ac\left(5cd^2+ae^2\right)+a\left(aCe^2+cd(Cd+2Be)\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16a^{7/2}c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] ((A*c*(5*c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x)/(16*a^3*c^2*(
a + c*x^2)) + (5*A*c^2*d^2*x + a*c*(C*d^2 + e*(2*B*d + A*e))*x - a^2*e*(12*
C*d + 6*B*e + 7*C*e*x))/(24*a^2*c^2*(a + c*x^2)^2) + (A*c^2*d^2*x + a^2*e*(
```

```
2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x)))/(
6*a*c^2*(a + c*x^2)^3) + ((A*c*(5*c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d +
2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(5/2))
```

Maple [A]
time = 0.10, size = 268, normalized size = 1.19

method	result
default	$\frac{\frac{\left(Aac\,e^{2}+5A\,c^{2}d^{2}+2acdeB+a^{2}C\,e^{2}+Cac\,d^{2}\right)x^{5}}{16a^{3}}+\frac{\left(Aac\,e^{2}+5A\,c^{2}d^{2}+2acdeB-a^{2}C\,e^{2}+Cac\,d^{2}\right)x^{3}}{6a^{2}c}-\frac{e(Be+2Cd)x^{2}}{4c}-\frac{\left(Aac\,e^{2}-11A\,c^{2}d^{2}+2acde\right.}{16c^{2}a}}{\left(c\,x^{2}+a\right)^{3}}$
risch	$\frac{\frac{\left(Aac\,e^{2}+5A\,c^{2}d^{2}+2acdeB+a^{2}C\,e^{2}+Cac\,d^{2}\right)x^{5}}{16a^{3}}+\frac{\left(Aac\,e^{2}+5A\,c^{2}d^{2}+2acdeB-a^{2}C\,e^{2}+Cac\,d^{2}\right)x^{3}}{6a^{2}c}-\frac{e(Be+2Cd)x^{2}}{4c}-\frac{\left(Aac\,e^{2}-11A\,c^{2}d^{2}+2acde\right.}{16c^{2}a}}{\left(c\,x^{2}+a\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*(A*a*c*e^2+5*A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d^2)/a^3*x^5+1/6*(
A*a*c*e^2+5*A*c^2*d^2+2*B*a*c*d*e-C*a^2*e^2+C*a*c*d^2)/a^2/c*x^3-1/4*e*(B*e
+2*C*d)*x^2/c-1/16*(A*a*c*e^2-11*A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d^2)
/c^2/a*x-1/12*(4*A*c*d*e+B*a*e^2+2*B*c*d^2+2*C*a*d*e)/c^2)/(c*x^2+a)^3+1/16
*(A*a*c*e^2+5*A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d^2)/a^3/c^2/(a*c)^(1/2
)*arctan(c*x/(a*c)^(1/2))
```

Maxima [A]
time = 0.50, size = 324, normalized size = 1.44

$$-\frac{8\,Ba^{3}cd^{2}-3\,(2\,Bac^{3}de+Ca^{2}c^{2}e^{2}+Aac^{3}e^{2}+(Cac^{3}+5\,Ac^{4})d^{2})x^{5}+4\,Ba^{4}e^{2}-8\,(2\,Ba^{2}c^{2}de-Ca^{3}ce^{2}+Aa^{2}c^{2}e^{2}+(Ca^{2}c^{2}+5\,Aac^{3})d^{2})x^{3}+12\,(2\,Ca^{3}cde+Ba^{3}ce^{2})x^{2}+8\,(Ca^{4}e+2\,Aa^{3}ce)d+3\,(2\,Ba^{3}cde+Ca^{4}e^{2}+Aa^{3}ce^{2}+(Ca^{3}c-11\,Aa^{2}c^{2})d^{2})x}{48\,(a^{3}c^{5}x^{6}+3\,a^{4}c^{4}x^{4}+3\,a^{5}c^{3}x^{2}+a^{6}c^{2})}+\frac{(2\,Bacde+Ca^{2}e^{2}+Aace^{2}+(Cac+5\,Ac^{2})d^{2})\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^{3}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(8*B*a^3*c*d^2 - 3*(2*B*a*c^3*d*e + C*a^2*c^2*e^2 + A*a*c^3*e^2 + (C*
a*c^3 + 5*A*c^4)*d^2)*x^5 + 4*B*a^4*e^2 - 8*(2*B*a^2*c^2*d*e - C*a^3*c*e^2
+ A*a^2*c^2*e^2 + (C*a^2*c^2 + 5*A*a*c^3)*d^2)*x^3 + 12*(2*C*a^3*c*d*e + B*
a^3*c*e^2)*x^2 + 8*(C*a^4*e + 2*A*a^3*c*e)*d + 3*(2*B*a^3*c*d*e + C*a^4*e^2
 + A*a^3*c*e^2 + (C*a^3*c - 11*A*a^2*c^2)*d^2)*x)/(a^3*c^5*x^6 + 3*a^4*c^4*
x^4 + 3*a^5*c^3*x^2 + a^6*c^2) + 1/16*(2*B*a*c*d*e + C*a^2*e^2 + A*a*c*e^2
+ (C*a*c + 5*A*c^2)*d^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 524 vs. 2(211) = 422.
time = 0.41, size = 1069, normalized size = 4.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*B*a^4*c^2*d^2 - 6*(C*a^2*c^4 + 5*A*a*c^5)*d^2*x^5 - 16*(C*a^3*c^
3 + 5*A*a^2*c^4)*d^2*x^3 + 6*(C*a^4*c^2 - 11*A*a^3*c^3)*d^2*x + 3*((C*a*c^4
 + 5*A*c^5)*d^2*x^6 + 3*(C*a^2*c^3 + 5*A*a*c^4)*d^2*x^4 + 3*(C*a^3*c^2 + 5*
A*a^2*c^3)*d^2*x^2 + (C*a^4*c + 5*A*a^3*c^2)*d^2 + ((C*a^2*c^3 + A*a*c^4)*x
^6 + C*a^5 + A*a^4*c + 3*(C*a^3*c^2 + A*a^2*c^3)*x^4 + 3*(C*a^4*c + A*a^3*c
^2)*x^2)*e^2 + 2*(B*a*c^4*d*x^6 + 3*B*a^2*c^3*d*x^4 + 3*B*a^3*c^2*d*x^2 + B
*a^4*c*d)*e)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(
12*B*a^4*c^2*x^2 + 4*B*a^5*c - 3*(C*a^3*c^3 + A*a^2*c^4)*x^5 + 8*(C*a^4*c^2
 - A*a^3*c^3)*x^3 + 3*(C*a^5*c + A*a^4*c^2)*x)*e^2 - 4*(3*B*a^2*c^4*d*x^5 +
 8*B*a^3*c^3*d*x^3 - 12*C*a^4*c^2*d*x^2 - 3*B*a^4*c^2*d*x - 4*(C*a^5*c + 2*
A*a^4*c^2)*d)*e)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3), -
1/48*(8*B*a^4*c^2*d^2 - 3*(C*a^2*c^4 + 5*A*a*c^5)*d^2*x^5 - 8*(C*a^3*c^3 +
5*A*a^2*c^4)*d^2*x^3 + 3*(C*a^4*c^2 - 11*A*a^3*c^3)*d^2*x - 3*((C*a*c^4 + 5
*A*c^5)*d^2*x^6 + 3*(C*a^2*c^3 + 5*A*a*c^4)*d^2*x^4 + 3*(C*a^3*c^2 + 5*A*a^
2*c^3)*d^2*x^2 + (C*a^4*c + 5*A*a^3*c^2)*d^2 + ((C*a^2*c^3 + A*a*c^4)*x^6 +
 C*a^5 + A*a^4*c + 3*(C*a^3*c^2 + A*a^2*c^3)*x^4 + 3*(C*a^4*c + A*a^3*c^2)*
x^2)*e^2 + 2*(B*a*c^4*d*x^6 + 3*B*a^2*c^3*d*x^4 + 3*B*a^3*c^2*d*x^2 + B*a^4
*c*d)*e)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (12*B*a^4*c^2*x^2 + 4*B*a^5*c -
3*(C*a^3*c^3 + A*a^2*c^4)*x^5 + 8*(C*a^4*c^2 - A*a^3*c^3)*x^3 + 3*(C*a^5*c
+ A*a^4*c^2)*x)*e^2 - 2*(3*B*a^2*c^4*d*x^5 + 8*B*a^3*c^3*d*x^3 - 12*C*a^4*c
^2*d*x^2 - 3*B*a^4*c^2*d*x - 4*(C*a^5*c + 2*A*a^4*c^2)*d)*e)/(a^4*c^6*x^6 +
 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.01, size = 328, normalized size = 1.46

$$\frac{\left(Cacd^2+5\,Ac^2d^2+2\,Bacde+Ca^2e^2+Aace^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3c^2}+\frac{3\,Cac^3d^2x^5+15\,Ac^4d^2x^5+6\,Bac^3dx^5e+3\,Ca^2c^2x^5e^2+3\,Aac^3x^5e^2+8\,Ca^2c^2d^2x^3+40\,Aac^3d^2x^3+16\,Ba^2c^2dx^3e-8\,Ca^3cx^3e^2+8\,Aa^2c^2x^3e^2-24\,Ca^3cdx^2e-3\,Ca^3cd^2x+33\,Aa^2c^2d^2x-12\,Ba^3cx^2e^2-6\,Ba^3cdxe-8\,Ba^3cd^2-3\,Ca^4xe^2-3\,Aa^3cxe^2-8\,Ca^4de-16\,Aa^3cde-4\,Ba^4e^2}{48\,{\left(cx^2+a\right)}^3a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*c*d^2 + 5*A*c^2*d^2 + 2*B*a*c*d*e + C*a^2*e^2 + A*a*c*e^2)*arctan
(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2) + 1/48*(3*C*a*c^3*d^2*x^5 + 15*A*c^4*d^
```

```
2*x^5 + 6*B*a*c^3*d*x^5*e + 3*C*a^2*c^2*x^5*e^2 + 3*A*a*c^3*x^5*e^2 + 8*C*a
^2*c^2*d^2*x^3 + 40*A*a*c^3*d^2*x^3 + 16*B*a^2*c^2*d*x^3*e - 8*C*a^3*c*x^3*
e^2 + 8*A*a^2*c^2*x^3*e^2 - 24*C*a^3*c*d*x^2*e - 3*C*a^3*c*d^2*x + 33*A*a^2
*c^2*d^2*x - 12*B*a^3*c*x^2*e^2 - 6*B*a^3*c*d*x*e - 8*B*a^3*c*d^2 - 3*C*a^4
*x*e^2 - 3*A*a^3*c*x*e^2 - 8*C*a^4*d*e - 16*A*a^3*c*d*e - 4*B*a^4*e^2)/((c*
x^2 + a)^3*a^3*c^2)
```

Mupad [B]
time = 0.23, size = 287, normalized size = 1.28

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2+5\,A\,c^2\,d^2\right)}{16\,a^{7/2}\,c^{5/2}}-\frac{\frac{B\,a\,e^2+2\,B\,c\,d^2+4\,A\,c\,d\,e+2\,C\,a\,d\,e}{12\,c^2}-\frac{x^5\,\left(C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2+5\,A\,c^2\,d^2\right)}{16\,a^3}+\frac{x^2\,\left(B\,e^2+2\,C\,d\,e\right)}{4\,c}+\frac{x\,\left(C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2-11\,A\,c^2\,d^2\right)}{16\,a\,c^2}-\frac{x^3\,\left(-C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2+5\,A\,c^2\,d^2\right)}{6\,a^2\,c}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(5*A*c^2*d^2 + C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^2
 + 2*B*a*c*d*e))/(16*a^(7/2)*c^(5/2)) - ((B*a*e^2 + 2*B*c*d^2 + 4*A*c*d*e +
 2*C*a*d*e)/(12*c^2) - (x^5*(5*A*c^2*d^2 + C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^
2 + 2*B*a*c*d*e))/(16*a^3) + (x^2*(B*e^2 + 2*C*d*e))/(4*c) + (x*(C*a^2*e^2
- 11*A*c^2*d^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(16*a*c^2) - (x^3*(5
*A*c^2*d^2 - C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(6*a^2*c))/(
a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4)
```

3.67 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4} dx$

Optimal. Leaf size=165

$$-\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3}-\frac{2a(2Ac+aC)e-c(5Acd+aCd+aBe)x}{24a^2c^2\left(a+cx^2\right)^2}+\frac{(5Acd+aCd+aBe)x}{16a^3c\left(a+cx^2\right)}+\frac{(5Acd$$

```
[Out] -1/6*(a*B-(A*c-C*a)*x)*(e*x+d)/a/c/(c*x^2+a)^3+1/24*(-2*a*(2*A*c+C*a)*e+c*(
5*A*c*d+B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)^2+1/16*(5*A*c*d+B*a*e+C*a*d)*x/a^
3/c/(c*x^2+a)+1/16*(5*A*c*d+B*a*e+C*a*d)*arctan(x*c^(1/2)/a^(1/2))/a^(7/2)/
c^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1659, 653, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd+5Acd)}{16a^{7/2}c^{3/2}}+\frac{x(aBe+aCd+5Acd)}{16a^3c\left(a+cx^2\right)}-\frac{2ae(aC+2Ac)-cx(aBe+aCd+5Acd)}{24a^2c^2\left(a+cx^2\right)^2}-\frac{(d+ex)(aB-x(Ac-aC))}{6ac\left(a+cx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -1/6*((a*B - (A*c - a*C)*x)*(d + e*x))/(a*c*(a + c*x^2)^3) - (2*a*(2*A*c +
a*C)*e - c*(5*A*c*d + a*C*d + a*B*e)*x)/(24*a^2*c^2*(a + c*x^2)^2) + ((5*A*
c*d + a*C*d + a*B*e)*x)/(16*a^3*c*(a + c*x^2)) + ((5*A*c*d + a*C*d + a*B*e)
*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
```

```
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx &= -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3}-\frac{\int \frac{-5Acd-a(Cd+Be)-2(2Ac+aC)ex}{\left(a+cx^2\right)^3}\,dx}{6ac}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3}-\frac{2a(2Ac+aC)e-c(5Acd+aCd+aB}{24a^2c^2\left(a+cx^2\right)^2}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3}-\frac{2a(2Ac+aC)e-c(5Acd+aCd+aB}{24a^2c^2\left(a+cx^2\right)^2}\\
&= -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3}-\frac{2a(2Ac+aC)e-c(5Acd+aCd+aB}{24a^2c^2\left(a+cx^2\right)^2}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 171, normalized size = 1.04

$$\frac{\frac{2a^{3/2}\left(-6a^2Ce+5Ac^2dx+ac(Cd+Be)x\right)}{\left(a+cx^2\right)^2}+\frac{3\sqrt{a}\,c(5Acd+aCd+aBe)x}{a+cx^2}+\frac{8a^{5/2}\left(a^2Ce+Ac^2dx-ac(Ae+Cdx+B(d+ex))\right)}{\left(a+cx^2\right)^3}+3\sqrt{c}\,(5Acd+aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{48a^{7/2}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] ((2*a^(3/2)*(-6*a^2*C*e + 5*A*c^2*d*x + a*c*(C*d + B*e)*x))/(a + c*x^2)^2 +
 (3*Sqrt[a]*c*(5*A*c*d + a*C*d + a*B*e)*x)/(a + c*x^2) + (8*a^(5/2)*(a^2*C*
e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x))))/(a + c*x^2)^3 + 3*Sqrt[c]
*(5*A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(48*a^(7/2)*c^2)
```

Maple [A]
time = 0.09, size = 149, normalized size = 0.90

method	result
default	$\frac{\frac{(5Acd+aBe+Cad)c\,x^5}{16a^3}+\frac{(5Acd+aBe+Cad)x^3}{6a^2}-\frac{Ce\,x^2}{4c}+\frac{(11Acd-aBe-Cad)x}{16ac}-\frac{2Ace+2Bcd+aCe}{12c^2}}{(c\,x^2+a)^3}+\frac{(5Acd+aBe+Cad)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16a^3c\sqrt{ac}}$
risch	$\frac{\frac{(5Acd+aBe+Cad)c\,x^5}{16a^3}+\frac{(5Acd+aBe+Cad)x^3}{6a^2}-\frac{Ce\,x^2}{4c}+\frac{(11Acd-aBe-Cad)x}{16ac}-\frac{2Ace+2Bcd+aCe}{12c^2}}{(c\,x^2+a)^3}-\frac{5\ln\left(cx+\sqrt{-ac}\right)Ad}{32\sqrt{-ac}\,a^3}-\frac{\ln\left(cx+\sqrt{\right.}}{32\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*(5*A*c*d+B*a*e+C*a*d)/a^3*c*x^5+1/6/a^2*(5*A*c*d+B*a*e+C*a*d)*x^3-1/4
*C*e*x^2/c+1/16*(11*A*c*d-B*a*e-C*a*d)/a/c*x-1/12*(2*A*c*e+2*B*c*d+C*a*e)/c
^2)/(c*x^2+a)^3+1/16*(5*A*c*d+B*a*e+C*a*d)/a^3/c/(a*c)^(1/2)*arctan(c*x/(a*
c)^(1/2))
```

Maxima [A]
time = 0.55, size = 214, normalized size = 1.30

$$-\frac{12\,Ca^3cx^2e+8\,Ba^3cd-3\,(Bac^3e+(Cac^3+5\,Ac^4)d)x^5+4\,Ca^4e+8\,Aa^3ce-8\,(Ba^2c^2e+(Ca^2c^2+5\,Aac^3)d)x^3+3\,(Ba^3ce+(Ca^3c-11\,Aa^2c^2)d)x}{48\,(a^3c^5x^6+3\,a^4c^4x^4+3\,a^5c^3x^2+a^6c^2)}+\frac{(Bae+(Ca+5\,Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(12*C*a^3*c*x^2*e + 8*B*a^3*c*d - 3*(B*a*c^3*e + (C*a*c^3 + 5*A*c^4)*
d)*x^5 + 4*C*a^4*e + 8*A*a^3*c*e - 8*(B*a^2*c^2*e + (C*a^2*c^2 + 5*A*a*c^3)
*d)*x^3 + 3*(B*a^3*c*e + (C*a^3*c - 11*A*a^2*c^2)*d)*x)/(a^3*c^5*x^6 + 3*a^
4*c^4*x^4 + 3*a^5*c^3*x^2 + a^6*c^2) + 1/16*(B*a*e + (C*a + 5*A*c)*d)*arcta
n(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(152) = 304$.
time = 0.51, size = 650, normalized size = 3.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*B*a^4*c*d - 6*(C*a^2*c^3 + 5*A*a*c^4)*d*x^5 - 16*(C*a^3*c^2 + 5*
A*a^2*c^3)*d*x^3 + 6*(C*a^4*c - 11*A*a^3*c^2)*d*x + 3*((C*a*c^3 + 5*A*c^4)*
d*x^6 + 3*(C*a^2*c^2 + 5*A*a*c^3)*d*x^4 + 3*(C*a^3*c + 5*A*a^2*c^2)*d*x^2 +
```

```
 (C*a^4 + 5*A*a^3*c)*d + (B*a*c^3*x^6 + 3*B*a^2*c^2*x^4 + 3*B*a^3*c*x^2 + B
*a^4)*e)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(3*B*
a^2*c^3*x^5 + 8*B*a^3*c^2*x^3 - 12*C*a^4*c*x^2 - 3*B*a^4*c*x - 4*C*a^5 - 8*
A*a^4*c)*e)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2), -1/48*
(8*B*a^4*c*d - 3*(C*a^2*c^3 + 5*A*a*c^4)*d*x^5 - 8*(C*a^3*c^2 + 5*A*a^2*c^3
)*d*x^3 + 3*(C*a^4*c - 11*A*a^3*c^2)*d*x - 3*((C*a*c^3 + 5*A*c^4)*d*x^6 + 3
*(C*a^2*c^2 + 5*A*a*c^3)*d*x^4 + 3*(C*a^3*c + 5*A*a^2*c^2)*d*x^2 + (C*a^4 +
 5*A*a^3*c)*d + (B*a*c^3*x^6 + 3*B*a^2*c^2*x^4 + 3*B*a^3*c*x^2 + B*a^4)*e)*
sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (3*B*a^2*c^3*x^5 + 8*B*a^3*c^2*x^3 - 12*C
*a^4*c*x^2 - 3*B*a^4*c*x - 4*C*a^5 - 8*A*a^4*c)*e)/(a^4*c^5*x^6 + 3*a^5*c^4
*x^4 + 3*a^6*c^3*x^2 + a^7*c^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.97, size = 194, normalized size = 1.18

$$\frac{(Cad+5\,Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3c}+\frac{3\,Cac^3dx^5+15\,Ac^4dx^5+3\,Bac^3x^5e+8\,Ca^2c^2dx^3+40\,Aac^3dx^3+8\,Ba^2c^2x^3e-12\,Ca^3cx^2e-3\,Ca^3cdx+33\,Aa^2c^2dx-3\,Ba^3cxe-8\,Ba^3cd-4\,Ca^4e-8\,Aa^3ce}{48\,(cx^2+a)^3a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*d + 5*A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c) + 1/
48*(3*C*a*c^3*d*x^5 + 15*A*c^4*d*x^5 + 3*B*a*c^3*x^5*e + 8*C*a^2*c^2*d*x^3
+ 40*A*a*c^3*d*x^3 + 8*B*a^2*c^2*x^3*e - 12*C*a^3*c*x^2*e - 3*C*a^3*c*d*x +
 33*A*a^2*c^2*d*x - 3*B*a^3*c*x*e - 8*B*a^3*c*d - 4*C*a^4*e - 8*A*a^3*c*e)/
((c*x^2 + a)^3*a^3*c^2)
```

Mupad [B]
time = 3.94, size = 164, normalized size = 0.99

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(5\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{16\,a^{7/2}\,c^{3/2}}-\frac{\frac{2\,A\,c\,e+2\,B\,c\,d+C\,a\,e}{12\,c^2}-\frac{x^3\,\left(5\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{6\,a^2}+\frac{C\,e\,x^2}{4\,c}+\frac{x\,\left(B\,a\,e-11\,A\,c\,d+C\,a\,d\right)}{16\,a\,c}-\frac{c\,x^5\,\left(5\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{16\,a^3}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(5*A*c*d + B*a*e + C*a*d))/(16*a^(7/2)*c^(3/2))
- ((2*A*c*e + 2*B*c*d + C*a*e)/(12*c^2) - (x^3*(5*A*c*d + B*a*e + C*a*d))/(
6*a^2) + (C*e*x^2)/(4*c) + (x*(B*a*e - 11*A*c*d + C*a*d))/(16*a*c) - (c*x^5
*(5*A*c*d + B*a*e + C*a*d))/(16*a^3))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^
2*x^4)
```

3.68 $\int \frac{A+Bx+Cx^2}{(a+cx^2)^4} dx$

Optimal. Leaf size=126

$$-\frac{aB-(Ac-aC)x}{6ac(a+cx^2)^3}+\frac{(5Ac+aC)x}{24a^2c(a+cx^2)^2}+\frac{(5Ac+aC)x}{16a^3c(a+cx^2)}+\frac{(5Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}$$

```
[Out] 1/6*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^3+1/24*(5*A*c+C*a)*x/a^2/c/(c*x^2+a)^2
+1/16*(5*A*c+C*a)*x/a^3/c/(c*x^2+a)+1/16*(5*A*c+C*a)*arctan(x*c^(1/2)/a^(1/
2))/a^(7/2)/c^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1828, 12, 205, 211}

$$\frac{(aC+5Ac)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}+\frac{x(aC+5Ac)}{16a^3c(a+cx^2)}+\frac{x(aC+5Ac)}{24a^2c(a+cx^2)^2}-\frac{aB-x(Ac-aC)}{6ac(a+cx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^4,x]
```

```
[Out] -1/6*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)^3) + ((5*A*c + a*C)*x)/(24*a^2*
c*(a + c*x^2)^2) + ((5*A*c + a*C)*x)/(16*a^3*c*(a + c*x^2)) + ((5*A*c + a*C
)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{(a+cx^2)^4}\,dx &= -\frac{aB-(Ac-aC)x}{6ac\,(a+cx^2)^3}-\frac{\int \frac{-5A-\frac{aC}{c}}{(a+cx^2)^3}\,dx}{6a}\\
&= -\frac{aB-(Ac-aC)x}{6ac\,(a+cx^2)^3}+\frac{(5Ac+aC)\int \frac{1}{(a+cx^2)^3}\,dx}{6ac}\\
&= -\frac{aB-(Ac-aC)x}{6ac\,(a+cx^2)^3}+\frac{(5Ac+aC)x}{24a^2c\,(a+cx^2)^2}+\frac{(5Ac+aC)\int \frac{1}{(a+cx^2)^2}\,dx}{8a^2c}\\
&= -\frac{aB-(Ac-aC)x}{6ac\,(a+cx^2)^3}+\frac{(5Ac+aC)x}{24a^2c\,(a+cx^2)^2}+\frac{(5Ac+aC)x}{16a^3c\,(a+cx^2)}+\frac{(5Ac+aC)\int \frac{1}{a+cx^2}\,dx}{16a^3c}\\
&= -\frac{aB-(Ac-aC)x}{6ac\,(a+cx^2)^3}+\frac{(5Ac+aC)x}{24a^2c\,(a+cx^2)^2}+\frac{(5Ac+aC)x}{16a^3c\,(a+cx^2)}+\frac{(5Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 112, normalized size = 0.89

$$\frac{15Ac^3x^5-a^3(8B+3Cx)+ac^2x^3(40A+3Cx^2)+a^2cx(33A+8Cx^2)}{48a^3c\,(a+cx^2)^3}+\frac{(5Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^4,x]
```

```
[Out] (15*A*c^3*x^5 - a^3*(8*B + 3*C*x) + a*c^2*x^3*(40*A + 3*C*x^2) + a^2*c*x*(3
3*A + 8*C*x^2))/(48*a^3*c*(a + c*x^2)^3) + ((5*A*c + a*C)*ArcTan[(Sqrt[c]*x
)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Maple [A]
time = 0.11, size = 100, normalized size = 0.79

method	result
default	$\frac{\frac{(5Ac+aC)c\,x^5}{16a^3}+\frac{(5Ac+aC)x^3}{6a^2}+\frac{(11Ac-aC)x}{16ac}-\frac{B}{6c}}{(c\,x^2+a)^3}+\frac{(5Ac+aC)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16a^3c\sqrt{ac}}$
risch	$\frac{\frac{(5Ac+aC)c\,x^5}{16a^3}+\frac{(5Ac+aC)x^3}{6a^2}+\frac{(11Ac-aC)x}{16ac}-\frac{B}{6c}}{(c\,x^2+a)^3}-\frac{5\ln\left(cx+\sqrt{-ac}\right)A}{32\sqrt{-ac}\,a^3}-\frac{\ln\left(cx+\sqrt{-ac}\right)C}{32\sqrt{-ac}\,c\,a^2}+\frac{5\ln\left(-cx+\sqrt{-ac}\right)A}{32\sqrt{-ac}\,a^3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*(5*A*c+C*a)/a^3*c*x^5+1/6/a^2*(5*A*c+C*a)*x^3+1/16*(11*A*c-C*a)/a/c*x
-1/6*B/c)/(c*x^2+a)^3+1/16*(5*A*c+C*a)/a^3/c/(a*c)^(1/2)*arctan(c*x/(a*c)^(
1/2))
```

Maxima [A]
time = 0.54, size = 133, normalized size = 1.06

$$\frac{3\,(Cac^2+5\,Ac^3)x^5-8\,Ba^3+8\,(Ca^2c+5\,Aac^2)x^3-3\,(Ca^3-11\,Aa^2c)x}{48\,(a^3c^4x^6+3\,a^4c^3x^4+3\,a^5c^2x^2+a^6c)}+\frac{(Ca+5\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(C*a*c^2 + 5*A*c^3)*x^5 - 8*B*a^3 + 8*(C*a^2*c + 5*A*a*c^2)*x^3 - 3
*(C*a^3 - 11*A*a^2*c)*x)/(a^3*c^4*x^6 + 3*a^4*c^3*x^4 + 3*a^5*c^2*x^2 + a^6
*c) + 1/16*(C*a + 5*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c)
```

Fricas [A]
time = 0.40, size = 430, normalized size = 3.41

$$\left[-\frac{16\,Ba^4c-6\,(Ca^2c^3+5\,Aac^4)x^5-16\,(Ca^3c^2+5\,Aa^2c^3)x^3+3\,((Cac^3+5\,Ac^4)x^6+Ca^4+5\,Aa^3c+3\,(Ca^2c^2+5\,Aac^3)x^4+3\,(Ca^3c+5\,Aa^2c^2)x^2)\sqrt{-ac}\,\log\left(\frac{cx^2-2\,\sqrt{-ac}\,x-a}{cx^2+a}\right)+6\,(Ca^4c-11\,Aa^3c^2)x}{96\,(a^4c^5x^6+3\,a^5c^4x^4+3\,a^6c^3x^2+a^7c^2)},-\frac{8\,Ba^4c-3\,(Ca^2c^3+5\,Aac^4)x^5-8\,(Ca^3c^2+5\,Aa^2c^3)x^3-3\,((Cac^3+5\,Ac^4)x^6+Ca^4+5\,Aa^3c+3\,(Ca^2c^2+5\,Aac^3)x^4+3\,(Ca^3c+5\,Aa^2c^2)x^2)\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,x}{a}\right)+3\,(Ca^4c-11\,Aa^3c^2)x}{48\,(a^4c^5x^6+3\,a^5c^4x^4+3\,a^6c^3x^2+a^7c^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*B*a^4*c - 6*(C*a^2*c^3 + 5*A*a*c^4)*x^5 - 16*(C*a^3*c^2 + 5*A*a^
2*c^3)*x^3 + 3*((C*a*c^3 + 5*A*c^4)*x^6 + C*a^4 + 5*A*a^3*c + 3*(C*a^2*c^2
+ 5*A*a*c^3)*x^4 + 3*(C*a^3*c + 5*A*a^2*c^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2
*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(C*a^4*c - 11*A*a^3*c^2)*x)/(a^4*c^5*x^
6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2), -1/48*(8*B*a^4*c - 3*(C*a^2*c
^3 + 5*A*a*c^4)*x^5 - 8*(C*a^3*c^2 + 5*A*a^2*c^3)*x^3 - 3*((C*a*c^3 + 5*A*c
^4)*x^6 + C*a^4 + 5*A*a^3*c + 3*(C*a^2*c^2 + 5*A*a*c^3)*x^4 + 3*(C*a^3*c +
```

```
5*A*a^2*c^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 3*(C*a^4*c - 11*A*a^3*c
^2)*x)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2)]
```

Sympy [A]
time = 1.08, size = 196, normalized size = 1.56

$$-\frac{\sqrt{-\frac{1}{a^7c^3}}\cdot(5Ac+Ca)\log\left(-a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^7c^3}}\cdot(5Ac+Ca)\log\left(a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32}+\frac{-8Ba^3+x^5\cdot(15Ac^3+3Cac^2)+x^3\cdot(40Aac^2+8Ca^2c)+x(33Aa^2c-3Ca^3)}{48a^6c+144a^5c^2x^2+144a^4c^3x^4+48a^3c^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] -sqrt(-1/(a**7*c**3))*(5*A*c + C*a)*log(-a**4*c*sqrt(-1/(a**7*c**3)) + x)/3
2 + sqrt(-1/(a**7*c**3))*(5*A*c + C*a)*log(a**4*c*sqrt(-1/(a**7*c**3)) + x)
/32 + (-8*B*a**3 + x**5*(15*A*c**3 + 3*C*a*c**2) + x**3*(40*A*a*c**2 + 8*C*
a**2*c) + x*(33*A*a**2*c - 3*C*a**3))/(48*a**6*c + 144*a**5*c**2*x**2 + 144
*a**4*c**3*x**4 + 48*a**3*c**4*x**6)
```

Giac [A]
time = 4.13, size = 109, normalized size = 0.87

$$\frac{(Ca+5\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3c}+\frac{3\,Cac^2x^5+15\,Ac^3x^5+8\,Ca^2cx^3+40\,Aac^2x^3-3\,Ca^3x+33\,Aa^2cx-8\,Ba^3}{48\,(cx^2+a)^3a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a + 5*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c) + 1/48*(3*C*a*c^
2*x^5 + 15*A*c^3*x^5 + 8*C*a^2*c*x^3 + 40*A*a*c^2*x^3 - 3*C*a^3*x + 33*A*a^
2*c*x - 8*B*a^3)/((c*x^2 + a)^3*a^3*c)
```

Mupad [B]
time = 3.89, size = 116, normalized size = 0.92

$$\frac{\frac{x^3\,(5\,A\,c+C\,a)}{6\,a^2}-\frac{B}{6\,c}+\frac{c\,x^5\,(5\,A\,c+C\,a)}{16\,a^3}+\frac{x\,(11\,A\,c-C\,a)}{16\,a\,c}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(5\,A\,c+C\,a)}{16\,a^{7/2}\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^4,x)
```

```
[Out] ((x^3*(5*A*c + C*a))/(6*a^2) - B/(6*c) + (c*x^5*(5*A*c + C*a))/(16*a^3) + (
x*(11*A*c - C*a))/(16*a*c))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4) + (
atan((c^(1/2)*x)/a^(1/2))*(5*A*c + C*a))/(16*a^(7/2)*c^(3/2))
```

3.69 $\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx$

Optimal. Leaf size=43

$$\frac{3x}{2}+\frac{x^2}{2}-\frac{x^3}{2(1+x^2)}-\frac{3}{2}\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] 3/2*x+1/2*x^2-1/2*x^3/(x^2+1)-3/2*arctan(x)-1/2*ln(x^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1818, 815, 649, 209, 266}

$$-\frac{3\text{ArcTan}(x)}{2}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-\frac{x^3}{2(x^2+1)}+\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] (3*x)/2 + x^2/2 - x^3/(2*(1 + x^2)) - (3*ArcTan[x])/2 - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx &= -\frac{x^3}{2(1+x^2)}-\frac{1}{2}\int \frac{(-3-2x)x^2}{1+x^2}\,dx\\ &= -\frac{x^3}{2(1+x^2)}-\frac{1}{2}\int \left(-3-2x+\frac{3+2x}{1+x^2}\right)dx\\ &= \frac{3x}{2}+\frac{x^2}{2}-\frac{x^3}{2(1+x^2)}-\frac{1}{2}\int \frac{3+2x}{1+x^2}\,dx\\ &= \frac{3x}{2}+\frac{x^2}{2}-\frac{x^3}{2(1+x^2)}-\frac{3}{2}\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\ &= \frac{3x}{2}+\frac{x^2}{2}-\frac{x^3}{2(1+x^2)}-\frac{3}{2}\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.67

$$\frac{1}{2}\left(x\left(2+x+\frac{1}{1+x^2}\right)-3\tan^{-1}(x)-\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + x + x^2))/(1 + x^2)^2,x]
[Out] (x*(2 + x + (1 + x^2)^(-1)) - 3*ArcTan[x] - Log[1 + x^2])/2
```

Maple [A]
time = 0.10, size = 30, normalized size = 0.70

method	result	size
default	$x+\frac{x^2}{2}+\frac{x}{2x^2+2}-\frac{\ln(x^2+1)}{2}-\frac{3\arctan(x)}{2}$	30
risch	$x+\frac{x^2}{2}+\frac{x}{2x^2+2}-\frac{\ln(x^2+1)}{2}-\frac{3\arctan(x)}{2}$	30
meijerg	$\frac{x^2(3x^2+6)}{6x^2+6}-\frac{\ln(x^2+1)}{2}+\frac{x(10x^2+15)}{10x^2+10}-\frac{3\arctan(x)}{2}-\frac{x^2}{2(x^2+1)}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x+1/2*x^2+1/2*x/(x^2+1)-1/2*ln(x^2+1)-3/2*arctan(x)
```

Maxima [**A**]
time = 0.52, size = 29, normalized size = 0.67

$$\frac{1}{2}\,x^2+x+\frac{x}{2\,(x^2+1)}-\frac{3}{2}\,\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x + 1/2*x/(x^2 + 1) - 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

Fricas [**A**]
time = 0.40, size = 46, normalized size = 1.07

$$\frac{x^4+2\,x^3+x^2-3\,(x^2+1)\arctan\left(x\right)-(x^2+1)\log\left(x^2+1\right)+3\,x}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^4 + 2*x^3 + x^2 - 3*(x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + 3
*x)/(x^2 + 1)
```

Sympy [**A**]
time = 0.04, size = 29, normalized size = 0.67

$$\frac{x^2}{2}+x+\frac{x}{2x^2+2}-\frac{\log\left(x^2+1\right)}{2}-\frac{3\,\mathrm{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+x+1)/(x**2+1)**2,x)
```

```
[Out] x**2/2 + x + x/(2*x**2 + 2) - log(x**2 + 1)/2 - 3*atan(x)/2
```

Giac [**A**]
time = 4.37, size = 29, normalized size = 0.67

$$\frac{1}{2}\,x^2+x+\frac{x}{2\,(x^2+1)}-\frac{3}{2}\,\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x + 1/2*x/(x^2 + 1) - 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

Mupad [B]
time = 0.04, size = 30, normalized size = 0.70

$$x-\frac{\ln\left(x^2+1\right)}{2}-\frac{3\,\mathrm{atan}(x)}{2}+\frac{x}{2\,\left(x^2+1\right)}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x + x^2 + 1))/(x^2 + 1)^2,x)
```

```
[Out] x - log(x^2 + 1)/2 - (3*atan(x))/2 + x/(2*(x^2 + 1)) + x^2/2
```

3.70 $\int \frac{x^2(1+x+x^2)}{(1+x^2)^2} dx$

Optimal. Leaf size=30

$$x - \frac{x^2}{2(1+x^2)} - \tan^{-1}(x) + \frac{1}{2}\log(1+x^2)$$

```
[Out] x-1/2*x^2/(x^2+1)-arctan(x)+1/2*ln(x^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1818, 788, 649, 209, 266}

$$-\text{ArcTan}(x) - \frac{x^2}{2(x^2+1)} + \frac{1}{2}\log(x^2+1) + x$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] x - x^2/(2*(1 + x^2)) - ArcTan[x] + Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx &= -\frac{x^2}{2(1+x^2)}-\frac{1}{2}\int \frac{(-2-2x)x}{1+x^2}\,dx\\ &= x-\frac{x^2}{2(1+x^2)}-\frac{1}{2}\int \frac{2-2x}{1+x^2}\,dx\\ &= x-\frac{x^2}{2(1+x^2)}-\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= x-\frac{x^2}{2(1+x^2)}-\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.90

$$x+\frac{1}{2(1+x^2)}-\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + x + x^2))/(1 + x^2)^2,x]
[Out] x + 1/(2*(1 + x^2)) - ArcTan[x] + Log[1 + x^2]/2
```

Maple [A]
time = 0.09, size = 24, normalized size = 0.80

method	result	size
default	$x+\frac{1}{2x^2+2}+\frac{\ln(x^2+1)}{2}-\arctan(x)$	24
risch	$x+\frac{1}{2x^2+2}+\frac{\ln(x^2+1)}{2}-\arctan(x)$	24
meijerg	$\frac{x(10x^2+15)}{10x^2+10}-\arctan(x)-\frac{x^2}{2(x^2+1)}+\frac{\ln(x^2+1)}{2}-\frac{x}{2(x^2+1)}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2+x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x+1/2/(x^2+1)+1/2*ln(x^2+1)-arctan(x)
```

Maxima [A]
time = 0.49, size = 23, normalized size = 0.77

$$x + \frac{1}{2\,(x^2+1)} - \arctan\left(x\right) + \frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] x + 1/2/(x^2 + 1) - arctan(x) + 1/2*log(x^2 + 1)
```

Fricas [A]
time = 0.45, size = 40, normalized size = 1.33

$$\frac{2\,x^3 - 2\,(x^2+1)\arctan\left(x\right) + (x^2+1)\log\left(x^2+1\right) + 2\,x + 1}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^3 - 2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) + 2*x + 1)/(x^2
 + 1)
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.67

$$x + \frac{\log\left(x^2+1\right)}{2} - \operatorname{atan}\left(x\right) + \frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+x+1)/(x**2+1)**2,x)
```

```
[Out] x + log(x**2 + 1)/2 - atan(x) + 1/(2*x**2 + 2)
```

Giac [A]
time = 4.90, size = 23, normalized size = 0.77

$$x + \frac{1}{2\,(x^2+1)} - \arctan\left(x\right) + \frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] x + 1/2/(x^2 + 1) - arctan(x) + 1/2*log(x^2 + 1)
```

Mupad [B]
time = 0.03, size = 23, normalized size = 0.77

$$x + \frac{\ln(x^2+1)}{2} - \operatorname{atan}(x) + \frac{1}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x + x^2 + 1))/(x^2 + 1)^2,x)
[Out] x + log(x^2 + 1)/2 - atan(x) + 1/(2*(x^2 + 1))
```

3.71 $\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx$

Optimal. Leaf size=29

$$-\frac{x}{2(1+x^2)}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)+1/2*ln(x^2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1818, 649, 209, 266}

$$\frac{\text{ArcTan}(x)}{2}-\frac{x}{2(x^2+1)}+\frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2 + Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
```

```
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx &= -\frac{x}{2(1+x^2)}-\frac{1}{2}\int \frac{-1-2x}{1+x^2}\,dx\\ &= -\frac{x}{2(1+x^2)}+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= -\frac{x}{2(1+x^2)}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.79

$$\frac{1}{2}\left(-\frac{x}{1+x^2}+\tan^{-1}(x)+\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + x + x^2))/(1 + x^2)^2,x]
[Out] (-(x/(1 + x^2)) + ArcTan[x] + Log[1 + x^2])/2
```

Maple [A]
time = 0.08, size = 24, normalized size = 0.83

method	result	size
default	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}+\frac{\ln(x^2+1)}{2}$	24
meijerg	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}+\frac{\ln(x^2+1)}{2}$	24
risch	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}+\frac{\ln(x^2+1)}{2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)+1/2*ln(x^2+1)
```

Maxima [A]
time = 0.51, size = 23, normalized size = 0.79

$$-\frac{x}{2\,(x^2+1)}+\frac{1}{2}\,\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

Fricas [A]
time = 0.37, size = 33, normalized size = 1.14

$$\frac{(x^2+1)\arctan(x)+(x^2+1)\log(x^2+1)-x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - x)/(x^2 + 1)
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.69

$$-\frac{x}{2x^2+2}+\frac{\log(x^2+1)}{2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+x+1)/(x**2+1)**2,x)
[Out] -x/(2*x**2 + 2) + log(x**2 + 1)/2 + atan(x)/2
```

Giac [A]
time = 4.28, size = 23, normalized size = 0.79

$$-\frac{x}{2(x^2+1)}+\frac{1}{2}\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.86

$$\frac{\ln(x^2+1)}{2}+\frac{\operatorname{atan}(x)}{2}-\frac{x}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + x^2 + 1))/(x^2 + 1)^2,x)
[Out] log(x^2 + 1)/2 + atan(x)/2 - x/(2*(x^2 + 1))
```

3.72 $\int \frac{1+x+x^2}{(1+x^2)^2}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2(1+x^2)}+\tan^{-1}(x)$$

```
[Out] -1/2/(x^2+1)+arctan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1828, 12, 209}

$$\text{ArcTan}(x)-\frac{1}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(1 + x^2)^2,x]
```

```
[Out] -1/2*1/(1 + x^2) + ArcTan[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1+x+x^2}{\left(1+x^2\right)^2}\,dx = -\frac{1}{2\left(1+x^2\right)} - \frac{1}{2}\int -\frac{2}{1+x^2}\,dx$$
$$= -\frac{1}{2\left(1+x^2\right)} + \int \frac{1}{1+x^2}\,dx$$
$$= -\frac{1}{2\left(1+x^2\right)} + \tan^{-1}(x)$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{1}{2\left(1+x^2\right)} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(1 + x^2)^2,x]
[Out] -1/2*1/(1 + x^2) + ArcTan[x]
```

Maple [A]
time = 0.08, size = 13, normalized size = 0.93

method	result	size
default	$-\frac{1}{2(x^2+1)} + \arctan(x)$	13
risch	$-\frac{1}{2(x^2+1)} + \arctan(x)$	13
meijerg	$-\frac{x}{2(x^2+1)} + \arctan(x) + \frac{x^2}{2x^2+2} + \frac{x}{2x^2+2}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] -1/2/(x^2+1)+arctan(x)
```

Maxima [A]
time = 0.53, size = 12, normalized size = 0.86

$$-\frac{1}{2\left(x^2+1\right)} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2/(x^2 + 1) + arctan(x)
```

Fricas [A]
time = 0.34, size = 20, normalized size = 1.43

$$\frac{2\,(x^2+1)\arctan(x)-1}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*(2*(x^2 + 1)*arctan(x) - 1)/(x^2 + 1)
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.71

$$\operatorname{atan}(x)-\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(x**2+1)**2,x)
[Out] atan(x) - 1/(2*x**2 + 2)
```

Giac [A]
time = 4.86, size = 12, normalized size = 0.86

$$-\frac{1}{2\,(x^2+1)}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2/(x^2 + 1) + arctan(x)
```

Mupad [B]
time = 3.80, size = 14, normalized size = 1.00

$$\operatorname{atan}(x)-\frac{1}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^2 + 1)^2,x)
[Out] atan(x) - 1/(2*(x^2 + 1))
```

3.73 $\int \frac{1+x+x^2}{x(1+x^2)^2} dx$

Optimal. Leaf size=31

$$\frac{x}{2(1+x^2)} + \frac{1}{2}\tan^{-1}(x) + \log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] 1/2*x/(x^2+1)+1/2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1819, 815, 649, 209, 266}

$$\frac{\text{ArcTan}(x)}{2} + \frac{x}{2(x^2+1)} - \frac{1}{2}\log\left(x^2+1\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x*(1 + x^2)^2),x]
```

```
[Out] x/(2*(1 + x^2)) + ArcTan[x]/2 + Log[x] - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx&=\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-2-x}{x\left(1+x^2\right)}\,dx\\&=\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \left(-\frac{2}{x}+\frac{-1+2x}{1+x^2}\right)dx\\&=\frac{x}{2\left(1+x^2\right)}+\log(x)-\frac{1}{2}\int \frac{-1+2x}{1+x^2}\,dx\\&=\frac{x}{2\left(1+x^2\right)}+\log(x)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\&=\frac{x}{2\left(1+x^2\right)}+\frac{1}{2}\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.90

$$\frac{1}{2}\left(\frac{x}{1+x^2}+\tan^{-1}(x)+2\log(x)-\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x*(1 + x^2)^2),x]
[Out] (x/(1 + x^2) + ArcTan[x] + 2*Log[x] - Log[1 + x^2])/2
```

Maple [A]
time = 0.10, size = 26, normalized size = 0.84

method	result	size
default	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}+\ln(x)-\frac{\ln(x^2+1)}{2}$	26
risch	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}+\ln(x)-\frac{\ln(x^2+1)}{2}$	26
meijerg	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}-\frac{\ln(x^2+1)}{2}+\frac{1}{2}+\ln(x)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/(x^2+1)+1/2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Maxima [A]
time = 0.54, size = 25, normalized size = 0.81

$$\frac{x}{2\,(x^2+1)}+\frac{1}{2}\,\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^2+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Fricas [A]
time = 0.37, size = 41, normalized size = 1.32

$$\frac{(x^2+1)\arctan\left(x\right)-(x^2+1)\log\left(x^2+1\right)+2\,(x^2+1)\log\left(x\right)+x}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + 2*(x^2 + 1)*log(x) + x)
/(x^2 + 1)
```

Sympy [A]
time = 0.05, size = 24, normalized size = 0.77

$$\frac{x}{2x^2+2}+\log\left(x\right)-\frac{\log\left(x^2+1\right)}{2}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x/(x**2+1)**2,x)
```

```
[Out] x/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 + atan(x)/2
```

Giac [A]
time = 4.18, size = 26, normalized size = 0.84

$$\frac{x}{2\,(x^2+1)}+\frac{1}{2}\,\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^2+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

Mupad [B]
time = 0.04, size = 32, normalized size = 1.03

$$\ln(x) + \frac{x}{2(x^2+1)} + \ln(x-\mathrm{i})\left(-\frac{1}{2} - \frac{1}{4}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{2} + \frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x*(x^2 + 1)^2),x)
```

```
[Out] log(x) - log(x + 1i)*(1/2 - 1i/4) - log(x - 1i)*(1/2 + 1i/4) + x/(2*(x^2 +
1))
```

3.74 $\int \frac{1+x+x^2}{x^2(1+x^2)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{x}+\frac{1}{2(1+x^2)}-\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -1/x+1/2/(x^2+1)-arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1819, 815, 649, 209, 266}

$$-\text{ArcTan}(x)+\frac{1}{2(x^2+1)}-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x^2*(1 + x^2)^2),x]
```

```
[Out] -x^(-1) + 1/(2*(1 + x^2)) - ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx &= \frac{1}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-2-2x}{x^2\left(1+x^2\right)}\,dx\\
&= \frac{1}{2\left(1+x^2\right)}-\frac{1}{2}\int \left(-\frac{2}{x^2}-\frac{2}{x}+\frac{2(1+x)}{1+x^2}\right)dx\\
&= -\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}+\log(x)-\int \frac{1+x}{1+x^2}\,dx\\
&= -\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}+\log(x)-\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= -\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}-\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$-\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}-\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x^2*(1 + x^2)^2),x]
[Out] -x^(-1) + 1/(2*(1 + x^2)) - ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Maple [A]
time = 0.08, size = 30, normalized size = 0.91

method	result	size
default	$-\frac{1}{x}+\frac{1}{2x^2+2}-\arctan\left(x\right)+\ln\left(x\right)-\frac{\ln\left(x^2+1\right)}{2}$	30
risch	$\frac{-x^2+\frac{1}{2}x-1}{x\left(x^2+1\right)}-\frac{\ln\left(x^2+1\right)}{2}-\arctan\left(x\right)+\ln\left(x\right)$	37
meijerg	$\frac{x}{2x^2+2}-\arctan\left(x\right)-\frac{x^2}{2x^2+2}-\frac{\ln\left(x^2+1\right)}{2}+\frac{1}{2}+\ln\left(x\right)-\frac{3x^2+2}{x\left(2x^2+2\right)}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x^2/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x+1/2/(x^2+1)-arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Maxima [A]
time = 0.50, size = 34, normalized size = 1.03

$$-\frac{2\,x^2-x+2}{2\,(x^3+x)}-\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^2+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*x^2 - x + 2)/(x^3 + x) - arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Fricas [A]
time = 0.36, size = 49, normalized size = 1.48

$$-\frac{2\,x^2+2\,(x^3+x)\arctan\left(x\right)+(x^3+x)\log\left(x^2+1\right)-2\,(x^3+x)\log\left(x\right)-x+2}{2\,(x^3+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^2 + 2*(x^3 + x)*arctan(x) + (x^3 + x)*log(x^2 + 1) - 2*(x^3 + x)*
log(x) - x + 2)/(x^3 + x)
```

Sympy [A]
time = 0.05, size = 31, normalized size = 0.94

$$\log\left(x\right)-\frac{\log\left(x^2+1\right)}{2}-\operatorname{atan}\left(x\right)+\frac{-2x^2+x-2}{2x^3+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**2/(x**2+1)**2,x)
```

```
[Out] log(x) - log(x**2 + 1)/2 - atan(x) + (-2*x**2 + x - 2)/(2*x**3 + 2*x)
```

Giac [A]
time = 3.68, size = 35, normalized size = 1.06

$$-\frac{2\,x^2-x+2}{2\,(x^3+x)}-\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^2+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*x^2 - x + 2)/(x^3 + x) - arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

Mupad [B]
time = 3.81, size = 38, normalized size = 1.15

$$\ln(x) - \frac{x^2 - \frac{x}{2} + 1}{x^3 + x} + \ln(x - \mathrm{i})\left(-\frac{1}{2} + \frac{1}{2}\mathrm{i}\right) + \ln(x + 1\mathrm{i})\left(-\frac{1}{2} - \frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^2*(x^2 + 1)^2),x)
```

```
[Out] log(x) - log(x + 1i)*(1/2 + 1i/2) - log(x - 1i)*(1/2 - 1i/2) - (x^2 - x/2 +
 1)/(x + x^3)
```

3.75 $\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{2x^2}-\frac{1}{x}-\frac{x}{2\left(1+x^2\right)}-\frac{3}{2}\tan^{-1}(x)-\log(x)+\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -1/2/x^2-1/x-1/2*x/(x^2+1)-3/2*arctan(x)-ln(x)+1/2*ln(x^2+1)
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1819, 1816, 649, 209, 266}

$$-\frac{3\text{ArcTan}(x)}{2}-\frac{x}{2\left(x^2+1\right)}-\frac{1}{2x^2}+\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x^3*(1 + x^2)^2),x]
```

```
[Out] -1/2*1/x^2 - x^(-1) - x/(2*(1 + x^2)) - (3*ArcTan[x])/2 - Log[x] + Log[1 +
x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2}{x^3(1+x^2)^2}\,dx &= -\frac{x}{2(1+x^2)} - \frac{1}{2}\int \frac{-2-2x+x^3}{x^3(1+x^2)}\,dx \\
&= -\frac{x}{2(1+x^2)} - \frac{1}{2}\int \left(-\frac{2}{x^3} - \frac{2}{x^2} + \frac{2}{x} + \frac{3-2x}{1+x^2}\right)dx \\
&= -\frac{1}{2x^2} - \frac{1}{x} - \frac{x}{2(1+x^2)} - \log(x) - \frac{1}{2}\int \frac{3-2x}{1+x^2}\,dx \\
&= -\frac{1}{2x^2} - \frac{1}{x} - \frac{x}{2(1+x^2)} - \log(x) - \frac{3}{2}\int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx \\
&= -\frac{1}{2x^2} - \frac{1}{x} - \frac{x}{2(1+x^2)} - \frac{3}{2}\tan^{-1}(x) - \log(x) + \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.87

$$\frac{1}{2}\left(-\frac{1}{x^2} - \frac{2}{x} - \frac{x}{1+x^2} - 3\tan^{-1}(x) - 2\log(x) + \log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x^3*(1 + x^2)^2),x]
[Out] (-x^(-2) - 2/x - x/(1 + x^2) - 3*ArcTan[x] - 2*Log[x] + Log[1 + x^2])/2
```

Maple [A]
time = 0.09, size = 38, normalized size = 0.84

method	result	size
default	$-\frac{1}{2x^2}-\frac{1}{x}-\frac{x}{2(x^2+1)}-\frac{3\arctan(x)}{2}-\ln(x)+\frac{\ln(x^2+1)}{2}$	38
risch	$\frac{-\frac{3}{2}x^3-\frac{1}{2}x^2-x-\frac{1}{2}}{x^2(x^2+1)}+\frac{\ln(x^2+1)}{2}-\frac{3\arctan(x)}{2}-\ln(x)$	44

meijerg	$-\frac{x^2}{2x^2+2}+\frac{\ln(x^2+1)}{2}-\ln(x)-\frac{3x^2+2}{x(2x^2+2)}-\frac{3\arctan(x)}{2}+\frac{3x^2}{2(3x^2+3)}-\frac{1}{2x^2}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x^3/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2-1/x-1/2*x/(x^2+1)-3/2*arctan(x)-ln(x)+1/2*ln(x^2+1)
```

Maxima [A]
time = 0.50, size = 41, normalized size = 0.91

$$-\frac{3\,x^3+x^2+2\,x+1}{2\,(x^4+x^2)}-\frac{3}{2}\,\arctan(x)+\frac{1}{2}\,\log\left(x^2+1\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*x^3 + x^2 + 2*x + 1)/(x^4 + x^2) - 3/2*arctan(x) + 1/2*log(x^2 + 1)
 - log(x)
```

Fricas [A]
time = 0.37, size = 61, normalized size = 1.36

$$-\frac{3\,x^3+x^2+3\,(x^4+x^2)\arctan(x)-(x^4+x^2)\log(x^2+1)+2\,(x^4+x^2)\log(x)+2\,x+1}{2\,(x^4+x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(3*x^3 + x^2 + 3*(x^4 + x^2)*arctan(x) - (x^4 + x^2)*log(x^2 + 1) + 2*
(x^4 + x^2)*log(x) + 2*x + 1)/(x^4 + x^2)
```

Sympy [A]
time = 0.07, size = 42, normalized size = 0.93

$$-\log(x)+\frac{\log(x^2+1)}{2}-\frac{3\operatorname{atan}(x)}{2}+\frac{-3x^3-x^2-2x-1}{2x^4+2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**3/(x**2+1)**2,x)
```

```
[Out] -log(x) + log(x**2 + 1)/2 - 3*atan(x)/2 + (-3*x**3 - x**2 - 2*x - 1)/(2*x**
4 + 2*x**2)
```

Giac [A]
time = 4.11, size = 43, normalized size = 0.96

$$-\frac{3\,x^3+x^2+2\,x+1}{2\,(x^2+1)x^2}-\frac{3}{2}\,\arctan(x)+\frac{1}{2}\,\log\left(x^2+1\right)-\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*x^3 + x^2 + 2*x + 1)/((x^2 + 1)*x^2) - 3/2*arctan(x) + 1/2*log(x^2
+ 1) - log(abs(x))
```

Mupad [B]
time = 0.04, size = 47, normalized size = 1.04

$$-\ln(x)-\frac{\frac{3x^3}{2}+\frac{x^2}{2}+x+\frac{1}{2}}{x^4+x^2}+\ln(x-\mathrm{i})\left(\frac{1}{2}+\frac{3}{4}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{1}{2}-\frac{3}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^3*(x^2 + 1)^2),x)
```

```
[Out] log(x - 1i)*(1/2 + 3i/4) + log(x + 1i)*(1/2 - 3i/4) - log(x) - (x + x^2/2 +
 (3*x^3)/2 + 1/2)/(x^2 + x^4)
```

3.76 $\int \frac{1+2x+x^2}{\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=12

$$-\frac{1}{1+x^2}+\tan^{-1}(x)$$

[Out] -1/(x^2+1)+arctan(x)

Rubi [A]

time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.83, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {27, 737, 209}

$$\text{ArcTan}(x)-\frac{(1-x)(x+1)}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x + x^2)/(1 + x^2)^2,x]

[Out] -1/2*((1 - x)*(1 + x))/(1 + x^2) + ArcTan[x]

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1+2x+x^2}{(1+x^2)^2}\,dx = \int \frac{(1+x)^2}{(1+x^2)^2}\,dx$$
$$= -\frac{(1-x)(1+x)}{2\,(1+x^2)} + \int \frac{1}{1+x^2}\,dx$$
$$= -\frac{(1-x)(1+x)}{2\,(1+x^2)} + \tan^{-1}(x)$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$-\frac{1}{1+x^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x + x^2)/(1 + x^2)^2,x]
[Out] -(1 + x^2)^(-1) + ArcTan[x]
```

Maple [A]
time = 0.08, size = 13, normalized size = 1.08

method	result	size
default	$-\frac{1}{x^2+1}+\arctan(x)$	13
risch	$-\frac{1}{x^2+1}+\arctan(x)$	13
meijerg	$-\frac{x}{2(x^2+1)}+\arctan(x)+\frac{x^2}{x^2+1}+\frac{x}{2x^2+2}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] -1/(x^2+1)+arctan(x)
```

Maxima [A]
time = 0.55, size = 12, normalized size = 1.00

$$-\frac{1}{x^2+1} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/(x^2 + 1) + arctan(x)
```

Fricas [A]
time = 0.35, size = 18, normalized size = 1.50

$$\frac{(x^2+1)\arctan(x)-1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] ((x^2 + 1)*arctan(x) - 1)/(x^2 + 1)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.67

$$\operatorname{atan}(x)-\frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x+1)/(x**2+1)**2,x)
[Out] atan(x) - 1/(x**2 + 1)
```

Giac [A]
time = 4.81, size = 12, normalized size = 1.00

$$-\frac{1}{x^2+1}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/(x^2 + 1) + arctan(x)
```

Mupad [B]
time = 0.03, size = 12, normalized size = 1.00

$$\mathrm{atan}(x)-\frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 + 1)/(x^2 + 1)^2,x)
[Out] atan(x) - 1/(x^2 + 1)
```

3.77 $\int \frac{2+12x+3x^2}{(4+x^2)^2} \, dx$

Optimal. Leaf size=27

$$-\frac{24+5x}{4\,(4+x^2)}+\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] 1/4*(-24-5*x)/(x^2+4)+7/8*arctan(1/2*x)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1828, 12, 209}

$$\frac{7}{8}\text{ArcTan}\left(\frac{x}{2}\right)-\frac{5x+24}{4\,(x^2+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 12*x + 3*x^2)/(4 + x^2)^2,x]
```

```
[Out] -1/4*(24 + 5*x)/(4 + x^2) + (7*ArcTan[x/2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx &= -\frac{24+5x}{4(4+x^2)} - \frac{1}{8}\int -\frac{14}{4+x^2}\,dx\\ &= -\frac{24+5x}{4(4+x^2)} + \frac{7}{4}\int \frac{1}{4+x^2}\,dx\\ &= -\frac{24+5x}{4(4+x^2)} + \frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.00

$$\frac{-24-5x}{4(4+x^2)} + \frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 12*x + 3*x^2)/(4 + x^2)^2,x]
```

```
[Out] (-24 - 5*x)/(4*(4 + x^2)) + (7*ArcTan[x/2])/8
```

Maple [A]
time = 0.08, size = 21, normalized size = 0.78

method	result	size
default	$\frac{-\frac{5x}{4}-6}{x^2+4}+\frac{7\arctan\left(\frac{x}{2}\right)}{8}$	21
risch	$\frac{-\frac{5x}{4}-6}{x^2+4}+\frac{7\arctan\left(\frac{x}{2}\right)}{8}$	21
meijerg	$\frac{x}{4x^2+16}+\frac{7\arctan\left(\frac{x}{2}\right)}{8}-\frac{3x}{8\left(\frac{x^2}{4}+1\right)}+\frac{3x^2}{8\left(\frac{x^2}{4}+1\right)}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+12*x+2)/(x^2+4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-5/4*x-6)/(x^2+4)+7/8*arctan(1/2*x)
```

Maxima [A]
time = 0.49, size = 21, normalized size = 0.78

$$-\frac{5\,x+24}{4\,(x^2+4)} + \frac{7}{8}\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(5*x + 24)/(x^2 + 4) + 7/8*arctan(1/2*x)
```

Fricas [A]
time = 0.34, size = 25, normalized size = 0.93

$$\frac{7\left(x^2+4\right)\arctan\left(\frac{1}{2}\,x\right)-10\,x-48}{8\left(x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(7*(x^2 + 4)*arctan(1/2*x) - 10*x - 48)/(x^2 + 4)
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.74

$$\frac{-5x-24}{4x^2+16}+\frac{7\,\mathrm{atan}\left(\frac{x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+12*x+2)/(x**2+4)**2,x)
```

```
[Out] (-5*x - 24)/(4*x**2 + 16) + 7*atan(x/2)/8
```

Giac [A]
time = 3.29, size = 21, normalized size = 0.78

$$-\frac{5\,x+24}{4\left(x^2+4\right)}+\frac{7}{8}\,\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(5*x + 24)/(x^2 + 4) + 7/8*arctan(1/2*x)
```

Mupad [B]
time = 3.83, size = 21, normalized size = 0.78

$$\frac{7\,\mathrm{atan}\left(\frac{x}{2}\right)}{8}-\frac{\frac{5\,x}{4}+6}{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x + 3*x^2 + 2)/(x^2 + 4)^2,x)
```

```
[Out] (7*atan(x/2))/8 - ((5*x)/4 + 6)/(x^2 + 4)
```

3.78 $\int (g+hx)^3\sqrt{a+cx^2}\,(d+ex+fx^2)\,dx$

Optimal. Leaf size=390

$$\frac{(8c^2dg^3+a^2h^2(3fg+eh)-2acg(fg^2+3h(eg+dh)))\,x\sqrt{a+cx^2}}{16c^2}-\frac{(8afh^2+c(3fg^2-7h(eg+2dh)))\,(g}{70c^2h}$$

```
[Out] -1/70*(8*a*f*h^2+c*(3*f*g^2-7*h*(2*d*h+e*g)))*(h*x+g)^2*(c*x^2+a)^(3/2)/c^2
/h-1/42*(-7*e*h+3*f*g)*(h*x+g)^3*(c*x^2+a)^(3/2)/c/h+1/7*f*(h*x+g)^4*(c*x^2
+a)^(3/2)/c/h+1/840*(64*a^2*f*h^4-16*a*c*h^2*(15*f*g^2+7*h*(d*h+3*e*g))-8*c
^2*g^2*(3*f*g^2-7*h*(12*d*h+e*g))-3*c*h*(a*h^2*(35*e*h+41*f*g)+2*c*g*(3*f*g
^2-7*h*(7*d*h+e*g)))*x)*(c*x^2+a)^(3/2)/c^3/h+1/16*a*(8*c^2*d*g^3+a^2*h^2*(
e*h+3*f*g)-2*a*c*g*(f*g^2+3*h*(d*h+e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2)
)/c^(5/2)+1/16*(8*c^2*d*g^3+a^2*h^2*(e*h+3*f*g)-2*a*c*g*(f*g^2+3*h*(d*h+e*g
)))*x*(c*x^2+a)^(1/2)/c^2
```

Rubi [A]
time = 0.49, antiderivative size = 387, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$,
Rules used = {1668, 847, 794, 201, 223, 212}

$$\frac{x\sqrt{a+cx^2}\,(a^2h^2(eh+3fg)-2acg(3h(dh+eg)+fg^2)+8c^2dg^3)}{16c^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(a^2h^2(eh+3fg)-2acg(3h(dh+eg)+fg^2)+8c^2dg^3)}{16c^{5/2}}+\frac{(a+cx^2)^{3/2}\,(8(8a^2fh^4-2ach^2(7h(dh+3eg)+15fg^2)-c^2(3fg^4-7g^2h(12dh+eg)))-3chx(ah^2(35eh+41fg)-14cgh(7dh+eg)+6cfg^3))}{840c^3h}-\frac{(a+cx^2)^{3/2}\,(g+hx)^2\,(8afh^2-7ch(2dh+eg)+3cfg^2)}{70c^2h}-\frac{(a+cx^2)^{3/2}\,(g+hx)^3(3fg-7eh)}{42ch}+\frac{f(a+cx^2)^{3/2}\,(g+hx)^4}{7ch}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((8*c^2*d*g^3 + a^2*h^2*(3*f*g + e*h) - 2*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*
x*Sqrt[a + c*x^2])/(16*c^2) - ((3*c*f*g^2 + 8*a*f*h^2 - 7*c*h*(e*g + 2*d*h)
)*(g + h*x)^2*(a + c*x^2)^(3/2))/(70*c^2*h) - ((3*f*g - 7*e*h)*(g + h*x)^3*
(a + c*x^2)^(3/2))/(42*c*h) + (f*(g + h*x)^4*(a + c*x^2)^(3/2))/(7*c*h) + (
(8*(8*a^2*f*h^4 - 2*a*c*h^2*(15*f*g^2 + 7*h*(3*e*g + d*h)) - c^2*(3*f*g^4 -
 7*g^2*h*(e*g + 12*d*h))) - 3*c*h*(6*c*f*g^3 - 14*c*g*h*(e*g + 7*d*h) + a*h
^2*(41*f*g + 35*e*h))*x)*(a + c*x^2)^(3/2))/(840*c^3*h) + (a*(8*c^2*d*g^3 +
 a^2*h^2*(3*f*g + e*h) - 2*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c
]*x)/Sqrt[a + c*x^2]])/(16*c^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{f(g+hx)^4\left(a+cx^2\right)^{3/2}}{7ch} + \frac{\int (g+hx)^3\left((7cd-4af)h^2-ch(3f\right.}{7ch^2}$$

$$= -\frac{(3fg-7eh)(g+hx)^3\left(a+cx^2\right)^{3/2}}{42ch} + \frac{f(g+hx)^4\left(a+cx^2\right)^{3/2}}{7ch}$$

$$= -\frac{\left(3cfg^2+8afh^2-7ch(eg+2dh)\right)(g+hx)^2\left(a+cx^2\right)^{3/2}}{70c^2h} - \Big($$

$$= -\frac{\left(3cfg^2+8afh^2-7ch(eg+2dh)\right)(g+hx)^2\left(a+cx^2\right)^{3/2}}{70c^2h} - \Big($$

$$= \frac{\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+}}{16c^2}$$

$$= \frac{\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+}}{16c^2}$$

$$= \frac{\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+}}{16c^2}$$

Mathematica [A]
time = 1.02, size = 358, normalized size = 0.92

$$\frac{\sqrt{a+cx^2}\left(128a^3fh^3-a^2ch\left(7h(96eg+32dh+15ehx)+f\left(672g^2+315ghx+64h^2x^2\right)\right)+2ac^2\left(7dh\left(120g^2+45ghx+8h^2x^2\right)+7e\left(40g^3+45g^2hx+24gh^2x^2+5h^3x^3\right)+3fx\left(35g^3+56g^2hx+35gh^2x^2+8h^3x^3\right)\right)+4c^3x\left(21d\left(10g^3+20g^2hx+15gh^2x^2+4h^3x^3\right)+x\left(7e\left(20g^3+45g^2hx+36gh^2x^2+10h^3x^3\right)+3fx\left(35g^3+84g^2hx+70gh^2x^2+20h^3x^3\right)\right)\right)\right)-105a\sqrt{c}\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg\left(fg^2+3h(eg+dh)\right)\right)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{1680c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(128*a^3*f*h^3 - a^2*c*h*(7*h*(96*e*g + 32*d*h + 15*e*h*x)
 + f*(672*g^2 + 315*g*h*x + 64*h^2*x^2)) + 2*a*c^2*(7*d*h*(120*g^2 + 45*g*h
*x + 8*h^2*x^2) + 7*e*(40*g^3 + 45*g^2*h*x + 24*g*h^2*x^2 + 5*h^3*x^3) + 3*
f*x*(35*g^3 + 56*g^2*h*x + 35*g*h^2*x^2 + 8*h^3*x^3)) + 4*c^3*x*(21*d*(10*g
^3 + 20*g^2*h*x + 15*g*h^2*x^2 + 4*h^3*x^3) + x*(7*e*(20*g^3 + 45*g^2*h*x +
 36*g*h^2*x^2 + 10*h^3*x^3) + 3*f*x*(35*g^3 + 84*g^2*h*x + 70*g*h^2*x^2 + 2
0*h^3*x^3)))) - 105*a*Sqrt[c]*(8*c^2*d*g^3 + a^2*h^2*(3*f*g + e*h) - 2*a*c*
g*(f*g^2 + 3*h*(e*g + d*h)))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(1680*c^3
)
```

Maple [A]
time = 0.10, size = 358, normalized size = 0.92

method	result

default	$f h^3\left(\frac{x^4(cx^2+a)^{\frac{3}{2}}}{7c}-\frac{4a\left(\frac{x^2(cx^2+a)^{\frac{3}{2}}}{5c}-\frac{2a(cx^2+a)^{\frac{3}{2}}}{15c^2}\right)}{7c}\right)+(e h^3+3fg h^2)\left(\frac{x^3(cx^2+a)^{\frac{3}{2}}}{6c}-\frac{a\left(\frac{x(cx^2+a)^{\frac{3}{2}}}{4c}-\frac{a\left(x \ldots\right.}{\ldots}\right.}{\ldots}\right.$
risch	$\frac{(240f h^3c^3x^6+280c^3e h^3x^5+840c^3fg h^2x^5+48a c^2f h^3x^4+336c^3d h^3x^4+1008c^3eg h^2x^4+1008c^3f g^2h x^4+70a c^2e h^3x^3+210a c^2fg h^2 \ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*h^3*(1/7*x^4*(c*x^2+a)^(3/2)/c-4/7*a/c*(1/5*x^2*(c*x^2+a)^(3/2)/c-2/15*a/
c^2*(c*x^2+a)^(3/2)))+(e*h^3+3*f*g*h^2)*(1/6*x^3*(c*x^2+a)^(3/2)/c-1/2*a/c*
(1/4*x*(c*x^2+a)^(3/2)/c-1/4*a/c*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*
c^(1/2)+(c*x^2+a)^(1/2)))))+(d*h^3+3*e*g*h^2+3*f*g^2*h)*(1/5*x^2*(c*x^2+a)^
(3/2)/c-2/15*a/c^2*(c*x^2+a)^(3/2))+(3*d*g*h^2+3*e*g^2*h+f*g^3)*(1/4*x*(c*x
^2+a)^(3/2)/c-1/4*a/c*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*
x^2+a)^(1/2))))+1/3*(3*d*g^2*h+e*g^3)*(c*x^2+a)^(3/2)/c+d*g^3*(1/2*x*(c*x^2
+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))
```

Maxima [A]

time = 0.28, size = 446, normalized size = 1.14

$$\frac{(cx^2+a)^{\frac{3}{2}}fh^3x^4}{7c}-\frac{4(cx^2+a)^{\frac{3}{2}}afh^3x^2}{35c^2}+\frac{1}{2}\sqrt{cx^2+a}\,dg^3x+\frac{adg^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}}+\frac{(cx^2+a)^{\frac{3}{2}}dg^2h}{c}+\frac{8(cx^2+a)^{\frac{3}{2}}a^2fh^3}{105c^3}+\frac{(3fgh^2+h^3e)(cx^2+a)^{\frac{3}{2}}x^3}{6c}+\frac{(cx^2+a)^{\frac{3}{2}}g^3e}{3c}+\frac{(3fg^2h+dh^3+3gh^2e)(cx^2+a)^{\frac{3}{2}}x^2}{5c}-\frac{(3fgh^2+h^3e)(cx^2+a)^{\frac{3}{2}}ax}{8c^2}+\frac{(3fgh^2+h^3e)\sqrt{cx^2+a}\,a^2x}{16c^2}+\frac{(fg^3+3dgh^2+3g^2he)(cx^2+a)^{\frac{3}{2}}x}{4c}-\frac{(fg^3+3dgh^2+3g^2he)\sqrt{cx^2+a}\,ax}{8c}+\frac{(3fgh^2+h^3e)a^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16c^{\frac{5}{2}}}-\frac{(fg^3+3dgh^2+3g^2he)a^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8c^{\frac{3}{2}}}-\frac{2(3fg^2h+dh^3+3gh^2e)(cx^2+a)^{\frac{3}{2}}a}{15c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(c*x^2 + a)^(3/2)*f*h^3*x^4/c - 4/35*(c*x^2 + a)^(3/2)*a*f*h^3*x^2/c^2
+ 1/2*sqrt(c*x^2 + a)*d*g^3*x + 1/2*a*d*g^3*arcsinh(c*x/sqrt(a*c))/sqrt(c)
+ (c*x^2 + a)^(3/2)*d*g^2*h/c + 8/105*(c*x^2 + a)^(3/2)*a^2*f*h^3/c^3 + 1/6
*(3*f*g*h^2 + h^3*e)*(c*x^2 + a)^(3/2)*x^3/c + 1/3*(c*x^2 + a)^(3/2)*g^3*e/
c + 1/5*(3*f*g^2*h + d*h^3 + 3*g*h^2*e)*(c*x^2 + a)^(3/2)*x^2/c - 1/8*(3*f*
g*h^2 + h^3*e)*(c*x^2 + a)^(3/2)*a*x/c^2 + 1/16*(3*f*g*h^2 + h^3*e)*sqrt(c*
x^2 + a)*a^2*x/c^2 + 1/4*(f*g^3 + 3*d*g*h^2 + 3*g^2*h*e)*(c*x^2 + a)^(3/2)*
x/c - 1/8*(f*g^3 + 3*d*g*h^2 + 3*g^2*h*e)*sqrt(c*x^2 + a)*a*x/c + 1/16*(3*f
*g*h^2 + h^3*e)*a^3*arcsinh(c*x/sqrt(a*c))/c^(5/2) - 1/8*(f*g^3 + 3*d*g*h^2
```

```
 + 3*g^2*h*e)*a^2*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/15*(3*f*g^2*h + d*h^3
+ 3*g*h^2*e)*(c*x^2 + a)^(3/2)*a/c^2
```

Fricas [A]
time = 0.58, size = 890, normalized size = 2.28

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3360*(105*(2*(4*a*c^2*d - a^2*c*f)*g^3 - 3*(2*a^2*c*d - a^3*f)*g*h^2 - (
6*a^2*c*g^2*h - a^3*h^3)*e)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c
)*x - a) + 2*(240*c^3*f*h^3*x^6 + 840*c^3*f*g*h^2*x^5 + 48*(21*c^3*f*g^2*h
+ (7*c^3*d + a*c^2*f)*h^3)*x^4 + 336*(5*a*c^2*d - 2*a^2*c*f)*g^2*h - 32*(7*
a^2*c*d - 4*a^3*f)*h^3 + 210*(2*c^3*f*g^3 + (6*c^3*d + a*c^2*f)*g*h^2)*x^3
+ 16*(21*(5*c^3*d + a*c^2*f)*g^2*h + (7*a*c^2*d - 4*a^2*c*f)*h^3)*x^2 + 105
*(2*(4*c^3*d + a*c^2*f)*g^3 + 3*(2*a*c^2*d - a^2*c*f)*g*h^2)*x + 7*(40*c^3*
h^3*x^5 + 144*c^3*g*h^2*x^4 + 80*a*c^2*g^3 - 96*a^2*c*g*h^2 + 10*(18*c^3*g^
2*h + a*c^2*h^3)*x^3 + 16*(5*c^3*g^3 + 3*a*c^2*g*h^2)*x^2 + 15*(6*a*c^2*g^2
*h - a^2*c*h^3)*x)*e)*sqrt(c*x^2 + a))/c^3, -1/1680*(105*(2*(4*a*c^2*d - a^
2*c*f)*g^3 - 3*(2*a^2*c*d - a^3*f)*g*h^2 - (6*a^2*c*g^2*h - a^3*h^3)*e)*sqr
t(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (240*c^3*f*h^3*x^6 + 840*c^3*f*g
*h^2*x^5 + 48*(21*c^3*f*g^2*h + (7*c^3*d + a*c^2*f)*h^3)*x^4 + 336*(5*a*c^2
*d - 2*a^2*c*f)*g^2*h - 32*(7*a^2*c*d - 4*a^3*f)*h^3 + 210*(2*c^3*f*g^3 + (
6*c^3*d + a*c^2*f)*g*h^2)*x^3 + 16*(21*(5*c^3*d + a*c^2*f)*g^2*h + (7*a*c^2
*d - 4*a^2*c*f)*h^3)*x^2 + 105*(2*(4*c^3*d + a*c^2*f)*g^3 + 3*(2*a*c^2*d -
a^2*c*f)*g*h^2)*x + 7*(40*c^3*h^3*x^5 + 144*c^3*g*h^2*x^4 + 80*a*c^2*g^3 -
96*a^2*c*g*h^2 + 10*(18*c^3*g^2*h + a*c^2*h^3)*x^3 + 16*(5*c^3*g^3 + 3*a*c^
2*g*h^2)*x^2 + 15*(6*a*c^2*g^2*h - a^2*c*h^3)*x)*e)*sqrt(c*x^2 + a))/c^3]
```

Sympy [A]
time = 17.99, size = 1088, normalized size = 2.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] -a**(5/2)*e*h**3*x/(16*c**2*sqrt(1 + c*x**2/a)) - 3*a**(5/2)*f*g*h**2*x/(16
*c**2*sqrt(1 + c*x**2/a)) + 3*a**(3/2)*d*g*h**2*x/(8*c*sqrt(1 + c*x**2/a))
+ 3*a**(3/2)*e*g**2*h*x/(8*c*sqrt(1 + c*x**2/a)) - a**(3/2)*e*h**3*x**3/(48
*c*sqrt(1 + c*x**2/a)) + a**(3/2)*f*g**3*x/(8*c*sqrt(1 + c*x**2/a)) - a**(3
/2)*f*g*h**2*x**3/(16*c*sqrt(1 + c*x**2/a)) + sqrt(a)*d*g**3*x*sqrt(1 + c*x
**2/a)/2 + 9*sqrt(a)*d*g*h**2*x**3/(8*sqrt(1 + c*x**2/a)) + 9*sqrt(a)*e*g**
2*h*x**3/(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*e*h**3*x**5/(24*sqrt(1 + c*x**2
```

```
/a)) + 3*sqrt(a)*f*g**3*x**3/(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*f*g*h**2*x*
*5/(8*sqrt(1 + c*x**2/a)) + a**3*e*h**3*asinh(sqrt(c)*x/sqrt(a))/(16*c**(5/
2)) + 3*a**3*f*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(5/2)) - 3*a**2*d*g*h
**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) - 3*a**2*e*g**2*h*asinh(sqrt(c)*x
/sqrt(a))/(8*c**(3/2)) - a**2*f*g**3*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2))
+ a*d*g**3*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + 3*d*g**2*h*Piecewise((sqr
t(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + d*h**3*Piecewi
se((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x
**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + e*g**3*Piecewi
se((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + 3*e*g*h
**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)
/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 3*f
*g**2*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x
**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) +
 f*h**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a
+ c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**
2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + 3*c*d*g*h**2*x**5/(4*sqrt(a)*sqr
t(1 + c*x**2/a)) + 3*c*e*g**2*h*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c*e*h
**3*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a)) + c*f*g**3*x**5/(4*sqrt(a)*sqrt(1 +
 c*x**2/a)) + c*f*g*h**2*x**7/(2*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [A]
time = 4.10, size = 475, normalized size = 1.22

$$\frac{1}{1680}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\left(6fh^3x+\frac{7(3c^5fgh^2+c^5h^3e)}{c^5}\right)x+\frac{6(21c^5fg^2h+7c^5dh^3+ac^4fh^3+21c^5gh^2e)}{c^5}\right)x+\frac{35(6c^5fg^3+18c^5dgh^2+3ac^4fgh^2+18c^5g^2he+ac^4h^3e)}{c^5}\right)x+\frac{8(105c^5dg^2h+21ac^4fg^2h+7ac^4dh^3-4a^2c^3fh^3+35c^5g^3e+21ac^4gh^2e)}{c^5}\right)x+\frac{105(8c^5dg^3+2ac^4fg^3+6ac^4dgh^2-3a^2c^3fgh^2+6ac^4g^2he-a^2c^3h^3e)}{c^5}\right)x+\frac{16(105ac^4dg^2h-42a^2c^3fg^2h-14a^2c^3dh^3+8a^3c^2fh^3+35ac^4g^3e-42a^2c^3gh^2e)}{c^5}\right)-\frac{(8ac^2dg^3-2a^2cfg^3-6a^2cdgh^2+3a^3fgh^2-6a^2cg^2he+a^3h^3e)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{16c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1680*sqrt(c*x^2 + a)*((2*((4*(5*(6*f*h^3*x + 7*(3*c^5*f*g*h^2 + c^5*h^3*e
)/c^5)*x + 6*(21*c^5*f*g^2*h + 7*c^5*d*h^3 + a*c^4*f*h^3 + 21*c^5*g*h^2*e)/
c^5)*x + 35*(6*c^5*f*g^3 + 18*c^5*d*g*h^2 + 3*a*c^4*f*g*h^2 + 18*c^5*g^2*h*
e + a*c^4*h^3*e)/c^5)*x + 8*(105*c^5*d*g^2*h + 21*a*c^4*f*g^2*h + 7*a*c^4*d
*h^3 - 4*a^2*c^3*f*h^3 + 35*c^5*g^3*e + 21*a*c^4*g*h^2*e)/c^5)*x + 105*(8*c
^5*d*g^3 + 2*a*c^4*f*g^3 + 6*a*c^4*d*g*h^2 - 3*a^2*c^3*f*g*h^2 + 6*a*c^4*g^
2*h*e - a^2*c^3*h^3*e)/c^5)*x + 16*(105*a*c^4*d*g^2*h - 42*a^2*c^3*f*g^2*h
- 14*a^2*c^3*d*h^3 + 8*a^3*c^2*f*h^3 + 35*a*c^4*g^3*e - 42*a^2*c^3*g*h^2*e)
/c^5) - 1/16*(8*a*c^2*d*g^3 - 2*a^2*c*f*g^3 - 6*a^2*c*d*g*h^2 + 3*a^3*f*g*h
^2 - 6*a^2*c*g^2*h*e + a^3*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^
(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g+h\,x)^3\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.79 $\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=280

$$\frac{(8c^2dg^2+a^2fh^2-2ac(fg^2+h(2eg+dh)))\,x\sqrt{a+cx^2}}{16c^2}-\frac{(fg-2eh)(g+hx)^2\,(a+cx^2)^{3/2}}{10ch}+\frac{f(g+hx)^3\,(a}{6ch}$$

```
[Out] -1/10*(-2*e*h+f*g)*(h*x+g)^2*(c*x^2+a)^(3/2)/c/h+1/6*f*(h*x+g)^3*(c*x^2+a)^
(3/2)/c/h-1/120*(16*a*h^2*(e*h+2*f*g)+8*c*g*(f*g^2-2*h*(5*d*h+e*g))-3*h*(5*
(-a*f+2*c*d)*h^2-2*c*g*(-2*e*h+f*g))*x)*(c*x^2+a)^(3/2)/c^2/h+1/16*a*(8*c^2
*d*g^2+a^2*f*h^2-2*a*c*(f*g^2+h*(d*h+2*e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(
1/2))/c^(5/2)+1/16*(8*c^2*d*g^2+a^2*f*h^2-2*a*c*(f*g^2+h*(d*h+2*e*g)))*x*(c
*x^2+a)^(1/2)/c^2
```

Rubi [A]
time = 0.27, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1668, 847, 794, 201, 223, 212}

$$\frac{x\sqrt{a+cx^2}\,(a^2fh^2-2ac(h(dh+2eg)+fg^2)+8c^2dg^2)}{16c^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(a^2fh^2-2ac(h(dh+2eg)+fg^2)+8c^2dg^2)}{16c^{5/2}}-\frac{(a+cx^2)^{3/2}\,(8(2ah^2(eh+2fg)-2cgh(5dh+eg)+cfg^3)-3hx(5h^2(2cd-af)-2cg(fg-2eh)))}{120c^2h}-\frac{(a+cx^2)^{3/2}\,(g+hx)^2(fg-2eh)}{10ch}+\frac{f(a+cx^2)^{3/2}\,(g+hx)^3}{6ch}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c*(f*g^2 + h*(2*e*g + d*h)))*x*Sqrt[a + c*x
^2])/(16*c^2) - ((f*g - 2*e*h)*(g + h*x)^2*(a + c*x^2)^(3/2))/(10*c*h) + (f
*(g + h*x)^3*(a + c*x^2)^(3/2))/(6*c*h) - ((8*(c*f*g^3 - 2*c*g*h*(e*g + 5*d
*h) + 2*a*h^2*(2*f*g + e*h)) - 3*h*(5*(2*c*d - a*f)*h^2 - 2*c*g*(f*g - 2*e*
h))*x)*(a + c*x^2)^(3/2))/(120*c^2*h) + (a*(8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c
*(f*g^2 + h*(2*e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(5/
2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch} + \frac{\int (g+hx)^2\left(3(2cd-af)h^2-3ch(fg\right.}{6ch^2}$$

$$= -\frac{(fg-2eh)(g+hx)^2\left(a+cx^2\right)^{3/2}}{10ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch} +$$

$$= -\frac{(fg-2eh)(g+hx)^2\left(a+cx^2\right)^{3/2}}{10ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch} -$$

$$= \frac{\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{(fg}{}$$

$$= \frac{\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{(fg}{}$$

$$= \frac{\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{(fg}{}$$

Mathematica [A]
time = 0.74, size = 245, normalized size = 0.88

$$\frac{\sqrt{a+cx^2}\left(-a^2h(64fg+32eh+15fhx)+2ac(5dh(16g+3hx)+fx(15g^2+16ghx+5h^2x^2)+e(40g^2+30ghx+8h^2x^2))+4c^2x(5d(6g^2+8ghx+3h^2x^2)+x(2e(10g^2+15ghx+6h^2x^2)+fx(15g^2+24ghx+10h^2x^2)))\right)}{240c^2} - \frac{a\left(8c^2dg^2+a^2fh^2-2ac(fg^2+h(2eg+dh))\right)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-(a^2*h*(64*f*g + 32*e*h + 15*f*h*x)) + 2*a*c*(5*d*h*(16*
g + 3*h*x) + f*x*(15*g^2 + 16*g*h*x + 5*h^2*x^2) + e*(40*g^2 + 30*g*h*x + 8
*h^2*x^2)) + 4*c^2*x*(5*d*(6*g^2 + 8*g*h*x + 3*h^2*x^2) + x*(2*e*(10*g^2 +
15*g*h*x + 6*h^2*x^2) + f*x*(15*g^2 + 24*g*h*x + 10*h^2*x^2)))))/(240*c^2)
- (a*(8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c*(f*g^2 + h*(2*e*g + d*h)))*Log[-(Sqrt
[c]*x) + Sqrt[a + c*x^2]])/(16*c^(5/2))
```

Maple [A]
time = 0.08, size = 272, normalized size = 0.97

method	result

default	$f\,h^{2}\left(\frac{x^{3}\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{6c}-\frac{a\left(\frac{x\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{4c}-\frac{a\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)}{4c}\right)}{2c}\right)+\left(e\,h^{2}+2fgh\right)\left(\frac{x^{2}(\right.$
risch	$-\frac{\left(-40f\,h^{2}c^{2}x^{5}-48c^{2}e\,h^{2}x^{4}-96c^{2}fgh\,x^{4}-10af\,h^{2}c\,x^{3}-60c^{2}d\,h^{2}x^{3}-120c^{2}egh\,x^{3}-60c^{2}f\,g^{2}x^{3}-16ae\,h^{2}c\,x^{2}-32afghc\,x^{2}-160c^{2}d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*h^2*(1/6*x^3*(c*x^2+a)^(3/2)/c-1/2*a/c*(1/4*x*(c*x^2+a)^(3/2)/c-1/4*a/c*(
1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))))+(e*h^2
+2*f*g*h)*(1/5*x^2*(c*x^2+a)^(3/2)/c-2/15*a/c^2*(c*x^2+a)^(3/2))+(d*h^2+2*e
*g*h+f*g^2)*(1/4*x*(c*x^2+a)^(3/2)/c-1/4*a/c*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c
^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))))+1/3*(2*d*g*h+e*g^2)*(c*x^2+a)^(3/2)/
c+d*g^2*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.29, size = 311, normalized size = 1.11

$$\frac{(cx^2+a)^{\frac{3}{2}}fh^2x^3}{6\,c}+\frac{1}{2}\,\sqrt{cx^2+a}\,dg^2x-\frac{(cx^2+a)^{\frac{3}{2}}afh^2x}{8\,c^2}+\frac{\sqrt{cx^2+a}\,a^2fh^2x}{16\,c^2}+\frac{adg^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}}+\frac{a^3fh^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,c^{\frac{5}{2}}}+\frac{2\,(cx^2+a)^{\frac{3}{2}}dgh}{3\,c}+\frac{(2\,fgh+h^2e)(cx^2+a)^{\frac{3}{2}}x^2}{5\,c}+\frac{(cx^2+a)^{\frac{3}{2}}g^2e}{3\,c}+\frac{(fg^2+dh^2+2\,ghe)(cx^2+a)^{\frac{3}{2}}x}{4\,c}-\frac{(fg^2+dh^2+2\,ghe)\sqrt{cx^2+a}\,ax}{8\,c}-\frac{(fg^2+dh^2+2\,ghe)a^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{3}{2}}}-\frac{2\,(2\,fgh+h^2e)(cx^2+a)^{\frac{3}{2}}a}{15\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2 + a)^(3/2)*f*h^2*x^3/c + 1/2*sqrt(c*x^2 + a)*d*g^2*x - 1/8*(c*x^
2 + a)^(3/2)*a*f*h^2*x/c^2 + 1/16*sqrt(c*x^2 + a)*a^2*f*h^2*x/c^2 + 1/2*a*d
*g^2*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 1/16*a^3*f*h^2*arcsinh(c*x/sqrt(a*c))
/c^(5/2) + 2/3*(c*x^2 + a)^(3/2)*d*g*h/c + 1/5*(2*f*g*h + h^2*e)*(c*x^2 + a
)^(3/2)*x^2/c + 1/3*(c*x^2 + a)^(3/2)*g^2*e/c + 1/4*(f*g^2 + d*h^2 + 2*g*h*
e)*(c*x^2 + a)^(3/2)*x/c - 1/8*(f*g^2 + d*h^2 + 2*g*h*e)*sqrt(c*x^2 + a)*a*
x/c - 1/8*(f*g^2 + d*h^2 + 2*g*h*e)*a^2*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/
15*(2*f*g*h + h^2*e)*(c*x^2 + a)^(3/2)*a/c^2
```

Fricas [A]
time = 0.41, size = 609, normalized size = 2.18

$$\left[\frac{15\,(4\,a^2cghe-2\,(4\,ac^2d-a^2cf)g^2+(2\,a^2cd-a^3f)h^2)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2\,(40\,c^3fh^2x^5+96\,c^3fghx^4+32\,(5\,c^3d+ac^2f)ghx^2+10\,(6\,c^3fg^2+(6\,c^3d+ac^2f)h^2)x^3+32\,(5\,ac^2d-2\,a^2cf)gh+15\,(2\,(4\,c^3d+ac^2f)g^2+(2\,ac^2d-a^2cf)h^2)x+4\,(12\,c^3h^2x^4+30\,c^3ghx^3+15\,ac^2ghx+20\,ac^2g^2-8\,a^2ch^2+4\,(5\,c^3g^2+ac^2h^2)x^2)e)\sqrt{cx^2+a}}{480\,c^3},\ \frac{15\,(4\,a^2cghe-2\,(4\,ac^2d-a^2cf)g^2+(2\,a^2cd-a^3f)h^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)+(40\,c^3fh^2x^5+96\,c^3fghx^4+32\,(5\,c^3d+ac^2f)ghx^2+10\,(6\,c^3fg^2+(6\,c^3d+ac^2f)h^2)x^3+32\,(5\,ac^2d-2\,a^2cf)gh+15\,(2\,(4\,c^3d+ac^2f)g^2+(2\,ac^2d-a^2cf)h^2)x+4\,(12\,c^3h^2x^4+30\,c^3ghx^3+15\,ac^2ghx+20\,ac^2g^2-8\,a^2ch^2+4\,(5\,c^3g^2+ac^2h^2)x^2)e)\sqrt{cx^2+a}}{240\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(15*(4*a^2*c*g*h*e - 2*(4*a*c^2*d - a^2*c*f)*g^2 + (2*a^2*c*d - a^3*
f)*h^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(40*c^3
*f*h^2*x^5 + 96*c^3*f*g*h*x^4 + 32*(5*c^3*d + a*c^2*f)*g*h*x^2 + 10*(6*c^3*
f*g^2 + (6*c^3*d + a*c^2*f)*h^2)*x^3 + 32*(5*a*c^2*d - 2*a^2*c*f)*g*h + 15*
(2*(4*c^3*d + a*c^2*f)*g^2 + (2*a*c^2*d - a^2*c*f)*h^2)*x + 4*(12*c^3*h^2*x
^4 + 30*c^3*g*h*x^3 + 15*a*c^2*g*h*x + 20*a*c^2*g^2 - 8*a^2*c*h^2 + 4*(5*c^
3*g^2 + a*c^2*h^2)*x^2)*e)*sqrt(c*x^2 + a))/c^3, 1/240*(15*(4*a^2*c*g*h*e -
 2*(4*a*c^2*d - a^2*c*f)*g^2 + (2*a^2*c*d - a^3*f)*h^2)*sqrt(-c)*arctan(sqr
t(-c)*x/sqrt(c*x^2 + a)) + (40*c^3*f*h^2*x^5 + 96*c^3*f*g*h*x^4 + 32*(5*c^3
*d + a*c^2*f)*g*h*x^2 + 10*(6*c^3*f*g^2 + (6*c^3*d + a*c^2*f)*h^2)*x^3 + 32
*(5*a*c^2*d - 2*a^2*c*f)*g*h + 15*(2*(4*c^3*d + a*c^2*f)*g^2 + (2*a*c^2*d -
 a^2*c*f)*h^2)*x + 4*(12*c^3*h^2*x^4 + 30*c^3*g*h*x^3 + 15*a*c^2*g*h*x + 20
*a*c^2*g^2 - 8*a^2*c*h^2 + 4*(5*c^3*g^2 + a*c^2*h^2)*x^2)*e)*sqrt(c*x^2 + a
))/c^3]
```

Sympy [A]
time = 12.62, size = 738, normalized size = 2.64

$$-\frac{a^{\frac{5}{2}}fh^2x}{16c^2\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}dh^2x}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}eghx}{4c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}fg^2x}{8c\sqrt{1+\frac{cx^2}{a}}}-\frac{a^{\frac{3}{2}}fh^2x^3}{48c\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}\,dg^2x\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{3\sqrt{a}\,dh^2x^3}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}\,eghx^3}{4\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}\,fg^2x^3}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{5\sqrt{a}\,fh^2x^5}{24\sqrt{1+\frac{cx^2}{a}}}+\frac{a^3fh^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16c^{\frac{5}{2}}}-\frac{a^2dh^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}}-\frac{a^2egh\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{4c^{\frac{3}{2}}}-\frac{a^2fg^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}}+\frac{adg^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\sqrt{c}}+2dgh\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\: c=0\\\frac{(a+cx^2)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)+eg^2\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\: c=0\\\frac{(a+cx^2)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)+eh^2\left(\begin{cases}-\frac{2a^2\sqrt{a+cx^2}}{15c^2}+\frac{ax^2\sqrt{a+cx^2}}{15c}+\frac{x^4\sqrt{a+cx^2}}{5} & \text{for}\: c\neq 0\\\frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+2fgh\left(\begin{cases}-\frac{2a^2\sqrt{a+cx^2}}{15c^2}+\frac{ax^2\sqrt{a+cx^2}}{15c}+\frac{x^4\sqrt{a+cx^2}}{5} & \text{for}\: c\neq 0\\\frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+\frac{cdh^2x^5}{4\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}+\frac{ceghx^5}{2\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}+\frac{cfg^2x^5}{4\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}+\frac{cfh^2x^7}{6\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] -a**(5/2)*f*h**2*x/(16*c**2*sqrt(1 + c*x**2/a)) + a**(3/2)*d*h**2*x/(8*c*sq
rt(1 + c*x**2/a)) + a**(3/2)*e*g*h*x/(4*c*sqrt(1 + c*x**2/a)) + a**(3/2)*f*
g**2*x/(8*c*sqrt(1 + c*x**2/a)) - a**(3/2)*f*h**2*x**3/(48*c*sqrt(1 + c*x**
2/a)) + sqrt(a)*d*g**2*x*sqrt(1 + c*x**2/a)/2 + 3*sqrt(a)*d*h**2*x**3/(8*sq
rt(1 + c*x**2/a)) + 3*sqrt(a)*e*g*h*x**3/(4*sqrt(1 + c*x**2/a)) + 3*sqrt(a)
*f*g**2*x**3/(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*f*h**2*x**5/(24*sqrt(1 + c*
x**2/a)) + a**3*f*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(5/2)) - a**2*d*h**2
*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) - a**2*e*g*h*asinh(sqrt(c)*x/sqrt(a)
)/(4*c**(3/2)) - a**2*f*g**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) + a*d*g*
*2*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + 2*d*g*h*Piecewise((sqrt(a)*x**2/2
, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + e*g**2*Piecewise((sqrt(a)
*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + e*h**2*Piecewise((
-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*
sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 2*f*g*h*Piecewise(
(-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4
*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c*d*h**2*x**5/(4*
sqrt(a)*sqrt(1 + c*x**2/a)) + c*e*g*h*x**5/(2*sqrt(a)*sqrt(1 + c*x**2/a)) +
 c*f*g**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c*f*h**2*x**7/(6*sqrt(a)*sq
rt(1 + c*x**2/a))
```

Giac [A]
time = 3.95, size = 321, normalized size = 1.15

$$\frac{1}{240}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\,fh^2x+\frac{6\,(2\,c^4fgh+c^4h^2e)}{c^4}\right)x+\frac{5\,(6\,c^4fg^2+6\,c^4dh^2+ac^3fh^2+12\,c^4ghe)}{c^4}\right)x+\frac{8\,(10\,c^4dgh+2\,ac^3fgh+5\,c^4g^2e+ac^3h^2e)}{c^4}\right)x+\frac{15\,(8\,c^4dg^2+2\,ac^3fg^2+2\,ac^3dh^2-a^2c^2fh^2+4\,ac^3ghe)}{c^4}\right)x+\frac{16\,(10\,ac^3dgh-4\,a^2c^2fgh+5\,ac^3g^2e-2\,a^2c^2h^2e)}{c^4}\right)-\frac{(8\,ac^2dg^2-2\,a^2cfg^2-2\,a^2cdh^2+a^3fh^2-4\,a^2cghe)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/240*sqrt(c*x^2 + a)*((2*((4*(5*f*h^2*x + 6*(2*c^4*f*g*h + c^4*h^2*e)/c^4)
*x + 5*(6*c^4*f*g^2 + 6*c^4*d*h^2 + a*c^3*f*h^2 + 12*c^4*g*h*e)/c^4)*x + 8*
(10*c^4*d*g*h + 2*a*c^3*f*g*h + 5*c^4*g^2*e + a*c^3*h^2*e)/c^4)*x + 15*(8*c
^4*d*g^2 + 2*a*c^3*f*g^2 + 2*a*c^3*d*h^2 - a^2*c^2*f*h^2 + 4*a*c^3*g*h*e)/c
^4)*x + 16*(10*a*c^3*d*g*h - 4*a^2*c^2*f*g*h + 5*a*c^3*g^2*e - 2*a^2*c^2*h^
2*e)/c^4) - 1/16*(8*a*c^2*d*g^2 - 2*a^2*c*f*g^2 - 2*a^2*c*d*h^2 + a^3*f*h^2
 - 4*a^2*c*g*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g+h\,x)^2\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.80 $\int (g+hx)\sqrt{a+cx^2}\,(d+ex+fx^2)\,dx$

Optimal. Leaf size=175

$$\frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c}+\frac{f(g+hx)^2\,(a+cx^2)^{3/2}}{5ch}-\frac{(4(2afh^2+c(3fg^2-5h(eg+dh)))+3ch(3fg-}{60c^2h}$$

```
[Out] 1/5*f*(h*x+g)^2*(c*x^2+a)^(3/2)/c/h-1/60*(8*a*f*h^2+4*c*(3*f*g^2-5*h*(d*h+e
*g))+3*c*h*(-5*e*h+3*f*g)*x)*(c*x^2+a)^(3/2)/c^2/h+1/8*a*(-a*e*h-a*f*g+4*c*
d*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+1/8*(4*c*d*g-a*(e*h+f*g))*x
*(c*x^2+a)^(1/2)/c
```

Rubi [A]
time = 0.14, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1668, 794, 201, 223, 212}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(-aeh-afg+4cdg)}{8c^{3/2}}-\frac{(a+cx^2)^{3/2}\,(4(2afh^2+c(3fg^2-5h(dh+eg)))+3chx(3fg-5eh))}{60c^2h}+\frac{x\sqrt{a+cx^2}\,(4cdg-a(eh+fg))}{8c}+\frac{f(a+cx^2)^{3/2}\,(g+hx)^2}{5ch}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((4*c*d*g - a*(f*g + e*h))*x*Sqrt[a + c*x^2])/(8*c) + (f*(g + h*x)^2*(a + c
*x^2)^(3/2))/(5*c*h) - ((4*(2*a*f*h^2 + c*(3*f*g^2 - 5*h*(e*g + d*h))) + 3*
c*h*(3*f*g - 5*e*h)*x)*(a + c*x^2)^(3/2))/(60*c^2*h) + (a*(4*c*d*g - a*f*g
- a*e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch} + \frac{\int (g+hx)\left((5cd-2af)h^2-ch(3fg \cdots\right.}{5ch^2}$$

$$= \frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch} - \frac{\left(4\left(2afh^2+c\left(3fg^2-5h(eg+dh)\right)\right) \cdots\right.}{60c^2h \cdots}$$

$$= \frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c} + \frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch} - \cdots$$

$$= \frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c} + \frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch} - \cdots$$

$$= \frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c} + \frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch} - \cdots$$

Mathematica [A]
time = 0.50, size = 145, normalized size = 0.83

$$\frac{\sqrt{a+cx^2}\left(-16a^2fh+ac(40dh+5e(8g+3hx)+fx(15g+8hx))+2c^2x(10d(3g+2hx)+x(5e(4g+3hx)+3fx(5g+4hx)))\right)+15a\sqrt{c}\left(-4cdg+afg+aeh\right)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{120c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-16*a^2*f*h + a*c*(40*d*h + 5*e*(8*g + 3*h*x) + f*x*(15*g
 + 8*h*x)) + 2*c^2*x*(10*d*(3*g + 2*h*x) + x*(5*e*(4*g + 3*h*x) + 3*f*x*(5*
g + 4*h*x)))) + 15*a*Sqrt[c]*(-4*c*d*g + a*f*g + a*e*h)*Log[-(Sqrt[c]*x) +
Sqrt[a + c*x^2]])/(120*c^2)
```

Maple [A]
time = 0.08, size = 162, normalized size = 0.93

method	result
default	$hf\left(\frac{x^{2}\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{5c}-\frac{2a\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{15c^{2}}\right)+\left(eh+gf\right)\left(\frac{x\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{4c}-\frac{a\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)}{4c}\right.$
risch	$-\frac{\left(-24hf\,c^{2}x^{4}-30c^{2}eh\,x^{3}-30c^{2}fg\,x^{3}-8acfh\,x^{2}-40c^{2}dh\,x^{2}-40c^{2}eg\,x^{2}-15aehxc-15afgxc-60c^{2}dgx+16a^{2}fh-40acdh-40aceg\right)\sqrt{}}{120c^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] h*f*(1/5*x^2*(c*x^2+a)^(3/2)/c-2/15*a/c^2*(c*x^2+a)^(3/2))+(e*h+f*g)*(1/4*x
*(c*x^2+a)^(3/2)/c-1/4*a/c*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2
)+(c*x^2+a)^(1/2))))+1/3*(d*h+e*g)*(c*x^2+a)^(3/2)/c+d*g*(1/2*x*(c*x^2+a)^(
1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.28, size = 173, normalized size = 0.99

$$\frac{(cx^2+a)^{\frac{3}{2}}fhx^2}{5\,c}+\frac{1}{2}\,\sqrt{cx^2+a}\,dgx+\frac{adg\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}}+\frac{(cx^2+a)^{\frac{3}{2}}dh}{3\,c}-\frac{2\,(cx^2+a)^{\frac{3}{2}}afh}{15\,c^2}+\frac{(cx^2+a)^{\frac{3}{2}}(fg+he)x}{4\,c}-\frac{\sqrt{cx^2+a}\,(fg+he)ax}{8\,c}-\frac{(fg+he)a^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{3}{2}}}+\frac{(cx^2+a)^{\frac{3}{2}}ge}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(c*x^2 + a)^(3/2)*f*h*x^2/c + 1/2*sqrt(c*x^2 + a)*d*g*x + 1/2*a*d*g*arc
sinh(c*x/sqrt(a*c))/sqrt(c) + 1/3*(c*x^2 + a)^(3/2)*d*h/c - 2/15*(c*x^2 + a
)^(3/2)*a*f*h/c^2 + 1/4*(c*x^2 + a)^(3/2)*(f*g + h*e)*x/c - 1/8*sqrt(c*x^2
+ a)*(f*g + h*e)*a*x/c - 1/8*(f*g + h*e)*a^2*arcsinh(c*x/sqrt(a*c))/c^(3/2)
 + 1/3*(c*x^2 + a)^(3/2)*g*e/c
```

Fricas [A]
time = 0.38, size = 339, normalized size = 1.94

$$\left[\frac{15\,(a^2he-(4\,acd-a^2f)g)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2\,(24\,c^2fhx^4+30\,c^2fgx^3+8\,(5\,c^2d+acf)hx^2+15\,(4\,c^2d+acf)gx+8\,(5\,acd-2\,a^2f)h+5\,(6\,c^2hx^3+8\,c^2gx^2+3\,achx+8\,acg)e)\sqrt{cx^2+a}}{240\,c^2},\ \frac{15\,(a^2he-(4\,acd-a^2f)g)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)+(24\,c^2fhx^4+30\,c^2fgx^3+8\,(5\,c^2d+acf)hx^2+15\,(4\,c^2d+acf)gx+8\,(5\,acd-2\,a^2f)h+5\,(6\,c^2hx^3+8\,c^2gx^2+3\,achx+8\,acg)e)\sqrt{cx^2+a}}{120\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*(a^2*h*e - (4*a*c*d - a^2*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*
x^2 + a)*sqrt(c)*x - a) + 2*(24*c^2*f*h*x^4 + 30*c^2*f*g*x^3 + 8*(5*c^2*d +
 a*c*f)*h*x^2 + 15*(4*c^2*d + a*c*f)*g*x + 8*(5*a*c*d - 2*a^2*f)*h + 5*(6*c
^2*h*x^3 + 8*c^2*g*x^2 + 3*a*c*h*x + 8*a*c*g)*e)*sqrt(c*x^2 + a))/c^2, 1/12
0*(15*(a^2*h*e - (4*a*c*d - a^2*f)*g)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2
 + a)) + (24*c^2*f*h*x^4 + 30*c^2*f*g*x^3 + 8*(5*c^2*d + a*c*f)*h*x^2 + 15*
(4*c^2*d + a*c*f)*g*x + 8*(5*a*c*d - 2*a^2*f)*h + 5*(6*c^2*h*x^3 + 8*c^2*g*
x^2 + 3*a*c*h*x + 8*a*c*g)*e)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A]
time = 5.76, size = 384, normalized size = 2.19

$$\frac{a^{\frac{3}{2}}ehx}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}fgx}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}\,dgx\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{3\sqrt{a}\,ehx^3}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}\,fgx^3}{8\sqrt{1+\frac{cx^2}{a}}}-\frac{a^2eh\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}}-\frac{a^2fg\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}}+\frac{adg\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\sqrt{c}}+dh\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for } c=0\\ \frac{(a+cx^2)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)+eg\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for } c=0\\ \frac{(a+cx^2)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)+fh\left(\begin{cases}-\frac{2a^2\sqrt{a+cx^2}}{15c^2}+\frac{ax^2\sqrt{a+cx^2}}{15c}+\frac{x^4\sqrt{a+cx^2}}{5} & \text{for } c\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+\frac{cehx^5}{4\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}+\frac{cfgx^5}{4\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] a**(3/2)*e*h*x/(8*c*sqrt(1 + c*x**2/a)) + a**(3/2)*f*g*x/(8*c*sqrt(1 + c*x*
*2/a)) + sqrt(a)*d*g*x*sqrt(1 + c*x**2/a)/2 + 3*sqrt(a)*e*h*x**3/(8*sqrt(1
+ c*x**2/a)) + 3*sqrt(a)*f*g*x**3/(8*sqrt(1 + c*x**2/a)) - a**2*e*h*asinh(s
qrt(c)*x/sqrt(a))/(8*c**(3/2)) - a**2*f*g*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3
/2)) + a*d*g*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + d*h*Piecewise((sqrt(a)*
x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + e*g*Piecewise((sqrt
(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + f*h*Piecewise((
-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*
sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c*e*h*x**5/(4*sqrt
(a)*sqrt(1 + c*x**2/a)) + c*f*g*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [A]
time = 3.88, size = 180, normalized size = 1.03

$$\frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(4\,fhx+\frac{5\,(c^3fg+c^3he)}{c^3}\right)x+\frac{4\,(5\,c^3dh+ac^2fh+5\,c^3ge)}{c^3}\right)x+\frac{15\,(4\,c^3dg+ac^2fg+ac^2he)}{c^3}\right)x+\frac{8\,(5\,ac^2dh-2\,a^2cfh+5\,ac^2ge)}{c^3}\right)-\frac{(4\,acdg-a^2fg-a^2he)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{8\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(c*x^2 + a)*((2*(3*(4*f*h*x + 5*(c^3*f*g + c^3*h*e)/c^3)*x + 4*(5
*c^3*d*h + a*c^2*f*h + 5*c^3*g*e)/c^3)*x + 15*(4*c^3*d*g + a*c^2*f*g + a*c^
2*h*e)/c^3)*x + 8*(5*a*c^2*d*h - 2*a^2*c*f*h + 5*a*c^2*g*e)/c^3) - 1/8*(4*a
*c*d*g - a^2*f*g - a^2*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g+h\,x)\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
[Out] int((g + h*x)*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.81 $\int \sqrt{a+cx^2}\,(d+ex+fx^2)\,dx$

Optimal. Leaf size=106

$$\frac{(4cd-af)x\sqrt{a+cx^2}}{8c}+\frac{e(a+cx^2)^{3/2}}{3c}+\frac{fx(a+cx^2)^{3/2}}{4c}+\frac{a(4cd-af)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}$$

```
[Out] 1/3*e*(c*x^2+a)^(3/2)/c+1/4*f*x*(c*x^2+a)^(3/2)/c+1/8*a*(-a*f+4*c*d)*arctan
h(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+1/8*(-a*f+4*c*d)*x*(c*x^2+a)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1829, 655, 201, 223, 212}

$$\frac{a(4cd-af)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}+\frac{x\sqrt{a+cx^2}\,(4cd-af)}{8c}+\frac{e(a+cx^2)^{3/2}}{3c}+\frac{fx(a+cx^2)^{3/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((4*c*d - a*f)*x*Sqrt[a + c*x^2])/(8*c) + (e*(a + c*x^2)^(3/2))/(3*c) + (f*
x*(a + c*x^2)^(3/2))/(4*c) + (a*(4*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/Sqrt[a +
c*x^2]])/(8*c^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{fx(a+cx^2)^{3/2}}{4c} + \frac{\int (4cd-af+4cex)\sqrt{a+cx^2}\,dx}{4c}$$

$$= \frac{e(a+cx^2)^{3/2}}{3c} + \frac{fx(a+cx^2)^{3/2}}{4c} + \frac{(4cd-af)\int \sqrt{a+cx^2}\,dx}{4c}$$

$$= \frac{(4cd-af)x\sqrt{a+cx^2}}{8c} + \frac{e(a+cx^2)^{3/2}}{3c} + \frac{fx(a+cx^2)^{3/2}}{4c} + \frac{(a(4cd-af)}{}$$

$$= \frac{(4cd-af)x\sqrt{a+cx^2}}{8c} + \frac{e(a+cx^2)^{3/2}}{3c} + \frac{fx(a+cx^2)^{3/2}}{4c} + \frac{(a(4cd-af)}{}$$

$$= \frac{(4cd-af)x\sqrt{a+cx^2}}{8c} + \frac{e(a+cx^2)^{3/2}}{3c} + \frac{fx(a+cx^2)^{3/2}}{4c} + \frac{a(4cd-af)}{}$$

Mathematica [A]
time = 0.23, size = 87, normalized size = 0.82

$$\frac{\sqrt{a+cx^2}\,(8ae+12cdx+3afx+8cex^2+6cfx^3)}{24c} + \frac{a(-4cd+af)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{8c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(8*a*e + 12*c*d*x + 3*a*f*x + 8*c*e*x^2 + 6*c*f*x^3))/(24*
c) + (a*(-4*c*d + a*f)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(8*c^(3/2))
```

Maple [A]
time = 0.07, size = 113, normalized size = 1.07

method	result
risch	$\frac{(6cf\,x^3+8ce\,x^2+3afx+12cdx+8ae)\sqrt{c\,x^2+a}}{24c}-\frac{a^2\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)f}{8c^{\frac{3}{2}}}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)d}{2\sqrt{c}}$
default	$f\left(\frac{x(c\,x^2+a)^{\frac{3}{2}}}{4c}-\frac{a\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)}{4c}\right)+\frac{e(c\,x^2+a)^{\frac{3}{2}}}{3c}+d\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/4*x*(c*x^2+a)^(3/2)/c-1/4*a/c*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(
x*c^(1/2)+(c*x^2+a)^(1/2))))+1/3*e*(c*x^2+a)^(3/2)/c+d*(1/2*x*(c*x^2+a)^(1/
2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.30, size = 97, normalized size = 0.92

$$\frac{1}{2}\sqrt{cx^2+a}\,dx+\frac{(cx^2+a)^{\frac{3}{2}}fx}{4\,c}-\frac{\sqrt{cx^2+a}\,afx}{8\,c}+\frac{ad\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}}-\frac{a^2f\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{3}{2}}}+\frac{(cx^2+a)^{\frac{3}{2}}e}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*d*x + 1/4*(c*x^2 + a)^(3/2)*f*x/c - 1/8*sqrt(c*x^2 + a)
*a*f*x/c + 1/2*a*d*arcsinh(c*x/sqrt(a*c))/sqrt(c) - 1/8*a^2*f*arcsinh(c*x/s
qrt(a*c))/c^(3/2) + 1/3*(c*x^2 + a)^(3/2)*e/c
```

Fricas [A]
time = 0.39, size = 192, normalized size = 1.81

$$\left[-\frac{3\,(4\,acd-a^2f)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\,(6\,c^2fx^3+3\,(4\,c^2d+acf)x+8\,(c^2x^2+ac)e)\sqrt{cx^2+a}}{48\,c^2},-\frac{3\,(4\,acd-a^2f)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)-(6\,c^2fx^3+3\,(4\,c^2d+acf)x+8\,(c^2x^2+ac)e)\sqrt{cx^2+a}}{24\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(4*a*c*d - a^2*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c
)*x - a) - 2*(6*c^2*f*x^3 + 3*(4*c^2*d + a*c*f)*x + 8*(c^2*x^2 + a*c)*e)*sq
rt(c*x^2 + a))/c^2, -1/24*(3*(4*a*c*d - a^2*f)*sqrt(-c)*arctan(sqrt(-c)*x/s
qrt(c*x^2 + a)) - (6*c^2*f*x^3 + 3*(4*c^2*d + a*c*f)*x + 8*(c^2*x^2 + a*c)*
e)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A]
time = 3.41, size = 170, normalized size = 1.60

$$\frac{a^{\frac{3}{2}}fx}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}\,dx\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{3\sqrt{a}\,fx^3}{8\sqrt{1+\frac{cx^2}{a}}}-\frac{a^2f\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}}+\frac{ad\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\sqrt{c}}+e\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for } c=0\\ \frac{(a+cx^2)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)+\frac{cfx^5}{4\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] a**(3/2)*f*x/(8*c*sqrt(1 + c*x**2/a)) + sqrt(a)*d*x*sqrt(1 + c*x**2/a)/2 +
3*sqrt(a)*f*x**3/(8*sqrt(1 + c*x**2/a)) - a**2*f*asinh(sqrt(c)*x/sqrt(a))/(
8*c**(3/2)) + a*d*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + e*Piecewise((sqrt(
a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + c*f*x**5/(4*sqrt
(a)*sqrt(1 + c*x**2/a))
```

Giac [A]
time = 2.57, size = 87, normalized size = 0.82

$$\frac{1}{24}\sqrt{cx^2+a}\left(\left(2\,(3\,fx+4\,e)x+\frac{3\,(4\,c^2d+acf)}{c^2}\right)x+\frac{8\,ae}{c}\right)-\frac{(4\,acd-a^2f)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{8\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + a)*((2*(3*f*x + 4*e)*x + 3*(4*c^2*d + a*c*f)/c^2)*x + 8*a
*e/c) - 1/8*(4*a*c*d - a^2*f)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.82 $\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{g+hx}\,dx$

Optimal. Leaf size=206

$$\frac{(2(fg^2-egh+dh^2)-h(fg-eh)x)\sqrt{a+cx^2}}{2h^3}+\frac{f(a+cx^2)^{3/2}}{3ch}-\frac{(2cdgh^2+(fg-eh)(2cg^2+ah^2))\tanh^{-}}{2\sqrt{c}\,h^4}$$

```
[Out] 1/3*f*(c*x^2+a)^(3/2)/c/h-1/2*(2*c*d*g*h^2+(-e*h+f*g)*(a*h^2+2*c*g^2))*arct
anh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^4/c^(1/2)-(d*h^2-e*g*h+f*g^2)*arctanh((-c*
g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))*(a*h^2+c*g^2)^(1/2)/h^4+1/2*(
2*d*h^2-2*e*g*h+2*f*g^2-h*(-e*h+f*g)*x)*(c*x^2+a)^(1/2)/h^3
```

Rubi [A]
time = 0.23, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1668, 829, 858, 223, 212, 739}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)((ah^2+2cg^2)(fg-eh)+2cdgh^2)}{2\sqrt{c}\,h^4}-\frac{\sqrt{ah^2+cg^2}\,(dh^2-egh+fg^2)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\,\sqrt{ah^2+cg^2}}\right)}{h^4}+\frac{\sqrt{a+cx^2}\,(2(dh^2-egh+fg^2)-hx(fg-eh))}{2h^3}+\frac{f(a+cx^2)^{3/2}}{3ch}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((2*(f*g^2 - e*g*h + d*h^2) - h*(f*g - e*h)*x)*Sqrt[a + c*x^2])/(2*h^3) + (
f*(a + c*x^2)^(3/2))/(3*c*h) - ((2*c*d*g*h^2 + (f*g - e*h)*(2*c*g^2 + a*h^2
))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*h^4) - (Sqrt[c*g^2 + a*
h^2]*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2])])/h^4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{g+hx}\,dx = \frac{f(a+cx^2)^{3/2}}{3ch} + \frac{\int \frac{(3cdh^2-3ch(fg-eh)x)\sqrt{a+cx^2}}{g+hx}\,dx}{3ch^2}$$

$$= \frac{(2(fg^2-egh+dh^2)-h(fg-eh)x)\sqrt{a+cx^2}}{2h^3} + \frac{f(a+cx^2)^{3/2}}{3ch} + \frac{\int \frac{3ac}{}}{}$$

$$= \frac{(2(fg^2-egh+dh^2)-h(fg-eh)x)\sqrt{a+cx^2}}{2h^3} + \frac{f(a+cx^2)^{3/2}}{3ch} + \frac{((cg}{}$$

$$= \frac{(2(fg^2-egh+dh^2)-h(fg-eh)x)\sqrt{a+cx^2}}{2h^3} + \frac{f(a+cx^2)^{3/2}}{3ch} - \frac{((cg}{}$$

$$= \frac{(2(fg^2-egh+dh^2)-h(fg-eh)x)\sqrt{a+cx^2}}{2h^3} + \frac{f(a+cx^2)^{3/2}}{3ch} - \frac{(2cd}{}$$

Mathematica [A]
time = 0.74, size = 213, normalized size = 1.03

$$\frac{\frac{h\sqrt{a+cx^2}\,(2afh^2+3ch(-2eg+2dh+ehx)+cf(6g^2-3ghx+2h^2x^2))}{c} + 12\sqrt{-cg^2-ah^2}\,(fg^2+h(-eg+dh))\tan^{-1}\left(\frac{\sqrt{c}\,(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right) + \frac{3(ah^2(fg-eh)+2c(fg^3+gh(-eg+dh)))\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{6h^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(2*a*f*h^2 + 3*c*h*(-2*e*g + 2*d*h + e*h*x) + c*f*(6*g^
2 - 3*g*h*x + 2*h^2*x^2)))/c + 12*Sqrt[-(c*g^2) - a*h^2]*(f*g^2 + h*(-(e*g)
 + d*h))*ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h
^2]] + (3*(a*h^2*(f*g - e*h) + 2*c*(f*g^3 + g*h*(-(e*g) + d*h)))*Log[-(Sqrt
[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/(6*h^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(183) = 366$.
time = 0.11, size = 376, normalized size = 1.83

method	result

default	$\frac{\frac{fh\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{3c}+eh\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)-gf\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)}{h^{2}}+$ — $(d$
risch	$\frac{\left(2f\,h^{2}c\,x^{2}+3ce\,h^{2}x-3cfghx+2af\,h^{2}+6cd\,h^{2}-6cegh+6cf\,g^{2}\right)\sqrt{c\,x^{2}+a}}{6c\,h^{3}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)ae}{2h\sqrt{c}}-\frac{\ln\left(x\sqrt{c}+\sqrt{c\right.}{2h^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] 1/h^2*(1/3*f*h*(c*x^2+a)^(3/2)/c+e*h*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*l
n(x*c^(1/2)+(c*x^2+a)^(1/2)))-g*f*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x
*c^(1/2)+(c*x^2+a)^(1/2))))+(d*h^2-e*g*h+f*g^2)/h^3*(((x+1/h*g)^2*c-2*c*g/h
*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(
1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^
2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+
2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/
h^2)^(1/2))/(x+1/h*g)))
```

Maxima [A]

time = 0.35, size = 367, normalized size = 1.78

$$-\frac{\sqrt{cx^2+a}\,fgx}{2\,h^2}-\frac{\sqrt{c}\,fg^3\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4}-\frac{\sqrt{c}\,dg\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^2}-\frac{afg\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}\,h^2}+\frac{\sqrt{a+\frac{cg^2}{h^2}}\,fg^2\,\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{h^3}+\frac{\sqrt{a+\frac{cg^2}{h^2}}\,d\,\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{h}+\frac{\sqrt{cx^2+a}\,xe}{2\,h}+\frac{\sqrt{c}\,g^2\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e}{h^3}+\frac{a\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e}{2\sqrt{c}\,h}-\frac{\sqrt{a+\frac{cg^2}{h^2}}\,g\,\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)e}{h^2}+\frac{\sqrt{cx^2+a}\,fg^2}{h^3}+\frac{\sqrt{cx^2+a}\,d}{h}+\frac{(cx^2+a)^{\frac{3}{2}}f}{3\,ch}-\frac{\sqrt{cx^2+a}\,ge}{h^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^2 + a)*f*g*x/h^2 - sqrt(c)*f*g^3*arcsinh(c*x/sqrt(a*c))/h^4 -
 sqrt(c)*d*g*arcsinh(c*x/sqrt(a*c))/h^2 - 1/2*a*f*g*arcsinh(c*x/sqrt(a*c))/
(sqrt(c)*h^2) + sqrt(a + c*g^2/h^2)*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x
+ g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 + sqrt(a + c*g^2/h^2)*d*arcsinh(c
*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h + 1/2*sqrt(
c*x^2 + a)*x*e/h + sqrt(c)*g^2*arcsinh(c*x/sqrt(a*c))*e/h^3 + 1/2*a*arcsinh
(c*x/sqrt(a*c))*e/(sqrt(c)*h) - sqrt(a + c*g^2/h^2)*g*arcsinh(c*g*x/(sqrt(a
*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))*e/h^2 + sqrt(c*x^2 + a)*f
*g^2/h^3 + sqrt(c*x^2 + a)*d/h + 1/3*(c*x^2 + a)^(3/2)*f/(c*h) - sqrt(c*x^2
 + a)*g*e/h^2
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}\,(d + ex + fx^2)}{g + hx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g),x)
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x), x)
```

Giac [**A**]
time = 3.59, size = 278, normalized size = 1.35

$$\frac{1}{6}\sqrt{cx^2+a}\left(\left(\frac{2\,fx}{h} - \frac{3\,(cfgh^8 - ch^9e)}{ch^{10}}\right)x + \frac{2\,(3\,cfg^2h^7 + 3\,cdh^9 + afh^9 - 3\,cgh^8e)}{ch^{10}}\right) + \frac{2\,(cfg^4 + cdg^2h^2 + afg^2h^2 + adh^4 - cg^3he - agh^3e)\arctan\left(-\frac{\left(\sqrt{c}\,x - \sqrt{cx^2+a}\right)h + \sqrt{c}\,g}{\sqrt{-cg^2 - ah^2}}\right)}{\sqrt{-cg^2 - ah^2}\,h^4} + \frac{\left(2\,c^{\frac{3}{2}}fg^3 + 2\,c^{\frac{3}{2}}dgh^2 + a\sqrt{c}\,fgh^2 - 2\,c^{\frac{3}{2}}g^2he - a\sqrt{c}\,h^3e\right)\log\left(\left|-\sqrt{c}\,x + \sqrt{cx^2+a}\right|\right)}{2\,ch^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="giac")
[Out] 1/6*sqrt(c*x^2 + a)*((2*f*x/h - 3*(c*f*g*h^8 - c*h^9*e)/(c*h^10))*x + 2*(3*
c*f*g^2*h^7 + 3*c*d*h^9 + a*f*h^9 - 3*c*g*h^8*e)/(c*h^10)) + 2*(c*f*g^4 + c
*d*g^2*h^2 + a*f*g^2*h^2 + a*d*h^4 - c*g^3*h*e - a*g*h^3*e)*arctan(-((sqrt(
c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 -
 a*h^2)*h^4) + 1/2*(2*c^(3/2)*f*g^3 + 2*c^(3/2)*d*g*h^2 + a*sqrt(c)*f*g*h^2
 - 2*c^(3/2)*g^2*h*e - a*sqrt(c)*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a
)))/(c*h^4)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x),x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

3.83 $\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^2}\,dx$

Optimal. Leaf size=308

$$-\frac{(2(ah^2(2fg-eh)+cg(3fg^2-h(2eg-dh)))-h(afh^2+c(3fg^2-2h(eg-dh)))\,x)\,\sqrt{a+cx^2}}{2h^3\,(cg^2+ah^2)}-\frac{(fg^2-e}{h\,(cg}$$

```
[Out] -(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)+1/2*(a*f*h^2+2
*c*(3*f*g^2-h*(-d*h+2*e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^4/c^(1/2)
+(a*h^2*(-e*h+2*f*g)+c*g*(3*f*g^2-h*(-d*h+2*e*g)))*arctanh((-c*g*x+a*h)/(a*
h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^4/(a*h^2+c*g^2)^(1/2)-1/2*(2*a*h^2*(-e*
h+2*f*g)+2*c*g*(3*f*g^2-h*(-d*h+2*e*g))-h*(a*f*h^2+c*(3*f*g^2-2*h*(-d*h+e*g
)))*x)*(c*x^2+a)^(1/2)/h^3/(a*h^2+c*g^2)
```

Rubi [A]
time = 0.30, antiderivative size = 303, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1665, 829, 858, 223, 212, 739}

$$-\frac{(a+cx^2)^{3/2}(dh^2-egh+fg^2)}{h(g+hx)(ah^2+cg^2)}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(afh^2-2ch(2eg-dh)+6cfg^2)}{2\sqrt{c}\,h^4}+\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(ah^2(2fg-eh)-cgh(2eg-dh)+3cfg^3)}{h^4\sqrt{ah^2+cg^2}}-\frac{\sqrt{a+cx^2}\,(2(ah^2(2fg-eh)-cgh(2eg-dh)+3cfg^3)-hx(afh^2-2ch(eg-dh)+3cfg^2))}{2h^3(ah^2+cg^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -1/2*((2*(3*c*f*g^3 - c*g*h*(2*e*g - d*h) + a*h^2*(2*f*g - e*h)) - h*(3*c*f
*g^2 + a*f*h^2 - 2*c*h*(e*g - d*h))*x)*Sqrt[a + c*x^2])/(h^3*(c*g^2 + a*h^2
)) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(3/2))/(h*(c*g^2 + a*h^2)*(g + h*
x)) + ((6*c*f*g^2 + a*f*h^2 - 2*c*h*(2*e*g - d*h))*ArcTanh[(Sqrt[c]*x)/Sqrt
[a + c*x^2]])/(2*Sqrt[c]*h^4) + ((3*c*f*g^3 - c*g*h*(2*e*g - d*h) + a*h^2*(
2*f*g - e*h))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])
/(h^4*Sqrt[c*g^2 + a*h^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^2}\,dx = -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{3/2}}{h\,(cg^2+ah^2)\,(g+hx)} - \frac{\int \frac{\left(-cdg+afg-aeh-\left(afh-c\left(2eg-\frac{3fg^2}{h}-2dh\right)\right)\cdots\right.}{g+hx}}{cg^2+ah^2}\cdots$$

$$= -\frac{(2(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh))-h(3cfg^2+afh^2-2ch(e\cdots}{2h^3\,(cg^2+ah^2)}$$

$$= -\frac{(2(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh))-h(3cfg^2+afh^2-2ch(e\cdots}{2h^3\,(cg^2+ah^2)}$$

$$= -\frac{(2(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh))-h(3cfg^2+afh^2-2ch(e\cdots}{2h^3\,(cg^2+ah^2)}$$

$$= -\frac{(2(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh))-h(3cfg^2+afh^2-2ch(e\cdots}{2h^3\,(cg^2+ah^2)}$$

Mathematica [A]
time = 1.07, size = 217, normalized size = 0.70

$$\frac{\frac{h\sqrt{a+cx^2}\,(2h(2eg-dh+ehx)+f(-6g^2-3ghx+h^2x^2))}{g+hx}+\frac{4(3cfg^3+cgh(-2eg+dh)+ah^2(2fg-eh))\tan^{-1}\left(\frac{\sqrt{c}\,(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{\sqrt{-cg^2-ah^2}}-\frac{(6cfg^2+afh^2+2ch(-2eg+dh))\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{2h^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(2*h*(2*e*g - d*h + e*h*x) + f*(-6*g^2 - 3*g*h*x + h^2*
x^2)))/(g + h*x) + (4*(3*c*f*g^3 + c*g*h*(-2*e*g + d*h) + a*h^2*(2*f*g - e*
h))*ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])
/Sqrt[-(c*g^2) - a*h^2] - ((6*c*f*g^2 + a*f*h^2 + 2*c*h*(-2*e*g + d*h))*Log
[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/(2*h^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 860 vs. 2(287) = 574.
time = 0.11, size = 861, normalized size = 2.80

method	result

default	$\frac{f\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)}{h^{2}}+\frac{(eh-2gf)\left(\sqrt{\left(x+\frac{g}{h}\right)^{2}c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{a\,h^{2}+c\,g^{2}}{h^{2}}}-\frac{\sqrt{c}\,g\ln\left(\cdots\right.}{\cdots}\right.}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^2*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))+1
/h^3*(e*h-2*f*g)*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)
-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*
g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln
((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h
*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/h^4*(d*h^
2-e*g*h+f*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/
h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g*h/(a*h^2+c*g^2)*(((x+1/h*g)^2*c-2*c*g/h*(
x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/
2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)
/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*
((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^
2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*
((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c
*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)
^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))))
```

<u>Maxima</u> [A]
time = 0.36, size = 483, normalized size = 1.57

$$-\frac{\sqrt{cx^2+a}\,fg^2}{h^4x+gh^3}+\frac{\sqrt{cx^2+a}\,ge}{h^3x+gh^2}-\frac{\sqrt{cx^2+a}\,d}{h^2x+gh}+\frac{\sqrt{cx^2+a}\,fx}{2\,h^2}+\frac{3\sqrt{c}\,fg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4}+\frac{\sqrt{c}\,d\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^2}+\frac{af\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}\,h^2}-\frac{cfg^3\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}\,h^5}-\frac{cdg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}\,h^3}-\frac{2\sqrt{a+\frac{cg^2}{h^2}}\,fg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{h^3}-\frac{2\sqrt{c}\,g\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e}{h^3}+\frac{cg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)e}{\sqrt{a+\frac{cg^2}{h^2}}\,h^4}+\frac{\sqrt{a+\frac{cg^2}{h^2}}\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)e}{h^2}-\frac{2\sqrt{cx^2+a}\,fg}{h^3}+\frac{\sqrt{cx^2+a}\,e}{h^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2 + a)*f*g^2/(h^4*x + g*h^3) + sqrt(c*x^2 + a)*g*e/(h^3*x + g*h^2
) - sqrt(c*x^2 + a)*d/(h^2*x + g*h) + 1/2*sqrt(c*x^2 + a)*f*x/h^2 + 3*sqrt(
c)*f*g^2*arcsinh(c*x/sqrt(a*c))/h^4 + sqrt(c)*d*arcsinh(c*x/sqrt(a*c))/h^2
+ 1/2*a*f*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h^2) - c*f*g^3*arcsinh(c*g*x/(sqr
t(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h
^5) - c*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
 + g)))/(sqrt(a + c*g^2/h^2)*h^3) - 2*sqrt(a + c*g^2/h^2)*f*g*arcsinh(c*g*x
/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 - 2*sqrt(c)*g
*arcsinh(c*x/sqrt(a*c))*e/h^3 + c*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)
) - a*h/(sqrt(a*c)*abs(h*x + g)))*e/(sqrt(a + c*g^2/h^2)*h^4) + sqrt(a + c*
g^2/h^2)*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x +
g)))*e/h^2 - 2*sqrt(c*x^2 + a)*f*g/h^3 + sqrt(c*x^2 + a)*e/h^2
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}\,(d + ex + fx^2)}{(g + hx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**2,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + a}\,\left(f\,x^2 + e\,x + d\right)}{{\left(g + h\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

3.84 $\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^3}\,dx$

Optimal. Leaf size=296

$$\frac{(2(3fg-eh)\left(cg^2+ah^2\right)+h(2afh^2+c(3fg^2-h(eg-dh)))\,x)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)}-\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{2h\left(cg^2+ah^2\right)(g+hx)^2}$$

```
[Out] -1/2*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^2-1/2*(2*a
^2*f*h^4+2*c^2*g^3*(-e*h+3*f*g)+a*c*h^2*(9*f*g^2-h*(-d*h+3*e*g)))*arctanh((
-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^4/(a*h^2+c*g^2)^(3/2)-(-
e*h+3*f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/h^4+1/2*(2*(-e*h+3*f*
g)*(a*h^2+c*g^2)+h*(2*a*f*h^2+c*(3*f*g^2-h*(-d*h+e*g)))*x)*(c*x^2+a)^(1/2)/
h^3/(a*h^2+c*g^2)/(h*x+g)
```

Rubi [A]
time = 0.30, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1665, 827, 858, 223, 212, 739}

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4+ach^2(9fg^2-h(3eg-dh))+2c^2g^3(3fg-eh)\right)}{2h^4\left(ah^2+cg^2\right)^{3/2}}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}+\frac{\sqrt{a+cx^2}\left(hx(2afh^2-ch(eg-dh)+3cfg^2)+2(ah^2+cg^2)(3fg-eh)\right)}{2h^3(g+hx)\left(ah^2+cg^2\right)}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(3fg-eh)}{h^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((2*(3*f*g - e*h)*(c*g^2 + a*h^2) + h*(3*c*f*g^2 + 2*a*f*h^2 - c*h*(e*g - d
*h))*x)*Sqrt[a + c*x^2])/(2*h^3*(c*g^2 + a*h^2)*(g + h*x)) - ((f*g^2 - e*g*
h + d*h^2)*(a + c*x^2)^(3/2))/(2*h*(c*g^2 + a*h^2)*(g + h*x)^2) - (Sqrt[c]*
(3*f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/h^4 - ((2*a^2*f*h^4 + 2
*c^2*g^3*(3*f*g - e*h) + a*c*h^2*(9*f*g^2 - h*(3*e*g - d*h)))*ArcTanh[(a*h
- c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*h^4*(c*g^2 + a*h^2)^(3/
2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^3}\,dx = -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{3/2}}{2h\,(cg^2+ah^2)\,(g+hx)^2} - \frac{\int \frac{\left(-2(cdg-afg+aeh)-\left(2afh-c\left(eg-\frac{3fg^2}{h}-dh\right.\right.\right.}{(g+hx)^2}}{2\,(cg^2+ah^2)}$$

$$= \frac{(2(3fg-eh)\,(cg^2+ah^2)+h(3cfg^2+2afh^2-ch(eg-dh))\,x)\,\sqrt{a+cx^2}}{2h^3\,(cg^2+ah^2)\,(g+hx)}$$

$$= \frac{(2(3fg-eh)\,(cg^2+ah^2)+h(3cfg^2+2afh^2-ch(eg-dh))\,x)\,\sqrt{a+cx^2}}{2h^3\,(cg^2+ah^2)\,(g+hx)}$$

$$= \frac{(2(3fg-eh)\,(cg^2+ah^2)+h(3cfg^2+2afh^2-ch(eg-dh))\,x)\,\sqrt{a+cx^2}}{2h^3\,(cg^2+ah^2)\,(g+hx)}$$

$$= \frac{(2(3fg-eh)\,(cg^2+ah^2)+h(3cfg^2+2afh^2-ch(eg-dh))\,x)\,\sqrt{a+cx^2}}{2h^3\,(cg^2+ah^2)\,(g+hx)}$$

Mathematica [A]
time = 1.87, size = 280, normalized size = 0.95

$$\frac{\frac{h\sqrt{a+cx^2}\,(ah^2(-h(eg+dh+2ehx)+f(5g^2+8ghx+2h^2x^2))+cg(dh^3x-egh(2g+3hx)+fg(6g^2+9ghx+2h^2x^2)))}{(cg^2+ah^2)(g+hx)^2}+\frac{2(2a^2fh^4+2c^2g^3(3fg-eh)+ach^2(9fg^2+h(-3eg+dh)))\tan^{-1}\left(\frac{\sqrt{c}\,(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{(-cg^2-ah^2)^{3/2}}+2\sqrt{c}\,(3fg-eh)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{2h^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(a*h^2*(-(h*(e*g + d*h + 2*e*h*x)) + f*(5*g^2 + 8*g*h*x
 + 2*h^2*x^2)) + c*g*(d*h^3*x - e*g*h*(2*g + 3*h*x) + f*g*(6*g^2 + 9*g*h*x
+ 2*h^2*x^2))))/((c*g^2 + a*h^2)*(g + h*x)^2) + (2*(2*a^2*f*h^4 + 2*c^2*g^3
*(3*f*g - e*h) + a*c*h^2*(9*f*g^2 + h*(-3*e*g + d*h)))*ArcTan[(Sqrt[c]*(g +
 h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(-(c*g^2) - a*h^2)^(3/2
) + 2*Sqrt[c]*(3*f*g - e*h)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2*h^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1720 vs. $2(274) = 548$.
time = 0.11, size = 1721, normalized size = 5.81

method	result	size
default	Expression too large to display	1721
risch	Expression too large to display	3596

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^3*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/
h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c
*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+
c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c
*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+(e*h-2*f*g)/h^4*(-1/(a
*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^
2)^(3/2)-c*g*h/(a*h^2+c*g^2)*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2
)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c
*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h
^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(
1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
+2*c/(a*h^2+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/
h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h
^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g
)+(a*h^2+c*g^2)/h^2)^(1/2))))+(d*h^2-e*g*h+f*g^2)/h^5*(-1/2/(a*h^2+c*g^2)*h
^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+1/
2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/
h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g*h/(a*h^2+c*g^2)*(((x+1/h*g)^2*c-2*
c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g)
)/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2
+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/
h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*
g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)-2*c*
g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(
a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x
+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2+c*g^
2)*h^2*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g
/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+
c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2
+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*
c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 934 vs. $2(278) = 556$.

time = 0.34, size = 934, normalized size = 3.16

$$-\frac{\sqrt{cx^2+a}\,cfg^3}{2\,(cg^2h^4x+ah^6x+cg^3h^3+agh^5)}-\frac{(cx^2+a)^{\frac{3}{2}}fg^2}{2\,(cg^2h^3x^2+ah^5x^2+2\,cg^3h^2x+2\,agh^4x+cg^4h+ag^2h^3)}+\frac{\sqrt{cx^2+a}\,cfg^2}{2\,(cg^2h^3+ah^5)}+\frac{\sqrt{cx^2+a}\,cg^2e}{2\,(cg^2h^3x+ah^5x+cg^3h^2+agh^4)}-\frac{\sqrt{cx^2+a}\,cdg}{2\,(cg^2h^2x+ah^4x+cg^3h+agh^3)}+\frac{(cx^2+a)^{\frac{3}{2}}ge}{2\,(cg^2h^2x^2+ah^4x^2+2\,cg^3hx+2\,agh^3x+cg^4+ag^2h^2)}-\frac{\sqrt{cx^2+a}\,cge}{2\,(cg^2h^2+ah^4)}-\frac{(cx^2+a)^{\frac{3}{2}}d}{2\left(cg^2hx^2+ah^3x^2+2\,cg^3x+2\,agh^2x+\frac{cg^4}{h}+ag^2h\right)}+\frac{\sqrt{cx^2+a}\,cd}{2\,(cg^2h+ah^3)}+\frac{2\sqrt{cx^2+a}\,fg}{h^4x+gh^3}-\frac{\sqrt{cx^2+a}\,e}{h^3x+gh^2}-\frac{3\sqrt{c}\,fg\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4}-\frac{c^2fg^4\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^7}-\frac{c^2dg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^5}+\frac{5\,cfg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{2\sqrt{a+\frac{cg^2}{h^2}}\,h^5}+\frac{cd\,\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{2\sqrt{a+\frac{cg^2}{h^2}}\,h^3}+\frac{\sqrt{a+\frac{cg^2}{h^2}}\,f\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{h^3}+\frac{\sqrt{c}\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e}{h^3}+\frac{c^2g^3\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)e}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^6}-\frac{3\,cg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)e}{2\sqrt{a+\frac{cg^2}{h^2}}\,h^4}+\frac{\sqrt{cx^2+a}\,f}{h^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^2 + a)*c*f*g^3/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3 + a*g*h^5)
- 1/2*(c*x^2 + a)^(3/2)*f*g^2/(c*g^2*h^3*x^2 + a*h^5*x^2 + 2*c*g^3*h^2*x +
2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 1/2*sqrt(c*x^2 + a)*c*f*g^2/(c*g^2*h^3
```

```
 + a*h^5) + 1/2*sqrt(c*x^2 + a)*c*g^2*e/(c*g^2*h^3*x + a*h^5*x + c*g^3*h^2
+ a*g*h^4) - 1/2*sqrt(c*x^2 + a)*c*d*g/(c*g^2*h^2*x + a*h^4*x + c*g^3*h + a
*g*h^3) + 1/2*(c*x^2 + a)^(3/2)*g*e/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h*
x + 2*a*g*h^3*x + c*g^4 + a*g^2*h^2) - 1/2*sqrt(c*x^2 + a)*c*g*e/(c*g^2*h^2
 + a*h^4) - 1/2*(c*x^2 + a)^(3/2)*d/(c*g^2*h*x^2 + a*h^3*x^2 + 2*c*g^3*x +
2*a*g*h^2*x + c*g^4/h + a*g^2*h) + 1/2*sqrt(c*x^2 + a)*c*d/(c*g^2*h + a*h^3
) + 2*sqrt(c*x^2 + a)*f*g/(h^4*x + g*h^3) - sqrt(c*x^2 + a)*e/(h^3*x + g*h^
2) - 3*sqrt(c)*f*g*arcsinh(c*x/sqrt(a*c))/h^4 - 1/2*c^2*f*g^4*arcsinh(c*g*x
/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^
(3/2)*h^7) - 1/2*c^2*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sq
rt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^5) + 5/2*c*f*g^2*arcsinh(c*
g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^
2/h^2)*h^5) + 1/2*c*d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*
c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^3) + sqrt(a + c*g^2/h^2)*f*arcsinh
(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 + sqrt(
c)*arcsinh(c*x/sqrt(a*c))*e/h^3 + 1/2*c^2*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(
h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))*e/((a + c*g^2/h^2)^(3/2)*h^6) - 3
/2*c*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)
))*e/(sqrt(a + c*g^2/h^2)*h^4) + sqrt(c*x^2 + a)*f/h^3
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**3,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(278) = 556$.
time = 3.19, size = 923, normalized size = 3.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="giac")

[Out] -(6*c^2*f*g^4 + 9*a*c*f*g^2*h^2 + a*c*d*h^4 + 2*a^2*f*h^4 - 2*c^2*g^3*h*e - 3*a*c*g*h^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c*g^2*h^4 + a*h^6)*sqrt(-c*g^2 - a*h^2)) + sqrt(c*x^2 + a)*f/h^3 + (6*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*f*g^4*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*d*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g^2*h^3 + (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*d*h^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*g^3*h^2*e - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*g*h^4*e + 10*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*d*g^3*h^2 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*f*g^3*h^2 - (sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*g^4*h*e - (sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*g^2*h^3*e + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*h^5*e - 14*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*f*g^4*h - 2*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2*h^3 - 11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^3 + (sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*d*h^5 + 8*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*g^3*h^2*e + 5*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*g*h^4*e + 5*a^2*c^(3/2)*f*g^3*h^2 + a^2*c^(3/2)*d*g*h^4 + 4*a^3*sqrt(c)*f*g*h^4 - 3*a^2*c^(3/2)*g^2*h^3*e - 2*a^3*sqrt(c)*h^5*e)/((c*g^2*h^4 + a*h^6)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2) + (3*sqrt(c)*f*g - sqrt(c)*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^4

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)

[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)

3.85 $\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^4}\,dx$

Optimal. Leaf size=314

$$-\frac{\left(2c^2fg^5+a^2eh^5+acgh^2(3fg^2+dh^2)+h(2a^2fh^4+acgh^2(6fg-eh)+c^2(3fg^4-dg^2h^2))\,x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)^2(g+hx)^2}-$$

```
[Out] -1/3*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^3+1/2*c*(2
*c^2*f*g^5+a^2*h^4*(-e*h+4*f*g)+a*c*g*h^2*(-d*h^2+5*f*g^2))*arctanh((-c*g*x
+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^4/(a*h^2+c*g^2)^(5/2)+f*arctan
h(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/h^4-1/2*(2*c^2*f*g^5+a^2*e*h^5+a*c*g*h
^2*(d*h^2+3*f*g^2)+h*(2*a^2*f*h^4+a*c*g*h^2*(-e*h+6*f*g)+c^2*(-d*g^2*h^2+3*
f*g^4))*x)*(c*x^2+a)^(1/2)/h^3/(a*h^2+c*g^2)^2/(h*x+g)^2
```

Rubi [A]
time = 0.29, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1665, 825, 858, 223, 212, 739}

$$\frac{c\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(a^2h^4(4fg-eh)+acgh^2(5fg^2-dh^2)+2c^2fg^5\right)}{2h^4\left(ah^2+cg^2\right)^{5/2}}-\frac{\sqrt{a+cx^2}\left(hx(2a^2fh^4+acgh^2(6fg-eh)+c^2(3fg^4-dg^2h^2))+a^2eh^5+acgh^2(dh^2+3fg^2)+2c^2fg^5\right)}{2h^3(g+hx)^2\left(ah^2+cg^2\right)^2}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}+\frac{\sqrt{c}\,f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{h^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -1/2*((2*c^2*f*g^5 + a^2*e*h^5 + a*c*g*h^2*(3*f*g^2 + d*h^2) + h*(2*a^2*f*h
^4 + a*c*g*h^2*(6*f*g - e*h) + c^2*(3*f*g^4 - d*g^2*h^2))*x)*Sqrt[a + c*x^2
])/(h^3*(c*g^2 + a*h^2)^2*(g + h*x)^2) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^
2)^(3/2))/(3*h*(c*g^2 + a*h^2)*(g + h*x)^3) + (Sqrt[c]*f*ArcTanh[(Sqrt[c]*x
)/Sqrt[a + c*x^2]])/h^4 + (c*(2*c^2*f*g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h
^2*(5*f*g^2 - d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c
*x^2])])/(2*h^4*(c*g^2 + a*h^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^4}\,dx = -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{3/2}}{3h\,(cg^2+ah^2)\,(g+hx)^3} - \frac{\int \frac{\left(-3(cdg-afg+aeh)-3f\left(\frac{cg^2}{h}+ah\right)x\right)\sqrt{a+}}{(g+hx)^3}}{3\,(cg^2+ah^2)}$$

$$= -\frac{(2c^2fg^5+a^2eh^5+acgh^2(3fg^2+dh^2)+h(2a^2fh^4+acgh^2(6fg-eh)+}{2h^3\,(cg^2+ah^2)^2\,(g+hx)^2}$$

$$= -\frac{(2c^2fg^5+a^2eh^5+acgh^2(3fg^2+dh^2)+h(2a^2fh^4+acgh^2(6fg-eh)+}{2h^3\,(cg^2+ah^2)^2\,(g+hx)^2}$$

$$= -\frac{(2c^2fg^5+a^2eh^5+acgh^2(3fg^2+dh^2)+h(2a^2fh^4+acgh^2(6fg-eh)+}{2h^3\,(cg^2+ah^2)^2\,(g+hx)^2}$$

$$= -\frac{(2c^2fg^5+a^2eh^5+acgh^2(3fg^2+dh^2)+h(2a^2fh^4+acgh^2(6fg-eh)+}{2h^3\,(cg^2+ah^2)^2\,(g+hx)^2}$$

Mathematica [A]
time = 10.53, size = 382, normalized size = 1.22

$$\frac{\frac{h\sqrt{a+cx^2}\left(-2(fg^2+h(-eg+dh))+\frac{(7cfg^3+cgh(-4eg+dh)-3ah^2(-2fg+eh))(g+hx)}{cg^2+ah^2}-\frac{(6a^2fh^4+c^2(11fg^4-g^2h(2eg+dh))+ach^2(20fg^2+h(-5eg+2dh)))(g+hx)^2}{(cg^2+ah^2)^2}\right)}{(g+hx)^3}-\frac{3c(2c^2fg^5+a^2h^4(4fg-eh)+acgh^2(5fg^2-dh^2))\log(g+hx)}{(cg^2+ah^2)^{5/2}}+6\sqrt{c}\,f\log\left(cx+\sqrt{c}\sqrt{a+cx^2}\right)+\frac{3c(2c^2fg^5+a^2h^4(4fg-eh)+acgh^2(5fg^2-dh^2))\log\left(ah-cgx+\sqrt{cg^2+ah^2}\sqrt{a+cx^2}\right)}{(cg^2+ah^2)^{5/2}}}{6h^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(-2*(f*g^2 + h*(-(e*g) + d*h)) + ((7*c*f*g^3 + c*g*h*(-
4*e*g + d*h) - 3*a*h^2*(-2*f*g + e*h))*(g + h*x))/(c*g^2 + a*h^2) - ((6*a^2
*f*h^4 + c^2*(11*f*g^4 - g^2*h*(2*e*g + d*h)) + a*c*h^2*(20*f*g^2 + h*(-5*e
*g + 2*d*h)))*(g + h*x)^2)/(c*g^2 + a*h^2)^2))/(g + h*x)^3 - (3*c*(2*c^2*f*
g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h^2*(5*f*g^2 - d*h^2))*Log[g + h*x])/(c
*g^2 + a*h^2)^(5/2) + 6*Sqrt[c]*f*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (3*c
*(2*c^2*f*g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h^2*(5*f*g^2 - d*h^2))*Log[a*
h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(5/2))/(6
*h^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2447 vs. $2(292) = 584$.
time = 0.11, size = 2448, normalized size = 7.80

method	result	size

default	Expression too large to display	2448

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^4*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h
^2+c*g^2)/h^2)^(3/2)-c*g*h/(a*h^2+c*g^2)*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+
(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/
h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*
h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c
*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))
/(x+1/h*g)))+2*c/(a*h^2+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g
)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2
-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*
g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))))+(d*h^2-e*g*h+f*g^2)/h^6*(-1/3/(a*
h^2+c*g^2)*h^2/(x+1/h*g)^3*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h
^2)^(3/2)+c*g*h/(a*h^2+c*g^2)*(-1/2/(a*h^2+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g
)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+1/2*c*g*h/(a*h^2+c*g^2)*(-
1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2
)/h^2)^(3/2)-c*g*h/(a*h^2+c*g^2)*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c
*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c
-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^
2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^
2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*
g)))+2*c/(a*h^2+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*
c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g
^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1
/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2+c*g^2)*h^2*(((x+1/h*g)^2*c-2
*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g
))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^
2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1
/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c
*g^2)/h^2)^(1/2))/(x+1/h*g)))))+(e*h-2*f*g)/h^5*(-1/2/(a*h^2+c*g^2)*h^2/(x+
1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+1/2*c*g*
h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1
/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g*h/(a*h^2+c*g^2)*(((x+1/h*g)^2*c-2*c*g/h*
(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1
/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2
)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2
*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h
^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c
*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+
c*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g
```

```
)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2+c*g^2)*h^2
*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln(
(-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)
/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2
)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*
(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1782 vs. $2(297) = 594$.
time = 0.37, size = 1782, normalized size = 5.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^2 + a)*c^2*f*g^4/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x
 + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) - 1/2*(c*x^2 + a)^(3/2)*c*f*g^3
/(c^2*g^4*h^3*x^2 + 2*a*c*g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a
*c*g^3*h^4*x + 2*a^2*g*h^6*x + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 1
/2*sqrt(c*x^2 + a)*c^2*f*g^3/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) + 1/2*
sqrt(c*x^2 + a)*c^2*g^3*e/(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^
2*g^5*h^2 + 2*a*c*g^3*h^4 + a^2*g*h^6) - 1/2*sqrt(c*x^2 + a)*c^2*d*g^2/(c^2
*g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3*h^3 + a^2*
g*h^5) + 1/2*(c*x^2 + a)^(3/2)*c*g^2*e/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2
 + a^2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6
+ 2*a*c*g^4*h^2 + a^2*g^2*h^4) - 1/2*sqrt(c*x^2 + a)*c^2*g^2*e/(c^2*g^4*h^2
 + 2*a*c*g^2*h^4 + a^2*h^6) - 1/2*(c*x^2 + a)^(3/2)*c*d*g/(c^2*g^4*h*x^2 +
2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 + 2*c^2*g^5*x + 4*a*c*g^3*h^2*x + 2*a^2*g*h
^4*x + c^2*g^6/h + 2*a*c*g^4*h + a^2*g^2*h^3) + 1/2*sqrt(c*x^2 + a)*c^2*d*g
/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2*h^5) - 1/3*(c*x^2 + a)^(3/2)*f*g^2/(c*g^2
*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*
a*g^2*h^4*x + c*g^5*h + a*g^3*h^3) + sqrt(c*x^2 + a)*c*f*g^2/(c*g^2*h^4*x +
 a*h^6*x + c*g^3*h^3 + a*g*h^5) + (c*x^2 + a)^(3/2)*f*g/(c*g^2*h^3*x^2 + a*
h^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) - sqrt(c*x^2 +
 a)*c*f*g/(c*g^2*h^3 + a*h^5) + 1/3*(c*x^2 + a)^(3/2)*g*e/(c*g^2*h^3*x^3 +
a*h^5*x^3 + 3*c*g^3*h^2*x^2 + 3*a*g*h^4*x^2 + 3*c*g^4*h*x + 3*a*g^2*h^3*x +
 c*g^5 + a*g^3*h^2) - 1/2*sqrt(c*x^2 + a)*c*g*e/(c*g^2*h^3*x + a*h^5*x + c*
g^3*h^2 + a*g*h^4) - 1/3*(c*x^2 + a)^(3/2)*d/(c*g^2*h^2*x^3 + a*h^4*x^3 + 3
*c*g^3*h*x^2 + 3*a*g*h^3*x^2 + 3*c*g^4*x + 3*a*g^2*h^2*x + c*g^5/h + a*g^3*
h) - 1/2*(c*x^2 + a)^(3/2)*e/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h*x + 2*a
*g*h^3*x + c*g^4 + a*g^2*h^2) + 1/2*sqrt(c*x^2 + a)*c*e/(c*g^2*h^2 + a*h^4)
 - sqrt(c*x^2 + a)*f/(h^4*x + g*h^3) + sqrt(c)*f*arcsinh(c*x/sqrt(a*c))/h^4
 - 1/2*c^3*f*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*ab
s(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^9) - 1/2*c^3*d*g^3*arcsinh(c*g*x/(sqr
```

```
t(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)
*h^7) + 3/2*c^2*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*
c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) + 1/2*c^2*d*g*arcsinh(c*g*x/(
sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3
/2)*h^5) - 2*c*f*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*
abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^5) + 1/2*c^3*g^4*arcsinh(c*g*x/(sqrt(
a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))*e/((a + c*g^2/h^2)^(5/2)
*h^8) - c^2*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs
(h*x + g)))*e/((a + c*g^2/h^2)^(3/2)*h^6) + 1/2*c*arcsinh(c*g*x/(sqrt(a*c)*
abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))*e/(sqrt(a + c*g^2/h^2)*h^4)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**4,x)
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1719 vs. 2(297) = 594.
time = 5.39, size = 1719, normalized size = 5.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="giac")
[Out] -(2*c^3*f*g^5 + 5*a*c^2*f*g^3*h^2 - a*c^2*d*g*h^4 + 4*a^2*c*f*g*h^4 - a^2*c
*h^5*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 -
 a*h^2))/((c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8)*sqrt(-c*g^2 - a*h^2)) - s
qrt(c)*f*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^4 - 1/3*(18*(sqrt(c)*x -
sqrt(c*x^2 + a))^5*c^3*f*g^5*h^2 + 33*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2
```

```
*f*g^3*h^4 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*d*g*h^6 + 12*(sqrt(c)*
x - sqrt(c*x^2 + a))^5*a^2*c*f*g*h^6 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^
3*g^4*h^3*e - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*g^2*h^5*e - 3*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*a^2*c*h^7*e + 54*(sqrt(c)*x - sqrt(c*x^2 + a))^4*
c^(7/2)*f*g^6*h - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*d*g^4*h^3 + 87*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*f*g^4*h^3 + 3*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a*c^(5/2)*d*g^2*h^5 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c
^(3/2)*f*g^2*h^5 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*d*h^7 - 6*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*sqrt(c)*f*h^7 - 12*(sqrt(c)*x - sqrt(c*
x^2 + a))^4*c^(7/2)*g^5*h^2*e - 24*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2
)*g^3*h^4*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*g*h^6*e + 44*(s
qrt(c)*x - sqrt(c*x^2 + a))^3*c^4*f*g^7 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))^3
*c^4*d*g^5*h^2 + 14*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*f*g^5*h^2 + 14*(s
qrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*d*g^3*h^4 - 96*(sqrt(c)*x - sqrt(c*x^2
+ a))^3*a^2*c^2*f*g^3*h^4 - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*d*g*
h^6 - 36*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c*f*g*h^6 - 8*(sqrt(c)*x - sqr
t(c*x^2 + a))^3*c^4*g^6*h*e - 8*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*g^4*h
^3*e + 30*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*g^2*h^5*e - 78*(sqrt(c)*x
 - sqrt(c*x^2 + a))^2*a*c^(7/2)*f*g^6*h + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^2
*a*c^(7/2)*d*g^4*h^3 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)*f*g^
4*h^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)*d*g^2*h^5 + 12*(sqrt
(c)*x - sqrt(c*x^2 + a))^2*a^4*sqrt(c)*f*h^7 + 12*(sqrt(c)*x - sqrt(c*x^2 +
 a))^2*a*c^(7/2)*g^5*h^2*e + 30*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)
*g^3*h^4*e - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*g*h^6*e + 48*(s
qrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*f*g^5*h^2 - 6*(sqrt(c)*x - sqrt(c*x^2 +
 a))*a^2*c^3*d*g^3*h^4 + 87*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*f*g^3*h^4
 + 9*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*d*g*h^6 + 24*(sqrt(c)*x - sqrt(c
*x^2 + a))*a^4*c*f*g*h^6 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*g^4*h^3*
e - 18*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*g^2*h^5*e + 3*(sqrt(c)*x - sqr
t(c*x^2 + a))*a^4*c*h^7*e - 11*a^3*c^(5/2)*f*g^4*h^3 + a^3*c^(5/2)*d*g^2*h^
5 - 20*a^4*c^(3/2)*f*g^2*h^5 - 2*a^4*c^(3/2)*d*h^7 - 6*a^5*sqrt(c)*f*h^7 +
2*a^3*c^(5/2)*g^3*h^4*e + 5*a^4*c^(3/2)*g*h^6*e)/((c^2*g^4*h^4 + 2*a*c*g^2*
h^6 + a^2*h^8)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x
^2 + a))*sqrt(c)*g - a*h)^3)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

3.86 $\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^5}\,dx$

Optimal. Leaf size=313

$$-\frac{(4c^2dg^2+4a^2fh^2-ac(fg^2-h(5eg-dh)))\,(ah-cgx)\sqrt{a+cx^2}}{8\,(cg^2+ah^2)^3\,(g+hx)^2}-\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{3/2}}{4h\,(cg^2+ah^2)\,(g+hx)^4}+\frac{(4a\cdots}{}$$

```
[Out] -1/4*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^4+1/12*(4*
a*h^2*(-e*h+2*f*g)+c*g*(3*f*g^2+h*(-5*d*h+e*g)))*(c*x^2+a)^(3/2)/h/(a*h^2+c
*g^2)^2/(h*x+g)^3-1/8*a*c*(4*c^2*d*g^2+4*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+5*e*g
)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)
^(7/2)-1/8*(4*c^2*d*g^2+4*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+5*e*g)))*(-c*g*x+a*h
)*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^3/(h*x+g)^2
```

Rubi [A]
time = 0.25, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1665, 821, 735, 739, 212}

$$-\frac{\sqrt{a+cx^2}\,(ah-cgx)\,(4a^2fh^2-ac(fg^2-h(5eg-dh))+4c^2dg^2)}{8(g+hx)^2\,(ah^2+cg^2)^3}-\frac{ac\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\,\sqrt{ah^2+cg^2}}\right)(4a^2fh^2-ac(fg^2-h(5eg-dh))+4c^2dg^2)}{8\,(ah^2+cg^2)^{7/2}}-\frac{(a+cx^2)^{3/2}\,(dh^2-egh+fg^2)}{4h(g+hx)^4\,(ah^2+cg^2)}+\frac{(a+cx^2)^{3/2}\,(4ah^2(2fg-eh)+cgh(eg-5dh)+3cfg^3)}{12h(g+hx)^3\,(ah^2+cg^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] -1/8*((4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 - h*(5*e*g - d*h)))*(a*h - c*
g*x)*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)^3*(g + h*x)^2) - ((f*g^2 - e*g*h + d
*h^2)*(a + c*x^2)^(3/2))/(4*h*(c*g^2 + a*h^2)*(g + h*x)^4) + ((3*c*f*g^3 +
c*g*h*(e*g - 5*d*h) + 4*a*h^2*(2*f*g - e*h))*(a + c*x^2)^(3/2))/(12*h*(c*g^
2 + a*h^2)^2*(g + h*x)^3) - (a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 -
h*(5*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2
])])/(8*(c*g^2 + a*h^2)^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(-2*a*e + (2*c*d)*x)*((a + c*x^2)^p/(2*(m + 1)*(c*d^2
+ a*e^2))), x] - Dist[4*a*c*(p/(2*(m + 1)*(c*d^2 + a*e^2))), Int[(d + e*x)^
(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
```

```
+ a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^5}\,dx = -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{3/2}}{4h\,(cg^2+ah^2)\,(g+hx)^4} - \frac{\int \frac{\left(-4(cdg-afg+aeh)-\left(4afh+c\left(eg+\frac{3fg^2}{h}-dh\right.\right.\right.}{(g+hx)^4}}{4\,(cg^2+ah^2)}$$

$$= -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{3/2}}{4h\,(cg^2+ah^2)\,(g+hx)^4} + \frac{(3cfg^3+cgh(eg-5dh)+4ah^2(2fg-}{12h\,(cg^2+ah^2)^2\,(g+h}$$

$$= -\frac{(4c^2dg^2+4a^2fh^2-ac(fg^2-h(5eg-dh)))\,(ah-cgx)\sqrt{a+cx^2}}{8\,(cg^2+ah^2)^3\,(g+hx)^2} - \frac{(f}{}$$

$$= -\frac{(4c^2dg^2+4a^2fh^2-ac(fg^2-h(5eg-dh)))\,(ah-cgx)\sqrt{a+cx^2}}{8\,(cg^2+ah^2)^3\,(g+hx)^2} - \frac{(f}{}$$

$$= -\frac{(4c^2dg^2+4a^2fh^2-ac(fg^2-h(5eg-dh)))\,(ah-cgx)\sqrt{a+cx^2}}{8\,(cg^2+ah^2)^3\,(g+hx)^2} - \frac{(f}{}$$

Mathematica [**A**]
time = 10.86, size = 439, normalized size = 1.40

$$\frac{1}{24}\left(-\frac{\sqrt{a+cx^2}\left(6(cg^2+ah^2)^3(fg^2+h(-eg+dh))-2(cg^2+ah^2)^2(9cfg^3+cgh(-5eg+dh)-4ah^2(-2fg+eh))(g+hx)+(cg^2+ah^2)(12a^2fh^4+2c^2(9fg^4-g^2h(eg+dh))+ach^2(35fg^2+h(-7eg+3dh)))(g+hx)^2-c(4a^2h^4(7fg-2eh)+acgh^2(19fg^2+h(9eg-13dh))+2c^2(3fg^5+g^3h(eg+dh)))(g+hx)^3\right)}{(cg^2h+ah^3)^3(g+hx)^4}+\frac{3ac(4c^2dg^2+4a^2fh^2-ac(fg^2+h(-5eg+dh)))\log(g+hx)}{(cg^2+ah^2)^{7/2}}-\frac{3ac(4c^2dg^2+4a^2fh^2-ac(fg^2+h(-5eg+dh)))\log\left(ah-cgx+\sqrt{cg^2+ah^2}\sqrt{a+cx^2}\right)}{(cg^2+ah^2)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (-((Sqrt[a + c*x^2]*(6*(c*g^2 + a*h^2)^3*(f*g^2 + h*(-(e*g) + d*h)) - 2*(c*
g^2 + a*h^2)^2*(9*c*f*g^3 + c*g*h*(-5*e*g + d*h) - 4*a*h^2*(-2*f*g + e*h))*
(g + h*x) + (c*g^2 + a*h^2)*(12*a^2*f*h^4 + 2*c^2*(9*f*g^4 - g^2*h*(e*g + d
*h)) + a*c*h^2*(35*f*g^2 + h*(-7*e*g + 3*d*h)))*(g + h*x)^2 - c*(4*a^2*h^4*
(7*f*g - 2*e*h) + a*c*g*h^2*(19*f*g^2 + h*(9*e*g - 13*d*h)) + 2*c^2*(3*f*g^
5 + g^3*h*(e*g + d*h)))*(g + h*x)^3))/((c*g^2*h + a*h^3)^3*(g + h*x)^4)) +
(3*a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 + h*(-5*e*g + d*h)))*Log[g +
 h*x])/(c*g^2 + a*h^2)^(7/2) - (3*a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g
^2 + h*(-5*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^
2]])/(c*g^2 + a*h^2)^(7/2))/24
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3902 vs. $2(293) = 586$.
time = 0.08, size = 3903, normalized size = 12.47

method	result	size
default	Expression too large to display	3903

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (d*h^2-e*g*h+f*g^2)/h^7*(-1/4/(a*h^2+c*g^2)*h^2/(x+1/h*g)^4*((x+1/h*g)^2*c-
2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+5/4*c*g*h/(a*h^2+c*g^2)*(-1/3/(a
*h^2+c*g^2)*h^2/(x+1/h*g)^3*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/
h^2)^(3/2)+c*g*h/(a*h^2+c*g^2)*(-1/2/(a*h^2+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*
g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+1/2*c*g*h/(a*h^2+c*g^2)*(
-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^
2)/h^2)^(3/2)-c*g*h/(a*h^2+c*g^2)*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+
c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*
c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g
^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h
^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h
*g)))+2*c/(a*h^2+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2
*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*
g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+
1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2+c*g^2)*h^2*(((x+1/h*g)^2*c-
2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*
```

```
g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h
^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+
1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+
c*g^2)/h^2)^(1/2))/(x+1/h*g)))))-1/4*c/(a*h^2+c*g^2)*h^2*(-1/2/(a*h^2+c*g^2
)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)
+1/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c
*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g*h/(a*h^2+c*g^2)*(((x+1/h*g)^2*c
-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h
*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*
h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x
+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2
+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)-2
*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*
c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+
((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2+c
*g^2)*h^2*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2
)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h
^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*
h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c
-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))))+(e*h-2*f*g)/h^6*
(-1/3/(a*h^2+c*g^2)*h^2/(x+1/h*g)^3*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2
+c*g^2)/h^2)^(3/2)+c*g*h/(a*h^2+c*g^2)*(-1/2/(a*h^2+c*g^2)*h^2/(x+1/h*g)^2*
((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+1/2*c*g*h/(a*h^2+
c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*
h^2+c*g^2)/h^2)^(3/2)-c*g*h/(a*h^2+c*g^2)*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)
+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1
/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a
*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+
c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)
)/(x+1/h*g)))+2*c/(a*h^2+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*
g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^
2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c
*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2+c*g^2)*h^2*(((x+1/h
*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c
*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/
2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c
*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)
+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))))+f/h^5*(-1/2/(a*h^2+c*g^2)*h^2/(x+1
/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+1/2*c*g*h
/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/
h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g*h/(a*h^2+c*g^2)*(((x+1/h*g)^2*c-2*c*g/h*(
x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/
2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)
/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*
((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^
```

```
2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*
((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c
*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)
^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2+c*g^2)*h^2*
(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((
-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*...
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3416 vs. $2(297) = 594$.
time = 0.43, size = 3416, normalized size = 10.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="maxima")
```

```
[Out] -5/8*sqrt(c*x^2 + a)*c^3*f*g^5/(c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c
*g^2*h^8*x + a^3*h^10*x + c^3*g^7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 +
 a^3*g*h^9) - 5/8*(c*x^2 + a)^(3/2)*c^2*f*g^4/(c^3*g^6*h^3*x^2 + 3*a*c^2*g^
4*h^5*x^2 + 3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x + 6*a*c^2*g
^5*h^4*x + 6*a^2*c*g^3*h^6*x + 2*a^3*g*h^8*x + c^3*g^8*h + 3*a*c^2*g^6*h^3
+ 3*a^2*c*g^4*h^5 + a^3*g^2*h^7) + 5/8*sqrt(c*x^2 + a)*c^3*f*g^4/(c^3*g^6*h
^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) + 5/8*sqrt(c*x^2 + a)*c^3
*g^4*e/(c^3*g^6*h^3*x + 3*a*c^2*g^4*h^5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x +
 c^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2*c*g^3*h^6 + a^3*g*h^8) - 5/8*sqrt(c*
x^2 + a)*c^3*d*g^3/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a^2*c*g^2*h^6*x +
 a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^7) + 5
/8*(c*x^2 + a)^(3/2)*c^2*g^3*e/(c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h^4*x^2 + 3*a
^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 + 2*c^3*g^7*h*x + 6*a*c^2*g^5*h^3*x + 6*a^2*
c*g^3*h^5*x + 2*a^3*g*h^7*x + c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*g^4*h^4 +
 a^3*g^2*h^6) - 5/8*sqrt(c*x^2 + a)*c^3*g^3*e/(c^3*g^6*h^2 + 3*a*c^2*g^4*h^
4 + 3*a^2*c*g^2*h^6 + a^3*h^8) - 5/8*(c*x^2 + a)^(3/2)*c^2*d*g^2/(c^3*g^6*h
*x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^2*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7*
x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g^3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a
*c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^3*g^2*h^5) + 5/8*sqrt(c*x^2 + a)*c^3*d*g^2
/(c^3*g^6*h + 3*a*c^2*g^4*h^3 + 3*a^2*c*g^2*h^5 + a^3*h^7) - 5/12*(c*x^2 +
a)^(3/2)*c*f*g^3/(c^2*g^4*h^4*x^3 + 2*a*c*g^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2
*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 + 3*a^2*g*h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a*
c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2*g^7*h + 2*a*c*g^5*h^3 + a^2*g^3*h^5) +
9/8*sqrt(c*x^2 + a)*c^2*f*g^3/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x
+ c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) + 9/8*(c*x^2 + a)^(3/2)*c*f*g^2/
(c^2*g^4*h^3*x^2 + 2*a*c*g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*
c*g^3*h^4*x + 2*a^2*g*h^6*x + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) - 9/
8*sqrt(c*x^2 + a)*c^2*f*g^2/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) + 5/12*
(c*x^2 + a)^(3/2)*c*g^2*e/(c^2*g^4*h^3*x^3 + 2*a*c*g^2*h^5*x^3 + a^2*h^7*x^
```

```
3 + 3*c^2*g^5*h^2*x^2 + 6*a*c*g^3*h^4*x^2 + 3*a^2*g*h^6*x^2 + 3*c^2*g^6*h*x
 + 6*a*c*g^4*h^3*x + 3*a^2*g^2*h^5*x + c^2*g^7 + 2*a*c*g^5*h^2 + a^2*g^3*h^
4) - 5/8*sqrt(c*x^2 + a)*c^2*g^2*e/(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h
^7*x + c^2*g^5*h^2 + 2*a*c*g^3*h^4 + a^2*g*h^6) - 5/12*(c*x^2 + a)^(3/2)*c*
d*g/(c^2*g^4*h^2*x^3 + 2*a*c*g^2*h^4*x^3 + a^2*h^6*x^3 + 3*c^2*g^5*h*x^2 +
6*a*c*g^3*h^3*x^2 + 3*a^2*g*h^5*x^2 + 3*c^2*g^6*x + 6*a*c*g^4*h^2*x + 3*a^2
*g^2*h^4*x + c^2*g^7/h + 2*a*c*g^5*h + a^2*g^3*h^3) + 1/8*sqrt(c*x^2 + a)*c
^2*d*g/(c^2*g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3
*h^3 + a^2*g*h^5) - 1/4*(c*x^2 + a)^(3/2)*f*g^2/(c*g^2*h^5*x^4 + a*h^7*x^4
+ 4*c*g^3*h^4*x^3 + 4*a*g*h^6*x^3 + 6*c*g^4*h^3*x^2 + 6*a*g^2*h^5*x^2 + 4*c
*g^5*h^2*x + 4*a*g^3*h^4*x + c*g^6*h + a*g^4*h^3) - 5/8*(c*x^2 + a)^(3/2)*c
*g*e/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2 + a^2*h^6*x^2 + 2*c^2*g^5*h*x + 4
*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6 + 2*a*c*g^4*h^2 + a^2*g^2*h^4) + 5
/8*sqrt(c*x^2 + a)*c^2*g*e/(c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) + 1/8*(c
*x^2 + a)^(3/2)*c*d/(c^2*g^4*h*x^2 + 2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 + 2*c^
2*g^5*x + 4*a*c*g^3*h^2*x + 2*a^2*g*h^4*x + c^2*g^6/h + 2*a*c*g^4*h + a^2*g
^2*h^3) - 1/8*sqrt(c*x^2 + a)*c^2*d/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2*h^5) +
 2/3*(c*x^2 + a)^(3/2)*f*g/(c*g^2*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3*x^2 + 3
*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*a*g^2*h^4*x + c*g^5*h + a*g^3*h^3) - 1/2*s
qrt(c*x^2 + a)*c*f*g/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3 + a*g*h^5) + 1/4*(c
*x^2 + a)^(3/2)*g*e/(c*g^2*h^4*x^4 + a*h^6*x^4 + 4*c*g^3*h^3*x^3 + 4*a*g*h^
5*x^3 + 6*c*g^4*h^2*x^2 + 6*a*g^2*h^4*x^2 + 4*c*g^5*h*x + 4*a*g^3*h^3*x + c
*g^6 + a*g^4*h^2) - 1/4*(c*x^2 + a)^(3/2)*d/(c*g^2*h^3*x^4 + a*h^5*x^4 + 4*
c*g^3*h^2*x^3 + 4*a*g*h^4*x^3 + 6*c*g^4*h*x^2 + 6*a*g^2*h^3*x^2 + 4*c*g^5*x
 + 4*a*g^3*h^2*x + c*g^6/h + a*g^4*h) - 1/2*(c*x^2 + a)^(3/2)*f/(c*g^2*h^3*
x^2 + a*h^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 1/2*
sqrt(c*x^2 + a)*c*f/(c*g^2*h^3 + a*h^5) - 1/3*(c*x^2 + a)^(3/2)*e/(c*g^2*h^
3*x^3 + a*h^5*x^3 + 3*c*g^3*h^2*x^2 + 3*a*g*h^4*x^2 + 3*c*g^4*h*x + 3*a*g^2
*h^3*x + c*g^5 + a*g^3*h^2) - 5/8*c^4*f*g^6*arcsinh(c*g*x/(sqrt(a*c)*abs(h*
x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^11) - 5/8*
c^4*d*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x +
 g)))/((a + c*g^2/h^2)^(7/2)*h^9) + 7/4*c^3*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*
abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^9) +
 3/4*c^3*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(
h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^7) - 13/8*c^2*f*g^2*arcsinh(c*g*x/(sqrt
(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*
h^7) - 1/8*c^2*d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*ab
s(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^5) + 1/2*...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1272 vs. $2(297) = 594$.

time = 53.46, size = 2571, normalized size = 8.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*((4*a*c^3*d - a^2*c^2*f)*g^6 - (a^2*c^2*d - 4*a^3*c*f)*g^4*h^2 + (
(4*a*c^3*d - a^2*c^2*f)*g^2*h^4 - (a^2*c^2*d - 4*a^3*c*f)*h^6)*x^4 + 4*((4*
a*c^3*d - a^2*c^2*f)*g^3*h^3 - (a^2*c^2*d - 4*a^3*c*f)*g*h^5)*x^3 + 6*((4*a
*c^3*d - a^2*c^2*f)*g^4*h^2 - (a^2*c^2*d - 4*a^3*c*f)*g^2*h^4)*x^2 + 4*((4*
a*c^3*d - a^2*c^2*f)*g^5*h - (a^2*c^2*d - 4*a^3*c*f)*g^3*h^3)*x + 5*(a^2*c^
2*g*h^5*x^4 + 4*a^2*c^2*g^2*h^4*x^3 + 6*a^2*c^2*g^3*h^3*x^2 + 4*a^2*c^2*g^4
*h^2*x + a^2*c^2*g^5*h)*e)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 -
 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h
)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) - 2*(6*a^4*d*h^7 + (28*a*c^3*
d - 13*a^2*c^2*f)*g^6*h + (47*a^2*c^2*d - 11*a^3*c*f)*g^4*h^3 + (25*a^3*c*d
 + 2*a^4*f)*g^2*h^5 - (6*c^4*f*g^7 + (2*c^4*d + 25*a*c^3*f)*g^5*h^2 - (11*a
*c^3*d - 47*a^2*c^2*f)*g^3*h^4 - (13*a^2*c^2*d - 28*a^3*c*f)*g*h^6)*x^3 - (
4*(2*c^4*d + a*c^3*f)*g^6*h - (32*a*c^3*d - 41*a^2*c^2*f)*g^4*h^3 - (43*a^2
*c^2*d - 25*a^3*c*f)*g^2*h^5 - 3*(a^3*c*d + 4*a^4*f)*h^7)*x^2 - (3*(4*c^4*d
 + a*c^3*f)*g^7 - (25*a*c^3*d - 43*a^2*c^2*f)*g^5*h^2 - (41*a^2*c^2*d - 32*
a^3*c*f)*g^3*h^4 - 4*(a^3*c*d + 2*a^4*f)*g*h^6)*x - (8*a*c^3*g^7 - a^2*c^2*
g^5*h^2 - 11*a^3*c*g^3*h^4 - 2*a^4*g*h^6 + (2*c^4*g^6*h + 11*a*c^3*g^4*h^3
+ a^2*c^2*g^2*h^5 - 8*a^3*c*h^7)*x^3 + (8*c^4*g^7 + 44*a*c^3*g^5*h^2 + 19*a
^2*c^2*g^3*h^4 - 17*a^3*c*g*h^6)*x^2 + (17*a*c^3*g^6*h - 19*a^2*c^2*g^4*h^3
 - 44*a^3*c*g^2*h^5 - 8*a^4*h^7)*x)*e)*sqrt(c*x^2 + a))/(c^4*g^12 + 4*a*c^3
*g^10*h^2 + 6*a^2*c^2*g^8*h^4 + 4*a^3*c*g^6*h^6 + a^4*g^4*h^8 + (c^4*g^8*h^
4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^12)*x^4
+ 4*(c^4*g^9*h^3 + 4*a*c^3*g^7*h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^3*c*g^3*h^9 +
a^4*g*h^11)*x^3 + 6*(c^4*g^10*h^2 + 4*a*c^3*g^8*h^4 + 6*a^2*c^2*g^6*h^6 + 4
*a^3*c*g^4*h^8 + a^4*g^2*h^10)*x^2 + 4*(c^4*g^11*h + 4*a*c^3*g^9*h^3 + 6*a^
2*c^2*g^7*h^5 + 4*a^3*c*g^5*h^7 + a^4*g^3*h^9)*x), -1/24*(3*((4*a*c^3*d - a
^2*c^2*f)*g^6 - (a^2*c^2*d - 4*a^3*c*f)*g^4*h^2 + ((4*a*c^3*d - a^2*c^2*f)*
g^2*h^4 - (a^2*c^2*d - 4*a^3*c*f)*h^6)*x^4 + 4*((4*a*c^3*d - a^2*c^2*f)*g^3
*h^3 - (a^2*c^2*d - 4*a^3*c*f)*g*h^5)*x^3 + 6*((4*a*c^3*d - a^2*c^2*f)*g^4*
h^2 - (a^2*c^2*d - 4*a^3*c*f)*g^2*h^4)*x^2 + 4*((4*a*c^3*d - a^2*c^2*f)*g^5
*h - (a^2*c^2*d - 4*a^3*c*f)*g^3*h^3)*x + 5*(a^2*c^2*g*h^5*x^4 + 4*a^2*c^2*
g^2*h^4*x^3 + 6*a^2*c^2*g^3*h^3*x^2 + 4*a^2*c^2*g^4*h^2*x + a^2*c^2*g^5*h)*
e)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^
2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*h^2)*x^2)) + (6*a^4*d*h^7 + (28*
a*c^3*d - 13*a^2*c^2*f)*g^6*h + (47*a^2*c^2*d - 11*a^3*c*f)*g^4*h^3 + (25*a
^3*c*d + 2*a^4*f)*g^2*h^5 - (6*c^4*f*g^7 + (2*c^4*d + 25*a*c^3*f)*g^5*h^2 -
 (11*a*c^3*d - 47*a^2*c^2*f)*g^3*h^4 - (13*a^2*c^2*d - 28*a^3*c*f)*g*h^6)*x
^3 - (4*(2*c^4*d + a*c^3*f)*g^6*h - (32*a*c^3*d - 41*a^2*c^2*f)*g^4*h^3 - (
43*a^2*c^2*d - 25*a^3*c*f)*g^2*h^5 - 3*(a^3*c*d + 4*a^4*f)*h^7)*x^2 - (3*(4
*c^4*d + a*c^3*f)*g^7 - (25*a*c^3*d - 43*a^2*c^2*f)*g^5*h^2 - (41*a^2*c^2*d
 - 32*a^3*c*f)*g^3*h^4 - 4*(a^3*c*d + 2*a^4*f)*g*h^6)*x - (8*a*c^3*g^7 - a^
2*c^2*g^5*h^2 - 11*a^3*c*g^3*h^4 - 2*a^4*g*h^6 + (2*c^4*g^6*h + 11*a*c^3*g^
4*h^3 + a^2*c^2*g^2*h^5 - 8*a^3*c*h^7)*x^3 + (8*c^4*g^7 + 44*a*c^3*g^5*h^2
```

```
+ 19*a^2*c^2*g^3*h^4 - 17*a^3*c*g*h^6)*x^2 + (17*a*c^3*g^6*h - 19*a^2*c^2*g
^4*h^3 - 44*a^3*c*g^2*h^5 - 8*a^4*h^7)*x)*e)*sqrt(c*x^2 + a))/(c^4*g^12 + 4
*a*c^3*g^10*h^2 + 6*a^2*c^2*g^8*h^4 + 4*a^3*c*g^6*h^6 + a^4*g^4*h^8 + (c^4*
g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^12
)*x^4 + 4*(c^4*g^9*h^3 + 4*a*c^3*g^7*h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^3*c*g^3*
h^9 + a^4*g*h^11)*x^3 + 6*(c^4*g^10*h^2 + 4*a*c^3*g^8*h^4 + 6*a^2*c^2*g^6*h
^6 + 4*a^3*c*g^4*h^8 + a^4*g^2*h^10)*x^2 + 4*(c^4*g^11*h + 4*a*c^3*g^9*h^3
+ 6*a^2*c^2*g^7*h^5 + 4*a^3*c*g^5*h^7 + a^4*g^3*h^9)*x)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^2}\,(d + ex + fx^2)}{(g + hx)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**5,x)
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

3.87 $\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^6}\,dx$

Optimal. Leaf size=433

$$-\frac{c(4c^2dg^3+a^2h^2(6fg-eh)-acg(fg^2-3h(2eg-dh)))\,(ah-cgx)\sqrt{a+cx^2}}{8\,(cg^2+ah^2)^4\,(g+hx)^2}-\frac{(fg^2-egh+dh^2)\,(a+c}{5h\,(cg^2+ah^2)\,(g+hx}$$

```
[Out] -1/5*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^5+1/20*(5*
a*h^2*(-e*h+2*f*g)+c*g*(3*f*g^2+h*(-7*d*h+2*e*g)))*(c*x^2+a)^(3/2)/h/(a*h^2
+c*g^2)^2/(h*x+g)^4-1/60*(20*a^2*f*h^4-c^2*g^2*(3*f*g^2+h*(-27*d*h+2*e*g))-
a*c*h^2*(18*f*g^2-h*(-8*d*h+33*e*g)))*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)^3/(h*
x+g)^3-1/8*a*c^2*(4*c^2*d*g^3+a^2*h^2*(-e*h+6*f*g)-a*c*g*(f*g^2-3*h*(-d*h+2
*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*
g^2)^(9/2)-1/8*c*(4*c^2*d*g^3+a^2*h^2*(-e*h+6*f*g)-a*c*g*(f*g^2-3*h*(-d*h+2
*e*g)))*(-c*g*x+a*h)*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^4/(h*x+g)^2
```

Rubi [A]
time = 0.45, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1665, 849, 821, 735, 739, 212}

$$-\frac{(a+cx^2)^{3/2}\,(20a^2fh^4-ach^2(18fg^2-h(33eg-8dh))-c^2(g^2h(2eg-27dh)+3fg^4))}{60h(g+hx)^3\,(ah^2+cg^2)^3}-\frac{c\sqrt{a+cx^2}\,(ah-cgx)\,(a^2h^2(6fg-eh)-acg(fg^2-3h(2eg-dh))+4c^2dg^3)}{8(g+hx)^2\,(ah^2+cg^2)^4}-\frac{ac^2\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\,\sqrt{ah^2+cg^2}}\right)(a^2h^2(6fg-eh)-acg(fg^2-3h(2eg-dh))+4c^2dg^3)}{8\,(ah^2+cg^2)^{9/2}}-\frac{(a+cx^2)^{3/2}\,(dh^2-egh+fg^2)}{5h(g+hx)^5\,(ah^2+cg^2)}+\frac{(a+cx^2)^{3/2}\,(5ah^2(2fg-eh)+cgh(2eg-7dh)+3cfg^3)}{20h(g+hx)^4\,(ah^2+cg^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -1/8*(c*(4*c^2*d*g^3 + a^2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(2*e*g -
d*h)))*(a*h - c*g*x)*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)^4*(g + h*x)^2) - ((f
*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(3/2))/(5*h*(c*g^2 + a*h^2)*(g + h*x)^5)
+ ((3*c*f*g^3 + c*g*h*(2*e*g - 7*d*h) + 5*a*h^2*(2*f*g - e*h))*(a + c*x^2)^
(3/2))/(20*h*(c*g^2 + a*h^2)^2*(g + h*x)^4) - ((20*a^2*f*h^4 - c^2*(3*f*g^4
 + g^2*h*(2*e*g - 27*d*h)) - a*c*h^2*(18*f*g^2 - h*(33*e*g - 8*d*h)))*(a +
c*x^2)^(3/2))/(60*h*(c*g^2 + a*h^2)^3*(g + h*x)^3) - (a*c^2*(4*c^2*d*g^3 +
a^2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(2*e*g - d*h)))*ArcTanh[(a*h - c
*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(8*(c*g^2 + a*h^2)^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(-2*a*e + (2*c*d)*x)*((a + c*x^2)^p/(2*(m + 1)*(c*d^2
+ a*e^2))), x] - Dist[4*a*c*(p/(2*(m + 1)*(c*d^2 + a*e^2))), Int[(d + e*x)^
(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
+ a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^6}\,dx = -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{3/2}}{5h\,(cg^2+ah^2)\,(g+hx)^5} - \frac{\int \frac{\left(-5(cdg-afg+aeh)-\left(5afh+c\left(2eg+\frac{3fg^2}{h}-\right.\right.\right.}{(g+hx)^5}}{5\,(cg^2+ah^2)}$$

$$= -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{3/2}}{5h\,(cg^2+ah^2)\,(g+hx)^5} + \frac{(3cfg^3+cgh(2eg-7dh)+5ah^2(2f}{20h\,(cg^2+ah^2)^2\,(g+}$$

$$= -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{3/2}}{5h\,(cg^2+ah^2)\,(g+hx)^5} + \frac{(3cfg^3+cgh(2eg-7dh)+5ah^2(2f}{20h\,(cg^2+ah^2)^2\,(g+}$$

$$= -\frac{c(4c^2dg^3+a^2h^2(6fg-eh)-acg(fg^2-3h(2eg-dh)))\,(ah-cgx)\sqrt{a}}{8\,(cg^2+ah^2)^4\,(g+hx)^2}$$

$$= -\frac{c(4c^2dg^3+a^2h^2(6fg-eh)-acg(fg^2-3h(2eg-dh)))\,(ah-cgx)\sqrt{a}}{8\,(cg^2+ah^2)^4\,(g+hx)^2}$$

$$= -\frac{c(4c^2dg^3+a^2h^2(6fg-eh)-acg(fg^2-3h(2eg-dh)))\,(ah-cgx)\sqrt{a}}{8\,(cg^2+ah^2)^4\,(g+hx)^2}$$

Mathematica [A]
time = 11.06, size = 583, normalized size = 1.35

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -1/120*(Sqrt[a + c*x^2]*(24*(c*g^2 + a*h^2)^4*(f*g^2 + h*(-(e*g) + d*h)) -
6*(c*g^2 + a*h^2)^3*(11*c*f*g^3 + c*g*h*(-6*e*g + d*h) - 5*a*h^2*(-2*f*g +
e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^2*(20*a^2*f*h^4 + c^2*(27*f*g^4 - g^2*h
*(2*e*g + 3*d*h)) + a*c*h^2*(54*f*g^2 + h*(-9*e*g + 4*d*h)))*(g + h*x)^2 -
c*(c*g^2 + a*h^2)*(5*a^2*h^4*(10*f*g - 3*e*h) + a*c*g*h^2*(21*f*g^2 + h*(24
*e*g - 29*d*h)) + c^2*(6*f*g^5 + 2*g^3*h*(2*e*g + 3*d*h)))*(g + h*x)^3 - c*
(-40*a^3*f*h^6 + a*c^2*g^2*h^2*(27*f*g^2 + h*(28*e*g - 83*d*h)) + c^3*(6*f*
g^6 + 2*g^4*h*(2*e*g + 3*d*h)) + a^2*c*h^4*(86*f*g^2 + h*(-81*e*g + 16*d*h)
))*(g + h*x)^4))/(h^3*(c*g^2 + a*h^2)^4*(g + h*x)^5) + (a*c^2*(4*c^2*d*g^3
+ a^2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 + 3*h*(-2*e*g + d*h)))*Log[g + h*x])
/(8*(c*g^2 + a*h^2)^(9/2)) - (a*c^2*(4*c^2*d*g^3 + a^2*h^2*(6*f*g - e*h) -
a*c*g*(f*g^2 + 3*h*(-2*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*S
qrt[a + c*x^2]])/(8*(c*g^2 + a*h^2)^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6084 vs. $2(409) = 818$.
time = 0.09, size = 6085, normalized size = 14.05

method	result	size
default	Expression too large to display	6085

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 5811 vs. $2(418) = 836$.
time = 0.59, size = 5811, normalized size = 13.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="maxima")
```

```
[Out] -7/8*sqrt(c*x^2 + a)*c^4*f*g^6/(c^4*g^8*h^4*x + 4*a*c^3*g^6*h^6*x + 6*a^2*c
^2*g^4*h^8*x + 4*a^3*c*g^2*h^10*x + a^4*h^12*x + c^4*g^9*h^3 + 4*a*c^3*g^7*
h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^3*c*g^3*h^9 + a^4*g*h^11) - 7/8*(c*x^2 + a)^(
3/2)*c^3*f*g^5/(c^4*g^8*h^3*x^2 + 4*a*c^3*g^6*h^5*x^2 + 6*a^2*c^2*g^4*h^7*x
^2 + 4*a^3*c*g^2*h^9*x^2 + a^4*h^11*x^2 + 2*c^4*g^9*h^2*x + 8*a*c^3*g^7*h^4
*x + 12*a^2*c^2*g^5*h^6*x + 8*a^3*c*g^3*h^8*x + 2*a^4*g*h^10*x + c^4*g^10*h
 + 4*a*c^3*g^8*h^3 + 6*a^2*c^2*g^6*h^5 + 4*a^3*c*g^4*h^7 + a^4*g^2*h^9) + 7
/8*sqrt(c*x^2 + a)*c^4*f*g^5/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4
*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) + 7/8*sqrt(c*x^2 + a)*c^4*g^5*e/(c^4*g^8
*h^3*x + 4*a*c^3*g^6*h^5*x + 6*a^2*c^2*g^4*h^7*x + 4*a^3*c*g^2*h^9*x + a^4*
h^11*x + c^4*g^9*h^2 + 4*a*c^3*g^7*h^4 + 6*a^2*c^2*g^5*h^6 + 4*a^3*c*g^3*h^
8 + a^4*g*h^10) - 7/8*sqrt(c*x^2 + a)*c^4*d*g^4/(c^4*g^8*h^2*x + 4*a*c^3*g^
6*h^4*x + 6*a^2*c^2*g^4*h^6*x + 4*a^3*c*g^2*h^8*x + a^4*h^10*x + c^4*g^9*h
+ 4*a*c^3*g^7*h^3 + 6*a^2*c^2*g^5*h^5 + 4*a^3*c*g^3*h^7 + a^4*g*h^9) + 7/8*
(c*x^2 + a)^(3/2)*c^3*g^4*e/(c^4*g^8*h^2*x^2 + 4*a*c^3*g^6*h^4*x^2 + 6*a^2*
c^2*g^4*h^6*x^2 + 4*a^3*c*g^2*h^8*x^2 + a^4*h^10*x^2 + 2*c^4*g^9*h*x + 8*a*
c^3*g^7*h^3*x + 12*a^2*c^2*g^5*h^5*x + 8*a^3*c*g^3*h^7*x + 2*a^4*g*h^9*x +
c^4*g^10 + 4*a*c^3*g^8*h^2 + 6*a^2*c^2*g^6*h^4 + 4*a^3*c*g^4*h^6 + a^4*g^2*
h^8) - 7/8*sqrt(c*x^2 + a)*c^4*g^4*e/(c^4*g^8*h^2 + 4*a*c^3*g^6*h^4 + 6*a^2
*c^2*g^4*h^6 + 4*a^3*c*g^2*h^8 + a^4*h^10) - 7/8*(c*x^2 + a)^(3/2)*c^3*d*g^
3/(c^4*g^8*h*x^2 + 4*a*c^3*g^6*h^3*x^2 + 6*a^2*c^2*g^4*h^5*x^2 + 4*a^3*c*g^
2*h^7*x^2 + a^4*h^9*x^2 + 2*c^4*g^9*x + 8*a*c^3*g^7*h^2*x + 12*a^2*c^2*g^5*
h^4*x + 8*a^3*c*g^3*h^6*x + 2*a^4*g*h^8*x + c^4*g^10/h + 4*a*c^3*g^8*h + 6*
a^2*c^2*g^6*h^3 + 4*a^3*c*g^4*h^5 + a^4*g^2*h^7) + 7/8*sqrt(c*x^2 + a)*c^4*
```

```
d*g^3/(c^4*g^8*h + 4*a*c^3*g^6*h^3 + 6*a^2*c^2*g^4*h^5 + 4*a^3*c*g^2*h^7 +
a^4*h^9) - 7/12*(c*x^2 + a)^(3/2)*c^2*f*g^4/(c^3*g^6*h^4*x^3 + 3*a*c^2*g^4*
h^6*x^3 + 3*a^2*c*g^2*h^8*x^3 + a^3*h^10*x^3 + 3*c^3*g^7*h^3*x^2 + 9*a*c^2*
g^5*h^5*x^2 + 9*a^2*c*g^3*h^7*x^2 + 3*a^3*g*h^9*x^2 + 3*c^3*g^8*h^2*x + 9*a
*c^2*g^6*h^4*x + 9*a^2*c*g^4*h^6*x + 3*a^3*g^2*h^8*x + c^3*g^9*h + 3*a*c^2*
g^7*h^3 + 3*a^2*c*g^5*h^5 + a^3*g^3*h^7) + 13/8*sqrt(c*x^2 + a)*c^3*f*g^4/(
c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c*g^2*h^8*x + a^3*h^10*x + c^3*g^
7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 + a^3*g*h^9) + 13/8*(c*x^2 + a)^(
3/2)*c^2*f*g^3/(c^3*g^6*h^3*x^2 + 3*a*c^2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7*x^2
 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x + 6*a*c^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*x +
2*a^3*g*h^8*x + c^3*g^8*h + 3*a*c^2*g^6*h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2*h^7
) - 13/8*sqrt(c*x^2 + a)*c^3*f*g^3/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c
*g^2*h^7 + a^3*h^9) + 7/12*(c*x^2 + a)^(3/2)*c^2*g^3*e/(c^3*g^6*h^3*x^3 + 3
*a*c^2*g^4*h^5*x^3 + 3*a^2*c*g^2*h^7*x^3 + a^3*h^9*x^3 + 3*c^3*g^7*h^2*x^2
+ 9*a*c^2*g^5*h^4*x^2 + 9*a^2*c*g^3*h^6*x^2 + 3*a^3*g*h^8*x^2 + 3*c^3*g^8*h
*x + 9*a*c^2*g^6*h^3*x + 9*a^2*c*g^4*h^5*x + 3*a^3*g^2*h^7*x + c^3*g^9 + 3*
a*c^2*g^7*h^2 + 3*a^2*c*g^5*h^4 + a^3*g^3*h^6) - sqrt(c*x^2 + a)*c^3*g^3*e/
(c^3*g^6*h^3*x + 3*a*c^2*g^4*h^5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c^3*g^
7*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2*c*g^3*h^6 + a^3*g*h^8) - 7/12*(c*x^2 + a)^(
3/2)*c^2*d*g^2/(c^3*g^6*h^2*x^3 + 3*a*c^2*g^4*h^4*x^3 + 3*a^2*c*g^2*h^6*x^3
 + a^3*h^8*x^3 + 3*c^3*g^7*h*x^2 + 9*a*c^2*g^5*h^3*x^2 + 9*a^2*c*g^3*h^5*x^
2 + 3*a^3*g*h^7*x^2 + 3*c^3*g^8*x + 9*a*c^2*g^6*h^2*x + 9*a^2*c*g^4*h^4*x +
 3*a^3*g^2*h^6*x + c^3*g^9/h + 3*a*c^2*g^7*h + 3*a^2*c*g^5*h^3 + a^3*g^3*h^
5) + 3/8*sqrt(c*x^2 + a)*c^3*d*g^2/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a
^2*c*g^2*h^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5
+ a^3*g*h^7) - 7/20*(c*x^2 + a)^(3/2)*c*f*g^3/(c^2*g^4*h^5*x^4 + 2*a*c*g^2*
h^7*x^4 + a^2*h^9*x^4 + 4*c^2*g^5*h^4*x^3 + 8*a*c*g^3*h^6*x^3 + 4*a^2*g*h^8
*x^3 + 6*c^2*g^6*h^3*x^2 + 12*a*c*g^4*h^5*x^2 + 6*a^2*g^2*h^7*x^2 + 4*c^2*g
^7*h^2*x + 8*a*c*g^5*h^4*x + 4*a^2*g^3*h^6*x + c^2*g^8*h + 2*a*c*g^6*h^3 +
a^2*g^4*h^5) - (c*x^2 + a)^(3/2)*c^2*g^2*e/(c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h
^4*x^2 + 3*a^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 + 2*c^3*g^7*h*x + 6*a*c^2*g^5*h^
3*x + 6*a^2*c*g^3*h^5*x + 2*a^3*g*h^7*x + c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2
*c*g^4*h^4 + a^3*g^2*h^6) + sqrt(c*x^2 + a)*c^3*g^2*e/(c^3*g^6*h^2 + 3*a*c^
2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) + 3/8*(c*x^2 + a)^(3/2)*c^2*d*g/(c^3
*g^6*h*x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^2*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^
3*g^7*x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g^3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h
 + 3*a*c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^3*g^2*h^5) - 3/8*sqrt(c*x^2 + a)*c^3
*d*g/(c^3*g^6*h + 3*a*c^2*g^4*h^3 + 3*a^2*c*g^2*h^5 + a^3*h^7) + 29/30*(c*x
^2 + a)^(3/2)*c*f*g^2/(c^2*g^4*h^4*x^3 + 2*a*c*g^2*h^6*x^3 + a^2*h^8*x^3 +
3*c^2*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 + 3*a^2*g*h^7*x^2 + 3*c^2*g^6*h^2*x +
 6*a*c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2*g^7*h + 2*a*c*g^5*h^3 + a^2*g^3*h^
5) - 3/4*sqrt(c*x^2 + a)*c^2*f*g^2/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h
^8*x + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7)...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1937 vs.

2(418) = 836.
time = 165.66, size = 3901, normalized size = 9.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="fricas")
```

```
[Out] [-1/240*(15*((4*a*c^4*d - a^2*c^3*f)*g^8 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^6*
h^2 + ((4*a*c^4*d - a^2*c^3*f)*g^3*h^5 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g*h^7)
*x^5 + 5*((4*a*c^4*d - a^2*c^3*f)*g^4*h^4 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^2
*h^6)*x^4 + 10*((4*a*c^4*d - a^2*c^3*f)*g^5*h^3 - 3*(a^2*c^3*d - 2*a^3*c^2*
f)*g^3*h^5)*x^3 + 10*((4*a*c^4*d - a^2*c^3*f)*g^6*h^2 - 3*(a^2*c^3*d - 2*a^
3*c^2*f)*g^4*h^4)*x^2 + 5*((4*a*c^4*d - a^2*c^3*f)*g^7*h - 3*(a^2*c^3*d - 2
*a^3*c^2*f)*g^5*h^3)*x + (6*a^2*c^3*g^7*h - a^3*c^2*g^5*h^3 + (6*a^2*c^3*g^
2*h^6 - a^3*c^2*h^8)*x^5 + 5*(6*a^2*c^3*g^3*h^5 - a^3*c^2*g*h^7)*x^4 + 10*(
6*a^2*c^3*g^4*h^4 - a^3*c^2*g^2*h^6)*x^3 + 10*(6*a^2*c^3*g^5*h^3 - a^3*c^2*
g^3*h^5)*x^2 + 5*(6*a^2*c^3*g^6*h^2 - a^3*c^2*g^4*h^4)*x)*e)*sqrt(c*g^2 + a
*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 +
2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g
^2)) + 2*(24*a^5*d*h^9 + 9*(20*a*c^4*d - 9*a^2*c^3*f)*g^8*h + (329*a^2*c^3*
d - 53*a^3*c^2*f)*g^6*h^3 + (247*a^3*c^2*d + 32*a^4*c*f)*g^4*h^5 + 2*(61*a^
4*c*d + 2*a^5*f)*g^2*h^7 - (6*c^5*f*g^8*h + 3*(2*c^5*d + 11*a*c^4*f)*g^6*h^
3 - (77*a*c^4*d - 113*a^2*c^3*f)*g^4*h^5 - (67*a^2*c^3*d - 46*a^3*c^2*f)*g^
2*h^7 + 8*(2*a^3*c^2*d - 5*a^4*c*f)*h^9)*x^4 - 5*(6*c^5*f*g^9 + 3*(2*c^5*d
+ 11*a*c^4*f)*g^7*h^2 - 5*(13*a*c^4*d - 22*a^2*c^3*f)*g^5*h^4 - (64*a^2*c^3
*d - 61*a^3*c^2*f)*g^3*h^6 + (7*a^3*c^2*d - 22*a^4*c*f)*g*h^8)*x^3 - (3*(20
*c^5*d + 9*a*c^4*f)*g^8*h - (503*a*c^4*d - 446*a^2*c^3*f)*g^6*h^3 - 141*(4*
a^2*c^3*d - a^3*c^2*f)*g^4*h^5 - 3*(3*a^3*c^2*d + 106*a^4*c*f)*g^2*h^7 - 8*
(a^4*c*d + 5*a^5*f)*h^9)*x^2 - 5*(3*(4*c^5*d + a*c^4*f)*g^9 - 3*(21*a*c^4*d
 - 22*a^2*c^3*f)*g^7*h^2 - 5*(16*a^2*c^3*d - 7*a^3*c^2*f)*g^5*h^4 - (7*a^3*
c^2*d + 32*a^4*c*f)*g^3*h^6 - 2*(a^4*c*d + 2*a^5*f)*g*h^8)*x - (40*a*c^4*g^
9 - 46*a^2*c^3*g^7*h^2 - 113*a^3*c^2*g^5*h^4 - 33*a^4*c*g^3*h^6 - 6*a^5*g*h
^8 + (4*c^5*g^7*h^2 + 32*a*c^4*g^5*h^4 - 53*a^2*c^3*g^3*h^6 - 81*a^3*c^2*g*
h^8)*x^4 + 5*(4*c^5*g^8*h + 32*a*c^4*g^6*h^3 - 35*a^2*c^3*g^4*h^5 - 66*a^3*
c^2*g^2*h^7 - 3*a^4*c*h^9)*x^3 + (40*c^5*g^9 + 318*a*c^4*g^7*h^2 - 141*a^2*
c^3*g^5*h^4 - 446*a^3*c^2*g^3*h^6 - 27*a^4*c*g*h^8)*x^2 + 5*(22*a*c^4*g^8*h
 - 61*a^2*c^3*g^6*h^3 - 110*a^3*c^2*g^4*h^5 - 33*a^4*c*g^2*h^7 - 6*a^5*h^9)
*x)*e)*sqrt(c*x^2 + a))/(c^5*g^15 + 5*a*c^4*g^13*h^2 + 10*a^2*c^3*g^11*h^4
+ 10*a^3*c^2*g^9*h^6 + 5*a^4*c*g^7*h^8 + a^5*g^5*h^10 + (c^5*g^10*h^5 + 5*a
*c^4*g^8*h^7 + 10*a^2*c^3*g^6*h^9 + 10*a^3*c^2*g^4*h^11 + 5*a^4*c*g^2*h^13
+ a^5*h^15)*x^5 + 5*(c^5*g^11*h^4 + 5*a*c^4*g^9*h^6 + 10*a^2*c^3*g^7*h^8 +
10*a^3*c^2*g^5*h^10 + 5*a^4*c*g^3*h^12 + a^5*g*h^14)*x^4 + 10*(c^5*g^12*h^3
 + 5*a*c^4*g^10*h^5 + 10*a^2*c^3*g^8*h^7 + 10*a^3*c^2*g^6*h^9 + 5*a^4*c*g^4
*h^11 + a^5*g^2*h^13)*x^3 + 10*(c^5*g^13*h^2 + 5*a*c^4*g^11*h^4 + 10*a^2*c^
```

```
3*g^9*h^6 + 10*a^3*c^2*g^7*h^8 + 5*a^4*c*g^5*h^10 + a^5*g^3*h^12)*x^2 + 5*(
c^5*g^14*h + 5*a*c^4*g^12*h^3 + 10*a^2*c^3*g^10*h^5 + 10*a^3*c^2*g^8*h^7 +
5*a^4*c*g^6*h^9 + a^5*g^4*h^11)*x), -1/120*(15*((4*a*c^4*d - a^2*c^3*f)*g^8
 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^6*h^2 + ((4*a*c^4*d - a^2*c^3*f)*g^3*h^5 -
 3*(a^2*c^3*d - 2*a^3*c^2*f)*g*h^7)*x^5 + 5*((4*a*c^4*d - a^2*c^3*f)*g^4*h^
4 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^2*h^6)*x^4 + 10*((4*a*c^4*d - a^2*c^3*f)*
g^5*h^3 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^3*h^5)*x^3 + 10*((4*a*c^4*d - a^2*c
^3*f)*g^6*h^2 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^4*h^4)*x^2 + 5*((4*a*c^4*d -
a^2*c^3*f)*g^7*h - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^5*h^3)*x + (6*a^2*c^3*g^7*
h - a^3*c^2*g^5*h^3 + (6*a^2*c^3*g^2*h^6 - a^3*c^2*h^8)*x^5 + 5*(6*a^2*c^3*
g^3*h^5 - a^3*c^2*g*h^7)*x^4 + 10*(6*a^2*c^3*g^4*h^4 - a^3*c^2*g^2*h^6)*x^3
 + 10*(6*a^2*c^3*g^5*h^3 - a^3*c^2*g^3*h^5)*x^2 + 5*(6*a^2*c^3*g^6*h^2 - a^
3*c^2*g^4*h^4)*x)*e)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*g^2 - a*h^2)*(c*g*
x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*h^2)*x^2)) + (
24*a^5*d*h^9 + 9*(20*a*c^4*d - 9*a^2*c^3*f)*g^8*h + (329*a^2*c^3*d - 53*a^3
*c^2*f)*g^6*h^3 + (247*a^3*c^2*d + 32*a^4*c*f)*g^4*h^5 + 2*(61*a^4*c*d + 2*
a^5*f)*g^2*h^7 - (6*c^5*f*g^8*h + 3*(2*c^5*d + 11*a*c^4*f)*g^6*h^3 - (77*a*
c^4*d - 113*a^2*c^3*f)*g^4*h^5 - (67*a^2*c^3*d - 46*a^3*c^2*f)*g^2*h^7 + 8*
(2*a^3*c^2*d - 5*a^4*c*f)*h^9)*x^4 - 5*(6*c^5*f*g^9 + 3*(2*c^5*d + 11*a*c^4
*f)*g^7*h^2 - 5*(13*a*c^4*d - 22*a^2*c^3*f)*g^5*h^4 - (64*a^2*c^3*d - 61*a^
3*c^2*f)*g^3*h^6 + (7*a^3*c^2*d - 22*a^4*c*f)*g*h^8)*x^3 - (3*(20*c^5*d + 9
*a*c^4*f)*g^8*h - (503*a*c^4*d - 446*a^2*c^3*f)*g^6*h^3 - 141*(4*a^2*c^3*d
- a^3*c^2*f)*g^4*h^5 - 3*(3*a^3*c^2*d + 106*a^4*c*f)*g^2*h^7 - 8*(a^4*c*d +
 5*a^5*f)*h^9)*x^2 - 5*(3*(4*c^5*d + a*c^4*f)*g^9 - 3*(21*a*c^4*d - 22*a^2*
c^3*f)*g^7*h^2 - 5*(16*a^2*c^3*d - 7*a^3*c^2*f)*g^5*h^4 - (7*a^3*c^2*d + 32
*a^4*c*f)*g^3*h^6 - 2*(a^4*c*d + 2*a^5*f)*g*h^8)*x - (40*a*c^4*g^9 - 46*a^2
*c^3*g^7*h^2 - 113*a^3*c^2*g^5*h^4 - 33*a^4*c*g^3*h^6 - 6*a^5*g*h^8 + (4*c^
5*g^7*h^2 + 32*a*c^4*g^5*h^4 - 53*a^2*c^3*g^3*h^6 - 81*a^3*c^2*g*h^8)*x^4 +
 5*(4*c^5*g^8*h + 32*a*c^4*g^6*h^3 - 35*a^2*c^3*g^4*h^5 - 66*a^3*c^2*g^2*h^
7 - 3*a^4*c*h^9)*x^3 + (40*c^5*g^9 + 318*a*c^4*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**6,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**6, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4212 vs. 2(418) = 836.
time = 5.71, size = 4212, normalized size = 9.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] -1/4*(4*a*c^4*d*g^3 - a^2*c^3*f*g^3 - 3*a^2*c^3*d*g*h^2 + 6*a^3*c^2*f*g*h^2
 + 6*a^2*c^3*g^2*h*e - a^3*c^2*h^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))
*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^4*g^8 + 4*a*c^3*g^6*h^2 + 6*a^2*c
^2*g^4*h^4 + 4*a^3*c*g^2*h^6 + a^4*h^8)*sqrt(-c*g^2 - a*h^2)) - 1/60*(60*(s
qrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*d*g^3*h^8 - 15*(sqrt(c)*x - sqrt(c*x^2
+ a))^9*a^2*c^3*f*g^3*h^8 - 45*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*d*g*
h^10 + 90*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^2*f*g*h^10 + 90*(sqrt(c)*x
- sqrt(c*x^2 + a))^9*a^2*c^3*g^2*h^9*e - 15*(sqrt(c)*x - sqrt(c*x^2 + a))^9
*a^3*c^2*h^11*e - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*f*g^8*h^3 -
480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9/2)*f*g^6*h^5 + 540*(sqrt(c)*x -
sqrt(c*x^2 + a))^8*a*c^(9/2)*d*g^4*h^7 - 855*(sqrt(c)*x - sqrt(c*x^2 + a))^
8*a^2*c^(7/2)*f*g^4*h^7 - 405*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*d
*g^2*h^9 + 330*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(5/2)*f*g^2*h^9 - 120*
(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(3/2)*f*h^11 + 810*(sqrt(c)*x - sqrt(
c*x^2 + a))^8*a^2*c^(7/2)*g^3*h^8*e - 135*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a
^3*c^(5/2)*g*h^10*e - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*f*g^9*h^2 - 9
60*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*f*g^7*h^4 + 1880*(sqrt(c)*x - sqrt
(c*x^2 + a))^7*a*c^5*d*g^5*h^6 - 1910*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c
^4*f*g^5*h^6 - 1690*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*d*g^3*h^8 + 193
0*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*f*g^3*h^8 + 210*(sqrt(c)*x - sqrt
(c*x^2 + a))^7*a^3*c^3*d*g*h^10 - 660*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c
^2*f*g*h^10 - 160*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*g^8*h^3*e - 640*(sqrt
(c)*x - sqrt(c*x^2 + a))^7*a*c^5*g^6*h^5*e + 1860*(sqrt(c)*x - sqrt(c*x^2 +
 a))^7*a^2*c^4*g^4*h^7*e - 1530*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*g^2
*h^9*e - 90*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^2*h^11*e - 240*(sqrt(c)*x
 - sqrt(c*x^2 + a))^6*c^(13/2)*f*g^10*h - 240*(sqrt(c)*x - sqrt(c*x^2 + a))
^6*c^(13/2)*d*g^8*h^3 - 720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*f*g^
8*h^3 + 2120*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*d*g^6*h^5 - 1250*(s
qrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*f*g^6*h^5 - 5710*(sqrt(c)*x - sqr
t(c*x^2 + a))^6*a^2*c^(9/2)*d*g^4*h^7 + 5590*(sqrt(c)*x - sqrt(c*x^2 + a))^
6*a^3*c^(7/2)*f*g^4*h^7 + 510*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*d
*g^2*h^9 - 2220*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*f*g^2*h^9 - 240
*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*d*h^11 + 240*(sqrt(c)*x - sqrt
(c*x^2 + a))^6*a^5*c^(3/2)*f*h^11 - 160*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(
13/2)*g^9*h^2*e - 640*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*g^7*h^4*e
+ 3660*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*g^5*h^6*e - 4350*(sqrt(c
)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*g^3*h^8*e + 330*(sqrt(c)*x - sqrt(c*x^
2 + a))^6*a^4*c^(5/2)*g*h^10*e - 96*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*f*g
^11 - 96*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*d*g^9*h^2 + 48*(sqrt(c)*x - sq
rt(c*x^2 + a))^5*a*c^6*f*g^9*h^2 + 1808*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c
^6*d*g^7*h^4 + 604*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*f*g^7*h^4 - 7076
```

*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*d*g^5*h^6 + 6710*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^4*f*g^5*h^6 + 3770*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^4*d*g^3*h^8 - 5780*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*f*g^3*h^8 - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*d*g*h^10 + 1200*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^2*f*g*h^10 - 64*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*g^10*h*e - 128*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^6*g^8*h^3*e + 3416*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*g^6*h^5*e - 7320*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^4*g^4*h^7*e + 2430*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*g^2*h^9*e + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*f*g^10*h + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*d*g^8*h^3 + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*f*g^8*h^3 - 5240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*d*g^6*h^5 + 2450*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(9/2)*f*g^6*h^5 + 5590*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(9/2)*d*g^4*h^7 - 7660*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*f*g^4*h^7 - 2240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*d*g^2*h^9 + 3440*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(5/2)*f*g^2*h^9 - 80*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(5/2)*d*h^11 - 160*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(3/2)*f*h^11 + 160*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*g^9*h^2*e + 1120*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*g^7*h^4*e - 6140*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(9/2)*g^5*h^6*e + 5650*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*g^3*h^8*e - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(5/2)*g*h^10*e - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*f*g^9*h^2 - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*d*g^7*h^4 - 960*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^5*f*g^7*h^4 + 5000*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^5*d*g^5*h^6 - 3890*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*f*g^5*h^6 - 2910*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*d*g^3*h^8 + 4710*(sqrt(c)...

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)

[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)

3.88 $\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=462

$$\frac{a\left(48c^2dg^3+3a^2h^2(3fg+eh)-8acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+cx^2}}{128c^2}+\frac{\left(48c^2dg^3+3a^2h^2(3fg+eh)-8acg\right.}{192}$$

[Out] 1/192*(48*c^2*d*g^3+3*a^2*h^2*(e*h+3*f*g)-8*a*c*g*(f*g^2+3*h*(d*h+e*g)))*x*(c*x^2+a)^(3/2)/c^2+1/504*(8*(-4*a*f+9*c*d)*h^2-3*c*g*(-9*e*h+5*f*g))*(h*x+g)^2*(c*x^2+a)^(5/2)/c^2/h-1/72*(-9*e*h+5*f*g)*(h*x+g)^3*(c*x^2+a)^(5/2)/c/h+1/9*f*(h*x+g)^4*(c*x^2+a)^(5/2)/c/h+1/5040*(128*a^2*f*h^4-32*a*c*h^2*(17*f*g^2+9*h*(d*h+3*e*g))-12*c^2*g^2*(5*f*g^2-3*h*(64*d*h+3*e*g))-5*c*h*(a*h^2*(63*e*h+61*f*g)+2*c*g*(5*f*g^2-9*h*(12*d*h+e*g)))*x)*(c*x^2+a)^(5/2)/c^3/h+1/128*a^2*(48*c^2*d*g^3+3*a^2*h^2*(e*h+3*f*g)-8*a*c*g*(f*g^2+3*h*(d*h+e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(5/2)+1/128*a*(48*c^2*d*g^3+3*a^2*h^2*(e*h+3*f*g)-8*a*c*g*(f*g^2+3*h*(d*h+e*g)))*x*(c*x^2+a)^(1/2)/c^2

Rubi [A]
time = 0.67, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1668, 847, 794, 201, 223, 212}

$$\frac{x\left(a+cx^2\right)^{3/2}\left(3a^2h^2(eh+3fg)-8acg\left(3h(dh+eg)+fg^2\right)+48c^2dg^3\right)}{192c^2}+\frac{ax\sqrt{a+cx^2}\left(3a^2h^2(eh+3fg)-8acg\left(3h(dh+eg)+fg^2\right)+48c^2dg^3\right)}{128c^2}+\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(3a^2h^2(eh+3fg)-8acg\left(3h(dh+eg)+fg^2\right)+48c^2dg^3\right)}{128c^{5/2}}+\frac{\left(a+cx^2\right)^{5/2}\left(4\left(32a^2fh^4-8ach^2\left(9h(dh+3eg)+17fg^2\right)-c^2\left(15fg^4-9g^2h(64dh+3eg)\right)\right)-5chx\left(ah^2(63eh+61fg)+2c\left(5fg^3-9gh(12dh+eg)\right)\right)\right)}{5040c^3h}+\frac{\left(a+cx^2\right)^{5/2}(g+hx)^2\left(8h^2(9cd-4af)-3cg(5fg-9eh)\right)}{504c^2h}-\frac{\left(a+cx^2\right)^{5/2}(g+hx)^3(5fg-9eh)}{72ch}+\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^4}{9ch}$$

Antiderivative was successfully verified.

[In] Int[(g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]

[Out] (a*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*x*Sqrt[a + c*x^2])/(128*c^2) + ((48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*x*(a + c*x^2)^(3/2))/(192*c^2) + ((8*(9*c*d - 4*a*f)*h^2 - 3*c*g*(5*f*g - 9*e*h))*(g + h*x)^2*(a + c*x^2)^(5/2))/(504*c^2*h) - ((5*f*g - 9*e*h)*(g + h*x)^3*(a + c*x^2)^(5/2))/(72*c*h) + (f*(g + h*x)^4*(a + c*x^2)^(5/2))/(9*c*h) + ((4*(32*a^2*f*h^4 - 8*a*c*h^2*(17*f*g^2 + 9*h*(3*e*g + d*h)) - c^2*(15*f*g^4 - 9*g^2*h*(3*e*g + 64*d*h))) - 5*c*h*(a*h^2*(61*f*g + 63*e*h) + 2*c*(5*f*g^3 - 9*g*h*(e*g + 12*d*h)))*x)*(a + c*x^2)^(5/2))/(5040*c^3*h) + (a^2*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*c^(5/2))

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \frac{f(g+hx)^4 \left(a+cx^2\right)^{5/2}}{9ch} + \frac{\int (g+hx)^3 \left((9cd-4af)h^2 - ch(5\right.}{9ch^2}$$

$$= -\frac{(5fg-9eh)(g+hx)^3 \left(a+cx^2\right)^{5/2}}{72ch} + \frac{f(g+hx)^4 \left(a+cx^2\right)^{5/}}{9ch}$$

$$= \frac{\left(8(9cd-4af)h^2 - 3cg(5fg-9eh)\right)(g+hx)^2 \left(a+cx^2\right)^{5/2}}{504c^2h} -$$

$$= \frac{\left(8(9cd-4af)h^2 - 3cg(5fg-9eh)\right)(g+hx)^2 \left(a+cx^2\right)^{5/2}}{504c^2h} -$$

$$= \frac{\left(48c^2dg^3 + 3a^2h^2(3fg+eh) - 8acg(fg^2+3h(eg+dh))\right)x(a}{192c^2}$$

$$= \frac{a\left(48c^2dg^3 + 3a^2h^2(3fg+eh) - 8acg(fg^2+3h(eg+dh))\right)x\sqrt{}}{128c^2}$$

$$= \frac{a\left(48c^2dg^3 + 3a^2h^2(3fg+eh) - 8acg(fg^2+3h(eg+dh))\right)x\sqrt{}}{128c^2}$$

$$= \frac{a\left(48c^2dg^3 + 3a^2h^2(3fg+eh) - 8acg(fg^2+3h(eg+dh))\right)x\sqrt{}}{128c^2}$$

Mathematica [A]
time = 1.44, size = 474, normalized size = 1.03

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(1024*a^4*f*h^3 - a^3*c*h*(9*h*(768*e*g + 256*d*h + 105*e*
h*x) + f*(6912*g^2 + 2835*g*h*x + 512*h^2*x^2)) + 6*a^2*c^2*(12*d*h*(336*g^
2 + 105*g*h*x + 16*h^2*x^2) + 3*e*(448*g^3 + 420*g^2*h*x + 192*g*h^2*x^2 +
35*h^3*x^3) + f*x*(420*g^3 + 576*g^2*h*x + 315*g*h^2*x^2 + 64*h^3*x^3)) + 1
6*c^4*x^3*(18*d*(35*g^3 + 84*g^2*h*x + 70*g*h^2*x^2 + 20*h^3*x^3) + x*(9*e*
(56*g^3 + 140*g^2*h*x + 120*g*h^2*x^2 + 35*h^3*x^3) + 5*f*x*(84*g^3 + 216*g
^2*h*x + 189*g*h^2*x^2 + 56*h^3*x^3))) + 8*a*c^3*x*(18*d*(175*g^3 + 336*g^2
*h*x + 245*g*h^2*x^2 + 64*h^3*x^3) + x*(9*e*(224*g^3 + 490*g^2*h*x + 384*g*
h^2*x^2 + 105*h^3*x^3) + f*x*(1470*g^3 + 3456*g^2*h*x + 2835*g*h^2*x^2 + 80
0*h^3*x^3)))) - 315*a^2*Sqrt[c]*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 8
*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(403
20*c^3)
```

Maple [A]
time = 0.08, size = 406, normalized size = 0.88

method	result
default	$f\,h^{3}\left(\frac{x^{4}\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{9c}-\frac{4a\left(\frac{x^{2}\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{7c}-\frac{2a\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{35c^{2}}\right)}{9c}\right)+\left(e\,h^{3}+3f g\,h^{2}\right)\left(\frac{x^{3}\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{8c}-\frac{3a\left(\frac{x\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{6c}-\cdots\right.}{}\right.$
risch	$\frac{(4480c^{4}f\,h^{3}x^{8}+5040c^{4}e\,h^{3}x^{7}+15120c^{4}f g\,h^{2}x^{7}+6400a\,c^{3}f\,h^{3}x^{6}+5760c^{4}d\,h^{3}x^{6}+17280c^{4}e g\,h^{2}x^{6}+17280c^{4}f\,g^{2}h\,x^{6}+7560a\,c^{3}e\,h^{3}x^{7}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] f*h^3*(1/9*x^4*(c*x^2+a)^(5/2)/c-4/9*a/c*(1/7*x^2*(c*x^2+a)^(5/2)/c-2/35*a/
c^2*(c*x^2+a)^(5/2)))+(e*h^3+3*f*g*h^2)*(1/8*x^3*(c*x^2+a)^(5/2)/c-3/8*a/c*
(1/6*x*(c*x^2+a)^(5/2)/c-1/6*a/c*(1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2
+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))))))+(d*h^3+3*e*g*h^2+
3*f*g^2*h)*(1/7*x^2*(c*x^2+a)^(5/2)/c-2/35*a/c^2*(c*x^2+a)^(5/2))+(3*d*g*h^
2+3*e*g^2*h+f*g^3)*(1/6*x*(c*x^2+a)^(5/2)/c-1/6*a/c*(1/4*x*(c*x^2+a)^(3/2)+
3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))))
+1/5*(3*d*g^2*h+e*g^3)*(c*x^2+a)^(5/2)/c+d*g^3*(1/4*x*(c*x^2+a)^(3/2)+3/4*a
*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.30, size = 537, normalized size = 1.16

$$\frac{(cx^2+a)^{\frac{5}{2}}fh^3x^4}{9c}-\frac{4(cx^2+a)^{\frac{5}{2}}afh^3x^2}{63c^2}+\frac{1}{4}(cx^2+a)^{\frac{3}{2}}dg^3x+\frac{3}{8}\sqrt{cx^2+a}\,adg^3x+\frac{3a^2dg^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{c}}+\frac{3(cx^2+a)^{\frac{5}{2}}dg^2h}{5c}+\frac{8(cx^2+a)^{\frac{5}{2}}a^2fh^3}{315c^3}+\frac{(3fgh^2+h^3e)(cx^2+a)^{\frac{5}{2}}x^3}{8c}+\frac{(cx^2+a)^{\frac{5}{2}}g^3e}{5c}+\frac{(3fg^2h+dh^3+3gh^2e)(cx^2+a)^{\frac{5}{2}}x^2}{7c}-\frac{(3fgh^2+h^3e)(cx^2+a)^{\frac{5}{2}}ax}{16c^2}+\frac{(3fgh^2+h^3e)(cx^2+a)^{\frac{3}{2}}a^2x}{64c^2}+\frac{3(3fgh^2+h^3e)\sqrt{cx^2+a}\,a^3x}{128c^2}+\frac{(fg^3+3dgh^2+3g^2he)(cx^2+a)^{\frac{5}{2}}x}{6c}-\frac{(fg^3+3dgh^2+3g^2he)(cx^2+a)^{\frac{3}{2}}ax}{24c}-\frac{(fg^3+3dgh^2+3g^2he)\sqrt{cx^2+a}\,a^2x}{16c}+\frac{3(3fgh^2+h^3e)a^4\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{128c^{\frac{5}{2}}}-\frac{(fg^3+3dgh^2+3g^2he)a^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16c^{\frac{3}{2}}}-\frac{2(3fg^2h+dh^3+3gh^2e)(cx^2+a)^{\frac{5}{2}}a}{35c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/9*(c*x^2 + a)^(5/2)*f*h^3*x^4/c - 4/63*(c*x^2 + a)^(5/2)*a*f*h^3*x^2/c^2
+ 1/4*(c*x^2 + a)^(3/2)*d*g^3*x + 3/8*sqrt(c*x^2 + a)*a*d*g^3*x + 3/8*a^2*d
*g^3*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 3/5*(c*x^2 + a)^(5/2)*d*g^2*h/c + 8/3
15*(c*x^2 + a)^(5/2)*a^2*f*h^3/c^3 + 1/8*(3*f*g*h^2 + h^3*e)*(c*x^2 + a)^(5
/2)*x^3/c + 1/5*(c*x^2 + a)^(5/2)*g^3*e/c + 1/7*(3*f*g^2*h + d*h^3 + 3*g*h^
2*e)*(c*x^2 + a)^(5/2)*x^2/c - 1/16*(3*f*g*h^2 + h^3*e)*(c*x^2 + a)^(5/2)*a
*x/c^2 + 1/64*(3*f*g*h^2 + h^3*e)*(c*x^2 + a)^(3/2)*a^2*x/c^2 + 3/128*(3*f*
g*h^2 + h^3*e)*sqrt(c*x^2 + a)*a^3*x/c^2 + 1/6*(f*g^3 + 3*d*g*h^2 + 3*g^2*h
*e)*(c*x^2 + a)^(5/2)*x/c - 1/24*(f*g^3 + 3*d*g*h^2 + 3*g^2*h*e)*(c*x^2 + a
)^(3/2)*a*x/c - 1/16*(f*g^3 + 3*d*g*h^2 + 3*g^2*h*e)*sqrt(c*x^2 + a)*a^2*x/
c + 3/128*(3*f*g*h^2 + h^3*e)*a^4*arcsinh(c*x/sqrt(a*c))/c^(5/2) - 1/16*(f*
g^3 + 3*d*g*h^2 + 3*g^2*h*e)*a^3*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/35*(3*f
*g^2*h + d*h^3 + 3*g*h^2*e)*(c*x^2 + a)^(5/2)*a/c^2
```

Fricas [A]
time = 0.66, size = 1228, normalized size = 2.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/80640*(315*(8*(6*a^2*c^2*d - a^3*c*f)*g^3 - 3*(8*a^3*c*d - 3*a^4*f)*g*h^
2 - 3*(8*a^3*c*g^2*h - a^4*h^3)*e)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)
*sqrt(c)*x - a) + 2*(4480*c^4*f*h^3*x^8 + 15120*c^4*f*g*h^2*x^7 + 640*(27*c
^4*f*g^2*h + (9*c^4*d + 10*a*c^3*f)*h^3)*x^6 + 840*(8*c^4*f*g^3 + 3*(8*c^4*
d + 9*a*c^3*f)*g*h^2)*x^5 + 384*(9*(7*c^4*d + 8*a*c^3*f)*g^2*h + (24*a*c^3*
d + a^2*c^2*f)*h^3)*x^4 + 3456*(7*a^2*c^2*d - 2*a^3*c*f)*g^2*h - 256*(9*a^3
*c*d - 4*a^4*f)*h^3 + 210*(8*(6*c^4*d + 7*a*c^3*f)*g^3 + 3*(56*a*c^3*d + 3*
a^2*c^2*f)*g*h^2)*x^3 + 128*(27*(14*a*c^3*d + a^2*c^2*f)*g^2*h + (9*a^2*c^2
*d - 4*a^3*c*f)*h^3)*x^2 + 315*(8*(10*a*c^3*d + a^2*c^2*f)*g^3 + 3*(8*a^2*c
^2*d - 3*a^3*c*f)*g*h^2)*x + 9*(560*c^4*h^3*x^7 + 1920*c^4*g*h^2*x^6 + 896*
a^2*c^2*g^3 - 768*a^3*c*g*h^2 + 280*(8*c^4*g^2*h + 3*a*c^3*h^3)*x^5 + 128*(
7*c^4*g^3 + 24*a*c^3*g*h^2)*x^4 + 70*(56*a*c^3*g^2*h + a^2*c^2*h^3)*x^3 + 1
28*(14*a*c^3*g^3 + 3*a^2*c^2*g*h^2)*x^2 + 105*(8*a^2*c^2*g^2*h - a^3*c*h^3)
*x)*e)*sqrt(c*x^2 + a))/c^3, -1/40320*(315*(8*(6*a^2*c^2*d - a^3*c*f)*g^3 -
 3*(8*a^3*c*d - 3*a^4*f)*g*h^2 - 3*(8*a^3*c*g^2*h - a^4*h^3)*e)*sqrt(-c)*ar
ctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (4480*c^4*f*h^3*x^8 + 15120*c^4*f*g*h^2*
x^7 + 640*(27*c^4*f*g^2*h + (9*c^4*d + 10*a*c^3*f)*h^3)*x^6 + 840*(8*c^4*f*
g^3 + 3*(8*c^4*d + 9*a*c^3*f)*g*h^2)*x^5 + 384*(9*(7*c^4*d + 8*a*c^3*f)*g^2
*h + (24*a*c^3*d + a^2*c^2*f)*h^3)*x^4 + 3456*(7*a^2*c^2*d - 2*a^3*c*f)*g^2
*h - 256*(9*a^3*c*d - 4*a^4*f)*h^3 + 210*(8*(6*c^4*d + 7*a*c^3*f)*g^3 + 3*(
```

```
56*a*c^3*d + 3*a^2*c^2*f)*g*h^2)*x^3 + 128*(27*(14*a*c^3*d + a^2*c^2*f)*g^2
*h + (9*a^2*c^2*d - 4*a^3*c*f)*h^3)*x^2 + 315*(8*(10*a*c^3*d + a^2*c^2*f)*g
^3 + 3*(8*a^2*c^2*d - 3*a^3*c*f)*g*h^2)*x + 9*(560*c^4*h^3*x^7 + 1920*c^4*g
*h^2*x^6 + 896*a^2*c^2*g^3 - 768*a^3*c*g*h^2 + 280*(8*c^4*g^2*h + 3*a*c^3*h
^3)*x^5 + 128*(7*c^4*g^3 + 24*a*c^3*g*h^2)*x^4 + 70*(56*a*c^3*g^2*h + a^2*c
^2*h^3)*x^3 + 128*(14*a*c^3*g^3 + 3*a^2*c^2*g*h^2)*x^2 + 105*(8*a^2*c^2*g^2
*h - a^3*c*h^3)*x)*e)*sqrt(c*x^2 + a))/c^3]
```

Sympy [**A**]
time = 77.04, size = 1916, normalized size = 4.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] -3*a**(7/2)*e*h**3*x/(128*c**2*sqrt(1 + c*x**2/a)) - 9*a**(7/2)*f*g*h**2*x/
(128*c**2*sqrt(1 + c*x**2/a)) + 3*a**(5/2)*d*g*h**2*x/(16*c*sqrt(1 + c*x**2
/a)) + 3*a**(5/2)*e*g**2*h*x/(16*c*sqrt(1 + c*x**2/a)) - a**(5/2)*e*h**3*x*
*3/(128*c*sqrt(1 + c*x**2/a)) + a**(5/2)*f*g**3*x/(16*c*sqrt(1 + c*x**2/a))
 - 3*a**(5/2)*f*g*h**2*x**3/(128*c*sqrt(1 + c*x**2/a)) + a**(3/2)*d*g**3*x*
sqrt(1 + c*x**2/a)/2 + a**(3/2)*d*g**3*x/(8*sqrt(1 + c*x**2/a)) + 17*a**(3/
2)*d*g*h**2*x**3/(16*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*e*g**2*h*x**3/(16*sq
rt(1 + c*x**2/a)) + 13*a**(3/2)*e*h**3*x**5/(64*sqrt(1 + c*x**2/a)) + 17*a*
*(3/2)*f*g**3*x**3/(48*sqrt(1 + c*x**2/a)) + 39*a**(3/2)*f*g*h**2*x**5/(64*
sqrt(1 + c*x**2/a)) + 3*sqrt(a)*c*d*g**3*x**3/(8*sqrt(1 + c*x**2/a)) + 11*s
qrt(a)*c*d*g*h**2*x**5/(8*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*e*g**2*h*x**5/
(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*c*e*h**3*x**7/(16*sqrt(1 + c*x**2/a)) +
11*sqrt(a)*c*f*g**3*x**5/(24*sqrt(1 + c*x**2/a)) + 15*sqrt(a)*c*f*g*h**2*x*
*7/(16*sqrt(1 + c*x**2/a)) + 3*a**4*e*h**3*asinh(sqrt(c)*x/sqrt(a))/(128*c*
*(5/2)) + 9*a**4*f*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(128*c**(5/2)) - 3*a**3*
d*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) - 3*a**3*e*g**2*h*asinh(sqr
t(c)*x/sqrt(a))/(16*c**(3/2)) - a**3*f*g**3*asinh(sqrt(c)*x/sqrt(a))/(16*c*
*(3/2)) + 3*a**2*d*g**3*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c)) + 3*a*d*g**2*h
*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) +
 a*d*h**3*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c
*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True))
 + a*e*g**3*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c
), True)) + 3*a*e*g*h**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*
x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)
*x**4/4, True)) + 3*a*f*g**2*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2
) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (s
qrt(a)*x**4/4, True)) + a*f*h**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c*
*3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35
*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + 3*c*d*g
```

```
**2*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**
2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c
*d*h**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a
+ c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**
2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + c*e*g**3*Piecewise((-2*a**2*sqrt
(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x
**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 3*c*e*g*h**2*Piecewise((8*a**3
*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*
x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)
*x**6/6, True)) + 3*c*f*g**2*h*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3
) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c
) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + c*f*h**3*
Piecewise((-16*a**4*sqrt(a + c*x**2)/(315*c**4) + 8*a**3*x**2*sqrt(a + c*x*
*2)/(315*c**3) - 2*a**2*x**4*sqrt(a + c*x**2)/(105*c**2) + a*x**6*sqrt(a +
c*x**2)/(63*c) + x**8*sqrt(a + c*x**2)/9, Ne(c, 0)), (sqrt(a)*x**8/8, True)
) + c**2*d*g**3*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*d*g*h**2*x**7/(2
*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*e*g**2*h*x**7/(2*sqrt(a)*sqrt(1 + c*x**
2/a)) + c**2*e*h**3*x**9/(8*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*f*g**3*x**7/
(6*sqrt(a)*sqrt(1 + c*x**2/a)) + 3*c**2*f*g*h**2*x**9/(8*sqrt(a)*sqrt(1 + c
*x**2/a))
```

Giac [A]
time = 4.77, size = 652, normalized size = 1.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/40320*sqrt(c*x^2 + a)*((2*((4*(5*(2*(7*(8*c*f*h^3*x + 9*(3*c^8*f*g*h^2 +
c^8*h^3*e)/c^7)*x + 8*(27*c^8*f*g^2*h + 9*c^8*d*h^3 + 10*a*c^7*f*h^3 + 27*c
^8*g*h^2*e)/c^7)*x + 21*(8*c^8*f*g^3 + 24*c^8*d*g*h^2 + 27*a*c^7*f*g*h^2 +
24*c^8*g^2*h*e + 9*a*c^7*h^3*e)/c^7)*x + 48*(63*c^8*d*g^2*h + 72*a*c^7*f*g^
2*h + 24*a*c^7*d*h^3 + a^2*c^6*f*h^3 + 21*c^8*g^3*e + 72*a*c^7*g*h^2*e)/c^7
)*x + 105*(48*c^8*d*g^3 + 56*a*c^7*f*g^3 + 168*a*c^7*d*g*h^2 + 9*a^2*c^6*f*
g*h^2 + 168*a*c^7*g^2*h*e + 3*a^2*c^6*h^3*e)/c^7)*x + 64*(378*a*c^7*d*g^2*h
 + 27*a^2*c^6*f*g^2*h + 9*a^2*c^6*d*h^3 - 4*a^3*c^5*f*h^3 + 126*a*c^7*g^3*e
 + 27*a^2*c^6*g*h^2*e)/c^7)*x + 315*(80*a*c^7*d*g^3 + 8*a^2*c^6*f*g^3 + 24*
a^2*c^6*d*g*h^2 - 9*a^3*c^5*f*g*h^2 + 24*a^2*c^6*g^2*h*e - 3*a^3*c^5*h^3*e)
/c^7)*x + 128*(189*a^2*c^6*d*g^2*h - 54*a^3*c^5*f*g^2*h - 18*a^3*c^5*d*h^3
+ 8*a^4*c^4*f*h^3 + 63*a^2*c^6*g^3*e - 54*a^3*c^5*g*h^2*e)/c^7) - 1/128*(48
*a^2*c^2*d*g^3 - 8*a^3*c*f*g^3 - 24*a^3*c*d*g*h^2 + 9*a^4*f*g*h^2 - 24*a^3*
c*g^2*h*e + 3*a^4*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (g+hx)^3 \left(cx^2+a\right)^{3/2} \left(fx^2+ex+d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.89 $\int (g+hx)^2 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=346

$$\frac{a(48c^2dg^2+3a^2fh^2-8ac(fg^2+h(2eg+dh)))\,x\sqrt{a+cx^2}}{128c^2}+\frac{(48c^2dg^2+3a^2fh^2-8ac(fg^2+h(2eg+dh)))}{192c^2}$$

```
[Out] 1/192*(48*c^2*d*g^2+3*a^2*f*h^2-8*a*c*(f*g^2+h*(d*h+2*e*g)))*x*(c*x^2+a)^(3
/2)/c^2-1/56*(-8*e*h+5*f*g)*(h*x+g)^2*(c*x^2+a)^(5/2)/c/h+1/8*f*(h*x+g)^3*(
c*x^2+a)^(5/2)/c/h-1/1680*(96*a*h^2*(e*h+2*f*g)+12*c*g*(5*f*g^2-8*h*(7*d*h+
e*g))-5*h*(7*(-3*a*f+8*c*d)*h^2-2*c*g*(-8*e*h+5*f*g))*x)*(c*x^2+a)^(5/2)/c^
2/h+1/128*a^2*(48*c^2*d*g^2+3*a^2*f*h^2-8*a*c*(f*g^2+h*(d*h+2*e*g)))*arctan
h(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(5/2)+1/128*a*(48*c^2*d*g^2+3*a^2*f*h^2-8*a*
c*(f*g^2+h*(d*h+2*e*g)))*x*(c*x^2+a)^(1/2)/c^2
```

Rubi [A]

time = 0.32, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1668, 847, 794, 201, 223, 212}

$$\frac{x(a+cx^2)^{3/2}(3a^2fh^2-8ac(h(dh+2eg)+fg^2)+48c^2dg^2)}{192c^2}+\frac{ax\sqrt{a+cx^2}(3a^2fh^2-8ac(h(dh+2eg)+fg^2)+48c^2dg^2)}{128c^2}+\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(3a^2fh^2-8ac(h(dh+2eg)+fg^2)+48c^2dg^2)}{128c^{5/2}}-\frac{(a+cx^2)^{5/2}(12(8ah^2(eh+2fg)-8cgh(7dh+eg)+5cfg^3)-5hx(7h^2(8cd-3af)-2cg(5fg-8eh)))}{1680c^2h}-\frac{(a+cx^2)^{5/2}(g+hx)^2(5fg-8eh)}{56ch}+\frac{f(a+cx^2)^{5/2}(g+hx)^3}{8ch}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*g + d*h)))*x*Sqrt[a
+ c*x^2])/(128*c^2) + ((48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*
g + d*h)))*x*(a + c*x^2)^(3/2))/(192*c^2) - ((5*f*g - 8*e*h)*(g + h*x)^2*(a
 + c*x^2)^(5/2))/(56*c*h) + (f*(g + h*x)^3*(a + c*x^2)^(5/2))/(8*c*h) - ((1
2*(5*c*f*g^3 - 8*c*g*h*(e*g + 7*d*h) + 8*a*h^2*(2*f*g + e*h)) - 5*h*(7*(8*c
*d - 3*a*f)*h^2 - 2*c*g*(5*f*g - 8*e*h))*x)*(a + c*x^2)^(5/2))/(1680*c^2*h)
 + (a^2*(48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*g + d*h)))*ArcT
anh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*c^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch} + \frac{\int (g+hx)^2\left((8cd-3af)h^2-ch(5\ldots\right.}{8ch^2}$$

$$= -\frac{(5fg-8eh)(g+hx)^2\left(a+cx^2\right)^{5/2}}{56ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{5/\ldots}}{8ch}$$

$$= -\frac{(5fg-8eh)(g+hx)^2\left(a+cx^2\right)^{5/2}}{56ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{5/\ldots}}{8ch}$$

$$= \frac{\left(48c^2dg^2+3a^2fh^2-8ac(fg^2+h(2eg+dh))\right)x\left(a+cx^2\right)^{3/2}}{192c^2}\ldots$$

$$= \frac{a\left(48c^2dg^2+3a^2fh^2-8ac(fg^2+h(2eg+dh))\right)x\sqrt{a+cx^2}}{128c^2}\ldots$$

$$= \frac{a\left(48c^2dg^2+3a^2fh^2-8ac(fg^2+h(2eg+dh))\right)x\sqrt{a+cx^2}}{128c^2}\ldots$$

$$= \frac{a\left(48c^2dg^2+3a^2fh^2-8ac(fg^2+h(2eg+dh))\right)x\sqrt{a+cx^2}}{128c^2}\ldots$$

Mathematica [A]
time = 1.09, size = 332, normalized size = 0.96

$$\frac{\sqrt{c}\sqrt{a+cx^2}\left(-3a^3h(512fg+256eh+105fhx)+6a^2c(28dh(32g+5hx)+8e(56g^2+35ghx+8h^2x^2)+fx(140g^2+128ghx+35h^2x^2))+16c^3x^3(14d(15g^2+24ghx+10h^2x^2)+x(8e(21g^2+35ghx+15h^2x^2)+5fx(28g^2+48ghx+21h^2x^2)))+8ac^2x(14d(75g^2+96ghx+35h^2x^2)+x(4e(168g^2+245ghx+96h^2x^2)+fx(490g^2+768ghx+315h^2x^2)))\right)-105a^2(48c^2dg^2+3a^2fh^2-8ac(fg^2+h(2eg+dh)))\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{13440c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + c*x^2]*(-3*a^3*h*(512*f*g + 256*e*h + 105*f*h*x) + 6*a^2*
c*(28*d*h*(32*g + 5*h*x) + 8*e*(56*g^2 + 35*g*h*x + 8*h^2*x^2) + f*x*(140*g
^2 + 128*g*h*x + 35*h^2*x^2)) + 16*c^3*x^3*(14*d*(15*g^2 + 24*g*h*x + 10*h^
2*x^2) + x*(8*e*(21*g^2 + 35*g*h*x + 15*h^2*x^2) + 5*f*x*(28*g^2 + 48*g*h*x
 + 21*h^2*x^2))) + 8*a*c^2*x*(14*d*(75*g^2 + 96*g*h*x + 35*h^2*x^2) + x*(4*
e*(168*g^2 + 245*g*h*x + 96*h^2*x^2) + f*x*(490*g^2 + 768*g*h*x + 315*h^2*x
^2)))) - 105*a^2*(48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*g + d*
h)))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(13440*c^(5/2))
```

Maple [A]
time = 0.11, size = 320, normalized size = 0.92

method	result

default	$f\,h^{2}\left(\frac{x^{3}\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{8c}-\frac{3a\left(\frac{x\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{6c}-\frac{a\left(\frac{x\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)}{4}\right)}{6c}\right)}{8c}\right)+(e\,h$
risch	$-\frac{(-1680c^{3}f\,h^{2}x^{7}-1920c^{3}e\,h^{2}x^{6}-3840c^{3}fgh\,x^{6}-2520a\,c^{2}f\,h^{2}x^{5}-2240c^{3}d\,h^{2}x^{5}-4480c^{3}egh\,x^{5}-2240c^{3}f\,g^{2}x^{5}-3072a\,c^{2}e\,h^{2}x^{4}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] f*h^2*(1/8*x^3*(c*x^2+a)^(5/2)/c-3/8*a/c*(1/6*x*(c*x^2+a)^(5/2)/c-1/6*a/c*(
1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/
2)+(c*x^2+a)^(1/2))))))+(e*h^2+2*f*g*h)*(1/7*x^2*(c*x^2+a)^(5/2)/c-2/35*a/c
^2*(c*x^2+a)^(5/2))+(d*h^2+2*e*g*h+f*g^2)*(1/6*x*(c*x^2+a)^(5/2)/c-1/6*a/c*
(1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1
/2)+(c*x^2+a)^(1/2)))))+1/5*(2*d*g*h+e*g^2)*(c*x^2+a)^(5/2)/c+d*g^2*(1/4*x*
(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*
x^2+a)^(1/2))))
```

Maxima [A]

time = 0.30, size = 387, normalized size = 1.12

$$\frac{(cx^2+a)^{\frac{5}{2}}fh^2x^3}{8\,c}+\frac{1}{4}\,(cx^2+a)^{\frac{3}{2}}dg^2x+\frac{3}{8}\,\sqrt{cx^2+a}\,adg^2x-\frac{(cx^2+a)^{\frac{5}{2}}afh^2x}{16\,c^2}+\frac{(cx^2+a)^{\frac{3}{2}}a^2fh^2x}{64\,c^2}+\frac{3\,\sqrt{cx^2+a}\,a^3fh^2x}{128\,c^2}+\frac{3\,a^2dg^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}}+\frac{3\,a^4fh^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{128\,c^{\frac{5}{2}}}+\frac{2\,(cx^2+a)^{\frac{5}{2}}dgh}{5\,c}+\frac{(2\,fgh+h^2e)(cx^2+a)^{\frac{5}{2}}x^2}{7\,c}+\frac{(cx^2+a)^{\frac{5}{2}}g^2e}{5\,c}+\frac{(fg^2+dh^2+2\,ghe)(cx^2+a)^{\frac{5}{2}}x}{6\,c}-\frac{(fg^2+dh^2+2\,ghe)(cx^2+a)^{\frac{3}{2}}ax}{24\,c}-\frac{(fg^2+dh^2+2\,ghe)\sqrt{cx^2+a}\,a^2x}{16\,c}-\frac{(fg^2+dh^2+2\,ghe)a^3\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,c^{\frac{3}{2}}}-\frac{2\,(2\,fgh+h^2e)(cx^2+a)^{\frac{5}{2}}a}{35\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^2 + a)^(5/2)*f*h^2*x^3/c + 1/4*(c*x^2 + a)^(3/2)*d*g^2*x + 3/8*sqr
t(c*x^2 + a)*a*d*g^2*x - 1/16*(c*x^2 + a)^(5/2)*a*f*h^2*x/c^2 + 1/64*(c*x^2
 + a)^(3/2)*a^2*f*h^2*x/c^2 + 3/128*sqrt(c*x^2 + a)*a^3*f*h^2*x/c^2 + 3/8*a
^2*d*g^2*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 3/128*a^4*f*h^2*arcsinh(c*x/sqrt(
a*c))/c^(5/2) + 2/5*(c*x^2 + a)^(5/2)*d*g*h/c + 1/7*(2*f*g*h + h^2*e)*(c*x^
2 + a)^(5/2)*x^2/c + 1/5*(c*x^2 + a)^(5/2)*g^2*e/c + 1/6*(f*g^2 + d*h^2 + 2
*g*h*e)*(c*x^2 + a)^(5/2)*x/c - 1/24*(f*g^2 + d*h^2 + 2*g*h*e)*(c*x^2 + a)^
(3/2)*a*x/c - 1/16*(f*g^2 + d*h^2 + 2*g*h*e)*sqrt(c*x^2 + a)*a^2*x/c - 1/16
*(f*g^2 + d*h^2 + 2*g*h*e)*a^3*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/35*(2*f*g
*h + h^2*e)*(c*x^2 + a)^(5/2)*a/c^2
```

Fricas [A]
time = 0.43, size = 851, normalized size = 2.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/26880*(105*(16*a^3*c*g*h*e - 8*(6*a^2*c^2*d - a^3*c*f)*g^2 + (8*a^3*c*d
 - 3*a^4*f)*h^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2
*(1680*c^4*f*h^2*x^7 + 3840*c^4*f*g*h*x^6 + 768*(7*c^4*d + 8*a*c^3*f)*g*h*x
^4 + 280*(8*c^4*f*g^2 + (8*c^4*d + 9*a*c^3*f)*h^2)*x^5 + 768*(14*a*c^3*d +
a^2*c^2*f)*g*h*x^2 + 70*(8*(6*c^4*d + 7*a*c^3*f)*g^2 + (56*a*c^3*d + 3*a^2*
c^2*f)*h^2)*x^3 + 768*(7*a^2*c^2*d - 2*a^3*c*f)*g*h + 105*(8*(10*a*c^3*d +
a^2*c^2*f)*g^2 + (8*a^2*c^2*d - 3*a^3*c*f)*h^2)*x + 16*(120*c^4*h^2*x^6 + 2
80*c^4*g*h*x^5 + 490*a*c^3*g*h*x^3 + 105*a^2*c^2*g*h*x + 168*a^2*c^2*g^2 -
48*a^3*c*h^2 + 24*(7*c^4*g^2 + 8*a*c^3*h^2)*x^4 + 24*(14*a*c^3*g^2 + a^2*c^
2*h^2)*x^2)*e)*sqrt(c*x^2 + a))/c^3, 1/13440*(105*(16*a^3*c*g*h*e - 8*(6*a^
2*c^2*d - a^3*c*f)*g^2 + (8*a^3*c*d - 3*a^4*f)*h^2)*sqrt(-c)*arctan(sqrt(-c
)*x/sqrt(c*x^2 + a)) + (1680*c^4*f*h^2*x^7 + 3840*c^4*f*g*h*x^6 + 768*(7*c^
4*d + 8*a*c^3*f)*g*h*x^4 + 280*(8*c^4*f*g^2 + (8*c^4*d + 9*a*c^3*f)*h^2)*x^
5 + 768*(14*a*c^3*d + a^2*c^2*f)*g*h*x^2 + 70*(8*(6*c^4*d + 7*a*c^3*f)*g^2
+ (56*a*c^3*d + 3*a^2*c^2*f)*h^2)*x^3 + 768*(7*a^2*c^2*d - 2*a^3*c*f)*g*h +
 105*(8*(10*a*c^3*d + a^2*c^2*f)*g^2 + (8*a^2*c^2*d - 3*a^3*c*f)*h^2)*x + 1
6*(120*c^4*h^2*x^6 + 280*c^4*g*h*x^5 + 490*a*c^3*g*h*x^3 + 105*a^2*c^2*g*h*
x + 168*a^2*c^2*g^2 - 48*a^3*c*h^2 + 24*(7*c^4*g^2 + 8*a*c^3*h^2)*x^4 + 24*
(14*a*c^3*g^2 + a^2*c^2*h^2)*x^2)*e)*sqrt(c*x^2 + a))/c^3]
```

Sympy [A]
time = 49.78, size = 1304, normalized size = 3.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] -3*a**(7/2)*f*h**2*x/(128*c**2*sqrt(1 + c*x**2/a)) + a**(5/2)*d*h**2*x/(16*
c*sqrt(1 + c*x**2/a)) + a**(5/2)*e*g*h*x/(8*c*sqrt(1 + c*x**2/a)) + a**(5/2
)*f*g**2*x/(16*c*sqrt(1 + c*x**2/a)) - a**(5/2)*f*h**2*x**3/(128*c*sqrt(1 +
 c*x**2/a)) + a**(3/2)*d*g**2*x*sqrt(1 + c*x**2/a)/2 + a**(3/2)*d*g**2*x/(8
*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*d*h**2*x**3/(48*sqrt(1 + c*x**2/a)) + 17
*a**(3/2)*e*g*h*x**3/(24*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*f*g**2*x**3/(48*
sqrt(1 + c*x**2/a)) + 13*a**(3/2)*f*h**2*x**5/(64*sqrt(1 + c*x**2/a)) + 3*s
qrt(a)*c*d*g**2*x**3/(8*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*d*h**2*x**5/(24*
sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*e*g*h*x**5/(12*sqrt(1 + c*x**2/a)) + 11*
sqrt(a)*c*f*g**2*x**5/(24*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*c*f*h**2*x**7/(16
*sqrt(1 + c*x**2/a)) + 3*a**4*f*h**2*asinh(sqrt(c)*x/sqrt(a))/(128*c**(5/2)
) - a**3*d*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) - a**3*e*g*h*asinh(s
qrt(c)*x/sqrt(a))/(8*c**(3/2)) - a**3*f*g**2*asinh(sqrt(c)*x/sqrt(a))/(16*c
**(3/2)) + 3*a**2*d*g**2*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c)) + 2*a*d*g*h*P
iecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + a
*e*g**2*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), T
rue)) + a*e*h**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqr
t(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4,
 True)) + 2*a*f*g*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*
sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4
/4, True)) + 2*c*d*g*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x*
*2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x
**4/4, True)) + c*e*g**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*
x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)
*x**4/4, True)) + c*e*h**2*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) -
4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) +
x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + 2*c*f*g*h*Pie
cewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(
105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c,
 0)), (sqrt(a)*x**6/6, True)) + c**2*d*g**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**2
/a)) + c**2*d*h**2*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*e*g*h*x**7/(3
*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*f*g**2*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/
a)) + c**2*f*h**2*x**9/(8*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [A]
time = 5.04, size = 452, normalized size = 1.31

$$\frac{1}{13440}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\left(6\left(7\,cfh^2x+\frac{8\,(2\,c^7fgh+c^7h^2e)}{c^6}\right)x+\frac{7\,(8\,c^7fg^2+8\,c^7dh^2+9\,ac^6fh^2+16\,c^7ghe)}{c^6}\right)x+\frac{48\,(14\,c^7dgh+16\,ac^6fgh+7\,c^7g^2e+8\,ac^6h^2e)}{c^6}\right)x+\frac{35\,(48\,c^7dg^2+56\,ac^6fg^2+56\,ac^6dh^2+3\,a^2c^5fh^2+112\,ac^6ghe)}{c^6}\right)x+\frac{192\,(28\,ac^6dgh+2\,a^2c^5fgh+14\,ac^6g^2e+a^2c^5h^2e)}{c^6}\right)x+\frac{105\,(80\,ac^6dg^2+8\,a^2c^5fg^2+8\,a^2c^5dh^2-3\,a^3c^4fh^2+16\,a^2c^5ghe)}{c^6}\right)x+\frac{384\,(14\,a^2c^5dgh-4\,a^3c^4fgh+7\,a^2c^5g^2e-2\,a^3c^4h^2e)}{c^6}\right)-\frac{(48\,a^2c^2dg^2-8\,a^3cfg^2-8\,a^3cdh^2+3\,a^4fh^2-16\,a^3cghe)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{128\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/13440*sqrt(c*x^2 + a)*((2*((4*(5*(6*(7*c*f*h^2*x + 8*(2*c^7*f*g*h + c^7*h
^2*e)/c^6)*x + 7*(8*c^7*f*g^2 + 8*c^7*d*h^2 + 9*a*c^6*f*h^2 + 16*c^7*g*h*e)
/c^6)*x + 48*(14*c^7*d*g*h + 16*a*c^6*f*g*h + 7*c^7*g^2*e + 8*a*c^6*h^2*e)/
```

```
c^6)*x + 35*(48*c^7*d*g^2 + 56*a*c^6*f*g^2 + 56*a*c^6*d*h^2 + 3*a^2*c^5*f*h
^2 + 112*a*c^6*g*h*e)/c^6)*x + 192*(28*a*c^6*d*g*h + 2*a^2*c^5*f*g*h + 14*a
*c^6*g^2*e + a^2*c^5*h^2*e)/c^6)*x + 105*(80*a*c^6*d*g^2 + 8*a^2*c^5*f*g^2
+ 8*a^2*c^5*d*h^2 - 3*a^3*c^4*f*h^2 + 16*a^2*c^5*g*h*e)/c^6)*x + 384*(14*a^
2*c^5*d*g*h - 4*a^3*c^4*f*g*h + 7*a^2*c^5*g^2*e - 2*a^3*c^4*h^2*e)/c^6) - 1
/128*(48*a^2*c^2*d*g^2 - 8*a^3*c*f*g^2 - 8*a^3*c*d*h^2 + 3*a^4*f*h^2 - 16*a
^3*c*g*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (g+h\,x)^2\,\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.90 $\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=213

$$\frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c}+\frac{(6cdg-a(fg+eh))x\left(a+cx^2\right)^{3/2}}{24c}+\frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch}-\frac{(6(2afh^2 \cdots}{}$$

```
[Out] 1/24*(6*c*d*g-a*(e*h+f*g))*x*(c*x^2+a)^(3/2)/c+1/7*f*(h*x+g)^2*(c*x^2+a)^(5
/2)/c/h-1/210*(12*a*f*h^2+6*c*(5*f*g^2-7*h*(d*h+e*g))+5*c*h*(-7*e*h+5*f*g)*
x)*(c*x^2+a)^(5/2)/c^2/h+1/16*a^2*(-a*e*h-a*f*g+6*c*d*g)*arctanh(x*c^(1/2)/
(c*x^2+a)^(1/2))/c^(3/2)+1/16*a*(-a*e*h-a*f*g+6*c*d*g)*x*(c*x^2+a)^(1/2)/c
```

Rubi [A]

time = 0.17, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1668, 794, 201, 223, 212}

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(-aeh-afg+6cdg)}{16c^{3/2}}-\frac{\left(a+cx^2\right)^{5/2}\left(6\left(2afh^2-7ch(dh+eg)+5cfg^2\right)+5chx(5fg-7eh)\right)}{210c^2h}+\frac{x\left(a+cx^2\right)^{3/2}(6cdg-a(eh+fg))}{24c}+\frac{ax\sqrt{a+cx^2}(-aeh-afg+6cdg)}{16c}+\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^2}{7ch}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(6*c*d*g - a*f*g - a*e*h)*x*Sqrt[a + c*x^2])/(16*c) + ((6*c*d*g - a*(f*g
 + e*h))*x*(a + c*x^2)^(3/2))/(24*c) + (f*(g + h*x)^2*(a + c*x^2)^(5/2))/(7
*c*h) - ((6*(5*c*f*g^2 + 2*a*f*h^2 - 7*c*h*(e*g + d*h)) + 5*c*h*(5*f*g - 7*
e*h)*x)*(a + c*x^2)^(5/2))/(210*c^2*h) + (a^2*(6*c*d*g - a*f*g - a*e*h)*Arc
Tanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} + \frac{\int (g+hx)\left((7cd-2af)h^2-ch(5fg\right.}{7ch^2}$$

$$= \frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} - \frac{(6(5cfg^2+2afh^2-7ch(eg+dh))}{210c^2}$$

$$= \frac{(6cdg-a(fg+eh))x(a+cx^2)^{3/2}}{24c} + \frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} -$$

$$= \frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c} + \frac{(6cdg-a(fg+eh))x(a+}{24c}$$

$$= \frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c} + \frac{(6cdg-a(fg+eh))x(a+}{24c}$$

$$= \frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c} + \frac{(6cdg-a(fg+eh))x(a+}{24c}$$

Mathematica [A]
time = 0.73, size = 197, normalized size = 0.92

$$\frac{\sqrt{a+cx^2}\left(-96a^3fh+3a^2c(112dh+7e(16g+5hx)+fx(35g+16hx))+4c^3x^3(21d(5g+4hx)+2x(7e(6g+5hx)+5fx(7g+6hx)))+2ac^2x(21d(25g+16hx)+x(7e(48g+35hx)+fx(245g+192hx)))\right)+105a^2\sqrt{c}\,(-6cdg+afg+aeh)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{1680c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-96*a^3*f*h + 3*a^2*c*(112*d*h + 7*e*(16*g + 5*h*x) + f*x
*(35*g + 16*h*x)) + 4*c^3*x^3*(21*d*(5*g + 4*h*x) + 2*x*(7*e*(6*g + 5*h*x)
+ 5*f*x*(7*g + 6*h*x))) + 2*a*c^2*x*(21*d*(25*g + 16*h*x) + x*(7*e*(48*g +
35*h*x) + f*x*(245*g + 192*h*x)))) + 105*a^2*Sqrt[c]*(-6*c*d*g + a*f*g + a*
e*h)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(1680*c^2)
```

Maple [A]
time = 0.09, size = 194, normalized size = 0.91

method	result
default	$hf\left(\frac{x^{2}\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{7c}-\frac{2a\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{35c^{2}}\right)+\left(eh+gf\right)\left(\frac{x\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{6c}-\frac{a\left(\frac{x\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\right.}{2}\right.}{4}\right.}{6c}\right.$
risch	$-\frac{\left(-240c^{3}fh\,x^{6}-280c^{3}eh\,x^{5}-280c^{3}fg\,x^{5}-384a\,c^{2}fh\,x^{4}-336c^{3}dh\,x^{4}-336c^{3}eg\,x^{4}-490a\,c^{2}eh\,x^{3}-490a\,c^{2}fg\,x^{3}-420c^{3}dg\,x^{3}-48a^{2}\right.}{1680c^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] h*f*(1/7*x^2*(c*x^2+a)^(5/2)/c-2/35*a/c^2*(c*x^2+a)^(5/2))+(e*h+f*g)*(1/6*x
*(c*x^2+a)^(5/2)/c-1/6*a/c*(1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1
/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))))+1/5*(d*h+e*g)*(c*x^2+a)^
(5/2)/c+d*g*(1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/
2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.30, size = 216, normalized size = 1.01

$$\frac{(cx^2+a)^{\frac{5}{2}}fhx^2}{7\,c}+\frac{1}{4}\,(cx^2+a)^{\frac{3}{2}}dgx+\frac{3}{8}\,\sqrt{cx^2+a}\,adgx+\frac{3\,a^2dg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}}+\frac{(cx^2+a)^{\frac{5}{2}}dh}{5\,c}-\frac{2\,(cx^2+a)^{\frac{5}{2}}afh}{35\,c^2}+\frac{(cx^2+a)^{\frac{5}{2}}(fg+he)x}{6\,c}-\frac{(cx^2+a)^{\frac{3}{2}}(fg+he)ax}{24\,c}-\frac{\sqrt{cx^2+a}\,(fg+he)a^2x}{16\,c}-\frac{(fg+he)a^3\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,c^{\frac{3}{2}}}+\frac{(cx^2+a)^{\frac{5}{2}}ge}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/7*(c*x^2 + a)^(5/2)*f*h*x^2/c + 1/4*(c*x^2 + a)^(3/2)*d*g*x + 3/8*sqrt(c*
x^2 + a)*a*d*g*x + 3/8*a^2*d*g*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 1/5*(c*x^2
```

```
+ a)^(5/2)*d*h/c - 2/35*(c*x^2 + a)^(5/2)*a*f*h/c^2 + 1/6*(c*x^2 + a)^(5/2)
*(f*g + h*e)*x/c - 1/24*(c*x^2 + a)^(3/2)*(f*g + h*e)*a*x/c - 1/16*sqrt(c*x
^2 + a)*(f*g + h*e)*a^2*x/c - 1/16*(f*g + h*e)*a^3*arcsinh(c*x/sqrt(a*c))/c
^(3/2) + 1/5*(c*x^2 + a)^(5/2)*g*e/c
```

Fricas [A]
time = 0.39, size = 487, normalized size = 2.29

$$\left[\frac{105\,(a^3he-(6\,a^2cd-a^3f)g)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}\,x-a\right)+2\,(240\,c^3fhx^6+280\,c^3fgx^5+48\,(7\,c^3d+8\,ac^2f)hx^4+70\,(6\,c^3d+7\,ac^2f)gx^3+48\,(14\,ac^2d+a^2cf)hx^2+105\,(10\,ac^2d+a^2cf)gx+48\,(7\,a^2cd-2\,a^3f)h+7\,(40\,c^3hx^5+48\,c^3gx^4+70\,ac^2hx^3+96\,ac^2gx^2+15\,a^2chx+48\,a^2cg)e)\sqrt{cx^2+a}}{3360\,c^2},\ \frac{105\,(a^3he-(6\,a^2cd-a^3f)g)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)+(240\,c^3fhx^6+280\,c^3fgx^5+48\,(7\,c^3d+8\,ac^2f)hx^4+70\,(6\,c^3d+7\,ac^2f)gx^3+48\,(14\,ac^2d+a^2cf)hx^2+105\,(10\,ac^2d+a^2cf)gx+48\,(7\,a^2cd-2\,a^3f)h+7\,(40\,c^3hx^5+48\,c^3gx^4+70\,ac^2hx^3+96\,ac^2gx^2+15\,a^2chx+48\,a^2cg)e)\sqrt{cx^2+a}}{1680\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/3360*(105*(a^3*h*e - (6*a^2*c*d - a^3*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sqr
t(c*x^2 + a)*sqrt(c)*x - a) + 2*(240*c^3*f*h*x^6 + 280*c^3*f*g*x^5 + 48*(7*
c^3*d + 8*a*c^2*f)*h*x^4 + 70*(6*c^3*d + 7*a*c^2*f)*g*x^3 + 48*(14*a*c^2*d
+ a^2*c*f)*h*x^2 + 105*(10*a*c^2*d + a^2*c*f)*g*x + 48*(7*a^2*c*d - 2*a^3*f
)*h + 7*(40*c^3*h*x^5 + 48*c^3*g*x^4 + 70*a*c^2*h*x^3 + 96*a*c^2*g*x^2 + 15
*a^2*c*h*x + 48*a^2*c*g)*e)*sqrt(c*x^2 + a))/c^2, 1/1680*(105*(a^3*h*e - (6
*a^2*c*d - a^3*f)*g)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (240*c^3
*f*h*x^6 + 280*c^3*f*g*x^5 + 48*(7*c^3*d + 8*a*c^2*f)*h*x^4 + 70*(6*c^3*d +
 7*a*c^2*f)*g*x^3 + 48*(14*a*c^2*d + a^2*c*f)*h*x^2 + 105*(10*a*c^2*d + a^2
*c*f)*g*x + 48*(7*a^2*c*d - 2*a^3*f)*h + 7*(40*c^3*h*x^5 + 48*c^3*g*x^4 + 7
0*a*c^2*h*x^3 + 96*a*c^2*g*x^2 + 15*a^2*c*h*x + 48*a^2*c*g)*e)*sqrt(c*x^2 +
 a))/c^2]
```

Sympy [A]
time = 18.98, size = 768, normalized size = 3.61

$$\frac{a^{\frac{5}{2}}ehx}{16c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{5}{2}}fgx}{16c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}dgx\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{a^{\frac{3}{2}}dgx}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{17a^{\frac{3}{2}}ehx^3}{48\sqrt{1+\frac{cx^2}{a}}}+\frac{17a^{\frac{3}{2}}fgx^3}{48\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}\,cdgx^3}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{11\sqrt{a}\,cehx^5}{24\sqrt{1+\frac{cx^2}{a}}}+\frac{11\sqrt{a}\,cfgx^5}{24\sqrt{1+\frac{cx^2}{a}}}-\frac{a^3eh\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16c^{\frac{3}{2}}}-\frac{a^3fg\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16c^{\frac{3}{2}}}+\frac{3a^2dg\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\sqrt{c}}+adh\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\: c=0\\\frac{(a+cx^2)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)+aeg\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\: c=0\\\frac{(a+cx^2)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)+afh\left(\begin{cases}-\frac{2a^2\sqrt{a+cx^2}}{15c^2}+\frac{ax^2\sqrt{a+cx^2}}{15c}+\frac{x^4\sqrt{a+cx^2}}{5} & \text{for}\: c\neq 0\\\frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+cdh\left(\begin{cases}-\frac{2a^2\sqrt{a+cx^2}}{15c^2}+\frac{ax^2\sqrt{a+cx^2}}{15c}+\frac{x^4\sqrt{a+cx^2}}{5} & \text{for}\: c\neq 0\\\frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+ceg\left(\begin{cases}-\frac{2a^2\sqrt{a+cx^2}}{15c^2}+\frac{ax^2\sqrt{a+cx^2}}{15c}+\frac{x^4\sqrt{a+cx^2}}{5} & \text{for}\: c\neq 0\\\frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+cfh\left(\begin{cases}\frac{8a^3\sqrt{a+cx^2}}{105c^3}-\frac{4a^2x^2\sqrt{a+cx^2}}{105c^2}+\frac{ax^4\sqrt{a+cx^2}}{35c}+\frac{x^6\sqrt{a+cx^2}}{7} & \text{for}\: c\neq 0\\\frac{\sqrt{a}\,x^6}{6} & \text{otherwise}\end{cases}\right)+\frac{c^2dgx^5}{4\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}+\frac{c^2ehx^7}{6\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}+\frac{c^2fgx^7}{6\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] a**(5/2)*e*h*x/(16*c*sqrt(1 + c*x**2/a)) + a**(5/2)*f*g*x/(16*c*sqrt(1 + c*
x**2/a)) + a**(3/2)*d*g*x*sqrt(1 + c*x**2/a)/2 + a**(3/2)*d*g*x/(8*sqrt(1 +
 c*x**2/a)) + 17*a**(3/2)*e*h*x**3/(48*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*f*
g*x**3/(48*sqrt(1 + c*x**2/a)) + 3*sqrt(a)*c*d*g*x**3/(8*sqrt(1 + c*x**2/a)
) + 11*sqrt(a)*c*e*h*x**5/(24*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*f*g*x**5/(
24*sqrt(1 + c*x**2/a)) - a**3*e*h*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) -
a**3*f*g*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) + 3*a**2*d*g*asinh(sqrt(c)*
x/sqrt(a))/(8*sqrt(c)) + a*d*h*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a +
c*x**2)**(3/2)/(3*c), True)) + a*e*g*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)),
((a + c*x**2)**(3/2)/(3*c), True)) + a*f*h*Piecewise((-2*a**2*sqrt(a + c*x*
*2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, N
e(c, 0)), (sqrt(a)*x**4/4, True)) + c*d*h*Piecewise((-2*a**2*sqrt(a + c*x**
```

```
2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne
(c, 0)), (sqrt(a)*x**4/4, True)) + c*e*g*Piecewise((-2*a**2*sqrt(a + c*x**2
)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(
c, 0)), (sqrt(a)*x**4/4, True)) + c*f*h*Piecewise((8*a**3*sqrt(a + c*x**2)/
(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x*
*2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) +
c**2*d*g*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*e*h*x**7/(6*sqrt(a)*sqr
t(1 + c*x**2/a)) + c**2*f*g*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [**A**]
time = 5.65, size = 264, normalized size = 1.24

$$\frac{1}{1680}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\left(6\,cfhx+\frac{7\,(c^6fg+c^6he)}{c^5}\right)x+\frac{6\,(7\,c^6dh+8\,ac^5fh+7\,c^6ge)}{c^5}\right)x+\frac{35\,(6\,c^6dg+7\,ac^5fg+7\,ac^5he)}{c^5}\right)x+\frac{24\,(14\,ac^5dh+a^2c^4fh+14\,ac^5ge)}{c^5}\right)x+\frac{105\,(10\,ac^5dg+a^2c^4fg+a^2c^4he)}{c^5}\right)x+\frac{48\,(7\,a^2c^4dh-2\,a^3c^3fh+7\,a^2c^4ge)}{c^5}\right)-\frac{(6\,a^2cdg-a^3fg-a^3he)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{16\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/1680*sqrt(c*x^2 + a)*((2*((4*(5*(6*c*f*h*x + 7*(c^6*f*g + c^6*h*e)/c^5)*x
 + 6*(7*c^6*d*h + 8*a*c^5*f*h + 7*c^6*g*e)/c^5)*x + 35*(6*c^6*d*g + 7*a*c^5
*f*g + 7*a*c^5*h*e)/c^5)*x + 24*(14*a*c^5*d*h + a^2*c^4*f*h + 14*a*c^5*g*e)
/c^5)*x + 105*(10*a*c^5*d*g + a^2*c^4*f*g + a^2*c^4*h*e)/c^5)*x + 48*(7*a^2
*c^4*d*h - 2*a^3*c^3*f*h + 7*a^2*c^4*g*e)/c^5) - 1/16*(6*a^2*c*d*g - a^3*f*
g - a^3*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g+h\,x)\,\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.91 $\int (a+cx^2)^{3/2} (d+ex+fx^2) \, dx$

Optimal. Leaf size=137

$$\frac{a(6cd-af)x\sqrt{a+cx^2}}{16c}+\frac{(6cd-af)x(a+cx^2)^{3/2}}{24c}+\frac{e(a+cx^2)^{5/2}}{5c}+\frac{fx(a+cx^2)^{5/2}}{6c}+\frac{a^2(6cd-af)\tanh^{-1}\left(\cdot\right.}{16c^{3/2}}$$

```
[Out] 1/24*(-a*f+6*c*d)*x*(c*x^2+a)^(3/2)/c+1/5*e*(c*x^2+a)^(5/2)/c+1/6*f*x*(c*x^
2+a)^(5/2)/c+1/16*a^2*(-a*f+6*c*d)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/
2)+1/16*a*(-a*f+6*c*d)*x*(c*x^2+a)^(1/2)/c
```

Rubi [A]
time = 0.05, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1829, 655, 201, 223, 212}

$$\frac{a^2(6cd-af)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}}+\frac{x(a+cx^2)^{3/2}(6cd-af)}{24c}+\frac{ax\sqrt{a+cx^2}\,(6cd-af)}{16c}+\frac{e(a+cx^2)^{5/2}}{5c}+\frac{fx(a+cx^2)^{5/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(6*c*d - a*f)*x*Sqrt[a + c*x^2])/(16*c) + ((6*c*d - a*f)*x*(a + c*x^2)^(
3/2))/(24*c) + (e*(a + c*x^2)^(5/2))/(5*c) + (f*x*(a + c*x^2)^(5/2))/(6*c)
 + (a^2*(6*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rubi steps

$$\int \left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \frac{fx(a+cx^2)^{5/2}}{6c}+\frac{\int (6cd-af+6cex)\left(a+cx^2\right)^{3/2}\,dx}{6c}$$

$$= \frac{e(a+cx^2)^{5/2}}{5c}+\frac{fx(a+cx^2)^{5/2}}{6c}+\frac{(6cd-af)\int \left(a+cx^2\right)^{3/2}\,dx}{6c}$$

$$= \frac{(6cd-af)x(a+cx^2)^{3/2}}{24c}+\frac{e(a+cx^2)^{5/2}}{5c}+\frac{fx(a+cx^2)^{5/2}}{6c}+\frac{(a(6cd-$$

$$= \frac{a(6cd-af)x\sqrt{a+cx^2}}{16c}+\frac{(6cd-af)x(a+cx^2)^{3/2}}{24c}+\frac{e(a+cx^2)^{5/2}}{5c}+$$

$$= \frac{a(6cd-af)x\sqrt{a+cx^2}}{16c}+\frac{(6cd-af)x(a+cx^2)^{3/2}}{24c}+\frac{e(a+cx^2)^{5/2}}{5c}+$$

$$= \frac{a(6cd-af)x\sqrt{a+cx^2}}{16c}+\frac{(6cd-af)x(a+cx^2)^{3/2}}{24c}+\frac{e(a+cx^2)^{5/2}}{5c}+$$

Mathematica [A]
time = 0.42, size = 117, normalized size = 0.85

$$\frac{\sqrt{c}\,\sqrt{a+cx^2}\,\left(3a^2(16e+5fx)+4c^2x^3(15d+2x(6e+5fx))+2acx(75d+x(48e+35fx))\right)+15a^2(-6cd+af)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{240c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + c*x^2]*(3*a^2*(16*e + 5*f*x) + 4*c^2*x^3*(15*d + 2*x*(6*e
 + 5*f*x)) + 2*a*c*x*(75*d + x*(48*e + 35*f*x))) + 15*a^2*(-6*c*d + a*f)*Lo
g[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(240*c^(3/2))
```

Maple [A]
time = 0.07, size = 145, normalized size = 1.06

method	result
risch	$\frac{(40c^2 f x^5+48c^2 e x^4+70acf x^3+60c^2 d x^3+96ace x^2+15a^2 fx+150acdx+48a^2e)\sqrt{cx^2+a}}{240c} - \frac{a^3 \ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right) f}{16c^{\frac{3}{2}}}$
default	$f\left(\frac{x(cx^2+a)^{\frac{5}{2}}}{6c} - \frac{a\left(\frac{x(cx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{cx^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{2\sqrt{c}}\right)}{4}\right)}{6c}\right)+\frac{e(cx^2+a)^{\frac{5}{2}}}{5c}+d\left(\frac{x(cx^2+a}{4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/6*x*(c*x^2+a)^(5/2)/c-1/6*a/c*(1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x
^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))))+1/5*e*(c*x^2+a)^
(5/2)/c+d*(1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)
*ln(x*c^(1/2)+(c*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.31, size = 132, normalized size = 0.96

$$\frac{1}{4}\,(cx^2+a)^{\frac{3}{2}}dx+\frac{3}{8}\,\sqrt{cx^2+a}\,adx+\frac{(cx^2+a)^{\frac{5}{2}}fx}{6\,c}-\frac{(cx^2+a)^{\frac{3}{2}}afx}{24\,c}-\frac{\sqrt{cx^2+a}\,a^2fx}{16\,c}+\frac{3\,a^2d\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}}-\frac{a^3f\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,c^{\frac{3}{2}}}+\frac{(cx^2+a)^{\frac{5}{2}}e}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2 + a)^(3/2)*d*x + 3/8*sqrt(c*x^2 + a)*a*d*x + 1/6*(c*x^2 + a)^(5/
2)*f*x/c - 1/24*(c*x^2 + a)^(3/2)*a*f*x/c - 1/16*sqrt(c*x^2 + a)*a^2*f*x/c
+ 3/8*a^2*d*arcsinh(c*x/sqrt(a*c))/sqrt(c) - 1/16*a^3*f*arcsinh(c*x/sqrt(a*
c))/c^(3/2) + 1/5*(c*x^2 + a)^(5/2)*e/c
```

Fricas [A]
time = 0.39, size = 262, normalized size = 1.91

$$\left[-\frac{15\,(6\,a^2cd-a^3f)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\,(40\,c^3fx^5+10\,(6\,c^3d+7\,ac^2f)x^3+15\,(10\,ac^2d+a^2cf)x+48\,(c^3x^4+2\,ac^2x^2+a^2c)e)\sqrt{cx^2+a}}{480\,c^2},\,-\frac{15\,(6\,a^2cd-a^3f)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)-(40\,c^3fx^5+10\,(6\,c^3d+7\,ac^2f)x^3+15\,(10\,ac^2d+a^2cf)x+48\,(c^3x^4+2\,ac^2x^2+a^2c)e)\sqrt{cx^2+a}}{240\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/480*(15*(6*a^2*c*d - a^3*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sq
rt(c)*x - a) - 2*(40*c^3*f*x^5 + 10*(6*c^3*d + 7*a*c^2*f)*x^3 + 15*(10*a*c^
2*d + a^2*c*f)*x + 48*(c^3*x^4 + 2*a*c^2*x^2 + a^2*c)*e)*sqrt(c*x^2 + a))/c
^2, -1/240*(15*(6*a^2*c*d - a^3*f)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 +
a)) - (40*c^3*f*x^5 + 10*(6*c^3*d + 7*a*c^2*f)*x^3 + 15*(10*a*c^2*d + a^2*c
*f)*x + 48*(c^3*x^4 + 2*a*c^2*x^2 + a^2*c)*e)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A]
time = 10.36, size = 348, normalized size = 2.54

$$\frac{a^{\frac{5}{2}}fx}{16c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}dx\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{a^{\frac{3}{2}}dx}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{17a^{\frac{3}{2}}fx^3}{48\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}\,cdx^3}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{11\sqrt{a}\,cfx^5}{24\sqrt{1+\frac{cx^2}{a}}}-\frac{a^3f\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16c^{\frac{3}{2}}}+\frac{3a^2d\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\sqrt{c}}+ae\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ c=0\\ \frac{(a+cx^2)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)+ce\left(\begin{cases}-\frac{2a^2\sqrt{a+cx^2}}{15c^2}+\frac{ax^2\sqrt{a+cx^2}}{15c}+\frac{x^4\sqrt{a+cx^2}}{5} & \text{for}\ c\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+\frac{c^2dx^5}{4\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}+\frac{c^2fx^7}{6\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] a**(5/2)*f*x/(16*c*sqrt(1 + c*x**2/a)) + a**(3/2)*d*x*sqrt(1 + c*x**2/a)/2
+ a**(3/2)*d*x/(8*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*f*x**3/(48*sqrt(1 + c*x
**2/a)) + 3*sqrt(a)*c*d*x**3/(8*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*f*x**5/(
24*sqrt(1 + c*x**2/a)) - a**3*f*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) + 3*
a**2*d*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c)) + a*e*Piecewise((sqrt(a)*x**2/2
, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + c*e*Piecewise((-2*a**2*sq
rt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c
*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c**2*d*x**5/(4*sqrt(a)*sqrt(
1 + c*x**2/a)) + c**2*f*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [A]
time = 4.27, size = 129, normalized size = 0.94

$$\frac{1}{240}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\,cfx+6\,ce\right)x+\frac{5\left(6\,c^5d+7\,ac^4f\right)}{c^4}\right)x+48\,ae\right)x+\frac{15\left(10\,ac^4d+a^2c^3f\right)}{c^4}\right)x+\frac{48\,a^2e}{c}\right)-\frac{\left(6\,a^2cd-a^3f\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{16\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/240*sqrt(c*x^2 + a)*((2*((4*(5*c*f*x + 6*c*e)*x + 5*(6*c^5*d + 7*a*c^4*f)
/c^4)*x + 48*a*e)*x + 15*(10*a*c^4*d + a^2*c^3*f)/c^4)*x + 48*a^2*e/c) - 1/
16*(6*a^2*c*d - a^3*f)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.92 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx} dx$

Optimal. Leaf size=326

$$\frac{(8(cg^2+ah^2)(fg^2-egh+dh^2)-h(4cdgh^2+(fg-eh)(4cg^2+3ah^2))x)\sqrt{a+cx^2}}{8h^5}+\frac{(4(fg^2-egh+dh^2)}{}$$

```
[Out] 1/12*(4*d*h^2-4*e*g*h+4*f*g^2-3*h*(-e*h+f*g)*x)*(c*x^2+a)^(3/2)/h^3+1/5*f*(
c*x^2+a)^(5/2)/c/h-(a*h^2+c*g^2)^(3/2)*(d*h^2-e*g*h+f*g^2)*arctanh((-c*g*x+
a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^6-1/8*(3*a^2*h^4*(-e*h+f*g)+8*c
^2*g^3*(f*g^2-h*(-d*h+e*g))+12*a*c*g*h^2*(f*g^2-h*(-d*h+e*g)))*arctanh(x*c^
(1/2)/(c*x^2+a)^(1/2))/h^6/c^(1/2)+1/8*(8*(a*h^2+c*g^2)*(d*h^2-e*g*h+f*g^2)
-h*(4*c*d*g*h^2+(-e*h+f*g)*(3*a*h^2+4*c*g^2))*x)*(c*x^2+a)^(1/2)/h^5
```

Rubi [A]
time = 0.47, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1668, 829, 858, 223, 212, 739}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(3a^2h^4(fg-eh)+12acgh^2(fg^2-h(eg-dh))+8c^2(fg^5-g^3h(eg-dh)))}{8\sqrt{c}\,h^6}-\frac{(ah^2+cg^2)^{3/2}(dh^2-egh+fg^2)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h^6}+\frac{\sqrt{a+cx^2}\,(8(ah^2+cg^2)(dh^2-egh+fg^2)-hx((3ah^2+4cg^2)(fg-eh)+4cdgh^2))}{8h^5}+\frac{(a+cx^2)^{3/2}(4(dh^2-egh+fg^2)-3hx(fg-eh))}{12h^3}+\frac{f(a+cx^2)^{5/2}}{5ch}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((8*(c*g^2 + a*h^2)*(f*g^2 - e*g*h + d*h^2) - h*(4*c*d*g*h^2 + (f*g - e*h)*
(4*c*g^2 + 3*a*h^2))*x)*Sqrt[a + c*x^2])/(8*h^5) + ((4*(f*g^2 - e*g*h + d*h
^2) - 3*h*(f*g - e*h)*x)*(a + c*x^2)^(3/2))/(12*h^3) + (f*(a + c*x^2)^(5/2)
)/(5*c*h) - ((3*a^2*h^4*(f*g - e*h) + 12*a*c*g*h^2*(f*g^2 - h*(e*g - d*h))
+ 8*c^2*(f*g^5 - g^3*h*(e*g - d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/
(8*Sqrt[c]*h^6) - ((c*g^2 + a*h^2)^(3/2)*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(a
*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/h^6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \frac{f(a+cx^2)^{5/2}}{5ch} + \frac{\int \frac{(5cdh^2-5ch(fg-eh)x)(a+cx^2)^{3/2}}{g+hx}\,dx}{5ch^2}$$

$$= \frac{(4(fg^2-egh+dh^2)-3h(fg-eh)x)(a+cx^2)^{3/2}}{12h^3} + \frac{f(a+cx^2)^{5/2}}{5ch} + \frac{\int}{}$$

$$= \frac{(8(cg^2+ah^2)(fg^2-egh+dh^2)-h(4cdgh^2+(fg-eh)(4cg^2+3ah^2)}{8h^5}$$

$$= \frac{(8(cg^2+ah^2)(fg^2-egh+dh^2)-h(4cdgh^2+(fg-eh)(4cg^2+3ah^2)}{8h^5}$$

$$= \frac{(8(cg^2+ah^2)(fg^2-egh+dh^2)-h(4cdgh^2+(fg-eh)(4cg^2+3ah^2)}{8h^5}$$

$$= \frac{(8(cg^2+ah^2)(fg^2-egh+dh^2)-h(4cdgh^2+(fg-eh)(4cg^2+3ah^2)}{8h^5}$$

Mathematica [A]
time = 1.39, size = 360, normalized size = 1.10

$$\frac{\frac{h\sqrt{a+cx^2}\,(24a^2fh^4+ach^2(5h(-32eg+32dh+15ehx)+f(160g^2-75ghx+48h^2x^2))+2c^2(f(60g^4-30g^3hx+20g^2h^2x^2-15gh^3x^3+12h^4x^4)+5h(2dh(6g^2-3ghx+2h^2x^2)+e(-12g^3+6g^2hx-4gh^2x^2+3h^3x^3))))}{c} - 240(-cg^2-ah^2)^{3/2}(fg^2+h(-eg+dh))\tan^{-1}\left(\frac{\sqrt{c}\,(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right) + \frac{15(3a^2h^4(fg-eh)+12acgh^2(fg^2+h(-eg+dh))+8c^2(fg^5+g^3h(-eg+dh)))\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{120h^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(24*a^2*f*h^4 + a*c*h^2*(5*h*(-32*e*g + 32*d*h + 15*e*h
*x) + f*(160*g^2 - 75*g*h*x + 48*h^2*x^2)) + 2*c^2*(f*(60*g^4 - 30*g^3*h*x
+ 20*g^2*h^2*x^2 - 15*g*h^3*x^3 + 12*h^4*x^4) + 5*h*(2*d*h*(6*g^2 - 3*g*h*x
 + 2*h^2*x^2) + e*(-12*g^3 + 6*g^2*h*x - 4*g*h^2*x^2 + 3*h^3*x^3)))))/c - 2
40*(-(c*g^2) - a*h^2)^(3/2)*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[c]*(g +
 h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]] + (15*(3*a^2*h^4*(f*g -
e*h) + 12*a*c*g*h^2*(f*g^2 + h*(-(e*g) + d*h)) + 8*c^2*(f*g^5 + g^3*h*(-(e*
g) + d*h)))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/(120*h^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 642 vs. 2(299) = 598.
time = 0.11, size = 643, normalized size = 1.97

method	result

default	$\frac{\frac{fh\left(c\,x^2+a\right)^{\frac{5}{2}}}{5c}+eh\left(\frac{x\left(c\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}\right)}{4}\right)-gf\left(\frac{x\left(c\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c\,x^2+a}}{2}\right.}{}\right.}{h^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] 1/h^2*(1/5*f*h*(c*x^2+a)^(5/2)/c+e*h*(1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c
*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))))-g*f*(1/4*x*(c*x
^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+
a)^(1/2)))))+(d*h^2-e*g*h+f*g^2)/h^3*(1/3*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+
(a*h^2+c*g^2)/h^2)^(3/2)-c*g/h*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*
c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2-4*c
^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*
(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)))+(a*h^2+c*g^2)/h^2*(((x+1/h*g)^2*c-2*c*
g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/
c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c
*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*
g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^
2)/h^2)^(1/2))/(x+1/h*g))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(305) = 610$.
time = 0.37, size = 642, normalized size = 1.97

$$-\frac{\sqrt{cx^2+a}\,cfg^3x}{2h^4}-\frac{\sqrt{cx^2+a}\,cdgx}{2h^2}-\frac{(cx^2+a)^{\frac{3}{2}}fgx}{4h^2}-\frac{3\sqrt{cx^2+a}\,afgx}{8h^2}-\frac{c^{\frac{3}{2}}fg^5\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^6}-\frac{c^{\frac{3}{2}}dg^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4}-\frac{3a\sqrt{c}\,fg^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2h^4}-\frac{3a\sqrt{c}\,dg\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2h^2}-\frac{3a^2fg\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{c}\,h^2}+\frac{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}fg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{h^3}+\frac{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}d\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{h}+\frac{\sqrt{cx^2+a}\,cg^2xe}{2h^3}+\frac{(cx^2+a)^{\frac{3}{2}}xe}{4h}+\frac{3\sqrt{cx^2+a}\,axe}{8h}+\frac{c^{\frac{3}{2}}g^4\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e}{h^5}+\frac{3a\sqrt{c}\,g^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e}{2h^3}+\frac{3a^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e}{8\sqrt{c}\,h}-\frac{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}g\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)e}{h^2}+\frac{\sqrt{cx^2+a}\,cfg^4}{h^5}+\frac{\sqrt{cx^2+a}\,cdg^2}{h^3}+\frac{(cx^2+a)^{\frac{3}{2}}fg^2}{3h^3}+\frac{\sqrt{cx^2+a}\,afg^2}{h^3}+\frac{(cx^2+a)^{\frac{3}{2}}d}{3h}+\frac{\sqrt{cx^2+a}\,ad}{h}+\frac{(cx^2+a)^{\frac{5}{2}}f}{5ch}-\frac{\sqrt{cx^2+a}\,cg^3e}{h^4}-\frac{(cx^2+a)^{\frac{3}{2}}ge}{3h^2}-\frac{\sqrt{cx^2+a}\,age}{h^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^2 + a)*c*f*g^3*x/h^4 - 1/2*sqrt(c*x^2 + a)*c*d*g*x/h^2 - 1/4*
(c*x^2 + a)^(3/2)*f*g*x/h^2 - 3/8*sqrt(c*x^2 + a)*a*f*g*x/h^2 - c^(3/2)*f*g
^5*arcsinh(c*x/sqrt(a*c))/h^6 - c^(3/2)*d*g^3*arcsinh(c*x/sqrt(a*c))/h^4 -
```

```
3/2*a*sqrt(c)*f*g^3*arcsinh(c*x/sqrt(a*c))/h^4 - 3/2*a*sqrt(c)*d*g*arcsinh(
c*x/sqrt(a*c))/h^2 - 3/8*a^2*f*g*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h^2) + (a
+ c*g^2/h^2)^(3/2)*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt
(a*c)*abs(h*x + g)))/h^3 + (a + c*g^2/h^2)^(3/2)*d*arcsinh(c*g*x/(sqrt(a*c)
*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h + 1/2*sqrt(c*x^2 + a)*c*g^
2*x*e/h^3 + 1/4*(c*x^2 + a)^(3/2)*x*e/h + 3/8*sqrt(c*x^2 + a)*a*x*e/h + c^(
3/2)*g^4*arcsinh(c*x/sqrt(a*c))*e/h^5 + 3/2*a*sqrt(c)*g^2*arcsinh(c*x/sqrt(
a*c))*e/h^3 + 3/8*a^2*arcsinh(c*x/sqrt(a*c))*e/(sqrt(c)*h) - (a + c*g^2/h^2
)^(3/2)*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x +
 g)))*e/h^2 + sqrt(c*x^2 + a)*c*f*g^4/h^5 + sqrt(c*x^2 + a)*c*d*g^2/h^3 + 1
/3*(c*x^2 + a)^(3/2)*f*g^2/h^3 + sqrt(c*x^2 + a)*a*f*g^2/h^3 + 1/3*(c*x^2 +
 a)^(3/2)*d/h + sqrt(c*x^2 + a)*a*d/h + 1/5*(c*x^2 + a)^(5/2)*f/(c*h) - sqr
t(c*x^2 + a)*c*g^3*e/h^4 - 1/3*(c*x^2 + a)^(3/2)*g*e/h^2 - sqrt(c*x^2 + a)*
a*g*e/h^2
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+cx^{2}\right)^{\frac{3}{2}}\left(d+ex+fx^{2}\right)}{g+hx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x), x)
```

Giac [**A**]
time = 3.79, size = 551, normalized size = 1.69

$$\frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(\frac{4cfx}{h}-\frac{5\left(c^4fgh^{19}-c^4h^{20}e\right)}{c^3h^{21}}\right)x+\frac{4\left(5c^4fg^2h^{18}+5c^4dh^{20}+6ac^3fh^{20}-5c^4gh^{19}e\right)}{c^3h^{21}}\right)x-\frac{15\left(4c^4fg^3h^{17}+4c^4dgh^{19}+5ac^3fgh^{19}-4c^4g^2h^{18}e-5ac^3h^{20}e\right)}{c^3h^{21}}\right)x+\frac{8\left(15c^4fg^4h^{16}+15c^4dg^2h^{18}+20ac^3fg^2h^{18}+20ac^3dh^{20}+3a^2c^2fh^{20}-15c^4g^3h^{17}e-20ac^3gh^{19}e\right)}{c^3h^{21}}\right)+\frac{2\left(c^2fg^6+c^2dg^4h^2+2acfg^4h^2+2acdg^2h^4+a^2fg^2h^4+a^2dh^6-c^2g^5he-2acg^3h^3e-a^2gh^5e\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)h+\sqrt{c}g}{\sqrt{-cg^2-ah^2}}\right)}{\sqrt{-cg^2-ah^2}\,h^6}+\frac{\left(8c^{\frac{5}{2}}fg^5+8c^{\frac{5}{2}}dg^3h^2+12ac^{\frac{3}{2}}fg^3h^2+12ac^{\frac{3}{2}}dgh^4+3a^2\sqrt{c}fgh^4-8c^{\frac{5}{2}}g^4he-12ac^{\frac{3}{2}}g^2h^3e-3a^2\sqrt{c}h^5e\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{8ch^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(c*x^2 + a)*((2*(3*(4*c*f*x/h - 5*(c^4*f*g*h^19 - c^4*h^20*e)/(c^
3*h^21))*x + 4*(5*c^4*f*g^2*h^18 + 5*c^4*d*h^20 + 6*a*c^3*f*h^20 - 5*c^4*g*
```

```
h^19*e)/(c^3*h^21))*x - 15*(4*c^4*f*g^3*h^17 + 4*c^4*d*g*h^19 + 5*a*c^3*f*g
*h^19 - 4*c^4*g^2*h^18*e - 5*a*c^3*h^20*e)/(c^3*h^21))*x + 8*(15*c^4*f*g^4*
h^16 + 15*c^4*d*g^2*h^18 + 20*a*c^3*f*g^2*h^18 + 20*a*c^3*d*h^20 + 3*a^2*c^
2*f*h^20 - 15*c^4*g^3*h^17*e - 20*a*c^3*g*h^19*e)/(c^3*h^21)) + 2*(c^2*f*g^
6 + c^2*d*g^4*h^2 + 2*a*c*f*g^4*h^2 + 2*a*c*d*g^2*h^4 + a^2*f*g^2*h^4 + a^2
*d*h^6 - c^2*g^5*h*e - 2*a*c*g^3*h^3*e - a^2*g*h^5*e)*arctan(-((sqrt(c)*x -
 sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 - a*h^2
)*h^6) + 1/8*(8*c^(5/2)*f*g^5 + 8*c^(5/2)*d*g^3*h^2 + 12*a*c^(3/2)*f*g^3*h^
2 + 12*a*c^(3/2)*d*g*h^4 + 3*a^2*sqrt(c)*f*g*h^4 - 8*c^(5/2)*g^4*h*e - 12*a
*c^(3/2)*g^2*h^3*e - 3*a^2*sqrt(c)*h^5*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
 a)))/(c*h^6)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{g+h\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

3.93 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2} \, dx$

Optimal. Leaf size=432

$$-\frac{(8(ah^2(2fg-eh)+cg(5fg^2-h(4eg-3dh)))-h(20cfg^2-16cegh+12cdh^2+3afh^2)\,x)\,\sqrt{a+cx^2}}{8h^5}-\frac{(4$$

```
[Out] -1/12*(4*a*h^2*(-e*h+2*f*g)+4*c*g*(5*f*g^2-h*(-3*d*h+4*e*g))-3*h*(a*f*h^2+c
*(5*f*g^2-4*h*(-d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^3/(a*h^2+c*g^2)-(d*h^2-e*g*
h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)+1/8*(3*a^2*f*h^4+8*c^2*g^2
*(5*f*g^2-h*(-3*d*h+4*e*g))+12*a*c*h^2*(3*f*g^2-h*(-d*h+2*e*g)))*arctanh(x*
c^(1/2)/(c*x^2+a)^(1/2))/h^6/c^(1/2)+(a*h^2*(-e*h+2*f*g)+c*g*(5*f*g^2-h*(-3
*d*h+4*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))*(a*
h^2+c*g^2)^(1/2)/h^6-1/8*(8*a*h^2*(-e*h+2*f*g)+8*c*g*(5*f*g^2-h*(-3*d*h+4*e
*g))-h*(3*a*f*h^2+12*c*d*h^2-16*c*e*g*h+20*c*f*g^2)*x)*(c*x^2+a)^(1/2)/h^5
```

Rubi [A]
time = 0.55, antiderivative size = 428, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1665, 829, 858, 223, 212, 739}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(3a^2fh^4+12ach^2(3fg^2-h(2eg-dh))+8c^2(5fg^4-g^2h(4eg-3dh)))}{8\sqrt{c}\,h^6}-\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{h(g+hx)(ah^2+cg^2)}+\frac{\sqrt{ah^2+cg^2}\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(ah^2(2fg-eh)-cgh(4eg-3dh)+5cfg^3)}{h^6}-\frac{\sqrt{a+cx^2}\,(8(ah^2(2fg-eh)-cgh(4eg-3dh)+5cfg^3)-hx(3afh^2+12cdh^2-16cegh+20cfg^2))}{8h^5}-\frac{(a+cx^2)^{3/2}(4(ah^2(2fg-eh)-cgh(4eg-3dh)+5cfg^3)-3hx(afh^2-4ch(eg-dh)+5cfg^2))}{12h^3(ah^2+cg^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -1/8*((8*(5*c*f*g^3 - c*g*h*(4*e*g - 3*d*h) + a*h^2*(2*f*g - e*h)) - h*(20*
c*f*g^2 - 16*c*e*g*h + 12*c*d*h^2 + 3*a*f*h^2)*x)*Sqrt[a + c*x^2])/h^5 - ((
4*(5*c*f*g^3 - c*g*h*(4*e*g - 3*d*h) + a*h^2*(2*f*g - e*h)) - 3*h*(5*c*f*g^
2 + a*f*h^2 - 4*c*h*(e*g - d*h))*x)*(a + c*x^2)^(3/2))/(12*h^3*(c*g^2 + a*h
^2)) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(h*(c*g^2 + a*h^2)*(g +
h*x)) + ((3*a^2*f*h^4 + 8*c^2*(5*f*g^4 - g^2*h*(4*e*g - 3*d*h)) + 12*a*c*h^
2*(3*f*g^2 - h*(2*e*g - d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqr
t[c]*h^6) + (Sqrt[c*g^2 + a*h^2]*(5*c*f*g^3 - c*g*h*(4*e*g - 3*d*h) + a*h^2
*(2*f*g - e*h))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])
])/h^6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = -\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{h(cg^2+ah^2)(g+hx)} - \frac{\int \frac{(-cdg+afg-aeh-(afh-c(4eg-\frac{5fg^2}{h}-4dh}{g+hx}}{cg^2+ah^2}$$

$$= -\frac{(4(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh))-3h(5cfg^2+afh^2-4}{12h^3(cg^2+ah^2)}$$

$$= -\frac{(8(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh))-h(20cfg^2-16cegh+}{8h^5}$$

$$= -\frac{(8(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh))-h(20cfg^2-16cegh+}{8h^5}$$

$$= -\frac{(8(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh))-h(20cfg^2-16cegh+}{8h^5}$$

$$= -\frac{(8(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh))-h(20cfg^2-16cegh+}{8h^5}$$

Mathematica [A]
time = 1.57, size = 364, normalized size = 0.84

$$\frac{\frac{h\sqrt{a+cx^2}(-2cf(60g^4+30g^3hx-10g^2h^2x^2+5gh^3x^3-3h^4x^4)+ah^2(8h(7eg-3dh+4ehx)+f(-88g^2-49ghx+15h^2x^2))+4ch(3dh(-6g^2-3ghx+h^2x^2)+2e(12g^3+6g^2hx-2gh^2x^2+h^3x^3)))}{g+hx}-48\sqrt{-cg^2-ah^2}(5cfg^3+cgh(-4eg+3dh)+ah^2(2fg-eh))\tan^{-1}\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)-\frac{3(3a^2fh^4+12ach^2(3fg^2+h(-2eg+dh))+8c^2(5fg^4+g^2h(-4eg+3dh)))\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{24h^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(-2*c*f*(60*g^4 + 30*g^3*h*x - 10*g^2*h^2*x^2 + 5*g*h^3
*x^3 - 3*h^4*x^4) + a*h^2*(8*h*(7*e*g - 3*d*h + 4*e*h*x) + f*(-88*g^2 - 49*
g*h*x + 15*h^2*x^2)) + 4*c*h*(3*d*h*(-6*g^2 - 3*g*h*x + h^2*x^2) + 2*e*(12*
g^3 + 6*g^2*h*x - 2*g*h^2*x^2 + h^3*x^3))))/(g + h*x) - 48*Sqrt[-(c*g^2) -
a*h^2]*(5*c*f*g^3 + c*g*h*(-4*e*g + 3*d*h) + a*h^2*(2*f*g - e*h))*ArcTan[(S
qrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]] - (3*(3*a^2*f
*h^4 + 12*a*c*h^2*(3*f*g^2 + h*(-2*e*g + d*h)) + 8*c^2*(5*f*g^4 + g^2*h*(-4
*e*g + 3*d*h)))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/(24*h^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1447 vs. $2(406) = 812$.
time = 0.12, size = 1448, normalized size = 3.35

method	result	size
default	Expression too large to display	1448
risch	Expression too large to display	3364

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^2*(1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(
x*c^(1/2)+(c*x^2+a)^(1/2))))+1/h^3*(e*h-2*f*g)*(1/3*((x+1/h*g)^2*c-2*c*g/h*
(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g/h*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x
+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^
2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*
c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)))+(a*h^2+c*g^2)/h^2*(((x+1/h*g
)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(
x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)
)-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g
/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(
a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))))+1/h^4*(d*h^2-e*g*h+f*g^2)*(-1/(a*h^2+
c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(5
/2)-3*c*g*h/(a*h^2+c*g^2)*(1/3*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^
2)/h^2)^(3/2)-c*g/h*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(
x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)
/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(
a*h^2+c*g^2)/h^2)^(1/2)))+(a*h^2+c*g^2)/h^2*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*
g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x
+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/(
(a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^
2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/
2))/(x+1/h*g))))+4*c/(a*h^2+c*g^2)*h^2*(1/8*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1
/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+3/16*(4*c*(a*h^2+c*g^2
)/h^2-4*c^2*g^2/h^2)/c*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/
h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h
^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g
)+(a*h^2+c*g^2)/h^2)^(1/2)))))
```

Maxima [A]
time = 0.34, size = 717, normalized size = 1.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="maxima")
```

```
[Out] -(c*x^2 + a)^(3/2)*f*g^2/(h^4*x + g*h^3) + (c*x^2 + a)^(3/2)*g*e/(h^3*x + g
*h^2) - (c*x^2 + a)^(3/2)*d/(h^2*x + g*h) + 5/2*sqrt(c*x^2 + a)*c*f*g^2*x/h
^4 + 3/2*sqrt(c*x^2 + a)*c*d*x/h^2 + 1/4*(c*x^2 + a)^(3/2)*f*x/h^2 + 3/8*sq
rt(c*x^2 + a)*a*f*x/h^2 + 5*c^(3/2)*f*g^4*arcsinh(c*x/sqrt(a*c))/h^6 + 3*c^
(3/2)*d*g^2*arcsinh(c*x/sqrt(a*c))/h^4 + 9/2*a*sqrt(c)*f*g^2*arcsinh(c*x/sq
rt(a*c))/h^4 + 3/2*a*sqrt(c)*d*arcsinh(c*x/sqrt(a*c))/h^2 + 3/8*a^2*f*arcsi
nh(c*x/sqrt(a*c))/(sqrt(c)*h^2) - 3*sqrt(a + c*g^2/h^2)*c*f*g^3*arcsinh(c*g
*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^5 - 3*sqrt(a
+ c*g^2/h^2)*c*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*
abs(h*x + g)))/h^3 - 2*(a + c*g^2/h^2)^(3/2)*f*g*arcsinh(c*g*x/(sqrt(a*c)*a
bs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 - 2*sqrt(c*x^2 + a)*c*g*x*
e/h^3 - 4*c^(3/2)*g^3*arcsinh(c*x/sqrt(a*c))*e/h^5 - 3*a*sqrt(c)*g*arcsinh(
c*x/sqrt(a*c))*e/h^3 + 3*sqrt(a + c*g^2/h^2)*c*g^2*arcsinh(c*g*x/(sqrt(a*c)
*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))*e/h^4 + (a + c*g^2/h^2)^(3/2
)*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))*e/
h^2 - 5*sqrt(c*x^2 + a)*c*f*g^3/h^5 - 3*sqrt(c*x^2 + a)*c*d*g/h^3 - 2/3*(c*
x^2 + a)^(3/2)*f*g/h^3 - 2*sqrt(c*x^2 + a)*a*f*g/h^3 + 4*sqrt(c*x^2 + a)*c*
g^2*e/h^4 + 1/3*(c*x^2 + a)^(3/2)*e/h^2 + sqrt(c*x^2 + a)*a*e/h^2
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^{\frac{3}{2}} (d + ex + fx^2)}{(g + hx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**2,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
```

```
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

3.94 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx$

Optimal. Leaf size=488

$$\frac{(2a^2fh^4+2c^2g^2(10fg^2-3h(2eg-dh))+ach^2(19fg^2-3h(3eg-dh))-ch(ah^2(7fg-3eh)+cg(10fg^2-}{2h^5\left(cg^2+ah^2\right)}$$

```
[Out] -1/6*(2*c*g*(6*e*g-10*f*g^2/h-3*d*h)-2*a*h*(-3*e*h+7*f*g)-(2*a*f*h^2+c*(5*f
*g^2-3*h*(-d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^2/(a*h^2+c*g^2)/(h*x+g)-1/2*(d*h
^2-e*g*h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^2-1/2*(3*a*h^2*(-e*
h+3*f*g)+2*c*g*(10*f*g^2-3*h*(-d*h+2*e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/
2))*c^(1/2)/h^6-1/2*(2*a^2*f*h^4+2*c^2*g^2*(10*f*g^2-3*h*(-d*h+2*e*g))+a*c*
h^2*(19*f*g^2-3*h*(-d*h+3*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(
c*x^2+a)^(1/2))/h^6/(a*h^2+c*g^2)^(1/2)+1/2*(2*a^2*f*h^4+2*c^2*g^2*(10*f*g^
2-3*h*(-d*h+2*e*g))+a*c*h^2*(19*f*g^2-3*h*(-d*h+3*e*g))-c*h*(a*h^2*(-3*e*h+
7*f*g)+c*g*(10*f*g^2-3*h*(-d*h+2*e*g)))*x)*(c*x^2+a)^(1/2)/h^5/(a*h^2+c*g^2
)
```

Rubi [A]
time = 0.57, antiderivative size = 480, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1665, 827, 829, 858, 223, 212, 739}

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(2a^2fh^4+ach^2(19fg^2-3h(3eg-dh))+2c^2(10fg^4-3g^2h(2eg-dh)))}{2h^6\sqrt{ah^2+cg^2}}+\frac{\sqrt{a+cx^2}\left(2a^2fh^3-cx(ah^2(7fg-3eh)-3cgh(2eg-dh)+10cfg^3)+ach(19fg^2-3h(3eg-dh))-2c^2g^2\left(-3dh+6eg-\frac{10fg^2}{h}\right)\right)}{2h^4(ah^2+cg^2)}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(3ah^2(3fg-eh)-6cgh(2eg-dh)+20cfg^3)}{2h^6}-\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{2h(g+hx)^2(ah^2+cg^2)}-\frac{(a+cx^2)^{3/2}\left(2\left(cg\left(-3dh+6eg-\frac{10fg^2}{h}\right)-ah(7fg-3eh)\right)-x(2afh^2-3ch(eg-dh)+5cfg^2)\right)}{6h^2(g+hx)(ah^2+cg^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((2*a^2*f*h^3 - 2*c^2*g^2*(6*e*g - (10*f*g^2)/h - 3*d*h) + a*c*h*(19*f*g^2
- 3*h*(3*e*g - d*h)) - c*(10*c*f*g^3 - 3*c*g*h*(2*e*g - d*h) + a*h^2*(7*f*g
 - 3*e*h))*x)*Sqrt[a + c*x^2])/(2*h^4*(c*g^2 + a*h^2)) - ((2*(c*g*(6*e*g -
(10*f*g^2)/h - 3*d*h) - a*h*(7*f*g - 3*e*h)) - (5*c*f*g^2 + 2*a*f*h^2 - 3*c
*h*(e*g - d*h))*x)*(a + c*x^2)^(3/2))/(6*h^2*(c*g^2 + a*h^2)*(g + h*x)) - (
(f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(2*h*(c*g^2 + a*h^2)*(g + h*x)^2
) - (Sqrt[c]*(20*c*f*g^3 - 6*c*g*h*(2*e*g - d*h) + 3*a*h^2*(3*f*g - e*h))*A
rcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*h^6) - ((2*a^2*f*h^4 + 2*c^2*(10*f*
g^4 - 3*g^2*h*(2*e*g - d*h)) + a*c*h^2*(19*f*g^2 - 3*h*(3*e*g - d*h)))*ArcT
anh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*h^6*Sqrt[c*g^2
 + a*h^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = -\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{2h(cg^2+ah^2)(g+hx)^2} - \frac{\int \frac{\left(-2(cdg-afg+aeh)-\left(2afh-c\left(3eg-\frac{5fg^2}{h}\right.\right.\right.}{(g+hx)^2}}{2(cg^2+ah^2)}$$

$$= -\frac{\left(2\left(cg\left(6eg-\frac{10fg^2}{h}-3dh\right)-ah(7fg-3eh)\right)-(5cfg^2+2afh^2-3c\right.}{6h^2(cg^2+ah^2)(g+hx)}$$

$$= \frac{\left(2a^2fh^3-2c^2g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)+ach(19fg^2-3h(3eg-dh))-c\right.}{2h^4(cg^2+ah}$$

$$= \frac{\left(2a^2fh^3-2c^2g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)+ach(19fg^2-3h(3eg-dh))-c\right.}{2h^4(cg^2+ah}$$

$$= \frac{\left(2a^2fh^3-2c^2g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)+ach(19fg^2-3h(3eg-dh))-c\right.}{2h^4(cg^2+ah}$$

$$= \frac{\left(2a^2fh^3-2c^2g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)+ach(19fg^2-3h(3eg-dh))-c\right.}{2h^4(cg^2+ah}$$

Mathematica [A]
time = 2.18, size = 361, normalized size = 0.74

$$\frac{\frac{h\sqrt{a+cx^2}\left(ah^2(-3h(eg+dh+2ehx)+f(17g^2+28ghx+8h^2x^2))+c(f(60g^4+90g^3hx+20g^2h^2x^2-5gh^3x^3+2h^4x^4)+3h(dh(6g^2+9ghx+2h^2x^2)+e(-12g^3-18g^2hx-4gh^2x^2+h^3x^3)))\right)}{(g+hx)^2}-\frac{6(2a^2fh^4+ach^2(19fg^2+3h(-3eg+dh))+2c^2(10fg^4+3g^2h(-2eg+dh)))\tan^{-1}\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{\sqrt{-cg^2-ah^2}}+3\sqrt{c}(20cfg^3+6cgh(-2eg+dh)-3ah^2(-3fg+eh))\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{6h^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(a*h^2*(-3*h*(e*g + d*h + 2*e*h*x) + f*(17*g^2 + 28*g*h
*x + 8*h^2*x^2)) + c*(f*(60*g^4 + 90*g^3*h*x + 20*g^2*h^2*x^2 - 5*g*h^3*x^3
```

```
 + 2*h^4*x^4) + 3*h*(d*h*(6*g^2 + 9*g*h*x + 2*h^2*x^2) + e*(-12*g^3 - 18*g^
2*h*x - 4*g*h^2*x^2 + h^3*x^3)))))/(g + h*x)^2 - (6*(2*a^2*f*h^4 + a*c*h^2*
(19*f*g^2 + 3*h*(-3*e*g + d*h)) + 2*c^2*(10*f*g^4 + 3*g^2*h*(-2*e*g + d*h))
)*ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/S
qrt[-(c*g^2) - a*h^2] + 3*Sqrt[c]*(20*c*f*g^3 + 6*c*g*h*(-2*e*g + d*h) - 3*
a*h^2*(-3*f*g + e*h))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(6*h^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2862 vs. $2(458) = 916$.
time = 0.14, size = 2863, normalized size = 5.87

method	result	size
default	Expression too large to display	2863
risch	Expression too large to display	5749

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^3*(1/3*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g/h*
(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^
2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+
c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1
/2)))+(a*h^2+c*g^2)/h^2*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2
)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*
(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(
1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*
((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))))+(e*
h-2*f*g)/h^4*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*
g)+(a*h^2+c*g^2)/h^2)^(5/2)-3*c*g*h/(a*h^2+c*g^2)*(1/3*((x+1/h*g)^2*c-2*c*g
/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g/h*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*
((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c
*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)
^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)))+(a*h^2+c*g^2)/h^2*(((x+1/
h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+
c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1
/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*
c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g
)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))))+4*c/(a*h^2+c*g^2)*h^2*(1/8*(2*c*(x
+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2
)+3/16*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c*(1/4*(2*c*(x+1/h*g)-2*c*g/h)
/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^
2+c*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h
*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)))))+(d*h^2-e*g*h+f*g^2)/
h^5*(-1/2/(a*h^2+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a
*h^2+c*g^2)/h^2)^(5/2)-1/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h
```

```
*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(5/2)-3*c*g*h/(a*h^
2+c*g^2)*(1/3*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g
/h*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c
*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g
/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)
^(1/2)))+(a*h^2+c*g^2)/h^2*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/
h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g
/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2
)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/
2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))))+
4*c/(a*h^2+c*g^2)*h^2*(1/8*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h
*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+3/16*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h
^2)/c*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^
2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-
c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h
^2)^(1/2)))))+3/2*c/(a*h^2+c*g^2)*h^2*(1/3*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)
+(a*h^2+c*g^2)/h^2)^(3/2)-c*g/h*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2
*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2-4*
c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h
*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)))+(a*h^2+c*g^2)/h^2*(((x+1/h*g)^2*c-2*c
*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))
/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+
c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h
*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g
^2)/h^2)^(1/2))/(x+1/h*g)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1311 vs. $2(464) = 928$.
time = 0.38, size = 1311, normalized size = 2.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(c*x^2 + a)*c^2*f*g^4/(c*g^2*h^5 + a*h^7) - 3/2*sqrt(c*x^2 + a)*c^2
*f*g^3*x/(c*g^2*h^4 + a*h^6) + 1/2*(c*x^2 + a)^(3/2)*c*f*g^3/(c*g^2*h^4*x +
 a*h^6*x + c*g^3*h^3 + a*g*h^5) - 3/2*sqrt(c*x^2 + a)*c^2*g^3*e/(c*g^2*h^4
+ a*h^6) + 3/2*sqrt(c*x^2 + a)*c^2*g^2*x*e/(c*g^2*h^3 + a*h^5) + 3/2*sqrt(c
*x^2 + a)*c^2*d*g^2/(c*g^2*h^3 + a*h^5) - 1/2*(c*x^2 + a)^(5/2)*f*g^2/(c*g^
2*h^3*x^2 + a*h^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3)
+ 1/2*(c*x^2 + a)^(3/2)*c*f*g^2/(c*g^2*h^3 + a*h^5) - 3/2*sqrt(c*x^2 + a)*c
^2*d*g*x/(c*g^2*h^2 + a*h^4) - 1/2*(c*x^2 + a)^(3/2)*c*g^2*e/(c*g^2*h^3*x +
 a*h^5*x + c*g^3*h^2 + a*g*h^4) + 1/2*(c*x^2 + a)^(3/2)*c*d*g/(c*g^2*h^2*x
+ a*h^4*x + c*g^3*h + a*g*h^3) + 1/2*(c*x^2 + a)^(5/2)*g*e/(c*g^2*h^2*x^2 +
```

```
a*h^4*x^2 + 2*c*g^3*h*x + 2*a*g*h^3*x + c*g^4 + a*g^2*h^2) - 1/2*(c*x^2 +
a)^(3/2)*c*g*e/(c*g^2*h^2 + a*h^4) - 1/2*(c*x^2 + a)^(5/2)*d/(c*g^2*h*x^2 +
 a*h^3*x^2 + 2*c*g^3*x + 2*a*g*h^2*x + c*g^4/h + a*g^2*h) + 1/2*(c*x^2 + a)
^(3/2)*c*d/(c*g^2*h + a*h^3) + 2*(c*x^2 + a)^(3/2)*f*g/(h^4*x + g*h^3) - (c
*x^2 + a)^(3/2)*e/(h^3*x + g*h^2) - 7/2*sqrt(c*x^2 + a)*c*f*g*x/h^4 - 10*c^
(3/2)*f*g^3*arcsinh(c*x/sqrt(a*c))/h^6 - 3*c^(3/2)*d*g*arcsinh(c*x/sqrt(a*c
))/h^4 - 9/2*a*sqrt(c)*f*g*arcsinh(c*x/sqrt(a*c))/h^4 + 3/2*c^2*f*g^4*arcsi
nh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a +
 c*g^2/h^2)*h^7) + 3/2*c^2*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a
*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^5) + 15/2*sqrt(a + c*g^
2/h^2)*c*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(
h*x + g)))/h^5 + 3/2*sqrt(a + c*g^2/h^2)*c*d*arcsinh(c*g*x/(sqrt(a*c)*abs(h
*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 + (a + c*g^2/h^2)^(3/2)*f*arcs
inh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 + 3/
2*sqrt(c*x^2 + a)*c*x*e/h^3 + 6*c^(3/2)*g^2*arcsinh(c*x/sqrt(a*c))*e/h^5 +
3/2*a*sqrt(c)*arcsinh(c*x/sqrt(a*c))*e/h^3 - 3/2*c^2*g^3*arcsinh(c*g*x/(sqr
t(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))*e/(sqrt(a + c*g^2/h^2)
*h^6) - 9/2*sqrt(a + c*g^2/h^2)*c*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g))
- a*h/(sqrt(a*c)*abs(h*x + g)))*e/h^4 + 17/2*sqrt(c*x^2 + a)*c*f*g^2/h^5 +
3/2*sqrt(c*x^2 + a)*c*d/h^3 + 1/3*(c*x^2 + a)^(3/2)*f/h^3 + sqrt(c*x^2 + a)
*a*f/h^3 - 9/2*sqrt(c*x^2 + a)*c*g*e/h^4
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^{\frac{3}{2}}\,(d + ex + fx^2)}{(g + hx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**3,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. $2(464) = 928$.
time = 5.55, size = 1036, normalized size = 2.12

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="giac")

[Out] 1/6*sqrt(c*x^2 + a)*(x*(2*c*f*x/h^3 - 3*(3*c^2*f*g*h^14 - c^2*h^15*e)/(c*h^18)) + 2*(18*c^2*f*g^2*h^13 + 3*c^2*d*h^15 + 4*a*c*f*h^15 - 9*c^2*g*h^14*e)/(c*h^18)) + 1/2*(20*c^(3/2)*f*g^3 + 6*c^(3/2)*d*g*h^2 + 9*a*sqrt(c)*f*g*h^2 - 12*c^(3/2)*g^2*h*e - 3*a*sqrt(c)*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^6 + (20*c^2*f*g^4 + 6*c^2*d*g^2*h^2 + 19*a*c*f*g^2*h^2 + 3*a*c*d*h^4 + 2*a^2*f*h^4 - 12*c^2*g^3*h*e - 9*a*c*g*h^3*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 - a*h^2)*h^6) + (10*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*f*g^4*h + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*d*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g^2*h^3 + (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*d*h^5 - 8*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*g^3*h^2*e - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*g*h^4*e + 18*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 + 10*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*d*g^3*h^2 - (sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*f*g^3*h^2 - 5*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 - 14*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*g^4*h*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*g^2*h^3*e + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*h^5*e - 26*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*f*g^4*h - 14*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2*h^3 - 11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^3 + (sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*d*h^5 + 20*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*g^3*h^2*e + 5*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*g*h^4*e + 9*a^2*c^(3/2)*f*g^3*h^2 + 5*a^2*c^(3/2)*d*g*h^4 + 4*a^3*sqrt(c)*f*g*h^4 - 7*a^2*c^(3/2)*g^2*h^3*e - 2*a^3*sqrt(c)*h^5*e)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2*h^6)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)

[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)

3.95 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx$

Optimal. Leaf size=475

$$-\frac{\left(\left(cg^2+ah^2\right)\left(3afh^2+2c(10fg^2-h(4eg-dh))\right)+ch(3ah^2(3fg-eh)+cg(10fg^2-h(4eg-dh)))\,x\right)\sqrt{}}{2h^5\left(cg^2+ah^2\right)(g+hx)}$$

```
[Out] -1/6*(c*g*(4*e*g-10*f*g^2/h-d*h)-3*a*h*(-e*h+3*f*g)-(3*a*f*h^2+c*(5*f*g^2-2
*h*(-d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^2/(a*h^2+c*g^2)/(h*x+g)^2-1/3*(d*h^2-e
*g*h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^3+1/2*c*(3*a^2*h^4*(-e*
h+4*f*g)+2*c^2*g^3*(10*f*g^2-h*(-d*h+4*e*g))+3*a*c*g*h^2*(11*f*g^2-h*(-d*h+
4*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^6/(a*h
^2+c*g^2)^(3/2)+1/2*(3*a*f*h^2+2*c*(10*f*g^2-h*(-d*h+4*e*g)))*arctanh(x*c^(
1/2)/(c*x^2+a)^(1/2))*c^(1/2)/h^6-1/2*((a*h^2+c*g^2)*(3*a*f*h^2+2*c*(10*f*g
^2-h*(-d*h+4*e*g)))+c*h*(3*a*h^2*(-e*h+3*f*g)+c*g*(10*f*g^2-h*(-d*h+4*e*g))
)*x)*(c*x^2+a)^(1/2)/h^5/(a*h^2+c*g^2)/(h*x+g)
```

Rubi [A]

time = 0.51, antiderivative size = 469, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1665, 827, 858, 223, 212, 739}

$$\frac{c\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(3a^2h^4(4fg-eh)+3acgh^2(11fg^2-h(4eg-dh))+2c^2(10fg^5-g^3h(4eg-dh))\right)}{2h^6\left(ah^2+cg^2\right)^{3/2}}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{3/2}\left(-x(3afh^2-2ch(eg-dh)+5cfg^2)-3ah(3fg-eh)+cg\left(-dh+4eg-\frac{10fg^2}{h}\right)\right)}{6h^2(g+hx)^2\left(ah^2+cg^2\right)}+\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(3afh^2-2ch(4eg-dh)+20cfg^2\right)}{2h^6}-\frac{\sqrt{a+cx^2}\left(chx(3ah^2(3fg-eh)-cgh(4eg-dh)+10cfg^3)+\left(ah^2+cg^2\right)\left(3afh^2-2ch(4eg-dh)+20cfg^2\right)\right)}{2h^5(g+hx)\left(ah^2+cg^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -1/2*(((c*g^2 + a*h^2)*(20*c*f*g^2 + 3*a*f*h^2 - 2*c*h*(4*e*g - d*h)) + c*h
*(10*c*f*g^3 - c*g*h*(4*e*g - d*h) + 3*a*h^2*(3*f*g - e*h))*x)*Sqrt[a + c*x
^2])/(h^5*(c*g^2 + a*h^2)*(g + h*x)) - ((c*g*(4*e*g - (10*f*g^2)/h - d*h) -
 3*a*h*(3*f*g - e*h) - (5*c*f*g^2 + 3*a*f*h^2 - 2*c*h*(e*g - d*h))*x)*(a +
c*x^2)^(3/2))/(6*h^2*(c*g^2 + a*h^2)*(g + h*x)^2) - ((f*g^2 - e*g*h + d*h^2
)*(a + c*x^2)^(5/2))/(3*h*(c*g^2 + a*h^2)*(g + h*x)^3) + (Sqrt[c]*(20*c*f*g
^2 + 3*a*f*h^2 - 2*c*h*(4*e*g - d*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])
/(2*h^6) + (c*(3*a^2*h^4*(4*f*g - e*h) + 3*a*c*g*h^2*(11*f*g^2 - h*(4*e*g -
 d*h)) + 2*c^2*(10*f*g^5 - g^3*h*(4*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqr
t[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*h^6*(c*g^2 + a*h^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = -\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{3h(cg^2+ah^2)(g+hx)^3} - \frac{\int \frac{\left(-3(cdg-afg+aeh)-\left(3afh-c\left(2eg-\frac{5fg^2}{h}\cdots\right.\right.\right.}{(g+hx)^3}}{3(cg^2+ah^2)\cdots}$$

$$= -\frac{\left(cg\left(4eg-\frac{10fg^2}{h}-dh\right)-3ah(3fg-eh)-(5cfg^2+3afh^2-2ch(e\cdots\right.}{6h^2(cg^2+ah^2)(g+hx)^2}$$

$$= -\frac{((cg^2+ah^2)(20cfg^2+3afh^2-2ch(4eg-dh))+ch(10cfg^3-cgh(\cdots}{2h^5(cg^2+ah^2)(g+hx)\cdots}$$

$$= -\frac{((cg^2+ah^2)(20cfg^2+3afh^2-2ch(4eg-dh))+ch(10cfg^3-cgh(\cdots}{2h^5(cg^2+ah^2)(g+hx)\cdots}$$

$$= -\frac{((cg^2+ah^2)(20cfg^2+3afh^2-2ch(4eg-dh))+ch(10cfg^3-cgh(\cdots}{2h^5(cg^2+ah^2)(g+hx)\cdots}$$

$$= -\frac{((cg^2+ah^2)(20cfg^2+3afh^2-2ch(4eg-dh))+ch(10cfg^3-cgh(\cdots}{2h^5(cg^2+ah^2)(g+hx)\cdots}$$

Mathematica [A]
time = 10.83, size = 517, normalized size = 1.09

$$-\frac{-\frac{h\sqrt{a+cx^2}\left(2(cg^2+ah^2)^2(fg^2+h(-eg+dh))-(cg^2+ah^2)(13cfg^3+cgh(-10eg+7dh)-3ah^2(-2fg+eh))(g+hx)+(6a^2fh^4+ach^2(50fg^2+h(-23eg+8dh))+c^2(47fg^4+g^2h(-26eg+11dh)))(g+hx)^2+6c(4fg-eh)(cg^2+ah^2)(g+hx)^3-3cfh(cg^2+ah^2)x(g+hx)^3\right)}{(cg^2+ah^2)(g+hx)^3}-\frac{3c(-3a^2h^4(-4fg+eh)+3acgh^2(11fg^2+h(-4eg+dh))+2c^2(10fg^5+g^3h(-4eg+dh)))\log(g+hx)}{(cg^2+ah^2)^{3/2}}+3\sqrt{c}(20cfg^2+3afh^2+2ch(-4eg+dh))\log\left(cx+\sqrt{c}\sqrt{a+cx^2}\right)+\frac{3c(-3a^2h^4(-4fg+eh)+3acgh^2(11fg^2+h(-4eg+dh))+2c^2(10fg^5+g^3h(-4eg+dh)))\log\left(ah-cgx+\sqrt{cg^2+ah^2}\sqrt{a+cx^2}\right)}{(cg^2+ah^2)^{3/2}}}{6h^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] (-((h*Sqrt[a + c*x^2]*(2*(c*g^2 + a*h^2)^2*(f*g^2 + h*(-(e*g) + d*h)) - (c*
g^2 + a*h^2)*(13*c*f*g^3 + c*g*h*(-10*e*g + 7*d*h) - 3*a*h^2*(-2*f*g + e*h)
)*(g + h*x) + (6*a^2*f*h^4 + a*c*h^2*(50*f*g^2 + h*(-23*e*g + 8*d*h)) + c^2
*(47*f*g^4 + g^2*h*(-26*e*g + 11*d*h)))*(g + h*x)^2 + 6*c*(4*f*g - e*h)*(c*
g^2 + a*h^2)*(g + h*x)^3 - 3*c*f*h*(c*g^2 + a*h^2)*x*(g + h*x)^3))/((c*g^2
+ a*h^2)*(g + h*x)^3)) - (3*c*(-3*a^2*h^4*(-4*f*g + e*h) + 3*a*c*g*h^2*(11*
f*g^2 + h*(-4*e*g + d*h)) + 2*c^2*(10*f*g^5 + g^3*h*(-4*e*g + d*h)))*Log[g
+ h*x])/(c*g^2 + a*h^2)^(3/2) + 3*Sqrt[c]*(20*c*f*g^2 + 3*a*f*h^2 + 2*c*h*(
-4*e*g + d*h))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (3*c*(-3*a^2*h^4*(-4*f*
g + e*h) + 3*a*c*g*h^2*(11*f*g^2 + h*(-4*e*g + d*h)) + 2*c^2*(10*f*g^5 + g^
3*h*(-4*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]]
)/(c*g^2 + a*h^2)^(3/2))/(6*h^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4812 vs. $2(447) = 894$.
time = 0.12, size = 4813, normalized size = 10.13

method	result	size
default	Expression too large to display	4813
risch	Expression too large to display	8514

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^4*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h
^2+c*g^2)/h^2)^(5/2)-3*c*g*h/(a*h^2+c*g^2)*(1/3*((x+1/h*g)^2*c-2*c*g/h*(x+1
/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g/h*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h
*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h
^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*
c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)))+(a*h^2+c*g^2)/h^2*(((x+1/h*g)^2*
c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/
h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a
*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(
x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^
2+c*g^2)/h^2)^(1/2))/(x+1/h*g))))+4*c/(a*h^2+c*g^2)*h^2*(1/8*(2*c*(x+1/h*g)
-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+3/16*
(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+
1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2
)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c
-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)))))+(d*h^2-e*g*h+f*g^2)/h^6*(-1
/3/(a*h^2+c*g^2)*h^2/(x+1/h*g)^3*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*
g^2)/h^2)^(5/2)+1/3*c*g*h/(a*h^2+c*g^2)*(-1/2/(a*h^2+c*g^2)*h^2/(x+1/h*g)^2
*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(5/2)-1/2*c*g*h/(a*h^2
+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a
*h^2+c*g^2)/h^2)^(5/2)-3*c*g*h/(a*h^2+c*g^2)*(1/3*((x+1/h*g)^2*c-2*c*g/h*(x
+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g/h*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1
/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)
/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-
2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)))+(a*h^2+c*g^2)/h^2*(((x+1/h*g)^
2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+
1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-
(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h
*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*
h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))))+4*c/(a*h^2+c*g^2)*h^2*(1/8*(2*c*(x+1/h*
g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+3/1
6*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((
x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g
```

```
^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2
*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)))))+3/2*c/(a*h^2+c*g^2)*h^2*(
1/3*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g/h*(1/4*(2
*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)
^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/
h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)))+(
a*h^2+c*g^2)/h^2*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)
-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*
g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln
((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h
*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))))+2/3*c/(a*
h^2+c*g^2)*h^2*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/
h*g)+(a*h^2+c*g^2)/h^2)^(5/2)-3*c*g*h/(a*h^2+c*g^2)*(1/3*((x+1/h*g)^2*c-2*c
*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g/h*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/
c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2
+c*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*
g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)))+(a*h^2+c*g^2)/h^2*(((x+
1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/
h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^
(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-
2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h
*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))))+4*c/(a*h^2+c*g^2)*h^2*(1/8*(2*c*
(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3
/2)+3/16*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c*(1/4*(2*c*(x+1/h*g)-2*c*g/
h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*
h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1
/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)))))+(e*h-2*f*g)/h^5*(
-1/2/(a*h^2+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+
c*g^2)/h^2)^(5/2)-1/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*(
(x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(5/2)-3*c*g*h/(a*h^2+c*g
^2)*(1/3*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g/h*(1
/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)
/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*
(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/...
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2431 vs. $2(447) = 894$.
time = 0.41, size = 2431, normalized size = 5.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*c^3*f*g^5/(c^2*g^4*h^5 + 2*a*c*g^2*h^7 + a^2*h^9) - 1/2
*sqrt(c*x^2 + a)*c^3*f*g^4*x/(c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) + 1/6*
```

(c*x^2 + a)^(3/2)*c^2*f*g^4/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) - 1/2*sqrt(c*x^2 + a)*c^3*g^4*e/(c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) + 1/2*sqrt(c*x^2 + a)*c^3*g^3*x*e/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) + 1/2*sqrt(c*x^2 + a)*c^3*d*g^3/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/6*(c*x^2 + a)^(5/2)*c*f*g^3/(c^2*g^4*h^3*x^2 + 2*a*c*g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6*x + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 1/6*(c*x^2 + a)^(3/2)*c^2*f*g^3/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/2*sqrt(c*x^2 + a)*c^3*d*g^2*x/(c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) - 1/6*(c*x^2 + a)^(3/2)*c^2*g^3*e/(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*c*g^3*h^4 + a^2*g*h^6) + 1/6*(c*x^2 + a)^(3/2)*c^2*d*g^2/(c^2*g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3*h^3 + a^2*g*h^5) - 9/2*sqrt(c*x^2 + a)*c^2*f*g^3/(c*g^2*h^5 + a*h^7) + 4*sqrt(c*x^2 + a)*c^2*f*g^2*x/(c*g^2*h^4 + a*h^6) + 1/6*(c*x^2 + a)^(5/2)*c*g^2*e/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2 + a^2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6 + 2*a*c*g^4*h^2 + a^2*g^2*h^4) - 1/6*(c*x^2 + a)^(3/2)*c^2*g^2*e/(c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) - 1/6*(c*x^2 + a)^(5/2)*c*d*g/(c^2*g^4*h*x^2 + 2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 + 2*c^2*g^5*x + 4*a*c*g^3*h^2*x + 2*a^2*g*h^4*x + c^2*g^6/h + 2*a*c*g^4*h + a^2*g^2*h^3) + 1/6*(c*x^2 + a)^(3/2)*c^2*d*g/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2*h^5) - 1/3*(c*x^2 + a)^(5/2)*f*g^2/(c*g^2*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*a*g^2*h^4*x + c*g^5*h + a*g^3*h^3) - 5/3*(c*x^2 + a)^(3/2)*c*f*g^2/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3 + a*g*h^5) + 3*sqrt(c*x^2 + a)*c^2*g^2*e/(c*g^2*h^4 + a*h^6) - 5/2*sqrt(c*x^2 + a)*c^2*g*x*e/(c*g^2*h^3 + a*h^5) - 3/2*sqrt(c*x^2 + a)*c^2*d*g/(c*g^2*h^3 + a*h^5) + (c*x^2 + a)^(5/2)*f*g/(c*g^2*h^3*x^2 + a*h^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) - (c*x^2 + a)^(3/2)*c*f*g/(c*g^2*h^3 + a*h^5) + sqrt(c*x^2 + a)*c^2*d*x/(c*g^2*h^2 + a*h^4) + 1/3*(c*x^2 + a)^(5/2)*g*e/(c*g^2*h^3*x^3 + a*h^5*x^3 + 3*c*g^3*h^2*x^2 + 3*a*g*h^4*x^2 + 3*c*g^4*h*x + 3*a*g^2*h^3*x + c*g^5 + a*g^3*h^2) + 7/6*(c*x^2 + a)^(3/2)*c*g*e/(c*g^2*h^3*x + a*h^5*x + c*g^3*h^2 + a*g*h^4) - 1/3*(c*x^2 + a)^(5/2)*d/(c*g^2*h^2*x^3 + a*h^4*x^3 + 3*c*g^3*h*x^2 + 3*a*g*h^3*x^2 + 3*c*g^4*x + 3*a*g^2*h^2*x + c*g^5/h + a*g^3*h) - 2/3*(c*x^2 + a)^(3/2)*c*d/(c*g^2*h^2*x + a*h^4*x + c*g^3*h + a*g*h^3) - 1/2*(c*x^2 + a)^(5/2)*e/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h*x + 2*a*g*h^3*x + c*g^4 + a*g^2*h^2) + 1/2*(c*x^2 + a)^(3/2)*c*e/(c*g^2*h^2 + a*h^4) - (c*x^2 + a)^(3/2)*f/(h^4*x + g*h^3) + 3/2*sqrt(c*x^2 + a)*c*f*x/h^4 + 10*c^(3/2)*f*g^2*arcsinh(c*x/sqrt(a*c))/h^6 + c^(3/2)*d*arcsinh(c*x/sqrt(a*c))/h^4 + 3/2*a*sqrt(c)*f*arcsinh(c*x/sqrt(a*c))/h^4 + 1/2*c^3*f*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^9) + 1/2*c^3*d*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) - 9/2*c^2*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^7) - 3/2*c^2*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^5) - 6*sqrt(a + c*g^2/h^2)*c*f*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)

```
*abs(h*x + g)))/h^5 - 4*c^(3/2)*g*arcsinh(c*x/sqrt(a*c))*e/h^5 - 1/2*c^3*g^
4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))*e/
((a + c*g^2/h^2)^(3/2)*h^8) + 3*c^2*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x +
g)) - a*h/(sqrt(a*c)*abs(h*x + g)))*e/(sqrt(a + c*g^2/h^2)*h^6) + 3/2*sqrt(
a + c*g^2/h^2)*c*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*ab
s(h*x + g)))*e/h^4 - 6*sqrt(c*x^2 + a)*c*f*g/h^5 + 3/2*sqrt(c*x^2 + a)*c*e/
h^4
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+cx^2)^{\frac{3}{2}}\,(d+ex+fx^2)}{(g+hx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**4,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1900 vs. 2(447) = 894.
time = 7.55, size = 1900, normalized size = 4.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*(c*f*x/h^4 - 2*(4*c*f*g*h^10 - c*h^11*e)/h^15) - (20*c^
3*f*g^5 + 2*c^3*d*g^3*h^2 + 33*a*c^2*f*g^3*h^2 + 3*a*c^2*d*g*h^4 + 12*a^2*c
*f*g*h^4 - 8*c^3*g^4*h*e - 12*a*c^2*g^2*h^3*e - 3*a^2*c*h^5*e)*arctan(-((sq
rt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c*g^2*h^6
 + a*h^8)*sqrt(-c*g^2 - a*h^2)) - 1/3*(60*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c
^3*f*g^5*h^2 + 18*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^3*d*g^3*h^4 + 69*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*a*c^2*f*g^3*h^4 + 15*(sqrt(c)*x - sqrt(c*x^2 + a)
)^5*a*c^2*d*g*h^6 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*f*g*h^6 - 36*(
```

```
sqrt(c)*x - sqrt(c*x^2 + a))^5*c^3*g^4*h^3*e - 36*(sqrt(c)*x - sqrt(c*x^2 +
 a))^5*a*c^2*g^2*h^5*e - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*h^7*e + 21
0*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*f*g^6*h + 54*(sqrt(c)*x - sqrt(c*
x^2 + a))^4*c^(7/2)*d*g^4*h^3 + 183*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/
2)*f*g^4*h^3 + 27*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*d*g^2*h^5 - 18*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*f*g^2*h^5 - 12*(sqrt(c)*x - sqr
t(c*x^2 + a))^4*a^2*c^(3/2)*d*h^7 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*s
qrt(c)*f*h^7 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*g^5*h^2*e - 84*(
sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*g^3*h^4*e + 21*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a^2*c^(3/2)*g*h^6*e + 188*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*
f*g^7 + 44*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*d*g^5*h^2 - 82*(sqrt(c)*x -
sqrt(c*x^2 + a))^3*a*c^3*f*g^5*h^2 - 34*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c
^3*d*g^3*h^4 - 276*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*f*g^3*h^4 - 48*(
sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*d*g*h^6 - 36*(sqrt(c)*x - sqrt(c*x^2
 + a))^3*a^3*c*f*g*h^6 - 104*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*g^6*h*e +
64*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*g^4*h^3*e + 138*(sqrt(c)*x - sqrt(
c*x^2 + a))^3*a^2*c^2*g^2*h^5*e - 354*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(
7/2)*f*g^6*h - 78*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(7/2)*d*g^4*h^3 - 276
*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)*f*g^4*h^3 - 36*(sqrt(c)*x - sq
rt(c*x^2 + a))^2*a^2*c^(5/2)*d*g^2*h^5 + 60*(sqrt(c)*x - sqrt(c*x^2 + a))^2
*a^3*c^(3/2)*f*g^2*h^5 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*d*h
^7 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*sqrt(c)*f*h^7 + 192*(sqrt(c)*x
- sqrt(c*x^2 + a))^2*a*c^(7/2)*g^5*h^2*e + 114*(sqrt(c)*x - sqrt(c*x^2 + a)
)^2*a^2*c^(5/2)*g^3*h^4*e - 48*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*
g*h^6*e + 222*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*f*g^5*h^2 + 48*(sqrt(c)
*x - sqrt(c*x^2 + a))*a^2*c^3*d*g^3*h^4 + 231*(sqrt(c)*x - sqrt(c*x^2 + a))
*a^3*c^2*f*g^3*h^4 + 33*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*d*g*h^6 + 24*
(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c*f*g*h^6 - 120*(sqrt(c)*x - sqrt(c*x^2 +
 a))*a^2*c^3*g^4*h^3*e - 102*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*g^2*h^5*
e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c*h^7*e - 47*a^3*c^(5/2)*f*g^4*h^3
- 11*a^3*c^(5/2)*d*g^2*h^5 - 50*a^4*c^(3/2)*f*g^2*h^5 - 8*a^4*c^(3/2)*d*h^7
 - 6*a^5*sqrt(c)*f*h^7 + 26*a^3*c^(5/2)*g^3*h^4*e + 23*a^4*c^(3/2)*g*h^6*e)
/((c*g^2*h^6 + a*h^8)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - s
qrt(c*x^2 + a))*sqrt(c)*g - a*h)^3) - 1/2*(20*c^(3/2)*f*g^2 + 2*c^(3/2)*d*h
^2 + 3*a*sqrt(c)*f*h^2 - 8*c^(3/2)*g*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
 a)))/h^6
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
```

```
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

3.96 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx$

Optimal. Leaf size=511

$$\frac{c\left(8(5fg-eh)\left(cg^2+ah^2\right)^2+h\left(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2\left(35fg^2-h(7eg-3dh)\right)\right)x\right)\sqrt{a+cx^2}}{8h^5\left(cg^2+ah^2\right)^2(g+hx)}+$$

```
[Out] 1/24*(4*a^2*h^4*(-2*e*h+f*g)-4*c^2*g^4*(-e*h+5*f*g)-a*c*g*h^2*(25*f*g^2-h*(
-9*d*h+5*e*g))-3*h*(4*a^2*f*h^4+a*c*h^2*(17*f*g^2-h*(-d*h+5*e*g))+2*c^2*g^2
*(5*f*g^2-h*(d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^3/(a*h^2+c*g^2)^2/(h*x+g)^3-1/
4*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^4-c^(3/2)*(-e
*h+5*f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^6-1/8*c*(12*a^3*f*h^6+8*c^3*
g^5*(-e*h+5*f*g)+20*a*c^2*g^3*h^2*(-e*h+5*f*g)+3*a^2*c*h^4*(25*f*g^2-h*(-d*
h+5*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^6/(a
*h^2+c*g^2)^(5/2)+1/8*c*(8*(-e*h+5*f*g)*(a*h^2+c*g^2)^2+h*(12*a^2*f*h^4+4*c
^2*g^3*(-e*h+5*f*g)+a*c*h^2*(35*f*g^2-h*(-3*d*h+7*e*g)))*x)*(c*x^2+a)^(1/2)
/h^5/(a*h^2+c*g^2)^2/(h*x+g)
```

Rubi [A]

time = 0.67, antiderivative size = 511, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1665, 825, 827, 858, 223, 212, 739}

$$\frac{\left(a+cx^2\right)^{3/2}\left(-3x\left(4a^2fh^4+ach^2(17fg^2-h(5eg-dh))+2c^2(5fg^4-g^2h(dh+eg))\right)+4a^2h^3(fg-2eh)-acgh(25fg^2-h(5eg-9dh))-\frac{4c^2g^4(5fg-eh)}{h}\right)}{24h^2(g+hx)^3\left(ah^2+cg^2\right)^2}+\frac{c\sqrt{a+cx^2}\left(hx\left(12a^2fh^4+ach^2(35fg^2-h(7eg-3dh))+4c^2g^3(5fg-eh)\right)+8\left(ah^2+cg^2\right)^2(5fg-eh)\right)}{8h^5(g+hx)\left(ah^2+cg^2\right)^2}-\frac{c\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(12a^3fh^6+3a^2ch^4(25fg^2-h(5eg-dh))+20ac^2g^3h^2(5fg-eh)+8c^3g^5(5fg-eh)\right)}{8h^6\left(ah^2+cg^2\right)^{5/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(5fg-eh)}{h^6}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (c*(8*(5*f*g - e*h)*(c*g^2 + a*h^2)^2 + h*(12*a^2*f*h^4 + 4*c^2*g^3*(5*f*g
- e*h) + a*c*h^2*(35*f*g^2 - h*(7*e*g - 3*d*h)))*x)*Sqrt[a + c*x^2])/(8*h^5
*(c*g^2 + a*h^2)^2*(g + h*x)) + ((4*a^2*h^3*(f*g - 2*e*h) - (4*c^2*g^4*(5*f
*g - e*h))/h - a*c*g*h*(25*f*g^2 - h*(5*e*g - 9*d*h)) - 3*(4*a^2*f*h^4 + a*
c*h^2*(17*f*g^2 - h*(5*e*g - d*h)) + 2*c^2*(5*f*g^4 - g^2*h*(e*g + d*h)))*x
)*(a + c*x^2)^(3/2))/(24*h^2*(c*g^2 + a*h^2)^2*(g + h*x)^3) - ((f*g^2 - e*g
*h + d*h^2)*(a + c*x^2)^(5/2))/(4*h*(c*g^2 + a*h^2)*(g + h*x)^4) - (c^(3/2)
*(5*f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/h^6 - (c*(12*a^3*f*h^6
 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h) + 3*a^2*c*h^4*(
25*f*g^2 - h*(5*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2]])])/(8*h^6*(c*g^2 + a*h^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = -\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{4h(cg^2+ah^2)(g+hx)^4} - \frac{\int \frac{\left(-4(cdg-afg+aeh)-\left(4afh-c\left(eg-\frac{5fg^2}{h}-\cdots\right.\right.\right.}{(g+hx)^4}}{4(cg^2+ah^2)}$$

$$= \frac{\left(4a^2h^3(fg-2eh)-\frac{4c^2g^4(5fg-eh)}{h}-acgh(25fg^2-h(5eg-9dh))-3(4a\cdots\right.}{24h^2(cg^2\cdots}$$

$$= \frac{c\left(8(5fg-eh)(cg^2+ah^2)^2+h(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2(35f\cdots\right.}{8h^5(cg^2+ah^2)^2(g+hx)}$$

$$= \frac{c\left(8(5fg-eh)(cg^2+ah^2)^2+h(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2(35f\cdots\right.}{8h^5(cg^2+ah^2)^2(g+hx)}$$

$$= \frac{c\left(8(5fg-eh)(cg^2+ah^2)^2+h(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2(35f\cdots\right.}{8h^5(cg^2+ah^2)^2(g+hx)}$$

$$= \frac{c\left(8(5fg-eh)(cg^2+ah^2)^2+h(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2(35f\cdots\right.}{8h^5(cg^2+ah^2)^2(g+hx)}$$

Mathematica [A]
time = 11.40, size = 575, normalized size = 1.13

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]

[Out] -1/24*((h*Sqrt[a + c*x^2]*(6*(c*g^2 + a*h^2)^3*(f*g^2 + h*(-(e*g) + d*h)) -
 2*(c*g^2 + a*h^2)^2*(17*c*f*g^3 + c*g*h*(-13*e*g + 9*d*h) - 4*a*h^2*(-2*f*
```

```
g + e*h))*(g + h*x) + (c*g^2 + a*h^2)*(12*a^2*f*h^4 + 2*c^2*(43*f*g^4 + g^2
*h*(-23*e*g + 9*d*h)) + a*c*h^2*(95*f*g^2 + h*(-43*e*g + 15*d*h)))*(g + h*x
)^2 - c*(4*a^2*h^4*(31*f*g - 8*e*h) + 2*c^2*(77*f*g^5 + g^3*h*(-25*e*g + 3*
d*h)) + a*c*g*h^2*(287*f*g^2 + h*(-91*e*g + 15*d*h)))*(g + h*x)^3 - 24*c*f*
(c*g^2 + a*h^2)^2*(g + h*x)^4))/((c*g^2 + a*h^2)^2*(g + h*x)^4) - (3*c*(12*
a^3*f*h^6 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h) + 3*a^
2*c*h^4*(25*f*g^2 + h*(-5*e*g + d*h)))*Log[g + h*x])/(c*g^2 + a*h^2)^(5/2)
+ 24*c^(3/2)*(5*f*g - e*h)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (3*c*(12*a^
3*f*h^6 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h) + 3*a^2*
c*h^4*(25*f*g^2 + h*(-5*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*
Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(5/2))/h^6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7960 vs. $2(485) = 970$.
time = 0.14, size = 7961, normalized size = 15.58

method	result	size
default	Expression too large to display	7961
risch	Expression too large to display	11887

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 4346 vs. $2(492) = 984$.
time = 0.51, size = 4346, normalized size = 8.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(c*x^2 + a)*c^4*f*g^6/(c^3*g^6*h^5 + 3*a*c^2*g^4*h^7 + 3*a^2*c*g^2*
h^9 + a^3*h^11) - 3/8*sqrt(c*x^2 + a)*c^4*f*g^5*x/(c^3*g^6*h^4 + 3*a*c^2*g^
4*h^6 + 3*a^2*c*g^2*h^8 + a^3*h^10) + 1/8*(c*x^2 + a)^(3/2)*c^3*f*g^5/(c^3*
g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c*g^2*h^8*x + a^3*h^10*x + c^3*g^7*h^
3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 + a^3*g*h^9) - 3/8*sqrt(c*x^2 + a)*c^
4*g^5*e/(c^3*g^6*h^4 + 3*a*c^2*g^4*h^6 + 3*a^2*c*g^2*h^8 + a^3*h^10) + 3/8*
sqrt(c*x^2 + a)*c^4*g^4*x*e/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^
7 + a^3*h^9) + 3/8*sqrt(c*x^2 + a)*c^4*d*g^4/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5
 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 1/8*(c*x^2 + a)^(5/2)*c^2*f*g^4/(c^3*g^6*h^
3*x^2 + 3*a*c^2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x^2 + 2*c^3*g^7
*h^2*x + 6*a*c^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*x + 2*a^3*g*h^8*x + c^3*g^8*h
```

+ 3*a*c^2*g^6*h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2*h^7) + 1/8*(c*x^2 + a)^(3/2)*
c^3*f*g^4/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 3/8
*sqrt(c*x^2 + a)*c^4*d*g^3*x/(c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h
^6 + a^3*h^8) - 1/8*(c*x^2 + a)^(3/2)*c^3*g^4*e/(c^3*g^6*h^3*x + 3*a*c^2*g^
4*h^5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3
*a^2*c*g^3*h^6 + a^3*g*h^8) + 1/8*(c*x^2 + a)^(3/2)*c^3*d*g^3/(c^3*g^6*h^2*
x + 3*a*c^2*g^4*h^4*x + 3*a^2*c*g^2*h^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2
*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^7) - 7/4*sqrt(c*x^2 + a)*c^3*f*g^4/(c^
2*g^4*h^5 + 2*a*c*g^2*h^7 + a^2*h^9) + 11/8*sqrt(c*x^2 + a)*c^3*f*g^3*x/(c^
2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) + 1/8*(c*x^2 + a)^(5/2)*c^2*g^3*e/(c^3
*g^6*h^2*x^2 + 3*a*c^2*g^4*h^4*x^2 + 3*a^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 + 2*
c^3*g^7*h*x + 6*a*c^2*g^5*h^3*x + 6*a^2*c*g^3*h^5*x + 2*a^3*g*h^7*x + c^3*g
^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*g^4*h^4 + a^3*g^2*h^6) - 1/8*(c*x^2 + a)^(3/
2)*c^3*g^3*e/(c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) -
1/8*(c*x^2 + a)^(5/2)*c^2*d*g^2/(c^3*g^6*h*x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^
2*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7*x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g
^3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a*c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^
3*g^2*h^5) + 1/8*(c*x^2 + a)^(3/2)*c^3*d*g^2/(c^3*g^6*h + 3*a*c^2*g^4*h^3 +
3*a^2*c*g^2*h^5 + a^3*h^7) - 1/4*(c*x^2 + a)^(5/2)*c*f*g^3/(c^2*g^4*h^4*x^
3 + 2*a*c*g^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2
+ 3*a^2*g*h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a*c*g^4*h^4*x + 3*a^2*g^2*h^6*x +
c^2*g^7*h + 2*a*c*g^5*h^3 + a^2*g^3*h^5) - 17/24*(c*x^2 + a)^(3/2)*c^2*f*g^
3/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x + c^2*g^5*h^3 + 2*a*c*g^3*h^
5 + a^2*g*h^7) + 5/4*sqrt(c*x^2 + a)*c^3*g^3*e/(c^2*g^4*h^4 + 2*a*c*g^2*h^6
+ a^2*h^8) - 7/8*sqrt(c*x^2 + a)*c^3*g^2*x*e/(c^2*g^4*h^3 + 2*a*c*g^2*h^5
+ a^2*h^7) - 3/4*sqrt(c*x^2 + a)*c^3*d*g^2/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a
^2*h^7) + 5/24*(c*x^2 + a)^(5/2)*c*f*g^2/(c^2*g^4*h^3*x^2 + 2*a*c*g^2*h^5*x
^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6*x + c^2*
g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) - 5/24*(c*x^2 + a)^(3/2)*c^2*f*g^2/(c^
2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) + 3/8*sqrt(c*x^2 + a)*c^3*d*g*x/(c^2*g
^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) + 1/4*(c*x^2 + a)^(5/2)*c*g^2*e/(c^2*g^4*
h^3*x^3 + 2*a*c*g^2*h^5*x^3 + a^2*h^7*x^3 + 3*c^2*g^5*h^2*x^2 + 6*a*c*g^3*h
^4*x^2 + 3*a^2*g*h^6*x^2 + 3*c^2*g^6*h*x + 6*a*c*g^4*h^3*x + 3*a^2*g^2*h^5*
x + c^2*g^7 + 2*a*c*g^5*h^2 + a^2*g^3*h^4) + 13/24*(c*x^2 + a)^(3/2)*c^2*g^
2*e/(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*c*g^3*
h^4 + a^2*g*h^6) - 1/4*(c*x^2 + a)^(5/2)*c*d*g/(c^2*g^4*h^2*x^3 + 2*a*c*g^2
*h^4*x^3 + a^2*h^6*x^3 + 3*c^2*g^5*h*x^2 + 6*a*c*g^3*h^3*x^2 + 3*a^2*g*h^5*
x^2 + 3*c^2*g^6*x + 6*a*c*g^4*h^2*x + 3*a^2*g^2*h^4*x + c^2*g^7/h + 2*a*c*g
^5*h + a^2*g^3*h^3) - 3/8*(c*x^2 + a)^(3/2)*c^2*d*g/(c^2*g^4*h^2*x + 2*a*c*
g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3*h^3 + a^2*g*h^5) - 1/4*(c*x^2
+ a)^(5/2)*f*g^2/(c*g^2*h^5*x^4 + a*h^7*x^4 + 4*c*g^3*h^4*x^3 + 4*a*g*h^6*
x^3 + 6*c*g^4*h^3*x^2 + 6*a*g^2*h^5*x^2 + 4*c*g^5*h^2*x + 4*a*g^3*h^4*x + c
*g^6*h + a*g^4*h^3) + 39/8*sqrt(c*x^2 + a)*c^2*f*g^2/(c*g^2*h^5 + a*h^7) -
7/2*sqrt(c*x^2 + a)*c^2*f*g*x/(c*g^2*h^4 + a*h^6) - 1/24*(c*x^2 + a)^(5/2)*
c*g*e/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2 + a^2*h^6*x^2 + 2*c^2*g^5*h*x +

```
4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6 + 2*a*c*g^4*h^2 + a^2*g^2*h^4) +
1/24*(c*x^2 + a)^(3/2)*c^2*g*e/(c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) - 1/
8*(c*x^2 + a)^(5/2)*c*d/(c^2*g^4*h*x^2 + 2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 +
2*c^2*g^5*x + 4*a*c*g^3*h^2*x + 2*a^2*g*h^4*x + c^2*g^6/h + 2*a*c*g^4*h + a
^2*g^2*h^3) + 1/8*(c*x^2 + a)^(3/2)*c^2*d/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2*
h^5) + 2/3*(c*x^2 + a)^(5/2)*f*g/(c*g^2*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3*x
^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*a*g^2*h^4*x + c*g^5*h + a*g^3*h^3) +
 11/6*(c*x^2 + a)^(3/2)*c*f*g/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3 + a*g*h^5)
 + 1/4*(c*x^2 + a)^(5/2)*g*e/(c*g^2*h^4*x^4 + a*h^6*x^4 + 4*c*g^3*h^3*x^3 +
 4*a*g*h^5*x^3 + 6*c*g^4*h^2*x^2 + 6*a*g^2*h^4*x^2 + 4*c*g^5*h*x + 4*a*g^3*
h^3*x + c*g^6 + a*g^4*h^2) - 15/8*sqrt(c*x^2 + ...
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^{\frac{3}{2}} (d + ex + fx^2)}{(g + hx)^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**5,x)
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x^2 + a)^{3/2}\,(f\,x^2 + e\,x + d)}{(g + h\,x)^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

3.97 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$

Optimal. Leaf size=507

$$-\frac{c(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4(13fg^2+3dh^2)+h(12c^3fg^6+8a^3fh^6+a^2cgh^4(34f}{8h^5\left(cg^2+ah^2\right)^3(g+hx)^2}$$

```
[Out] -1/12*(4*c^2*f*g^5-a^2*h^4*(-3*e*h+2*f*g)+a*c*g*h^2*(3*d*h^2+5*f*g^2)+h*(4*
a^2*f*h^4+a*c*g*h^2*(-3*e*h+14*f*g)+c^2*(-3*d*g^2*h^2+7*f*g^4))*x)*(c*x^2+a
)^(3/2)/h^3/(a*h^2+c*g^2)^2/(h*x+g)^4-1/5*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(5/
2)/h/(a*h^2+c*g^2)/(h*x+g)^5+c^(3/2)*f*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h
^6+1/8*c^2*(8*c^3*f*g^7+28*a*c^2*f*g^5*h^2+3*a^3*h^6*(-e*h+6*f*g)+a^2*c*g*h
^4*(-3*d*h^2+35*f*g^2))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^
(1/2))/h^6/(a*h^2+c*g^2)^(7/2)-1/8*c*(8*c^3*f*g^7+20*a*c^2*f*g^5*h^2-a^3*h^
6*(-3*e*h+2*f*g)+a^2*c*g*h^4*(3*d*h^2+13*f*g^2)+h*(12*c^3*f*g^6+8*a^3*f*h^6
+a^2*c*g*h^4*(-3*e*h+34*f*g)+a*c^2*g^2*h^2*(-3*d*h^2+35*f*g^2))*x)*(c*x^2+a
)^(1/2)/h^5/(a*h^2+c*g^2)^3/(h*x+g)^2
```

Rubi [A]

time = 0.53, antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1665, 825, 858, 223, 212, 739}

$$-\frac{\left(a+cx^2\right)^{3/2}\left(hx\left(4a^2fh^4+acgh^2(14fg-3eh)+c^2\left(7fg^4-3dg^2h^2\right)\right)-a^2h^4(2fg-3eh)+acgh^2\left(3dh^2+5fg^2\right)+4c^2fg^5\right)}{12h^3(g+hx)^4\left(ah^2+cg^2\right)^2}+\frac{c^2\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(3a^3h^6(6fg-eh)+a^2cgh^4\left(35fg^2-3dh^2\right)+28ac^2fg^5h^2+8c^3fg^7\right)}{8h^6\left(ah^2+cg^2\right)^{7/2}}-\frac{c\sqrt{a+cx^2}\left(-a^3h^6(2fg-3eh)+a^2cgh^4\left(3dh^2+13fg^2\right)+hx\left(8a^3fh^6+a^2cgh^4(34fg-3eh)+ac^2g^2h^2\left(35fg^2-3dh^2\right)+12c^3fg^6\right)+20ac^2fg^5h^2+8c^3fg^7\right)}{8h^5(g+hx)^2\left(ah^2+cg^2\right)^3}+\frac{c^{3/2}f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{h^6}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -1/8*(c*(8*c^3*f*g^7 + 20*a*c^2*f*g^5*h^2 - a^3*h^6*(2*f*g - 3*e*h) + a^2*c
*g*h^4*(13*f*g^2 + 3*d*h^2) + h*(12*c^3*f*g^6 + 8*a^3*f*h^6 + a^2*c*g*h^4*(
34*f*g - 3*e*h) + a*c^2*g^2*h^2*(35*f*g^2 - 3*d*h^2))*x)*Sqrt[a + c*x^2])/(
h^5*(c*g^2 + a*h^2)^3*(g + h*x)^2) - ((4*c^2*f*g^5 - a^2*h^4*(2*f*g - 3*e*h
) + a*c*g*h^2*(5*f*g^2 + 3*d*h^2) + h*(4*a^2*f*h^4 + a*c*g*h^2*(14*f*g - 3*
e*h) + c^2*(7*f*g^4 - 3*d*g^2*h^2))*x)*(a + c*x^2)^(3/2))/(12*h^3*(c*g^2 +
a*h^2)^2*(g + h*x)^4) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(5*h*(c
*g^2 + a*h^2)*(g + h*x)^5) + (c^(3/2)*f*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]
])/h^6 + (c^2*(8*c^3*f*g^7 + 28*a*c^2*f*g^5*h^2 + 3*a^3*h^6*(6*f*g - e*h) +
 a^2*c*g*h^4*(35*f*g^2 - 3*d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^
2]*Sqrt[a + c*x^2])])/(8*h^6*(c*g^2 + a*h^2)^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5} - \frac{\int \frac{\left(-5(cdg-afg+aeh)-5f\left(\frac{cg^2}{h}+ah\right)x\right)(a+\cdots}{(g+hx)^5}}{5\left(cg^2+ah^2\right)}$$

$$= -\frac{(4c^2fg^5 - a^2h^4(2fg-3eh) + acgh^2(5fg^2+3dh^2) + h(4a^2fh^4 + acg\cdots}{12h^3\left(cg^2+ah^2\right)^2(g\cdots}$$

$$= -\frac{c(8c^3fg^7 + 20ac^2fg^5h^2 - a^3h^6(2fg-3eh) + a^2cgh^4(13fg^2+3dh^2)\cdots}{8h^5(\cdots}$$

$$= -\frac{c(8c^3fg^7 + 20ac^2fg^5h^2 - a^3h^6(2fg-3eh) + a^2cgh^4(13fg^2+3dh^2)\cdots}{8h^5(\cdots}$$

$$= -\frac{c(8c^3fg^7 + 20ac^2fg^5h^2 - a^3h^6(2fg-3eh) + a^2cgh^4(13fg^2+3dh^2)\cdots}{8h^5(\cdots}$$

$$= -\frac{c(8c^3fg^7 + 20ac^2fg^5h^2 - a^3h^6(2fg-3eh) + a^2cgh^4(13fg^2+3dh^2)\cdots}{8h^5(\cdots}$$

Mathematica [A]
time = 11.38, size = 639, normalized size = 1.26

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] (-((h*Sqrt[a + c*x^2]*(24*(c*g^2 + a*h^2)^4*(f*g^2 + h*(-(e*g) + d*h)) - 6*
(c*g^2 + a*h^2)^3*(21*c*f*g^3 + c*g*h*(-16*e*g + 11*d*h) - 5*a*h^2*(-2*f*g
+ e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^2*(20*a^2*f*h^4 + c^2*(137*f*g^4 + 9*
g^2*h*(-8*e*g + 3*d*h)) + a*c*h^2*(154*f*g^2 + 3*h*(-23*e*g + 8*d*h)))*(g +
 h*x)^2 - c*(c*g^2 + a*h^2)*(5*a^2*h^4*(58*f*g - 15*e*h) + c^2*(326*f*g^5 +
 6*g^3*h*(-16*e*g + d*h)) + a*c*g*h^2*(631*f*g^2 + 3*h*(-62*e*g + 7*d*h)))*
(g + h*x)^3 + c*(160*a^3*f*h^6 + c^3*(274*f*g^6 - 6*g^4*h*(4*e*g + d*h)) +
3*a^2*c*h^4*(238*f*g^2 + h*(-33*e*g + 8*d*h)) + 3*a*c^2*g^2*h^2*(261*f*g^2
- h*(26*e*g + 9*d*h)))*(g + h*x)^4))/((c*g^2 + a*h^2)^3*(g + h*x)^5)) - (15
*c^2*(8*c^3*f*g^7 + 28*a*c^2*f*g^5*h^2 - 3*a^3*h^6*(-6*f*g + e*h) + a^2*c*g
*h^4*(35*f*g^2 - 3*d*h^2))*Log[g + h*x])/(c*g^2 + a*h^2)^(7/2) + 120*c^(3/2
)*f*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (15*c^2*(8*c^3*f*g^7 + 28*a*c^2*f*
```

```
g^5*h^2 - 3*a^3*h^6*(-6*f*g + e*h) + a^2*c*g*h^4*(35*f*g^2 - 3*d*h^2))*Log[
a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(7/2))/
(120*h^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10597 vs. $2(481) = 962$.
time = 0.09, size = 10598, normalized size = 20.90

method	result	size
default	Expression too large to display	10598

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 6673 vs. $2(488) = 976$.
time = 0.56, size = 6673, normalized size = 13.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(c*x^2 + a)*c^5*f*g^7/(c^4*g^8*h^5 + 4*a*c^3*g^6*h^7 + 6*a^2*c^2*g^
4*h^9 + 4*a^3*c*g^2*h^11 + a^4*h^13) - 3/8*sqrt(c*x^2 + a)*c^5*f*g^6*x/(c^4
*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^1
2) + 1/8*(c*x^2 + a)^(3/2)*c^4*f*g^6/(c^4*g^8*h^4*x + 4*a*c^3*g^6*h^6*x + 6
*a^2*c^2*g^4*h^8*x + 4*a^3*c*g^2*h^10*x + a^4*h^12*x + c^4*g^9*h^3 + 4*a*c^
3*g^7*h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^3*c*g^3*h^9 + a^4*g*h^11) - 3/8*sqrt(c*
x^2 + a)*c^5*g^6*e/(c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a
^3*c*g^2*h^10 + a^4*h^12) + 3/8*sqrt(c*x^2 + a)*c^5*g^5*x*e/(c^4*g^8*h^3 +
4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) + 3/8*sqr
t(c*x^2 + a)*c^5*d*g^5/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 +
 4*a^3*c*g^2*h^9 + a^4*h^11) - 1/8*(c*x^2 + a)^(5/2)*c^3*f*g^5/(c^4*g^8*h^3
*x^2 + 4*a*c^3*g^6*h^5*x^2 + 6*a^2*c^2*g^4*h^7*x^2 + 4*a^3*c*g^2*h^9*x^2 +
a^4*h^11*x^2 + 2*c^4*g^9*h^2*x + 8*a*c^3*g^7*h^4*x + 12*a^2*c^2*g^5*h^6*x +
 8*a^3*c*g^3*h^8*x + 2*a^4*g*h^10*x + c^4*g^10*h + 4*a*c^3*g^8*h^3 + 6*a^2*
c^2*g^6*h^5 + 4*a^3*c*g^4*h^7 + a^4*g^2*h^9) + 1/8*(c*x^2 + a)^(3/2)*c^4*f*
g^5/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 +
a^4*h^11) - 3/8*sqrt(c*x^2 + a)*c^5*d*g^4*x/(c^4*g^8*h^2 + 4*a*c^3*g^6*h^4
+ 6*a^2*c^2*g^4*h^6 + 4*a^3*c*g^2*h^8 + a^4*h^10) - 1/8*(c*x^2 + a)^(3/2)*c
^4*g^5*e/(c^4*g^8*h^3*x + 4*a*c^3*g^6*h^5*x + 6*a^2*c^2*g^4*h^7*x + 4*a^3*c
*g^2*h^9*x + a^4*h^11*x + c^4*g^9*h^2 + 4*a*c^3*g^7*h^4 + 6*a^2*c^2*g^5*h^6
```

+ 4*a^3*c*g^3*h^8 + a^4*g*h^10) + 1/8*(c*x^2 + a)^(3/2)*c^4*d*g^4/(c^4*g^8
*h^2*x + 4*a*c^3*g^6*h^4*x + 6*a^2*c^2*g^4*h^6*x + 4*a^3*c*g^2*h^8*x + a^4*
h^10*x + c^4*g^9*h + 4*a*c^3*g^7*h^3 + 6*a^2*c^2*g^5*h^5 + 4*a^3*c*g^3*h^7
+ a^4*g*h^9) - 3/2*sqrt(c*x^2 + a)*c^4*f*g^5/(c^3*g^6*h^5 + 3*a*c^2*g^4*h^7
+ 3*a^2*c*g^2*h^9 + a^3*h^11) + 9/8*sqrt(c*x^2 + a)*c^4*f*g^4*x/(c^3*g^6*h
^4 + 3*a*c^2*g^4*h^6 + 3*a^2*c*g^2*h^8 + a^3*h^10) + 1/8*(c*x^2 + a)^(5/2)*
c^3*g^4*e/(c^4*g^8*h^2*x^2 + 4*a*c^3*g^6*h^4*x^2 + 6*a^2*c^2*g^4*h^6*x^2 +
4*a^3*c*g^2*h^8*x^2 + a^4*h^10*x^2 + 2*c^4*g^9*h*x + 8*a*c^3*g^7*h^3*x + 12
*a^2*c^2*g^5*h^5*x + 8*a^3*c*g^3*h^7*x + 2*a^4*g*h^9*x + c^4*g^10 + 4*a*c^3
*g^8*h^2 + 6*a^2*c^2*g^6*h^4 + 4*a^3*c*g^4*h^6 + a^4*g^2*h^8) - 1/8*(c*x^2
+ a)^(3/2)*c^4*g^4*e/(c^4*g^8*h^2 + 4*a*c^3*g^6*h^4 + 6*a^2*c^2*g^4*h^6 + 4
*a^3*c*g^2*h^8 + a^4*h^10) - 1/8*(c*x^2 + a)^(5/2)*c^3*d*g^3/(c^4*g^8*h*x^2
+ 4*a*c^3*g^6*h^3*x^2 + 6*a^2*c^2*g^4*h^5*x^2 + 4*a^3*c*g^2*h^7*x^2 + a^4*
h^9*x^2 + 2*c^4*g^9*x + 8*a*c^3*g^7*h^2*x + 12*a^2*c^2*g^5*h^4*x + 8*a^3*c*
g^3*h^6*x + 2*a^4*g*h^8*x + c^4*g^10/h + 4*a*c^3*g^8*h + 6*a^2*c^2*g^6*h^3
+ 4*a^3*c*g^4*h^5 + a^4*g^2*h^7) + 1/8*(c*x^2 + a)^(3/2)*c^4*d*g^3/(c^4*g^8
*h + 4*a*c^3*g^6*h^3 + 6*a^2*c^2*g^4*h^5 + 4*a^3*c*g^2*h^7 + a^4*h^9) - 1/4
*(c*x^2 + a)^(5/2)*c^2*f*g^4/(c^3*g^6*h^4*x^3 + 3*a*c^2*g^4*h^6*x^3 + 3*a^2
*c*g^2*h^8*x^3 + a^3*h^10*x^3 + 3*c^3*g^7*h^3*x^2 + 9*a*c^2*g^5*h^5*x^2 + 9
*a^2*c*g^3*h^7*x^2 + 3*a^3*g*h^9*x^2 + 3*c^3*g^8*h^2*x + 9*a*c^2*g^6*h^4*x
+ 9*a^2*c*g^4*h^6*x + 3*a^3*g^2*h^8*x + c^3*g^9*h + 3*a*c^2*g^7*h^3 + 3*a^2
*c*g^5*h^5 + a^3*g^3*h^7) - 5/8*(c*x^2 + a)^(3/2)*c^3*f*g^4/(c^3*g^6*h^4*x
+ 3*a*c^2*g^4*h^6*x + 3*a^2*c*g^2*h^8*x + a^3*h^10*x + c^3*g^7*h^3 + 3*a*c^
2*g^5*h^5 + 3*a^2*c*g^3*h^7 + a^3*g*h^9) + 9/8*sqrt(c*x^2 + a)*c^4*g^4*e/(c
^3*g^6*h^4 + 3*a*c^2*g^4*h^6 + 3*a^2*c*g^2*h^8 + a^3*h^10) - 3/4*sqrt(c*x^2
+ a)*c^4*g^3*x*e/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^
9) - 3/4*sqrt(c*x^2 + a)*c^4*d*g^3/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c
*g^2*h^7 + a^3*h^9) + 1/8*(c*x^2 + a)^(5/2)*c^2*f*g^3/(c^3*g^6*h^3*x^2 + 3*
a*c^2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x + 6
*a*c^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*x + 2*a^3*g*h^8*x + c^3*g^8*h + 3*a*c^2*
g^6*h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2*h^7) - 1/8*(c*x^2 + a)^(3/2)*c^3*f*g^3/
(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) + 3/8*sqrt(c*x^
2 + a)*c^4*d*g^2*x/(c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h
^8) + 1/4*(c*x^2 + a)^(5/2)*c^2*g^3*e/(c^3*g^6*h^3*x^3 + 3*a*c^2*g^4*h^5*x^
3 + 3*a^2*c*g^2*h^7*x^3 + a^3*h^9*x^3 + 3*c^3*g^7*h^2*x^2 + 9*a*c^2*g^5*h^4
*x^2 + 9*a^2*c*g^3*h^6*x^2 + 3*a^3*g*h^8*x^2 + 3*c^3*g^8*h*x + 9*a*c^2*g^6*
h^3*x + 9*a^2*c*g^4*h^5*x + 3*a^3*g^2*h^7*x + c^3*g^9 + 3*a*c^2*g^7*h^2 + 3
*a^2*c*g^5*h^4 + a^3*g^3*h^6) + 1/2*(c*x^2 + a)^(3/2)*c^3*g^3*e/(c^3*g^6*h^
3*x + 3*a*c^2*g^4*h^5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c^3*g^7*h^2 + 3*a
*c^2*g^5*h^4 + 3*a^2*c*g^3*h^6 + a^3*g*h^8) - 1/4*(c*x^2 + a)^(5/2)*c^2*d*g
^2/(c^3*g^6*h^2*x^3 + 3*a*c^2*g^4*h^4*x^3 + 3*a^2*c*g^2*h^6*x^3 + a^3*h^8*x
^3 + 3*c^3*g^7*h*x^2 + 9*a*c^2*g^5*h^3*x^2 + 9*a^2*c*g^3*h^5*x^2 + 3*a^3*g*
h^7*x^2 + 3*c^3*g^8*x + 9*a*c^2*g^6*h^2*x + 9*a^2*c*g^4*h^4*x + 3*a^3*g^2*h
^6*x + c^3*g^9/h + 3*a*c^2*g^7*h + 3*a^2*c*g^5*h^3 + a^3*g^3*h^5) - 3/8*(c*
x^2 + a)^(3/2)*c^3*d*g^2/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a^2*c*g^2*h

```
^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^
7) - 1/4*(c*x^2 + a)^(5/2)*c*f*g^3/(c^2*g^4*h^5...
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + cx^2)^{\frac{3}{2}} (d + ex + fx^2)}{(g + hx)^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**6,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**6, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4408 vs. 2(488) = 976.
time = 5.12, size = 4408, normalized size = 8.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] -1/4*(8*c^5*f*g^7 + 28*a*c^4*f*g^5*h^2 + 35*a^2*c^3*f*g^3*h^4 - 3*a^2*c^3*d
*g*h^6 + 18*a^3*c^2*f*g*h^6 - 3*a^3*c^2*h^7*e)*arctan(-((sqrt(c)*x - sqrt(c
*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^3*g^6*h^6 + 3*a*c^2*g^4
*h^8 + 3*a^2*c*g^2*h^10 + a^3*h^12)*sqrt(-c*g^2 - a*h^2)) - c^(3/2)*f*log(a
bs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^6 - 1/60*(600*(sqrt(c)*x - sqrt(c*x^2 +
 a))^9*c^5*f*g^7*h^4 + 1740*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*f*g^5*h^6
 + 1635*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*f*g^3*h^8 + 45*(sqrt(c)*x -
 sqrt(c*x^2 + a))^9*a^2*c^3*d*g*h^10 + 450*(sqrt(c)*x - sqrt(c*x^2 + a))^9*
a^3*c^2*f*g*h^10 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^5*g^6*h^5*e - 360*
(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*g^4*h^7*e - 360*(sqrt(c)*x - sqrt(c*x
^2 + a))^9*a^2*c^3*g^2*h^9*e - 75*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^2*h
^11*e + 3600*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*f*g^8*h^3 - 120*(sqrt
(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*d*g^6*h^5 + 10020*(sqrt(c)*x - sqrt(c*x
```

^2 + a))^8*a*c^(9/2)*f*g^6*h^5 - 360*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9
/2)*d*g^4*h^7 + 8595*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*f*g^4*h^7
+ 45*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*d*g^2*h^9 + 1530*(sqrt(c)*
x - sqrt(c*x^2 + a))^8*a^3*c^(5/2)*f*g^2*h^9 - 120*(sqrt(c)*x - sqrt(c*x^2
+ a))^8*a^3*c^(5/2)*d*h^11 - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(3/2
)*f*h^11 - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*g^7*h^4*e - 1440*(s
qrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9/2)*g^5*h^6*e - 1440*(sqrt(c)*x - sqrt(
c*x^2 + a))^8*a^2*c^(7/2)*g^3*h^8*e - 75*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^
3*c^(5/2)*g*h^10*e + 8800*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*f*g^9*h^2 - 2
40*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*d*g^7*h^4 + 21240*(sqrt(c)*x - sqrt(
c*x^2 + a))^7*a*c^5*f*g^7*h^4 - 720*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*d
*g^5*h^6 + 11670*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*f*g^5*h^6 + 690*(s
qrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*d*g^3*h^8 - 4970*(sqrt(c)*x - sqrt(c*
x^2 + a))^7*a^3*c^3*f*g^3*h^8 - 450*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3
*d*g*h^10 - 2580*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^2*f*g*h^10 - 960*(sq
rt(c)*x - sqrt(c*x^2 + a))^7*c^6*g^8*h^3*e - 2640*(sqrt(c)*x - sqrt(c*x^2 +
a))^7*a*c^5*g^6*h^5*e - 2160*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*g^4*h
^7*e + 1170*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*g^2*h^9*e + 30*(sqrt(c)
*x - sqrt(c*x^2 + a))^7*a^4*c^2*h^11*e + 10000*(sqrt(c)*x - sqrt(c*x^2 + a)
)^6*c^(13/2)*f*g^10*h - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(13/2)*d*g^8*
h^3 + 14040*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*f*g^8*h^3 - 720*(sqr
t(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*d*g^6*h^5 - 14430*(sqrt(c)*x - sqrt(
c*x^2 + a))^6*a^2*c^(9/2)*f*g^6*h^5 + 1590*(sqrt(c)*x - sqrt(c*x^2 + a))^6*
a^2*c^(9/2)*d*g^4*h^7 - 28790*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*f
*g^4*h^7 - 1710*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*d*g^2*h^9 - 582
0*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*f*g^2*h^9 + 720*(sqrt(c)*x -
sqrt(c*x^2 + a))^6*a^5*c^(3/2)*f*h^11 - 960*(sqrt(c)*x - sqrt(c*x^2 + a))^6
*c^(13/2)*g^9*h^2*e - 1680*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*g^7*h
^4*e + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*g^5*h^6*e + 4950*(sq
rt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*g^3*h^8*e - 270*(sqrt(c)*x - sqrt(
c*x^2 + a))^6*a^4*c^(5/2)*g*h^10*e + 4384*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c
^7*f*g^11 - 96*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*d*g^9*h^2 - 9392*(sqrt(c
)*x - sqrt(c*x^2 + a))^5*a*c^6*f*g^9*h^2 + 48*(sqrt(c)*x - sqrt(c*x^2 + a))
^5*a*c^6*d*g^7*h^4 - 42996*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*f*g^7*h^
4 + 2364*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*d*g^5*h^6 - 31070*(sqrt(c)
*x - sqrt(c*x^2 + a))^5*a^3*c^4*f*g^5*h^6 - 2730*(sqrt(c)*x - sqrt(c*x^2 +
a))^5*a^3*c^4*d*g^3*h^8 + 8620*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*f*g^
3*h^8 + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*d*g*h^10 + 4800*(sqrt(c
)*x - sqrt(c*x^2 + a))^5*a^5*c^2*f*g*h^10 - 384*(sqrt(c)*x - sqrt(c*x^2 + a
))^5*c^7*g^10*h*e + 672*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^6*g^8*h^3*e + 3
936*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*g^6*h^5*e + 5580*(sqrt(c)*x - s
qrt(c*x^2 + a))^5*a^3*c^4*g^4*h^7*e - 2970*(sqrt(c)*x - sqrt(c*x^2 + a))^5*
a^4*c^3*g^2*h^9*e - 11920*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*f*g^10
h + 240(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*d*g^8*h^3 - 15720*(sqrt
(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*f*g^8*h^3 + 720*(sqrt(c)*x - sqrt(c

```
*x^2 + a))^4*a^2*c^(11/2)*d*g^6*h^5 + 19670*(sqrt(c)*x - sqrt(c*x^2 + a))^4
*a^3*c^(9/2)*f*g^6*h^5 - 3510*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(9/2)*d
*g^4*h^7 + 36260*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*f*g^4*h^7 + 14
40*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*d*g^2*h^9 + 6240*(sqrt(c)*x
- sqrt(c*x^2 + a))^4*a^5*c^(5/2)*f*g^2*h^9 - 240*(sqrt(c)*x - sqrt(c*x^2 +
a))^4*a^5*c^(5/2)*d*h^11 - 880*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(3/2)*
f*h^11 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*g^9*h^2*e + 1680*(s
qrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*g^7*h^4*e - 480*(sqrt(c)*x - sqr
t(c*x^2 + a))^4*a^3*c^(9/2)*g^5*h^6*e - 6150*(s...
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

3.98 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx$

Optimal. Leaf size=404

$$-\frac{ac(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))\,(ah-cgx)\sqrt{a+cx^2}}{16\,(cg^2+ah^2)^4\,(g+hx)^2}-\frac{(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7e}{24\,(cg^2+ah^2}$$

```
[Out] -1/24*(6*c^2*d*g^2+6*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+7*e*g)))*(-c*g*x+a*h)*(c*
x^2+a)^(3/2)/(a*h^2+c*g^2)^3/(h*x+g)^4-1/6*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(5
/2)/h/(a*h^2+c*g^2)/(h*x+g)^6+1/30*(6*a*h^2*(-e*h+2*f*g)+c*g*(5*f*g^2+h*(-7
*d*h+e*g)))*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)^2/(h*x+g)^5-1/16*a^2*c^2*(6*c^2
*d*g^2+6*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+7*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+
c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(9/2)-1/16*a*c*(6*c^2*d*g^2+6*a
^2*f*h^2-a*c*(f*g^2-h*(-d*h+7*e*g)))*(-c*g*x+a*h)*(c*x^2+a)^(1/2)/(a*h^2+c*
g^2)^4/(h*x+g)^2
```

Rubi [A]

time = 0.36, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1665, 821, 735, 739, 212}

$$-\frac{(a+cx^2)^{3/2}(ah-cgx)(6a^2fh^2-ac(fg^2-h(7eg-dh))+6c^2dg^2)}{24(g+hx)^4(ah^2+cg^2)^3}-\frac{ac\sqrt{a+cx^2}(ah-cgx)(6a^2fh^2-ac(fg^2-h(7eg-dh))+6c^2dg^2)}{16(g+hx)^2(ah^2+cg^2)^4}-\frac{a^2c^2\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(6a^2fh^2-ac(fg^2-h(7eg-dh))+6c^2dg^2)}{16(ah^2+cg^2)^{9/2}}-\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{6h(g+hx)^6(ah^2+cg^2)}+\frac{(a+cx^2)^{5/2}(6ah^2(2fg-eh)+cgh(eg-7dh)+5cfg^3)}{30h(g+hx)^5(ah^2+cg^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] -1/16*(a*c*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d*h)))*(a*h
 - c*g*x)*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)^4*(g + h*x)^2) - ((6*c^2*d*g^2
+ 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d*h)))*(a*h - c*g*x)*(a + c*x^2)^(3
/2))/(24*(c*g^2 + a*h^2)^3*(g + h*x)^4) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x
^2)^(5/2))/(6*h*(c*g^2 + a*h^2)*(g + h*x)^6) + ((5*c*f*g^3 + c*g*h*(e*g - 7
*d*h) + 6*a*h^2*(2*f*g - e*h))*(a + c*x^2)^(5/2))/(30*h*(c*g^2 + a*h^2)^2*(
g + h*x)^5) - (a^2*c^2*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g -
 d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(16*(
c*g^2 + a*h^2)^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(-2*a*e + (2*c*d)*x)*((a + c*x^2)^p/(2*(m + 1)*(c*d^2
+ a*e^2))), x] - Dist[4*a*c*(p/(2*(m + 1)*(c*d^2 + a*e^2))), Int[(d + e*x)^
(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
+ a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^7}\,dx = -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{5/2}}{6h\,(cg^2+ah^2)\,(g+hx)^6} - \frac{\int \frac{\left(-6(cdg-afg+aeh)-\left(6afh+c\left(eg+\frac{5fg^2}{h}\right.\right.\right.}{(g+hx)^6}}{6\,(cg^2+ah^2)}$$

$$= -\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{5/2}}{6h\,(cg^2+ah^2)\,(g+hx)^6} + \frac{(5cfg^3+cgh(eg-7dh)+6ah^2(2\ldots}{30h\,(cg^2+ah^2)^2\,(g\ldots}$$

$$= -\frac{(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))\,(ah-cgx)\,(a+cx^2)^{3/2}}{24\,(cg^2+ah^2)^3\,(g+hx)^4}$$

$$= -\frac{ac(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))\,(ah-cgx)\sqrt{a+cx^2}}{16\,(cg^2+ah^2)^4\,(g+hx)^2}$$

$$= -\frac{ac(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))\,(ah-cgx)\sqrt{a+cx^2}}{16\,(cg^2+ah^2)^4\,(g+hx)^2}$$

$$= -\frac{ac(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))\,(ah-cgx)\sqrt{a+cx^2}}{16\,(cg^2+ah^2)^4\,(g+hx)^2}$$

Mathematica [A]
time = 11.55, size = 696, normalized size = 1.72

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]

[Out] (-((Sqrt[a + c*x^2]*(40*(c*g^2 + a*h^2)^5*(f*g^2 + h*(-(e*g) + d*h)) - 8*(c
*g^2 + a*h^2)^4*(25*c*f*g^3 + c*g*h*(-19*e*g + 13*d*h) - 6*a*h^2*(-2*f*g +
e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^3*(30*a^2*f*h^4 + 2*c^2*(100*f*g^4 + g^
2*h*(-52*e*g + 19*d*h)) + a*c*h^2*(227*f*g^2 + h*(-101*e*g + 35*d*h)))*(g +
 h*x)^2 - 2*c*(c*g^2 + a*h^2)^2*(6*a^2*h^4*(31*f*g - 8*e*h) + 2*c^2*(100*f*
g^5 + g^3*h*(-28*e*g + d*h)) + 3*a*c*g*h^2*(131*f*g^2 + h*(-37*e*g + 3*d*h)
))*(g + h*x)^3 + c*(c*g^2 + a*h^2)*(150*a^3*f*h^6 + 4*c^3*(50*f*g^6 - g^4*h
*(2*e*g + d*h)) + 6*a*c^2*g^2*h^2*(99*f*g^2 - h*(5*e*g + 4*d*h)) + 3*a^2*c*
h^4*(193*f*g^2 + h*(-19*e*g + 5*d*h)))*(g + h*x)^4 - c^2*(6*a^3*h^6*(41*f*g
 - 8*e*h) + 3*a^2*c*g*h^4*(89*f*g^2 + h*(29*e*g - 27*d*h)) + 4*c^3*(10*f*g^
7 + g^5*h*(2*e*g + d*h)) + 2*a*c^2*g^3*h^2*(83*f*g^2 + h*(19*e*g + 14*d*h))
)*(g + h*x)^5))/(h^5*(c*g^2 + a*h^2)^4*(g + h*x)^6)) + (15*a^2*c^2*(6*c^2*d
*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 + h*(-7*e*g + d*h)))*Log[g + h*x])/(c*g^2 +
 a*h^2)^(9/2) - (15*a^2*c^2*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 + h*(-7
```

```
*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^
2 + a*h^2)^(9/2))/240
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 16382 vs. 2(380) = 760.
time = 0.10, size = 16383, normalized size = 40.55

method	result	size
default	Expression too large to display	16383

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 10749 vs. 2(383) = 766.
time = 0.75, size = 10749, normalized size = 26.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="maxima")
```

```
[Out] 7/16*sqrt(c*x^2 + a)*c^6*f*g^8/(c^5*g^10*h^5 + 5*a*c^4*g^8*h^7 + 10*a^2*c^3
*g^6*h^9 + 10*a^3*c^2*g^4*h^11 + 5*a^4*c*g^2*h^13 + a^5*h^15) - 7/16*sqrt(c
*x^2 + a)*c^6*f*g^7*x/(c^5*g^10*h^4 + 5*a*c^4*g^8*h^6 + 10*a^2*c^3*g^6*h^8
+ 10*a^3*c^2*g^4*h^10 + 5*a^4*c*g^2*h^12 + a^5*h^14) + 7/48*(c*x^2 + a)^(3/
2)*c^5*f*g^7/(c^5*g^10*h^4*x + 5*a*c^4*g^8*h^6*x + 10*a^2*c^3*g^6*h^8*x + 1
0*a^3*c^2*g^4*h^10*x + 5*a^4*c*g^2*h^12*x + a^5*h^14*x + c^5*g^11*h^3 + 5*a
*c^4*g^9*h^5 + 10*a^2*c^3*g^7*h^7 + 10*a^3*c^2*g^5*h^9 + 5*a^4*c*g^3*h^11 +
 a^5*g*h^13) - 7/16*sqrt(c*x^2 + a)*c^6*g^7*e/(c^5*g^10*h^4 + 5*a*c^4*g^8*h
^6 + 10*a^2*c^3*g^6*h^8 + 10*a^3*c^2*g^4*h^10 + 5*a^4*c*g^2*h^12 + a^5*h^14
) + 7/16*sqrt(c*x^2 + a)*c^6*g^6*x*e/(c^5*g^10*h^3 + 5*a*c^4*g^8*h^5 + 10*a
^2*c^3*g^6*h^7 + 10*a^3*c^2*g^4*h^9 + 5*a^4*c*g^2*h^11 + a^5*h^13) + 7/16*s
qrt(c*x^2 + a)*c^6*d*g^6/(c^5*g^10*h^3 + 5*a*c^4*g^8*h^5 + 10*a^2*c^3*g^6*h
^7 + 10*a^3*c^2*g^4*h^9 + 5*a^4*c*g^2*h^11 + a^5*h^13) - 7/48*(c*x^2 + a)^(
5/2)*c^4*f*g^6/(c^5*g^10*h^3*x^2 + 5*a*c^4*g^8*h^5*x^2 + 10*a^2*c^3*g^6*h^7
*x^2 + 10*a^3*c^2*g^4*h^9*x^2 + 5*a^4*c*g^2*h^11*x^2 + a^5*h^13*x^2 + 2*c^5
*g^11*h^2*x + 10*a*c^4*g^9*h^4*x + 20*a^2*c^3*g^7*h^6*x + 20*a^3*c^2*g^5*h^
8*x + 10*a^4*c*g^3*h^10*x + 2*a^5*g*h^12*x + c^5*g^12*h + 5*a*c^4*g^10*h^3
+ 10*a^2*c^3*g^8*h^5 + 10*a^3*c^2*g^6*h^7 + 5*a^4*c*g^4*h^9 + a^5*g^2*h^11)
 + 7/48*(c*x^2 + a)^(3/2)*c^5*f*g^6/(c^5*g^10*h^3 + 5*a*c^4*g^8*h^5 + 10*a^
2*c^3*g^6*h^7 + 10*a^3*c^2*g^4*h^9 + 5*a^4*c*g^2*h^11 + a^5*h^13) - 7/16*sq
rt(c*x^2 + a)*c^6*d*g^5*x/(c^5*g^10*h^2 + 5*a*c^4*g^8*h^4 + 10*a^2*c^3*g^6*
```

h^6 + 10*a^3*c^2*g^4*h^8 + 5*a^4*c*g^2*h^10 + a^5*h^12) - 7/48*(c*x^2 + a)^
(3/2)*c^5*g^6*e/(c^5*g^10*h^3*x + 5*a*c^4*g^8*h^5*x + 10*a^2*c^3*g^6*h^7*x
+ 10*a^3*c^2*g^4*h^9*x + 5*a^4*c*g^2*h^11*x + a^5*h^13*x + c^5*g^11*h^2 + 5
*a*c^4*g^9*h^4 + 10*a^2*c^3*g^7*h^6 + 10*a^3*c^2*g^5*h^8 + 5*a^4*c*g^3*h^10
 + a^5*g*h^12) + 7/48*(c*x^2 + a)^(3/2)*c^5*d*g^5/(c^5*g^10*h^2*x + 5*a*c^4
*g^8*h^4*x + 10*a^2*c^3*g^6*h^6*x + 10*a^3*c^2*g^4*h^8*x + 5*a^4*c*g^2*h^10
*x + a^5*h^12*x + c^5*g^11*h + 5*a*c^4*g^9*h^3 + 10*a^2*c^3*g^7*h^5 + 10*a^
3*c^2*g^5*h^7 + 5*a^4*c*g^3*h^9 + a^5*g*h^11) - 27/16*sqrt(c*x^2 + a)*c^5*f
*g^6/(c^4*g^8*h^5 + 4*a*c^3*g^6*h^7 + 6*a^2*c^2*g^4*h^9 + 4*a^3*c*g^2*h^11
+ a^4*h^13) + 5/4*sqrt(c*x^2 + a)*c^5*f*g^5*x/(c^4*g^8*h^4 + 4*a*c^3*g^6*h^
6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^12) + 7/48*(c*x^2 + a)^(5/
2)*c^4*g^5*e/(c^5*g^10*h^2*x^2 + 5*a*c^4*g^8*h^4*x^2 + 10*a^2*c^3*g^6*h^6*x
^2 + 10*a^3*c^2*g^4*h^8*x^2 + 5*a^4*c*g^2*h^10*x^2 + a^5*h^12*x^2 + 2*c^5*g
^11*h*x + 10*a*c^4*g^9*h^3*x + 20*a^2*c^3*g^7*h^5*x + 20*a^3*c^2*g^5*h^7*x
+ 10*a^4*c*g^3*h^9*x + 2*a^5*g*h^11*x + c^5*g^12 + 5*a*c^4*g^10*h^2 + 10*a^
2*c^3*g^8*h^4 + 10*a^3*c^2*g^6*h^6 + 5*a^4*c*g^4*h^8 + a^5*g^2*h^10) - 7/48
*(c*x^2 + a)^(3/2)*c^5*g^5*e/(c^5*g^10*h^2 + 5*a*c^4*g^8*h^4 + 10*a^2*c^3*g
^6*h^6 + 10*a^3*c^2*g^4*h^8 + 5*a^4*c*g^2*h^10 + a^5*h^12) - 7/48*(c*x^2 +
a)^(5/2)*c^4*d*g^4/(c^5*g^10*h*x^2 + 5*a*c^4*g^8*h^3*x^2 + 10*a^2*c^3*g^6*h
^5*x^2 + 10*a^3*c^2*g^4*h^7*x^2 + 5*a^4*c*g^2*h^9*x^2 + a^5*h^11*x^2 + 2*c^
5*g^11*x + 10*a*c^4*g^9*h^2*x + 20*a^2*c^3*g^7*h^4*x + 20*a^3*c^2*g^5*h^6*x
 + 10*a^4*c*g^3*h^8*x + 2*a^5*g*h^10*x + c^5*g^12/h + 5*a*c^4*g^10*h + 10*a
^2*c^3*g^8*h^3 + 10*a^3*c^2*g^6*h^5 + 5*a^4*c*g^4*h^7 + a^5*g^2*h^9) + 7/48
*(c*x^2 + a)^(3/2)*c^5*d*g^4/(c^5*g^10*h + 5*a*c^4*g^8*h^3 + 10*a^2*c^3*g^6
*h^5 + 10*a^3*c^2*g^4*h^7 + 5*a^4*c*g^2*h^9 + a^5*h^11) - 7/24*(c*x^2 + a)^
(5/2)*c^3*f*g^5/(c^4*g^8*h^4*x^3 + 4*a*c^3*g^6*h^6*x^3 + 6*a^2*c^2*g^4*h^8*
x^3 + 4*a^3*c*g^2*h^10*x^3 + a^4*h^12*x^3 + 3*c^4*g^9*h^3*x^2 + 12*a*c^3*g^
7*h^5*x^2 + 18*a^2*c^2*g^5*h^7*x^2 + 12*a^3*c*g^3*h^9*x^2 + 3*a^4*g*h^11*x^
2 + 3*c^4*g^10*h^2*x + 12*a*c^3*g^8*h^4*x + 18*a^2*c^2*g^6*h^6*x + 12*a^3*c
*g^4*h^8*x + 3*a^4*g^2*h^10*x + c^4*g^11*h + 4*a*c^3*g^9*h^3 + 6*a^2*c^2*g^
7*h^5 + 4*a^3*c*g^5*h^7 + a^4*g^3*h^9) - 17/24*(c*x^2 + a)^(3/2)*c^4*f*g^5/
(c^4*g^8*h^4*x + 4*a*c^3*g^6*h^6*x + 6*a^2*c^2*g^4*h^8*x + 4*a^3*c*g^2*h^10
*x + a^4*h^12*x + c^4*g^9*h^3 + 4*a*c^3*g^7*h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^3
*c*g^3*h^9 + a^4*g*h^11) + 21/16*sqrt(c*x^2 + a)*c^5*g^5*e/(c^4*g^8*h^4 + 4
*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^12) - 7/8*sqr
t(c*x^2 + a)*c^5*g^4*x*e/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7
 + 4*a^3*c*g^2*h^9 + a^4*h^11) - 15/16*sqrt(c*x^2 + a)*c^5*d*g^4/(c^4*g^8*h
^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) + 1/
8*(c*x^2 + a)^(5/2)*c^3*f*g^4/(c^4*g^8*h^3*x^2 + 4*a*c^3*g^6*h^5*x^2 + 6*a^
2*c^2*g^4*h^7*x^2 + 4*a^3*c*g^2*h^9*x^2 + a^4*h^11*x^2 + 2*c^4*g^9*h^2*x +
8*a*c^3*g^7*h^4*x + 12*a^2*c^2*g^5*h^6*x + 8*a^3*c*g^3*h^8*x + 2*a^4*g*h^10
*x + c^4*g^10*h + 4*a*c^3*g^8*h^3 + 6*a^2*c^2*g^6*h^5 + 4*a^3*c*g^4*h^7 + a
^4*g^2*h^9) - 1/8*(c*x^2 + a)^(3/2)*c^4*f*g^4/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^
5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) + 1/2*sqrt(c*x^2 + a)*c
^5*d*g^3*x/(c^4*g^8*h^2 + 4*a*c^3*g^6*h^4 + 6*a^2*c^2*g^4*h^6 + 4*a^3*c*g^2

```
*h^8 + a^4*h^10) + 7/24*(c*x^2 + a)^(5/2)*c^3*g...
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**7,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6122 vs. 2(383) = 766.
time = 4.68, size = 6122, normalized size = 15.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="giac")
```

```
[Out] 1/8*(6*a^2*c^4*d*g^2 - a^3*c^3*f*g^2 - a^3*c^3*d*h^2 + 6*a^4*c^2*f*h^2 + 7*
a^3*c^3*g*h*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-
c*g^2 - a*h^2))/((c^4*g^8 + 4*a*c^3*g^6*h^2 + 6*a^2*c^2*g^4*h^4 + 4*a^3*c*g
^2*h^6 + a^4*h^8)*sqrt(-c*g^2 - a*h^2)) + 1/120*(240*(sqrt(c)*x - sqrt(c*x^
2 + a))^11*c^6*f*g^8*h^5 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a*c^5*f*g^6
*h^7 + 1440*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^2*c^4*f*g^4*h^9 - 90*(sqrt(c
)*x - sqrt(c*x^2 + a))^11*a^2*c^4*d*g^2*h^11 + 975*(sqrt(c)*x - sqrt(c*x^2
+ a))^11*a^3*c^3*f*g^2*h^11 + 15*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^3*d
*h^13 + 150*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^4*c^2*f*h^13 - 105*(sqrt(c)*
x - sqrt(c*x^2 + a))^11*a^3*c^3*g*h^12*e + 1200*(sqrt(c)*x - sqrt(c*x^2 + a
))^10*c^(13/2)*f*g^9*h^4 + 4800*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(11/2)
*f*g^7*h^6 + 7200*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(9/2)*f*g^5*h^8 -
990*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(9/2)*d*g^3*h^10 + 4965*(sqrt(c)
*x - sqrt(c*x^2 + a))^10*a^3*c^(7/2)*f*g^3*h^10 + 165*(sqrt(c)*x - sqrt(c*x
^2 + a))^10*a^3*c^(7/2)*d*g*h^12 + 210*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^4
```

*c^(5/2)*f*g*h^12 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^10*c^(13/2)*g^8*h^5*e
 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(11/2)*g^6*h^7*e + 1440*(sqrt(c
)*x - sqrt(c*x^2 + a))^10*a^2*c^(9/2)*g^4*h^9*e - 195*(sqrt(c)*x - sqrt(c*x
^2 + a))^10*a^3*c^(7/2)*g^2*h^11*e + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a
^4*c^(5/2)*h^13*e + 3200*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^7*f*g^10*h^3 + 3
20*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^7*d*g^8*h^5 + 12080*(sqrt(c)*x - sqrt(
c*x^2 + a))^9*a*c^6*f*g^8*h^5 + 1280*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^6*
d*g^6*h^7 + 16320*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^5*f*g^6*h^7 - 2520*
(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^5*d*g^4*h^9 + 9220*(sqrt(c)*x - sqrt(
c*x^2 + a))^9*a^3*c^4*f*g^4*h^9 + 2530*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*
c^4*d*g^2*h^11 - 4205*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^3*f*g^2*h^11 +
235*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^3*d*h^13 - 210*(sqrt(c)*x - sqrt(
c*x^2 + a))^9*a^5*c^2*f*h^13 + 640*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^7*g^9*
h^4*e + 2560*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^6*g^7*h^6*e + 3840*(sqrt(c
)*x - sqrt(c*x^2 + a))^9*a^2*c^5*g^5*h^8*e - 2620*(sqrt(c)*x - sqrt(c*x^2 +
 a))^9*a^3*c^4*g^3*h^10*e + 1235*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^3*g*
h^12*e + 4800*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(15/2)*f*g^11*h^2 + 480*(sq
rt(c)*x - sqrt(c*x^2 + a))^8*c^(15/2)*d*g^9*h^4 + 15120*(sqrt(c)*x - sqrt(c
*x^2 + a))^8*a*c^(13/2)*f*g^9*h^4 + 1920*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*
c^(13/2)*d*g^7*h^6 + 12480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(11/2)*f*g
^7*h^6 - 7380*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(11/2)*d*g^5*h^8 - 3570
*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(9/2)*f*g^5*h^8 + 8220*(sqrt(c)*x -
sqrt(c*x^2 + a))^8*a^3*c^(9/2)*d*g^3*h^10 - 22545*(sqrt(c)*x - sqrt(c*x^2 +
 a))^8*a^4*c^(7/2)*f*g^3*h^10 - 285*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(
7/2)*d*g*h^12 + 510*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(5/2)*f*g*h^12 +
960*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(15/2)*g^10*h^3*e + 3600*(sqrt(c)*x -
 sqrt(c*x^2 + a))^8*a*c^(13/2)*g^8*h^5*e + 4800*(sqrt(c)*x - sqrt(c*x^2 + a
))^8*a^2*c^(11/2)*g^6*h^7*e - 9570*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(9
/2)*g^4*h^9*e + 5355*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(7/2)*g^2*h^11*e
 - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(5/2)*h^13*e + 3840*(sqrt(c)*x
 - sqrt(c*x^2 + a))^7*c^8*f*g^12*h + 384*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^
8*d*g^10*h^3 + 6336*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^7*f*g^10*h^3 + 1728
*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^7*d*g^8*h^5 - 11808*(sqrt(c)*x - sqrt(
c*x^2 + a))^7*a^2*c^6*f*g^8*h^5 - 9456*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*
c^6*d*g^6*h^7 - 31704*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^5*f*g^6*h^7 + 2
0760*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^5*d*g^4*h^9 - 39960*(sqrt(c)*x -
 sqrt(c*x^2 + a))^7*a^4*c^4*f*g^4*h^9 - 2700*(sqrt(c)*x - sqrt(c*x^2 + a))^
7*a^4*c^4*d*g^2*h^11 + 12150*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^3*f*g^2*
h^11 + 390*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^3*d*h^13 + 60*(sqrt(c)*x -
 sqrt(c*x^2 + a))^7*a^6*c^2*f*h^13 + 768*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^
8*g^11*h^2*e + 1728*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^7*g^9*h^4*e - 768*(
sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^6*g^7*h^6*e - 19608*(sqrt(c)*x - sqrt(
c*x^2 + a))^7*a^3*c^5*g^5*h^8*e + 14040*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4
*c^4*g^3*h^10*e - 2730*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^3*g*h^12*e + 1
280*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(17/2)*f*g^13 + 128*(sqrt(c)*x - sqrt

```
(c*x^2 + a))^6*c^(17/2)*d*g^11*h^2 - 4288*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a
*c^(15/2)*f*g^11*h^2 - 64*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(15/2)*d*g^9*
h^4 - 24096*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(13/2)*f*g^9*h^4 - 8592*(
sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(13/2)*d*g^7*h^6 - 26728*(sqrt(c)*x -
sqrt(c*x^2 + a))^6*a^3*c^(11/2)*f*g^7*h^6 + 24440*(sqrt(c)*x - sqrt(c*x^2 +
 a))^6*a^3*c^(11/2)*d*g^5*h^8 - 12640*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c
^(9/2)*f*g^5*h^8 - 14860*(sqrt(c)*x - sqrt(c*x^...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x)
```

```
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7, x)
```

3.99 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx$

Optimal. Leaf size=532

$$-\frac{ac^2(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))\,(ah-cgx)\sqrt{a+cx^2}}{16\,(cg^2+ah^2)^5\,(g+hx)^2}-\frac{c(6c^2dg^3+a^2h^2(8fg-$$

```
[Out] -1/24*c*(6*c^2*d*g^3+a^2*h^2*(-e*h+8*f*g)-a*c*g*(f*g^2-h*(-3*d*h+8*e*g)))*(
-c*g*x+a*h)*(c*x^2+a)^(3/2)/(a*h^2+c*g^2)^4/(h*x+g)^4-1/7*(d*h^2-e*g*h+f*g^
2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^7+1/42*(7*a*h^2*(-e*h+2*f*g)+c*g
*(5*f*g^2+h*(-9*d*h+2*e*g)))*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)^2/(h*x+g)^6-1/
210*(42*a^2*f*h^4-c^2*g^2*(5*f*g^2+h*(-51*d*h+2*e*g))-a*c*h^2*(26*f*g^2-h*(
-12*d*h+61*e*g)))*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)^3/(h*x+g)^5-1/16*a^2*c^3*
(6*c^2*d*g^3+a^2*h^2*(-e*h+8*f*g)-a*c*g*(f*g^2-h*(-3*d*h+8*e*g)))*arctanh((
-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(11/2)-1/16*
a*c^2*(6*c^2*d*g^3+a^2*h^2*(-e*h+8*f*g)-a*c*g*(f*g^2-h*(-3*d*h+8*e*g)))*(-c
*g*x+a*h)*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^5/(h*x+g)^2
```

Rubi [A]

time = 0.55, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1665, 849, 821, 735, 739, 212}

$$-\frac{(a+cx^2)^{5/2}(42a^2fh^4-ach^2(26fg^2-h(61eg-12dh))-c^2(g^2h(2eg-51dh)+5fg^4))}{210h(g+hx)^5(ah^2+cg^2)^3}-\frac{ac^2\sqrt{a+cx^2}(ah-cgx)(a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh))+6c^2dg^3)}{16(g+hx)^2(ah^2+cg^2)^5}-\frac{c(a+cx^2)^{3/2}(ah-cgx)(a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh))+6c^2dg^3)}{24(g+hx)^4(ah^2+cg^2)^4}-\frac{a^2c^3\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh))+6c^2dg^3)}{16(ah^2+cg^2)^{11/2}}-\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{7h(g+hx)^7(ah^2+cg^2)}+\frac{(a+cx^2)^{5/2}(7ah^2(2fg-eh)+cgh(2eg-9dh)+5cfg^3)}{42h(g+hx)^6(ah^2+cg^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -1/16*(a*c^2*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g
 - 3*d*h)))*(a*h - c*g*x)*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)^5*(g + h*x)^2)
- (c*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g - 3*d*h
)))*(a*h - c*g*x)*(a + c*x^2)^(3/2))/(24*(c*g^2 + a*h^2)^4*(g + h*x)^4) - (
(f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(7*h*(c*g^2 + a*h^2)*(g + h*x)^7
) + ((5*c*f*g^3 + c*g*h*(2*e*g - 9*d*h) + 7*a*h^2*(2*f*g - e*h))*(a + c*x^2
)^(5/2))/(42*h*(c*g^2 + a*h^2)^2*(g + h*x)^6) - ((42*a^2*f*h^4 - c^2*(5*f*g
^4 + g^2*h*(2*e*g - 51*d*h)) - a*c*h^2*(26*f*g^2 - h*(61*e*g - 12*d*h)))*(a
 + c*x^2)^(5/2))/(210*h*(c*g^2 + a*h^2)^3*(g + h*x)^5) - (a^2*c^3*(6*c^2*d*
g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g - 3*d*h)))*ArcTanh[(a
*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(16*(c*g^2 + a*h^2)^(11
/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(-2*a*e + (2*c*d)*x)*((a + c*x^2)^p/(2*(m + 1)*(c*d^2
+ a*e^2))), x] - Dist[4*a*c*(p/(2*(m + 1)*(c*d^2 + a*e^2))), Int[(d + e*x)^
(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
+ a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = -\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{7h(cg^2+ah^2)(g+hx)^7} - \frac{\int \frac{\left(-7(cdg-afg+aeh)-\left(7afh+c\left(2eg+\frac{5fg^2}{h}\right.\right.\right.}{(g+hx)^7}}{7(cg^2+ah^2)}$$

$$= -\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{7h(cg^2+ah^2)(g+hx)^7} + \frac{(5cfg^3+cgh(2eg-9dh)+7ah^2(2}{42h(cg^2+ah^2)^2(g}$$

$$= -\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{7h(cg^2+ah^2)(g+hx)^7} + \frac{(5cfg^3+cgh(2eg-9dh)+7ah^2(2}{42h(cg^2+ah^2)^2(g}$$

$$= -\frac{c(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))(ah-cgx)(}{24(cg^2+ah^2)^4(g+hx)^4}$$

$$= -\frac{ac^2(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))(ah-cgx}{16(cg^2+ah^2)^5(g+hx)^2}$$

$$= -\frac{ac^2(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))(ah-cgx}{16(cg^2+ah^2)^5(g+hx)^2}$$

$$= -\frac{ac^2(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))(ah-cgx}{16(cg^2+ah^2)^5(g+hx)^2}$$

Mathematica [A]
time = 11.61, size = 863, normalized size = 1.62

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -1/1680*(Sqrt[a + c*x^2]*(240*(c*g^2 + a*h^2)^6*(f*g^2 + h*(-(e*g) + d*h))
- 40*(c*g^2 + a*h^2)^5*(29*c*f*g^3 + c*g*h*(-22*e*g + 15*d*h) - 7*a*h^2*(-2
*f*g + e*h))*(g + h*x) + 8*(c*g^2 + a*h^2)^4*(42*a^2*f*h^4 + a*c*h^2*(314*f
*g^2 + h*(-139*e*g + 48*d*h)) + c^2*(275*f*g^4 + g^2*h*(-142*e*g + 51*d*h))
)*(g + h*x)^2 - 2*c*(c*g^2 + a*h^2)^3*(7*a^2*h^4*(136*f*g - 35*e*h) + 2*c^2
*(500*f*g^5 + g^3*h*(-136*e*g + 3*d*h)) + a*c*g*h^2*(1979*f*g^2 + h*(-544*e
*g + 33*d*h)))*(g + h*x)^3 + 2*c*(c*g^2 + a*h^2)^2*(336*a^3*f*h^6 + c^3*(40
0*f*g^6 - 2*g^4*h*(4*e*g + 3*d*h)) + 3*a^2*c*h^4*(400*f*g^2 + h*(-29*e*g +
8*d*h)) + a*c^2*g^2*h^2*(1201*f*g^2 - h*(32*e*g + 45*d*h)))*(g + h*x)^4 - c
^2*(c*g^2 + a*h^2)*(21*a^3*h^6*(24*f*g - 5*e*h) + 2*a*c^2*g^3*h^2*(89*f*g^2
```

```
 + 44*e*g*h + 54*d*h^2) + 3*a^2*c*g*h^4*(109*f*g^2 + h*(94*e*g - 73*d*h)) +
 4*c^3*(10*f*g^7 + g^5*h*(4*e*g + 3*d*h)))*(g + h*x)^5 - c^2*(-336*a^4*f*h^
8 + 2*a*c^3*g^4*h^2*(109*f*g^2 + 52*e*g*h + 60*d*h^2) + a^2*c^2*g^2*h^4*(50
5*f*g^2 + h*(370*e*g - 741*d*h)) + 4*c^4*(10*f*g^8 + g^6*h*(4*e*g + 3*d*h))
 + 3*a^3*c*h^6*(312*f*g^2 + h*(-221*e*g + 32*d*h)))*(g + h*x)^6))/((c*g^2*h
 + a*h^3)^5*(g + h*x)^7) + (a^2*c^3*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) -
a*c*g*(f*g^2 + h*(-8*e*g + 3*d*h)))*Log[g + h*x])/(16*(c*g^2 + a*h^2)^(11/2
)) - (a^2*c^3*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 + h*(-8*e
*g + 3*d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(16*(
c*g^2 + a*h^2)^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 24804 vs. $2(504) = 1008$.
time = 0.10, size = 24805, normalized size = 46.63

method	result	size
default	Expression too large to display	24805

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 16286 vs. $2(514) = 1028$.
time = 0.92, size = 16286, normalized size = 30.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="maxima")
```

```
[Out] 9/16*sqrt(c*x^2 + a)*c^7*f*g^9/(c^6*g^12*h^5 + 6*a*c^5*g^10*h^7 + 15*a^2*c^
4*g^8*h^9 + 20*a^3*c^3*g^6*h^11 + 15*a^4*c^2*g^4*h^13 + 6*a^5*c*g^2*h^15 +
a^6*h^17) - 9/16*sqrt(c*x^2 + a)*c^7*f*g^8*x/(c^6*g^12*h^4 + 6*a*c^5*g^10*h
^6 + 15*a^2*c^4*g^8*h^8 + 20*a^3*c^3*g^6*h^10 + 15*a^4*c^2*g^4*h^12 + 6*a^5
*c*g^2*h^14 + a^6*h^16) + 3/16*(c*x^2 + a)^(3/2)*c^6*f*g^8/(c^6*g^12*h^4*x
+ 6*a*c^5*g^10*h^6*x + 15*a^2*c^4*g^8*h^8*x + 20*a^3*c^3*g^6*h^10*x + 15*a^
4*c^2*g^4*h^12*x + 6*a^5*c*g^2*h^14*x + a^6*h^16*x + c^6*g^13*h^3 + 6*a*c^5
*g^11*h^5 + 15*a^2*c^4*g^9*h^7 + 20*a^3*c^3*g^7*h^9 + 15*a^4*c^2*g^5*h^11 +
 6*a^5*c*g^3*h^13 + a^6*g*h^15) - 9/16*sqrt(c*x^2 + a)*c^7*g^8*e/(c^6*g^12*
h^4 + 6*a*c^5*g^10*h^6 + 15*a^2*c^4*g^8*h^8 + 20*a^3*c^3*g^6*h^10 + 15*a^4*
c^2*g^4*h^12 + 6*a^5*c*g^2*h^14 + a^6*h^16) + 9/16*sqrt(c*x^2 + a)*c^7*g^7*
x*e/(c^6*g^12*h^3 + 6*a*c^5*g^10*h^5 + 15*a^2*c^4*g^8*h^7 + 20*a^3*c^3*g^6*
h^9 + 15*a^4*c^2*g^4*h^11 + 6*a^5*c*g^2*h^13 + a^6*h^15) + 9/16*sqrt(c*x^2
```

+ a)*c^7*d*g^7/(c^6*g^12*h^3 + 6*a*c^5*g^10*h^5 + 15*a^2*c^4*g^8*h^7 + 20*a
^3*c^3*g^6*h^9 + 15*a^4*c^2*g^4*h^11 + 6*a^5*c*g^2*h^13 + a^6*h^15) - 3/16*
(c*x^2 + a)^(5/2)*c^5*f*g^7/(c^6*g^12*h^3*x^2 + 6*a*c^5*g^10*h^5*x^2 + 15*a
^2*c^4*g^8*h^7*x^2 + 20*a^3*c^3*g^6*h^9*x^2 + 15*a^4*c^2*g^4*h^11*x^2 + 6*a
^5*c*g^2*h^13*x^2 + a^6*h^15*x^2 + 2*c^6*g^13*h^2*x + 12*a*c^5*g^11*h^4*x +
 30*a^2*c^4*g^9*h^6*x + 40*a^3*c^3*g^7*h^8*x + 30*a^4*c^2*g^5*h^10*x + 12*a
^5*c*g^3*h^12*x + 2*a^6*g*h^14*x + c^6*g^14*h + 6*a*c^5*g^12*h^3 + 15*a^2*c
^4*g^10*h^5 + 20*a^3*c^3*g^8*h^7 + 15*a^4*c^2*g^6*h^9 + 6*a^5*c*g^4*h^11 +
a^6*g^2*h^13) + 3/16*(c*x^2 + a)^(3/2)*c^6*f*g^7/(c^6*g^12*h^3 + 6*a*c^5*g^
10*h^5 + 15*a^2*c^4*g^8*h^7 + 20*a^3*c^3*g^6*h^9 + 15*a^4*c^2*g^4*h^11 + 6*
a^5*c*g^2*h^13 + a^6*h^15) - 9/16*sqrt(c*x^2 + a)*c^7*d*g^6*x/(c^6*g^12*h^2
 + 6*a*c^5*g^10*h^4 + 15*a^2*c^4*g^8*h^6 + 20*a^3*c^3*g^6*h^8 + 15*a^4*c^2*
g^4*h^10 + 6*a^5*c*g^2*h^12 + a^6*h^14) - 3/16*(c*x^2 + a)^(3/2)*c^6*g^7*e/
(c^6*g^12*h^3*x + 6*a*c^5*g^10*h^5*x + 15*a^2*c^4*g^8*h^7*x + 20*a^3*c^3*g^
6*h^9*x + 15*a^4*c^2*g^4*h^11*x + 6*a^5*c*g^2*h^13*x + a^6*h^15*x + c^6*g^1
3*h^2 + 6*a*c^5*g^11*h^4 + 15*a^2*c^4*g^9*h^6 + 20*a^3*c^3*g^7*h^8 + 15*a^4
*c^2*g^5*h^10 + 6*a^5*c*g^3*h^12 + a^6*g*h^14) + 3/16*(c*x^2 + a)^(3/2)*c^6
*d*g^6/(c^6*g^12*h^2*x + 6*a*c^5*g^10*h^4*x + 15*a^2*c^4*g^8*h^6*x + 20*a^3
*c^3*g^6*h^8*x + 15*a^4*c^2*g^4*h^10*x + 6*a^5*c*g^2*h^12*x + a^6*h^14*x +
c^6*g^13*h + 6*a*c^5*g^11*h^3 + 15*a^2*c^4*g^9*h^5 + 20*a^3*c^3*g^7*h^7 + 1
5*a^4*c^2*g^5*h^9 + 6*a^5*c*g^3*h^11 + a^6*g*h^13) - 35/16*sqrt(c*x^2 + a)*
c^6*f*g^7/(c^5*g^10*h^5 + 5*a*c^4*g^8*h^7 + 10*a^2*c^3*g^6*h^9 + 10*a^3*c^2
*g^4*h^11 + 5*a^4*c*g^2*h^13 + a^5*h^15) + 13/8*sqrt(c*x^2 + a)*c^6*f*g^6*x
/(c^5*g^10*h^4 + 5*a*c^4*g^8*h^6 + 10*a^2*c^3*g^6*h^8 + 10*a^3*c^2*g^4*h^10
 + 5*a^4*c*g^2*h^12 + a^5*h^14) + 3/16*(c*x^2 + a)^(5/2)*c^5*g^6*e/(c^6*g^1
2*h^2*x^2 + 6*a*c^5*g^10*h^4*x^2 + 15*a^2*c^4*g^8*h^6*x^2 + 20*a^3*c^3*g^6*
h^8*x^2 + 15*a^4*c^2*g^4*h^10*x^2 + 6*a^5*c*g^2*h^12*x^2 + a^6*h^14*x^2 + 2
*c^6*g^13*h*x + 12*a*c^5*g^11*h^3*x + 30*a^2*c^4*g^9*h^5*x + 40*a^3*c^3*g^7
*h^7*x + 30*a^4*c^2*g^5*h^9*x + 12*a^5*c*g^3*h^11*x + 2*a^6*g*h^13*x + c^6*
g^14 + 6*a*c^5*g^12*h^2 + 15*a^2*c^4*g^10*h^4 + 20*a^3*c^3*g^8*h^6 + 15*a^4
*c^2*g^6*h^8 + 6*a^5*c*g^4*h^10 + a^6*g^2*h^12) - 3/16*(c*x^2 + a)^(3/2)*c^
6*g^6*e/(c^6*g^12*h^2 + 6*a*c^5*g^10*h^4 + 15*a^2*c^4*g^8*h^6 + 20*a^3*c^3*
g^6*h^8 + 15*a^4*c^2*g^4*h^10 + 6*a^5*c*g^2*h^12 + a^6*h^14) - 3/16*(c*x^2
+ a)^(5/2)*c^5*d*g^5/(c^6*g^12*h*x^2 + 6*a*c^5*g^10*h^3*x^2 + 15*a^2*c^4*g^
8*h^5*x^2 + 20*a^3*c^3*g^6*h^7*x^2 + 15*a^4*c^2*g^4*h^9*x^2 + 6*a^5*c*g^2*h
^11*x^2 + a^6*h^13*x^2 + 2*c^6*g^13*x + 12*a*c^5*g^11*h^2*x + 30*a^2*c^4*g^
9*h^4*x + 40*a^3*c^3*g^7*h^6*x + 30*a^4*c^2*g^5*h^8*x + 12*a^5*c*g^3*h^10*x
 + 2*a^6*g*h^12*x + c^6*g^14/h + 6*a*c^5*g^12*h + 15*a^2*c^4*g^10*h^3 + 20*
a^3*c^3*g^8*h^5 + 15*a^4*c^2*g^6*h^7 + 6*a^5*c*g^4*h^9 + a^6*g^2*h^11) + 3/
16*(c*x^2 + a)^(3/2)*c^6*d*g^5/(c^6*g^12*h + 6*a*c^5*g^10*h^3 + 15*a^2*c^4*
g^8*h^5 + 20*a^3*c^3*g^6*h^7 + 15*a^4*c^2*g^4*h^9 + 6*a^5*c*g^2*h^11 + a^6*
h^13) - 3/8*(c*x^2 + a)^(5/2)*c^4*f*g^6/(c^5*g^10*h^4*x^3 + 5*a*c^4*g^8*h^6
*x^3 + 10*a^2*c^3*g^6*h^8*x^3 + 10*a^3*c^2*g^4*h^10*x^3 + 5*a^4*c*g^2*h^12*
x^3 + a^5*h^14*x^3 + 3*c^5*g^11*h^3*x^2 + 15*a*c^4*g^9*h^5*x^2 + 30*a^2*c^3
*g^7*h^7*x^2 + 30*a^3*c^2*g^5*h^9*x^2 + 15*a^4*c*g^3*h^11*x^2 + 3*a^5*g*h^1

```
3*x^2 + 3*c^5*g^12*h^2*x + 15*a*c^4*g^10*h^4*x + 30*a^2*c^3*g^8*h^6*x + 30*
a^3*c^2*g^6*h^8*x + 15*a^4*c*g^4*h^10*x + 3*a^5*g^2*h^12*x + c^5*g^13*h + 5
*a*c^4*g^11*h^3 + 10*a^2*c^3*g^9*h^5 + 10*a^3*c^2*g^7*h^7 + 5*a^4*c*g^5*h^9
 + a^5*g^3*h^11) - 11/12*(c*x^2 + a)^(3/2)*c^5*f*g^6/(c^5*g^10*h^4*x + 5*a*
c^4*g^8*h^6*x + 10*a^2*c^3*g^6*h^8*x + 10*a^3*c^2*g^4*h^10*x + 5*a^4*c*g^2*
h^12*x + a^5*h^14*x + c^5*g^11*h^3 + 5*a*c^4*g^9*h^5 + 10*a^2*c^3*g^7*h^7 +
 10*a^3*c^2*g^5*h^9 + 5*a^4*c*g^3*h^11 + a^5*g*h^13) + 7/4*sqrt(c*x^2 + a)*
c^6*g^6*e/(c^5*g^10*h^4 + 5*a*c^4*g^8*h^6 + 10*a^2*c^3*g^6*h^8 + 10*a^3*c^2
*g^4*h^10 + 5*a^4*c*g^2*h^12 + a^5*h^14) - 19/1...
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**8,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 7936 vs. $2(514) = 1028$.
time = 7.84, size = 7936, normalized size = 14.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="giac")
```

```
[Out] -1/8*(6*a^2*c^5*d*g^3 - a^3*c^4*f*g^3 - 3*a^3*c^4*d*g*h^2 + 8*a^4*c^3*f*g*h
^2 + 8*a^3*c^4*g^2*h*e - a^4*c^3*h^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a
))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^5*g^10 + 5*a*c^4*g^8*h^2 + 10*a
^2*c^3*g^6*h^4 + 10*a^3*c^2*g^4*h^6 + 5*a^4*c*g^2*h^8 + a^5*h^10)*sqrt(-c*g
^2 - a*h^2)) - 1/840*(630*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^2*c^5*d*g^3*h^
12 - 105*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^3*c^4*f*g^3*h^12 - 315*(sqrt(c)
*x - sqrt(c*x^2 + a))^13*a^3*c^4*d*g*h^14 + 840*(sqrt(c)*x - sqrt(c*x^2 + a
```

))^13*a^4*c^3*f*g*h^14 + 840*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^3*c^4*g^2*h
^13*e - 105*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^4*c^3*h^15*e - 1680*(sqrt(c)
*x - sqrt(c*x^2 + a))^12*c^(15/2)*f*g^10*h^5 - 8400*(sqrt(c)*x - sqrt(c*x^2
 + a))^12*a*c^(13/2)*f*g^8*h^7 - 16800*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^2
*c^(11/2)*f*g^6*h^9 + 8190*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^2*c^(11/2)*d*
g^4*h^11 - 18165*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^3*c^(9/2)*f*g^4*h^11 -
4095*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^3*c^(9/2)*d*g^2*h^13 + 2520*(sqrt(c
)*x - sqrt(c*x^2 + a))^12*a^4*c^(7/2)*f*g^2*h^13 - 1680*(sqrt(c)*x - sqrt(c
*x^2 + a))^12*a^5*c^(5/2)*f*h^15 + 10920*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a
^3*c^(9/2)*g^3*h^12*e - 1365*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^4*c^(7/2)*g
*h^14*e - 5600*(sqrt(c)*x - sqrt(c*x^2 + a))^11*c^8*f*g^11*h^4 - 28000*(sqr
t(c)*x - sqrt(c*x^2 + a))^11*a*c^7*f*g^9*h^6 - 56000*(sqrt(c)*x - sqrt(c*x^
2 + a))^11*a^2*c^6*f*g^7*h^8 + 44940*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^2*c
^6*d*g^5*h^10 - 63490*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^5*f*g^5*h^10 -
 26670*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^5*d*g^3*h^12 + 32620*(sqrt(c)
*x - sqrt(c*x^2 + a))^11*a^4*c^4*f*g^3*h^12 + 2100*(sqrt(c)*x - sqrt(c*x^2
+ a))^11*a^4*c^4*d*g*h^14 - 11200*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^5*c^3*
f*g*h^14 - 2240*(sqrt(c)*x - sqrt(c*x^2 + a))^11*c^8*g^10*h^5*e - 11200*(sq
rt(c)*x - sqrt(c*x^2 + a))^11*a*c^7*g^8*h^7*e - 22400*(sqrt(c)*x - sqrt(c*x
^2 + a))^11*a^2*c^6*g^6*h^9*e + 37520*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*
c^5*g^4*h^11*e - 24290*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^4*c^4*g^2*h^13*e
- 1540*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^5*c^3*h^15*e - 11200*(sqrt(c)*x -
 sqrt(c*x^2 + a))^10*c^(17/2)*f*g^12*h^3 - 3360*(sqrt(c)*x - sqrt(c*x^2 + a
))^10*c^(17/2)*d*g^10*h^5 - 52640*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(15/
2)*f*g^10*h^5 - 16800*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(15/2)*d*g^8*h^7
 - 95200*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(13/2)*f*g^8*h^7 + 100380*(
sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(13/2)*d*g^6*h^9 - 100730*(sqrt(c)*x
- sqrt(c*x^2 + a))^10*a^3*c^(11/2)*f*g^6*h^9 - 146790*(sqrt(c)*x - sqrt(c*x
^2 + a))^10*a^3*c^(11/2)*d*g^4*h^11 + 163940*(sqrt(c)*x - sqrt(c*x^2 + a))^
10*a^4*c^(9/2)*f*g^4*h^11 + 6300*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^4*c^(9/
2)*d*g^2*h^13 - 56000*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^5*c^(7/2)*f*g^2*h^
13 - 3360*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^5*c^(7/2)*d*h^15 + 3360*(sqrt(
c)*x - sqrt(c*x^2 + a))^10*a^6*c^(5/2)*f*h^15 - 4480*(sqrt(c)*x - sqrt(c*x^
2 + a))^10*c^(17/2)*g^11*h^4*e - 22400*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c
^(15/2)*g^9*h^6*e - 44800*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(13/2)*g^7
*h^8*e + 133840*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^3*c^(11/2)*g^5*h^10*e -
106330*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^4*c^(9/2)*g^3*h^12*e + 3220*(sqrt
(c)*x - sqrt(c*x^2 + a))^10*a^5*c^(7/2)*g*h^14*e - 13440*(sqrt(c)*x - sqrt(
c*x^2 + a))^9*c^9*f*g^13*h^2 - 4032*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^9*d*g
^11*h^4 - 50848*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^8*f*g^11*h^4 - 20160*(s
qrt(c)*x - sqrt(c*x^2 + a))^9*a*c^8*d*g^9*h^6 - 52640*(sqrt(c)*x - sqrt(c*x
^2 + a))^9*a^2*c^7*f*g^9*h^6 + 191016*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c
^7*d*g^7*h^8 - 9436*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^6*f*g^7*h^8 - 363
216*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^6*d*g^5*h^10 + 439306*(sqrt(c)*x
- sqrt(c*x^2 + a))^9*a^4*c^5*f*g^5*h^10 + 95340*(sqrt(c)*x - sqrt(c*x^2 + a

```
))^9*a^4*c^5*d*g^3*h^12 - 209965*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^5*c^4*f*
g^3*h^12 - 9975*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^5*c^4*d*g*h^14 + 32200*(s
qrt(c)*x - sqrt(c*x^2 + a))^9*a^6*c^3*f*g*h^14 - 5376*(sqrt(c)*x - sqrt(c*x
^2 + a))^9*c^9*g^12*h^3*e - 25984*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^8*g^1
0*h^5*e - 49280*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^7*g^8*h^7*e + 263648*
(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^6*g^6*h^9*e - 332780*(sqrt(c)*x - sqr
t(c*x^2 + a))^9*a^4*c^5*g^4*h^11*e + 49490*(sqrt(c)*x - sqrt(c*x^2 + a))^9*
a^5*c^4*g^2*h^13*e - 1085*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^6*c^3*h^15*e -
8960*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(19/2)*f*g^14*h - 2688*(sqrt(c)*x -
sqrt(c*x^2 + a))^8*c^(19/2)*d*g^12*h^3 - 15232*(sqrt(c)*x - sqrt(c*x^2 + a)
)^8*a*c^(17/2)*f*g^12*h^3 - 16800*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(17/2
)*d*g^10*h^5 + 53200*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(15/2)*f*g^10*h^
5 + 181104*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(15/2)*d*g^8*h^7 + 143416*
(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(13/2)*f*g^8*h^7 - 651924*(sqrt(c)*x
- sqrt(c*x^2 + a))^8*a^3*c^(13/2)*d*g^6*h^9 + 5...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8, x)
```

3.100 $\int (a+cx^2)^{5/2} (A+Bx+Cx^2)\, dx$

Optimal. Leaf size=168

$$\frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c}+\frac{5a(8Ac-aC)x(a+cx^2)^{3/2}}{192c}+\frac{(8Ac-aC)x(a+cx^2)^{5/2}}{48c}+\frac{B(a+cx^2)^{7/2}}{7c}+\frac{Cx(\cdots}{}$$

```
[Out] 5/192*a*(8*A*c-C*a)*x*(c*x^2+a)^(3/2)/c+1/48*(8*A*c-C*a)*x*(c*x^2+a)^(5/2)/
c+1/7*B*(c*x^2+a)^(7/2)/c+1/8*C*x*(c*x^2+a)^(7/2)/c+5/128*a^3*(8*A*c-C*a)*a
rctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+5/128*a^2*(8*A*c-C*a)*x*(c*x^2+a)
^(1/2)/c
```

Rubi [A]
time = 0.06, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1829, 655, 201, 223, 212}

$$\frac{5a^3(8Ac-aC)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{128c^{3/2}}+\frac{5a^2x\sqrt{a+cx^2}\,(8Ac-aC)}{128c}+\frac{x(a+cx^2)^{5/2}(8Ac-aC)}{48c}+\frac{5ax(a+cx^2)^{3/2}(8Ac-aC)}{192c}+\frac{B(a+cx^2)^{7/2}}{7c}+\frac{Cx(a+cx^2)^{7/2}}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)*(A + B*x + C*x^2),x]
```

```
[Out] (5*a^2*(8*A*c - a*C)*x*Sqrt[a + c*x^2])/(128*c) + (5*a*(8*A*c - a*C)*x*(a +
 c*x^2)^(3/2))/(192*c) + ((8*A*c - a*C)*x*(a + c*x^2)^(5/2))/(48*c) + (B*(a
 + c*x^2)^(7/2))/(7*c) + (C*x*(a + c*x^2)^(7/2))/(8*c) + (5*a^3*(8*A*c - a*
C)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*c^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+cx^2\right)^{5/2}\left(A+Bx+Cx^2\right)\,dx &= \frac{Cx\left(a+cx^2\right)^{7/2}}{8c}+\frac{\int (8Ac-aC+8Bcx)\left(a+cx^2\right)^{5/2}\,dx}{8c}\\
&= \frac{B\left(a+cx^2\right)^{7/2}}{7c}+\frac{Cx\left(a+cx^2\right)^{7/2}}{8c}+\frac{(8Ac-aC)\int \left(a+cx^2\right)^{5/2}\,dx}{8c}\\
&= \frac{(8Ac-aC)x\left(a+cx^2\right)^{5/2}}{48c}+\frac{B\left(a+cx^2\right)^{7/2}}{7c}+\frac{Cx\left(a+cx^2\right)^{7/2}}{8c}+\frac{(5a(8
\\
&= \frac{5a(8Ac-aC)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{(8Ac-aC)x\left(a+cx^2\right)^{5/2}}{48c}+\frac{B(a+cx^2
}{7c}\\
&= \frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c}+\frac{5a(8Ac-aC)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{(8Ac-
\\
&= \frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c}+\frac{5a(8Ac-aC)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{(8Ac-
\\
&= \frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c}+\frac{5a(8Ac-aC)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{(8Ac-
\end{aligned}$$

Mathematica [A]
time = 0.52, size = 142, normalized size = 0.85

$$\frac{\sqrt{c}\,\sqrt{a+cx^2}\,(3a^3(128B+35Cx)+16c^3x^5(28A+3x(8B+7Cx))+8ac^2x^3(182A+x(144B+119Cx))+2a^2cx(924A+x(576B+413Cx)))+105a^3(-8Ac+aC)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{2688c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + c*x^2]*(3*a^3*(128*B + 35*C*x) + 16*c^3*x^5*(28*A + 3*x*(
8*B + 7*C*x)) + 8*a*c^2*x^3*(182*A + x*(144*B + 119*C*x)) + 2*a^2*c*x*(924*
A + x*(576*B + 413*C*x))) + 105*a^3*(-8*A*c + a*C)*Log[-(Sqrt[c]*x) + Sqrt[
a + c*x^2]])/(2688*c^(3/2))
```

Maple [A]

time = 0.09, size = 177, normalized size = 1.05

method	result
risch	$\frac{(336c^3Cx^7+384c^3Bx^6+448c^3Ax^5+952c^2aCx^5+1152c^2aBx^4+1456Aac^2x^3+826Ca^2cx^3+1152x^2a^2cB+1848a^2cAx+105a^3Cx+}{2688c}$
default	$C\left(\frac{x(cx^2+a)^{\frac{7}{2}}}{8c}-\frac{a\left(\frac{x(cx^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(cx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{cx^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{2\sqrt{c}}\right)}{4}\right)}{6}\right)}{8c}\right)+\frac{B(cx^2+}{7c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] C*(1/8*x*(c*x^2+a)^(7/2)/c-1/8*a/c*(1/6*x*(c*x^2+a)^(5/2)+5/6*a*(1/4*x*(c*x
^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+
a)^(1/2))))))+1/7*B*(c*x^2+a)^(7/2)/c+A*(1/6*x*(c*x^2+a)^(5/2)+5/6*a*(1/4*x
*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c
*x^2+a)^(1/2)))))
```

Maxima [A]

time = 0.29, size = 166, normalized size = 0.99

$$\frac{1}{6}\left(cx^2+a\right)^{\frac{5}{2}}Ax+\frac{5}{24}\left(cx^2+a\right)^{\frac{3}{2}}Aax+\frac{5}{16}\sqrt{cx^2+a}\,Aa^2x+\frac{(cx^2+a)^{\frac{7}{2}}Cx}{8\,c}-\frac{(cx^2+a)^{\frac{5}{2}}Cax}{48\,c}-\frac{5\,(cx^2+a)^{\frac{3}{2}}Ca^2x}{192\,c}-\frac{5\,\sqrt{cx^2+a}\,Ca^3x}{128\,c}-\frac{5\,Ca^4\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{128\,c^{\frac{3}{2}}}+\frac{5\,Aa^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{c}}+\frac{(cx^2+a)^{\frac{7}{2}}B}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2 + a)^(5/2)*A*x + 5/24*(c*x^2 + a)^(3/2)*A*a*x + 5/16*sqrt(c*x^2
+ a)*A*a^2*x + 1/8*(c*x^2 + a)^(7/2)*C*x/c - 1/48*(c*x^2 + a)^(5/2)*C*a*x/c
 - 5/192*(c*x^2 + a)^(3/2)*C*a^2*x/c - 5/128*sqrt(c*x^2 + a)*C*a^3*x/c - 5/
128*C*a^4*arcsinh(c*x/sqrt(a*c))/c^(3/2) + 5/16*A*a^3*arcsinh(c*x/sqrt(a*c)
)/sqrt(c) + 1/7*(c*x^2 + a)^(7/2)*B/c
```

Fricas [A]
time = 0.41, size = 333, normalized size = 1.98

$$\left[-\frac{105\,(Ca^4-8\,Aa^3c)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\,(336\,Cc^4x^7+384\,Bc^4x^6+1152\,Bac^3x^4+1152\,Ba^2c^2x^2+56\,(17\,Cac^3+8\,Ac^4)x^5+384\,Ba^3c+14\,(59\,Ca^2c^2+104\,Aac^3)x^3+21\,(5\,Ca^3c+88\,Aa^2c^2)x)\sqrt{cx^2+a}}{5376\,c^2},\ \frac{105\,(Ca^4-8\,Aa^3c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)+(336\,Cc^4x^7+384\,Bc^4x^6+1152\,Bac^3x^4+1152\,Ba^2c^2x^2+56\,(17\,Cac^3+8\,Ac^4)x^5+384\,Ba^3c+14\,(59\,Ca^2c^2+104\,Aac^3)x^3+21\,(5\,Ca^3c+88\,Aa^2c^2)x)\sqrt{cx^2+a}}{2688\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] [-1/5376*(105*(C*a^4 - 8*A*a^3*c)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*
sqrt(c)*x - a) - 2*(336*C*c^4*x^7 + 384*B*c^4*x^6 + 1152*B*a*c^3*x^4 + 1152
*B*a^2*c^2*x^2 + 56*(17*C*a*c^3 + 8*A*c^4)*x^5 + 384*B*a^3*c + 14*(59*C*a^2
*c^2 + 104*A*a*c^3)*x^3 + 21*(5*C*a^3*c + 88*A*a^2*c^2)*x)*sqrt(c*x^2 + a))
/c^2, 1/2688*(105*(C*a^4 - 8*A*a^3*c)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2
 + a)) + (336*C*c^4*x^7 + 384*B*c^4*x^6 + 1152*B*a*c^3*x^4 + 1152*B*a^2*c^2
*x^2 + 56*(17*C*a*c^3 + 8*A*c^4)*x^5 + 384*B*a^3*c + 14*(59*C*a^2*c^2 + 104
*A*a*c^3)*x^3 + 21*(5*C*a^3*c + 88*A*a^2*c^2)*x)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A]
time = 35.05, size = 510, normalized size = 3.04

$$\frac{Aa^{\frac{5}{2}}x\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{3Aa^{\frac{5}{2}}x}{16\sqrt{1+\frac{cx^2}{a}}}+\frac{35Aa^{\frac{3}{2}}cx^3}{48\sqrt{1+\frac{cx^2}{a}}}+\frac{17A\sqrt{a}\,c^2x^5}{24\sqrt{1+\frac{cx^2}{a}}}+\frac{5Aa^3\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16\sqrt{c}}+\frac{Ac^3x^7}{6\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}+Ba^2\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ c=0\\ \frac{(a+cx^2)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)+2Bac\left(\begin{cases}-\frac{2a^2\sqrt{a+cx^2}}{15c^2}+\frac{ax^2\sqrt{a+cx^2}}{15c}+\frac{x^4\sqrt{a+cx^2}}{5} & \text{for}\ c\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+Bc^2\left(\begin{cases}\frac{8a^3\sqrt{a+cx^2}}{105c^3}-\frac{4a^2x^2\sqrt{a+cx^2}}{105c^2}+\frac{ax^4\sqrt{a+cx^2}}{35c}+\frac{x^6\sqrt{a+cx^2}}{7} & \text{for}\ c\neq 0\\ \frac{\sqrt{a}\,x^6}{6} & \text{otherwise}\end{cases}\right)+\frac{5Ca^{\frac{7}{2}}x}{128c\sqrt{1+\frac{cx^2}{a}}}+\frac{133Ca^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{cx^2}{a}}}+\frac{127Ca^{\frac{3}{2}}cx^5}{192\sqrt{1+\frac{cx^2}{a}}}+\frac{23C\sqrt{a}\,c^2x^7}{48\sqrt{1+\frac{cx^2}{a}}}-\frac{5Ca^4\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{128c^{\frac{3}{2}}}+\frac{Cc^3x^9}{8\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)*(C*x**2+B*x+A),x)
```

```
[Out] A*a**(5/2)*x*sqrt(1 + c*x**2/a)/2 + 3*A*a**(5/2)*x/(16*sqrt(1 + c*x**2/a))
+ 35*A*a**(3/2)*c*x**3/(48*sqrt(1 + c*x**2/a)) + 17*A*sqrt(a)*c**2*x**5/(24
*sqrt(1 + c*x**2/a)) + 5*A*a**3*asinh(sqrt(c)*x/sqrt(a))/(16*sqrt(c)) + A*c
**3*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a)) + B*a**2*Piecewise((sqrt(a)*x**2/2,
 Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + 2*B*a*c*Piecewise((-2*a**2
*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a
+ c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + B*c**2*Piecewise((8*a**3*
sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x
**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*
x**6/6, True)) + 5*C*a**(7/2)*x/(128*c*sqrt(1 + c*x**2/a)) + 133*C*a**(5/2)
*x**3/(384*sqrt(1 + c*x**2/a)) + 127*C*a**(3/2)*c*x**5/(192*sqrt(1 + c*x**2
```

```
/a)) + 23*C*sqrt(a)*c**2*x**7/(48*sqrt(1 + c*x**2/a)) - 5*C*a**4*asinh(sqrt
(c)*x/sqrt(a))/(128*c**(3/2)) + C*c**3*x**9/(8*sqrt(a)*sqrt(1 + c*x**2/a))
```

Giac [A]
time = 6.37, size = 168, normalized size = 1.00

$$\frac{1}{2688}\left(\frac{384\,Ba^{3}}{c}+\left(2\left(576\,Ba^{2}+\left(4\left(144\,Bac+\left(6\left(7\,Cc^{2}x+8\,Bc^{2}\right)x+\frac{7\left(17\,Cac^{7}+8\,Ac^{8}\right)}{c^{6}}\right)x\right)x+\frac{7\left(59\,Ca^{2}c^{6}+104\,Aac^{7}\right)}{c^{6}}\right)x\right)x+\frac{21\left(5\,Ca^{3}c^{5}+88\,Aa^{2}c^{6}\right)}{c^{6}}\right)x\right)\sqrt{cx^{2}+a}+\frac{5\left(Ca^{4}-8\,Aa^{3}c\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^{2}+a}\right|\right)}{128\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/2688*(384*B*a^3/c + (2*(576*B*a^2 + (4*(144*B*a*c + (6*(7*C*c^2*x + 8*B*c
^2)*x + 7*(17*C*a*c^7 + 8*A*c^8)/c^6)*x)*x + 7*(59*C*a^2*c^6 + 104*A*a*c^7)
/c^6)*x)*x + 21*(5*C*a^3*c^5 + 88*A*a^2*c^6)/c^6)*x)*sqrt(c*x^2 + a) + 5/12
8*(C*a^4 - 8*A*a^3*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,x^2+a\right)^{5/2}\,\left(C\,x^2+B\,x+A\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(5/2)*(A + B*x + C*x^2),x)
```

```
[Out] int((a + c*x^2)^(5/2)*(A + B*x + C*x^2), x)
```

3.101 $\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=325

$$\frac{(4(5cd-4af)h^2-3cg(fg-5eh))\,(g+hx)^2\sqrt{a+cx^2}}{60c^2h}-\frac{(fg-5eh)(g+hx)^3\sqrt{a+cx^2}}{20ch}+\frac{f(g+hx)^4\sqrt{a+}}{5ch}$$

```
[Out] 1/8*(8*c^2*d*g^3+3*a^2*h^2*(e*h+3*f*g)-4*a*c*g*(f*g^2+3*h*(d*h+e*g)))*arcta
nh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(5/2)+1/60*(4*(-4*a*f+5*c*d)*h^2-3*c*g*(-5*
e*h+f*g))*(h*x+g)^2*(c*x^2+a)^(1/2)/c^2/h-1/20*(-5*e*h+f*g)*(h*x+g)^3*(c*x^
2+a)^(1/2)/c/h+1/5*f*(h*x+g)^4*(c*x^2+a)^(1/2)/c/h+1/120*(64*a^2*f*h^4-16*a
*c*h^2*(13*f*g^2+5*h*(d*h+3*e*g))-4*c^2*g^2*(3*f*g^2-5*h*(16*d*h+3*e*g))-c*
h*(a*h^2*(45*e*h+71*f*g)+2*c*g*(3*f*g^2-5*h*(10*d*h+3*e*g)))*x)*(c*x^2+a)^(
1/2)/c^3/h
```

Rubi [A]

time = 0.39, antiderivative size = 323, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1668, 847, 794, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(3a^2h^2(eh+3fg)-4acg(3h(dh+eg)+fg^2)+8c^2dg^3)}{8c^{5/2}}+\frac{\sqrt{a+cx^2}\,(4(16a^2fh^4-4ach^2(5h(dh+3eg)+13fg^2)-c^2g^2(3fg^2-5h(16dh+3eg)))-chx(ah^2(45eh+71fg)-10cgh(10dh+3eg)+6cfg^3))}{120c^3h}+\frac{\sqrt{a+cx^2}\,(g+hx)^2\,(4h^2(5cd-4af)-3cg(fg-5eh))}{60c^2h}-\frac{\sqrt{a+cx^2}\,(g+hx)^3(fg-5eh)}{20ch}+\frac{f\sqrt{a+cx^2}\,(g+hx)^4}{5ch}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] ((4*(5*c*d - 4*a*f)*h^2 - 3*c*g*(f*g - 5*e*h))*(g + h*x)^2*Sqrt[a + c*x^2])
/(60*c^2*h) - ((f*g - 5*e*h)*(g + h*x)^3*Sqrt[a + c*x^2])/(20*c*h) + (f*(g
+ h*x)^4*Sqrt[a + c*x^2])/(5*c*h) + ((4*(16*a^2*f*h^4 - 4*a*c*h^2*(13*f*g^2
 + 5*h*(3*e*g + d*h)) - c^2*g^2*(3*f*g^2 - 5*h*(3*e*g + 16*d*h))) - c*h*(6*
c*f*g^3 - 10*c*g*h*(3*e*g + 10*d*h) + a*h^2*(71*f*g + 45*e*h))*x)*Sqrt[a +
c*x^2])/(120*c^3*h) + ((8*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 4*a*c*g*(f*
g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx = \frac{f(g+hx)^4\sqrt{a+cx^2}}{5ch} + \frac{\int \frac{(g+hx)^3\left((5cd-4af)h^2-ch(fg-5eh)x\right)}{\sqrt{a+cx^2}}\,dx}{5ch^2}$$

$$= -\frac{(fg-5eh)(g+hx)^3\sqrt{a+cx^2}}{20ch} + \frac{f(g+hx)^4\sqrt{a+cx^2}}{5ch} + \frac{\int \frac{(g+hx)^2\left(ch^2(\right.}{}}{}$$

$$= \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h} - \frac{(fg-5eh)(g+}{20}$$

$$= \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h} - \frac{(fg-5eh)(g+}{20}$$

$$= \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h} - \frac{(fg-5eh)(g+}{20}$$

$$= \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h} - \frac{(fg-5eh)(g+}{20}$$

Mathematica [A]
time = 0.79, size = 244, normalized size = 0.75

$$\frac{\sqrt{a+cx^2}\left(64a^2fh^3-ach(5h(48eg+16dh+9ehx)+f(240g^2+135ghx+32h^2x^2))+2c^2(10dh(18g^2+9ghx+2h^2x^2)+15e(4g^3+6g^2hx+4gh^2x^2+h^3x^3)+3fx(10g^3+20g^2hx+15gh^2x^2+4h^3x^3))\right)-15\sqrt{c}\left(8c^2dg^3+3a^2h^2(3fg+eh)-4acg(fg^2+3h(eg+dh))\right)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{120c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[a + c*x^2]*(64*a^2*f*h^3 - a*c*h*(5*h*(48*e*g + 16*d*h + 9*e*h*x) + f
*(240*g^2 + 135*g*h*x + 32*h^2*x^2)) + 2*c^2*(10*d*h*(18*g^2 + 9*g*h*x + 2*
h^2*x^2) + 15*e*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3) + 3*f*x*(10*g^3
 + 20*g^2*h*x + 15*g*h^2*x^2 + 4*h^3*x^3))) - 15*Sqrt[c]*(8*c^2*d*g^3 + 3*a
^2*h^2*(3*f*g + e*h) - 4*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*Log[-(Sqrt[c]*x)
+ Sqrt[a + c*x^2]])/(120*c^3)
```

Maple [A]
time = 0.09, size = 303, normalized size = 0.93

method	result

default	$f\,h^3\left(\frac{x^4\sqrt{c\,x^2+a}}{5c}-\frac{4a\left(\frac{x^2\sqrt{c\,x^2+a}}{3c}-\frac{2a\sqrt{c\,x^2+a}}{3c^2}\right)}{5c}\right)+(e\,h^3+3fg\,h^2)\left(\frac{x^3\sqrt{c\,x^2+a}}{4c}-\frac{3a\left(\right.}{}\right.$
risch	$\frac{(24f\,h^3c^2x^4+30c^2e\,h^3x^3+90c^2fg\,h^2x^3-32acf\,h^3x^2+40c^2d\,h^3x^2+120c^2eg\,h^2x^2+120c^2f\,g^2h\,x^2-45ace\,h^3x-135acfg\,h^2x+180c^2d}{120c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*h^3*(1/5*x^4/c*(c*x^2+a)^(1/2)-4/5*a/c*(1/3*x^2/c*(c*x^2+a)^(1/2)-2/3*a/c
^2*(c*x^2+a)^(1/2)))+(e*h^3+3*f*g*h^2)*(1/4*x^3/c*(c*x^2+a)^(1/2)-3/4*a/c*(
1/2*x/c*(c*x^2+a)^(1/2)-1/2*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))))+(d*h^
3+3*e*g*h^2+3*f*g^2*h)*(1/3*x^2/c*(c*x^2+a)^(1/2)-2/3*a/c^2*(c*x^2+a)^(1/2)
)+(3*d*g*h^2+3*e*g^2*h+f*g^3)*(1/2*x/c*(c*x^2+a)^(1/2)-1/2*a/c^(3/2)*ln(x*c
^(1/2)+(c*x^2+a)^(1/2)))+(3*d*g^2*h+e*g^3)/c*(c*x^2+a)^(1/2)+d*g^3*ln(x*c^(
1/2)+(c*x^2+a)^(1/2))/c^(1/2)
```

Maxima [A]
time = 0.30, size = 357, normalized size = 1.10

$$\frac{\sqrt{cx^2+a}\,fh^3x^4}{5\,c}-\frac{4\,\sqrt{cx^2+a}\,afh^3x^2}{15\,c^2}+\frac{dg^3\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}+\frac{3\,\sqrt{cx^2+a}\,dg^2h}{c}+\frac{8\,\sqrt{cx^2+a}\,a^2fh^3}{15\,c^3}+\frac{(3\,fgh^2+h^3e)\sqrt{cx^2+a}\,x^3}{4\,c}+\frac{\sqrt{cx^2+a}\,g^3e}{c}+\frac{(3\,fg^2h+dh^3+3\,gh^2e)\sqrt{cx^2+a}\,x^2}{3\,c}-\frac{3\,(3\,fgh^2+h^3e)\sqrt{cx^2+a}\,ax}{8\,c^2}+\frac{(fg^3+3\,dgh^2+3\,g^2he)\sqrt{cx^2+a}\,x}{2\,c}+\frac{3\,(3\,fgh^2+h^3e)a^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{5}{2}}}-\frac{(fg^3+3\,dgh^2+3\,g^2he)a\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}}-\frac{2\,(3\,fg^2h+dh^3+3\,gh^2e)\sqrt{cx^2+a}\,a}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(c*x^2 + a)*f*h^3*x^4/c - 4/15*sqrt(c*x^2 + a)*a*f*h^3*x^2/c^2 + d*
g^3*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 3*sqrt(c*x^2 + a)*d*g^2*h/c + 8/15*sqr
t(c*x^2 + a)*a^2*f*h^3/c^3 + 1/4*(3*f*g*h^2 + h^3*e)*sqrt(c*x^2 + a)*x^3/c
+ sqrt(c*x^2 + a)*g^3*e/c + 1/3*(3*f*g^2*h + d*h^3 + 3*g*h^2*e)*sqrt(c*x^2
+ a)*x^2/c - 3/8*(3*f*g*h^2 + h^3*e)*sqrt(c*x^2 + a)*a*x/c^2 + 1/2*(f*g^3 +
 3*d*g*h^2 + 3*g^2*h*e)*sqrt(c*x^2 + a)*x/c + 3/8*(3*f*g*h^2 + h^3*e)*a^2*a
rcsinh(c*x/sqrt(a*c))/c^(5/2) - 1/2*(f*g^3 + 3*d*g*h^2 + 3*g^2*h*e)*a*arcsi
nh(c*x/sqrt(a*c))/c^(3/2) - 2/3*(3*f*g^2*h + d*h^3 + 3*g*h^2*e)*sqrt(c*x^2
+ a)*a/c^2
```

Fricas [A]
time = 0.58, size = 580, normalized size = 1.78

$$\left[\frac{15\,(4\,(2\,c^2d-acf)g^3-3\,(4\,acd-3\,a^2f)gh^2-3\,(4\,acg^2h-a^2h^3)e)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2\,(24\,c^2fh^3x^4+90\,c^2fgh^2x^3+120\,(3\,c^2d-2\,acf)g^2h-16\,(5\,acd-4\,a^2f)h^3+8\,(15\,c^2fg^2h+(5\,c^2d-4\,acf)h^3)x^2+15\,(4\,c^2fg^3+3\,(4\,c^2d-3\,acf)gh^2)x+15\,(2\,c^2h^3x^3+8\,c^2gh^2x^2+8\,c^2g^3-16\,acgh^2+3\,(4\,c^2g^2h-ach^3)x)e)\sqrt{cx^2+a}}{240\,c^3},\ -\frac{15\,(4\,(2\,c^2d-acf)g^3-3\,(4\,acd-3\,a^2f)gh^2-3\,(4\,acg^2h-a^2h^3)e)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)-(24\,c^2fh^3x^4+90\,c^2fgh^2x^3+120\,(3\,c^2d-2\,acf)g^2h-16\,(5\,acd-4\,a^2f)h^3+8\,(15\,c^2fg^2h+(5\,c^2d-4\,acf)h^3)x^2+15\,(4\,c^2fg^3+3\,(4\,c^2d-3\,acf)gh^2)x+15\,(2\,c^2h^3x^3+8\,c^2gh^2x^2+8\,c^2g^3-16\,acgh^2+3\,(4\,c^2g^2h-ach^3)x)e)\sqrt{cx^2+a}}{120\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*(4*(2*c^2*d - a*c*f)*g^3 - 3*(4*a*c*d - 3*a^2*f)*g*h^2 - 3*(4*a*
c*g^2*h - a^2*h^3)*e)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x -
a) + 2*(24*c^2*f*h^3*x^4 + 90*c^2*f*g*h^2*x^3 + 120*(3*c^2*d - 2*a*c*f)*g^2
*h - 16*(5*a*c*d - 4*a^2*f)*h^3 + 8*(15*c^2*f*g^2*h + (5*c^2*d - 4*a*c*f)*h
^3)*x^2 + 15*(4*c^2*f*g^3 + 3*(4*c^2*d - 3*a*c*f)*g*h^2)*x + 15*(2*c^2*h^3*
x^3 + 8*c^2*g*h^2*x^2 + 8*c^2*g^3 - 16*a*c*g*h^2 + 3*(4*c^2*g^2*h - a*c*h^3
)*x)*e)*sqrt(c*x^2 + a))/c^3, -1/120*(15*(4*(2*c^2*d - a*c*f)*g^3 - 3*(4*a*
c*d - 3*a^2*f)*g*h^2 - 3*(4*a*c*g^2*h - a^2*h^3)*e)*sqrt(-c)*arctan(sqrt(-c
)*x/sqrt(c*x^2 + a)) - (24*c^2*f*h^3*x^4 + 90*c^2*f*g*h^2*x^3 + 120*(3*c^2*
d - 2*a*c*f)*g^2*h - 16*(5*a*c*d - 4*a^2*f)*h^3 + 8*(15*c^2*f*g^2*h + (5*c^
2*d - 4*a*c*f)*h^3)*x^2 + 15*(4*c^2*f*g^3 + 3*(4*c^2*d - 3*a*c*f)*g*h^2)*x
+ 15*(2*c^2*h^3*x^3 + 8*c^2*g*h^2*x^2 + 8*c^2*g^3 - 16*a*c*g*h^2 + 3*(4*c^2
*g^2*h - a*c*h^3)*x)*e)*sqrt(c*x^2 + a))/c^3]
```

Sympy [A]
time = 10.86, size = 796, normalized size = 2.45

$$- \frac{3 a^{\frac{3}{2}} e h^{3} x}{8 c^{2} \sqrt{1 + \frac{c x^{2}}{a}}} - \frac{9 a^{\frac{3}{2}} f g h^{2} x}{8 c^{2} \sqrt{1 + \frac{c x^{2}}{a}}} + \frac{3 \sqrt{a} d g h^{2} x \sqrt{1 + \frac{c x^{2}}{a}}}{2 c} + \frac{3 \sqrt{a} e g^{2} h x \sqrt{1 + \frac{c x^{2}}{a}}}{2 c} - \frac{\sqrt{a} e h^{3} x^{3}}{8 c \sqrt{1 + \frac{c x^{2}}{a}}} + \frac{\sqrt{a} f g^{3} x \sqrt{1 + \frac{c x^{2}}{a}}}{2 c} - \frac{3 \sqrt{a} f g h^{2} x^{3}}{8 c \sqrt{1 + \frac{c x^{2}}{a}}} + \frac{3 a^{2} e h^{3} \operatorname{asinh}{\left(\frac{\sqrt{c} x}{\sqrt{a}} \right)}}{8 c^{\frac{5}{2}}} + \frac{9 a^{2} f g h^{2} \operatorname{asinh}{\left(\frac{\sqrt{c} x}{\sqrt{a}} \right)}}{8 c^{\frac{5}{2}}} - \frac{3 a d g h^{2} \operatorname{asinh}{\left(\frac{\sqrt{c} x}{\sqrt{a}} \right)}}{2 c^{\frac{3}{2}}} - \frac{3 a e g^{2} h \operatorname{asinh}{\left(\frac{\sqrt{c} x}{\sqrt{a}} \right)}}{2 c^{\frac{3}{2}}} - \frac{a f g^{3} \operatorname{asinh}{\left(\frac{\sqrt{c} x}{\sqrt{a}} \right)}}{2 c^{\frac{3}{2}}} + d g^{3} \left(\begin{cases} \frac{\sqrt{- \frac{a}{c}} \operatorname{asin}{\left(x \sqrt{- \frac{c}{a}} \right)}}{\sqrt{a}} & \text{for}\: a > 0 \wedge c < 0 \\ \frac{\sqrt{\frac{a}{c}} \operatorname{asinh}{\left(x \sqrt{\frac{c}{a}} \right)}}{\sqrt{a}} & \text{for}\: a > 0 \wedge c > 0 \\ \frac{\sqrt{- \frac{a}{c}} \operatorname{acosh}{\left(x \sqrt{- \frac{c}{a}} \right)}}{\sqrt{- a}} & \text{for}\: c > 0 \wedge a < 0 \end{cases}\right) + 3 d g^{2} h \left(\begin{cases} \frac{x^{2}}{2 \sqrt{a}} & \text{for}\: c = 0 \\ \frac{\sqrt{a + c x^{2}}}{c} & \text{otherwise} \end{cases}\right) + d h^{3} \left(\begin{cases} - \frac{2 a \sqrt{a + c x^{2}}}{3 c^{2}} + \frac{x^{2} \sqrt{a + c x^{2}}}{3 c} & \text{for}\: c \neq 0 \\ \frac{x^{4}}{4 \sqrt{a}} & \text{otherwise} \end{cases}\right) + e g^{3} \left(\begin{cases} \frac{x^{2}}{2 \sqrt{a}} & \text{for}\: c = 0 \\ \frac{\sqrt{a + c x^{2}}}{c} & \text{otherwise} \end{cases}\right) + 3 e g h^{2} \left(\begin{cases} - \frac{2 a \sqrt{a + c x^{2}}}{3 c^{2}} + \frac{x^{2} \sqrt{a + c x^{2}}}{3 c} & \text{for}\: c \neq 0 \\ \frac{x^{4}}{4 \sqrt{a}} & \text{otherwise} \end{cases}\right) + 3 f g^{2} h \left(\begin{cases} - \frac{2 a \sqrt{a + c x^{2}}}{3 c^{2}} + \frac{x^{2} \sqrt{a + c x^{2}}}{3 c} & \text{for}\: c \neq 0 \\ \frac{x^{4}}{4 \sqrt{a}} & \text{otherwise} \end{cases}\right) + f h^{3} \left(\begin{cases} \frac{8 a^{2} \sqrt{a + c x^{2}}}{15 c^{3}} - \frac{4 a x^{2} \sqrt{a + c x^{2}}}{15 c^{2}} + \frac{x^{4} \sqrt{a + c x^{2}}}{5 c} & \text{for}\: c \neq 0 \\ \frac{x^{6}}{6 \sqrt{a}} & \text{otherwise} \end{cases}\right) + \frac{e h^{3} x^{5}}{4 \sqrt{a} \sqrt{1 + \frac{c x^{2}}{a}}} + \frac{3 f g h^{2} x^{5}}{4 \sqrt{a} \sqrt{1 + \frac{c x^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*e*h**3*x/(8*c**2*sqrt(1 + c*x**2/a)) - 9*a**(3/2)*f*g*h**2*x/(8
*c**2*sqrt(1 + c*x**2/a)) + 3*sqrt(a)*d*g*h**2*x*sqrt(1 + c*x**2/a)/(2*c) +
 3*sqrt(a)*e*g**2*h*x*sqrt(1 + c*x**2/a)/(2*c) - sqrt(a)*e*h**3*x**3/(8*c*s
qrt(1 + c*x**2/a)) + sqrt(a)*f*g**3*x*sqrt(1 + c*x**2/a)/(2*c) - 3*sqrt(a)*
f*g*h**2*x**3/(8*c*sqrt(1 + c*x**2/a)) + 3*a**2*e*h**3*asinh(sqrt(c)*x/sqrt
(a))/(8*c**(5/2)) + 9*a**2*f*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(5/2)) -
 3*a*d*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(2*c**(3/2)) - 3*a*e*g**2*h*asinh(sq
rt(c)*x/sqrt(a))/(2*c**(3/2)) - a*f*g**3*asinh(sqrt(c)*x/sqrt(a))/(2*c**(3/
2)) + d*g**3*Piecewise((sqrt(-a/c)*asin(x*sqrt(-c/a))/sqrt(a), (a > 0) & (c
 < 0)), (sqrt(a/c)*asinh(x*sqrt(c/a))/sqrt(a), (a > 0) & (c > 0)), (sqrt(-a
/c)*acosh(x*sqrt(-c/a))/sqrt(-a), (c > 0) & (a < 0))) + 3*d*g**2*h*Piecewis
e((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True)) + d*h**3*Piecew
ise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c), Ne(c, 0)
), (x**4/(4*sqrt(a)), True)) + e*g**3*Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)
), (sqrt(a + c*x**2)/c, True)) + 3*e*g*h**2*Piecewise((-2*a*sqrt(a + c*x**2
)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c), Ne(c, 0)), (x**4/(4*sqrt(a)), Tru
e)) + 3*f*g**2*h*Piecewise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a +
c*x**2)/(3*c), Ne(c, 0)), (x**4/(4*sqrt(a)), True)) + f*h**3*Piecewise((8*a
**2*sqrt(a + c*x**2)/(15*c**3) - 4*a*x**2*sqrt(a + c*x**2)/(15*c**2) + x**4
*sqrt(a + c*x**2)/(5*c), Ne(c, 0)), (x**6/(6*sqrt(a)), True)) + e*h**3*x**5
/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + 3*f*g*h**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**
2/a))
```

Giac [A]
time = 6.66, size = 314, normalized size = 0.97

$$\frac{1}{120} \sqrt{c x^2 + a} \left(\left(2 \left(3 \left(\frac{4 f h^3 x}{c} + \frac{5 (3 c^4 f g h^2 + c^4 h^3 e)}{c^5}\right) x + \frac{4 (15 c^4 f g^2 h + 5 c^4 d h^3 - 4 a c^3 f h^3 + 15 c^4 g h^2 e)}{c^5}\right) x + \frac{15 (4 c^4 f g^3 + 12 c^4 d g h^2 - 9 a c^3 f g h^2 + 12 c^4 g^2 h e - 3 a c^3 h^3 e)}{c^5}\right) x + \frac{8 (45 c^4 d g^2 h - 30 a c^3 f g^2 h - 10 a c^3 d h^3 + 8 a^2 c^2 f h^3 + 15 c^4 g^3 e - 30 a c^3 g h^2 e)}{c^5}\right) - \frac{(8 c^2 d g^3 - 4 a c f g^3 - 12 a c d g h^2 + 9 a^2 f g h^2 - 12 a c g^2 h e + 3 a^2 h^3 e) \log\left(\left|-\sqrt{c} x + \sqrt{c x^2 + a}\right|\right)}{8 c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(c*x^2 + a)*((2*(3*(4*f*h^3*x/c + 5*(3*c^4*f*g*h^2 + c^4*h^3*e)/c
^5)*x + 4*(15*c^4*f*g^2*h + 5*c^4*d*h^3 - 4*a*c^3*f*h^3 + 15*c^4*g*h^2*e)/c
^5)*x + 15*(4*c^4*f*g^3 + 12*c^4*d*g*h^2 - 9*a*c^3*f*g*h^2 + 12*c^4*g^2*h*e
 - 3*a*c^3*h^3*e)/c^5)*x + 8*(45*c^4*d*g^2*h - 30*a*c^3*f*g^2*h - 10*a*c^3*
d*h^3 + 8*a^2*c^2*f*h^3 + 15*c^4*g^3*e - 30*a*c^3*g*h^2*e)/c^5) - 1/8*(8*c^
2*d*g^3 - 4*a*c*f*g^3 - 12*a*c*d*g*h^2 + 9*a^2*f*g*h^2 - 12*a*c*g^2*h*e + 3
*a^2*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g + h\,x)^3\,(f\,x^2 + e\,x + d)}{\sqrt{c\,x^2 + a}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(1/2),x)
```

```
[Out] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(1/2), x)
```

3.102 $\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=223

$$-\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch}+\frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch}-\frac{(4(4ah^2(2fg+eh)+cg(fg^2-4h(eg+3dh)))-}{24c}$$

```
[Out] 1/8*(8*c^2*d*g^2+3*a^2*f*h^2-4*a*c*(f*g^2+h*(d*h+2*e*g)))*arctanh(x*c^(1/2)
/(c*x^2+a)^(1/2))/c^(5/2)-1/12*(-4*e*h+f*g)*(h*x+g)^2*(c*x^2+a)^(1/2)/c/h+1
/4*f*(h*x+g)^3*(c*x^2+a)^(1/2)/c/h-1/24*(16*a*h^2*(e*h+2*f*g)+4*c*g*(f*g^2-
4*h*(3*d*h+e*g))-h*(3*(-3*a*f+4*c*d)*h^2-2*c*g*(-4*e*h+f*g))*x)*(c*x^2+a)^(
1/2)/c^2/h
```

Rubi [A]
time = 0.22, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1668, 847, 794, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(3a^2fh^2-4ac(h(dh+2eg)+fg^2)+8c^2dg^2)}{8c^{5/2}}-\frac{\sqrt{a+cx^2}\,(4(4ah^2(eh+2fg)-4cgh(3dh+eg)+cfg^3)-hx(3h^2(4cd-3af)-2cg(fg-4eh)))}{24c^2h}-\frac{\sqrt{a+cx^2}\,(g+hx)^2(fg-4eh)}{12ch}+\frac{f\sqrt{a+cx^2}\,(g+hx)^3}{4ch}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] -1/12*((f*g - 4*e*h)*(g + h*x)^2*Sqrt[a + c*x^2])/(c*h) + (f*(g + h*x)^3*Sq
rt[a + c*x^2])/(4*c*h) - ((4*(c*f*g^3 - 4*c*g*h*(e*g + 3*d*h) + 4*a*h^2*(2*
f*g + e*h)) - h*(3*(4*c*d - 3*a*f)*h^2 - 2*c*g*(f*g - 4*e*h))*x)*Sqrt[a + c
*x^2])/(24*c^2*h) + ((8*c^2*d*g^2 + 3*a^2*f*h^2 - 4*a*c*(f*g^2 + h*(2*e*g +
 d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
```

```
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx = \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} + \frac{\int \frac{(g+hx)^2\left((4cd-3af)h^2-ch(fg-4eh)x\right)}{\sqrt{a+cx^2}}\,dx}{4ch^2}$$

$$= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} + \frac{\int \frac{(g+hx)(ch^2(\cdots}{\cdots}}{\cdots}$$

$$= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} - \frac{(4(cfg^3-4\cdots}{\cdots}$$

$$= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} - \frac{(4(cfg^3-4\cdots}{\cdots}$$

$$= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} - \frac{(4(cfg^3-4\cdots}{\cdots}$$

Mathematica [A]
time = 0.55, size = 165, normalized size = 0.74

$$\frac{\sqrt{a+cx^2}\left(-ah(32fg+16eh+9fhx)+2c(6dh(4g+hx)+4e(3g^2+3ghx+h^2x^2)+fx(6g^2+8ghx+3h^2x^2))\right)}{24c^2}-\frac{(8c^2dg^2+3a^2fh^2-4ac(fg^2+h(2eg+dh)))\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{8c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[a + c*x^2]*(-(a*h*(32*f*g + 16*e*h + 9*f*h*x)) + 2*c*(6*d*h*(4*g + h*
x) + 4*e*(3*g^2 + 3*g*h*x + h^2*x^2) + f*x*(6*g^2 + 8*g*h*x + 3*h^2*x^2))))
/(24*c^2) - ((8*c^2*d*g^2 + 3*a^2*f*h^2 - 4*a*c*(f*g^2 + h*(2*e*g + d*h)))*
Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(8*c^(5/2))
```

Maple [A]
time = 0.10, size = 217, normalized size = 0.97

method	result
default	$f\,h^2\left(\frac{x^3\sqrt{c\,x^2+a}}{4c}-\frac{3a\left(\frac{x\sqrt{c\,x^2+a}}{2c}-\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}\right)+(e\,h^2+2fgh)\left(\frac{x^2\sqrt{c\,x^2+a}}{3c}\right.$
risch	$-\frac{\left(-6f\,h^2c\,x^3-8ce\,h^2x^2-16cfgh\,x^2+9xaf\,h^2-12d\,h^2xc-24eghxc-12f\,g^2xc+16ae\,h^2+32afgh-48cdgh-24ce\,g^2\right)\sqrt{c\,x^2+a}}{24c^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*h^2*(1/4*x^3/c*(c*x^2+a)^(1/2)-3/4*a/c*(1/2*x/c*(c*x^2+a)^(1/2)-1/2*a/c^(
3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))))+(e*h^2+2*f*g*h)*(1/3*x^2/c*(c*x^2+a)^(
1/2)-2/3*a/c^2*(c*x^2+a)^(1/2))+(d*h^2+2*e*g*h+f*g^2)*(1/2*x/c*(c*x^2+a)^(1
/2)-1/2*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))+(2*d*g*h+e*g^2)/c*(c*x^2+a
)^(1/2)+d*g^2*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)
```

Maxima [A]
time = 0.28, size = 235, normalized size = 1.05

$$\frac{\sqrt{cx^2+a}\,fh^2x^3}{4\,c}-\frac{3\,\sqrt{cx^2+a}\,afh^2x}{8\,c^2}+\frac{dg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}+\frac{3\,a^2fh^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{5}{2}}}+\frac{2\,\sqrt{cx^2+a}\,dgh}{c}+\frac{(2\,fgh+h^2e)\sqrt{cx^2+a}\,x^2}{3\,c}+\frac{\sqrt{cx^2+a}\,g^2e}{c}+\frac{(fg^2+dh^2+2\,ghe)\sqrt{cx^2+a}\,x}{2\,c}-\frac{(fg^2+dh^2+2\,ghe)a\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}}-\frac{2\,(2\,fgh+h^2e)\sqrt{cx^2+a}\,a}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c*x^2 + a)*f*h^2*x^3/c - 3/8*sqrt(c*x^2 + a)*a*f*h^2*x/c^2 + d*g^2
*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 3/8*a^2*f*h^2*arcsinh(c*x/sqrt(a*c))/c^(5
```

```
/2) + 2*sqrt(c*x^2 + a)*d*g*h/c + 1/3*(2*f*g*h + h^2*e)*sqrt(c*x^2 + a)*x^2
/c + sqrt(c*x^2 + a)*g^2*e/c + 1/2*(f*g^2 + d*h^2 + 2*g*h*e)*sqrt(c*x^2 + a
)*x/c - 1/2*(f*g^2 + d*h^2 + 2*g*h*e)*a*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/
3*(2*f*g*h + h^2*e)*sqrt(c*x^2 + a)*a/c^2
```

Fricas [A]
time = 0.44, size = 387, normalized size = 1.74

$$\left[\frac{3\,(8\,acghe - 4\,(2\,c^2d - acf)g^2 + (4\,acd - 3\,a^2f)h^2)\sqrt{c}\,\log\left(-2\,cx^2 + 2\,\sqrt{cx^2+a}\,\sqrt{c}\,x - a\right) + 2\,(6\,c^2fh^2x^3 + 16\,c^2fghx^2 + 16\,(3\,c^2d - 2\,acf)gh + 3\,(4\,c^2fg^2 + (4\,c^2d - 3\,acf)h^2)x + 8\,(c^2h^2x^2 + 3\,c^2ghx + 3\,c^2g^2 - 2\,ach^2)e)\sqrt{cx^2+a}}{48\,c^3}, \frac{3\,(8\,acghe - 4\,(2\,c^2d - acf)g^2 + (4\,acd - 3\,a^2f)h^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right) + (6\,c^2fh^2x^3 + 16\,c^2fghx^2 + 16\,(3\,c^2d - 2\,acf)gh + 3\,(4\,c^2fg^2 + (4\,c^2d - 3\,acf)h^2)x + 8\,(c^2h^2x^2 + 3\,c^2ghx + 3\,c^2g^2 - 2\,ach^2)e)\sqrt{cx^2+a}}{24\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(8*a*c*g*h*e - 4*(2*c^2*d - a*c*f)*g^2 + (4*a*c*d - 3*a^2*f)*h^2)*
sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(6*c^2*f*h^2*x^
3 + 16*c^2*f*g*h*x^2 + 16*(3*c^2*d - 2*a*c*f)*g*h + 3*(4*c^2*f*g^2 + (4*c^2
*d - 3*a*c*f)*h^2)*x + 8*(c^2*h^2*x^2 + 3*c^2*g*h*x + 3*c^2*g^2 - 2*a*c*h^2
)*e)*sqrt(c*x^2 + a))/c^3, 1/24*(3*(8*a*c*g*h*e - 4*(2*c^2*d - a*c*f)*g^2 +
 (4*a*c*d - 3*a^2*f)*h^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (6*
c^2*f*h^2*x^3 + 16*c^2*f*g*h*x^2 + 16*(3*c^2*d - 2*a*c*f)*g*h + 3*(4*c^2*f*
g^2 + (4*c^2*d - 3*a*c*f)*h^2)*x + 8*(c^2*h^2*x^2 + 3*c^2*g*h*x + 3*c^2*g^2
 - 2*a*c*h^2)*e)*sqrt(c*x^2 + a))/c^3]
```

Sympy [A]
time = 7.65, size = 518, normalized size = 2.32

$$-\frac{3a^{\frac{3}{2}}fh^2x}{8c^2\sqrt{1+\frac{cx^2}{a}}} + \frac{\sqrt{a}\,dh^2x\sqrt{1+\frac{cx^2}{a}}}{2c} + \frac{\sqrt{a}\,eghx\sqrt{1+\frac{cx^2}{a}}}{c} + \frac{\sqrt{a}\,fg^2x\sqrt{1+\frac{cx^2}{a}}}{2c} - \frac{\sqrt{a}\,fh^2x^3}{8c\sqrt{1+\frac{cx^2}{a}}} + \frac{3a^2fh^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8c^{\frac{5}{2}}} - \frac{adh^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}} - \frac{aegh\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}} - \frac{afg^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}} + dg^2\left(\begin{cases}\frac{\sqrt{-\frac{a}{c}}\operatorname{asin}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0 \wedge c<0\\ \frac{\sqrt{\frac{a}{c}}\operatorname{asinh}\left(x\sqrt{\frac{c}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0 \wedge c>0\\ \frac{\sqrt{-\frac{a}{c}}\operatorname{acosh}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{-a}} & \text{for}\ c>0 \wedge a<0\end{cases}\right) + 2dgh\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for}\ c=0\\ \frac{\sqrt{a+cx^2}}{c} & \text{otherwise}\end{cases}\right) + eg^2\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for}\ c=0\\ \frac{\sqrt{a+cx^2}}{c} & \text{otherwise}\end{cases}\right) + eh^2\left(\begin{cases}-\frac{2a\sqrt{a+cx^2}}{3c^2} + \frac{x^2\sqrt{a+cx^2}}{3c} & \text{for}\ c\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise}\end{cases}\right) + 2fgh\left(\begin{cases}-\frac{2a\sqrt{a+cx^2}}{3c^2} + \frac{x^2\sqrt{a+cx^2}}{3c} & \text{for}\ c\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise}\end{cases}\right) + \frac{fh^2x^5}{4\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*f*h**2*x/(8*c**2*sqrt(1 + c*x**2/a)) + sqrt(a)*d*h**2*x*sqrt(1
+ c*x**2/a)/(2*c) + sqrt(a)*e*g*h*x*sqrt(1 + c*x**2/a)/c + sqrt(a)*f*g**2*x
*sqrt(1 + c*x**2/a)/(2*c) - sqrt(a)*f*h**2*x**3/(8*c*sqrt(1 + c*x**2/a)) +
3*a**2*f*h**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(5/2)) - a*d*h**2*asinh(sqrt(c
)*x/sqrt(a))/(2*c**(3/2)) - a*e*g*h*asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - a*f
*g**2*asinh(sqrt(c)*x/sqrt(a))/(2*c**(3/2)) + d*g**2*Piecewise((sqrt(-a/c)*
asin(x*sqrt(-c/a))/sqrt(a), (a > 0) & (c < 0)), (sqrt(a/c)*asinh(x*sqrt(c/a
))/sqrt(a), (a > 0) & (c > 0)), (sqrt(-a/c)*acosh(x*sqrt(-c/a))/sqrt(-a), (
c > 0) & (a < 0))) + 2*d*g*h*Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(
a + c*x**2)/c, True)) + e*g**2*Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqr
t(a + c*x**2)/c, True)) + e*h**2*Piecewise((-2*a*sqrt(a + c*x**2)/(3*c**2)
+ x**2*sqrt(a + c*x**2)/(3*c), Ne(c, 0)), (x**4/(4*sqrt(a)), True)) + 2*f*g
*h*Piecewise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c),
 Ne(c, 0)), (x**4/(4*sqrt(a)), True)) + f*h**2*x**5/(4*sqrt(a)*sqrt(1 + c*x
**2/a))
```

Giac [A]
time = 5.17, size = 206, normalized size = 0.92

$$\frac{1}{24}\sqrt{cx^2+a}\left(\left(2\left(\frac{3\,fh^2x}{c}+\frac{4\,(2\,c^3fgh+c^3h^2e)}{c^4}\right)x+\frac{3\,(4\,c^3fg^2+4\,c^3dh^2-3\,ac^2fh^2+8\,c^3ghe)}{c^4}\right)x+\frac{8\,(6\,c^3dgh-4\,ac^2fgh+3\,c^3g^2e-2\,ac^2h^2e)}{c^4}\right)-\frac{(8\,c^2dg^2-4\,acfg^2-4\,acdh^2+3\,a^2fh^2-8\,acghe)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{8\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + a)*((2*(3*f*h^2*x/c + 4*(2*c^3*f*g*h + c^3*h^2*e)/c^4)*x
+ 3*(4*c^3*f*g^2 + 4*c^3*d*h^2 - 3*a*c^2*f*h^2 + 8*c^3*g*h*e)/c^4)*x + 8*(6
*c^3*d*g*h - 4*a*c^2*f*g*h + 3*c^3*g^2*e - 2*a*c^2*h^2*e)/c^4) - 1/8*(8*c^2
*d*g^2 - 4*a*c*f*g^2 - 4*a*c*d*h^2 + 3*a^2*f*h^2 - 8*a*c*g*h*e)*log(abs(-sq
rt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g+h\,x)^2\,(f\,x^2+e\,x+d)}{\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(1/2),x)
```

```
[Out] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(1/2), x)
```

3.103 $\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=136

$$\frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch}-\frac{(2(2afh^2+c(fg^2-3h(eg+dh)))+ch(fg-3eh)x)\sqrt{a+cx^2}}{6c^2h}+\frac{(2cdg-a(fg+e}{}$$

```
[Out] 1/2*(2*c*d*g-a*(e*h+f*g))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+1/3*f*
(h*x+g)^2*(c*x^2+a)^(1/2)/c/h-1/6*(4*a*f*h^2+2*c*(f*g^2-3*h*(d*h+e*g))+c*h*
(-3*e*h+f*g)*x)*(c*x^2+a)^(1/2)/c^2/h
```

Rubi [A]
time = 0.10, antiderivative size = 135, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1668, 794, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(2cdg-a(eh+fg))}{2c^{3/2}}-\frac{\sqrt{a+cx^2}\,(2(2afh^2-3ch(dh+eg)+cfg^2)+chx(fg-3eh))}{6c^2h}+\frac{f\sqrt{a+cx^2}\,(g+hx)^2}{3ch}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (f*(g + h*x)^2*Sqrt[a + c*x^2])/(3*c*h) - ((2*(c*f*g^2 + 2*a*f*h^2 - 3*c*h*
(e*g + d*h)) + c*h*(f*g - 3*e*h)*x)*Sqrt[a + c*x^2])/(6*c^2*h) + ((2*c*d*g
- a*(f*g + e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx = \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} + \frac{\int \frac{(g+hx)\left((3cd-2af)h^2-ch(fg-3eh)x\right)}{\sqrt{a+cx^2}}\,dx}{3ch^2}$$

$$= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} - \frac{(2(cfg^2+2afh^2-3ch(eg+dh))+ch(fg-3eh)}{6c^2h}$$

$$= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} - \frac{(2(cfg^2+2afh^2-3ch(eg+dh))+ch(fg-3eh)}{6c^2h}$$

$$= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} - \frac{(2(cfg^2+2afh^2-3ch(eg+dh))+ch(fg-3eh)}{6c^2h}$$

Mathematica [A]
time = 0.41, size = 96, normalized size = 0.71

$$\frac{\sqrt{a+cx^2}\left(-4afh+c(6eg+6dh+3fgx+3ehx+2fhx^2)\right)+3\sqrt{c}\,(-2cdg+afg+aeh)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[a + c*x^2]*(-4*a*f*h + c*(6*e*g + 6*d*h + 3*f*g*x + 3*e*h*x + 2*f*h*x
^2)) + 3*Sqrt[c]*(-2*c*d*g + a*f*g + a*e*h)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x
^2]])/(6*c^2)
```

Maple [A]
time = 0.09, size = 126, normalized size = 0.93

method	result
risch	$-\frac{(-2hfc\,x^2-3ehxc-3fgxc+4afh-6cdh-6ceg)\sqrt{c\,x^2+a}}{6c^2}-\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)aeh}{2c^{\frac{3}{2}}}-\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a\right.}}{2c^{\frac{3}{2}}}$
default	$hf\left(\frac{x^2\sqrt{c\,x^2+a}}{3c}-\frac{2a\sqrt{c\,x^2+a}}{3c^2}\right)+(eh+gf)\left(\frac{x\sqrt{c\,x^2+a}}{2c}-\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}\right)+\frac{(dh}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] h*f*(1/3*x^2/c*(c*x^2+a)^(1/2)-2/3*a/c^2*(c*x^2+a)^(1/2))+(e*h+f*g)*(1/2*x/
c*(c*x^2+a)^(1/2)-1/2*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))+(d*h+e*g)/c*
(c*x^2+a)^(1/2)+d*g*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)
```

Maxima [A]
time = 0.27, size = 129, normalized size = 0.95

$$\frac{\sqrt{cx^2+a}\,fhx^2}{3\,c}+\frac{dg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}+\frac{\sqrt{cx^2+a}\,dh}{c}-\frac{2\,\sqrt{cx^2+a}\,afh}{3\,c^2}+\frac{\sqrt{cx^2+a}\,(fg+he)x}{2\,c}-\frac{(fg+he)a\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^2+a}\,ge}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c*x^2 + a)*f*h*x^2/c + d*g*arcsinh(c*x/sqrt(a*c))/sqrt(c) + sqrt(c
*x^2 + a)*d*h/c - 2/3*sqrt(c*x^2 + a)*a*f*h/c^2 + 1/2*sqrt(c*x^2 + a)*(f*g
+ h*e)*x/c - 1/2*(f*g + h*e)*a*arcsinh(c*x/sqrt(a*c))/c^(3/2) + sqrt(c*x^2
+ a)*g*e/c
```

Fricas [A]
time = 0.39, size = 205, normalized size = 1.51

$$\left[\frac{3\,(ahe-(2\,cd-af)g)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2\,(2\,cfhx^2+3\,cfgx+2\,(3\,cd-2\,af)h+3\,(chx+2\,cg)e)\sqrt{cx^2+a}}{12\,c^2},\ \frac{3\,(ahe-(2\,cd-af)g)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)+(2\,cfhx^2+3\,cfgx+2\,(3\,cd-2\,af)h+3\,(chx+2\,cg)e)\sqrt{cx^2+a}}{6\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*h*e - (2*c*d - a*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)
*sqrt(c)*x - a) + 2*(2*c*f*h*x^2 + 3*c*f*g*x + 2*(3*c*d - 2*a*f)*h + 3*(c*h
*x + 2*c*g)*e)*sqrt(c*x^2 + a))/c^2, 1/6*(3*(a*h*e - (2*c*d - a*f)*g)*sqrt(
-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (2*c*f*h*x^2 + 3*c*f*g*x + 2*(3*c*
d - 2*a*f)*h + 3*(c*h*x + 2*c*g)*e)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A]
time = 3.83, size = 282, normalized size = 2.07

$$\frac{\sqrt{a}\,ehx\sqrt{1+\frac{cx^2}{a}}}{2c}+\frac{\sqrt{a}\,fgx\sqrt{1+\frac{cx^2}{a}}}{2c}-\frac{aeh\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}}-\frac{afg\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}}+dg\left(\begin{cases}\frac{\sqrt{-\frac{a}{c}}\operatorname{asin}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge c<0\\ \frac{\sqrt{\frac{a}{c}}\operatorname{asinh}\left(x\sqrt{\frac{c}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge c>0\\ \frac{\sqrt{-\frac{a}{c}}\operatorname{acosh}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{-a}} & \text{for } c>0\wedge a<0\end{cases}\right)+dh\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for } c=0\\ \frac{\sqrt{a+cx^2}}{c} & \text{otherwise}\end{cases}\right)+eg\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for } c=0\\ \frac{\sqrt{a+cx^2}}{c} & \text{otherwise}\end{cases}\right)+fh\left(\begin{cases}-\frac{2a\sqrt{a+cx^2}}{3c^2}+\frac{x^2\sqrt{a+cx^2}}{3c} & \text{for } c\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*e*h*x*sqrt(1 + c*x**2/a)/(2*c) + sqrt(a)*f*g*x*sqrt(1 + c*x**2/a)/(
2*c) - a*e*h*asinh(sqrt(c)*x/sqrt(a))/(2*c**(3/2)) - a*f*g*asinh(sqrt(c)*x/
sqrt(a))/(2*c**(3/2)) + d*g*Piecewise((sqrt(-a/c)*asin(x*sqrt(-c/a))/sqrt(a
), (a > 0) & (c < 0)), (sqrt(a/c)*asinh(x*sqrt(c/a))/sqrt(a), (a > 0) & (c
> 0)), (sqrt(-a/c)*acosh(x*sqrt(-c/a))/sqrt(-a), (c > 0) & (a < 0))) + d*h*
Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True)) + e*g*P
iecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True)) + f*h*Pi
ecewise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c), Ne(c
, 0)), (x**4/(4*sqrt(a)), True))
```

Giac [A]
time = 4.33, size = 110, normalized size = 0.81

$$\frac{1}{6}\sqrt{cx^2+a}\left(\left(\frac{2\,fhx}{c}+\frac{3\,(c^2fg+c^2he)}{c^3}\right)x+\frac{2\,(3\,c^2dh-2\,acfh+3\,c^2ge)}{c^3}\right)-\frac{(2\,cdg-afg-ahe)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(c*x^2 + a)*((2*f*h*x/c + 3*(c^2*f*g + c^2*h*e)/c^3)*x + 2*(3*c^2*d
*h - 2*a*c*f*h + 3*c^2*g*e)/c^3) - 1/2*(2*c*d*g - a*f*g - a*h*e)*log(abs(-s
qrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

Mupad [B]
time = 5.17, size = 227, normalized size = 1.67

$$\begin{cases}\frac{2\,f\,g\,x^3+3\,e\,g\,x^2+6\,d\,g\,x}{6\,\sqrt{a}}+\frac{3\,f\,h\,x^4+4\,e\,h\,x^3+6\,d\,h\,x^2}{12\,\sqrt{a}} & \text{if } c=0\\ \frac{d\,g\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}+\frac{d\,h\,\sqrt{c\,x^2+a}}{c}+\frac{e\,g\,\sqrt{c\,x^2+a}}{c}+\frac{e\,h\,x\,\sqrt{c\,x^2+a}}{2\,c}+\frac{f\,g\,x\,\sqrt{c\,x^2+a}}{2\,c}-\frac{f\,h\,\sqrt{c\,x^2+a}\,\left(2\,a-c\,x^2\right)}{3\,c^2}-\frac{a\,e\,h\,\ln\left(2\,\sqrt{c}\,x+2\,\sqrt{c\,x^2+a}\right)}{2\,c^{3/2}}-\frac{a\,f\,g\,\ln\left(2\,\sqrt{c}\,x+2\,\sqrt{c\,x^2+a}\right)}{2\,c^{3/2}} & \text{if } c\neq 0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(1/2),x)
```

```
[Out] piecewise(c == 0, (3*e*g*x^2 + 2*f*g*x^3 + 6*d*g*x)/(6*a^(1/2)) + (6*d*h*x^
2 + 4*e*h*x^3 + 3*f*h*x^4)/(12*a^(1/2)), c ~= 0, (d*g*log(c^(1/2)*x + (a +
c*x^2)^(1/2)))/c^(1/2) + (d*h*(a + c*x^2)^(1/2))/c + (e*g*(a + c*x^2)^(1/2)
)/c + (e*h*x*(a + c*x^2)^(1/2))/(2*c) + (f*g*x*(a + c*x^2)^(1/2))/(2*c) - (
f*h*(a + c*x^2)^(1/2)*(2*a - c*x^2))/(3*c^2) - (a*e*h*log(2*c^(1/2)*x + 2*(
a + c*x^2)^(1/2)))/(2*c^(3/2)) - (a*f*g*log(2*c^(1/2)*x + 2*(a + c*x^2)^(1/
2)))/(2*c^(3/2)))
```

3.104 $\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=74

$$\frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}+\frac{(2cd-af)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}$$

```
[Out] 1/2*(-a*f+2*c*d)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+e*(c*x^2+a)^(1/2)/c+1/2*f*x*(c*x^2+a)^(1/2)/c
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1829, 655, 223, 212}

$$\frac{(2cd-af)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}+\frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/Sqrt[a + c*x^2],x]
```

```
[Out] (e*Sqrt[a + c*x^2])/c + (f*x*Sqrt[a + c*x^2])/(2*c) + ((2*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
```

```
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx &= \frac{fx\sqrt{a+cx^2}}{2c}+\frac{\int \frac{2cd-af+2cex}{\sqrt{a+cx^2}}\,dx}{2c}\\
&= \frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}+\frac{(2cd-af)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2c}\\
&= \frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}+\frac{(2cd-af)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{2c}\\
&= \frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}+\frac{(2cd-af)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.22, size = 64, normalized size = 0.86

$$\frac{(2e+fx)\sqrt{a+cx^2}}{2c}+\frac{(-2cd+af)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/Sqrt[a + c*x^2],x]
[Out] ((2*e + f*x)*Sqrt[a + c*x^2])/(2*c) + ((-2*c*d + a*f)*Log[-(Sqrt[c]*x) + Sq
rt[a + c*x^2]])/(2*c^(3/2))
```

Maple [A]
time = 0.08, size = 77, normalized size = 1.04

method	result	size
risch	$\frac{(fx+2e)\sqrt{c\,x^2+a}}{2c}-\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)fa}{2c^{\frac{3}{2}}}+\frac{d\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}$	67
default	$f\left(\frac{x\sqrt{c\,x^2+a}}{2c}-\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}\right)+\frac{e\sqrt{c\,x^2+a}}{c}+\frac{d\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/2*x/c*(c*x^2+a)^(1/2)-1/2*a/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))+e*(
c*x^2+a)^(1/2)/c+d*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)
```

Maxima [A]
time = 0.27, size = 62, normalized size = 0.84

$$\frac{\sqrt{cx^2+a}\,fx}{2\,c}+\frac{d\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}-\frac{af\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^2+a}\,e}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*f*x/c + d*arcsinh(c*x/sqrt(a*c))/sqrt(c) - 1/2*a*f*arcs
inh(c*x/sqrt(a*c))/c^(3/2) + sqrt(c*x^2 + a)*e/c
```

Fricas [A]
time = 0.43, size = 126, normalized size = 1.70

$$\left[-\frac{(2\,cd-af)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\,(cfx+2\,ce)\sqrt{cx^2+a}}{4\,c^2},-\frac{(2\,cd-af)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)-(cfx+2\,ce)\sqrt{cx^2+a}}{2\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*c*d - a*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a
) - 2*(c*f*x + 2*c*e)*sqrt(c*x^2 + a))/c^2, -1/2*((2*c*d - a*f)*sqrt(-c)*ar
ctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c*f*x + 2*c*e)*sqrt(c*x^2 + a))/c^2]
```

Sympy [A]
time = 1.62, size = 150, normalized size = 2.03

$$\frac{\sqrt{a}\,fx\sqrt{1+\frac{cx^2}{a}}}{2c}-\frac{af\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}}+d\left(\begin{cases}\frac{\sqrt{-\frac{a}{c}}\,\mathrm{asin}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge c<0\\ \frac{\sqrt{\frac{a}{c}}\,\mathrm{asinh}\left(x\sqrt{\frac{c}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge c>0\\ \frac{\sqrt{-\frac{a}{c}}\,\mathrm{acosh}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{-a}} & \text{for } c>0\wedge a<0\end{cases}\right)+e\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for } c=0\\ \frac{\sqrt{a+cx^2}}{c} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*f*x*sqrt(1 + c*x**2/a)/(2*c) - a*f*asinh(sqrt(c)*x/sqrt(a))/(2*c**(
3/2)) + d*Piecewise((sqrt(-a/c)*asin(x*sqrt(-c/a))/sqrt(a), (a > 0) & (c <
```

```
0)), (sqrt(a/c)*asinh(x*sqrt(c/a))/sqrt(a), (a > 0) & (c > 0)), (sqrt(-a/c)
*acosh(x*sqrt(-c/a))/sqrt(-a), (c > 0) & (a < 0))) + e*Piecewise((x**2/(2*s
qrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True))
```

Giac [A]
time = 4.68, size = 58, normalized size = 0.78

$$\frac{1}{2}\sqrt{cx^2+a}\left(\frac{fx}{c}+\frac{2e}{c}\right)-\frac{(2cd-af)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*(f*x/c + 2*e/c) - 1/2*(2*c*d - a*f)*log(abs(-sqrt(c)*x
+ sqrt(c*x^2 + a)))/c^(3/2)
```

Mupad [B]
time = 4.56, size = 107, normalized size = 1.45

$$\begin{cases} \frac{2\,f\,x^3+3\,e\,x^2+6\,d\,x}{6\,\sqrt{a}} & \text{if } c=0 \\ \frac{e\,\sqrt{c\,x^2+a}}{c}+\frac{d\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}-\frac{a\,f\,\ln\left(2\,\sqrt{c}\,x+2\,\sqrt{c\,x^2+a}\right)}{2\,c^{3/2}}+\frac{f\,x\,\sqrt{c\,x^2+a}}{2\,c} & \text{if } c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + c*x^2)^(1/2),x)
```

```
[Out] piecewise(c == 0, (6*d*x + 3*e*x^2 + 2*f*x^3)/(6*a^(1/2)), c ~= 0, (e*(a +
c*x^2)^(1/2))/c + (d*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/c^(1/2) - (a*f*log
(2*c^(1/2)*x + 2*(a + c*x^2)^(1/2)))/(2*c^(3/2)) + (f*x*(a + c*x^2)^(1/2))/
(2*c))
```

3.105 $\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=130

$$\frac{f\sqrt{a+cx^2}}{ch}-\frac{(fg-eh)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,h^2}-\frac{(fg^2-egh+dh^2)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^2\sqrt{cg^2+ah^2}}$$

```
[Out] -(-e*h+f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^2/c^(1/2)-(d*h^2-e*g*h+f*g
^2)*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^2/(a*h^2+c*
g^2)^(1/2)+f*(c*x^2+a)^(1/2)/c/h
```

Rubi [A]

time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1668, 858, 223, 212, 739}

$$-\frac{(dh^2-egh+fg^2)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h^2\sqrt{ah^2+cg^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(fg-eh)}{\sqrt{c}\,h^2}+\frac{f\sqrt{a+cx^2}}{ch}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (f*Sqrt[a + c*x^2])/(c*h) - ((f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2
]])/(Sqrt[c]*h^2) - ((f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(Sqrt[c*
g^2 + a*h^2]*Sqrt[a + c*x^2])])/(h^2*Sqrt[c*g^2 + a*h^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx &= \frac{f\sqrt{a+cx^2}}{ch}+\frac{\int \frac{cdh^2-ch(fg-eh)x}{(g+hx)\sqrt{a+cx^2}}\,dx}{ch^2}\\
&= \frac{f\sqrt{a+cx^2}}{ch}-\frac{(fg-eh)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{h^2}+\frac{(fg^2-egh+dh^2)\int \frac{1}{(g+hx)\sqrt{a+cx}}}{h^2}\\
&= \frac{f\sqrt{a+cx^2}}{ch}-\frac{(fg-eh)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{h^2}-\frac{(fg^2-egh+dh^2}{}\\
&= \frac{f\sqrt{a+cx^2}}{ch}-\frac{(fg-eh)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,h^2}-\frac{(fg^2-egh+dh^2)\tanh^{-1}\left(\right.}{h^2\sqrt{cg^2+}}
\end{aligned}$$

Mathematica [A]
time = 0.48, size = 137, normalized size = 1.05

$$\frac{\frac{fh\sqrt{a+cx^2}}{c}-\frac{2\left(fg^2+h(-eg+dh)\right)\tan^{-1}\left(\frac{\sqrt{c}\,(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{\sqrt{-cg^2-ah^2}}+\frac{(fg-eh)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((f*h*Sqrt[a + c*x^2])/c - (2*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[c]*(g
 + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/Sqrt[-(c*g^2) - a*h^2
] + ((f*g - e*h)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/h^2
```

Maple [A]
time = 0.11, size = 209, normalized size = 1.61

method	result
default	$\frac{\frac{fh\sqrt{c\,x^2+a}}{c}+\frac{eh\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}-\frac{gf\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}}{h^2}-\frac{(d\,h^2-egh+f\,g^2)\ln\left(\frac{\frac{2a\,h^2+2c\,g^2}{h^2}-\frac{2cg\left(x+\frac{g}{h}\right)}{h}\ldots}{\ldots}\right)}{\ldots}$
risch	$\frac{f\sqrt{c\,x^2+a}}{ch}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)e}{h\sqrt{c}}-\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)gf}{h^2\sqrt{c}}-\frac{\ln\left(\frac{\frac{2a\,h^2+2c\,g^2}{h^2}-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a\,h^2+\ldots}{h^2}}\ldots}{\ldots}\right)}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/h^2*(f*h/c*(c*x^2+a)^(1/2)+e*h*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-g*f*
ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2))-(d*h^2-e*g*h+f*g^2)/h^3/((a*h^2+c*g^
2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^
2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*
g))
```

Maxima [A]
time = 0.30, size = 220, normalized size = 1.69

$$-\frac{fg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,h^2}+\frac{fg^2\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}\,h^3}+\frac{d\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}\,h}+\frac{\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)e}{\sqrt{c}\,h}-\frac{g\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)e}{\sqrt{a+\frac{cg^2}{h^2}}\,h^2}+\frac{\sqrt{cx^2+a}\,f}{ch}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -f*g*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h^2) + f*g^2*arcsinh(c*g*x/(sqrt(a*c)*
abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^3) + d
*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sq
rt(a + c*g^2/h^2)*h) + arcsinh(c*x/sqrt(a*c))*e/(sqrt(c)*h) - g*arcsinh(c*g
```

```
*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))*e/(sqrt(a + c*g
^2/h^2)*h^2) + sqrt(c*x^2 + a)*f/(c*h)
```

Fricas [A]
time = 147.24, size = 889, normalized size = 6.84

$$\left[-\frac{\left(cfg^3+afgh^2-\left(cg^2h+ah^3\right)e\right)\sqrt{c}\log\left(-2\,cx^2-2\sqrt{cx^2+a}\sqrt{c}\,x-a\right)+\left(cfg^2+cdh^2-cghe\right)\sqrt{cg^2+ah^2}\log\left(\frac{2\,acghx-acg^2-2\,a^2h^2-\left(2\,c^2g^2+ach^2\right)x^2+2\sqrt{cg^2+ah^2}\left(cgx-ah\right)\sqrt{cx^2+a}}{h^2x^2+2\,ghx+g^2}\right)-2\left(cfg^2h+afh^3\right)\sqrt{cx^2+a}}{2\left(c^2g^2h^2+ach^4\right)},\ -\frac{2\left(cfg^2+cdh^2-cghe\right)\sqrt{-cg^2-ah^2}\arctan\left(\frac{\sqrt{-cg^2-ah^2}\left(cgx-ah\right)\sqrt{cx^2+a}}{acg^2+a^2h^2+\left(c^2g^2+ach^2\right)x^2}\right)+\left(cfg^3+afgh^2-\left(cg^2h+ah^3\right)e\right)\sqrt{c}\log\left(-2\,cx^2-2\sqrt{cx^2+a}\sqrt{c}\,x-a\right)-2\left(cfg^2h+afh^3\right)\sqrt{cx^2+a}}{2\left(c^2g^2h^2+ach^4\right)},\ \frac{2\left(cfg^3+afgh^2-\left(cg^2h+ah^3\right)e\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)-\left(cfg^2+cdh^2-cghe\right)\sqrt{cg^2+ah^2}\log\left(\frac{2\,acghx-acg^2-2\,a^2h^2-\left(2\,c^2g^2+ach^2\right)x^2+2\sqrt{cg^2+ah^2}\left(cgx-ah\right)\sqrt{cx^2+a}}{h^2x^2+2\,ghx+g^2}\right)+2\left(cfg^2h+afh^3\right)\sqrt{cx^2+a}}{2\left(c^2g^2h^2+ach^4\right)},\ -\frac{\left(cfg^2+cdh^2-cghe\right)\sqrt{-cg^2-ah^2}\arctan\left(\frac{\sqrt{-cg^2-ah^2}\left(cgx-ah\right)\sqrt{cx^2+a}}{acg^2+a^2h^2+\left(c^2g^2+ach^2\right)x^2}\right)-\left(cfg^3+afgh^2-\left(cg^2h+ah^3\right)e\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)-\left(cfg^2h+afh^3\right)\sqrt{cx^2+a}}{c^2g^2h^2+ach^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((c*f*g^3 + a*f*g*h^2 - (c*g^2*h + a*h^3)*e)*sqrt(c)*log(-2*c*x^2 - 2
*sqrt(c*x^2 + a)*sqrt(c)*x - a) + (c*f*g^2 + c*d*h^2 - c*g*h*e)*sqrt(c*g^2
+ a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2
 + 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x
+ g^2)) - 2*(c*f*g^2*h + a*f*h^3)*sqrt(c*x^2 + a))/(c^2*g^2*h^2 + a*c*h^4),
 -1/2*(2*(c*f*g^2 + c*d*h^2 - c*g*h*e)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*
g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 +
a*c*h^2)*x^2)) + (c*f*g^3 + a*f*g*h^2 - (c*g^2*h + a*h^3)*e)*sqrt(c)*log(-2
*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(c*f*g^2*h + a*f*h^3)*sqrt(c*
x^2 + a))/(c^2*g^2*h^2 + a*c*h^4), 1/2*(2*(c*f*g^3 + a*f*g*h^2 - (c*g^2*h +
 a*h^3)*e)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c*f*g^2 + c*d*h^2
 - c*g*h*e)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2
*c^2*g^2 + a*c*h^2)*x^2 + 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 +
a))/(h^2*x^2 + 2*g*h*x + g^2)) + 2*(c*f*g^2*h + a*f*h^3)*sqrt(c*x^2 + a))/(
c^2*g^2*h^2 + a*c*h^4), -((c*f*g^2 + c*d*h^2 - c*g*h*e)*sqrt(-c*g^2 - a*h^2
)*arctan(sqrt(-c*g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*
h^2 + (c^2*g^2 + a*c*h^2)*x^2)) - (c*f*g^3 + a*f*g*h^2 - (c*g^2*h + a*h^3)*
e)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c*f*g^2*h + a*f*h^3)*sqrt
(c*x^2 + a))/(c^2*g^2*h^2 + a*c*h^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{d+ex+fx^2}{\sqrt{a+cx^2}\,\left(g+hx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)), x)
```

Giac [A]
time = 5.83, size = 138, normalized size = 1.06

$$\frac{\sqrt{cx^2+a}\,f}{ch}+\frac{2\left(fg^2+dh^2-ghe\right)\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)h+\sqrt{c}\,g}{\sqrt{-cg^2-ah^2}}\right)}{\sqrt{-cg^2-ah^2}\,h^2}+\frac{\left(\sqrt{c}\,fg-\sqrt{c}\,he\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{ch^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x^2 + a)*f/(c*h) + 2*(f*g^2 + d*h^2 - g*h*e)*arctan(-((sqrt(c)*x - s
qrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 - a*h^2)*
h^2) + (sqrt(c)*f*g - sqrt(c)*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/(
c*h^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2+e\,x+d}{\left(g+h\,x\right)\,\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(1/2)), x)
```

3.106 $\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=168

$$-\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{h(cg^2+ah^2)(g+hx)}+\frac{f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,h^2}+\frac{(ah^2(2fg-eh)+c(fg^3-dgh^2))\tanh^{-1}\left(\frac{}{\sqrt{cg^2}}\right.}{h^2(cg^2+ah^2)^{3/2}}$$

```
[Out] (a*h^2*(-e*h+2*f*g)+c*(-d*g*h^2+f*g^3))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^
(1/2)/(c*x^2+a)^(1/2))/h^2/(a*h^2+c*g^2)^(3/2)+f*arctanh(x*c^(1/2)/(c*x^2+a
)^(1/2))/h^2/c^(1/2)-(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(1/2)/h/(a*h^2+c*g^2)/(h
*x+g)
```

Rubi [A]
time = 0.14, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1665, 858, 223, 212, 739}

$$-\frac{\sqrt{a+cx^2}\,(dh^2-egh+fg^2)}{h(g+hx)\,(ah^2+cg^2)}+\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\,\sqrt{ah^2+cg^2}}\right)(ah^2(2fg-eh)+c(fg^3-dgh^2))}{h^2\,(ah^2+cg^2)^{3/2}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,h^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -(((f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/(h*(c*g^2 + a*h^2)*(g + h*x)))
+ (f*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*h^2) + ((a*h^2*(2*f*g -
 e*h) + c*(f*g^3 - d*g*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2])])/(h^2*(c*g^2 + a*h^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{h(cg^2+ah^2)(g+hx)} - \frac{\int \frac{-cdg+afg-aeh-f\left(\frac{cg^2}{h}+ah\right)x}{(g+hx)\sqrt{a+cx^2}}\,dx}{cg^2+ah^2}$$

$$= -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{h(cg^2+ah^2)(g+hx)} + \frac{f\int \frac{1}{\sqrt{a+cx^2}}\,dx}{h^2} + \frac{\left(cdg-2afg-\frac{cfg^3}{h^2}+\right.}{cg}$$

$$= -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{h(cg^2+ah^2)(g+hx)} + \frac{f\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{h^2} - \frac{\left(cdg\right.}{}$$

$$= -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{h(cg^2+ah^2)(g+hx)} + \frac{f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,h^2} - \frac{\left(cdg-2afg-\right.}{}$$

Mathematica [A]
time = 0.92, size = 177, normalized size = 1.05

$$-\frac{\frac{h(fg^2+h(-eg+dh))\sqrt{a+cx^2}}{(cg^2+ah^2)(g+hx)} + \frac{2(ah^2(2fg-eh)+c(fg^3-dgh^2))\tan^{-1}\left(\frac{\sqrt{c}\,(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{(-cg^2-ah^2)^{3/2}} + \frac{f\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -(((h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)*(g + h*x
)) + (2*(a*h^2*(2*f*g - e*h) + c*(f*g^3 - d*g*h^2))*ArcTan[(Sqrt[c]*(g + h*
x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(-(c*g^2) - a*h^2)^(3/2) +
 (f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/h^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(154) = 308$.
time = 0.09, size = 390, normalized size = 2.32

method	result
default	$\frac{f \ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{h^{2}\sqrt{c}}-\frac{(eh-2gf)\ln\left(\frac{\frac{2a\,h^{2}+2c\,g^{2}}{h^{2}}-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a\,h^{2}+c\,g^{2}}{h^{2}}}\,\sqrt{\left(x+\frac{g}{h}\right)^{2}c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{a\,h^{2}}{h}}}{x+\frac{g}{h}}\right.}{h^{3}\sqrt{\frac{a\,h^{2}+c\,g^{2}}{h^{2}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f/h^2*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-1/h^3*(e*h-2*f*g)/((a*h^2+c*g^2
)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2
)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g
))+1/h^4*(d*h^2-e*g*h+f*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c
-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g
^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h
^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h
*g)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(157) = 314$.
time = 0.31, size = 422, normalized size = 2.51

$$-\frac{\sqrt{cx^{2}+a}\,fg^{2}}{cg^{2}h^{2}x+ah^{4}x+cg^{3}h+agh^{3}}+\frac{\sqrt{cx^{2}+a}\,ge}{cg^{2}hx+ah^{3}x+cg^{3}+agh^{2}}-\frac{\sqrt{cx^{2}+a}\,d}{cg^{2}x+ah^{2}x+\frac{cg^{3}}{h}+agh}+\frac{f\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,h^{2}}+\frac{cfg^{3}\,\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{\left(a+\frac{cg^{2}}{h^{2}}\right)^{\frac{3}{2}}h^{5}}+\frac{cdg\,\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{\left(a+\frac{cg^{2}}{h^{2}}\right)^{\frac{3}{2}}h^{3}}-\frac{2\,fg\,\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{\sqrt{a+\frac{cg^{2}}{h^{2}}}\,h^{3}}-\frac{cg^{2}\,\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)e}{\left(a+\frac{cg^{2}}{h^{2}}\right)^{\frac{3}{2}}h^{4}}+\frac{\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)e}{\sqrt{a+\frac{cg^{2}}{h^{2}}}\,h^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2 + a)*f*g^2/(c*g^2*h^2*x + a*h^4*x + c*g^3*h + a*g*h^3) + sqrt(c
*x^2 + a)*g*e/(c*g^2*h*x + a*h^3*x + c*g^3 + a*g*h^2) - sqrt(c*x^2 + a)*d/(
```

```
c*g^2*x + a*h^2*x + c*g^3/h + a*g*h) + f*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h^
2) + c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*
x + g)))/((a + c*g^2/h^2)^(3/2)*h^5) + c*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h
*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^3) - 2*f*
g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(s
qrt(a + c*g^2/h^2)*h^3) - c*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*
h/(sqrt(a*c)*abs(h*x + g)))*e/((a + c*g^2/h^2)^(3/2)*h^4) + arcsinh(c*g*x/(
sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))*e/(sqrt(a + c*g^2/h
^2)*h^2)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2}{\sqrt{a + cx^2}\,(g + hx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2 + e\,x + d}{{\left(g + h\,x\right)}^2\,\sqrt{c\,x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(1/2)), x)
```

3.107 $\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=225

$$-\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{2h\,(cg^2+ah^2)\,(g+hx)^2}+\frac{(2ah^2(2fg-eh)+cg(fg^2+h(eg-3dh)))\sqrt{a+cx^2}}{2h\,(cg^2+ah^2)^2\,(g+hx)}-\frac{(2c^2dg^2+2a^2fh^2}{}$$

```
[Out] -1/2*(2*c^2*d*g^2+2*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+3*e*g)))*arctanh((-c*g*x+a
*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(5/2)-1/2*(d*h^2-e*g
*h+f*g^2)*(c*x^2+a)^(1/2)/h/(a*h^2+c*g^2)/(h*x+g)^2+1/2*(2*a*h^2*(-e*h+2*f*
g)+c*g*(f*g^2+h*(-3*d*h+e*g)))*(c*x^2+a)^(1/2)/h/(a*h^2+c*g^2)^2/(h*x+g)
```

Rubi [A]
time = 0.18, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1665, 821, 739, 212}

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)(2a^2fh^2-ac(fg^2-h(3eg-dh))+2c^2dg^2)}{2\,(ah^2+cg^2)^{5/2}}-\frac{\sqrt{a+cx^2}\,(dh^2-egh+fg^2)}{2h(g+hx)^2\,(ah^2+cg^2)}+\frac{\sqrt{a+cx^2}\,(2ah^2(2fg-eh)+cgh(eg-3dh)+cfg^3)}{2h(g+hx)\,(ah^2+cg^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] -1/2*((f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/(h*(c*g^2 + a*h^2)*(g + h*x)
^2) + ((c*f*g^3 + c*g*h*(e*g - 3*d*h) + 2*a*h^2*(2*f*g - e*h))*Sqrt[a + c*x
^2])/(2*h*(c*g^2 + a*h^2)^2*(g + h*x)) - ((2*c^2*d*g^2 + 2*a^2*f*h^2 - a*c*
(f*g^2 - h*(3*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[
a + c*x^2]])])/(2*(c*g^2 + a*h^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
```

```
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx = -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{2h(cg^2+ah^2)(g+hx)^2} - \frac{\int \frac{-2(cdg-afg+aeh)-\left(2afh+c\left(eg+\frac{fg^2}{h}-dh\right)\right)x}{(g+hx)^2\sqrt{a+cx^2}}\,dx}{2(cg^2+ah^2)}$$

$$= -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{2h(cg^2+ah^2)(g+hx)^2} + \frac{(cfg^3+cgh(eg-3dh)+2ah^2(2fg-eh))\sqrt{}}{2h(cg^2+ah^2)^2(g+hx)}$$

$$= -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{2h(cg^2+ah^2)(g+hx)^2} + \frac{(cfg^3+cgh(eg-3dh)+2ah^2(2fg-eh))\sqrt{}}{2h(cg^2+ah^2)^2(g+hx)}$$

$$= -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{2h(cg^2+ah^2)(g+hx)^2} + \frac{(cfg^3+cgh(eg-3dh)+2ah^2(2fg-eh))\sqrt{}}{2h(cg^2+ah^2)^2(g+hx)}$$

Mathematica [A]
time = 1.20, size = 203, normalized size = 0.90

$$\frac{\sqrt{a+cx^2}\,(cg(fg^2x+eg(2g+hx)-dh(4g+3hx))-ah(-fg(3g+4hx)+h(dh+e(g+2hx))))}{2(cg^2+ah^2)^2(g+hx)^2} - \frac{(2c^2dg^2+2a^2fh^2-ac(fg^2+h(-3eg+dh)))\tan^{-1}\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{(-cg^2-ah^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[a + c*x^2]*(c*g*(f*g^2*x + e*g*(2*g + h*x) - d*h*(4*g + 3*h*x)) - a*h
*(-(f*g*(3*g + 4*h*x)) + h*(d*h + e*(g + 2*h*x)))))/(2*(c*g^2 + a*h^2)^2*(g
 + h*x)^2) - ((2*c^2*d*g^2 + 2*a^2*f*h^2 - a*c*(f*g^2 + h*(-3*e*g + d*h)))*
```

```
ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(-(
c*g^2) - a*h^2)^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 809 vs. $2(209) = 418$.

time = 0.09, size = 810, normalized size = 3.60

method	result
default	$-\frac{f \ln\left(\frac{\frac{2a h^2+2c g^2}{h^2}-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a h^2+c g^2}{h^2}}\,\sqrt{\left(x+\frac{g}{h}\right)^2 c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{a h^2+c g^2}{h^2}}}{x+\frac{g}{h}}\right)}{h^3\sqrt{\frac{a h^2+c g^2}{h^2}}}+\frac{(eh-2gf)\left(-\frac{h^2\sqrt{(x+ \cdots}}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -f/h^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+
2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/
h^2)^(1/2))/(x+1/h*g))+(e*h-2*f*g)/h^4*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+
1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/(
(a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^
2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/
2))/(x+1/h*g)))+(d*h^2-e*g*h+f*g^2)/h^5*(-1/2/(a*h^2+c*g^2)*h^2/(x+1/h*g)^2
*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+3/2*c*g*h/(a*h^2
+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a
*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*
(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^
2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/2*c/(a*h^2+c*
g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g
)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2
)/h^2)^(1/2))/(x+1/h*g)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(214) = 428$.

time = 0.32, size = 901, normalized size = 4.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(c*x^2 + a)*c*f*g^3/(c^2*g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x +
 c^2*g^5*h + 2*a*c*g^3*h^3 + a^2*g*h^5) + 3/2*sqrt(c*x^2 + a)*c*g^2*e/(c^2*
g^4*h*x + 2*a*c*g^2*h^3*x + a^2*h^5*x + c^2*g^5 + 2*a*c*g^3*h^2 + a^2*g*h^4
) - 3/2*sqrt(c*x^2 + a)*c*d*g/(c^2*g^4*x + 2*a*c*g^2*h^2*x + a^2*h^4*x + c^
2*g^5/h + 2*a*c*g^3*h + a^2*g*h^3) - 1/2*sqrt(c*x^2 + a)*f*g^2/(c*g^2*h^3*x
^2 + a*h^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 2*sqr
t(c*x^2 + a)*f*g/(c*g^2*h^2*x + a*h^4*x + c*g^3*h + a*g*h^3) + 1/2*sqrt(c*x
^2 + a)*g*e/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h*x + 2*a*g*h^3*x + c*g^4
+ a*g^2*h^2) - 1/2*sqrt(c*x^2 + a)*d/(c*g^2*h*x^2 + a*h^3*x^2 + 2*c*g^3*x +
 2*a*g*h^2*x + c*g^4/h + a*g^2*h) - sqrt(c*x^2 + a)*e/(c*g^2*h*x + a*h^3*x
+ c*g^3 + a*g*h^2) + 3/2*c^2*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
 a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^7) + 3/2*c^2*d*g^2*
arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a
+ c*g^2/h^2)^(5/2)*h^5) - 5/2*c*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)
) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^5) - 1/2*c*d*arc
sinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c
*g^2/h^2)^(3/2)*h^3) + f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt
(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^3) - 3/2*c^2*g^3*arcsinh(c*g*x/
(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))*e/((a + c*g^2/h^2)
^(5/2)*h^6) + 3/2*c*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*
c)*abs(h*x + g)))*e/((a + c*g^2/h^2)^(3/2)*h^4)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(214) = 428$.
time = 3.96, size = 1095, normalized size = 4.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*c^2*d - a*c*f)*g^4 - (a*c*d - 2*a^2*f)*g^2*h^2 + ((2*c^2*d - a*c*
f)*g^2*h^2 - (a*c*d - 2*a^2*f)*h^4)*x^2 + 2*((2*c^2*d - a*c*f)*g^3*h - (a*c
*d - 2*a^2*f)*g*h^3)*x + 3*(a*c*g*h^3*x^2 + 2*a*c*g^2*h^2*x + a*c*g^3*h)*e)
*sqrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 +
a*c*h^2)*x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^
2 + 2*g*h*x + g^2)) - 2*(a^2*d*h^5 + (4*c^2*d - 3*a*c*f)*g^4*h + (5*a*c*d -
 3*a^2*f)*g^2*h^3 - (c^2*f*g^5 - (3*c^2*d - 5*a*c*f)*g^3*h^2 - (3*a*c*d - 4
```

```
*a^2*f)*g*h^4)*x - (2*c^2*g^5 + a*c*g^3*h^2 - a^2*g*h^4 + (c^2*g^4*h - a*c*
g^2*h^3 - 2*a^2*h^5)*x)*e)*sqrt(c*x^2 + a))/(c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*
a^2*c*g^4*h^4 + a^3*g^2*h^6 + (c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*
h^6 + a^3*h^8)*x^2 + 2*(c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3
*g*h^7)*x), -1/2*(((2*c^2*d - a*c*f)*g^4 - (a*c*d - 2*a^2*f)*g^2*h^2 + ((2*
c^2*d - a*c*f)*g^2*h^2 - (a*c*d - 2*a^2*f)*h^4)*x^2 + 2*((2*c^2*d - a*c*f)*
g^3*h - (a*c*d - 2*a^2*f)*g*h^3)*x + 3*(a*c*g*h^3*x^2 + 2*a*c*g^2*h^2*x + a
*c*g^3*h)*e)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*g^2 - a*h^2)*(c*g*x - a*h)
*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*h^2)*x^2)) + (a^2*d*h^
5 + (4*c^2*d - 3*a*c*f)*g^4*h + (5*a*c*d - 3*a^2*f)*g^2*h^3 - (c^2*f*g^5 -
(3*c^2*d - 5*a*c*f)*g^3*h^2 - (3*a*c*d - 4*a^2*f)*g*h^4)*x - (2*c^2*g^5 + a
*c*g^3*h^2 - a^2*g*h^4 + (c^2*g^4*h - a*c*g^2*h^3 - 2*a^2*h^5)*x)*e)*sqrt(c
*x^2 + a))/(c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*g^4*h^4 + a^3*g^2*h^6 + (c^
3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8)*x^2 + 2*(c^3*g^7*h
 + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^7)*x)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}\,(g+hx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+a)**(1/2),x)
[Out] Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)**3), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 848 vs. 2(214) = 428.
time = 2.83, size = 848, normalized size = 3.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] -(2*c^2*d*g^2 - a*c*f*g^2 - a*c*d*h^2 + 2*a^2*f*h^2 + 3*a*c*g*h*e)*arctan((
(sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^2*g^
4 + 2*a*c*g^2*h^2 + a^2*h^4)*sqrt(-c*g^2 - a*h^2)) + (2*(sqrt(c)*x - sqrt(c
*x^2 + a))^3*c^2*f*g^4*h - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*d*g^2*h^3
+ 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g^2*h^3 + (sqrt(c)*x - sqrt(c*x^2
 + a))^3*a*c*d*h^5 - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*g*h^4*e + 2*(sqr
t(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))
^2*c^(5/2)*d*g^3*h^2 + 7*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*f*g^3*h^
2 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)*x - sq
rt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^
```

```
(5/2)*g^4*h*e - 5*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*g^2*h^3*e + 2*(
sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*h^5*e - 2*(sqrt(c)*x - sqrt(c*x^
2 + a))*a*c^2*f*g^4*h + 10*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2*h^3 -
11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^3 + (sqrt(c)*x - sqrt(c*x^2
+ a))*a^2*c*d*h^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*g^3*h^2*e + 5*(sq
rt(c)*x - sqrt(c*x^2 + a))*a^2*c*g*h^4*e + a^2*c^(3/2)*f*g^3*h^2 - 3*a^2*c^
(3/2)*d*g*h^4 + 4*a^3*sqrt(c)*f*g*h^4 + a^2*c^(3/2)*g^2*h^3*e - 2*a^3*sqrt(
c)*h^5*e)/((c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6)*((sqrt(c)*x - sqrt(c*x^2
 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^3\,\sqrt{c\,x^2+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(1/2)), x)
```

3.108 $\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=229

$$-\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}}-\frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2}-\frac{h(4(3c^2dg^2+4a^2fh^2-ac(7fg^2+3h(3$$

```
[Out] -1/2*(3*a*h^2*(e*h+3*f*g)-2*c*g*(f*g^2+3*h*(d*h+e*g)))*arctanh(x*c^(1/2)/(c
*x^2+a)^(1/2))/c^(5/2)-(a*e-(-a*f+c*d)*x)*(h*x+g)^3/a/c/(c*x^2+a)^(1/2)-1/3
*(-4*a*f+3*c*d)*h*(h*x+g)^2*(c*x^2+a)^(1/2)/a/c^2-1/6*h*(12*c^2*d*g^2+16*a^
2*f*h^2-4*a*c*(7*f*g^2+3*h*(d*h+3*e*g))+c*h*(-9*a*e*h-11*a*f*g+6*c*d*g)*x)*
(c*x^2+a)^(1/2)/a/c^3
```

Rubi [A]
time = 0.20, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1659, 847, 794, 223, 212}

$$-\frac{h\sqrt{a+cx^2}\,(4(4a^2fh^2-ac(3h(dh+3eg)+7fg^2)+3c^2dg^2)+chx(-9aeh-11afg+6cdg))}{6ac^3}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(-3ah^2(eh+3fg)+6cgh(dh+eg)+2cfg^3)}{2c^{5/2}}-\frac{h\sqrt{a+cx^2}\,(g+hx)^2(3cd-4af)}{3ac^2}-\frac{(g+hx)^3(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x)^3)/(a*c*Sqrt[a + c*x^2])) - ((3*c*d - 4*
a*f)*h*(g + h*x)^2*Sqrt[a + c*x^2])/(3*a*c^2) - (h*(4*(3*c^2*d*g^2 + 4*a^2*
f*h^2 - a*c*(7*f*g^2 + 3*h*(3*e*g + d*h))) + c*h*(6*c*d*g - 11*a*f*g - 9*a*
e*h)*x)*Sqrt[a + c*x^2])/(6*a*c^3) + ((2*c*f*g^3 + 6*c*g*h*(e*g + d*h) - 3*
a*h^2*(3*f*g + e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
```

```
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{\int \frac{(g+hx)^2(-a(fg+3eh)+(3cd-4af)hx)}{\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} - \frac{\int \frac{(g\cdots}{\cdots}}{\cdots}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} - \frac{h(4(\cdots}{\cdots}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} - \frac{h(4(\cdots}{\cdots}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} - \frac{h(4(\cdots}{\cdots}$$

Mathematica [A]
time = 0.93, size = 256, normalized size = 1.12

$$\frac{-16a^3fh^3+6c^3dg^3x+ac^2(6dh(-3g^2-3ghx+h^2x^2)-3e(2g^3+6g^2hx-6gh^2x^2-h^3x^3)+fx(-6g^3+18g^2hx+9gh^2x^2+2h^3x^3))+a^2ch(f(36g^2+27ghx-8h^2x^2)+3h(4dh+3e(4g+hx)))+3a\sqrt{c}\,(3ah^2(3fg+eh)-2c(fg^3+3gh(eg+dh)))\sqrt{a+cx^2}\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{6ac^3\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] (-16*a^3*f*h^3 + 6*c^3*d*g^3*x + a*c^2*(6*d*h*(-3*g^2 - 3*g*h*x + h^2*x^2)
- 3*e*(2*g^3 + 6*g^2*h*x - 6*g*h^2*x^2 - h^3*x^3) + f*x*(-6*g^3 + 18*g^2*h*
x + 9*g*h^2*x^2 + 2*h^3*x^3)) + a^2*c*h*(f*(36*g^2 + 27*g*h*x - 8*h^2*x^2)
+ 3*h*(4*d*h + 3*e*(4*g + h*x))) + 3*a*Sqrt[c]*(3*a*h^2*(3*f*g + e*h) - 2*c
*(f*g^3 + 3*g*h*(e*g + d*h)))*Sqrt[a + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[a + c
*x^2]])/(6*a*c^3*Sqrt[a + c*x^2])
```

Maple [A]
time = 0.10, size = 292, normalized size = 1.28

method	result
default	$f\,h^3\left(\frac{x^4}{3c\sqrt{c\,x^2+a}}-\frac{4a\left(\frac{x^2}{c\sqrt{c\,x^2+a}}+\frac{2a}{c^2\sqrt{c\,x^2+a}}\right)}{3c}\right)+(e\,h^3+3f\,g\,h^2)\left(\frac{x^3}{2c\sqrt{c\,x^2+a}}-\right.$
risch	$-\frac{h(-2f\,h^2x^2c-3ce\,h^2x-9cf\,ghx+10af\,h^2-6cd\,h^2-18cegh-18cf\,g^2)\sqrt{c\,x^2+a}}{6c^3}+\frac{xae\,h^3}{c^2\sqrt{c\,x^2+a}}+\frac{3xaf\,g\,h^2}{c^2\sqrt{c\,x^2+a}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*h^3*(1/3*x^4/c/(c*x^2+a)^(1/2)-4/3*a/c*(x^2/c/(c*x^2+a)^(1/2)+2*a/c^2/(c*
x^2+a)^(1/2)))+(e*h^3+3*f*g*h^2)*(1/2*x^3/c/(c*x^2+a)^(1/2)-3/2*a/c*(-x/c/(
c*x^2+a)^(1/2)+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))))+(d*h^3+3*e*g*h^2+3
*f*g^2*h)*(x^2/c/(c*x^2+a)^(1/2)+2*a/c^2/(c*x^2+a)^(1/2))+(3*d*g*h^2+3*e*g^
2*h+f*g^3)*(-x/c/(c*x^2+a)^(1/2)+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))-(
3*d*g^2*h+e*g^3)/c/(c*x^2+a)^(1/2)+d*g^3*x/a/(c*x^2+a)^(1/2)
```

Maxima [A]
time = 0.28, size = 354, normalized size = 1.55

$$\frac{fh^3x^4}{3\sqrt{cx^2+a}\,c}-\frac{4afh^3x^2}{3\sqrt{cx^2+a}\,c^2}+\frac{dg^3x}{\sqrt{cx^2+a}\,a}-\frac{3dg^2h}{\sqrt{cx^2+a}\,c}-\frac{8a^2fh^3}{3\sqrt{cx^2+a}\,c^3}+\frac{(3fgh^2+h^3e)x^3}{2\sqrt{cx^2+a}\,c}-\frac{g^3e}{\sqrt{cx^2+a}\,c}+\frac{(3fg^2h+dh^3+3gh^2e)x^2}{\sqrt{cx^2+a}\,c}+\frac{3(3fgh^2+h^3e)ax}{2\sqrt{cx^2+a}\,c^2}-\frac{(fg^3+3dgh^2+3g^2he)x}{\sqrt{cx^2+a}\,c}-\frac{3(3fgh^2+h^3e)a\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2c^{\frac{5}{2}}}+\frac{(fg^3+3dgh^2+3g^2he)\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}}+\frac{2(3fg^2h+dh^3+3gh^2e)a}{\sqrt{cx^2+a}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*f*h^3*x^4/(sqrt(c*x^2 + a)*c) - 4/3*a*f*h^3*x^2/(sqrt(c*x^2 + a)*c^2) +
 d*g^3*x/(sqrt(c*x^2 + a)*a) - 3*d*g^2*h/(sqrt(c*x^2 + a)*c) - 8/3*a^2*f*h^
3/(sqrt(c*x^2 + a)*c^3) + 1/2*(3*f*g*h^2 + h^3*e)*x^3/(sqrt(c*x^2 + a)*c) -
 g^3*e/(sqrt(c*x^2 + a)*c) + (3*f*g^2*h + d*h^3 + 3*g*h^2*e)*x^2/(sqrt(c*x^
2 + a)*c) + 3/2*(3*f*g*h^2 + h^3*e)*a*x/(sqrt(c*x^2 + a)*c^2) - (f*g^3 + 3*
d*g*h^2 + 3*g^2*h*e)*x/(sqrt(c*x^2 + a)*c) - 3/2*(3*f*g*h^2 + h^3*e)*a*arcs
inh(c*x/sqrt(a*c))/c^(5/2) + (f*g^3 + 3*d*g*h^2 + 3*g^2*h*e)*arcsinh(c*x/sq
rt(a*c))/c^(3/2) + 2*(3*f*g^2*h + d*h^3 + 3*g*h^2*e)*a/(sqrt(c*x^2 + a)*c^2
)
```

Fricas [A]
time = 0.42, size = 782, normalized size = 3.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*a^2*c*f*g^3 + 3*(2*a^2*c*d - 3*a^3*f)*g*h^2 + (2*a*c^2*f*g^3 +
 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x^2 + 3*(2*a^2*c*g^2*h - a^3*h^3 + (2*a*c
^2*g^2*h - a^2*c*h^3)*x^2)*e)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt
(c)*x - a) - 2*(2*a*c^2*f*h^3*x^4 + 9*a*c^2*f*g*h^2*x^3 - 18*(a*c^2*d - 2*a
^2*c*f)*g^2*h + 4*(3*a^2*c*d - 4*a^3*f)*h^3 + 2*(9*a*c^2*f*g^2*h + (3*a*c^2
*d - 4*a^2*c*f)*h^3)*x^2 + 3*(2*(c^3*d - a*c^2*f)*g^3 - 3*(2*a*c^2*d - 3*a^
2*c*f)*g*h^2)*x + 3*(a*c^2*h^3*x^3 + 6*a*c^2*g*h^2*x^2 - 2*a*c^2*g^3 + 12*a
^2*c*g*h^2 - 3*(2*a*c^2*g^2*h - a^2*c*h^3)*x)*e)*sqrt(c*x^2 + a))/(a*c^4*x^
2 + a^2*c^3), -1/6*(3*(2*a^2*c*f*g^3 + 3*(2*a^2*c*d - 3*a^3*f)*g*h^2 + (2*a
*c^2*f*g^3 + 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x^2 + 3*(2*a^2*c*g^2*h - a^3*
h^3 + (2*a*c^2*g^2*h - a^2*c*h^3)*x^2)*e)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) - (2*a*c^2*f*h^3*x^4 + 9*a*c^2*f*g*h^2*x^3 - 18*(a*c^2*d - 2*a^2
*c*f)*g^2*h + 4*(3*a^2*c*d - 4*a^3*f)*h^3 + 2*(9*a*c^2*f*g^2*h + (3*a*c^2*d
 - 4*a^2*c*f)*h^3)*x^2 + 3*(2*(c^3*d - a*c^2*f)*g^3 - 3*(2*a*c^2*d - 3*a^2*
c*f)*g*h^2)*x + 3*(a*c^2*h^3*x^3 + 6*a*c^2*g*h^2*x^2 - 2*a*c^2*g^3 + 12*a^2
*c*g*h^2 - 3*(2*a*c^2*g^2*h - a^2*c*h^3)*x)*e)*sqrt(c*x^2 + a))/(a*c^4*x^2
+ a^2*c^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**3*(d + e*x + f*x**2)/(a + c*x**2)**(3/2), x)
```

Giac [A]
time = 6.01, size = 339, normalized size = 1.48

$$\frac{\left(\left(\left(\frac{2\,fh^3x}{c}+\frac{3\,\left(3\,ac^4fgh^2+ac^4h^3e\right)}{ac^5}\right)x+\frac{2\,\left(9\,ac^4fg^2h+3\,ac^4dh^3-4\,a^2c^3fh^3+9\,ac^4gh^2e\right)}{ac^5}\right)x+\frac{3\,\left(2\,c^5dg^3-2\,ac^4fg^3-6\,ac^4dgh^2+9\,a^2c^3fgh^2-6\,ac^4g^2he+3\,a^2c^3h^3e\right)}{ac^5}\right)x-\frac{2\,\left(9\,ac^4dg^2h-18\,a^2c^3fg^2h-6\,a^2c^3dh^3+8\,a^3c^2fh^3+3\,ac^4g^3e-18\,a^2c^3gh^2e\right)}{ac^5}}{6\,\sqrt{cx^2+a}}-\frac{\left(2\,cfg^3+6\,cdgh^2-9\,afgh^2+6\,cg^2he-3\,ah^3e\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{2\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*(((((2*f*h^3*x/c + 3*(3*a*c^4*f*g*h^2 + a*c^4*h^3*e)/(a*c^5))*x + 2*(9*a
*c^4*f*g^2*h + 3*a*c^4*d*h^3 - 4*a^2*c^3*f*h^3 + 9*a*c^4*g*h^2*e)/(a*c^5))*
x + 3*(2*c^5*d*g^3 - 2*a*c^4*f*g^3 - 6*a*c^4*d*g*h^2 + 9*a^2*c^3*f*g*h^2 -
6*a*c^4*g^2*h*e + 3*a^2*c^3*h^3*e)/(a*c^5))*x - 2*(9*a*c^4*d*g^2*h - 18*a^2
*c^3*f*g^2*h - 6*a^2*c^3*d*h^3 + 8*a^3*c^2*f*h^3 + 3*a*c^4*g^3*e - 18*a^2*c
^3*g*h^2*e)/(a*c^5))/sqrt(c*x^2 + a) - 1/2*(2*c*f*g^3 + 6*c*d*g*h^2 - 9*a*f
*g*h^2 + 6*c*g^2*h*e - 3*a*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^
(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g+h\,x\right)}^3\,\left(f\,x^2+e\,x+d\right)}{{\left(c\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2), x)
```

3.109 $\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}}-\frac{h(4(cdg-a(2fg+eh))+(2cd-3af)hx)\sqrt{a+cx^2}}{2ac^2}+\frac{((2cd-3af)h^2+2cg}{}$$

```
[Out] 1/2*((-3*a*f+2*c*d)*h^2+2*c*g*(2*e*h+f*g))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2
))/c^(5/2)-(a*e-(-a*f+c*d)*x)*(h*x+g)^2/a/c/(c*x^2+a)^(1/2)-1/2*h*(4*c*d*g-
4*a*(e*h+2*f*g)+(-3*a*f+2*c*d)*h*x)*(c*x^2+a)^(1/2)/a/c^2
```

Rubi [A]
time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1659, 794, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(h^2(2cd-3af)+2cg(2eh+fg)\right)}{2c^{5/2}}-\frac{h\sqrt{a+cx^2}\,(4(cdg-a(eh+2fg))+hx(2cd-3af))}{2ac^2}-\frac{(g+hx)^2(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x)^2)/(a*c*Sqrt[a + c*x^2])) - (h*(4*(c*d*g
 - a*(2*f*g + e*h)) + (2*c*d - 3*a*f)*h*x)*Sqrt[a + c*x^2])/(2*a*c^2) + (((
2*c*d - 3*a*f)*h^2 + 2*c*g*(f*g + 2*e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^
2]])/(2*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} - \frac{\int \frac{(g+hx)(-a(fg+2eh)+(2cd-3af)hx)}{\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} - \frac{h(4(cdg-a(2fg+eh))+(2cd-3af)h}{2ac^2}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} - \frac{h(4(cdg-a(2fg+eh))+(2cd-3af)h}{2ac^2}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} - \frac{h(4(cdg-a(2fg+eh))+(2cd-3af)h}{2ac^2}$$

Mathematica [**A**]
time = 0.68, size = 165, normalized size = 1.11

$$\frac{\frac{\sqrt{c}\left(2c^2dg^2x+a^2h(8fg+4eh+3fhx)+ac\left(-2dh(2g+hx)-2e\left(g^2+2ghx-h^2x^2\right)+fx\left(-2g^2+4ghx+h^2x^2\right)\right)\right)}{a\sqrt{a+cx^2}}+\left(3afh^2-2c\left(fg^2+h(2eg+dh)\right)\right)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] ((Sqrt[c]*(2*c^2*d*g^2*x + a^2*h*(8*f*g + 4*e*h + 3*f*h*x) + a*c*(-2*d*h*(2
*g + h*x) - 2*e*(g^2 + 2*g*h*x - h^2*x^2) + f*x*(-2*g^2 + 4*g*h*x + h^2*x^2
))))/(a*Sqrt[a + c*x^2]) + (3*a*f*h^2 - 2*c*(f*g^2 + h*(2*e*g + d*h)))*Log[
-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2*c^(5/2))
```

Maple [A]
time = 0.08, size = 207, normalized size = 1.39

method	result
default	$f\,h^{2}\left(\frac{x^{3}}{2c\sqrt{c\,x^{2}+a}}-\frac{3a\left(-\frac{x}{c\sqrt{c\,x^{2}+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{c^{\frac{3}{2}}}\right)}{2c}\right)+\left(e\,h^{2}+2fgh\right)\left(\frac{x^{2}}{c\sqrt{c\,x^{2}+a}}+\right.$
risch	$\frac{h(fxh+2eh+4gf)\sqrt{c\,x^{2}+a}}{2c^{2}}+\frac{xaf\,h^{2}}{c^{2}\sqrt{c\,x^{2}+a}}-\frac{xd\,h^{2}}{c\sqrt{c\,x^{2}+a}}-\frac{2xegh}{c\sqrt{c\,x^{2}+a}}-\frac{xf\,g^{2}}{c\sqrt{c\,x^{2}+a}}-\frac{3\ln\left(x\sqrt{c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*h^2*(1/2*x^3/c/(c*x^2+a)^(1/2)-3/2*a/c*(-x/c/(c*x^2+a)^(1/2)+1/c^(3/2)*ln
(x*c^(1/2)+(c*x^2+a)^(1/2))))+(e*h^2+2*f*g*h)*(x^2/c/(c*x^2+a)^(1/2)+2*a/c^
2/(c*x^2+a)^(1/2))+(d*h^2+2*e*g*h+f*g^2)*(-x/c/(c*x^2+a)^(1/2)+1/c^(3/2)*ln
(x*c^(1/2)+(c*x^2+a)^(1/2)))-(2*d*g*h+e*g^2)/c/(c*x^2+a)^(1/2)+d*g^2*x/a/(c
*x^2+a)^(1/2)
```

Maxima [A]
time = 0.27, size = 232, normalized size = 1.56

$$\frac{fh^{2}x^{3}}{2\sqrt{cx^{2}+a}\,c}+\frac{dg^{2}x}{\sqrt{cx^{2}+a}\,a}+\frac{3\,afh^{2}x}{2\sqrt{cx^{2}+a}\,c^{2}}-\frac{3\,afh^{2}\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{5}{2}}}-\frac{2\,dgh}{\sqrt{cx^{2}+a}\,c}+\frac{(2\,fgh+h^{2}e)x^{2}}{\sqrt{cx^{2}+a}\,c}-\frac{g^{2}e}{\sqrt{cx^{2}+a}\,c}-\frac{(fg^{2}+dh^{2}+2\,ghe)x}{\sqrt{cx^{2}+a}\,c}+\frac{(fg^{2}+dh^{2}+2\,ghe)\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}}+\frac{2\,(2\,fgh+h^{2}e)a}{\sqrt{cx^{2}+a}\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*f*h^2*x^3/(sqrt(c*x^2 + a)*c) + d*g^2*x/(sqrt(c*x^2 + a)*a) + 3/2*a*f*h
^2*x/(sqrt(c*x^2 + a)*c^2) - 3/2*a*f*h^2*arcsinh(c*x/sqrt(a*c))/c^(5/2) - 2
*d*g*h/(sqrt(c*x^2 + a)*c) + (2*f*g*h + h^2*e)*x^2/(sqrt(c*x^2 + a)*c) - g^
2*e/(sqrt(c*x^2 + a)*c) - (f*g^2 + d*h^2 + 2*g*h*e)*x/(sqrt(c*x^2 + a)*c) +
 (f*g^2 + d*h^2 + 2*g*h*e)*arcsinh(c*x/sqrt(a*c))/c^(3/2) + 2*(2*f*g*h + h^
2*e)*a/(sqrt(c*x^2 + a)*c^2)
```

Fricas [A]
time = 0.38, size = 542, normalized size = 3.64

$$\left[\frac{(2\,a^{2}cfg^{2}+(2\,a^{2}cd-3\,a^{3}f)h^{2}+(2\,ac^{2}fg^{2}+(2\,ac^{2}d-3\,a^{2}cf)h^{2})x^{2}+4\,(ac^{2}ghx^{2}+a^{2}cgh)e)\sqrt{c}\,\log\left(-2\,cx^{2}-2\sqrt{cx^{2}+a}\,\sqrt{c}\,x-a\right)+2\,(ac^{2}fh^{2}x^{3}+4\,ac^{2}fghx^{2}-4\,(ac^{2}d-2\,a^{2}cf)gh+(2\,(c^{3}d-ac^{2}f)g^{2}-(2\,ac^{2}d-3\,a^{2}cf)h^{2})x+2\,(ac^{2}h^{2}x^{2}-2\,ac^{2}ghx-ac^{2}g^{2}+2\,a^{2}ch^{2})e)\sqrt{cx^{2}+a}}{4\,(ac^{4}x^{2}+a^{2}c^{3})},\ -\frac{(2\,a^{2}cfg^{2}+(2\,a^{2}cd-3\,a^{3}f)h^{2}+(2\,ac^{2}fg^{2}+(2\,ac^{2}d-3\,a^{2}cf)h^{2})x^{2}+4\,(ac^{2}ghx^{2}+a^{2}cgh)e)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^{2}+a}}\right)-(ac^{2}fh^{2}x^{3}+4\,ac^{2}fghx^{2}-4\,(ac^{2}d-2\,a^{2}cf)gh+(2\,(c^{3}d-ac^{2}f)g^{2}-(2\,ac^{2}d-3\,a^{2}cf)h^{2})x+2\,(ac^{2}h^{2}x^{2}-2\,ac^{2}ghx-ac^{2}g^{2}+2\,a^{2}ch^{2})e)\sqrt{cx^{2}+a}}{2\,(ac^{4}x^{2}+a^{2}c^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*a^2*c*f*g^2 + (2*a^2*c*d - 3*a^3*f)*h^2 + (2*a*c^2*f*g^2 + (2*a*c^
2*d - 3*a^2*c*f)*h^2)*x^2 + 4*(a*c^2*g*h*x^2 + a^2*c*g*h)*e)*sqrt(c)*log(-2
*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(a*c^2*f*h^2*x^3 + 4*a*c^2*f*
g*h*x^2 - 4*(a*c^2*d - 2*a^2*c*f)*g*h + (2*(c^3*d - a*c^2*f)*g^2 - (2*a*c^2
*d - 3*a^2*c*f)*h^2)*x + 2*(a*c^2*h^2*x^2 - 2*a*c^2*g*h*x - a*c^2*g^2 + 2*a
^2*c*h^2)*e)*sqrt(c*x^2 + a))/(a*c^4*x^2 + a^2*c^3), -1/2*((2*a^2*c*f*g^2 +
 (2*a^2*c*d - 3*a^3*f)*h^2 + (2*a*c^2*f*g^2 + (2*a*c^2*d - 3*a^2*c*f)*h^2)*
x^2 + 4*(a*c^2*g*h*x^2 + a^2*c*g*h)*e)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^
2 + a)) - (a*c^2*f*h^2*x^3 + 4*a*c^2*f*g*h*x^2 - 4*(a*c^2*d - 2*a^2*c*f)*g*
h + (2*(c^3*d - a*c^2*f)*g^2 - (2*a*c^2*d - 3*a^2*c*f)*h^2)*x + 2*(a*c^2*h^
2*x^2 - 2*a*c^2*g*h*x - a*c^2*g^2 + 2*a^2*c*h^2)*e)*sqrt(c*x^2 + a))/(a*c^4
*x^2 + a^2*c^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**2*(d + e*x + f*x**2)/(a + c*x**2)**(3/2), x)
```

Giac [A]
time = 5.34, size = 219, normalized size = 1.47

$$\frac{\left(\left(\frac{fh^2x}{c}+\frac{2\,(2\,ac^3fgh+ac^3h^2e)}{ac^4}\right)x+\frac{2\,c^4dg^2-2\,ac^3fg^2-2\,ac^3dh^2+3\,a^2c^2fh^2-4\,ac^3ghe}{ac^4}\right)x-\frac{2\,(2\,ac^3dgh-4\,a^2c^2fgh+ac^3g^2e-2\,a^2c^2h^2e)}{ac^4}}{2\,\sqrt{cx^2+a}}-\frac{(2\,cfg^2+2\,cdh^2-3\,afh^2+4\,cghe)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{2\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(((f*h^2*x/c + 2*(2*a*c^3*f*g*h + a*c^3*h^2*e)/(a*c^4))*x + (2*c^4*d*g^
2 - 2*a*c^3*f*g^2 - 2*a*c^3*d*h^2 + 3*a^2*c^2*f*h^2 - 4*a*c^3*g*h*e)/(a*c^4
))*x - 2*(2*a*c^3*d*g*h - 4*a^2*c^2*f*g*h + a*c^3*g^2*e - 2*a^2*c^2*h^2*e)/
(a*c^4))/sqrt(c*x^2 + a) - 1/2*(2*c*f*g^2 + 2*c*d*h^2 - 3*a*f*h^2 + 4*c*g*h
*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g+h\,x\right)}^2\,\left(f\,x^2+e\,x+d\right)}{{\left(c\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2), x)
```

3.110 $\int \frac{(g+hx)(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}}-\frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2}+\frac{(fg+eh)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}$$

```
[Out] (e*h+f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)-(a*e-(-a*f+c*d)*x)*(h*
x+g)/a/c/(c*x^2+a)^(1/2)-(-2*a*f+c*d)*h*(c*x^2+a)^(1/2)/a/c^2
```

Rubi [A]
time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1659, 655, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(eh+fg)}{c^{3/2}}-\frac{h\sqrt{a+cx^2}\,(cd-2af)}{ac^2}-\frac{(g+hx)(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x))/(a*c*Sqrt[a + c*x^2])) - ((c*d - 2*a*f)
*h*Sqrt[a + c*x^2])/(a*c^2) + ((f*g + e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x
^2]])/c^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{\int \frac{-a(fg+eh)+(cd-2af)hx}{\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2} + \frac{(fg+eh)\int \frac{}{\sqrt{a}}}{c}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2} + \frac{(fg+eh)\text{Subst}}{}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2} + \frac{(fg+eh)\tanh}{c}$$

Mathematica [A]
time = 0.49, size = 104, normalized size = 1.04

$$\frac{-aceg-acdh+2a^2fh+c^2dgx-acfgx-acehx+acfhx^2}{ac^2\sqrt{a+cx^2}}+\frac{(-fg-eh)\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] (-(a*c*e*g) - a*c*d*h + 2*a^2*f*h + c^2*d*g*x - a*c*f*g*x - a*c*e*h*x + a*c
*f*h*x^2)/(a*c^2*Sqrt[a + c*x^2]) + ((-(f*g) - e*h)*Log[-(Sqrt[c]*x) + Sqrt
[a + c*x^2]])/c^(3/2)
```

Maple [A]
time = 0.08, size = 118, normalized size = 1.18

method	result
default	$hf\left(\frac{x^{2}}{c\sqrt{c\,x^{2}+a}}+\frac{2a}{c^{2}\sqrt{c\,x^{2}+a}}\right)+\left(eh+gf\right)\left(-\frac{x}{c\sqrt{c\,x^{2}+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{c^{\frac{3}{2}}}\right)-\frac{dh}{c\sqrt{c\,x}}$
risch	$\frac{hf\sqrt{c\,x^{2}+a}}{c^{2}}-\frac{xeh}{c\sqrt{c\,x^{2}+a}}-\frac{xfg}{c\sqrt{c\,x^{2}+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)eh}{c^{\frac{3}{2}}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)fg}{c^{\frac{3}{2}}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] h*f*(x^2/c/(c*x^2+a)^(1/2)+2*a/c^2/(c*x^2+a)^(1/2))+(e*h+f*g)*(-x/c/(c*x^2+
a)^(1/2)+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))-(d*h+e*g)/c/(c*x^2+a)^(1/
2)+d*g*x/a/(c*x^2+a)^(1/2)
```

Maxima [A]
time = 0.28, size = 129, normalized size = 1.29

$$\frac{fhx^{2}}{\sqrt{cx^{2}+a}\,c}+\frac{dgx}{\sqrt{cx^{2}+a}\,a}-\frac{dh}{\sqrt{cx^{2}+a}\,c}+\frac{2\,afh}{\sqrt{cx^{2}+a}\,c^{2}}-\frac{(fg+he)x}{\sqrt{cx^{2}+a}\,c}+\frac{(fg+he)\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}}-\frac{ge}{\sqrt{cx^{2}+a}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] f*h*x^2/(sqrt(c*x^2 + a)*c) + d*g*x/(sqrt(c*x^2 + a)*a) - d*h/(sqrt(c*x^2 +
 a)*c) + 2*a*f*h/(sqrt(c*x^2 + a)*c^2) - (f*g + h*e)*x/(sqrt(c*x^2 + a)*c)
+ (f*g + h*e)*arcsinh(c*x/sqrt(a*c))/c^(3/2) - g*e/(sqrt(c*x^2 + a)*c)
```

Fricas [A]
time = 0.44, size = 282, normalized size = 2.82

$$\left[\frac{(acfgx^{2}+a^{2}fg+(achx^{2}+a^{2}h)e)\sqrt{c}\,\log\left(-2\,cx^{2}-2\,\sqrt{cx^{2}+a}\,\sqrt{c}\,x-a\right)+2\,(acfhx^{2}+(c^{2}d-acf)gx-(acd-2\,a^{2}f)h-(achx+acg)e)\sqrt{cx^{2}+a}}{2\,(ac^{3}x^{2}+a^{2}c^{2})},-\frac{(acfgx^{2}+a^{2}fg+(achx^{2}+a^{2}h)e)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^{2}+a}}\right)-(acfhx^{2}+(c^{2}d-acf)gx-(acd-2\,a^{2}f)h-(achx+acg)e)\sqrt{cx^{2}+a}}{ac^{3}x^{2}+a^{2}c^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c*f*g*x^2 + a^2*f*g + (a*c*h*x^2 + a^2*h)*e)*sqrt(c)*log(-2*c*x^2
- 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(a*c*f*h*x^2 + (c^2*d - a*c*f)*g*x -
 (a*c*d - 2*a^2*f)*h - (a*c*h*x + a*c*g)*e)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a
^2*c^2), -((a*c*f*g*x^2 + a^2*f*g + (a*c*h*x^2 + a^2*h)*e)*sqrt(-c)*arctan(
sqrt(-c)*x/sqrt(c*x^2 + a)) - (a*c*f*h*x^2 + (c^2*d - a*c*f)*g*x - (a*c*d -
 2*a^2*f)*h - (a*c*h*x + a*c*g)*e)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2*c^2)]
```

Sympy [A]
time = 7.60, size = 209, normalized size = 2.09

$$dh\left(\begin{cases} -\frac{1}{c\sqrt{a+cx^2}} & \text{for } c\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)+eg\left(\begin{cases} -\frac{1}{c\sqrt{a+cx^2}} & \text{for } c\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)+eh\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,c\sqrt{1+\frac{cx^2}{a}}}\right)+fg\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,c\sqrt{1+\frac{cx^2}{a}}}\right)+fh\left(\begin{cases} \frac{2a}{c^2\sqrt{a+cx^2}}+\frac{x^2}{c\sqrt{a+cx^2}} & \text{for } c\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)+\frac{dgx}{a^{\frac{3}{2}}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] d*h*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)), True
)) + e*g*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)),
 True)) + e*h*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(1 + c*
x**2/a))) + f*g*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(1 +
c*x**2/a))) + f*h*Piecewise((2*a/(c**2*sqrt(a + c*x**2)) + x**2/(c*sqrt(a +
 c*x**2)), Ne(c, 0)), (x**4/(4*a**(3/2)), True)) + d*g*x/(a**(3/2)*sqrt(1 +
 c*x**2/a))
```

Giac [A]
time = 4.85, size = 116, normalized size = 1.16

$$\frac{\left(\frac{fhx}{c}+\frac{c^3dg-ac^2fg-ac^2he}{ac^3}\right)x-\frac{ac^2dh-2\,a^2cfh+ac^2ge}{ac^3}}{\sqrt{cx^2+a}}-\frac{(fg+he)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((f*h*x/c + (c^3*d*g - a*c^2*f*g - a*c^2*h*e)/(a*c^3))*x - (a*c^2*d*h - 2*a
^2*c*f*h + a*c^2*g*e)/(a*c^3))/sqrt(c*x^2 + a) - (f*g + h*e)*log(abs(-sqrt(
c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

Mupad [B]
time = 5.28, size = 151, normalized size = 1.51

$$\frac{e\,h\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{c^{3/2}}+\frac{f\,g\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{c^{3/2}}-\frac{d\,h}{c\,\sqrt{c\,x^2+a}}-\frac{e\,g}{c\,\sqrt{c\,x^2+a}}+\frac{d\,g\,x}{a\,\sqrt{c\,x^2+a}}-\frac{e\,h\,x}{c\,\sqrt{c\,x^2+a}}-\frac{f\,g\,x}{c\,\sqrt{c\,x^2+a}}+\frac{f\,h\,(c\,x^2+2\,a)}{c^2\,\sqrt{c\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x)
```

```
[Out] (e*h*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/c^(3/2) + (f*g*log(c^(1/2)*x + (a
+ c*x^2)^(1/2)))/c^(3/2) - (d*h)/(c*(a + c*x^2)^(1/2)) - (e*g)/(c*(a + c*x^
2)^(1/2)) + (d*g*x)/(a*(a + c*x^2)^(1/2)) - (e*h*x)/(c*(a + c*x^2)^(1/2)) -
 (f*g*x)/(c*(a + c*x^2)^(1/2)) + (f*h*(2*a + c*x^2))/(c^2*(a + c*x^2)^(1/2)
)
```

3.111 $\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}$$

```
[Out] f*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+(-a*e+(-a*f+c*d)*x)/a/c/(c*x^2
+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1828, 12, 223, 212}

$$\frac{f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}-\frac{ae-x(cd-af)}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + c*x^2)^(3/2),x]
```

```
[Out] -((a*e - (c*d - a*f)*x)/(a*c*Sqrt[a + c*x^2])) + (f*ArcTanh[(Sqrt[c]*x)/Sqr
t[a + c*x^2]])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}}\,dx &= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{\int \frac{af}{c\sqrt{a+cx^2}}\,dx}{a}\\ &= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{f\int \frac{1}{\sqrt{a+cx^2}}\,dx}{c}\\ &= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{f\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{c}\\ &= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.35, size = 62, normalized size = 1.02

$$\frac{-ae+cdx-afx}{ac\sqrt{a+cx^2}}-\frac{f\log\left(-\sqrt{c}\,x+\sqrt{a+cx^2}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + c*x^2)^(3/2),x]
```

```
[Out] (-(a*e) + c*d*x - a*f*x)/(a*c*Sqrt[a + c*x^2]) - (f*Log[-(Sqrt[c]*x) + Sqrt
[a + c*x^2]])/c^(3/2)
```

Maple [A]
time = 0.07, size = 70, normalized size = 1.15

method	result	size
default	$f\left(-\frac{x}{c\sqrt{c\,x^2+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{c^{\frac{3}{2}}}\right)-\frac{e}{c\sqrt{c\,x^2+a}}+\frac{dx}{a\sqrt{c\,x^2+a}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(-x/c/(c*x^2+a)^(1/2)+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))-e/c/(c*x^2
+a)^(1/2)+d*x/a/(c*x^2+a)^(1/2)
```

Maxima [A]
time = 0.29, size = 62, normalized size = 1.02

$$\frac{dx}{\sqrt{cx^2+a}\,a}-\frac{fx}{\sqrt{cx^2+a}\,c}+\frac{f\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}}-\frac{e}{\sqrt{cx^2+a}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] d*x/(sqrt(c*x^2 + a)*a) - f*x/(sqrt(c*x^2 + a)*c) + f*arcsinh(c*x/sqrt(a*c)
)/c^(3/2) - e/(sqrt(c*x^2 + a)*c)
```

Fricas [A]
time = 0.36, size = 183, normalized size = 3.00

$$\left[\frac{(acfx^2+a^2f)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\,\sqrt{cx^2+a}\,(ace-(c^2d-acf)x)}{2\,(ac^3x^2+a^2c^2)},\ -\frac{(acfx^2+a^2f)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right)+\sqrt{cx^2+a}\,(ace-(c^2d-acf)x)}{ac^3x^2+a^2c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c*f*x^2 + a^2*f)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*
x - a) - 2*sqrt(c*x^2 + a)*(a*c*e - (c^2*d - a*c*f)*x))/(a*c^3*x^2 + a^2*c^
2), -((a*c*f*x^2 + a^2*f)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + sqr
t(c*x^2 + a)*(a*c*e - (c^2*d - a*c*f)*x))/(a*c^3*x^2 + a^2*c^2)]
```

Sympy [A]
time = 3.50, size = 87, normalized size = 1.43

$$e\left(\begin{cases}-\frac{1}{c\sqrt{a+cx^2}} & \text{for}\ c\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+f\left(\frac{\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,c\sqrt{1+\frac{cx^2}{a}}}\right)+\frac{dx}{a^{\frac{3}{2}}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] e*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)), True))
 + f*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(1 + c*x**2/a)))
 + d*x/(a**(3/2)*sqrt(1 + c*x**2/a))
```

Giac [A]
time = 3.89, size = 63, normalized size = 1.03

$$-\frac{\frac{e}{c} - \frac{(c^2d - acf)x}{ac^2}}{\sqrt{cx^2 + a}} - \frac{f \log\left(\left|-\sqrt{c}\,x + \sqrt{cx^2 + a}\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(e/c - (c^2*d - a*c*f)*x/(a*c^2))/sqrt(c*x^2 + a) - f*log(abs(-sqrt(c)*x +
 sqrt(c*x^2 + a)))/c^(3/2)
```

Mupad [B]
time = 4.33, size = 68, normalized size = 1.11

$$\frac{f \ln\left(\sqrt{c}\,x + \sqrt{c\,x^2 + a}\right)}{c^{3/2}} - \frac{e}{c\,\sqrt{c\,x^2 + a}} + \frac{d\,x}{a\,\sqrt{c\,x^2 + a}} - \frac{f\,x}{c\,\sqrt{c\,x^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + c*x^2)^(3/2),x)
```

```
[Out] (f*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/c^(3/2) - e/(c*(a + c*x^2)^(1/2)) +
(d*x)/(a*(a + c*x^2)^(1/2)) - (f*x)/(c*(a + c*x^2)^(1/2))
```

3.112 $\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}}-\frac{\left(fg^2-egh+dh^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{\left(cg^2+ah^2\right)^{3/2}}$$

```
[Out] -(d*h^2-e*g*h+f*g^2)*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/
2))/(a*h^2+c*g^2)^(3/2)+(-a*(a*f*h-c*d*h+c*e*g)+c*(a*e*h-a*f*g+c*d*g)*x)/a/
c/(a*h^2+c*g^2)/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1661, 12, 739, 212}

$$-\frac{a(afh-cdh+ceg)-cx(aeh-afg+cdg)}{ac\sqrt{a+cx^2}\left(ah^2+cg^2\right)}-\frac{\left(dh^2-egh+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{\left(ah^2+cg^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((a*(c*e*g - c*d*h + a*f*h) - c*(c*d*g - a*f*g + a*e*h)*x)/(a*c*(c*g^2 + a
*h^2)*Sqrt[a + c*x^2])) - ((f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(S
qrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(c*g^2 + a*h^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}}+\frac{\int \frac{ac(fg^2-egh+dh^2)}{(cg^2+ah^2)(g+hx)\sqrt{a+cx^2}}\,d}{ac}$$

$$= -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}}+\frac{(fg^2-egh+dh^2)\int \frac{}{(g+hx)\sqrt{}}}{cg^2+ah^2}$$

$$= -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}}-\frac{(fg^2-egh+dh^2)\,\text{Subst}\Big(\int}{cg^2}$$

$$= -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}}-\frac{(fg^2-egh+dh^2)\tanh^{-1}\Big(}{(cg^2+a}$$

Mathematica [A]
time = 0.69, size = 147, normalized size = 1.07

$$\frac{-a^2fh+c^2dgx+ac(-eg+dh-fgx+ehx)}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}}+\frac{2(fg^2+h(-eg+dh))\tan^{-1}\left(\frac{\sqrt{c}\,(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{\left(-cg^2-ah^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-(a^2*f*h) + c^2*d*g*x + a*c*(-(e*g) + d*h - f*g*x + e*h*x))/(a*c*(c*g^2 +
 a*h^2)*Sqrt[a + c*x^2]) + (2*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[c]*(g
 + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(-(c*g^2) - a*h^2)^(3
/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(129) = 258$.
time = 0.11, size = 386, normalized size = 2.80

method	result
default	$\dfrac{-\frac{fh}{c\sqrt{c\,x^2+a}}+\frac{ehx}{a\sqrt{c\,x^2+a}}-\frac{gfx}{a\sqrt{c\,x^2+a}}}{h^2}+\dfrac{(d\,h^2-egh+f\,g^2)\left(\frac{h^2}{(a\,h^2+c\,g^2)\sqrt{\left(x+\frac{g}{h}\right)^2 c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{a\,h^2+}{h^2}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/h^2*(-f*h/c/(c*x^2+a)^(1/2)+e*h*x/a/(c*x^2+a)^(1/2)-g*f*x/a/(c*x^2+a)^(1/
2))+(d*h^2-e*g*h+f*g^2)/h^3*(1/(a*h^2+c*g^2)*h^2/((x+1/h*g)^2*c-2*c*g/h*(x+
1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+2*c*g*h/(a*h^2+c*g^2)*(2*c*(x+1/h*g)-2*c*g/
h)/(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(
a*h^2+c*g^2)/h^2)^(1/2)-1/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2
*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)
^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(133) = 266$.
time = 0.31, size = 457, normalized size = 3.31

$$\frac{cfg^3x}{\sqrt{cx^2+a}\,acg^2h^2+\sqrt{cx^2+a}\,a^2h^4}-\frac{cg^2xe}{\sqrt{cx^2+a}\,acg^2h+\sqrt{cx^2+a}\,a^2h^3}+\frac{cdgx}{\sqrt{cx^2+a}\,acg^2+\sqrt{cx^2+a}\,a^2h^2}+\frac{fg^2}{\sqrt{cx^2+a}\,cg^2h+\sqrt{cx^2+a}\,ah^3}-\frac{ge}{\sqrt{cx^2+a}\,cg^2+\sqrt{cx^2+a}\,ah^2}+\frac{d}{\frac{\sqrt{cx^2+a}\,cg^2}{h}+\sqrt{cx^2+a}\,ah}-\frac{fgx}{\sqrt{cx^2+a}\,ah^2}+\frac{fg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^3}+\frac{d\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h}+\frac{xe}{\sqrt{cx^2+a}\,ah}-\frac{g\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}\,|hx+g|}-\frac{ah}{\sqrt{ac}\,|hx+g|}\right)e}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^2}-\frac{f}{\sqrt{cx^2+a}\,ch}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] c*f*g^3*x/(sqrt(c*x^2 + a)*a*c*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) - c*g^2*x
*e/(sqrt(c*x^2 + a)*a*c*g^2*h + sqrt(c*x^2 + a)*a^2*h^3) + c*d*g*x/(sqrt(c*
x^2 + a)*a*c*g^2 + sqrt(c*x^2 + a)*a^2*h^2) + f*g^2/(sqrt(c*x^2 + a)*c*g^2*
h + sqrt(c*x^2 + a)*a*h^3) - g*e/(sqrt(c*x^2 + a)*c*g^2 + sqrt(c*x^2 + a)*a
*h^2) + d/(sqrt(c*x^2 + a)*c*g^2/h + sqrt(c*x^2 + a)*a*h) - f*g*x/(sqrt(c*x
^2 + a)*a*h^2) + f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a
*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^3) + d*arcsinh(c*g*x/(sqrt(a*c)
*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h) +
x*e/(sqrt(c*x^2 + a)*a*h) - g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/
(sqrt(a*c)*abs(h*x + g)))*e/((a + c*g^2/h^2)^(3/2)*h^2) - f/(sqrt(c*x^2 + a
)*c*h)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(133) = 266$.
time = 1.20, size = 734, normalized size = 5.32

$$\left[-\frac{\left(a^2cfg^2+a^2cdh^2+(ac^2fg^2+ac^2dh^2)x^2-(ac^2ghx^2+a^2cgh)e\right)\sqrt{cg^2+ah^2}\,\log\left(\frac{2\,acghx-acg^2-2\,a^2h^2-(2\,c^2g^2+ach^2)x^2+2\,\sqrt{cg^2+ah^2}\,(cgx-ah)\sqrt{cx^2+a}}{h^2x^2+2\,ghx+g^2}\right)-2\left((ac^2d-a^2cf)g^2h+(a^2cd-a^3f)h^3+((c^3d-ac^2f)g^3+(ac^2d-a^2cf)gh^2)x-(ac^2g^3+a^2cgh^2-(ac^2g^2h+a^2ch^3)x)e\right)\sqrt{cx^2+a}}{2\left(a^2c^3g^4+2\,a^3c^2g^2h^2+a^4ch^4+(ac^4g^4+2\,a^2c^3g^2h^2+a^3c^2h^4)x^2\right)},\;-\frac{\left(a^2cfg^2+a^2cdh^2+(ac^2fg^2+ac^2dh^2)x^2-(ac^2ghx^2+a^2cgh)e\right)\sqrt{-cg^2-ah^2}\,\arctan\left(\frac{\sqrt{-cg^2-ah^2}\,(cgx-ah)\sqrt{cx^2+a}}{acg^2+a^2h^2+(c^2g^2+ach^2)x^2}\right)-\left((ac^2d-a^2cf)g^2h+(a^2cd-a^3f)h^3+((c^3d-ac^2f)g^3+(ac^2d-a^2cf)gh^2)x-(ac^2g^3+a^2cgh^2-(ac^2g^2h+a^2ch^3)x)e\right)\sqrt{cx^2+a}}{a^2c^3g^4+2\,a^3c^2g^2h^2+a^4ch^4+(ac^4g^4+2\,a^2c^3g^2h^2+a^3c^2h^4)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*c*f*g^2 + a^2*c*d*h^2 + (a*c^2*f*g^2 + a*c^2*d*h^2)*x^2 - (a*c^
2*g*h*x^2 + a^2*c*g*h)*e)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 -
2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 + 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)
*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) - 2*((a*c^2*d - a^2*c*f)*g^2*h
 + (a^2*c*d - a^3*f)*h^3 + ((c^3*d - a*c^2*f)*g^3 + (a*c^2*d - a^2*c*f)*g*h
^2)*x - (a*c^2*g^3 + a^2*c*g*h^2 - (a*c^2*g^2*h + a^2*c*h^3)*x)*e)*sqrt(c*x
^2 + a))/(a^2*c^3*g^4 + 2*a^3*c^2*g^2*h^2 + a^4*c*h^4 + (a*c^4*g^4 + 2*a^2*
c^3*g^2*h^2 + a^3*c^2*h^4)*x^2), -((a^2*c*f*g^2 + a^2*c*d*h^2 + (a*c^2*f*g^
2 + a*c^2*d*h^2)*x^2 - (a*c^2*g*h*x^2 + a^2*c*g*h)*e)*sqrt(-c*g^2 - a*h^2)*
arctan(sqrt(-c*g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^
2 + (c^2*g^2 + a*c*h^2)*x^2)) - ((a*c^2*d - a^2*c*f)*g^2*h + (a^2*c*d - a^3
*f)*h^3 + ((c^3*d - a*c^2*f)*g^3 + (a*c^2*d - a^2*c*f)*g*h^2)*x - (a*c^2*g^
3 + a^2*c*g*h^2 - (a*c^2*g^2*h + a^2*c*h^3)*x)*e)*sqrt(c*x^2 + a))/(a^2*c^3
*g^4 + 2*a^3*c^2*g^2*h^2 + a^4*c*h^4 + (a*c^4*g^4 + 2*a^2*c^3*g^2*h^2 + a^3
*c^2*h^4)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2}{(a + cx^2)^{\frac{3}{2}}\,(g + hx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((a + c*x**2)**(3/2)*(g + h*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(133) = 266.
time = 5.77, size = 294, normalized size = 2.13

$$\frac{\frac{(c^3dg^3 - ac^2fg^3 + ac^2dgh^2 - a^2cfgh^2 + ac^2g^2he + a^2ch^3e)x}{ac^3g^4 + 2\,a^2c^2g^2h^2 + a^3ch^4} + \frac{ac^2dg^2h - a^2cfg^2h + a^2cdh^3 - a^3fh^3 - ac^2g^3e - a^2cgh^2e}{ac^3g^4 + 2\,a^2c^2g^2h^2 + a^3ch^4}}{\sqrt{cx^2 + a}} - \frac{2\,(fg^2 + dh^2 - ghe)\arctan\left(\frac{\left(\sqrt{c}\,x - \sqrt{cx^2 + a}\right)h + \sqrt{c}\,g}{\sqrt{-cg^2 - ah^2}}\right)}{(cg^2 + ah^2)\sqrt{-cg^2 - ah^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((c^3*d*g^3 - a*c^2*f*g^3 + a*c^2*d*g*h^2 - a^2*c*f*g*h^2 + a*c^2*g^2*h*e +
 a^2*c*h^3*e)*x/(a*c^3*g^4 + 2*a^2*c^2*g^2*h^2 + a^3*c*h^4) + (a*c^2*d*g^2*
h - a^2*c*f*g^2*h + a^2*c*d*h^3 - a^3*f*h^3 - a*c^2*g^3*e - a^2*c*g*h^2*e)/
(a*c^3*g^4 + 2*a^2*c^2*g^2*h^2 + a^3*c*h^4))/sqrt(c*x^2 + a) - 2*(f*g^2 + d
```

```
*h^2 - g*h*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*
g^2 - a*h^2))/((c*g^2 + a*h^2)*sqrt(-c*g^2 - a*h^2))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2+e\,x+d}{\left(g+h\,x\right)\,{\left(c\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)), x)
```

3.113 $\int \frac{d+ex+fx^2}{(g+hx)^2 (a+cx^2)^{3/2}} \, dx$

Optimal. Leaf size=239

$$-\frac{a(cg(eg-2dh)+ah(2fg-eh))-(c^2dg^2+a^2fh^2-ac(fg^2-h(2eg-dh)))\,x}{a\,(cg^2+ah^2)^2\sqrt{a+cx^2}}-\frac{h(fg^2-egh+dh^2)\sqrt{a}}{(cg^2+ah^2)^2\,(g+h}$$

```
[Out] (a*h^2*(-e*h+2*f*g)-c*g*(f*g^2-h*(-3*d*h+2*e*g)))*arctanh((-c*g*x+a*h)/(a*h
^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(5/2)+(-a*(c*g*(-2*d*h+e*g)+
a*h*(-e*h+2*f*g))+(c^2*d*g^2+a^2*f*h^2-a*c*(f*g^2-h*(-d*h+2*e*g)))*x)/a/(a*
h^2+c*g^2)^2/(c*x^2+a)^(1/2)-h*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(1/2)/(a*h^2+c
*g^2)^2/(h*x+g)
```

Rubi [A]

time = 0.26, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1661, 821, 739, 212}

$$-\frac{a(ah(2fg-eh)+cg(eg-2dh))-x(a^2fh^2-ac(fg^2-h(2eg-dh))+c^2dg^2)}{a\sqrt{a+cx^2}\,(ah^2+cg^2)^2}-\frac{h\sqrt{a+cx^2}\,(dh^2-egh+fg^2)}{(g+hx)\,(ah^2+cg^2)^2}-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\,\sqrt{ah^2+cg^2}}\right)(-ah^2(2fg-eh)-cgh(2eg-3dh)+cfg^3)}{(ah^2+cg^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((a*(c*g*(e*g - 2*d*h) + a*h*(2*f*g - e*h)) - (c^2*d*g^2 + a^2*f*h^2 - a*c
*(f*g^2 - h*(2*e*g - d*h)))*x)/(a*(c*g^2 + a*h^2)^2*Sqrt[a + c*x^2])) - (h*
(f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)^2*(g + h*x)) - ((
c*f*g^3 - c*g*h*(2*e*g - 3*d*h) - a*h^2*(2*f*g - e*h))*ArcTanh[(a*h - c*g*x
)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(c*g^2 + a*h^2)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
```

```
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+cx^2\right)^{3/2}}\,dx = -\frac{a(cg(eg-2dh)+ah(2fg-eh))-(c^2dg^2+a^2fh^2-ac(fg^2-h(2eg-dh)}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}}$$

$$= -\frac{a(cg(eg-2dh)+ah(2fg-eh))-(c^2dg^2+a^2fh^2-ac(fg^2-h(2eg-dh)}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}}$$

$$= -\frac{a(cg(eg-2dh)+ah(2fg-eh))-(c^2dg^2+a^2fh^2-ac(fg^2-h(2eg-dh)}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}}$$

$$= -\frac{a(cg(eg-2dh)+ah(2fg-eh))-(c^2dg^2+a^2fh^2-ac(fg^2-h(2eg-dh)}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}}$$

Mathematica [A]
time = 1.44, size = 248, normalized size = 1.04

$$\frac{c^2dg^2x(g+hx)+a^2h(h(2eg-dh+ehx)+f(-3g^2-ghx+h^2x^2))+ac(-fg^2x(g+2hx)+dh(2g^2+ghx-2h^2x^2)+eg(-g^2+ghx+3h^2x^2))}{a\left(cg^2+ah^2\right)^2(g+hx)\sqrt{a+cx^2}}-\frac{2(cfg^3+cgh(-2eg+3dh)+ah^2(-2fg+eh))\tan^{-1}\left(\frac{\sqrt{c}\,(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{\left(-cg^2-ah^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x]
```

```
[Out] (c^2*d*g^2*x*(g + h*x) + a^2*h*(h*(2*e*g - d*h + e*h*x) + f*(-3*g^2 - g*h*x
 + h^2*x^2)) + a*c*(-(f*g^2*x*(g + 2*h*x)) + d*h*(2*g^2 + g*h*x - 2*h^2*x^2
```

```
) + e*g*(-g^2 + g*h*x + 3*h^2*x^2)))/(a*(c*g^2 + a*h^2)^2*(g + h*x)*Sqrt[a
+ c*x^2]) - (2*(c*f*g^3 + c*g*h*(-2*e*g + 3*d*h) + a*h^2*(-2*f*g + e*h))*Ar
cTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(-(c*
g^2) - a*h^2)^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(228) = 456$.

time = 0.10, size = 872, normalized size = 3.65

method	result
default	$\frac{fx}{h^2 a\sqrt{c x^2+a}} + \frac{(eh-2gf)\left(\frac{h^2}{(a h^2+c g^2)\sqrt{\left(x+\frac{g}{h}\right)^2 c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{a h^2+c g^2}{h^2}}}+\frac{2cg \cdots}{(a h^2+c g^2)\left(\frac{4c\left(a h^2+c g^2\right)}{h^2}-\frac{4c^2 g^2}{h^2} \cdots\right.} \cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f/h^2*x/a/(c*x^2+a)^(1/2)+1/h^3*(e*h-2*f*g)*(1/(a*h^2+c*g^2)*h^2/((x+1/h*g)
^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+2*c*g*h/(a*h^2+c*g^2)*(2*c*
(x+1/h*g)-2*c*g/h)/(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2*c
*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-1/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/
h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^
(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))
)+1/h^4*(d*h^2-e*g*h+f*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)/((x+1/h*g)^2*c-
2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+3*c*g*h/(a*h^2+c*g^2)*(1/(a*h^2+
c*g^2)*h^2/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+2*c*g*
h/(a*h^2+c*g^2)*(2*c*(x+1/h*g)-2*c*g/h)/(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^
2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-1/(a*h^2+c*g^2
)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2
*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h
^2)^(1/2))/(x+1/h*g)))-4*c/(a*h^2+c*g^2)*h^2*(2*c*(x+1/h*g)-2*c*g/h)/(4*c*(
a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g
^2)/h^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1092 vs. $2(233) = 466$.

time = 0.39, size = 1092, normalized size = 4.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 3*c^2*f*g^4*x/(sqrt(c*x^2 + a)*a*c^2*g^4*h^2 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*
h^4 + sqrt(c*x^2 + a)*a^3*h^6) - 3*c^2*g^3*x*e/(sqrt(c*x^2 + a)*a*c^2*g^4*h
 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^3 + sqrt(c*x^2 + a)*a^3*h^5) + 3*c^2*d*g^2
*x/(sqrt(c*x^2 + a)*a*c^2*g^4 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^2 + sqrt(c*x^
2 + a)*a^3*h^4) + 3*c*f*g^3/(sqrt(c*x^2 + a)*c^2*g^4*h + 2*sqrt(c*x^2 + a)*
a*c*g^2*h^3 + sqrt(c*x^2 + a)*a^2*h^5) - 4*c*f*g^2*x/(sqrt(c*x^2 + a)*a*c*g
^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) - 3*c*g^2*e/(sqrt(c*x^2 + a)*c^2*g^4 + 2*
sqrt(c*x^2 + a)*a*c*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) + 3*c*g*x*e/(sqrt(c*
x^2 + a)*a*c*g^2*h + sqrt(c*x^2 + a)*a^2*h^3) + 3*c*d*g/(sqrt(c*x^2 + a)*c^
2*g^4/h + 2*sqrt(c*x^2 + a)*a*c*g^2*h + sqrt(c*x^2 + a)*a^2*h^3) - f*g^2/(s
qrt(c*x^2 + a)*c*g^2*h^2*x + sqrt(c*x^2 + a)*a*h^4*x + sqrt(c*x^2 + a)*c*g^
3*h + sqrt(c*x^2 + a)*a*g*h^3) - 2*c*d*x/(sqrt(c*x^2 + a)*a*c*g^2 + sqrt(c*
x^2 + a)*a^2*h^2) - 2*f*g/(sqrt(c*x^2 + a)*c*g^2*h + sqrt(c*x^2 + a)*a*h^3)
 + g*e/(sqrt(c*x^2 + a)*c*g^2*h*x + sqrt(c*x^2 + a)*a*h^3*x + sqrt(c*x^2 +
a)*c*g^3 + sqrt(c*x^2 + a)*a*g*h^2) - d/(sqrt(c*x^2 + a)*c*g^2*x + sqrt(c*x
^2 + a)*a*h^2*x + sqrt(c*x^2 + a)*c*g^3/h + sqrt(c*x^2 + a)*a*g*h) + e/(sqr
t(c*x^2 + a)*c*g^2 + sqrt(c*x^2 + a)*a*h^2) + f*x/(sqrt(c*x^2 + a)*a*h^2) +
 3*c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
+ g)))/((a + c*g^2/h^2)^(5/2)*h^5) + 3*c*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h
*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^3) - 2*f*
g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((
a + c*g^2/h^2)^(3/2)*h^3) - 3*c*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g))
- a*h/(sqrt(a*c)*abs(h*x + g)))*e/((a + c*g^2/h^2)^(5/2)*h^4) + arcsinh(c*g
*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))*e/((a + c*g^2/h
^2)^(3/2)*h^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(233) = 466$.
time = 1.71, size = 1609, normalized size = 6.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*c*f*g^4 + (3*a^2*c*d - 2*a^3*f)*g^2*h^2 + (a*c^2*f*g^3*h + (3*a*
c^2*d - 2*a^2*c*f)*g*h^3)*x^3 + (a*c^2*f*g^4 + (3*a*c^2*d - 2*a^2*c*f)*g^2*
h^2)*x^2 + (a^2*c*f*g^3*h + (3*a^2*c*d - 2*a^3*f)*g*h^3)*x - (2*a^2*c*g^3*h
```

```
 - a^3*g*h^3 + (2*a*c^2*g^2*h^2 - a^2*c*h^4)*x^3 + (2*a*c^2*g^3*h - a^2*c*g
*h^3)*x^2 + (2*a^2*c*g^2*h^2 - a^3*h^4)*x)*e)*sqrt(c*g^2 + a*h^2)*log((2*a*
c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 - 2*sqrt(c*g^2 +
a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) - 2*(a^3*d
*h^5 - (2*a*c^2*d - 3*a^2*c*f)*g^4*h - (a^2*c*d - 3*a^3*f)*g^2*h^3 - ((c^3*
d - 2*a*c^2*f)*g^4*h - (a*c^2*d + a^2*c*f)*g^2*h^3 - (2*a^2*c*d - a^3*f)*h^
5)*x^2 - ((c^3*d - a*c^2*f)*g^5 + 2*(a*c^2*d - a^2*c*f)*g^3*h^2 + (a^2*c*d
- a^3*f)*g*h^4)*x + (a*c^2*g^5 - a^2*c*g^3*h^2 - 2*a^3*g*h^4 - 3*(a*c^2*g^3
*h^2 + a^2*c*g*h^4)*x^2 - (a*c^2*g^4*h + 2*a^2*c*g^2*h^3 + a^3*h^5)*x)*e)*s
qrt(c*x^2 + a))/(a^2*c^3*g^7 + 3*a^3*c^2*g^5*h^2 + 3*a^4*c*g^3*h^4 + a^5*g*
h^6 + (a*c^4*g^6*h + 3*a^2*c^3*g^4*h^3 + 3*a^3*c^2*g^2*h^5 + a^4*c*h^7)*x^3
 + (a*c^4*g^7 + 3*a^2*c^3*g^5*h^2 + 3*a^3*c^2*g^3*h^4 + a^4*c*g*h^6)*x^2 +
(a^2*c^3*g^6*h + 3*a^3*c^2*g^4*h^3 + 3*a^4*c*g^2*h^5 + a^5*h^7)*x), -((a^2*
c*f*g^4 + (3*a^2*c*d - 2*a^3*f)*g^2*h^2 + (a*c^2*f*g^3*h + (3*a*c^2*d - 2*a
^2*c*f)*g*h^3)*x^3 + (a*c^2*f*g^4 + (3*a*c^2*d - 2*a^2*c*f)*g^2*h^2)*x^2 +
(a^2*c*f*g^3*h + (3*a^2*c*d - 2*a^3*f)*g*h^3)*x - (2*a^2*c*g^3*h - a^3*g*h^
3 + (2*a*c^2*g^2*h^2 - a^2*c*h^4)*x^3 + (2*a*c^2*g^3*h - a^2*c*g*h^3)*x^2 +
 (2*a^2*c*g^2*h^2 - a^3*h^4)*x)*e)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*g^2
- a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*
h^2)*x^2)) + (a^3*d*h^5 - (2*a*c^2*d - 3*a^2*c*f)*g^4*h - (a^2*c*d - 3*a^3*
f)*g^2*h^3 - ((c^3*d - 2*a*c^2*f)*g^4*h - (a*c^2*d + a^2*c*f)*g^2*h^3 - (2*
a^2*c*d - a^3*f)*h^5)*x^2 - ((c^3*d - a*c^2*f)*g^5 + 2*(a*c^2*d - a^2*c*f)*
g^3*h^2 + (a^2*c*d - a^3*f)*g*h^4)*x + (a*c^2*g^5 - a^2*c*g^3*h^2 - 2*a^3*g
*h^4 - 3*(a*c^2*g^3*h^2 + a^2*c*g*h^4)*x^2 - (a*c^2*g^4*h + 2*a^2*c*g^2*h^3
 + a^3*h^5)*x)*e)*sqrt(c*x^2 + a))/(a^2*c^3*g^7 + 3*a^3*c^2*g^5*h^2 + 3*a^4
*c*g^3*h^4 + a^5*g*h^6 + (a*c^4*g^6*h + 3*a^2*c^3*g^4*h^3 + 3*a^3*c^2*g^2*h
^5 + a^4*c*h^7)*x^3 + (a*c^4*g^7 + 3*a^2*c^3*g^5*h^2 + 3*a^3*c^2*g^3*h^4 +
a^4*c*g*h^6)*x^2 + (a^2*c^3*g^6*h + 3*a^3*c^2*g^4*h^3 + 3*a^4*c*g^2*h^5 + a
^5*h^7)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^2\,{\left(c\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)), x)
```

3.114 $\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=374

$$\frac{a(a^2fh^3-c^2g^2(eg-3dh)-ach(3fg^2-h(3eg-dh)))+c(c^2dg^3+a^2h^2(3fg-eh)-acg(fg^2-3h(eg-d$$
$$a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}$$

```
[Out] -1/2*(2*a^2*f*h^4-a*c*h^2*(3*d*h^2-9*e*g*h+11*f*g^2)+2*c^2*g^2*(6*d*h^2-3*e
*g*h+f*g^2))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h
^2+c*g^2)^(7/2)+(a*(a^2*f*h^3-c^2*g^2*(-3*d*h+e*g)-a*c*h*(3*f*g^2-h*(-d*h+3
*e*g)))+c*(c^2*d*g^3+a^2*h^2*(-e*h+3*f*g)-a*c*g*(f*g^2-3*h*(-d*h+e*g)))*x)/
a/(a*h^2+c*g^2)^3/(c*x^2+a)^(1/2)-1/2*h*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(1/2)
/(a*h^2+c*g^2)^2/(h*x+g)^2+1/2*h*(2*a*h^2*(-e*h+2*f*g)-c*g*(3*f*g^2-h*(-7*d
*h+5*e*g)))*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^3/(h*x+g)
```

Rubi [A]
time = 0.69, antiderivative size = 372, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1661, 1665, 821, 739, 212}

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4-ach^2(3dh^2-9egh+11fg^2)+2c^2g^2(6dh^2-3egh+fg^2)\right)}{2\left(ah^2+cg^2\right)^{7/2}}+\frac{a(a^2fh^3-ach(3fg^2-h(3eg-dh))-c^2g^2(eg-3dh))+cx(a^2h^2(3fg-eh)-acg(fg^2-3h(eg-dh))+c^2dg^3)}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^3}-\frac{h\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{2(g+hx)^2\left(ah^2+cg^2\right)^2}-\frac{h\sqrt{a+cx^2}\left(-2ah^2(2fg-eh)-cgh(5eg-7dh)+3cfg^3\right)}{2(g+hx)\left(ah^2+cg^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*(a^2*f*h^3 - c^2*g^2*(e*g - 3*d*h) - a*c*h*(3*f*g^2 - h*(3*e*g - d*h)))
+ c*(c^2*d*g^3 + a^2*h^2*(3*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(e*g - d*h)))*x
)/(a*(c*g^2 + a*h^2)^3*Sqrt[a + c*x^2]) - (h*(f*g^2 - e*g*h + d*h^2)*Sqrt[a
 + c*x^2])/(2*(c*g^2 + a*h^2)^2*(g + h*x)^2) - (h*(3*c*f*g^3 - c*g*h*(5*e*g
 - 7*d*h) - 2*a*h^2*(2*f*g - e*h))*Sqrt[a + c*x^2])/(2*(c*g^2 + a*h^2)^3*(g
 + h*x)) - ((2*a^2*f*h^4 - a*c*h^2*(11*f*g^2 - 9*e*g*h + 3*d*h^2) + 2*c^2*g
^2*(f*g^2 - 3*e*g*h + 6*d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*
Sqrt[a + c*x^2]])])/(2*(c*g^2 + a*h^2)^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx = \frac{a(a^2fh^3-c^2g^2(eg-3dh)-ach(3fg^2-h(3eg-dh)))+c(c^2dg^3+a^2h^2(3f}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

$$= \frac{a(a^2fh^3-c^2g^2(eg-3dh)-ach(3fg^2-h(3eg-dh)))+c(c^2dg^3+a^2h^2(3f}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

$$= \frac{a(a^2fh^3-c^2g^2(eg-3dh)-ach(3fg^2-h(3eg-dh)))+c(c^2dg^3+a^2h^2(3f}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

$$= \frac{a(a^2fh^3-c^2g^2(eg-3dh)-ach(3fg^2-h(3eg-dh)))+c(c^2dg^3+a^2h^2(3f}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

$$= \frac{a(a^2fh^3-c^2g^2(eg-3dh)-ach(3fg^2-h(3eg-dh)))+c(c^2dg^3+a^2h^2(3f}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1537 vs. $2(374) = 748$.
time = 11.20, size = 1537, normalized size = 4.11

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a^4*h^3*(f*(5*g^2 + 8*g*h*x + 2*h^2*x^2) - h*(d*h + e*(g + 2*h*x))) - 4*c^
(7/2)*g^2*x^2*(-(Sqrt[c]*x) + Sqrt[a + c*x^2])*(d*(-g^3 + 4*g^2*h*x + 18*g*
h^2*x^2 + 12*h^3*x^3) + g*x*(f*g*x*(3*g + 2*h*x) - e*(2*g^2 + 9*g*h*x + 6*h
^2*x^2))) - a^3*(Sqrt[c]*h^2*Sqrt[a + c*x^2]*(-3*d*h^3*x + e*h*(4*g^2 + 5*g
*h*x - 2*h^2*x^2) + f*(-10*g^3 - 5*g^2*h*x + 14*g*h^2*x^2 + 6*h^3*x^3)) + c
*h*(f*(10*g^4 + 39*g^3*h*x + 26*g^2*h^2*x^2 - 20*g*h^3*x^3 - 10*h^4*x^4) +
h*(d*h*(10*g^2 + 11*g*h*x + 8*h^2*x^2) - e*(12*g^3 + 27*g^2*h*x + 20*g*h^2*
x^2 - 2*h^3*x^3)))) + a^2*(c^2*(d*h*(6*g^4 + 45*g^3*h*x + 14*g^2*h^2*x^2 -
29*g*h^3*x^3 - 19*h^4*x^4) + e*g*(-2*g^4 - 31*g^3*h*x + 10*g^2*h^2*x^2 + 81
*g*h^3*x^3 + 57*h^4*x^4) + f*x*(13*g^5 - 28*g^4*h*x - 105*g^3*h^2*x^2 - 75*
g^2*h^3*x^3 + 12*g*h^4*x^4 + 8*h^5*x^5)) + c^(3/2)*Sqrt[a + c*x^2]*(f*(-5*g
^5 + 20*g^4*h*x + 72*g^3*h^2*x^2 + 53*g^2*h^3*x^3 - 12*g*h^4*x^4 - 8*h^5*x^
5) + h*(e*g*(11*g^3 - 14*g^2*h*x - 54*g*h^2*x^2 - 39*h^3*x^3) + d*h*(-13*g^
```

```
3 + 4*g^2*h*x + 20*g*h^2*x^2 + 13*h^3*x^3)))) + a*(c^(5/2)*Sqrt[a + c*x^2]*
(d*(2*g^5 - 14*g^4*h*x - 81*g^3*h^2*x^2 - 48*g^2*h^3*x^3 + 18*g*h^4*x^4 + 1
2*h^5*x^5) + g*x*(f*g*x*(-19*g^3 + 14*g^2*h*x + 66*g*h^2*x^2 + 44*h^3*x^3)
+ e*(6*g^4 + 49*g^3*h*x + 14*g^2*h^2*x^2 - 54*g*h^3*x^3 - 36*h^4*x^4))) - c
^3*x*(d*(4*g^5 - 22*g^4*h*x - 117*g^3*h^2*x^2 - 72*g^2*h^3*x^3 + 18*g*h^4*x
^4 + 12*h^5*x^5) + g*x*(f*g*x*(-25*g^3 + 10*g^2*h*x + 66*g*h^2*x^2 + 44*h^3
*x^3) + e*(10*g^4 + 67*g^3*h*x + 26*g^2*h^2*x^2 - 54*g*h^3*x^3 - 36*h^4*x^4
)))))/(2*(c*g^2 + a*h^2)^3*(g + h*x)^2*(a + c*x^2)*(a*(-3*Sqrt[c]*x + Sqrt[
a + c*x^2]) + 4*c*x^2*(-(Sqrt[c]*x) + Sqrt[a + c*x^2]))) - (15*a*f*h^2*ArcT
an[(-(Sqrt[c]*(g + h*x)) + h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(-(c
*g^2) - a*h^2)^(5/2) + (21*a*e*h^3*ArcTan[(-(Sqrt[c]*(g + h*x)) + h*Sqrt[a
+ c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(g*(-(c*g^2) - a*h^2)^(5/2)) - (2*f*ArcT
an[(-(Sqrt[c]*(g + h*x)) + h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(-(c
*g^2) - a*h^2)^(3/2) + (6*e*h*ArcTan[(-(Sqrt[c]*(g + h*x)) + h*Sqrt[a + c*x
^2])/Sqrt[-(c*g^2) - a*h^2]])/(g*(-(c*g^2) - a*h^2)^(3/2)) - (12*d*h^2*ArcT
an[(-(Sqrt[c]*(g + h*x)) + h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(g^2
*(-(c*g^2) - a*h^2)^(3/2)) - (27*a*d*h^4*ArcTan[(-(Sqrt[c]*(g + h*x)) + h*S
qrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(Sqrt[-(c*g^2) - a*h^2]*(c*g^3 + a
*g*h^2)^2) - (15*a^2*h^4*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(-(Sqrt[c]*(g +
h*x)) + h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(g^2*(-(c*g^2) - a*h^2)
^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1786 vs. $2(356) = 712$.
time = 0.08, size = 1787, normalized size = 4.78

method	result	size
default	Expression too large to display	1787

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f/h^3*(1/(a*h^2+c*g^2)*h^2/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h
^2)^(1/2)+2*c*g*h/(a*h^2+c*g^2)*(2*c*(x+1/h*g)-2*c*g/h)/(4*c*(a*h^2+c*g^2)/
h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2
)-1/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c
*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)
+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+(e*h-2*f*g)/h^4*(-1/(a*h^2+c*g^2)*h^
2/(x+1/h*g)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+3*c*g
*h/(a*h^2+c*g^2)*(1/(a*h^2+c*g^2)*h^2/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h
^2+c*g^2)/h^2)^(1/2)+2*c*g*h/(a*h^2+c*g^2)*(2*c*(x+1/h*g)-2*c*g/h)/(4*c*(a*
h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2
)/h^2)^(1/2)-1/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g
^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/
h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))-4*c/(a*h^2+c*g^2)*h^2*(2*
```

```
c*(x+1/h*g)-2*c*g/h)/(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2
*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))+(d*h^2-e*g*h+f*g^2)/h^5*(-1/2/(a
*h^2+c*g^2)*h^2/(x+1/h*g)^2/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/
h^2)^(1/2)+5/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)/((x+1/h*
g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+3*c*g*h/(a*h^2+c*g^2)*(1/
(a*h^2+c*g^2)*h^2/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)
+2*c*g*h/(a*h^2+c*g^2)*(2*c*(x+1/h*g)-2*c*g/h)/(4*c*(a*h^2+c*g^2)/h^2-4*c^2
*g^2/h^2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-1/(a*h^
2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1
/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c
*g^2)/h^2)^(1/2))/(x+1/h*g)))-4*c/(a*h^2+c*g^2)*h^2*(2*c*(x+1/h*g)-2*c*g/h)
/(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*
h^2+c*g^2)/h^2)^(1/2))-3/2*c/(a*h^2+c*g^2)*h^2*(1/(a*h^2+c*g^2)*h^2/((x+1/h
*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+2*c*g*h/(a*h^2+c*g^2)*(2
*c*(x+1/h*g)-2*c*g/h)/(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-
2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-1/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^
2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^
2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*
g))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2264 vs. 2(363) = 726.
time = 0.39, size = 2264, normalized size = 6.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 15/2*c^3*f*g^5*x/(sqrt(c*x^2 + a)*a*c^3*g^6*h^2 + 3*sqrt(c*x^2 + a)*a^2*c^2
*g^4*h^4 + 3*sqrt(c*x^2 + a)*a^3*c*g^2*h^6 + sqrt(c*x^2 + a)*a^4*h^8) - 15/
2*c^3*g^4*x*e/(sqrt(c*x^2 + a)*a*c^3*g^6*h + 3*sqrt(c*x^2 + a)*a^2*c^2*g^4*
h^3 + 3*sqrt(c*x^2 + a)*a^3*c*g^2*h^5 + sqrt(c*x^2 + a)*a^4*h^7) + 15/2*c^3
*d*g^3*x/(sqrt(c*x^2 + a)*a*c^3*g^6 + 3*sqrt(c*x^2 + a)*a^2*c^2*g^4*h^2 + 3
*sqrt(c*x^2 + a)*a^3*c*g^2*h^4 + sqrt(c*x^2 + a)*a^4*h^6) + 15/2*c^2*f*g^4/
(sqrt(c*x^2 + a)*c^3*g^6*h + 3*sqrt(c*x^2 + a)*a*c^2*g^4*h^3 + 3*sqrt(c*x^2
 + a)*a^2*c*g^2*h^5 + sqrt(c*x^2 + a)*a^3*h^7) - 25/2*c^2*f*g^3*x/(sqrt(c*x
^2 + a)*a*c^2*g^4*h^2 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^4 + sqrt(c*x^2 + a)*a
^3*h^6) - 15/2*c^2*g^3*e/(sqrt(c*x^2 + a)*c^3*g^6 + 3*sqrt(c*x^2 + a)*a*c^2
*g^4*h^2 + 3*sqrt(c*x^2 + a)*a^2*c*g^2*h^4 + sqrt(c*x^2 + a)*a^3*h^6) + 19/
2*c^2*g^2*x*e/(sqrt(c*x^2 + a)*a*c^2*g^4*h + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^
3 + sqrt(c*x^2 + a)*a^3*h^5) + 15/2*c^2*d*g^2/(sqrt(c*x^2 + a)*c^3*g^6/h +
3*sqrt(c*x^2 + a)*a*c^2*g^4*h + 3*sqrt(c*x^2 + a)*a^2*c*g^2*h^3 + sqrt(c*x^
2 + a)*a^3*h^5) - 5/2*c*f*g^3/(sqrt(c*x^2 + a)*c^2*g^4*h^2*x + 2*sqrt(c*x^2
 + a)*a*c*g^2*h^4*x + sqrt(c*x^2 + a)*a^2*h^6*x + sqrt(c*x^2 + a)*c^2*g^5*h
```

```
 + 2*sqrt(c*x^2 + a)*a*c*g^3*h^3 + sqrt(c*x^2 + a)*a^2*g*h^5) - 13/2*c^2*d*
g*x/(sqrt(c*x^2 + a)*a*c^2*g^4 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^2 + sqrt(c*x
^2 + a)*a^3*h^4) - 15/2*c*f*g^2/(sqrt(c*x^2 + a)*c^2*g^4*h + 2*sqrt(c*x^2 +
 a)*a*c*g^2*h^3 + sqrt(c*x^2 + a)*a^2*h^5) + 5*c*f*g*x/(sqrt(c*x^2 + a)*a*c
*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) + 5/2*c*g^2*e/(sqrt(c*x^2 + a)*c^2*g^4*
h*x + 2*sqrt(c*x^2 + a)*a*c*g^2*h^3*x + sqrt(c*x^2 + a)*a^2*h^5*x + sqrt(c*
x^2 + a)*c^2*g^5 + 2*sqrt(c*x^2 + a)*a*c*g^3*h^2 + sqrt(c*x^2 + a)*a^2*g*h^
4) - 5/2*c*d*g/(sqrt(c*x^2 + a)*c^2*g^4*x + 2*sqrt(c*x^2 + a)*a*c*g^2*h^2*x
 + sqrt(c*x^2 + a)*a^2*h^4*x + sqrt(c*x^2 + a)*c^2*g^5/h + 2*sqrt(c*x^2 + a
)*a*c*g^3*h + sqrt(c*x^2 + a)*a^2*g*h^3) - 1/2*f*g^2/(sqrt(c*x^2 + a)*c*g^2
*h^3*x^2 + sqrt(c*x^2 + a)*a*h^5*x^2 + 2*sqrt(c*x^2 + a)*c*g^3*h^2*x + 2*sq
rt(c*x^2 + a)*a*g*h^4*x + sqrt(c*x^2 + a)*c*g^4*h + sqrt(c*x^2 + a)*a*g^2*h
^3) + 9/2*c*g*e/(sqrt(c*x^2 + a)*c^2*g^4 + 2*sqrt(c*x^2 + a)*a*c*g^2*h^2 +
sqrt(c*x^2 + a)*a^2*h^4) - 2*c*x*e/(sqrt(c*x^2 + a)*a*c*g^2*h + sqrt(c*x^2
+ a)*a^2*h^3) - 3/2*c*d/(sqrt(c*x^2 + a)*c^2*g^4/h + 2*sqrt(c*x^2 + a)*a*c*
g^2*h + sqrt(c*x^2 + a)*a^2*h^3) + 2*f*g/(sqrt(c*x^2 + a)*c*g^2*h^2*x + sqr
t(c*x^2 + a)*a*h^4*x + sqrt(c*x^2 + a)*c*g^3*h + sqrt(c*x^2 + a)*a*g*h^3) +
 1/2*g*e/(sqrt(c*x^2 + a)*c*g^2*h^2*x^2 + sqrt(c*x^2 + a)*a*h^4*x^2 + 2*sqr
t(c*x^2 + a)*c*g^3*h*x + 2*sqrt(c*x^2 + a)*a*g*h^3*x + sqrt(c*x^2 + a)*c*g^
4 + sqrt(c*x^2 + a)*a*g^2*h^2) - 1/2*d/(sqrt(c*x^2 + a)*c*g^2*h*x^2 + sqrt(
c*x^2 + a)*a*h^3*x^2 + 2*sqrt(c*x^2 + a)*c*g^3*x + 2*sqrt(c*x^2 + a)*a*g*h^
2*x + sqrt(c*x^2 + a)*c*g^4/h + sqrt(c*x^2 + a)*a*g^2*h) + f/(sqrt(c*x^2 +
a)*c*g^2*h + sqrt(c*x^2 + a)*a*h^3) - e/(sqrt(c*x^2 + a)*c*g^2*h*x + sqrt(c
*x^2 + a)*a*h^3*x + sqrt(c*x^2 + a)*c*g^3 + sqrt(c*x^2 + a)*a*g*h^2) + 15/2
*c^2*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
+ g)))/((a + c*g^2/h^2)^(7/2)*h^7) + 15/2*c^2*d*g^2*arcsinh(c*g*x/(sqrt(a*c
)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^5)
 - 15/2*c*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs
(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^5) - 3/2*c*d*arcsinh(c*g*x/(sqrt(a*c)*
abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^3) +
 f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(
(a + c*g^2/h^2)^(3/2)*h^3) - 15/2*c^2*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x
+ g)) - a*h/(sqrt(a*c)*abs(h*x + g)))*e/((a + c*g^2/h^2)^(7/2)*h^6) + 9/2*c
*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))*e
/((a + c*g^2/h^2)^(5/2)*h^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1433 vs. $2(363) = 726$.
time = 5.94, size = 2893, normalized size = 7.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

[Out] [1/4*((2*a^2*c^2*f*g^6 + (12*a^2*c^2*d - 11*a^3*c*f)*g^4*h^2 - (3*a^3*c*d - 2*a^4*f)*g^2*h^4 + (2*a*c^3*f*g^4*h^2 + (12*a*c^3*d - 11*a^2*c^2*f)*g^2*h^4 - (3*a^2*c^2*d - 2*a^3*c*f)*h^6)*x^4 + 2*(2*a*c^3*f*g^5*h + (12*a*c^3*d - 11*a^2*c^2*f)*g^3*h^3 - (3*a^2*c^2*d - 2*a^3*c*f)*g*h^5)*x^3 + (2*a*c^3*f*g^6 + 3*(4*a*c^3*d - 3*a^2*c^2*f)*g^4*h^2 + 9*(a^2*c^2*d - a^3*c*f)*g^2*h^4 - (3*a^3*c*d - 2*a^4*f)*h^6)*x^2 + 2*(2*a^2*c^2*f*g^5*h + (12*a^2*c^2*d - 11*a^3*c*f)*g^3*h^3 - (3*a^3*c*d - 2*a^4*f)*g*h^5)*x - 3*(2*a^2*c^2*g^5*h - 3*a^3*c*g^3*h^3 + (2*a*c^3*g^3*h^3 - 3*a^2*c^2*g*h^5)*x^4 + 2*(2*a*c^3*g^4*h^2 - 3*a^2*c^2*g^2*h^4)*x^3 + (2*a*c^3*g^5*h - a^2*c^2*g^3*h^3 - 3*a^3*c*g*h^5)*x^2 + 2*(2*a^2*c^2*g^4*h^2 - 3*a^3*c*g^2*h^4)*x)*e)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) - 2*(a^4*d*h^7 - 2*(3*a*c^3*d - 5*a^2*c^2*f)*g^6*h + (4*a^2*c^2*d + 5*a^3*c*f)*g^4*h^3 + (11*a^3*c*d - 5*a^4*f)*g^2*h^5 - ((2*c^4*d - 5*a*c^3*f)*g^5*h^2 - (11*a*c^3*d - 5*a^2*c^2*f)*g^3*h^4 - (13*a^2*c^2*d - 10*a^3*c*f)*g*h^6)*x^3 - (4*(c^4*d - 2*a*c^3*f)*g^6*h - (10*a*c^3*d - a^2*c^2*f)*g^4*h^3 - (17*a^2*c^2*d - 11*a^3*c*f)*g^2*h^5 - (3*a^3*c*d - 2*a^4*f)*h^7)*x^2 - (2*(c^4*d - a*c^3*f)*g^7 + (8*a*c^3*d - 11*a^2*c^2*f)*g^5*h^2 - (5*a^2*c^2*d + a^3*c*f)*g^3*h^4 - (11*a^3*c*d - 8*a^4*f)*g*h^6)*x + (2*a*c^3*g^7 - 10*a^2*c^2*g^5*h^2 - 11*a^3*c*g^3*h^4 + a^4*g*h^6 - (11*a*c^3*g^4*h^3 + 7*a^2*c^2*g^2*h^5 - 4*a^3*c*h^7)*x^3 - (16*a*c^3*g^5*h^2 + 17*a^2*c^2*g^3*h^4 + a^3*c*g*h^6)*x^2 - (2*a*c^3*g^6*h + 17*a^2*c^2*g^4*h^3 + 13*a^3*c*g^2*h^5 - 2*a^4*h^7)*x)*e)*sqrt(c*x^2 + a))/(a^2*c^4*g^10 + 4*a^3*c^3*g^8*h^2 + 6*a^4*c^2*g^6*h^4 + 4*a^5*c*g^4*h^6 + a^6*g^2*h^8 + (a*c^5*g^8*h^2 + 4*a^2*c^4*g^6*h^4 + 6*a^3*c^3*g^4*h^6 + 4*a^4*c^2*g^2*h^8 + a^5*c*h^10)*x^4 + 2*(a*c^5*g^9*h + 4*a^2*c^4*g^7*h^3 + 6*a^3*c^3*g^5*h^5 + 4*a^4*c^2*g^3*h^7 + a^5*c*g*h^9)*x^3 + (a*c^5*g^10 + 5*a^2*c^4*g^8*h^2 + 10*a^3*c^3*g^6*h^4 + 10*a^4*c^2*g^4*h^6 + 5*a^5*c*g^2*h^8 + a^6*h^10)*x^2 + 2*(a^2*c^4*g^9*h + 4*a^3*c^3*g^7*h^3 + 6*a^4*c^2*g^5*h^5 + 4*a^5*c*g^3*h^7 + a^6*g*h^9)*x), -1/2*((2*a^2*c^2*f*g^6 + (12*a^2*c^2*d - 11*a^3*c*f)*g^4*h^2 - (3*a^3*c*d - 2*a^4*f)*g^2*h^4 + (2*a*c^3*f*g^4*h^2 + (12*a*c^3*d - 11*a^2*c^2*f)*g^2*h^4 - (3*a^2*c^2*d - 2*a^3*c*f)*h^6)*x^4 + 2*(2*a*c^3*f*g^5*h + (12*a*c^3*d - 11*a^2*c^2*f)*g^3*h^3 - (3*a^2*c^2*d - 2*a^3*c*f)*g*h^5)*x^3 + (2*a*c^3*f*g^6 + 3*(4*a*c^3*d - 3*a^2*c^2*f)*g^4*h^2 + 9*(a^2*c^2*d - a^3*c*f)*g^2*h^4 - (3*a^3*c*d - 2*a^4*f)*h^6)*x^2 + 2*(2*a^2*c^2*f*g^5*h + (12*a^2*c^2*d - 11*a^3*c*f)*g^3*h^3 - (3*a^3*c*d - 2*a^4*f)*g*h^5)*x - 3*(2*a^2*c^2*g^5*h - 3*a^3*c*g^3*h^3 + (2*a*c^3*g^3*h^3 - 3*a^2*c^2*g*h^5)*x^4 + 2*(2*a*c^3*g^4*h^2 - 3*a^2*c^2*g^2*h^4)*x^3 + (2*a*c^3*g^5*h - a^2*c^2*g^3*h^3 - 3*a^3*c*g*h^5)*x^2 + 2*(2*a^2*c^2*g^4*h^2 - 3*a^3*c*g^2*h^4)*x)*e)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*h^2)*x^2)) + (a^4*d*h^7 - 2*(3*a*c^3*d - 5*a^2*c^2*f)*g^6*h + (4*a^2*c^2*d + 5*a^3*c*f)*g^4*h^3 + (11*a^3*c*d - 5*a^4*f)*g^2*h^5 - ((2*c^4*d - 5*a*c^3*f)*g^5*h^2 - (11*a*c^3*d - 5*a^2*c^2*f)*g^3*h^4 - (13*a^2*c^2*d - 10*a^3*c*f)*g*h^6)*x^3 - (4*(c^4*d - 2*a*c^3*f)*g^6*h - (10*a*c^3*d - a^2*c^2*f)*g^4*h^3 - (17*a^2*c^2*d - 11*a^3*c*f)*g^2*h^5 - (3*a^3*c*d - 2*a^

```
4*f)*h^7)*x^2 - (2*(c^4*d - a*c^3*f)*g^7 + (8*a*c^3*d - 11*a^2*c^2*f)*g^5*h
^2 - (5*a^2*c^2*d + a^3*c*f)*g^3*h^4 - (11*a^3*c*d - 8*a^4*f)*g*h^6)*x + (2
*a*c^3*g^7 - 10*a^2*c^2*g^5*h^2 - 11*a^3*c*g^3*h^4 + a^4*g*h^6 - (11*a*c^3*
g^4*h^3 + 7*a^2*c^2*g^2*h^5 - 4*a^3*c*h^7)*x^3 - (16*a*c^3*g^5*h^2 + 17*a^2
*c^2*g^3*h^4 + a^3*c*g*h^6)*x^2 - (2*a*c^3*g^6*h + 17*a^2*c^2*g^4*h^3 + 13*
a^3*c*g^2*h^5 - 2*a^4*h^7)*x)*e)*sqrt(c*x^2 + a))/(a^2*c^4*g^10 + 4*a^3*c^3
*g^8*h^2 + 6*a^4*c^2*g^6*h^4 + 4*a^5*c*g^4*h^6 + a^6*g^2*h^8 + (a*c^5*g^8*h
^2 + 4*a^2*c^4*g^6*h^4 + 6*a^3*c^3*g^4*h^6 + 4*a^4*c^2*g^2*h^8 + a^5*c*h^10
)*x^4 + 2*(a*c^5*g^9*h + 4*a^2*c^4*g^7*h^3 + 6*a^3*c^3*g^5*h^5 + 4*a^4*c^2*
g^3*h^7 + a^5*c*g*h^9)*x^3 + (a*c^5*g^10 + 5*a^2*c^4*g^8*h^2 + 10*a^3*c^3*g
^6*h^4 + 10*a^4*c^2*g^4*h^6 + 5*a^5*c*g^2*h^8 + a^6*h^10)*x^2 + 2*(a^2*c^4*
g^9*h + 4*a^3*c^3*g^7*h^3 + 6*a^4*c^2*g^5*h^5 + 4*a^5*c*g^3*h^7 + a^6*g*h^9
)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1440 vs. $2(363) = 726$.
time = 6.49, size = 1440, normalized size = 3.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((c^6*d*g^9 - a*c^5*f*g^9 - 6*a^2*c^4*d*g^5*h^4 + 6*a^3*c^3*f*g^5*h^4 - 8*a
^3*c^3*d*g^3*h^6 + 8*a^4*c^2*f*g^3*h^6 - 3*a^4*c^2*d*g*h^8 + 3*a^5*c*f*g*h^
8 + 3*a*c^5*g^8*h*e + 8*a^2*c^4*g^6*h^3*e + 6*a^3*c^3*g^4*h^5*e - a^5*c*h^9
*e)*x/(a*c^6*g^12 + 6*a^2*c^5*g^10*h^2 + 15*a^3*c^4*g^8*h^4 + 20*a^4*c^3*g^
6*h^6 + 15*a^5*c^2*g^4*h^8 + 6*a^6*c*g^2*h^10 + a^7*h^12) + (3*a*c^5*d*g^8*
h - 3*a^2*c^4*f*g^8*h + 8*a^2*c^4*d*g^6*h^3 - 8*a^3*c^3*f*g^6*h^3 + 6*a^3*c
^3*d*g^4*h^5 - 6*a^4*c^2*f*g^4*h^5 - a^5*c*d*h^9 + a^6*f*h^9 - a*c^5*g^9*e
+ 6*a^3*c^3*g^5*h^4*e + 8*a^4*c^2*g^3*h^6*e + 3*a^5*c*g*h^8*e)/(a*c^6*g^12
+ 6*a^2*c^5*g^10*h^2 + 15*a^3*c^4*g^8*h^4 + 20*a^4*c^3*g^6*h^6 + 15*a^5*c^2
*g^4*h^8 + 6*a^6*c*g^2*h^10 + a^7*h^12))/sqrt(c*x^2 + a) - (2*c^2*f*g^4 + 1
2*c^2*d*g^2*h^2 - 11*a*c*f*g^2*h^2 - 3*a*c*d*h^4 + 2*a^2*f*h^4 - 6*c^2*g^3*
h*e + 9*a*c*g*h^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/s
qrt(-c*g^2 - a*h^2))/((c^3*g^6 + 3*a*c^2*g^4*h^2 + 3*a^2*c*g^2*h^4 + a^3*h^
```

```
6)*sqrt(-c*g^2 - a*h^2)) - (2*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*f*g^4*h +
 6*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*d*g^2*h^3 - 5*(sqrt(c)*x - sqrt(c*x^
2 + a))^3*a*c*f*g^2*h^3 - (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*d*h^5 - 4*(sq
rt(c)*x - sqrt(c*x^2 + a))^3*c^2*g^3*h^2*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a)
)^3*a*c*g*h^4*e + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 + 14*(sqr
t(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*d*g^3*h^2 - 11*(sqrt(c)*x - sqrt(c*x^2
+ a))^2*a*c^(3/2)*f*g^3*h^2 - 7*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d
*g*h^4 + 4*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 - 10*(sqrt(c
)*x - sqrt(c*x^2 + a))^2*c^(5/2)*g^4*h*e + 9*(sqrt(c)*x - sqrt(c*x^2 + a))^
2*a*c^(3/2)*g^2*h^3*e - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*h^5*e
 - 10*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*f*g^4*h - 22*(sqrt(c)*x - sqrt(c*
x^2 + a))*a*c^2*d*g^2*h^3 + 11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^
3 - (sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*d*h^5 + 16*(sqrt(c)*x - sqrt(c*x^2
+ a))*a*c^2*g^3*h^2*e - 5*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*g*h^4*e + 3*a
^2*c^(3/2)*f*g^3*h^2 + 7*a^2*c^(3/2)*d*g*h^4 - 4*a^3*sqrt(c)*f*g*h^4 - 5*a^
2*c^(3/2)*g^2*h^3*e + 2*a^3*sqrt(c)*h^5*e)/((c^3*g^6 + 3*a*c^2*g^4*h^2 + 3*
a^2*c*g^2*h^4 + a^3*h^6)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x
- sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^3\,{\left(c\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)), x)
```

3.115 $\int \frac{A+Bx+Cx^2}{(a+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{aB-(Ac-aC)x}{3ac\,(a+cx^2)^{3/2}}+\frac{(2Ac+aC)x}{3a^2c\sqrt{a+cx^2}}$$

```
[Out] 1/3*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^(3/2)+1/3*(2*A*c+C*a)*x/a^2/c/(c*x^2+a
)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1828, 12, 197}

$$\frac{x(aC+2Ac)}{3a^2c\sqrt{a+cx^2}}-\frac{aB-x(Ac-aC)}{3ac\,(a+cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^(5/2),x]
```

```
[Out] -1/3*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)^(3/2)) + ((2*A*c + a*C)*x)/(3*a
^2*c*Sqrt[a + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{5/2}}\,dx = -\frac{aB-(Ac-aC)x}{3ac\,(a+cx^2)^{3/2}} - \frac{\int \frac{-2A-\frac{aC}{c}}{(a+cx^2)^{3/2}}\,dx}{3a}$$

$$= -\frac{aB-(Ac-aC)x}{3ac\,(a+cx^2)^{3/2}} + \frac{(2Ac+aC)\int \frac{1}{(a+cx^2)^{3/2}}\,dx}{3ac}$$

$$= -\frac{aB-(Ac-aC)x}{3ac\,(a+cx^2)^{3/2}} + \frac{(2Ac+aC)x}{3a^2c\sqrt{a+cx^2}}$$

Mathematica [A]
time = 0.45, size = 50, normalized size = 0.75

$$\frac{-a^2B+3aAcx+2Ac^2x^3+acCx^3}{3a^2c\,(a+cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(5/2),x]
```

```
[Out] (-(a^2*B) + 3*a*A*c*x + 2*A*c^2*x^3 + a*c*C*x^3)/(3*a^2*c*(a + c*x^2)^(3/2)
)
```

Maple [A]
time = 0.07, size = 105, normalized size = 1.57

method	result
gosper	$\frac{2A\,c^2x^3+Cac\,x^3+3Axac-a^2B}{3(c\,x^2+a)^{\frac{3}{2}}\,a^2c}$
trager	$\frac{2A\,c^2x^3+Cac\,x^3+3Axac-a^2B}{3(c\,x^2+a)^{\frac{3}{2}}\,a^2c}$
default	$C\left(-\frac{x}{2c(c\,x^2+a)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(c\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{c\,x^2+a}}\right)}{2c}\right)-\frac{B}{3c(c\,x^2+a)^{\frac{3}{2}}}+A\left(\frac{x}{3a(c\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{c\,x^2+a}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-1/2*x/c/(c*x^2+a)^(3/2)+1/2*a/c*(1/3*x/a/(c*x^2+a)^(3/2)+2/3/a^2*x/(c*x
^2+a)^(1/2)))-1/3*B/c/(c*x^2+a)^(3/2)+A*(1/3*x/a/(c*x^2+a)^(3/2)+2/3/a^2*x/
(c*x^2+a)^(1/2))
```

Maxima [A]
time = 0.29, size = 83, normalized size = 1.24

$$\frac{2\,Ax}{3\,\sqrt{cx^2+a}\,a^2}+\frac{Ax}{3\,(cx^2+a)^{\frac{3}{2}}a}-\frac{Cx}{3\,(cx^2+a)^{\frac{3}{2}}c}+\frac{Cx}{3\,\sqrt{cx^2+a}\,ac}-\frac{B}{3\,(cx^2+a)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*A*x/(sqrt(c*x^2 + a)*a^2) + 1/3*A*x/((c*x^2 + a)^(3/2)*a) - 1/3*C*x/((c
*x^2 + a)^(3/2)*c) + 1/3*C*x/(sqrt(c*x^2 + a)*a*c) - 1/3*B/((c*x^2 + a)^(3/
2)*c)
```

Fricas [A]
time = 0.32, size = 68, normalized size = 1.01

$$\frac{(3\,Aacx+(Cac+2\,Ac^2)x^3-Ba^2)\sqrt{cx^2+a}}{3\,(a^2c^3x^4+2\,a^3c^2x^2+a^4c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*A*a*c*x + (C*a*c + 2*A*c^2)*x^3 - B*a^2)*sqrt(c*x^2 + a)/(a^2*c^3*x^
4 + 2*a^3*c^2*x^2 + a^4*c)
```

Sympy [A]
time = 5.93, size = 194, normalized size = 2.90

$$A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^2}{a}}+3a^{\frac{5}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}}+\frac{2cx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^2}{a}}+3a^{\frac{5}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}}\right)+B\left(\begin{cases}-\frac{1}{3ac\sqrt{a+cx^2}+3c^2x^2\sqrt{a+cx^2}} & \text{for } c\neq 0\\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)+\frac{Cx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{cx^2}{a}}+3a^{\frac{3}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a*x/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 + c*x**2
/a)) + 2*c*x**3/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 +
 c*x**2/a))) + B*Piecewise((-1/(3*a*c*sqrt(a + c*x**2) + 3*c**2*x**2*sqrt(a
 + c*x**2)), Ne(c, 0)), (x**2/(2*a**(5/2)), True)) + C*x**3/(3*a**(5/2)*sqr
t(1 + c*x**2/a) + 3*a**(3/2)*c*x**2*sqrt(1 + c*x**2/a))
```

Giac [A]
time = 8.35, size = 48, normalized size = 0.72

$$\frac{x\left(\frac{3\,A}{a}+\frac{(Cac+2\,Ac^2)x^2}{a^2c}\right)-\frac{B}{c}}{3\,(cx^2+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(x*(3*A/a + (C*a*c + 2*A*c^2)*x^2/(a^2*c)) - B/c)/(c*x^2 + a)^(3/2)
```

Mupad [B]
time = 4.22, size = 59, normalized size = 0.88

$$\frac{2\,A\,c\,x\,\left(c\,x^2+a\right)-C\,a^2\,x-B\,a^2+C\,a\,x\,\left(c\,x^2+a\right)+A\,a\,c\,x}{3\,a^2\,c\,{\left(c\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^(5/2),x)
```

```
[Out] (2*A*c*x*(a + c*x^2) - C*a^2*x - B*a^2 + C*a*x*(a + c*x^2) + A*a*c*x)/(3*a^
2*c*(a + c*x^2)^(3/2))
```

3.116 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{aB-(Ac-aC)x}{5ac\left(a+cx^2\right)^{5/2}}+\frac{(4Ac+aC)x}{15a^2c\left(a+cx^2\right)^{3/2}}+\frac{2(4Ac+aC)x}{15a^3c\sqrt{a+cx^2}}$$

```
[Out] 1/5*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^(5/2)+1/15*(4*A*c+C*a)*x/a^2/c/(c*x^2+
a)^(3/2)+2/15*(4*A*c+C*a)*x/a^3/c/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1828, 12, 198, 197}

$$\frac{2x(aC+4Ac)}{15a^3c\sqrt{a+cx^2}}+\frac{x(aC+4Ac)}{15a^2c\left(a+cx^2\right)^{3/2}}-\frac{aB-x(Ac-aC)}{5ac\left(a+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^(7/2),x]
```

```
[Out] -1/5*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)^(5/2)) + ((4*A*c + a*C)*x)/(15*
a^2*c*(a + c*x^2)^(3/2)) + (2*(4*A*c + a*C)*x)/(15*a^3*c*Sqrt[a + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
```

```
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(a+cx^2)^{7/2}}\,dx &= -\frac{aB-(Ac-aC)x}{5ac\,(a+cx^2)^{5/2}}-\frac{\int \frac{-4A-\frac{aC}{c}}{(a+cx^2)^{5/2}}\,dx}{5a}\\ &= -\frac{aB-(Ac-aC)x}{5ac\,(a+cx^2)^{5/2}}+\frac{(4Ac+aC)\int \frac{1}{(a+cx^2)^{5/2}}\,dx}{5ac}\\ &= -\frac{aB-(Ac-aC)x}{5ac\,(a+cx^2)^{5/2}}+\frac{(4Ac+aC)x}{15a^2c\,(a+cx^2)^{3/2}}+\frac{(2(4Ac+aC))\int \frac{1}{(a+cx^2)^{3/2}}\,dx}{15a^2c}\\ &= -\frac{aB-(Ac-aC)x}{5ac\,(a+cx^2)^{5/2}}+\frac{(4Ac+aC)x}{15a^2c\,(a+cx^2)^{3/2}}+\frac{2(4Ac+aC)x}{15a^3c\sqrt{a+cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.51, size = 71, normalized size = 0.73

$$\frac{-3a^3B+8Ac^3x^5+5a^2cx(3A+Cx^2)+2ac^2x^3(10A+Cx^2)}{15a^3c\,(a+cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(7/2),x]
[Out] (-3*a^3*B + 8*A*c^3*x^5 + 5*a^2*c*x*(3*A + C*x^2) + 2*a*c^2*x^3*(10*A + C*x
^2))/(15*a^3*c*(a + c*x^2)^(5/2))
```

Maple [A]
time = 0.09, size = 147, normalized size = 1.52

method	result
gosper	$\frac{8c^3Ax^5+2c^2aCx^5+20Aac^2x^3+5Ca^2cx^3+15a^2cAx-3a^3B}{15(cx^2+a)^{\frac{5}{2}}a^3c}$
trager	$\frac{8c^3Ax^5+2c^2aCx^5+20Aac^2x^3+5Ca^2cx^3+15a^2cAx-3a^3B}{15(cx^2+a)^{\frac{5}{2}}a^3c}$

default	$C\left(-\frac{x}{4c(cx^2+a)^{\frac{5}{2}}}+\frac{a\left(\frac{x}{5a(cx^2+a)^{\frac{5}{2}}}+\frac{\frac{4x}{15a(cx^2+a)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{cx^2+a}}}{a}\right)}{4c}\right)-\frac{B}{5c(cx^2+a)^{\frac{5}{2}}}+A\left(\frac{x}{5a(cx^2+a)^{\frac{5}{2}}}+\frac{\quad}{15a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-1/4*x/c/(c*x^2+a)^(5/2)+1/4*a/c*(1/5*x/a/(c*x^2+a)^(5/2)+4/5/a*(1/3*x/a
/(c*x^2+a)^(3/2)+2/3/a^2*x/(c*x^2+a)^(1/2))))-1/5*B/c/(c*x^2+a)^(5/2)+A*(1/
5*x/a/(c*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(c*x^2+a)^(3/2)+2/3/a^2*x/(c*x^2+a)^(1
/2)))
```

Maxima [A]
time = 0.28, size = 118, normalized size = 1.22

$$\frac{8\,Ax}{15\,\sqrt{cx^2+a}\,a^3}+\frac{4\,Ax}{15\,(cx^2+a)^{\frac{3}{2}}a^2}+\frac{Ax}{5\,(cx^2+a)^{\frac{5}{2}}a}-\frac{Cx}{5\,(cx^2+a)^{\frac{5}{2}}c}+\frac{2\,Cx}{15\,\sqrt{cx^2+a}\,a^2c}+\frac{Cx}{15\,(cx^2+a)^{\frac{3}{2}}ac}-\frac{B}{5\,(cx^2+a)^{\frac{5}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*A*x/(sqrt(c*x^2 + a)*a^3) + 4/15*A*x/((c*x^2 + a)^(3/2)*a^2) + 1/5*A*x
/((c*x^2 + a)^(5/2)*a) - 1/5*C*x/((c*x^2 + a)^(5/2)*c) + 2/15*C*x/(sqrt(c*x
^2 + a)*a^2*c) + 1/15*C*x/((c*x^2 + a)^(3/2)*a*c) - 1/5*B/((c*x^2 + a)^(5/2
)*c)
```

Fricas [A]
time = 0.35, size = 103, normalized size = 1.06

$$\frac{(2\,(Cac^2+4\,Ac^3)x^5+15\,Aa^2cx-3\,Ba^3+5\,(Ca^2c+4\,Aac^2)x^3)\sqrt{cx^2+a}}{15\,(a^3c^4x^6+3\,a^4c^3x^4+3\,a^5c^2x^2+a^6c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(C*a*c^2 + 4*A*c^3)*x^5 + 15*A*a^2*c*x - 3*B*a^3 + 5*(C*a^2*c + 4*A
*a*c^2)*x^3)*sqrt(c*x^2 + a)/(a^3*c^4*x^6 + 3*a^4*c^3*x^4 + 3*a^5*c^2*x^2 +
 a^6*c)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(87) = 174.
time = 13.37, size = 638, normalized size = 6.58

$$A\left(\frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}+\frac{35a^4cx^3}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}+\frac{28a^3c^2x^5}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}+\frac{8a^2c^3x^7}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}\right)+B\left(\begin{cases}-\frac{1}{5a^2c\sqrt{a+cx^2}+10ac^2x^2\sqrt{a+cx^2}+5c^3x^4\sqrt{a+cx^2}} & \text{for}\ c\neq 0\\ \frac{x^2}{2a^{\frac{7}{2}}} & \text{otherwise}\end{cases}\right)+C\left(\frac{5ax^3}{15a^{\frac{9}{2}}\sqrt{1+\frac{cx^2}{a}}+30a^{\frac{7}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{5}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}}+\frac{2cx^5}{15a^{\frac{9}{2}}\sqrt{1+\frac{cx^2}{a}}+30a^{\frac{7}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{5}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**(7/2),x)
```

```
[Out] A*(15*a**5*x/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1
+ c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3
*x**6*sqrt(1 + c*x**2/a)) + 35*a**4*c*x**3/(15*a**(17/2)*sqrt(1 + c*x**2/a)
 + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 +
 c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a)) + 28*a**3*c**2*x**5
/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a)
+ 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1
 + c*x**2/a)) + 8*a**2*c**3*x**7/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(
15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a)
 + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a))) + B*Piecewise((-1/(5*a**2*c*
sqrt(a + c*x**2) + 10*a*c**2*x**2*sqrt(a + c*x**2) + 5*c**3*x**4*sqrt(a + c
*x**2)), Ne(c, 0)), (x**2/(2*a**(7/2)), True)) + C*(5*a*x**3/(15*a**(9/2)*s
qrt(1 + c*x**2/a) + 30*a**(7/2)*c*x**2*sqrt(1 + c*x**2/a) + 15*a**(5/2)*c**
2*x**4*sqrt(1 + c*x**2/a)) + 2*c*x**5/(15*a**(9/2)*sqrt(1 + c*x**2/a) + 30*
a**(7/2)*c*x**2*sqrt(1 + c*x**2/a) + 15*a**(5/2)*c**2*x**4*sqrt(1 + c*x**2/
a)))
```

Giac [A]
time = 5.27, size = 80, normalized size = 0.82

$$\frac{\left(x^2\left(\frac{2\,(Cac^3+4\,Ac^4)x^2}{a^3c^2}+\frac{5\,(Ca^2c^2+4\,Aac^3)}{a^3c^2}\right)+\frac{15\,A}{a}\right)x-\frac{3\,B}{c}}{15\,(cx^2+a)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((x^2*(2*(C*a*c^3 + 4*A*c^4)*x^2/(a^3*c^2) + 5*(C*a^2*c^2 + 4*A*a*c^3)
/(a^3*c^2)) + 15*A/a)*x - 3*B/c)/(c*x^2 + a)^(5/2)
```

Mupad [B]
time = 4.28, size = 93, normalized size = 0.96

$$\frac{8\,A\,c\,x\,{\left(c\,x^2+a\right)}^2-3\,C\,a^3\,x-3\,B\,a^3+2\,C\,a\,x\,{\left(c\,x^2+a\right)}^2+C\,a^2\,x\,\left(c\,x^2+a\right)+3\,A\,a^2\,c\,x+4\,A\,a\,c\,x\,\left(c\,x^2+a\right)}{15\,a^3\,c\,{\left(c\,x^2+a\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^(7/2),x)
```

```
[Out] (8*A*c*x*(a + c*x^2)^2 - 3*C*a^3*x - 3*B*a^3 + 2*C*a*x*(a + c*x^2)^2 + C*a^
2*x*(a + c*x^2) + 3*A*a^2*c*x + 4*A*a*c*x*(a + c*x^2))/(15*a^3*c*(a + c*x^2
)^(5/2))
```

3.117 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}}+\frac{(6Ac+aC)x}{35a^2c\left(a+cx^2\right)^{5/2}}+\frac{4(6Ac+aC)x}{105a^3c\left(a+cx^2\right)^{3/2}}+\frac{8(6Ac+aC)x}{105a^4c\sqrt{a+cx^2}}$$

```
[Out] 1/7*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^(7/2)+1/35*(6*A*c+C*a)*x/a^2/c/(c*x^2+
a)^(5/2)+4/105*(6*A*c+C*a)*x/a^3/c/(c*x^2+a)^(3/2)+8/105*(6*A*c+C*a)*x/a^4/
c/(c*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1828, 12, 198, 197}

$$\frac{8x(aC+6Ac)}{105a^4c\sqrt{a+cx^2}}+\frac{4x(aC+6Ac)}{105a^3c\left(a+cx^2\right)^{3/2}}+\frac{x(aC+6Ac)}{35a^2c\left(a+cx^2\right)^{5/2}}-\frac{aB-x(Ac-aC)}{7ac\left(a+cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^(9/2),x]
```

```
[Out] -1/7*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)^(7/2)) + ((6*A*c + a*C)*x)/(35*
a^2*c*(a + c*x^2)^(5/2)) + (4*(6*A*c + a*C)*x)/(105*a^3*c*(a + c*x^2)^(3/2)
) + (8*(6*A*c + a*C)*x)/(105*a^4*c*Sqrt[a + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{(a+cx^2)^{9/2}}\,dx &= -\frac{aB-(Ac-aC)x}{7ac\,(a+cx^2)^{7/2}}-\frac{\int \frac{-6A-\frac{aC}{c}}{(a+cx^2)^{7/2}}\,dx}{7a}\\
&= -\frac{aB-(Ac-aC)x}{7ac\,(a+cx^2)^{7/2}}+\frac{(6Ac+aC)\int \frac{1}{(a+cx^2)^{7/2}}\,dx}{7ac}\\
&= -\frac{aB-(Ac-aC)x}{7ac\,(a+cx^2)^{7/2}}+\frac{(6Ac+aC)x}{35a^2c\,(a+cx^2)^{5/2}}+\frac{(4(6Ac+aC))\int \frac{1}{(a+cx^2)^{5/2}}\,dx}{35a^2c}\\
&= -\frac{aB-(Ac-aC)x}{7ac\,(a+cx^2)^{7/2}}+\frac{(6Ac+aC)x}{35a^2c\,(a+cx^2)^{5/2}}+\frac{4(6Ac+aC)x}{105a^3c\,(a+cx^2)^{3/2}}+\frac{(8(6Ac+aC))\ldots}{105a\ldots}\\
&= -\frac{aB-(Ac-aC)x}{7ac\,(a+cx^2)^{7/2}}+\frac{(6Ac+aC)x}{35a^2c\,(a+cx^2)^{5/2}}+\frac{4(6Ac+aC)x}{105a^3c\,(a+cx^2)^{3/2}}+\frac{8(6Ac+aC)x}{105a^4c\sqrt{a+cx\ldots}}
\end{aligned}$$

Mathematica [A]
time = 0.61, size = 92, normalized size = 0.72

$$\frac{-15a^4B+48Ac^4x^7+35a^3cx(3A+Cx^2)+8ac^3x^5(21A+Cx^2)+14a^2c^2x^3(15A+2Cx^2)}{105a^4c\,(a+cx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(9/2),x]
```

```
[Out] (-15*a^4*B + 48*A*c^4*x^7 + 35*a^3*c*x*(3*A + C*x^2) + 8*a*c^3*x^5*(21*A +
C*x^2) + 14*a^2*c^2*x^3*(15*A + 2*C*x^2))/(105*a^4*c*(a + c*x^2)^(7/2))
```

Maple [A]
time = 0.11, size = 189, normalized size = 1.49

method	result
gosper	$\frac{48A\,c^4x^7+8Ca\,c^3x^7+168Aa\,c^3x^5+28C\,a^2c^2x^5+210A\,a^2c^2x^3+35C\,a^3c\,x^3+105Ax\,a^3c-15B\,a^4}{105(c\,x^2+a)^{\frac{7}{2}}a^4c}$

trager	$\frac{48A\,c^{4}x^{7}+8Ca\,c^{3}x^{7}+168Aa\,c^{3}x^{5}+28C\,a^{2}c^{2}x^{5}+210A\,a^{2}c^{2}x^{3}+35C\,a^{3}c\,x^{3}+105Ax\,a^{3}c-15B\,a^{4}}{105(c\,x^{2}+a)^{\frac{7}{2}}a^{4}c}$
default	$C\left(-\frac{x}{6c(c\,x^{2}+a)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(c\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(c\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(c\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{c\,x^{2}+a}}\right)}{7a}}{a}\right)}{6c}\right)-\frac{B}{7c(c\,x^{2}+a)^{\frac{7}{2}}}+A\left(\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-1/6*x/c/(c*x^2+a)^(7/2)+1/6*a/c*(1/7*x/a/(c*x^2+a)^(7/2)+6/7/a*(1/5*x/a
/(c*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(c*x^2+a)^(3/2)+2/3/a^2*x/(c*x^2+a)^(1/2)))
))-1/7*B/c/(c*x^2+a)^(7/2)+A*(1/7*x/a/(c*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(c*x^2
+a)^(5/2)+4/5/a*(1/3*x/a/(c*x^2+a)^(3/2)+2/3/a^2*x/(c*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.28, size = 153, normalized size = 1.20

$$\frac{16\,Ax}{35\,\sqrt{cx^2+a}\,a^4}+\frac{8\,Ax}{35\,(cx^2+a)^{\frac{3}{2}}a^3}+\frac{6\,Ax}{35\,(cx^2+a)^{\frac{5}{2}}a^2}+\frac{Ax}{7\,(cx^2+a)^{\frac{7}{2}}a}-\frac{Cx}{7\,(cx^2+a)^{\frac{7}{2}}c}+\frac{8\,Cx}{105\,\sqrt{cx^2+a}\,a^3c}+\frac{4\,Cx}{105\,(cx^2+a)^{\frac{3}{2}}a^2c}+\frac{Cx}{35\,(cx^2+a)^{\frac{5}{2}}ac}-\frac{B}{7\,(cx^2+a)^{\frac{7}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*A*x/(sqrt(c*x^2 + a)*a^4) + 8/35*A*x/((c*x^2 + a)^(3/2)*a^3) + 6/35*A
*x/((c*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((c*x^2 + a)^(7/2)*a) - 1/7*C*x/((c*x^
2 + a)^(7/2)*c) + 8/105*C*x/(sqrt(c*x^2 + a)*a^3*c) + 4/105*C*x/((c*x^2 + a
)^(3/2)*a^2*c) + 1/35*C*x/((c*x^2 + a)^(5/2)*a*c) - 1/7*B/((c*x^2 + a)^(7/2
)*c)
```

Fricas [A]
time = 0.35, size = 137, normalized size = 1.08

$$\frac{(8\,(Cac^3+6\,Ac^4)x^7+105\,Aa^3cx+28\,(Ca^2c^2+6\,Aac^3)x^5-15\,Ba^4+35\,(Ca^3c+6\,Aa^2c^2)x^3)\sqrt{cx^2+a}}{105\,(a^4c^5x^8+4\,a^5c^4x^6+6\,a^6c^3x^4+4\,a^7c^2x^2+a^8c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(C*a*c^3 + 6*A*c^4)*x^7 + 105*A*a^3*c*x + 28*(C*a^2*c^2 + 6*A*a*c^
3)*x^5 - 15*B*a^4 + 35*(C*a^3*c + 6*A*a^2*c^2)*x^3)*sqrt(c*x^2 + a)/(a^4*c^
5*x^8 + 4*a^5*c^4*x^6 + 6*a^6*c^3*x^4 + 4*a^7*c^2*x^2 + a^8*c)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. $2(117) = 234$.

time = 26.93, size = 1880, normalized size = 14.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(
1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*
c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) +
 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt
(1 + c*x**2/a)) + 175*a**13*c*x**3/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a
**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**
2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8
*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(
25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 371*a**12*c**2*x**5/(35*a**(37/2)*sq
rt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*
c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) +
 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt
(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 429*a**11*c*
*3*x**7/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*
x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x
**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a
**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c
*x**2/a)) + 286*a**10*c**4*x**9/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(
35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a
) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sq
rt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/
2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 104*a**9*c**5*x**11/(35*a**(37/2)*sqrt(
1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**
2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 52
5*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1
+ c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 16*a**8*c**6*x*
*13/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2
/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*
sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(2
7/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**
2/a))) + B*Piecewise((-1/(7*a**3*c*sqrt(a + c*x**2) + 21*a**2*c**2*x**2*sqr
```

```
t(a + c*x**2) + 21*a*c**3*x**4*sqrt(a + c*x**2) + 7*c**4*x**6*sqrt(a + c*x*
*2)), Ne(c, 0)), (x**2/(2*a**(9/2)), True)) + C*(35*a**5*x**3/(105*a**(19/2
)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**2/a) + 630*a**(15
/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6*sqrt(1 + c*x**2/
a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)) + 63*a**4*c*x**5/(105*a**(
19/2)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**2/a) + 630*a*
*(15/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6*sqrt(1 + c*x
**2/a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)) + 36*a**3*c**2*x**7/(1
05*a**(19/2)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**2/a) +
 630*a**(15/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6*sqrt(
1 + c*x**2/a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)) + 8*a**2*c**3*x
**9/(105*a**(19/2)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**
2/a) + 630*a**(15/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6
*sqrt(1 + c*x**2/a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)))
```

Giac [A]
time = 2.79, size = 112, normalized size = 0.88

$$\frac{\left(\left(4\,x^2\left(\frac{2\,(Cac^5+6\,Ac^6)x^2}{a^4c^3}+\frac{7\,(Ca^2c^4+6\,Aac^5)}{a^4c^3}\right)+\frac{35\,(Ca^3c^3+6\,Aa^2c^4)}{a^4c^3}\right)x^2+\frac{105\,A}{a}\right)x-\frac{15\,B}{c}}{105\,(cx^2+a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*x^2*(2*(C*a*c^5 + 6*A*c^6)*x^2/(a^4*c^3) + 7*(C*a^2*c^4 + 6*A*a*
c^5)/(a^4*c^3)) + 35*(C*a^3*c^3 + 6*A*a^2*c^4)/(a^4*c^3))*x^2 + 105*A/a)*x
- 15*B/c)/(c*x^2 + a)^(7/2)
```

Mupad [B]
time = 4.37, size = 115, normalized size = 0.91

$$\frac{x\,(6\,A\,c+C\,a)}{35\,a^2\,c\,{(c\,x^2+a)}^{5/2}}-\frac{\frac{B}{7\,c}-x\,\left(\frac{A}{7\,a}-\frac{C}{7\,c}\right)}{{(c\,x^2+a)}^{7/2}}+\frac{x\,(24\,A\,c+4\,C\,a)}{105\,a^3\,c\,{(c\,x^2+a)}^{3/2}}+\frac{x\,(48\,A\,c+8\,C\,a)}{105\,a^4\,c\,\sqrt{c\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^(9/2),x)
```

```
[Out] (x*(6*A*c + C*a))/(35*a^2*c*(a + c*x^2)^(5/2)) - (B/(7*c) - x*(A/(7*a) - C/
(7*c)))/(a + c*x^2)^(7/2) + (x*(24*A*c + 4*C*a))/(105*a^3*c*(a + c*x^2)^(3/
2)) + (x*(48*A*c + 8*C*a))/(105*a^4*c*(a + c*x^2)^(1/2))
```

3.118 $\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=106

$$-\frac{19}{540}(1+2x)^2\sqrt{2+3x^2}+\frac{13}{60}(1+2x)^3\sqrt{2+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2+3x^2}-\frac{1}{810}(3937+2073x)\sqrt{2+3x^2}+\frac{5\,\mathrm{s}}{}$$

```
[Out] 5/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)-19/540*(1+2*x)^2*(3*x^2+2)^(1/2)+13/60*(
1+2*x)^3*(3*x^2+2)^(1/2)+2/15*(1+2*x)^4*(3*x^2+2)^(1/2)-1/810*(3937+2073*x)
*(3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1668, 847, 794, 221}

$$\frac{2}{15}\sqrt{3x^2+2}\,(2x+1)^4+\frac{13}{60}\sqrt{3x^2+2}\,(2x+1)^3-\frac{19}{540}\sqrt{3x^2+2}\,(2x+1)^2-\frac{1}{810}(2073x+3937)\sqrt{3x^2+2}+\frac{5\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (-19*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/540 + (13*(1 + 2*x)^3*Sqrt[2 + 3*x^2])/60
 + (2*(1 + 2*x)^4*Sqrt[2 + 3*x^2])/15 - ((3937 + 2073*x)*Sqrt[2 + 3*x^2])/8
10 + (5*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
```

```
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} + \frac{1}{60}\int \frac{(1+2x)^3(-68+156x)}{\sqrt{2+3x^2}}\,dx$$

$$= \frac{13}{60}(1+2x)^3\sqrt{2+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} + \frac{1}{720}\int \frac{(-2688-228x)}{\sqrt{2+3x}}$$

$$= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2} + \frac{13}{60}(1+2x)^3\sqrt{2+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2+3x}$$

$$= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2} + \frac{13}{60}(1+2x)^3\sqrt{2+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2+3x}$$

$$= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2} + \frac{13}{60}(1+2x)^3\sqrt{2+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2+3x}$$

Mathematica [A]
time = 0.20, size = 66, normalized size = 0.62

$$\frac{1}{405}\sqrt{2+3x^2}\left(-1841-135x+2292x^2+2430x^3+864x^4\right) - \frac{5\log\left(-\sqrt{3}\,x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (Sqrt[2 + 3*x^2]*(-1841 - 135*x + 2292*x^2 + 2430*x^3 + 864*x^4))/405 - (5*
Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

Maple [A]
time = 0.12, size = 79, normalized size = 0.75

method	result
risch	$\frac{(864x^4+2430x^3+2292x^2-135x-1841)\sqrt{3x^2+2}}{405}+\frac{5\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$
trager	$\left(\frac{32}{15}x^4+6x^3+\frac{764}{135}x^2-\frac{1}{3}x-\frac{1841}{405}\right)\sqrt{3x^2+2}-\frac{5\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
default	$\frac{32x^4\sqrt{3x^2+2}}{15}+\frac{764x^2\sqrt{3x^2+2}}{135}-\frac{1841\sqrt{3x^2+2}}{405}+6x^3\sqrt{3x^2+2}-\frac{x\sqrt{3x^2+2}}{3}+\frac{5\,\mathrm{arcsinh}\left(\frac{x\sqrt{}}{2}\right.}{9}$
meijerg	$\frac{\sqrt{3}\,\mathrm{arcsinh}\left(\frac{x\sqrt{3}\sqrt{2}}{2}\right)}{3}+\frac{34\sqrt{3}\left(\frac{\sqrt{\pi}\,x\sqrt{3}\sqrt{2}\sqrt{\frac{3x^2}{2}+1}}{2}-\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{3}\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{3\sqrt{2}\left(-2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 32/15*x^4*(3*x^2+2)^(1/2)+764/135*x^2*(3*x^2+2)^(1/2)-1841/405*(3*x^2+2)^(1
/2)+6*x^3*(3*x^2+2)^(1/2)-1/3*x*(3*x^2+2)^(1/2)+5/9*arcsinh(1/2*x*6^(1/2))*
3^(1/2)
```

Maxima [A]
time = 0.50, size = 78, normalized size = 0.74

$$\frac{32}{15}\sqrt{3x^2+2}\,x^4+6\sqrt{3x^2+2}\,x^3+\frac{764}{135}\sqrt{3x^2+2}\,x^2-\frac{1}{3}\sqrt{3x^2+2}\,x+\frac{5}{9}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}\,x\right)-\frac{1841}{405}\sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/15*sqrt(3*x^2 + 2)*x^4 + 6*sqrt(3*x^2 + 2)*x^3 + 764/135*sqrt(3*x^2 + 2)
*x^2 - 1/3*sqrt(3*x^2 + 2)*x + 5/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 1841/40
5*sqrt(3*x^2 + 2)
```

Fricas [A]
time = 0.35, size = 60, normalized size = 0.57

$$\frac{1}{405}\left(864\,x^4+2430\,x^3+2292\,x^2-135\,x-1841\right)\sqrt{3\,x^2+2}+\frac{5}{18}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}\,x-3\,x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/405*(864*x^4 + 2430*x^3 + 2292*x^2 - 135*x - 1841)*sqrt(3*x^2 + 2) + 5/18
*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A]
time = 0.35, size = 94, normalized size = 0.89

$$\frac{32x^4\sqrt{3x^2+2}}{15} + 6x^3\sqrt{3x^2+2} + \frac{764x^2\sqrt{3x^2+2}}{135} - \frac{x\sqrt{3x^2+2}}{3} - \frac{1841\sqrt{3x^2+2}}{405} + \frac{5\sqrt{3}\,\operatorname{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

```
[Out] 32*x**4*sqrt(3*x**2 + 2)/15 + 6*x**3*sqrt(3*x**2 + 2) + 764*x**2*sqrt(3*x**
2 + 2)/135 - x*sqrt(3*x**2 + 2)/3 - 1841*sqrt(3*x**2 + 2)/405 + 5*sqrt(3)*a
sinh(sqrt(6)*x/2)/9
```

Giac [A]
time = 2.96, size = 54, normalized size = 0.51

$$\frac{1}{405}\left(3\left(2\left(9\left(16\,x+45\right)x+382\right)x-45\right)x-1841\right)\sqrt{3\,x^2+2} - \frac{5}{9}\,\sqrt{3}\,\log\left(-\sqrt{3}\,x+\sqrt{3\,x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/405*(3*(2*(9*(16*x + 45)*x + 382)*x - 45)*x - 1841)*sqrt(3*x^2 + 2) - 5/9
*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2))
```

Mupad [B]
time = 0.05, size = 45, normalized size = 0.42

$$\frac{5\,\sqrt{3}\,\operatorname{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9} + \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{32\,x^4}{5}+18\,x^3+\frac{764\,x^2}{45}-x-\frac{1841}{135}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

```
[Out] (5*3^(1/2)*asinh((6^(1/2)*x)/2))/9 + (3^(1/2)*(x^2 + 2/3)^(1/2)*((764*x^2)/
45 - x + 18*x^3 + (32*x^4)/5 - 1841/135))/3
```

3.119 $\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=82

$$\frac{5}{18}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2+3x^2}-\frac{1}{27}(61+3x)\sqrt{2+3x^2}-\sqrt{3}\,\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)$$

```
[Out] -arcsinh(1/2*x*6^(1/2))*3^(1/2)+5/18*(1+2*x)^2*(3*x^2+2)^(1/2)+1/6*(1+2*x)^
3*(3*x^2+2)^(1/2)-1/27*(61+3*x)*(3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1668, 847, 794, 221}

$$\frac{1}{6}\sqrt{3x^2+2}\,(2x+1)^3+\frac{5}{18}\sqrt{3x^2+2}\,(2x+1)^2-\frac{1}{27}(3x+61)\sqrt{3x^2+2}-\sqrt{3}\,\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (5*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/18 + ((1 + 2*x)^3*Sqrt[2 + 3*x^2])/6 - ((61
 + 3*x)*Sqrt[2 + 3*x^2])/27 - Sqrt[3]*ArcSinh[Sqrt[3/2]*x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
```

```
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx &= \frac{1}{6}(1+2x)^3\sqrt{2+3x^2}+\frac{1}{48}\int \frac{(1+2x)^2(-48+120x)}{\sqrt{2+3x^2}}\,dx\\ &= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2+3x^2}+\frac{1}{432}\int \frac{(-1392-144x)(}{\sqrt{2+3x^2}}\\ &= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2+3x^2}-\frac{1}{27}(61+3x)\sqrt{2+3x^2}\;-\\ &= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2+3x^2}-\frac{1}{27}(61+3x)\sqrt{2+3x^2}\;-\end{aligned}$$

Mathematica [A]
time = 0.16, size = 58, normalized size = 0.71

$$\frac{1}{27}\sqrt{2+3x^2}\left(-49+54x+84x^2+36x^3\right)+\sqrt{3}\,\log\left(-\sqrt{3}\,x+\sqrt{2+3x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (Sqrt[2 + 3*x^2]*(-49 + 54*x + 84*x^2 + 36*x^3))/27 + Sqrt[3]*Log[-(Sqrt[3]
*x) + Sqrt[2 + 3*x^2]]
```

Maple [A]
time = 0.09, size = 65, normalized size = 0.79

method	result
risch	$\frac{(36x^3+84x^2+54x-49)\sqrt{3x^2+2}}{27}-\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}$
trager	$\left(\frac{4}{3}x^3+\frac{28}{9}x^2+2x-\frac{49}{27}\right)\sqrt{3x^2+2}-\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right.$
default	$\frac{4x^3\sqrt{3x^2+2}}{3}+2x\sqrt{3x^2+2}-\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}+\frac{28x^2\sqrt{3x^2+2}}{9}-\frac{49\sqrt{3x^2+2}}{27}$
meijerg	$\frac{\sqrt{3}\operatorname{arcsinh}\left(\frac{x\sqrt{3}\sqrt{2}}{2}\right)}{3}+\frac{20\sqrt{3}\left(\frac{\sqrt{\pi}\,x\sqrt{3}\sqrt{2}\sqrt{\frac{3x^2}{2}+1}}{2}-\sqrt{\pi}\operatorname{arcsinh}\left(\frac{x\sqrt{3}\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{7\sqrt{2}\left(-2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*x^3*(3*x^2+2)^(1/2)+2*x*(3*x^2+2)^(1/2)-arcsinh(1/2*x*6^(1/2))*3^(1/2)+
28/9*x^2*(3*x^2+2)^(1/2)-49/27*(3*x^2+2)^(1/2)
```

Maxima [A]
time = 0.49, size = 64, normalized size = 0.78

$$\frac{4}{3}\sqrt{3x^2+2}\,x^3+\frac{28}{9}\sqrt{3x^2+2}\,x^2+2\sqrt{3x^2+2}\,x-\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}\,x\right)-\frac{49}{27}\sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3*x^2 + 2)*x^3 + 28/9*sqrt(3*x^2 + 2)*x^2 + 2*sqrt(3*x^2 + 2)*x -
sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 49/27*sqrt(3*x^2 + 2)
```

Fricas [A]
time = 0.37, size = 54, normalized size = 0.66

$$\frac{1}{27}\left(36x^3+84x^2+54x-49\right)\sqrt{3x^2+2}+\frac{1}{2}\sqrt{3}\log\left(\sqrt{3}\sqrt{3x^2+2}\,x-3x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(36*x^3 + 84*x^2 + 54*x - 49)*sqrt(3*x^2 + 2) + 1/2*sqrt(3)*log(sqrt(3
)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A]
time = 0.22, size = 75, normalized size = 0.91

$$\frac{4x^3\sqrt{3x^2+2}}{3}+\frac{28x^2\sqrt{3x^2+2}}{9}+2x\sqrt{3x^2+2}-\frac{49\sqrt{3x^2+2}}{27}-\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

```
[Out] 4*x**3*sqrt(3*x**2 + 2)/3 + 28*x**2*sqrt(3*x**2 + 2)/9 + 2*x*sqrt(3*x**2 +
2) - 49*sqrt(3*x**2 + 2)/27 - sqrt(3)*asinh(sqrt(6)*x/2)
```

Giac [A]
time = 4.97, size = 48, normalized size = 0.59

$$\frac{1}{27}\left(6\left(2\left(3x+7\right)x+9\right)x-49\right)\sqrt{3x^2+2}+\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/27*(6*(2*(3*x + 7)*x + 9)*x - 49)*sqrt(3*x^2 + 2) + sqrt(3)*log(-sqrt(3)*
x + sqrt(3*x^2 + 2))
```

Mupad [B]
time = 4.10, size = 40, normalized size = 0.49

$$\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(4x^3+\frac{28x^2}{3}+6x-\frac{49}{9}\right)}{3}-\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

```
[Out] (3^(1/2)*(x^2 + 2/3)^(1/2)*(6*x + (28*x^2)/3 + 4*x^3 - 49/9))/3 - 3^(1/2)*a
sinh((6^(1/2)*x)/2)
```

3.120 $\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=62

$$\frac{2}{9}(1+2x)^2\sqrt{2+3x^2}+\frac{7}{27}(1+3x)\sqrt{2+3x^2}-\frac{7\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

```
[Out] -7/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+2/9*(1+2*x)^2*(3*x^2+2)^(1/2)+7/27*(1+3
*x)*(3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1668, 794, 221}

$$\frac{2}{9}\sqrt{3x^2+2}\,(2x+1)^2+\frac{7}{27}(3x+1)\sqrt{3x^2+2}-\frac{7\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (2*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/9 + (7*(1 + 3*x)*Sqrt[2 + 3*x^2])/27 - (7*A
rcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
```

```
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx &= \frac{2}{9}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{36}\int \frac{(1+2x)(-28+84x)}{\sqrt{2+3x^2}}\,dx\\ &= \frac{2}{9}(1+2x)^2\sqrt{2+3x^2}+\frac{7}{27}(1+3x)\sqrt{2+3x^2}-\frac{7}{3}\int \frac{1}{\sqrt{2+3x^2}}\,dx\\ &= \frac{2}{9}(1+2x)^2\sqrt{2+3x^2}+\frac{7}{27}(1+3x)\sqrt{2+3x^2}-\frac{7\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 56, normalized size = 0.90

$$\frac{1}{27}\sqrt{2+3x^2}\left(13+45x+24x^2\right)+\frac{7\log\left(-\sqrt{3}\,x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
[Out] (Sqrt[2 + 3*x^2]*(13 + 45*x + 24*x^2))/27 + (7*Log[-(Sqrt[3]*x) + Sqrt[2 +
3*x^2]])/(3*Sqrt[3])
```

Maple [A]
time = 0.07, size = 51, normalized size = 0.82

method	result
risch	$\frac{(24x^2+45x+13)\sqrt{3x^2+2}}{27}-\frac{7\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$
default	$\frac{8x^2\sqrt{3x^2+2}}{9}+\frac{13\sqrt{3x^2+2}}{27}+\frac{5x\sqrt{3x^2+2}}{3}-\frac{7\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$

trager	$\left(\frac{8}{9}x^2+\frac{5}{3}x+\frac{13}{27}\right)\sqrt{3x^2+2}-\frac{7\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
meijerg	$\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{x\sqrt{3}\,\sqrt{2}}{2}\right)}{3}+\frac{10\sqrt{3}\left(\frac{\sqrt{\pi}\,x\sqrt{3}\,\sqrt{2}\,\sqrt{\frac{3x^2}{2}+1}}{2}-\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x\sqrt{3}\,\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{5\sqrt{2}\,\left(-2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/9*x^2*(3*x^2+2)^(1/2)+13/27*(3*x^2+2)^(1/2)+5/3*x*(3*x^2+2)^(1/2)-7/9*arc
sinh(1/2*x*6^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.49, size = 50, normalized size = 0.81

$$\frac{8}{9}\sqrt{3\,x^2+2}\,x^2+\frac{5}{3}\sqrt{3\,x^2+2}\,x-\frac{7}{9}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}\,x\right)+\frac{13}{27}\sqrt{3\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*sqrt(3*x^2 + 2)*x^2 + 5/3*sqrt(3*x^2 + 2)*x - 7/9*sqrt(3)*arcsinh(1/2*s
qrt(6)*x) + 13/27*sqrt(3*x^2 + 2)
```

Fricas [A]
time = 0.35, size = 49, normalized size = 0.79

$$\frac{1}{27}\left(24\,x^2+45\,x+13\right)\sqrt{3\,x^2+2}+\frac{7}{18}\sqrt{3}\,\log\left(\sqrt{3}\,\sqrt{3\,x^2+2}\,x-3\,x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(24*x^2 + 45*x + 13)*sqrt(3*x^2 + 2) + 7/18*sqrt(3)*log(sqrt(3)*sqrt(3
*x^2 + 2)*x - 3*x^2 - 1)
```

Sympy [A]
time = 0.14, size = 63, normalized size = 1.02

$$\frac{8x^2\sqrt{3x^2+2}}{9}+\frac{5x\sqrt{3x^2+2}}{3}+\frac{13\sqrt{3x^2+2}}{27}-\frac{7\sqrt{3}\,\operatorname{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

```
[Out] 8*x**2*sqrt(3*x**2 + 2)/9 + 5*x*sqrt(3*x**2 + 2)/3 + 13*sqrt(3*x**2 + 2)/27
 - 7*sqrt(3)*asinh(sqrt(6)*x/2)/9
```

Giac [A]
time = 4.37, size = 44, normalized size = 0.71

$$\frac{1}{27}\left(3\left(8\,x+15\right)x+13\right)\sqrt{3\,x^2+2}+\frac{7}{9}\,\sqrt{3}\,\log\left(-\sqrt{3}\,x+\sqrt{3\,x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/27*(3*(8*x + 15)*x + 13)*sqrt(3*x^2 + 2) + 7/9*sqrt(3)*log(-sqrt(3)*x + s
qrt(3*x^2 + 2))
```

Mupad [B]
time = 0.03, size = 35, normalized size = 0.56

$$\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{8\,x^2}{3}+5\,x+\frac{13}{9}\right)}{3}-\frac{7\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

```
[Out] (3^(1/2)*(x^2 + 2/3)^(1/2)*(5*x + (8*x^2)/3 + 13/9))/3 - (7*3^(1/2)*asinh((
6^(1/2)*x)/2))/9
```

3.121 $\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=67

$$\frac{2}{3}\sqrt{2+3x^2}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{2\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\,\sqrt{2+3x^2}}\right)}{2\sqrt{11}}$$

```
[Out] 1/6*arcsinh(1/2*x*6^(1/2))*3^(1/2)-1/22*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^
2+2)^(1/2))*11^(1/2)+2/3*(3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1668, 858, 221, 739, 212}

$$\frac{2}{3}\sqrt{3x^2+2}-\frac{\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\,\sqrt{3x^2+2}}\right)}{2\sqrt{11}}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (2*Sqrt[2 + 3*x^2])/3 + ArcSinh[Sqrt[3/2]*x]/(2*Sqrt[3]) - ArcTanh[(4 - 3*x
)/(Sqrt[11]*Sqrt[2 + 3*x^2])]/(2*Sqrt[11])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx &= \frac{2}{3}\sqrt{2+3x^2}+\frac{1}{12}\int \frac{12+12x}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= \frac{2}{3}\sqrt{2+3x^2}+\frac{1}{2}\int \frac{1}{\sqrt{2+3x^2}}\,dx+\frac{1}{2}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= \frac{2}{3}\sqrt{2+3x^2}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{2\sqrt{3}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)\\
&= \frac{2}{3}\sqrt{2+3x^2}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{2\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{2\sqrt{11}}\end{aligned}$$

Mathematica [A]
time = 0.24, size = 89, normalized size = 1.33

$$\frac{2}{3}\sqrt{2+3x^2}+\frac{\tanh^{-1}\left(\sqrt{\frac{3}{11}}+2\sqrt{\frac{3}{11}}\,x-\frac{2\sqrt{2+3x^2}}{\sqrt{11}}\right)}{\sqrt{11}}-\frac{\log\left(-\sqrt{3}\,x+\sqrt{2+3x^2}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 + 3*x^2]),x]

[Out] (2*Sqrt[2 + 3*x^2])/3 + ArcTanh[Sqrt[3/11] + 2*Sqrt[3/11]*x - (2*Sqrt[2 + 3*x^2])/Sqrt[11]]/Sqrt[11] - Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]]/(2*Sqrt[3])

Maple [A]
time = 0.14, size = 55, normalized size = 0.82

method	result
default	$\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{6}+\frac{2\sqrt{3x^2+2}}{3}-\frac{\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{22}$
risch	$\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{6}+\frac{2\sqrt{3x^2+2}}{3}-\frac{\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{22}$
trager	$\frac{2\sqrt{3x^2+2}}{3}-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{6}+\frac{\operatorname{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^2-\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2+3*x+1)/(2*x+1)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/6*arcsinh(1/2*x*6^(1/2))*3^(1/2)+2/3*(3*x^2+2)^(1/2)-1/22*11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))

Maxima [A]
time = 0.50, size = 58, normalized size = 0.87

$$\frac{1}{6}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}\,x\right)+\frac{1}{22}\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{2}{3}\sqrt{3\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="maxima")

[Out] 1/6*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 1/22*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1)) + 2/3*sqrt(3*x^2 + 2)

Fricas [A]
time = 0.35, size = 88, normalized size = 1.31

$$\frac{1}{12}\sqrt{3}\,\log\left(-\sqrt{3}\,\sqrt{3\,x^2+2}\,x-3\,x^2-1\right)+\frac{1}{44}\sqrt{11}\,\log\left(-\frac{\sqrt{11}\,\sqrt{3\,x^2+2}\,(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right)+\frac{2}{3}\sqrt{3\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + 1/44*sqrt(11)*lo
g(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x +
 1)) + 2/3*sqrt(3*x^2 + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)\sqrt{3x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*sqrt(3*x**2 + 2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(49) = 98.
time = 5.26, size = 99, normalized size = 1.48

$$-\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\,x + \sqrt{3\,x^2+2}\right) + \frac{1}{22}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}\,x - \sqrt{11} - \sqrt{3} + 2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}\,x - \sqrt{11} + \sqrt{3} - 2\sqrt{3\,x^2+2}}\right) + \frac{2}{3}\sqrt{3\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/22*sqrt(11)*log(-abs(-2*
sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(11)
 + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 2/3*sqrt(3*x^2 + 2)
```

Mupad [B]
time = 0.19, size = 61, normalized size = 0.91

$$\frac{\sqrt{11}\left(2\,\ln\left(x+\frac{1}{2}\right) - 2\,\ln\left(x - \frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3} - \frac{4}{3}\right)\right)}{44} + \frac{2\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{3} + \frac{\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(1/2)),x)
```

```
[Out] (11^(1/2)*(2*log(x + 1/2) - 2*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/
3 - 4/3)))/44 + (2*3^(1/2)*(x^2 + 2/3)^(1/2))/3 + (3^(1/2)*asinh((2^(1/2)*3
^(1/2)*x)/2))/6
```

3.122 $\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=71

$$-\frac{\sqrt{2+3x^2}}{11(1+2x)}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{3}}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\,\sqrt{2+3x^2}}\right)}{11\sqrt{11}}$$

```
[Out] 1/3*arcsinh(1/2*x*6^(1/2))*3^(1/2)+4/121*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x
^2+2)^(1/2))*11^(1/2)-1/11*(3*x^2+2)^(1/2)/(1+2*x)
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1665, 858, 221, 739, 212}

$$-\frac{\sqrt{3x^2+2}}{11(2x+1)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\,\sqrt{3x^2+2}}\right)}{11\sqrt{11}}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 + 3*x^2]),x]
```

```
[Out] -1/11*Sqrt[2 + 3*x^2]/(1 + 2*x) + ArcSinh[Sqrt[3/2]*x]/Sqrt[3] + (4*ArcTanh
[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(11*Sqrt[11])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx &= -\frac{\sqrt{2+3x^2}}{11(1+2x)} - \frac{1}{11}\int \frac{-7-22x}{(1+2x)\sqrt{2+3x^2}}\,dx \\
&= -\frac{\sqrt{2+3x^2}}{11(1+2x)} - \frac{4}{11}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx + \int \frac{1}{\sqrt{2+3x^2}}\,dx \\
&= -\frac{\sqrt{2+3x^2}}{11(1+2x)} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{3}} + \frac{4}{11}\text{Subst}\left(\int \frac{1}{11-x^2}\,dx, x, \frac{4-3x}{\sqrt{2+3x^2}}\right) \\
&= -\frac{\sqrt{2+3x^2}}{11(1+2x)} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{3}} + \frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{11\sqrt{11}}
\end{aligned}$$

Mathematica [A]
time = 0.32, size = 92, normalized size = 1.30

$$-\frac{\sqrt{2+3x^2}}{11+22x} - \frac{8\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}\,x-2\sqrt{2+3x^2}}{\sqrt{11}}\right)}{11\sqrt{11}} - \frac{\log\left(-\sqrt{3}\,x+\sqrt{2+3x^2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 + 3*x^2]),x]
[Out] -(Sqrt[2 + 3*x^2]/(11 + 22*x)) - (8*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt
[2 + 3*x^2])/Sqrt[11]])/(11*Sqrt[11]) - Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]]
/Sqrt[3]
```

Maple [A]
time = 0.15, size = 65, normalized size = 0.92

method	result
risch	$-\frac{\sqrt{3x^{2}+2}}{11(2x+1)}+\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{3}+\frac{4\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^{2}-12x+5}}\right)}{121}$
default	$\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{3}+\frac{4\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^{2}-12x+5}}\right)}{121}-\frac{\sqrt{3\left(x+\frac{1}{2}\right)^{2}-3x+\frac{5}{4}}}{22\left(x+\frac{1}{2}\right)}$
trager	$-\frac{\sqrt{3x^{2}+2}}{11(2x+1)}+\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}-3\right)\sqrt{3x^{2}+2}+3x\right)}{3}+\frac{4\operatorname{RootOf}\left(\_Z^{2}-11\right)\ln\left(\frac{-3\operatorname{RootOf}\left(\_Z\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^2/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arcsinh(1/2*x*6^(1/2))*3^(1/2)+4/121*11^(1/2)*arctanh(2/11*(4-3*x)*11^(
1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))-1/22/(x+1/2)*(3*(x+1/2)^2-3*x+5/4)^(1/2)
```

Maxima [A]
time = 0.49, size = 65, normalized size = 0.92

$$\frac{1}{3}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}\,x\right)-\frac{4}{121}\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)-\frac{\sqrt{3\,x^{2}+2}}{11\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 4/121*sqrt(11)*arcsinh(1/2*sqrt(6)*x/a
bs(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1)) - 1/11*sqrt(3*x^2 + 2)/(2*x + 1)
```

Fricas [A]
time = 0.36, size = 106, normalized size = 1.49

$$\frac{121\sqrt{3}\,(2\,x+1)\log\left(-\sqrt{3}\,\sqrt{3\,x^{2}+2}\,x-3\,x^{2}-1\right)+12\sqrt{11}\,(2\,x+1)\log\left(\frac{\sqrt{11}\,\sqrt{3\,x^{2}+2}\,(3\,x-4)-21\,x^{2}+12\,x-19}{4\,x^{2}+4\,x+1}\right)-66\sqrt{3\,x^{2}+2}}{726\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/726*(121*sqrt(3)*(2*x + 1)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
12*sqrt(11)*(2*x + 1)*log((sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) - 21*x^2 + 12
*x - 19)/(4*x^2 + 4*x + 1)) - 66*sqrt(3*x^2 + 2))/(2*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x+1)^2 \sqrt{3x^2+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*sqrt(3*x**2 + 2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(56) = 112.
time = 5.34, size = 191, normalized size = 2.69

$$\frac{4\sqrt{11}\log\left(\sqrt{11}\left(\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}+\frac{\sqrt{11}}{2x+1}\right)-3\right)}{121\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)} - \frac{\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}+\frac{2\sqrt{11}}{2x+1}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}+\frac{\sqrt{11}}{2x+1}\right)}\right)}{3\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)} - \frac{\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}}{22\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/121*sqrt(11)*log(sqrt(11)*(sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3) + sqrt
(11)/(2*x + 1)) - 3)/sgn(1/(2*x + 1)) - 1/3*sqrt(3)*log(1/2*abs(-2*sqrt(3)
+ 2*sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3) + 2*sqrt(11)/(2*x + 1))/(sqrt(3
) + sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3) + sqrt(11)/(2*x + 1)))/sgn(1/(2
*x + 1)) - 1/22*sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3)/sgn(1/(2*x + 1))
```

Mupad [B]
time = 0.11, size = 68, normalized size = 0.96

$$\frac{\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{3} - \frac{4\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{121} + \frac{4\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{121} - \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{22\,\left(x+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(1/2)),x)
```

```
[Out] (3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/3 - (4*11^(1/2)*log(x + 1/2))/121 +
(4*11^(1/2)*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/121 - (3
^(1/2)*(x^2 + 2/3)^(1/2))/(22*(x + 1/2))
```

3.123 $\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=77

$$-\frac{\sqrt{2+3x^2}}{22(1+2x)^2}+\frac{13\sqrt{2+3x^2}}{242(1+2x)}-\frac{103\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}$$

```
[Out] -103/1331*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)-1/22*(3*x
^2+2)^(1/2)/(1+2*x)^2+13/242*(3*x^2+2)^(1/2)/(1+2*x)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1665, 821, 739, 212}

$$\frac{13\sqrt{3x^2+2}}{242(2x+1)}-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}-\frac{103\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 + 3*x^2]),x]
```

```
[Out] -1/22*Sqrt[2 + 3*x^2]/(1 + 2*x)^2 + (13*Sqrt[2 + 3*x^2])/(242*(1 + 2*x)) -
(103*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx &= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2}-\frac{1}{22}\int \frac{-14-41x}{(1+2x)^2\sqrt{2+3x^2}}\,dx\\ &= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2}+\frac{13\sqrt{2+3x^2}}{242(1+2x)}+\frac{103}{121}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\ &= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2}+\frac{13\sqrt{2+3x^2}}{242(1+2x)}-\frac{103}{121}\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)\\ &= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2}+\frac{13\sqrt{2+3x^2}}{242(1+2x)}-\frac{103\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 71, normalized size = 0.92

$$\frac{\frac{11(1+13x)\sqrt{2+3x^2}}{(1+2x)^2}+206\sqrt{11}\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{11}}\right)}{1331}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 + 3*x^2]),x]
```

```
[Out] ((11*(1 + 13*x)*Sqrt[2 + 3*x^2])/(1 + 2*x)^2 + 206*Sqrt[11]*ArcTanh[(Sqrt[3
] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[11]])/1331
```

Maple [A]
time = 0.09, size = 74, normalized size = 0.96

method	result

risch	$\frac{39x^3+3x^2+26x+2}{121(2x+1)^2\sqrt{3x^2+2}}-\frac{103\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{1331}$
trager	$\frac{(13x+1)\sqrt{3x^2+2}}{121(2x+1)^2}-\frac{103\,\mathrm{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{-3\,\mathrm{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}+4\,\mathrm{RootOf}\left(\_Z^2-11\right)}{2x+1}\right)}{1331}$
default	$-\frac{\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}{88\left(x+\frac{1}{2}\right)^2}+\frac{13\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}{484\left(x+\frac{1}{2}\right)}-\frac{103\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x}}\right.}{1331}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^3/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/88/(x+1/2)^2*(3*(x+1/2)^2-3*x+5/4)^(1/2)+13/484/(x+1/2)*(3*(x+1/2)^2-3*x
+5/4)^(1/2)-103/1331*11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-1
2*x+5)^(1/2))
```

Maxima [A]
time = 0.50, size = 76, normalized size = 0.99

$$\frac{103}{1331}\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)-\frac{\sqrt{3\,x^2+2}}{22\,(4\,x^2+4\,x+1)}+\frac{13\,\sqrt{3\,x^2+2}}{242\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 103/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
+ 1)) - 1/22*sqrt(3*x^2 + 2)/(4*x^2 + 4*x + 1) + 13/242*sqrt(3*x^2 + 2)/(2*
x + 1)
```

Fricas [A]
time = 0.37, size = 89, normalized size = 1.16

$$\frac{103\,\sqrt{11}\,(4\,x^2+4\,x+1)\log\left(-\frac{\sqrt{11}\,\sqrt{3\,x^2+2}\,(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right)+22\,\sqrt{3\,x^2+2}\,(13\,x+1)}{2662\,(4\,x^2+4\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2662*(103*sqrt(11)*(4*x^2 + 4*x + 1)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x
- 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 22*sqrt(3*x^2 + 2)*(13*x +
1))/(4*x^2 + 4*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)^3\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*sqrt(3*x**2 + 2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(62) = 124.
time = 4.48, size = 180, normalized size = 2.34

$$\frac{103}{1331}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{72\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^3-13\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2-168\sqrt{3}x+104\sqrt{3}+168\sqrt{3x^2+2}}{484\left(\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2+\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)-2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 103/1331*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2
 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/484*(72*
(sqrt(3)*x - sqrt(3*x^2 + 2))^3 - 13*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^
2 - 168*sqrt(3)*x + 104*sqrt(3) + 168*sqrt(3*x^2 + 2))/((sqrt(3)*x - sqrt(3
*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 2)^2
```

Mupad [B]
time = 0.11, size = 77, normalized size = 1.00

$$\frac{103\sqrt{11}\ \ln\left(x+\frac{1}{2}\right)}{1331}-\frac{103\sqrt{11}\ \ln\left(x-\frac{\sqrt{3}\ \sqrt{11}\ \sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{1331}-\frac{\sqrt{3}\ \sqrt{x^2+\frac{2}{3}}}{88\ \left(x^2+x+\frac{1}{4}\right)}+\frac{13\sqrt{3}\ \sqrt{x^2+\frac{2}{3}}}{484\ \left(x+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(1/2)),x)
```

```
[Out] (103*11^(1/2)*log(x + 1/2))/1331 - (103*11^(1/2)*log(x - (3^(1/2)*11^(1/2)*
(x^2 + 2/3)^(1/2))/3 - 4/3))/1331 - (3^(1/2)*(x^2 + 2/3)^(1/2))/(88*(x + x^
2 + 1/4)) + (13*3^(1/2)*(x^2 + 2/3)^(1/2))/(484*(x + 1/2))
```

3.124 $\int \frac{(1+2x)^3 (1+3x+4x^2)}{(2+3x^2)^{3/2}} \, dx$

Optimal. Leaf size=87

$$\frac{398+279x}{54\sqrt{2+3x^2}}+\frac{292}{81}\sqrt{2+3x^2}+4x\sqrt{2+3x^2}+\frac{32}{27}x^2\sqrt{2+3x^2}-\frac{38\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

```
[Out] -38/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/54*(398+279*x)/(3*x^2+2)^(1/2)+292/8
1*(3*x^2+2)^(1/2)+4*x*(3*x^2+2)^(1/2)+32/27*x^2*(3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1828, 1829, 655, 221}

$$\frac{32}{27}\sqrt{3x^2+2}\,x^2+4\sqrt{3x^2+2}\,x+\frac{292}{81}\sqrt{3x^2+2}+\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{38\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (398 + 279*x)/(54*Sqrt[2 + 3*x^2]) + (292*Sqrt[2 + 3*x^2])/81 + 4*x*Sqrt[2
+ 3*x^2] + (32*x^2*Sqrt[2 + 3*x^2])/27 - (38*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[
3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
```

```
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx &= \frac{398+279x}{54\sqrt{2+3x^2}}-\frac{1}{2}\int \frac{\frac{28}{3}-\frac{280x}{9}-48x^2-\frac{64x^3}{3}}{\sqrt{2+3x^2}}\,dx\\ &= \frac{398+279x}{54\sqrt{2+3x^2}}+\frac{32}{27}x^2\sqrt{2+3x^2}-\frac{1}{18}\int \frac{84-\frac{584x}{3}-432x^2}{\sqrt{2+3x^2}}\,dx\\ &= \frac{398+279x}{54\sqrt{2+3x^2}}+4x\sqrt{2+3x^2}+\frac{32}{27}x^2\sqrt{2+3x^2}-\frac{1}{108}\int \frac{1368-1168x}{\sqrt{2+3x^2}}\,dx\\ &= \frac{398+279x}{54\sqrt{2+3x^2}}+\frac{292}{81}\sqrt{2+3x^2}+4x\sqrt{2+3x^2}+\frac{32}{27}x^2\sqrt{2+3x^2}-\frac{38}{3}\int\\ &= \frac{398+279x}{54\sqrt{2+3x^2}}+\frac{292}{81}\sqrt{2+3x^2}+4x\sqrt{2+3x^2}+\frac{32}{27}x^2\sqrt{2+3x^2}-\frac{38\sinh}{}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 66, normalized size = 0.76

$$\frac{2362+2133x+2136x^2+1944x^3+576x^4}{162\sqrt{2+3x^2}}+\frac{38\log\left(-\sqrt{3}\,x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (2362 + 2133*x + 2136*x^2 + 1944*x^3 + 576*x^4)/(162*Sqrt[2 + 3*x^2]) + (38
*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

Maple [A]
time = 0.10, size = 79, normalized size = 0.91

method	result
risch	$\frac{576x^4+1944x^3+2136x^2+2133x+2362}{162\sqrt{3x^2+2}}-\frac{38\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$
trager	$\frac{576x^4+1944x^3+2136x^2+2133x+2362}{162\sqrt{3x^2+2}}+\frac{38\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
default	$\frac{32x^4}{9\sqrt{3x^2+2}}+\frac{356x^2}{27\sqrt{3x^2+2}}+\frac{1181}{81\sqrt{3x^2+2}}+\frac{12x^3}{\sqrt{3x^2+2}}+\frac{79x}{6\sqrt{3x^2+2}}-\frac{38\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$
meijerg	$\frac{\sqrt{2}\,x}{4\sqrt{\frac{3x^2}{2}+1}}+\frac{34\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{3}\,\sqrt{2}}{2\sqrt{\frac{3x^2}{2}+1}}+\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{3}\,\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{3\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{\frac{3x^2}{2}+1}}\right)}{2\sqrt{\pi}}+$ 6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 32/9*x^4/(3*x^2+2)^(1/2)+356/27*x^2/(3*x^2+2)^(1/2)+1181/81/(3*x^2+2)^(1/2)
+12*x^3/(3*x^2+2)^(1/2)+79/6*x/(3*x^2+2)^(1/2)-38/9*arcsinh(1/2*x*6^(1/2))*
3^(1/2)
```

Maxima [A]
time = 0.51, size = 78, normalized size = 0.90

$$\frac{32\,x^4}{9\,\sqrt{3\,x^2+2}}+\frac{12\,x^3}{\sqrt{3\,x^2+2}}+\frac{356\,x^2}{27\,\sqrt{3\,x^2+2}}-\frac{38}{9}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}\,x\right)+\frac{79\,x}{6\,\sqrt{3\,x^2+2}}+\frac{1181}{81\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 32/9*x^4/sqrt(3*x^2 + 2) + 12*x^3/sqrt(3*x^2 + 2) + 356/27*x^2/sqrt(3*x^2 +
 2) - 38/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 79/6*x/sqrt(3*x^2 + 2) + 1181/8
1/sqrt(3*x^2 + 2)
```

Fricas [A]
time = 0.36, size = 76, normalized size = 0.87

$$\frac{342\,\sqrt{3}\,(3\,x^2+2)\log\left(\sqrt{3}\,\sqrt{3\,x^2+2}\,x-3\,x^2-1\right)+(576\,x^4+1944\,x^3+2136\,x^2+2133\,x+2362)\sqrt{3\,x^2+2}}{162\,(3\,x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/162*(342*sqrt(3)*(3*x^2 + 2)*log(sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
 (576*x^4 + 1944*x^3 + 2136*x^2 + 2133*x + 2362)*sqrt(3*x^2 + 2))/(3*x^2 +
2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^3 \cdot (4x^2+3x+1)}{(3x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(3/2), x)
```

Giac [A]
time = 4.67, size = 54, normalized size = 0.62

$$\frac{38}{9}\,\sqrt{3}\,\log\left(-\sqrt{3}\,x+\sqrt{3\,x^2+2}\right)+\frac{3\,(8\,(3\,(8\,x+27)x+89)x+711)x+2362}{162\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] 38/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/162*(3*(8*(3*(8*x + 27)*
x + 89)*x + 711)*x + 2362)/sqrt(3*x^2 + 2)
```

Mupad [B]
time = 0.06, size = 110, normalized size = 1.26

$$\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{32\,x^2}{9}+12\,x+\frac{292}{27}\right)}{3}-\frac{38\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{9}-\frac{\sqrt{3}\,\sqrt{6}\,\left(-1194+\sqrt{6}\,279\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{\sqrt{3}\,\sqrt{6}\,\left(1194+\sqrt{6}\,279\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
```

```
[Out] (3^(1/2)*(x^2 + 2/3)^(1/2)*(12*x + (32*x^2)/9 + 292/27))/3 - (38*3^(1/2)*as
inh((2^(1/2)*3^(1/2)*x)/2))/9 - (3^(1/2)*6^(1/2)*(6^(1/2)*279i - 1194)*(x^2
 + 2/3)^(1/2)*1i)/(1944*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*2
79i + 1194)*(x^2 + 2/3)^(1/2)*1i)/(1944*(x - (6^(1/2)*1i)/3))
```

3.125 $\int \frac{(1+2x)^2 (1+3x+4x^2)}{(2+3x^2)^{3/2}} \, dx$

Optimal. Leaf size=71

$$\frac{70-47x}{18\sqrt{2+3x^2}} + \frac{28}{9}\sqrt{2+3x^2} + \frac{8}{9}x\sqrt{2+3x^2} + \frac{4\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

```
[Out] 4/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/18*(70-47*x)/(3*x^2+2)^(1/2)+28/9*(3*x
^2+2)^(1/2)+8/9*x*(3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1828, 1829, 655, 221}

$$\frac{70-47x}{18\sqrt{3x^2+2}} + \frac{8}{9}x\sqrt{3x^2+2} + \frac{28}{9}\sqrt{3x^2+2} + \frac{4\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (70 - 47*x)/(18*Sqrt[2 + 3*x^2]) + (28*Sqrt[2 + 3*x^2])/9 + (8*x*Sqrt[2 + 3
*x^2])/9 + (4*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx &= \frac{70-47x}{18\sqrt{2+3x^2}} - \frac{1}{2}\int \frac{-\frac{56}{9}-\frac{56x}{3}-\frac{32x^2}{3}}{\sqrt{2+3x^2}}\,dx \\
&= \frac{70-47x}{18\sqrt{2+3x^2}} + \frac{8}{9}x\sqrt{2+3x^2} - \frac{1}{12}\int \frac{-16-112x}{\sqrt{2+3x^2}}\,dx \\
&= \frac{70-47x}{18\sqrt{2+3x^2}} + \frac{28}{9}\sqrt{2+3x^2} + \frac{8}{9}x\sqrt{2+3x^2} + \frac{4}{3}\int \frac{1}{\sqrt{2+3x^2}}\,dx \\
&= \frac{70-47x}{18\sqrt{2+3x^2}} + \frac{28}{9}\sqrt{2+3x^2} + \frac{8}{9}x\sqrt{2+3x^2} + \frac{4\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 61, normalized size = 0.86

$$\frac{182-15x+168x^2+48x^3}{18\sqrt{2+3x^2}} - \frac{4\log\left(-\sqrt{3}\,x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (182 - 15*x + 168*x^2 + 48*x^3)/(18*Sqrt[2 + 3*x^2]) - (4*Log[-(Sqrt[3]*x)
+ Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

Maple [A]
time = 0.08, size = 65, normalized size = 0.92

method	result

risch	$\frac{48x^3+168x^2-15x+182}{18\sqrt{3x^2+2}}+\frac{4\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$
trager	$\frac{48x^3+168x^2-15x+182}{18\sqrt{3x^2+2}}+\frac{4\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
default	$\frac{8x^3}{3\sqrt{3x^2+2}}-\frac{5x}{6\sqrt{3x^2+2}}+\frac{4\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}+\frac{28x^2}{3\sqrt{3x^2+2}}+\frac{91}{9\sqrt{3x^2+2}}$
meijerg	$\frac{\sqrt{2}\,x}{4\sqrt{\frac{3x^2}{2}+1}}+\frac{20\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{3}\,\sqrt{2}}{2\sqrt{\frac{3x^2}{2}+1}}+\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{3}\,\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{7\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{\frac{3x^2}{2}+1}}\right)}{6\sqrt{\pi}}+ -$ 3

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/3*x^3/(3*x^2+2)^(1/2)-5/6*x/(3*x^2+2)^(1/2)+4/9*arcsinh(1/2*x*6^(1/2))*3^
(1/2)+28/3*x^2/(3*x^2+2)^(1/2)+91/9/(3*x^2+2)^(1/2)
```

Maxima [A]
time = 0.50, size = 64, normalized size = 0.90

$$\frac{8\,x^3}{3\,\sqrt{3\,x^2+2}}+\frac{28\,x^2}{3\,\sqrt{3\,x^2+2}}+\frac{4}{9}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}\,x\right)-\frac{5\,x}{6\,\sqrt{3\,x^2+2}}+\frac{91}{9\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^3/sqrt(3*x^2 + 2) + 28/3*x^2/sqrt(3*x^2 + 2) + 4/9*sqrt(3)*arcsinh(1/
2*sqrt(6)*x) - 5/6*x/sqrt(3*x^2 + 2) + 91/9/sqrt(3*x^2 + 2)
```

Fricas [A]
time = 0.34, size = 72, normalized size = 1.01

$$\frac{4\,\sqrt{3}\,(3\,x^2+2)\log\left(-\sqrt{3}\,\sqrt{3\,x^2+2}\,x-3\,x^2-1\right)+(48\,x^3+168\,x^2-15\,x+182)\sqrt{3\,x^2+2}}{18\,(3\,x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/18*(4*sqrt(3)*(3*x^2 + 2)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + (
48*x^3 + 168*x^2 - 15*x + 182)*sqrt(3*x^2 + 2))/(3*x^2 + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^2 \cdot (4x^2+3x+1)}{(3x^2+2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(3/2), x)
```

Giac [A]
time = 4.47, size = 49, normalized size = 0.69

$$-\frac{4}{9}\sqrt{3}\log\left(-\sqrt{3}\,x+\sqrt{3\,x^2+2}\right)+\frac{3\,(8\,(2\,x+7)x-5)x+182}{18\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] -4/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/18*(3*(8*(2*x + 7)*x - 5
)*x + 182)/sqrt(3*x^2 + 2)
```

Mupad [B]
time = 4.07, size = 105, normalized size = 1.48

$$\frac{4\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{9}+\frac{\sqrt{3}\,\left(\frac{8x}{3}+\frac{28}{3}\right)\sqrt{x^2+\frac{2}{3}}}{3}+\frac{\sqrt{3}\,\sqrt{6}\,\left(-630+\sqrt{6}\,141\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{\sqrt{3}\,\sqrt{6}\,\left(630+\sqrt{6}\,141\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
```

```
[Out] (4*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/9 + (3^(1/2)*((8*x)/3 + 28/3)*(x^2
 + 2/3)^(1/2))/3 + (3^(1/2)*6^(1/2)*(6^(1/2)*141i - 630)*(x^2 + 2/3)^(1/2)*
1i)/(1944*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*141i + 630)*(x^
2 + 2/3)^(1/2)*1i)/(1944*(x + (6^(1/2)*1i)/3))
```

3.126 $\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{2-51x}{18\sqrt{2+3x^2}}+\frac{8}{9}\sqrt{2+3x^2}+\frac{10\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

```
[Out] 10/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/18*(2-51*x)/(3*x^2+2)^(1/2)+8/9*(3*x^
2+2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1828, 655, 221}

$$\frac{2-51x}{18\sqrt{3x^2+2}}+\frac{8}{9}\sqrt{3x^2+2}+\frac{10\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (2 - 51*x)/(18*Sqrt[2 + 3*x^2]) + (8*Sqrt[2 + 3*x^2])/9 + (10*ArcSinh[Sqrt[
3/2]*x])/(3*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx &= \frac{2-51x}{18\sqrt{2+3x^2}}-\frac{1}{2}\int \frac{-\frac{20}{3}-\frac{16x}{3}}{\sqrt{2+3x^2}}\,dx\\ &= \frac{2-51x}{18\sqrt{2+3x^2}}+\frac{8}{9}\sqrt{2+3x^2}+\frac{10}{3}\int \frac{1}{\sqrt{2+3x^2}}\,dx\\ &= \frac{2-51x}{18\sqrt{2+3x^2}}+\frac{8}{9}\sqrt{2+3x^2}+\frac{10\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 56, normalized size = 1.02

$$\frac{34-51x+48x^2}{18\sqrt{2+3x^2}}-\frac{10\log\left(-\sqrt{3}\,x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
[Out] (34 - 51*x + 48*x^2)/(18*Sqrt[2 + 3*x^2]) - (10*Log[-(Sqrt[3]*x) + Sqrt[2 +
 3*x^2]])/(3*Sqrt[3])
```

Maple [A]
time = 0.08, size = 51, normalized size = 0.93

method	result
risch	$\frac{48x^2-51x+34}{18\sqrt{3x^2+2}}+\frac{10\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$
default	$\frac{8x^2}{3\sqrt{3x^2+2}}+\frac{17}{9\sqrt{3x^2+2}}-\frac{17x}{6\sqrt{3x^2+2}}+\frac{10\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$
trager	$\frac{48x^2-51x+34}{18\sqrt{3x^2+2}}+\frac{10\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$

meijerg	$\frac{\sqrt{2}\,x}{4\sqrt{\frac{3x^2}{2}+1}}+\frac{10\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{3}\,\sqrt{2}}{2\sqrt{\frac{3x^2}{2}+1}}+\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x\sqrt{3}\,\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{5\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{\frac{3x^2}{2}+1}}\right)}{6\sqrt{\pi}}+\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/3*x^2/(3*x^2+2)^(1/2)+17/9/(3*x^2+2)^(1/2)-17/6*x/(3*x^2+2)^(1/2)+10/9*ar
csinh(1/2*x*6^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.48, size = 50, normalized size = 0.91

$$\frac{8\,x^2}{3\,\sqrt{3\,x^2+2}}+\frac{10}{9}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\,\sqrt{6}\,x\right)-\frac{17\,x}{6\,\sqrt{3\,x^2+2}}+\frac{17}{9\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^2/sqrt(3*x^2 + 2) + 10/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 17/6*x/sqrt
(3*x^2 + 2) + 17/9/sqrt(3*x^2 + 2)
```

Fricas [A]
time = 0.36, size = 67, normalized size = 1.22

$$\frac{10\,\sqrt{3}\,(3\,x^2+2)\log\left(-\sqrt{3}\,\sqrt{3\,x^2+2}\,x-3\,x^2-1\right)+(48\,x^2-51\,x+34)\sqrt{3\,x^2+2}}{18\,(3\,x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/18*(10*sqrt(3)*(3*x^2 + 2)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
(48*x^2 - 51*x + 34)*sqrt(3*x^2 + 2))/(3*x^2 + 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(49) = 98$.
time = 6.07, size = 114, normalized size = 2.07

$$\frac{30\sqrt{3}\,x^2\operatorname{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{27x^2+18}+\frac{8x^2}{3\sqrt{3x^2+2}}-\frac{30x\sqrt{3x^2+2}}{27x^2+18}+\frac{x}{2\sqrt{3x^2+2}}+\frac{20\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{27x^2+18}+\frac{17}{9\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

```
[Out] 30*sqrt(3)*x**2*asinh(sqrt(6)*x/2)/(27*x**2 + 18) + 8*x**2/(3*sqrt(3*x**2 +
 2)) - 30*x*sqrt(3*x**2 + 2)/(27*x**2 + 18) + x/(2*sqrt(3*x**2 + 2)) + 20*s
qrt(3)*asinh(sqrt(6)*x/2)/(27*x**2 + 18) + 17/(9*sqrt(3*x**2 + 2))
```

Giac [A]
time = 4.79, size = 44, normalized size = 0.80

$$-\frac{10}{9}\sqrt{3}\,\log\left(-\sqrt{3}\,x+\sqrt{3\,x^2+2}\right)+\frac{3\,(16\,x-17)x+34}{18\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] -10/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/18*(3*(16*x - 17)*x + 3
4)/sqrt(3*x^2 + 2)
```

Mupad [B]
time = 0.04, size = 100, normalized size = 1.82

$$\frac{8\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{9}+\frac{10\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{9}+\frac{\sqrt{3}\,\sqrt{6}\,\left(-6+\sqrt{6}\,51\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{648\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{\sqrt{3}\,\sqrt{6}\,\left(6+\sqrt{6}\,51\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{648\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
```

```
[Out] (8*3^(1/2)*(x^2 + 2/3)^(1/2))/9 + (10*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))
/9 + (3^(1/2)*6^(1/2)*(6^(1/2)*51i - 6)*(x^2 + 2/3)^(1/2)*1i)/(648*(x - (6^
(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*51i + 6)*(x^2 + 2/3)^(1/2)*1i)/(6
48*(x + (6^(1/2)*1i)/3))
```

3.127 $\int \frac{1+3x+4x^2}{(1+2x)(2+3x^2)^{3/2}} dx$

Optimal. Leaf size=53

$$\frac{-38+21x}{66\sqrt{2+3x^2}}-\frac{2\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{11\sqrt{11}}$$

```
[Out] -2/121*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)+1/66*(-38+21
*x)/(3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1661, 12, 739, 212}

$$-\frac{38-21x}{66\sqrt{3x^2+2}}-\frac{2\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] -1/66*(38 - 21*x)/Sqrt[2 + 3*x^2] - (2*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 +
 3*x^2])])/(11*Sqrt[11])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx &= -\frac{38-21x}{66\sqrt{2+3x^2}}-\frac{1}{6}\int -\frac{12}{11(1+2x)\sqrt{2+3x^2}}\,dx\\ &= -\frac{38-21x}{66\sqrt{2+3x^2}}+\frac{2}{11}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\ &= -\frac{38-21x}{66\sqrt{2+3x^2}}-\frac{2}{11}\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)\\ &= -\frac{38-21x}{66\sqrt{2+3x^2}}-\frac{2\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{11\sqrt{11}}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 51, normalized size = 0.96

$$\frac{-418+231x-12\sqrt{22+33x^2}\tanh^{-1}\left(\frac{4-3x}{\sqrt{22+33x^2}}\right)}{726\sqrt{2+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(3/2)),x]
[Out] (-418 + 231*x - 12*Sqrt[22 + 33*x^2]*ArcTanh[(4 - 3*x)/Sqrt[22 + 33*x^2]])/
(726*Sqrt[2 + 3*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(42) = 84$.
time = 0.09, size = 88, normalized size = 1.66

method	result

risch	$\frac{-38+21x}{66\sqrt{3x^2+2}}-\frac{2\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{121}$
trager	$\frac{-38+21x}{66\sqrt{3x^2+2}}-\frac{2\operatorname{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{-3\operatorname{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}+4\operatorname{RootOf}\left(\_Z^2-11\right)}{2x+1}\right)}{121}$
default	$\frac{x}{4\sqrt{3x^2+2}}-\frac{2}{3\sqrt{3x^2+2}}+\frac{1}{11\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}+\frac{3x}{44\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}-\frac{2\sqrt{11}\,\operatorname{arct}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x/(3*x^2+2)^(1/2)-2/3/(3*x^2+2)^(1/2)+1/11/(3*(x+1/2)^2-3*x+5/4)^(1/2)+
3/44*x/(3*(x+1/2)^2-3*x+5/4)^(1/2)-2/121*11^(1/2)*arctanh(2/11*(4-3*x)*11^(
1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

Maxima [A]
time = 0.50, size = 58, normalized size = 1.09

$$\frac{2}{121}\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{7\,x}{22\,\sqrt{3\,x^2+2}}-\frac{19}{33\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/121*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1
)) + 7/22*x/sqrt(3*x^2 + 2) - 19/33/sqrt(3*x^2 + 2)
```

Fricas [A]
time = 0.33, size = 83, normalized size = 1.57

$$\frac{6\,\sqrt{11}\,(3\,x^2+2)\log\left(-\frac{\sqrt{11}\,\sqrt{3\,x^2+2}\,(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right)+11\,\sqrt{3\,x^2+2}\,(21\,x-38)}{726\,(3\,x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/726*(6*sqrt(11)*(3*x^2 + 2)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21
*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*sqrt(3*x^2 + 2)*(21*x - 38))/(3*x
^2 + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 + 2)**(3/2)), x)
```

Giac [A]
time = 4.01, size = 82, normalized size = 1.55

$$\frac{2}{121}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{21x-38}{66\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/121*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 +
2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/66*(21*x -
38)/sqrt(3*x^2 + 2)
```

Mupad [B]
time = 0.14, size = 106, normalized size = 2.00

$$\frac{\sqrt{11}\left(2\ln\left(x+\frac{1}{2}\right)-2\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{121}-\frac{\sqrt{3}\sqrt{6}\left(-114+\sqrt{6}\,21\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2376\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{\sqrt{3}\sqrt{6}\left(114+\sqrt{6}\,21\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2376\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(3/2)),x)
```

```
[Out] (11^(1/2)*(2*log(x + 1/2) - 2*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/
3 - 4/3)))/121 - (3^(1/2)*6^(1/2)*(6^(1/2)*21i - 114)*(x^2 + 2/3)^(1/2)*1i)
/(2376*(x - (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*21i + 114)*(x^2 +
2/3)^(1/2)*1i)/(2376*(x + (6^(1/2)*1i)/3))
```

3.128 $\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=75

$$\frac{-10+97x}{242\sqrt{2+3x^2}}-\frac{4\sqrt{2+3x^2}}{121(1+2x)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}$$

```
[Out] 4/1331*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)+1/242*(-10+9
7*x)/(3*x^2+2)^(1/2)-4/121*(3*x^2+2)^(1/2)/(1+2*x)
```

Rubi [A]
time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1661, 821, 739, 212}

$$-\frac{10-97x}{242\sqrt{3x^2+2}}-\frac{4\sqrt{3x^2+2}}{121(2x+1)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] -1/242*(10 - 97*x)/Sqrt[2 + 3*x^2] - (4*Sqrt[2 + 3*x^2])/(121*(1 + 2*x)) +
(4*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx &= -\frac{10-97x}{242\sqrt{2+3x^2}}-\frac{1}{6}\int \frac{-\frac{72}{121}+\frac{120x}{121}}{(1+2x)^2\sqrt{2+3x^2}}\,dx\\
&= -\frac{10-97x}{242\sqrt{2+3x^2}}-\frac{4\sqrt{2+3x^2}}{121(1+2x)}-\frac{4}{121}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= -\frac{10-97x}{242\sqrt{2+3x^2}}-\frac{4\sqrt{2+3x^2}}{121(1+2x)}+\frac{4}{121}\mathrm{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)\\
&= -\frac{10-97x}{242\sqrt{2+3x^2}}-\frac{4\sqrt{2+3x^2}}{121(1+2x)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}
\end{aligned}$$

Mathematica [A]
time = 0.44, size = 71, normalized size = 0.95

$$\frac{11\left(-26+77x+170x^2\right)+8(1+2x)\sqrt{22+33x^2}\,\tanh^{-1}\left(\frac{4-3x}{\sqrt{22+33x^2}}\right)}{2662(1+2x)\sqrt{2+3x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] (11*(-26 + 77*x + 170*x^2) + 8*(1 + 2*x)*Sqrt[22 + 33*x^2]*ArcTanh[(4 - 3*x
)/Sqrt[22 + 33*x^2]])/(2662*(1 + 2*x)*Sqrt[2 + 3*x^2])
```

Maple [A]
time = 0.09, size = 98, normalized size = 1.31

method	result

risch	$\frac{170x^2+77x-26}{242(2x+1)\sqrt{3x^2+2}}+\frac{4\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{1331}$
trager	$\frac{(170x^2+77x-26)\sqrt{3x^2+2}}{1452x^3+726x^2+968x+484}+\frac{4\operatorname{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{-3\operatorname{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}+4\operatorname{RootOf}\left(\_Z^2-11\right)}{2x+1}\right)}{1331}$
default	$\frac{x}{2\sqrt{3x^2+2}}-\frac{2}{121\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}-\frac{18x}{121\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}+\frac{4\sqrt{11}\,\operatorname{arctanh}\left(\frac{}{11\sqrt{12\,(}}\right)}{133}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^2/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/(3*x^2+2)^(1/2)-2/121/(3*(x+1/2)^2-3*x+5/4)^(1/2)-18/121*x/(3*(x+1/2)
^2-3*x+5/4)^(1/2)+4/1331*11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)
^2-12*x+5)^(1/2))-1/22/(x+1/2)/(3*(x+1/2)^2-3*x+5/4)^(1/2)
```

Maxima [A]
time = 0.50, size = 84, normalized size = 1.12

$$-\frac{4}{1331}\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{85\,x}{242\,\sqrt{3\,x^2+2}}-\frac{2}{121\,\sqrt{3\,x^2+2}}-\frac{1}{11\,\left(2\,\sqrt{3\,x^2+2}\,x+\sqrt{3\,x^2+2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x +
 1)) + 85/242*x/sqrt(3*x^2 + 2) - 2/121/sqrt(3*x^2 + 2) - 1/11/(2*sqrt(3*x^
2 + 2)*x + sqrt(3*x^2 + 2))
```

Fricas [A]
time = 0.36, size = 103, normalized size = 1.37

$$\frac{4\,\sqrt{11}\,(6\,x^3+3\,x^2+4\,x+2)\log\left(\frac{\sqrt{11}\,\sqrt{3\,x^2+2}\,(3\,x-4)-21\,x^2+12\,x-19}{4\,x^2+4\,x+1}\right)+11\,(170\,x^2+77\,x-26)\sqrt{3\,x^2+2}}{2662\,(6\,x^3+3\,x^2+4\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2662*(4*sqrt(11)*(6*x^3 + 3*x^2 + 4*x + 2)*log((sqrt(11)*sqrt(3*x^2 + 2)*
(3*x - 4) - 21*x^2 + 12*x - 19)/(4*x^2 + 4*x + 1)) + 11*(170*x^2 + 77*x - 2
6)*sqrt(3*x^2 + 2))/(6*x^3 + 3*x^2 + 4*x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x+1)^2 (3x^2+2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 + 2)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(60) = 120.
time = 6.08, size = 168, normalized size = 2.24

$$-\frac{1}{7986}\sqrt{11}\left(85\sqrt{11}\sqrt{3} + 24\log\left(\sqrt{11}\sqrt{3} - 3\right)\right)\mathrm{sgn}\left(\frac{1}{2x+1}\right) - \frac{\frac{\frac{93}{\mathrm{sgn}\left(\frac{1}{2x+1}\right)} + \frac{44}{(2x+1)\mathrm{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1} - \frac{85}{\mathrm{sgn}\left(\frac{1}{2x+1}\right)}}{242\sqrt{-\frac{6}{2x+1} + \frac{11}{(2x+1)^2} + 3}} + \frac{4\sqrt{11}\log\left(\sqrt{11}\left(\sqrt{-\frac{6}{2x+1} + \frac{11}{(2x+1)^2} + 3} + \frac{\sqrt{11}}{2x+1}\right) - 3\right)}{1331\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/7986*sqrt(11)*(85*sqrt(11)*sqrt(3) + 24*log(sqrt(11)*sqrt(3) - 3))*sgn(1
/(2*x + 1)) - 1/242*((93/sgn(1/(2*x + 1)) + 44/((2*x + 1)*sgn(1/(2*x + 1)))
)/(2*x + 1) - 85/sgn(1/(2*x + 1)))/sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3)
+ 4/1331*sqrt(11)*log(sqrt(11)*(sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3) + s
qrt(11)/(2*x + 1)) - 3)/sgn(1/(2*x + 1))
```

Mupad [B]
time = 4.14, size = 157, normalized size = 2.09

$$\frac{4\sqrt{11}\,\ln\left(x - \frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3} - \frac{4}{3}\right)}{1331} - \frac{4\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{1331} + \frac{97\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1452\left(x - \frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{97\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1452\left(x + \frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{2\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{121\left(x+\frac{1}{2}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,5\mathrm{i}}{1452\left(x - \frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,5\mathrm{i}}{1452\left(x + \frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(3/2)),x)
```

```
[Out] (4*11^(1/2)*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/1331 - (
4*11^(1/2)*log(x + 1/2))/1331 + (97*3^(1/2)*(x^2 + 2/3)^(1/2))/(1452*(x - (
6^(1/2)*1i)/3)) + (97*3^(1/2)*(x^2 + 2/3)^(1/2))/(1452*(x + (6^(1/2)*1i)/3)
) - (2*3^(1/2)*(x^2 + 2/3)^(1/2))/(121*(x + 1/2)) + (3^(1/2)*6^(1/2)*(x^2 +
 2/3)^(1/2)*5i)/(1452*(x - (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(x^2 + 2/3)^
(1/2)*5i)/(1452*(x + (6^(1/2)*1i)/3))
```

3.129 $\int \frac{1+3x+4x^2}{(1+2x)^3(2+3x^2)^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{358+351x}{2662\sqrt{2+3x^2}}-\frac{2\sqrt{2+3x^2}}{121(1+2x)^2}+\frac{2\sqrt{2+3x^2}}{1331(1+2x)}-\frac{322\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{1331\sqrt{11}}$$

```
[Out] -322/14641*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)+1/2662*(
358+351*x)/(3*x^2+2)^(1/2)-2/121*(3*x^2+2)^(1/2)/(1+2*x)^2+2/1331*(3*x^2+2)
^(1/2)/(1+2*x)
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1661, 1665, 821, 739, 212}

$$\frac{351x+358}{2662\sqrt{3x^2+2}}+\frac{2\sqrt{3x^2+2}}{1331(2x+1)}-\frac{2\sqrt{3x^2+2}}{121(2x+1)^2}-\frac{322\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] (358 + 351*x)/(2662*Sqrt[2 + 3*x^2]) - (2*Sqrt[2 + 3*x^2])/(121*(1 + 2*x)^2
) + (2*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)) - (322*ArcTanh[(4 - 3*x)/(Sqrt[11]
*Sqrt[2 + 3*x^2])])/(1331*Sqrt[11])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
```

```
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx &= \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{1}{6}\int \frac{-\frac{2940}{1331}-\frac{7272x}{1331}-\frac{8592x^2}{1331}}{(1+2x)^3\sqrt{2+3x^2}}\,dx \\
&= \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{2\sqrt{2+3x^2}}{121(1+2x)^2} + \frac{1}{132}\int \frac{\frac{3768}{121}+\frac{7800x}{121}}{(1+2x)^2\sqrt{2+3x^2}}\,dx \\
&= \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{2\sqrt{2+3x^2}}{121(1+2x)^2} + \frac{2\sqrt{2+3x^2}}{1331(1+2x)} + \frac{322\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx}{1331} \\
&= \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{2\sqrt{2+3x^2}}{121(1+2x)^2} + \frac{2\sqrt{2+3x^2}}{1331(1+2x)} - \frac{322\text{Subst}\left(\int \frac{1}{11-x^2}\,dx, x,\right.}{1331} \\
&= \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{2\sqrt{2+3x^2}}{121(1+2x)^2} + \frac{2\sqrt{2+3x^2}}{1331(1+2x)} - \frac{322\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3}}\right.}{1331\sqrt{11}}
\end{aligned}$$

Mathematica [A]

time = 0.48, size = 81, normalized size = 0.84

$$\frac{\frac{11(278+1799x+2716x^2+1428x^3)}{(1+2x)^2\sqrt{2+3x^2}}+1288\sqrt{11}\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{11}}\right)}{29282}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] ((11*(278 + 1799*x + 2716*x^2 + 1428*x^3))/((1 + 2*x)^2*Sqrt[2 + 3*x^2]) +
1288*Sqrt[11]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[11]]
)/29282
```

Maple [A]
time = 0.09, size = 107, normalized size = 1.10

method	result
risch	$\frac{1428x^3+2716x^2+1799x+278}{2662(2x+1)^2\sqrt{3x^2+2}}-\frac{322\sqrt{11}\,\mathrm{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{14641}$
trager	$\frac{1428x^3+2716x^2+1799x+278}{2662(2x+1)^2\sqrt{3x^2+2}}+\frac{322\,\mathrm{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}-4\,\mathrm{RootOf}\left(\_Z^2-11\right)}{2x+1}\right)}{14641}$
default	$-\frac{1}{88\left(x+\frac{1}{2}\right)^2\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}+\frac{7}{484\left(x+\frac{1}{2}\right)\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}+\frac{161}{1331\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^3/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/88/(x+1/2)^2/(3*(x+1/2)^2-3*x+5/4)^(1/2)+7/484/(x+1/2)/(3*(x+1/2)^2-3*x+
5/4)^(1/2)+161/1331/(3*(x+1/2)^2-3*x+5/4)^(1/2)+357/2662*x/(3*(x+1/2)^2-3*x
+5/4)^(1/2)-322/14641*11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-
12*x+5)^(1/2))
```

Maxima [A]
time = 0.51, size = 124, normalized size = 1.28

$$\frac{322}{14641}\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{357\,x}{2662\sqrt{3\,x^2+2}}+\frac{161}{1331\sqrt{3\,x^2+2}}-\frac{1}{22\left(4\sqrt{3\,x^2+2}\,x^2+4\sqrt{3\,x^2+2}\,x+\sqrt{3\,x^2+2}\right)}+\frac{7}{242\left(2\sqrt{3\,x^2+2}\,x+\sqrt{3\,x^2+2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 322/14641*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
 + 1)) + 357/2662*x/sqrt(3*x^2 + 2) + 161/1331/sqrt(3*x^2 + 2) - 1/22/(4*sq
rt(3*x^2 + 2)*x^2 + 4*sqrt(3*x^2 + 2)*x + sqrt(3*x^2 + 2)) + 7/242/(2*sqrt(
3*x^2 + 2)*x + sqrt(3*x^2 + 2))
```

Fricas [A]
time = 0.35, size = 119, normalized size = 1.23

$$\frac{322\,\sqrt{11}\,(12\,x^4+12\,x^3+11\,x^2+8\,x+2)\log\left(-\frac{\sqrt{11}\,\sqrt{3\,x^2+2}\,(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right)+11\,(1428\,x^3+2716\,x^2+1799\,x+278)\sqrt{3\,x^2+2}}{29282\,(12\,x^4+12\,x^3+11\,x^2+8\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/29282*(322*sqrt(11)*(12*x^4 + 12*x^3 + 11*x^2 + 8*x + 2)*log(-(sqrt(11)*s
qrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*(142
8*x^3 + 2716*x^2 + 1799*x + 278)*sqrt(3*x^2 + 2))/(12*x^4 + 12*x^3 + 11*x^2
 + 8*x + 2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(78) = 156.
time = 3.61, size = 196, normalized size = 2.02

$$\frac{322}{14641}\sqrt{11}\log\left(-\frac{\left|-2\,\sqrt{3}\,x-\sqrt{11}-\sqrt{3}+2\,\sqrt{3\,x^2+2}\right|}{2\,\sqrt{3}\,x-\sqrt{11}+\sqrt{3}-2\,\sqrt{3\,x^2+2}}\right)+\frac{351\,x+358}{2662\,\sqrt{3\,x^2+2}}+\frac{36\left(\sqrt{3}\,x-\sqrt{3\,x^2+2}\right)^3-\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^2+2}\right)^2+48\,\sqrt{3}\,x+8\,\sqrt{3}-48\,\sqrt{3\,x^2+2}}{1331\left(\left(\sqrt{3}\,x-\sqrt{3\,x^2+2}\right)^2+\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^2+2}\right)-2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] 322/14641*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^
2 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/2662*(3
51*x + 358)/sqrt(3*x^2 + 2) + 1/1331*(36*(sqrt(3)*x - sqrt(3*x^2 + 2))^3 -
sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 48*sqrt(3)*x + 8*sqrt(3) - 48*sqr
t(3*x^2 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(
3*x^2 + 2)) - 2)^2
```

Mupad [B]
time = 4.17, size = 180, normalized size = 1.86

$$\frac{322\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{14641}-\frac{322\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{14641}+\frac{117\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{5324\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{117\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{5324\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{242\left(x^2+x+\frac{1}{4}\right)}+\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{1331\left(x+\frac{1}{2}\right)}-\frac{\sqrt{3}\,\sqrt{6}\,\sqrt{x^2+\frac{2}{3}}\,179\mathrm{i}}{15972\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{\sqrt{3}\,\sqrt{6}\,\sqrt{x^2+\frac{2}{3}}\,179\mathrm{i}}{15972\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(3/2)),x)
```

```
[Out] (322*11^(1/2)*log(x + 1/2))/14641 - (322*11^(1/2)*log(x - (3^(1/2)*11^(1/2)
*(x^2 + 2/3)^(1/2))/3 - 4/3))/14641 + (117*3^(1/2)*(x^2 + 2/3)^(1/2))/(5324
*(x - (6^(1/2)*1i)/3)) + (117*3^(1/2)*(x^2 + 2/3)^(1/2))/(5324*(x + (6^(1/2
)*1i)/3)) - (3^(1/2)*(x^2 + 2/3)^(1/2))/(242*(x + x^2 + 1/4)) + (3^(1/2)*(x
^2 + 2/3)^(1/2))/(1331*(x + 1/2)) - (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)*179i
)/(15972*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)*179i)/(
15972*(x + (6^(1/2)*1i)/3))
```

3.130 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{398+279x}{162\,(2+3x^2)^{3/2}}-\frac{152+465x}{54\sqrt{2+3x^2}}+\frac{32}{27}\sqrt{2+3x^2}+\frac{8\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{3}}$$

```
[Out] 1/162*(398+279*x)/(3*x^2+2)^(3/2)+8/3*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/54*(
-152-465*x)/(3*x^2+2)^(1/2)+32/27*(3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1828, 655, 221}

$$\frac{279x+398}{162\,(3x^2+2)^{3/2}}+\frac{32}{27}\sqrt{3x^2+2}-\frac{465x+152}{54\sqrt{3x^2+2}}+\frac{8\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (398 + 279*x)/(162*(2 + 3*x^2)^(3/2)) - (152 + 465*x)/(54*Sqrt[2 + 3*x^2])
+ (32*Sqrt[2 + 3*x^2])/27 + (8*ArcSinh[Sqrt[3/2]*x])/Sqrt[3]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{398+279x}{162(2+3x^2)^{3/2}} - \frac{1}{6}\int \frac{\frac{22}{3}-\frac{280x}{3}-144x^2-64x^3}{(2+3x^2)^{3/2}}\,dx$$
$$= \frac{398+279x}{162(2+3x^2)^{3/2}} - \frac{152+465x}{54\sqrt{2+3x^2}} + \frac{1}{12}\int \frac{96+\frac{128x}{3}}{\sqrt{2+3x^2}}\,dx$$
$$= \frac{398+279x}{162(2+3x^2)^{3/2}} - \frac{152+465x}{54\sqrt{2+3x^2}} + \frac{32}{27}\sqrt{2+3x^2} + 8\int \frac{1}{\sqrt{2+3x^2}}\,dx$$
$$= \frac{398+279x}{162(2+3x^2)^{3/2}} - \frac{152+465x}{54\sqrt{2+3x^2}} + \frac{32}{27}\sqrt{2+3x^2} + \frac{8\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.33, size = 64, normalized size = 0.88

$$\frac{254-2511x+936x^2-4185x^3+1728x^4}{162(2+3x^2)^{3/2}} - \frac{8\log\left(-\sqrt{3}\,x+\sqrt{2+3x^2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
[Out] (254 - 2511*x + 936*x^2 - 4185*x^3 + 1728*x^4)/(162*(2 + 3*x^2)^(3/2)) - (8
*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/Sqrt[3]
```

Maple [A]
time = 0.10, size = 91, normalized size = 1.25

method	result
risch	$\frac{1728x^4-4185x^3+936x^2-2511x+254}{162(3x^2+2)^{\frac{3}{2}}}+\frac{8\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{3}$
trager	$\frac{1728x^4-4185x^3+936x^2-2511x+254}{162(3x^2+2)^{\frac{3}{2}}}-\frac{8\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{3}$
default	$\frac{32x^4}{3(3x^2+2)^{\frac{3}{2}}}+\frac{52x^2}{9(3x^2+2)^{\frac{3}{2}}}+\frac{127}{81(3x^2+2)^{\frac{3}{2}}}-\frac{8x^3}{(3x^2+2)^{\frac{3}{2}}}-\frac{107x}{18\sqrt{3x^2+2}}+\frac{8\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{3}-\frac{65x}{18(3x^2+2)^{\frac{3}{2}}}$

meijerg	$\frac{\sqrt{2}\,x(3x^2+3)}{24\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{17\sqrt{2}\,x^3}{12\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{\sqrt{2}\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{2\sqrt{\pi}}+\frac{64\sqrt{2}\left(-4\sqrt{\pi}+\frac{\sqrt{\pi}\left(\frac{27}{2}x^4+36x^2+16\right)}{4\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{81\sqrt{\pi}}+\frac{16\sqrt{3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 32/3*x^4/(3*x^2+2)^(3/2)+52/9*x^2/(3*x^2+2)^(3/2)+127/81/(3*x^2+2)^(3/2)-8*
x^3/(3*x^2+2)^(3/2)-107/18*x/(3*x^2+2)^(1/2)+8/3*arcsinh(1/2*x*6^(1/2))*3^(
1/2)-65/18/(3*x^2+2)^(3/2)*x
```

Maxima [A]
time = 0.49, size = 105, normalized size = 1.44

$$\frac{32\,x^4}{3\,(3\,x^2+2)^{\frac{3}{2}}}-\frac{8}{3}\,x\left(\frac{9\,x^2}{(3\,x^2+2)^{\frac{3}{2}}}+\frac{4}{(3\,x^2+2)^{\frac{3}{2}}}\right)+\frac{8}{3}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\,\sqrt{6}\,x\right)-\frac{11\,x}{18\,\sqrt{3\,x^2+2}}+\frac{52\,x^2}{9\,(3\,x^2+2)^{\frac{3}{2}}}-\frac{65\,x}{18\,(3\,x^2+2)^{\frac{3}{2}}}+\frac{127}{81\,(3\,x^2+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/3*x^4/(3*x^2 + 2)^(3/2) - 8/3*x*(9*x^2/(3*x^2 + 2)^(3/2) + 4/(3*x^2 + 2)
^(3/2)) + 8/3*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 11/18*x/sqrt(3*x^2 + 2) + 52
/9*x^2/(3*x^2 + 2)^(3/2) - 65/18*x/(3*x^2 + 2)^(3/2) + 127/81/(3*x^2 + 2)^(
3/2)
```

Fricas [A]
time = 0.37, size = 87, normalized size = 1.19

$$\frac{216\,\sqrt{3}\,(9\,x^4+12\,x^2+4)\log\left(-\sqrt{3}\,\sqrt{3\,x^2+2}\,x-3\,x^2-1\right)+(1728\,x^4-4185\,x^3+936\,x^2-2511\,x+254)\sqrt{3\,x^2+2}}{162\,(9\,x^4+12\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/162*(216*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*
x^2 - 1) + (1728*x^4 - 4185*x^3 + 936*x^2 - 2511*x + 254)*sqrt(3*x^2 + 2))/
(9*x^4 + 12*x^2 + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(2x+1)^3\cdot(4x^2+3x+1)}{(3x^2+2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(5/2), x)
```

Giac [A]
time = 5.82, size = 53, normalized size = 0.73

$$-\frac{8}{3}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)+\frac{9\left(\left(3\left(64x-155\right)x+104\right)x-279\right)x+254}{162\left(3x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] -8/3*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/162*(9*((3*(64*x - 155)*
x + 104)*x - 279)*x + 254)/(3*x^2 + 2)^(3/2)
```

Mupad [B]
time = 0.06, size = 212, normalized size = 2.90

$$\frac{32\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{27}+\frac{8\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}x}{2}\right)}{3}-\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{31}{16}+\frac{\sqrt{6}\,199\mathrm{i}}{144}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(-\frac{31}{24}+\frac{\sqrt{6}\,199\mathrm{i}}{216}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}+\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{31}{16}+\frac{\sqrt{6}\,199\mathrm{i}}{144}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(\frac{31}{24}+\frac{\sqrt{6}\,199\mathrm{i}}{216}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}+\frac{\sqrt{3}\sqrt{6}\left(-1824+\sqrt{6}\,1953\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{7776\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{\sqrt{3}\sqrt{6}\left(1824+\sqrt{6}\,1953\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{7776\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

```
[Out] (32*3^(1/2)*(x^2 + 2/3)^(1/2))/27 + (8*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2)
)/3 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*199i)/144 - 31/16)/(x - (6^(1/2
)*1i)/3) - (6^(1/2)*((6^(1/2)*199i)/216 - 31/24)*1i)/(2*(x - (6^(1/2)*1i)/3
)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*199i)/144 + 31/16)/(x + (
6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*199i)/216 + 31/24)*1i)/(2*(x + (6^(1/2)
*1i)/3)^2)))/27 + (3^(1/2)*6^(1/2)*(6^(1/2)*1953i - 1824)*(x^2 + 2/3)^(1/2)
*1i)/(7776*(x + (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*1953i + 1824)*
(x^2 + 2/3)^(1/2)*1i)/(7776*(x - (6^(1/2)*1i)/3))
```

3.131 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx$

Optimal. Leaf size=60

$$\frac{70-47x}{54\,(2+3x^2)^{3/2}}-\frac{168+59x}{54\sqrt{2+3x^2}}+\frac{16\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{9\sqrt{3}}$$

```
[Out] 1/54*(70-47*x)/(3*x^2+2)^(3/2)+16/27*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/54*(-
168-59*x)/(3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1828, 12, 221}

$$\frac{70-47x}{54\,(3x^2+2)^{3/2}}-\frac{59x+168}{54\sqrt{3x^2+2}}+\frac{16\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (70 - 47*x)/(54*(2 + 3*x^2)^(3/2)) - (168 + 59*x)/(54*Sqrt[2 + 3*x^2]) + (1
6*ArcSinh[Sqrt[3/2]*x])/(9*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx &= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{1}{6}\int \frac{-\frac{74}{9}-56x-32x^2}{\left(2+3x^2\right)^{3/2}}\,dx \\
&= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{168+59x}{54\sqrt{2+3x^2}}+\frac{1}{12}\int \frac{64}{3\sqrt{2+3x^2}}\,dx \\
&= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{168+59x}{54\sqrt{2+3x^2}}+\frac{16}{9}\int \frac{1}{\sqrt{2+3x^2}}\,dx \\
&= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{168+59x}{54\sqrt{2+3x^2}}+\frac{16\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{9\sqrt{3}}
\end{aligned}
$$

Mathematica [A]
time = 0.29, size = 61, normalized size = 1.02

$$\frac{-266-165x-504x^2-177x^3}{54\left(2+3x^2\right)^{3/2}}-\frac{16\log\left(-\sqrt{3}\,x+\sqrt{2+3x^2}\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
[Out] (-266 - 165*x - 504*x^2 - 177*x^3)/(54*(2 + 3*x^2)^(3/2)) - (16*Log[-(Sqrt[
3]*x) + Sqrt[2 + 3*x^2]])/(9*Sqrt[3])
```

Maple [A]
time = 0.09, size = 77, normalized size = 1.28

method	result
risch	$-\frac{177x^3+504x^2+165x+266}{54(3x^2+2)^{\frac{3}{2}}}+\frac{16\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{27}$
trager	$-\frac{177x^3+504x^2+165x+266}{54(3x^2+2)^{\frac{3}{2}}}+\frac{16\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{27}$
default	$-\frac{16x^3}{9(3x^2+2)^{\frac{3}{2}}}-\frac{x}{2\sqrt{3x^2+2}}+\frac{16\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{27}-\frac{28x^2}{3(3x^2+2)^{\frac{3}{2}}}-\frac{133}{27(3x^2+2)^{\frac{3}{2}}}-\frac{37x}{18(3x^2+2)^{\frac{3}{2}}}$

meijerg	$\frac{\sqrt{2}\,x(3x^2+3)}{24\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{5\sqrt{2}\,x^3}{6\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{7\sqrt{2}\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{18\sqrt{\pi}}+\frac{32\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{3}\,\sqrt{2}\,(30x^2+15)}{20\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+3\sqrt{\pi}\,\operatorname{arcsinh}\right.}{81\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/9*x^3/(3*x^2+2)^(3/2)-1/2*x/(3*x^2+2)^(1/2)+16/27*arcsinh(1/2*x*6^(1/2)
)*3^(1/2)-28/3*x^2/(3*x^2+2)^(3/2)-133/27/(3*x^2+2)^(3/2)-37/18/(3*x^2+2)^(
3/2)*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(45) = 90.
time = 0.52, size = 91, normalized size = 1.52

$$-\frac{16}{27}\,x\left(\frac{9\,x^2}{(3\,x^2+2)^{\frac{3}{2}}}+\frac{4}{(3\,x^2+2)^{\frac{3}{2}}}\right)+\frac{16}{27}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\,\sqrt{6}\,x\right)+\frac{37\,x}{54\,\sqrt{3\,x^2+2}}-\frac{28\,x^2}{3\,(3\,x^2+2)^{\frac{3}{2}}}-\frac{37\,x}{18\,(3\,x^2+2)^{\frac{3}{2}}}-\frac{133}{27\,(3\,x^2+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] -16/27*x*(9*x^2/(3*x^2 + 2)^(3/2) + 4/(3*x^2 + 2)^(3/2)) + 16/27*sqrt(3)*ar
csinh(1/2*sqrt(6)*x) + 37/54*x/sqrt(3*x^2 + 2) - 28/3*x^2/(3*x^2 + 2)^(3/2)
 - 37/18*x/(3*x^2 + 2)^(3/2) - 133/27/(3*x^2 + 2)^(3/2)
```

Fricas [A]
time = 0.36, size = 83, normalized size = 1.38

$$\frac{16\,\sqrt{3}\,(9\,x^4+12\,x^2+4)\log\left(-\sqrt{3}\,\sqrt{3\,x^2+2}\,x-3\,x^2-1\right)-(177\,x^3+504\,x^2+165\,x+266)\sqrt{3\,x^2+2}}{54\,(9\,x^4+12\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/54*(16*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^
2 - 1) - (177*x^3 + 504*x^2 + 165*x + 266)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x^2
 + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(2x+1)^2\cdot(4x^2+3x+1)}{(3x^2+2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(5/2), x)
```

Giac [A]
time = 4.73, size = 48, normalized size = 0.80

$$-\frac{16}{27}\sqrt{3}\,\log\left(-\sqrt{3}\,x+\sqrt{3\,x^2+2}\right)-\frac{3\,((59\,x+168)x+55)x+266}{54\,(3\,x^2+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] -16/27*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) - 1/54*(3*((59*x + 168)*x
+ 55)*x + 266)/(3*x^2 + 2)^(3/2)
```

Mupad [B]
time = 0.05, size = 200, normalized size = 3.33

$$\frac{16\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{27}+\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{-\frac{47}{48}+\frac{\sqrt{6}\,35\mathrm{i}}{48}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\,\left(-\frac{47}{72}+\frac{\sqrt{6}\,35\mathrm{i}}{72}\right)\,1\mathrm{i}}{2\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}-\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{\frac{47}{48}+\frac{\sqrt{6}\,35\mathrm{i}}{48}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\,\left(\frac{47}{72}+\frac{\sqrt{6}\,35\mathrm{i}}{72}\right)\,1\mathrm{i}}{2\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}+\frac{\sqrt{3}\,\sqrt{6}\,\left(-672+\sqrt{6}\,63\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{\sqrt{3}\,\sqrt{6}\,\left(672+\sqrt{6}\,63\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

```
[Out] (16*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(
((6^(1/2)*35i)/48 - 47/48)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*35i)/7
2 - 47/72)*1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*(x^2 + 2/3)^(1/2)
*(((6^(1/2)*35i)/48 + 47/48)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*35i)
/72 + 47/72)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*6^(1/2)*(6^(1/2
)*63i - 672)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x + (6^(1/2)*1i)/3)) + (3^(1/2)*6
^(1/2)*(6^(1/2)*63i + 672)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x - (6^(1/2)*1i)/3)
)
```

3.132 $\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2-51x}{54(2+3x^2)^{3/2}}-\frac{16-13x}{18\sqrt{2+3x^2}}$$

```
[Out] 1/54*(2-51*x)/(3*x^2+2)^(3/2)+1/18*(-16+13*x)/(3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1828, 651}

$$\frac{2-51x}{54(3x^2+2)^{3/2}}-\frac{16-13x}{18\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (2 - 51*x)/(54*(2 + 3*x^2)^(3/2)) - (16 - 13*x)/(18*Sqrt[2 + 3*x^2])
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx &= \frac{2-51x}{54(2+3x^2)^{3/2}}-\frac{1}{6}\int \frac{-\frac{26}{3}-16x}{(2+3x^2)^{3/2}}\,dx\\ &= \frac{2-51x}{54(2+3x^2)^{3/2}}-\frac{16-13x}{18\sqrt{2+3x^2}}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 30, normalized size = 0.73

$$\frac{-94+27x-144x^2+117x^3}{54\left(2+3x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
[Out] (-94 + 27*x - 144*x^2 + 117*x^3)/(54*(2 + 3*x^2)^(3/2))
```

Maple [A]
time = 0.07, size = 51, normalized size = 1.24

method	result	size
gosper	$\frac{117x^3-144x^2+27x-94}{54\left(3x^2+2\right)^{\frac{3}{2}}}$	27
trager	$\frac{117x^3-144x^2+27x-94}{54\left(3x^2+2\right)^{\frac{3}{2}}}$	27
risch	$\frac{117x^3-144x^2+27x-94}{54\left(3x^2+2\right)^{\frac{3}{2}}}$	27
default	$-\frac{8x^2}{3\left(3x^2+2\right)^{\frac{3}{2}}}-\frac{47}{27\left(3x^2+2\right)^{\frac{3}{2}}}-\frac{17x}{18\left(3x^2+2\right)^{\frac{3}{2}}}+\frac{13x}{18\sqrt{3x^2+2}}$	51
meijerg	$\frac{\sqrt{2}\,x\left(3x^2+3\right)}{24\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{5\sqrt{2}\,x^3}{12\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{5\sqrt{2}\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{18\sqrt{\pi}}+\frac{8\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}\left(18x^2+8\right)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{27\sqrt{\pi}}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -8/3*x^2/(3*x^2+2)^(3/2)-47/27/(3*x^2+2)^(3/2)-17/18/(3*x^2+2)^(3/2)*x+13/1
8*x/(3*x^2+2)^(1/2)
```

Maxima [A]
time = 0.30, size = 50, normalized size = 1.22

$$\frac{13\,x}{18\,\sqrt{3\,x^2+2}}-\frac{8\,x^2}{3\,\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{17\,x}{18\,\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{47}{27\,\left(3\,x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
[Out] 13/18*x/sqrt(3*x^2 + 2) - 8/3*x^2/(3*x^2 + 2)^(3/2) - 17/18*x/(3*x^2 + 2)^(
3/2) - 47/27/(3*x^2 + 2)^(3/2)
```

Fricas [A]
time = 0.37, size = 40, normalized size = 0.98

$$\frac{\left(117\,x^{3}-144\,x^{2}+27\,x-94\right)\sqrt{3\,x^{2}+2}}{54\,\left(9\,x^{4}+12\,x^{2}+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/54*(117*x^3 - 144*x^2 + 27*x - 94)*sqrt(3*x^2 + 2)/(9*x^4 + 12*x^2 + 4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(36) = 72$.
time = 19.00, size = 180, normalized size = 4.39

$$\frac{10x^{3}}{18x^{2}\sqrt{3x^{2}+2}+12\sqrt{3x^{2}+2}}+\frac{x^{3}}{6x^{2}\sqrt{3x^{2}+2}+4\sqrt{3x^{2}+2}}-\frac{72x^{2}}{81x^{2}\sqrt{3x^{2}+2}+54\sqrt{3x^{2}+2}}+\frac{x}{6x^{2}\sqrt{3x^{2}+2}+4\sqrt{3x^{2}+2}}-\frac{32}{81x^{2}\sqrt{3x^{2}+2}+54\sqrt{3x^{2}+2}}-\frac{5}{27x^{2}\sqrt{3x^{2}+2}+18\sqrt{3x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

```
[Out] 10*x**3/(18*x**2*sqrt(3*x**2 + 2) + 12*sqrt(3*x**2 + 2)) + x**3/(6*x**2*sqr
t(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) - 72*x**2/(81*x**2*sqrt(3*x**2 + 2) + 5
4*sqrt(3*x**2 + 2)) + x/(6*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) - 32
/(81*x**2*sqrt(3*x**2 + 2) + 54*sqrt(3*x**2 + 2)) - 5/(27*x**2*sqrt(3*x**2
+ 2) + 18*sqrt(3*x**2 + 2))
```

Giac [A]
time = 4.83, size = 25, normalized size = 0.61

$$\frac{9\left(\left(13\,x-16\right)x+3\right)x-94}{54\,\left(3\,x^{2}+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/54*(9*((13*x - 16)*x + 3)*x - 94)/(3*x^2 + 2)^(3/2)
```

Mupad [B]
time = 4.11, size = 185, normalized size = 4.51

$$\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{-\frac{17}{16}+\frac{\sqrt{6}\,1{}\mathrm{i}}{48}}{x+\frac{\sqrt{6}\,1{}\mathrm{i}}{3}}+\frac{\sqrt{6}\,\left(-\frac{17}{24}+\frac{\sqrt{6}\,1{}\mathrm{i}}{72}\right)\,1{}\mathrm{i}}{2\,{\left(x+\frac{\sqrt{6}\,1{}\mathrm{i}}{3}\right)}^2}\right)}{27}-\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{\frac{17}{16}+\frac{\sqrt{6}\,1{}\mathrm{i}}{48}}{x-\frac{\sqrt{6}\,1{}\mathrm{i}}{3}}-\frac{\sqrt{6}\,\left(\frac{17}{24}+\frac{\sqrt{6}\,1{}\mathrm{i}}{72}\right)\,1{}\mathrm{i}}{2\,{\left(x-\frac{\sqrt{6}\,1{}\mathrm{i}}{3}\right)}^2}\right)}{27}-\frac{\sqrt{3}\,\sqrt{6}\,\left(-192+\sqrt{6}\,69{}\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1{}\mathrm{i}}{2592\,\left(x-\frac{\sqrt{6}\,1{}\mathrm{i}}{3}\right)}-\frac{\sqrt{3}\,\sqrt{6}\,\left(192+\sqrt{6}\,69{}\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1{}\mathrm{i}}{2592\,\left(x+\frac{\sqrt{6}\,1{}\mathrm{i}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

```
[Out] (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*1i)/48 - 17/16)/(x + (6^(1/2)*1i)/3)
+ (6^(1/2)*((6^(1/2)*1i)/72 - 17/24)*1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27 -
(3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*1i)/48 + 17/16)/(x - (6^(1/2)*1i)/3)
- (6^(1/2)*((6^(1/2)*1i)/72 + 17/24)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 -
(3^(1/2)*6^(1/2)*(6^(1/2)*69i - 192)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x - (6^(1
/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*69i + 192)*(x^2 + 2/3)^(1/2)*1i)/(2
592*(x + (6^(1/2)*1i)/3))
```

3.133 $\int \frac{1+3x+4x^2}{(1+2x)(2+3x^2)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{-38+21x}{198\,(2+3x^2)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}-\frac{8\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\,\sqrt{2+3x^2}}\right)}{121\sqrt{11}}$$

```
[Out] 1/198*(-38+21*x)/(3*x^2+2)^(3/2)-8/1331*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^
2+2)^(1/2))*11^(1/2)+1/726*(24+95*x)/(3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1661, 837, 12, 739, 212}

$$-\frac{38-21x}{198\,(3x^2+2)^{3/2}}+\frac{95x+24}{726\sqrt{3x^2+2}}-\frac{8\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\,\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] -1/198*(38 - 21*x)/(2 + 3*x^2)^(3/2) + (24 + 95*x)/(726*Sqrt[2 + 3*x^2]) -
(8*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx &= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}-\frac{1}{18}\int \frac{-\frac{78}{11}-\frac{84x}{11}}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx\\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}+\frac{\int \frac{864}{11(1+2x)\sqrt{2+3x^2}}\,dx}{1188}\\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}+\frac{8}{121}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}-\frac{8}{121}\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right.\\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}-\frac{8\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}
\end{aligned}$$

Mathematica [A]
time = 0.60, size = 58, normalized size = 0.79

$$\frac{-274+801x+216x^2+855x^3}{2178\left(2+3x^2\right)^{3/2}}-\frac{8\tanh^{-1}\left(\frac{4-3x}{\sqrt{22+33x^2}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] (-274 + 801*x + 216*x^2 + 855*x^3)/(2178*(2 + 3*x^2)^(3/2)) - (8*ArcTanh[(4
 - 3*x)/Sqrt[22 + 33*x^2]])/(121*Sqrt[11])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(58) = 116$.
time = 0.09, size = 133, normalized size = 1.82

method	result
trager	$\frac{855x^3+216x^2+801x-274}{2178(3x^2+2)^{\frac{3}{2}}}+\frac{8\,\mathrm{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}-4\,\mathrm{RootOf}\left(\_Z^2-11\right)}{2x+1}\right)}{1331}$
default	$\frac{x}{12(3x^2+2)^{\frac{3}{2}}}+\frac{x}{12\sqrt{3x^2+2}}-\frac{2}{9(3x^2+2)^{\frac{3}{2}}}+\frac{1}{33\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{x}{44\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{23x}{484\sqrt{3\left(x+\frac{1}{2}\right)^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/(3*x^2+2)^(3/2)*x+1/12*x/(3*x^2+2)^(1/2)-2/9/(3*x^2+2)^(3/2)+1/33/(3*(
x+1/2)^2-3*x+5/4)^(3/2)+1/44*x/(3*(x+1/2)^2-3*x+5/4)^(3/2)+23/484*x/(3*(x+1
/2)^2-3*x+5/4)^(1/2)+4/121/(3*(x+1/2)^2-3*x+5/4)^(1/2)-8/1331*11^(1/2)*arct
anh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

Maxima [A]
time = 0.50, size = 81, normalized size = 1.11

$$\frac{8}{1331}\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{95\,x}{726\,\sqrt{3\,x^2+2}}+\frac{4}{121\,\sqrt{3\,x^2+2}}+\frac{7\,x}{66\,(3\,x^2+2)^{\frac{3}{2}}}-\frac{19}{99\,(3\,x^2+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x +
1)) + 95/726*x/sqrt(3*x^2 + 2) + 4/121/sqrt(3*x^2 + 2) + 7/66*x/(3*x^2 + 2)
^(3/2) - 19/99/(3*x^2 + 2)^(3/2)
```

Fricas [A]
time = 0.33, size = 103, normalized size = 1.41

$$\frac{72\,\sqrt{11}\,(9\,x^4+12\,x^2+4)\log\left(-\frac{\sqrt{11}\,\sqrt{3\,x^2+2}\,(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right)+11\,(855\,x^3+216\,x^2+801\,x-274)\sqrt{3\,x^2+2}}{23958\,(9\,x^4+12\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/23958*(72*sqrt(11)*(9*x^4 + 12*x^2 + 4)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3
*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*(855*x^3 + 216*x^2 +
801*x - 274)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x^2 + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)\,(3x^2+2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 + 2)**(5/2)), x)
```

Giac [A]
time = 3.98, size = 91, normalized size = 1.25

$$\frac{8}{1331}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}\,x-\sqrt{11}-\sqrt{3}+2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}\,x-\sqrt{11}+\sqrt{3}-2\sqrt{3\,x^2+2}}\right)+\frac{9\,((95\,x+24)x+89)x-274}{2178\,(3\,x^2+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] 8/1331*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 +
 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/2178*(9*((
95*x + 24)*x + 89)*x - 274)/(3*x^2 + 2)^(3/2)
```

Mupad [B]
time = 0.13, size = 218, normalized size = 2.99

$$\frac{\sqrt{11}\left(8\ln\left(x+\frac{1}{2}\right)-8\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{1331}-\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{21}{176}+\frac{\sqrt{6}\,19\mathrm{i}}{176}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(-\frac{7}{88}+\frac{\sqrt{6}\,19\mathrm{i}}{264}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}+\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{21}{176}+\frac{\sqrt{6}\,19\mathrm{i}}{176}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(\frac{7}{88}+\frac{\sqrt{6}\,19\mathrm{i}}{264}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}-\frac{\sqrt{3}\sqrt{6}\left(-288+\sqrt{6}\,303\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{104544\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{\sqrt{3}\sqrt{6}\left(288+\sqrt{6}\,303\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{104544\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(5/2)),x)
```

```
[Out] (11^(1/2)*(8*log(x + 1/2) - 8*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/
3 - 4/3)))/1331 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*19i)/176 - 21/176)/
(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*19i)/264 - 7/88)*1i)/(2*(x + (6^(
1/2)*1i)/3)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*19i)/176 + 21/1
76)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*19i)/264 + 7/88)*1i)/(2*(x -
(6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*6^(1/2)*(6^(1/2)*303i - 288)*(x^2 + 2/3)
^(1/2)*1i)/(104544*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*303i +
 288)*(x^2 + 2/3)^(1/2)*1i)/(104544*(x - (6^(1/2)*1i)/3))
```

3.134 $\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{5/2}}\,dx$

Optimal. Leaf size=95

$$\frac{-10+97x}{726\,(2+3x^2)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}-\frac{16\sqrt{2+3x^2}}{1331(1+2x)}-\frac{32\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{1331\sqrt{11}}$$

```
[Out] 1/726*(-10+97*x)/(3*x^2+2)^(3/2)-32/14641*arctanh(1/11*(4-3*x)*11^(1/2)/(3*
x^2+2)^(1/2))*11^(1/2)+1/7986*(24+887*x)/(3*x^2+2)^(1/2)-16/1331*(3*x^2+2)^
(1/2)/(1+2*x)
```

Rubi [A]
time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1661, 821, 739, 212}

$$-\frac{10-97x}{726\,(3x^2+2)^{3/2}}-\frac{16\sqrt{3x^2+2}}{1331(2x+1)}+\frac{887x+24}{7986\sqrt{3x^2+2}}-\frac{32\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] -1/726*(10 - 97*x)/(2 + 3*x^2)^(3/2) + (24 + 887*x)/(7986*Sqrt[2 + 3*x^2])
- (16*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)) - (32*ArcTanh[(4 - 3*x)/(Sqrt[11]*S
qrt[2 + 3*x^2])])/(1331*Sqrt[11])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
```

```
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{5/2}}\,dx &= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}-\frac{1}{18}\int \frac{-\frac{798}{121}-\frac{1968x}{121}-\frac{2328x^2}{121}}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx \\
&= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}+\frac{1}{108}\int \frac{\frac{10368}{1331}+\frac{1728x}{1331}}{(1+2x)^2\sqrt{2+3x^2}}\,dx \\
&= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}-\frac{16\sqrt{2+3x^2}}{1331(1+2x)}+\frac{32\int \frac{1}{(1+2x)\sqrt{2+3x^2}}}{1331} \\
&= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}-\frac{16\sqrt{2+3x^2}}{1331(1+2x)}-\frac{32\text{Subst}\left(\int \frac{1}{11-x^2}\,dx\right.}{133} \\
&= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}-\frac{16\sqrt{2+3x^2}}{1331(1+2x)}-\frac{32\tanh^{-1}\left(\frac{4-}{\sqrt{11}\sqrt{}}\right.}{1331\sqrt{11}}
\end{aligned}
$$

Mathematica [A]
time = 0.72, size = 91, normalized size = 0.96

$$
\frac{11\left(-446+2717x+4602x^2+2805x^3+4458x^4\right)-192\sqrt{22+33x^2}\left(2+4x+3x^2+6x^3\right)\tanh^{-1}\left(\frac{4-3x}{\sqrt{22+33x^2}}\right)}{87846(1+2x)\left(2+3x^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] (11*(-446 + 2717*x + 4602*x^2 + 2805*x^3 + 4458*x^4) - 192*Sqrt[22 + 33*x^2
]*(2 + 4*x + 3*x^2 + 6*x^3)*ArcTanh[(4 - 3*x)/Sqrt[22 + 33*x^2]])/(87846*(1
 + 2*x)*(2 + 3*x^2)^(3/2))
```

Maple [A]
time = 0.09, size = 143, normalized size = 1.51

method	result
risch	$\frac{4458x^4+2805x^3+4602x^2+2717x-446}{7986(3x^2+2)^{\frac{3}{2}}(2x+1)}-\frac{32\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{14641}$
trager	$\frac{4458x^4+2805x^3+4602x^2+2717x-446}{7986(3x^2+2)^{\frac{3}{2}}(2x+1)}+\frac{32\,\mathrm{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}-4\,\mathrm{RootOf}\left(\_Z^2-11\right)}{2x+1}\right)}{14641}$
default	$\frac{x}{6(3x^2+2)^{\frac{3}{2}}}+\frac{x}{6\sqrt{3x^2+2}}+\frac{4}{363\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}}-\frac{10x}{121\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}}-\frac{98x}{1331\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^2/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/(3*x^2+2)^(3/2)*x+1/6*x/(3*x^2+2)^(1/2)+4/363/(3*(x+1/2)^2-3*x+5/4)^(3/
2)-10/121*x/(3*(x+1/2)^2-3*x+5/4)^(3/2)-98/1331*x/(3*(x+1/2)^2-3*x+5/4)^(1/
2)+16/1331/(3*(x+1/2)^2-3*x+5/4)^(1/2)-32/14641*11^(1/2)*arctanh(2/11*(4-3*
x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))-1/22/(x+1/2)/(3*(x+1/2)^2-3*x+5/4)
^(3/2)
```

Maxima [A]
time = 0.50, size = 107, normalized size = 1.13

$$\frac{32}{14641}\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{743\,x}{7986\,\sqrt{3\,x^2+2}}+\frac{16}{1331\,\sqrt{3\,x^2+2}}+\frac{61\,x}{726\,(3\,x^2+2)^{\frac{3}{2}}}-\frac{1}{11\left(2\,(3\,x^2+2)^{\frac{3}{2}}x+(3\,x^2+2)^{\frac{3}{2}}\right)}+\frac{4}{363\,(3\,x^2+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/14641*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
+ 1)) + 743/7986*x/sqrt(3*x^2 + 2) + 16/1331/sqrt(3*x^2 + 2) + 61/726*x/(3*
x^2 + 2)^(3/2) - 1/11/(2*(3*x^2 + 2)^(3/2)*x + (3*x^2 + 2)^(3/2)) + 4/363/(
3*x^2 + 2)^(3/2)
```

Fricas [A]
time = 0.36, size = 134, normalized size = 1.41

$$\frac{96\sqrt{11}\left(18x^5+9x^4+24x^3+12x^2+8x+4\right)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}\,(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right)+11\left(4458x^4+2805x^3+4602x^2+2717x-446\right)\sqrt{3x^2+2}}{87846\left(18x^5+9x^4+24x^3+12x^2+8x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/87846*(96*sqrt(11)*(18*x^5 + 9*x^4 + 24*x^3 + 12*x^2 + 8*x + 4)*log(-(sqr
t(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) +
11*(4458*x^4 + 2805*x^3 + 4602*x^2 + 2717*x - 446)*sqrt(3*x^2 + 2))/(18*x^5
 + 9*x^4 + 24*x^3 + 12*x^2 + 8*x + 4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(76) = 152.
time = 5.20, size = 233, normalized size = 2.45

$$-\frac{1}{263538}\sqrt{11}\left(743\sqrt{11}\sqrt{3}-576\log\left(\sqrt{11}\sqrt{3}-3\right)\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)-\frac{32\sqrt{11}\log\left(\sqrt{11}\left(\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}+\frac{\sqrt{11}}{2x+1}\right)-3\right)}{14641\operatorname{sgn}\left(\frac{1}{2x+1}\right)}+\frac{\frac{\frac{\frac{11\left(\frac{731}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}+\frac{528}{(2x+1)\operatorname{sgn}\left(\frac{1}{2x+1}\right)}\right)}{2x+1}-\frac{14163}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}+\frac{6111}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}-\frac{2229}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{7986\left(\frac{6}{2x+1}-\frac{11}{(2x+1)^2}-3\right)\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/263538*sqrt(11)*(743*sqrt(11)*sqrt(3) - 576*log(sqrt(11)*sqrt(3) - 3))*s
gn(1/(2*x + 1)) - 32/14641*sqrt(11)*log(sqrt(11)*(sqrt(-6/(2*x + 1) + 11/(2
*x + 1)^2 + 3) + sqrt(11)/(2*x + 1)) - 3)/sgn(1/(2*x + 1)) + 1/7986*(((11*(
731/sgn(1/(2*x + 1)) + 528/((2*x + 1)*sgn(1/(2*x + 1))))/(2*x + 1) - 14163/
sgn(1/(2*x + 1)))/(2*x + 1) + 6111/sgn(1/(2*x + 1)))/(2*x + 1) - 2229/sgn(1
/(2*x + 1)))/((6/(2*x + 1) - 11/(2*x + 1)^2 - 3)*sqrt(-6/(2*x + 1) + 11/(2*
x + 1)^2 + 3))
```

Mupad [B]
time = 4.31, size = 270, normalized size = 2.84

$$\frac{\sqrt{11}\left(8\ln\left(x+\frac{1}{2}\right)-8\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{14641}+\frac{\sqrt{11}\left(\frac{48\ln\left(x+\frac{1}{2}\right)}{1331}-\frac{48\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{1331}\right)}{22}-\frac{8\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1331\left(x+\frac{1}{2}\right)}-\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{291}{1936}+\frac{\sqrt{6}\,15i}{1936}}{x+\frac{\sqrt{6}\,1i}{3}}+\frac{\sqrt{6}\left(-\frac{97}{968}+\frac{\sqrt{6}\,5i}{968}\right)1i}{2\left(x+\frac{\sqrt{6}\,1i}{3}\right)^2}\right)}{27}+\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{291}{1936}+\frac{\sqrt{6}\,15i}{1936}}{x-\frac{\sqrt{6}\,1i}{3}}-\frac{\sqrt{6}\left(\frac{97}{968}+\frac{\sqrt{6}\,5i}{968}\right)1i}{2\left(x-\frac{\sqrt{6}\,1i}{3}\right)^2}\right)}{27}-\frac{\sqrt{3}\sqrt{6}\left(-288+\sqrt{6}\,2481i\right)\sqrt{x^2+\frac{2}{3}}\,1i}{1149984\left(x+\frac{\sqrt{6}\,1i}{3}\right)}-\frac{\sqrt{3}\sqrt{6}\left(288+\sqrt{6}\,2481i\right)\sqrt{x^2+\frac{2}{3}}\,1i}{1149984\left(x-\frac{\sqrt{6}\,1i}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(5/2)),x)
```

```
[Out] (11^(1/2)*(8*log(x + 1/2) - 8*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/
3 - 4/3)))/14641 + (11^(1/2)*((48*log(x + 1/2))/1331 - (48*log(x - (3^(1/2)
*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/1331))/22 - (8*3^(1/2)*(x^2 + 2/3)^(
1/2))/(1331*(x + 1/2)) - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*15i)/1936 -
291/1936)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*5i)/968 - 97/968)*1i)/(
2*(x + (6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*15i)
/1936 + 291/1936)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*5i)/968 + 97/96
8)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*6^(1/2)*(6^(1/2)*2481i -
288)*(x^2 + 2/3)^(1/2)*1i)/(1149984*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2
)*(6^(1/2)*2481i + 288)*(x^2 + 2/3)^(1/2)*1i)/(1149984*(x - (6^(1/2)*1i)/3)
)
```

3.135 $\int \frac{1+3x+4x^2}{(1+2x)^3(2+3x^2)^{5/2}} dx$

Optimal. Leaf size=117

$$\frac{358+351x}{7986\,(2+3x^2)^{3/2}}+\frac{1216+2133x}{29282\sqrt{2+3x^2}}-\frac{8\sqrt{2+3x^2}}{1331(1+2x)^2}-\frac{8\sqrt{2+3x^2}}{1331(1+2x)}-\frac{1216\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\,\sqrt{2+3x^2}}\right)}{14641\sqrt{11}}$$

```
[Out] 1/7986*(358+351*x)/(3*x^2+2)^(3/2)-1216/161051*arctanh(1/11*(4-3*x)*11^(1/2
)/(3*x^2+2)^(1/2))*11^(1/2)+1/29282*(1216+2133*x)/(3*x^2+2)^(1/2)-8/1331*(3
*x^2+2)^(1/2)/(1+2*x)^2-8/1331*(3*x^2+2)^(1/2)/(1+2*x)
```

Rubi [A]
time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1661, 1665, 821, 739, 212}

$$\frac{351x+358}{7986\,(3x^2+2)^{3/2}}-\frac{8\sqrt{3x^2+2}}{1331(2x+1)}-\frac{8\sqrt{3x^2+2}}{1331(2x+1)^2}+\frac{2133x+1216}{29282\sqrt{3x^2+2}}-\frac{1216\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\,\sqrt{3x^2+2}}\right)}{14641\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] (358 + 351*x)/(7986*(2 + 3*x^2)^(3/2)) + (1216 + 2133*x)/(29282*Sqrt[2 + 3*
x^2]) - (8*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)^2) - (8*Sqrt[2 + 3*x^2])/(1331*
(1 + 2*x)) - (1216*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(14641*Sq
rt[11])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
```

```
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx=\frac{358+351x}{7986\left(2+3x^2\right)^{3/2}}-\frac{1}{18}\int\frac{-\frac{10926}{1331}-\frac{3132x}{121}-\frac{51048x^2}{1331}-\frac{16848x^3}{1331}}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx$$

$$=\frac{358+351x}{7986\left(2+3x^2\right)^{3/2}}+\frac{1216+2133x}{29282\sqrt{2+3x^2}}+\frac{1}{108}\int\frac{\frac{245376}{14641}+\frac{544320x}{14641}+\frac{525312x^2}{14641}}{(1+2x)^3\sqrt{2+3x^2}}\,dx$$

$$=\frac{358+351x}{7986\left(2+3x^2\right)^{3/2}}+\frac{1216+2133x}{29282\sqrt{2+3x^2}}-\frac{8\sqrt{2+3x^2}}{1331(1+2x)^2}-\frac{\int\frac{-\frac{338688}{1331}-\frac{468288x}{1331}}{(1+2x)^2\sqrt{2+3x^2}}}{2376}$$

$$=\frac{358+351x}{7986\left(2+3x^2\right)^{3/2}}+\frac{1216+2133x}{29282\sqrt{2+3x^2}}-\frac{8\sqrt{2+3x^2}}{1331(1+2x)^2}-\frac{8\sqrt{2+3x^2}}{1331(1+2x)}+$$

$$=\frac{358+351x}{7986\left(2+3x^2\right)^{3/2}}+\frac{1216+2133x}{29282\sqrt{2+3x^2}}-\frac{8\sqrt{2+3x^2}}{1331(1+2x)^2}-\frac{8\sqrt{2+3x^2}}{1331(1+2x)}-$$

$$=\frac{358+351x}{7986\left(2+3x^2\right)^{3/2}}+\frac{1216+2133x}{29282\sqrt{2+3x^2}}-\frac{8\sqrt{2+3x^2}}{1331(1+2x)^2}-\frac{8\sqrt{2+3x^2}}{1331(1+2x)}-$$

Mathematica [A]
time = 0.62, size = 91, normalized size = 0.78

$$\frac{\frac{11\left(7010+57371x+109844x^2+116937x^3+111060x^4+67284x^5\right)}{(1+2x)^2\left(2+3x^2\right)^{3/2}}+14592\sqrt{11}\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}\,x-2\sqrt{2+3x^2}}{\sqrt{11}}\right)}{966306}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(5/2)),x]
[Out] ((11*(7010 + 57371*x + 109844*x^2 + 116937*x^3 + 111060*x^4 + 67284*x^5))/(
(1 + 2*x)^2*(2 + 3*x^2)^(3/2)) + 14592*Sqrt[11]*ArcTanh[(Sqrt[3] + 2*Sqrt[3
]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[11]])/966306
```

Maple [A]
time = 0.11, size = 140, normalized size = 1.20

method	result
risch	$\frac{67284x^5+111060x^4+116937x^3+109844x^2+57371x+7010}{87846(3x^2+2)^{\frac{3}{2}}(2x+1)^2}-\frac{1216\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{161051}$

trager	$\frac{(67284x^5+111060x^4+116937x^3+109844x^2+57371x+7010)\sqrt{3x^2+2}}{87846(6x^3+3x^2+4x+2)^2}+\frac{1216\,\mathrm{RootOf}(\_Z^2-11)\ln\left(\frac{3\,\mathrm{RootOf}(\_Z^2-11)x+1}{}\right.}{161051}$
default	$-\frac{1}{88(x+\frac{1}{2})^2(3(x+\frac{1}{2})^2-3x+\frac{5}{4})^{\frac{3}{2}}}+\frac{1}{484(x+\frac{1}{2})(3(x+\frac{1}{2})^2-3x+\frac{5}{4})^{\frac{3}{2}}}+\frac{152}{3993(3(x+\frac{1}{2})^2-3x+\frac{5}{4})^{\frac{3}{2}}}+\frac{87x}{2662(3(x+\frac{1}{2})^2-3x+\frac{5}{4})^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^3/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/88/(x+1/2)^2/(3*(x+1/2)^2-3*x+5/4)^(3/2)+1/484/(x+1/2)/(3*(x+1/2)^2-3*x+
5/4)^(3/2)+152/3993/(3*(x+1/2)^2-3*x+5/4)^(3/2)+87/2662*x/(3*(x+1/2)^2-3*x+
5/4)^(3/2)+1869/29282*x/(3*(x+1/2)^2-3*x+5/4)^(1/2)+608/14641/(3*(x+1/2)^2-
3*x+5/4)^(1/2)-1216/161051*11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/
2)^2-12*x+5)^(1/2))
```

Maxima [A]
time = 0.49, size = 147, normalized size = 1.26

$$\frac{1216}{161051}\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{1869\,x}{29282\,\sqrt{3\,x^2+2}}+\frac{608}{14641\,\sqrt{3\,x^2+2}}+\frac{87\,x}{2662\,(3\,x^2+2)^{\frac{3}{2}}}-\frac{1}{22\left(4\,(3\,x^2+2)^{\frac{3}{2}}x^2+4\,(3\,x^2+2)^{\frac{3}{2}}x+(3\,x^2+2)^{\frac{3}{2}}\right)}+\frac{1}{242\left(2\,(3\,x^2+2)^{\frac{3}{2}}x+(3\,x^2+2)^{\frac{3}{2}}\right)}+\frac{152}{3993\,(3\,x^2+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1216/161051*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2
*x + 1)) + 1869/29282*x/sqrt(3*x^2 + 2) + 608/14641/sqrt(3*x^2 + 2) + 87/26
62*x/(3*x^2 + 2)^(3/2) - 1/22/(4*(3*x^2 + 2)^(3/2)*x^2 + 4*(3*x^2 + 2)^(3/2
)*x + (3*x^2 + 2)^(3/2)) + 1/242/(2*(3*x^2 + 2)^(3/2)*x + (3*x^2 + 2)^(3/2)
) + 152/3993/(3*x^2 + 2)^(3/2)
```

Fricas [A]
time = 0.34, size = 149, normalized size = 1.27

$$\frac{3648\,\sqrt{11}\,(36\,x^6+36\,x^5+57\,x^4+48\,x^3+28\,x^2+16\,x+4)\log\left(-\frac{\sqrt{11}\,\sqrt{3\,x^2+2}\,(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right)+11\,(67284\,x^5+111060\,x^4+116937\,x^3+109844\,x^2+57371\,x+7010)\sqrt{3\,x^2+2}}{966306\,(36\,x^6+36\,x^5+57\,x^4+48\,x^3+28\,x^2+16\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/966306*(3648*sqrt(11)*(36*x^6 + 36*x^5 + 57*x^4 + 48*x^3 + 28*x^2 + 16*x
+ 4)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2
+ 4*x + 1)) + 11*(67284*x^5 + 111060*x^4 + 116937*x^3 + 109844*x^2 + 57371*
```

```
x + 7010)*sqrt(3*x^2 + 2))/(36*x^6 + 36*x^5 + 57*x^4 + 48*x^3 + 28*x^2 + 16
*x + 4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.72, size = 183, normalized size = 1.56

$$\frac{1216}{161051}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}\,x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}\,x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{9\left((2133x+1216)x+1851\right)x+11234}{87846\left(3x^2+2\right)^{\frac{3}{2}}}+\frac{4\left(\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3x^2+2}\right)^2+24\sqrt{3}\,x-8\sqrt{3}-24\sqrt{3x^2+2}\right)}{1331\left(\left(\sqrt{3}\,x-\sqrt{3x^2+2}\right)^2+\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3x^2+2}\right)-2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1216/161051*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*
x^2 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/87846
*(9*((2133*x + 1216)*x + 1851)*x + 11234)/(3*x^2 + 2)^(3/2) + 4/1331*(sqrt(
3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 24*sqrt(3)*x - 8*sqrt(3) - 24*sqrt(3*x
^2 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(3*x^2
 + 2)) - 2)^2
```

Mupad [**B**]
time = 4.19, size = 301, normalized size = 2.57

$$\frac{1216\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{161051}-\frac{1216\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{161051}-\frac{179\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{95832\left(x^2+\frac{2\mathrm{i}\,\sqrt{6}\,x}{3}-\frac{2}{3}\right)}+\frac{711\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{58564\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{711\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{58564\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{2\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{1331\left(x^2+x+\frac{1}{4}\right)}+\frac{179\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{95832\left(-x^2+\frac{2\mathrm{i}\,\sqrt{6}\,x}{3}+\frac{2}{3}\right)}-\frac{4\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{1331\left(x+\frac{1}{2}\right)}+\frac{\sqrt{3}\,\sqrt{6}\,\sqrt{x^2+\frac{2}{3}}\,13\mathrm{i}}{21296\left(x^2+\frac{2\mathrm{i}\,\sqrt{6}\,x}{3}-\frac{2}{3}\right)}-\frac{\sqrt{3}\,\sqrt{6}\,\sqrt{x^2+\frac{2}{3}}\,9265\mathrm{i}}{2108304\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{\sqrt{3}\,\sqrt{6}\,\sqrt{x^2+\frac{2}{3}}\,9265\mathrm{i}}{2108304\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{\sqrt{3}\,\sqrt{6}\,\sqrt{x^2+\frac{2}{3}}\,13\mathrm{i}}{21296\left(-x^2+\frac{2\mathrm{i}\,\sqrt{6}\,x}{3}+\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(5/2)),x)
```

```
[Out] (1216*11^(1/2)*log(x + 1/2))/161051 - (1216*11^(1/2)*log(x - (3^(1/2)*11^(1
/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/161051 - (179*3^(1/2)*(x^2 + 2/3)^(1/2))/(
95832*((6^(1/2)*x*2i)/3 + x^2 - 2/3)) + (711*3^(1/2)*(x^2 + 2/3)^(1/2))/(58
564*(x - (6^(1/2)*1i)/3)) + (711*3^(1/2)*(x^2 + 2/3)^(1/2))/(58564*(x + (6^
(1/2)*1i)/3)) - (2*3^(1/2)*(x^2 + 2/3)^(1/2))/(1331*(x + x^2 + 1/4)) + (179
*3^(1/2)*(x^2 + 2/3)^(1/2))/(95832*((6^(1/2)*x*2i)/3 - x^2 + 2/3)) - (4*3^(
1/2)*(x^2 + 2/3)^(1/2))/(1331*(x + 1/2)) + (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/
2)*13i)/(21296*((6^(1/2)*x*2i)/3 + x^2 - 2/3)) - (3^(1/2)*6^(1/2)*(x^2 + 2/
3)^(1/2)*9265i)/(2108304*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(x^2 + 2/
3)^(1/2)*9265i)/(2108304*(x + (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(x^2 + 2/
3)^(1/2)*13i)/(21296*((6^(1/2)*x*2i)/3 - x^2 + 2/3))
```

3.136 $\int (g+hx)^m (a+cx^2)^p (d+ex+fx^2)\, dx$

Optimal. Leaf size=420

$$\frac{f(g+hx)^{1+m}(a+cx^2)^{1+p}}{ch(3+m+2p)} - \frac{(afh^2(1+m) - c(2fg^2(1+p) - h(eg-dh)(3+m+2p)))(g+hx)^{1+m}(a+c}{ch^3(}$$

```
[Out] f*(h*x+g)^(1+m)*(c*x^2+a)^(1+p)/c/h/(3+m+2*p)-(a*f*h^2*(1+m)-c*(2*f*g^2*(1+
p)-h*(-d*h+e*g)*(3+m+2*p)))*(h*x+g)^(1+m)*(c*x^2+a)^p*AppellF1(1+m,-p,-p,2+
m,(h*x+g)/(g-h*(-a)^(1/2)/c^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))/c/h^3/
(1+m)/(3+m+2*p)/((1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)))^p)/((1+(-h*x-g)/(g+h
*(-a)^(1/2)/c^(1/2)))^p)-(2*f*g*(1+p)-e*h*(3+m+2*p))*(h*x+g)^(2+m)*(c*x^2+a
)^p*AppellF1(2+m,-p,-p,3+m,(h*x+g)/(g-h*(-a)^(1/2)/c^(1/2)),(h*x+g)/(g+h*(-
a)^(1/2)/c^(1/2)))/h^3/(2+m)/(3+m+2*p)/((1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)
))^p)/((1+(-h*x-g)/(g+h*(-a)^(1/2)/c^(1/2)))^p)
```

Rubi [A]

time = 0.36, antiderivative size = 417, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1668, 858, 774, 138}

$$-\frac{(a+cx^2)^p (g+hx)^{m+1}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}\,h}{\sqrt{c}}+g}\right)^{-p} F_1\left(m+1;-p,-p;m+2;\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)(afh^2(m+1)+ch(m+2p+3)(eg-dh)-2cfg^2(p+1))}{ch^3(m+1)(m+2p+3)} - \frac{(a+cx^2)^p (g+hx)^{m+2}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}\,h}{\sqrt{c}}+g}\right)^{-p}(2fg(p+1)-eh(m+2p+3))F_1\left(m+2;-p,-p;m+3;\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)}{h^3(m+2)(m+2p+3)} + \frac{f(a+cx^2)^{p+1}(g+hx)^{m+1}}{ch(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*h*(3 + m + 2*p)) - ((a*f*h^2*(
1 + m) - 2*c*f*g^2*(1 + p) + c*h*(e*g - d*h)*(3 + m + 2*p))*(g + h*x)^(1 +
m)*(a + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (g + h*x)/(g - (Sqrt[-a]*h)
/Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(c*h^3*(1 + m)*(3 + m + 2
*p)*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1 - (g + h*x)/(g + (Sqrt[
-a]*h)/Sqrt[c]))^p) - ((2*f*g*(1 + p) - e*h*(3 + m + 2*p))*(g + h*x)^(2 + m
)*(a + c*x^2)^p*AppellF1[2 + m, -p, -p, 3 + m, (g + h*x)/(g - (Sqrt[-a]*h)/
Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(h^3*(2 + m)*(3 + m + 2*p)
*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1 - (g + h*x)/(g + (Sqrt[-a]
*h)/Sqrt[c]))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right) dx = \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\int (g+hx)^m \left(-h^2(af(1+m)-c\right.}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\left(eh-\frac{2fg(1+p)}{3+m+2p}\right)\int (g+hx)^{1+m}(a}{h^2}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\left(\left(eh-\frac{2fg(1+p)}{3+m+2p}\right)\left(a+cx^2\right)^p\left(1\right.\right.}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} - \frac{(afh^2(1+m)-2cfg^2(1+p)+}{}$$

Mathematica [**F**]
time = 0.78, size = 0, normalized size = 0.00

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right) dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] Integrate[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2), x]
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int (hx+g)^m \left(c\,x^2+a\right)^p \left(f\,x^2+ex+d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x)
```

```
[Out] int((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + x*e + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] integral((f*x^2 + x*e + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**m*(c*x**2+a)**p*(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + x*e + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g+h\,x)^m\,\left(c\,x^2+a\right)^p\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2), x)
```

3.137 $\int (g+hx)^m \sqrt{a+cx^2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=403

$$\frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)}-\frac{\left(afh^2(1+m)-c\left(3fg^2-h(eg-dh)(4+m)\right)\right)(g+hx)^{1+m}\sqrt{a+cx^2}\,F_1\left(1+m\right.}{ch^3(1+m)(4+m)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}}}\sqrt{1-\frac{g}{g+}}}$$

```
[Out] f*(h*x+g)^(1+m)*(c*x^2+a)^(3/2)/c/h/(4+m)-(a*f*h^2*(1+m)-c*(3*f*g^2-h*(-d*h
+e*g)*(4+m)))*(h*x+g)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,(h*x+g)/(g-h*(-a)^(1
/2)/c^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))*(c*x^2+a)^(1/2)/c/h^3/(1+m)/
(4+m)/(1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)))^(1/2)/(1+(-h*x-g)/(g+h*(-a)^(1/
2)/c^(1/2)))^(1/2)-(3*f*g-e*h*(4+m))*(h*x+g)^(2+m)*AppellF1(2+m,-1/2,-1/2,3
+m,(h*x+g)/(g-h*(-a)^(1/2)/c^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))*(c*x^
2+a)^(1/2)/h^3/(2+m)/(4+m)/(1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)))^(1/2)/(1+(
-h*x-g)/(g+h*(-a)^(1/2)/c^(1/2)))^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1668, 858, 774, 138}

$$\frac{\sqrt{a+cx^2}(g+hx)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)\left(-afh^2(m+1)-ch(m+4)(eg-dh)+3cfg^2\right)}{ch^3(m+1)(m+4)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{\frac{\sqrt{-a}\,h}{\sqrt{c}}+g}}}-\frac{\sqrt{a+cx^2}(g+hx)^{m+2}(3fg-eh(m+4))F_1\left(m+2;-\frac{1}{2},-\frac{1}{2};m+3;\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)}{h^3(m+2)(m+4)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{\frac{\sqrt{-a}\,h}{\sqrt{c}}+g}}}+\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + c*x^2)^(3/2))/(c*h*(4 + m)) + ((3*c*f*g^2 - a*f*h
^2*(1 + m) - c*h*(e*g - d*h)*(4 + m))*(g + h*x)^(1 + m)*Sqrt[a + c*x^2]*App
ellF1[1 + m, -1/2, -1/2, 2 + m, (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g +
h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(c*h^3*(1 + m)*(4 + m)*Sqrt[1 - (g + h*x)
/(g - (Sqrt[-a]*h)/Sqrt[c])]*Sqrt[1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])]
) - ((3*f*g - e*h*(4 + m))*(g + h*x)^(2 + m)*Sqrt[a + c*x^2]*AppellF1[2 + m
, -1/2, -1/2, 3 + m, (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g + h*x)/(g + (
Sqrt[-a]*h)/Sqrt[c])])/(h^3*(2 + m)*(4 + m)*Sqrt[1 - (g + h*x)/(g - (Sqrt[-
a]*h)/Sqrt[c])]*Sqrt[1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
```

```
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^m \sqrt{a+cx^2} \left(d+ex+fx^2\right) dx = \frac{f(g+hx)^{1+m} \left(a+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\int (g+hx)^m \left(-h^2(af(1+m)-cd \ldots\right.}{}$$

$$= \frac{f(g+hx)^{1+m} \left(a+cx^2\right)^{3/2}}{ch(4+m)} - \frac{(3fg-eh(4+m)) \int (g+hx)^{1+m} \ldots}{h^2(4+m)}$$

$$= \frac{f(g+hx)^{1+m} \left(a+cx^2\right)^{3/2}}{ch(4+m)} - \frac{\left((3fg-eh(4+m))\sqrt{a+cx^2}\right) \mathrm{S} \ldots}{h^3(4+ \ldots}$$

$$= \frac{f(g+hx)^{1+m} \left(a+cx^2\right)^{3/2}}{ch(4+m)} + \frac{(3cfg^2-afh^2(1+m)-ch(eg- \ldots}{ch^3 \ldots}$$

Mathematica [F]
time = 0.76, size = 0, normalized size = 0.00

$$\int (g+hx)^m \sqrt{a+cx^2} \left(d+ex+fx^2\right) dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2), x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (hx+g)^m \left(f\,x^2+ex+d\right) \sqrt{c\,x^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
[Out] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*(f*x^2 + x*e + d)*(h*x + g)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2 + a)*(f*x^2 + x*e + d)*(h*x + g)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + cx^2}\,(g + hx)^m \left(d + ex + fx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**m*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
[Out] Integral(sqrt(a + c*x**2)*(g + h*x)**m*(d + e*x + f*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)*(f*x^2 + x*e + d)*(h*x + g)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g + h\,x)^m\,\sqrt{c\,x^2 + a}\,\left(f\,x^2 + e\,x + d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^m*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^m*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.138 $\int (g+hx)^{-3-2p} (a+cx^2)^p (d+ex+fx^2)\, dx$

Optimal. Leaf size=474

$$-\frac{(fg^2-egh+dh^2)(g+hx)^{-2(1+p)}(a+cx^2)^{1+p}}{2h(cg^2+ah^2)(1+p)}-\frac{f(g+hx)^{-2p}(a+cx^2)^p\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+h}{g+\frac{\sqrt{-}}{\sqrt{}}}\right.}{2h}$$

```
[Out] -1/2*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(1+p)/h/(a*h^2+c*g^2)/(1+p)/((h*x+g)^(2+
2*p))-1/2*f*(c*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(h*x+g)/(g-h*(-a)^(1/2)/c
^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))/h^3/p/((h*x+g)^(2*p))/((1+(-h*x-g
)/(g-h*(-a)^(1/2)/c^(1/2)))^p)/((1+(-h*x-g)/(g+h*(-a)^(1/2)/c^(1/2)))^p)+(a
*h^2*(-e*h+2*f*g)+c*(-d*g*h^2+f*g^3))*(h*x+g)^(-1-2*p)*(c*x^2+a)^p*hypergeo
m([-p, -1-2*p],[-2*p],2*(h*x+g)*(-a)^(1/2)*c^(1/2)/(-h*(-a)^(1/2)+g*c^(1/2)
)/((-a)^(1/2)-x*c^(1/2)))*((-a)^(1/2)-x*c^(1/2))/h^2/(a*h^2+c*g^2)/(1+2*p)/
(h*(-a)^(1/2)+g*c^(1/2))/((-(h*(-a)^(1/2)+g*c^(1/2))*((-a)^(1/2)+x*c^(1/2))
/(-h*(-a)^(1/2)+g*c^(1/2))/((-a)^(1/2)-x*c^(1/2)))^p)
```

Rubi [A]
time = 0.32, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1670, 774, 138, 821, 741}

$$-\frac{f(a+cx^2)^p(g+hx)^{-2p}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}\,h}{\sqrt{c}}+g}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)}{2h^3p}-\frac{(a+cx^2)^{p+1}(g+hx)^{-2(p+1)}(dh^2-egh+fg^2)}{2h(p+1)(ah^2+cg^2)}+\frac{(\sqrt{-a}-\sqrt{c}\,x)(a+cx^2)^p(g+hx)^{-2p-1}\left(-\frac{(\sqrt{-a}+\sqrt{c}\,x)(\sqrt{-a}\,h+\sqrt{c}\,g)}{(\sqrt{-a}-\sqrt{c}\,x)(\sqrt{c}\,g-\sqrt{-a}\,h)}\right)^{-p}(ah^2(2fg-eh)+c(fg^3-dgh^2))\,{}_2F_1\left(-2p-1,-p;-2p;\frac{2\sqrt{-a}\sqrt{c}(g+hx)}{(\sqrt{c}\,g-\sqrt{-a}\,h)(\sqrt{-a}-\sqrt{c}\,x)}\right)}{h^2(2p+1)(\sqrt{-a}\,h+\sqrt{c}\,g)(ah^2+cg^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] -1/2*((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(1 + p))/(h*(c*g^2 + a*h^2)*(1 +
p)*(g + h*x)^(2*(1 + p))) - (f*(a + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p
, (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c]
)])/(2*h^3*p*(g + h*x)^(2*p)*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(
1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c]))^p) + ((a*h^2*(2*f*g - e*h) + c*(f
*g^3 - d*g*h^2))*(Sqrt[-a] - Sqrt[c]*x)*(g + h*x)^(-1 - 2*p)*(a + c*x^2)^p*
Hypergeometric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sqrt[c]*(g + h*x))/((Sqrt
[c]*g - Sqrt[-a]*h)*(Sqrt[-a] - Sqrt[c]*x))])/(h^2*(Sqrt[c]*g + Sqrt[-a]*h)
*(c*g^2 + a*h^2)*(1 + 2*p)*(-(((Sqrt[c]*g + Sqrt[-a]*h)*(Sqrt[-a] + Sqrt[c]
*x))/((Sqrt[c]*g - Sqrt[-a]*h)*(Sqrt[-a] - Sqrt[c]*x))))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
```

```
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(Rt[(-a)*c, 2] - c*x)*(d + e*x)^(m + 1)*((a + c*x^2)^p/((m + 1)*(c*d + e*Rt
[(-a)*c, 2])*((c*d + e*Rt[(-a)*c, 2])*((Rt[(-a)*c, 2] + c*x)/((c*d - e*Rt[(
-a)*c, 2])*(-Rt[(-a)*c, 2] + c*x))))^p))*Hypergeometric2F1[m + 1, -p, m + 2
, 2*c*Rt[(-a)*c, 2]*((d + e*x)/((c*d - e*Rt[(-a)*c, 2])*(Rt[(-a)*c, 2] - c*
x)))], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Int
egerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1670

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> With[{q = Expon[Pq, x]}, Dist[Coeff[Pq, x, q]/e^q, Int[(d + e*x)^(m + q)
*(a + c*x^2)^p, x], x] + Dist[1/e^q, Int[(d + e*x)^m*(a + c*x^2)^p*ExpandTo
Sum[e^q*Pq - Coeff[Pq, x, q]*(d + e*x)^q, x], x], x]] /; FreeQ[{a, c, d, e,
 m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(IGtQ[m, 0] && Rat
ionalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx = \frac{\int (g+hx)^{-3-2p}\left(-fg^2+dh^2-h(2fg-eh)x\right)\left(a+cx^2\right)^p\,d}{h^2}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)(g+hx)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{2h\left(cg^2+ah^2\right)(1+p)} - \frac{\left(ah^2(2\right.}{}$$

$$f(g+$$

$$= -\frac{\left(fg^2-egh+dh^2\right)(g+hx)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{2h\left(cg^2+ah^2\right)(1+p)} - \frac{}{}$$

Mathematica [F]
time = 2.57, size = 0, normalized size = 0.00

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2), x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (hx+g)^{-3-2p}\left(c\,x^2+a\right)^p\left(f\,x^2+ex+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x)
[Out] int((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + x*e + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
[Out] integral((f*x^2 + x*e + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**(-3-2*p)*(c*x**2+a)**p*(f*x**2+e*x+d),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
[Out] integrate((f*x^2 + x*e + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x^2+a)^p\,(f\,x^2+e\,x+d)}{(g+h\,x)^{2\,p+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3),x)
[Out] int(((a + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3), x)
```

3.139 $\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p ((-cd+be)$

Optimal. Leaf size=222

$$\frac{g(d+ex)^{-1+m}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{2+p}}{ce^2(3+m+2p)}-\frac{(beg(1+m+p)+c(dg(1-m)-ef(3+m+2p)))(d+e$$

```
[Out] g*(e*x+d)^(-1+m)*(-d*(-b*e+c*d)+b*e^2*x+c*e^2*x^2)^(2+p)/c/e^2/(3+m+2*p)-(b
*e*g*(1+m+p)+c*(d*g*(1-m)-e*f*(3+m+2*p)))*(e*x+d)^m*(c*(e*x+d)/(-b*e+2*c*d)
)^(-m-p)*(-c*e*x-b*e+c*d)^2*(-d*(-b*e+c*d)+b*e^2*x+c*e^2*x^2)^p*hypergeom([
2+p, -m-p],[3+p],(-c*e*x-b*e+c*d)/(-b*e+2*c*d))/c^2/e^2/(2+p)/(3+m+2*p)
```

Rubi [A]
time = 0.25, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 69, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1646, 808, 693, 691, 72, 71}

$$\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)}-\frac{(d+ex)^m(-be+cd-cex)^2\left(-d(cd-be)+be^2x+ce^2x^2\right)^p\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-p}(beg(m+p+1)+c(dg(1-m)-ef(m+2p+3)))\,{}_2F_1\left(-m-p,p+2;p+3;\frac{cd-be-cex}{2cd-be}\right)}{c^2e^2(p+2)(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(-(c*d^2) + b*d*e + b*e^2*x + c*e^2*x^2)^p*((-(c*d) + b*e)*
f + (c*e*f - c*d*g + b*e*g)*x + c*e*g*x^2),x]
```

```
[Out] (g*(d + e*x)^(-1 + m)*(-(d*(c*d - b*e)) + b*e^2*x + c*e^2*x^2)^(2 + p))/(c*
e^2*(3 + m + 2*p)) - ((b*e*g*(1 + m + p) + c*(d*g*(1 - m) - e*f*(3 + m + 2*
p)))*(d + e*x)^m*((c*(d + e*x))/(2*c*d - b*e))^(-m - p)*(c*d - b*e - c*e*x)
^2*(-(d*(c*d - b*e)) + b*e^2*x + c*e^2*x^2)^p*Hypergeometric2F1[-m - p, 2 +
 p, 3 + p, (c*d - b*e - c*e*x)/(2*c*d - b*e)])/(c^2*e^2*(2 + p)*(3 + m + 2*
p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 691

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p])), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) &&  !(
IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[d^IntPart[m]*((d + e*x)^FracPart[m]/(1 + e*(x/d))^FracPart[m]
), Int[(1 + e*(x/d))^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !Intege
rQ[p] &&  !(IntegerQ[m] || GtQ[d, 0])
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 1646

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e
 + c*d*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rubi steps

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left(-(cd-be)f+(cef-cdg+beg)x+cegx^2\right)\,dx = (de)\int (d+e$$

$$= \frac{g(d+ex)^{-1+}}{}$$

$$= \frac{g(d+ex)^{-1+}}{}$$

$$= \frac{g(d+ex)^{-1+}}{}$$

$$= \frac{g(d+ex)^{-1+}}{}$$

$$= \frac{g(d+ex)^{-1+}}{}$$

Mathematica [A]
time = 0.42, size = 165, normalized size = 0.74

$$\frac{(d+ex)^m(-cd+be+cex)^2(-((d+ex)(-be+c(d-ex))))^p\left(ceg(d+ex)+\frac{e(cdg(-1+m)-beg(1+m+p)+cef(3+m+2p))\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-p}{}_2F_1\left(-m-p,2+p;3+p;\frac{-cd+be+cex}{-2cd+be}\right)}{2+p}\right)}{c^2e^3(3+m+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(-(c*d^2) + b*d*e + b*e^2*x + c*e^2*x^2)^p*((-(c*d) +
 b*e)*f + (c*e*f - c*d*g + b*e*g)*x + c*e*g*x^2),x]
```

```
[Out] ((d + e*x)^m*(-(c*d) + b*e + c*e*x)^2*(-((d + e*x)*(-(b*e) + c*(d - e*x))))
^p*(c*e*g*(d + e*x) + (e*(c*d*g*(-1 + m) - b*e*g*(1 + m + p) + c*e*f*(3 + m
 + 2*p))*((c*(d + e*x))/(2*c*d - b*e))^(-m - p)*Hypergeometric2F1[-m - p, 2
 + p, 3 + p, (-(c*d) + b*e + c*e*x)/(-2*c*d + b*e)])/(2 + p)))/(c^2*e^3*(3
+ m + 2*p))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (ex+d)^m \left(c\,e^2x^2+b\,e^2x+deb-c\,d^2\right)^p \left(-(-eb+cd)\,f+(beg-dgc+cef)\,x+ceg\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g-c*d*g
+c*e*f)*x+c*e*g*x^2),x)
```

```
[Out] int((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g-c*d*g
+c*e*f)*x+c*e*g*x^2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g
-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="maxima")
```

```
[Out] integrate((c*g*x^2*e - (c*d - b*e)*f - (c*d*g - c*f*e - b*g*e)*x)*(c*x^2*e^
2 - c*d^2 + b*x*e^2 + b*d*e)^p*(x*e + d)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g
-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="fricas")
```

```
[Out] integral(-(c*d*g*x + c*d*f - (c*g*x^2 + b*f + (c*f + b*g)*x)*e)*(-c*d^2 + b
*d*e + (c*x^2 + b*x)*e^2)^p*(x*e + d)^m, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*e**2*x**2+b*e**2*x+b*d*e-c*d**2)**p*(-(-b*e+c*d)*f+
(b*e*g-c*d*g+c*e*f)*x+c*e*g*x**2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g
-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="giac")
```

```
[Out] integrate((c*g*x^2*e - (c*d - b*e)*f - (c*d*g - c*f*e - b*g*e)*x)*(c*x^2*e^
2 - c*d^2 + b*x*e^2 + b*d*e)^p*(x*e + d)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d+e\,x)^m\,\left(c\,e\,g\,x^2+\left(b\,e\,g-c\,d\,g+c\,e\,f\right)\,x+f\,\left(b\,e-c\,d\right)\right)\,{\left(-c\,d^2+b\,d\,e+c\,e^2\,x^2+b\,e^2\,x\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m*(f*(b*e - c*d) + x*(b*e*g - c*d*g + c*e*f) + c*e*g*x^2)*(c*
e^2*x^2 - c*d^2 + b*d*e + b*e^2*x)^p,x)
```

```
[Out] int((d + e*x)^m*(f*(b*e - c*d) + x*(b*e*g - c*d*g + c*e*f) + c*e*g*x^2)*(c*
e^2*x^2 - c*d^2 + b*d*e + b*e^2*x)^p, x)
```

3.140 $\int \left(a+bx+cx^2\right)^4 \left(A+Cx^2\right) \, dx$

Optimal. Leaf size=254

$$a^4Ax+2a^3Abx^2+\frac{1}{3}a^2\left(6Ab^2+4aAc+a^2C\right)x^3+ab\left(A\left(b^2+3ac\right)+a^2C\right)x^4+\frac{1}{5}\left(A\left(b^4+12ab^2c+6a^2c^2\right)+2a\right.$$

```
[Out] a^4*A*x+2*a^3*A*b*x^2+1/3*a^2*(4*A*a*c+6*A*b^2+C*a^2)*x^3+a*b*(A*(3*a*c+b^2
)+a^2*C)*x^4+1/5*(A*(6*a^2*c^2+12*a*b^2*c+b^4)+2*a^2*(2*a*c+3*b^2)*C)*x^5+2
/3*b*(3*a*c+b^2)*(A*c+C*a)*x^6+1/7*(2*A*c^2*(2*a*c+3*b^2)+(6*a^2*c^2+12*a*b
^2*c+b^4)*C)*x^7+1/2*b*c*(A*c^2+(3*a*c+b^2)*C)*x^8+1/9*c^2*(A*c^2+4*C*a*c+6
*C*b^2)*x^9+2/5*b*c^3*C*x^10+1/11*c^4*C*x^11
```

Rubi [A]
time = 0.20, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1671}

$$a^4Ax+2a^3Abx^2+abx^4\left(a^2C+A\left(3ac+b^2\right)\right)+\frac{1}{3}a^2x^3\left(a^2C+4aAc+6Ab^2\right)+\frac{1}{7}x^7\left(C\left(6a^2c^2+12ab^2c+b^4\right)+2Ac^2\left(2ac+3b^2\right)\right)+\frac{1}{5}x^5\left(A\left(6a^2c^2+12ab^2c+b^4\right)+2a^2C\left(2ac+3b^2\right)\right)+\frac{1}{9}c^2x^9\left(4acC+Ac^2+6b^2C\right)+\frac{1}{2}bcx^8\left(C\left(3ac+b^2\right)+Ac^2\right)+\frac{2}{3}bx^6\left(3ac+b^2\right)\left(aC+Ac\right)+\frac{2}{5}bc^3Cx^{10}+\frac{1}{11}c^4Cx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4*(A + C*x^2),x]
```

```
[Out] a^4*A*x + 2*a^3*A*b*x^2 + (a^2*(6*A*b^2 + 4*a*A*c + a^2*C)*x^3)/3 + a*b*(A*
(b^2 + 3*a*c) + a^2*C)*x^4 + ((A*(b^4 + 12*a*b^2*c + 6*a^2*c^2) + 2*a^2*(3*
b^2 + 2*a*c)*C)*x^5)/5 + (2*b*(b^2 + 3*a*c)*(A*c + a*C)*x^6)/3 + ((2*A*c^2*
(3*b^2 + 2*a*c) + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*C)*x^7)/7 + (b*c*(A*c^2 +
(b^2 + 3*a*c)*C)*x^8)/2 + (c^2*(A*c^2 + 6*b^2*C + 4*a*c*C)*x^9)/9 + (2*b*c^
3*C*x^10)/5 + (c^4*C*x^11)/11
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^4 \left(A+Cx^2\right) \, dx = \int \left(a^4A+4a^3Abx+a^2\left(6Ab^2+4aAc+a^2C\right)x^2+4ab\left(A\left(b^2+3ac\right)+\right.\right.$$

$$=a^4Ax+2a^3Abx^2+\frac{1}{3}a^2\left(6Ab^2+4aAc+a^2C\right)x^3+ab\left(A\left(b^2+3ac\right)+a\right.$$

Mathematica [A]
time = 0.06, size = 256, normalized size = 1.01

$$a^4Ax+2a^3Abx^2+\frac{1}{3}a^2\left(6Ab^2+4aAc+a^2C\right)x^3+ab\left(Ab^2+3aAc+a^2C\right)x^4+\frac{1}{5}\left(Ab^4+12aAb^2c+6a^2Ac^2+6a^2b^2C+4a^3cC\right)x^5+\frac{2}{3}b\left(b^2+3ac\right)(Ac+aC)x^6+\frac{1}{7}\left(6Ab^2c^2+4aAc^3+b^4C+12ab^2cC+6a^2c^2C\right)x^7+\frac{1}{2}bc\left(Ac^2+b^2C+3acC\right)x^8+\frac{1}{9}c^2\left(Ac^2+6b^2C+4acC\right)x^9+\frac{2}{5}bc^3Cx^{10}+\frac{1}{11}c^4Cx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4*(A + C*x^2),x]
```

```
[Out] a^4*A*x + 2*a^3*A*b*x^2 + (a^2*(6*A*b^2 + 4*a*A*c + a^2*C)*x^3)/3 + a*b*(A*
b^2 + 3*a*A*c + a^2*C)*x^4 + ((A*b^4 + 12*a*A*b^2*c + 6*a^2*A*c^2 + 6*a^2*b
^2*C + 4*a^3*c*C)*x^5)/5 + (2*b*(b^2 + 3*a*c)*(A*c + a*C)*x^6)/3 + ((6*A*b^
2*c^2 + 4*a*A*c^3 + b^4*C + 12*a*b^2*c*C + 6*a^2*c^2*C)*x^7)/7 + (b*c*(A*c^
2 + b^2*C + 3*a*c*C)*x^8)/2 + (c^2*(A*c^2 + 6*b^2*C + 4*a*c*C)*x^9)/9 + (2*
b*c^3*C*x^10)/5 + (c^4*C*x^11)/11
```

Maple [A]
time = 0.14, size = 343, normalized size = 1.35

method	result
norman	$\frac{c^4C\,x^{11}}{11}+\frac{2b\,c^3C\,x^{10}}{5}+\left(\frac{1}{9}c^4A+\frac{4}{9}Ca\,c^3+\frac{2}{3}C\,b^2c^2\right)x^9+\left(\frac{1}{2}b\,c^3A+\frac{3}{2}Cab\,c^2+\frac{1}{2}C\,b^3c\right)x^8+\left(\frac{4}{7}Aa\,c^3+\right.$
gosper	$\frac{1}{11}c^4C\,x^{11}+2x^6C\,a^2bc+\frac{12}{5}x^5Aa\,b^2c+3A\,a^2bc\,x^4+\frac{12}{7}x^7Ca\,b^2c+2x^6Aab\,c^2+\frac{3}{2}x^8Cab\,c^2+\frac{4}{5}x^5C$
risch	$\frac{1}{11}c^4C\,x^{11}+2x^6C\,a^2bc+\frac{12}{5}x^5Aa\,b^2c+3A\,a^2bc\,x^4+\frac{12}{7}x^7Ca\,b^2c+2x^6Aab\,c^2+\frac{3}{2}x^8Cab\,c^2+\frac{4}{5}x^5C$
default	$\frac{c^4C\,x^{11}}{11}+\frac{2b\,c^3C\,x^{10}}{5}+\frac{\left(\left(2\left(2ac+b^2\right)c^2+4b^2c^2\right)C+c^4A\right)x^9}{9}+\frac{\left(\left(4ab\,c^2+4\left(2ac+b^2\right)bc\right)C+4b\,c^3A\right)x^8}{8}+\frac{\left(\left(2a^2c^2+8a\,b^2c+(2ac+\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4*(C*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c^4*C*x^11+2/5*b*c^3*C*x^10+1/9*((2*(2*a*c+b^2)*c^2+4*b^2*c^2)*C+c^4*A
)*x^9+1/8*((4*a*b*c^2+4*(2*a*c+b^2)*b*c)*C+4*b*c^3*A)*x^8+1/7*((2*a^2*c^2+8
*a*b^2*c+(2*a*c+b^2)^2)*C+(2*(2*a*c+b^2)*c^2+4*b^2*c^2)*A)*x^7+1/6*((4*a^2*
b*c+4*a*b*(2*a*c+b^2))*C+(4*a*b*c^2+4*(2*a*c+b^2)*b*c)*A)*x^6+1/5*((2*a^2*(
2*a*c+b^2)+4*a^2*b^2)*C+(2*a^2*c^2+8*a*b^2*c+(2*a*c+b^2)^2)*A)*x^5+1/4*(4*a
^3*b*C+(4*a^2*b*c+4*a*b*(2*a*c+b^2))*A)*x^4+1/3*(a^4*C+(2*a^2*(2*a*c+b^2)+4
*a^2*b^2)*A)*x^3+2*a^3*A*b*x^2+a^4*A*x
```

Maxima [A]
time = 0.28, size = 263, normalized size = 1.04

$$\frac{1}{11}Cc^4x^{11}+\frac{2}{5}Cbc^3x^{10}+\frac{1}{9}\left(6Cb^2c^2+4Cac^3+Ac^4\right)x^9+\frac{1}{2}\left(Cb^3c+3Cabc^2+Abc^3\right)x^8+\frac{1}{7}\left(Cb^4+12Cab^2c+4Aac^3+6\left(Ca^2+Ab^2\right)c^2\right)x^7+2Aa^3bx^2+\frac{2}{3}\left(Cab^3+3Aabc^2+\left(3Ca^2b+Ab^3\right)c\right)x^6+Aa^4x+\frac{1}{5}\left(6Ca^2b^2+Ab^4+6Aa^2c^2+4\left(Ca^3+3Aab^2\right)c\right)x^5+\left(Ca^3b+Aab^3+3Aa^2bc\right)x^4+\frac{1}{3}\left(Ca^4+6Aa^2b^2+4Aa^3c\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/11*C*c^4*x^11 + 2/5*C*b*c^3*x^10 + 1/9*(6*C*b^2*c^2 + 4*C*a*c^3 + A*c^4)*
x^9 + 1/2*(C*b^3*c + 3*C*a*b*c^2 + A*b*c^3)*x^8 + 1/7*(C*b^4 + 12*C*a*b^2*c
 + 4*A*a*c^3 + 6*(C*a^2 + A*b^2)*c^2)*x^7 + 2*A*a^3*b*x^2 + 2/3*(C*a*b^3 +
3*A*a*b*c^2 + (3*C*a^2*b + A*b^3)*c)*x^6 + A*a^4*x + 1/5*(6*C*a^2*b^2 + A*b
^4 + 6*A*a^2*c^2 + 4*(C*a^3 + 3*A*a*b^2)*c)*x^5 + (C*a^3*b + A*a*b^3 + 3*A*
a^2*b*c)*x^4 + 1/3*(C*a^4 + 6*A*a^2*b^2 + 4*A*a^3*c)*x^3
```

Fricas [A]
time = 0.33, size = 263, normalized size = 1.04

$$\frac{1}{11}Cc^4x^{11}+\frac{2}{5}Cbc^3x^{10}+\frac{1}{9}\left(6Cb^2c^2+4Cac^3+Ac^4\right)x^9+\frac{1}{2}\left(Cb^3c+3Cabc^2+Abc^3\right)x^8+\frac{1}{7}\left(Cb^4+12Cab^2c+4Aac^3+6\left(Ca^2+Ab^2\right)c^2\right)x^7+2Aa^3bx^2+\frac{2}{3}\left(Cab^3+3Aabc^2+\left(3Ca^2b+Ab^3\right)c\right)x^6+Aa^4x+\frac{1}{5}\left(6Ca^2b^2+Ab^4+6Aa^2c^2+4\left(Ca^3+3Aab^2\right)c\right)x^5+\left(Ca^3b+Aab^3+3Aa^2bc\right)x^4+\frac{1}{3}\left(Ca^4+6Aa^2b^2+4Aa^3c\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/11*C*c^4*x^11 + 2/5*C*b*c^3*x^10 + 1/9*(6*C*b^2*c^2 + 4*C*a*c^3 + A*c^4)*
x^9 + 1/2*(C*b^3*c + 3*C*a*b*c^2 + A*b*c^3)*x^8 + 1/7*(C*b^4 + 12*C*a*b^2*c
 + 4*A*a*c^3 + 6*(C*a^2 + A*b^2)*c^2)*x^7 + 2*A*a^3*b*x^2 + 2/3*(C*a*b^3 +
3*A*a*b*c^2 + (3*C*a^2*b + A*b^3)*c)*x^6 + A*a^4*x + 1/5*(6*C*a^2*b^2 + A*b
^4 + 6*A*a^2*c^2 + 4*(C*a^3 + 3*A*a*b^2)*c)*x^5 + (C*a^3*b + A*a*b^3 + 3*A*
a^2*b*c)*x^4 + 1/3*(C*a^4 + 6*A*a^2*b^2 + 4*A*a^3*c)*x^3
```

Sympy [A]
time = 0.03, size = 320, normalized size = 1.26

$$Aa^4x+2Aa^3bx^2+\frac{2Cbc^3x^{10}}{5}+\frac{Cc^4x^{11}}{11}+x^9\left(\frac{Ac^4}{9}+\frac{4Cac^3}{9}+\frac{2Cb^2c^2}{3}\right)+x^8\left(\frac{Abc^3}{2}+\frac{3Cabc^2}{2}+\frac{Cb^3c}{2}\right)+x^7\cdot\left(\frac{4Aac^3}{7}+\frac{6Ab^2c^2}{7}+\frac{6Ca^2c^2}{7}+\frac{12Cab^2c}{7}+\frac{Cb^4}{7}\right)+x^6\cdot\left(2Aabc^2+\frac{2Ab^3c}{3}+2Ca^2bc+\frac{2Cab^3}{3}\right)+x^5\cdot\left(\frac{6Aa^2c^2}{5}+\frac{12Aab^2c}{5}+\frac{Ab^4}{5}+\frac{4Ca^3c}{5}+\frac{6Ca^2b^2}{5}\right)+x^4\cdot\left(3Aa^2bc+Aab^3+Ca^3b\right)+x^3\cdot\left(\frac{4Aa^3c}{3}+2Aa^2b^2+\frac{Ca^4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4*(C*x**2+A),x)
```

```
[Out] A*a**4*x + 2*A*a**3*b*x**2 + 2*C*b*c**3*x**10/5 + C*c**4*x**11/11 + x**9*(A
*c**4/9 + 4*C*a*c**3/9 + 2*C*b**2*c**2/3) + x**8*(A*b*c**3/2 + 3*C*a*b*c**2
/2 + C*b**3*c/2) + x**7*(4*A*a*c**3/7 + 6*A*b**2*c**2/7 + 6*C*a**2*c**2/7 +
 12*C*a*b**2*c/7 + C*b**4/7) + x**6*(2*A*a*b*c**2 + 2*A*b**3*c/3 + 2*C*a**2
*b*c + 2*C*a*b**3/3) + x**5*(6*A*a**2*c**2/5 + 12*A*a*b**2*c/5 + A*b**4/5 +
 4*C*a**3*c/5 + 6*C*a**2*b**2/5) + x**4*(3*A*a**2*b*c + A*a*b**3 + C*a**3*b
) + x**3*(4*A*a**3*c/3 + 2*A*a**2*b**2 + C*a**4/3)
```

Giac [A]
time = 3.28, size = 308, normalized size = 1.21

$$\frac{1}{11}Cc^4x^{11}+\frac{2}{5}Cbc^3x^{10}+\frac{2}{3}Cb^2c^2x^9+\frac{4}{9}Cac^3x^9+\frac{1}{9}Ac^4x^9+\frac{1}{2}Cb^3cx^8+\frac{3}{2}Cabc^2x^8+\frac{1}{2}Abc^3x^8+\frac{1}{7}Cb^4x^7+\frac{12}{7}Cab^2cx^7+\frac{6}{7}Ca^2c^2x^7+\frac{6}{7}Ab^2c^2x^7+\frac{4}{7}Aac^3x^7+\frac{2}{3}Cab^3x^6+2Ca^2bcx^6+\frac{2}{3}Ab^3cx^6+2Aabc^2x^6+\frac{6}{5}Ca^2b^2x^5+\frac{1}{5}Ab^4x^5+\frac{4}{5}Ca^3cx^5+\frac{12}{5}Aab^2cx^5+\frac{6}{5}Aa^2c^2x^5+Ca^3bx^4+Aab^3x^4+3Aa^2bcx^4+\frac{1}{3}Ca^4x^3+2Aa^2b^2x^3+\frac{4}{3}Aa^3cx^3+2Aa^3bx^2+Aa^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/11*C*c^4*x^11 + 2/5*C*b*c^3*x^10 + 2/3*C*b^2*c^2*x^9 + 4/9*C*a*c^3*x^9 +
1/9*A*c^4*x^9 + 1/2*C*b^3*c*x^8 + 3/2*C*a*b*c^2*x^8 + 1/2*A*b*c^3*x^8 + 1/7
*C*b^4*x^7 + 12/7*C*a*b^2*c*x^7 + 6/7*C*a^2*c^2*x^7 + 6/7*A*b^2*c^2*x^7 + 4
/7*A*a*c^3*x^7 + 2/3*C*a*b^3*x^6 + 2*C*a^2*b*c*x^6 + 2/3*A*b^3*c*x^6 + 2*A*
a*b*c^2*x^6 + 6/5*C*a^2*b^2*x^5 + 1/5*A*b^4*x^5 + 4/5*C*a^3*c*x^5 + 12/5*A*
a*b^2*c*x^5 + 6/5*A*a^2*c^2*x^5 + C*a^3*b*x^4 + A*a*b^3*x^4 + 3*A*a^2*b*c*x
^4 + 1/3*C*a^4*x^3 + 2*A*a^2*b^2*x^3 + 4/3*A*a^3*c*x^3 + 2*A*a^3*b*x^2 + A*
a^4*x
```

Mupad [B]
time = 0.13, size = 244, normalized size = 0.96

$$x^5\left(\frac{4Ca^3c}{5}+\frac{6Ca^2b^2}{5}+\frac{6Aa^2c^2}{5}+\frac{12Aab^2c}{5}+\frac{Ab^4}{5}\right)+x^7\left(\frac{6Ca^2c^2}{7}+\frac{12Cab^2c}{7}+\frac{4Aac^3}{7}+\frac{Cb^4}{7}+\frac{6Ab^2c^2}{7}\right)+x^3\left(\frac{Ca^4}{3}+\frac{4Aca^3}{3}+2Aa^2b^2\right)+x^9\left(\frac{2Cb^2c^2}{3}+\frac{Ac^4}{9}+\frac{4Cac^3}{9}\right)+\frac{Cc^4x^{11}}{11}+Aa^4x+\frac{2bx^6\left(b^2+3ac\right)\left(Ac+Ca\right)}{3}+abx^4\left(Ca^2+3Aca+Ab^2\right)+\frac{bcx^8\left(Cb^2+Ac^2+3Cac\right)}{2}+2Aa^3bx^2+\frac{2Cbc^3x^{10}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^4,x)
```

```
[Out] x^5*((A*b^4)/5 + (6*A*a^2*c^2)/5 + (6*C*a^2*b^2)/5 + (4*C*a^3*c)/5 + (12*A*
a*b^2*c)/5) + x^7*((C*b^4)/7 + (6*A*b^2*c^2)/7 + (6*C*a^2*c^2)/7 + (4*A*a*c
^3)/7 + (12*C*a*b^2*c)/7) + x^3*((C*a^4)/3 + 2*A*a^2*b^2 + (4*A*a^3*c)/3) +
 x^9*((A*c^4)/9 + (2*C*b^2*c^2)/3 + (4*C*a*c^3)/9) + (C*c^4*x^11)/11 + A*a^
4*x + (2*b*x^6*(3*a*c + b^2)*(A*c + C*a))/3 + a*b*x^4*(A*b^2 + C*a^2 + 3*A*
a*c) + (b*c*x^8*(A*c^2 + C*b^2 + 3*C*a*c))/2 + 2*A*a^3*b*x^2 + (2*C*b*c^3*x
^10)/5
```

3.141 $\int \left(a+bx+cx^2\right)^3 \left(A+Cx^2\right) \, dx$

Optimal. Leaf size=161

$$a^3Ax+\frac{3}{2}a^2Abx^2+\frac{1}{3}a\left(3A\left(b^2+ac\right)+a^2C\right)x^3+\frac{1}{4}b\left(A\left(b^2+6ac\right)+3a^2C\right)x^4+\frac{3}{5}\left(b^2+ac\right)(Ac+aC)x^5+\frac{1}{6}b(3A$$

```
[Out] a^3*A*x+3/2*a^2*A*b*x^2+1/3*a*(3*A*(a*c+b^2)+a^2*C)*x^3+1/4*b*(A*(6*a*c+b^2
)+3*a^2*C)*x^4+3/5*(a*c+b^2)*(A*c+C*a)*x^5+1/6*b*(3*A*c^2+(6*a*c+b^2)*C)*x^
6+1/7*c*(A*c^2+3*(a*c+b^2)*C)*x^7+3/8*b*c^2*C*x^8+1/9*c^3*C*x^9
```

Rubi [A]
time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{1671\}$

$$a^3Ax+\frac{1}{4}bx^4\left(3a^2C+A\left(6ac+b^2\right)\right)+\frac{1}{3}ax^3\left(a^2C+3A\left(ac+b^2\right)\right)+\frac{3}{2}a^2Abx^2+\frac{1}{7}cx^7\left(3C\left(ac+b^2\right)+Ac^2\right)+\frac{1}{6}bx^6\left(C\left(6ac+b^2\right)+3Ac^2\right)+\frac{3}{5}x^5\left(ac+b^2\right)(aC+Ac)+\frac{3}{8}bc^2Cx^8+\frac{1}{9}c^3Cx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3*(A + C*x^2),x]
```

```
[Out] a^3*A*x + (3*a^2*A*b*x^2)/2 + (a*(3*A*(b^2 + a*c) + a^2*C)*x^3)/3 + (b*(A*(
b^2 + 6*a*c) + 3*a^2*C)*x^4)/4 + (3*(b^2 + a*c)*(A*c + a*C)*x^5)/5 + (b*(3*
A*c^2 + (b^2 + 6*a*c)*C)*x^6)/6 + (c*(A*c^2 + 3*(b^2 + a*c)*C)*x^7)/7 + (3*
b*c^2*C*x^8)/8 + (c^3*C*x^9)/9
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)^3 \left(A+Cx^2\right) \, dx &= \int \left(a^3A+3a^2Abx+a\left(3A\left(b^2+ac\right)+a^2C\right)x^2+b\left(A\left(b^2+6ac\right)+3a^2C\right.\right.\\ &= a^3Ax+\frac{3}{2}a^2Abx^2+\frac{1}{3}a\left(3A\left(b^2+ac\right)+a^2C\right)x^3+\frac{1}{4}b\left(A\left(b^2+6ac\right)+3a\right.\end{aligned}$$

Mathematica [A]
time = 0.03, size = 163, normalized size = 1.01

$$a^3Ax+\frac{3}{2}a^2Abx^2+\frac{1}{3}a\left(3Ab^2+3aAc+a^2C\right)x^3+\frac{1}{4}b\left(Ab^2+6aAc+3a^2C\right)x^4+\frac{3}{5}\left(b^2+ac\right)(Ac+aC)x^5+\frac{1}{6}b\left(3Ac^2+b^2C+6acC\right)x^6+\frac{1}{7}c\left(Ac^2+3b^2C+3acC\right)x^7+\frac{3}{8}bc^2Cx^8+\frac{1}{9}c^3Cx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3*(A + C*x^2),x]
```

```
[Out] a^3*A*x + (3*a^2*A*b*x^2)/2 + (a*(3*A*b^2 + 3*a*A*c + a^2*C)*x^3)/3 + (b*(A
*b^2 + 6*a*A*c + 3*a^2*C)*x^4)/4 + (3*(b^2 + a*c)*(A*c + a*C)*x^5)/5 + (b*(
3*A*c^2 + b^2*C + 6*a*c*C)*x^6)/6 + (c*(A*c^2 + 3*b^2*C + 3*a*c*C)*x^7)/7 +
 (3*b*c^2*C*x^8)/8 + (c^3*C*x^9)/9
```

Maple [A]
time = 0.12, size = 223, normalized size = 1.39

method	result
norman	$\frac{c^3 C x^9}{9}+\frac{3b c^2 C x^8}{8}+\left(\frac{1}{7}c^3 A+\frac{3}{7}c^2 aC+\frac{3}{7}C b^2 c\right) x^7+\left(\frac{1}{2}b c^2 A+Cabc+\frac{1}{6}C b^3\right) x^6+\left(\frac{3}{5}c^2 aA+\frac{3}{5}A b^2 c\right.$
gosper	$\frac{1}{9}c^3 C x^9+\frac{3}{8}b c^2 C x^8+\frac{1}{7}x^7 c^3 A+\frac{3}{7}x^7 c^2 aC+\frac{3}{7}x^7 C b^2 c+\frac{1}{2}x^6 b c^2 A+x^6 Cabc+\frac{1}{6}x^6 C b^3+\frac{3}{5}x^5 c^2 aA$
risch	$\frac{1}{9}c^3 C x^9+\frac{3}{8}b c^2 C x^8+\frac{1}{7}x^7 c^3 A+\frac{3}{7}x^7 c^2 aC+\frac{3}{7}x^7 C b^2 c+\frac{1}{2}x^6 b c^2 A+x^6 Cabc+\frac{1}{6}x^6 C b^3+\frac{3}{5}x^5 c^2 aA$
default	$\frac{c^3 C x^9}{9}+\frac{3b c^2 C x^8}{8}+\frac{\left(\left(c^2 a+2b^2 c+c\left(2ac+b^2\right)\right)C+c^3 A\right)x^7}{7}+\frac{\left(\left(4abc+b\left(2ac+b^2\right)\right)C+3b c^2 A\right)x^6}{6}+\frac{\left(\left(a\left(2ac+b^2\right)+2a b^2+a^2 c\right)C\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3*(C*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*c^3*C*x^9+3/8*b*c^2*C*x^8+1/7*((c^2*a+2*b^2*c+c*(2*a*c+b^2))*C+c^3*A)*x
^7+1/6*((4*a*b*c+b*(2*a*c+b^2))*C+3*b*c^2*A)*x^6+1/5*((a*(2*a*c+b^2)+2*a*b^
2+a^2*c)*C+(c^2*a+2*b^2*c+c*(2*a*c+b^2))*A)*x^5+1/4*(3*a^2*b*C+(4*a*b*c+b*(
2*a*c+b^2))*A)*x^4+1/3*(a^3*C+(a*(2*a*c+b^2)+2*a*b^2+a^2*c)*A)*x^3+3/2*a^2*
A*b*x^2+a^3*A*x
```

Maxima [A]
time = 0.28, size = 165, normalized size = 1.02

$$\frac{1}{9}Cc^3x^9+\frac{3}{8}Cbc^2x^8+\frac{1}{7}\left(3Cb^2c+3Cac^2+Ac^3\right)x^7+\frac{1}{6}\left(Cb^3+6Cabc+3Abc^2\right)x^6+\frac{3}{2}Aa^2bx^2+\frac{3}{5}\left(Cab^2+Aac^2+\left(Ca^2+Ab^2\right)c\right)x^5+Aa^3x+\frac{1}{4}\left(3Ca^2b+Ab^3+6Aabc\right)x^4+\frac{1}{3}\left(Ca^3+3Aab^2+3Aa^2c\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/9*C*c^3*x^9 + 3/8*C*b*c^2*x^8 + 1/7*(3*C*b^2*c + 3*C*a*c^2 + A*c^3)*x^7 +
 1/6*(C*b^3 + 6*C*a*b*c + 3*A*b*c^2)*x^6 + 3/2*A*a^2*b*x^2 + 3/5*(C*a*b^2 +
 A*a*c^2 + (C*a^2 + A*b^2)*c)*x^5 + A*a^3*x + 1/4*(3*C*a^2*b + A*b^3 + 6*A*
a*b*c)*x^4 + 1/3*(C*a^3 + 3*A*a*b^2 + 3*A*a^2*c)*x^3
```

Fricas [A]
time = 0.33, size = 165, normalized size = 1.02

$$\frac{1}{9}Cc^3x^9+\frac{3}{8}Cbc^2x^8+\frac{1}{7}\left(3Cb^2c+3Cac^2+Ac^3\right)x^7+\frac{1}{6}\left(Cb^3+6Cabc+3Abc^2\right)x^6+\frac{3}{2}Aa^2bx^2+\frac{3}{5}\left(Cab^2+Aac^2+\left(Ca^2+Ab^2\right)c\right)x^5+Aa^3x+\frac{1}{4}\left(3Ca^2b+Ab^3+6Aabc\right)x^4+\frac{1}{3}\left(Ca^3+3Aab^2+3Aa^2c\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/9*C*c^3*x^9 + 3/8*C*b*c^2*x^8 + 1/7*(3*C*b^2*c + 3*C*a*c^2 + A*c^3)*x^7 +
 1/6*(C*b^3 + 6*C*a*b*c + 3*A*b*c^2)*x^6 + 3/2*A*a^2*b*x^2 + 3/5*(C*a*b^2 +
 A*a*c^2 + (C*a^2 + A*b^2)*c)*x^5 + A*a^3*x + 1/4*(3*C*a^2*b + A*b^3 + 6*A*
a*b*c)*x^4 + 1/3*(C*a^3 + 3*A*a*b^2 + 3*A*a^2*c)*x^3
```

Sympy [A]
time = 0.02, size = 197, normalized size = 1.22

$$Aa^3x + \frac{3Aa^2bx^2}{2} + \frac{3Cbc^2x^8}{8} + \frac{Cc^3x^9}{9} + x^7\left(\frac{Ac^3}{7} + \frac{3Cac^2}{7} + \frac{3Cb^2c}{7}\right) + x^6\left(\frac{Abc^2}{2} + Cabc + \frac{Cb^3}{6}\right) + x^5\cdot\left(\frac{3Aac^2}{5} + \frac{3Ab^2c}{5} + \frac{3Ca^2c}{5} + \frac{3Cab^2}{5}\right) + x^4\cdot\left(\frac{3Aabc}{2} + \frac{Ab^3}{4} + \frac{3Ca^2b}{4}\right) + x^3\left(Aa^2c + Aab^2 + \frac{Ca^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3*(C*x**2+A),x)
```

```
[Out] A*a**3*x + 3*A*a**2*b*x**2/2 + 3*C*b*c**2*x**8/8 + C*c**3*x**9/9 + x**7*(A*
c**3/7 + 3*C*a*c**2/7 + 3*C*b**2*c/7) + x**6*(A*b*c**2/2 + C*a*b*c + C*b**3
/6) + x**5*(3*A*a*c**2/5 + 3*A*b**2*c/5 + 3*C*a**2*c/5 + 3*C*a*b**2/5) + x*
*4*(3*A*a*b*c/2 + A*b**3/4 + 3*C*a**2*b/4) + x**3*(A*a**2*c + A*a*b**2 + C*
a**3/3)
```

Giac [A]
time = 2.54, size = 187, normalized size = 1.16

$$\frac{1}{9}Cc^3x^9 + \frac{3}{8}Cbc^2x^8 + \frac{3}{7}Cb^2cx^7 + \frac{3}{7}Cac^2x^7 + \frac{1}{7}Ac^3x^7 + \frac{1}{6}Cb^3x^6 + Cabcx^6 + \frac{1}{2}Abc^2x^6 + \frac{3}{5}Cab^2x^5 + \frac{3}{5}Ca^2cx^5 + \frac{3}{5}Ab^2cx^5 + \frac{3}{5}Aac^2x^5 + \frac{3}{4}Ca^2bx^4 + \frac{1}{4}Ab^3x^4 + \frac{3}{2}Aabcx^4 + \frac{1}{3}Ca^3x^3 + Aab^2x^3 + Aa^2cx^3 + \frac{3}{2}Aa^2bx^2 + Aa^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/9*C*c^3*x^9 + 3/8*C*b*c^2*x^8 + 3/7*C*b^2*c*x^7 + 3/7*C*a*c^2*x^7 + 1/7*A
*c^3*x^7 + 1/6*C*b^3*x^6 + C*a*b*c*x^6 + 1/2*A*b*c^2*x^6 + 3/5*C*a*b^2*x^5
+ 3/5*C*a^2*c*x^5 + 3/5*A*b^2*c*x^5 + 3/5*A*a*c^2*x^5 + 3/4*C*a^2*b*x^4 + 1
/4*A*b^3*x^4 + 3/2*A*a*b*c*x^4 + 1/3*C*a^3*x^3 + A*a*b^2*x^3 + A*a^2*c*x^3
+ 3/2*A*a^2*b*x^2 + A*a^3*x
```

Mupad [B]
time = 0.07, size = 149, normalized size = 0.93

$$x^3\left(\frac{C\,a^3}{3} + A\,c\,a^2 + A\,a\,b^2\right) + x^7\left(\frac{3\,C\,b^2\,c}{7} + \frac{A\,c^3}{7} + \frac{3\,C\,a\,c^2}{7}\right) + \frac{b\,x^4\,\left(3\,C\,a^2 + 6\,A\,c\,a + A\,b^2\right)}{4} + \frac{b\,x^6\,\left(C\,b^2 + 3\,A\,c^2 + 6\,C\,a\,c\right)}{6} + \frac{C\,c^3\,x^9}{9} + A\,a^3\,x + \frac{3\,x^5\,\left(b^2 + a\,c\right)\,\left(A\,c + C\,a\right)}{5} + \frac{3\,A\,a^2\,b\,x^2}{2} + \frac{3\,C\,b\,c^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^3,x)
```

```
[Out] x^3*((C*a^3)/3 + A*a*b^2 + A*a^2*c) + x^7*((A*c^3)/7 + (3*C*a*c^2)/7 + (3*C
*b^2*c)/7) + (b*x^4*(A*b^2 + 3*C*a^2 + 6*A*a*c))/4 + (b*x^6*(3*A*c^2 + C*b^
2 + 6*C*a*c))/6 + (C*c^3*x^9)/9 + A*a^3*x + (3*x^5*(a*c + b^2)*(A*c + C*a))
/5 + (3*A*a^2*b*x^2)/2 + (3*C*b*c^2*x^8)/8
```

3.142 $\int (a+bx+cx^2)^2 (A+Cx^2)\, dx$

Optimal. Leaf size=96

$$a^2Ax+aAbx^2+\frac{1}{3}\left(A\left(b^2+2ac\right)+a^2C\right)x^3+\frac{1}{2}b(Ac+aC)x^4+\frac{1}{5}\left(Ac^2+\left(b^2+2ac\right)C\right)x^5+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7$$

```
[Out] a^2*A*x+a*A*b*x^2+1/3*(A*(2*a*c+b^2)+a^2*C)*x^3+1/2*b*(A*c+C*a)*x^4+1/5*(A*
c^2+(2*a*c+b^2)*C)*x^5+1/3*b*c*C*x^6+1/7*c^2*C*x^7
```

Rubi [A]
time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{1671\}$

$$\frac{1}{3}x^3\left(a^2C+A\left(2ac+b^2\right)\right)+a^2Ax+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+Ac^2\right)+\frac{1}{2}bx^4(aC+Ac)+aAbx^2+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2*(A + C*x^2),x]
```

```
[Out] a^2*A*x + a*A*b*x^2 + ((A*(b^2 + 2*a*c) + a^2*C)*x^3)/3 + (b*(A*c + a*C)*x^
4)/2 + ((A*c^2 + (b^2 + 2*a*c)*C)*x^5)/5 + (b*c*C*x^6)/3 + (c^2*C*x^7)/7
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^2\left(A+Cx^2\right)\,dx=\int \left(a^2A+2aAbx+\left(A\left(b^2+2ac\right)+a^2C\right)x^2+2b(Ac+aC)x^3+\left(Ac^2+\right.\right.$$

$$=a^2Ax+aAbx^2+\frac{1}{3}\left(A\left(b^2+2ac\right)+a^2C\right)x^3+\frac{1}{2}b(Ac+aC)x^4+\frac{1}{5}\left(Ac^2\right.$$

Mathematica [A]
time = 0.02, size = 96, normalized size = 1.00

$$a^2Ax+aAbx^2+\frac{1}{3}\left(Ab^2+2aAc+a^2C\right)x^3+\frac{1}{2}b(Ac+aC)x^4+\frac{1}{5}\left(Ac^2+b^2C+2acC\right)x^5+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2*(A + C*x^2),x]
```

```
[Out] a^2*A*x + a*A*b*x^2 + ((A*b^2 + 2*a*A*c + a^2*C)*x^3)/3 + (b*(A*c + a*C)*x^
4)/2 + ((A*c^2 + b^2*C + 2*a*c*C)*x^5)/5 + (b*c*C*x^6)/3 + (c^2*C*x^7)/7
```

Maple [A]
time = 0.11, size = 90, normalized size = 0.94

method	result
default	$\frac{c^2 C x^7}{7}+\frac{bcC x^6}{3}+\frac{(A c^2+(2ac+b^2)C)x^5}{5}+\frac{(2bcA+2abC)x^4}{4}+\frac{(A(2ac+b^2)+a^2C)x^3}{3}+aAb x^2+a^2Ax$
norman	$\frac{c^2 C x^7}{7}+\frac{bcC x^6}{3}+\left(\frac{1}{5}A c^2+\frac{2}{5}acC+\frac{1}{5}C b^2\right)x^5+\left(\frac{1}{2}bcA+\frac{1}{2}abC\right)x^4+\left(\frac{2}{3}acA+\frac{1}{3}A b^2+\frac{1}{3}a^2C\right)x^3$
gosper	$\frac{1}{7}c^2C x^7+\frac{1}{3}bcC x^6+\frac{1}{5}x^5A c^2+\frac{2}{5}x^5acC+\frac{1}{5}x^5C b^2+\frac{1}{2}x^4bcA+\frac{1}{2}x^4abC+\frac{2}{3}x^3acA+\frac{1}{3}x^3A b^2+$
risch	$\frac{1}{7}c^2C x^7+\frac{1}{3}bcC x^6+\frac{1}{5}x^5A c^2+\frac{2}{5}x^5acC+\frac{1}{5}x^5C b^2+\frac{1}{2}x^4bcA+\frac{1}{2}x^4abC+\frac{2}{3}x^3acA+\frac{1}{3}x^3A b^2+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2*(C*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*c^2*C*x^7+1/3*b*c*C*x^6+1/5*(A*c^2+(2*a*c+b^2)*C)*x^5+1/4*(2*A*b*c+2*C*
a*b)*x^4+1/3*(A*(2*a*c+b^2)+a^2*C)*x^3+a*A*b*x^2+a^2*A*x
```

Maxima [A]
time = 0.28, size = 87, normalized size = 0.91

$$\frac{1}{7}Cc^2x^7+\frac{1}{3}Cbcx^6+\frac{1}{5}(Cb^2+2Cac+Ac^2)x^5+Aabx^2+\frac{1}{2}(Cab+Abc)x^4+Aa^2x+\frac{1}{3}(Ca^2+Ab^2+2Aac)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/7*C*c^2*x^7 + 1/3*C*b*c*x^6 + 1/5*(C*b^2 + 2*C*a*c + A*c^2)*x^5 + A*a*b*x
^2 + 1/2*(C*a*b + A*b*c)*x^4 + A*a^2*x + 1/3*(C*a^2 + A*b^2 + 2*A*a*c)*x^3
```

Fricas [A]
time = 0.34, size = 87, normalized size = 0.91

$$\frac{1}{7}Cc^2x^7+\frac{1}{3}Cbcx^6+\frac{1}{5}(Cb^2+2Cac+Ac^2)x^5+Aabx^2+\frac{1}{2}(Cab+Abc)x^4+Aa^2x+\frac{1}{3}(Ca^2+Ab^2+2Aac)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/7*C*c^2*x^7 + 1/3*C*b*c*x^6 + 1/5*(C*b^2 + 2*C*a*c + A*c^2)*x^5 + A*a*b*x
^2 + 1/2*(C*a*b + A*b*c)*x^4 + A*a^2*x + 1/3*(C*a^2 + A*b^2 + 2*A*a*c)*x^3
```

Sympy [A]
time = 0.02, size = 102, normalized size = 1.06

$$Aa^2x+Aabx^2+\frac{Cbcx^6}{3}+\frac{Cc^2x^7}{7}+x^5\left(\frac{Ac^2}{5}+\frac{2Cac}{5}+\frac{Cb^2}{5}\right)+x^4\left(\frac{Abc}{2}+\frac{Cab}{2}\right)+x^3\cdot\left(\frac{2Aac}{3}+\frac{Ab^2}{3}+\frac{Ca^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2*(C*x**2+A),x)
```

```
[Out] A*a**2*x + A*a*b*x**2 + C*b*c*x**6/3 + C*c**2*x**7/7 + x**5*(A*c**2/5 + 2*C
*a*c/5 + C*b**2/5) + x**4*(A*b*c/2 + C*a*b/2) + x**3*(2*A*a*c/3 + A*b**2/3
+ C*a**2/3)
```

Giac [A]
time = 3.76, size = 99, normalized size = 1.03

$$\frac{1}{7}Cc^2x^7+\frac{1}{3}Cbcx^6+\frac{1}{5}Cb^2x^5+\frac{2}{5}Cacx^5+\frac{1}{5}Ac^2x^5+\frac{1}{2}Cabx^4+\frac{1}{2}Abcx^4+\frac{1}{3}Ca^2x^3+\frac{1}{3}Ab^2x^3+\frac{2}{3}Aacx^3+Aabx^2+Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/7*C*c^2*x^7 + 1/3*C*b*c*x^6 + 1/5*C*b^2*x^5 + 2/5*C*a*c*x^5 + 1/5*A*c^2*x
^5 + 1/2*C*a*b*x^4 + 1/2*A*b*c*x^4 + 1/3*C*a^2*x^3 + 1/3*A*b^2*x^3 + 2/3*A*
a*c*x^3 + A*a*b*x^2 + A*a^2*x
```

Mupad [B]
time = 4.09, size = 88, normalized size = 0.92

$$x^3\left(\frac{C\,a^2}{3}+\frac{2\,A\,c\,a}{3}+\frac{A\,b^2}{3}\right)+x^5\left(\frac{C\,b^2}{5}+\frac{A\,c^2}{5}+\frac{2\,C\,a\,c}{5}\right)+\frac{C\,c^2\,x^7}{7}+A\,a^2\,x+\frac{b\,x^4\,\left(A\,c+C\,a\right)}{2}+A\,a\,b\,x^2+\frac{C\,b\,c\,x^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^2,x)
```

```
[Out] x^3*((A*b^2)/3 + (C*a^2)/3 + (2*A*a*c)/3) + x^5*((A*c^2)/5 + (C*b^2)/5 + (2
*C*a*c)/5) + (C*c^2*x^7)/7 + A*a^2*x + (b*x^4*(A*c + C*a))/2 + A*a*b*x^2 +
(C*b*c*x^6)/3
```

3.143 $\int (a+bx+cx^2)(A+Cx^2)\,dx$

Optimal. Leaf size=46

$$aAx+\frac{1}{2}Abx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

```
[Out] a*A*x+1/2*A*b*x^2+1/3*(A*c+C*a)*x^3+1/4*b*C*x^4+1/5*c*C*x^5
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1671}

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}Abx^2+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(A + C*x^2),x]
[Out] a*A*x + (A*b*x^2)/2 + ((A*c + a*C)*x^3)/3 + (b*C*x^4)/4 + (c*C*x^5)/5
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx &= \int \left(aA+Abx+(Ac+aC)x^2+bCx^3+cCx^4\right)\,dx\\ &= aAx+\frac{1}{2}Abx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.00

$$aAx+\frac{1}{2}Abx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(A + C*x^2),x]
[Out] a*A*x + (A*b*x^2)/2 + ((A*c + a*C)*x^3)/3 + (b*C*x^4)/4 + (c*C*x^5)/5
```

Maple [A]
time = 0.05, size = 39, normalized size = 0.85

method	result	size
default	$aAx+\frac{Ab\,x^2}{2}+\frac{(Ac+aC)x^3}{3}+\frac{bC\,x^4}{4}+\frac{cC\,x^5}{5}$	39
norman	$\frac{cC\,x^5}{5}+\frac{bC\,x^4}{4}+\left(\frac{Ac}{3}+\frac{aC}{3}\right)x^3+\frac{Ab\,x^2}{2}+aAx$	40
gosper	$\frac{1}{5}cC\,x^5+\frac{1}{4}bC\,x^4+\frac{1}{3}x^3Ac+\frac{1}{3}x^3aC+\frac{1}{2}Ab\,x^2+aAx$	41
risch	$\frac{1}{5}cC\,x^5+\frac{1}{4}bC\,x^4+\frac{1}{3}x^3Ac+\frac{1}{3}x^3aC+\frac{1}{2}Ab\,x^2+aAx$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(C*x^2+A),x,method=_RETURNVERBOSE)
[Out] a*A*x+1/2*A*b*x^2+1/3*(A*c+C*a)*x^3+1/4*b*C*x^4+1/5*c*C*x^5
```

Maxima [A]
time = 0.28, size = 38, normalized size = 0.83

$$\frac{1}{5}\,Ccx^5+\frac{1}{4}\,Cbx^4+\frac{1}{2}\,Abx^2+\frac{1}{3}\,(Ca+Ac)x^3+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="maxima")
[Out] 1/5*C*c*x^5 + 1/4*C*b*x^4 + 1/2*A*b*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

Fricas [A]
time = 0.35, size = 38, normalized size = 0.83

$$\frac{1}{5}\,Ccx^5+\frac{1}{4}\,Cbx^4+\frac{1}{2}\,Abx^2+\frac{1}{3}\,(Ca+Ac)x^3+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="fricas")
[Out] 1/5*C*c*x^5 + 1/4*C*b*x^4 + 1/2*A*b*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

Sympy [A]
time = 0.01, size = 42, normalized size = 0.91

$$Aax+\frac{Abx^2}{2}+\frac{Cbx^4}{4}+\frac{Ccx^5}{5}+x^3\left(\frac{Ac}{3}+\frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(C*x**2+A),x)
```

```
[Out] A*a*x + A*b*x**2/2 + C*b*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 + C*a/3)
```

Giac [A]
time = 4.25, size = 40, normalized size = 0.87

$$\frac{1}{5}Ccx^5 + \frac{1}{4}Cbx^4 + \frac{1}{3}Cax^3 + \frac{1}{3}Acx^3 + \frac{1}{2}Abx^2 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="giac")
[Out] 1/5*C*c*x^5 + 1/4*C*b*x^4 + 1/3*C*a*x^3 + 1/3*A*c*x^3 + 1/2*A*b*x^2 + A*a*x
```

Mupad [B]
time = 0.03, size = 39, normalized size = 0.85

$$\frac{C\,c\,x^5}{5} + \frac{C\,b\,x^4}{4} + \left(\frac{A\,c}{3} + \frac{C\,a}{3}\right)\,x^3 + \frac{A\,b\,x^2}{2} + A\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2),x)
[Out] x^3*((A*c)/3 + (C*a)/3) + A*a*x + (A*b*x^2)/2 + (C*b*x^4)/4 + (C*c*x^5)/5
```

3.144 $\int \frac{A+Cx^2}{a+bx+cx^2} dx$

Optimal. Leaf size=81

$$\frac{Cx}{c} - \frac{(2Ac^2 + (b^2 - 2ac)C)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{bC\log\left(a+bx+cx^2\right)}{2c^2}$$

```
[Out] C*x/c-1/2*b*C*ln(c*x^2+b*x+a)/c^2-(2*A*c^2+(-2*a*c+b^2)*C)*arctanh((2*c*x+b
)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1671, 648, 632, 212, 642}

$$-\frac{(C(b^2-2ac) + 2Ac^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{bC\log\left(a+bx+cx^2\right)}{2c^2} + \frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (C*x)/c - ((2*A*c^2 + (b^2 - 2*a*c)*C)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/(c^2*Sqrt[b^2 - 4*a*c]) - (b*C*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Cx^2}{a+bx+cx^2}\,dx &= \int \left(\frac{C}{c}+\frac{Ac-aC-bCx}{c\,(a+bx+cx^2)}\right)dx\\
&= \frac{Cx}{c}+\frac{\int \frac{Ac-aC-bCx}{a+bx+cx^2}\,dx}{c}\\
&= \frac{Cx}{c}-\frac{(bC)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{1}{2}\left(2A+\frac{(b^2-2ac)\,C}{c^2}\right)\int \frac{1}{a+bx+cx^2}\,dx\\
&= \frac{Cx}{c}-\frac{bC\log(a+bx+cx^2)}{2c^2}+\left(-2A-\frac{(b^2-2ac)\,C}{c^2}\right)\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x\right.\\
&= \frac{Cx}{c}-\frac{\left(2A+\frac{(b^2-2ac)C}{c^2}\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{bC\log(a+bx+cx^2)}{2c^2}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 84, normalized size = 1.04

$$\frac{Cx}{c}+\frac{(2Ac^2+b^2C-2acC)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}}-\frac{bC\log(a+bx+cx^2)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (C*x)/c + ((2*A*c^2 + b^2*C - 2*a*c*C)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c
]])/(c^2*Sqrt[-b^2 + 4*a*c]) - (b*C*Log[a + b*x + c*x^2])/(2*c^2)
```

Maple [A]
time = 0.18, size = 82, normalized size = 1.01

method	result
default	$\frac{Cx}{c}+\frac{-\frac{bC\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(Ac-aC+\frac{b^{2}C}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{c}$
risch	$\frac{Cx}{c}-\frac{2\ln\left(8Aa\,c^{3}-2A\,b^{2}c^{2}-8C\,a^{2}c^{2}+6Ca\,b^{2}c-C\,b^{4}-2\sqrt{-\left(4ac-b^{2}\right)\left(2A\,c^{2}-2acC+C\,b^{2}\right)^{2}}\,cx-\sqrt{-\left(4ac-\right.}\right.}{c\left(4ac-b^{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] C*x/c+1/c*(-1/2*b*C/c*ln(c*x^2+b*x+a)+2*(A*c-a*C+1/2*b^2*C/c)/(4*a*c-b^2)^(
1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.35, size = 265, normalized size = 3.27

$$\left[\frac{\left(Cb^{2}-2\,Cac+2\,Ac^{2}\right)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac-\sqrt{b^{2}-4\,ac}\,(2\,cx+b)}{cx^{2}+bx+a}\right)+2\,(Cb^{2}c-4\,Cac^{2})x-(Cb^{3}-4\,Cabc)\log\left(cx^{2}+bx+a\right)}{2\left(b^{2}c^{2}-4\,ac^{3}\right)},-\frac{2\left(Cb^{2}-2\,Cac+2\,Ac^{2}\right)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}\,(2\,cx+b)}{b^{2}-4\,ac}\right)-2\,(Cb^{2}c-4\,Cac^{2})x+(Cb^{3}-4\,Cabc)\log\left(cx^{2}+bx+a\right)}{2\left(b^{2}c^{2}-4\,ac^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*((C*b^2 - 2*C*a*c + 2*A*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*
x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(C*
b^2*c - 4*C*a*c^2)*x - (C*b^3 - 4*C*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 -
 4*a*c^3), -1/2*(2*(C*b^2 - 2*C*a*c + 2*A*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(C*b^2*c - 4*C*a*c^2)*x +
(C*b^3 - 4*C*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(76) = 152$.

time = 0.70, size = 413, normalized size = 5.10

$$\frac{Cx}{c} + \left(-\frac{Cb}{2c^2} - \frac{\sqrt{-4ac+b^2}\,(-2Ac^2+2Cac-Cb^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x + \frac{-Abc - Cab - 4ac^2\left(-\frac{Cb}{2c^2} - \frac{\sqrt{-4ac+b^2}\,(-2Ac^2+2Cac-Cb^2)}{2c^2\cdot(4ac-b^2)}\right) + b^2c\left(-\frac{Cb}{2c^2} - \frac{\sqrt{-4ac+b^2}\,(-2Ac^2+2Cac-Cb^2)}{2c^2\cdot(4ac-b^2)}\right)}{-2Ac^2+2Cac-Cb^2}\right) + \left(-\frac{Cb}{2c^2} + \frac{\sqrt{-4ac+b^2}\,(-2Ac^2+2Cac-Cb^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x + \frac{-Abc - Cab - 4ac^2\left(-\frac{Cb}{2c^2} + \frac{\sqrt{-4ac+b^2}\,(-2Ac^2+2Cac-Cb^2)}{2c^2\cdot(4ac-b^2)}\right) + b^2c\left(-\frac{Cb}{2c^2} + \frac{\sqrt{-4ac+b^2}\,(-2Ac^2+2Cac-Cb^2)}{2c^2\cdot(4ac-b^2)}\right)}{-2Ac^2+2Cac-Cb^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a),x)
```

```
[Out] C*x/c + (-C*b/(2*c**2) - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)
/(2*c**2*(4*a*c - b**2)))*log(x + (-A*b*c - C*a*b - 4*a*c**2*(-C*b/(2*c**2)
 - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*a*c - b**2
))) + b**2*c*(-C*b/(2*c**2) - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*
b**2)/(2*c**2*(4*a*c - b**2))))/(-2*A*c**2 + 2*C*a*c - C*b**2)) + (-C*b/(2*
c**2) + sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*a*c -
 b**2)))*log(x + (-A*b*c - C*a*b - 4*a*c**2*(-C*b/(2*c**2) + sqrt(-4*a*c +
b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*a*c - b**2))) + b**2*c*(-C*
b/(2*c**2) + sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*
a*c - b**2))))/(-2*A*c**2 + 2*C*a*c - C*b**2))
```

Giac [A]

time = 3.97, size = 78, normalized size = 0.96

$$\frac{Cx}{c} - \frac{Cb\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{\left(Cb^2 - 2\,Cac + 2\,Ac^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] C*x/c - 1/2*C*b*log(c*x^2 + b*x + a)/c^2 + (C*b^2 - 2*C*a*c + 2*A*c^2)*arct
an((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B]

time = 0.19, size = 224, normalized size = 2.77

$$\frac{2\,A\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}} + \frac{C\,x}{c} + \frac{C\,b^3\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)} - \frac{2\,C\,a\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}} + \frac{C\,b^2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c^2\,\sqrt{4\,a\,c-b^2}} - \frac{2\,C\,a\,b\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^3-b^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2),x)
```

```
[Out] (2*A*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^
2)^(1/2) + (C*x)/c + (C*b^3*log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) -
 (2*C*a*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*
c - b^2)^(1/2)) + (C*b^2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)
^(1/2)))/(c^2*(4*a*c - b^2)^(1/2)) - (2*C*a*b*c*log(a + b*x + c*x^2))/(4*a*
c^3 - b^2*c^2)
```

3.145 $\int \frac{A+Cx^2}{(a+bx+cx^2)^2}\,dx$

Optimal. Leaf size=100

$$-\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4(Ac+aC)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (-b*c*(A+a*C/c)-(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*
(A*c+C*a)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1674, 12, 632, 212}

$$\frac{4(aC+Ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] -((b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(c*(b^2 - 4*a*c)*(a +
 b*x + c*x^2))) + (4*(A*c + a*C)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b
^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Cx^2}{(a+bx+cx^2)^2}\,dx &= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{2(Ac+aC)}{a+bx+cx^2}\,dx}{-b^2+4ac}\\
&= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2(Ac+aC))\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}\\
&= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{(4(Ac+aC))\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b\right.}{b^2-4ac}\\
&= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4(Ac+aC)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.05, size = 98, normalized size = 0.98

$$\frac{b^2Cx+aC(b-2cx)+Ac(b+2cx)}{c\left(-b^2+4ac\right)\left(a+x(b+cx)\right)}+\frac{4(Ac+aC)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b +
 c*x))) + (4*(A*c + a*C)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*
a*c)^(3/2)
```

Maple [**A**]
time = 0.12, size = 115, normalized size = 1.15

method	result

default	$\frac{\frac{(2Ac^2-2acC+Cb^2)x}{c(4ac-b^2)}+\frac{b(Ac+aC)}{c(4ac-b^2)}}{cx^2+bx+a}+\frac{4(Ac+aC)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{\frac{(2Ac^2-2acC+Cb^2)x}{c(4ac-b^2)}+\frac{b(Ac+aC)}{c(4ac-b^2)}}{cx^2+bx+a}+\frac{2\ln\left((-8c^2a+2b^2c)x+(-4ac+b^2)^{\frac{3}{2}}-4abc+b^3\right)Ac}{(-4ac+b^2)^{\frac{3}{2}}}+\frac{2\ln\left((-8c^2a+2b^2c)x+(-4ac+b^2)^{\frac{3}{2}}-4\right.}{(-4ac+b^2)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((2*A*c^2-2*C*a*c+C*b^2)/c/(4*a*c-b^2)*x+b/c*(A*c+C*a)/(4*a*c-b^2))/(c*x^2+
b*x+a)+4*(A*c+C*a)/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(96) = 192$.
time = 0.36, size = 511, normalized size = 5.11

$$\left[-\frac{Cab^3-4\,Aabc^2+2\,(Ca^2c+Aac^2+(Cac^2+Ac^3)x^2+(Cabc+Abc^2)x)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-(4\,Ca^2b-Ab^3)c+(Cb^4-6\,Cab^2c-8\,Aac^3+2\,(4\,Ca^2+Ab^2)c^2)x}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^2+(b^5c-8\,ab^3c^2+16\,a^2bc^3)x},-\frac{Cab^3-4\,Aabc^2-4\,(Ca^2c+Aac^2+(Cac^2+Ac^3)x^2+(Cabc+Abc^2)x)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)-(4\,Ca^2b-Ab^3)c+(Cb^4-6\,Cab^2c-8\,Aac^3+2\,(4\,Ca^2+Ab^2)c^2)x}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^2+(b^5c-8\,ab^3c^2+16\,a^2bc^3)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-(C*a*b^3 - 4*A*a*b*c^2 + 2*(C*a^2*c + A*a*c^2 + (C*a*c^2 + A*c^3)*x^2 + (
C*a*b*c + A*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*
a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (4*C*a^2*b - A*b^
3)*c + (C*b^4 - 6*C*a*b^2*c - 8*A*a*c^3 + 2*(4*C*a^2 + A*b^2)*c^2)*x)/(a*b^
4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2
 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), -(C*a*b^3 - 4*A*a*b*c^2 - 4*(C*
a^2*c + A*a*c^2 + (C*a*c^2 + A*c^3)*x^2 + (C*a*b*c + A*b*c^2)*x)*sqrt(-b^2
+ 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (4*C*a^2*b
 - A*b^3)*c + (C*b^4 - 6*C*a*b^2*c - 8*A*a*c^3 + 2*(4*C*a^2 + A*b^2)*c^2)*x
```

```
)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(92) = 184$.
time = 0.68, size = 376, normalized size = 3.76

$$-2\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(Ac+Ca)\log\left(x+\frac{2Abc+2Cab-32a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(Ac+Ca)+16ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(Ac+Ca)-2b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(Ac+Ca)}{4Ac^2+4Cac}\right)+2\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(Ac+Ca)\log\left(x+\frac{2Abc+2Cab+32a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(Ac+Ca)-16ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(Ac+Ca)+2b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(Ac+Ca)}{4Ac^2+4Cac}\right)+\frac{Abc+Cab+x(2Ac^2-2Cac+Cb^2)}{4a^2c^2-ab^2c+x^2\cdot(4ac^3-b^2c^2)+x\,(4abc^2-b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**2,x)
```

```
[Out] -2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a)*log(x + (2*A*b*c + 2*C*a*b - 32*a
**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a) + 16*a*b**2*c*sqrt(-1/(4*a*
c - b**2)**3)*(A*c + C*a) - 2*b**4*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a))/
(4*A*c**2 + 4*C*a*c)) + 2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a)*log(x + (2
*A*b*c + 2*C*a*b + 32*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a) - 16
*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a) + 2*b**4*sqrt(-1/(4*a*c -
b**2)**3)*(A*c + C*a))/(4*A*c**2 + 4*C*a*c)) + (A*b*c + C*a*b + x*(2*A*c**2
 - 2*C*a*c + C*b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2)
 + x*(4*a*b*c**2 - b**3*c))
```

Giac [A]
time = 3.87, size = 108, normalized size = 1.08

$$-\frac{4\,(Ca+Ac)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{Cb^2x-2\,Cacx+2\,Ac^2x+Cab+Abc}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4*(C*a + A*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-
b^2 + 4*a*c)) - (C*b^2*x - 2*C*a*c*x + 2*A*c^2*x + C*a*b + A*b*c)/((b^2*c -
 4*a*c^2)*(c*x^2 + b*x + a))
```

Mupad [B]
time = 4.53, size = 172, normalized size = 1.72

$$\frac{\frac{A\,b\,c+C\,a\,b}{c\,(4\,a\,c-b^2)}+\frac{x\,(C\,b^2+2\,A\,c^2-2\,C\,a\,c)}{c\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a}-\frac{4\,\mathrm{atan}\left(\frac{\left(\frac{2\,(A\,c+C\,a)\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{4\,c\,x\,(A\,c+C\,a)}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\,(4\,a\,c-b^2)}{2\,A\,c+2\,C\,a}\right)\,(A\,c+C\,a)}{(4\,a\,c-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^2,x)
```

```
[Out] ((A*b*c + C*a*b)/(c*(4*a*c - b^2)) + (x*(2*A*c^2 + C*b^2 - 2*C*a*c))/(c*(4*
a*c - b^2)))/(a + b*x + c*x^2) - (4*atan((((2*(A*c + C*a)*(b^3 - 4*a*b*c))/
(4*a*c - b^2)^(5/2) - (4*c*x*(A*c + C*a))/(4*a*c - b^2)^(3/2))*(4*a*c - b^2
))/(2*A*c + 2*C*a))*(A*c + C*a))/(4*a*c - b^2)^(3/2)
```

3.146 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^3} \, dx$

Optimal. Leaf size=161

$$-\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{2\left(6Ac^2+\left(b^2+2ac\right)C\right)\tanh^{-1}\left(\right.}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] 1/2*(-b*c*(A+a*C/c)-(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^2+1/2*(6*A*c+2*a*C+b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)-2*(6*A
*c^2+(2*a*c+b^2)*C)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2
)
```

Rubi [A]

time = 0.07, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1674, 12, 628, 632, 212}

$$-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{2\left(C\left(2ac+b^2\right)+6Ac^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{(b+2cx)\left(2aC+6Ac+\frac{b^2C}{c}\right)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -1/2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(c*(b^2 - 4*a*c)*(
a + b*x + c*x^2)^2) + ((6*A*c + 2*a*C + (b^2*C)/c)*(b + 2*c*x))/(2*(b^2 - 4
*a*c)^2*(a + b*x + c*x^2)) - (2*(6*A*c^2 + (b^2 + 2*a*c)*C)*ArcTanh[(b + 2*
c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{6Ac+2aC+\frac{b^2C}{c}}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{(6Ac^2 \ldots}{\ldots}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{(2(6Ac \ldots}{\ldots}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{2(6Ac^2 \ldots}{\ldots}$$

Mathematica [A]
time = 0.13, size = 160, normalized size = 0.99

$$\frac{1}{2}\left(\frac{\left(6Ac^2+\left(b^2+2ac\right)C\right)(b+2cx)}{c\left(b^2-4ac\right)^2\left(a+x(b+cx)\right)}+\frac{b^2Cx+aC(b-2cx)+Ac(b+2cx)}{c\left(-b^2+4ac\right)\left(a+x(b+cx)\right)^2}+\frac{4\left(6Ac^2+\left(b^2+2ac\right)C\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] (((6*A*c^2 + (b^2 + 2*a*c)*C)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c
*x))) + (b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*a*c)*(a
+ x*(b + c*x))^2) + (4*(6*A*c^2 + (b^2 + 2*a*c)*C)*ArcTan[(b + 2*c*x)/Sqrt[
-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

Maple [A]
time = 0.14, size = 269, normalized size = 1.67

method	result
default	$\frac{\frac{c\left(6A\,c^2+2acC+C\,b^2\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{3b\left(6A\,c^2+2acC+C\,b^2\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(10c^2aA+2A\,b^2c-2a^2cC+5Ca\,b^2\right)x}{16a^2c^2-8a\,b^2c+b^4}+\frac{b\left(10acA-A\,b^2+6a^2C\right)}{32a^2c^2-16a\,b^2c+2b^4}}{\left(c\,x^2+bx+a\right)^2}+\frac{2(6A\,c^2+2acC+C\,b}{(16a^2c^2-8a\,b}$
risch	$\frac{\frac{c\left(6A\,c^2+2acC+C\,b^2\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{3b\left(6A\,c^2+2acC+C\,b^2\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(10c^2aA+2A\,b^2c-2a^2cC+5Ca\,b^2\right)x}{16a^2c^2-8a\,b^2c+b^4}+\frac{b\left(10acA-A\,b^2+6a^2C\right)}{32a^2c^2-16a\,b^2c+2b^4}}{\left(c\,x^2+bx+a\right)^2}-\frac{6\ln\left(\left(32a^2c^3-16a\,b\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (c*(6*A*c^2+2*C*a*c+C*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/2*b*(6*A*c^2+2*
C*a*c+C*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+(10*A*a*c^2+2*A*b^2*c-2*C*a^2*c
+5*C*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/2*b*(10*A*a*c-A*b^2+6*C*a^2)/(16
*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+2*(6*A*c^2+2*C*a*c+C*b^2)/(16*a^2*
c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 589 vs. 2(153) = 306.
time = 0.37, size = 1199, normalized size = 7.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(6*C*a^2*b^3 - A*b^5 - 40*A*a^2*b*c^2 + 2*(C*b^4*c - 2*C*a*b^2*c^2 - 2
4*A*a*c^4 - 2*(4*C*a^2 - 3*A*b^2)*c^3)*x^3 + 3*(C*b^5 - 2*C*a*b^3*c - 24*A*
a*b*c^3 - 2*(4*C*a^2*b - 3*A*b^3)*c^2)*x^2 + 2*(C*a^2*b^2 + 2*C*a^3*c + 6*A
*a^2*c^2 + (C*b^2*c^2 + 2*C*a*c^3 + 6*A*c^4)*x^4 + 2*(C*b^3*c + 2*C*a*b*c^2
 + 6*A*b*c^3)*x^3 + (C*b^4 + 4*C*a*b^2*c + 12*A*a*c^3 + 2*(2*C*a^2 + 3*A*b^
2)*c^2)*x^2 + 2*(C*a*b^3 + 2*C*a^2*b*c + 6*A*a*b*c^2)*x)*sqrt(b^2 - 4*a*c)*
log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) - 2*(12*C*a^3*b - 7*A*a*b^3)*c + 2*(5*C*a*b^4 - 40*A*a^2*c^
3 + 2*(4*C*a^3 + A*a*b^2)*c^2 - 2*(11*C*a^2*b^2 - A*b^4)*c)*x)/(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128
*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)
, 1/2*(6*C*a^2*b^3 - A*b^5 - 40*A*a^2*b*c^2 + 2*(C*b^4*c - 2*C*a*b^2*c^2 -
24*A*a*c^4 - 2*(4*C*a^2 - 3*A*b^2)*c^3)*x^3 + 3*(C*b^5 - 2*C*a*b^3*c - 24*A
*a*b*c^3 - 2*(4*C*a^2*b - 3*A*b^3)*c^2)*x^2 - 4*(C*a^2*b^2 + 2*C*a^3*c + 6*
A*a^2*c^2 + (C*b^2*c^2 + 2*C*a*c^3 + 6*A*c^4)*x^4 + 2*(C*b^3*c + 2*C*a*b*c^
2 + 6*A*b*c^3)*x^3 + (C*b^4 + 4*C*a*b^2*c + 12*A*a*c^3 + 2*(2*C*a^2 + 3*A*b
^2)*c^2)*x^2 + 2*(C*a*b^3 + 2*C*a^2*b*c + 6*A*a*b*c^2)*x)*sqrt(-b^2 + 4*a*c
)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(12*C*a^3*b - 7
*A*a*b^3)*c + 2*(5*C*a*b^4 - 40*A*a^2*c^3 + 2*(4*C*a^3 + A*a*b^2)*c^2 - 2*(
11*C*a^2*b^2 - A*b^4)*c)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a
^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(150) = 300$.
time = 1.44, size = 774, normalized size = 4.81

$$-\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(6Ac^2+2Cac+Cb^2)\log\left(x+\frac{6Abc^2+2Cabc+Cb^3-64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(6Ac^2+2Cac+Cb^2)+48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(6Ac^2+2Cac+Cb^2)-12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(6Ac^2+2Cac+Cb^2)+b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(6Ac^2+2Cac+Cb^2)}{12Ac^3+4Cac^2+2Cb^2c}\right)+\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(6Ac^2+2Cac+Cb^2)\log\left(x+\frac{6Abc^2+2Cabc+Cb^3+64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(6Ac^2+2Cac+Cb^2)-48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(6Ac^2+2Cac+Cb^2)+12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(6Ac^2+2Cac+Cb^2)-b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(6Ac^2+2Cac+Cb^2)}{12Ac^3+4Cac^2+2Cb^2c}\right)+\frac{10Aabc-Ab^3+6Ca^2b+x^3\cdot(12Ac^3+4Cac^2+2Cb^2c)+x^2\cdot(18Abc^2+6Cabc+3Cb^3)+x(20Aac^2+4Ab^2c-4Ca^2c+10Cab^2)}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\cdot(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3\cdot(64a^2bc^3-32ab^3c^2+4b^5c)+x^2\cdot(64a^3c^3-12ab^4c+2b^6)+x\,(64a^3bc^2-32a^2b^3c+4ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**3,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2)*log(x + (6*A*b*c*
*2 + 2*C*a*b*c + C*b**3 - 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2
 + 2*C*a*c + C*b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c*
*2 + 2*C*a*c + C*b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 +
 2*C*a*c + C*b**2) + b**6*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c +
C*b**2))/(12*A*c**3 + 4*C*a*c**2 + 2*C*b**2*c)) + sqrt(-1/(4*a*c - b**2)**5
```

```
)*(6*A*c**2 + 2*C*a*c + C*b**2)*log(x + (6*A*b*c**2 + 2*C*a*b*c + C*b**3 +
64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2) - 48*
a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2) + 1
2*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2) - b**6*
sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2))/(12*A*c**3 + 4*C*
a*c**2 + 2*C*b**2*c)) + (10*A*a*b*c - A*b**3 + 6*C*a**2*b + x**3*(12*A*c**3
 + 4*C*a*c**2 + 2*C*b**2*c) + x**2*(18*A*b*c**2 + 6*C*a*b*c + 3*C*b**3) + x
*(20*A*a*c**2 + 4*A*b**2*c - 4*C*a**2*c + 10*C*a*b**2))/(32*a**4*c**2 - 16*
a**3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c*
*2) + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**
3 - 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Giac [A]
time = 4.44, size = 217, normalized size = 1.35

$$\frac{2\,(Cb^2 + 2\,Cac + 6\,Ac^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4 - 8\,ab^2c + 16\,a^2c^2)\sqrt{-b^2+4\,ac}} + \frac{2\,Cb^2cx^3 + 4\,Cac^2x^3 + 12\,Ac^3x^3 + 3\,Cb^3x^2 + 6\,Cabcx^2 + 18\,Abc^2x^2 + 10\,Cab^2x - 4\,Ca^2cx + 4\,Ab^2cx + 20\,Aac^2x + 6\,Ca^2b - Ab^3 + 10\,Aabc}{2\,(b^4 - 8\,ab^2c + 16\,a^2c^2)(cx^2+bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2*(C*b^2 + 2*C*a*c + 6*A*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4
- 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(2*C*b^2*c*x^3 + 4*C*a*
c^2*x^3 + 12*A*c^3*x^3 + 3*C*b^3*x^2 + 6*C*a*b*c*x^2 + 18*A*b*c^2*x^2 + 10*
C*a*b^2*x - 4*C*a^2*c*x + 4*A*b^2*c*x + 20*A*a*c^2*x + 6*C*a^2*b - A*b^3 +
10*A*a*b*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

Mupad [B]
time = 4.17, size = 401, normalized size = 2.49

$$\frac{\frac{6\,C\,a^2\,b+10\,A\,c\,a\,b-A\,b^3}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)} + \frac{x\,(-2\,C\,a^2\,c+5\,C\,a\,b^2+10\,A\,a\,c^2+2\,A\,b^2\,c)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4} + \frac{3\,b\,x^2\,(C\,b^2+6\,A\,c^2+2\,C\,a\,c)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)} + \frac{c\,x^3\,(C\,b^2+6\,A\,c^2+2\,C\,a\,c)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,(b^2+2\,a\,c)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} + \frac{2\,\mathrm{atan}\left(\frac{\left(\frac{(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5)\,(C\,b^2+6\,A\,c^2+2\,C\,a\,c)}{(4\,a\,c-b^2)^{5/2}\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)} + \frac{2\,c\,x\,(C\,b^2+6\,A\,c^2+2\,C\,a\,c)}{(4\,a\,c-b^2)^{5/2}}\right)(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}{C\,b^2+6\,A\,c^2+2\,C\,a\,c}\right)(C\,b^2+6\,A\,c^2+2\,C\,a\,c)}{(4\,a\,c-b^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^3,x)
```

```
[Out] ((6*C*a^2*b - A*b^3 + 10*A*a*b*c)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(
10*A*a*c^2 + 2*A*b^2*c + 5*C*a*b^2 - 2*C*a^2*c))/(b^4 + 16*a^2*c^2 - 8*a*b^
2*c) + (3*b*x^2*(6*A*c^2 + C*b^2 + 2*C*a*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2
*c)) + (c*x^3*(6*A*c^2 + C*b^2 + 2*C*a*c))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/
(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (2*atan(((((b^5
 + 16*a^2*b*c^2 - 8*a*b^3*c)*(6*A*c^2 + C*b^2 + 2*C*a*c))/((4*a*c - b^2)^(5
/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (2*c*x*(6*A*c^2 + C*b^2 + 2*C*a*c))/(
4*a*c - b^2)^(5/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*A*c^2 + C*b^2 + 2*C*
a*c))*(6*A*c^2 + C*b^2 + 2*C*a*c))/(4*a*c - b^2)^(5/2)
```

3.147 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^4} \, dx$

Optimal. Leaf size=206

$$-\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}-\frac{2\left(5Ac^2+\left(b^2+ac\right)C\right)(b+2cx)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}+$$

```
[Out] 1/3*(-b*c*(A+a*C/c)-(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^3+1/3*(5*A*c+(a+b^2/c)*C)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^2-2*(5*A
*c^2+(a*c+b^2)*C)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2+b*x+a)+8*c*(5*A*c^2+(a*c+
b^2)*C)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(7/2)
```

Rubi [A]
time = 0.11, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1674, 12, 628, 632, 212}

$$-\frac{2(b+2cx)\left(C\left(ac+b^2\right)+5Ac^2\right)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{8c\left(C\left(ac+b^2\right)+5Ac^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{7/2}}+\frac{(b+2cx)\left(C\left(a+\frac{b^2}{c}\right)+5Ac\right)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^4,x]
```

```
[Out] -1/3*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(c*(b^2 - 4*a*c)*(
a + b*x + c*x^2)^3) + ((5*A*c + (a + b^2/c)*C)*(b + 2*c*x))/(3*(b^2 - 4*a*c
)^2*(a + b*x + c*x^2)^2) - (2*(5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/((b^2
- 4*a*c)^3*(a + b*x + c*x^2)) + (8*c*(5*A*c^2 + (b^2 + a*c)*C)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(7/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^4}\,dx = -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{\int \frac{2\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)}{(a+bx+cx^2)^3}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{\left(2\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)\right)\int \frac{1}{(a+bx+cx^2)^3}}{3\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} + \frac{(2(5}{}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{2(5A}{(b}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{2(5A}{(b}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{2(5A}{(b}$$

Mathematica [A]

time = 0.23, size = 204, normalized size = 0.99

$$\frac{1}{3}\left(\frac{(5Ac^2+(b^2+ac)C)(b+2cx)}{c(b^2-4ac)^2(a+x(b+cx))^2}-\frac{6(5Ac^2+(b^2+ac)C)(b+2cx)}{(b^2-4ac)^3(a+x(b+cx))}+\frac{b^2Cx+aC(b-2cx)+Ac(b+2cx)}{c(-b^2+4ac)(a+x(b+cx))^3}+\frac{24c(5Ac^2+(b^2+ac)C)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^4,x]
```

```
[Out] (((5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x
))^2) - (6*(5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/((b^2 - 4*a*c)^3*(a + x*(
b + c*x))) + (b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*a*c
)*(a + x*(b + c*x))^3) + (24*c*(5*A*c^2 + (b^2 + a*c)*C)*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2))/3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(200) = 400$.
time = 0.15, size = 510, normalized size = 2.48

method	result
default	$\frac{\frac{4c^3(5Ac^2+acC+Cb^2)x^5}{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}+\frac{10c^2(5Ac^2+acC+Cb^2)bx^4}{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}+\frac{2(16ac+11b^2)c(5Ac^2+acC+Cb^2)x^3}{3(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{b(16ac+b^2)(5Ac^2+acC+Cb^2)x^2}{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}+\cdots}{(cx^2+bx+a)^3}$
risch	$\frac{\frac{4c^3(5Ac^2+acC+Cb^2)x^5}{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}+\frac{10c^2(5Ac^2+acC+Cb^2)bx^4}{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}+\frac{2(16ac+11b^2)c(5Ac^2+acC+Cb^2)x^3}{3(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{b(16ac+b^2)(5Ac^2+acC+Cb^2)x^2}{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}+\cdots}{(cx^2+bx+a)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (4*c^3*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^5
+10*c^2*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*b*
x^4+2/3*(16*a*c+11*b^2)*c*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+
12*a*b^4*c-b^6)*x^3+b*(16*a*c+b^2)*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2
*b^2*c^2+12*a*b^4*c-b^6)*x^2+(44*A*a^2*c^3+18*A*a*b^2*c^2-A*b^4*c-4*C*a^3*c
^2+22*C*a^2*b^2*c+C*a*b^4)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x+1/3
*(66*A*a^2*c^2-13*A*a*b^2*c+A*b^4+26*C*a^3*c+C*a^2*b^2)*b/(64*a^3*c^3-48*a^
2*b^2*c^2+12*a*b^4*c-b^6))/(c*x^2+b*x+a)^3+8*c*(5*A*c^2+C*a*c+C*b^2)/(64*a^
3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*
a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(198) = 396$.
time = 0.37, size = 2103, normalized size = 10.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/3*(C*a^2*b^5 + A*b^7 - 264*A*a^3*b*c^3 + 12*(C*b^4*c^3 - 3*C*a*b^2*c^4
- 20*A*a*c^6 - (4*C*a^2 - 5*A*b^2)*c^5)*x^5 + 30*(C*b^5*c^2 - 3*C*a*b^3*c^3
 - 20*A*a*b*c^5 - (4*C*a^2*b - 5*A*b^3)*c^4)*x^4 + 2*(11*C*b^6*c - 17*C*a*b
^4*c^2 - 320*A*a^2*c^5 - 4*(16*C*a^3 + 35*A*a*b^2)*c^4 - (92*C*a^2*b^2 - 55
*A*b^4)*c^3)*x^3 - 2*(52*C*a^4*b - 59*A*a^2*b^3)*c^2 + 3*(C*b^7 + 13*C*a*b^
5*c - 320*A*a^2*b*c^4 - 4*(16*C*a^3*b - 15*A*a*b^3)*c^3 - (52*C*a^2*b^3 - 5
*A*b^5)*c^2)*x^2 + 12*(C*a^3*b^2*c + C*a^4*c^2 + 5*A*a^3*c^3 + (C*b^2*c^4 +
 C*a*c^5 + 5*A*c^6)*x^6 + 3*(C*b^3*c^3 + C*a*b*c^4 + 5*A*b*c^5)*x^5 + 3*(C*
b^4*c^2 + 2*C*a*b^2*c^3 + 5*A*a*c^5 + (C*a^2 + 5*A*b^2)*c^4)*x^4 + (C*b^5*c
 + 7*C*a*b^3*c^2 + 30*A*a*b*c^4 + (6*C*a^2*b + 5*A*b^3)*c^3)*x^3 + 3*(C*a*b
^4*c + 2*C*a^2*b^2*c^2 + 5*A*a^2*c^4 + (C*a^3 + 5*A*a*b^2)*c^3)*x^2 + 3*(C*
a^2*b^3*c + C*a^3*b*c^2 + 5*A*a^2*b*c^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^
2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a
)) + (22*C*a^3*b^3 - 17*A*a*b^5)*c + 3*(C*a*b^6 - 176*A*a^3*c^4 + 4*(4*C*a^
4 - 7*A*a^2*b^2)*c^3 - 2*(46*C*a^3*b^2 - 11*A*a*b^4)*c^2 + (18*C*a^2*b^4 -
A*b^6)*c)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3 + 2
56*a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6 + 2
56*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256*a^3*b^3*
c^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*b^6*c^3 - 160*
a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3*b^5*c^3 - 128
0*a^4*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*c + 80*a^3*b^6
*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a^3*b^7*c + 96*
a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x), -1/3*(C*a^2*b^5 + A*b^7
- 264*A*a^3*b*c^3 + 12*(C*b^4*c^3 - 3*C*a*b^2*c^4 - 20*A*a*c^6 - (4*C*a^2 -
 5*A*b^2)*c^5)*x^5 + 30*(C*b^5*c^2 - 3*C*a*b^3*c^3 - 20*A*a*b*c^5 - (4*C*a^
2*b - 5*A*b^3)*c^4)*x^4 + 2*(11*C*b^6*c - 17*C*a*b^4*c^2 - 320*A*a^2*c^5 -
4*(16*C*a^3 + 35*A*a*b^2)*c^4 - (92*C*a^2*b^2 - 55*A*b^4)*c^3)*x^3 - 2*(52*
C*a^4*b - 59*A*a^2*b^3)*c^2 + 3*(C*b^7 + 13*C*a*b^5*c - 320*A*a^2*b*c^4 - 4
*(16*C*a^3*b - 15*A*a*b^3)*c^3 - (52*C*a^2*b^3 - 5*A*b^5)*c^2)*x^2 - 24*(C*
a^3*b^2*c + C*a^4*c^2 + 5*A*a^3*c^3 + (C*b^2*c^4 + C*a*c^5 + 5*A*c^6)*x^6 +
```

```
3*(C*b^3*c^3 + C*a*b*c^4 + 5*A*b*c^5)*x^5 + 3*(C*b^4*c^2 + 2*C*a*b^2*c^3 +
 5*A*a*c^5 + (C*a^2 + 5*A*b^2)*c^4)*x^4 + (C*b^5*c + 7*C*a*b^3*c^2 + 30*A*a
*b*c^4 + (6*C*a^2*b + 5*A*b^3)*c^3)*x^3 + 3*(C*a*b^4*c + 2*C*a^2*b^2*c^2 +
5*A*a^2*c^4 + (C*a^3 + 5*A*a*b^2)*c^3)*x^2 + 3*(C*a^2*b^3*c + C*a^3*b*c^2 +
 5*A*a^2*b*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b
)/(b^2 - 4*a*c)) + (22*C*a^3*b^3 - 17*A*a*b^5)*c + 3*(C*a*b^6 - 176*A*a^3*c
^4 + 4*(4*C*a^4 - 7*A*a^2*b^2)*c^3 - 2*(46*C*a^3*b^2 - 11*A*a*b^4)*c^2 + (1
8*C*a^2*b^4 - A*b^6)*c)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a
^6*b^2*c^3 + 256*a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a
^3*b^2*c^6 + 256*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4
- 256*a^3*b^3*c^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*
b^6*c^3 - 160*a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3
*b^5*c^3 - 1280*a^4*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*
c + 80*a^3*b^6*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a
^3*b^7*c + 96*a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1224 vs. $2(196) = 392$.
time = 2.50, size = 1224, normalized size = 5.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**4,x)
```

```
[Out] -4*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2)*log(x + (20*A*b
*c**3 + 4*C*a*b*c**2 + 4*C*b**3*c - 1024*a**4*c**5*sqrt(-1/(4*a*c - b**2)**
7)*(5*A*c**2 + C*a*c + C*b**2) + 1024*a**3*b**2*c**4*sqrt(-1/(4*a*c - b**2)
**7)*(5*A*c**2 + C*a*c + C*b**2) - 384*a**2*b**4*c**3*sqrt(-1/(4*a*c - b**2
)**7)*(5*A*c**2 + C*a*c + C*b**2) + 64*a*b**6*c**2*sqrt(-1/(4*a*c - b**2)**
7)*(5*A*c**2 + C*a*c + C*b**2) - 4*b**8*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c
**2 + C*a*c + C*b**2))/(40*A*c**4 + 8*C*a*c**3 + 8*C*b**2*c**2)) + 4*c*sqrt
(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2)*log(x + (20*A*b*c**3 + 4
*C*a*b*c**2 + 4*C*b**3*c + 1024*a**4*c**5*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c
**2 + C*a*c + C*b**2) - 1024*a**3*b**2*c**4*sqrt(-1/(4*a*c - b**2)**7)*(5*A
*c**2 + C*a*c + C*b**2) + 384*a**2*b**4*c**3*sqrt(-1/(4*a*c - b**2)**7)*(5*
A*c**2 + C*a*c + C*b**2) - 64*a*b**6*c**2*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c
**2 + C*a*c + C*b**2) + 4*b**8*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a
*c + C*b**2))/(40*A*c**4 + 8*C*a*c**3 + 8*C*b**2*c**2)) + (66*A*a**2*b*c**2
 - 13*A*a*b**3*c + A*b**5 + 26*C*a**3*b*c + C*a**2*b**3 + x**5*(60*A*c**5 +
 12*C*a*c**4 + 12*C*b**2*c**3) + x**4*(150*A*b*c**4 + 30*C*a*b*c**3 + 30*C*
b**3*c**2) + x**3*(160*A*a*c**4 + 110*A*b**2*c**3 + 32*C*a**2*c**3 + 54*C*a
*b**2*c**2 + 22*C*b**4*c) + x**2*(240*A*a*b*c**3 + 15*A*b**3*c**2 + 48*C*a*
*2*b*c**2 + 51*C*a*b**3*c + 3*C*b**5) + x*(132*A*a**2*c**3 + 54*A*a*b**2*c*
*2 - 3*A*b**4*c - 12*C*a**3*c**2 + 66*C*a**2*b**2*c + 3*C*a*b**4))/(192*a**
```

```
6*c**3 - 144*a**5*b**2*c**2 + 36*a**4*b**4*c - 3*a**3*b**6 + x**6*(192*a**3
*c**6 - 144*a**2*b**2*c**5 + 36*a*b**4*c**4 - 3*b**6*c**3) + x**5*(576*a**3
*b*c**5 - 432*a**2*b**3*c**4 + 108*a*b**5*c**3 - 9*b**7*c**2) + x**4*(576*a
**4*c**5 + 144*a**3*b**2*c**4 - 324*a**2*b**4*c**3 + 99*a*b**6*c**2 - 9*b**
8*c) + x**3*(1152*a**4*b*c**4 - 672*a**3*b**3*c**3 + 72*a**2*b**5*c**2 + 18
*a*b**7*c - 3*b**9) + x**2*(576*a**5*c**4 + 144*a**4*b**2*c**3 - 324*a**3*b
**4*c**2 + 99*a**2*b**6*c - 9*a*b**8) + x*(576*a**5*b*c**3 - 432*a**4*b**3*
c**2 + 108*a**3*b**5*c - 9*a**2*b**7))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(198) = 396.
time = 3.49, size = 407, normalized size = 1.98

$$-\frac{8\,(Cb^2c+Cac^2+5\,Ac^3)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3)\sqrt{-b^2+4\,ac}}-\frac{12\,Cb^2c^3x^5+12\,Cac^4x^5+60\,Ac^5x^5+30\,Cb^3c^2x^4+30\,Cabc^3x^4+150\,Abc^4x^4+22\,Cb^4cx^3+54\,Cab^2c^2x^3+32\,Ca^2c^3x^3+110\,Ab^2c^3x^3+160\,Aac^4x^3+3\,Cb^5x^2+51\,Cab^3cx^2+48\,Ca^2bc^2x^2+15\,Ab^3c^2x^2+240\,Aabc^3x^2+3\,Cab^4x+66\,Ca^2b^2cx-3\,Ab^4cx-12\,Ca^3c^2x+54\,Aab^2c^2x+132\,Aa^2c^3x+Ca^2b^3+Ab^5+26\,Ca^3bc-13\,Aab^3c+66\,Aa^2bc^2}{3\,(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3)(cx^2+bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] -8*(C*b^2*c + C*a*c^2 + 5*A*c^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b
^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) - 1/3*(1
2*C*b^2*c^3*x^5 + 12*C*a*c^4*x^5 + 60*A*c^5*x^5 + 30*C*b^3*c^2*x^4 + 30*C*a
*b*c^3*x^4 + 150*A*b*c^4*x^4 + 22*C*b^4*c*x^3 + 54*C*a*b^2*c^2*x^3 + 32*C*a
^2*c^3*x^3 + 110*A*b^2*c^3*x^3 + 160*A*a*c^4*x^3 + 3*C*b^5*x^2 + 51*C*a*b^3
*c*x^2 + 48*C*a^2*b*c^2*x^2 + 15*A*b^3*c^2*x^2 + 240*A*a*b*c^3*x^2 + 3*C*a*
b^4*x + 66*C*a^2*b^2*c*x - 3*A*b^4*c*x - 12*C*a^3*c^2*x + 54*A*a*b^2*c^2*x
+ 132*A*a^2*c^3*x + C*a^2*b^3 + A*b^5 + 26*C*a^3*b*c - 13*A*a*b^3*c + 66*A*
a^2*b*c^2)/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*(c*x^2 + b*x +
 a)^3)
```

Mupad [B]
time = 4.36, size = 698, normalized size = 3.39

$$-\frac{\frac{26\,C\,a^3\,b\,c+C\,a^2\,b^3+66\,A\,a^2\,b\,c^2-13\,A\,a\,b^3\,c+A\,b^5}{3\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6\right)}+\frac{x\,\left(-4\,C\,a^3\,c^2+22\,C\,a^2\,b^2\,c+44\,A\,a^2\,c^3+C\,a\,b^4+18\,A\,a\,b^2\,c^2-A\,b^4\,c\right)}{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}+\frac{2\,x^3\,\left(11\,b^2\,c+16\,a\,c^2\right)\,\left(C\,b^2+5\,A\,c^2+C\,a\,c\right)}{3\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6\right)}+\frac{x^2\,\left(b^3+16\,a\,c\,b\right)\,\left(C\,b^2+5\,A\,c^2+C\,a\,c\right)}{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}+\frac{4\,c^3\,x^5\,\left(C\,b^2+5\,A\,c^2+C\,a\,c\right)}{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}+\frac{10\,b\,c^2\,x^4\,\left(C\,b^2+5\,A\,c^2+C\,a\,c\right)}{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}}{x^2\,\left(3\,c\,a^2+3\,a\,b^2\right)+x^4\,\left(3\,b^2\,c+3\,a\,c^2\right)+a^3+x^3\,\left(b^3+6\,a\,c\,b\right)+c^3\,x^6+3\,b\,c^2\,x^5+3\,a^2\,b\,x}-\frac{8\,c\,\mathrm{atan}\left(\frac{\left(\frac{8\,c^2\,x\,\left(C\,b^2+5\,A\,c^2+C\,a\,c\right)}{{\left(4\,a\,c-b^2\right)}^{7/2}}+\frac{4\,c\,\left(C\,b^2+5\,A\,c^2+C\,a\,c\right)\,\left(-64\,a^3\,b\,c^3+48\,a^2\,b^3\,c^2-12\,a\,b^5\,c+b^7\right)}{{\left(4\,a\,c-b^2\right)}^{7/2}\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6\right)}\right)\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6\right)}{4\,C\,b^2\,c+20\,A\,c^3+4\,C\,a\,c^2}\right)\,\left(C\,b^2+5\,A\,c^2+C\,a\,c\right)}{{\left(4\,a\,c-b^2\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^4,x)
```

```
[Out] - ((A*b^5 + C*a^2*b^3 - 13*A*a*b^3*c + 26*C*a^3*b*c + 66*A*a^2*b*c^2)/(3*(b
^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(44*A*a^2*c^3 - 4*C*a^
3*c^2 - A*b^4*c + C*a*b^4 + 18*A*a*b^2*c^2 + 22*C*a^2*b^2*c))/(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (2*x^3*(16*a*c^2 + 11*b^2*c)*(5*A*c^2
 + C*b^2 + C*a*c))/(3*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (
x^2*(b^3 + 16*a*b*c)*(5*A*c^2 + C*b^2 + C*a*c))/(b^6 - 64*a^3*c^3 + 48*a^2*
b^2*c^2 - 12*a*b^4*c) + (4*c^3*x^5*(5*A*c^2 + C*b^2 + C*a*c))/(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (10*b*c^2*x^4*(5*A*c^2 + C*b^2 + C*a*
```

```
c))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))/(x^2*(3*a*b^2 + 3*a^2
*c) + x^4*(3*a*c^2 + 3*b^2*c) + a^3 + x^3*(b^3 + 6*a*b*c) + c^3*x^6 + 3*b*c
^2*x^5 + 3*a^2*b*x) - (8*c*atan((((8*c^2*x*(5*A*c^2 + C*b^2 + C*a*c))/(4*a*
c - b^2)^(7/2) + (4*c*(5*A*c^2 + C*b^2 + C*a*c)*(b^7 - 64*a^3*b*c^3 + 48*a^
2*b^3*c^2 - 12*a*b^5*c))/((4*a*c - b^2)^(7/2)*(b^6 - 64*a^3*c^3 + 48*a^2*b^
2*c^2 - 12*a*b^4*c)))*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))/(20
*A*c^3 + 4*C*a*c^2 + 4*C*b^2*c))*(5*A*c^2 + C*b^2 + C*a*c))/(4*a*c - b^2)^(
7/2)
```

3.148 $\int \frac{(d+ex)^3(f+gx+hx^2)}{a+bx+cx^2}\,dx$

Optimal. Leaf size=591

$$-\frac{(b^3e^3h - c^3d(3e^2f + 3deg + d^2h) - bce^2(beg + 3bdh + 2aeh) + c^2e(ae(eg + 3dh) + b(e^2f + 3deg + 3d^2h))}{c^4}$$

```
[Out] -(b^3*e^3*h-c^3*d*(d^2*h+3*d*e*g+3*e^2*f)-b*c*e^2*(2*a*e*h+3*b*d*h+b*e*g)+c
^2*e*(a*e*(3*d*h+e*g)+b*(3*d^2*h+3*d*e*g+e^2*f)))*x/c^4+1/2*e*(b^2*e^2*h+c^
2*(3*d^2*h+3*d*e*g+e^2*f)-c*e*(a*e*h+3*b*d*h+b*e*g))*x^2/c^3+1/3*e^2*(-b*e*
h+3*c*d*h+c*e*g)*x^3/c^2+1/4*e^3*h*x^4/c+1/2*(c^4*d^2*(d*g+3*e*f)+b^4*e^3*h
-b^2*c*e^2*(3*a*e*h+3*b*d*h+b*e*g)+c^2*e*(a^2*e^2*h+2*a*b*e*(3*d*h+e*g)+b^2
*(3*d^2*h+3*d*e*g+e^2*f))-c^3*(b*d*(d^2*h+3*d*e*g+3*e^2*f)+a*e*(3*d^2*h+3*d
*e*g+e^2*f)))*ln(c*x^2+b*x+a)/c^5-(2*c^5*d^3*f-b^5*e^3*h+b^3*c*e^2*(5*a*e*h
+3*b*d*h+b*e*g)-c^4*d*(b*d*(d*g+3*e*f)+2*a*(d^2*h+3*d*e*g+3*e^2*f))-b*c^2*e
*(5*a^2*e^2*h+4*a*b*e*(3*d*h+e*g)+b^2*(3*d^2*h+3*d*e*g+e^2*f))+c^3*(2*a^2*e
^2*(3*d*h+e*g)+b^2*d*(d^2*h+3*d*e*g+3*e^2*f)+3*a*b*e*(3*d^2*h+3*d*e*g+e^2*f
)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.93, antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1642, 648, 632, 212, 642}

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] -(((b^3*e^3*h - c^3*d*(3*e^2*f + 3*d*e*g + d^2*h) - b*c*e^2*(b*e*g + 3*b*d*
h + 2*a*e*h) + c^2*e*(a*e*(e*g + 3*d*h) + b*(e^2*f + 3*d*e*g + 3*d^2*h)))*x
)/c^4) + (e*(b^2*e^2*h + c^2*(e^2*f + 3*d*e*g + 3*d^2*h) - c*e*(b*e*g + 3*b
*d*h + a*e*h))*x^2)/(2*c^3) + (e^2*(c*e*g + 3*c*d*h - b*e*h)*x^3)/(3*c^2) +
 (e^3*h*x^4)/(4*c) - ((2*c^5*d^3*f - b^5*e^3*h + b^3*c*e^2*(b*e*g + 3*b*d*h
 + 5*a*e*h) - c^4*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f + 3*d*e*g + d^2*h)) -
 b*c^2*e*(5*a^2*e^2*h + 4*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f + 3*d*e*g + 3*d^
2*h)) + c^3*(2*a^2*e^2*(e*g + 3*d*h) + b^2*d*(3*e^2*f + 3*d*e*g + d^2*h) +
3*a*b*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]
])/(c^5*Sqrt[b^2 - 4*a*c]) + ((c^4*d^2*(3*e*f + d*g) + b^4*e^3*h - b^2*c*e^
2*(b*e*g + 3*b*d*h + 3*a*e*h) + c^2*e*(a^2*e^2*h + 2*a*b*e*(e*g + 3*d*h) +
b^2*(e^2*f + 3*d*e*g + 3*d^2*h)) - c^3*(b*d*(3*e^2*f + 3*d*e*g + d^2*h) + a
*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*Log[a + b*x + c*x^2])/(2*c^5)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \int \left(-\frac{b^3e^3h-c^3d(3e^2f+3deg+d^2h)-bce^2(beg+3bdh+2aeh)+c^2e(a}{c^4}\right.$$

$$= -\frac{(b^3e^3h-c^3d(3e^2f+3deg+d^2h)-bce^2(beg+3bdh+2aeh)+c^2e(ae(eg}{c^4}$$

$$= -\frac{(b^3e^3h-c^3d(3e^2f+3deg+d^2h)-bce^2(beg+3bdh+2aeh)+c^2e(ae(eg}{c^4}$$

$$= -\frac{(b^3e^3h-c^3d(3e^2f+3deg+d^2h)-bce^2(beg+3bdh+2aeh)+c^2e(ae(eg}{c^4}$$

$$= -\frac{(b^3e^3h-c^3d(3e^2f+3deg+d^2h)-bce^2(beg+3bdh+2aeh)+c^2e(ae(eg}{c^4}$$

Mathematica [A]
time = 0.37, size = 585, normalized size = 0.99

$$\frac{12c(-b^3e^3h+c^3d(3e^2f+3deg+d^2h)+bce^2(beg+3bdh+2aeh)-c^2e(ae(eg+3dh)+b(e^2f+3deg+3d^2h)))x+6c^2e(b^2e^2h+c^2(e^2f+3deg+3d^2h)-ce(beg+3bdh+aeh))x^2+4c^3e^2(ceg+3cdh-beh)x^3+3c^4e^3hx^4+\frac{12(2c^5d^3f-b^5e^3h+b^3ce^2(beg+3bdh+5aeh)-c^4d(bd(3ef+dg)+2a(3e^2f+3deg+d^2h))-bc^2e(5a^2e^2h+4abe(eg+3dh)+b^2(e^2f+3deg+3d^2h))+c^3(2a^2e^2(eg+3dh)+b^2d(3e^2f+3deg+d^2h)+3abe(e^2f+3deg+3d^2h)))\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+6(c^4d^2(3ef+dg)+b^4e^3h-b^2ce^2(beg+3bdh+3aeh)+c^2e(a^2e^2h+2abe(eg+3dh)+b^2(e^2f+3deg+3d^2h))-c^3(bd(3e^2f+3deg+d^2h)+ae(e^2f+3deg+3d^2h)))\log(a+x(b+cx))}{12c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] (12*c*(-(b^3*e^3*h) + c^3*d*(3*e^2*f + 3*d*e*g + d^2*h) + b*c*e^2*(b*e*g +
3*b*d*h + 2*a*e*h) - c^2*e*(a*e*(e*g + 3*d*h) + b*(e^2*f + 3*d*e*g + 3*d^2*
h)))*x + 6*c^2*e*(b^2*e^2*h + c^2*(e^2*f + 3*d*e*g + 3*d^2*h) - c*e*(b*e*g
+ 3*b*d*h + a*e*h))*x^2 + 4*c^3*e^2*(c*e*g + 3*c*d*h - b*e*h)*x^3 + 3*c^4*e
^3*h*x^4 + (12*(2*c^5*d^3*f - b^5*e^3*h + b^3*c*e^2*(b*e*g + 3*b*d*h + 5*a*
e*h) - c^4*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f + 3*d*e*g + d^2*h)) - b*c^2*
e*(5*a^2*e^2*h + 4*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f + 3*d*e*g + 3*d^2*h)) +
 c^3*(2*a^2*e^2*(e*g + 3*d*h) + b^2*d*(3*e^2*f + 3*d*e*g + d^2*h) + 3*a*b*e
*(e^2*f + 3*d*e*g + 3*d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] + 6*(c^4*d^2*(3*e*f + d*g) + b^4*e^3*h - b^2*c*e^2*(b*e*g +
3*b*d*h + 3*a*e*h) + c^2*e*(a^2*e^2*h + 2*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f
+ 3*d*e*g + 3*d^2*h)) - c^3*(b*d*(3*e^2*f + 3*d*e*g + d^2*h) + a*e*(e^2*f +
 3*d*e*g + 3*d^2*h)))*Log[a + x*(b + c*x)])/(12*c^5)
```

Maple [A]
time = 0.25, size = 869, normalized size = 1.47

method	result
default	$\frac{\frac{1}{4}he^3x^4c^3-\frac{1}{3}bc^2e^3hx^3+c^3de^2hx^3+\frac{1}{3}c^3e^3gx^3-\frac{1}{2}ac^2e^3hx^2+\frac{1}{2}b^2ce^3hx^2-\frac{3}{2}bc^2de^2hx^2-\frac{1}{2}bc^2e^3gx^2+\frac{3}{2}c^3d^2ehx^2+\frac{3}{2}c^3de^2gx^2+ \cdots}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/4*h*e^3*x^4*c^3-1/3*b*c^2*e^3*h*x^3+c^3*d*e^2*h*x^3+1/3*c^3*e^3*g*
x^3-1/2*a*c^2*e^3*h*x^2+1/2*b^2*c*e^3*h*x^2-3/2*b*c^2*d*e^2*h*x^2-1/2*b*c^2
*e^3*g*x^2+3/2*c^3*d^2*e*h*x^2+3/2*c^3*d*e^2*g*x^2+1/2*c^3*e^3*f*x^2+2*a*b*
c*e^3*h*x-3*a*c^2*d*e^2*h*x-a*c^2*e^3*g*x-b^3*e^3*h*x+3*b^2*c*d*e^2*h*x+b^2
*c*e^3*g*x-3*b*c^2*d^2*e*h*x-3*b*c^2*d*e^2*g*x-b*c^2*e^3*f*x+c^3*d^3*h*x+3*
c^3*d^2*e*g*x+3*c^3*d*e^2*f*x)+1/c^4*(1/2*(a^2*c^2*e^3*h-3*a*b^2*c*e^3*h+6*
a*b*c^2*d*e^2*h+2*a*b*c^2*e^3*g-3*a*c^3*d^2*e*h-3*a*c^3*d*e^2*g-a*c^3*e^3*f
+b^4*e^3*h-3*b^3*c*d*e^2*h-b^3*c*e^3*g+3*b^2*c^2*d^2*e*h+3*b^2*c^2*d*e^2*g+
b^2*c^2*e^3*f-b*c^3*d^3*h-3*b*c^3*d^2*e*g-3*b*c^3*d*e^2*f+c^4*d^3*g+3*c^4*d
^2*e*f)/c*ln(c*x^2+b*x+a)+2*(-2*a^2*b*c*e^3*h+3*a^2*c^2*d*e^2*h+a^2*c^2*e^3
```

```
*g+a*b^3*e^3*h-3*a*b^2*c*d*e^2*h-a*b^2*c*e^3*g+3*a*b*c^2*d^2*e*h+3*a*b*c^2*
d*e^2*g+a*b*c^2*e^3*f-a*c^3*d^3*h-3*a*c^3*d^2*e*g-3*a*c^3*d*e^2*f+c^4*d^3*f
-1/2*(a^2*c^2*e^3*h-3*a*b^2*c*e^3*h+6*a*b*c^2*d*e^2*h+2*a*b*c^2*e^3*g-3*a*c
^3*d^2*e*h-3*a*c^3*d*e^2*g-a*c^3*e^3*f+b^4*e^3*h-3*b^3*c*d*e^2*h-b^3*c*e^3*
g+3*b^2*c^2*d^2*e*h+3*b^2*c^2*d*e^2*g+b^2*c^2*e^3*f-b*c^3*d^3*h-3*b*c^3*d^2
*e*g-3*b*c^3*d*e^2*f+c^4*d^3*g+3*c^4*d^2*e*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.76, size = 2072, normalized size = 3.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/12*(12*(b^2*c^4 - 4*a*c^5)*d^3*h*x - 6*(2*c^5*d^3*f - b*c^4*d^3*g + (b^2
*c^3 - 2*a*c^4)*d^3*h - ((b^3*c^2 - 3*a*b*c^3)*f - (b^4*c - 4*a*b^2*c^2 + 2
*a^2*c^3)*g + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*h)*e^3 + 3*((b^2*c^3 - 2*a*c^
4)*d*f - (b^3*c^2 - 3*a*b*c^3)*d*g + (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*h)
*e^2 - 3*(b*c^4*d^2*f - (b^2*c^3 - 2*a*c^4)*d^2*g + (b^3*c^2 - 3*a*b*c^3)*d
^2*h)*e)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (3*(b^2*c^4 - 4*a*c^5)*h*x^4 +
 4*((b^2*c^4 - 4*a*c^5)*g - (b^3*c^3 - 4*a*b*c^4)*h)*x^3 + 6*((b^2*c^4 - 4*
a*c^5)*f - (b^3*c^3 - 4*a*b*c^4)*g + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*h)
*x^2 - 12*((b^3*c^3 - 4*a*b*c^4)*f - (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*g
+ (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*h)*x)*e^3 + 6*(2*(b^2*c^4 - 4*a*c^5)*
d*h*x^3 + 3*((b^2*c^4 - 4*a*c^5)*d*g - (b^3*c^3 - 4*a*b*c^4)*d*h)*x^2 + 6*(
(b^2*c^4 - 4*a*c^5)*d*f - (b^3*c^3 - 4*a*b*c^4)*d*g + (b^4*c^2 - 5*a*b^2*c^
3 + 4*a^2*c^4)*d*h)*x)*e^2 + 18*((b^2*c^4 - 4*a*c^5)*d^2*h*x^2 + 2*((b^2*c^
4 - 4*a*c^5)*d^2*g - (b^3*c^3 - 4*a*b*c^4)*d^2*h)*x)*e + 6*((b^2*c^4 - 4*a*
c^5)*d^3*g - (b^3*c^3 - 4*a*b*c^4)*d^3*h + ((b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*
c^4)*f - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*g + (b^6 - 7*a*b^4*c + 13*a^2*
```

```
b^2*c^2 - 4*a^3*c^3)*h)*e^3 - 3*((b^3*c^3 - 4*a*b*c^4)*d*f - (b^4*c^2 - 5*a
*b^2*c^3 + 4*a^2*c^4)*d*g + (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d*h)*e^2 +
3*((b^2*c^4 - 4*a*c^5)*d^2*f - (b^3*c^3 - 4*a*b*c^4)*d^2*g + (b^4*c^2 - 5*a
*b^2*c^3 + 4*a^2*c^4)*d^2*h)*e)*log(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6),
1/12*(12*(b^2*c^4 - 4*a*c^5)*d^3*h*x - 12*(2*c^5*d^3*f - b*c^4*d^3*g + (b^2
*c^3 - 2*a*c^4)*d^3*h - ((b^3*c^2 - 3*a*b*c^3)*f - (b^4*c - 4*a*b^2*c^2 + 2
*a^2*c^3)*g + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*h)*e^3 + 3*((b^2*c^3 - 2*a*c^
4)*d*f - (b^3*c^2 - 3*a*b*c^3)*d*g + (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*h)
*e^2 - 3*(b*c^4*d^2*f - (b^2*c^3 - 2*a*c^4)*d^2*g + (b^3*c^2 - 3*a*b*c^3)*d
^2*h)*e)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4
*a*c)) + (3*(b^2*c^4 - 4*a*c^5)*h*x^4 + 4*((b^2*c^4 - 4*a*c^5)*g - (b^3*c^3
 - 4*a*b*c^4)*h)*x^3 + 6*((b^2*c^4 - 4*a*c^5)*f - (b^3*c^3 - 4*a*b*c^4)*g +
 (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*h)*x^2 - 12*((b^3*c^3 - 4*a*b*c^4)*f -
 (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*g + (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3
)*h)*x)*e^3 + 6*(2*(b^2*c^4 - 4*a*c^5)*d*h*x^3 + 3*((b^2*c^4 - 4*a*c^5)*d*g
 - (b^3*c^3 - 4*a*b*c^4)*d*h)*x^2 + 6*((b^2*c^4 - 4*a*c^5)*d*f - (b^3*c^3 -
 4*a*b*c^4)*d*g + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d*h)*x)*e^2 + 18*((b^
2*c^4 - 4*a*c^5)*d^2*h*x^2 + 2*((b^2*c^4 - 4*a*c^5)*d^2*g - (b^3*c^3 - 4*a*
b*c^4)*d^2*h)*x)*e + 6*((b^2*c^4 - 4*a*c^5)*d^3*g - (b^3*c^3 - 4*a*b*c^4)*d
^3*h + ((b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*f - (b^5*c - 6*a*b^3*c^2 + 8*a^
2*b*c^3)*g + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*h)*e^3 - 3*((b^
3*c^3 - 4*a*b*c^4)*d*f - (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d*g + (b^5*c -
 6*a*b^3*c^2 + 8*a^2*b*c^3)*d*h)*e^2 + 3*((b^2*c^4 - 4*a*c^5)*d^2*f - (b^3*
c^3 - 4*a*b*c^4)*d^2*g + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2*h)*e)*log(
c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4972 vs. $2(619) = 1238$.
time = 73.11, size = 4972, normalized size = 8.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] x**3*(-b*e**3*h/(3*c**2) + d*e**2*h/c + e**3*g/(3*c)) + x**2*(-a*e**3*h/(2*
c**2) + b**2*e**3*h/(2*c**3) - 3*b*d*e**2*h/(2*c**2) - b*e**3*g/(2*c**2) +
3*d**2*e*h/(2*c) + 3*d*e**2*g/(2*c) + e**3*f/(2*c)) + x*(2*a*b*e**3*h/c**3
- 3*a*d*e**2*h/c**2 - a*e**3*g/c**2 - b**3*e**3*h/c**4 + 3*b**2*d*e**2*h/c*
*3 + b**2*e**3*g/c**3 - 3*b*d**2*e*h/c**2 - 3*b*d*e**2*g/c**2 - b*e**3*f/c*
*2 + d**3*h/c + 3*d**2*e*g/c + 3*d*e**2*f/c) + (-sqrt(-4*a*c + b**2)*(5*a**
2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e*
*3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h
 - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**
2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g
```

```
 + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c*
*3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3
*b*c**4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e**3
*h - 3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*c**3
*d**2*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*d*e*
*2*h - b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b**2*c
**2*e**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c**4*d
**3*g + 3*c**4*d**2*e*f)/(2*c**5))*log(x + (2*a**3*c**2*e**3*h - 4*a**2*b**
2*c*e**3*h + 9*a**2*b*c**2*d*e**2*h + 3*a**2*b*c**2*e**3*g - 6*a**2*c**3*d*
*2*e*h - 6*a**2*c**3*d*e**2*g - 2*a**2*c**3*e**3*f + a*b**4*e**3*h - 3*a*b*
*3*c*d*e**2*h - a*b**3*c*e**3*g + 3*a*b**2*c**2*d**2*e*h + 3*a*b**2*c**2*d*
e**2*g + a*b**2*c**2*e**3*f - a*b*c**3*d**3*h - 3*a*b*c**3*d**2*e*g - 3*a*b
*c**3*d*e**2*f - 4*a*c**5*(-sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e**3*h - 6*a
**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a*b**2*c**2
*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c**3*d*e**2*
g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a*c**4*d*e*
*2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c**2*d**2*e
*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c**3*d**3*h - 3*b**2*c*
*3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**2*e*f - 2*
c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e**3*h - 3*a*b**2*c*e**3*
h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*c**3*d**2*e*h - 3*a*c**3*
d*e**2*g - a*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*d*e**2*h - b**3*c*e**3*g
 + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b**2*c**2*e**3*f - b*c**3*d
**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c**4*d**3*g + 3*c**4*d**2*e
*f)/(2*c**5)) + 2*a*c**4*d**3*g + 6*a*c**4*d**2*e*f + b**2*c**4*(-sqrt(-4*a
*c + b**2)*(5*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*
g - 5*a*b**3*c*e**3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*
a*b*c**3*d**2*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3
*h + 6*a*c**4*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*
h - b**4*c*e**3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2
*e**3*f - b**2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f +
b*c**4*d**3*g + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2))
 + (a**2*c**2*e**3*h - 3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*
e**3*g - 3*a*c**3*d**2*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*
h - 3*b**3*c*d*e**2*h - b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*
d*e**2*g + b**2*c**2*e**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*
d*e**2*f + c**4*d**3*g + 3*c**4*d**2*e*f)/(2*c**5)) - b*c**4*d**3*f)/(5*a**
2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e*
*3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h
 - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**
2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g
 + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c*
*3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3
*b*c**4*d**2*e*f - 2*c**5*d**3*f)) + (sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e*
*3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a
```

```
*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c*
*3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a
*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c
**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c**3*d**3*h -
 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**
2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e**3*h - 3*a*b*
*2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*c**3*d**2*e*h -
 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*d*e**2*h - b**3
*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b**2*c**2*e**3*f
- b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c**4*d**3*g + 3*c
**4*d**2*e*f)/(2*c**5))*log(x + (2*a**3*c**2*e**3*h - 4*a**2*b**2*c*e**3*h
+ 9*a**2*b*c**2*d*e**2*h + 3*a**2*b*c**2*e**3*g...
```

Giac [A]
time = 3.85, size = 771, normalized size = 1.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*(3*c^3*h*x^4*e^3 + 12*c^3*d*h*x^3*e^2 + 18*c^3*d^2*h*x^2*e + 12*c^3*d^
3*h*x + 4*c^3*g*x^3*e^3 - 4*b*c^2*h*x^3*e^3 + 18*c^3*d*g*x^2*e^2 - 18*b*c^2
*d*h*x^2*e^2 + 36*c^3*d^2*g*x*e - 36*b*c^2*d^2*h*x*e + 6*c^3*f*x^2*e^3 - 6*
b*c^2*g*x^2*e^3 + 6*b^2*c*h*x^2*e^3 - 6*a*c^2*h*x^2*e^3 + 36*c^3*d*f*x*e^2
- 36*b*c^2*d*g*x*e^2 + 36*b^2*c*d*h*x*e^2 - 36*a*c^2*d*h*x*e^2 - 12*b*c^2*f
*x*e^3 + 12*b^2*c*g*x*e^3 - 12*a*c^2*g*x*e^3 - 12*b^3*h*x*e^3 + 24*a*b*c*h*
x*e^3)/c^4 + 1/2*(c^4*d^3*g - b*c^3*d^3*h + 3*c^4*d^2*f*e - 3*b*c^3*d^2*g*e
 + 3*b^2*c^2*d^2*h*e - 3*a*c^3*d^2*h*e - 3*b*c^3*d*f*e^2 + 3*b^2*c^2*d*g*e^
2 - 3*a*c^3*d*g*e^2 - 3*b^3*c*d*h*e^2 + 6*a*b*c^2*d*h*e^2 + b^2*c^2*f*e^3 -
 a*c^3*f*e^3 - b^3*c*g*e^3 + 2*a*b*c^2*g*e^3 + b^4*h*e^3 - 3*a*b^2*c*h*e^3
+ a^2*c^2*h*e^3)*log(c*x^2 + b*x + a)/c^5 + (2*c^5*d^3*f - b*c^4*d^3*g + b^
2*c^3*d^3*h - 2*a*c^4*d^3*h - 3*b*c^4*d^2*f*e + 3*b^2*c^3*d^2*g*e - 6*a*c^4
*d^2*g*e - 3*b^3*c^2*d^2*h*e + 9*a*b*c^3*d^2*h*e + 3*b^2*c^3*d*f*e^2 - 6*a*
c^4*d*f*e^2 - 3*b^3*c^2*d*g*e^2 + 9*a*b*c^3*d*g*e^2 + 3*b^4*c*d*h*e^2 - 12*
a*b^2*c^2*d*h*e^2 + 6*a^2*c^3*d*h*e^2 - b^3*c^2*f*e^3 + 3*a*b*c^3*f*e^3 + b
^4*c*g*e^3 - 4*a*b^2*c^2*g*e^3 + 2*a^2*c^3*g*e^3 - b^5*h*e^3 + 5*a*b^3*c*h*
e^3 - 5*a^2*b*c^2*h*e^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
+ 4*a*c)*c^5)
```

Mupad [B]
time = 5.46, size = 967, normalized size = 1.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

```
[Out] x^3*((e^3*g + 3*d*e^2*h)/(3*c) - (b*e^3*h)/(3*c^2)) + x*((d^3*h + 3*d*e^2*f
 + 3*d^2*e*g)/c + (b*((b*((e^3*g + 3*d*e^2*h)/c - (b*e^3*h)/c^2))/c - (e^3*
f + 3*d*e^2*g + 3*d^2*e*h)/c + (a*e^3*h)/c^2))/c - (a*((e^3*g + 3*d*e^2*h)/
c - (b*e^3*h)/c^2))/c) - x^2*((b*((e^3*g + 3*d*e^2*h)/c - (b*e^3*h)/c^2))/(
2*c) - (e^3*f + 3*d*e^2*g + 3*d^2*e*h)/(2*c) + (a*e^3*h)/(2*c^2)) - (log(a
+ b*x + c*x^2)*(b^6*e^3*h + 4*a^2*c^4*e^3*f + b^2*c^4*d^3*g + b^4*c^2*e^3*f
 - 4*a^3*c^3*e^3*h - b^3*c^3*d^3*h - 4*a*c^5*d^3*g - b^5*c*e^3*g + 4*a*b*c^
4*d^3*h - 7*a*b^4*c*e^3*h - 12*a*c^5*d^2*e*f - 3*b^5*c*d*e^2*h - 5*a*b^2*c^
3*e^3*f + 6*a*b^3*c^2*e^3*g - 8*a^2*b*c^3*e^3*g + 12*a^2*c^4*d*e^2*g + 3*b^
2*c^4*d^2*e*f - 3*b^3*c^3*d*e^2*f + 12*a^2*c^4*d^2*e*h - 3*b^3*c^3*d^2*e*g
+ 3*b^4*c^2*d*e^2*g + 3*b^4*c^2*d^2*e*h + 13*a^2*b^2*c^2*e^3*h + 12*a*b*c^4
*d*e^2*f + 12*a*b*c^4*d^2*e*g - 15*a*b^2*c^3*d*e^2*g - 15*a*b^2*c^3*d^2*e*h
 + 18*a*b^3*c^2*d*e^2*h - 24*a^2*b*c^3*d*e^2*h))/(2*(4*a*c^6 - b^2*c^5)) +
(e^3*h*x^4)/(4*c) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/
2))*(2*c^5*d^3*f - b^5*e^3*h + 2*a^2*c^3*e^3*g - b^3*c^2*e^3*f + b^2*c^3*d^
3*h - 2*a*c^4*d^3*h - b*c^4*d^3*g + b^4*c*e^3*g + 3*a*b*c^3*e^3*f + 5*a*b^3
*c*e^3*h - 6*a*c^4*d*e^2*f - 6*a*c^4*d^2*e*g - 3*b*c^4*d^2*e*f + 3*b^4*c*d*
e^2*h - 4*a*b^2*c^2*e^3*g - 5*a^2*b*c^2*e^3*h + 3*b^2*c^3*d*e^2*f + 6*a^2*c
^3*d*e^2*h + 3*b^2*c^3*d^2*e*g - 3*b^3*c^2*d*e^2*g - 3*b^3*c^2*d^2*e*h + 9*
a*b*c^3*d*e^2*g + 9*a*b*c^3*d^2*e*h - 12*a*b^2*c^2*d*e^2*h))/(c^5*(4*a*c -
b^2)^(1/2))
```

3.149 $\int \frac{(d+ex)^2(f+gx+hx^2)}{a+bx+cx^2}\,dx$

Optimal. Leaf size=348

$$\frac{\left(b^2e^2h+c^2(e^2f+2deg+d^2h)-ce(beg+2bdh+aeh)\right)x}{c^3}+\frac{e(ceg+2cdh-beh)x^2}{2c^2}+\frac{e^2hx^3}{3c}-\frac{(2c^4d^2f+b^4e^2}{}$$

```
[Out] (b^2*e^2*h+c^2*(d^2*h+2*d*e*g+e^2*f)-c*e*(a*e*h+2*b*d*h+b*e*g))*x/c^3+1/2*e
*(-b*e*h+2*c*d*h+c*e*g)*x^2/c^2+1/3*e^2*h*x^3/c+1/2*(c^3*d*(d*g+2*e*f)-b^3*
e^2*h+b*c*e*(2*a*e*h+2*b*d*h+b*e*g)-c^2*(a*e*(2*d*h+e*g)+b*(d^2*h+2*d*e*g+e
^2*f)))*ln(c*x^2+b*x+a)/c^4-(2*c^4*d^2*f+b^4*e^2*h-b^2*c*e*(4*a*e*h+2*b*d*h
+b*e*g)-c^3*(b*d*(d*g+2*e*f)+2*a*(d^2*h+2*d*e*g+e^2*f))+c^2*(2*a^2*e^2*h+3*
a*b*e*(2*d*h+e*g)+b^2*(d^2*h+2*d*e*g+e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2
)^(1/2))/c^4/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1642, 648, 632, 212, 642}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^2(2a^2e^2h+3abe(2dh+eg)+b^2(d^2h+2deg+e^2f))-b^2ce(4aeh+2bdh+beg)-c^3(2a(d^2h+2deg+e^2f)+bd(dg+2ef))+b^4e^2h+2c^4d^2f\right)}{c^4\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)\left(-c^2(ae(2dh+eg)+b(d^2h+2deg+e^2f))+bce(2aeh+2bdh+beg)+b^3(-e^2)h+c^3d(dg+2ef)\right)}{2c^4}+\frac{x\left(-ce(aeh+2bdh+beg)+b^2e^2h+c^2(d^2h+2deg+e^2f)\right)}{c^3}+\frac{ex^2(-beh+2cdh+ceg)}{2c^2}+\frac{e^2hx^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] ((b^2*e^2*h + c^2*(e^2*f + 2*d*e*g + d^2*h) - c*e*(b*e*g + 2*b*d*h + a*e*h)
)*x)/c^3 + (e*(c*e*g + 2*c*d*h - b*e*h)*x^2)/(2*c^2) + (e^2*h*x^3)/(3*c) -
((2*c^4*d^2*f + b^4*e^2*h - b^2*c*e*(b*e*g + 2*b*d*h + 4*a*e*h) - c^3*(b*d*
(2*e*f + d*g) + 2*a*(e^2*f + 2*d*e*g + d^2*h)) + c^2*(2*a^2*e^2*h + 3*a*b*e
*(e*g + 2*d*h) + b^2*(e^2*f + 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) + ((c^3*d*(2*e*f + d*g) - b^3*e^2*h +
 b*c*e*(b*e*g + 2*b*d*h + 2*a*e*h) - c^2*(a*e*(e*g + 2*d*h) + b*(e^2*f + 2*
d*e*g + d^2*h)))*Log[a + b*x + c*x^2])/(2*c^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \int \left(\frac{b^2e^2h+c^2(e^2f+2deg+d^2h)-ce(beg+2bdh+aeh)}{c^3} + \frac{e(ceg+2cd\right.$$

$$= \frac{(b^2e^2h+c^2(e^2f+2deg+d^2h)-ce(beg+2bdh+aeh))\,x}{c^3} + \frac{e(ceg+2cdh}{2c^2}$$

$$= \frac{(b^2e^2h+c^2(e^2f+2deg+d^2h)-ce(beg+2bdh+aeh))\,x}{c^3} + \frac{e(ceg+2cdh}{2c^2}$$

$$= \frac{(b^2e^2h+c^2(e^2f+2deg+d^2h)-ce(beg+2bdh+aeh))\,x}{c^3} + \frac{e(ceg+2cdh}{2c^2}$$

$$= \frac{(b^2e^2h+c^2(e^2f+2deg+d^2h)-ce(beg+2bdh+aeh))\,x}{c^3} + \frac{e(ceg+2cdh}{2c^2}$$

Mathematica [A]
time = 0.22, size = 345, normalized size = 0.99

$$\frac{6c(b^2e^2h+c^2(e^2f+2deg+d^2h)-ce(beg+2bdh+aeh))\,x+3c^2e(ceg+2cdh-beh)x^2+2c^3e^2hx^3+\frac{6\left(2c^4d^2f+b^4e^2h-b^2ce(beg+2bdh+4aeh)-c^3\left(bd(2ef+dg)+2a\left(e^2f+2deg+d^2h\right)\right)+c^2\left(2a^2e^2h+3abe(eg+2dh)+b^2\left(e^2f+2deg+d^2h\right)\right)\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+3(c^3d(2ef+dg)-b^3e^2h+bce(beg+2bdh+2aeh)-c^2(ae(eg+2dh)+b(e^2f+2deg+d^2h)))\log(a+x(b+cx))}{6c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] (6*c*(b^2*e^2*h + c^2*(e^2*f + 2*d*e*g + d^2*h) - c*e*(b*e*g + 2*b*d*h + a*
e*h))*x + 3*c^2*e*(c*e*g + 2*c*d*h - b*e*h)*x^2 + 2*c^3*e^2*h*x^3 + (6*(2*c
^4*d^2*f + b^4*e^2*h - b^2*c*e*(b*e*g + 2*b*d*h + 4*a*e*h) - c^3*(b*d*(2*e*
f + d*g) + 2*a*(e^2*f + 2*d*e*g + d^2*h)) + c^2*(2*a^2*e^2*h + 3*a*b*e*(e*g
 + 2*d*h) + b^2*(e^2*f + 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
4*a*c]])/Sqrt[-b^2 + 4*a*c] + 3*(c^3*d*(2*e*f + d*g) - b^3*e^2*h + b*c*e*(b
*e*g + 2*b*d*h + 2*a*e*h) - c^2*(a*e*(e*g + 2*d*h) + b*(e^2*f + 2*d*e*g + d
^2*h)))*Log[a + x*(b + c*x)])/(6*c^4)
```

Maple [A]
time = 0.21, size = 453, normalized size = 1.30

method	result
default	$-\frac{-\frac{1}{3}he^2x^3c^2+\frac{1}{2}bce^2hx^2-c^2dehx^2-\frac{1}{2}c^2e^2gx^2+ace^2hx-b^2e^2hx+2bcdehx+bce^2gx-c^2d^2hx-2c^2degx-c^2e^2fx}{c^3}+\frac{\left(2abce^2h-2c\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3*(-1/3*h*e^2*x^3*c^2+1/2*b*c*e^2*h*x^2-c^2*d*e*h*x^2-1/2*c^2*e^2*g*x^
2+a*c*e^2*h*x-b^2*e^2*h*x+2*b*c*d*e*h*x+b*c*e^2*g*x-c^2*d^2*h*x-2*c^2*d*e*g
*x-c^2*e^2*f*x)+1/c^3*(1/2*(2*a*b*c*e^2*h-2*a*c^2*d*e*h-a*c^2*e^2*g-b^3*e^2
*h+2*b^2*c*d*e*h+b^2*c*e^2*g-b*c^2*d^2*h-2*b*c^2*d*e*g-b*c^2*e^2*f+c^3*d^2*
g+2*c^3*d*e*f)/c*ln(c*x^2+b*x+a)+2*(a^2*c*e^2*h-a*b^2*e^2*h+2*a*b*c*d*e*h+a
*b*c*e^2*g-a*c^2*d^2*h-2*a*c^2*d*e*g-a*c^2*e^2*f+c^3*d^2*f-1/2*(2*a*b*c*e^2
*h-2*a*c^2*d*e*h-a*c^2*e^2*g-b^3*e^2*h+2*b^2*c*d*e*h+b^2*c*e^2*g-b*c^2*d^2*
h-2*b*c^2*d*e*g-b*c^2*e^2*f+c^3*d^2*g+2*c^3*d*e*f)*b/c)/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.51, size = 1241, normalized size = 3.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b^2*c^3 - 4*a*c^4)*d^2*h*x + 3*(2*c^4*d^2*f - b*c^3*d^2*g + (b^2*c
^2 - 2*a*c^3)*d^2*h + ((b^2*c^2 - 2*a*c^3)*f - (b^3*c - 3*a*b*c^2)*g + (b^4
 - 4*a*b^2*c + 2*a^2*c^2)*h)*e^2 - 2*(b*c^3*d*f - (b^2*c^2 - 2*a*c^3)*d*g +
 (b^3*c - 3*a*b*c^2)*d*h)*e)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b
^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (2*(b^2*c^
3 - 4*a*c^4)*h*x^3 + 3*((b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^
2 + 6*((b^2*c^3 - 4*a*c^4)*f - (b^3*c^2 - 4*a*b*c^3)*g + (b^4*c - 5*a*b^2*c
^2 + 4*a^2*c^3)*h)*x)*e^2 + 6*((b^2*c^3 - 4*a*c^4)*d*h*x^2 + 2*((b^2*c^3 -
4*a*c^4)*d*g - (b^3*c^2 - 4*a*b*c^3)*d*h)*x)*e + 3*((b^2*c^3 - 4*a*c^4)*d^2
*g - (b^3*c^2 - 4*a*b*c^3)*d^2*h - ((b^3*c^2 - 4*a*b*c^3)*f - (b^4*c - 5*a*
b^2*c^2 + 4*a^2*c^3)*g + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*h)*e^2 + 2*((b^2*c
^3 - 4*a*c^4)*d*f - (b^3*c^2 - 4*a*b*c^3)*d*g + (b^4*c - 5*a*b^2*c^2 + 4*a^
2*c^3)*d*h)*e)*log(c*x^2 + b*x + a))/(b^2*c^4 - 4*a*c^5), 1/6*(6*(b^2*c^3 -
 4*a*c^4)*d^2*h*x - 6*(2*c^4*d^2*f - b*c^3*d^2*g + (b^2*c^2 - 2*a*c^3)*d^2*
h + ((b^2*c^2 - 2*a*c^3)*f - (b^3*c - 3*a*b*c^2)*g + (b^4 - 4*a*b^2*c + 2*a
^2*c^2)*h)*e^2 - 2*(b*c^3*d*f - (b^2*c^2 - 2*a*c^3)*d*g + (b^3*c - 3*a*b*c^
2)*d*h)*e)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 -
 4*a*c)) + (2*(b^2*c^3 - 4*a*c^4)*h*x^3 + 3*((b^2*c^3 - 4*a*c^4)*g - (b^3*c
^2 - 4*a*b*c^3)*h)*x^2 + 6*((b^2*c^3 - 4*a*c^4)*f - (b^3*c^2 - 4*a*b*c^3)*g
 + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*h)*x)*e^2 + 6*((b^2*c^3 - 4*a*c^4)*d*h
*x^2 + 2*((b^2*c^3 - 4*a*c^4)*d*g - (b^3*c^2 - 4*a*b*c^3)*d*h)*x)*e + 3*((b
^2*c^3 - 4*a*c^4)*d^2*g - (b^3*c^2 - 4*a*b*c^3)*d^2*h - ((b^3*c^2 - 4*a*b*c
^3)*f - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*g + (b^5 - 6*a*b^3*c + 8*a^2*b*c^
2)*h)*e^2 + 2*((b^2*c^3 - 4*a*c^4)*d*f - (b^3*c^2 - 4*a*b*c^3)*d*g + (b^4*c
 - 5*a*b^2*c^2 + 4*a^2*c^3)*d*h)*e)*log(c*x^2 + b*x + a))/(b^2*c^4 - 4*a*c^
5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2839 vs. $2(359) = 718$.
time = 27.38, size = 2839, normalized size = 8.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] x**2*(-b*e**2*h/(2*c**2) + d*e*h/c + e**2*g/(2*c)) + x*(-a*e**2*h/c**2 + b*
*2*e**2*h/c**3 - 2*b*d*e*h/c**2 - b*e**2*g/c**2 + d**2*h/c + 2*d*e*g/c + e*
```

```
*2*f/c) + (-sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6
*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*
a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d*
*2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*
f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e
*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*
d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c*
*4))*log(x + (-3*a**2*b*c*e**2*h + 4*a**2*c**2*d*e*h + 2*a**2*c**2*e**2*g +
 a*b**3*e**2*h - 2*a*b**2*c*d*e*h - a*b**2*c*e**2*g + a*b*c**2*d**2*h + 2*a
*b*c**2*d*e*g + a*b*c**2*e**2*f + 4*a*c**4*(-sqrt(-4*a*c + b**2)*(2*a**2*c*
*2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*
c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e
*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*
f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2))
 + (2*a*b*c*e**2*h - 2*a*c**2*d*e*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*
c*d*e*h + b**2*c*e**2*g - b*c**2*d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f +
c**3*d**2*g + 2*c**3*d*e*f)/(2*c**4)) - 2*a*c**3*d**2*g - 4*a*c**3*d*e*f -
b**2*c**3*(-sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6
*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*
a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d*
*2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*
f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e
*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*
d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c*
*4)) + b*c**3*d**2*f)/(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*
d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**
2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b
**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4
*d**2*f)) + (sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h +
6*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2
*a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d
**2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e
*f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*
e*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2
*d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c
**4))*log(x + (-3*a**2*b*c*e**2*h + 4*a**2*c**2*d*e*h + 2*a**2*c**2*e**2*g
+ a*b**3*e**2*h - 2*a*b**2*c*d*e*h - a*b**2*c*e**2*g + a*b*c**2*d**2*h + 2*
a*b*c**2*d*e*g + a*b*c**2*e**2*f + 4*a*c**4*(sqrt(-4*a*c + b**2)*(2*a**2*c*
*2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*
c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e
*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*
f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2))
 + (2*a*b*c*e**2*h - 2*a*c**2*d*e*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*
c*d*e*h + b**2*c*e**2*g - b*c**2*d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f +
c**3*d**2*g + 2*c**3*d*e*f)/(2*c**4)) - 2*a*c**3*d**2*g - 4*a*c**3*d*e*f -
```

```
b**2*c**3*(sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*
a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a
*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**
2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f
 + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e*
h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*d
**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c**
4)) + b*c**3*d**2*f)/(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*d
*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**2
*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b*
*2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4*
d**2*f)) + e**2*h*x**3/(3*c)
```

Giac [A]
time = 3.51, size = 426, normalized size = 1.22

$$\frac{2c^2hx^3e^2+6c^2dhx^2e+6c^2d^2hx+3c^2gx^2e^2-3bchx^2e^2+12c^2dgxe-12bcdhxe+6c^2fxe^2-6bcgxe^2+6b^2hxe^2-6achxe^2}{6c^3}+\frac{(c^3d^2g-bc^2d^2h+2c^3dfe-2bc^2dge+2b^2cdhe-2ac^2dhe-bc^2fe^2+b^2cge^2-ac^2ge^2-b^3he^2+2abche^2)\log(cx^2+bx+a)}{2c^4}+\frac{(2c^4d^2f-bc^3d^2g+b^2c^2d^2h-2ac^3d^2h-2bc^3dfe+2b^2c^2dge-4ac^3dge-2b^3cdhe+6abc^2dhe+b^2c^2fe^2-2ac^3fe^2-b^3cge^2+3abc^2ge^2+b^4he^2-4ab^2che^2+2a^2c^2he^2)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(2*c^2*h*x^3*e^2 + 6*c^2*d*h*x^2*e + 6*c^2*d^2*h*x + 3*c^2*g*x^2*e^2 -
3*b*c*h*x^2*e^2 + 12*c^2*d*g*x*e - 12*b*c*d*h*x*e + 6*c^2*f*x*e^2 - 6*b*c*g
*x*e^2 + 6*b^2*h*x*e^2 - 6*a*c*h*x*e^2)/c^3 + 1/2*(c^3*d^2*g - b*c^2*d^2*h
+ 2*c^3*d*f*e - 2*b*c^2*d*g*e + 2*b^2*c*d*h*e - 2*a*c^2*d*h*e - b*c^2*f*e^2
 + b^2*c*g*e^2 - a*c^2*g*e^2 - b^3*h*e^2 + 2*a*b*c*h*e^2)*log(c*x^2 + b*x +
 a)/c^4 + (2*c^4*d^2*f - b*c^3*d^2*g + b^2*c^2*d^2*h - 2*a*c^3*d^2*h - 2*b*
c^3*d*f*e + 2*b^2*c^2*d*g*e - 4*a*c^3*d*g*e - 2*b^3*c*d*h*e + 6*a*b*c^2*d*h
*e + b^2*c^2*f*e^2 - 2*a*c^3*f*e^2 - b^3*c*g*e^2 + 3*a*b*c^2*g*e^2 + b^4*h*
e^2 - 4*a*b^2*c*h*e^2 + 2*a^2*c^2*h*e^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a
*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

Mupad [B]
time = 4.68, size = 557, normalized size = 1.60

$$x^2\left(\frac{ge^2+2dhe}{2c}-\frac{be^2h}{2c^2}\right)-x\left(\frac{b\left(\frac{ge^2+2dhe}{c}-\frac{be^2h}{c^2}\right)}{c}-\frac{hd^2+2gde+fe^2}{c}+\frac{ae^2h}{c^2}\right)-\frac{\ln(cx^2+bx+a)\,(-8ha^2bc^2e^2+8ha^2c^3de+4ga^2c^3e^2+6hab^3ce^2-10hab^2c^2de-5gab^2c^2e^2+4habc^3d^2+8gabc^3de+4fabc^3e^2-4gac^4d^2-8fac^4de-hb^5e^2+2hb^4cde+gb^4ce^2-hb^3c^2d^2-2gb^3c^2de-fb^3c^2e^2+gb^2c^3d^2+2fb^2c^3de)}{2(4ac^5-b^2c^4)}+\frac{e^2hx^3}{3c}+\frac{\mathrm{atan}\left(\frac{b}{\sqrt{4ac-b^2}}+\frac{2cx}{\sqrt{4ac-b^2}}\right)(2ha^2c^2e^2-4hab^2ce^2+6habc^2de+3gabc^2e^2-2hac^3d^2-4gac^3de-2fac^3e^2+hb^4e^2-2hb^3cde-gb^3ce^2+hb^2c^2d^2+2gb^2c^2de+fb^2c^2e^2-gbc^3d^2-2fbc^3de+2fc^4d^2)}{c^4\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

```
[Out] x^2*((e^2*g + 2*d*e*h)/(2*c) - (b*e^2*h)/(2*c^2)) - x*((b*((e^2*g + 2*d*e*h
)/c - (b*e^2*h)/c^2))/c - (e^2*f + d^2*h + 2*d*e*g)/c + (a*e^2*h)/c^2) - (l
og(a + b*x + c*x^2)*(4*a^2*c^3*e^2*g - b^5*e^2*h + b^2*c^3*d^2*g - b^3*c^2*
e^2*f - b^3*c^2*d^2*h - 4*a*c^4*d^2*g + b^4*c*e^2*g + 4*a*b*c^3*e^2*f + 4*a
*b*c^3*d^2*h + 6*a*b^3*c*e^2*h + 2*b^2*c^3*d*e*f + 8*a^2*c^3*d*e*h - 2*b^3*
c^2*d*e*g - 5*a*b^2*c^2*e^2*g - 8*a^2*b*c^2*e^2*h - 8*a*c^4*d*e*f + 2*b^4*c
```

```
*d*e*h + 8*a*b*c^3*d*e*g - 10*a*b^2*c^2*d*e*h))/(2*(4*a*c^5 - b^2*c^4)) + (
e^2*h*x^3)/(3*c) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2
))*(2*c^4*d^2*f + b^4*e^2*h + b^2*c^2*e^2*f + 2*a^2*c^2*e^2*h + b^2*c^2*d^2
*h - 2*a*c^3*e^2*f - 2*a*c^3*d^2*h - b*c^3*d^2*g - b^3*c*e^2*g + 3*a*b*c^2*
e^2*g - 4*a*b^2*c*e^2*h + 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - 2*b*c^3*d*e*f -
 2*b^3*c*d*e*h + 6*a*b*c^2*d*e*h))/(c^4*(4*a*c - b^2)^(1/2))
```

3.150 $\int \frac{(d+ex)(f+gx+hx^2)}{a+bx+cx^2} \, dx$

Optimal. Leaf size=177

$$\frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}-\frac{(2c^3df-b^3eh-c^2(bef+bdg+2aeg+2adh)+bc(beg+bdh+3aeh))\tanh^{-1}\left(\right.}{c^3\sqrt{b^2-4ac}}$$

```
[Out] (-b*e*h+c*d*h+c*e*g)*x/c^2+1/2*e*h*x^2/c+1/2*(c^2*(d*g+e*f)+b^2*e*h-c*(a*e*
h+b*d*h+b*e*g))*ln(c*x^2+b*x+a)/c^3-(2*c^3*d*f-b^3*e*h-c^2*(2*a*d*h+2*a*e*g
+b*d*g+b*e*f)+b*c*(3*a*e*h+b*d*h+b*e*g))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/
2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1642, 648, 632, 212, 642}

$$\frac{\log(a+bx+cx^2)(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef))}{2c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(-c^2(2adh+2aeg+bdg+bef)+bc(3aeh+bdh+beg)+b^3(-e)h+2c^3df)}{c^3\sqrt{b^2-4ac}}+\frac{x(-beh+cdh+ceg)}{c^2}+\frac{ehx^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] ((c*e*g + c*d*h - b*e*h)*x)/c^2 + (e*h*x^2)/(2*c) - ((2*c^3*d*f - b^3*e*h -
 c^2*(b*e*f + b*d*g + 2*a*e*g + 2*a*d*h) + b*c*(b*e*g + b*d*h + 3*a*e*h))*A
rcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*(e*f
 + d*g) + b^2*e*h - c*(b*e*g + b*d*h + a*e*h))*Log[a + b*x + c*x^2])/(2*c^3
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = \int \left(\frac{ceg+cdh-beh}{c^2}+\frac{ehx}{c}+\frac{c^2df+abeh-ac(eg+dh)+(c^2(ef+dg)}{c^2\left(a+bx+c\right.}\right.$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}+\frac{\int \frac{c^2df+abeh-ac(eg+dh)+(c^2(ef+dg)+b^2eh-c(beg+bdh}{a+bx+cx^2}}{c^2}$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}+\frac{(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh))}{2c^3}$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}+\frac{(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh))}{2c^3}$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}-\frac{(2c^3df-b^3eh-c^2(bef+bdg+2aeg+2a}{c^3}$$

Mathematica [A]
time = 0.12, size = 173, normalized size = 0.98

$$\frac{2c(ceg+cdh-beh)x+c^2ehx^2-\frac{2\left(-2c^3df+b^3eh+c^2(bef+bdg+2aeg+2adh)-bc(beg+bdh+3aeh)\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\left(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh)\right)\log(a+x(b+cx))}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*c*(c*e*g + c*d*h - b*e*h)*x + c^2*e*h*x^2 - (2*(-2*c^3*d*f + b^3*e*h + c
^2*(b*e*f + b*d*g + 2*a*e*g + 2*a*d*h) - b*c*(b*e*g + b*d*h + 3*a*e*h))*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c^2*(e*f + d*g)
+ b^2*e*h - c*(b*e*g + b*d*h + a*e*h))*Log[a + x*(b + c*x)])/(2*c^3)
```

Maple [A]
time = 0.17, size = 192, normalized size = 1.08

method	result
default	$-\frac{-\frac{1}{2}he\,x^2c+behx-cdhx-cegx}{c^2}+\frac{\frac{\left(-aceh+b^2eh-bcdh-bceg+c^2dg+c^2ef\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(abhe-acdh-aceg+c^2df-\frac{\left(-aceh+b^2eh-bcd\right.}{}\right.}{\sqrt{4c}}}{c^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2*(-1/2*h*e*x^2*c+b*e*h*x-c*d*h*x-c*e*g*x)+1/c^2*(1/2*(-a*c*e*h+b^2*e*
h-b*c*d*h-b*c*e*g+c^2*d*g+c^2*e*f)/c*ln(c*x^2+b*x+a)+2*(a*b*h*e-a*c*d*h-a*c
*e*g+c^2*d*f-1/2*(-a*c*e*h+b^2*e*h-b*c*d*h-b*c*e*g+c^2*d*g+c^2*e*f)*b/c)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.43, size = 648, normalized size = 3.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c^2 - 4*a*c^3)*d*h*x + (2*c^3*d*f - b*c^2*d*g + (b^2*c - 2*a*c
^2)*d*h - (b*c^2*f - (b^2*c - 2*a*c^2)*g + (b^3 - 3*a*b*c)*h)*e)*sqrt(b^2 -
 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x +
 b))/(c*x^2 + b*x + a)) + ((b^2*c^2 - 4*a*c^3)*h*x^2 + 2*((b^2*c^2 - 4*a*c^
3)*g - (b^3*c - 4*a*b*c^2)*h)*x)*e + ((b^2*c^2 - 4*a*c^3)*d*g - (b^3*c - 4*
a*b*c^2)*d*h + ((b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b*c^2)*g + (b^4 - 5*a*
```

```
b^2*c + 4*a^2*c^2)*h)*e)*log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*(2*
(b^2*c^2 - 4*a*c^3)*d*h*x - 2*(2*c^3*d*f - b*c^2*d*g + (b^2*c - 2*a*c^2)*d*
h - (b*c^2*f - (b^2*c - 2*a*c^2)*g + (b^3 - 3*a*b*c)*h)*e)*sqrt(-b^2 + 4*a*
c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + ((b^2*c^2 - 4*a*
c^3)*h*x^2 + 2*((b^2*c^2 - 4*a*c^3)*g - (b^3*c - 4*a*b*c^2)*h)*x)*e + ((b^2
*c^2 - 4*a*c^3)*d*g - (b^3*c - 4*a*b*c^2)*d*h + ((b^2*c^2 - 4*a*c^3)*f - (b
^3*c - 4*a*b*c^2)*g + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*h)*e)*log(c*x^2 + b*x +
 a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. $2(182) = 364$.
time = 8.17, size = 1265, normalized size = 7.15

$$x\left(-\frac{beh}{c^2}+\frac{dh}{c}+\frac{eg}{c}\right)+\left(-\frac{\sqrt{-4ac+b^2}\cdot(3abceh-2ac^2dh-2ac^2eg-b^3eh+b^2cdh+b^2ceg-bc^2dg-bc^2ef+2c^3df)}{2c^3\cdot(4ac-b^2)}-\frac{aceh-b^2eh+bcdh+bceg-c^2dg-c^2ef}{2c^3}\right)\log\left(x+\frac{2a^2ceh-ab^2eh+abcdh+abceg+4ac^3\left(-\frac{\sqrt{-4ac+b^2}\cdot(3abceh-2ac^2dh-2ac^2eg-b^3eh+b^2cdh+b^2ceg-bc^2dg-bc^2ef+2c^3df)}{2c^3\cdot(4ac-b^2)}-\frac{aceh-b^2eh+bcdh+bceg-c^2dg-c^2ef}{2c^3}\right)-2ac^2dg-2ac^2ef-b^2c^2\left(-\frac{\sqrt{-4ac+b^2}\cdot(3abceh-2ac^2dh-2ac^2eg-b^3eh+b^2cdh+b^2ceg-bc^2dg-bc^2ef+2c^3df)}{2c^3\cdot(4ac-b^2)}-\frac{aceh-b^2eh+bcdh+bceg-c^2dg-c^2ef}{2c^3}\right)+bc^2df}{3abceh-2ac^2dh-2ac^2eg-b^3eh+b^2cdh+b^2ceg-bc^2dg-bc^2ef+2c^3df}\right)+\left(\frac{\sqrt{-4ac+b^2}\cdot(3abceh-2ac^2dh-2ac^2eg-b^3eh+b^2cdh+b^2ceg-bc^2dg-bc^2ef+2c^3df)}{2c^3\cdot(4ac-b^2)}-\frac{aceh-b^2eh+bcdh+bceg-c^2dg-c^2ef}{2c^3}\right)\log\left(x+\frac{2a^2ceh-ab^2eh+abcdh+abceg+4ac^3\left(\frac{\sqrt{-4ac+b^2}\cdot(3abceh-2ac^2dh-2ac^2eg-b^3eh+b^2cdh+b^2ceg-bc^2dg-bc^2ef+2c^3df)}{2c^3\cdot(4ac-b^2)}-\frac{aceh-b^2eh+bcdh+bceg-c^2dg-c^2ef}{2c^3}\right)-2ac^2dg-2ac^2ef-b^2c^2\left(\frac{\sqrt{-4ac+b^2}\cdot(3abceh-2ac^2dh-2ac^2eg-b^3eh+b^2cdh+b^2ceg-bc^2dg-bc^2ef+2c^3df)}{2c^3\cdot(4ac-b^2)}-\frac{aceh-b^2eh+bcdh+bceg-c^2dg-c^2ef}{2c^3}\right)+bc^2df}{3abceh-2ac^2dh-2ac^2eg-b^3eh+b^2cdh+b^2ceg-bc^2dg-bc^2ef+2c^3df}\right)+\frac{ehx^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] x*(-b*e*h/c**2 + d*h/c + e*g/c) + (-sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*
c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g -
 b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b
*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3))*log(x + (2*a**2*c*e*h - a
*b**2*e*h + a*b*c*d*h + a*b*c*e*g + 4*a*c**3*(-sqrt(-4*a*c + b**2)*(3*a*b*c
*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b
*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b
**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) - 2*a*c**2*d*g
 - 2*a*c**2*e*f - b**2*c**2*(-sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*c**2*d
*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**
2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h
 + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) + b*c**2*d*f)/(3*a*b*c*e*h - 2*
a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g
 - b*c**2*e*f + 2*c**3*d*f)) + (sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*c**2
*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c
**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d
*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3))*log(x + (2*a**2*c*e*h - a*b**
2*e*h + a*b*c*d*h + a*b*c*e*g + 4*a*c**3*(sqrt(-4*a*c + b**2)*(3*a*b*c*e*h
- 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2
*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e
*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) - 2*a*c**2*d*g - 2*
a*c**2*e*f - b**2*c**2*(sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*c**2*d*h - 2
*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f
+ 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h + b*c
*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) + b*c**2*d*f)/(3*a*b*c*e*h - 2*a*c**2
*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c
**2*e*f + 2*c**3*d*f)) + e*h*x**2/(2*c)
```

Giac [A]
time = 3.98, size = 201, normalized size = 1.14

$$\frac{chx^2e + 2\,cdhx + 2\,cgxe - 2\,bhxe}{2\,c^2} + \frac{(c^2dg - bcdh + c^2fe - bcge + b^2he - ache)\log(cx^2 + bx + a)}{2\,c^3} + \frac{(2\,c^3df - bc^2dg + b^2cdh - 2\,ac^2dh - bc^2fe + b^2cge - 2\,ac^2ge - b^3he + 3\,abche)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(c*h*x^2*e + 2*c*d*h*x + 2*c*g*x*e - 2*b*h*x*e)/c^2 + 1/2*(c^2*d*g - b*
c*d*h + c^2*f*e - b*c*g*e + b^2*h*e - a*c*h*e)*log(c*x^2 + b*x + a)/c^3 + (
2*c^3*d*f - b*c^2*d*g + b^2*c*d*h - 2*a*c^2*d*h - b*c^2*f*e + b^2*c*g*e - 2
*a*c^2*g*e - b^3*h*e + 3*a*b*c*h*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B]
time = 0.53, size = 273, normalized size = 1.54

$$x\left(\frac{d\,h+e\,g}{c}-\frac{b\,e\,h}{c^2}\right)-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(b^4\,e\,h-4\,a\,c^3\,d\,g-4\,a\,c^3\,e\,f-b^3\,c\,d\,h-b^3\,c\,e\,g+b^2\,c^2\,d\,g+b^2\,c^2\,e\,f+4\,a^2\,c^2\,e\,h+4\,a\,b\,c^2\,d\,h+4\,a\,b\,c^2\,e\,g-5\,a\,b^2\,c\,e\,h\right)}{2\,\left(4\,a\,c^4-b^2\,c^3\right)}-\frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(b^3\,e\,h-2\,c^3\,d\,f+2\,a\,c^2\,d\,h+2\,a\,c^2\,e\,g+b\,c^2\,d\,g+b\,c^2\,e\,f-b^2\,c\,d\,h-b^2\,c\,e\,g-3\,a\,b\,c\,e\,h\right)}{c^3\,\sqrt{4\,a\,c-b^2}}+\frac{e\,h\,x^2}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

```
[Out] x*((d*h + e*g)/c - (b*e*h)/c^2) - (log(a + b*x + c*x^2)*(b^4*e*h - 4*a*c^3*
d*g - 4*a*c^3*e*f - b^3*c*d*h - b^3*c*e*g + b^2*c^2*d*g + b^2*c^2*e*f + 4*a
^2*c^2*e*h + 4*a*b*c^2*d*h + 4*a*b*c^2*e*g - 5*a*b^2*c*e*h))/(2*(4*a*c^4 -
b^2*c^3)) - (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*(b^3
*e*h - 2*c^3*d*f + 2*a*c^2*d*h + 2*a*c^2*e*g + b*c^2*d*g + b*c^2*e*f - b^2*
c*d*h - b^2*c*e*g - 3*a*b*c*e*h))/(c^3*(4*a*c - b^2)^(1/2)) + (e*h*x^2)/(2*
c)
```

3.151 $\int \frac{f+gx+hx^2}{a+bx+cx^2} \, dx$

Optimal. Leaf size=92

$$\frac{hx}{c}-\frac{(2c^2f-bcg+b^2h-2ach)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}$$

```
[Out] h*x/c+1/2*(-b*h+c*g)*ln(c*x^2+b*x+a)/c^2-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*arc
tanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1671, 648, 632, 212, 642}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(-2ach+b^2h-bcg+2c^2f)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{hx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (h*x)/c - ((2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2
 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + ((c*g - b*h)*Log[a + b*x + c*x^2])/(2
*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx &= \int \left(\frac{h}{c}+\frac{cf-ah+(cg-bh)x}{c\left(a+bx+cx^2\right)}\right)\,dx \\
&= \frac{hx}{c}+\frac{\int \frac{cf-ah+(cg-bh)x}{a+bx+cx^2}\,dx}{c} \\
&= \frac{hx}{c}+\frac{(cg-bh)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{\left(2c^2f-bcg+b^2h-2ach\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2} \\
&= \frac{hx}{c}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}-\frac{\left(2c^2f-bcg+b^2h-2ach\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,\right.}{c^2} \\
&= \frac{hx}{c}-\frac{\left(2c^2f-bcg+b^2h-2ach\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right.}{2c^2}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 95, normalized size = 1.03

$$\frac{hx}{c}+\frac{\left(2c^2f-bcg+b^2h-2ach\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (h*x)/c + ((2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]) + ((c*g - b*h)*Log[a + b*x + c*x^2])/(
2*c^2)
```

Maple [A]
time = 0.16, size = 93, normalized size = 1.01

method	result	size
default	$\frac{hx}{c}+\frac{\frac{(-bh+cg)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-ah+cf-\frac{(-bh+cg)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{c}$	93
risch	Expression too large to display	1649

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] h*x/c+1/c*(1/2*(-b*h+c*g)/c*ln(c*x^2+b*x+a)+2*(-a*h+c*f-1/2*(-b*h+c*g)*b/c)
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 302, normalized size = 3.28

$$\left[\frac{2\,(b^{2}c-4\,ac^{2})hx-(2\,c^{2}f-bcg+(b^{2}-2\,ac)h)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac+\sqrt{b^{2}-4\,ac}\,(2\,cx+b)}{cx^{2}+bx+a}\right)+((b^{2}c-4\,ac^{2})g-(b^{3}-4\,abc)h)\log\left(cx^{2}+bx+a\right)}{2\,(b^{2}c^{2}-4\,ac^{3})},\ \frac{2\,(b^{2}c-4\,ac^{2})hx-2\,(2\,c^{2}f-bcg+(b^{2}-2\,ac)h)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}\,(2\,cx+b)}{b^{2}-4\,ac}\right)+((b^{2}c-4\,ac^{2})g-(b^{3}-4\,abc)h)\log\left(cx^{2}+bx+a\right)}{2\,(b^{2}c^{2}-4\,ac^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c - 4*a*c^2)*h*x - (2*c^2*f - b*c*g + (b^2 - 2*a*c)*h)*sqrt(b^
2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*
x + b))/(c*x^2 + b*x + a)) + ((b^2*c - 4*a*c^2)*g - (b^3 - 4*a*b*c)*h)*log(
c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), 1/2*(2*(b^2*c - 4*a*c^2)*h*x - 2*(2*
c^2*f - b*c*g + (b^2 - 2*a*c)*h)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a
*c)*(2*c*x + b)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*g - (b^3 - 4*a*b*c)*h)*
log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(88) = 176$.
time = 1.29, size = 488, normalized size = 5.30

$$\left(-\frac{\sqrt{-4ac+b^{2}}\cdot(2ach-b^{2}h+bcg-2c^{2}f)}{2c^{2}\cdot(4ac-b^{2})}-\frac{bh-cg}{2c^{2}}\right)\log\left(x+\frac{-abh-4ac^{2}\left(-\frac{\sqrt{-4ac+b^{2}}\cdot(2ach-b^{2}h+bcg-2c^{2}f)}{2c^{2}\cdot(4ac-b^{2})}-\frac{bh-cg}{2c^{2}}\right)+2acg+b^{2}c\left(-\frac{\sqrt{-4ac+b^{2}}\cdot(2ach-b^{2}h+bcg-2c^{2}f)}{2c^{2}\cdot(4ac-b^{2})}-\frac{bh-cg}{2c^{2}}\right)-bcf}{2ach-b^{2}h+bcg-2c^{2}f}\right)+\left(\frac{\sqrt{-4ac+b^{2}}\cdot(2ach-b^{2}h+bcg-2c^{2}f)}{2c^{2}\cdot(4ac-b^{2})}-\frac{bh-cg}{2c^{2}}\right)\log\left(x+\frac{-abh-4ac^{2}\left(\frac{\sqrt{-4ac+b^{2}}\cdot(2ach-b^{2}h+bcg-2c^{2}f)}{2c^{2}\cdot(4ac-b^{2})}-\frac{bh-cg}{2c^{2}}\right)+2acg+b^{2}c\left(\frac{\sqrt{-4ac+b^{2}}\cdot(2ach-b^{2}h+bcg-2c^{2}f)}{2c^{2}\cdot(4ac-b^{2})}-\frac{bh-cg}{2c^{2}}\right)-bcf}{2ach-b^{2}h+bcg-2c^{2}f}\right)+\frac{hx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c
- b**2)) - (b*h - c*g)/(2*c**2))*log(x + (-a*b*h - 4*a*c**2*(-sqrt(-4*a*c +
 b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h
 - c*g)/(2*c**2)) + 2*a*c*g + b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*
h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)) - b*c
*f)/(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)) + (sqrt(-4*a*c + b**2)*(2*a*c*h
- b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)
)*log(x + (-a*b*h - 4*a*c**2*(sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g
 - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)) + 2*a*c*g + b*
*2*c*(sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*
a*c - b**2)) - (b*h - c*g)/(2*c**2)) - b*c*f)/(2*a*c*h - b**2*h + b*c*g - 2
*c**2*f)) + h*x/c
```

Giac [A]
time = 3.62, size = 89, normalized size = 0.97

$$\frac{hx}{c}+\frac{(cg-bh)\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{\left(2\,c^2f-bcg+b^2h-2\,ach\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] h*x/c + 1/2*(c*g - b*h)*log(c*x^2 + b*x + a)/c^2 + (2*c^2*f - b*c*g + b^2*h
 - 2*a*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B]
time = 0.25, size = 132, normalized size = 1.43

$$\frac{h\,x}{c}+\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(h\,b^3-g\,b^2\,c-4\,a\,h\,b\,c+4\,a\,g\,c^2\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)}+\frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(h\,b^2-g\,b\,c+2\,f\,c^2-2\,a\,h\,c\right)}{c^2\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/(a + b*x + c*x^2),x)
```

```
[Out] (h*x)/c + (log(a + b*x + c*x^2)*(b^3*h + 4*a*c^2*g - b^2*c*g - 4*a*b*c*h))/
(2*(4*a*c^3 - b^2*c^2)) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^
2)^(1/2))*(2*c^2*f + b^2*h - 2*a*c*h - b*c*g))/(c^2*(4*a*c - b^2)^(1/2))
```

3.152 $\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)}\,dx$

Optimal. Leaf size=196

$$-\frac{(2c^2df+b(bd-ae)h-c(bef+bdg-2aeg+2adh))\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)}+\frac{(e^2f-deg+d^2h)\log(d+}{e\,(cd^2-bde+ae^2)}$$

```
[Out] (d^2*h-d*e*g+e^2*f)*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)-1/2*(-a*e*h+b*d*h-c*d*g
+c*e*f)*ln(c*x^2+b*x+a)/c/(a*e^2-b*d*e+c*d^2)-(2*c^2*d*f+b*(-a*e+b*d)*h-c*(
2*a*d*h-2*a*e*g+b*d*g+b*e*f))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c/(a*e^
2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1642, 648, 632, 212, 642}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df)}{c\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}-\frac{\log\left(a+bx+cx^2\right)(-aeh+bdh-cdg+cef)}{2c\left(ae^2-bde+cd^2\right)}+\frac{\log(d+ex)\left(d^2h-deg+e^2f\right)}{e\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x]
```

```
[Out] -(((2*c^2*d*f + b*(b*d - a*e)*h - c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d*h))*Ar
cTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e +
 a*e^2))) + ((e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(e*(c*d^2 - b*d*e + a*e^
2)) - ((c*e*f - c*d*g + b*d*h - a*e*h)*Log[a + b*x + c*x^2])/(2*c*(c*d^2 -
b*d*e + a*e^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)\,(a+bx+cx^2)}\,dx = \int \left(\frac{e^2f-deg+d^2h}{(cd^2-bde+ae^2)\,(d+ex)} + \frac{cdf-bef+aeg-adh-(cef-cdg+bd}{(cd^2-bde+ae^2)\,(a+bx+cx^2}\right.$$

$$= \frac{(e^2f-deg+d^2h)\log(d+ex)}{e\,(cd^2-bde+ae^2)} + \frac{\int \frac{cdf-bef+aeg-adh-(cef-cdg+bdh-aeh)x}{a+bx+cx^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{(e^2f-deg+d^2h)\log(d+ex)}{e\,(cd^2-bde+ae^2)} - \frac{(cef-cdg+bdh-aeh)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c\,(cd^2-bde+ae^2)} + \cdots$$

$$= \frac{(e^2f-deg+d^2h)\log(d+ex)}{e\,(cd^2-bde+ae^2)} - \frac{(cef-cdg+bdh-aeh)\log\left(a+bx+cx^2\right.}{2c\,(cd^2-bde+ae^2)}$$

$$= -\frac{(2c^2df+b(bd-ae)h-c(bef+bdg-2aeg+2adh))\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right.}{c\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)}$$

Mathematica [A]
time = 0.13, size = 193, normalized size = 0.98

$$\frac{-2e(-2c^2df+b(-bd+ae)h+c(bef+bdg-2aeg+2adh))\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)+2c\sqrt{-b^2+4ac}\,(e^2f-deg+d^2h)\log(d+ex)-\sqrt{-b^2+4ac}\,e(cef-cdg+bdh-aeh)\log(a+x(b+cx))}{2c\sqrt{-b^2+4ac}\,e\,(cd^2+e(-bd+ae))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*e*(-2*c^2*d*f + b*(-(b*d) + a*e)*h + c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d
*h))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + 2*c*Sqrt[-b^2 + 4*a*c]*(e^2*f
 - d*e*g + d^2*h)*Log[d + e*x] - Sqrt[-b^2 + 4*a*c]*e*(c*e*f - c*d*g + b*d*
```

```
h - a*e*h)*Log[a + x*(b + c*x)])/(2*c*Sqrt[-b^2 + 4*a*c]*e*(c*d^2 + e*(-(b*
d) + a*e)))
```

Maple [A]
time = 0.16, size = 179, normalized size = 0.91

method	result
default	$\frac{(d^2h-gde+e^2f)\ln(ex+d)}{e(a\,e^2-deb+c\,d^2)}+\frac{\frac{(aeh-bdh+dgc-cef)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-adh+aeg-bef+cdf-\frac{(aeh-bdh+dgc-cef)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-}}\right.}{\sqrt{4ac-b^2}}}{a\,e^2-deb+c\,d^2}$
risch	$\frac{\ln(ex+d)d^2h}{e(a\,e^2-deb+c\,d^2)}-\frac{\ln(ex+d)gd}{a\,e^2-deb+c\,d^2}+\frac{\ln(ex+d)ef}{a\,e^2-deb+c\,d^2}+\Bigg(\quad\_R=\text{RootOf}\left((4a^2c^2e^2-a\,b^2c\,e^2-4ab\,c^2de+4a\,c^3d^2+b^3cde-b^2c^2d^2)\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] (d^2*h-d*e*g+e^2*f)*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)+1/(a*e^2-b*d*e+c*d^2)*(
1/2*(a*e*h-b*d*h+c*d*g-c*e*f)/c*ln(c*x^2+b*x+a)+2*(-a*d*h+a*e*g-b*e*f+c*d*f
-1/2*(a*e*h-b*d*h+c*d*g-c*e*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 58.36, size = 609, normalized size = 3.11

$$\left[\frac{\sqrt{b^2-4\,ac}\,((bcf-2\,acg+abh)e^2-(2\,c^2df-bcdg+(b^2-2\,ac)dh)e)\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-(((b^2c-4\,ac^2)f-(ab^2-4\,a^2c)h)e^2-((b^2c-4\,ac^2)dg-(b^3-4\,abc)dh)e)\log(cx^2+bx+a)+2\,((b^2c-4\,ac^2)d^2h-(b^2c-4\,ac^2)dge+(b^2c-4\,ac^2)fe^2)\log(xe+d)}{2\,((b^2c^2-4\,ac^3)d^2e-(b^3c-4\,abc^2)de^2+(ab^2c-4\,a^2c^2)e^3)},\ \frac{2\,\sqrt{-b^2+4\,ac}\,((bcf-2\,acg+abh)e^2-(2\,c^2df-bcdg+(b^2-2\,ac)dh)e)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)-(((b^2c-4\,ac^2)f-(ab^2-4\,a^2c)h)e^2-((b^2c-4\,ac^2)dg-(b^3-4\,abc)dh)e)\log(cx^2+bx+a)+2\,((b^2c-4\,ac^2)d^2h-(b^2c-4\,ac^2)dge+(b^2c-4\,ac^2)fe^2)\log(xe+d)}{2\,((b^2c^2-4\,ac^3)d^2e-(b^3c-4\,abc^2)de^2+(ab^2c-4\,a^2c^2)e^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*((b*c*f - 2*a*c*g + a*b*h)*e^2 - (2*c^2*d*f - b*c*d
*g + (b^2 - 2*a*c)*d*h)*e)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^
```

```
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (((b^2*c - 4*a*c^2)*f - (a*b^2
 - 4*a^2*c)*h)*e^2 - ((b^2*c - 4*a*c^2)*d*g - (b^3 - 4*a*b*c)*d*h)*e)*log(c
*x^2 + b*x + a) + 2*((b^2*c - 4*a*c^2)*d^2*h - (b^2*c - 4*a*c^2)*d*g*e + (b
^2*c - 4*a*c^2)*f*e^2)*log(x*e + d))/((b^2*c^2 - 4*a*c^3)*d^2*e - (b^3*c -
4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), 1/2*(2*sqrt(-b^2 + 4*a*c)*((
b*c*f - 2*a*c*g + a*b*h)*e^2 - (2*c^2*d*f - b*c*d*g + (b^2 - 2*a*c)*d*h)*e)
*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (((b^2*c - 4*a*c^2
)*f - (a*b^2 - 4*a^2*c)*h)*e^2 - ((b^2*c - 4*a*c^2)*d*g - (b^3 - 4*a*b*c)*d
*h)*e)*log(c*x^2 + b*x + a) + 2*((b^2*c - 4*a*c^2)*d^2*h - (b^2*c - 4*a*c^2
)*d*g*e + (b^2*c - 4*a*c^2)*f*e^2)*log(x*e + d))/((b^2*c^2 - 4*a*c^3)*d^2*e
 - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.89, size = 204, normalized size = 1.04

$$\frac{(cdg-bdh-cfe+ahe)\log\left(cx^2+bx+a\right)}{2\left(c^2d^2-bcde+ace^2\right)}+\frac{\left(d^2h-dge+fe^2\right)\log\left(|xe+d|\right)}{cd^2e-bde^2+ae^3}+\frac{\left(2\,c^2df-bcdg+b^2dh-2\,acdh-bcfe+2\,acge-abhe\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(c^2d^2-bcde+ace^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(c*d*g - b*d*h - c*f*e + a*h*e)*log(c*x^2 + b*x + a)/(c^2*d^2 - b*c*d*e
 + a*c*e^2) + (d^2*h - d*g*e + f*e^2)*log(abs(x*e + d))/(c*d^2*e - b*d*e^2
+ a*e^3) + (2*c^2*d*f - b*c*d*g + b^2*d*h - 2*a*c*d*h - b*c*f*e + 2*a*c*g*e
 - a*b*h*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^2 - b*c*d*e + a*
c*e^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [B]
time = 10.45, size = 2467, normalized size = 12.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x)
```

```
[Out] (log(a^2*b*e^4*g - 2*a*b^2*e^4*f - 2*a^3*e^4*h + 6*a^2*c*e^4*f - 4*a*c^2*d^
4*h + b^2*c*d^4*h + b^3*d^3*e*h - 2*b^3*e^4*f*x + a^2*e^4*g*(b^2 - 4*a*c)^(
```

1/2) + a*b^2*d*e^3*g + 6*a*c^2*d^3*e*g + b*c^2*d^3*e*f + 3*a^2*b*d*e^3*h - 10*a^2*c*d*e^3*g - 2*b^2*c*d^3*e*g + a*b^2*e^4*g*x - a^2*b*e^4*h*x - 2*a^2*c*e^4*g*x + b^3*d*e^3*g*x + 2*c^3*d^3*e*f*x - 3*a^2*d*e^3*h*(b^2 - 4*a*c)^(1/2) - c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) - b^2*d^3*e*h*(b^2 - 4*a*c)^(1/2) - 2*b^2*e^4*f*x*(b^2 - 4*a*c)^(1/2) - a^2*e^4*h*x*(b^2 - 4*a*c)^(1/2) - 2*c^2*d^4*h*x*(b^2 - 4*a*c)^(1/2) - 10*a*c^2*d^2*e^2*f - 4*a*b^2*d^2*e^2*h + b^2*c*d^2*e^2*f + 10*a^2*c*d^2*e^2*h - b^3*d^2*e^2*h*x - 2*a*b*e^4*f*(b^2 - 4*a*c)^(1/2) - b*c*d^4*h*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d*e^3*f - 3*a*b*c*d^3*e*h + 7*a*b*c*e^4*f*x - 5*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^(1/2) - b^2*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2) + a*b*d*e^3*g*(b^2 - 4*a*c)^(1/2) + 7*a*c*d*e^3*f*(b^2 - 4*a*c)^(1/2) + 5*a*c*d^3*e*h*(b^2 - 4*a*c)^(1/2) + 2*b*c*d^3*e*g*(b^2 - 4*a*c)^(1/2) + a*b*e^4*g*x*(b^2 - 4*a*c)^(1/2) + 3*a*c*e^4*f*x*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d^2*e^2*g - 14*a*c^2*d*e^3*f*x + 5*b^2*c*d*e^3*f*x - 10*a*c^2*d^3*e*h*x - b*c^2*d^3*e*g*x + 6*a^2*c*d*e^3*h*x + 3*b^2*c*d^3*e*h*x + 2*a*b*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) - 7*a*c*d^2*e^2*g*(b^2 - 4*a*c)^(1/2) - b*c*d^2*e^2*f*(b^2 - 4*a*c)^(1/2) + b^2*d*e^3*g*x*(b^2 - 4*a*c)^(1/2) + 3*c^2*d^3*e*g*x*(b^2 - 4*a*c)^(1/2) + 14*a*c^2*d^2*e^2*g*x - 3*b*c^2*d^2*e^2*f*x - 2*b^2*c*d^2*e^2*g*x + 5*a*c*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2) - 2*b*c*d^2*e^2*g*x*(b^2 - 4*a*c)^(1/2) - 7*a*b*c*d*e^3*g*x - 5*a*c*d*e^3*g*x*(b^2 - 4*a*c)^(1/2) + 5*b*c*d*e^3*f*x*(b^2 - 4*a*c)^(1/2) + b*c*d^3*e*h*x*(b^2 - 4*a*c)^(1/2) + a*b*c*d^2*e^2*h*x)*(b^3*d*h + 4*a*c^2*d*g - 4*a*c^2*e*f - a*b^2*e*h - b^2*c*d*g + b^2*c*e*f + 4*a^2*c*e*h - 2*c^2*d*f*(b^2 - 4*a*c)^(1/2) - b^2*d*h*(b^2 - 4*a*c)^(1/2) - 4*a*b*c*d*h + a*b*e*h*(b^2 - 4*a*c)^(1/2) + 2*a*c*d*h*(b^2 - 4*a*c)^(1/2) - 2*a*c*e*g*(b^2 - 4*a*c)^(1/2) + b*c*d*g*(b^2 - 4*a*c)^(1/2) + b*c*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^2 + 4*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a*b*c^2*d*e)) - (log(a^2*b*e^4*g - 2*a*b^2*e^4*f - 2*a^3*e^4*h + 6*a^2*c*e^4*f - 4*a*c^2*d^4*h + b^2*c*d^4*h + b^3*d^3*e*h - 2*b^3*e^4*f*x - a^2*e^4*g*(b^2 - 4*a*c)^(1/2) + a*b^2*d*e^3*g + 6*a*c^2*d^3*e*g + b*c^2*d^3*e*f + 3*a^2*b*d*e^3*h - 10*a^2*c*d*e^3*g - 2*b^2*c*d^3*e*g + a*b^2*e^4*g*x - a^2*b*e^4*h*x - 2*a^2*c*e^4*g*x + b^3*d*e^3*g*x + 2*c^3*d^3*e*f*x + 3*a^2*d*e^3*h*(b^2 - 4*a*c)^(1/2) + c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) + b^2*d^3*e*h*(b^2 - 4*a*c)^(1/2) + 2*b^2*e^4*f*x*(b^2 - 4*a*c)^(1/2) + a^2*e^4*h*x*(b^2 - 4*a*c)^(1/2) + 2*c^2*d^4*h*x*(b^2 - 4*a*c)^(1/2) - 10*a*c^2*d^2*e^2*f - 4*a*b^2*d^2*e^2*h + b^2*c*d^2*e^2*f + 10*a^2*c*d^2*e^2*h - b^3*d^2*e^2*h*x + 2*a*b*e^4*f*(b^2 - 4*a*c)^(1/2) + b*c*d^4*h*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d*e^3*f - 3*a*b*c*d^3*e*h + 7*a*b*c*e^4*f*x + 5*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^(1/2) + b^2*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2) - a*b*d*e^3*g*(b^2 - 4*a*c)^(1/2) - 7*a*c*d*e^3*f*(b^2 - 4*a*c)^(1/2) - 5*a*c*d^3*e*h*(b^2 - 4*a*c)^(1/2) - 2*b*c*d^3*e*g*(b^2 - 4*a*c)^(1/2) - a*b*e^4*g*x*(b^2 - 4*a*c)^(1/2) - 3*a*c*e^4*f*x*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d^2*e^2*g - 14*a*c^2*d*e^3*f*x + 5*b^2*c*d*e^3*f*x - 10*a*c^2*d^3*e*h*x - b*c^2*d^3*e*g*x + 6*a^2*c*d*e^3*h*x + 3*b^2*c*d^3*e*h*x - 2*a*b*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) + 7*a*c*d^2*e^2*g*(b^2 - 4*a*c)^(1/2) + b*c*d^2*e^2*f*(b^2 - 4*a*c)^(1/2) - b^2*d*e^3*g*x*(b^2 - 4*a*c)^(1/2) - 3*c^2*d^3*e*g*x*(b^2 - 4*a*c)^(1/2) + 14*a*c^2*d^2*e^2*g*x - 3*b*c^2*d^2*e

```
^2*f*x - 2*b^2*c*d^2*e^2*g*x - 5*a*c*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2) + 2*b*
c*d^2*e^2*g*x*(b^2 - 4*a*c)^(1/2) - 7*a*b*c*d*e^3*g*x + 5*a*c*d*e^3*g*x*(b^
2 - 4*a*c)^(1/2) - 5*b*c*d*e^3*f*x*(b^2 - 4*a*c)^(1/2) - b*c*d^3*e*h*x*(b^2
 - 4*a*c)^(1/2) + a*b*c*d^2*e^2*h*x)*(4*a*c^2*e*f - 4*a*c^2*d*g - b^3*d*h +
 a*b^2*e*h + b^2*c*d*g - b^2*c*e*f - 4*a^2*c*e*h - 2*c^2*d*f*(b^2 - 4*a*c)^
(1/2) - b^2*d*h*(b^2 - 4*a*c)^(1/2) + 4*a*b*c*d*h + a*b*e*h*(b^2 - 4*a*c)^(
1/2) + 2*a*c*d*h*(b^2 - 4*a*c)^(1/2) - 2*a*c*e*g*(b^2 - 4*a*c)^(1/2) + b*c*
d*g*(b^2 - 4*a*c)^(1/2) + b*c*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^2 + 4
*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a*b*c^2*d*e)) + (l
og(d + e*x)*(e^2*f + d^2*h - d*e*g))/(a*e^3 - b*d*e^2 + c*d^2*e)
```

3.153 $\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)}\,dx$

Optimal. Leaf size=316

$$-\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}-\frac{\left(2c^2d^2f+2a^2e^2h-abe(eg+2dh)+b^2\left(e^2f+d^2h\right)-c\left(bd(2ef+dg)+2a(\right.\right.}{\sqrt{b^2-4ac}\,\left(cd^2-bde+ae^2\right)^2}$$

```
[Out] (-d^2*h+d*e*g-e^2*f)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)+(c*d*(-d*g+2*e*f)+a*e*(-
2*d*h+e*g)-b*(-d^2*h+e^2*f))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2-1/2*(c*d*(-d*g
+2*e*f)+a*e*(-2*d*h+e*g)-b*(-d^2*h+e^2*f))*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d
^2)^2-(2*c^2*d^2*f+2*a^2*e^2*h-a*b*e*(2*d*h+e*g)+b^2*(d^2*h+e^2*f)-c*(b*d*(
d*g+2*e*f)+2*a*(d^2*h-2*d*e*g+e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)/(a*e^2-b*d*e+c*d^2)^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1642, 648, 632, 212, 642}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)}{\sqrt{b^2-4ac}\,\left(ae^2-bde+cd^2\right)^2}-\frac{\log\left(a+bx+cx^2\right)\left(ae(eg-2dh)-b\left(e^2f-d^2h\right)+cd(2ef-dg)\right)}{2\left(ae^2-bde+cd^2\right)^2}-\frac{d^2h-deg+e^2f}{e(d+ex)\left(ae^2-bde+cd^2\right)}+\frac{\log(d+ex)\left(ae(eg-2dh)-b\left(e^2f-d^2h\right)+cd(2ef-dg)\right)}{\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x]
```

```
[Out] -((e^2*f - d*e*g + d^2*h)/(e*(c*d^2 - b*d*e + a*e^2)*(d + e*x))) - ((2*c^2*
d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c*(b*d*(2
*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2
- 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^2) + ((c*d*(2*e*f - d
*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))*Log[d + e*x])/(c*d^2 - b*d*e +
 a*e^2)^2 - ((c*d*(2*e*f - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))*Lo
g[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)}\,dx &= \int \left(\frac{e^2f-deg+d^2h}{\left(cd^2-bde+ae^2\right)(d+ex)^2}+\frac{e(cd(2ef-dg)+ae(eg-2dh)-b(e^2f \cdots}{\left(cd^2-bde+ae^2\right)^2(d+ex)} \cdots\right. \\
&= -\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{(cd(2ef-dg)+ae(eg-2dh)-b(e^2f- \cdots}{\left(cd^2-bde+ae^2\right)^2} \cdots \\
&= -\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{(cd(2ef-dg)+ae(eg-2dh)-b(e^2f- \cdots}{\left(cd^2-bde+ae^2\right)^2} \cdots \\
&= -\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{(cd(2ef-dg)+ae(eg-2dh)-b(e^2f- \cdots}{\left(cd^2-bde+ae^2\right)^2} \cdots \\
&= -\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}-\frac{(2c^2d^2f+2a^2e^2h-abe(eg+2dh)+b^2(e^2 \cdots}{\cdots} \cdots
\end{aligned}$$

Mathematica [A]
time = 0.34, size = 281, normalized size = 0.89

$$\frac{-\frac{2\left(cd^2+e(-bd+ae)\right)\left(e^2f-deg+d^2h\right)}{e(d+ex)}+\frac{2\left(2c^2d^2f+2a^2e^2h-abe(eg+2dh)+b^2\left(e^2f+d^2h\right)-c\left(bd(2ef+dg)+2a\left(e^2f-2deg+d^2h\right)\right)\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+2(cd(2ef-dg)+ae(eg-2dh)+b(-e^2f+d^2h))\log(d+ex)+(cd(-2ef+dg)+ae(-eg+2dh)+b(e^2f-d^2h))\log(a+x(b+cx))}{2\left(cd^2+e(-bd+ae)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x]
```

[Out] ((-2*(c*d^2 + e*(-(b*d) + a*e))*(e^2*f - d*e*g + d^2*h))/(e*(d + e*x)) + (2 *(2*c^2*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c *(b*d*(2*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sq rt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(c*d*(2*e*f - d*g) + a*e*(e*g - 2 *d*h) + b*(-(e^2*f) + d^2*h))*Log[d + e*x] + (c*d*(-2*e*f + d*g) + a*e*(-(e *g) + 2*d*h) + b*(e^2*f - d^2*h))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b* d) + a*e))^2)

Maple [A]
time = 0.28, size = 346, normalized size = 1.09

method	result
default	$-\frac{d^{2} h-g d e+e^{2} f}{\left(a \,e^{2}-d e b+c \,d^{2}\right) e \left(e x+d\right)}-\frac{\left(2 a d e h-a \,e^{2} g-b \,d^{2} h+b \,e^{2} f+c \,d^{2} g-2 c d e f\right) \ln \left(e x+d\right)}{\left(a \,e^{2}-d e b+c \,d^{2}\right)^{2}}+\frac{\frac{\left(2 a c d h e-a c \,e^{2} g-b c \,d^{2} h+b c \,e^{2} f+c^{2} d^{2} g-2 c^{2} d}{2 c}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)

[Out] -(d^2*h-d*e*g+e^2*f)/(a*e^2-b*d*e+c*d^2)/e/(e*x+d)-(2*a*d*e*h-a*e^2*g-b*d^2 *h+b*e^2*f+c*d^2*g-2*c*d*e*f)/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)+1/(a*e^2-b*d* e+c*d^2)^2*(1/2*(2*a*c*d*e*h-a*c*e^2*g-b*c*d^2*h+b*c*e^2*f+c^2*d^2*g-2*c^2* d*e*f)/c*ln(c*x^2+b*x+a)+2*(a^2*e^2*h-a*b*e^2*g-a*c*d^2*h+2*a*c*d*e*g-a*c*e ^2*f+b^2*e^2*f-2*b*c*d*e*f+c^2*d^2*f-1/2*(2*a*c*d*e*h-a*c*e^2*g-b*c*d^2*h+b *c*e^2*f+c^2*d^2*g-2*c^2*d*e*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4* a*c-b^2)^(1/2)))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* h elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo re deta

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)**2/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.40, size = 449, normalized size = 1.42

$$\frac{\left(2\,c^{2}d^{2}fe^{2}-bcd^{2}ge^{2}+b^{2}d^{2}he^{2}-2\,acd^{2}he^{2}-2\,bcdfe^{3}+4\,acdge^{3}-2\,abdhe^{3}+b^{2}fe^{4}-2\,acfe^{4}-abge^{4}+2\,a^{2}he^{4}\right)\arctan\left(\frac{\left(2\,cd-\frac{2\,cd^{2}}{xe+d}-be+\frac{2\,bde}{xe+d}-\frac{2\,ae^{2}}{xe+d}\right)e^{(-1)}}{\sqrt{-b^{2}+4\,ac}}\right)e^{(-2)}}{\left(c^{2}d^{4}-2\,bcd^{3}e+b^{2}d^{2}e^{2}+2\,acd^{2}e^{2}-2\,abde^{3}+a^{2}e^{4}\right)\sqrt{-b^{2}+4\,ac}}+\frac{\left(cd^{2}g-bd^{2}h-2\,cdfe+2\,adhe+bfe^{2}-age^{2}\right)\log\left(c-\frac{2\,cd}{xe+d}+\frac{cd^{2}}{(xe+d)^{2}}+\frac{be}{xe+d}-\frac{bde}{(xe+d)^{2}}+\frac{ae^{2}}{(xe+d)^{2}}\right)}{2\left(c^{2}d^{4}-2\,bcd^{3}e+b^{2}d^{2}e^{2}+2\,acd^{2}e^{2}-2\,abde^{3}+a^{2}e^{4}\right)}-\frac{\frac{d^{2}he}{xe+d}-\frac{dge^{2}}{xe+d}+\frac{fe^{3}}{xe+d}}{cd^{2}e^{2}-bde^{3}+ae^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] (2*c^2*d^2*f*e^2 - b*c*d^2*g*e^2 + b^2*d^2*h*e^2 - 2*a*c*d^2*h*e^2 - 2*b*c*
d*f*e^3 + 4*a*c*d*g*e^3 - 2*a*b*d*h*e^3 + b^2*f*e^4 - 2*a*c*f*e^4 - a*b*g*e
^4 + 2*a^2*h*e^4)*arctan((2*c*d - 2*c*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e +
d) - 2*a*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((c^2*d^4 - 2*b*c
*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(-b^2 + 4
*a*c)) + 1/2*(c*d^2*g - b*d^2*h - 2*c*d*f*e + 2*a*d*h*e + b*f*e^2 - a*g*e^2
)*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e
+ d)^2 + a*e^2/(x*e + d)^2)/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^
2*e^2 - 2*a*b*d*e^3 + a^2*e^4) - (d^2*h*e/(x*e + d) - d*g*e^2/(x*e + d) + f
*e^3/(x*e + d))/(c*d^2*e^2 - b*d*e^3 + a*e^4)
```

Mupad [B]
time = 14.71, size = 2500, normalized size = 7.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x)
```

```
[Out] (log(d + e*x)*(e^2*(a*g - b*f) + d^2*(b*h - c*g) - d*e*(2*a*h - 2*c*f)))/(a
^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2)
 + (log(2*a*b^3*e^4*f - 2*b^2*c^2*d^4*g - 2*a^2*b^2*e^4*g + 6*a*c^3*d^4*g +
 b*c^3*d^4*f + a^3*b*e^4*h + 6*a^3*c*e^4*g + 2*b^3*c*d^4*h + 2*b^4*e^4*f*x
```

+ 2*c^4*d^4*f*x - c^3*d^4*f*(b^2 - 4*a*c)^(1/2) + a^3*e^4*h*(b^2 - 4*a*c)^(
1/2) - 7*a^2*b*c*e^4*f - 7*a*b*c^2*d^4*h - 16*a*c^3*d^3*e*f - 16*a^3*c*d*e^
3*h - 2*a*b^3*e^4*g*x - 2*a*c^3*d^4*h*x - b*c^3*d^4*g*x - 2*a^3*c*e^4*h*x +
 2*a*b^2*e^4*f*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*e^4*g*(b^2 - 4*a*c)^(1/2) - a^
2*c*e^4*f*(b^2 - 4*a*c)^(1/2) + a*c^2*d^4*h*(b^2 - 4*a*c)^(1/2) + 2*b*c^2*d
^4*g*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d^4*h*(b^2 - 4*a*c)^(1/2) + 2*b^3*e^4*f*
x*(b^2 - 4*a*c)^(1/2) + 3*c^3*d^4*g*x*(b^2 - 4*a*c)^(1/2) + 16*a^2*c^2*d*e^
3*f - a*b^3*d^2*e^2*h + 2*a^2*b^2*d*e^3*h + 2*b^2*c^2*d^3*e*f - b^3*c*d^2*e
^2*f + 16*a^2*c^2*d^3*e*h + 2*a^2*c^2*e^4*f*x + a^2*b^2*e^4*h*x + b^2*c^2*d
^4*h*x - b^4*d^2*e^2*h*x - 20*a^2*c^2*d^2*e^2*g + 14*a*c^2*d^2*e^2*f*(b^2 -
 4*a*c)^(1/2) - a*b^2*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) + b^2*c*d^2*e^2*f*(b^2
- 4*a*c)^(1/2) - 14*a^2*c*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) - b^3*d^2*e^2*h*x*(
b^2 - 4*a*c)^(1/2) + 10*b^2*c^2*d^2*e^2*f*x + 28*a^2*c^2*d^2*e^2*h*x - 6*a*
b^2*c*d*e^3*f + 4*a*b*c^2*d^3*e*g + 4*a^2*b*c*d*e^3*g - 6*a*b^2*c*d^3*e*h -
 8*a*b^2*c*e^4*f*x + 7*a^2*b*c*e^4*g*x + 2*a*b^3*d*e^3*h*x + 16*a*c^3*d^3*e
*g*x - 4*b*c^3*d^3*e*f*x - 8*b^3*c*d*e^3*f*x + 2*b^3*c*d^3*e*h*x - 8*a*c^2*
d^3*e*g*(b^2 - 4*a*c)^(1/2) - 2*b*c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) + 2*a^2*b
*d*e^3*h*(b^2 - 4*a*c)^(1/2) + 8*a^2*c*d*e^3*g*(b^2 - 4*a*c)^(1/2) - 2*a*b^
2*e^4*g*x*(b^2 - 4*a*c)^(1/2) + a^2*b*e^4*h*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*c
*e^4*g*x*(b^2 - 4*a*c)^(1/2) - 3*b*c^2*d^4*h*x*(b^2 - 4*a*c)^(1/2) - 8*c^3*
d^3*e*f*x*(b^2 - 4*a*c)^(1/2) + 10*a*b*c^2*d^2*e^2*f + 2*a*b^2*c*d^2*e^2*g
+ 10*a^2*b*c*d^2*e^2*h - 28*a*c^3*d^2*e^2*f*x - 16*a^2*c^2*d*e^3*g*x - 2*b^
2*c^2*d^3*e*g*x + b^3*c*d^2*e^2*g*x + 8*a*c^2*d*e^3*f*x*(b^2 - 4*a*c)^(1/2)
 + 2*a*b^2*d*e^3*h*x*(b^2 - 4*a*c)^(1/2) - 8*b^2*c*d*e^3*f*x*(b^2 - 4*a*c)^
(1/2) + 8*a*c^2*d^3*e*h*x*(b^2 - 4*a*c)^(1/2) - 2*b*c^2*d^3*e*g*x*(b^2 - 4*
a*c)^(1/2) - 8*a^2*c*d*e^3*h*x*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*d^3*e*h*x*(b^2
 - 4*a*c)^(1/2) - 10*a*b*c^2*d^2*e^2*g*x - 10*a*c^2*d^2*e^2*g*x*(b^2 - 4*a*
c)^(1/2) + 12*b*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^(1/2) + b^2*c*d^2*e^2*g*x*(b^
2 - 4*a*c)^(1/2) - 10*a*b*c*d*e^3*f*(b^2 - 4*a*c)^(1/2) + 10*a*b*c*d^3*e*h*
(b^2 - 4*a*c)^(1/2) - 4*a*b*c*e^4*f*x*(b^2 - 4*a*c)^(1/2) + 28*a*b*c^2*d*e^
3*f*x + 6*a*b^2*c*d*e^3*g*x - 12*a*b*c^2*d^3*e*h*x - 12*a^2*b*c*d*e^3*h*x +
 6*a*b*c*d*e^3*g*x*(b^2 - 4*a*c)^(1/2) - 2*a*b*c*d^2*e^2*h*x*(b^2 - 4*a*c)^
(1/2))*(b^3*d^2*h - b^3*e^2*f + a*b^2*e^2*g + 4*a*c^2*d^2*g - 4*a^2*c*e^2*g
 - b^2*c*d^2*g - b^2*e^2*f*(b^2 - 4*a*c)^(1/2) - 2*c^2*d^2*f*(b^2 - 4*a*c)^
(1/2) - 2*a^2*e^2*h*(b^2 - 4*a*c)^(1/2) - b^2*d^2*h*(b^2 - 4*a*c)^(1/2) + 4
*a*b*c*e^2*f - 4*a*b*c*d^2*h - 8*a*c^2*d*e*f - 2*a*b^2*d*e*h + 2*b^2*c*d*e*
f + 8*a^2*c*d*e*h + a*b*e^2*g*(b^2 - 4*a*c)^(1/2) + 2*a*c*e^2*f*(b^2 - 4*a*
c)^(1/2) + 2*a*c*d^2*h*(b^2 - 4*a*c)^(1/2) + b*c*d^2*g*(b^2 - 4*a*c)^(1/2)
+ 2*a*b*d*e*h*(b^2 - 4*a*c)^(1/2) - 4*a*c*d*e*g*(b^2 - 4*a*c)^(1/2) + 2*b*c
*d*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 -
b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3
*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2)) - (log(2*a*b^3
*e^4*f - 2*b^2*c^2*d^4*g - 2*a^2*b^2*e^4*g + 6*a*c^3*d^4*g + b*c^3*d^4*f +
a^3*b*e^4*h + 6*a^3*c*e^4*g + 2*b^3*c*d^4*h + 2*b^4*e^4*f*x + 2*c^4*d^4*f*x
 + c^3*d^4*f*(b^2 - 4*a*c)^(1/2) - a^3*e^4*h*(b^2 - 4*a*c)^(1/2) - 7*a^2*b*

```
c*e^4*f - 7*a*b*c^2*d^4*h - 16*a*c^3*d^3*e*f - 16*a^3*c*d*e^3*h - 2*a*b^3*e
^4*g*x - 2*a*c^3*d^4*h*x - b*c^3*d^4*g*x - 2*a^3*c*e^4*h*x - 2*a*b^2*e^4*f*
(b^2 - 4*a*c)^(1/2) + 2*a^2*b*e^4*g*(b^2 - 4*a*c)^(1/2) + a^2*c*e^4*f*(b^2
- 4*a*c)^(1/2) - a*c^2*d^4*h*(b^2 - 4*a*c)^(1/2) - 2*b*c^2*d^4*g*(b^2 - 4*a
*c)^(1/2) + 2*b^2*c*d^4*h*(b^2 - 4*a*c)^(1/2) - 2*b^3*e^4*f*x*(b^2 - 4*a*c)
^(1/2) - 3*c^3*d^4*g*x*(b^2 - 4*a*c)^(1/2) + 16*a^2*c^2*d*e^3*f - a*b^3*d^2
*e^2*h + 2*a^2*b^2*d*e^3*h + 2*b^2*c^2*d^3*e*f - b^3*c*d^2*e^2*f + 16*a^2*c
^2*d^3*e*h + 2*a^2*c^2*e^4*f*x + a^2*b^2*e^4*h*x + b^2*c^2*d^4*h*x - b^4*d^
2*e^2*h*x - 20*a^2*c^2*d^2*e^2*g - 14*a*c^2*d^2*e^2*f*(b^2 - 4*a*c)^(1/2) +
 a*b^2*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) - b^2*c*d^2*e^2*f*(b^2 - 4*a*c)^(1/2)
+ 14*a^2*c*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) + b^3*d^2*e^2*h*x*(b^2 - 4*a*c)^(1
/2) + 10*b^2*c^2*d^2*e^2*f*x + 28*a^2*c^2*d^2*e^2*h*x - 6*a*b^2*c*d*e^3*f +
 4*a*b*c^2*d^3*e*g + 4*a^2*b*c*d*e^3*g - 6*a*b^2*c*d^3*e*h - 8*a*b^2*c*e^4*
f*x + 7*a^2*b*c*e^4*g*x + 2*a*b^3*d*e^3*h*x + 16*a*c^3*d^3*e*g*x - 4*b*c^3*
d^3*e*f*x - 8*b^3*c*d*e^3*f*x + 2*b^3*c*d^3*e*h*x + 8*a*c^2*d^3*e*g*(b^2 -
4*a*c)^(1/2) + 2*b*c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*d*e^3*h*(b^2 -
 4*a*c)^(1/2) - 8*a^2*c*d*e^3*g*(b^2 - 4*a*c)^(1/2) + 2*a*b^2*e^4*g*x*(b^2
- 4*a*c)^(1/2) - a^2*b*e^4*h*x*(b^2 - 4*a*c)^(1...
```

3.154 $\int \frac{f+gx+hx^2}{(d+ex)^3(a+bx+cx^2)}\,dx$

Optimal. Leaf size=509

$$-\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2}-\frac{cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h)}{\left(cd^2-bde+ae^2\right)^2(d+ex)}-\frac{(2c^3d^3f-be^3(b^2f-abg-}{}$$

```
[Out] 1/2*(-d^2*h+d*e*g-e^2*f)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^2+(-c*d*(-d*g+2*e*f)
-a*e*(-2*d*h+e*g)+b*(-d^2*h+e^2*f))/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)+(c^2*d^2*
(-d*g+3*e*f)+e^3*(a^2*h-a*b*g+b^2*f)-c*(a*e*(3*d^2*h-3*d*e*g+e^2*f)+b*(-d^3
*h+3*d*e^2*f)))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^3-1/2*(c^2*d^2*(-d*g+3*e*f)+e
^3*(a^2*h-a*b*g+b^2*f)-c*(a*e*(3*d^2*h-3*d*e*g+e^2*f)+b*(-d^3*h+3*d*e^2*f))
)*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d^2)^3-(2*c^3*d^3*f-b*e^3*(a^2*h-a*b*g+b^2
*f)-c^2*d*(b*d*(d*g+3*e*f)+2*a*(d^2*h-3*d*e*g+3*e^2*f))-c*(2*a^2*e^2*(-3*d*
h+e*g)-3*a*b*e*(-d^2*h-d*e*g+e^2*f)-b^2*(d^3*h+3*d*e^2*f)))*arctanh((2*c*x+
b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.76, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1642, 648, 632, 212, 642}

$$-\frac{\log\left(a+bx+cx^2\right)\left(e^3(a^2h-abg+b^2f)-ace(3d^2h-3deg+e^2f)-bc(3de^2f-d^3h)+c^2d^2(3ef-dg)\right)}{2\left(ae^2-bde+cd^2\right)^3}+\frac{\log(d+ex)\left(e^3(a^2h-abg+b^2f)-ace(3d^2h-3deg+e^2f)-bc(3de^2f-d^3h)+c^2d^2(3ef-dg)\right)}{\left(ae^2-bde+cd^2\right)^3}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2a^2e^2(eg-3dh)-3abe(d^2(-h)-deg+e^2f)-(b^2(d^3h+3de^2f)))-be^3(a^2h-abg+b^2f)-c^2d(2a(d^2h-3deg+3e^2f)+bd(dg+3ef))+2c^3d^3f\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^3}-\frac{d^2h-deg+e^2f}{2e(d+ex)^2\left(ae^2-bde+cd^2\right)}-\frac{ae(eg-2dh)-b(e^2f-d^2h)+cd(2ef-dg)}{(d+ex)\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x]
```

```
[Out] -1/2*(e^2*f - d*e*g + d^2*h)/(e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (c*d
*(2*e*f - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))/((c*d^2 - b*d*e + a
*e^2)^2*(d + e*x)) - ((2*c^3*d^3*f - b*e^3*(b^2*f - a*b*g + a^2*h) - c^2*d*
(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f - 3*d*e*g + d^2*h)) - c*(2*a^2*e^2*(e*g -
 3*d*h) - 3*a*b*e*(e^2*f - d*e*g - d^2*h) - b^2*(3*d*e^2*f + d^3*h)))*ArcTa
nh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^
2)^3) + ((c^2*d^2*(3*e*f - d*g) + e^3*(b^2*f - a*b*g + a^2*h) - a*c*e*(e^2*
f - 3*d*e*g + 3*d^2*h) - b*c*(3*d*e^2*f - d^3*h))*Log[d + e*x])/(c*d^2 - b*
d*e + a*e^2)^3 - ((c^2*d^2*(3*e*f - d*g) + e^3*(b^2*f - a*b*g + a^2*h) - a*
c*e*(e^2*f - 3*d*e*g + 3*d^2*h) - b*c*(3*d*e^2*f - d^3*h))*Log[a + b*x + c*
x^2])/(2*(c*d^2 - b*d*e + a*e^2)^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)^3\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{e^2f-deg+d^2h}{\left(cd^2-bde+ae^2\right)(d+ex)^3} + \frac{e(cd(2ef-dg)+ae(eg-2dh)-b(e^2f}{\left(cd^2-bde+ae^2\right)^2(d+ex)^2}\right.$$

$$= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{cd(2ef-dg)+ae(eg-2dh)-b(e^2f-}{\left(cd^2-bde+ae^2\right)^2(d+ex)}$$

$$= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{cd(2ef-dg)+ae(eg-2dh)-b(e^2f-}{\left(cd^2-bde+ae^2\right)^2(d+ex)}$$

$$= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{cd(2ef-dg)+ae(eg-2dh)-b(e^2f-}{\left(cd^2-bde+ae^2\right)^2(d+ex)}$$

$$= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{cd(2ef-dg)+ae(eg-2dh)-b(e^2f-}{\left(cd^2-bde+ae^2\right)^2(d+ex)}$$

Mathematica [A]
time = 0.45, size = 504, normalized size = 0.99

$$-\frac{e^2f-deg+d^2h}{2e\left(cd^2+e(-bd+ae)\right)(d+ex)^2} + \frac{cd(-2ef+dg)+ae(-eg+2dh)+b\left(e^2f-d^2h\right)}{\left(cd^2+e(-bd+ae)\right)^2(d+ex)} + \frac{\left(-2c^3d^3f+be^3\left(b^2f-abg+a^2h\right)+c^2d\left(bd(3ef+dg)+2a\left(3e^2f-3deg+d^2h\right)\right)-c\left(-2a^2e^2(eg-3dh)+3abe\left(e^2f-deg-d^2h\right)+b^2\left(3de^2f+d^3h\right)\right)\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}\left(-cd^2+e(bd-ae)\right)^3} - \frac{\left(c^2d^2(-3ef+dg)-e^3\left(b^2f-abg+a^2h\right)+ace\left(e^2f-3deg+3d^2h\right)+bc\left(3de^2f-d^3h\right)\right)\log(d+ex)}{\left(cd^2+e(-bd+ae)\right)^3} + \frac{\left(c^2d^2(-3ef+dg)-e^3\left(b^2f-abg+a^2h\right)+ace\left(e^2f-3deg+3d^2h\right)+bc\left(3de^2f-d^3h\right)\right)\log(a+x(b+cx))}{2\left(cd^2+e(-bd+ae)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x]
```

```
[Out] -1/2*(e^2*f - d*e*g + d^2*h)/(e*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) + (
c*d*(-2*e*f + d*g) + a*e*(-(e*g) + 2*d*h) + b*(e^2*f - d^2*h))/((c*d^2 + e*
(-(b*d) + a*e))^2*(d + e*x)) + ((-2*c^3*d^3*f + b*e^3*(b^2*f - a*b*g + a^2*
h) + c^2*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f - 3*d*e*g + d^2*h)) - c*(-2*a^
2*e^2*(e*g - 3*d*h) + 3*a*b*e*(e^2*f - d*e*g - d^2*h) + b^2*(3*d*e^2*f + d^
3*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*(-(c*d^2
) + e*(b*d - a*e))^3) - ((c^2*d^2*(-3*e*f + d*g) - e^3*(b^2*f - a*b*g + a^2
*h) + a*c*e*(e^2*f - 3*d*e*g + 3*d^2*h) + b*c*(3*d*e^2*f - d^3*h))*Log[d +
e*x])/(c*d^2 + e*(-(b*d) + a*e))^3 + ((c^2*d^2*(-3*e*f + d*g) - e^3*(b^2*f
- a*b*g + a^2*h) + a*c*e*(e^2*f - 3*d*e*g + 3*d^2*h) + b*c*(3*d*e^2*f - d^3
*h))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^3)
```

Maple [A]
time = 0.25, size = 629, normalized size = 1.24

method	result
default	$-\frac{d^2h-gde+e^2f}{2(a e^2-deb+c d^2)e(ex+d)^2}+\frac{(a^2e^3h-ab e^3g-3ac d^2eh+3acd e^2g-ac e^3f+b^2e^3f+bc d^3h-3bcd e^2f-c^2d^3g+3c^2d^2ef)\ln(ex+d}{(a e^2-deb+c d^2)^3}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(d^2*h-d*e*g+e^2*f)/(a*e^2-b*d*e+c*d^2)/e/(e*x+d)^2+(a^2*e^3*h-a*b*e^3
*g-3*a*c*d^2*e*h+3*a*c*d*e^2*g-a*c*e^3*f+b^2*e^3*f+b*c*d^3*h-3*b*c*d*e^2*f-
c^2*d^3*g+3*c^2*d^2*e*f)/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)+(2*a*d*e*h-a*e^2*g
-b*d^2*h+b*e^2*f+c*d^2*g-2*c*d*e*f)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)+1/(a*e^2-
b*d*e+c*d^2)^3*(1/2*(-a^2*c*e^3*h+a*b*c*e^3*g+3*a*c^2*d^2*e*h-3*a*c^2*d*e^2
*g+a*c^2*e^3*f-b^2*c*e^3*f-b*c^2*d^3*h+3*b*c^2*d*e^2*f+c^3*d^3*g-3*c^3*d^2*
e*f)/c*ln(c*x^2+b*x+a)+2*(-a^2*b*e^3*h+3*a^2*c*d*e^2*h-a^2*c*e^3*g+a*b^2*e^
3*g-3*a*b*c*d*e^2*g+2*a*b*c*e^3*f-a*c^2*d^3*h+3*a*c^2*d^2*e*g-3*a*c^2*d*e^2
*f-b^3*e^3*f+3*b^2*c*d*e^2*f-3*b*c^2*d^2*e*f+c^3*d^3*f-1/2*(-a^2*c*e^3*h+a*
b*c*e^3*g+3*a*c^2*d^2*e*h-3*a*c^2*d*e^2*g+a*c^2*e^3*f-b^2*c*e^3*f-b*c^2*d^3
*h+3*b*c^2*d*e^2*f+c^3*d^3*g-3*c^3*d^2*e*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)**3/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.92, size = 1002, normalized size = 1.97

$$\frac{\left(c^2d^3g - bcd^3h - 3c^2d^2fe + 3acd^2he + 3bcdfe^2 - 3acdge^2 - b^2fe^3 + acfe^3 + abge^3 - a^2he^3\right)\log\left(cx^2 + bx + a\right)}{2\left(c^3d^6 - 3bc^2d^5e + 3b^2cd^4e^2 + 3ac^2d^4e^2 - b^3d^3e^3 - 6abcd^3e^3 + 3ab^2d^2e^4 + 3a^2cd^2e^4 - 3a^2bde^5 + a^3e^6\right)} - \frac{\left(c^2d^3ge - bcd^3he - 3c^2d^2fe^2 + 3acd^2he^2 + 3bcdfe^3 - 3acdge^3 - b^2fe^4 + acfe^4 + abge^4 - a^2he^4\right)\log\left(|xe + d|\right)}{c^3d^6e - 3bc^2d^5e^2 + 3b^2cd^4e^3 + 3ac^2d^4e^3 - b^3d^3e^4 - 6abcd^3e^4 + 3ab^2d^2e^5 + 3a^2cd^2e^5 - 3a^2bde^6 + a^3e^7} + \text{[illegible]}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(c^2*d^3*g - b*c*d^3*h - 3*c^2*d^2*f*e + 3*a*c*d^2*h*e + 3*b*c*d*f*e^2
- 3*a*c*d*g*e^2 - b^2*f*e^3 + a*c*f*e^3 + a*b*g*e^3 - a^2*h*e^3)*log(c*x^2
+ b*x + a)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b
^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*
d*e^5 + a^3*e^6) - (c^2*d^3*g*e - b*c*d^3*h*e - 3*c^2*d^2*f*e^2 + 3*a*c*d^2
*h*e^2 + 3*b*c*d*f*e^3 - 3*a*c*d*g*e^3 - b^2*f*e^4 + a*c*f*e^4 + a*b*g*e^4
- a^2*h*e^4)*log(abs(x*e + d))/(c^3*d^6*e - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e
^3 + 3*a*c^2*d^4*e^3 - b^3*d^3*e^4 - 6*a*b*c*d^3*e^4 + 3*a*b^2*d^2*e^5 + 3*
a^2*c*d^2*e^5 - 3*a^2*b*d*e^6 + a^3*e^7) + (2*c^3*d^3*f - b*c^2*d^3*g + b^2
*c*d^3*h - 2*a*c^2*d^3*h - 3*b*c^2*d^2*f*e + 6*a*c^2*d^2*g*e - 3*a*b*c*d^2*
```

```
h*e + 3*b^2*c*d*f*e^2 - 6*a*c^2*d*f*e^2 - 3*a*b*c*d*g*e^2 + 6*a^2*c*d*h*e^2
 - b^3*f*e^3 + 3*a*b*c*f*e^3 + a*b^2*g*e^3 - 2*a^2*c*g*e^3 - a^2*b*h*e^3)*a
rctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d
^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4
+ 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*sqrt(-b^2 + 4*a*c)) - 1/2*(c^2
*d^6*h - 3*c^2*d^5*g*e + 5*c^2*d^4*f*e^2 + 4*b*c*d^4*g*e^2 - b^2*d^4*h*e^2
- 2*a*c*d^4*h*e^2 - 8*b*c*d^3*f*e^3 - b^2*d^3*g*e^3 - 2*a*c*d^3*g*e^3 + 4*a
*b*d^3*h*e^3 + 3*b^2*d^2*f*e^4 + 6*a*c*d^2*f*e^4 - 3*a^2*d^2*h*e^4 - 4*a*b*
d*f*e^5 + a^2*d*g*e^5 + a^2*f*e^6 - 2*(c^2*d^4*g*e^2 - b*c*d^4*h*e^2 - 2*c^
2*d^3*f*e^3 - b*c*d^3*g*e^3 + b^2*d^3*h*e^3 + 2*a*c*d^3*h*e^3 + 3*b*c*d^2*f
*e^4 - 3*a*b*d^2*h*e^4 - b^2*d*f*e^5 - 2*a*c*d*f*e^5 + a*b*d*g*e^5 + 2*a^2*
d*h*e^5 + a*b*f*e^6 - a^2*g*e^6)*x)*e^(-1)/((c*d^2 - b*d*e + a*e^2)^3*(x*e
+ d)^2)
```

Mupad [B]
time = 6.82, size = 2500, normalized size = 4.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x)
```

```
[Out] symsum(log(root(24*a^6*b*c*d*e^11*z^3 + 24*a*b*c^6*d^11*e*z^3 + 240*a^4*b*c
^3*d^5*e^7*z^3 + 240*a^3*b*c^4*d^7*e^5*z^3 + 120*a^5*b*c^2*d^3*e^9*z^3 + 12
0*a^2*b*c^5*d^9*e^3*z^3 - 54*a^5*b^2*c*d^2*e^10*z^3 - 54*a*b^2*c^5*d^10*e^2
*z^3 + 50*a^4*b^3*c*d^3*e^9*z^3 + 50*a*b^3*c^4*d^9*e^3*z^3 - 36*a^2*b^5*c*d
^5*e^7*z^3 - 36*a*b^5*c^2*d^7*e^5*z^3 + 26*a*b^6*c*d^6*e^6*z^3 - 340*a^3*b^
2*c^3*d^6*e^6*z^3 - 225*a^4*b^2*c^2*d^4*e^8*z^3 - 225*a^2*b^2*c^4*d^8*e^4*z
^3 + 180*a^3*b^3*c^2*d^5*e^7*z^3 + 180*a^2*b^3*c^3*d^7*e^5*z^3 - 30*a^2*b^4
*c^2*d^6*e^6*z^3 - 6*b^7*c*d^7*e^5*z^3 - 6*b^3*c^5*d^11*e*z^3 - 6*a^5*b^3*d
*e^11*z^3 - 6*a*b^7*d^5*e^7*z^3 - 20*b^5*c^3*d^9*e^3*z^3 + 15*b^6*c^2*d^8*e
^4*z^3 + 15*b^4*c^4*d^10*e^2*z^3 - 80*a^4*c^4*d^6*e^6*z^3 - 60*a^5*c^3*d^4*
e^8*z^3 - 60*a^3*c^5*d^8*e^4*z^3 - 24*a^6*c^2*d^2*e^10*z^3 - 24*a^2*c^6*d^1
0*e^2*z^3 - 20*a^3*b^5*d^3*e^9*z^3 + 15*a^4*b^4*d^2*e^10*z^3 + 15*a^2*b^6*d
^4*e^8*z^3 - 4*a^7*c*e^12*z^3 - 4*a*c^7*d^12*z^3 + b^8*d^6*e^6*z^3 + b^2*c^
6*d^12*z^3 + a^6*b^2*e^12*z^3 - 9*a^3*b^2*c*d*e^5*g*h*z - 9*a*b^2*c^3*d^5*e
*g*h*z - 30*a^3*b*c^2*d*e^5*f*h*z + 9*a^2*b^3*c*d*e^5*f*h*z + 3*a*b^4*c*d^2
*e^4*f*h*z + 27*a*b*c^4*d^4*e^2*f*g*z + 6*a^2*b^2*c^2*d^3*e^3*g*h*z - 33*a^
2*b^2*c^2*d^2*e^4*f*h*z + 18*a*b*c^4*d^5*e*f*h*z - 12*a*b^4*c*d*e^5*f*g*z +
 27*a^3*b*c^2*d^2*e^4*g*h*z + 27*a^2*b*c^3*d^4*e^2*g*h*z - 3*a^2*b^3*c*d^2*
e^4*g*h*z - 3*a*b^3*c^2*d^4*e^2*g*h*z + 52*a^2*b*c^3*d^3*e^3*f*h*z - 4*a*b^
3*c^2*d^3*e^3*f*h*z - 3*a*b^2*c^3*d^4*e^2*f*h*z - 93*a^2*b*c^3*d^2*e^4*f*g*
z + 51*a^2*b^2*c^2*d*e^5*f*g*z - 34*a*b^2*c^3*d^3*e^3*f*g*z + 27*a*b^3*c^2*
d^2*e^4*f*g*z - 24*a*c^5*d^5*e*f*g*z - 7*a^4*b*c*e^6*g*h*z - 7*a*b*c^4*d^6*
g*h*z + a*b^4*c*d^3*e^3*g*h*z - 80*a^3*c^3*d^3*e^3*g*h*z + 3*b^4*c^2*d^4*e^
```

2*f*h*z - 66*a^2*c^4*d^4*e^2*f*h*z + 54*a^3*c^3*d^2*e^4*f*h*z - 3*b^3*c^3*d
^4*e^2*f*g*z + 80*a^2*c^4*d^3*e^3*f*g*z - 21*a^2*b*c^3*d^5*e*h^2*z + 6*a*b^
3*c^2*d^5*e*h^2*z - 21*a^3*b*c^2*d*e^5*g^2*z + 6*a^2*b^3*c*d*e^5*g^2*z - 66
*a*b*c^4*d^3*e^3*f^2*z - 30*a*b^3*c^2*d*e^5*f^2*z + 27*a^2*b*c^3*d*e^5*f^2*
z - 12*a^2*b^2*c^2*d^4*e^2*h^2*z - 12*a^2*b^2*c^2*d^2*e^4*g^2*z + 24*a^4*c^
2*d*e^5*g*h*z + 24*a^2*c^4*d^5*e*g*h*z - 3*b^3*c^3*d^5*e*f*h*z - b^5*c*d^3*
e^3*f*h*z + 3*b^2*c^4*d^5*e*f*g*z - 24*a^3*c^3*d*e^5*f*g*z + 9*a^3*b^2*c*e^
6*f*h*z - 10*a^2*b^3*c*e^6*f*g*z + 9*a^3*b*c^2*e^6*f*g*z + 3*a^4*b*c*d*e^5*
h^2*z + 3*a*b*c^4*d^5*e*g^2*z + 14*a^3*b*c^2*d^3*e^3*h^2*z + 3*a^3*b^2*c*d^
2*e^4*h^2*z - a^2*b^3*c*d^3*e^3*h^2*z + 14*a^2*b*c^3*d^3*e^3*g^2*z + 3*a*b^
2*c^3*d^4*e^2*g^2*z - a*b^3*c^2*d^3*e^3*g^2*z + 63*a*b^2*c^3*d^2*e^4*f^2*z
+ 2*b^3*c^3*d^6*g*h*z - 6*a^4*c^2*e^6*f*h*z + 2*a^3*b^3*e^6*g*h*z - b^2*c^4
*d^6*f*h*z - 2*a^2*b^4*e^6*f*h*z + 6*b^5*c*d*e^5*f^2*z + 3*b*c^5*d^5*e*f^2*
z + 6*a*b^4*c*e^6*f^2*z + b^4*c^2*d^3*e^3*f*g*z + 33*a^3*c^3*d^4*e^2*h^2*z
- 27*a^4*c^2*d^2*e^4*h^2*z + 33*a^3*c^3*d^2*e^4*g^2*z - 27*a^2*c^4*d^4*e^2*
g^2*z + 19*b^3*c^3*d^3*e^3*f^2*z - 15*b^4*c^2*d^2*e^4*f^2*z - 12*b^2*c^4*d^
4*e^2*f^2*z - 27*a^2*c^4*d^2*e^4*f^2*z - 9*a^2*b^2*c^2*e^6*f^2*z + 2*a*c^5*
d^6*f*h*z + 2*a*b^5*e^6*f*g*z + 33*a*c^5*d^4*e^2*f^2*z + 4*a^3*b^2*c*e^6*g^
2*z + 4*a*b^2*c^3*d^6*h^2*z - b^4*c^2*d^6*h^2*z - b^2*c^4*d^6*g^2*z - a^4*c
^2*e^6*g^2*z - a^4*b^2*e^6*h^2*z - a^2*c^4*d^6*h^2*z + 3*a^3*c^3*e^6*f^2*z
- a^2*b^4*e^6*g^2*z + b*c^5*d^6*f*g*z + 3*a^5*c*e^6*h^2*z + 3*a*c^5*d^6*g^2
*z - c^6*d^6*f^2*z - b^6*e^6*f^2*z + 6*a*b^2*c^2*d*e^2*f*g*h - 2*a*b^3*c*e^
3*f*g*h + 3*a^2*b*c^2*d^2*e*g*h^2 - 3*a^2*b*c^2*d*e^2*g^2*h - 3*a^2*b*c^2*d
*e^2*f*h^2 - 3*a*b^2*c^2*d^2*e*f*h^2 - 6*a^2*c^3*d*e^2*f*g*h + 2*a^2*b*c^2*
e^3*f*g*h + 6*a*b*c^3*d*e^2*f^2*h - 6*a*b*c^3*d*e^2*f*g^2 - 2*b^2*c^3*d^3*f
*g*h - 9*a*c^4*d^2*e*f^2*h - 3*b*c^4*d^2*e*f^2*g + 3*a*c^4*d^2*e*f*g^2 + 3*
a*c^4*d*e^2*f^2*g - 2*a^3*b*c*e^3*g*h^2 + 2*a*b*c^3*d^3*g^2*h - 2*a*b*c^3*d
^3*f*h^2 + 2*a*c^4*d^3*f*g*h - 3*b^3*c^2*d*e^2*f^2*h + 3*b^2*c^3*d^2*e*f^2*
h + 3*a^3*c^2*d*e^2*g*h^2 - 3*a^2*c^3*d^2*e*g^2*h + 9*a^2*c^3*d^2*e*f*h^2 +
3*b^2*c^3*d*e^2*f^2*g - 3*a*b^2*c^2*e^3*f^2*h + 2*a^2*b^2*c*e^3*f*h^2 - a*
b^2*c^2*d^3*g*h^2 + 2*a*b^2*c^2*e^3*f*g^2 - 3*a^3*c^2*e^3*f*h^2 + 3*a^2*c^3
*e^3*f^2*h - b^3*c^2*e^3*f^2*g - a^2*c^3*d^3*g*h^2 - a^2*c^3*e^3*f*g^2 - 3*
a^3*c^2*d^2*e*h^3 + 3*a^2*c^3*d*e^2*g^3 - a^2*b*c^2*e^3*g^3 - 3*b*c^4*d*e^2
*f^3 + a^2*b^2*c*e^3*g^2*h + a^3*c^2*e^3*g^2*h + b^3*c^2*d^3*f*h^2 + a^2*b*
c^2*d^3*h^3 + b^4*c*e^3*f^2*h + b*c^4*d^3*f^2*h + b*c^4*d^3*f*g^2 - c^5*d^3
*f^2*g + 3*c^5*d^2*e*f^3 - a*c^4*e^3*f^3 - a*c^4*d^3*g^3 + b^2*c^3*e^3*f^3
+ a^4*c*e^3*h^3, z, k)*(root(24*a^6*b*c*d*e^11*z^3 + 24*a*b*c^6*d^11*e*z^3
+ 240*a^4*b*c^3*d^5*e^7*z^3 + 240*a^3*b*c^4*d^7*e^5*z^3 + 120*a^5*b*c^2*d^3
*e^9*z^3 + 120*a^2*b*c^5*d^9*e^3*z^3 - 54*a^5*b^2*c*d^2*e^10*z^3 - 54*a*b^2
*c^5*d^10*e^2*z^3 + 50*a^4*b^3*c*d^3*e^9*z^3 + 50*a*b^3*c^4*d^9*e^3*z^3 - 3
6*a^2*b^5*c*d^5*e^7*z^3 - 36*a*b^5*c^2*d^7*e^5*z^3 + 26*a*b^6*c*d^6*e^6*z^3
- 340*a^3*b^2*c^3*d^6*e^6*z^3 - 225*a^4*b^2*c^2*d^4*e^8*z^3 - 225*a^2*b^2*
c^4*d^8*e^4*z^3 + 180*a^3*b^3*c^2*d^5*e^7*z^3 + 180*a^2*b^3*c^3*d^7*e^5*z^3
- 30*a^2*b^4*c^2*d^6*e^6*z^3 - 6*b^7*c*d^7*e^5...

3.155 $\int \frac{(d+ex)^2(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx$

Optimal. Leaf size=288

$$\frac{e^2(2c^2f-bcg+2b^2h-6ach)\,x}{c^2\,(b^2-4ac)}+\frac{(d+ex)^2\,\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-(2c^2f-bcg+b^2h-2ach)\,x\right)}{c\,(b^2-4ac)\,(a+bx+cx^2)}+\frac{(4c^4d^2f}{}$$

```
[Out] e^2*(2*c^2*f+2*b^2*h-c*(6*a*h+b*g))*x/c^2/(-4*a*c+b^2)+(e*x+d)^2*(c*(2*a*g-
b*(f+a*h/c))-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)
+(4*c^4*d^2*f-2*b^4*e^2*h-6*a*c^2*e*(2*a*e*h+2*b*d*h+b*e*g)+b^2*c*e*(12*a*e
*h+2*b*d*h+b*e*g)-c^3*(2*b*d*(d*g+2*e*f)-4*a*(d^2*h+2*d*e*g+e^2*f)))*arctan
h((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)+1/2*e*(-2*b*e*h+2*c*
d*h+c*e*g)*ln(c*x^2+b*x+a)/c^3
```

Rubi [A]
time = 0.45, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1658, 787, 648, 632, 212, 642}

$$\frac{(d+ex)^2\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x(-2ach+b^2h-bcg+2c^2f)\right)}{c\,(b^2-4ac)\,(a+bx+cx^2)}+\frac{e^2x(-6ach+2b^2h-bcg+2c^2f)}{c^2\,(b^2-4ac)}+\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(b^2ce(12aeh+2bdh+beg)-c^3(2bd(dg+2ef)-4a(d^2h+2deg+e^2f))-6ac^2e(2aeh+2bdh+beg)-2b^4e^2h+4c^4d^2f)}{c^3\,(b^2-4ac)^{3/2}}+\frac{e\log\left(a+bx+cx^2\right)(-2beh+2cdh+ceg)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

```
[Out] (e^2*(2*c^2*f - b*c*g + 2*b^2*h - 6*a*c*h)*x)/(c^2*(b^2 - 4*a*c)) + ((d + e
*x)^2*(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*x)
)/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((4*c^4*d^2*f - 2*b^4*e^2*h - 6*a*c
^2*e*(b*e*g + 2*b*d*h + 2*a*e*h) + b^2*c*e*(b*e*g + 2*b*d*h + 12*a*e*h) - c
^3*(2*b*d*(2*e*f + d*g) - 4*a*(e^2*f + 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) + (e*(c*e*g + 2*c*d*h - 2*
b*e*h)*Log[a + b*x + c*x^2])/(2*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx = \frac{(d+ex)^2 \left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-(2c^2f-bcg+b^2h-2ach)\,x\right)}{c\,(b^2-4ac)\,(a+bx+cx^2)} + \frac{\int \frac{(d+ex}{}}{}$$

$$= \frac{e^2(2c^2f-bcg+2b^2h-6ach)\,x}{c^2\,(b^2-4ac)} + \frac{(d+ex)^2 \left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-(2c^2\right.}{c\,(b^2-4ac)\,(a+bx}$$

$$= \frac{e^2(2c^2f-bcg+2b^2h-6ach)\,x}{c^2\,(b^2-4ac)} + \frac{(d+ex)^2 \left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-(2c^2\right.}{c\,(b^2-4ac)\,(a+bx}$$

$$= \frac{e^2(2c^2f-bcg+2b^2h-6ach)\,x}{c^2\,(b^2-4ac)} + \frac{(d+ex)^2 \left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-(2c^2\right.}{c\,(b^2-4ac)\,(a+bx}$$

$$= \frac{e^2(2c^2f-bcg+2b^2h-6ach)\,x}{c^2\,(b^2-4ac)} + \frac{(d+ex)^2 \left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-(2c^2\right.}{c\,(b^2-4ac)\,(a+bx}$$

Mathematica [A]
time = 0.52, size = 398, normalized size = 1.38

$$\frac{2ce^2hx-\frac{2\left(b^4e^2hx+b^3e(aeh-c(eg+2dh)x)+b^2c\left(c(e^2f+2deg+d^2h)x-ae(eg+2dh+4ehx)\right)+2c^2\left(c^2d^2fx-ac\left(e^2fx+2de(f+gx)+d^2(g+hx)\right)+a^2e(2dh+e(g+hx))\right)+bc\left(-3a^2e^2h+c^2d(-2efx+d(f-gx))+ac\left(d^2h+e^2(f+3gx)+2de(g+3hx)\right)\right)\right)}{(b^2-4ac)(a+x(b+cx))}+\frac{2\left(4c^4d^2f-2b^4e^2h-6ac^2e(beg+2bdh+2aeh)+b^2ce(beg+2bdh+12aeh)+c^3\left(-2bd(2ef+dg)+4a(e^2f+2deg+d^2h)\right)\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+e(ceg+2cdh-2beh)\log(a+x(b+cx))}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

```
[Out] (2*c*e^2*h*x - (2*(b^4*e^2*h*x + b^3*e*(a*e*h - c*(e*g + 2*d*h)*x) + b^2*c*
(c*(e^2*f + 2*d*e*g + d^2*h)*x - a*e*(e*g + 2*d*h + 4*e*h*x)) + 2*c^2*(c^2*
d^2*f*x - a*c*(e^2*f*x + 2*d*e*(f + g*x) + d^2*(g + h*x)) + a^2*e*(2*d*h +
e*(g + h*x))) + b*c*(-3*a^2*e^2*h + c^2*d*(-2*e*f*x + d*(f - g*x)) + a*c*(d
^2*h + e^2*(f + 3*g*x) + 2*d*e*(g + 3*h*x)))))/((b^2 - 4*a*c)*(a + x*(b + c
*x))) + (2*(4*c^4*d^2*f - 2*b^4*e^2*h - 6*a*c^2*e*(b*e*g + 2*b*d*h + 2*a*e*
h) + b^2*c*e*(b*e*g + 2*b*d*h + 12*a*e*h) + c^3*(-2*b*d*(2*e*f + d*g) + 4*a
*(e^2*f + 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2
+ 4*a*c)^(3/2) + e*(c*e*g + 2*c*d*h - 2*b*e*h)*Log[a + x*(b + c*x)])/(2*c^3
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 627 vs. 2(283) = 566.
time = 0.20, size = 628, normalized size = 2.18

method	result

default	$\frac{h e^2 x}{c^2} - \frac{-\frac{\left(2a^2c^2e^2h-4a b^2c e^2h+6ab c^2deh+3ab c^2e^2g-2a c^3d^2h-4a c^3deg-2a c^3e^2f+b^4e^2h-2b^3cdeh-b^3c e^2g+b^2c^2d^2h+2b^2c^2deg+b^2c^2e^2f}{c\left(4ac-b^2\right)}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] h*e^2/c^2*x-1/c^2*((-(2*a^2*c^2*e^2*h-4*a*b^2*c*e^2*h+6*a*b*c^2*d*e*h+3*a*b
*c^2*e^2*g-2*a*c^3*d^2*h-4*a*c^3*d*e*g-2*a*c^3*e^2*f+b^4*e^2*h-2*b^3*c*d*e*
h-b^3*c*e^2*g+b^2*c^2*d^2*h+2*b^2*c^2*d*e*g+b^2*c^2*e^2*f-b*c^3*d^2*g-2*b*c
^3*d*e*f+2*c^4*d^2*f)/c/(4*a*c-b^2)*x+(3*a^2*b*c*e^2*h-4*a^2*c^2*d*e*h-2*a^
2*c^2*e^2*g-a*b^3*e^2*h+2*a*b^2*c*d*e*h+a*b^2*c*e^2*g-a*b*c^2*d^2*h-2*a*b*c
^2*d*e*g-a*b*c^2*e^2*f+2*a*c^3*d^2*g+4*a*c^3*d*e*f-b*c^3*d^2*f)/c/(4*a*c-b^
2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(8*a*b*c*e^2*h-8*a*c^2*d*e*h-4*a*c^2*e
^2*g-2*b^3*e^2*h+2*b^2*c*d*e*h+b^2*c*e^2*g)/c*ln(c*x^2+b*x+a)+2*(6*a^2*c*e^
2*h-2*a*b^2*e^2*h+2*a*b*c*d*e*h+a*b*c*e^2*g-2*a*c^2*d^2*h-4*a*c^2*d*e*g-2*a
*c^2*e^2*f+b*c^2*d^2*g+2*b*c^2*d*e*f-2*c^3*d^2*f-1/2*(8*a*b*c*e^2*h-8*a*c^2
*d*e*h-4*a*c^2*e^2*g-2*b^3*e^2*h+2*b^2*c*d*e*h+b^2*c*e^2*g)*b/c)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

<u>Maxima</u> [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1335 vs. 2(290) = 580.
time = 0.56, size = 2690, normalized size = 9.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^3*c^3 - 4*a*b*c^4)*d^2*f - 4*(a*b^2*c^3 - 4*a^2*c^4)*d^2*g + 2*
(a*b^3*c^2 - 4*a^2*b*c^3)*d^2*h - (4*a*c^4*d^2*f - 2*a*b*c^3*d^2*g + 4*a^2*
c^3*d^2*h + 2*(2*c^5*d^2*f - b*c^4*d^2*g + 2*a*c^4*d^2*h)*x^2 + 2*(2*b*c^4*
d^2*f - b^2*c^3*d^2*g + 2*a*b*c^3*d^2*h)*x + (4*a^2*c^3*f + (4*a*c^4*f + (b
^3*c^2 - 6*a*b*c^3)*g - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*h)*x^2 + (a*b^3
*c - 6*a^2*b*c^2)*g - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*h + (4*a*b*c^3*f
+ (b^4*c - 6*a*b^2*c^2)*g - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*h)*x)*e^2 - 2
*(2*a*b*c^3*d*f - 4*a^2*c^3*d*g - (a*b^3*c - 6*a^2*b*c^2)*d*h + (2*b*c^4*d*
f - 4*a*c^4*d*g - (b^3*c^2 - 6*a*b*c^3)*d*h)*x^2 + (2*b^2*c^3*d*f - 4*a*b*c
^3*d*g - (b^4*c - 6*a*b^2*c^2)*d*h)*x)*e)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2
+ 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a))
 + 2*(2*(b^2*c^4 - 4*a*c^5)*d^2*f - (b^3*c^3 - 4*a*b*c^4)*d^2*g + (b^4*c^2
- 6*a*b^2*c^3 + 8*a^2*c^4)*d^2*h)*x - 2*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*h*x^3 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*h*x^2 - (a*b^3*c^2 - 4*a^2*
b*c^3)*f + (a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*g - (a*b^5 - 7*a^2*b^3*c +
 12*a^3*b*c^2)*h - ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*f - (b^5*c - 7*a*b^
3*c^2 + 12*a^2*b*c^3)*g + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*h
)*x)*e^2 - 4*(2*(a*b^2*c^3 - 4*a^2*c^4)*d*f - (a*b^3*c^2 - 4*a^2*b*c^3)*d*g
 + (a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*d*h + ((b^3*c^3 - 4*a*b*c^4)*d*f -
 (b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d*g + (b^5*c - 7*a*b^3*c^2 + 12*a^2*b*
c^3)*d*h)*x)*e - ((((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*g - 2*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*h)*x^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*g
- 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*h + ((b^5*c - 8*a*b^3*c^2 + 16*a^2
*b*c^3)*g - 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*h)*x)*e^2 + 2*((b^4*c^2 -
8*a*b^2*c^3 + 16*a^2*c^4)*d*h*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*
h*x + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*d*h)*e)*log(c*x^2 + b*x + a))/
(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c
^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x), -1/2*(2*(b^3*c^3 - 4*a
*b*c^4)*d^2*f - 4*(a*b^2*c^3 - 4*a^2*c^4)*d^2*g + 2*(a*b^3*c^2 - 4*a^2*b*c^
3)*d^2*h - 2*(4*a*c^4*d^2*f - 2*a*b*c^3*d^2*g + 4*a^2*c^3*d^2*h + 2*(2*c^5*
d^2*f - b*c^4*d^2*g + 2*a*c^4*d^2*h)*x^2 + 2*(2*b*c^4*d^2*f - b^2*c^3*d^2*g
 + 2*a*b*c^3*d^2*h)*x + (4*a^2*c^3*f + (4*a*c^4*f + (b^3*c^2 - 6*a*b*c^3)*g
 - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*h)*x^2 + (a*b^3*c - 6*a^2*b*c^2)*g -
 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*h + (4*a*b*c^3*f + (b^4*c - 6*a*b^2*c^
2)*g - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*h)*x)*e^2 - 2*(2*a*b*c^3*d*f - 4*a
^2*c^3*d*g - (a*b^3*c - 6*a^2*b*c^2)*d*h + (2*b*c^4*d*f - 4*a*c^4*d*g - (b^
3*c^2 - 6*a*b*c^3)*d*h)*x^2 + (2*b^2*c^3*d*f - 4*a*b*c^3*d*g - (b^4*c - 6*a
*b^2*c^2)*d*h)*x)*e)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x +
 b)/(b^2 - 4*a*c)) + 2*(2*(b^2*c^4 - 4*a*c^5)*d^2*f - (b^3*c^3 - 4*a*b*c^4)
*d^2*g + (b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2*h)*x - 2*((b^4*c^2 - 8*a*b
^2*c^3 + 16*a^2*c^4)*h*x^3 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*h*x^2 - (
a*b^3*c^2 - 4*a^2*b*c^3)*f + (a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*g - (a*b
^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*h - ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*f
 - (b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*g + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c
^2 - 24*a^3*c^3)*h)*x)*e^2 - 4*(2*(a*b^2*c^3 - 4*a^2*c^4)*d*f - (a*b^3*c^2
```

```
- 4*a^2*b*c^3)*d*g + (a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*d*h + ((b^3*c^3
- 4*a*b*c^4)*d*f - (b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d*g + (b^5*c - 7*a*b
^3*c^2 + 12*a^2*b*c^3)*d*h)*x)*e - ((((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*
g - 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*h)*x^2 + (a*b^4*c - 8*a^2*b^2*c^
2 + 16*a^3*c^3)*g - 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*h + ((b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*g - 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*h)*x)*e^
2 + 2*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*h*x^2 + (b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3)*d*h*x + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*d*h)*e)*log(
c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*
b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
[Out] Timed out
```

Giac [A]
time = 4.30, size = 540, normalized size = 1.88

$$\frac{hxe^2}{c^2} - \frac{(4c^4d^2f - 2bc^3d^2g + 4ac^3d^2h - 4bc^3dfe + 8ac^3dge + 2b^3cdhe - 12abc^2dhe + 4ac^3fe^2 + b^3cge^2 - 6abc^2ge^2 - 2b^4he^2 + 12ab^2che^2 - 12a^2c^2he^2)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2c^3-4ac^4)\sqrt{-b^2+4ac}} + \frac{(2cdhe + cge^2 - 2bhe^2)\log(cx^2+bx+a)}{2c^3} - \frac{\frac{(2c^4d^2f - bc^3d^2g + b^2c^2d^2h - 2ac^3d^2h - 2bc^3dfe + 2b^2c^2dge - 4ac^3dge - 2b^3cdhe + 6abc^2dhe + b^2c^2fe^2 - 2ac^3fe^2 - b^3cge^2 + 3abc^2ge^2 + b^4he^2 - 4ab^2che^2 + 2a^2c^2he^2)x}{c} + \frac{bc^3d^2f - 2ac^3d^2g + abc^2d^2h - 4ac^3dfe + 2abc^2dge - 2ab^2cdhe + 4a^2c^2dhe + abc^2fe^2 - ab^2cge^2 + 2a^2c^2ge^2 + ab^3he^2 - 3a^2bche^2}{c}}{(cx^2+bx+a)(b^2-4ac)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
[Out] h*x*e^2/c^2 - (4*c^4*d^2*f - 2*b*c^3*d^2*g + 4*a*c^3*d^2*h - 4*b*c^3*d*f*e
+ 8*a*c^3*d*g*e + 2*b^3*c*d*h*e - 12*a*b*c^2*d*h*e + 4*a*c^3*f*e^2 + b^3*c*
g*e^2 - 6*a*b*c^2*g*e^2 - 2*b^4*h*e^2 + 12*a*b^2*c*h*e^2 - 12*a^2*c^2*h*e^2
)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4
*a*c)) + 1/2*(2*c*d*h*e + c*g*e^2 - 2*b*h*e^2)*log(c*x^2 + b*x + a)/c^3 - (
(2*c^4*d^2*f - b*c^3*d^2*g + b^2*c^2*d^2*h - 2*a*c^3*d^2*h - 2*b*c^3*d*f*e
+ 2*b^2*c^2*d*g*e - 4*a*c^3*d*g*e - 2*b^3*c*d*h*e + 6*a*b*c^2*d*h*e + b^2*c
^2*f*e^2 - 2*a*c^3*f*e^2 - b^3*c*g*e^2 + 3*a*b*c^2*g*e^2 + b^4*h*e^2 - 4*a*
b^2*c*h*e^2 + 2*a^2*c^2*h*e^2)*x/c + (b*c^3*d^2*f - 2*a*c^3*d^2*g + a*b*c^2
*d^2*h - 4*a*c^3*d*f*e + 2*a*b*c^2*d*g*e - 2*a*b^2*c*d*h*e + 4*a^2*c^2*d*h*
e + a*b*c^2*f*e^2 - a*b^2*c*g*e^2 + 2*a^2*c^2*g*e^2 + a*b^3*h*e^2 - 3*a^2*b
*c*h*e^2)/c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B]
time = 5.78, size = 742, normalized size = 2.58

$$[\text{illegible}] + \frac{\ln(cx^2+bx+a)\,(-128ha^3bc^3e^2 + 64ga^3c^4e^2 + 128dha^3c^4e + 96ha^2b^3c^2e^2 - 48ga^2b^2c^3e^2 - 96dha^2b^2c^3e - 24hab^5ce^2 + 12gab^4c^2e^2 + 24dhab^4c^2e + 2hb^7e^2 - gb^6ce^2 - 2dhb^6ce)}{2(64a^3c^6 - 48a^2b^2c^5 + 12ab^4c^4 - b^6c^3)} + [\text{illegible}] + \frac{e^2hx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x)
```

```
[Out] ((2*a^2*c^2*e^2*g - 2*a*c^3*d^2*g + b*c^3*d^2*f + a*b^3*e^2*h + a*b*c^2*e^2
*f + a*b*c^2*d^2*h - a*b^2*c*e^2*g - 3*a^2*b*c*e^2*h + 4*a^2*c^2*d*e*h - 4*
a*c^3*d*e*f + 2*a*b*c^2*d*e*g - 2*a*b^2*c*d*e*h)/(c*(4*a*c - b^2)) + (x*(2*
c^4*d^2*f + b^4*e^2*h + b^2*c^2*e^2*f + 2*a^2*c^2*e^2*h + b^2*c^2*d^2*h - 2
*a*c^3*e^2*f - 2*a*c^3*d^2*h - b*c^3*d^2*g - b^3*c*e^2*g + 3*a*b*c^2*e^2*g
- 4*a*b^2*c*e^2*h + 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - 2*b*c^3*d*e*f - 2*b^3
*c*d*e*h + 6*a*b*c^2*d*e*h))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x)
 + (log(a + b*x + c*x^2)*(2*b^7*e^2*h + 64*a^3*c^4*e^2*g - b^6*c*e^2*g - 24
*a*b^5*c*e^2*h + 128*a^3*c^4*d*e*h + 12*a*b^4*c^2*e^2*g - 128*a^3*b*c^3*e^2
*h - 2*b^6*c*d*e*h - 48*a^2*b^2*c^3*e^2*g + 96*a^2*b^3*c^2*e^2*h + 24*a*b^4
*c^2*d*e*h - 96*a^2*b^2*c^3*d*e*h))/(2*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4
 - 48*a^2*b^2*c^5)) + (atan((2*c*x)/(4*a*c - b^2)^(1/2) - (b^3*c^2 - 4*a*b*
c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(4*c^4*d^2*f - 2*b^4*e^2*h - 12*a^2*c^2*e^2
*h + 4*a*c^3*e^2*f + 4*a*c^3*d^2*h - 2*b*c^3*d^2*g + b^3*c*e^2*g - 6*a*b*c^
2*e^2*g + 12*a*b^2*c*e^2*h + 8*a*c^3*d*e*g - 4*b*c^3*d*e*f + 2*b^3*c*d*e*h
- 12*a*b*c^2*d*e*h))/(c^3*(4*a*c - b^2)^(3/2)) + (e^2*h*x)/c^2
```

3.156 $\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=178

$$\frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\left(4c^3df+b^3eh-6abceh-2c^2(b(ef+dg)-\right.}{c^2\left(b^2-4ac\right.}$$

```
[Out] (e*x+d)*(c*(2*a*g-b*(f+a*h/c))-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*x)/c/(-4*a*c+
b^2)/(c*x^2+b*x+a)+(4*c^3*d*f+b^3*e*h-6*a*b*c*e*h-2*c^2*(b*(d*g+e*f)-2*a*(d
*h+e*g)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/2*
e*h*ln(c*x^2+b*x+a)/c^2
```

Rubi [A]
time = 0.17, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1658, 648, 632, 212, 642}

$$\frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2c^2(b(dg+ef)-2a(dh+eg))-6abceh+b^3eh+4c^3df\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{eh\log\left(a+bx+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

```
[Out] ((d + e*x)*(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*
h)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((4*c^3*d*f + b^3*e*h - 6*a*b*
c*e*h - 2*c^2*(b*(e*f + d*g) - 2*a*(e*g + d*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(c^2*(b^2 - 4*a*c)^(3/2)) + (e*h*Log[a + b*x + c*x^2])/(2*c^2
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{2cdf-b(\cdots}{\cdots}}{\cdots}$$

$$= \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{(eh)\int \frac{\cdots}{a\cdots}}{2\cdots}$$

$$= \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{eh\log(a\cdots}{\cdots}$$

$$= \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\left(4c^3df+\cdots\right.}{\cdots}$$

Mathematica [A]

time = 0.29, size = 225, normalized size = 1.26

$$\frac{-\frac{2\left(-b^3ehx+b^2(-aeh+c(eg+dh)x)+bc(adh-cefx+cd(f-gx)+ae(g+3hx))+2c\left(a^2eh+c^2dfx-ac(e(f+gx)+d(g+hx))\right)\right)}{\left(b^2-4ac\right)(a+x(b+cx))}+\frac{2\left(4c^3df+b^3eh-6abceh-2c^2(b(ef+dg)-2a(eg+dh))\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{3/2}}+eh\log(a+x(b+cx))}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

```
[Out] ((-2*(-(b^3*e*h*x) + b^2*(-(a*e*h) + c*(e*g + d*h)*x) + b*c*(a*d*h - c*e*f*
x + c*d*(f - g*x) + a*e*(g + 3*h*x)) + 2*c*(a^2*e*h + c^2*d*f*x - a*c*(e*(f
 + g*x) + d*(g + h*x)))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*(4*c^3*d*f
 + b^3*e*h - 6*a*b*c*e*h - 2*c^2*(b*(e*f + d*g) - 2*a*(e*g + d*h)))*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + e*h*Log[a + x*(b +
c*x)])/(2*c^2)
```

Maple [A]
time = 0.16, size = 308, normalized size = 1.73

method	result
default	$\frac{\frac{\left(3abceh-2a\,c^{2}dh-2a\,c^{2}eg-b^{3}eh+b^{2}cdh+b^{2}ceg-b\,c^{2}dg-b\,c^{2}ef+2c^{3}df\right)x}{c^{2}\left(4ac-b^{2}\right)}+\frac{2a^{2}ceh-a\,b^{2}eh+abcdh+abceg-2a\,c^{2}dg-2a\,c^{2}ef+b\,c^{2}df}{\left(4ac-b^{2}\right)c^{2}}}{c\,x^{2}+bx+a}+\frac{\left(4aceh-b\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((3*a*b*c*e*h-2*a*c^2*d*h-2*a*c^2*e*g-b^3*e*h+b^2*c*d*h+b^2*c*e*g-b*c^2*d*g
-b*c^2*e*f+2*c^3*d*f)/c^2/(4*a*c-b^2)*x+(2*a^2*c*e*h-a*b^2*e*h+a*b*c*d*h+a*
b*c*e*g-2*a*c^2*d*g-2*a*c^2*e*f+b*c^2*d*f)/(4*a*c-b^2)/c^2)/(c*x^2+b*x+a)+1
/c/(4*a*c-b^2)*(1/2*(4*a*c*e*h-b^2*e*h)/c*ln(c*x^2+b*x+a)+2*(-a*b*h*e+2*a*c
*d*h+2*a*c*e*g-b*c*d*g-b*c*e*f+2*c^2*d*f-1/2*(4*a*c*e*h-b^2*e*h)*b/c)/(4*a*
c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(178) = 356$.
time = 0.38, size = 1392, normalized size = 7.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^3*c^2 - 4*a*b*c^3)*d*f - 4*(a*b^2*c^2 - 4*a^2*c^3)*d*g + 2*(a*b
^3*c - 4*a^2*b*c^2)*d*h - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*h*x^2 + (b^5
- 8*a*b^3*c + 16*a^2*b*c^2)*h*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*h)*e*l
og(c*x^2 + b*x + a) - (4*a*c^3*d*f - 2*a*b*c^2*d*g + 4*a^2*c^2*d*h + 2*(2*c
^4*d*f - b*c^3*d*g + 2*a*c^3*d*h)*x^2 + 2*(2*b*c^3*d*f - b^2*c^2*d*g + 2*a*
b*c^2*d*h)*x - (2*a*b*c^2*f - 4*a^2*c^2*g + (2*b*c^3*f - 4*a*c^3*g - (b^3*c
 - 6*a*b*c^2)*h)*x^2 - (a*b^3 - 6*a^2*b*c)*h + (2*b^2*c^2*f - 4*a*b*c^2*g -
 (b^4 - 6*a*b^2*c)*h)*x)*e)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^
2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(2*(b^2*c
^3 - 4*a*c^4)*d*f - (b^3*c^2 - 4*a*b*c^3)*d*g + (b^4*c - 6*a*b^2*c^2 + 8*a^
2*c^3)*d*h)*x - 2*(2*(a*b^2*c^2 - 4*a^2*c^3)*f - (a*b^3*c - 4*a^2*b*c^2)*g
+ (a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*h + ((b^3*c^2 - 4*a*b*c^3)*f - (b^4*c -
 6*a*b^2*c^2 + 8*a^2*c^3)*g + (b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*h)*x)*e)/(a*
b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x), -1/2*(2*(b^3*c^2 - 4*a*b*
c^3)*d*f - 4*(a*b^2*c^2 - 4*a^2*c^3)*d*g + 2*(a*b^3*c - 4*a^2*b*c^2)*d*h -
((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*h*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2
)*h*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*h)*e*log(c*x^2 + b*x + a) - 2*(4
*a*c^3*d*f - 2*a*b*c^2*d*g + 4*a^2*c^2*d*h + 2*(2*c^4*d*f - b*c^3*d*g + 2*a
*c^3*d*h)*x^2 + 2*(2*b*c^3*d*f - b^2*c^2*d*g + 2*a*b*c^2*d*h)*x - (2*a*b*c^
2*f - 4*a^2*c^2*g + (2*b*c^3*f - 4*a*c^3*g - (b^3*c - 6*a*b*c^2)*h)*x^2 - (
a*b^3 - 6*a^2*b*c)*h + (2*b^2*c^2*f - 4*a*b*c^2*g - (b^4 - 6*a*b^2*c)*h)*x)
*e)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)
) + 2*(2*(b^2*c^3 - 4*a*c^4)*d*f - (b^3*c^2 - 4*a*b*c^3)*d*g + (b^4*c - 6*a
*b^2*c^2 + 8*a^2*c^3)*d*h)*x - 2*(2*(a*b^2*c^2 - 4*a^2*c^3)*f - (a*b^3*c -
4*a^2*b*c^2)*g + (a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*h + ((b^3*c^2 - 4*a*b*c^
3)*f - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*g + (b^5 - 7*a*b^3*c + 12*a^2*b*c^
2)*h)*x)*e)/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^
4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1535 vs. $2(170) = 340$.
time = 28.79, size = 1535, normalized size = 8.62

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] (e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*
c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64
*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-16*a**2*c*
```

```
*3*(e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4
*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*
(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 8*a**2*c*e*h +
8*a*b**2*c**2*(e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c
**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*
f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - a*b*
*2*e*h - 2*a*b*c*d*h - 2*a*b*c*e*g - b**4*c*(e*h/(2*c**2) - sqrt(-(4*a*c -
b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*
d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6))) + b**2*c*d*g + b**2*c*e*f - 2*b*c**2*d*f)/(6*a*b*c*
e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f
- 4*c**3*d*f)) + (e*h/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*
a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3
*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log
(x + (-16*a**2*c**3*(e*h/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h -
 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c
**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))
+ 8*a**2*c*e*h + 8*a*b**2*c**2*(e*h/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*
a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**
2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6))) - a*b**2*e*h - 2*a*b*c*d*h - 2*a*b*c*e*g - b**4*c*(e*h/(2*c**2)
+ sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**
3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 4
8*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + b**2*c*d*g + b**2*c*e*f - 2*b*c*
*2*d*f)/(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*
g + 2*b*c**2*e*f - 4*c**3*d*f)) + (2*a**2*c*e*h - a*b**2*e*h + a*b*c*d*h +
a*b*c*e*g - 2*a*c**2*d*g - 2*a*c**2*e*f + b*c**2*d*f + x*(3*a*b*c*e*h - 2*a
*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g
- b*c**2*e*f + 2*c**3*d*f))/(4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b
**2*c**3) + x*(4*a*b*c**3 - b**3*c**2))
```

Giac [A]
time = 5.49, size = 285, normalized size = 1.60

$$\frac{he\log\left(cx^2+bx+a\right)}{2\,c^2}-\frac{\left(4\,c^3df-2\,bc^2dg+4\,ac^2dh-2\,bc^2fe+4\,ac^2ge+b^3he-6\,abche\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^2-4\,ac^3\right)\sqrt{-b^2+4\,ac}}-\frac{bc^2df-2\,ac^2dg+abcdh-2\,ac^2fe+abcge-ab^2he+2\,a^2che+\left(2\,c^3df-bc^2dg+b^2cdh-2\,ac^2dh-bc^2fe+b^2cge-2\,ac^2ge-b^3he+3\,abche\right)x}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*h*e*log(c*x^2 + b*x + a)/c^2 - (4*c^3*d*f - 2*b*c^2*d*g + 4*a*c^2*d*h -
 2*b*c^2*f*e + 4*a*c^2*g*e + b^3*h*e - 6*a*b*c*h*e)*arctan((2*c*x + b)/sqrt
(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) - (b*c^2*d*f - 2*a
*c^2*d*g + a*b*c*d*h - 2*a*c^2*f*e + a*b*c*g*e - a*b^2*h*e + 2*a^2*c*h*e +
(2*c^3*d*f - b*c^2*d*g + b^2*c*d*h - 2*a*c^2*d*h - b*c^2*f*e + b^2*c*g*e -
2*a*c^2*g*e - b^3*h*e + 3*a*b*c*h*e)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^
2)
```

Mupad [B]
time = 5.04, size = 376, normalized size = 2.11

$$\frac{\frac{b\,c^2\,d\,f-2\,a\,c^2\,e\,f-2\,a\,c^2\,d\,g-a\,b^2\,e\,h+2\,a^2\,c\,e\,h+a\,b\,c\,d\,h+a\,b\,c\,e\,g}{c^2\,\left(4\,a\,c-b^2\right)}-\frac{x\,\left(b^3\,e\,h-2\,c^3\,d\,f+2\,a\,c^2\,d\,h+2\,a\,c^2\,e\,g+b\,c^2\,d\,g+b\,c^2\,e\,f-b^2\,c\,d\,h-b^2\,c\,e\,g-3\,a\,b\,c\,e\,h\right)}{c^2\,\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a}-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-64\,e\,h\,a^3\,c^3+48\,e\,h\,a^2\,b^2\,c^2-12\,e\,h\,a\,b^4\,c+e\,h\,b^6\right)}{2\,\left(64\,a^3\,c^5-48\,a^2\,b^2\,c^4+12\,a\,b^4\,c^3-b^6\,c^2\right)}+\frac{\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c-4\,a\,b\,c^2}{c\,{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,c^3\,d\,f+b^3\,e\,h+4\,a\,c^2\,d\,h+4\,a\,c^2\,e\,g-2\,b\,c^2\,d\,g-2\,b\,c^2\,e\,f-6\,a\,b\,c\,e\,h\right)}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x)
```

```
[Out] ((b*c^2*d*f - 2*a*c^2*e*f - 2*a*c^2*d*g - a*b^2*e*h + 2*a^2*c*e*h + a*b*c*d
*h + a*b*c*e*g)/(c^2*(4*a*c - b^2)) - (x*(b^3*e*h - 2*c^3*d*f + 2*a*c^2*d*h
 + 2*a*c^2*e*g + b*c^2*d*g + b*c^2*e*f - b^2*c*d*h - b^2*c*e*g - 3*a*b*c*e*
h))/(c^2*(4*a*c - b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6*e*h
 - 64*a^3*c^3*e*h + 48*a^2*b^2*c^2*e*h - 12*a*b^4*c*e*h))/(2*(64*a^3*c^5 -
b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)) + (atan((2*c*x)/(4*a*c - b^2)^(1/
2) - (b^3*c - 4*a*b*c^2)/(c*(4*a*c - b^2)^(3/2)))*(4*c^3*d*f + b^3*e*h + 4*
a*c^2*d*h + 4*a*c^2*e*g - 2*b*c^2*d*g - 2*b*c^2*e*f - 6*a*b*c*e*h))/(c^2*(4
*a*c - b^2)^(3/2))
```

3.157 $\int \frac{f+gx+hx^2}{\left(a+bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=118

$$\frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{2(2cf-bg+2ah)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (c*(2*a*g-b*(f+a*h/c))-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*x)/c/(-4*a*c+b^2)/(c*
x^2+b*x+a)+2*(2*a*h-b*g+2*c*f)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1674, 12, 632, 212}

$$\frac{c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2ah-bg+2cf)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*x)/(c*(b
^2 - 4*a*c)*(a + b*x + c*x^2)) + (2*(2*c*f - b*g + 2*a*h)*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx &= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-(2c^2f-bcg+b^2h-2ach)\,x}{c\,(b^2-4ac)\,(a+bx+cx^2)}+\frac{\int \frac{2cf-bg+2ah}{a+bx+cx^2}\,dx}{-b^2+4ac}\\ &= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-(2c^2f-bcg+b^2h-2ach)\,x}{c\,(b^2-4ac)\,(a+bx+cx^2)}-\frac{(2cf-bg+2ah)\int \frac{1}{a+bx+cx^2}\\ &= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-(2c^2f-bcg+b^2h-2ach)\,x}{c\,(b^2-4ac)\,(a+bx+cx^2)}+\frac{(2(2cf-bg+2ah))\text{Subst}\big(\\ &= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-(2c^2f-bcg+b^2h-2ach)\,x}{c\,(b^2-4ac)\,(a+bx+cx^2)}+\frac{2(2cf-bg+2ah)\tanh^{-1}\Big(}{(b^2-4ac)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 114, normalized size = 0.97

$$\frac{abh+2c^2fx+b^2hx+bc(f-gx)-2ac(g+hx)}{c\,(-b^2+4ac)\,(a+x(b+cx))}-\frac{2(-2cf+bg-2ah)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x]
[Out] (a*b*h + 2*c^2*f*x + b^2*h*x + b*c*(f - g*x) - 2*a*c*(g + h*x))/(c*(-b^2 +
4*a*c)*(a + x*(b + c*x))) - (2*(-2*c*f + b*g - 2*a*h)*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A]
time = 0.16, size = 133, normalized size = 1.13

method	result

default	$\frac{-\frac{\left(2ach-b^{2}h+bcg-2c^{2}f\right)x}{c\left(4ac-b^{2}\right)}+\frac{abh-2acg+bcf}{c\left(4ac-b^{2}\right)}}{c\,x^{2}+bx+a}+\frac{2(2ah-bg+2cf)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{(4ac-b^{2})^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(2ach-b^{2}h+bcg-2c^{2}f\right)x}{c\left(4ac-b^{2}\right)}+\frac{abh-2acg+bcf}{c\left(4ac-b^{2}\right)}}{c\,x^{2}+bx+a}+\frac{2\ln\left(\left(-8c^{2}a+2b^{2}c\right)x+\left(-4ac+b^{2}\right)^{\frac{3}{2}}-4abc+b^{3}\right)ah}{(-4ac+b^{2})^{\frac{3}{2}}}-\frac{\ln\left(\left(-8c^{2}a+2b^{2}c\right)x+\left(-4ac+b^{2}\right.\right.}{(-4ac+b^{2})^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-(2*a*c*h-b^2*h+b*c*g-2*c^2*f)/c/(4*a*c-b^2)*x+1/c*(a*b*h-2*a*c*g+b*c*f)/(
4*a*c-b^2))/(c*x^2+b*x+a)+2*(2*a*h-b*g+2*c*f)/(4*a*c-b^2)^(3/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(113) = 226$.
time = 0.36, size = 632, normalized size = 5.36

$$\left[-\frac{\left(2\,ac^{2}f-abcg+2\,a^{2}ch+\left(2\,c^{3}f-bc^{2}g+2\,ac^{2}h\right)x^{2}+\left(2\,bc^{2}f-b^{2}cg+2\,abch\right)x\right)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac-\sqrt{b^{2}-4\,ac}\,(2\,cx+b)}{cx^{2}+bx+a}\right)+\left(b^{3}c-4\,abc^{2}\right)f-2\left(ab^{2}c-4\,a^{2}c^{2}\right)g+\left(ab^{3}-4\,a^{2}bc\right)h+\left(2\left(b^{2}c^{2}-4\,ac^{3}\right)f-\left(b^{3}c-4\,abc^{2}\right)g+\left(b^{4}-6\,ab^{2}c+8\,a^{2}c^{2}\right)h\right)x}{ab^{4}c-8\,a^{2}b^{2}c^{2}+16\,a^{3}c^{3}+\left(b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4}\right)x^{2}+\left(b^{5}c-8\,ab^{3}c^{2}+16\,a^{2}bc^{3}\right)x},\ \frac{2\left(2\,ac^{2}f-abcg+2\,a^{2}ch+\left(2\,c^{3}f-bc^{2}g+2\,ac^{2}h\right)x^{2}+\left(2\,bc^{2}f-b^{2}cg+2\,abch\right)x\right)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}\,(2\,cx+b)}{b^{2}-4\,ac}\right)-\left(b^{3}c-4\,abc^{2}\right)f+2\left(ab^{2}c-4\,a^{2}c^{2}\right)g-\left(ab^{3}-4\,a^{2}bc\right)h-\left(2\left(b^{2}c^{2}-4\,ac^{3}\right)f-\left(b^{3}c-4\,abc^{2}\right)g+\left(b^{4}-6\,ab^{2}c+8\,a^{2}c^{2}\right)h\right)x}{ab^{4}c-8\,a^{2}b^{2}c^{2}+16\,a^{3}c^{3}+\left(b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4}\right)x^{2}+\left(b^{5}c-8\,ab^{3}c^{2}+16\,a^{2}bc^{3}\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-((2*a*c^2*f - a*b*c*g + 2*a^2*c*h + (2*c^3*f - b*c^2*g + 2*a*c^2*h)*x^2 +
 (2*b*c^2*f - b^2*c*g + 2*a*b*c*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*
b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (
b^3*c - 4*a*b*c^2)*f - 2*(a*b^2*c - 4*a^2*c^2)*g + (a*b^3 - 4*a^2*b*c)*h +
(2*(b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b*c^2)*g + (b^4 - 6*a*b^2*c + 8*a^2
*c^2)*h)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3
+ 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), (2*(2*a*c^2*f
- a*b*c*g + 2*a^2*c*h + (2*c^3*f - b*c^2*g + 2*a*c^2*h)*x^2 + (2*b*c^2*f -
```

```
b^2*c*g + 2*a*b*c*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*
x + b)/(b^2 - 4*a*c)) - (b^3*c - 4*a*b*c^2)*f + 2*(a*b^2*c - 4*a^2*c^2)*g -
 (a*b^3 - 4*a^2*b*c)*h - (2*(b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b*c^2)*g +
 (b^4 - 6*a*b^2*c + 8*a^2*c^2)*h)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3
+ (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(107) = 214$.

time = 1.29, size = 459, normalized size = 3.89

$$-\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)\log\left(x+\frac{-16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)+8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)+2abh-b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)-b^2g+2bcf}{4ach-2bcg+4c^2f}\right)+\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)\log\left(x+\frac{16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)-8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)+2abh+b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)-b^2g+2bcf}{4ach-2bcg+4c^2f}\right)+\frac{abh-2acg+bcf+x(-2ach+b^2h-bcg+2c^2f)}{4a^2c^2-ab^2c+x^2\cdot(4ac^3-b^2c^2)+x(4abc^2-b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f)*log(x + (-16*a**2*c**2*sq
rt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f) + 8*a*b**2*c*sqrt(-1/(4*a*c
- b**2)**3)*(2*a*h - b*g + 2*c*f) + 2*a*b*h - b**4*sqrt(-1/(4*a*c - b**2)**
3)*(2*a*h - b*g + 2*c*f) - b**2*g + 2*b*c*f)/(4*a*c*h - 2*b*c*g + 4*c**2*f)
) + sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f)*log(x + (16*a**2*c**2*
sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f) - 8*a*b**2*c*sqrt(-1/(4*a*
c - b**2)**3)*(2*a*h - b*g + 2*c*f) + 2*a*b*h + b**4*sqrt(-1/(4*a*c - b**2)
**3)*(2*a*h - b*g + 2*c*f) - b**2*g + 2*b*c*f)/(4*a*c*h - 2*b*c*g + 4*c**2*
f)) + (a*b*h - 2*a*c*g + b*c*f + x*(-2*a*c*h + b**2*h - b*c*g + 2*c**2*f))/
(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*(4*a*b*c**2 - b**
3*c))
```

Giac [A]

time = 4.22, size = 125, normalized size = 1.06

$$-\frac{2\,(2\,cf-bg+2\,ah)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{2\,c^2fx-bcgx+b^2hx-2\,achx+bcf-2\,acg+abh}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*(2*c*f - b*g + 2*a*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a
*c)*sqrt(-b^2 + 4*a*c)) - (2*c^2*f*x - b*c*g*x + b^2*h*x - 2*a*c*h*x + b*c*
f - 2*a*c*g + a*b*h)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

Mupad [B]

time = 3.90, size = 203, normalized size = 1.72

$$\frac{\frac{a\,b\,h-2\,a\,c\,g+b\,c\,f}{c\,(4\,a\,c-b^2)}+\frac{x\,(h\,b^2-g\,b\,c+2\,f\,c^2-2\,a\,h\,c)}{c\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a}-\frac{2\,\mathrm{atan}\left(\frac{\left(\frac{(b^3-4\,a\,b\,c)\,(2\,a\,h-b\,g+2\,c\,f)}{(4\,a\,c-b^2)^{5/2}}-\frac{2\,c\,x\,(2\,a\,h-b\,g+2\,c\,f)}{(4\,a\,c-b^2)^{3/2}}\right)(4\,a\,c-b^2)}{2\,a\,h-b\,g+2\,c\,f}\right)(2\,a\,h-b\,g+2\,c\,f)}{(4\,a\,c-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x)
```

```
[Out] ((a*b*h - 2*a*c*g + b*c*f)/(c*(4*a*c - b^2)) + (x*(2*c^2*f + b^2*h - 2*a*c*
h - b*c*g))/(c*(4*a*c - b^2)))/(a + b*x + c*x^2) - (2*atan(((((b^3 - 4*a*b*
c)*(2*a*h - b*g + 2*c*f))/(4*a*c - b^2)^(5/2) - (2*c*x*(2*a*h - b*g + 2*c*f
))/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a*h - b*g + 2*c*f))*(2*a*h - b*g
+ 2*c*f))/(4*a*c - b^2)^(3/2)
```

3.158 $\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)^2} \, dx$

Optimal. Leaf size=407

$$\frac{b^2ef - b(cdf + aeg + adh) - 2a(cef - cdg - aeh) - (2c^2df + b(bd - ae)h - c(bef + bdg - 2aeg + 2adh))}{(b^2 - 4ac)(cd^2 - bde + ae^2)(a + bx + cx^2)}$$

```
[Out] (b^2*e*f-b*(a*d*h+a*e*g+c*d*f)-2*a*(-a*e*h-c*d*g+c*e*f)-(2*c^2*d*f+b*(-a*e+
b*d)*h-c*(2*a*d*h-2*a*e*g+b*d*g+b*e*f))*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)
/(c*x^2+b*x+a)+(4*c^3*d^3*f+b*e*(4*a*b*d*e*h-2*a^2*e^2*h+b^2*(-d^2*h-d*e*g+
e^2*f))-2*c^2*d*(b*d*(d*g+3*e*f)-2*a*(d^2*h-d*e*g+3*e^2*f))+2*c*e*(2*b^2*d^
2*g+2*a^2*e*(-d*h+e*g)-a*b*(d^2*h+d*e*g+3*e^2*f)))*arctanh((2*c*x+b)/(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(a*e^2-b*d*e+c*d^2)^2+e*(d^2*h-d*e*g+e^2*f
)*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2-1/2*e*(d^2*h-d*e*g+e^2*f)*ln(c*x^2+b*x+a)
/(a*e^2-b*d*e+c*d^2)^2
```

Rubi [A]
time = 0.67, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1660, 814, 648, 632, 212, 642}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ce\left(2a^2e(eg-dh)-ab\left(d^2h+deg+3e^2f\right)+2b^2d^2g\right)+be\left(-2a^2e^2h+4abdeh+b^2\left(d^2(-h)-deg+e^2f\right)\right)-2c^2d\left(bd(dg+3ef)-2a\left(d^2h-deg+3e^2f\right)\right)+4c^3d^3f\right)}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)^2}+\frac{-x\left(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df\right)-b(adh+aeg+cdf)-2a(-aeh-cdg+cef)+b^2ef}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)}-\frac{e\log\left(a+bx+cx^2\right)\left(d^2h-deg+e^2f\right)}{2\left(ae^2-bde+cd^2\right)^2}+\frac{e\log(d+ex)\left(d^2h-deg+e^2f\right)}{\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (b^2*e*f - b*(c*d*f + a*e*g + a*d*h) - 2*a*(c*e*f - c*d*g - a*e*h) - (2*c^2
*d*f + b*(b*d - a*e)*h - c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d*h))*x)/((b^2 -
4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)) + ((4*c^3*d^3*f + b*e*(4*
a*b*d*e*h - 2*a^2*e^2*h + b^2*(e^2*f - d*e*g - d^2*h)) - 2*c^2*d*(b*d*(3*e*
f + d*g) - 2*a*(3*e^2*f - d*e*g + d^2*h)) + 2*c*e*(2*b^2*d^2*g + 2*a^2*e*(e
*g - d*h) - a*b*(3*e^2*f + d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^2) + (e*(e^2*f - d*e*
g + d^2*h)*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^2 - (e*(e^2*f - d*e*g + d^
2*h)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx = \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-(2c^2df+b(bd-ae)h}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(a+bx+cx}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-(2c^2df+b(bd-ae)h}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(a+bx+cx}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-(2c^2df+b(bd-ae)h}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(a+bx+cx}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-(2c^2df+b(bd-ae)h}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(a+bx+cx}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-(2c^2df+b(bd-ae)h}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(a+bx+cx}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-(2c^2df+b(bd-ae)h}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)(a+bx+cx}$$

Mathematica [A]
time = 0.55, size = 405, normalized size = 1.00

$$\frac{-2a^2eh+2c^2dfx+b^2(-ef+dhx)+bc(-efx+d(f-gx))+ab(dh+e(g-hx))+2ac(e(f+gx)-d(g+hx))}{\left(b^2-4ac\right)\left(-cd^2+e(bd-ae)\right)(a+x(b+cx))} - \frac{\left(-4c^3d^3f+2c^2d(bd(3ef+dg)-2a(3e^2f-deg+d^2h))+be(-4abdeh+2a^2e^2h+b^2(-e^2f+deg+d^2h))+2ce(-2b^2d^2g+2a^2e(-eg+dh)+ab(3e^2f+deg+d^2h))\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{3/2}\left(cd^2+e(-bd+ae)\right)^2} + \frac{e(e^2f-deg+d^2h)\log(d+ex)}{\left(cd^2+e(-bd+ae)\right)^2} - \frac{e(e^2f-deg+d^2h)\log(a+x(b+cx))}{2\left(cd^2+e(-bd+ae)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-2*a^2*e*h + 2*c^2*d*f*x + b^2*(-(e*f) + d*h*x) + b*c*(-(e*f*x) + d*(f - g
*x)) + a*b*(d*h + e*(g - h*x)) + 2*a*c*(e*(f + g*x) - d*(g + h*x)))/((b^2 -
 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*(a + x*(b + c*x))) - ((-4*c^3*d^3*f + 2*
c^2*d*(b*d*(3*e*f + d*g) - 2*a*(3*e^2*f - d*e*g + d^2*h)) + b*e*(-4*a*b*d*e
*h + 2*a^2*e^2*h + b^2*(-(e^2*f) + d*e*g + d^2*h)) + 2*c*e*(-2*b^2*d^2*g +
2*a^2*e*(-(e*g) + d*h) + a*b*(3*e^2*f + d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^2) +
 (e*(e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^2 - (e
*(e^2*f - d*e*g + d^2*h)*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e)
)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(401) = 802$.
time = 0.19, size = 809, normalized size = 1.99

method	result
default	$\frac{e(d^2h-gde+e^2f)\ln(ex+d)}{(a\,e^2-deb+c\,d^2)^2}-\frac{\frac{(a^2b\,e^3h+2a^2cd\,e^2h-2a^2c\,e^3g-2a\,b^2d\,e^2h-abc\,d^2eh+3abcd\,e^2g+abc\,e^3f+2a\,c^2d^3h-2a\,c^2d^2eg-2a\,c^2d\,e^2f+b^3}{4ac-b^2}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e*(d^2*h-d*e*g+e^2*f)*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2-1/(a*e^2-b*d*e+c*d^2)
^2*(((a^2*b*e^3*h+2*a^2*c*d*e^2*h-2*a^2*c*e^3*g-2*a*b^2*d*e^2*h-a*b*c*d^2*e
*h+3*a*b*c*d*e^2*g+a*b*c*e^3*f+2*a*c^2*d^3*h-2*a*c^2*d^2*e*g-2*a*c^2*d*e^2*
f+b^3*d^2*e*h-b^2*c*d^3*h-b^2*c*d^2*e*g-b^2*c*d*e^2*f+b*c^2*d^3*g+3*b*c^2*d
^2*e*f-2*c^3*d^3*f)/(4*a*c-b^2)*x+(2*a^3*e^3*h-3*a^2*b*d*e^2*h-a^2*b*e^3*g+
2*a^2*c*d^2*e*h+2*a^2*c*d*e^2*g-2*a^2*c*e^3*f+a*b^2*d^2*e*h+a*b^2*d*e^2*g+a
*b^2*e^3*f-a*b*c*d^3*h-3*a*b*c*d^2*e*g+a*b*c*d*e^2*f+2*a*c^2*d^3*g-2*a*c^2*
d^2*e*f-b^3*d*e^2*f+2*b^2*c*d^2*e*f-b*c^2*d^3*f)/(4*a*c-b^2))/(c*x^2+b*x+a)
+1/(4*a*c-b^2)*(1/2*(4*a*c^2*d^2*e*h-4*a*c^2*d*e^2*g+4*a*c^2*e^3*f-b^2*c*d^
2*e*h+b^2*c*d*e^2*g-b^2*c*e^3*f)/c*ln(c*x^2+b*x+a)+2*(a^2*b*e^3*h+2*a^2*c*d
*e^2*h-2*a^2*c*e^3*g-2*a*b^2*d*e^2*h+3*a*b*c*d^2*e*h-a*b*c*d*e^2*g+5*a*b*c*
e^3*f-2*a*c^2*d^3*h+2*a*c^2*d^2*e*g-6*a*c^2*d*e^2*f+b^3*d*e^2*g-b^3*e^3*f-2
*b^2*c*d^2*e*g+b*c^2*d^3*g+3*b*c^2*d^2*e*f-2*c^3*d^3*f-1/2*(4*a*c^2*d^2*e*h
-4*a*c^2*d*e^2*g+4*a*c^2*e^3*f-b^2*c*d^2*e*h+b^2*c*d*e^2*g-b^2*c*e^3*f)*b/c
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 860 vs. 2(415) = 830.
time = 4.01, size = 860, normalized size = 2.11

$$-\frac{(d^2he-dge^2+fe^3)\log(cx^2+bx+a)}{2\,(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4)}+\frac{(d^2he^2-dge^3+fe^4)\log(|xe+d|)}{c^2d^4e-2\,bcd^3e^2+b^2d^2e^3+2\,acd^2e^3-2\,abde^4+a^2e^5}-\frac{(4\,c^3d^3f-2\,bc^2d^3g+4\,ac^2d^3h-6\,bc^2d^2fe+4\,b^2cd^2ge-4\,ac^2d^2ge-b^3d^2he-2\,abcd^2he+12\,ac^2dfe^2-b^3dge^2-2\,abcdge^2+4\,ab^2dhe^2-4\,a^2cdhe^2+b^3fe^3-6\,abcfe^3+4\,a^2cge^3-2\,a^2bhe^3)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4)\sqrt{-b^2+4\,ac}}-\frac{bc^2d^3f-2\,ac^2d^3g+abcd^3h-2\,b^2cd^2fe+2\,ac^2d^2fe+3\,abcd^2ge-ab^2d^2he-2\,a^2cd^2he+b^3dfe^2-abcdfe^2-ab^2dge^2-2\,a^2cdge^2+3\,a^2bdhe^2-ab^2fe^3+2\,a^2cfe^3+a^2bge^3-2\,a^3he^3+(2\,c^3d^3f-bc^2d^3g+b^2cd^3h-2\,ac^2d^3h-3\,bc^2d^2fe+b^2cd^2ge+2\,ac^2d^2ge-b^3d^2he+abcd^2he+b^2cdfe^2+2\,ac^2dfe^2-3\,abcdge^2+2\,ab^2dhe^2-2\,a^2cdhe^2-abcfe^3+2\,a^2cge^3-a^2bhe^3)x}{(cd^2-bde+ae^2)^2(cx^2+bx+a)(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(d^2*h*e - d*g*e^2 + f*e^3)*log(c*x^2 + b*x + a)/(c^2*d^4 - 2*b*c*d^3*
e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4) + (d^2*h*e^2 - d*g
*e^3 + f*e^4)*log(abs(x*e + d))/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 +
2*a*c*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5) - (4*c^3*d^3*f - 2*b*c^2*d^3*g + 4*a
*c^2*d^3*h - 6*b*c^2*d^2*f*e + 4*b^2*c*d^2*g*e - 4*a*c^2*d^2*g*e - b^3*d^2*
h*e - 2*a*b*c*d^2*h*e + 12*a*c^2*d*f*e^2 - b^3*d*g*e^2 - 2*a*b*c*d*g*e^2 +
4*a*b^2*d*h*e^2 - 4*a^2*c*d*h*e^2 + b^3*f*e^3 - 6*a*b*c*f*e^3 + 4*a^2*c*g*e
^3 - 2*a^2*b*h*e^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2*d^4 -
4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2
*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 -
4*a^3*c*e^4)*sqrt(-b^2 + 4*a*c)) - (b*c^2*d^3*f - 2*a*c^2*d^3*g + a*b*c*d^3
*h - 2*b^2*c*d^2*f*e + 2*a*c^2*d^2*f*e + 3*a*b*c*d^2*g*e - a*b^2*d^2*h*e -
2*a^2*c*d^2*h*e + b^3*d*f*e^2 - a*b*c*d*f*e^2 - a*b^2*d*g*e^2 - 2*a^2*c*d*g
*e^2 + 3*a^2*b*d*h*e^2 - a*b^2*f*e^3 + 2*a^2*c*f*e^3 + a^2*b*g*e^3 - 2*a^3*
h*e^3 + (2*c^3*d^3*f - b*c^2*d^3*g + b^2*c*d^3*h - 2*a*c^2*d^3*h - 3*b*c^2*
d^2*f*e + b^2*c*d^2*g*e + 2*a*c^2*d^2*g*e - b^3*d^2*h*e + a*b*c*d^2*h*e + b
^2*c*d*f*e^2 + 2*a*c^2*d*f*e^2 - 3*a*b*c*d*g*e^2 + 2*a*b^2*d*h*e^2 - 2*a^2*
c*d*h*e^2 - a*b*c*f*e^3 + 2*a^2*c*g*e^3 - a^2*b*h*e^3)*x)/((c*d^2 - b*d*e +
 a*e^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [**B**]
time = 6.70, size = 2500, normalized size = 6.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x)

[Out] symsum(log(root(768*a^5*b*c^4*d^3*e^5*z^3 + 768*a^4*b*c^5*d^5*e^3*z^3 - 192*a^5*b^3*c^2*d*e^7*z^3 - 192*a^2*b^3*c^5*d^7*e*z^3 - 68*a^3*b^6*c*d^2*e^6*z^3 - 68*a*b^6*c^3*d^6*e^2*z^3 + 36*a^2*b^7*c*d^3*e^5*z^3 + 36*a*b^7*c^2*d^5*e^3*z^3 + 256*a^6*b*c^3*d*e^7*z^3 + 256*a^3*b*c^6*d^7*e*z^3 + 48*a^4*b^5*c*d*e^7*z^3 + 48*a*b^5*c^4*d^7*e*z^3 - 480*a^4*b^2*c^4*d^4*e^4*z^3 + 440*a^3*b^4*c^3*d^4*e^4*z^3 - 320*a^4*b^3*c^3*d^3*e^5*z^3 - 320*a^3*b^3*c^4*d^5*e^3*z^3 + 240*a^4*b^4*c^2*d^2*e^6*z^3 + 240*a^2*b^4*c^4*d^6*e^2*z^3 - 192*a^5*b^2*c^3*d^2*e^6*z^3 - 192*a^3*b^2*c^5*d^6*e^2*z^3 - 90*a^2*b^6*c^2*d^4*e^4*z^3 - 48*a^3*b^5*c^2*d^3*e^5*z^3 - 48*a^2*b^5*c^3*d^5*e^3*z^3 - 4*b^9*c*d^5*e^3*z^3 - 4*b^7*c^3*d^7*e*z^3 - 4*a^3*b^7*d*e^7*z^3 - 4*a*b^9*d^3*e^5*z^3 - 12*a^5*b^4*c*e^8*z^3 - 12*a*b^4*c^5*d^8*z^3 + 6*b^8*c^2*d^6*e^2*z^3 - 384*a^5*c^5*d^4*e^4*z^3 - 256*a^6*c^4*d^2*e^6*z^3 - 256*a^4*c^6*d^6*e^2*z^3 + 6*a^2*b^8*d^2*e^6*z^3 + 48*a^6*b^2*c^2*e^8*z^3 + 48*a^2*b^2*c^6*d^8*z^3 - 64*a^7*c^3*e^8*z^3 - 64*a^3*c^7*d^8*z^3 + b^10*d^4*e^4*z^3 + b^6*c^4*d^8*z^3 + a^4*b^6*e^8*z^3 - 28*a*b^4*c*d^3*e^3*g*h*z - 10*a^3*b^2*c*d*e^5*g*h*z - 10*a*b^2*c^3*d^5*e*g*h*z + 16*a*b^4*c*d^2*e^4*f*h*z + 14*a^2*b^3*c*d*e^5*f*h*z + 4*a*b*c^4*d^4*e^2*f*g*z + 84*a^2*b^2*c^2*d^3*e^3*g*h*z - 108*a^2*b^2*c^2*d^2*e^4*f*h*z + 16*a*b*c^4*d^5*e*f*h*z - 20*a*b^4*c*d*e^5*f*g*z + 8*a^2*b^3*c*d^2*e^4*g*h*z + 8*a*b^3*c^2*d^4*e^2*g*h*z - 4*a^3*b*c^2*d^2*e^4*g*h*z - 4*a^2*b*c^3*d^4*e^2*g*h*z + 16*a^2*b*c^3*d^3*e^3*f*h*z + 16*a*b^3*c^2*d^3*e^3*f*h*z - 14*a*b^2*c^3*d^4*e^2*f*h*z + 66*a^2*b^2*c^2*d*e^5*f*g*z - 36*a*b^2*c^3*d^3*e^3*f*g*z + 20*a*b^3*c^2*d^2*e^4*f*g*z + 12*a^2*b*c^3*d^2*e^4*f*g*z + 8*a*c^5*d^5*e*f*g*z + 4*a^4*b*c*e^6*g*h*z - 2*a*b^5*d*e^5*f*h*z + 4*a*b*c^4*d^6*g*h*z - 112*a^3*c^3*d^3*e^3*g*h*z - 3*b^4*c^2*d^4*e^2*f*h*z + 120*a^3*c^3*d^2*e^4*f*h*z - 16*a^2*c^4*d^4*e^2*f*h*z + 14*b^3*c^3*d^4*e^2*f*g*z - 2*b^4*c^2*d^3*e^3*f*g*z + 16*a^2*c^4*d^3*e^3*f*g*z + 8*a*b^4*c*d^4*e^2*h^2*z + 4*a^2*b*c^3*d^5*e*h^2*z + 2*a*b^3*c^2*d^5*e*h^2*z + 8*a*b^4*c*d^2*e^4*g^2*z + 4*a^3*b*c^2*d*e^5*g^2*z + 2*a^2*b^3*c*d*e^5*g^2*z + 48*a*b*c^4*d^3*e^3*f^2*z + 36*a^2*b*c^3*d*e^5*f^2*z - 6*a*b^3*c^2*d*e^5*f^2*z - 45*a^2*b^2*c^2*d^4*e^2*h^2*z - 45*a^2*b^2*c^2*d^2*e^4*g^2*z + 2*b^5*c*d^4*e^2*g*h*z - b^4*c^2*d^5*e*g*h*z + 8*a^4*c^2*d*e^5*g*h*z + 8*a^2*c^4*d^5*e*g*h*z + 2*b^3*c^3*d^5*e*f*h*z - 14*b^2*c^4*d^5*e*f*g*z - 2*b^5*c*d^2*e^4*f*g*z + 2*a*b^5*d^2*e^4*g*h*z - a^2*b^4*d*e^5*g*h*z - 120*a^3*c^3*d*e^5*f*g*z - 6*a^3*b^2*c*e^6*f*h*z + 12*a^3*b*c^2*e^6*f*g*z - 2*a^2*b^3*c*e^6*f*g*z - 4*a^4*b*c*d*e^5*h^2*z - 4*a*b*c^4*d^5*e*g^2*z + 6*a^3*b^2*c*d^2*e^4*h^2*z + 2*a^2*b^3*c*d^3*e^3*h^2*z + 6*a*b^2*c^3*d^4*e^2*g^2*z + 2*a*b^3*c^2*d^3*e^3*g^2*z - 18*a*b^2*c^3*d^2*e^4*f^2*z - b^6*d^2*e^4*f*h*z + 12*b*c^5*d^5*e*f^2*z + 12*a*b^4*c*e^6*f^2*z + 56*a^3*c^3*d^4*e^2*h^2*z - 5*b^4*c^2*d^4*e^2*g^2*z - 4*a^4*c^2*d^2*e^4*h^2*z + 56*a^3*c^3*d^2*e^4*g^2*z - 9*b^2*c^4*d^4*e^2*f^2*z - 5*a^2*b^4*d^2*e^4*h^2*z - 4*a^2*c^4*d^4*e^2*g^2*z + 3*b^4*c^2*d^2*e^4*f^2*z - 2*b^3*c^3*d^3*e^3*f^2*z - 36*a^2*c^4*d^2*e^4*f^2*z - 45*a^2*b^

2*c^2*e^6*f^2*z + 2*b^6*d*e^5*f*g*z - 8*a*c^5*d^6*f*h*z + 4*b*c^5*d^6*f*g*z + 4*b^3*c^3*d^5*e*g^2*z + 2*b^5*c*d^3*e^3*g^2*z + 4*a^3*b^3*d*e^5*h^2*z + 2*a*b^5*d^3*e^3*h^2*z - 24*a*c^5*d^4*e^2*f^2*z + b^6*d^3*e^3*g*h*z + a^2*b^4*e^6*f*h*z - b^6*d^4*e^2*h^2*z - b^6*d^2*e^4*g^2*z - 4*a^4*c^2*e^6*g^2*z - 4*a^2*c^4*d^6*h^2*z - b^2*c^4*d^6*g^2*z - a^4*b^2*e^6*h^2*z + 48*a^3*c^3*e^6*f^2*z - 4*c^6*d^6*f^2*z - b^6*e^6*f^2*z - 16*a*b*c^2*d^2*e^3*f*g*h - 4*a*b^2*c*d*e^4*f*g*h - 4*b*c^3*d^4*e*f*g*h - 4*a^2*b*c*e^5*f*g*h + 6*b^2*c^2*d^3*e^2*f*g*h - 8*a^2*b*c*d^2*e^3*g*h^2 + 8*a*b*c^2*d^3*e^2*g^2*h + 2*a*b^2*c*d^3*e^2*g*h^2 - 2*a*b^2*c*d^2*e^3*g^2*h + 6*a*b^2*c*d^2*e^3*f*h^2 + 4*b^3*c*d^2*e^3*f*g*h - 16*a*c^3*d^3*e^2*f*g*h - 8*a^2*c^2*d*e^4*f*g*h + 4*a^2*b*c*d*e^4*g^2*h - 4*a*b*c^2*d^4*e*g*h^2 + 4*a^2*b*c*d*e^4*f*h^2 + 16*a*b*c^2*d*e^4*f*g^2 - 2*b^3*c*d*e^4*f^2*h + 8*a*c^3*d^4*e*f*h^2 - 4*b^3*c*d*e^4*f*g^2 - 24*a*c^3*d*e^4*f^2*g - 2*a*b^3*d*e^4*f*h^2 + 6*a*b^2*c*e^5*f^2*h - 12*a*b*c^2*e^5*f^2*g - 12*a^2*c^2*d^3*e^2*g*h^2 + 12*a^2*c^2*d^2*e^3*g^2*h - 3*b^2*c^2*d^2*e^3*f^2*h - 5*b^2*c^2*d^2*e^3*f*g^2 + 4*a^2*c^2*d^2*e^3*f*h^2 + 2*b^4*d*e^4*f*g*h - 2*b^3*c*d^3*e^2*g^2*h - 4*b*c^3*d^3*e^2*f^2*h - 2*b^3*c*d^3*e^2*f*h^2 + 24*a*c^3*d^2*e^3*f^2*h + 9*b^2*c^2*d*e^4*f^2*g + 4*b*c^3*d^3*e^2*f*g^2 + 2*a*b^3*d^2*e^3*g*h^2 - a^2*b^2*d*e^4*g*h^2 + 8*a*c^3*d^2*e^3*f*g^2 + 4*a^2*b*c*d^3*e^2*h^3 - 4*a*b*c^2*d^2*e^3*g^3 - b^4*d^2*e^3*g^2*h - 4*c^4*d^3*e^2*f^2*g - b^4*d^2*e^3*f*h^2 + 4*a^2*c^2*e^5*f*g^2 + 4*a^2*c^2*d^4*e*h^3 + 2*b^3*c*d^2*e^3*g^3 - 4*a^2*c^2*d*e^4*g^3 - 2*a*b^3*d^3*e^2*h^3 + 4*c^4*d^4*e*f^2*h + 2*b^3*c*e^5*f^2*g - 4*b*c^3*d*e^4*f^3 + b^2*c^2*d^4*e*g^2*h - b^2*c^2*d^3*e^2*g^3 + b^4*d^3*e^2*g*h^2 + a^2*b^2*e^5*f*h^2 + 4*c^4*d^2*e^3*f^3 - 3*b^2*c^2*e^5*f^3 + a^2*b^2*d^2*e^3*h^3 - b^4*e^5*f^2*h + 16*a*c^3*e^5*f^3, z, k)*((a*b^5*c*e^6*f - 8*a^4*c^3*e^6*g + 8*a*c^6*d^5*e*f - b^6*c*d*e^5*f + 20*a^3*b*c^3*e^6*f - a...

3.159 $\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2} \, dx$

Optimal. Leaf size=673

$$-\frac{e(e^2f-deg+d^2h)}{\left(cd^2-bde+ae^2\right)^2(d+ex)}-\frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+ae(eg-2dh))+b(c^2d^2f+a^2}{}$$

```
[Out] -e*(d^2*h-d*e*g+e^2*f)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)+(-b^3*e^2*f+b^2*e*(a*e
*g+2*c*d*f)-2*a*c*(c*d*(-d*g+2*e*f)+a*e*(-2*d*h+e*g))-b*(c^2*d^2*f+a^2*e^2*
h-a*c*(-d^2*h-2*d*e*g+3*e^2*f))-c*(2*c^2*d^2*f+2*a^2*e^2*h-a*b*e*(2*d*h+e*g
)+b^2*(d^2*h+e^2*f)-c*(b*d*(d*g+2*e*f)+2*a*(d^2*h-2*d*e*g+e^2*f)))*x)/(-4*a
*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)+(4*c^4*d^4*f-b^3*e^3*(2*a*d*h-a
*e*g-b*d*g+2*b*e*f)-2*c^3*d^2*(b*d*(d*g+4*e*f)-2*a*(d^2*h-2*d*e*g+6*e^2*f))
-6*c^2*e*(4*a*b*d*e^2*f-b^2*d^3*g+2*a^2*e*(2*d^2*h-2*d*e*g+e^2*f))-c*e*(6*a
^2*b*e^3*g-4*a^3*e^3*h-b^3*d*(-2*d^2*h-3*d*e*g+4*e^2*f)-6*a*b^2*e*(2*d^2*h-
d*e*g+2*e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(
a*e^2-b*d*e+c*d^2)^3-e*(e^2*(2*a*d*h-a*e*g-b*d*g+2*b*e*f)-c*d*(2*d^2*h-3*d*
e*g+4*e^2*f))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^3+1/2*e*(e^2*(2*a*d*h-a*e*g-b*d
*g+2*b*e*f)-c*d*(2*d^2*h-3*d*e*g+4*e^2*f))*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d
^2)^3
```

Rubi [A]
time = 1.67, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1660, 1642, 648, 632, 212, 642}

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((e*(e^2*f - d*e*g + d^2*h))/((c*d^2 - b*d*e + a*e^2)^2*(d + e*x))) - (b^3
*e^2*f - b^2*e*(2*c*d*f + a*e*g) + 2*a*c*(c*d*(2*e*f - d*g) + a*e*(e*g - 2*
d*h)) + b*(c^2*d^2*f + a^2*e^2*h - a*c*(3*e^2*f - 2*d*e*g - d^2*h)) + c*(2*
c^2*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c*(b*
d*(2*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*x)/((b^2 - 4*a*c)*(c*d^2
 - b*d*e + a*e^2)^2*(a + b*x + c*x^2)) + ((4*c^4*d^4*f - b^3*e^3*(2*b*e*f -
b*d*g - a*e*g + 2*a*d*h) - 2*c^3*d^2*(b*d*(4*e*f + d*g) - 2*a*(6*e^2*f - 2*
d*e*g + d^2*h)) - 6*c^2*e*(4*a*b*d*e^2*f - b^2*d^3*g + 2*a^2*e*(e^2*f - 2*d
*e*g + 2*d^2*h)) - c*e*(6*a^2*b*e^3*g - 4*a^3*e^3*h - b^3*d*(4*e^2*f - 3*d*
e*g - 2*d^2*h) - 6*a*b^2*e*(2*e^2*f - d*e*g + 2*d^2*h)))*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^3) - (e*
(e^2*(2*b*e*f - b*d*g - a*e*g + 2*a*d*h) - c*d*(4*e^2*f - 3*d*e*g + 2*d^2*h
```

```
))*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^3 + (e*(e^2*(2*b*e*f - b*d*g - a*e
*g + 2*a*d*h) - c*d*(4*e^2*f - 3*d*e*g + 2*d^2*h))*Log[a + b*x + c*x^2])/(2
*(c*d^2 - b*d*e + a*e^2)^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx = -\frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+ae(eg-2dh))+b(c^2d^2f}{}$$

$$= -\frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+ae(eg-2dh))+b(c^2d^2f}{}$$

$$= -\frac{e(e^2f-deg+d^2h)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-}{}$$

$$= -\frac{e(e^2f-deg+d^2h)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-}{}$$

$$= -\frac{e(e^2f-deg+d^2h)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-}{}$$

$$= -\frac{e(e^2f-deg+d^2h)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-}{}$$

Mathematica [A]
time = 1.34, size = 650, normalized size = 0.97

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((e*(e^2*f - d*e*g + d^2*h))/((c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x))) + (
-(b^3*e^2*f) + b^2*(a*e^2*g - c*(-2*d*e*f + e^2*f*x + d^2*h*x)) + b*(-(a^2*
e^2*h) + c^2*d*(-(d*f) + 2*e*f*x + d*g*x) + a*c*(-(d^2*h) + e^2*(3*f + g*x)
 - 2*d*e*(g - h*x))) + 2*c*(-(c^2*d^2*f*x) + a*c*(e^2*f*x - 2*d*e*(f + g*x)
 + d^2*(g + h*x)) - a^2*e*(-2*d*h + e*(g + h*x))))/((b^2 - 4*a*c)*(c*d^2 +
e*(-(b*d) + a*e))^2*(a + x*(b + c*x))) - ((4*c^4*d^4*f + b^3*e^3*(-2*b*e*f
+ b*d*g + a*e*g - 2*a*d*h) - 2*c^3*d^2*(b*d*(4*e*f + d*g) - 2*a*(6*e^2*f -
2*d*e*g + d^2*h)) - 6*c^2*e*(4*a*b*d*e^2*f - b^2*d^3*g + 2*a^2*e*(e^2*f - 2
*d*e*g + 2*d^2*h)) + c*e*(-6*a^2*b*e^3*g + 4*a^3*e^3*h + b^3*d*(4*e^2*f - 3
*d*e*g - 2*d^2*h) + 6*a*b^2*e*(2*e^2*f - d*e*g + 2*d^2*h)))*ArcTan[(b + 2*c
*x)/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(-(c*d^2) + e*(b*d - a*e))^3
) + ((e^3*(-2*b*e*f + b*d*g + a*e*g - 2*a*d*h) + c*d*e*(4*e^2*f - 3*d*e*g +
 2*d^2*h))*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^3 - ((e^3*(-2*b*e*f + b
```

```
*d*g + a*e*g - 2*a*d*h) + c*d*e*(4*e^2*f - 3*d*e*g + 2*d^2*h))*Log[a + x*(b
 + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1344 vs. $2(668) = 1336$.

time = 0.51, size = 1345, normalized size = 2.00

method	result
default	$-\frac{e(d^2h-gde+e^2f)}{(ae^2-deb+cd^2)^2(ex+d)}-\frac{e(2ad\,e^2h-a\,e^3g-bd\,e^2g+2b\,e^3f-2c\,d^3h+3c\,d^2eg-4cd\,e^2f)\ln(ex+d)}{(a\,e^2-deb+c\,d^2)^3}+\frac{c\left(2a^3e^4h-4a^2bd\,e^3h-a^2b\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -e*(d^2*h-d*e*g+e^2*f)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)-e*(2*a*d*e^2*h-a*e^3*g
-b*d*e^2*g+2*b*e^3*f-2*c*d^3*h+3*c*d^2*e*g-4*c*d*e^2*f)/(a*e^2-b*d*e+c*d^2)
^3*ln(e*x+d)+1/(a*e^2-b*d*e+c*d^2)^3*((c*(2*a^3*e^4*h-4*a^2*b*d*e^3*h-a^2*b
*e^4*g+4*a^2*c*d*e^3*g-2*a^2*c*e^4*f+3*a*b^2*d^2*e^2*h+a*b^2*d*e^3*g+a*b^2*
e^4*f-6*a*b*c*d^2*e^2*g-2*a*c^2*d^4*h+4*a*c^2*d^3*e*g-b^3*d^3*e*h-b^3*d*e^3
*f+b^2*c*d^4*h+b^2*c*d^3*e*g+3*b^2*c*d^2*e^2*f-b*c^2*d^4*g-4*b*c^2*d^3*e*f+
2*c^3*d^4*f)/(4*a*c-b^2)*x+(a^3*b*e^4*h-4*a^3*c*d*e^3*h+2*a^3*c*e^4*g-a^2*b
^2*d*e^3*h-a^2*b^2*e^4*g+6*a^2*b*c*d^2*e^2*h-3*a^2*b*c*e^4*f-4*a^2*c^2*d^3*
e*h+4*a^2*c^2*d*e^3*f+a*b^3*d*e^3*g+a*b^3*e^4*f-a*b^2*c*d^3*e*h-3*a*b^2*c*d
^2*e^2*g+a*b^2*c*d*e^3*f+a*b*c^2*d^4*h+4*a*b*c^2*d^3*e*g-6*a*b*c^2*d^2*e^2*
f-2*a*c^3*d^4*g+4*a*c^3*d^3*e*f-b^4*d*e^3*f+3*b^3*c*d^2*e^2*f-3*b^2*c^2*d^3
*e*f+b*c^3*d^4*f)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(8*a^2*c^2*
d*e^3*h-4*a^2*c^2*e^4*g-2*a*b^2*c*d*e^3*h+a*b^2*c*e^4*g-4*a*b*c^2*d*e^3*g+8
*a*b*c^2*e^4*f-8*a*c^3*d^3*e*h+12*a*c^3*d^2*e^2*g-16*a*c^3*d*e^3*f+b^3*c*d*
e^3*g-2*b^3*c*e^4*f+2*b^2*c^2*d^3*e*h-3*b^2*c^2*d^2*e^2*g+4*b^2*c^2*d*e^3*f
)/c*ln(c*x^2+b*x+a)+2*(6*a*b^2*c*d^2*e^2*h-4*a*b*c^2*d^3*e*h+6*a*b*c^2*d^2*
e^2*g-20*a*b*c^2*d*e^3*f+4*a^2*b*c*d*e^3*h-5*a*b^2*c*d*e^3*g+b^4*d*e^3*g-b*
c^3*d^4*g+a*b^3*e^4*g+2*a*c^3*d^4*h+2*a^3*c*e^4*h-6*a^2*c^2*e^4*f-5*a^2*b*c
*e^4*g+12*a^2*c^2*d*e^3*g+10*a*b^2*c*e^4*f-4*a*c^3*d^3*e*g+4*b^3*c*d*e^3*f-
4*b*c^3*d^3*e*f+2*c^4*d^4*f-1/2*(8*a^2*c^2*d*e^3*h-4*a^2*c^2*e^4*g-2*a*b^2*
c*d*e^3*h+a*b^2*c*e^4*g-4*a*b*c^2*d*e^3*g+8*a*b*c^2*e^4*f-8*a*c^3*d^3*e*h+1
2*a*c^3*d^2*e^2*g-16*a*c^3*d*e^3*f+b^3*c*d*e^3*g-2*b^3*c*e^4*f+2*b^2*c^2*d^
3*e*h-3*b^2*c^2*d^2*e^2*g+4*b^2*c^2*d*e^3*f)*b/c-2*b^4*e^4*f-12*a^2*c^2*d^2
*e^2*h-2*a*b^3*d*e^3*h+12*a*c^3*d^2*e^2*f-3*b^3*c*d^2*e^2*g+3*b^2*c^2*d^3*e
*g)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)**2/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1437 vs. 2(667) = 1334.
time = 3.42, size = 1437, normalized size = 2.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(4*c^4*d^4*f*e^2 - 2*b*c^3*d^4*g*e^2 + 4*a*c^3*d^4*h*e^2 - 8*b*c^3*d^3*f*e
^3 + 6*b^2*c^2*d^3*g*e^3 - 8*a*c^3*d^3*g*e^3 - 2*b^3*c*d^3*h*e^3 + 24*a*c^3
*d^2*f*e^4 - 3*b^3*c*d^2*g*e^4 + 12*a*b^2*c*d^2*h*e^4 - 24*a^2*c^2*d^2*h*e^
4 + 4*b^3*c*d*f*e^5 - 24*a*b*c^2*d*f*e^5 + b^4*d*g*e^5 - 6*a*b^2*c*d*g*e^5
+ 24*a^2*c^2*d*g*e^5 - 2*a*b^3*d*h*e^5 - 2*b^4*f*e^6 + 12*a*b^2*c*f*e^6 - 1
```

```
2*a^2*c^2*f*e^6 + a*b^3*g*e^6 - 6*a^2*b*c*g*e^6 + 4*a^3*c*h*e^6)*arctan((2*
c*d - 2*c*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*a*e^2/(x*e + d))*e^(-
1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((b^2*c^3*d^6 - 4*a*c^4*d^6 - 3*b^3*c^2*d^5*e
 + 12*a*b*c^3*d^5*e + 3*b^4*c*d^4*e^2 - 9*a*b^2*c^2*d^4*e^2 - 12*a^2*c^3*d^
4*e^2 - b^5*d^3*e^3 - 2*a*b^3*c*d^3*e^3 + 24*a^2*b*c^2*d^3*e^3 + 3*a*b^4*d^
2*e^4 - 9*a^2*b^2*c*d^2*e^4 - 12*a^3*c^2*d^2*e^4 - 3*a^2*b^3*d*e^5 + 12*a^3
*b*c*d*e^5 + a^3*b^2*e^6 - 4*a^4*c*e^6)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c*d^3*
h*e - 3*c*d^2*g*e^2 + 4*c*d*f*e^3 + b*d*g*e^3 - 2*a*d*h*e^3 - 2*b*f*e^4 + a
*g*e^4)*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e
/(x*e + d)^2 + a*e^2/(x*e + d)^2)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^
2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a
^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6) - (d^2*h*e^5/(x*e + d) - d*g*e^6/(x
*e + d) + f*e^7/(x*e + d))/(c^2*d^4*e^4 - 2*b*c*d^3*e^5 + b^2*d^2*e^6 + 2*a
*c*d^2*e^6 - 2*a*b*d*e^7 + a^2*e^8) - ((2*c^4*d^3*f*e - b*c^3*d^3*g*e + b^2
*c^2*d^3*h*e - 2*a*c^3*d^3*h*e - 3*b*c^3*d^2*f*e^2 + 6*a*c^3*d^2*g*e^2 - 3*
a*b*c^2*d^2*h*e^2 + 3*b^2*c^2*d*f*e^3 - 6*a*c^3*d*f*e^3 - 3*a*b*c^2*d*g*e^3
 + 6*a^2*c^2*d*h*e^3 - b^3*c*f*e^4 + 3*a*b*c^2*f*e^4 + a*b^2*c*g*e^4 - 2*a^
2*c^2*g*e^4 - a^2*b*c*h*e^4)/(c*d^2 - b*d*e + a*e^2) - (2*c^4*d^4*f*e^2 - b
*c^3*d^4*g*e^2 + b^2*c^2*d^4*h*e^2 - 2*a*c^3*d^4*h*e^2 - 4*b*c^3*d^3*f*e^3
+ 8*a*c^3*d^3*g*e^3 - 4*a*b*c^2*d^3*h*e^3 + 6*b^2*c^2*d^2*f*e^4 - 12*a*c^3*
d^2*f*e^4 - 6*a*b*c^2*d^2*g*e^4 + 12*a^2*c^2*d^2*h*e^4 - 4*b^3*c*d*f*e^5 +
12*a*b*c^2*d*f*e^5 + 4*a*b^2*c*d*g*e^5 - 8*a^2*c^2*d*g*e^5 - 4*a^2*b*c*d*h*
e^5 + b^4*f*e^6 - 4*a*b^2*c*f*e^6 + 2*a^2*c^2*f*e^6 - a*b^3*g*e^6 + 3*a^2*b
*c*g*e^6 + a^2*b^2*h*e^6 - 2*a^3*c*h*e^6)*e^(-1)/((c*d^2 - b*d*e + a*e^2)*(
x*e + d)))/((c*d^2 - b*d*e + a*e^2)^2*(b^2 - 4*a*c)*(c - 2*c*d/(x*e + d) +
c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + a*e^2/(x*e + d)^2))
```

Mupad [B]
time = 8.93, size = 2500, normalized size = 3.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x)
```

```
[Out] ((a*b^2*e^3*f - 2*a*c^2*d^3*g + b*c^2*d^3*f - 4*a^2*c*e^3*f + b^3*d*e^2*f -
 2*a*b^2*d*e^2*g + 4*a*c^2*d^2*e*f + a*b^2*d^2*e*h + a^2*b*d*e^2*h + 6*a^2*
c*d*e^2*g - 2*b^2*c*d^2*e*f - 8*a^2*c*d^2*e*h + a*b*c*d^3*h - 3*a*b*c*d*e^2
*f + 2*a*b*c*d^2*e*g)/(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^
4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b
*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2) + (x*(2*b^3*e^3*f + 2*c^3
*d^3*f - a*b^2*e^3*g - 2*a*c^2*d^3*h - b*c^2*d^3*g + a^2*b*e^3*h + 2*a^2*c*
e^3*g + b^2*c*d^3*h - b^3*d*e^2*g + b^3*d^2*e*h + 2*a*c^2*d*e^2*f + 2*a*c^2
*d^2*e*g - b*c^2*d^2*e*f - b^2*c*d*e^2*f - 2*a^2*c*d*e^2*h - 7*a*b*c*e^3*f
+ 5*a*b*c*d*e^2*g - 5*a*b*c*d^2*e*h))/(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*
```

e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3
*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2) - (x^2*(6
*a*c^2*e^3*f - 2*b^2*c*e^3*f - 2*a^2*c*e^3*h - 2*c^3*d^2*e*f - 8*a*c^2*d*e^
2*g + 2*b*c^2*d*e^2*f + 6*a*c^2*d^2*e*h + b*c^2*d^2*e*g + b^2*c*d*e^2*g - 2
*b^2*c*d^2*e*h + a*b*c*e^3*g + 2*a*b*c*d*e^2*h))/(4*a*c^3*d^4 + 4*a^3*c*e^4
 - a^2*b^2*e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*
e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2
))/(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3) + symsum(log((x*(36*a^
2*c^5*e^7*f^2 + 4*b^4*c^3*e^7*f^2 + 4*a^4*c^3*e^7*h^2 + 4*c^7*d^4*e^3*f^2 +
 a^2*b^2*c^3*e^7*g^2 + 64*a^2*c^5*d^2*e^5*g^2 + 12*b^2*c^5*d^2*e^5*f^2 + 36
*a^2*c^5*d^4*e^3*h^2 - 24*a^3*c^4*d^2*e^5*h^2 + b^2*c^5*d^4*e^3*g^2 + 2*b^3
*c^4*d^3*e^4*g^2 + b^4*c^3*d^2*e^5*g^2 + 4*b^4*c^3*d^4*e^3*h^2 - 24*a^3*c^4
*e^7*f*h - 24*a*b^2*c^4*e^7*f^2 - 24*a*c^6*d^2*e^5*f^2 - 8*b*c^6*d^3*e^4*f^
2 - 8*b^3*c^4*d*e^6*f^2 - 16*a*b*c^5*d^3*e^4*g^2 + 2*a*b^3*c^3*d*e^6*g^2 -
16*a^2*b*c^4*d*e^6*g^2 - 8*a^3*b*c^3*d*e^6*h^2 + 8*a^2*b^2*c^3*e^7*f*h + 80
*a^2*c^5*d^2*e^5*f*h - 96*a^2*c^5*d^3*e^4*g*h + 8*b^2*c^5*d^4*e^3*f*h - 8*b
^3*c^4*d^3*e^4*f*h + 8*b^4*c^3*d^2*e^5*f*h - 4*b^3*c^4*d^4*e^3*g*h - 4*b^4*
c^3*d^3*e^4*g*h - 14*a*b^2*c^4*d^2*e^5*g^2 - 24*a*b^2*c^4*d^4*e^3*h^2 - 8*a
*b^3*c^3*d^3*e^4*h^2 + 24*a^2*b*c^4*d^3*e^4*h^2 + 24*a*b*c^5*d*e^6*f^2 - 4*
a*b^3*c^3*e^7*f*g + 12*a^2*b*c^4*e^7*f*g + 32*a*c^6*d^3*e^4*f*g - 96*a^2*c^
5*d*e^6*f*g - 4*a^3*b*c^3*e^7*g*h - 24*a*c^6*d^4*e^3*f*h - 4*b*c^6*d^4*e^3*
f*g - 4*b^4*c^3*d*e^6*f*g + 32*a^3*c^4*d*e^6*g*h + 12*a^2*b^2*c^3*d^2*e^5*h
^2 - 24*a*b*c^5*d^2*e^5*f*g + 48*a*b^2*c^4*d*e^6*f*g + 16*a*b*c^5*d^3*e^4*f
*h - 8*a*b^3*c^3*d*e^6*f*h + 16*a^2*b*c^4*d*e^6*f*h + 12*a*b*c^5*d^4*e^3*g*
h - 40*a*b^2*c^4*d^2*e^5*f*h + 48*a*b^2*c^4*d^3*e^4*g*h - 24*a^2*b*c^4*d^2*
e^5*g*h))/(16*a^2*c^6*d^8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^
8*d^4*e^4 - 8*a*b^2*c^5*d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5
*d*e^7 - 4*b^5*c^3*d^7*e - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5
*d^6*e^2 + 96*a^4*c^4*d^4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64
*a^2*b^2*c^4*d^6*e^2 + 32*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 14
4*a^3*b^2*c^3*d^4*e^4 + 32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 3
2*a*b^3*c^4*d^7*e + 4*a*b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d
*e^7 - 64*a^5*b*c^2*d*e^7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 2
0*a^2*b^5*c*d^3*e^5 - 192*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^
4*b*c^3*d^3*e^5) - root(3840*a^6*b*c^5*d^5*e^7*z^3 + 3840*a^5*b*c^6*d^7*e^5
*z^3 + 1920*a^7*b*c^4*d^3*e^9*z^3 + 1920*a^4*b*c^7*d^9*e^3*z^3 - 288*a^7*b^
3*c^2*d*e^11*z^3 - 288*a^2*b^3*c^7*d^11*e*z^3 + 210*a^4*b^7*c*d^3*e^9*z^3 +
 210*a*b^7*c^4*d^9*e^3*z^3 - 174*a^5*b^6*c*d^2*e^10*z^3 - 174*a*b^6*c^5*d^1
0*e^2*z^3 - 120*a^3*b^8*c*d^4*e^8*z^3 - 120*a*b^8*c^3*d^8*e^4*z^3 + 12*a^2*
b^9*c*d^5*e^7*z^3 + 12*a*b^9*c^2*d^7*e^5*z^3 + 384*a^8*b*c^3*d*e^11*z^3 + 3
84*a^3*b*c^8*d^11*e*z^3 + 72*a^6*b^5*c*d*e^11*z^3 + 72*a*b^5*c^6*d^11*e*z^3
 + 18*a*b^10*c*d^6*e^6*z^3 - 4800*a^5*b^2*c^5*d^6*e^6*z^3 - 3120*a^6*b^2*c^
4*d^4*e^8*z^3 - 3120*a^4*b^2*c^6*d^8*e^4*z^3 + 2160*a^4*b^4*c^4*d^6*e^6*z^3
 - 1776*a^4*b^5*c^3*d^5*e^7*z^3 - 1776*a^3*b^5*c^4*d^7*e^5*z^3 + 1740*a^5*b
^4*c^3*d^4*e^8*z^3 + 1740*a^3*b^4*c^5*d^8*e^4*z^3 + 960*a^5*b^3*c^4*d^5*e^7

```
*z^3 + 960*a^4*b^3*c^5*d^7*e^5*z^3 - 672*a^7*b^2*c^3*d^2*e^10*z^3 - 672*a^3
*b^2*c^7*d^10*e^2*z^3 + 648*a^6*b^4*c^2*d^2*e^10*z^3 + 648*a^2*b^4*c^6*d^10
*e^2*z^3 - 600*a^5*b^5*c^2*d^3*e^9*z^3 - 600*a^2*b^5*c^5*d^9*e^3*z^3 + 372*
a^3*b^7*c^2*d^5*e^7*z^3 + 372*a^2*b^7*c^3*d^7*e^5*z^3 + 316*a^3*b^6*c^3*d^6
*e^6*z^3 - 222*a^2*b^8*c^2*d^6*e^6*z^3 - 160*a^6*b^3*c^3*d^3*e^9*z^3 - 160*
a^3*b^3*c^6*d^9*e^3*z^3 + 15*a^4*b^6*c^2*d^4*e^8*z^3 + 15*a^2*b^6*c^4*d^8*e
^4*z^3 - 6*b^11*c*d^7*e^5*z^3 - 6*b^7*c^5*d^11*e*z^3 - 6*a^5*b^7*d*e^11*z^3
 - 6*a*b^11*d^5*e^7*z^3 - 12*a^7*b^4*c*e^12*z^3 - 12*a*b^4*c^7*d^12*z^3 - 2
0*b^9*c^3*d^9*e^3*z^3 + 15*b^10*c^2*d^8*e^4*z^3 + 15*b^8*c^4*d^10*e^2*z^3 -
 1280*a^6*c^6*d^6*e^6*z^3 - 960*a^7*c^5*d^4*e^8*z^3 - 960*a^5*c^7*d^8*e^4*z
^3 - 384*a^8*c^4*d^2*e^10*z^3 - 384*a^4*c^8*d^10*e^2*z^3 - 20*a^3*b^9*d^3*e
^9*z^3 + 15*a^4*b^8*d^2*e^10*z^3 + 15*a^2*b^10*...
```

3.160 $\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx$

Optimal. Leaf size=62

$$3x+\frac{x^2}{2}+\frac{2(2-x)}{3(1-x+x^2)}+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+2\log\left(1-x+x^2\right)$$

```
[Out] 3*x+1/2*x^2+2/3*(2-x)/(x^2-x+1)+2*ln(x^2-x+1)+10/9*arctan(1/3*(1-2*x)*3^(1/
2))*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1674, 1671, 648, 632, 210, 642}

$$\frac{10\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{x^2}{2}+\frac{2(2-x)}{3(x^2-x+1)}+2\log\left(x^2-x+1\right)+3x$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] 3*x + x^2/2 + (2*(2 - x))/(3*(1 - x + x^2)) + (10*ArcTan[(1 - 2*x)/Sqrt[3]]
)/(3*Sqrt[3]) + 2*Log[1 - x + x^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx &= \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{-2+6x+6x^2+3x^3}{1-x+x^2}\,dx\\
&= \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(9+3x-\frac{11-12x}{1-x+x^2}\right)dx\\
&= 3x+\frac{x^2}{2}+\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{1}{3}\int \frac{11-12x}{1-x+x^2}\,dx\\
&= 3x+\frac{x^2}{2}+\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{5}{3}\int \frac{1}{1-x+x^2}\,dx+2\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= 3x+\frac{x^2}{2}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+2\log\left(1-x+x^2\right)+\frac{10}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+\right.\\
&= 3x+\frac{x^2}{2}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+2\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 60, normalized size = 0.97

$$3x+\frac{x^2}{2}-\frac{2(-2+x)}{3\left(1-x+x^2\right)}-\frac{10\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+2\log\left(1-x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] 3*x + x^2/2 - (2*(-2 + x))/(3*(1 - x + x^2)) - (10*ArcTan[(-1 + 2*x)/Sqrt[3
]])/(3*Sqrt[3]) + 2*Log[1 - x + x^2]
```

Maple [A]
time = 0.14, size = 53, normalized size = 0.85

method	result	size
default	$3x+\frac{x^2}{2}+\frac{-\frac{2x}{3}+\frac{4}{3}}{x^2-x+1}+2\ln\left(x^2-x+1\right)-\frac{10\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	53
risch	$3x+\frac{x^2}{2}+\frac{-\frac{2x}{3}+\frac{4}{3}}{x^2-x+1}+2\ln\left(4x^2-4x+4\right)-\frac{10\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+x+1)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3*x+1/2*x^2+(-2/3*x+4/3)/(x^2-x+1)+2*ln(x^2-x+1)-10/9*3^(1/2)*arctan(1/3*(2
*x-1)*3^(1/2))
```

Maxima [A]
time = 0.51, size = 51, normalized size = 0.82

$$\frac{1}{2}\,x^2-\frac{10}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+3\,x-\frac{2\,(x-2)}{3\,(x^2-x+1)}+2\,\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 2/3*(x - 2)/(x
^2 - x + 1) + 2*log(x^2 - x + 1)
```

Fricas [A]
time = 0.63, size = 75, normalized size = 1.21

$$\frac{9\,x^4+45\,x^3-20\,\sqrt{3}\,(x^2-x+1)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-45\,x^2+36\,(x^2-x+1)\log\left(x^2-x+1\right)+42\,x+24}{18\,(x^2-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(9*x^4 + 45*x^3 - 20*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1
)) - 45*x^2 + 36*(x^2 - x + 1)*log(x^2 - x + 1) + 42*x + 24)/(x^2 - x + 1)
```

Sympy [A]
time = 0.06, size = 60, normalized size = 0.97

$$\frac{x^2}{2} + 3x + \frac{4 - 2x}{3x^2 - 3x + 3} + 2\log\left(x^2 - x + 1\right) - \frac{10\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] x**2/2 + 3*x + (4 - 2*x)/(3*x**2 - 3*x + 3) + 2*log(x**2 - x + 1) - 10*sqrt
(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Giac [A]
time = 5.33, size = 51, normalized size = 0.82

$$\frac{1}{2}\,x^2 - \frac{10}{9}\,\sqrt{3}\ \arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x - 1)\right) + 3\,x - \frac{2\,(x - 2)}{3\,(x^2 - x + 1)} + 2\,\log\left(x^2 - x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 2/3*(x - 2)/(x
^2 - x + 1) + 2*log(x^2 - x + 1)
```

Mupad [B]
time = 0.04, size = 55, normalized size = 0.89

$$3\,x + 2\,\ln\left(x^2 - x + 1\right) - \frac{\frac{2\,x}{3} - \frac{4}{3}}{x^2 - x + 1} - \frac{10\,\sqrt{3}\ \mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3} - \frac{\sqrt{3}}{3}\right)}{9} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

```
[Out] 3*x + 2*log(x^2 - x + 1) - ((2*x)/3 - 4/3)/(x^2 - x + 1) - (10*3^(1/2)*atan
((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9 + x^2/2
```

3.161 $\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2} dx$

Optimal. Leaf size=55

$$x + \frac{2(1-2x)}{3(1-x+x^2)} - \frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{3}{2}\log\left(1-x+x^2\right)$$

```
[Out] x+2/3*(1-2*x)/(x^2-x+1)+3/2*ln(x^2-x+1)-7/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(
1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1674, 1671, 648, 632, 210, 642}

$$-\frac{7\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{2(1-2x)}{3(x^2-x+1)} + \frac{3}{2}\log\left(x^2-x+1\right) + x$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] x + (2*(1 - 2*x))/(3*(1 - x + x^2)) - (7*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt
[3]) + (3*Log[1 - x + x^2])/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx &= \frac{2(1-2x)}{3(1-x+x^2)}+\frac{1}{3}\int \frac{2+6x+3x^2}{1-x+x^2}\,dx\\
&= \frac{2(1-2x)}{3(1-x+x^2)}+\frac{1}{3}\int \left(3-\frac{1-9x}{1-x+x^2}\right)dx\\
&= x+\frac{2(1-2x)}{3(1-x+x^2)}-\frac{1}{3}\int \frac{1-9x}{1-x+x^2}\,dx\\
&= x+\frac{2(1-2x)}{3(1-x+x^2)}+\frac{7}{6}\int \frac{1}{1-x+x^2}\,dx+\frac{3}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= x+\frac{2(1-2x)}{3(1-x+x^2)}+\frac{3}{2}\log\left(1-x+x^2\right)-\frac{7}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= x+\frac{2(1-2x)}{3(1-x+x^2)}-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{3}{2}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 55, normalized size = 1.00

$$x-\frac{2(-1+2x)}{3\left(1-x+x^2\right)}+\frac{7\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{3}{2}\log\left(1-x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] x - (2*(-1 + 2*x))/(3*(1 - x + x^2)) + (7*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sq
rt[3]) + (3*Log[1 - x + x^2])/2
```

Maple [A]
time = 0.13, size = 46, normalized size = 0.84

method	result	size
default	$x+\frac{-\frac{4x}{3}+\frac{2}{3}}{x^2-x+1}+\frac{3\ln\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	46
risch	$x+\frac{-\frac{4x}{3}+\frac{2}{3}}{x^2-x+1}+\frac{3\ln\left(4x^2-4x+4\right)}{2}+\frac{7\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2+x+1)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x+(-4/3*x+2/3)/(x^2-x+1)+3/2*ln(x^2-x+1)+7/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(
1/2))
```

Maxima [A]
time = 0.50, size = 46, normalized size = 0.84

$$\frac{7}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+x-\frac{2(2x-1)}{3\left(x^2-x+1\right)}+\frac{3}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 2/3*(2*x - 1)/(x^2 - x + 1)
 + 3/2*log(x^2 - x + 1)
```

Fricas [A]
time = 0.64, size = 70, normalized size = 1.27

$$\frac{18x^3+14\sqrt{3}\left(x^2-x+1\right)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-18x^2+27\left(x^2-x+1\right)\log\left(x^2-x+1\right)-6x+12}{18\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(18*x^3 + 14*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) - 18*
x^2 + 27*(x^2 - x + 1)*log(x^2 - x + 1) - 6*x + 12)/(x^2 - x + 1)
```

Sympy [A]
time = 0.06, size = 54, normalized size = 0.98

$$x+\frac{2-4x}{3x^2-3x+3}+\frac{3\log\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] x + (2 - 4*x)/(3*x**2 - 3*x + 3) + 3*log(x**2 - x + 1)/2 + 7*sqrt(3)*atan(2
*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Giac [A]
time = 2.84, size = 46, normalized size = 0.84

$$\frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+x-\frac{2\,(2\,x-1)}{3\,(x^2-x+1)}+\frac{3}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 2/3*(2*x - 1)/(x^2 - x + 1)
 + 3/2*log(x^2 - x + 1)
```

Mupad [B]
time = 0.04, size = 48, normalized size = 0.87

$$x+\frac{3\,\ln\left(x^2-x+1\right)}{2}-\frac{\frac{4\,x}{3}-\frac{2}{3}}{x^2-x+1}+\frac{7\,\sqrt{3}\,\operatorname{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

```
[Out] x + (3*log(x^2 - x + 1))/2 - ((4*x)/3 - 2/3)/(x^2 - x + 1) + (7*3^(1/2)*ata
n((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9
```

3.162 $\int \frac{x(1+x+x^2)}{(1-x+x^2)^2} \, dx$

Optimal. Leaf size=52

$$-\frac{2(1+x)}{3(1-x+x^2)}-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)$$

```
[Out] -2/3*(1+x)/(x^2-x+1)+1/2*ln(x^2-x+1)-11/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/
2)
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1674, 648, 632, 210, 642}

$$-\frac{11\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2(x+1)}{3(x^2-x+1)}+\frac{1}{2}\log\left(x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] (-2*(1 + x))/(3*(1 - x + x^2)) - (11*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3])
 + Log[1 - x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx &= -\frac{2(1+x)}{3(1-x+x^2)}+\frac{1}{3}\int \frac{4+3x}{1-x+x^2}\,dx\\
&= -\frac{2(1+x)}{3(1-x+x^2)}+\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{11}{6}\int \frac{1}{1-x+x^2}\,dx\\
&= -\frac{2(1+x)}{3(1-x+x^2)}+\frac{1}{2}\log\left(1-x+x^2\right)-\frac{11}{3}\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= -\frac{2(1+x)}{3(1-x+x^2)}-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 1.00

$$
-\frac{2(1+x)}{3(1-x+x^2)}+\frac{11\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] (-2*(1 + x))/(3*(1 - x + x^2)) + (11*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]
) + Log[1 - x + x^2]/2
```

Maple [A]
time = 0.12, size = 45, normalized size = 0.87

method	result	size
default	$\frac{-\frac{2x}{3}-\frac{2}{3}}{x^2-x+1}+\frac{\ln(x^2-x+1)}{2}+\frac{11\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	45
risch	$\frac{-\frac{2x}{3}-\frac{2}{3}}{x^2-x+1}+\frac{\ln(4x^2-4x+4)}{2}+\frac{11\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+x+1)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-2/3*x-2/3)/(x^2-x+1)+1/2*ln(x^2-x+1)+11/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))
```

Maxima [A]
time = 0.50, size = 43, normalized size = 0.83

$$\frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{2\,(x+1)}{3\,(x^2-x+1)}+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 2/3*(x + 1)/(x^2 - x + 1) + 1/
2*log(x^2 - x + 1)
```

Fricas [A]
time = 0.53, size = 60, normalized size = 1.15

$$\frac{22\sqrt{3}\,(x^2-x+1)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+9\,(x^2-x+1)\log\left(x^2-x+1\right)-12\,x-12}{18\,(x^2-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(22*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9*(x^2 - x +
 1)*log(x^2 - x + 1) - 12*x - 12)/(x^2 - x + 1)
```

Sympy [A]
time = 0.05, size = 53, normalized size = 1.02

$$\frac{-2x-2}{3x^2-3x+3}+\frac{\log\left(x^2-x+1\right)}{2}+\frac{11\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] (-2*x - 2)/(3*x**2 - 3*x + 3) + log(x**2 - x + 1)/2 + 11*sqrt(3)*atan(2*sqr
t(3)*x/3 - sqrt(3)/3)/9
```

Giac [A]
time = 3.74, size = 43, normalized size = 0.83

$$\frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)-\frac{2\left(x+1\right)}{3\left(x^{2}-x+1\right)}+\frac{1}{2}\log\left(x^{2}-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 2/3*(x + 1)/(x^2 - x + 1) + 1/
2*log(x^2 - x + 1)
```

Mupad [B]
time = 3.84, size = 59, normalized size = 1.13

$$\frac{\ln\left(x^{2}-x+1\right)}{2}-\frac{2x}{3\left(x^{2}-x+1\right)}-\frac{2}{3\left(x^{2}-x+1\right)}+\frac{11\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

```
[Out] log(x^2 - x + 1)/2 - (2*x)/(3*(x^2 - x + 1)) - 2/(3*(x^2 - x + 1)) + (11*3^
(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9
```

3.163 $\int \frac{1+x+x^2}{(1-x+x^2)^2} \, dx$

Optimal. Leaf size=41

$$-\frac{2(2-x)}{3(1-x+x^2)}-\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -2/3*(2-x)/(x^2-x+1)-10/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1674, 12, 632, 210}

$$-\frac{10\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2(2-x)}{3(x^2-x+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(1 - x + x^2)^2,x]
[Out] (-2*(2 - x))/(3*(1 - x + x^2)) - (10*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
```

```
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x+x^2}{(1-x+x^2)^2}\,dx &= -\frac{2(2-x)}{3(1-x+x^2)}+\frac{1}{3}\int \frac{5}{1-x+x^2}\,dx \\
&= -\frac{2(2-x)}{3(1-x+x^2)}+\frac{5}{3}\int \frac{1}{1-x+x^2}\,dx \\
&= -\frac{2(2-x)}{3(1-x+x^2)}-\frac{10}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right) \\
&= -\frac{2(2-x)}{3(1-x+x^2)}-\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.95

$$
\frac{2(-2+x)}{3(1-x+x^2)}+\frac{10\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(1 - x + x^2)^2,x]
```

```
[Out] (2*(-2 + x))/(3*(1 - x + x^2)) + (10*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]
)
```

Maple [A]
time = 0.13, size = 34, normalized size = 0.83

method	result	size
default	$\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}+\frac{10\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	34
risch	$\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}+\frac{10\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (2/3*x-4/3)/(x^2-x+1)+10/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.51, size = 32, normalized size = 0.78

$$\frac{10}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{2\,(x-2)}{3\,(x^2-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(x - 2)/(x^2 - x + 1)
```

Fricas [A]
time = 0.59, size = 41, normalized size = 1.00

$$\frac{2\left(5\sqrt{3}\,(x^2-x+1)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+3\,x-6\right)}{9\,(x^2-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 2/9*(5*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 6)/(x^2
- x + 1)
```

Sympy [A]
time = 0.05, size = 41, normalized size = 1.00

$$\frac{2x-4}{3x^2-3x+3}+\frac{10\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] (2*x - 4)/(3*x**2 - 3*x + 3) + 10*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Giac [A]
time = 3.73, size = 32, normalized size = 0.78

$$\frac{10}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{2\,(x-2)}{3\,(x^2-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(x - 2)/(x^2 - x + 1)
```

Mupad [B]
time = 3.83, size = 35, normalized size = 0.85

$$\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}+\frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^2 - x + 1)^2,x)
```

```
[Out] ((2*x)/3 - 4/3)/(x^2 - x + 1) + (10*3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/
3))/9
```

3.164 $\int \frac{1+x+x^2}{x(1-x+x^2)^2} \, dx$

Optimal. Leaf size=56

$$-\frac{2(1-2x)}{3(1-x+x^2)}-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{2}\log\left(1-x+x^2\right)$$

```
[Out] -2/3*(1-2*x)/(x^2-x+1)+ln(x)-1/2*ln(x^2-x+1)-11/9*arctan(1/3*(1-2*x)*3^(1/2
))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1660, 814, 648, 632, 210, 642}

$$-\frac{11\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2(1-2x)}{3(x^2-x+1)}-\frac{1}{2}\log\left(x^2-x+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x*(1 - x + x^2)^2),x]
```

```
[Out] (-2*(1 - 2*x))/(3*(1 - x + x^2)) - (11*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3
]) + Log[x] - Log[1 - x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx &= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{3+4x}{x\left(1-x+x^2\right)}\,dx\\
&= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(\frac{3}{x}+\frac{7-3x}{1-x+x^2}\right)dx\\
&= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\log(x)+\frac{1}{3}\int \frac{7-3x}{1-x+x^2}\,dx\\
&= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\log(x)-\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{11}{6}\int \frac{1}{1-x+x^2}\,dx\\
&= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\log(x)-\frac{1}{2}\log\left(1-x+x^2\right)-\frac{11}{3}\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+\right.\\
&= -\frac{2(1-2x)}{3\left(1-x+x^2\right)}-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{2}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 1.00

$$\frac{2(-1+2x)}{3\left(1-x+x^2\right)}+\frac{11\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{2}\log\left(1-x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x*(1 - x + x^2)^2),x]
```

```
[Out] (2*(-1 + 2*x))/(3*(1 - x + x^2)) + (11*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[
3]) + Log[x] - Log[1 - x + x^2]/2
```

Maple [A]
time = 0.11, size = 48, normalized size = 0.86

method	result	size
default	$\ln\left(x\right)-\frac{-\frac{4x}{3}+\frac{2}{3}}{x^2-x+1}-\frac{\ln\left(x^2-x+1\right)}{2}+\frac{11\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	48
risch	$\frac{\frac{4x}{3}-\frac{2}{3}}{x^2-x+1}+\ln\left(x\right)-\frac{\ln\left(121x^2-121x+121\right)}{2}+\frac{11\sqrt{3}\,\arctan\left(\frac{2\left(11x-\frac{11}{2}\right)\sqrt{3}}{33}\right)}{9}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-(-4/3*x+2/3)/(x^2-x+1)-1/2*ln(x^2-x+1)+11/9*3^(1/2)*arctan(1/3*(2*x-1
)*3^(1/2))
```

Maxima [A]
time = 0.51, size = 47, normalized size = 0.84

$$\frac{11}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{2\,(2x-1)}{3\,(x^2-x+1)}-\frac{1}{2}\log\left(x^2-x+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(2*x - 1)/(x^2 - x + 1) -
1/2*log(x^2 - x + 1) + log(x)
```

Fricas [A]
time = 0.70, size = 72, normalized size = 1.29

$$\frac{22\sqrt{3}\,(x^2-x+1)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-9\,(x^2-x+1)\log\left(x^2-x+1\right)+18\,(x^2-x+1)\log\left(x\right)+24x-12}{18\,(x^2-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(22*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) - 9*(x^2 - x +
 1)*log(x^2 - x + 1) + 18*(x^2 - x + 1)*log(x) + 24*x - 12)/(x^2 - x + 1)
```

Sympy [A]
time = 0.07, size = 54, normalized size = 0.96

$$\frac{4x-2}{3x^2-3x+3}+\log\left(x\right)-\frac{\log\left(x^2-x+1\right)}{2}+\frac{11\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x/(x**2-x+1)**2,x)
```

```
[Out] (4*x - 2)/(3*x**2 - 3*x + 3) + log(x) - log(x**2 - x + 1)/2 + 11*sqrt(3)*at
an(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Giac [A]
time = 4.89, size = 48, normalized size = 0.86

$$\frac{11}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{2\,(2\,x-1)}{3\,(x^2-x+1)}-\frac{1}{2}\,\log\left(x^2-x+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(2*x - 1)/(x^2 - x + 1) -
1/2*log(x^2 - x + 1) + log(abs(x))
```

Mupad [B]
time = 0.10, size = 58, normalized size = 1.04

$$\ln\left(x\right)+\frac{\frac{4\,x}{3}-\frac{2}{3}}{x^2-x+1}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,11{}\mathrm{i}}{18}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,11{}\mathrm{i}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x*(x^2 - x + 1)^2),x)
```

```
[Out] log(x) + ((4*x)/3 - 2/3)/(x^2 - x + 1) - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*11i)/18 + 1/2) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*11i)/18 - 1/
2)
```

3.165 $\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx$

Optimal. Leaf size=61

$$-\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)$$

```
[Out] -1/x+2/3*(1+x)/(x^2-x+1)+3*ln(x)-3/2*ln(x^2-x+1)-7/9*arctan(1/3*(1-2*x)*3^(
1/2))*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {1660, 1642, 648, 632, 210, 642}

$$-\frac{7\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2(x+1)}{3\left(x^2-x+1\right)}-\frac{3}{2}\log\left(x^2-x+1\right)-\frac{1}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x^2*(1 - x + x^2)^2),x]
```

```
[Out] -x^(-1) + (2*(1 + x))/(3*(1 - x + x^2)) - (7*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*
Sqrt[3]) + 3*Log[x] - (3*Log[1 - x + x^2])/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx &= \frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{3+6x+2x^2}{x^2\left(1-x+x^2\right)}\,dx\\
&= \frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(\frac{3}{x^2}+\frac{9}{x}+\frac{8-9x}{1-x+x^2}\right)\,dx\\
&= -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+3\log(x)+\frac{1}{3}\int \frac{8-9x}{1-x+x^2}\,dx\\
&= -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+3\log(x)+\frac{7}{6}\int \frac{1}{1-x+x^2}\,dx-\frac{3}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)-\frac{7}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x\right.\\
&= -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 61, normalized size = 1.00

$$-\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{7\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x^2*(1 - x + x^2)^2),x]
```

```
[Out] -x^(-1) + (2*(1 + x))/(3*(1 - x + x^2)) + (7*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3
*Sqrt[3]) + 3*Log[x] - (3*Log[1 - x + x^2])/2
```

Maple [A]
time = 0.11, size = 55, normalized size = 0.90

method	result	size
default	$-\frac{1}{x}+3\ln\left(x\right)-\frac{-\frac{2x}{3}-\frac{2}{3}}{x^2-x+1}-\frac{3\ln\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	55
risch	$\frac{-\frac{1}{3}x^2+\frac{5}{3}x-1}{x(x^2-x+1)}-\frac{3\ln\left(4x^2-4x+4\right)}{2}+\frac{7\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}+3\ln\left(x\right)$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x^2/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x+3*ln(x)-(-2/3*x-2/3)/(x^2-x+1)-3/2*ln(x^2-x+1)+7/9*3^(1/2)*arctan(1/3*
(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.53, size = 54, normalized size = 0.89

$$\frac{7}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{x^2-5\,x+3}{3\,(x^3-x^2+x)}-\frac{3}{2}\,\log\left(x^2-x+1\right)+3\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/3*(x^2 - 5*x + 3)/(x^3 - x^2
+ x) - 3/2*log(x^2 - x + 1) + 3*log(x)
```

Fricas [A]
time = 0.53, size = 85, normalized size = 1.39

$$\frac{14\sqrt{3}\,(x^3-x^2+x)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-6\,x^2-27\,(x^3-x^2+x)\log\left(x^2-x+1\right)+54\,(x^3-x^2+x)\log\left(x\right)+30\,x-18}{18\,(x^3-x^2+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(14*sqrt(3)*(x^3 - x^2 + x)*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*x^2 - 27
*(x^3 - x^2 + x)*log(x^2 - x + 1) + 54*(x^3 - x^2 + x)*log(x) + 30*x - 18)/
(x^3 - x^2 + x)
```

Sympy [A]
time = 0.08, size = 65, normalized size = 1.07

$$\frac{-x^2+5x-3}{3x^3-3x^2+3x}+3\log(x)-\frac{3\log(x^2-x+1)}{2}+\frac{7\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**2/(x**2-x+1)**2,x)
```

```
[Out] (-x**2 + 5*x - 3)/(3*x**3 - 3*x**2 + 3*x) + 3*log(x) - 3*log(x**2 - x + 1)/
2 + 7*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Giac [A]
time = 4.20, size = 55, normalized size = 0.90

$$\frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{x^2-5x+3}{3(x^3-x^2+x)}-\frac{3}{2}\log\left(x^2-x+1\right)+3\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/3*(x^2 - 5*x + 3)/(x^3 - x^2
+ x) - 3/2*log(x^2 - x + 1) + 3*log(abs(x))
```

Mupad [B]
time = 4.13, size = 68, normalized size = 1.11

$$3\ln(x)-\frac{\frac{x^2}{3}-\frac{5x}{3}+1}{x^3-x^2+x}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,7\mathrm{i}}{18}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{3}{2}+\frac{\sqrt{3}\,7\mathrm{i}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^2*(x^2 - x + 1)^2),x)
```

```
[Out] 3*log(x) - (x^2/3 - (5*x)/3 + 1)/(x - x^2 + x^3) - log(x - (3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*7i)/18 + 3/2) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*7i)
/18 - 3/2)
```

3.166 $\int \frac{1+x+x^2}{x^3(1-x+x^2)^2} \, dx$

Optimal. Leaf size=68

$$-\frac{1}{2x^2} - \frac{3}{x} + \frac{2(2-x)}{3(1-x+x^2)} + \frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + 4\log(x) - 2\log\left(1-x+x^2\right)$$

```
[Out] -1/2/x^2-3/x+2/3*(2-x)/(x^2-x+1)+4*ln(x)-2*ln(x^2-x+1)+10/9*arctan(1/3*(1-2
*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1660, 1642, 648, 632, 210, 642}

$$\frac{10\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{2(2-x)}{3(x^2-x+1)} - \frac{1}{2x^2} - 2\log\left(x^2-x+1\right) - \frac{3}{x} + 4\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x^3*(1 - x + x^2)^2),x]
```

```
[Out] -1/2*1/x^2 - 3/x + (2*(2 - x))/(3*(1 - x + x^2)) + (10*ArcTan[(1 - 2*x)/Sqr
t[3]])/(3*Sqrt[3]) + 4*Log[x] - 2*Log[1 - x + x^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx &= \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{3+6x+6x^2-2x^3}{x^3\left(1-x+x^2\right)}\,dx \\
&= \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(\frac{3}{x^3}+\frac{9}{x^2}+\frac{12}{x}+\frac{1-12x}{1-x+x^2}\right)\,dx \\
&= -\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+4\log(x)+\frac{1}{3}\int \frac{1-12x}{1-x+x^2}\,dx \\
&= -\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+4\log(x)-\frac{5}{3}\int \frac{1}{1-x+x^2}\,dx-2\int \frac{-1+2x}{1-x+x^2}\,dx \\
&= -\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+4\log(x)-2\log\left(1-x+x^2\right)+\frac{10}{3}\mathrm{Subst}\left(\int \frac{1}{-3-\cdots}\right. \\
&= -\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+4\log(x)-2\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [**A**]

time = 0.02, size = 66, normalized size = 0.97

$$-\frac{1}{2x^2}-\frac{3}{x}-\frac{2(-2+x)}{3\left(1-x+x^2\right)}-\frac{10\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+4\log(x)-2\log\left(1-x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x^3*(1 - x + x^2)^2),x]
```

```
[Out] -1/2*1/x^2 - 3/x - (2*(-2 + x))/(3*(1 - x + x^2)) - (10*ArcTan[(-1 + 2*x)/S
qrt[3]])/(3*Sqrt[3]) + 4*Log[x] - 2*Log[1 - x + x^2]
```

Maple [A]
time = 0.10, size = 60, normalized size = 0.88

method	result	size
default	$-\frac{1}{2x^2}-\frac{3}{x}+4\ln(x)-\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}-2\ln\left(x^2-x+1\right)-\frac{10\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	60
risch	$\frac{-\frac{11}{3}x^3+\frac{23}{6}x^2-\frac{5}{2}x-\frac{1}{2}}{x^2(x^2-x+1)}+4\ln(x)-2\ln\left(100x^2-100x+100\right)-\frac{10\sqrt{3}\arctan\left(\frac{(10x-5)\sqrt{3}}{15}\right)}{9}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x^3/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2-3/x+4*ln(x)-(2/3*x-4/3)/(x^2-x+1)-2*ln(x^2-x+1)-10/9*3^(1/2)*arcta
n(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.51, size = 63, normalized size = 0.93

$$-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{22x^3-23x^2+15x+3}{6\left(x^4-x^3+x^2\right)}-2\log\left(x^2-x+1\right)+4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] -10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*(22*x^3 - 23*x^2 + 15*x +
 3)/(x^4 - x^3 + x^2) - 2*log(x^2 - x + 1) + 4*log(x)
```

Fricas [A]
time = 0.42, size = 98, normalized size = 1.44

$$-\frac{66x^3+20\sqrt{3}\left(x^4-x^3+x^2\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-69x^2+36\left(x^4-x^3+x^2\right)\log\left(x^2-x+1\right)-72\left(x^4-x^3+x^2\right)\log(x)+45x+9}{18\left(x^4-x^3+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(66*x^3 + 20*sqrt(3)*(x^4 - x^3 + x^2)*arctan(1/3*sqrt(3)*(2*x - 1))
- 69*x^2 + 36*(x^4 - x^3 + x^2)*log(x^2 - x + 1) - 72*(x^4 - x^3 + x^2)*log
(x) + 45*x + 9)/(x^4 - x^3 + x^2)
```

Sympy [A]
time = 0.09, size = 71, normalized size = 1.04

$$4\log(x) - 2\log\left(x^2 - x + 1\right) - \frac{10\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3} - \frac{\sqrt{3}}{3}\right)}{9} + \frac{-22x^3 + 23x^2 - 15x - 3}{6x^4 - 6x^3 + 6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**3/(x**2-x+1)**2,x)
```

```
[Out] 4*log(x) - 2*log(x**2 - x + 1) - 10*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)
/9 + (-22*x**3 + 23*x**2 - 15*x - 3)/(6*x**4 - 6*x**3 + 6*x**2)
```

Giac [A]
time = 4.26, size = 63, normalized size = 0.93

$$-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right) - \frac{22\,x^3 - 23\,x^2 + 15\,x + 3}{6\,(x^2 - x + 1)x^2} - 2\log\left(x^2 - x + 1\right) + 4\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] -10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*(22*x^3 - 23*x^2 + 15*x +
 3)/((x^2 - x + 1)*x^2) - 2*log(x^2 - x + 1) + 4*log(abs(x))
```

Mupad [B]
time = 0.10, size = 75, normalized size = 1.10

$$4\ln(x) + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-2 + \frac{\sqrt{3}\,5\mathrm{i}}{9}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(2 + \frac{\sqrt{3}\,5\mathrm{i}}{9}\right) - \frac{\frac{11\,x^3}{3} - \frac{23\,x^2}{6} + \frac{5\,x}{2} + \frac{1}{2}}{x^4 - x^3 + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^3*(x^2 - x + 1)^2),x)
```

```
[Out] 4*log(x) + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*5i)/9 - 2) - log(x + (3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*5i)/9 + 2) - ((5*x)/2 - (23*x^2)/6 + (11*x^3)/
3 + 1/2)/(x^2 - x^3 + x^4)
```

3.167 $\int \frac{1-x^2}{(1+x+x^2)^2} \, dx$

Optimal. Leaf size=10

$$\frac{x}{1+x+x^2}$$

[Out] x/(x^2+x+1)

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1602}

$$\frac{x}{x^2+x+1}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + x + x^2)^2,x]

[Out] x/(1 + x + x^2)

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1-x^2}{(1+x+x^2)^2} \, dx = \frac{x}{1+x+x^2}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{x}{1+x+x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + x + x^2)^2,x]

[Out] x/(1 + x + x^2)

Maple [A]
time = 0.15, size = 11, normalized size = 1.10

method	result	size
gosper	$\frac{x}{x^2+x+1}$	11
default	$\frac{x}{x^2+x+1}$	11
norman	$\frac{x}{x^2+x+1}$	11
risch	$\frac{x}{x^2+x+1}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^2+x+1)^2,x,method=_RETURNVERBOSE)
[Out] x/(x^2+x+1)
```

Maxima [A]
time = 0.29, size = 10, normalized size = 1.00

$$\frac{x}{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="maxima")
[Out] x/(x^2 + x + 1)
```

Fricas [A]
time = 0.34, size = 10, normalized size = 1.00

$$\frac{x}{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="fricas")
[Out] x/(x^2 + x + 1)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.70

$$\frac{x}{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**2+x+1)**2,x)
[Out] x/(x**2 + x + 1)
```

Giac [A]
time = 3.40, size = 8, normalized size = 0.80

$$\frac{1}{x+\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="giac")
```

```
[Out] 1/(x + 1/x + 1)
```

Mupad [B]
time = 0.05, size = 10, normalized size = 1.00

$$\frac{x}{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x + x^2 + 1)^2,x)
```

```
[Out] x/(x + x^2 + 1)
```

3.168 $\int \frac{1+x^2}{1+x+x^2} \, dx$

Optimal. Leaf size=31

$$x+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(1+x+x^2\right)$$

```
[Out] x-1/2*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1671, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2+x+1\right)+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x + x^2),x]
```

```
[Out] x + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1+x+x^2}\,dx &= \int \left(1-\frac{x}{1+x+x^2}\right)\,dx\\ &= x-\int \frac{x}{1+x+x^2}\,dx\\ &= x+\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx\\ &= x-\frac{1}{2}\log\left(1+x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\ &= x+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$x+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(1+x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + x + x^2),x]
[Out] x + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

Maple [A]
time = 0.12, size = 28, normalized size = 0.90

method	result	size
default	$x-\frac{\ln\left(x^2+x+1\right)}{2}+\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	28

risch	$x-\frac{\ln(4x^2+4x+4)}{2}+\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2+x+1),x,method=_RETURNVERBOSE)
[Out] x-1/2*ln(x^2+x+1)+1/3*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.52, size = 27, normalized size = 0.87

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x-\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

Fricas [A]
time = 0.37, size = 27, normalized size = 0.87

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x-\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

Sympy [A]
time = 0.04, size = 36, normalized size = 1.16

$$x-\frac{\log\left(x^2+x+1\right)}{2}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2+x+1),x)
[Out] x - log(x**2 + x + 1)/2 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

Giac [A]
time = 3.60, size = 27, normalized size = 0.87

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x-\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

Mupad [B]
time = 0.03, size = 29, normalized size = 0.94

$$x-\frac{\ln\left(x^2+x+1\right)}{2}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x + x^2 + 1),x)
```

```
[Out] x - log(x + x^2 + 1)/2 + (3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3))/3
```

3.169 $\int \frac{-1+x^2}{25-6x+x^2} dx$

Optimal. Leaf size=23

$$x - 2\tan^{-1}\left(\frac{1}{4}(-3+x)\right) + 3\log\left(25 - 6x + x^2\right)$$

```
[Out] x-2*arctan(-3/4+1/4*x)+3*ln(x^2-6*x+25)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1671, 648, 632, 210, 642}

$$-2\text{ArcTan}\left(\frac{x-3}{4}\right) + 3\log\left(x^2 - 6x + 25\right) + x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(25 - 6*x + x^2),x]
```

```
[Out] x - 2*ArcTan[(-3 + x)/4] + 3*Log[25 - 6*x + x^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^2}{25-6x+x^2}\,dx &= \int \left(1-\frac{2(13-3x)}{25-6x+x^2}\right)\,dx\\
&= x-2\int \frac{13-3x}{25-6x+x^2}\,dx\\
&= x+3\int \frac{-6+2x}{25-6x+x^2}\,dx-8\int \frac{1}{25-6x+x^2}\,dx\\
&= x+3\log\left(25-6x+x^2\right)+16\text{Subst}\left(\int \frac{1}{-64-x^2}\,dx,x,-6+2x\right)\\
&= x-2\tan^{-1}\left(\frac{1}{4}(-3+x)\right)+3\log\left(25-6x+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$x-2\tan^{-1}\left(\frac{1}{4}(-3+x)\right)+3\log\left(25-6x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(25 - 6*x + x^2),x]
```

```
[Out] x - 2*ArcTan[(-3 + x)/4] + 3*Log[25 - 6*x + x^2]
```

Maple [A]
time = 0.12, size = 22, normalized size = 0.96

method	result	size
default	$x-2\arctan\left(\frac{x}{4}-\frac{3}{4}\right)+3\ln\left(x^2-6x+25\right)$	22
risch	$x-2\arctan\left(\frac{x}{4}-\frac{3}{4}\right)+3\ln\left(x^2-6x+25\right)$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2-6*x+25),x,method=_RETURNVERBOSE)
```

```
[Out] x-2*arctan(1/4*x-3/4)+3*ln(x^2-6*x+25)
```

Maxima [A]
time = 0.52, size = 21, normalized size = 0.91

$$x - 2\ \arctan\left(\frac{1}{4}\,x - \frac{3}{4}\right) + 3\ \log\left(x^2 - 6\,x + 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2-6*x+25),x, algorithm="maxima")
[Out] x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

Fricas [A]
time = 0.36, size = 21, normalized size = 0.91

$$x - 2\ \arctan\left(\frac{1}{4}\,x - \frac{3}{4}\right) + 3\ \log\left(x^2 - 6\,x + 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2-6*x+25),x, algorithm="fricas")
[Out] x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

Sympy [A]
time = 0.03, size = 22, normalized size = 0.96

$$x + 3\log\left(x^2 - 6x + 25\right) - 2\,\mathrm{atan}\left(\frac{x}{4} - \frac{3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2-6*x+25),x)
[Out] x + 3*log(x**2 - 6*x + 25) - 2*atan(x/4 - 3/4)
```

Giac [A]
time = 3.88, size = 21, normalized size = 0.91

$$x - 2\ \arctan\left(\frac{1}{4}\,x - \frac{3}{4}\right) + 3\ \log\left(x^2 - 6\,x + 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2-6*x+25),x, algorithm="giac")
[Out] x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

Mupad [B]
time = 0.04, size = 21, normalized size = 0.91

$$x + 3\ \ln\left(x^2 - 6\,x + 25\right) - 2\,\mathrm{atan}\left(\frac{x}{4} - \frac{3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^2 - 6*x + 25),x)
```

```
[Out] x + 3*log(x^2 - 6*x + 25) - 2*atan(x/4 - 3/4)
```

3.170 $\int \frac{-10+3x^2}{4-4x+x^2} \, dx$

Optimal. Leaf size=21

$$\frac{2}{2-x} + 3x + 12\log(2-x)$$

```
[Out] 2/(2-x)+3*x+12*ln(2-x)
```

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {27, 711}

$$3x + \frac{2}{2-x} + 12\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(-10 + 3*x^2)/(4 - 4*x + x^2),x]
[Out] 2/(2 - x) + 3*x + 12*Log[2 - x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-10+3x^2}{4-4x+x^2} \, dx &= \int \frac{-10+3x^2}{(-2+x)^2} \, dx \\
&= \int \left(3 + \frac{2}{(-2+x)^2} + \frac{12}{-2+x}\right) dx \\
&= \frac{2}{2-x} + 3x + 12\log(2-x)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 19, normalized size = 0.90

$$-\frac{2}{-2+x} + 3(-2+x) + 12\log(-2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-10 + 3*x^2)/(4 - 4*x + x^2),x]
```

```
[Out] -2/(-2 + x) + 3*(-2 + x) + 12*Log[-2 + x]
```

Maple [A]
time = 0.10, size = 18, normalized size = 0.86

method	result	size
default	$3x+12\ln\left(x-2\right)-\frac{2}{x-2}$	18
risch	$3x+12\ln\left(x-2\right)-\frac{2}{x-2}$	18
norman	$\frac{3x^{2}-14}{x-2}+12\ln\left(x-2\right)$	21
meijerg	$-\frac{5x}{2\left(1-\frac{x}{2}\right)}+\frac{x\left(-\frac{3x}{2}+6\right)}{1-\frac{x}{2}}+12\ln\left(1-\frac{x}{2}\right)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-10)/(x^2-4*x+4),x,method=_RETURNVERBOSE)
```

```
[Out] 3*x+12*ln(x-2)-2/(x-2)
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.81

$$3\,x-\frac{2}{x-2}+12\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="maxima")
```

```
[Out] 3*x - 2/(x - 2) + 12*log(x - 2)
```

Fricas [A]
time = 0.45, size = 25, normalized size = 1.19

$$\frac{3\,x^{2}+12\,(x-2)\log\left(x-2\right)-6\,x-2}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="fricas")
```

```
[Out] (3*x^2 + 12*(x - 2)*log(x - 2) - 6*x - 2)/(x - 2)
```

Sympy [A]
time = 0.02, size = 14, normalized size = 0.67

$$3x+12\log\left(x-2\right)-\frac{2}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-10)/(x**2-4*x+4),x)
```

```
[Out] 3*x + 12*log(x - 2) - 2/(x - 2)
```

Giac [A]
time = 3.58, size = 18, normalized size = 0.86

$$3\,x-\frac{2}{x-2}+12\,\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="giac")
```

```
[Out] 3*x - 2/(x - 2) + 12*log(abs(x - 2))
```

Mupad [B]
time = 0.04, size = 17, normalized size = 0.81

$$3\,x+12\,\ln\left(x-2\right)-\frac{2}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 10)/(x^2 - 4*x + 4),x)
```

```
[Out] 3*x + 12*log(x - 2) - 2/(x - 2)
```

3.171 $\int \frac{8+x^2}{6-5x+x^2}\,dx$

Optimal. Leaf size=18

$$x - 12\log(2-x) + 17\log(3-x)$$

[Out] x-12*ln(2-x)+17*ln(3-x)

Rubi [A]

time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1671, 646, 31}

$$x - 12\log(2-x) + 17\log(3-x)$$

Antiderivative was successfully verified.

[In] Int[(8 + x^2)/(6 - 5*x + x^2),x]

[Out] x - 12*Log[2 - x] + 17*Log[3 - x]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{8+x^2}{6-5x+x^2}\,dx &= \int \left(1+\frac{2+5x}{6-5x+x^2}\right) dx\\ &= x+\int \frac{2+5x}{6-5x+x^2}\,dx\\ &= x-12\int \frac{1}{-2+x}\,dx+17\int \frac{1}{-3+x}\,dx\\ &= x-12\log(2-x)+17\log(3-x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$x-12\log(2-x)+17\log(3-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + x^2)/(6 - 5*x + x^2),x]
[Out] x - 12*Log[2 - x] + 17*Log[3 - x]
```

Maple [A]
time = 0.12, size = 15, normalized size = 0.83

method	result	size
default	$x+17\ln(x-3)-12\ln(x-2)$	15
norman	$x+17\ln(x-3)-12\ln(x-2)$	15
risch	$x+17\ln(x-3)-12\ln(x-2)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+8)/(x^2-5*x+6),x,method=_RETURNVERBOSE)
[Out] x+17*ln(x-3)-12*ln(x-2)
```

Maxima [A]
time = 0.29, size = 14, normalized size = 0.78

$$x-12\,\log(x-2)+17\,\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+8)/(x^2-5*x+6),x, algorithm="maxima")
[Out] x - 12*log(x - 2) + 17*log(x - 3)
```

Fricas [A]
time = 0.56, size = 14, normalized size = 0.78

$$x - 12 \log(x - 2) + 17 \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+8)/(x^2-5*x+6),x, algorithm="fricas")
[Out] x - 12*log(x - 2) + 17*log(x - 3)
```

Sympy [A]
time = 0.04, size = 14, normalized size = 0.78

$$x + 17\log(x - 3) - 12\log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+8)/(x**2-5*x+6),x)
[Out] x + 17*log(x - 3) - 12*log(x - 2)
```

Giac [A]
time = 4.19, size = 16, normalized size = 0.89

$$x - 12 \log(|x - 2|) + 17 \log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+8)/(x^2-5*x+6),x, algorithm="giac")
[Out] x - 12*log(abs(x - 2)) + 17*log(abs(x - 3))
```

Mupad [B]
time = 3.92, size = 14, normalized size = 0.78

$$x - 12 \ln(x - 2) + 17 \ln(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 8)/(x^2 - 5*x + 6),x)
[Out] x - 12*log(x - 2) + 17*log(x - 3)
```

3.172 $\int \frac{-4+3x+x^2}{-8-2x+x^2} \, dx$

Optimal. Leaf size=14

$$x + 4\log(4 - x) + \log(2 + x)$$

[Out] x+4*ln(4-x)+ln(2+x)

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1671, 646, 31}

$$x + 4\log(4 - x) + \log(x + 2)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 3*x + x^2)/(-8 - 2*x + x^2),x]
```

```
[Out] x + 4*Log[4 - x] + Log[2 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{-4+3x+x^2}{-8-2x+x^2}\,dx = \int \left(1+\frac{4+5x}{-8-2x+x^2}\right)dx$$
$$= x+\int \frac{4+5x}{-8-2x+x^2}\,dx$$
$$= x+4\int \frac{1}{-4+x}\,dx+\int \frac{1}{2+x}\,dx$$
$$= x+4\log(4-x)+\log(2+x)$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$x+4\log(4-x)+\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 3*x + x^2)/(-8 - 2*x + x^2),x]
```

```
[Out] x + 4*Log[4 - x] + Log[2 + x]
```

Maple [A]
time = 0.10, size = 13, normalized size = 0.93

method	result	size
default	$x+\ln(x+2)+4\ln(x-4)$	13
norman	$x+\ln(x+2)+4\ln(x-4)$	13
risch	$x+\ln(x+2)+4\ln(x-4)$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x-4)/(x^2-2*x-8),x,method=_RETURNVERBOSE)
```

```
[Out] x+ln(x+2)+4*ln(x-4)
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.86

$$x+\log(x+2)+4\,\log(x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="maxima")
```

```
[Out] x + log(x + 2) + 4*log(x - 4)
```

Fricas [A]
time = 0.46, size = 12, normalized size = 0.86

$$x + \log(x + 2) + 4 \log(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="fricas")
[Out] x + log(x + 2) + 4*log(x - 4)
```

Sympy [A]
time = 0.04, size = 12, normalized size = 0.86

$$x + 4\log(x - 4) + \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x-4)/(x**2-2*x-8),x)
[Out] x + 4*log(x - 4) + log(x + 2)
```

Giac [A]
time = 3.28, size = 14, normalized size = 1.00

$$x + \log(|x + 2|) + 4 \log(|x - 4|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="giac")
[Out] x + log(abs(x + 2)) + 4*log(abs(x - 4))
```

Mupad [B]
time = 3.85, size = 12, normalized size = 0.86

$$x + \ln(x + 2) + 4 \ln(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x + x^2 - 4)/(2*x - x^2 + 8),x)
[Out] x + log(x + 2) + 4*log(x - 4)
```

3.173 $\int \frac{7+5x+4x^2}{5+4x+4x^2} \, dx$

Optimal. Leaf size=27

$$x+\frac{3}{8}\tan^{-1}\left(\frac{1}{2}+x\right)+\frac{1}{8}\log\left(5+4x+4x^2\right)$$

```
[Out] x+3/8*arctan(1/2+x)+1/8*ln(4*x^2+4*x+5)
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1671, 648, 632, 210, 642}

$$\frac{3}{8}\text{ArcTan}\left(x+\frac{1}{2}\right)+\frac{1}{8}\log\left(4x^2+4x+5\right)+x$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
```

```
[Out] x + (3*ArcTan[1/2 + x])/8 + Log[5 + 4*x + 4*x^2]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx &= \int \left(1+\frac{2+x}{5+4x+4x^2}\right)dx\\ &= x+\int \frac{2+x}{5+4x+4x^2}\,dx\\ &= x+\frac{1}{8}\int \frac{4+8x}{5+4x+4x^2}\,dx+\frac{3}{2}\int \frac{1}{5+4x+4x^2}\,dx\\ &= x+\frac{1}{8}\log\left(5+4x+4x^2\right)-3\text{Subst}\left(\int \frac{1}{-64-x^2}\,dx,x,4+8x\right)\\ &= x+\frac{3}{8}\tan^{-1}\left(\frac{1}{2}+x\right)+\frac{1}{8}\log\left(5+4x+4x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.15

$$x+\frac{3}{8}\tan^{-1}\left(\frac{1}{2}(1+2x)\right)+\frac{1}{8}\log\left(5+4x+4x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
[Out] x + (3*ArcTan[(1 + 2*x)/2])/8 + Log[5 + 4*x + 4*x^2]/8
```

Maple [A]
time = 0.11, size = 22, normalized size = 0.81

method	result	size
default	$x+\frac{3\arctan\left(x+\frac{1}{2}\right)}{8}+\frac{\ln\left(4x^2+4x+5\right)}{8}$	22
risch	$x+\frac{3\arctan\left(x+\frac{1}{2}\right)}{8}+\frac{\ln\left(4x^2+4x+5\right)}{8}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+5*x+7)/(4*x^2+4*x+5),x,method=_RETURNVERBOSE)
[Out] x+3/8*arctan(x+1/2)+1/8*ln(4*x^2+4*x+5)
```

Maxima [A]
time = 0.52, size = 21, normalized size = 0.78

$$x+\frac{3}{8}\arctan\left(x+\frac{1}{2}\right)+\frac{1}{8}\log\left(4x^2+4x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="maxima")
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

Fricas [A]
time = 0.45, size = 21, normalized size = 0.78

$$x+\frac{3}{8}\arctan\left(x+\frac{1}{2}\right)+\frac{1}{8}\log\left(4x^2+4x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="fricas")
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

Sympy [A]
time = 0.04, size = 22, normalized size = 0.81

$$x+\frac{\log\left(x^2+x+\frac{5}{4}\right)}{8}+\frac{3\operatorname{atan}\left(x+\frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+5*x+7)/(4*x**2+4*x+5),x)
[Out] x + log(x**2 + x + 5/4)/8 + 3*atan(x + 1/2)/8
```

Giac [A]
time = 3.28, size = 21, normalized size = 0.78

$$x+\frac{3}{8}\arctan\left(x+\frac{1}{2}\right)+\frac{1}{8}\log\left(4x^2+4x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="giac")
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

Mupad [B]
time = 3.80, size = 17, normalized size = 0.63

$$x+\frac{\ln\left(x^2+x+\frac{5}{4}\right)}{8}+\frac{3\operatorname{atan}\left(x+\frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 4*x^2 + 7)/(4*x + 4*x^2 + 5),x)
[Out] x + log(x + x^2 + 5/4)/8 + (3*atan(x + 1/2))/8
```

3.174 $\int \frac{2-x+x^2}{-5+2x+x^2} \, dx$

Optimal. Leaf size=48

$$x-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(1-\sqrt{6}+x\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(1+\sqrt{6}+x\right)$$

```
[Out] x-1/6*ln(1+x-6^(1/2))*(9-5*6^(1/2))-1/6*ln(1+x+6^(1/2))*(9+5*6^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1671, 646, 31}

$$x-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(x-\sqrt{6}+1\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(x+\sqrt{6}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + x^2)/(-5 + 2*x + x^2),x]
```

```
[Out] x - ((9 - 5*Sqrt[6])*Log[1 - Sqrt[6] + x])/6 - ((9 + 5*Sqrt[6])*Log[1 + Sqr
t[6] + x])/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{2-x+x^2}{-5+2x+x^2}\,dx &= \int \left(1+\frac{7-3x}{-5+2x+x^2}\right)dx\\ &= x+\int \frac{7-3x}{-5+2x+x^2}\,dx\\ &= x+\frac{1}{6}\left(-9+5\sqrt{6}\right)\int \frac{1}{1-\sqrt{6}+x}\,dx-\frac{1}{6}\left(9+5\sqrt{6}\right)\int \frac{1}{1+\sqrt{6}+x}\,dx\\ &= x-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(1-\sqrt{6}+x\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(1+\sqrt{6}+x\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 1.00

$$x+\frac{1}{6}\left(-9+5\sqrt{6}\right)\log\left(-1+\sqrt{6}-x\right)+\frac{1}{6}\left(-9-5\sqrt{6}\right)\log\left(1+\sqrt{6}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^2)/(-5 + 2*x + x^2),x]
[Out] x + ((-9 + 5*Sqrt[6])*Log[-1 + Sqrt[6] - x])/6 + ((-9 - 5*Sqrt[6])*Log[1 +
Sqrt[6] + x])/6
```

Maple [A]
time = 0.13, size = 30, normalized size = 0.62

method	result	size
default	$x-\frac{3\ln\left(x^2+2x-5\right)}{2}-\frac{5\sqrt{6}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{6}}{12}\right)}{3}$	30
risch	$x-\frac{3\ln\left(1+x-\sqrt{6}\right)}{2}+\frac{5\ln\left(1+x-\sqrt{6}\right)\sqrt{6}}{6}-\frac{3\ln\left(1+x+\sqrt{6}\right)}{2}-\frac{5\ln\left(1+x+\sqrt{6}\right)\sqrt{6}}{6}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+2)/(x^2+2*x-5),x,method=_RETURNVERBOSE)
[Out] x-3/2*ln(x^2+2*x-5)-5/3*6^(1/2)*arctanh(1/12*(2*x+2)*6^(1/2))
```

Maxima [A]
time = 0.51, size = 36, normalized size = 0.75

$$\frac{5}{6}\sqrt{6}\log\left(\frac{x-\sqrt{6}+1}{x+\sqrt{6}+1}\right)+x-\frac{3}{2}\log\left(x^2+2\,x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="maxima")
```

```
[Out] 5/6*sqrt(6)*log((x - sqrt(6) + 1)/(x + sqrt(6) + 1)) + x - 3/2*log(x^2 + 2*
x - 5)
```

Fricas [A]
time = 0.36, size = 55, normalized size = 1.15

$$\frac{5}{6}\sqrt{3}\sqrt{2}\log\left(-\frac{2\sqrt{3}\sqrt{2}(x+1)-x^2-2x-7}{x^2+2x-5}\right)+x-\frac{3}{2}\log\left(x^2+2x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="fricas")
```

```
[Out] 5/6*sqrt(3)*sqrt(2)*log(-(2*sqrt(3)*sqrt(2)*(x + 1) - x^2 - 2*x - 7)/(x^2 +
 2*x - 5)) + x - 3/2*log(x^2 + 2*x - 5)
```

Sympy [A]
time = 0.04, size = 46, normalized size = 0.96

$$x+\left(-\frac{5\sqrt{6}}{6}-\frac{3}{2}\right)\log\left(x+1+\sqrt{6}\right)+\left(-\frac{3}{2}+\frac{5\sqrt{6}}{6}\right)\log\left(x-\sqrt{6}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+2)/(x**2+2*x-5),x)
```

```
[Out] x + (-5*sqrt(6)/6 - 3/2)*log(x + 1 + sqrt(6)) + (-3/2 + 5*sqrt(6)/6)*log(x
- sqrt(6) + 1)
```

Giac [A]
time = 3.65, size = 45, normalized size = 0.94

$$\frac{5}{6}\sqrt{6}\log\left(\frac{\left|2x-2\sqrt{6}+2\right|}{\left|2x+2\sqrt{6}+2\right|}\right)+x-\frac{3}{2}\log\left(\left|x^2+2x-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="giac")
```

```
[Out] 5/6*sqrt(6)*log(abs(2*x - 2*sqrt(6) + 2)/abs(2*x + 2*sqrt(6) + 2)) + x - 3/
2*log(abs(x^2 + 2*x - 5))
```

Mupad [B]
time = 0.11, size = 35, normalized size = 0.73

$$x-\ln\left(x+\sqrt{6}+1\right)\left(\frac{5\sqrt{6}}{6}+\frac{3}{2}\right)+\ln\left(x-\sqrt{6}+1\right)\left(\frac{5\sqrt{6}}{6}-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x + 2)/(2*x + x^2 - 5),x)
```

```
[Out] x - log(x + 6^(1/2) + 1)*((5*6^(1/2))/6 + 3/2) + log(x - 6^(1/2) + 1)*((5*6
^(1/2))/6 - 3/2)
```

3.175 $\int \frac{1+4x+3x^2}{\left(4+7x+2x^2\right)^2} \, dx$

Optimal. Leaf size=21

$$-\frac{2+3x}{2\left(4+7x+2x^2\right)}$$

[Out] 1/2*(-2-3*x)/(2*x^2+7*x+4)

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1674, 8}

$$-\frac{3x+2}{2\left(2x^2+7x+4\right)}$$

Antiderivative was successfully verified.

[In] Int[(1 + 4*x + 3*x^2)/(4 + 7*x + 2*x^2)^2,x]

[Out] -1/2*(2 + 3*x)/(4 + 7*x + 2*x^2)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+4x+3x^2}{\left(4+7x+2x^2\right)^2} \, dx &= -\frac{2+3x}{2\left(4+7x+2x^2\right)}-\frac{\int 0 \, dx}{17}\\
&= -\frac{2+3x}{2\left(4+7x+2x^2\right)}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$\frac{-2-3x}{2\left(4+7x+2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 3*x^2)/(4 + 7*x + 2*x^2)^2,x]
```

```
[Out] (-2 - 3*x)/(2*(4 + 7*x + 2*x^2))
```

Maple [A]
time = 0.10, size = 17, normalized size = 0.81

method	result	size
default	$\frac{-\frac{3x}{4}-\frac{1}{2}}{x^2+\frac{7}{2}x+2}$	17
risch	$\frac{-\frac{3x}{4}-\frac{1}{2}}{x^2+\frac{7}{2}x+2}$	17
norman	$\frac{-\frac{3x}{2}-1}{2x^2+7x+4}$	19
gosper	$-\frac{2+3x}{2(2x^2+7x+4)}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/4*x-1/2)/(x^2+7/2*x+2)
```

Maxima [A]
time = 0.29, size = 19, normalized size = 0.90

$$-\frac{3x+2}{2\left(2x^2+7x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

Fricas [A]
time = 0.35, size = 19, normalized size = 0.90

$$-\frac{3x+2}{2\left(2x^2+7x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

Sympy [A]
time = 0.04, size = 15, normalized size = 0.71

$$\frac{-3x-2}{4x^2+14x+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+4*x+1)/(2*x**2+7*x+4)**2,x)
```

```
[Out] (-3*x - 2)/(4*x**2 + 14*x + 8)
```

Giac [A]
time = 2.81, size = 19, normalized size = 0.90

$$-\frac{3\,x+2}{2\,(2\,x^2+7\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

Mupad [B]
time = 3.84, size = 17, normalized size = 0.81

$$-\frac{\frac{3\,x}{4}+\frac{1}{2}}{x^2+\frac{7\,x}{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 3*x^2 + 1)/(7*x + 2*x^2 + 4)^2,x)
```

```
[Out] -((3*x)/4 + 1/2)/((7*x)/2 + x^2 + 2)
```

3.176 $\int \frac{1+x+x^2}{(3+2x+x^2)^2} \, dx$

Optimal. Leaf size=39

$$\frac{1-x}{4(3+2x+x^2)}+\frac{3\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] 1/4*(1-x)/(x^2+2*x+3)+3/8*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1674, 12, 632, 210}

$$\frac{3\text{ArcTan}\left(\frac{x+1}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{1-x}{4(x^2+2x+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(3 + 2*x + x^2)^2,x]
```

```
[Out] (1 - x)/(4*(3 + 2*x + x^2)) + (3*ArcTan[(1 + x)/Sqrt[2]])/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
```

```
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^2}{(3+2x+x^2)^2}\,dx &= \frac{1-x}{4(3+2x+x^2)}+\frac{1}{8}\int \frac{6}{3+2x+x^2}\,dx\\ &= \frac{1-x}{4(3+2x+x^2)}+\frac{3}{4}\int \frac{1}{3+2x+x^2}\,dx\\ &= \frac{1-x}{4(3+2x+x^2)}-\frac{3}{2}\mathrm{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2+2x\right)\\ &= \frac{1-x}{4(3+2x+x^2)}+\frac{3\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.00

$$\frac{1-x}{4(3+2x+x^2)}+\frac{3\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(3 + 2*x + x^2)^2,x]
[Out] (1 - x)/(4*(3 + 2*x + x^2)) + (3*ArcTan[(1 + x)/Sqrt[2]])/(4*Sqrt[2])
```

Maple [A]
time = 0.24, size = 34, normalized size = 0.87

method	result	size
risch	$\frac{-\frac{x}{4}+\frac{1}{4}}{x^2+2x+3}+\frac{3\arctan\left(\frac{(1+x)\sqrt{2}}{2}\right)\sqrt{2}}{8}$	32
default	$\frac{-\frac{x}{4}+\frac{1}{4}}{x^2+2x+3}+\frac{3\sqrt{2}\,\arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{8}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/(x^2+2*x+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*x+1/4)/(x^2+2*x+3)+3/8*2^(1/2)*arctan(1/4*(2*x+2)*2^(1/2))
```

Maxima [A]
time = 0.51, size = 30, normalized size = 0.77

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\frac{x-1}{4(x^2+2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="maxima")
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - 1/4*(x - 1)/(x^2 + 2*x + 3)
```

Fricas [A]
time = 0.37, size = 39, normalized size = 1.00

$$\frac{3\sqrt{2}(x^2+2x+3)\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-2x+2}{8(x^2+2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="fricas")
[Out] 1/8*(3*sqrt(2)*(x^2 + 2*x + 3)*arctan(1/2*sqrt(2)*(x + 1)) - 2*x + 2)/(x^2
+ 2*x + 3)
```

Sympy [A]
time = 0.05, size = 37, normalized size = 0.95

$$\frac{1-x}{4x^2+8x+12}+\frac{3\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}+\frac{\sqrt{2}}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(x**2+2*x+3)**2,x)
[Out] (1 - x)/(4*x**2 + 8*x + 12) + 3*sqrt(2)*atan(sqrt(2)*x/2 + sqrt(2)/2)/8
```

Giac [A]
time = 3.64, size = 30, normalized size = 0.77

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\frac{x-1}{4(x^2+2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - 1/4*(x - 1)/(x^2 + 2*x + 3)
```

Mupad [B]
time = 3.84, size = 36, normalized size = 0.92

$$\frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}+\frac{\sqrt{2}}{2}\right)}{8}-\frac{\frac{x}{4}-\frac{1}{4}}{x^2+2\,x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(2*x + x^2 + 3)^2,x)
[Out] (3*2^(1/2)*atan((2^(1/2)*x)/2 + 2^(1/2)/2))/8 - (x/4 - 1/4)/(2*x + x^2 + 3)
```

3.177 $\int \frac{-1+2x+5x^2}{(1+x+x^2)^4}\,dx$

Optimal. Leaf size=11

$$-\frac{x}{(1+x+x^2)^3}$$

```
[Out] -x/(x^2+x+1)^3
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1602}

$$-\frac{x}{(x^2+x+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x + 5*x^2)/(1 + x + x^2)^4,x]
```

```
[Out] -(x/(1 + x + x^2)^3)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+2x+5x^2}{(1+x+x^2)^4}\,dx = -\frac{x}{(1+x+x^2)^3}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$-\frac{x}{(1+x+x^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x + 5*x^2)/(1 + x + x^2)^4,x]
```

```
[Out] -(x/(1 + x + x^2)^3)
```

Maple [A]
time = 0.12, size = 12, normalized size = 1.09

method	result	size
gosper	$-\frac{x}{(x^2+x+1)^3}$	12
default	$-\frac{x}{(x^2+x+1)^3}$	12
norman	$-\frac{x}{(x^2+x+1)^3}$	12
risch	$-\frac{x}{(x^2+x+1)^3}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x-1)/(x^2+x+1)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -x/(x^2+x+1)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(11) = 22$.
time = 0.28, size = 33, normalized size = 3.00

$$-\frac{x}{x^6 + 3\,x^5 + 6\,x^4 + 7\,x^3 + 6\,x^2 + 3\,x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="maxima")
```

```
[Out] -x/(x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(11) = 22$.
time = 0.34, size = 33, normalized size = 3.00

$$-\frac{x}{x^6 + 3\,x^5 + 6\,x^4 + 7\,x^3 + 6\,x^2 + 3\,x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="fricas")
```

```
[Out] -x/(x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(10) = 20$.
time = 0.04, size = 31, normalized size = 2.82

$$-\frac{x}{x^6 + 3x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x-1)/(x**2+x+1)**4,x)
```

```
[Out] -x/(x**6 + 3*x**5 + 6*x**4 + 7*x**3 + 6*x**2 + 3*x + 1)
```

Giac [A]
time = 4.57, size = 11, normalized size = 1.00

$$-\frac{x}{(x^2+x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="giac")
```

```
[Out] -x/(x^2 + x + 1)^3
```

Mupad [B]
time = 3.80, size = 11, normalized size = 1.00

$$-\frac{x}{(x^2+x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5*x^2 - 1)/(x + x^2 + 1)^4,x)
```

```
[Out] -x/(x + x^2 + 1)^3
```

3.178 $\int (a+bx+cx^2)^{5/2} (A+Cx^2)\,dx$

Optimal. Leaf size=267

$$\frac{5(b^2-4ac)^2(32Ac^2+9b^2C-4acC)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5}-\frac{5(b^2-4ac)(32Ac^2+9b^2C-4acC)(b+2cx}{6144c^4}$$

```
[Out] -5/6144*(-4*a*c+b^2)*(32*A*c^2-4*C*a*c+9*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(3/
2)/c^4+1/384*(32*A*c^2-4*C*a*c+9*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(5/2)/c^3-9
/112*b*C*(c*x^2+b*x+a)^(7/2)/c^2+1/8*C*x*(c*x^2+b*x+a)^(7/2)/c-5/32768*(-4*
a*c+b^2)^3*(32*A*c^2-4*C*a*c+9*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+
b*x+a)^(1/2))/c^(11/2)+5/16384*(-4*a*c+b^2)^2*(32*A*c^2-4*C*a*c+9*C*b^2)*(2
*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

Rubi [A]
time = 0.15, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1675, 654, 626, 635, 212}

$$-\frac{5(b^2-4ac)^3(-4acC+32Ac^2+9b^2C)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{11/2}}+\frac{5(b^2-4ac)^2(b+2cx)\sqrt{a+bx+cx^2}(-4acC+32Ac^2+9b^2C)}{16384c^5}-\frac{5(b^2-4ac)(b+2cx)(a+bx+cx^2)^{3/2}(-4acC+32Ac^2+9b^2C)}{6144c^4}+\frac{(b+2cx)(a+bx+cx^2)^{5/2}(-4acC+32Ac^2+9b^2C)}{384c^3}-\frac{9bC(a+bx+cx^2)^{7/2}}{112c^2}+\frac{Cx(a+bx+cx^2)^{7/2}}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)*(A + C*x^2),x]
```

```
[Out] (5*(b^2 - 4*a*c)^2*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b*x
+ c*x^2])/(16384*c^5) - (5*(b^2 - 4*a*c)*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/2))/(6144*c^4) + ((32*A*c^2 + 9*b^2*C - 4*a*c
*C)*(b + 2*c*x)*(a + b*x + c*x^2)^(5/2))/(384*c^3) - (9*b*C*(a + b*x + c*x^
2)^(7/2))/(112*c^2) + (C*x*(a + b*x + c*x^2)^(7/2))/(8*c) - (5*(b^2 - 4*a*c
)^3*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])])/(32768*c^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{5/2}\left(A+Cx^2\right)\,dx=\frac{Cx\left(a+bx+cx^2\right)^{7/2}}{8c}+\frac{\int \left(8Ac-aC-\frac{9bCx}{2}\right)\left(a+bx+cx^2\right)^{5/2}\,dx}{8c}$$

$$=-\frac{9bC\left(a+bx+cx^2\right)^{7/2}}{112c^2}+\frac{Cx\left(a+bx+cx^2\right)^{7/2}}{8c}+\frac{\left(\frac{9b^2C}{2}+2c(8Ac-\right.}{}$$

$$=\frac{\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{384c^3}-\frac{9bC(a+bx+}{112c^2}$$

$$=-\frac{5\left(b^2-4ac\right)\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{6144c^4}+$$

$$=\frac{5\left(b^2-4ac\right)^2\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5}-\frac{5(b}{}$$

$$=\frac{5\left(b^2-4ac\right)^2\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5}-\frac{5(b}{}$$

$$=\frac{5\left(b^2-4ac\right)^2\left(32Ac^2+9b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5}-\frac{5(b}{}$$

Mathematica [A]
time = 1.50, size = 357, normalized size = 1.34

$$\frac{2\sqrt{c}\,\sqrt{a+x(b+cx)}\,(224Ac^2(b+2cx)\,(15b^4-40b^3cx+32bc^2x(13a+8cx^2)+8b^2c(-20a+11cx^2)+16c^2(33a^2+26acx^2+8c^2x^4))+C(945b^7-630b^6cx+8b^4c^2x(791a-54cx^2)+84b^5c(-125a+6cx^2)+16b^3c^2(2359a^2-284acx^2+24c^2x^4)+96b^2c^3x(-199a^2+36acx^2+648c^2x^4)+896c^4x(15a^3+118a^2cx^2+136ac^2x^4+48c^3x^6)+64bc^3(-663a^3+174a^2cx^2+2456ac^2x^4+1584c^3x^6)))+105(b^2-4ac)^3\,(32Ac^2+9b^2C-4acC)\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)}{688128c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)*(A + C*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(224*A*c^2*(b + 2*c*x)*(15*b^4 - 40*b^3*c*
x + 32*b*c^2*x*(13*a + 8*c*x^2) + 8*b^2*c*(-20*a + 11*c*x^2) + 16*c^2*(33*a
^2 + 26*a*c*x^2 + 8*c^2*x^4)) + C*(945*b^7 - 630*b^6*c*x + 8*b^4*c^2*x*(791
*a - 54*c*x^2) + 84*b^5*c*(-125*a + 6*c*x^2) + 16*b^3*c^2*(2359*a^2 - 284*a
*c*x^2 + 24*c^2*x^4) + 96*b^2*c^3*x*(-199*a^2 + 36*a*c*x^2 + 648*c^2*x^4) +
 896*c^4*x*(15*a^3 + 118*a^2*c*x^2 + 136*a*c^2*x^4 + 48*c^3*x^6) + 64*b*c^3
*(-663*a^3 + 174*a^2*c*x^2 + 2456*a*c^2*x^4 + 1584*c^3*x^6))) + 105*(b^2 -
4*a*c)^3*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a +
x*(b + c*x)]])/(688128*c^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(237) = 474$.
time = 0.15, size = 487, normalized size = 1.82

method	result

default	C	$\left(\frac{x(cx^2+bx+a)^{\frac{7}{2}}}{8c} - \frac{9b\left(\frac{(cx^2+bx+a)^{\frac{7}{2}}}{7c} - \frac{b\left(\frac{(2cx+b)(cx^2+bx+a)^{\frac{5}{2}}}{12c} + \frac{5(4ac-b^2)\left(\frac{(2cx+b)(cx^2+bx+a)^{\frac{3}{2}}}{8c} + \frac{3(4ac-b^2)\left(\frac{(2cx+b)\sqrt{\ldots}}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.}{2c}\right.}{16c}\right.$

risch	$\frac{(43008c^7Cx^7+101376bc^6Cx^6+57344Ac^7x^5+121856Cac^6x^5+62208Cb^2c^5x^5+143360Abc^6x^4+157184Cabc^5x^4+384Cb^3c^4x^4+186}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] C*(1/8*x*(c*x^2+b*x+a)^(7/2)/c-9/16*b/c*(1/7*(c*x^2+b*x+a)^(7/2)/c-1/2*b/c*
(1/12*(2*c*x+b)/c*(c*x^2+b*x+a)^(5/2)+5/24*(4*a*c-b^2)/c*(1/8*(2*c*x+b)/c*(
c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+
1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))-1/8
*a/c*(1/12*(2*c*x+b)/c*(c*x^2+b*x+a)^(5/2)+5/24*(4*a*c-b^2)/c*(1/8*(2*c*x+b
)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(
1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))
)+A*(1/12*(2*c*x+b)/c*(c*x^2+b*x+a)^(5/2)+5/24*(4*a*c-b^2)/c*(1/8*(2*c*x+b)
/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1
/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(237) = 474$.
time = 0.52, size = 953, normalized size = 3.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] [1/1376256*(105*(9*C*b^8 - 112*C*a*b^6*c - 2048*A*a^3*c^5 + 256*(C*a^4 + 6*
A*a^2*b^2)*c^4 - 384*(2*C*a^3*b^2 + A*a*b^4)*c^3 + 32*(15*C*a^2*b^4 + A*b^6
)*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*
c*x + b)*sqrt(c) - 4*a*c) + 4*(43008*C*c^8*x^7 + 101376*C*b*c^7*x^6 + 945*C
*b^7*c - 10500*C*a*b^5*c^2 + 118272*A*a^2*b*c^5 + 256*(243*C*b^2*c^6 + 476*
```

```
C*a*c^7 + 224*A*c^8)*x^5 - 64*(663*C*a^3*b + 560*A*a*b^3)*c^4 + 128*(3*C*b^
3*c^5 + 1228*C*a*b*c^6 + 1120*A*b*c^7)*x^4 + 112*(337*C*a^2*b^3 + 30*A*b^5)
*c^3 - 16*(27*C*b^4*c^4 - 216*C*a*b^2*c^5 - 11648*A*a*c^7 - 112*(59*C*a^2 +
 54*A*b^2)*c^6)*x^3 + 8*(63*C*b^5*c^3 - 568*C*a*b^3*c^4 + 34944*A*a*b*c^6 +
 16*(87*C*a^2*b + 14*A*b^3)*c^5)*x^2 - 2*(315*C*b^6*c^2 - 3164*C*a*b^4*c^3
- 118272*A*a^2*c^6 - 1344*(5*C*a^3 + 8*A*a*b^2)*c^5 + 16*(597*C*a^2*b^2 + 7
0*A*b^4)*c^4)*x)*sqrt(c*x^2 + b*x + a))/c^6, 1/688128*(105*(9*C*b^8 - 112*C
*a*b^6*c - 2048*A*a^3*c^5 + 256*(C*a^4 + 6*A*a^2*b^2)*c^4 - 384*(2*C*a^3*b^
2 + A*a*b^4)*c^3 + 32*(15*C*a^2*b^4 + A*b^6)*c^2)*sqrt(-c)*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(43008*C
*c^8*x^7 + 101376*C*b*c^7*x^6 + 945*C*b^7*c - 10500*C*a*b^5*c^2 + 118272*A*
a^2*b*c^5 + 256*(243*C*b^2*c^6 + 476*C*a*c^7 + 224*A*c^8)*x^5 - 64*(663*C*a
^3*b + 560*A*a*b^3)*c^4 + 128*(3*C*b^3*c^5 + 1228*C*a*b*c^6 + 1120*A*b*c^7)
*x^4 + 112*(337*C*a^2*b^3 + 30*A*b^5)*c^3 - 16*(27*C*b^4*c^4 - 216*C*a*b^2*
c^5 - 11648*A*a*c^7 - 112*(59*C*a^2 + 54*A*b^2)*c^6)*x^3 + 8*(63*C*b^5*c^3
- 568*C*a*b^3*c^4 + 34944*A*a*b*c^6 + 16*(87*C*a^2*b + 14*A*b^3)*c^5)*x^2 -
 2*(315*C*b^6*c^2 - 3164*C*a*b^4*c^3 - 118272*A*a^2*c^6 - 1344*(5*C*a^3 + 8
*A*a*b^2)*c^5 + 16*(597*C*a^2*b^2 + 70*A*b^4)*c^4)*x)*sqrt(c*x^2 + b*x + a)
)/c^6]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A+Cx^2\right)\left(a+bx+cx^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)*(C*x**2+A),x)
```

```
[Out] Integral((A + C*x**2)*(a + b*x + c*x**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(237) = 474.
time = 3.76, size = 482, normalized size = 1.81

$$\frac{1}{344064}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(2\left(12\left(14Cc^2x+33Cbc\right)x+\frac{243Cb^2c^7+476Cac^8+224Ac^9}{c^7}\right)x+\frac{3Cb^3c^6+1228Cabc^7+1120Abc^8}{c^7}\right)x-\frac{27Cb^4c^5-216Cab^2c^6-6608Ca^2c^7-6048Ab^2c^7-11648Aac^8}{c^7}\right)x+\frac{63Cb^5c^4-568Cab^3c^5+1392Ca^2bc^6+224Ab^3c^6+34944Aabc^7}{c^7}\right)x-\frac{315Cb^6c^3-3164Cab^4c^4+9552Ca^2b^2c^5+1120Ab^4c^5-6720Ca^3c^6-10752Aab^2c^6-118272Aa^2c^7}{c^7}\right)x+\frac{945Cb^7c^2-10500Cab^5c^3+37744Ca^2b^3c^4+3360Ab^5c^4-42432Ca^3bc^5-35840Aab^3c^5+118272Aa^2bc^6}{c^7}\right)+\frac{5\left(9Cb^8-112Cab^6c+480Ca^2b^4c^2+32Ab^6c^2-768Ca^3b^2c^3-384Aab^4c^3+256Ca^4c^4+1536Aa^2b^2c^4-2048Aa^3c^5\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{32768c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/344064*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(12*(14*C*c^2*x + 33*C*b*c)*x
 + (243*C*b^2*c^7 + 476*C*a*c^8 + 224*A*c^9)/c^7)*x + (3*C*b^3*c^6 + 1228*C
*a*b*c^7 + 1120*A*b*c^8)/c^7)*x - (27*C*b^4*c^5 - 216*C*a*b^2*c^6 - 6608*C*
a^2*c^7 - 6048*A*b^2*c^7 - 11648*A*a*c^8)/c^7)*x + (63*C*b^5*c^4 - 568*C*a*
b^3*c^5 + 1392*C*a^2*b*c^6 + 224*A*b^3*c^6 + 34944*A*a*b*c^7)/c^7)*x - (315
*C*b^6*c^3 - 3164*C*a*b^4*c^4 + 9552*C*a^2*b^2*c^5 + 1120*A*b^4*c^5 - 6720*
```

```
C*a^3*c^6 - 10752*A*a*b^2*c^6 - 118272*A*a^2*c^7)/c^7)*x + (945*C*b^7*c^2 -
 10500*C*a*b^5*c^3 + 37744*C*a^2*b^3*c^4 + 3360*A*b^5*c^4 - 42432*C*a^3*b*c
^5 - 35840*A*a*b^3*c^5 + 118272*A*a^2*b*c^6)/c^7) + 5/32768*(9*C*b^8 - 112*
C*a*b^6*c + 480*C*a^2*b^4*c^2 + 32*A*b^6*c^2 - 768*C*a^3*b^2*c^3 - 384*A*a*
b^4*c^3 + 256*C*a^4*c^4 + 1536*A*a^2*b^2*c^4 - 2048*A*a^3*c^5)*log(abs(-2*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(C\,x^2+A\right)\,\left(c\,x^2+b\,x+a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] int((A + C*x^2)*(a + b*x + c*x^2)^(5/2), x)
```

3.179 $\int (a+bx+cx^2)^{3/2} (A+Cx^2) \, dx$

Optimal. Leaf size=212

$$-\frac{(b^2-4ac)(24Ac^2+7b^2C-4acC)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{(24Ac^2+7b^2C-4acC)(b+2cx)(a+bx-}{192c^3}$$

```
[Out] 1/192*(24*A*c^2-4*C*a*c+7*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^3-7/60*b*C
*(c*x^2+b*x+a)^(5/2)/c^2+1/6*C*x*(c*x^2+b*x+a)^(5/2)/c+1/1024*(-4*a*c+b^2)^
2*(24*A*c^2-4*C*a*c+7*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/c^(9/2)-1/512*(-4*a*c+b^2)*(24*A*c^2-4*C*a*c+7*C*b^2)*(2*c*x+b)*(c*x^2
+b*x+a)^(1/2)/c^4
```

Rubi [A]
time = 0.11, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1675, 654, 626, 635, 212}

$$\frac{(b^2-4ac)^2(-4acC+24Ac^2+7b^2C)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}}-\frac{(b^2-4ac)(b+2cx)\sqrt{a+bx+cx^2}(-4acC+24Ac^2+7b^2C)}{512c^4}+\frac{(b+2cx)(a+bx+cx^2)^{3/2}(-4acC+24Ac^2+7b^2C)}{192c^3}-\frac{7bC(a+bx+cx^2)^{5/2}}{60c^2}+\frac{Cx(a+bx+cx^2)^{5/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(A + C*x^2),x]
```

```
[Out] -1/512*((b^2 - 4*a*c)*(24*A*c^2 + 7*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b
*x + c*x^2])/c^4 + ((24*A*c^2 + 7*b^2*C - 4*a*c*C)*(b + 2*c*x)*(a + b*x + c
*x^2)^(3/2))/(192*c^3) - (7*b*C*(a + b*x + c*x^2)^(5/2))/(60*c^2) + (C*x*(a
 + b*x + c*x^2)^(5/2))/(6*c) + ((b^2 - 4*a*c)^2*(24*A*c^2 + 7*b^2*C - 4*a*c
*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{3/2}\left(A+Cx^2\right)\,dx=\frac{Cx(a+bx+cx^2)^{5/2}}{6c}+\frac{\int \left(6Ac-aC-\frac{7bCx}{2}\right)\left(a+bx+cx^2\right)^{3/2}\,dx}{6c}$$

$$=-\frac{7bC(a+bx+cx^2)^{5/2}}{60c^2}+\frac{Cx(a+bx+cx^2)^{5/2}}{6c}+\frac{\left(\frac{7b^2C}{2}+2c(6Ac-a\right.}{}$$

$$=\frac{\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3}-\frac{7bC(a+bx+c}{60c^2}$$

$$=-\frac{\left(b^2-4ac\right)\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{(24A}{}$$

$$=-\frac{\left(b^2-4ac\right)\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{(24A}{}$$

$$=-\frac{\left(b^2-4ac\right)\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{(24A}{}$$

Mathematica [A]
time = 0.88, size = 227, normalized size = 1.07

$$\frac{2\sqrt{c}\,\sqrt{a+x(b+cx)}\left(120Ac^2(b+2cx)\left(-3b^2+8bcx+4c(5a+2cx^2)\right)+C\left(-105b^5+70b^4cx+8b^3c(95a-7cx^2)+48b^2c^2x(-9a+cx^2)+160c^3x(3a^2+14acx^2+8c^2x^4)+16bc^2(-81a^2+18acx^2+104c^2x^4)\right)\right)-15(b^2-4ac)^2\left(24Ac^2+7b^2C-4acC\right)\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)}{15360c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(A + C*x^2),x]
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(120*A*c^2*(b + 2*c*x)*(-3*b^2 + 8*b*c*x +
 4*c*(5*a + 2*c*x^2)) + C*(-105*b^5 + 70*b^4*c*x + 8*b^3*c*(95*a - 7*c*x^2)
 + 48*b^2*c^2*x*(-9*a + c*x^2) + 160*c^3*x*(3*a^2 + 14*a*c*x^2 + 8*c^2*x^4)
 + 16*b*c^2*(-81*a^2 + 18*a*c*x^2 + 104*c^2*x^4))) - 15*(b^2 - 4*a*c)^2*(24
*A*c^2 + 7*b^2*C - 4*a*c*C)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]
])/(15360*c^(9/2))
```

Maple [A]
time = 0.13, size = 370, normalized size = 1.75

method	result
default	$C\left(\frac{x(cx^2+bx+a)^{\frac{5}{2}}}{6c}-\frac{7b\left(\frac{(cx^2+bx+a)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(2cx+b)(cx^2+bx+a)^{\frac{3}{2}}}{8c}+\frac{3(4ac-b^2)\left(\frac{(2cx+b)\sqrt{cx^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{b}{2} \ldots\right.}{\ldots}\right.}{16c}\right.}{2c}\right.}{12c}\right.$
risch	$\frac{(1280c^5Cx^5+1664bCc^4x^4+1920Ac^5x^3+2240Cac^4x^3+48Cb^2c^3x^3+2880Abc^4x^2+288Cabc^3x^2-56Cb^3c^2x^2+4800Aac^4x+240Ab \ldots}{7680 \ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] C*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*
(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c
*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)
/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+A*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+
3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(
3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 0.49, size = 605, normalized size = 2.85

$$\left[\frac{15\,(7\,Cb^6 - 60\,Cab^4c + 384\,Aa^2c^4 - 64\,(Ca^3 + 3\,Aab^2)c^3 + 24\,(6\,Ca^2b^2 + Ab^4)c^2)\sqrt{c}\log\left(-8\,c^2x^2 - 8\,bcx - b^2 - 4\,\sqrt{cx^2 + bx + a}\,(2\,cx + b)\sqrt{c} - 4\,ac\right) + 4\,(1280\,Cc^6x^5 + 1664\,Cbc^5x^4 - 105\,Cb^5c + 760\,Cab^3c^2 + 2400\,Aabc^4 - 72\,(18\,Ca^2b + 5\,Ab^3)c^3 + 16\,(3\,Cb^2c^4 + 140\,Cac^5 + 120\,Ac^6)x^3 - 8\,(7\,Cb^3c^3 - 36\,Cabc^4 - 360\,Abc^5)x^2 + 2\,(35\,Cb^4c^2 - 216\,Cab^2c^3 + 2400\,Aac^5 + 120\,(2\,Ca^2 + Ab^2)c^4)x)\sqrt{cx^2 + bx + a}}{30720\,c^5}, -\frac{15\,(7\,Cb^6 - 60\,Cab^4c + 384\,Aa^2c^4 - 64\,(Ca^3 + 3\,Aab^2)c^3 + 24\,(6\,Ca^2b^2 + Ab^4)c^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2 + bx + a}\,(2\,cx + b)\sqrt{-c}}{2\,(c^2x^2 + bcx + ac)}\right) - 2\,(1280\,Cc^6x^5 + 1664\,Cbc^5x^4 - 105\,Cb^5c + 760\,Cab^3c^2 + 2400\,Aabc^4 - 72\,(18\,Ca^2b + 5\,Ab^3)c^3 + 16\,(3\,Cb^2c^4 + 140\,Cac^5 + 120\,Ac^6)x^3 - 8\,(7\,Cb^3c^3 - 36\,Cabc^4 - 360\,Abc^5)x^2 + 2\,(35\,Cb^4c^2 - 216\,Cab^2c^3 + 2400\,Aac^5 + 120\,(2\,Ca^2 + Ab^2)c^4)x)\sqrt{cx^2 + bx + a}}{15360\,c^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] [1/30720*(15*(7*C*b^6 - 60*C*a*b^4*c + 384*A*a^2*c^4 - 64*(C*a^3 + 3*A*a*b^
2)*c^3 + 24*(6*C*a^2*b^2 + A*b^4)*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b
^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(1280*C*c^6*x
^5 + 1664*C*b*c^5*x^4 - 105*C*b^5*c + 760*C*a*b^3*c^2 + 2400*A*a*b*c^4 - 72
*(18*C*a^2*b + 5*A*b^3)*c^3 + 16*(3*C*b^2*c^4 + 140*C*a*c^5 + 120*A*c^6)*x^
3 - 8*(7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*b*c^5)*x^2 + 2*(35*C*b^4*c^2 - 21
6*C*a*b^2*c^3 + 2400*A*a*c^5 + 120*(2*C*a^2 + A*b^2)*c^4)*x)*sqrt(c*x^2 + b
*x + a))/c^5, -1/15360*(15*(7*C*b^6 - 60*C*a*b^4*c + 384*A*a^2*c^4 - 64*(C*
a^3 + 3*A*a*b^2)*c^3 + 24*(6*C*a^2*b^2 + A*b^4)*c^2)*sqrt(-c)*arctan(1/2*sq
rt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1280
*C*c^6*x^5 + 1664*C*b*c^5*x^4 - 105*C*b^5*c + 760*C*a*b^3*c^2 + 2400*A*a*b*
c^4 - 72*(18*C*a^2*b + 5*A*b^3)*c^3 + 16*(3*C*b^2*c^4 + 140*C*a*c^5 + 120*A
*c^6)*x^3 - 8*(7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*b*c^5)*x^2 + 2*(35*C*b^4*
c^2 - 216*C*a*b^2*c^3 + 2400*A*a*c^5 + 120*(2*C*a^2 + A*b^2)*c^4)*x)*sqrt(c
*x^2 + b*x + a))/c^5]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + Cx^2\right)\left(a + bx + cx^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(C*x**2+A),x)
```

```
[Out] Integral((A + C*x**2)*(a + b*x + c*x**2)**(3/2), x)
```

Giac [A]
time = 3.97, size = 297, normalized size = 1.40

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\,(10\,Ccx+13\,Cb)x+\frac{3\,Cb^2c^4+140\,Cac^5+120\,Ac^6}{c^5}\right)x-\frac{7\,Cb^3c^3-36\,Cabc^4-360\,Abc^5}{c^5}\right)x+\frac{35\,Cb^4c^2-216\,Cab^2c^3+240\,Ca^2c^4+120\,Ab^2c^4+2400\,Aac^5}{c^5}\right)x-\frac{105\,Cb^5c-760\,Cab^3c^2+1296\,Ca^2bc^3+360\,Ab^3c^3-2400\,Aabc^4}{c^5}\right)-\frac{\left(7\,Cb^6-60\,Cab^4c+144\,Ca^2b^2c^2+24\,Ab^4c^2-64\,Ca^3c^3-192\,Aab^2c^3+384\,Aa^2c^4\right)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{1024\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*C*c*x + 13*C*b)*x + (3*C*b^2*c
^4 + 140*C*a*c^5 + 120*A*c^6)/c^5)*x - (7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*
b*c^5)/c^5)*x + (35*C*b^4*c^2 - 216*C*a*b^2*c^3 + 240*C*a^2*c^4 + 120*A*b^2
*c^4 + 2400*A*a*c^5)/c^5)*x - (105*C*b^5*c - 760*C*a*b^3*c^2 + 1296*C*a^2*b
*c^3 + 360*A*b^3*c^3 - 2400*A*a*b*c^4)/c^5) - 1/1024*(7*C*b^6 - 60*C*a*b^4*
c + 144*C*a^2*b^2*c^2 + 24*A*b^4*c^2 - 64*C*a^3*c^3 - 192*A*a*b^2*c^3 + 384
*A*a^2*c^4)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^
(9/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(C\,x^2+A\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int((A + C*x^2)*(a + b*x + c*x^2)^(3/2), x)
```

3.180 $\int \sqrt{a+bx+cx^2}\,(A+Cx^2)\,dx$

Optimal. Leaf size=157

$$\frac{(16Ac^2+5b^2C-4acC)\,(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}-\frac{5bC(a+bx+cx^2)^{3/2}}{24c^2}+\frac{Cx(a+bx+cx^2)^{3/2}}{4c}-\frac{(b^2-4ac)}{}$$

```
[Out] -5/24*b*C*(c*x^2+b*x+a)^(3/2)/c^2+1/4*C*x*(c*x^2+b*x+a)^(3/2)/c-1/128*(-4*a
*c+b^2)*(16*A*c^2-4*C*a*c+5*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x
+a)^(1/2))/c^(7/2)+1/64*(16*A*c^2-4*C*a*c+5*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^
(1/2)/c^3
```

Rubi [A]
time = 0.09, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1675, 654, 626, 635, 212}

$$-\frac{(b^2-4ac)\,(-4acC+16Ac^2+5b^2C)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}+\frac{(b+2cx)\sqrt{a+bx+cx^2}\,(-4acC+16Ac^2+5b^2C)}{64c^3}-\frac{5bC(a+bx+cx^2)^{3/2}}{24c^2}+\frac{Cx(a+bx+cx^2)^{3/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]*(A + C*x^2),x]
```

```
[Out] ((16*A*c^2 + 5*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(64*c^3)
 - (5*b*C*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (C*x*(a + b*x + c*x^2)^(3/2))
/(4*c) - ((b^2 - 4*a*c)*(16*A*c^2 + 5*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)\,dx = \frac{Cx\left(a+bx+cx^2\right)^{3/2}}{4c}+\frac{\int \left(4Ac-aC-\frac{5bCx}{2}\right)\sqrt{a+bx+cx^2}\,dx}{4c}$$

$$= -\frac{5bC\left(a+bx+cx^2\right)^{3/2}}{24c^2}+\frac{Cx\left(a+bx+cx^2\right)^{3/2}}{4c}+\frac{\left(\frac{5b^2C}{2}+2c(4Ac-aC\right.}{}$$

$$= \frac{\left(16Ac^2+5b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}-\frac{5bC\left(a+bx+cx^2\right)}{24c^2}$$

$$= \frac{\left(16Ac^2+5b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}-\frac{5bC\left(a+bx+cx^2\right)}{24c^2}$$

$$= \frac{\left(16Ac^2+5b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}-\frac{5bC\left(a+bx+cx^2\right)}{24c^2}$$

Mathematica [A]
time = 0.37, size = 142, normalized size = 0.90

$$\frac{2\sqrt{c}\,\sqrt{a+x(b+cx)}\left(48Ac^2(b+2cx)+C\left(15b^3-10b^2cx+24c^2x\left(a+2cx^2\right)+b\left(-52ac+8c^2x^2\right)\right)\right)+3\left(b^2-4ac\right)\left(16Ac^2+5b^2C-4acC\right)\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(A + C*x^2),x]

[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(48*A*c^2*(b + 2*c*x) + C*(15*b^3 - 10*b^2
*c*x + 24*c^2*x*(a + 2*c*x^2) + b*(-52*a*c + 8*c^2*x^2))) + 3*(b^2 - 4*a*c)
```

```
*(16*A*c^2 + 5*b^2*C - 4*a*c*C)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c
*x)]])/(384*c^(7/2))
```

Maple [A]
time = 0.13, size = 253, normalized size = 1.61

method	result
risch	$\frac{\left(48 c^{3} C x^{3}+8 b c^{2} C x^{2}+96 c^{3} A x+24 c^{2} a C x-10 C b^{2} c x+48 b c^{2} A-52 C a b c+15 C b^{3}\right) \sqrt{c x^{2}+b x+a}}{192 c^{3}}+\frac{\ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c x^{2}+b x}\right.}{2 \sqrt{c}}$
default	$C \left(\frac{x\left(c x^{2}+b x+a\right)^{\frac{3}{2}}}{4 c}-\frac{5 b \left(\frac{\left(c x^{2}+b x+a\right)^{\frac{3}{2}}}{3 c}-\frac{b \left(\frac{(2 c x+b) \sqrt{c x^{2}+b x+a}}{4 c}+\frac{\left(4 a c-b^{2}\right) \ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c x^{2}+b x+a}\right)}{8 c^{\frac{3}{2}}}\right)}{2 c}\right)}{8 c}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x,method=_RETURNVERBOSE)
```

```
[Out] C*(1/4*x*(c*x^2+b*x+a)^(3/2)/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(
1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)
+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+A*(1
/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x+a)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.42, size = 355, normalized size = 2.26

$$\left[-\frac{3\,(5\,Cb^4-24\,Cab^2c-64\,Aac^3+16\,(Ca^2+Ab^2)c^2)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{c}-4\,ac\right)-4\,(48\,Cc^4x^3+8\,Cbc^3x^2+15\,Cb^3c-52\,Cabc^2+48\,Abc^3-2\,(5\,Cb^2c^2-12\,Cac^3-48\,Ac^4)x)\sqrt{cx^2+bx+a}}{768\,c^4},\ \frac{3\,(5\,Cb^4-24\,Cab^2c-64\,Aac^3+16\,(Ca^2+Ab^2)c^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)+2\,(48\,Cc^4x^3+8\,Cbc^3x^2+15\,Cb^3c-52\,Cabc^2+48\,Abc^3-2\,(5\,Cb^2c^2-12\,Cac^3-48\,Ac^4)x)\sqrt{cx^2+bx+a}}{384\,c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(5*C*b^4 - 24*C*a*b^2*c - 64*A*a*c^3 + 16*(C*a^2 + A*b^2)*c^2)*s
qrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)
*sqrt(c) - 4*a*c) - 4*(48*C*c^4*x^3 + 8*C*b*c^3*x^2 + 15*C*b^3*c - 52*C*a*b
*c^2 + 48*A*b*c^3 - 2*(5*C*b^2*c^2 - 12*C*a*c^3 - 48*A*c^4)*x)*sqrt(c*x^2 +
 b*x + a))/c^4, 1/384*(3*(5*C*b^4 - 24*C*a*b^2*c - 64*A*a*c^3 + 16*(C*a^2 +
 A*b^2)*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)
/(c^2*x^2 + b*c*x + a*c)) + 2*(48*C*c^4*x^3 + 8*C*b*c^3*x^2 + 15*C*b^3*c -
52*C*a*b*c^2 + 48*A*b*c^3 - 2*(5*C*b^2*c^2 - 12*C*a*c^3 - 48*A*c^4)*x)*sqrt
(c*x^2 + b*x + a))/c^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A+Cx^2\right)\sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(C*x**2+A),x)
```

```
[Out] Integral((A + C*x**2)*sqrt(a + b*x + c*x**2), x)
```

Giac [A]
time = 4.16, size = 160, normalized size = 1.02

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,Cx+\frac{Cb}{c}\right)x-\frac{5\,Cb^2c-12\,Cac^2-48\,Ac^3}{c^3}\right)x+\frac{15\,Cb^3-52\,Cabc+48\,Abc^2}{c^3}\right)+\frac{(5\,Cb^4-24\,Cab^2c+16\,Ca^2c^2+16\,Ab^2c^2-64\,Aac^3)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{128\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*C*x + C*b/c)*x - (5*C*b^2*c - 12*C*a*c
^2 - 48*A*c^3)/c^3)*x + (15*C*b^3 - 52*C*a*b*c + 48*A*b*c^2)/c^3) + 1/128*(
5*C*b^4 - 24*C*a*b^2*c + 16*C*a^2*c^2 + 16*A*b^2*c^2 - 64*A*a*c^3)*log(abs(
-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

Mupad [B]
time = 4.26, size = 240, normalized size = 1.53

$$A\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a}-\frac{C\,a\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}+\frac{A\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}-\frac{5\,C\,b\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{8\,c}+\frac{C\,x\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] A*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (C*a*((x/2 + b/(4*c))*(a + b*x
+ c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c -
 b^2/4))/(2*c^(3/2))))/(4*c) + (A*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^
2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) - (5*C*b*((log((b + 2*c*x)/c^(1/2) + 2
*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2)
 - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) + (C*x*(a + b
*x + c*x^2)^(3/2))/(4*c)
```

3.181 $\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=104

$$-\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}+\frac{(8Ac^2+3b^2C-4acC)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}$$

```
[Out] 1/8*(8*A*c^2-4*C*a*c+3*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(5/2)-3/4*b*C*(c*x^2+b*x+a)^(1/2)/c^2+1/2*C*x*(c*x^2+b*x+a)^(1/2)/c
```

Rubi [A]
time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1675, 654, 635, 212}

$$\frac{(-4acC+8Ac^2+3b^2C)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}-\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-3*b*C*Sqrt[a + b*x + c*x^2])/(4*c^2) + (C*x*Sqrt[a + b*x + c*x^2])/(2*c)
+ ((8*A*c^2 + 3*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(8*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{Cx\sqrt{a+bx+cx^2}}{2c}+\frac{\int \frac{2Ac-aC-\frac{3bCx}{2}}{\sqrt{a+bx+cx^2}}\,dx}{2c}$$

$$= -\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}+\frac{\left(\frac{3b^2C}{2}+2c(2Ac-aC)\right)\int \frac{1}{\sqrt{a+bx+\cdots}}$$

$$= -\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}+\frac{\left(\frac{3b^2C}{2}+2c(2Ac-aC)\right)\text{Subst}\left(\int \frac{}{4c\cdots}\right.}{2c^2}$$

$$= -\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}+\frac{(8Ac^2+3b^2C-4acC)\tanh^{-1}\left(\frac{}{2\sqrt{c}\cdots}\right.}{8c^{5/2}}$$

Mathematica [A]
time = 0.35, size = 88, normalized size = 0.85

$$\frac{C(-3b+2cx)\sqrt{a+x(b+cx)}}{4c^2}-\frac{(8Ac^2+3b^2C-4acC)\log\left(c^2\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)}{8c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (C*(-3*b + 2*c*x)*Sqrt[a + x*(b + c*x)])/(4*c^2) - ((8*A*c^2 + 3*b^2*C - 4*
a*c*C)*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(8*c^(5/2))
```

Maple [A]
time = 0.14, size = 138, normalized size = 1.33

method	result
risch	$-\frac{C(-2cx+3b)\sqrt{c\,x^2+bx+a}}{4c^2}+\frac{A\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}-\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)aC}{2c^{\frac{3}{2}}}+\frac{3\ln\left(\cdots\right.}{}$

default	$C\left(\frac{x\sqrt{c\,x^{2}+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^{2}+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+b\right.}{2c^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+A*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))/c^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.46, size = 203, normalized size = 1.95

$$\left[\frac{(3\,Cb^{2}-4\,Cac+8\,Ac^{2})\sqrt{c}\,\log\left(-8\,c^{2}x^{2}-8\,bcx-b^{2}-4\,\sqrt{cx^{2}+bx+a}\,(2\,cx+b)\sqrt{c}-4\,ac\right)+4\,(2\,Cc^{2}x-3\,Cbc)\sqrt{cx^{2}+bx+a}}{16\,c^{3}},-\frac{(3\,Cb^{2}-4\,Cac+8\,Ac^{2})\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{2}+bx+a}\,(2\,cx+b)\sqrt{-c}}{2\,(c^{2}x^{2}+bcx+ac)}\right)-2\,(2\,Cc^{2}x-3\,Cbc)\sqrt{cx^{2}+bx+a}}{8\,c^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*C*b^2 - 4*C*a*c + 8*A*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2
 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2*C*c^2*x - 3*
C*b*c)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((3*C*b^2 - 4*C*a*c + 8*A*c^2)*sqrt
(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x
 + a*c)) - 2*(2*C*c^2*x - 3*C*b*c)*sqrt(c*x^2 + b*x + a))/c^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Cx^2}{\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((A + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

Giac [A]
time = 3.99, size = 84, normalized size = 0.81

$$\frac{1}{4}\,\sqrt{cx^2 + bx + a}\left(\frac{2\,Cx}{c} - \frac{3\,Cb}{c^2}\right) - \frac{(3\,Cb^2 - 4\,Cac + 8\,Ac^2)\log\left(\left|-2\left(\sqrt{c}\,x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} - b\right|\right)}{8\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*C*x/c - 3*C*b/c^2) - 1/8*(3*C*b^2 - 4*C*a*c +
8*A*c^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/
2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\,x^2 + A}{\sqrt{c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((A + C*x^2)/(a + b*x + c*x^2)^(1/2), x)
```

3.182 $\int \frac{A+Cx^2}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{C\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

```
[Out] C*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)-2*(b*c*(A+a*C/
c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1674, 12, 635, 212}

$$\frac{C\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(c*(b^2 - 4*a*c)*S
qrt[a + b*x + c*x^2]) + (C*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx &= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{\left(b^2-4ac\right)C}{2c\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{C\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{c}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{(2C)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2c}{\sqrt{a+bx}}\right.}{c}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{C\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.49, size = 103, normalized size = 1.05

$$\frac{\frac{2\sqrt{c}\left(b^2Cx+aC(b-2cx)+Ac(b+2cx)\right)}{\sqrt{a+x(b+cx)}}+\left(b^2-4ac\right)C\log\left(c\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{c^{3/2}\left(-b^2+4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x)))/Sqrt[a + x*(b +
c*x)] + (b^2 - 4*a*c)*C*Log[c*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])
])/(c^(3/2)*(-b^2 + 4*a*c))
```

Maple [A]
time = 0.13, size = 145, normalized size = 1.48

method	result
default	$C\left(-\frac{x}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right.}{c^{\frac{3}{2}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)
/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2)))+2*A*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(88) = 176$.
time = 0.51, size = 403, normalized size = 4.11

$$\left[\frac{(Cab^{2}-4\,Ca^{2}c+(Cb^{2}c-4\,Cac^{2})x^{2}+(Cb^{3}-4\,Cabc)x)\sqrt{c}\,\log\left(-8\,c^{2}x^{2}-8\,bcx-b^{2}-4\,\sqrt{cx^{2}+bx+a}\,(2\,cx+b)\sqrt{c}-4\,ac\right)-4\,(Cabc+Abc^{2}+(Cb^{2}c-2\,Cac^{2}+2\,Ac^{3})x)\sqrt{cx^{2}+bx+a}}{2\,(ab^{2}c^{2}-4\,a^{2}c^{3}+(b^{2}c^{3}-4\,ac^{4})x^{2}+(b^{3}c^{2}-4\,abc^{3})x)},\ -\frac{(Cab^{2}-4\,Ca^{2}c+(Cb^{2}c-4\,Cac^{2})x^{2}+(Cb^{3}-4\,Cabc)x)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{2}+bx+a}\,(2\,cx+b)\sqrt{-c}}{2\,(c^{2}x^{2}+bcx+ac)}\right)+2\,(Cabc+Abc^{2}+(Cb^{2}c-2\,Cac^{2}+2\,Ac^{3})x)\sqrt{cx^{2}+bx+a}}{ab^{2}c^{2}-4\,a^{2}c^{3}+(b^{2}c^{3}-4\,ac^{4})x^{2}+(b^{3}c^{2}-4\,abc^{3})x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((C*a*b^2 - 4*C*a^2*c + (C*b^2*c - 4*C*a*c^2)*x^2 + (C*b^3 - 4*C*a*b*c
)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*
x + b)*sqrt(c) - 4*a*c) - 4*(C*a*b*c + A*b*c^2 + (C*b^2*c - 2*C*a*c^2 + 2*A
*c^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4
)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -((C*a*b^2 - 4*C*a^2*c + (C*b^2*c - 4*C*a
*c^2)*x^2 + (C*b^3 - 4*C*a*b*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a
)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(C*a*b*c + A*b*c^2 + (C
*b^2*c - 2*C*a*c^2 + 2*A*c^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*
c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((A + C*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Giac [A]
time = 4.43, size = 110, normalized size = 1.12

$$-\frac{2\left(\frac{(Cb^2-2\,Cac+2\,Ac^2)x}{b^2c-4\,ac^2}+\frac{Cab+Abc}{b^2c-4\,ac^2}\right)}{\sqrt{cx^2+bx+a}}-\frac{C\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}\,-b\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((C*b^2 - 2*C*a*c + 2*A*c^2)*x/(b^2*c - 4*a*c^2) + (C*a*b + A*b*c)/(b^2*
c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - C*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*sqrt(c) - b))/c^(3/2)
```

Mupad [B]
time = 4.21, size = 108, normalized size = 1.10

$$\frac{C\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{c^{3/2}}+\frac{A\,\left(\frac{b}{2}+c\,x\right)}{\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}+\frac{C\,\left(\frac{a\,b}{2}-x\,\left(a\,c-\frac{b^2}{2}\right)\right)}{c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] (C*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) + (A*(b/2 +
c*x))/((a*c - b^2/4)*(a + b*x + c*x^2)^(1/2)) + (C*((a*b)/2 - x*(a*c - b^2/
2)))/(c*(a*c - b^2/4)*(a + b*x + c*x^2)^(1/2))
```

3.183 $\int \frac{A+Cx^2}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{2\left(8Ac+4aC+\frac{b^2C}{c}\right)(b+2cx)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

```
[Out] -2/3*(b*c*(A+a*C/c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^(3/2)+2/3*(8*A*c+4*a*C+b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^(1
/2)
```

Rubi [A]
time = 0.04, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1674, 12, 627}

$$\frac{2(b+2cx)\left(4aC+8Ac+\frac{b^2C}{c}\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(3*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(3/2)) + (2*(8*A*c + 4*a*C + (b^2*C)/c)*(b + 2*c*x))/(3*
(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
```

```
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Cx^2}{(a+bx+cx^2)^{5/2}}\,dx &= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{2\int \frac{8Ac+4aC+\frac{b^2C}{c}}{2(a+bx+cx^2)^{3/2}}\,dx}{3\left(b^2-4ac\right)}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{\left(8Ac+4aC+\frac{b^2C}{c}\right)\int \frac{1}{(a+bx+cx^2)^{3/2}}}{3\left(b^2-4ac\right)}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{2\left(8Ac+4aC+\frac{b^2C}{c}\right)(b+2cx)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.78, size = 107, normalized size = 0.94

$$
\frac{-2A(b+2cx)\left(b^2-8bcx-4c(3a+2cx^2)\right)+2C\left(8a^2b+b^2x^2(3b+2cx)+4ax(3b^2+3bcx+2c^2x^2)\right)}{3\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(5/2),x]
[Out] (-2*A*(b + 2*c*x)*(b^2 - 8*b*c*x - 4*c*(3*a + 2*c*x^2)) + 2*C*(8*a^2*b + b^
2*x^2*(3*b + 2*c*x) + 4*a*x*(3*b^2 + 3*b*c*x + 2*c^2*x^2)))/(3*(b^2 - 4*a*c
)^2*(a + x*(b + c*x))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(106) = 212.
time = 0.14, size = 259, normalized size = 2.27

method	result
trager	$\frac{\frac{32}{3}Ac^3x^3+\frac{16}{3}Cac^2x^3+\frac{4}{3}Cb^2cx^3+16Abc^2x^2+8Cabcx^2+2Cb^3x^2+16Aac^2x+4Ab^2cx+8Cab^2x+8Aabc-\frac{2}{3}Ab^3+\frac{16}{3}a^2bC}{(4ac-b^2)^2(cx^2+bx+a)^{\frac{3}{2}}}$
gosper	$\frac{\frac{32}{3}Ac^3x^3+\frac{16}{3}Cac^2x^3+\frac{4}{3}Cb^2cx^3+16Abc^2x^2+8Cabcx^2+2Cb^3x^2+16Aac^2x+4Ab^2cx+8Cab^2x+8Aabc-\frac{2}{3}Ab^3+\frac{16}{3}a^2bC}{(cx^2+bx+a)^{\frac{3}{2}}(16a^2c^2-8ab^2c+b^4)}$

default	$C\left(-\frac{x}{2c(cx^2+bx+a)^{\frac{3}{2}}}-\frac{b\left(-\frac{1}{3c(cx^2+bx+a)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{(4ac-b^2)(cx^2+bx+a)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3(4ac-b^2)^2\sqrt{cx^2+bx+a}}\right)}{2c}\right)}{4c}+\frac{a\left(\frac{}{(4ac}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] C*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+A*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more deta

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(106) = 212$.
time = 0.95, size = 242, normalized size = 2.12

$$\frac{2\,(8\,Ca^2b-Ab^3+12\,Aabc+2\,(Cb^2c+4\,Cac^2+8\,Ac^3)x^3+3\,(Cb^3+4\,Cabc+8\,Abc^2)x^2+6\,(2\,Cab^2+Ab^2c+4\,Aac^2)x)\sqrt{cx^2+bx+a}}{3\,(a^2b^4-8\,a^3b^2c+16\,a^4c^2+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^4+2\,(b^5c-8\,ab^3c^2+16\,a^2bc^3)x^3+(b^6-6\,ab^4c+32\,a^3c^3)x^2+2\,(ab^5-8\,a^2b^3c+16\,a^3bc^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")

[Out] 2/3*(8*C*a^2*b - A*b^3 + 12*A*a*b*c + 2*(C*b^2*c + 4*C*a*c^2 + 8*A*c^3)*x^3 + 3*(C*b^3 + 4*C*a*b*c + 8*A*b*c^2)*x^2 + 6*(2*C*a*b^2 + A*b^2*c + 4*A*a*c^2)*x)*sqrt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2

```
 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x
^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b
*c^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Cx^2}{(a + bx + cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral((A + C*x**2)/(a + b*x + c*x**2)**(5/2), x)
```

Giac [A]
time = 3.85, size = 193, normalized size = 1.69

$$\frac{2\left(\left(\left(\frac{2\,(Cb^2c+4\,Cac^2+8\,Ac^3)x}{b^4-8\,ab^2c+16\,a^2c^2}+\frac{3\,(Cb^3+4\,Cabc+8\,Abc^2)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{6\,(2\,Cab^2+Ab^2c+4\,Aac^2)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{8\,Ca^2b-Ab^3+12\,Aabc}{b^4-8\,ab^2c+16\,a^2c^2}\right)}{3\,(cx^2+bx+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(((2*(C*b^2*c + 4*C*a*c^2 + 8*A*c^3)*x/(b^4 - 8*a*b^2*c + 16*a^2*c^2) +
 3*(C*b^3 + 4*C*a*b*c + 8*A*b*c^2)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + 6*(2
*C*a*b^2 + A*b^2*c + 4*A*a*c^2)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + (8*C*a^
2*b - A*b^3 + 12*A*a*b*c)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c*x^2 + b*x + a)
^(3/2)
```

Mupad [B]
time = 4.14, size = 127, normalized size = 1.11

$$\frac{2\,(8\,C\,a^2\,b+12\,C\,a\,b^2\,x+12\,C\,a\,b\,c\,x^2+12\,A\,a\,b\,c+8\,C\,a\,c^2\,x^3+24\,A\,a\,c^2\,x+3\,C\,b^3\,x^2-A\,b^3+2\,C\,b^2\,c\,x^3+6\,A\,b^2\,c\,x+24\,A\,b\,c^2\,x^2+16\,A\,c^3\,x^3)}{3\,(4\,a\,c-b^2)^2\,(c\,x^2+b\,x+a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] (2*(16*A*c^3*x^3 - A*b^3 + 3*C*b^3*x^2 + 8*C*a^2*b + 24*A*a*c^2*x + 6*A*b^2
*c*x + 12*C*a*b^2*x + 24*A*b*c^2*x^2 + 8*C*a*c^2*x^3 + 2*C*b^2*c*x^3 + 12*A
*a*b*c + 12*C*a*b*c*x^2))/(3*(4*a*c - b^2)^2*(a + b*x + c*x^2)^(3/2))
```

3.184 $\int \frac{A+Cx^2}{(a+bx+cx^2)^{7/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}+\frac{2\left(16Ac+4aC+\frac{3b^2C}{c}\right)(b+2cx)}{15\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{3/2}}-\frac{16(16Ac^2+3b^2C+4acC}{15\left(b^2-4ac\right)^3\sqrt{a+bx}}$$

```
[Out] -2/5*(b*c*(A+a*C/c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^(5/2)+2/15*(16*A*c+4*a*C+3*b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a
)^(3/2)-16/15*(16*A*c^2+4*C*a*c+3*C*b^2)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2+b*
x+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1674, 12, 628, 627}

$$-\frac{16(b+2cx)\left(4acC+16Ac^2+3b^2C\right)}{15\left(b^2-4ac\right)^3\sqrt{a+bx+cx^2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}+\frac{2(b+2cx)\left(4aC+16Ac+\frac{3b^2C}{c}\right)}{15\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(7/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(5*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(5/2)) + (2*(16*A*c + 4*a*C + (3*b^2*C)/c)*(b + 2*c*x))/
(15*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(3/2)) - (16*(16*A*c^2 + 3*b^2*C + 4*
a*c*C)*(b + 2*c*x))/(15*(b^2 - 4*a*c)^3*Sqrt[a + b*x + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
```

```
egerQ[4*p]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{7/2}}\,dx = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+(b^2-2ac)\,C\right)x\right)}{5c\,(b^2-4ac)\,(a+bx+cx^2)^{5/2}} - \frac{2\int \frac{16Ac+4aC+\frac{3b^2C}{c}}{2(a+bx+cx^2)^{5/2}}\,dx}{5\,(b^2-4ac)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+(b^2-2ac)\,C\right)x\right)}{5c\,(b^2-4ac)\,(a+bx+cx^2)^{5/2}} - \frac{\left(16Ac+4aC+\frac{3b^2C}{c}\right)\int \frac{1}{(a+bx+cx^2)^{5}}}{5\,(b^2-4ac)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+(b^2-2ac)\,C\right)x\right)}{5c\,(b^2-4ac)\,(a+bx+cx^2)^{5/2}} + \frac{2\left(16Ac+4aC+\frac{3b^2C}{c}\right)(b+2cx)}{15\,(b^2-4ac)^2\,(a+bx+cx^2)^{3/2}} +$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+(b^2-2ac)\,C\right)x\right)}{5c\,(b^2-4ac)\,(a+bx+cx^2)^{5/2}} + \frac{2\left(16Ac+4aC+\frac{3b^2C}{c}\right)(b+2cx)}{15\,(b^2-4ac)^2\,(a+bx+cx^2)^{3/2}} -$$

Mathematica [A]
time = 1.76, size = 234, normalized size = 1.40

$$-\frac{2(A(b+2cx)(3b^4-16b^3cx+64bc^2x(5a+4cx^2)+8b^2c(-5a+14cx^2)+16c^2(15a^2+20acx^2+8c^2x^4))+C(96a^3bc+3b^2x^2(5b^3+30b^2cx+40bc^2x^2+16c^3x^3)+8a^2(b^3+30b^2cx+30bc^2x^2+20c^3x^3)+4ax(5b^4+50b^3cx+60b^2c^2x^2+40bc^3x^3+16c^4x^4)))}{15\,(b^2-4ac)^3\,(a+x(b+cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(7/2),x]
```

```
[Out] (-2*(A*(b + 2*c*x)*(3*b^4 - 16*b^3*c*x + 64*b*c^2*x*(5*a + 4*c*x^2) + 8*b^2
*c*(-5*a + 14*c*x^2) + 16*c^2*(15*a^2 + 20*a*c*x^2 + 8*c^2*x^4)) + C*(96*a^
3*b*c + 3*b^2*x^2*(5*b^3 + 30*b^2*c*x + 40*b*c^2*x^2 + 16*c^3*x^3) + 8*a^2*
(b^3 + 30*b^2*c*x + 30*b*c^2*x^2 + 20*c^3*x^3) + 4*a*x*(5*b^4 + 50*b^3*c*x
+ 60*b^2*c^2*x^2 + 40*b*c^3*x^3 + 16*c^4*x^4))))/(15*(b^2 - 4*a*c)^3*(a + x
*(b + c*x))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(155) = 310$.

time = 0.12, size = 403, normalized size = 2.41

method	result
trager	$\frac{2C b^5 x^2+\frac{16}{15}C a^2 b^3+\frac{512}{15}A c^5 x^5+32Aa b^2 c^2 x+32C a^2 b^2 cx+\frac{80}{3}Ca b^3 c x^2+32C a^2 b c^2 x^2+\frac{64}{3}Cab c^3 x^4+32Ca b^2 c^2 x^3+128Aab c^3 x^2+}{}$
gosper	$\frac{2C b^5 x^2+\frac{16}{15}C a^2 b^3+\frac{512}{15}A c^5 x^5+32Aa b^2 c^2 x+32C a^2 b^2 cx+\frac{80}{3}Ca b^3 c x^2+32C a^2 b c^2 x^2+\frac{64}{3}Cab c^3 x^4+32Ca b^2 c^2 x^3+128Aab c^3 x^2+}{}$
default	$C\left(-\frac{x}{4c(c x^2+bx+a)^{\frac{5}{2}}}-\frac{3b\left(-\frac{1}{5c\left(c x^2+bx+a\right)^{\frac{5}{2}}}-\frac{b\left(\frac{\frac{4cx}{5}+\frac{2b}{5}}{\left(4ac-b^2\right)\left(c x^2+bx+a\right)^{\frac{5}{2}}}+\frac{16c\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{\left(4ac-b^2\right)\left(c x^2+bx+a\right)^{\frac{3}{2}}}+\frac{16c(2cx}{3\left(4ac-b^2\right)^2\sqrt{c x^2}}\right.}{5\left(4ac-b^2\right)}\right.}{2c}\right.}{8c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-1/4*x/c/(c*x^2+b*x+a)^(5/2)-3/8*b/c*(-1/5/c/(c*x^2+b*x+a)^(5/2)-1/2*b/c
*(2/5*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(5/2)+16/5*c/(4*a*c-b^2)*(2/3*(2*
c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^
2+b*x+a)^(1/2))))+1/4*a/c*(2/5*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(5/2)+16
/5*c/(4*a*c-b^2)*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a
*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2))))+A*(2/5*(2*c*x+b)/(4*a*c-b^2)/(c*
x^2+b*x+a)^(5/2)+16/5*c/(4*a*c-b^2)*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a
)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(155) = 310$.
time = 5.28, size = 563, normalized size = 3.37

$$-\frac{2\left(8\,Ca^2b^3+3\,Ab^5+240\,Aa^2bc^2+16\left(3\,Cb^2c^3+4\,Cac^4+16\,Ac^5\right)x^5+40\left(3\,Cb^3c^2+4\,Cabc^3+16\,Abc^4\right)x^4+10\left(9\,Cb^4c+24\,Cab^2c^2+64\,Aac^4+16\left(Ca^2+3\,Ab^2\right)c^3\right)x^3+5\left(3\,Cb^5+40\,Cab^3c+192\,Aabc^3+16\left(3\,Ca^2b+Ab^3\right)c^2\right)x^2+8\left(12\,Ca^3b-5\,Aab^3\right)c+10\left(2\,Cab^4+24\,Aab^2c^2+48\,Aa^2c^3+\left(24\,Ca^2b^2-Ab^4\right)c\right)x\right)\sqrt{cx^2+bx+a}}{15\left(a^3b^6-12\,a^4b^4c+48\,a^5b^2c^2-64\,a^6c^3+\left(b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6\right)x^6+3\left(b^7c^2-12\,ab^5c^3+48\,a^2b^3c^4-64\,a^3bc^5\right)x^5+3\left(b^8c-11\,ab^6c^2+36\,a^2b^4c^3-16\,a^3b^2c^4-64\,a^4c^5\right)x^4+\left(b^9-6\,ab^7c-24\,a^2b^5c^2+224\,a^3b^3c^3-384\,a^4bc^4\right)x^3+3\left(ab^8-11\,a^2b^6c+36\,a^3b^4c^2-16\,a^4b^2c^3-64\,a^5c^4\right)x^2+3\left(a^2b^7-12\,a^3b^5c+48\,a^4b^3c^2-64\,a^5bc^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*C*a^2*b^3 + 3*A*b^5 + 240*A*a^2*b*c^2 + 16*(3*C*b^2*c^3 + 4*C*a*c^
4 + 16*A*c^5)*x^5 + 40*(3*C*b^3*c^2 + 4*C*a*b*c^3 + 16*A*b*c^4)*x^4 + 10*(9
*C*b^4*c + 24*C*a*b^2*c^2 + 64*A*a*c^4 + 16*(C*a^2 + 3*A*b^2)*c^3)*x^3 + 5*
(3*C*b^5 + 40*C*a*b^3*c + 192*A*a*b*c^3 + 16*(3*C*a^2*b + A*b^3)*c^2)*x^2 +
 8*(12*C*a^3*b - 5*A*a*b^3)*c + 10*(2*C*a*b^4 + 24*A*a*b^2*c^2 + 48*A*a^2*c
^3 + (24*C*a^2*b^2 - A*b^4)*c)*x)*sqrt(c*x^2 + b*x + a)/(a^3*b^6 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c
^5 - 64*a^3*c^6)*x^6 + 3*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*
b*c^5)*x^5 + 3*(b^8*c - 11*a*b^6*c^2 + 36*a^2*b^4*c^3 - 16*a^3*b^2*c^4 - 64
*a^4*c^5)*x^4 + (b^9 - 6*a*b^7*c - 24*a^2*b^5*c^2 + 224*a^3*b^3*c^3 - 384*a
^4*b*c^4)*x^3 + 3*(a*b^8 - 11*a^2*b^6*c + 36*a^3*b^4*c^2 - 16*a^4*b^2*c^3 -
 64*a^5*c^4)*x^2 + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^
3)*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(155) = 310$.
time = 3.55, size = 452, normalized size = 2.71

$$-\frac{2\left(\left(\left(2\left(4\left(\frac{2\left(3\,Cb^2c^3+4\,Cac^4+16\,Ac^5\right)x}{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}+\frac{5\left(3\,Cb^3c^2+4\,Cabc^3+16\,Abc^4\right)}{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}\right)x+\frac{5\left(9\,Cb^4c+24\,Cab^2c^2+16\,Ca^2c^3+48\,Ab^2c^3+64\,Aac^4\right)}{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}\right)x+\frac{5\left(3\,Cb^5+40\,Cab^3c+48\,Ca^2bc^2+16\,Ab^3c^2+192\,Aabc^3\right)}{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}\right)x+\frac{10\left(2\,Cab^4+24\,Ca^2b^2c-Ab^4c+24\,Aab^2c^2+48\,Aa^2c^3\right)}{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}\right)x+\frac{8\,Ca^2b^3+3\,Ab^5+96\,Ca^3bc-40\,Aab^3c+240\,Aa^2bc^2}{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}\right)}{15\left(cx^2+bx+a\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(((2*(4*(2*(3*C*b^2*c^3 + 4*C*a*c^4 + 16*A*c^5)*x/(b^6 - 12*a*b^4*c +
 48*a^2*b^2*c^2 - 64*a^3*c^3) + 5*(3*C*b^3*c^2 + 4*C*a*b*c^3 + 16*A*b*c^4)/
(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))*x + 5*(9*C*b^4*c + 24*C*a
*b^2*c^2 + 16*C*a^2*c^3 + 48*A*b^2*c^3 + 64*A*a*c^4)/(b^6 - 12*a*b^4*c + 48
*a^2*b^2*c^2 - 64*a^3*c^3))*x + 5*(3*C*b^5 + 40*C*a*b^3*c + 48*C*a^2*b*c^2
+ 16*A*b^3*c^2 + 192*A*a*b*c^3)/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3
*c^3))*x + 10*(2*C*a*b^4 + 24*C*a^2*b^2*c - A*b^4*c + 24*A*a*b^2*c^2 + 48*A
*a^2*c^3)/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))*x + (8*C*a^2*b^
3 + 3*A*b^5 + 96*C*a^3*b*c - 40*A*a*b^3*c + 240*A*a^2*b*c^2)/(b^6 - 12*a*b^
4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))/(c*x^2 + b*x + a)^(5/2)
```

Mupad [B]
time = 4.53, size = 578, normalized size = 3.46

$$\frac{\frac{b\,c\,\left(56\,C\,b^2+256\,A\,c^2+32\,C\,a\,c\right)}{15\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}+\frac{2\,c^2\,x\,\left(56\,C\,b^2+256\,A\,c^2+32\,C\,a\,c\right)}{15\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}}{\sqrt{c\,x^2+b\,x+a}}+\frac{\frac{8\,C\,b\,c}{15\,\left(4\,a\,c^2-b^2\,c\right)\,\left(4\,a\,c-b^2\right)}+\frac{16\,C\,c^2\,x}{15\,\left(4\,a\,c^2-b^2\,c\right)\,\left(4\,a\,c-b^2\right)}}{\sqrt{c\,x^2+b\,x+a}}-\frac{\frac{4\,C\,x}{15\,\left(4\,a\,c-b^2\right)}-\frac{2\,C\,b}{15\,c\,\left(4\,a\,c-b^2\right)}}{{\left(c\,x^2+b\,x+a\right)}^{3/2}}+\frac{x\,\left(\frac{4\,A\,c^2}{5\,\left(4\,a\,c^2-b^2\,c\right)}+\frac{2\,C\,b^2}{5\,\left(4\,a\,c^2-b^2\,c\right)}-\frac{4\,C\,a\,c}{5\,\left(4\,a\,c^2-b^2\,c\right)}\right)+\frac{2\,A\,b\,c}{5\,\left(4\,a\,c^2-b^2\,c\right)}+\frac{2\,C\,a\,b}{5\,\left(4\,a\,c^2-b^2\,c\right)}}{{\left(c\,x^2+b\,x+a\right)}^{5/2}}+\frac{x\,\left(\frac{2\,c\,\left(8\,C\,b^2+32\,A\,c^2+8\,C\,a\,c\right)}{15\,\left(4\,a\,c^2-b^2\,c\right)\,\left(4\,a\,c-b^2\right)}+\frac{16\,C\,a\,c^2}{15\,\left(4\,a\,c^2-b^2\,c\right)\,\left(4\,a\,c-b^2\right)}-\frac{8\,C\,b^2\,c}{15\,\left(4\,a\,c^2-b^2\,c\right)\,\left(4\,a\,c-b^2\right)}\right)+\frac{b\,\left(8\,C\,b^2+32\,A\,c^2+8\,C\,a\,c\right)}{15\,\left(4\,a\,c^2-b^2\,c\right)\,\left(4\,a\,c-b^2\right)}-\frac{8\,C\,a\,b\,c}{15\,\left(4\,a\,c^2-b^2\,c\right)\,\left(4\,a\,c-b^2\right)}}{{\left(c\,x^2+b\,x+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(7/2),x)
```

```
[Out] ((b*c*(256*A*c^2 + 56*C*b^2 + 32*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2
)^2) + (2*c^2*x*(256*A*c^2 + 56*C*b^2 + 32*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4
*a*c - b^2)^2))/(a + b*x + c*x^2)^(1/2) + ((8*C*b*c)/(15*(4*a*c^2 - b^2*c)*
(4*a*c - b^2)) + (16*C*c^2*x)/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*
x + c*x^2)^(1/2) - ((4*C*x)/(15*(4*a*c - b^2)) - (2*C*b)/(15*c*(4*a*c - b^2
)))/(a + b*x + c*x^2)^(3/2) + (x*((4*A*c^2)/(5*(4*a*c^2 - b^2*c)) + (2*C*b^
2)/(5*(4*a*c^2 - b^2*c)) - (4*C*a*c)/(5*(4*a*c^2 - b^2*c))) + (2*A*b*c)/(5*
(4*a*c^2 - b^2*c)) + (2*C*a*b)/(5*(4*a*c^2 - b^2*c)))/(a + b*x + c*x^2)^(5/
2) + (x*((2*c*(32*A*c^2 + 8*C*b^2 + 8*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4*a*c
- b^2)) + (16*C*a*c^2)/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*b^2*c)/(
15*(4*a*c^2 - b^2*c)*(4*a*c - b^2))) + (b*(32*A*c^2 + 8*C*b^2 + 8*C*a*c))/(
15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*a*b*c)/(15*(4*a*c^2 - b^2*c)*(4*
a*c - b^2)))/(a + b*x + c*x^2)^(3/2)
```

3.185 $\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx$

Optimal. Leaf size=220

$$-\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}+\frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}}-\frac{32(24Ac^2+5b^2C+4acC)\,(}{105\left(b^2-4ac\right)^3(a+bx+}$$

```
[Out] -2/7*(b*c*(A+a*C/c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^(7/2)+2/35*(24*A*c+4*a*C+5*b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a
)^(5/2)-32/105*(24*A*c^2+4*C*a*c+5*C*b^2)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2+b
*x+a)^(3/2)+256/105*c*(24*A*c^2+4*C*a*c+5*C*b^2)*(2*c*x+b)/(-4*a*c+b^2)^4/(
c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1674, 12, 628, 627}

$$\frac{256c(b+2cx)\left(4acC+24Ac^2+5b^2C\right)}{105\left(b^2-4ac\right)^4\sqrt{a+bx+cx^2}}-\frac{32(b+2cx)\left(4acC+24Ac^2+5b^2C\right)}{105\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^{3/2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}+\frac{2(b+2cx)\left(4aC+24Ac+\frac{5b^2C}{c}\right)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(9/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(7*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(7/2)) + (2*(24*A*c + 4*a*C + (5*b^2*C)/c)*(b + 2*c*x))/
(35*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(5/2)) - (32*(24*A*c^2 + 5*b^2*C + 4*
a*c*C)*(b + 2*c*x))/(105*(b^2 - 4*a*c)^3*(a + b*x + c*x^2)^(3/2)) + (256*c*
(24*A*c^2 + 5*b^2*C + 4*a*c*C)*(b + 2*c*x))/(105*(b^2 - 4*a*c)^4*Sqrt[a + b
*x + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
```

```
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx &= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}-\frac{2\int\frac{24Ac+4aC+\frac{5b^2C}{c}}{2(a+bx+cx^2)^{7/2}}\,dx}{7\left(b^2-4ac\right)}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}-\frac{\left(24Ac+4aC+\frac{5b^2C}{c}\right)\int\frac{1}{(a+bx+cx^2}}{7\left(b^2-4ac\right)}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}+\frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}+\frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}+\frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 3.41, size = 410, normalized size = 1.86

$$\frac{-6A(b+2cx)\left(5b^6-24b^5cx+64b^3c^2x(7a-12cx^2)+4b^4c(-21a+26cx^2)-128bc^3x(35a^2+56acx^2+24c^2x^4)-16b^2c^2(-35a^2+196acx^2+184c^2x^4)-64c^3(35a^3+70a^2cx^2+56ac^2x^4+16c^3x^6)\right)+2C\left(1920a^4bc^2+320a^3c(b^3+21b^2cx+21bc^2x^2+14c^3x^3)+5b^2x^2(-7b^5+70b^4cx+560b^3c^2x^2+1120b^2c^3x^3+896bc^4x^4+256c^5x^5)+8a^2(-b^5+140b^4cx+1190b^3c^2x^2+1540b^2c^3x^3+1120bc^4x^4+448c^5x^5)+4ax(-7b^6+343b^5cx+2170b^4c^2x^2+3360b^3c^3x^3+2240b^2c^4x^4+896bc^5x^5+256c^6x^6)\right)}{105\left(b^2-4ac\right)^4\left(a+x(b+cx)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(9/2),x]
```

```
[Out] (-6*A*(b + 2*c*x)*(5*b^6 - 24*b^5*c*x + 64*b^3*c^2*x*(7*a - 12*c*x^2) + 4*b
^4*c*(-21*a + 26*c*x^2) - 128*b*c^3*x*(35*a^2 + 56*a*c*x^2 + 24*c^2*x^4) -
```

```
16*b^2*c^2*(-35*a^2 + 196*a*c*x^2 + 184*c^2*x^4) - 64*c^3*(35*a^3 + 70*a^2*
c*x^2 + 56*a*c^2*x^4 + 16*c^3*x^6)) + 2*C*(1920*a^4*b*c^2 + 320*a^3*c*(b^3
+ 21*b^2*c*x + 21*b*c^2*x^2 + 14*c^3*x^3) + 5*b^2*x^2*(-7*b^5 + 70*b^4*c*x
+ 560*b^3*c^2*x^2 + 1120*b^2*c^3*x^3 + 896*b*c^4*x^4 + 256*c^5*x^5) + 8*a^2
*(-b^5 + 140*b^4*c*x + 1190*b^3*c^2*x^2 + 1540*b^2*c^3*x^3 + 1120*b*c^4*x^4
 + 448*c^5*x^5) + 4*a*x*(-7*b^6 + 343*b^5*c*x + 2170*b^4*c^2*x^2 + 3360*b^3
*c^3*x^3 + 2240*b^2*c^4*x^4 + 896*b*c^5*x^5 + 256*c^6*x^6)))/(105*(b^2 - 4*
a*c)^4*(a + x*(b + c*x))^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(204) = 408$.

time = 0.13, size = 547, normalized size = 2.49

method	result
trager	$\frac{\frac{4096}{35} A\,c^{7} x^{7}-\frac{2}{3} C\,b^{7} x^{2}-\frac{16}{105} C\,a^{2} b^{5}+\frac{64}{3} C\,a^{2} b^{4} c x+128 C\,a^{3} b\,c^{3} x^{2}+\frac{544}{3} C\,a^{2} b^{3} c^{2} x^{2}+\frac{392}{15} C a\,b^{5} c\,x^{2}+192 A\,a^{2} b^{2} c^{3} x-16 A a\,b^{4} c^{2} x+128 C\,a}{}$

default	$C\left(-\frac{x}{6c(c\,x^2+bx+a)^{\frac{7}{2}}}-\frac{5b\left(-\frac{1}{7c\left(c\,x^2+bx+a\right)^{\frac{7}{2}}}-\frac{b\left(\frac{\frac{4cx}{7}+\frac{2b}{7}}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)^{\frac{7}{2}}}+\frac{24c\left(\frac{\frac{4cx}{5}+\frac{2b}{5}}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)^{\frac{5}{2}}}+\frac{16c\left(\frac{\frac{4cx}{3}+\cdots}{\left(4ac-b^2\right)\left(c\,x^2\cdots\right.}\right.}{\cdots}\right)}{7\left(4ac\cdots\right.}\right)}{2c}\right)}{12c}\right.$
gosper	$\frac{\frac{4096}{35}A\,c^7x^7-\frac{2}{3}C\,b^7x^2-\frac{16}{105}C\,a^2b^5+\frac{64}{3}C\,a^2b^4cx+128C\,a^3b\,c^3x^2+\frac{544}{3}C\,a^2b^3c^2x^2+\frac{392}{15}Ca\,b^5c\,x^2+192A\,a^2b^2c^3x-16Aa\,b^4c^2x+128C\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-1/6*x/c/(c*x^2+b*x+a)^(7/2)-5/12*b/c*(-1/7/c/(c*x^2+b*x+a)^(7/2)-1/2*b/
c*(2/7*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(7/2)+24/7*c/(4*a*c-b^2)*(2/5*(2
*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(5/2)+16/5*c/(4*a*c-b^2)*(2/3*(2*c*x+b)/(
4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)
^(1/2)))))+1/6*a/c*(2/7*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(7/2)+24/7*c/(4
*a*c-b^2)*(2/5*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(5/2)+16/5*c/(4*a*c-b^2)
```

```
*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x
+b)/(c*x^2+b*x+a)^(1/2)))))+A*(2/7*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(7/2
)+24/7*c/(4*a*c-b^2)*(2/5*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(5/2)+16/5*c/
(4*a*c-b^2)*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^
2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 978 vs. $2(204) = 408$.
time = 14.39, size = 978, normalized size = 4.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(8*C*a^2*b^5 + 15*A*b^7 - 6720*A*a^3*b*c^3 - 256*(5*C*b^2*c^5 + 4*C*
a*c^6 + 24*A*c^7)*x^7 - 896*(5*C*b^3*c^4 + 4*C*a*b*c^5 + 24*A*b*c^6)*x^6 -
224*(25*C*b^4*c^3 + 40*C*a*b^2*c^4 + 96*A*a*c^6 + 8*(2*C*a^2 + 15*A*b^2)*c^
5)*x^5 - 560*(5*C*b^5*c^2 + 24*C*a*b^3*c^3 + 96*A*a*b*c^5 + 8*(2*C*a^2*b +
3*A*b^3)*c^4)*x^4 - 70*(5*C*b^6*c + 124*C*a*b^4*c^2 + 384*A*a^2*c^5 + 64*(C
*a^3 + 9*A*a*b^2)*c^4 + 8*(22*C*a^2*b^2 + 3*A*b^4)*c^3)*x^3 - 240*(8*C*a^4*
b - 7*A*a^2*b^3)*c^2 + 7*(5*C*b^7 - 196*C*a*b^5*c - 5760*A*a^2*b*c^4 - 960*
(C*a^3*b + A*a*b^3)*c^3 - 8*(170*C*a^2*b^3 - 3*A*b^5)*c^2)*x^2 - 4*(80*C*a^
3*b^3 + 63*A*a*b^5)*c + 14*(2*C*a*b^6 - 720*A*a^2*b^2*c^3 - 960*A*a^3*c^4 -
 60*(8*C*a^3*b^2 - A*a*b^4)*c^2 - (80*C*a^2*b^4 + 3*A*b^6)*c)*x)*sqrt(c*x^2
 + b*x + a)/(a^4*b^8 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3 + 25
6*a^8*c^4 + (b^8*c^4 - 16*a*b^6*c^5 + 96*a^2*b^4*c^6 - 256*a^3*b^2*c^7 + 25
6*a^4*c^8)*x^8 + 4*(b^9*c^3 - 16*a*b^7*c^4 + 96*a^2*b^5*c^5 - 256*a^3*b^3*c
^6 + 256*a^4*b*c^7)*x^7 + 2*(3*b^10*c^2 - 46*a*b^8*c^3 + 256*a^2*b^6*c^4 -
576*a^3*b^4*c^5 + 256*a^4*b^2*c^6 + 512*a^5*c^7)*x^6 + 4*(b^11*c - 13*a*b^9
*c^2 + 48*a^2*b^7*c^3 + 32*a^3*b^5*c^4 - 512*a^4*b^3*c^5 + 768*a^5*b*c^6)*x
^5 + (b^12 - 4*a*b^10*c - 90*a^2*b^8*c^2 + 800*a^3*b^6*c^3 - 2240*a^4*b^4*c
^4 + 1536*a^5*b^2*c^5 + 1536*a^6*c^6)*x^4 + 4*(a*b^11 - 13*a^2*b^9*c + 48*a
```

```
^3*b^7*c^2 + 32*a^4*b^5*c^3 - 512*a^5*b^3*c^4 + 768*a^6*b*c^5)*x^3 + 2*(3*a
^2*b^10 - 46*a^3*b^8*c + 256*a^4*b^6*c^2 - 576*a^5*b^4*c^3 + 256*a^6*b^2*c^
4 + 512*a^7*c^5)*x^2 + 4*(a^3*b^9 - 16*a^4*b^7*c + 96*a^5*b^5*c^2 - 256*a^6
*b^3*c^3 + 256*a^7*b*c^4)*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(9/2),x)
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 805 vs. 2(204) = 408.
time = 4.56, size = 805, normalized size = 3.66

$$\frac{2\left(\left(\left(2\left(8\left(2\left(4\left(\frac{2(5Cb^2c^5+4Cac^6+24Ac^7)x}{b^8-16ab^6c+96a^2b^4c^2-256a^3b^2c^3+256a^4c^4}+\frac{7(5Cb^3c^4+4Cabc^5+24Abc^6)}{b^8-16ab^6c+96a^2b^4c^2-256a^3b^2c^3+256a^4c^4}\right)x+\frac{7(25Cb^4c^3+40Cab^2c^4+16Ca^2c^5+120Ab^2c^5+96Aac^6)}{b^8-16ab^6c+96a^2b^4c^2-256a^3b^2c^3+256a^4c^4}\right)x+\frac{35(5Cb^5c^2+24Cab^3c^3+16Ca^2bc^4+24Ab^3c^4+96Aabc^5)}{b^8-16ab^6c+96a^2b^4c^2-256a^3b^2c^3+256a^4c^4}\right)x+\frac{35(5Cb^6c+124Cab^4c^2+176Ca^2b^2c^3+24Ab^4c^3+64Ca^3c^4+576Aab^2c^4+384Aa^2c^5)}{b^8-16ab^6c+96a^2b^4c^2-256a^3b^2c^3+256a^4c^4}\right)x-\frac{7(5Cb^7-196Cab^5c-1360Ca^2b^3c^2+24Ab^5c^2-960Ca^3bc^3-960Aab^3c^3-5760Aa^2bc^4)}{b^8-16ab^6c+96a^2b^4c^2-256a^3b^2c^3+256a^4c^4}\right)x-\frac{14(2Cab^6-80Ca^2b^4c-3Ab^6c-480Ca^3b^2c^2+60Aab^4c^2-720Aa^2b^2c^3-960Aa^3c^4)}{b^8-16ab^6c+96a^2b^4c^2-256a^3b^2c^3+256a^4c^4}\right)x-\frac{8Ca^2b^5+15Ab^7-320Ca^3b^3c-252Aab^5c-1920Ca^4bc^2+1680Aa^2b^3c^2-6720Aa^3bc^3}{b^8-16ab^6c+96a^2b^4c^2-256a^3b^2c^3+256a^4c^4}\right)}{105\,(cx^2+bx+a)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="giac")
[Out] 2/105*(((2*(8*(2*(4*(2*(5*C*b^2*c^5 + 4*C*a*c^6 + 24*A*c^7)*x/(b^8 - 16*a*b
^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4) + 7*(5*C*b^3*c^4 + 4
*C*a*b*c^5 + 24*A*b*c^6)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c
^3 + 256*a^4*c^4))*x + 7*(25*C*b^4*c^3 + 40*C*a*b^2*c^4 + 16*C*a^2*c^5 + 12
0*A*b^2*c^5 + 96*A*a*c^6)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*
c^3 + 256*a^4*c^4))*x + 35*(5*C*b^5*c^2 + 24*C*a*b^3*c^3 + 16*C*a^2*b*c^4 +
 24*A*b^3*c^4 + 96*A*a*b*c^5)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*
b^2*c^3 + 256*a^4*c^4))*x + 35*(5*C*b^6*c + 124*C*a*b^4*c^2 + 176*C*a^2*b^2
*c^3 + 24*A*b^4*c^3 + 64*C*a^3*c^4 + 576*A*a*b^2*c^4 + 384*A*a^2*c^5)/(b^8
- 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*x - 7*(5*C*
b^7 - 196*C*a*b^5*c - 1360*C*a^2*b^3*c^2 + 24*A*b^5*c^2 - 960*C*a^3*b*c^3 -
 960*A*a*b^3*c^3 - 5760*A*a^2*b*c^4)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 2
56*a^3*b^2*c^3 + 256*a^4*c^4))*x - 14*(2*C*a*b^6 - 80*C*a^2*b^4*c - 3*A*b^6
*c - 480*C*a^3*b^2*c^2 + 60*A*a*b^4*c^2 - 720*A*a^2*b^2*c^3 - 960*A*a^3*c^4
)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*x -
(8*C*a^2*b^5 + 15*A*b^7 - 320*C*a^3*b^3*c - 252*A*a*b^5*c - 1920*C*a^4*b*c^
2 + 1680*A*a^2*b^3*c^2 - 6720*A*a^3*b*c^3)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c
^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))/(c*x^2 + b*x + a)^(7/2)
```

Mupad [B]
time = 5.06, size = 1018, normalized size = 4.63

$$\frac{x\left(\frac{2c^2(160Cb^2+768Ac^2+96Cac)}{105(4ac^2-b^2c)(4ac-b^2)^2}-\frac{64Cac^3}{105(4ac^2-b^2c)(4ac-b^2)^2}+\frac{32Cb^2c^2}{105(4ac^2-b^2c)(4ac-b^2)^2}\right)+\frac{bc(160Cb^2+768Ac^2+96Cac)}{105(4ac^2-b^2c)(4ac-b^2)^2}+\frac{32Cabc^2}{105(4ac^2-b^2c)(4ac-b^2)^2}}{(cx^2+bx+a)^{3/2}}-\frac{\frac{8Cb}{105(4ac-b^2)^2}-\frac{16Ccx}{105(4ac-b^2)^2}}{(cx^2+bx+a)^{3/2}}+\frac{\frac{8Cbc}{105(4ac^2-b^2c)(4ac-b^2)}+\frac{16Cc^2x}{105(4ac^2-b^2c)(4ac-b^2)}}{(cx^2+bx+a)^{3/2}}-\frac{\frac{4Cx}{35(4ac-b^2)}-\frac{2Cb}{35c(4ac-b^2)}}{(cx^2+bx+a)^{5/2}}+\frac{\frac{bc(1312Cb^2c+6144Ac^3+896Cac^2)}{105(4ac^2-b^2c)(4ac-b^2)^3}+\frac{2c^2x(1312Cb^2c+6144Ac^3+896Cac^2)}{105(4ac^2-b^2c)(4ac-b^2)^3}}{\sqrt{cx^2+bx+a}}+\frac{x\left(\frac{4Ac^2}{7(4ac^2-b^2c)}+\frac{2Cb^2}{7(4ac^2-b^2c)}-\frac{4Cac}{7(4ac^2-b^2c)}\right)+\frac{2Abc}{7(4ac^2-b^2c)}+\frac{2Cab}{7(4ac^2-b^2c)}}{(cx^2+bx+a)^{7/2}}+\frac{x\left(\frac{2c(12Cb^2+48Ac^2+8Cac)}{35(4ac^2-b^2c)(4ac-b^2)}+\frac{16Cac^2}{35(4ac^2-b^2c)(4ac-b^2)}-\frac{8Cb^2c}{35(4ac^2-b^2c)(4ac-b^2)}\right)+\frac{b(12Cb^2+48Ac^2+8Cac)}{35(4ac^2-b^2c)(4ac-b^2)}-\frac{8Cabc}{35(4ac^2-b^2c)(4ac-b^2)}}{(cx^2+bx+a)^{5/2}}-\frac{\frac{32Cbc^2}{105(4ac^2-b^2c)(4ac-b^2)^2}+\frac{64Cc^3x}{105(4ac^2-b^2c)(4ac-b^2)^2}}{\sqrt{cx^2+bx+a}}+\frac{\frac{64Cbc^2}{105(4ac^2-b^2c)(4ac-b^2)^2}+\frac{128Cc^3x}{105(4ac^2-b^2c)(4ac-b^2)^2}}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(9/2),x)
```

```
[Out] (x*((2*c^2*(768*A*c^2 + 160*C*b^2 + 96*C*a*c))/(105*(4*a*c^2 - b^2*c)*(4*a*
c - b^2)^2) - (64*C*a*c^3)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (32*C*
b^2*c^2)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2)) + (b*c*(768*A*c^2 + 160*C
*b^2 + 96*C*a*c))/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (32*C*a*b*c^2)/
(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x + c*x^2)^(3/2) - ((8*C*b)
/(105*(4*a*c - b^2)^2) - (16*C*c*x)/(105*(4*a*c - b^2)^2))/(a + b*x + c*x^2
)^(3/2) + ((8*C*b*c)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) + (16*C*c^2*x)/(
105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*x + c*x^2)^(3/2) - ((4*C*x)/(3
5*(4*a*c - b^2)) - (2*C*b)/(35*c*(4*a*c - b^2)))/(a + b*x + c*x^2)^(5/2) +
((b*c*(6144*A*c^3 + 896*C*a*c^2 + 1312*C*b^2*c))/(105*(4*a*c^2 - b^2*c)*(4*
a*c - b^2)^3) + (2*c^2*x*(6144*A*c^3 + 896*C*a*c^2 + 1312*C*b^2*c))/(105*(4
*a*c^2 - b^2*c)*(4*a*c - b^2)^3))/(a + b*x + c*x^2)^(1/2) + (x*((4*A*c^2)/(
7*(4*a*c^2 - b^2*c)) + (2*C*b^2)/(7*(4*a*c^2 - b^2*c)) - (4*C*a*c)/(7*(4*a*
c^2 - b^2*c))) + (2*A*b*c)/(7*(4*a*c^2 - b^2*c)) + (2*C*a*b)/(7*(4*a*c^2 -
b^2*c)))/(a + b*x + c*x^2)^(7/2) + (x*((2*c*(48*A*c^2 + 12*C*b^2 + 8*C*a*c)
)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) + (16*C*a*c^2)/(35*(4*a*c^2 - b^2*c)
*(4*a*c - b^2)) - (8*C*b^2*c)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2))) + (b*(4
8*A*c^2 + 12*C*b^2 + 8*C*a*c))/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*
a*b*c)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*x + c*x^2)^(5/2) - ((32
*C*b*c^2)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (64*C*c^3*x)/(105*(4*a*
c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x + c*x^2)^(1/2) + ((64*C*b*c^2)/(105
*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (128*C*c^3*x)/(105*(4*a*c^2 - b^2*c)*
(4*a*c - b^2)^2))/(a + b*x + c*x^2)^(1/2)
```

3.186 $\int (g+hx)^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)\,dx$

Optimal. Leaf size=930

$(256c^5dg^3 - 33b^5fh^3 + 6b^3ch^2(20afh + 7b(3fg + eh)) - 8bc^2h(10a^2fh^2 + 14abh(3fg + eh) + 7b^2(3fg^2 + 3$

```
[Out] 1/280*(33*b^2*f*h^2-2*c*h*(16*a*f*h+21*b*e*h+8*b*f*g)-4*c^2*(3*f*g^2-7*h*(2
*d*h+e*g)))*(h*x+g)^2*(c*x^2+b*x+a)^(3/2)/c^3/h-1/84*(11*b*f*h-14*c*e*h+6*c
*f*g)*(h*x+g)^3*(c*x^2+b*x+a)^(3/2)/c^2/h+1/7*f*(h*x+g)^4*(c*x^2+b*x+a)^(3/
2)/c/h+1/13440*(1155*b^4*f*h^4-128*c^4*g^2*(3*f*g^2-7*h*(12*d*h+e*g))-42*b^
2*c*h^3*(78*a*f*h+35*b*(e*h+3*f*g))+8*c^2*h^2*(128*a^2*f*h^2+343*a*b*h*(e*h
+3*f*g)+b^2*(537*f*g^2+245*h*(d*h+3*e*g)))-16*c^3*h*(16*a*h*(15*f*g^2+7*h*(
d*h+3*e*g))+b*g*(17*f*g^2+21*h*(25*d*h+19*e*g)))-6*c*h*(231*b^3*f*h^3-6*b*c
*h^2*(74*a*f*h+49*b*e*h+59*b*f*g)+16*c^3*g*(3*f*g^2-7*h*(7*d*h+e*g))+8*c^2*
h*(a*h*(35*e*h+41*f*g)+b*(5*f*g^2+7*h*(7*d*h+9*e*g))))*x)*(c*x^2+b*x+a)^(3/
2)/c^5/h-1/2048*(-4*a*c+b^2)*(256*c^5*d*g^3-33*b^5*f*h^3+6*b^3*c*h^2*(20*a*
f*h+7*b*(e*h+3*f*g))-8*b*c^2*h*(10*a^2*f*h^2+14*a*b*h*(e*h+3*f*g)+7*b^2*(d*
h^2+3*e*g*h+3*f*g^2))-64*c^4*g*(2*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+
16*c^3*(2*a^2*h^2*(e*h+3*f*g)+5*b^2*g*(f*g^2+3*h*(d*h+e*g))+6*a*b*h*(3*f*g^
2+h*(d*h+3*e*g))))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(13
/2)+1/1024*(256*c^5*d*g^3-33*b^5*f*h^3+6*b^3*c*h^2*(20*a*f*h+7*b*(e*h+3*f*g
))-8*b*c^2*h*(10*a^2*f*h^2+14*a*b*h*(e*h+3*f*g)+7*b^2*(d*h^2+3*e*g*h+3*f*g^
2))-64*c^4*g*(2*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+16*c^3*(2*a^2*h^2*
(e*h+3*f*g)+5*b^2*g*(f*g^2+3*h*(d*h+e*g))+6*a*b*h*(3*f*g^2+h*(d*h+3*e*g))))
*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^6
```

Rubi [A]

time = 1.66, antiderivative size = 927, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 846, 793, 626, 635, 212}

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((256*c^5*d*g^3 - 33*b^5*f*h^3 - 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2*
b*g*(e*g + 3*d*h)) + 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) - 8*b*c^2*h
*(10*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)
) + 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 6
*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(10
24*c^6) + ((33*b^2*f*h^2 - 2*c*h*(8*b*f*g + 21*b*e*h + 16*a*f*h) - 4*c^2*(3
*f*g^2 - 7*h*(e*g + 2*d*h)))*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(280*c^3*
```

```
h) - ((6*c*f*g - 14*c*e*h + 11*b*f*h)*(g + h*x)^3*(a + b*x + c*x^2)^(3/2))/
(84*c^2*h) + (f*(g + h*x)^4*(a + b*x + c*x^2)^(3/2))/(7*c*h) + ((1155*b^4*f
*h^4 - 128*c^4*(3*f*g^4 - 7*g^2*h*(e*g + 12*d*h)) - 42*b^2*c*h^3*(78*a*f*h
+ 35*b*(3*f*g + e*h)) + 8*c^2*h^2*(128*a^2*f*h^2 + 343*a*b*h*(3*f*g + e*h)
+ b^2*(537*f*g^2 + 245*h*(3*e*g + d*h))) - 16*c^3*h*(16*a*h*(15*f*g^2 + 7*h
*(3*e*g + d*h)) + b*g*(17*f*g^2 + 21*h*(19*e*g + 25*d*h))) - 6*c*h*(231*b^3
*f*h^3 - 6*b*c*h^2*(59*b*f*g + 49*b*e*h + 74*a*f*h) + 16*c^3*(3*f*g^3 - 7*g
*h*(e*g + 7*d*h)) + 8*c^2*h*(5*b*f*g^2 + 7*b*h*(9*e*g + 7*d*h) + a*h*(41*f*
g + 35*e*h)))*x)*(a + b*x + c*x^2)^(3/2))/(13440*c^5*h) - ((b^2 - 4*a*c)*(2
56*c^5*d*g^3 - 33*b^5*f*h^3 - 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2*b*g
*(e*g + 3*d*h)) + 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) - 8*b*c^2*h*(1
0*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) +
 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 6*a*
b*h*(3*f*g^2 + h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(2048*c^(13/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
```

```
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}+\frac{\int (g+hx)^3\left(-\frac{1}{2}h(3bfg-\right.}{}$$

$$= -\frac{(6cfg-14ceh+11bfh)(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{84c^2h}+$$

$$= \frac{(33b^2fh^2-2ch(8bfg+21beh+16afh)-4c^2(3fg^2-7h}{280c^3h}$$

$$= \frac{(33b^2fh^2-2ch(8bfg+21beh+16afh)-4c^2(3fg^2-7h}{280c^3h}$$

$$= \frac{(256c^5dg^3-33b^5fh^3-64c^4g(afg^2+3ah(eg+dh)+2bg}{}$$

$$= \frac{(256c^5dg^3-33b^5fh^3-64c^4g(afg^2+3ah(eg+dh)+2bg}{}$$

$$= \frac{(256c^5dg^3-33b^5fh^3-64c^4g(afg^2+3ah(eg+dh)+2bg}{}$$

Mathematica [A]

time = 8.38, size = 1091, normalized size = 1.17

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^6*f*h^3 + 210*b^5*c*h^2*(63*f*g +
 21*e*h + 11*f*h*x) - 84*b^4*c*h*(-260*a*f*h^2 + 35*c*h*(6*e*g + 2*d*h + e*
h*x) + c*f*(210*g^2 + 105*g*h*x + 22*h^2*x^2)) - 16*b^2*c^2*(2163*a^2*f*h^3
 - 2*a*c*h*(7*h*(345*e*g + 115*d*h + 56*e*h*x) + 3*f*(805*g^2 + 392*g*h*x +
 81*h^2*x^2)) + 2*c^2*(7*d*h*(180*g^2 + 75*g*h*x + 14*h^2*x^2) + 21*e*(20*g
^3 + 25*g^2*h*x + 14*g*h^2*x^2 + 3*h^3*x^3) + f*x*(175*g^3 + 294*g^2*h*x +
189*g*h^2*x^2 + 44*h^3*x^3))) + 16*b^3*c^2*(-42*a*h^2*(35*e*h + 3*f*(35*g +
 6*h*x)) + c*(f*(525*g^3 + 735*g^2*h*x + 441*g*h^2*x^2 + 99*h^3*x^3) + 7*h*
(5*d*h*(45*g + 7*h*x) + 3*e*(75*g^2 + 35*g*h*x + 7*h^2*x^2)))) + 32*b*c^3*(
a^2*h^2*(2373*f*g + 791*e*h + 397*f*h*x) - 2*a*c*(f*(455*g^3 + 609*g^2*h*x
+ 357*g*h^2*x^2 + 79*h^3*x^3) + 7*h*(d*h*(195*g + 29*h*x) + e*(195*g^2 + 87
*g*h*x + 17*h^2*x^2))) + 4*c^2*(21*d*(10*g^3 + 10*g^2*h*x + 5*g*h^2*x^2 + h
^3*x^3) + x*(7*e*(10*g^3 + 15*g^2*h*x + 9*g*h^2*x^2 + 2*h^3*x^3) + f*x*(35*
g^3 + 63*g^2*h*x + 42*g*h^2*x^2 + 10*h^3*x^3)))) + 64*c^3*(128*a^3*f*h^3 -
a^2*c*h*(7*h*(96*e*g + 32*d*h + 15*e*h*x) + f*(672*g^2 + 315*g*h*x + 64*h^2
*x^2)) + 2*a*c^2*(7*d*h*(120*g^2 + 45*g*h*x + 8*h^2*x^2) + 7*e*(40*g^3 + 45
*g^2*h*x + 24*g*h^2*x^2 + 5*h^3*x^3) + 3*f*x*(35*g^3 + 56*g^2*h*x + 35*g*h^
2*x^2 + 8*h^3*x^3)) + 4*c^3*x*(21*d*(10*g^3 + 20*g^2*h*x + 15*g*h^2*x^2 + 4
*h^3*x^3) + x*(7*e*(20*g^3 + 45*g^2*h*x + 36*g*h^2*x^2 + 10*h^3*x^3) + 3*f*
x*(35*g^3 + 84*g^2*h*x + 70*g*h^2*x^2 + 20*h^3*x^3))))) - 105*(b^2 - 4*a*c)
*(-256*c^5*d*g^3 + 33*b^5*f*h^3 + 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2
*b*g*(e*g + 3*d*h)) - 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) + 8*b*c^2*
h*(10*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2
)) - 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) +
6*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*
(b + c*x)]])/(215040*c^(13/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2097 vs. $2(900) = 1800$.
time = 0.14, size = 2098, normalized size = 2.26

method	result	size
default	Expression too large to display	2098
risch	Expression too large to display	2635

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] f*h^3*(1/7*x^4*(c*x^2+b*x+a)^(3/2)/c-11/14*b/c*(1/6*x^3*(c*x^2+b*x+a)^(3/2)
/c-3/4*b/c*(1/5*x^2*(c*x^2+b*x+a)^(3/2)/c-7/10*b/c*(1/4*x*(c*x^2+b*x+a)^(3/
2)/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x
+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-2/5*a/c*(1/3*(c*x^2+b*x+a)^(3/
2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/2*a/c*(1/4*x*(c*x^2+b*x+a)^
(3/2)/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+
b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-4/7*a/c*(1/5*x^2*(c*x^2+b*
x+a)^(3/2)/c-7/10*b/c*(1/4*x*(c*x^2+b*x+a)^(3/2)/c-5/8*b/c*(1/3*(c*x^2+b*x+
a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(
3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-1/4*a/c*(1/4*(2*c*x+b)/c
*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))))-2/5*a/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c
*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))))))+(e*h^3+3*f*g*h^2)*(1/6*x^3*(c*x^2+b*x+a)^(3/2)/c-3/4*b/c*
(1/5*x^2*(c*x^2+b*x+a)^(3/2)/c-7/10*b/c*(1/4*x*(c*x^2+b*x+a)^(3/2)/c-5/8*b/
c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1
/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-1/4*a/
c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*
x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-2/5*a/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/
c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*
x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/2*a/c*(1/4*x*(c*x^2+b*x+a)^(3/2)/c-5/8
b/c(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2
)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-1/4
a/c(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+(d*h^3+3*e*g*h^2+3*f*g^2*h)*(1/5*x^2*
(c*x^2+b*x+a)^(3/2)/c-7/10*b/c*(1/4*x*(c*x^2+b*x+a)^(3/2)/c-5/8*b/c*(1/3*(c
*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c
-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-1/4*a/c*(1/4*(2
*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))))-2/5*a/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2
*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2)))))+(3*d*g*h^2+3*e*g^2*h+f*g^3)*(1/4*x*(c*x^2+b*x+a)^
(3/2)/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+
b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+(3*d*g^2*h+e*g^3)*(1/3*(c*x
^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b
^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+d*g^3*(1/4*(2*c*x
+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2)))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 1.45, size = 2975, normalized size = 3.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/430080*(105*(16*(16*(b^2*c^5 - 4*a*c^6)*d + (5*b^4*c^3 - 24*a*b^2*c^4 +
 16*a^2*c^5)*f)*g^3 - 24*(16*(b^3*c^4 - 4*a*b*c^5)*d + (7*b^5*c^2 - 40*a*b^
3*c^3 + 48*a^2*b*c^4)*f)*g^2*h + 6*(8*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^
5)*d + (21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*f)*g*h^2 -
 (8*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*d + (33*b^7 - 252*a*b^5*c + 5
60*a^2*b^3*c^2 - 320*a^3*b*c^3)*f)*h^3 - 2*(64*(b^3*c^4 - 4*a*b*c^5)*g^3 -
24*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*g^2*h + 12*(7*b^5*c^2 - 40*a*b^3
*c^3 + 48*a^2*b*c^4)*g*h^2 - (21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 -
64*a^3*c^4)*h^3)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 +
 b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(15360*c^7*f*h^3*x^6 + 1280*(42*
c^7*f*g*h^2 + b*c^6*f*h^3)*x^5 + 128*(504*c^7*f*g^2*h + 42*b*c^6*f*g*h^2 +
(168*c^7*d - (11*b^2*c^5 - 24*a*c^6)*f)*h^3)*x^4 + 560*(48*b*c^6*d + (15*b^
3*c^4 - 52*a*b*c^5)*f)*g^3 - 168*(80*(3*b^2*c^5 - 8*a*c^6)*d + (105*b^4*c^3
 - 460*a*b^2*c^4 + 256*a^2*c^5)*f)*g^2*h + 42*(40*(15*b^3*c^4 - 52*a*b*c^5)
*d + (315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*f)*g*h^2 - (56*(105*b^
4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*d + (3465*b^6*c - 21840*a*b^4*c^2 + 34
608*a^2*b^2*c^3 - 8192*a^3*c^4)*f)*h^3 + 16*(1680*c^7*f*g^3 + 504*b*c^6*f*g
^2*h + 42*(120*c^7*d - (9*b^2*c^5 - 20*a*c^6)*f)*g*h^2 + (168*b*c^6*d + (99
*b^3*c^4 - 316*a*b*c^5)*f)*h^3)*x^3 + 8*(560*b*c^6*f*g^3 + 168*(80*c^7*d -
(7*b^2*c^5 - 16*a*c^6)*f)*g^2*h + 42*(40*b*c^6*d + (21*b^3*c^4 - 68*a*b*c^5
)*f)*g*h^2 - (56*(7*b^2*c^5 - 16*a*c^6)*d + (231*b^4*c^3 - 972*a*b^2*c^4 +
512*a^2*c^5)*f)*h^3)*x^2 + 2*(560*(48*c^7*d - (5*b^2*c^5 - 12*a*c^6)*f)*g^3
```

+ 168*(80*b*c^6*d + (35*b^3*c^4 - 116*a*b*c^5)*f)*g^2*h - 42*(40*(5*b^2*c^5 - 12*a*c^6)*d + (105*b^4*c^3 - 448*a*b^2*c^4 + 240*a^2*c^5)*f)*g*h^2 + (56*(35*b^3*c^4 - 116*a*b*c^5)*d + (1155*b^5*c^2 - 6048*a*b^3*c^3 + 6352*a^2*b*c^4)*f)*h^3)*x + 14*(1280*c^7*h^3*x^5 + 128*(36*c^7*g*h^2 + b*c^6*h^3)*x^4 - 320*(3*b^2*c^5 - 8*a*c^6)*g^3 + 120*(15*b^3*c^4 - 52*a*b*c^5)*g^2*h - 12*(105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*g*h^2 + (315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*h^3 + 16*(360*c^7*g^2*h + 36*b*c^6*g*h^2 - (9*b^2*c^5 - 20*a*c^6)*h^3)*x^3 + 8*(320*c^7*g^3 + 120*b*c^6*g^2*h - 12*(7*b^2*c^5 - 16*a*c^6)*g*h^2 + (21*b^3*c^4 - 68*a*b*c^5)*h^3)*x^2 + 2*(320*b*c^6*g^3 - 120*(5*b^2*c^5 - 12*a*c^6)*g^2*h + 12*(35*b^3*c^4 - 116*a*b*c^5)*g*h^2 - (105*b^4*c^3 - 448*a*b^2*c^4 + 240*a^2*c^5)*h^3)*x)*e)*sqrt(c*x^2 + b*x + a))/c^7, 1/215040*(105*(16*(16*(b^2*c^5 - 4*a*c^6)*d + (5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*f)*g^3 - 24*(16*(b^3*c^4 - 4*a*b*c^5)*d + (7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*f)*g^2*h + 6*(8*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*d + (21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*f)*g*h^2 - (8*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*d + (33*b^7 - 252*a*b^5*c + 560*a^2*b^3*c^2 - 320*a^3*b*c^3)*f)*h^3 - 2*(64*(b^3*c^4 - 4*a*b*c^5)*g^3 - 24*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*g^2*h + 12*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*g*h^2 - (21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*h^3)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(15360*c^7*f*h^3*x^6 + 1280*(42*c^7*f*g*h^2 + b*c^6*f*h^3)*x^5 + 128*(504*c^7*f*g^2*h + 42*b*c^6*f*g*h^2 + (168*c^7*d - (11*b^2*c^5 - 24*a*c^6)*f)*h^3)*x^4 + 560*(48*b*c^6*d + (15*b^3*c^4 - 52*a*b*c^5)*f)*g^3 - 168*(80*(3*b^2*c^5 - 8*a*c^6)*d + (105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*f)*g^2*h + 42*(40*(15*b^3*c^4 - 52*a*b*c^5)*d + (315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*f)*g*h^2 - (56*(105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*d + (3465*b^6*c - 21840*a*b^4*c^2 + 34608*a^2*b^2*c^3 - 8192*a^3*c^4)*f)*h^3 + 16*(1680*c^7*f*g^3 + 504*b*c^6*f*g^2*h + 42*(120*c^7*d - (9*b^2*c^5 - 20*a*c^6)*f)*g*h^2 + (168*b*c^6*d + (99*b^3*c^4 - 316*a*b*c^5)*f)*h^3)*x^3 + 8*(560*b*c^6*f*g^3 + 168*(80*c^7*d - (7*b^2*c^5 - 16*a*c^6)*f)*g^2*h + 42*(40*b*c^6*d + (21*b^3*c^4 - 68*a*b*c^5)*f)*g*h^2 - (56*(7*b^2*c^5 - 16*a*c^6)*d + (231*b^4*c^3 - 972*a*b^2*c^4 + 512*a^2*c^5)*f)*h^3)*x^2 + 2*(560*(48*c^7*d - (5*b^2*c^5 - 12*a*c^6)*f)*g^3 + 168*(80*b*c^6*d + (35*b^3*c^4 - 116*a*b*c^5)*f)*g^2*h - 42*(40*(5*b^2*c^5 - 12*a*c^6)*d + (105*b^4*c^3 - 448*a*b^2*c^4 + 240*a^2*c^5)*f)*g*h^2 + (56*(35*b^3*c^4 - 116*a*b*c^5)*d + (1155*b^5*c^2 - 6048*a*b^3*c^3 + 6352*a^2*b*c^4)*f)*h^3)*x + 14*(1280*c^7*h^3*x^5 + 128*(36*c^7*g*h^2 + b*c^6*h^3)*x^4 - 320*(3*b^2*c^5 - 8*a*c^6)*g^3 + 120*(15*b^3*c^4 - 52*a*b*c^5)*g^2*h - 12*(105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*g*h^2 + (315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*h^3 + 16*(360*c^7*g^2*h + 36*b*c^6*g*h^2 - (9*b^2*c^5 - 20*a*c^6)*h^3)*x^3 + 8*(320*c^7*g^3 + 120*b*c^6*g^2*h - 12*(7*b^2*c^5 - 16*a*c^6)*g*h^2 + (21*b^3*c^4 - 68*a*b*c^5)*h^3)*x^2 + 2*(320*b*c^6*g^3 - 120*(5*b^2*c^5 - 12*a*c^6)*g^2*h + 12*(35*b^3*c^4 - 116*a*b*c^5)*g*h^2 - (105*b^4*c^3 - 448*a*b^2*c^4 + 240*a^2*c^5)*h^3)*x)*e)*sqrt(c*x^2 + b*x + a))/c^7]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (g+hx)^3 \sqrt{a+bx+cx^2}\,(d+ex+fx^2)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**3*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

Giac [A]
time = 2.98, size = 1702, normalized size = 1.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/107520*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*f*h^3*x + (42*c^6*f*g*h^
2 + b*c^5*f*h^3 + 14*c^6*h^3*e)/c^6)*x + (504*c^6*f*g^2*h + 42*b*c^5*f*g*h^
2 + 168*c^6*d*h^3 - 11*b^2*c^4*f*h^3 + 24*a*c^5*f*h^3 + 504*c^6*g*h^2*e + 1
4*b*c^5*h^3*e)/c^6)*x + (1680*c^6*f*g^3 + 504*b*c^5*f*g^2*h + 5040*c^6*d*g*
h^2 - 378*b^2*c^4*f*g*h^2 + 840*a*c^5*f*g*h^2 + 168*b*c^5*d*h^3 + 99*b^3*c^
3*f*h^3 - 316*a*b*c^4*f*h^3 + 5040*c^6*g^2*h*e + 504*b*c^5*g*h^2*e - 126*b^
2*c^4*h^3*e + 280*a*c^5*h^3*e)/c^6)*x + (560*b*c^5*f*g^3 + 13440*c^6*d*g^2*
h - 1176*b^2*c^4*f*g^2*h + 2688*a*c^5*f*g^2*h + 1680*b*c^5*d*g*h^2 + 882*b^
3*c^3*f*g*h^2 - 2856*a*b*c^4*f*g*h^2 - 392*b^2*c^4*d*h^3 + 896*a*c^5*d*h^3
- 231*b^4*c^2*f*h^3 + 972*a*b^2*c^3*f*h^3 - 512*a^2*c^4*f*h^3 + 4480*c^6*g^
3*e + 1680*b*c^5*g^2*h*e - 1176*b^2*c^4*g*h^2*e + 2688*a*c^5*g*h^2*e + 294*
b^3*c^3*h^3*e - 952*a*b*c^4*h^3*e)/c^6)*x + (26880*c^6*d*g^3 - 2800*b^2*c^4
*f*g^3 + 6720*a*c^5*f*g^3 + 13440*b*c^5*d*g^2*h + 5880*b^3*c^3*f*g^2*h - 19
488*a*b*c^4*f*g^2*h - 8400*b^2*c^4*d*g*h^2 + 20160*a*c^5*d*g*h^2 - 4410*b^4
*c^2*f*g*h^2 + 18816*a*b^2*c^3*f*g*h^2 - 10080*a^2*c^4*f*g*h^2 + 1960*b^3*c
^3*d*h^3 - 6496*a*b*c^4*d*h^3 + 1155*b^5*c*f*h^3 - 6048*a*b^3*c^2*f*h^3 + 6
352*a^2*b*c^3*f*h^3 + 4480*b*c^5*g^3*e - 8400*b^2*c^4*g^2*h*e + 20160*a*c^5
*g^2*h*e + 5880*b^3*c^3*g*h^2*e - 19488*a*b*c^4*g*h^2*e - 1470*b^4*c^2*h^3*
e + 6272*a*b^2*c^3*h^3*e - 3360*a^2*c^4*h^3*e)/c^6)*x + (26880*b*c^5*d*g^3
+ 8400*b^3*c^3*f*g^3 - 29120*a*b*c^4*f*g^3 - 40320*b^2*c^4*d*g^2*h + 107520
*a*c^5*d*g^2*h - 17640*b^4*c^2*f*g^2*h + 77280*a*b^2*c^3*f*g^2*h - 43008*a^
2*c^4*f*g^2*h + 25200*b^3*c^3*d*g*h^2 - 87360*a*b*c^4*d*g*h^2 + 13230*b^5*c
*f*g*h^2 - 70560*a*b^3*c^2*f*g*h^2 + 75936*a^2*b*c^3*f*g*h^2 - 5880*b^4*c^2
*d*h^3 + 25760*a*b^2*c^3*d*h^3 - 14336*a^2*c^4*d*h^3 - 3465*b^6*f*h^3 + 218
40*a*b^4*c*f*h^3 - 34608*a^2*b^2*c^2*f*h^3 + 8192*a^3*c^3*f*h^3 - 13440*b^2
*c^4*g^3*e + 35840*a*c^5*g^3*e + 25200*b^3*c^3*g^2*h*e - 87360*a*b*c^4*g^2*
```

```
h*e - 17640*b^4*c^2*g*h^2*e + 77280*a*b^2*c^3*g*h^2*e - 43008*a^2*c^4*g*h^2
*e + 4410*b^5*c*h^3*e - 23520*a*b^3*c^2*h^3*e + 25312*a^2*b*c^3*h^3*e)/c^6)
 + 1/2048*(256*b^2*c^5*d*g^3 - 1024*a*c^6*d*g^3 + 80*b^4*c^3*f*g^3 - 384*a*
b^2*c^4*f*g^3 + 256*a^2*c^5*f*g^3 - 384*b^3*c^4*d*g^2*h + 1536*a*b*c^5*d*g^
2*h - 168*b^5*c^2*f*g^2*h + 960*a*b^3*c^3*f*g^2*h - 1152*a^2*b*c^4*f*g^2*h
+ 240*b^4*c^3*d*g*h^2 - 1152*a*b^2*c^4*d*g*h^2 + 768*a^2*c^5*d*g*h^2 + 126*
b^6*c*f*g*h^2 - 840*a*b^4*c^2*f*g*h^2 + 1440*a^2*b^2*c^3*f*g*h^2 - 384*a^3*
c^4*f*g*h^2 - 56*b^5*c^2*d*h^3 + 320*a*b^3*c^3*d*h^3 - 384*a^2*b*c^4*d*h^3
- 33*b^7*f*h^3 + 252*a*b^5*c*f*h^3 - 560*a^2*b^3*c^2*f*h^3 + 320*a^3*b*c^3*
f*h^3 - 128*b^3*c^4*g^3*e + 512*a*b*c^5*g^3*e + 240*b^4*c^3*g^2*h*e - 1152*
a*b^2*c^4*g^2*h*e + 768*a^2*c^5*g^2*h*e - 168*b^5*c^2*g*h^2*e + 960*a*b^3*c
^3*g*h^2*e - 1152*a^2*b*c^4*g*h^2*e + 42*b^6*c*h^3*e - 280*a*b^4*c^2*h^3*e
+ 480*a^2*b^2*c^3*h^3*e - 128*a^3*c^4*h^3*e)*log(abs(-2*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2)
```

Mupad [B]
time = 14.70, size = 2500, normalized size = 2.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*g^3*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (8*a^3*f*h^3*(a + b*x + c*x
^2)^(1/2))/(105*c^3) - (33*b^6*f*h^3*(a + b*x + c*x^2)^(1/2))/(1024*c^6) +
(d*h^3*x^2*(a + b*x + c*x^2)^(3/2))/(5*c) + (e*h^3*x^3*(a + b*x + c*x^2)^(3
/2))/(6*c) + (f*h^3*x^4*(a + b*x + c*x^2)^(3/2))/(7*c) - (a*f*g^3*((x/2 + b
/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x
^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) + (d*g^3*log((b/2 + c*x)/c^(1
/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*g^3*log((b +
 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))
- (2*a*d*h^3*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*
x^2)^(1/2))/(24*c^2)))/(5*c) - (5*b*f*g^3*((log((b + 2*c*x)/c^(1/2) + 2*(a
+ b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3
*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) + (e*g^3*(8*c*(a
+ c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2) + (33*b^7*f*h
^3*log(b + 2*c^(1/2)*(a + b*x + c*x^2)^(1/2) + 2*c*x))/(2048*c^(13/2)) + (f
*g^3*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*e*h^3*((5*b*((log((b + 2*c*x)/c^
(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a
 + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x
*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/
2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*
c^(3/2))))/(4*c)))/(2*c) + (7*b*d*h^3*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(
a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) -
```

3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (3*b*e*h^3*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (2*a*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(4*c) + (3*d*g*h^2*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (3*e*g^2*h*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (3*a*f*g*h^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(2*c) + (21*b*e*g*h^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) + (21*b*f*g^2*h*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (9*b*f*g*h^2*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (2*a*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(4*c) + (35*a^2*b^3*f*h^3*log(b + 2*c^(1/2)*(a + b*x + c*x^2)^(1/2) + 2*c*x))/(128*c^(9/2)) + (13*a*b^4*f*h^3*(a + b*x + c*x^2)^(1/2))/(64*c^5) - (4*a*f*h^3*x^2*(a + b*x + c*x^2)^(3/2))/(35*c^2) - (11*b*f*h^3*x^3*(a + b*x + c*x^2)^(3/2))/(84*c^2) - (33*b^3*f*h^3*x*(a + b*x + c*x^2)^(3/2))/(320*c^4) + (11*b^5*f*h^3*x*(a + b*x + c*x^2)^(1/2))/(512*c^5) + (3*e*g*h^2*x^2*(a + b*x + c*x^2)^(3/2))/(5*c) + (3*f*g^2*h*x^2*(a + b*x + c*x^2)^(3/2))/(5*c) + (f*g*h^2*x^3*(a + b*x + c*x^2)^(3/2))/(2*c) - (3*a*d*g*h^2*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (3*a*e*g^2*h*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c...

3.187 $\int (g+hx)^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)\,dx$

Optimal. Leaf size=584

$$\frac{(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3(2bg(eg+2dh)+a(fg^2+2egh+dh^2))+8c^2(2a}{512c^5}$$

```
[Out] -1/20*(3*b*f*h-4*c*e*h+2*c*f*g)*(h*x+g)^2*(c*x^2+b*x+a)^(3/2)/c^2/h+1/6*f*(
h*x+g)^3*(c*x^2+b*x+a)^(3/2)/c/h-1/960*(105*b^3*f*h^3+64*c^3*g*(f*g^2-2*h*(
5*d*h+e*g))-28*b*c*h^2*(7*a*f*h+5*b*(e*h+2*f*g))+8*c^2*h*(16*a*h*(e*h+2*f*g
)+b*(7*f*g^2+25*h*(d*h+2*e*g)))-6*c*h*(21*b^2*f*h^2-4*c*h*(5*a*f*h+7*b*e*h+
2*b*f*g)-8*c^2*(f*g^2-h*(5*d*h+2*e*g)))*x)*(c*x^2+b*x+a)^(3/2)/c^4/h-1/1024
*(-4*a*c+b^2)*(128*c^4*d*g^2+21*b^4*f*h^2-28*b^2*c*h*(2*a*f*h+b*e*h+2*b*f*g
)-32*c^3*(2*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+8*c^2*(2*a^2*f*h^2+6*a
*b*h*(e*h+2*f*g)+5*b^2*(d*h^2+2*e*g*h+f*g^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2
)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/512*(128*c^4*d*g^2+21*b^4*f*h^2-28*b^2*c*
h*(2*a*f*h+b*e*h+2*b*f*g)-32*c^3*(2*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2)
)+8*c^2*(2*a^2*f*h^2+6*a*b*h*(e*h+2*f*g)+5*b^2*(d*h^2+2*e*g*h+f*g^2)))*(2*c
*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

Rubi [A]
time = 0.86, antiderivative size = 581, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 846, 793, 626, 635, 212}

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((128*c^4*d*g^2 + 21*b^4*f*h^2 - 28*b^2*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 3
2*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 2*b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*
h^2 + 6*a*b*h*(2*f*g + e*h) + 5*b^2*(f*g^2 + h*(2*e*g + d*h))))*(b + 2*c*x)
*Sqrt[a + b*x + c*x^2])/(512*c^5) - ((2*c*f*g - 4*c*e*h + 3*b*f*h)*(g + h*x
)^2*(a + b*x + c*x^2)^(3/2))/(20*c^2*h) + (f*(g + h*x)^3*(a + b*x + c*x^2)^
(3/2))/(6*c*h) - ((105*b^3*f*h^3 + 64*c^3*(f*g^3 - 2*g*h*(e*g + 5*d*h)) - 2
8*b*c*h^2*(7*a*f*h + 5*b*(2*f*g + e*h)) + 8*c^2*h*(7*b*f*g^2 + 25*b*h*(2*e*
g + d*h) + 16*a*h*(2*f*g + e*h)) - 6*c*h*(21*b^2*f*h^2 - 4*c*h*(2*b*f*g + 7
*b*e*h + 5*a*f*h) - 8*c^2*(f*g^2 - h*(2*e*g + 5*d*h)))*x)*(a + b*x + c*x^2)
^(3/2))/(960*c^4*h) - ((b^2 - 4*a*c)*(128*c^4*d*g^2 + 21*b^4*f*h^2 - 28*b^2
*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 32*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 2*
b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*h^2 + 6*a*b*h*(2*f*g + e*h) + 5*b^2*(f*
g^2 + h*(2*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(1024*c^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
```

```
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch} + \frac{\int (g+hx)^2\left(-\frac{3}{2}h(bfg-\right.}{}$$

$$= -\frac{(2cfg-4ceh+3bfh)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{20c^2h} + \frac{f(}{}$$

$$= -\frac{(2cfg-4ceh+3bfh)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{20c^2h} + \frac{f(}{}$$

$$= \frac{(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c}{}$$

$$= \frac{(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c}{}$$

$$= \frac{(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c}{}$$

Mathematica [**A**]
time = 3.89, size = 654, normalized size = 1.12

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(315*b^5*f*h^2 - 210*b^4*c*h*(4*f*g + 2*e*
h + f*h*x) + 8*b^3*c*(-210*a*f*h^2 + 5*c*h*(30*e*g + 15*d*h + 7*e*h*x) + c*
f*(75*g^2 + 70*g*h*x + 21*h^2*x^2)) - 16*b^2*c^2*(-(a*h*(230*f*g + 115*e*h
+ 56*f*h*x)) + c*(5*d*h*(24*g + 5*h*x) + 2*e*(30*g^2 + 25*g*h*x + 7*h^2*x^2
) + f*x*(25*g^2 + 28*g*h*x + 9*h^2*x^2))) + 16*b*c^2*(113*a^2*f*h^2 - 2*a*c
*(h*(130*e*g + 65*d*h + 29*e*h*x) + f*(65*g^2 + 58*g*h*x + 17*h^2*x^2)) + 4
*c^2*(5*d*(6*g^2 + 4*g*h*x + h^2*x^2) + x*(f*x*(5*g^2 + 6*g*h*x + 2*h^2*x^2
) + e*(10*g^2 + 10*g*h*x + 3*h^2*x^2)))) - 32*c^3*(a^2*h*(64*f*g + 32*e*h +
 15*f*h*x) - 2*a*c*(5*d*h*(16*g + 3*h*x) + f*x*(15*g^2 + 16*g*h*x + 5*h^2*x
```

```
^2) + e*(40*g^2 + 30*g*h*x + 8*h^2*x^2)) - 4*c^2*x*(5*d*(6*g^2 + 8*g*h*x +
3*h^2*x^2) + x*(2*e*(10*g^2 + 15*g*h*x + 6*h^2*x^2) + f*x*(15*g^2 + 24*g*h*
x + 10*h^2*x^2))))) + 15*(b^2 - 4*a*c)*(128*c^4*d*g^2 + 21*b^4*f*h^2 - 28*b
^2*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 32*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) +
2*b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*h^2 + 6*a*b*h*(2*f*g + e*h) + 5*b^2*(
f*g^2 + h*(2*e*g + d*h))))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]
)/(15360*c^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1211 vs. $2(558) = 1116$.
time = 0.14, size = 1212, normalized size = 2.08

method	result	size
default	Expression too large to display	1212
risch	Expression too large to display	1630

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*h^2*(1/6*x^3*(c*x^2+b*x+a)^(3/2)/c-3/4*b/c*(1/5*x^2*(c*x^2+b*x+a)^(3/2)/c
-7/10*b/c*(1/4*x*(c*x^2+b*x+a)^(3/2)/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1
/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2
*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+
a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)
)))-2/5*a/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+
a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)
))))-1/2*a/c*(1/4*x*(c*x^2+b*x+a)^(3/2)/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/
c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b
*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1
/2)))))+(e*h^2+2*f*g*h)*(1/5*x^2*(c*x^2+b*x+a)^(3/2)/c-7/10*b/c*(1/4*x*(c*x
^2+b*x+a)^(3/2)/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)
/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^
2+b*x+a)^(1/2))))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b
^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-2/5*a/c*(1/3*(c*x
^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b
^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+(d*h^2+2*e*g*h+f
*g^2)*(1/4*x*(c*x^2+b*x+a)^(3/2)/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b
/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(
1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+
(2*d*g*h+e*g^2)*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+
b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))))+d*g^2*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.78, size = 1881, normalized size = 3.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/30720*(15*(8*(16*(b^2*c^4 - 4*a*c^5)*d + (5*b^4*c^2 - 24*a*b^2*c^3 + 16
*a^2*c^4)*f)*g^2 - 8*(16*(b^3*c^3 - 4*a*b*c^4)*d + (7*b^5*c - 40*a*b^3*c^2
+ 48*a^2*b*c^3)*f)*g*h + (8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*d + (21
*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*f)*h^2 - 4*(16*(b^3*c^3
- 4*a*b*c^4)*g^2 - 4*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*g*h + (7*b^5*c
 - 40*a*b^3*c^2 + 48*a^2*b*c^3)*h^2)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x -
b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*
h^2*x^5 + 128*(24*c^6*f*g*h + b*c^5*f*h^2)*x^4 + 16*(120*c^6*f*g^2 + 24*b*c
^5*f*g*h + (120*c^6*d - (9*b^2*c^4 - 20*a*c^5)*f)*h^2)*x^3 + 40*(48*b*c^5*d
 + (15*b^3*c^3 - 52*a*b*c^4)*f)*g^2 - 8*(80*(3*b^2*c^4 - 8*a*c^5)*d + (105*
b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*f)*g*h + (40*(15*b^3*c^3 - 52*a*b*c^
4)*d + (315*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f)*h^2 + 8*(40*b*c^5*f
*g^2 + 8*(80*c^6*d - (7*b^2*c^4 - 16*a*c^5)*f)*g*h + (40*b*c^5*d + (21*b^3*
c^3 - 68*a*b*c^4)*f)*h^2)*x^2 + 2*(40*(48*c^6*d - (5*b^2*c^4 - 12*a*c^5)*f)
*g^2 + 8*(80*b*c^5*d + (35*b^3*c^3 - 116*a*b*c^4)*f)*g*h - (40*(5*b^2*c^4 -
 12*a*c^5)*d + (105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*f)*h^2)*x + 4*(3
84*c^6*h^2*x^4 + 48*(20*c^6*g*h + b*c^5*h^2)*x^3 - 80*(3*b^2*c^4 - 8*a*c^5)
*g^2 + 20*(15*b^3*c^3 - 52*a*b*c^4)*g*h - (105*b^4*c^2 - 460*a*b^2*c^3 + 25
6*a^2*c^4)*h^2 + 8*(80*c^6*g^2 + 20*b*c^5*g*h - (7*b^2*c^4 - 16*a*c^5)*h^2)
*x^2 + 2*(80*b*c^5*g^2 - 20*(5*b^2*c^4 - 12*a*c^5)*g*h + (35*b^3*c^3 - 116*
a*b*c^4)*h^2)*x)*e)*sqrt(c*x^2 + b*x + a))/c^6, 1/15360*(15*(8*(16*(b^2*c^4
 - 4*a*c^5)*d + (5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*f)*g^2 - 8*(16*(b^3
```

```
*c^3 - 4*a*b*c^4)*d + (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*f)*g*h + (8*(
5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*d + (21*b^6 - 140*a*b^4*c + 240*a^2*
b^2*c^2 - 64*a^3*c^3)*f)*h^2 - 4*(16*(b^3*c^3 - 4*a*b*c^4)*g^2 - 4*(5*b^4*c
^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*g*h + (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^
3)*h^2)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(
c^2*x^2 + b*c*x + a*c)) + 2*(1280*c^6*f*h^2*x^5 + 128*(24*c^6*f*g*h + b*c^5
*f*h^2)*x^4 + 16*(120*c^6*f*g^2 + 24*b*c^5*f*g*h + (120*c^6*d - (9*b^2*c^4
- 20*a*c^5)*f)*h^2)*x^3 + 40*(48*b*c^5*d + (15*b^3*c^3 - 52*a*b*c^4)*f)*g^2
 - 8*(80*(3*b^2*c^4 - 8*a*c^5)*d + (105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c
^4)*f)*g*h + (40*(15*b^3*c^3 - 52*a*b*c^4)*d + (315*b^5*c - 1680*a*b^3*c^2
+ 1808*a^2*b*c^3)*f)*h^2 + 8*(40*b*c^5*f*g^2 + 8*(80*c^6*d - (7*b^2*c^4 - 1
6*a*c^5)*f)*g*h + (40*b*c^5*d + (21*b^3*c^3 - 68*a*b*c^4)*f)*h^2)*x^2 + 2*(
40*(48*c^6*d - (5*b^2*c^4 - 12*a*c^5)*f)*g^2 + 8*(80*b*c^5*d + (35*b^3*c^3
- 116*a*b*c^4)*f)*g*h - (40*(5*b^2*c^4 - 12*a*c^5)*d + (105*b^4*c^2 - 448*a
*b^2*c^3 + 240*a^2*c^4)*f)*h^2)*x + 4*(384*c^6*h^2*x^4 + 48*(20*c^6*g*h + b
*c^5*h^2)*x^3 - 80*(3*b^2*c^4 - 8*a*c^5)*g^2 + 20*(15*b^3*c^3 - 52*a*b*c^4)
*g*h - (105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*h^2 + 8*(80*c^6*g^2 + 20
*b*c^5*g*h - (7*b^2*c^4 - 16*a*c^5)*h^2)*x^2 + 2*(80*b*c^5*g^2 - 20*(5*b^2*
c^4 - 12*a*c^5)*g*h + (35*b^3*c^3 - 116*a*b*c^4)*h^2)*x)*e)*sqrt(c*x^2 + b*
x + a))/c^6]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (g + hx)^2 \sqrt{a + bx + cx^2} \left(d + ex + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**2*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

Giac [A]
time = 4.51, size = 1012, normalized size = 1.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f*h^2*x + (24*c^5*f*g*h + b*c^
4*f*h^2 + 12*c^5*h^2*e)/c^5)*x + (120*c^5*f*g^2 + 24*b*c^4*f*g*h + 120*c^5*
d*h^2 - 9*b^2*c^3*f*h^2 + 20*a*c^4*f*h^2 + 240*c^5*g*h*e + 12*b*c^4*h^2*e)/
c^5)*x + (40*b*c^4*f*g^2 + 640*c^5*d*g*h - 56*b^2*c^3*f*g*h + 128*a*c^4*f*g
*h + 40*b*c^4*d*h^2 + 21*b^3*c^2*f*h^2 - 68*a*b*c^3*f*h^2 + 320*c^5*g^2*e +
 80*b*c^4*g*h*e - 28*b^2*c^3*h^2*e + 64*a*c^4*h^2*e)/c^5)*x + (1920*c^5*d*g
```

```
^2 - 200*b^2*c^3*f*g^2 + 480*a*c^4*f*g^2 + 640*b*c^4*d*g*h + 280*b^3*c^2*f*
g*h - 928*a*b*c^3*f*g*h - 200*b^2*c^3*d*h^2 + 480*a*c^4*d*h^2 - 105*b^4*c*f
*h^2 + 448*a*b^2*c^2*f*h^2 - 240*a^2*c^3*f*h^2 + 320*b*c^4*g^2*e - 400*b^2*
c^3*g*h*e + 960*a*c^4*g*h*e + 140*b^3*c^2*h^2*e - 464*a*b*c^3*h^2*e)/c^5)*x
 + (1920*b*c^4*d*g^2 + 600*b^3*c^2*f*g^2 - 2080*a*b*c^3*f*g^2 - 1920*b^2*c^
3*d*g*h + 5120*a*c^4*d*g*h - 840*b^4*c*f*g*h + 3680*a*b^2*c^2*f*g*h - 2048*
a^2*c^3*f*g*h + 600*b^3*c^2*d*h^2 - 2080*a*b*c^3*d*h^2 + 315*b^5*f*h^2 - 16
80*a*b^3*c*f*h^2 + 1808*a^2*b*c^2*f*h^2 - 960*b^2*c^3*g^2*e + 2560*a*c^4*g^
2*e + 1200*b^3*c^2*g*h*e - 4160*a*b*c^3*g*h*e - 420*b^4*c*h^2*e + 1840*a*b^
2*c^2*h^2*e - 1024*a^2*c^3*h^2*e)/c^5) + 1/1024*(128*b^2*c^4*d*g^2 - 512*a*
c^5*d*g^2 + 40*b^4*c^2*f*g^2 - 192*a*b^2*c^3*f*g^2 + 128*a^2*c^4*f*g^2 - 12
8*b^3*c^3*d*g*h + 512*a*b*c^4*d*g*h - 56*b^5*c*f*g*h + 320*a*b^3*c^2*f*g*h
 - 384*a^2*b*c^3*f*g*h + 40*b^4*c^2*d*h^2 - 192*a*b^2*c^3*d*h^2 + 128*a^2*c^
4*d*h^2 + 21*b^6*f*h^2 - 140*a*b^4*c*f*h^2 + 240*a^2*b^2*c^2*f*h^2 - 64*a^3
*c^3*f*h^2 - 64*b^3*c^3*g^2*e + 256*a*b*c^4*g^2*e + 80*b^4*c^2*g*h*e - 384*
a*b^2*c^3*g*h*e + 256*a^2*c^4*g*h*e - 28*b^5*c*h^2*e + 160*a*b^3*c^2*h^2*e
 - 192*a^2*b*c^3*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(
c) - b))/c^(11/2)
```

Mupad [B]
time = 7.91, size = 1881, normalized size = 3.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*g^2*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (e*h^2*x^2*(a + b*x + c*x^2
)^(3/2))/(5*c) + (f*h^2*x^3*(a + b*x + c*x^2)^(3/2))/(6*c) - (a*d*h^2*((x/2
 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x +
 c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (a*f*g^2*((x/2 + b/(4*c
))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(
1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) + (d*g^2*log((b/2 + c*x)/c^(1/2) +
 (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*g^2*log((b + 2*c*
x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (2*
a*e*h^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b
*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^
(1/2))/(24*c^2)))/(5*c) - (5*b*d*h^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x
 + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2
+ 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (5*b*f*g^2*((log((b
+ 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))
 + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))
/(8*c) + (e*g^2*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2)
)/(24*c^2) + (d*h^2*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (f*g^2*x*(a + b*x +
c*x^2)^(3/2))/(4*c) + (a*f*h^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x
```

+ c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(2*c) + (7*b*e*h^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (3*b*f*h^2*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (2*a*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(4*c) + (2*f*g*h*x^2*(a + b*x + c*x^2)^(3/2))/(5*c) - (a*e*g*h*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(2*c) + (d*g*h*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(8*c^(5/2)) - (4*a*f*g*h*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (5*b*e*g*h*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(4*c) + (d*g*h*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(12*c^2) + (e*g*h*x*(a + b*x + c*x^2)^(3/2))/(2*c) + (7*b*f*g*h*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(5*c)

3.188 $\int (g+hx)\sqrt{a+bx+cx^2}\,(d+ex+fx^2)\,dx$

Optimal. Leaf size=322

$$\frac{(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh+5b(fg+eh)))\,(b+2cx)\sqrt{a+bx+cx^2}}{128c^4}+$$

```
[Out] 1/5*f*(h*x+g)^2*(c*x^2+b*x+a)^(3/2)/c/h+1/240*(35*b^2*f*h^2-16*c^2*(3*f*g^2
-5*h*(d*h+e*g))-2*c*h*(16*a*f*h+25*b*(e*h+f*g))-6*c*h*(7*b*f*h-10*c*e*h+6*c
*f*g)*x)*(c*x^2+b*x+a)^(3/2)/c^3/h-1/256*(-4*a*c+b^2)*(32*c^3*d*g-7*b^3*f*h
-8*c^2*(a*e*h+a*f*g+2*b*d*h+2*b*e*g)+2*b*c*(6*a*f*h+5*b*(e*h+f*g)))*arctanh
(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)+1/128*(32*c^3*d*g-7*b^3
*f*h-8*c^2*(a*e*h+a*f*g+2*b*d*h+2*b*e*g)+2*b*c*(6*a*f*h+5*b*(e*h+f*g)))*(2*
c*x+b)*(c*x^2+b*x+a)^(1/2)/c^4
```

Rubi [A]
time = 0.29, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1667, 793, 626, 635, 212}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\,(-8c^2(aeh+afg+2bdh+2beg)+2bc(6afh+5b(eh+fg))-7b^3fh+32c^3dg)}{128c^4}+\frac{(a+bx+cx^2)^{3/2}\,(-2ch(16afh+25b(eh+fg))+35b^2fh^2-6chx(7bfh-10ceh+6cfg)-(c^2(48fg^2-80h(dh+eg))))}{240c^3h}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-8c^2(aeh+afg+2bdh+2beg)+2bc(6afh+5b(eh+fg))-7b^3fh+32c^3dg)}{256c^{9/2}}+\frac{f(g+hx)^2\,(a+bx+cx^2)^{3/2}}{5ch}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((32*c^3*d*g - 7*b^3*f*h - 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a*e*h) + 2*b*
c*(6*a*f*h + 5*b*(f*g + e*h)))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^4)
 + (f*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(5*c*h) + ((35*b^2*f*h^2 - c^2*(
48*f*g^2 - 80*h*(e*g + d*h)) - 2*c*h*(16*a*f*h + 25*b*(f*g + e*h)) - 6*c*h*
(6*c*f*g - 10*c*e*h + 7*b*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(240*c^3*h) - ((
b^2 - 4*a*c)*(32*c^3*d*g - 7*b^3*f*h - 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a
*e*h) + 2*b*c*(6*a*f*h + 5*b*(f*g + e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(256*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch} + \frac{\int (g+hx)\left(-\frac{1}{2}h(3bfg-1\right.}{}$$

$$= \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch} + \frac{(35b^2fh^2-c^2(48fg^2-80}{}$$

$$= \frac{(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6}{128c^4}$$

$$= \frac{(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6}{128c^4}$$

$$= \frac{(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6}{128c^4}$$

Mathematica [A]
time = 1.56, size = 342, normalized size = 1.06

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(-105b^4fh+10b^3c(15fg+15eh+7fhx)-4b^2c(-115afh+c(60eg+60dh+25fgx+25ehx+14fhx^2))+8bc^2(20cd(3g+hx)-a(65fg+65eh+29fhx)+2cx(5e(2g+hx)+fx(5g+3hx)))+16c^2(-16a^2fh+ac(40dh+5e(8g+3hx)+fx(15g+8hx))+2c^2x(10d(3g+2hx)+x(5e(4g+3hx)+3fx(5g+4hx)))))-15(b^2-4ac)\left(-32c^3dg+7b^3fh+8c^2(2beg+afg+2bdh+aeh)-2bc(6afh+5b(fg+eh))\right)\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{3840c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^4*f*h + 10*b^3*c*(15*f*g + 15*e*h
+ 7*f*h*x) - 4*b^2*c*(-115*a*f*h + c*(60*e*g + 60*d*h + 25*f*g*x + 25*e*h*x
 + 14*f*h*x^2)) + 8*b*c^2*(20*c*d*(3*g + h*x) - a*(65*f*g + 65*e*h + 29*f*h
*x) + 2*c*x*(5*e*(2*g + h*x) + f*x*(5*g + 3*h*x))) + 16*c^2*(-16*a^2*f*h +
a*c*(40*d*h + 5*e*(8*g + 3*h*x) + f*x*(15*g + 8*h*x)) + 2*c^2*x*(10*d*(3*g
+ 2*h*x) + x*(5*e*(4*g + 3*h*x) + 3*f*x*(5*g + 4*h*x))))) - 15*(b^2 - 4*a*c
)*(-32*c^3*d*g + 7*b^3*f*h + 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a*e*h) - 2*
b*c*(6*a*f*h + 5*b*(f*g + e*h)))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b +
c*x)]])/(3840*c^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(300) = 600$.
time = 0.12, size = 663, normalized size = 2.06

method	result

default	$hf\left(\frac{x^2(cx^2+bx+a)^{\frac{3}{2}}}{5c}-\frac{7b\left(\frac{x(cx^2+bx+a)^{\frac{3}{2}}}{4c}-\frac{5b\left(\frac{(cx^2+bx+a)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{cx^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2}\right.}{8c^{\frac{3}{2}}}\right.}{2c}\right.}{8c}\right.}{1}\right.$
risch	$-\frac{(-384hfc^4x^4-48bc^3fhx^3-480c^4ehx^3-480c^4fgx^3-128ac^3fhx^2+56b^2c^2fhx^2-80bc^3ehx^2-80bc^3fgx^2-640c^4dhx^2-640c^4eg}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] h*f*(1/5*x^2*(c*x^2+b*x+a)^(3/2)/c-7/10*b/c*(1/4*x*(c*x^2+b*x+a)^(3/2)/c-5/
8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/
2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-1/
4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-2/5*a/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/
2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+(e*h+f*g)*(1/4*x*(c*x^2+b*x+a)^(3/2)
/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a
```

```
)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln(
(1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+(d*h+e*g)*(1/3*(c*x^2+b*x+a)^(3/
2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+d*g*(1/4*(2*c*x+b)/c*(c*x^2+b*
x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.49, size = 1043, normalized size = 3.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(2*(16*(b^2*c^3 - 4*a*c^4)*d + (5*b^4*c - 24*a*b^2*c^2 + 16*a^2
*c^3)*f)*g - (16*(b^3*c^2 - 4*a*b*c^3)*d + (7*b^5 - 40*a*b^3*c + 48*a^2*b*c
^2)*f)*h - 2*(8*(b^3*c^2 - 4*a*b*c^3)*g - (5*b^4*c - 24*a*b^2*c^2 + 16*a^2*
c^3)*h)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)
*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(384*c^5*f*h*x^4 + 48*(10*c^5*f*g + b*c^4
*f*h)*x^3 + 8*(10*b*c^4*f*g + (80*c^5*d - (7*b^2*c^3 - 16*a*c^4)*f)*h)*x^2
+ 10*(48*b*c^4*d + (15*b^3*c^2 - 52*a*b*c^3)*f)*g - (80*(3*b^2*c^3 - 8*a*c^
4)*d + (105*b^4*c - 460*a*b^2*c^2 + 256*a^2*c^3)*f)*h + 2*(10*(48*c^5*d - (
5*b^2*c^3 - 12*a*c^4)*f)*g + (80*b*c^4*d + (35*b^3*c^2 - 116*a*b*c^3)*f)*h)
*x + 10*(48*c^5*h*x^3 + 8*(8*c^5*g + b*c^4*h)*x^2 - 8*(3*b^2*c^3 - 8*a*c^4)
*g + (15*b^3*c^2 - 52*a*b*c^3)*h + 2*(8*b*c^4*g - (5*b^2*c^3 - 12*a*c^4)*h)
*x)*e)*sqrt(c*x^2 + b*x + a))/c^5, 1/3840*(15*(2*(16*(b^2*c^3 - 4*a*c^4)*d
+ (5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*g - (16*(b^3*c^2 - 4*a*b*c^3)*d
+ (7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*f)*h - 2*(8*(b^3*c^2 - 4*a*b*c^3)*g -
 (5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*h)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(384*c^5*f*h*
x^4 + 48*(10*c^5*f*g + b*c^4*f*h)*x^3 + 8*(10*b*c^4*f*g + (80*c^5*d - (7*b^
```

```
2*c^3 - 16*a*c^4)*f)*h)*x^2 + 10*(48*b*c^4*d + (15*b^3*c^2 - 52*a*b*c^3)*f)
*g - (80*(3*b^2*c^3 - 8*a*c^4)*d + (105*b^4*c - 460*a*b^2*c^2 + 256*a^2*c^3
)*f)*h + 2*(10*(48*c^5*d - (5*b^2*c^3 - 12*a*c^4)*f)*g + (80*b*c^4*d + (35*
b^3*c^2 - 116*a*b*c^3)*f)*h)*x + 10*(48*c^5*h*x^3 + 8*(8*c^5*g + b*c^4*h)*x
^2 - 8*(3*b^2*c^3 - 8*a*c^4)*g + (15*b^3*c^2 - 52*a*b*c^3)*h + 2*(8*b*c^4*g
 - (5*b^2*c^3 - 12*a*c^4)*h)*x)*e)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

Giac [A]
time = 4.47, size = 495, normalized size = 1.54

$$\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(8\,fhx+\frac{10\,c^4fg+bc^3fh+10\,c^4he}{c^4}\right)x+\frac{10\,bc^3fg+80\,c^4dh-7\,b^2c^2fh+16\,ac^3fh+80\,c^4ge+10\,bc^3he}{c^4}\right)x+\frac{480\,c^4dg-50\,b^2c^2fg+120\,ac^3fg+80\,bc^3dh+35\,b^3cfh-116\,abc^2fh+80\,bc^3ge-50\,b^2c^2he+120\,ac^3he}{c^4}\right)x+\frac{480\,bc^3dg+150\,b^3cfg-520\,abc^2fg-240\,b^2c^2dh+640\,ac^3dh-105\,b^4fh+460\,ab^2cfh-256\,a^2c^2fh-240\,b^2c^2ge+640\,ac^3ge+150\,b^3che-520\,abc^2he}{c^4}\right)+\frac{\left(32\,b^2c^3dg-128\,ac^4dg+10\,b^4cfg-48\,ab^2c^2fg+32\,a^2c^3fg-16\,b^3c^2dh+64\,abc^3dh-7\,b^5fh+40\,ab^3cfh-48\,a^2bc^2fh-16\,b^3c^2ge+64\,abc^3ge+10\,b^4che-48\,ab^2c^2he+32\,a^2c^3he\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{256\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*f*h*x + (10*c^4*f*g + b*c^3*f*h +
10*c^4*h*e)/c^4)*x + (10*b*c^3*f*g + 80*c^4*d*h - 7*b^2*c^2*f*h + 16*a*c^3*
f*h + 80*c^4*g*e + 10*b*c^3*h*e)/c^4)*x + (480*c^4*d*g - 50*b^2*c^2*f*g + 1
20*a*c^3*f*g + 80*b*c^3*d*h + 35*b^3*c*f*h - 116*a*b*c^2*f*h + 80*b*c^3*g*e
 - 50*b^2*c^2*h*e + 120*a*c^3*h*e)/c^4)*x + (480*b*c^3*d*g + 150*b^3*c*f*g
- 520*a*b*c^2*f*g - 240*b^2*c^2*d*h + 640*a*c^3*d*h - 105*b^4*f*h + 460*a*b
^2*c*f*h - 256*a^2*c^2*f*h - 240*b^2*c^2*g*e + 640*a*c^3*g*e + 150*b^3*c*h*
e - 520*a*b*c^2*h*e)/c^4) + 1/256*(32*b^2*c^3*d*g - 128*a*c^4*d*g + 10*b^4*
c*f*g - 48*a*b^2*c^2*f*g + 32*a^2*c^3*f*g - 16*b^3*c^2*d*h + 64*a*b*c^3*d*h
 - 7*b^5*f*h + 40*a*b^3*c*f*h - 48*a^2*b*c^2*f*h - 16*b^3*c^2*g*e + 64*a*b*
c^3*g*e + 10*b^4*c*h*e - 48*a*b^2*c^2*h*e + 32*a^2*c^3*h*e)*log(abs(-2*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

Mupad [B]
time = 5.62, size = 877, normalized size = 2.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*g*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (2*a*f*h*((log((b + 2*c*x)/c^
(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a
 + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (5
*b*e*h*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*
c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(
1/2))/(24*c^2)))/(8*c) - (5*b*f*g*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x +
c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2
*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) + (d*h*(8*c*(a + c*x^2) -
 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2) + (e*g*(8*c*(a + c*x^2)
 - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2) + (e*h*x*(a + b*x + c
*x^2)^(3/2))/(4*c) + (f*g*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (7*b*f*h*((5*b
*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(1
6*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/
(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*
(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2
))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) + (f*h*x^2*(a + b*x + c*x^2)
^(3/2))/(5*c) - (a*e*h*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2
 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*
c) - (a*f*g*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(
1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) + (d*g*l
og((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)
) + (d*h*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*
c))/(16*c^(5/2)) + (e*g*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2)
)*(b^3 - 4*a*b*c))/(16*c^(5/2))
```

3.189 $\int \sqrt{a+bx+cx^2}\,(d+ex+fx^2)\,dx$

Optimal. Leaf size=175

$$\frac{(16c^2d-8bce+5b^2f-4acf)\,(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{(8ce-5bf)\,(a+bx+cx^2)^{3/2}}{24c^2}+\frac{fx(a+bx+cx^2)^{3/2}}{4c}$$

```
[Out] 1/24*(-5*b*f+8*c*e)*(c*x^2+b*x+a)^(3/2)/c^2+1/4*f*x*(c*x^2+b*x+a)^(3/2)/c-1
/128*(-4*a*c+b^2)*(16*c^2*d+5*b^2*f-4*c*(a*f+2*b*e))*arctanh(1/2*(2*c*x+b)/
c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1/64*(-4*a*c*f+5*b^2*f-8*b*c*e+16*c^2*
d)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^3
```

Rubi [A]
time = 0.10, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1675, 654, 626, 635, 212}

$$-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)(-4c(af+2be)+5b^2f+16c^2d)}{128c^{7/2}}+\frac{(b+2cx)\sqrt{a+bx+cx^2}\,(-4acf+5b^2f-8bce+16c^2d)}{64c^3}+\frac{(a+bx+cx^2)^{3/2}\,(8ce-5bf)}{24c^2}+\frac{fx(a+bx+cx^2)^{3/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((16*c^2*d - 8*b*c*e + 5*b^2*f - 4*a*c*f)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]
)/(64*c^3) + ((8*c*e - 5*b*f)*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (f*x*(a +
 b*x + c*x^2)^(3/2))/(4*c) - ((b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*
e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7
/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx &= \frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}+\frac{\int \left(4cd-af+\frac{1}{2}(8ce-5bf)x\right)\sqrt{a+bx+c}}{4c}\\ &= \frac{(8ce-5bf)\left(a+bx+cx^2\right)^{3/2}}{24c^2}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}+\frac{\left(16c^2d-8\right.}{}\\ &= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{(8ce-5}{}\\ &= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{(8ce-5}{}\\ &= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}+\frac{(8ce-5}{}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 171, normalized size = 0.98

$$\frac{2\sqrt{c}\,\sqrt{a+x(b+cx)}\,\left(15b^3f-2b^2c(12e+5fx)+4bc\left(-13af+2c\left(6d+2ex+fx^2\right)\right)+8c^2\left(a(8e+3fx)+2cx\left(6d+4ex+3fx^2\right)\right)\right)+3\left(b^2-4ac\right)\left(16c^2d+5b^2f-4c(2be+af)\right)\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^3*f - 2*b^2*c*(12*e + 5*f*x) + 4*b*c
*(-13*a*f + 2*c*(6*d + 2*e*x + f*x^2)) + 8*c^2*(a*(8*e + 3*f*x) + 2*c*x*(6*
d + 4*e*x + 3*f*x^2))) + 3*(b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*e +
 a*f))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(153) = 306$.
time = 0.12, size = 343, normalized size = 1.96

method	result
default	$f\left(\frac{x(cx^2+bx+a)^{\frac{3}{2}}}{4c}-\frac{5b\left(\frac{(cx^2+bx+a)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{cx^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{2c}\right)}{8c}-\right.$
risch	$-\frac{(-48c^3fx^3-8bc^2fx^2-64c^3ex^2-24ac^2fx+10b^2cfx-16bc^2ex-96c^3dx+52abcf-64ac^2e-15b^3f+24b^2ce-48bc^2d)\sqrt{cx^2+bx}}{192c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/4*x*(c*x^2+b*x+a)^(3/2)/c-5/8*b/c*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(
1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))))-1/4*a/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)
+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+e*(1
/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(
4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+d*(1/4*(2*
c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.46, size = 469, normalized size = 2.68

$$\left[\frac{3\,(16\,(b^2c^2-4\,ac^3)d+(5\,b^4-24\,ab^2c+16\,a^2c^2)f-8\,(b^3c-4\,abc^2)e)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{c}-4\,ac\right)+4\,(48\,c^4fx^3+8\,bc^3fx^2+48\,bc^3d+(15\,b^3c-52\,abc^2)f+2\,(48\,c^4d-(5\,b^2c^2-12\,ac^3)f)x+8\,(8\,c^4x^2+2\,bc^3x-3\,b^2c^2+8\,ac^3)e)\sqrt{cx^2+bx+a}}{768\,c^4}, \frac{3\,(16\,(b^2c^2-4\,ac^3)d+(5\,b^4-24\,ab^2c+16\,a^2c^2)f-8\,(b^3c-4\,abc^2)e)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)+2\,(48\,c^4fx^3+8\,bc^3fx^2+48\,bc^3d+(15\,b^3c-52\,abc^2)f+2\,(48\,c^4d-(5\,b^2c^2-12\,ac^3)f)x+8\,(8\,c^4x^2+2\,bc^3x-3\,b^2c^2+8\,ac^3)e)\sqrt{cx^2+bx+a}}{384\,c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(16*(b^2*c^2 - 4*a*c^3)*d + (5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*f -
 8*(b^3*c - 4*a*b*c^2)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*x^3 + 8*b*c^3*f*
x^2 + 48*b*c^3*d + (15*b^3*c - 52*a*b*c^2)*f + 2*(48*c^4*d - (5*b^2*c^2 - 1
2*a*c^3)*f)*x + 8*(8*c^4*x^2 + 2*b*c^3*x - 3*b^2*c^2 + 8*a*c^3)*e)*sqrt(c*x
^2 + b*x + a))/c^4, 1/384*(3*(16*(b^2*c^2 - 4*a*c^3)*d + (5*b^4 - 24*a*b^2*
c + 16*a^2*c^2)*f - 8*(b^3*c - 4*a*b*c^2)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*f*x^3
 + 8*b*c^3*f*x^2 + 48*b*c^3*d + (15*b^3*c - 52*a*b*c^2)*f + 2*(48*c^4*d - (
5*b^2*c^2 - 12*a*c^3)*f)*x + 8*(8*c^4*x^2 + 2*b*c^3*x - 3*b^2*c^2 + 8*a*c^3
)*e)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx + cx^2}\,\left(d + ex + fx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

Giac [A]
time = 4.21, size = 212, normalized size = 1.21

$$\frac{1}{192}\,\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,fx+\frac{bc^2f+8\,c^3e}{c^3}\right)x+\frac{48\,c^3d-5\,b^2cf+12\,ac^2f+8\,bc^2e}{c^3}\right)x+\frac{48\,bc^2d+15\,b^3f-52\,abcf-24\,b^2ce+64\,ac^2e}{c^3}\right)+\frac{(16\,b^2c^2d-64\,ac^3d+5\,b^4f-24\,ab^2cf+16\,a^2c^2f-8\,b^3ce+32\,abc^2e)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{128\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*x + (b*c^2*f + 8*c^3*e)/c^3)*x + (48
*c^3*d - 5*b^2*c*f + 12*a*c^2*f + 8*b*c^2*e)/c^3)*x + (48*b*c^2*d + 15*b^3*
```

```
f - 52*a*b*c*f - 24*b^2*c*e + 64*a*c^2*e)/c^3) + 1/128*(16*b^2*c^2*d - 64*a
*c^3*d + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2*f - 8*b^3*c*e + 32*a*b*c^2*e)*
log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

Mupad [B]
time = 4.24, size = 320, normalized size = 1.83

$$d\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,x^2+b\,x+a}-\frac{a\,f\,\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}+\frac{d\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}+\frac{e\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}-\frac{5\,b\,f\,\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{8\,c}+\frac{e\,\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}+\frac{f\,x\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (a*f*((x/2 + b/(4*c))*(a + b*x
 + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c -
 b^2/4))/(2*c^(3/2))))/(4*c) + (d*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^
2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*log((b + 2*c*x)/c^(1/2) + 2*(a +
b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (5*b*f*((log((b + 2*c*x
)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*
c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c)
 + (e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)
 + (f*x*(a + b*x + c*x^2)^(3/2))/(4*c)
```

3.190 $\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{g+hx}\,dx$

Optimal. Leaf size=321

$$-\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+bfh)x)\sqrt{a+bx+cx^2}}{8c^2h^3}+\frac{f(a+$$

```
[Out] 1/3*f*(c*x^2+b*x+a)^(3/2)/c/h+1/16*(4*c*h*(-b*h+2*c*g)*(b*f*g-2*c*d*h)-(b*f
*h-2*c*e*h+2*c*f*g)*(8*c^2*g^2-b^2*h^2-4*c*h*(-a*h+b*g)))*arctanh(1/2*(2*c*
x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/h^4+(d*h^2-e*g*h+f*g^2)*arctanh(1
/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2)
)*(a*h^2-b*g*h+c*g^2)^(1/2)/h^4-1/8*(4*c*h*(b*f*g-2*c*d*h)-(-b*h+4*c*g)*(b*
f*h-2*c*e*h+2*c*f*g)+2*c*h*(b*f*h-2*c*e*h+2*c*f*g)*x)*(c*x^2+b*x+a)^(1/2)/c
^2/h^3
```

Rubi [A]
time = 0.46, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 828, 857, 635, 212, 738}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(4ch(2cg-bh)(bfg-2cdh)-(-4ch(bg-ah)-b^2h^2+8c^2g^2)(bfh-2ceh+2cfg))}{16c^{5/2}h^4}-\frac{\sqrt{a+bx+cx^2}\,(4ch(bfg-2cdh)+2chx(bfh-2ceh+2cfg)-(4cg-bh)(bfh-2ceh+2cfg))}{8c^2h^3}+\frac{\sqrt{ah^2-bgh+cg^2}\,(dh^2-egh+fg^2)\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{h^4}+\frac{f(a+bx+cx^2)^{3/2}}{3ch}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] -1/8*((4*c*h*(b*f*g - 2*c*d*h) - (4*c*g - b*h)*(2*c*f*g - 2*c*e*h + b*f*h)
+ 2*c*h*(2*c*f*g - 2*c*e*h + b*f*h)*x)*Sqrt[a + b*x + c*x^2])/(c^2*h^3) + (
f*(a + b*x + c*x^2)^(3/2))/(3*c*h) + ((4*c*h*(2*c*g - b*h)*(b*f*g - 2*c*d*h
) - (2*c*f*g - 2*c*e*h + b*f*h)*(8*c^2*g^2 - b^2*h^2 - 4*c*h*(b*g - a*h)))*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*h^4) +
(Sqrt[c*g^2 - b*g*h + a*h^2]*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(b*g - 2*a*h +
 (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/h
^4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{g+hx}\,dx = \frac{f(a+bx+cx^2)^{3/2}}{3ch} + \frac{\int \frac{\left(-\frac{3}{2}h(bfg-2cdh)-\frac{3}{2}h(2cfg-2ceh+bfh)x\right)\sqrt{a+bx+}}{g+hx}}{3ch^2}$$

$$= -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg}{8c^2h^3}$$

$$= -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg}{8c^2h^3}$$

$$= -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg}{8c^2h^3}$$

$$= -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg}{8c^2h^3}$$

Mathematica [A]
time = 1.53, size = 318, normalized size = 0.99

$$\frac{\frac{2h\sqrt{a+x(b+cx)}\,(-3b^2fh^2+2ch(4afh+b(-3fg+3eh+fhx))+4c^2(3h(-2eg+2dh+ehx)+f(6g^2-3ghx+2h^2x^2)))}{c^2}+96\sqrt{-cg^2+bgh-ah^2}\,(fg^2+h(-eg+dh))\tan^{-1}\left(\frac{\sqrt{c}\,(g+hx)-h\sqrt{a+x(b+cx)}}{\sqrt{-cg^2+h(bg-ah)}}\right)+\frac{3(-b^3fh^3+2bch^2(-bfg+beh+2afh)+16c^3(fg^3+gh(-eg+dh))-8c^2h(bfg^2+bh(-eg+dh)+ah(-fg+eh)))\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{c^{5/2}}}{48h^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((2*h*Sqrt[a + x*(b + c*x)]*(-3*b^2*f*h^2 + 2*c*h*(4*a*f*h + b*(-3*f*g + 3*
e*h + f*h*x)) + 4*c^2*(3*h*(-2*e*g + 2*d*h + e*h*x) + f*(6*g^2 - 3*g*h*x +
2*h^2*x^2))))/c^2 + 96*Sqrt[-(c*g^2) + b*g*h - a*h^2]*(f*g^2 + h*(-(e*g) +
d*h))*ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*g^2) +
h*(b*g - a*h)]] + (3*(-(b^3*f*h^3) + 2*b*c*h^2*(-(b*f*g) + b*e*h + 2*a*f*h)
 + 16*c^3*(f*g^3 + g*h*(-(e*g) + d*h)) - 8*c^2*h*(b*f*g^2 + b*h*(-(e*g) + d
*h) + a*h*(-(f*g) + e*h)))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]
)/c^(5/2))/(48*h^4)
```

Maple [A]
time = 0.14, size = 577, normalized size = 1.80

method	result

default	$$\frac{fh\left(\frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{2c}\right)+eh\left(\frac{(2cx+b)\sqrt{c\,x^2+bx}}{4c}\right.}{}$$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] 1/h^2*(f*h*(1/3*(c*x^2+b*x+a)^(3/2)/c-1/2*b/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a
)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
))+e*h*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2
*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-g*f*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1
/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+(
d*h^2-e*g*h+f*g^2)/h^3*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(
1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))
/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h
^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/
2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(
x+1/h*g)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re deta
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{g+hx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

```
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

3.191 $\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^2}\,dx$

Optimal. Leaf size=459

$$-\frac{\left(bfh^2(bg-ah)+4c^2g(3fg^2-h(2eg-dh))+ch(4ah(2fg-eh)-b(13fg^2-8egh+4dh^2))+2ch^2\left(2ceg\right.\right.}{4ch^3\left(cg^2-bgh+ah^2\right)}$$

```
[Out] -(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)-1/8
*(b^2*f*h^2+4*c*h*(-a*f*h-b*e*h+2*b*f*g)-8*c^2*(3*f*g^2-h*(-d*h+2*e*g)))*ar
ctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/h^4-1/2*(2*c*g*(3*
f*g^2-h*(-d*h+2*e*g))+h*(2*a*h*(-e*h+2*f*g)-b*(d*h^2-3*e*g*h+5*f*g^2)))*arc
tanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)
^(1/2))/h^4/(a*h^2-b*g*h+c*g^2)^(1/2)-1/4*(b*f*h^2*(-a*h+b*g)+4*c^2*g*(3*f*
g^2-h*(-d*h+2*e*g))+c*h*(4*a*h*(-e*h+2*f*g)-b*(4*d*h^2-8*e*g*h+13*f*g^2))+2
*c*h^2*(2*c*e*g+b*f*g-3*c*f*g^2/h-2*c*d*h-a*f*h)*x)*(c*x^2+b*x+a)^(1/2)/c/h
^3/(a*h^2-b*g*h+c*g^2)
```

Rubi [A]

time = 0.64, antiderivative size = 453, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1664, 828, 857, 635, 212, 738}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(-afh-beh+2bfg)+b^2fh^2-8c^2(3fg^2-h(2eg-dh))\right)}{8c^{3/2}h^4}-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(-afh+bfg-2cdh+2ceg-\frac{3cfg^2}{h}\right)+bfh(bg-ah)+4ach(2fg-eh)-bc(4dh^2-8egh+13fg^2)-4c^2g\left(-dh+2eg-\frac{3fg^2}{h}\right)\right)}{4ch^2\left(ah^2-bgh+cg^2\right)}-\frac{(a+bx+cx^2)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}-\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(2c(3fg^3-gh(2eg-dh))-h(-2ah(2fg-eh)-bh(3eg-dh)+5bfg^2)\right)}{2h^4\sqrt{ah^2-bgh+cg^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -1/4*((b*f*h*(b*g - a*h) - 4*c^2*g*(2*e*g - (3*f*g^2)/h - d*h) + 4*a*c*h*(2
*f*g - e*h) - b*c*(13*f*g^2 - 8*e*g*h + 4*d*h^2) + 2*c*h*(2*c*e*g + b*f*g -
 (3*c*f*g^2)/h - 2*c*d*h - a*f*h)*x)*Sqrt[a + b*x + c*x^2])/(c*h^2*(c*g^2 -
 b*g*h + a*h^2)) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(h*(c*
g^2 - b*g*h + a*h^2)*(g + h*x)) - ((b^2*f*h^2 + 4*c*h*(2*b*f*g - b*e*h - a*
f*h) - 8*c^2*(3*f*g^2 - h*(2*e*g - d*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sq
rt[a + b*x + c*x^2])])/(8*c^(3/2)*h^4) - ((2*c*(3*f*g^3 - g*h*(2*e*g - d*h)
) - h*(5*b*f*g^2 - b*h*(3*e*g - d*h) - 2*a*h*(2*f*g - e*h)))*ArcTanh[(b*g -
 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x
^2])])/(2*h^4*Sqrt[c*g^2 - b*g*h + a*h^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^2}\,dx = -\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{3/2}}{h\,(cg^2-bgh+ah^2)\,(g+hx)} - \frac{\int \frac{\left(\frac{1}{2}\left(-2cdg+3beg+2afg-\frac{3bfg^2}{h}\right.\right.}{}}{}$$

$$= -\frac{\left(bfh(bg-ah)-4c^2g\left(2eg-\frac{3fg^2}{h}-dh\right)+4ach(2fg-eh)-bc(1\right.}{4ch}$$

$$= -\frac{\left(bfh(bg-ah)-4c^2g\left(2eg-\frac{3fg^2}{h}-dh\right)+4ach(2fg-eh)-bc(1\right.}{4ch}$$

$$= -\frac{\left(bfh(bg-ah)-4c^2g\left(2eg-\frac{3fg^2}{h}-dh\right)+4ach(2fg-eh)-bc(1\right.}{4ch}$$

$$= -\frac{\left(bfh(bg-ah)-4c^2g\left(2eg-\frac{3fg^2}{h}-dh\right)+4ach(2fg-eh)-bc(1\right.}{4ch}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1473 vs. $2(459) = 918$.
time = 8.38, size = 1473, normalized size = 3.21

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] ((d + x*(e + f*x))*(2*a*Sqrt[c] + 2*b*Sqrt[c]*x + 2*c^(3/2)*x^2 - b*Sqrt[a
+ x*(b + c*x)] - 2*c*x*Sqrt[a + x*(b + c*x)]))/(g*h*(b + 2*c*x - 2*Sqrt[c]*
Sqrt[a + x*(b + c*x)])) - (x*(d + x*(e + f*x))*(2*a*Sqrt[c] + 2*b*Sqrt[c]*x
 + 2*c^(3/2)*x^2 - b*Sqrt[a + x*(b + c*x)] - 2*c*x*Sqrt[a + x*(b + c*x)]))/
(g*(g + h*x)*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])) + (6*c*f*g^3*Ar
cTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*g^2) + h*(b*g -
 a*h)]])/(h^4*Sqrt[-(c*g^2) + h*(b*g - a*h)]) + ((4*c*e + 5*b*f)*g^2*ArcTan
[(-(Sqrt[c]*(g + h*x)) + h*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*g^2) + h*(b*g -
a*h)]])/(h^3*Sqrt[-(c*g^2) + h*(b*g - a*h)]) + ((b*d + 2*a*e)*ArcTan[(-(Sqr
t[c]*(g + h*x)) + h*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*g^2) + h*(b*g - a*h)]])
/(h*Sqrt[-(c*g^2) + h*(b*g - a*h)]) - (3*Sqrt[c]*f*g^2*Log[c*(b + 2*c*x - 2
*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/h^4 + (g*(-(b^2*f) + 12*a*c*f + 2*b*c*(e
+ 5*f*x) - 4*b*Sqrt[c]*f*Sqrt[a + x*(b + c*x)] + 4*c^(3/2)*(e + 3*f*x)*(Sqr
t[c]*x - Sqrt[a + x*(b + c*x)]) + 2*(2*c*e + b*f)*(b + 2*c*x - 2*Sqrt[c]*Sq
```

```
rt[a + x*(b + c*x)])*Log[c*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))
/(2*Sqrt[c]*h^3*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])) + (-32*c^(3/
2)*(2*c*d + 3*b*e + 4*a*f)*g*(b^2 + b*(8*c*x - 4*Sqrt[c]*Sqrt[a + x*(b + c*
x)]) + 4*c*(a + 2*c*x^2 - 2*Sqrt[c]*x*Sqrt[a + x*(b + c*x)]))*ArcTan[(-(Sqr
t[c]*(g + h*x)) + h*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*g^2) + h*(b*g - a*h)]]
+ Sqrt[-(c*g^2) + b*g*h - a*h^2]*(-32*b^2*c^2*d - 128*a*c^3*d + 8*b^3*c*e -
 224*a*b*c^2*e + b^4*f - 28*a*b^2*c*f - 256*b*c^3*d*x - 192*b^2*c^2*e*x - 5
12*a*c^3*e*x - 24*b^3*c*f*x - 256*a*b*c^2*f*x - 256*c^4*d*x^2 - 704*b*c^3*e
*x^2 - 280*b^2*c^2*f*x^2 - 384*a*c^3*f*x^2 - 512*c^4*e*x^3 - 640*b*c^3*f*x^
3 - 384*c^4*f*x^4 + 128*b*c^(5/2)*d*Sqrt[a + x*(b + c*x)] + 32*b^2*c^(3/2)*
e*Sqrt[a + x*(b + c*x)] + 256*a*c^(5/2)*e*Sqrt[a + x*(b + c*x)] + 4*b^3*Sqr
t[c]*f*Sqrt[a + x*(b + c*x)] + 32*a*b*c^(3/2)*f*Sqrt[a + x*(b + c*x)] + 256
*c^(7/2)*d*x*Sqrt[a + x*(b + c*x)] + 448*b*c^(5/2)*e*x*Sqrt[a + x*(b + c*x)
] + 104*b^2*c^(3/2)*f*x*Sqrt[a + x*(b + c*x)] + 192*a*c^(5/2)*f*x*Sqrt[a +
x*(b + c*x)] + 512*c^(7/2)*e*x^2*Sqrt[a + x*(b + c*x)] + 448*b*c^(5/2)*f*x^
2*Sqrt[a + x*(b + c*x)] + 384*c^(7/2)*f*x^3*Sqrt[a + x*(b + c*x)] + 4*(-8*c
^2*d + b^2*f - 4*c*(b*e + a*f))*(b^2 + b*(8*c*x - 4*Sqrt[c]*Sqrt[a + x*(b +
 c*x)]) + 4*c*(a + 2*c*x^2 - 2*Sqrt[c]*x*Sqrt[a + x*(b + c*x)]))*Log[c*(b +
 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(32*c^(3/2)*h^2*Sqrt[-(c*g^2)
+ h*(b*g - a*h)]*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. $2(433) = 866$.

time = 0.17, size = 1097, normalized size = 2.39

method	result
default	$\frac{f\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{h^2}+\frac{(eh-2gf)\left(\sqrt{\left(x+\frac{g}{h}\right)^2 c+\frac{(bh-2cg)(x}{h}}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^2*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+1/h^3*(e*h-2*f*g)*(((x+1/h*g)^2*c+(b*h
-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/
2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)
+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*
g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)
+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(
a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/h^4*(d*h^2-e*g*h+f*g^2)*(-1/(a
*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h
^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(((x+1/h*g
)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g
)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h
*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/
((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h
*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(
x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2-b*g*h+c*g^2
)*h^2*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+
1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h
-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g
)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re deta
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**2,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
```

```
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

3.192 $\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^3}\,dx$

Optimal. Leaf size=448

$$\frac{\left(\frac{4cg^2(3fg-eh)}{h}+4ah(3fg-eh)-b(11fg^2-3egh-dh^2)-2h\left(ceg+2bfg-\frac{3cfg^2}{h}-cdh-2afh\right)x\right)\sqrt{a+b}}{4h^2\left(cg^2-bgh+ah^2\right)(g+hx)}$$

```
[Out] -1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)
^2+1/8*(8*c^2*g^3*(-e*h+3*f*g)-4*c*h*(b*g^2*(-3*e*h+10*f*g)-a*h*(d*h^2-3*e*
g*h+9*f*g^2))+h^2*(8*a^2*f*h^2-4*a*b*h*(-e*h+6*f*g)+b^2*(15*f*g^2-h*(d*h+3*
e*g))))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c
*x^2+b*x+a)^(1/2))/h^4/(a*h^2-b*g*h+c*g^2)^(3/2)-1/2*(-b*f*h-2*c*e*h+6*c*f*
g)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/h^4/c^(1/2)+1/4*(4*c*
g^2*(-e*h+3*f*g)/h+4*a*h*(-e*h+3*f*g)-b*(-d*h^2-3*e*g*h+11*f*g^2)-2*h*(c*e*
g+2*b*f*g-3*c*f*g^2/h-c*d*h-2*a*f*h)*x)*(c*x^2+b*x+a)^(1/2)/h^2/(a*h^2-b*g*
h+c*g^2)/(h*x+g)
```

Rubi [A]

time = 0.52, antiderivative size = 446, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1664, 826, 857, 635, 212, 738}

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h^2(8a^2fh^2-4abh(6fg-eh)+b^2(15fg^2-h(dh+3eg)))-4ch(bg^2(10fg-3eh)-ah(dh^2-3egh+9fg^2))+8c^2g^3(3fg-eh)\right)}{8h^4\left(ah^2-bgh+cg^2\right)^{3/2}}-\frac{(a+bx+cx^2)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}-\frac{\sqrt{a+bx+cx^2}\left(2hx\left(-2afh+2bfg-cdh+ceg-\frac{3cfg^2}{h}\right)-4ah(3fg-eh)-bh(dh+3eg)+11bfg^2-\frac{4cg^2(3fg-eh)}{h}\right)}{4h^2(g+hx)\left(ah^2-bgh+cg^2\right)}-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-bfh-2ceh+6cfg)}{2\sqrt{c}\,h^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] -1/4*((11*b*f*g^2 - b*h*(3*e*g + d*h) - (4*c*g^2*(3*f*g - e*h))/h - 4*a*h*(
3*f*g - e*h) + 2*h*(c*e*g + 2*b*f*g - (3*c*f*g^2)/h - c*d*h - 2*a*f*h)*x)*S
qrt[a + b*x + c*x^2])/(h^2*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) - ((f*g^2 - h
*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(2*h*(c*g^2 - b*g*h + a*h^2)*(g + h*
x)^2) - ((6*c*f*g - 2*c*e*h - b*f*h)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ b*x + c*x^2])])/(2*Sqrt[c]*h^4) + ((8*c^2*g^3*(3*f*g - e*h) - 4*c*h*(b*g^
2*(10*f*g - 3*e*h) - a*h*(9*f*g^2 - 3*e*g*h + d*h^2)) + h^2*(8*a^2*f*h^2 -
4*a*b*h*(6*f*g - e*h) + b^2*(15*f*g^2 - h*(3*e*g + d*h))))*ArcTanh[(b*g - 2
*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2
]])])/(8*h^4*(c*g^2 - b*g*h + a*h^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^3}\,dx = -\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{3/2}}{2h\,(cg^2-bgh+ah^2)\,(g+hx)^2} - \frac{\int \left(\frac{1}{2}\left(-4cdg+3beg+4afg-\frac{3bfg^2}{h} \ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(11bfg^2-bh(3eg+dh)-\frac{4cg^2(3fg-eh)}{h}-4ah(3fg-eh)+2h\left(ceg \ldots\right.\right.}{4h^2\,(cg^2-bgh+ah^2)\,(\ldots}$$

$$= -\frac{\left(11bfg^2-bh(3eg+dh)-\frac{4cg^2(3fg-eh)}{h}-4ah(3fg-eh)+2h\left(ceg \ldots\right.\right.}{4h^2\,(cg^2-bgh+ah^2)\,(\ldots}$$

$$= -\frac{\left(11bfg^2-bh(3eg+dh)-\frac{4cg^2(3fg-eh)}{h}-4ah(3fg-eh)+2h\left(ceg \ldots\right.\right.}{4h^2\,(cg^2-bgh+ah^2)\,(\ldots}$$

$$= -\frac{\left(11bfg^2-bh(3eg+dh)-\frac{4cg^2(3fg-eh)}{h}-4ah(3fg-eh)+2h\left(ceg \ldots\right.\right.}{4h^2\,(cg^2-bgh+ah^2)\,(\ldots}$$

Mathematica [**A**]
time = 11.10, size = 500, normalized size = 1.12

$$\frac{2h\sqrt{a+x(b+cx)}\left(4f-\frac{2(fg^2+h(-eg+dh))}{(g+hx)^2}+\frac{2c(5fg^3+gh(-3eg+dh))-h(9bfg^2+bh(-5eg+dh)+4ah(-2fg+eh))}{(cg^2+h(-bg+ah))(g+hx)}\right)+\frac{(8c^2g^3(3fg-eh)+4ch(bg^2(-10fg+3eh)+ah(9fg^2-3egh+dh^2))+h^2(8a^2fh^2+4abh(-6fg+eh)+b^2(15fg^2-h(3eg+dh))))\log(g+hx)}{(cg^2+h(-bg+ah))^{3/2}}-\frac{4(6cfg-2ceh-bfh)\log\left(b+2cx+2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{\sqrt{c}}-\frac{(8c^2g^3(3fg-eh)+4ch(bg^2(-10fg+3eh)+ah(9fg^2-3egh+dh^2))+h^2(8a^2fh^2+4abh(-6fg+eh)+b^2(15fg^2-h(3eg+dh))))\log\left(-bg+2ah-2cgx+bhx+2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b+cx)}\right)}{(cg^2+h(-bg+ah))^{3/2}}}{8h^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] (2*h*Sqrt[a + x*(b + c*x)]*(4*f - (2*(f*g^2 + h*(-(e*g) + d*h))))/(g + h*x)^
2 + (2*c*(5*f*g^3 + g*h*(-3*e*g + d*h)) - h*(9*b*f*g^2 + b*h*(-5*e*g + d*h)
 + 4*a*h*(-2*f*g + e*h)))/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x))) + ((8*c^2
*g^3*(3*f*g - e*h) + 4*c*h*(b*g^2*(-10*f*g + 3*e*h) + a*h*(9*f*g^2 - 3*e*g*
h + d*h^2)) + h^2*(8*a^2*f*h^2 + 4*a*b*h*(-6*f*g + e*h) + b^2*(15*f*g^2 - h
*(3*e*g + d*h))))*Log[g + h*x])/(c*g^2 + h*(-(b*g) + a*h))^(3/2) - (4*(6*c*
f*g - 2*c*e*h - b*f*h)*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/Sq
rt[c] - ((8*c^2*g^3*(3*f*g - e*h) + 4*c*h*(b*g^2*(-10*f*g + 3*e*h) + a*h*(9
*f*g^2 - 3*e*g*h + d*h^2)) + h^2*(8*a^2*f*h^2 + 4*a*b*h*(-6*f*g + e*h) + b^
2*(15*f*g^2 - h*(3*e*g + d*h))))*Log[-(b*g) + 2*a*h - 2*c*g*x + b*h*x + 2*S
qrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)]])/(c*g^2 + h*(-(b*g) +
a*h))^(3/2))/(8*h^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2165 vs. $2(420) = 840$.
time = 0.16, size = 2166, normalized size = 4.83

method	result	size
default	Expression too large to display	2166
risch	Expression too large to display	10239

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^3*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2
*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b
*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h
^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c
+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+(e*h-2
*f*g)/h^4*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/
h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c
*g^2)*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2
*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b
*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h
^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c
+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a
*h^2-b*g*h+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(
b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h
+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c
^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
))))+(d*h^2-e*g*h+f*g^2)/h^5*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+
1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)-1/4*(b*h-
2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*
g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*
g)*h/(a*h^2-b*g*h+c*g^2)*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c
^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a
*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(
1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))
/(x+1/h*g)))+2*c/(a*h^2-b*g*h+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)
/c*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/
8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*
g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2-b*g*h+c*g^2)*h^2*(((x+1/h*g)^2*c+(b*h-
2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2
*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g
```

```
*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+
2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a
*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}\,(d + ex + fx^2)}{(g + hx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**3,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[6,0,0,7,0,0,0,0]%%%}+%%%{%%{[-6,0]:[1,0,%%%{-1,[1]%%%}]%%}
,[5,0,
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}\,\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
```

```
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

3.193 $\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^4}\,dx$

Optimal. Leaf size=603

$$-\frac{(8c^2fg^5-2cgh(7bfg^3-6afg^2h+bdgh^2-2adh^3)+h^2(4a^2eh^3+b^2g(5fg^2+egh+dh^2)-2abh(3fg^2+2\cdots}{\cdots}$$

```
[Out] -1/3*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)
^3-1/16*(16*c^3*f*g^5-8*c^2*g*h*(a*d*h^3-5*a*f*g^2*h+5*b*f*g^3)-b*h^3*(8*a^
2*f*h^2-2*a*b*h*(e*h+6*f*g)+b^2*(d*h^2+e*g*h+5*f*g^2))+2*c*h^2*(4*a^2*h^2*(
-e*h+4*f*g)-2*a*b*h*(-d*h^2-e*g*h+15*f*g^2)+b^2*(d*g*h^2+15*f*g^3)))*arctan
h(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1
/2))/h^4/(a*h^2-b*g*h+c*g^2)^(5/2)+f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b
*x+a)^(1/2))*c^(1/2)/h^4-1/8*(8*c^2*f*g^5-2*c*g*h*(-2*a*d*h^3-6*a*f*g^2*h+b
*d*g*h^2+7*b*f*g^3)+h^2*(4*a^2*e*h^3+b^2*g*(d*h^2+e*g*h+5*f*g^2)-2*a*b*h*(d
*h^2+2*e*g*h+3*f*g^2))+h*(4*c^2*(-d*g^2*h^2+3*f*g^4)+h^2*(8*a^2*f*h^2-2*a*b
*h*(-e*h+10*f*g)+b^2*(11*f*g^2-h*(d*h+e*g)))+2*c*g*h*(2*a*h*(-e*h+6*f*g)-b*
(12*f*g^2-h*(2*d*h+e*g))))*x)*(c*x^2+b*x+a)^(1/2)/h^3/(a*h^2-b*g*h+c*g^2)^2
/(h*x+g)^2
```

Rubi [A]
time = 0.87, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1664, 824, 857, 635, 212, 738}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -1/8*(((8*c^2*f*g^5)/h + 4*a^2*e*h^4 + 4*a*c*g*h*(3*f*g^2 + d*h^2) + b^2*g*
h*(5*f*g^2 + h*(e*g + d*h)) - 2*b*(a*h^2*(3*f*g^2 + 2*e*g*h + d*h^2) + c*(7
*f*g^4 + d*g^2*h^2)) + h*(8*a^2*f*h^3 + 4*a*c*g*h*(6*f*g - e*h) + c^2*((12*
f*g^4)/h - 4*d*g^2*h) + b^2*h*(11*f*g^2 - h*(e*g + d*h)) - 2*b*(12*c*f*g^3
 - c*g*h*(e*g + 2*d*h) + a*h^2*(10*f*g - e*h)))*x)*Sqrt[a + b*x + c*x^2])/(h
^2*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b
*x + c*x^2)^(3/2))/(3*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) + (Sqrt[c]*f*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h^4 - ((16*c^3*f*g^5
 - 8*c^2*g*h*(5*b*f*g^3 - 5*a*f*g^2*h + a*d*h^3) - b*h^3*(8*a^2*f*h^2 - 2*a
*b*h*(6*f*g + e*h) + b^2*(5*f*g^2 + e*g*h + d*h^2)) + 2*c*h^2*(4*a^2*h^2*(4
*f*g - e*h) - 2*a*b*h*(15*f*g^2 - e*g*h - d*h^2) + b^2*(15*f*g^3 + d*g*h^2)
))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*S
qrt[a + b*x + c*x^2])])/(16*h^4*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
```

```
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^4}\,dx = -\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{3/2}}{3h\,(cg^2-bgh+ah^2)\,(g+hx)^3} - \frac{\int \frac{\left(-\frac{3}{2}\left(2cdg-beg-2afg+\frac{bfg^2}{h}-b\right.\right.}{\cdots}}{3\cdots}$$

$$= -\frac{\left(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh(3fg^2+dh^2)+b^2gh(5fg^2+h(eg+dh))\cdots\right.}{\cdots}$$

$$= -\frac{\left(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh(3fg^2+dh^2)+b^2gh(5fg^2+h(eg+dh))\cdots\right.}{\cdots}$$

$$= -\frac{\left(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh(3fg^2+dh^2)+b^2gh(5fg^2+h(eg+dh))\cdots\right.}{\cdots}$$

$$= -\frac{\left(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh(3fg^2+dh^2)+b^2gh(5fg^2+h(eg+dh))\cdots\right.}{\cdots}$$

Mathematica [A]
time = 12.31, size = 732, normalized size = 1.21

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] ((-2*h*Sqrt[a + x*(b + c*x)]*(8*(c*g^2 + h*(-(b*g) + a*h))^2*(f*g^2 + h*(-(
e*g) + d*h)) - 2*(c*g^2 + h*(-(b*g) + a*h))*(2*c*(7*f*g^3 + g*h*(-4*e*g + d
*h)) - h*(13*b*f*g^2 + b*h*(-7*e*g + d*h) + 6*a*h*(-2*f*g + e*h)))*(g + h*x
) + (c^2*(44*f*g^4 - 4*g^2*h*(2*e*g + d*h)) - 3*h^2*(-8*a^2*f*h^2 - 2*a*b*h
*(-10*f*g + e*h) + b^2*(-11*f*g^2 + e*g*h + d*h^2)) + 2*c*h*(2*a*h*(20*f*g^
2 - 5*e*g*h + 2*d*h^2) + b*g*(-40*f*g^2 + 7*e*g*h + 2*d*h^2)))*(g + h*x)^2)
)/((c*g^2 + h*(-(b*g) + a*h))^2*(g + h*x)^3) - (3*(16*c^3*f*g^5 - 8*c^2*g*h
```

```
*(5*b*f*g^3 - 5*a*f*g^2*h + a*d*h^3) - b*h^3*(8*a^2*f*h^2 - 2*a*b*h*(6*f*g
+ e*h) + b^2*(5*f*g^2 + e*g*h + d*h^2)) + 2*c*h^2*(-4*a^2*h^2*(-4*f*g + e*h
) + 2*a*b*h*(-15*f*g^2 + e*g*h + d*h^2) + b^2*(15*f*g^3 + d*g*h^2)))*Log[g
+ h*x])/(c*g^2 + h*(-(b*g) + a*h))^(5/2) + 48*Sqrt[c]*f*Log[b + 2*c*x + 2*S
qrt[c]*Sqrt[a + x*(b + c*x)]] + (3*(16*c^3*f*g^5 - 8*c^2*g*h*(5*b*f*g^3 - 5
*a*f*g^2*h + a*d*h^3) - b*h^3*(8*a^2*f*h^2 - 2*a*b*h*(6*f*g + e*h) + b^2*(5
*f*g^2 + e*g*h + d*h^2)) + 2*c*h^2*(-4*a^2*h^2*(-4*f*g + e*h) + 2*a*b*h*(-1
5*f*g^2 + e*g*h + d*h^2) + b^2*(15*f*g^3 + d*g*h^2)))*Log[-(b*g) + 2*a*h -
2*c*g*x + b*h*x + 2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)]])/
(c*g^2 + h*(-(b*g) + a*h))^(5/2))/(48*h^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3089 vs. $2(577) = 1154$.
time = 0.13, size = 3090, normalized size = 5.12

method	result	size
default	Expression too large to display	3090

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^4*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x
+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2
)*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/
2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(
b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h
+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(
b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*
h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2
-b*g*h+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-
2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g
^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/
2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))))
+(e*h-2*f*g)/h^5*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(
b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)-1/4*(b*h-2*c*g)*h/(a*
h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-
2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)*h/(a*h^2-
b*g*h+c*g^2)*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^
2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1
/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(
a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c
*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h
*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
+2*c/(a*h^2-b*g*h+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g
)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^
```

```
2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/
h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^
2)^(1/2))))+1/2*c/(a*h^2-b*g*h+c*g^2)*h^2*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+
1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)
/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h
+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^
2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*
g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c
*g^2)/h^2)^(1/2))/(x+1/h*g))))+(d*h^2-e*g*h+f*g^2)/h^6*(-1/3/(a*h^2-b*g*h+c
*g^2)*h^2/(x+1/h*g)^3*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c
*g^2)/h^2)^(3/2)-1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/2/(a*h^2-b*g*h+c
*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c
*g^2)/h^2)^(3/2)-1/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g
^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2
)/h^2)^(3/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(((x+1/h*g)^2*c+(b*h-2*c
*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b
*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*
h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+
c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*(
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^
2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2-b*g*h+c*g^2)*h^2*(1/4*(2*
c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-
b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)
/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c
*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2-b*g*h+c*g^2)
*h^2*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*
c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*
g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^
2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+
(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**4,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 15.38Unable to divid
e, perhaps due to rounding error%%%{%%{[-1,0]:[1,0,%%%{-1,[1]%%%}]%%},[8,0,
0,0,0,
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
```

```
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

3.194 $\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^5}\,dx$

Optimal. Leaf size=497

$$\frac{(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)+b^2(5fg^2+3egh+5dh^2)-4c(2bg(eg+2dh)+a(fg^2-5egh+dh^2}{64\,(cg^2-bgh+ah^2)^3\,(g+hx)^2}$$

```
[Out] -1/4*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)
^4+1/24*(2*c*g*(3*f*g^2+h*(-5*d*h+e*g))+h*(8*a*h*(-e*h+2*f*g)-b*(-5*d*h^2-3
*e*g*h+11*f*g^2)))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^3-1/
128*(-4*a*c+b^2)*(16*c^2*d*g^2+16*a^2*f*h^2-8*a*b*h*(e*h+2*f*g)+b^2*(5*d*h^
2+3*e*g*h+5*f*g^2)-4*c*(2*b*g*(2*d*h+e*g)+a*(d*h^2-5*e*g*h+f*g^2)))*arctanh
(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/
2))/(a*h^2-b*g*h+c*g^2)^(7/2)+1/64*(16*c^2*d*g^2+16*a^2*f*h^2-8*a*b*h*(e*h+
2*f*g)+b^2*(5*d*h^2+3*e*g*h+5*f*g^2)-4*c*(2*b*g*(2*d*h+e*g)+a*(d*h^2-5*e*g*
h+f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^
2)^3/(h*x+g)^2
```

Rubi [A]
time = 0.50, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$,
Rules used = {1664, 820, 734, 738, 212}

$$\frac{\sqrt{a+bx+cx^2}\,(-2ah+x(2cg-bh)+bg)\,(16a^2fh^2-4c(-ah(5eg-dh)+afg^2+2bg(2dh+eg))-8abh(eh+2fg)+b^2(h(5dh+3eg)+5fg^2)+16c^2dg^2)}{64(g+hx)^2\,(ah^2-bgh+cg^2)^3}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\,\sqrt{ah^2-bgh+cg^2}}\right)(16a^2fh^2-4c(-ah(5eg-dh)+afg^2+2bg(2dh+eg))-8abh(eh+2fg)+b^2(h(5dh+3eg)+5fg^2)+16c^2dg^2)}{128\,(ah^2-bgh+cg^2)^{7/2}}-\frac{(a+bx+cx^2)^{3/2}\,(fg^2-h(eg-dh))}{4h(g+hx)^4\,(ah^2-bgh+cg^2)}+\frac{(a+bx+cx^2)^{3/2}\,(8ah^2(2fg-eh)-bh(11fg^2-h(5dh+3eg))+2cgh(eg-5dh)+6cfg^3)}{24h(g+hx)^3\,(ah^2-bgh+cg^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] ((16*c^2*d*g^2 + 16*a^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*
(5*e*g - d*h) + 2*b*g*(e*g + 2*d*h)) + b^2*(5*f*g^2 + h*(3*e*g + 5*d*h)))*(
b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(64*(c*g^2 - b*g*h +
a*h^2)^3*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(
4*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) + ((6*c*f*g^3 + 2*c*g*h*(e*g - 5*d
*h) + 8*a*h^2*(2*f*g - e*h) - b*h*(11*f*g^2 - h*(3*e*g + 5*d*h)))*(a + b*x
+ c*x^2)^(3/2))/(24*h*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^3) - ((b^2 - 4*a*
c)*(16*c^2*d*g^2 + 16*a^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*
h*(5*e*g - d*h) + 2*b*g*(e*g + 2*d*h)) + b^2*(5*f*g^2 + h*(3*e*g + 5*d*h)))
*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqr
t[a + b*x + c*x^2])])/(128*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^5}\,dx = -\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{3/2}}{4h\,(cg^2-bgh+ah^2)\,(g+hx)^4} - \int \frac{\left(\frac{1}{2}\left(-8cdg+3beg+8afg-\frac{3bfg^2}{h}\cdots\right.\right.}{\cdots}$$

$$= -\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{3/2}}{4h\,(cg^2-bgh+ah^2)\,(g+hx)^4} + \frac{(6cfg^3+2cgh(eg-5dh)\cdots}{\cdots}$$

$$= \frac{(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c(afg^2-ah(5eg-dh)+\cdots}{64\,(cg^2\cdots}$$

$$= \frac{(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c(afg^2-ah(5eg-dh)+\cdots}{64\,(cg^2\cdots}$$

$$= \frac{(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c(afg^2-ah(5eg-dh)+\cdots}{64\,(cg^2\cdots}$$

Mathematica [A]
time = 13.87, size = 813, normalized size = 1.64

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] ((-2*Sqrt[a + x*(b + c*x)]*(48*(c*g^2 + h*(-(b*g) + a*h))^3*(f*g^2 + h*(-(e
*g) + d*h)) - 8*(c*g^2 + h*(-(b*g) + a*h))^2*(2*c*(9*f*g^3 + g*h*(-5*e*g +
d*h)) - h*(17*b*f*g^2 + b*h*(-9*e*g + d*h) + 8*a*h*(-2*f*g + e*h)))*(g + h*
x) + 2*(c*g^2 + h*(-(b*g) + a*h))*(8*c^2*(9*f*g^4 - g^2*h*(e*g + d*h)) + h^
2*(48*a^2*f*h^2 + 8*a*b*h*(-14*f*g + e*h) + b^2*(59*f*g^2 - 3*e*g*h - 5*d*h
^2)) + 4*c*h*(2*b*g*(-17*f*g^2 + 2*e*g*h + d*h^2) + a*h*(35*f*g^2 - 7*e*g*h
 + 3*d*h^2)))*(g + h*x)^2 - (16*c^3*(3*f*g^5 + g^3*h*(e*g + d*h)) - 3*b*h^3
*(16*a^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) + b^2*(5*f*g^2 + 3*e*g*h + 5*d*h^2))
 - 8*c^2*g*h*(b*g*(17*f*g^2 + 5*e*g*h + 3*d*h^2) + a*h*(-19*f*g^2 - 9*e*g*h
 + 13*d*h^2)) + 2*c*h^2*(16*a^2*h^2*(7*f*g - 2*e*h) + 2*a*b*h*(-75*f*g^2 -
5*e*g*h + 13*d*h^2) + b^2*g*(59*f*g^2 + 9*e*g*h + 19*d*h^2)))*(g + h*x)^3))
/(3*h^3*(c*g^2 + h*(-(b*g) + a*h))^3*(g + h*x)^4) - ((b^2 - 4*a*c)*(16*c^2*
d*g^2 + 16*a^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 + a*h*(-5*e*g +
 d*h) + 2*b*g*(e*g + 2*d*h)) + b^2*(5*f*g^2 + h*(3*e*g + 5*d*h)))*Log[g + h
*x])/(c*g^2 + h*(-(b*g) + a*h))^(7/2) + ((b^2 - 4*a*c)*(16*c^2*d*g^2 + 16*a
^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 + a*h*(-5*e*g + d*h) + 2*b*
g*(e*g + 2*d*h)) + b^2*(5*f*g^2 + h*(3*e*g + 5*d*h)))*Log[-(b*g) + 2*a*h -
```

```
2*c*g*x + b*h*x + 2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)]])/
(c*g^2 + h*(-(b*g) + a*h))^(7/2))/128
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4939 vs. $2(475) = 950$.
time = 0.12, size = 4940, normalized size = 9.94

method	result	size
default	Expression too large to display	4940

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^5*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(b*h-2*c*g)/
h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)-1/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c
*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x
+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2
)*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/
2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(
b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h
+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(
b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*
h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2
-b*g*h+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-
2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g
^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/
2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))))
+1/2*c/(a*h^2-b*g*h+c*g^2)*h^2*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h
^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h
*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2
)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln
((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^
(1/2))/(x+1/h*g))))+(d*h^2-e*g*h+f*g^2)/h^7*(-1/4/(a*h^2-b*g*h+c*g^2)*h^2/(
x+1/h*g)^4*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^
(3/2)-5/8*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/3/(a*h^2-b*g*h+c*g^2)*h^2/(
x+1/h*g)^3*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^
(3/2)-1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(
x+1/h*g)^2*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^
(3/2)-1/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+
1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2
)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/
h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h
+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c
*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)
```

```
^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*
h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g
^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2-b*g*h+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)
+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2
)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln
((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/
h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2-b*g*h+c*g^2)*h^2*(((x+1
/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2
*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*
g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/
h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*
g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)
/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))))-1/4*c/(a*h^2-b*g
*h+c*g^2)*h^2*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(b*h
-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)-1/4*(b*h-2*c*g)*h/(a*h^2
-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c
*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g
*h+c*g^2)*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^
(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*
g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h
^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^
2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)
^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*
c/(a*h^2-b*g*h+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2
*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b
*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g
))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^
(1/2))))+1/2*c/(a*h^2-b*g*h+c*g^2)*h^2*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h
*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+
c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*
g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^
(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^
2)/h^2)^(1/2))/(x+1/h*g)))))+(e*h-2*f*g)/h^6*(-1/3/(a*h^2-b*g*h+c*g^2)*h^2/
(x+1/h*g)^3*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h...
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}\,(d + ex + fx^2)}{(g + hx)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**5,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2 + b\,x + a}\,\left(f\,x^2 + e\,x + d\right)}{(g + h\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
```

```
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

3.195 $\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^6}\,dx$

Optimal. Leaf size=824

$$\frac{(32c^3dg^3-8c^2g(2bg(eg+3dh)+a(fg^2-6egh+3dh^2))-bh(16a^2fh^2-2abh(6fg+5eh)+b^2(3fg^2+3eg}{128}$$

```
[Out] -1/5*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)
^5+1/40*(2*c*g*(3*f*g^2+h*(-7*d*h+2*e*g))+h*(10*a*h*(-e*h+2*f*g)-b*(-7*d*h^
2-3*e*g*h+13*f*g^2)))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^4
+1/240*(4*c^2*g^2*(3*f*g^2+h*(-27*d*h+2*e*g))-5*h^2*(16*a^2*f*h^2-2*a*b*h*(
5*e*h+6*f*g)+b^2*(7*d*h^2+3*e*g*h+3*f*g^2))-2*c*h*(b*g*(-54*d*h^2-21*e*g*h+
16*f*g^2)-2*a*h*(8*d*h^2-33*e*g*h+18*f*g^2)))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-
b*g*h+c*g^2)^3/(h*x+g)^3-1/256*(-4*a*c+b^2)*(32*c^3*d*g^3-8*c^2*g*(2*b*g*(3
*d*h+e*g)+a*(3*d*h^2-6*e*g*h+f*g^2))-b*h*(16*a^2*f*h^2-2*a*b*h*(5*e*h+6*f*g
)+b^2*(7*d*h^2+3*e*g*h+3*f*g^2))+2*c*(4*a^2*h^2*(-e*h+6*f*g)-6*a*b*h*(-d*h^
2+3*e*g*h+3*f*g^2)+b^2*g*(15*d*h^2+6*e*g*h+5*f*g^2)))*arctanh(1/2*(b*g-2*a*
h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g
*h+c*g^2)^(9/2)+1/128*(32*c^3*d*g^3-8*c^2*g*(2*b*g*(3*d*h+e*g)+a*(3*d*h^2-6
*e*g*h+f*g^2))-b*h*(16*a^2*f*h^2-2*a*b*h*(5*e*h+6*f*g)+b^2*(7*d*h^2+3*e*g*h
+3*f*g^2))+2*c*(4*a^2*h^2*(-e*h+6*f*g)-6*a*b*h*(-d*h^2+3*e*g*h+3*f*g^2)+b^2
*g*(15*d*h^2+6*e*g*h+5*f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1
/2)/(a*h^2-b*g*h+c*g^2)^4/(h*x+g)^2
```

Rubi [A]
time = 1.34, antiderivative size = 826, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1664, 848, 820, 734, 738, 212}

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] ((32*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - 3*a*h*(2*e*g - d*h) + 2*b*g*(e*g + 3*d*
h)) + 2*c*(4*a^2*h^2*(6*f*g - e*h) - 6*a*b*h*(3*f*g^2 + h*(3*e*g - d*h)) +
b^2*(5*f*g^3 + 3*g*h*(2*e*g + 5*d*h))) - b*h*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g
 + 5*e*h) + b^2*(3*f*g^2 + h*(3*e*g + 7*d*h))))*(b*g - 2*a*h + (2*c*g - b*h
)*x)*Sqrt[a + b*x + c*x^2])/(128*(c*g^2 - b*g*h + a*h^2)^4*(g + h*x)^2) - (
(f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(5*h*(c*g^2 - b*g*h + a*h^
2)*(g + h*x)^5) + ((6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(2*f*g -
 e*h) - b*h*(13*f*g^2 - h*(3*e*g + 7*d*h)))*(a + b*x + c*x^2)^(3/2))/(40*h*
(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^4) + ((4*c^2*(3*f*g^4 + g^2*h*(2*e*g -
```

```
27*d*h)) - 5*h^2*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g + 5*e*h) + b^2*(3*f*g^2 + 3
*e*g*h + 7*d*h^2)) - 2*c*h*(b*g*(16*f*g^2 - 21*e*g*h - 54*d*h^2) - 2*a*h*(1
8*f*g^2 - 33*e*g*h + 8*d*h^2)))*(a + b*x + c*x^2)^(3/2))/(240*h*(c*g^2 - b*
g*h + a*h^2)^3*(g + h*x)^3) - ((b^2 - 4*a*c)*(32*c^3*d*g^3 - 8*c^2*g*(a*f*g
^2 - 3*a*h*(2*e*g - d*h) + 2*b*g*(e*g + 3*d*h)) + 2*c*(4*a^2*h^2*(6*f*g - e
*h) - 6*a*b*h*(3*f*g^2 + h*(3*e*g - d*h)) + b^2*(5*f*g^3 + 3*g*h*(2*e*g + 5
*d*h))) - b*h*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g + 5*e*h) + b^2*(3*f*g^2 + h*(3
*e*g + 7*d*h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*
g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(256*(c*g^2 - b*g*h + a*h^2)^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
```

```
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^6}\,dx = -\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{3/2}}{5h\,(cg^2-bgh+ah^2)\,(g+hx)^5} - \frac{\int \frac{\left(\frac{1}{2}\left(-10cdg+3beg+10afg-\frac{3bfg}{h}\cdots\right.\right.}{\cdots}}{\cdots}$$

$$= -\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{3/2}}{5h\,(cg^2-bgh+ah^2)\,(g+hx)^5} + \frac{(6cfg^3+2cgh(2eg-7dh)\cdots}{\cdots}$$

$$= -\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{3/2}}{5h\,(cg^2-bgh+ah^2)\,(g+hx)^5} + \frac{(6cfg^3+2cgh(2eg-7dh)\cdots}{\cdots}$$

$$= \frac{(32c^3dg^3-8c^2g(afg^2-3ah(2eg-dh)+2bg(eg+3dh))+2c(4a^2h^2\cdots}{\cdots}$$

$$= \frac{(32c^3dg^3-8c^2g(afg^2-3ah(2eg-dh)+2bg(eg+3dh))+2c(4a^2h^2\cdots}{\cdots}$$

$$= \frac{(32c^3dg^3-8c^2g(afg^2-3ah(2eg-dh)+2bg(eg+3dh))+2c(4a^2h^2\cdots}{\cdots}$$

Mathematica [A]

time = 16.50, size = 1633, normalized size = 1.98

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] Sqrt[a + x*(b + c*x)]*((-(f*g^2) + e*g*h - d*h^2)/(5*h^3*(g + h*x)^5) + (22
*c*f*g^3 - 12*c*e*g^2*h - 21*b*f*g^2*h + 2*c*d*g*h^2 + 11*b*e*g*h^2 + 20*a*
f*g*h^2 - b*d*h^3 - 10*a*e*h^3)/(40*h^3*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4
) + (-108*c^2*f*g^4 + 8*c^2*e*g^3*h + 208*b*c*f*g^3*h + 12*c^2*d*g^2*h^2 -
18*b*c*e*g^2*h^2 - 93*b^2*f*g^2*h^2 - 216*a*c*f*g^2*h^2 - 12*b*c*d*g*h^3 +
3*b^2*e*g*h^3 + 36*a*c*e*g*h^3 + 180*a*b*f*g*h^3 + 7*b^2*d*h^4 - 16*a*c*d*h
^4 - 10*a*b*e*h^4 - 80*a^2*f*h^4)/(240*h^3*(c*g^2 - b*g*h + a*h^2)^2*(g + h
*x)^3) + (48*c^3*f*g^5 + 32*c^3*e*g^4*h - 152*b*c^2*f*g^4*h + 48*c^3*d*g^3*
h^2 - 88*b*c^2*e*g^3*h^2 + 154*b^2*c*f*g^3*h^2 + 168*a*c^2*f*g^3*h^2 - 72*b
*c^2*d*g^2*h^3 + 36*b^2*c*e*g^2*h^3 + 192*a*c^2*e*g^2*h^3 - 15*b^3*f*g^2*h^
3 - 444*a*b*c*f*g^2*h^3 + 94*b^2*c*d*g*h^4 - 232*a*c^2*d*g*h^4 - 15*b^3*e*g
*h^4 - 76*a*b*c*e*g*h^4 + 60*a*b^2*f*g*h^4 + 400*a^2*c*f*g*h^4 - 35*b^3*d*h
^5 + 116*a*b*c*d*h^5 + 50*a*b^2*e*h^5 - 120*a^2*c*e*h^5 - 80*a^2*b*f*h^5)/(
960*h^3*(c*g^2 - b*g*h + a*h^2)^3*(g + h*x)^2) + (96*c^4*f*g^6 + 64*c^4*e*g
^5*h - 352*b*c^3*f*g^5*h + 96*c^4*d*g^4*h^2 - 208*b*c^3*e*g^4*h^2 + 436*b^2
*c^2*f*g^4*h^2 + 432*a*c^3*f*g^4*h^2 - 192*b*c^3*d*g^3*h^3 + 144*b^2*c^2*e*
g^3*h^3 + 448*a*c^3*e*g^3*h^3 - 120*b^3*c*f*g^3*h^3 - 1312*a*b*c^2*f*g^3*h^
3 + 476*b^2*c^2*d*g^2*h^4 - 1328*a*c^3*d*g^2*h^4 - 150*b^3*c*e*g^2*h^4 - 8*
a*b*c^2*e*g^2*h^4 + 45*b^4*f*g^2*h^4 + 300*a*b^2*c*f*g^2*h^4 + 1376*a^2*c^2
*f*g^2*h^4 - 380*b^3*c*d*g*h^5 + 1328*a*b*c^2*d*g*h^5 + 45*b^4*e*g*h^5 + 32
0*a*b^2*c*e*g*h^5 - 1296*a^2*c^2*e*g*h^5 - 180*a*b^3*f*g*h^5 - 80*a^2*b*c*f
*g*h^5 + 105*b^4*d*h^6 - 460*a*b^2*c*d*h^6 + 256*a^2*c^2*d*h^6 - 150*a*b^3*
e*h^6 + 520*a^2*b*c*e*h^6 + 240*a^2*b^2*f*h^6 - 640*a^3*c*f*h^6)/(1920*h^3*
(c*g^2 - b*g*h + a*h^2)^4*(g + h*x))) + ((b^2 - 4*a*c)*(-32*c^3*d*g^3 + 16*
b*c^2*e*g^3 - 10*b^2*c*f*g^3 + 8*a*c^2*f*g^3 + 48*b*c^2*d*g^2*h - 12*b^2*c*
e*g^2*h - 48*a*c^2*e*g^2*h + 3*b^3*f*g^2*h + 36*a*b*c*f*g^2*h - 30*b^2*c*d*
g*h^2 + 24*a*c^2*d*g*h^2 + 3*b^3*e*g*h^2 + 36*a*b*c*e*g*h^2 - 12*a*b^2*f*g*
h^2 - 48*a^2*c*f*g*h^2 + 7*b^3*d*h^3 - 12*a*b*c*d*h^3 - 10*a*b^2*e*h^3 + 8*
a^2*c*e*h^3 + 16*a^2*b*f*h^3)*Sqrt[a + x*(b + c*x)]*Log[g + h*x])/(256*(c*g
^2 - b*g*h + a*h^2)^(9/2)*Sqrt[a + b*x + c*x^2]) - ((b^2 - 4*a*c)*(-32*c^3*
d*g^3 + 16*b*c^2*e*g^3 - 10*b^2*c*f*g^3 + 8*a*c^2*f*g^3 + 48*b*c^2*d*g^2*h
 - 12*b^2*c*e*g^2*h - 48*a*c^2*e*g^2*h + 3*b^3*f*g^2*h + 36*a*b*c*f*g^2*h -
30*b^2*c*d*g*h^2 + 24*a*c^2*d*g*h^2 + 3*b^3*e*g*h^2 + 36*a*b*c*e*g*h^2 - 12
*a*b^2*f*g*h^2 - 48*a^2*c*f*g*h^2 + 7*b^3*d*h^3 - 12*a*b*c*d*h^3 - 10*a*b^2
*e*h^3 + 8*a^2*c*e*h^3 + 16*a^2*b*f*h^3)*Sqrt[a + x*(b + c*x)]*Log[-(b*g) +
 2*a*h - 2*c*g*x + b*h*x + 2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x
^2]])/(256*(c*g^2 - b*g*h + a*h^2)^(9/2)*Sqrt[a + b*x + c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7713 vs. $2(798) = 1596$.
time = 0.16, size = 7714, normalized size = 9.36

method	result	size
default	Expression too large to display	7714

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2}\,(d + ex + fx^2)}{(g + hx)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**6,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**6, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 28857 vs. $2(822) = 1644$.
time = 6.87, size = 28857, normalized size = 35.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] -1/128*(32*b^2*c^3*d*g^3 - 128*a*c^4*d*g^3 + 10*b^4*c*f*g^3 - 48*a*b^2*c^2*
f*g^3 + 32*a^2*c^3*f*g^3 - 48*b^3*c^2*d*g^2*h + 192*a*b*c^3*d*g^2*h - 3*b^5
*f*g^2*h - 24*a*b^3*c*f*g^2*h + 144*a^2*b*c^2*f*g^2*h + 30*b^4*c*d*g*h^2 -
144*a*b^2*c^2*d*g*h^2 + 96*a^2*c^3*d*g*h^2 + 12*a*b^4*f*g*h^2 - 192*a^3*c^2
*f*g*h^2 - 7*b^5*d*h^3 + 40*a*b^3*c*d*h^3 - 48*a^2*b*c^2*d*h^3 - 16*a^2*b^3
*f*h^3 + 64*a^3*b*c*f*h^3 - 16*b^3*c^2*g^3*e + 64*a*b*c^3*g^3*e + 12*b^4*c*
g^2*h*e - 192*a^2*c^3*g^2*h*e - 3*b^5*g*h^2*e - 24*a*b^3*c*g*h^2*e + 144*a^
2*b*c^2*g*h^2*e + 10*a*b^4*h^3*e - 48*a^2*b^2*c*h^3*e + 32*a^3*c^2*h^3*e)*a
rctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*
g*h - a*h^2))/((c^4*g^8 - 4*b*c^3*g^7*h + 6*b^2*c^2*g^6*h^2 + 4*a*c^3*g^6*h
^2 - 4*b^3*c*g^5*h^3 - 12*a*b*c^2*g^5*h^3 + b^4*g^4*h^4 + 12*a*b^2*c*g^4*h^
4 + 6*a^2*c^2*g^4*h^4 - 4*a*b^3*g^3*h^5 - 12*a^2*b*c*g^3*h^5 + 6*a^2*b^2*g^
2*h^6 + 4*a^3*c*g^2*h^6 - 4*a^3*b*g*h^7 + a^4*h^8)*sqrt(-c*g^2 + b*g*h - a*
h^2)) + 1/1920*(480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^2*c^3*d*g^3*h^8
 - 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*c^4*d*g^3*h^8 + 150*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^9*b^4*c*f*g^3*h^8 - 720*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^9*a*b^2*c^2*f*g^3*h^8 + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^9*a^2*c^3*f*g^3*h^8 - 720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^3*c^2*
d*g^2*h^9 + 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b*c^3*d*g^2*h^9 -
45*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*f*g^2*h^9 - 360*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^9*a*b^3*c*f*g^2*h^9 + 2160*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^9*a^2*b*c^2*f*g^2*h^9 + 450*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9
*b^4*c*d*g*h^10 - 2160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^2*c^2*d*g*
h^10 + 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*c^3*d*g*h^10 + 180*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^4*f*g*h^10 - 2880*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^9*a^3*c^2*f*g*h^10 - 105*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^9*b^5*d*h^11 + 600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c*d*h^1
1 - 720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b*c^2*d*h^11 - 240*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^3*f*h^11 + 960*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^9*a^3*b*c*f*h^11 - 240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*
b^3*c^2*g^3*h^8*e + 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b*c^3*g^3*h
^8*e + 180*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^4*c*g^2*h^9*e - 2880*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*c^3*g^2*h^9*e - 45*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^9*b^5*g*h^10*e - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
```

```
^9*a*b^3*c*g*h^10*e + 2160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b*c^2*
g*h^10*e + 150*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^4*h^11*e - 720*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^2*c*h^11*e + 480*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^9*a^3*c^2*h^11*e + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^8*c^(11/2)*f*g^8*h^3 - 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b*c^
(9/2)*f*g^7*h^4 + 23040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^2*c^(7/2)*f
*g^6*h^5 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*c^(9/2)*f*g^6*h^5
- 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^3*c^(5/2)*f*g^5*h^6 - 46080
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b*c^(7/2)*f*g^5*h^6 + 4320*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^8*b^2*c^(7/2)*d*g^4*h^7 - 17280*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^8*a*c^(9/2)*d*g^4*h^7 + 5190*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^8*b^4*c^(3/2)*f*g^4*h^7 + 39600*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^8*a*b^2*c^(5/2)*f*g^4*h^7 + 27360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
8*a^2*c^(7/2)*f*g^4*h^7 - 6480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^3*c^
(5/2)*d*g^3*h^8 + 25920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b*c^(7/2)*d
*g^3*h^8 - 405*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^5*sqrt(c)*f*g^3*h^8
- 18600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^3*c^(3/2)*f*g^3*h^8 - 266
40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b*c^(5/2)*f*g^3*h^8 + 4050*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^4*c^(3/2)*d*g^2*h^9 - 19440*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^8*a*b^2*c^(5/2)*d*g^2*h^9 + 12960*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^8*a^2*c^(7/2)*d*g^2*h^9 + 1620*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^8*a*b^4*sqrt(c)*f*g^2*h^9 + 23040*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^8*a^2*b^2*c^(3/2)*f*g^2*h^9 - 10560*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^8*a^3*c^(5/2)*f*g^2*h^9 - 945*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*b^
5*sqrt(c)*d*g*h^10 + 5400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a*b^3*c^(3/
2)*d*g*h^10 - 6480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b*c^(5/2)*d*g*
h^10 - 2160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^2*b^3*sqrt(c)*f*g*h^10
- 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^3*b*c^(3/2)*f*g*h^10 + 3840*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^8*a^4*c^(3/2)*f*h^11 - 2160*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^8*b^3*c^(5/2)*g^4*h^7*e + 8640*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^8*a*b*c^(7/2)*g^4*h^7*e + 1620*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^8*b^4*c^(3/2)*g^3*h^8*e - 25920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^8*a^2*c^(7/2)*g^3*h^8*e - 405*(sqrt(c)*x - sq...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

3.196 $\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=1169

$$-\frac{(b^2-4ac)\left(1536c^5dg^3-143b^5fh^3+22b^3ch^2(20afh+9b(3fg+eh))-48bc^2h(5a^2fh^2+9abh(3fg+eh)\right.}{}$$

```
[Out] 1/12288*(1536*c^5*d*g^3-143*b^5*f*h^3+22*b^3*c*h^2*(20*a*f*h+9*b*(e*h+3*f*g
))-48*b*c^2*h*(5*a^2*f*h^2+9*a*b*h*(e*h+3*f*g)+6*b^2*(d*h^2+3*e*g*h+3*f*g^2
))-256*c^4*g*(3*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+32*c^3*(3*a^2*h^2*
(e*h+3*f*g)+14*b^2*g*(f*g^2+3*h*(d*h+e*g))+12*a*b*h*(3*f*g^2+h*(d*h+3*e*g))
))*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^6+1/2016*(143*b^2*f*h^2-2*c*h*(64*a*f*h+
99*b*e*h+24*b*f*g)-12*c^2*(5*f*g^2-3*h*(8*d*h+3*e*g)))*(h*x+g)^2*(c*x^2+b*x
+a)^(5/2)/c^3/h-1/144*(13*b*f*h-18*c*e*h+10*c*f*g)*(h*x+g)^3*(c*x^2+b*x+a)^
(5/2)/c^2/h+1/9*f*(h*x+g)^4*(c*x^2+b*x+a)^(5/2)/c/h+1/80640*(3003*b^4*f*h^4
-192*c^4*g^2*(5*f*g^2-3*h*(64*d*h+3*e*g))-198*b^2*c*h^3*(38*a*f*h+21*b*(e*h
+3*f*g))+8*c^2*h^2*(256*a^2*f*h^2+837*a*b*h*(e*h+3*f*g)+b^2*(1553*f*g^2+756
*h*(d*h+3*e*g)))-16*c^3*h*(32*a*h*(17*f*g^2+9*h*(d*h+3*e*g))+b*g*(13*f*g^2+
9*h*(196*d*h+141*e*g)))-10*c*h*(429*b^3*f*h^3-22*b*c*h^2*(34*a*f*h+27*b*e*h
+29*b*f*g)+16*c^3*g*(5*f*g^2-9*h*(12*d*h+e*g))+8*c^2*h*(a*h*(63*e*h+61*f*g)
+3*b*(f*g^2+6*h*(6*d*h+7*e*g))))*x)*(c*x^2+b*x+a)^(5/2)/c^5/h+1/65536*(-4*a
*c+b^2)^2*(1536*c^5*d*g^3-143*b^5*f*h^3+22*b^3*c*h^2*(20*a*f*h+9*b*(e*h+3*f
*g))-48*b*c^2*h*(5*a^2*f*h^2+9*a*b*h*(e*h+3*f*g)+6*b^2*(d*h^2+3*e*g*h+3*f*g
^2))-256*c^4*g*(3*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+32*c^3*(3*a^2*h^
2*(e*h+3*f*g)+14*b^2*g*(f*g^2+3*h*(d*h+e*g))+12*a*b*h*(3*f*g^2+h*(d*h+3*e*g
))))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(15/2)-1/32768*(-
4*a*c+b^2)*(1536*c^5*d*g^3-143*b^5*f*h^3+22*b^3*c*h^2*(20*a*f*h+9*b*(e*h+3*
f*g))-48*b*c^2*h*(5*a^2*f*h^2+9*a*b*h*(e*h+3*f*g)+6*b^2*(d*h^2+3*e*g*h+3*f*
g^2))-256*c^4*g*(3*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+32*c^3*(3*a^2*h
^2*(e*h+3*f*g)+14*b^2*g*(f*g^2+3*h*(d*h+e*g))+12*a*b*h*(3*f*g^2+h*(d*h+3*e*
g))))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^7
```

Rubi [A]
time = 2.09, antiderivative size = 1166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 846, 793, 626, 635, 212}

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -1/32768*((b^2 - 4*a*c)*(1536*c^5*d*g^3 - 143*b^5*f*h^3 - 256*c^4*g*(a*f*g^
2 + 3*a*h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h)) + 22*b^3*c*h^2*(20*a*f*h + 9*b
```

```
*(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) + 6*b^2*(
3*f*g^2 + 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*b^2*g*(f
*g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b + 2*c*x
)*Sqrt[a + b*x + c*x^2])/c^7 + ((1536*c^5*d*g^3 - 143*b^5*f*h^3 - 256*c^4*g
*(a*f*g^2 + 3*a*h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h)) + 22*b^3*c*h^2*(20*a*f
*h + 9*b*(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) +
 6*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*
b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b
 + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(12288*c^6) + ((143*b^2*f*h^2 - 2*c*h*(2
4*b*f*g + 99*b*e*h + 64*a*f*h) - 12*c^2*(5*f*g^2 - 3*h*(3*e*g + 8*d*h)))*(g
 + h*x)^2*(a + b*x + c*x^2)^(5/2))/(2016*c^3*h) - ((10*c*f*g - 18*c*e*h + 1
3*b*f*h)*(g + h*x)^3*(a + b*x + c*x^2)^(5/2))/(144*c^2*h) + (f*(g + h*x)^4*
(a + b*x + c*x^2)^(5/2))/(9*c*h) + ((3003*b^4*f*h^4 - 192*c^4*(5*f*g^4 - 3*
g^2*h*(3*e*g + 64*d*h)) - 198*b^2*c*h^3*(38*a*f*h + 21*b*(3*f*g + e*h)) + 8
*c^2*h^2*(256*a^2*f*h^2 + 837*a*b*h*(3*f*g + e*h) + b^2*(1553*f*g^2 + 756*h
*(3*e*g + d*h))) - 16*c^3*h*(32*a*h*(17*f*g^2 + 9*h*(3*e*g + d*h)) + b*g*(1
3*f*g^2 + 9*h*(141*e*g + 196*d*h))) - 10*c*h*(429*b^3*f*h^3 - 22*b*c*h^2*(2
9*b*f*g + 27*b*e*h + 34*a*f*h) + 16*c^3*(5*f*g^3 - 9*g*h*(e*g + 12*d*h)) +
8*c^2*h*(a*h*(61*f*g + 63*e*h) + 3*b*(f*g^2 + 6*h*(7*e*g + 6*d*h))))*x)*(a
+ b*x + c*x^2)^(5/2))/(80640*c^5*h) + ((b^2 - 4*a*c)^2*(1536*c^5*d*g^3 - 14
3*b^5*f*h^3 - 256*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h))
 + 22*b^3*c*h^2*(20*a*f*h + 9*b*(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 +
9*a*b*h*(3*f*g + e*h) + 6*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*
h^2*(3*f*g + e*h) + 14*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2
+ h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(65536*c^(15/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \frac{f(g+hx)^4 \left(a+bx+cx^2\right)^{5/2}}{9ch} + \frac{\int (g+hx)^3 \left(-\frac{1}{2}h(5bfg\right.}{}$$

$$= -\frac{(10cfg-18ceh+13bfh)(g+hx)^3 \left(a+bx+cx^2\right)^{5/2}}{144c^2h}$$

$$= \frac{(143b^2fh^2-2ch(24bfg+99beh+64afh)-12c^2(5fg^2}{2016c^3h}$$

$$= \frac{(143b^2fh^2-2ch(24bfg+99beh+64afh)-12c^2(5fg^2}{2016c^3h}$$

$$= \frac{(1536c^5dg^3-143b^5fh^3-256c^4g(afg^2+3ah(eg+dh)}{}$$

$$= -\frac{(b^2-4ac)\,(1536c^5dg^3-143b^5fh^3-256c^4g(afg^2+3a}{}$$

$$= -\frac{(b^2-4ac)\,(1536c^5dg^3-143b^5fh^3-256c^4g(afg^2+3a}{}$$

$$= -\frac{(b^2-4ac)\,(1536c^5dg^3-143b^5fh^3-256c^4g(afg^2+3a}{}$$

Mathematica [A]
time = 10.75, size = 1799, normalized size = 1.54

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(45045*b^8*f*h^3 - 2310*b^7*c*h^2*(81*f*g
+ 27*e*h + 13*f*h*x) + 84*b^6*c*h*(-5225*a*f*h^2 + 45*c*h*(72*e*g + 24*d*h
+ 11*e*h*x) + c*f*(3240*g^2 + 1485*g*h*x + 286*h^2*x^2)) - 72*b^5*c^2*(-7*a
*h^2*(3285*f*g + 1095*e*h + 517*f*h*x) + 2*c*f*(980*g^3 + 1260*g^2*h*x + 69
3*g*h^2*x^2 + 143*h^3*x^3) + 42*c*h*(20*d*h*(7*g + h*x) + e*(140*g^2 + 60*g
*h*x + 11*h^2*x^2))) + 16*b^4*c^2*(86499*a^2*f*h^3 - 9*a*c*h*(21*h*(720*e*g
 + 240*d*h + 107*e*h*x) + f*(15120*g^2 + 6741*g*h*x + 1276*h^2*x^2)) + 2*c^
2*(252*d*h*(90*g^2 + 35*g*h*x + 6*h^2*x^2) + 9*e*(840*g^3 + 980*g^2*h*x + 5
04*g*h^2*x^2 + 99*h^3*x^3) + f*x*(2940*g^3 + 4536*g^2*h*x + 2673*g*h^2*x^2
+ 572*h^3*x^3))) + 192*b^2*c^3*(-7641*a^3*f*h^3 + a^2*c*h*(3*h*(8232*e*g +
2744*d*h + 1181*e*h*x) + f*(24696*g^2 + 10629*g*h*x + 1970*h^2*x^2)) - 4*a*
c^2*(12*d*h*(525*g^2 + 189*g*h*x + 31*h^2*x^2) + 3*e*(700*g^3 + 756*g^2*h*x
```

```
 + 372*g*h^2*x^2 + 71*h^3*x^3) + f*x*(756*g^3 + 1116*g^2*h*x + 639*g*h^2*x^
2 + 134*h^3*x^3)) + 8*c^3*x*(6*d*(35*g^3 + 42*g^2*h*x + 21*g*h^2*x^2 + 4*h^
3*x^3) + x*(3*e*(28*g^3 + 42*g^2*h*x + 24*g*h^2*x^2 + 5*h^3*x^3) + f*x*(42*
g^3 + 72*g^2*h*x + 45*g*h^2*x^2 + 10*h^3*x^3)))) - 32*b^3*c^3*(9*a^2*h^2*(1
5309*f*g + 5103*e*h + 2353*f*h*x) - 4*a*c*(f*(7980*g^3 + 9828*g^2*h*x + 526
5*g*h^2*x^2 + 1067*h^3*x^3) + 9*h*(28*d*h*(95*g + 13*h*x) + e*(2660*g^2 + 1
092*g*h*x + 195*h^2*x^2))) + 8*c^2*(18*d*(105*g^3 + 105*g^2*h*x + 49*g*h^2*
x^2 + 9*h^3*x^3) + x*(9*e*(70*g^3 + 98*g^2*h*x + 54*g*h^2*x^2 + 11*h^3*x^3)
 + f*x*(294*g^3 + 486*g^2*h*x + 297*g*h^2*x^2 + 65*h^3*x^3)))) + 128*b*c^4*
(a^3*h^2*(24813*f*g + 8271*e*h + 3701*f*h*x) - 6*a^2*c*(f*(2268*g^3 + 2628*
g^2*h*x + 1359*g*h^2*x^2 + 269*h^3*x^3) + 3*h*(4*d*h*(567*g + 73*h*x) + e*(
2268*g^2 + 876*g*h*x + 151*h^2*x^2))) + 24*a*c^2*(6*d*(175*g^3 + 147*g^2*h*
x + 63*g*h^2*x^2 + 11*h^3*x^3) + x*(3*e*(98*g^3 + 126*g^2*h*x + 66*g*h^2*x^
2 + 13*h^3*x^3) + f*x*(126*g^3 + 198*g^2*h*x + 117*g*h^2*x^2 + 25*h^3*x^3))
) + 16*c^3*x^2*(18*d*(105*g^3 + 231*g^2*h*x + 182*g*h^2*x^2 + 50*h^3*x^3) +
 x*(9*e*(154*g^3 + 364*g^2*h*x + 300*g*h^2*x^2 + 85*h^3*x^3) + f*x*(1092*g^
3 + 2700*g^2*h*x + 2295*g*h^2*x^2 + 665*h^3*x^3)))) + 256*c^4*(1024*a^4*f*h
^3 - a^3*c*h*(9*h*(768*e*g + 256*d*h + 105*e*h*x) + f*(6912*g^2 + 2835*g*h*
x + 512*h^2*x^2)) + 6*a^2*c^2*(12*d*h*(336*g^2 + 105*g*h*x + 16*h^2*x^2) +
3*e*(448*g^3 + 420*g^2*h*x + 192*g*h^2*x^2 + 35*h^3*x^3) + f*x*(420*g^3 + 5
76*g^2*h*x + 315*g*h^2*x^2 + 64*h^3*x^3)) + 16*c^4*x^3*(18*d*(35*g^3 + 84*g
^2*h*x + 70*g*h^2*x^2 + 20*h^3*x^3) + x*(9*e*(56*g^3 + 140*g^2*h*x + 120*g*
h^2*x^2 + 35*h^3*x^3) + 5*f*x*(84*g^3 + 216*g^2*h*x + 189*g*h^2*x^2 + 56*h^
3*x^3))) + 8*a*c^3*x*(18*d*(175*g^3 + 336*g^2*h*x + 245*g*h^2*x^2 + 64*h^3*
x^3) + x*(9*e*(224*g^3 + 490*g^2*h*x + 384*g*h^2*x^2 + 105*h^3*x^3) + f*x*(
1470*g^3 + 3456*g^2*h*x + 2835*g*h^2*x^2 + 800*h^3*x^3))))) + 315*(b^2 - 4*
a*c)^2*(-1536*c^5*d*g^3 + 143*b^5*f*h^3 + 256*c^4*g*(a*f*g^2 + 3*a*h*(e*g +
 d*h) + 3*b*g*(e*g + 3*d*h)) - 22*b^3*c*h^2*(20*a*f*h + 9*b*(3*f*g + e*h))
+ 48*b*c^2*h*(5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) + 6*b^2*(3*f*g^2 + 3*e*g*
h + d*h^2)) - 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*b^2*g*(f*g^2 + 3*h*(e*g
+ d*h)) + 12*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*Log[b + 2*c*x - 2*Sqrt[c]*
Sqrt[a + x*(b + c*x)]])/(20643840*c^(15/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2877 vs. $2(1135) = 2270$.
time = 0.14, size = 2878, normalized size = 2.46

method	result	size
default	Expression too large to display	2878
risch	Expression too large to display	4324

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] f*h^3*(1/9*x^4*(c*x^2+b*x+a)^(5/2)/c-13/18*b/c*(1/8*x^3*(c*x^2+b*x+a)^(5/2)
```

/c-11/16*b/c*(1/7*x^2*(c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/6*x*(c*x^2+b*x+a)^(
5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+
b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4
*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1
/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x
^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2)))))-2/7*a/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*
x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/
8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))-3/8*a
/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c
(1/8(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(
c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2
)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))-4/9*a/c*(1/7*x^2*(c*x^2+b*x+a)^(5/2)/
c-9/14*b/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c
-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*
x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4
*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-2/7*a/c*(1/5*(c*x^2+b*x+a)^(5/
2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(
2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2)))))))+(e*h^3+3*f*g*h^2)*(1/8*x^3*(c*x^2+b*x+a)^(5/2)
/c-11/16*b/c*(1/7*x^2*(c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/6*x*(c*x^2+b*x+a)^(
5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+
b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4
*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1
/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x
^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2)))))-2/7*a/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*
x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/
8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))-3/8*a
/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c
(1/8(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(
c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2
)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))+(d*h^3+3*e*g*h^2+3*f*g^2*h)*(1/7*x^2*
(c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(
c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a
c-b^2)/c(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+
b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4
*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-2/7*a/c*(1

```
/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*
(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))+(3*d*g*h^2+3*e*g^2*h+f*g^3)
*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(
1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*
x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/
c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*
x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+(3*d*g^2*h+e*g^3)*(1/5*(c*x^2+b*x+a)^(5/
2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(
2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2)))))+d*g^3*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*
(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2478 vs. $2(1172) = 2344$.
time = 4.73, size = 4959, normalized size = 4.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas"
)
```

```
[Out] [1/41287680*(315*(64*(24*(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d + (7*b^6*c^
3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*f)*g^3 - 288*(8*(b^5*c^4 -
 8*a*b^3*c^5 + 16*a^2*b*c^6)*d + (3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4
 - 64*a^3*b*c^5)*f)*g^2*h + 6*(32*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c
^5 - 64*a^3*c^6)*d + 3*(33*b^8*c - 336*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*
a^3*b^2*c^4 + 256*a^4*c^5)*f)*g*h^2 - (96*(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^
2*b^3*c^4 - 64*a^3*b*c^5)*d + (143*b^9 - 1584*a*b^7*c + 6048*a^2*b^5*c^2 -
```

8960*a^3*b^3*c^3 + 3840*a^4*b*c^4)*f)*h^3 - 6*(128*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*g^3 - 32*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*g^2*h + 48*(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*g*h^2 - (33*b^8*c - 336*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*h^3)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(1146880*c^9*f*h^3*x^8 + 71680*(54*c^9*f*g*h^2 + 19*b*c^8*f*h^3)*x^7 + 5120*(864*c^9*f*g^2*h + 918*b*c^8*f*g*h^2 + (288*c^9*d + (3*b^2*c^7 + 320*a*c^8)*f)*h^3)*x^6 + 1280*(1344*c^9*f*g^3 + 4320*b*c^8*f*g^2*h + 18*(224*c^9*d + 3*(b^2*c^7 + 84*a*c^8)*f)*g*h^2 + (1440*b*c^8*d - (13*b^3*c^6 - 60*a*b*c^7)*f)*h^3)*x^5 + 128*(17472*b*c^8*f*g^3 + 864*(56*c^9*d + (b^2*c^7 + 64*a*c^8)*f)*g^2*h + 18*(2912*b*c^8*d - 3*(11*b^3*c^6 - 52*a*b*c^7)*f)*g*h^2 + (288*(b^2*c^7 + 64*a*c^8)*d + (143*b^4*c^5 - 804*a*b^2*c^6 + 768*a^2*c^7)*f)*h^3)*x^4 - 1344*(120*(3*b^3*c^6 - 20*a*b*c^7)*d + (105*b^5*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*f)*g^3 + 864*(56*(15*b^4*c^5 - 100*a*b^2*c^6 + 128*a^2*c^7)*d + (315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2*b^2*c^5 - 2048*a^3*c^6)*f)*g^2*h - 18*(224*(105*b^5*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*d + 3*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 81648*a^2*b^3*c^4 - 58816*a^3*b*c^5)*f)*g*h^2 + (288*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2*b^2*c^5 - 2048*a^3*c^6)*d + (45045*b^8*c - 438900*a*b^6*c^2 + 1383984*a^2*b^4*c^3 - 1467072*a^3*b^2*c^4 + 262144*a^4*c^5)*f)*h^3 + 16*(1344*(120*c^9*d + (3*b^2*c^7 + 140*a*c^8)*f)*g^3 + 864*(616*b*c^8*d - (9*b^3*c^6 - 44*a*b*c^7)*f)*g^2*h + 18*(224*(3*b^2*c^7 + 140*a*c^8)*d + 3*(99*b^4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*f)*g*h^2 - (288*(9*b^3*c^6 - 44*a*b*c^7)*d + (1287*b^5*c^4 - 8536*a*b^3*c^5 + 12912*a^2*b*c^6)*f)*h^3)*x^3 + 8*(1344*(360*b*c^8*d - (7*b^3*c^6 - 36*a*b*c^7)*f)*g^3 + 864*(56*(b^2*c^7 + 32*a*c^8)*d + (21*b^4*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*f)*g^2*h - 18*(224*(7*b^3*c^6 - 36*a*b*c^7)*d + 3*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2416*a^2*b*c^6)*f)*g*h^2 + (288*(21*b^4*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*d + (3003*b^6*c^3 - 22968*a*b^4*c^4 + 47280*a^2*b^2*c^5 - 16384*a^3*c^6)*f)*h^3)*x^2 + 2*(1344*(120*(b^2*c^7 + 20*a*c^8)*d + (35*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2*c^7)*f)*g^3 - 864*(56*(5*b^3*c^6 - 28*a*b*c^7)*d + (105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*f)*g^2*h + 18*(224*(35*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2*c^7)*d + 3*(1155*b^6*c^3 - 8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*f)*g*h^2 - (288*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*d + (15015*b^7*c^2 - 130284*a*b^5*c^3 + 338832*a^2*b^3*c^4 - 236864*a^3*b*c^5)*f)*h^3)*x + 18*(71680*c^9*h^3*x^7 + 5120*(48*c^9*g*h^2 + 17*b*c^8*h^3)*x^6 + 1280*(224*c^9*g^2*h + 240*b*c^8*g*h^2 + (b^2*c^7 + 84*a*c^8)*h^3)*x^5 + 128*(896*c^9*g^3 + 2912*b*c^8*g^2*h + 48*(b^2*c^7 + 64*a*c^8)*g*h^2 - (11*b^3*c^6 - 52*a*b*c^7)*h^3)*x^4 + 896*(15*b^4*c^5 - 100*a*b^2*c^6 + 128*a^2*c^7)*g^3 - 224*(105*b^5*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*g^2*h + 48*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2*b^2*c^5 - 2048*a^3*c^6)*g*h^2 - (3465*b^7*c^2 - 30660*a*b^5*c^3 + 81648*a^2*b^3*c^4 - 58816*a^3*b*c^5)*h^3 + 16*(9856*b*c^8*g^3 + 224*(3*b^2*c^7 + 140*a*c^8)*g^2*h - 48*(9*b^3*c^6 - 44*a*b*c^7)*g*h^2 + (99*b^4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*h^3)*x^3 + 8*(896*(b^2*c^7 + 32*a*c^8)*g^3 - 224*(7*b^3*c^6 - 36*a*b*c^7)*g^2*h + 48*(21*

```
b^4*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*g*h^2 - (231*b^5*c^4 - 1560*a*b^3*c^
5 + 2416*a^2*b*c^6)*h^3)*x^2 - 2*(896*(5*b^3*c^6 - 28*a*b*c^7)*g^3 - 224*(3
5*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2*c^7)*g^2*h + 48*(105*b^5*c^4 - 728*a*b^
3*c^5 + 1168*a^2*b*c^6)*g*h^2 - (1155*b^6*c^3 - 8988*a*b^4*c^4 + 18896*a^2*
b^2*c^5 - 6720*a^3*c^6)*h^3)*x)*e)*sqrt(c*x^2 + b*x + a))/c^8, -1/20643840*
(315*(64*(24*(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d + (7*b^6*c^3 - 60*a*b^4
*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*f)*g^3 - 288*(8*(b^5*c^4 - 8*a*b^3*c^5
 + 16*a^2*b*c^6)*d + (3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*
c^5)*f)*g^2*h + 6*(32*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*
c^6)*d + 3*(33*b^8*c - 336*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4
+ 256*a^4*c^5)*f)*g*h^2 - (96*(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 -
64*a^3*b*c^5)*d + (143*b^9 - 1584*a*b^7*c + 6048*a^2*b^5*c^2 - 8960*a^3*b^3
*c^3 + 3840*a^4*b*c^4)*f)*h^3 - 6*(128*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^
6)*g^3 - 32*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*g^2*h
 + 48*(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{\frac{3}{2}} \left(d+ex+fx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((g + h*x)**3*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2977 vs. 2(1172) = 2344.
time = 5.94, size = 2977, normalized size = 2.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/10321920*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*(14*(16*c*f*h^3*x + (54
*c^9*f*g*h^2 + 19*b*c^8*f*h^3 + 18*c^9*h^3*e)/c^8)*x + (864*c^9*f*g^2*h + 9
18*b*c^8*f*g*h^2 + 288*c^9*d*h^3 + 3*b^2*c^7*f*h^3 + 320*a*c^8*f*h^3 + 864*
c^9*g*h^2*e + 306*b*c^8*h^3*e)/c^8)*x + (1344*c^9*f*g^3 + 4320*b*c^8*f*g^2*
h + 4032*c^9*d*g*h^2 + 54*b^2*c^7*f*g*h^2 + 4536*a*c^8*f*g*h^2 + 1440*b*c^8
*d*h^3 - 13*b^3*c^6*f*h^3 + 60*a*b*c^7*f*h^3 + 4032*c^9*g^2*h*e + 4320*b*c^
8*g*h^2*e + 18*b^2*c^7*h^3*e + 1512*a*c^8*h^3*e)/c^8)*x + (17472*b*c^8*f*g^
3 + 48384*c^9*d*g^2*h + 864*b^2*c^7*f*g^2*h + 55296*a*c^8*f*g^2*h + 52416*b
*c^8*d*g*h^2 - 594*b^3*c^6*f*g*h^2 + 2808*a*b*c^7*f*g*h^2 + 288*b^2*c^7*d*h
^3 + 18432*a*c^8*d*h^3 + 143*b^4*c^5*f*h^3 - 804*a*b^2*c^6*f*h^3 + 768*a^2*
```

c^7*f*h^3 + 16128*c^9*g^3*e + 52416*b*c^8*g^2*h*e + 864*b^2*c^7*g*h^2*e + 5
5296*a*c^8*g*h^2*e - 198*b^3*c^6*h^3*e + 936*a*b*c^7*h^3*e)/c^8)*x + (16128
0*c^9*d*g^3 + 4032*b^2*c^7*f*g^3 + 188160*a*c^8*f*g^3 + 532224*b*c^8*d*g^2*
h - 7776*b^3*c^6*f*g^2*h + 38016*a*b*c^7*f*g^2*h + 12096*b^2*c^7*d*g*h^2 +
564480*a*c^8*d*g*h^2 + 5346*b^4*c^5*f*g*h^2 - 30672*a*b^2*c^6*f*g*h^2 + 302
40*a^2*c^7*f*g*h^2 - 2592*b^3*c^6*d*h^3 + 12672*a*b*c^7*d*h^3 - 1287*b^5*c^
4*f*h^3 + 8536*a*b^3*c^5*f*h^3 - 12912*a^2*b*c^6*f*h^3 + 177408*b*c^8*g^3*e
 + 12096*b^2*c^7*g^2*h*e + 564480*a*c^8*g^2*h*e - 7776*b^3*c^6*g*h^2*e + 38
016*a*b*c^7*g*h^2*e + 1782*b^4*c^5*h^3*e - 10224*a*b^2*c^6*h^3*e + 10080*a^
2*c^7*h^3*e)/c^8)*x + (483840*b*c^8*d*g^3 - 9408*b^3*c^6*f*g^3 + 48384*a*b*
c^7*f*g^3 + 48384*b^2*c^7*d*g^2*h + 1548288*a*c^8*d*g^2*h + 18144*b^4*c^5*f
*g^2*h - 107136*a*b^2*c^6*f*g^2*h + 110592*a^2*c^7*f*g^2*h - 28224*b^3*c^6*
d*g*h^2 + 145152*a*b*c^7*d*g*h^2 - 12474*b^5*c^4*f*g*h^2 + 84240*a*b^3*c^5*
f*g*h^2 - 130464*a^2*b*c^6*f*g*h^2 + 6048*b^4*c^5*d*h^3 - 35712*a*b^2*c^6*d
*h^3 + 36864*a^2*c^7*d*h^3 + 3003*b^6*c^3*f*h^3 - 22968*a*b^4*c^4*f*h^3 + 4
7280*a^2*b^2*c^5*f*h^3 - 16384*a^3*c^6*f*h^3 + 16128*b^2*c^7*g^3*e + 516096
*a*c^8*g^3*e - 28224*b^3*c^6*g^2*h*e + 145152*a*b*c^7*g^2*h*e + 18144*b^4*c
^5*g*h^2*e - 107136*a*b^2*c^6*g*h^2*e + 110592*a^2*c^7*g*h^2*e - 4158*b^5*c
^4*h^3*e + 28080*a*b^3*c^5*h^3*e - 43488*a^2*b*c^6*h^3*e)/c^8)*x + (161280*
b^2*c^7*d*g^3 + 3225600*a*c^8*d*g^3 + 47040*b^4*c^5*f*g^3 - 290304*a*b^2*c^
6*f*g^3 + 322560*a^2*c^7*f*g^3 - 241920*b^3*c^6*d*g^2*h + 1354752*a*b*c^7*d
*g^2*h - 90720*b^5*c^4*f*g^2*h + 628992*a*b^3*c^5*f*g^2*h - 1009152*a^2*b*c
^6*f*g^2*h + 141120*b^4*c^5*d*g*h^2 - 870912*a*b^2*c^6*d*g*h^2 + 967680*a^2
*c^7*d*g*h^2 + 62370*b^6*c^3*f*g*h^2 - 485352*a*b^4*c^4*f*g*h^2 + 1020384*a
^2*b^2*c^5*f*g*h^2 - 362880*a^3*c^6*f*g*h^2 - 30240*b^5*c^4*d*h^3 + 209664*
a*b^3*c^5*d*h^3 - 336384*a^2*b*c^6*d*h^3 - 15015*b^7*c^2*f*h^3 + 130284*a*b
^5*c^3*f*h^3 - 338832*a^2*b^3*c^4*f*h^3 + 236864*a^3*b*c^5*f*h^3 - 80640*b^
3*c^6*g^3*e + 451584*a*b*c^7*g^3*e + 141120*b^4*c^5*g^2*h*e - 870912*a*b^2*
c^6*g^2*h*e + 967680*a^2*c^7*g^2*h*e - 90720*b^5*c^4*g*h^2*e + 628992*a*b^3
*c^5*g*h^2*e - 1009152*a^2*b*c^6*g*h^2*e + 20790*b^6*c^3*h^3*e - 161784*a*b
^4*c^4*h^3*e + 340128*a^2*b^2*c^5*h^3*e - 120960*a^3*c^6*h^3*e)/c^8)*x - (4
83840*b^3*c^6*d*g^3 - 3225600*a*b*c^7*d*g^3 + 141120*b^5*c^4*f*g^3 - 102144
0*a*b^3*c^5*f*g^3 + 1741824*a^2*b*c^6*f*g^3 - 725760*b^4*c^5*d*g^2*h + 4838
400*a*b^2*c^6*d*g^2*h - 6193152*a^2*c^7*d*g^2*h - 272160*b^6*c^3*f*g^2*h +
2177280*a*b^4*c^4*f*g^2*h - 4741632*a^2*b^2*c^5*f*g^2*h + 1769472*a^3*c^6*f
*g^2*h + 423360*b^5*c^4*d*g*h^2 - 3064320*a*b^3*c^5*d*g*h^2 + 5225472*a^2*b
*c^6*d*g*h^2 + 187110*b^7*c^2*f*g*h^2 - 1655640*a*b^5*c^3*f*g*h^2 + 4408992
*a^2*b^3*c^4*f*g*h^2 - 3176064*a^3*b*c^5*f*g*h^2 - 90720*b^6*c^3*d*h^3 + 72
5760*a*b^4*c^4*d*h^3 - 1580544*a^2*b^2*c^5*d*h^3 + 589824*a^3*c^6*d*h^3 - 4
5045*b^8*c*f*h^3 + 438900*a*b^6*c^2*f*h^3 - 1383984*a^2*b^4*c^3*f*h^3 + 146
7072*a^3*b^2*c^4*f*h^3 - 262144*a^4*c^5*f*h^3 - 241920*b^4*c^5*g^3*e + 1612
800*a*b^2*c^6*g^3*e - 2064384*a^2*c^7*g^3*e + 423360*b^5*c^4*g^2*h*e - 3064
320*a*b^3*c^5*g^2*h*e + 5225472*a^2*b*c^6*g^2*h*e - 272160*b^6*c^3*g*h^2*e
+ 2177280*a*b^4*c^4*g*h^2*e - 4741632*a^2*b^2*c^5*g*h^2*e + 1769472*a^3*c^6
*g*h^2*e + 62370*b^7*c^2*h^3*e - 551880*a*b^5*c^3*h^3*e + 1469664*a^2*b^3*c

```
^4*h^3*e - 1058688*a^3*b*c^5*h^3*e)/c^8) - 1/65536*(1536*b^4*c^5*d*g^3 - 12
288*a*b^2*c^6*d*g^3 + 24576*a^2*c^7*d*g^3 + 448*b^6*c^3*f*g^3 - 3840*a*b^4*
c^4*f*g^3 + 9216*a^2*b^2*c^5*f*g^3 - 4096*a^3*c^6*f*g^3 - 2304*b^5*c^4*d*g^
2*h + 18432*a*b^3*c^5*d*g^2*h - 36864*a^2*b*c^6*d*g^2*h - 864*b^7*c^2*f*g^2
*h + 8064*a*b^5*c^3*f*g^2*h - 23040*a^2*b^3*c^4*f*g^2*h + 18432*a^3*b*c^5*f
*g^2*h + 1344*b^6*c^3*d*g*h^2 - 11520*a*b^4*c^4*d*g*h^2 + 27648*a^2*b^2*c^5
*d*g*h^2 - 12288*a^3*c^6*d*g*h^2 + 594*b^8*c*f*g*h^2 - 6048*a*b^6*c^2*f*g*h
^2 + 20160*a^2*b^4*c^3*f*g*h^2 - 23040*a^3*b^2*c^4*f*g*h^2 + 4608*a^4*c^5*f
*g*h^2 - 288*b^7*c^2*d*h^3 + 2688*a*b^5*c^3*d*h^3 - 7680*a^2*b^3*c^4*d*h^3
+ 6144*a^3*b*c^5*d*h^3 - 143*b^9*f*h^3 + 1584*a*b^7*c*f*h^3 - 6048*a^2*b^5*
c^2*f*h^3 + 8960*a^3*b^3*c^3*f*h^3 - 3840*a^4*b*c^4*f*h^3 - 768*b^5*c^4*g^3
*e + 6144*a*b^3*c^5*g^3*e - 12288*a^2*b*c^6*g^3...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (g+h\,x)^3\,\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.197 $\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=753

$$-\frac{(b^2-4ac)\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(3bg(eg+2dh)+a(fg^2+2egh+\ldots\right.}{16384c^6}$$

```
[Out] 1/6144*(768*c^4*d*g^2+99*b^4*f*h^2-72*b^2*c*h*(3*a*f*h+2*b*e*h+4*b*f*g)-128
*c^3*(3*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+16*c^2*(3*a^2*f*h^2+12*a*b
*h*(e*h+2*f*g)+14*b^2*(d*h^2+2*e*g*h+f*g^2)))*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)
/c^5-1/112*(11*b*f*h-16*c*e*h+10*c*f*g)*(h*x+g)^2*(c*x^2+b*x+a)^(5/2)/c^2/h
+1/8*f*(h*x+g)^3*(c*x^2+b*x+a)^(5/2)/c/h-1/13440*(693*b^3*f*h^3+96*c^3*g*(5
*f*g^2-8*h*(7*d*h+e*g))-36*b*c*h^2*(31*a*f*h+28*b*(e*h+2*f*g))+8*c^2*h*(96*
a*h*(e*h+2*f*g)+b*(31*f*g^2+196*h*(d*h+2*e*g)))-10*c*h*(99*b^2*f*h^2-8*c^2*
(5*f*g^2-4*h*(7*d*h+2*e*g))-12*c*h*(7*a*f*h+2*b*(6*e*h+f*g)))*x)*(c*x^2+b*x
+a)^(5/2)/c^4/h+1/32768*(-4*a*c+b^2)^2*(768*c^4*d*g^2+99*b^4*f*h^2-72*b^2*c
*h*(3*a*f*h+2*b*e*h+4*b*f*g)-128*c^3*(3*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*
g^2))+16*c^2*(3*a^2*f*h^2+12*a*b*h*(e*h+2*f*g)+14*b^2*(d*h^2+2*e*g*h+f*g^2)
))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(13/2)-1/16384*(-4*
a*c+b^2)*(768*c^4*d*g^2+99*b^4*f*h^2-72*b^2*c*h*(3*a*f*h+2*b*e*h+4*b*f*g)-1
28*c^3*(3*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+16*c^2*(3*a^2*f*h^2+12*a
*b*h*(e*h+2*f*g)+14*b^2*(d*h^2+2*e*g*h+f*g^2)))*(2*c*x+b)*(c*x^2+b*x+a)^(1/
2)/c^6
```

Rubi [A]
time = 1.16, antiderivative size = 749, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 846, 793, 626, 635, 212}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -1/16384*((b^2 - 4*a*c)*(768*c^4*d*g^2 + 99*b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g
 + 2*b*e*h + 3*a*f*h) - 128*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 3*b*g*(e*g +
 2*d*h)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b*h*(2*f*g + e*h) + 14*b^2*(f*g^2 + h
*(2*e*g + d*h))))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/c^6 + ((768*c^4*d*g^2
+ 99*b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a*f*h) - 128*c^3*(a*f*g^
2 + a*h*(2*e*g + d*h) + 3*b*g*(e*g + 2*d*h)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b
*h*(2*f*g + e*h) + 14*b^2*(f*g^2 + h*(2*e*g + d*h))))*(b + 2*c*x)*(a + b*x
+ c*x^2)^(3/2))/(6144*c^5) - ((10*c*f*g - 16*c*e*h + 11*b*f*h)*(g + h*x)^2*
(a + b*x + c*x^2)^(5/2))/(112*c^2*h) + (f*(g + h*x)^3*(a + b*x + c*x^2)^(5/
2))/(8*c*h) - ((693*b^3*f*h^3 + 96*c^3*(5*f*g^3 - 8*g*h*(e*g + 7*d*h)) - 36
```

```
*b*c*h^2*(31*a*f*h + 28*b*(2*f*g + e*h)) + 8*c^2*h*(31*b*f*g^2 + 196*b*h*(2
*e*g + d*h) + 96*a*h*(2*f*g + e*h)) - 10*c*h*(99*b^2*f*h^2 - 8*c^2*(5*f*g^2
 - 4*h*(2*e*g + 7*d*h)) - 12*c*h*(7*a*f*h + 2*b*(f*g + 6*e*h)))*x)*(a + b*x
 + c*x^2)^(5/2))/(13440*c^4*h) + ((b^2 - 4*a*c)^2*(768*c^4*d*g^2 + 99*b^4*f
*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a*f*h) - 128*c^3*(a*f*g^2 + a*h*(2
*e*g + d*h) + 3*b*g*(e*g + 2*d*h)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b*h*(2*f*g
+ e*h) + 14*b^2*(f*g^2 + h*(2*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(32768*c^(13/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx = \frac{f(g+hx)^3 \left(a+bx+cx^2\right)^{5/2}}{8ch} + \frac{\int (g+hx)^2 \left(-\frac{1}{2}h(5bfg\right.}{}$$

$$= -\frac{(10cfg-16ceh+11bfh)(g+hx)^2 \left(a+bx+cx^2\right)^{5/2}}{112c^2h}$$

$$= -\frac{(10cfg-16ceh+11bfh)(g+hx)^2 \left(a+bx+cx^2\right)^{5/2}}{112c^2h}$$

$$= \frac{(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-1}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh\right.}{}$$

Mathematica [A]
time = 11.25, size = 1126, normalized size = 1.50

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(10395*b^7*f*h^2 - 630*b^6*c*h*(48*f*g +
24*e*h + 11*f*h*x) + 84*b^5*c*(-1095*a*f*h^2 + 40*c*h*(14*e*g + 7*d*h + 3*e
*h*x) + 2*c*f*(140*g^2 + 120*g*h*x + 33*h^2*x^2)) - 8*b^4*c^2*(-63*a*h*(480
*f*g + 240*e*h + 107*f*h*x) + 2*c*(140*d*h*(36*g + 7*h*x) + 56*e*(45*g^2 +
35*g*h*x + 9*h^2*x^2) + f*x*(980*g^2 + 1008*g*h*x + 297*h^2*x^2))) - 96*b^2
*c^3*(a^2*h*(5488*f*g + 2744*e*h + 1181*f*h*x) - 4*a*c*(28*d*h*(50*g + 9*h*
x) + 4*e*(175*g^2 + 126*g*h*x + 31*h^2*x^2) + f*x*(252*g^2 + 248*g*h*x + 71
*h^2*x^2)) + 8*c^2*x*(14*d*(5*g^2 + 4*g*h*x + h^2*x^2) + x*(4*e*(7*g^2 + 7*
g*h*x + 2*h^2*x^2) + f*x*(14*g^2 + 16*g*h*x + 5*h^2*x^2)))) + 16*b^3*c^2*(1
5309*a^2*f*h^2 - 4*a*c*(28*h*(190*e*g + 95*d*h + 39*e*h*x) + f*(2660*g^2 +
2184*g*h*x + 585*h^2*x^2)) + 8*c^2*(14*d*(45*g^2 + 30*g*h*x + 7*h^2*x^2) +
x*(2*e*(105*g^2 + 98*g*h*x + 27*h^2*x^2) + f*x*(98*g^2 + 108*g*h*x + 33*h^2
*x^2)))) - 64*b*c^3*(2757*a^3*f*h^2 - 6*a^2*c*(4*h*(378*e*g + 189*d*h + 73*
e*h*x) + f*(756*g^2 + 584*g*h*x + 151*h^2*x^2)) + 24*a*c^2*(14*d*(25*g^2 +
14*g*h*x + 3*h^2*x^2) + x*(f*x*(42*g^2 + 44*g*h*x + 13*h^2*x^2) + e*(98*g^2
 + 84*g*h*x + 22*h^2*x^2))) + 16*c^3*x^2*(14*d*(45*g^2 + 66*g*h*x + 26*h^2*
x^2) + x*(f*x*(364*g^2 + 600*g*h*x + 255*h^2*x^2) + e*(462*g^2 + 728*g*h*x
+ 300*h^2*x^2)))) - 128*c^4*(-3*a^3*h*(512*f*g + 256*e*h + 105*f*h*x) + 6*a
^2*c*(28*d*h*(32*g + 5*h*x) + 8*e*(56*g^2 + 35*g*h*x + 8*h^2*x^2) + f*x*(14
0*g^2 + 128*g*h*x + 35*h^2*x^2)) + 16*c^3*x^3*(14*d*(15*g^2 + 24*g*h*x + 10
*h^2*x^2) + x*(8*e*(21*g^2 + 35*g*h*x + 15*h^2*x^2) + 5*f*x*(28*g^2 + 48*g*
h*x + 21*h^2*x^2))) + 8*a*c^2*x*(14*d*(75*g^2 + 96*g*h*x + 35*h^2*x^2) + x*
(4*e*(168*g^2 + 245*g*h*x + 96*h^2*x^2) + f*x*(490*g^2 + 768*g*h*x + 315*h^
2*x^2))))) + 105*(b^2 - 4*a*c)^2*(768*c^4*d*g^2 + 99*b^4*f*h^2 - 72*b^2*c*h
*(4*b*f*g + 2*b*e*h + 3*a*f*h) - 128*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 3*b
*g*(e*g + 2*d*h)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b*h*(2*f*g + e*h) + 14*b^2*(
f*g^2 + h*(2*e*g + d*h))))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]
)/(3440640*c^(13/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1679 vs. $2(723) = 1446$.
time = 0.15, size = 1680, normalized size = 2.23

method	result	size
default	Expression too large to display	1680
risch	Expression too large to display	2761

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] f*h^2*(1/8*x^3*(c*x^2+b*x+a)^(5/2)/c-11/16*b/c*(1/7*x^2*(c*x^2+b*x+a)^(5/2)
/c-9/14*b/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/
c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c
```

```
*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+
(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(
4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-2/7*a/c*(1/5*(c*x^2+b*x+a)^(5
/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*
(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))))))-3/8*a/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*
(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/1
6*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(
c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+
1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))+(e*
h^2+2*f*g*h)*(1/7*x^2*(c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/6*x*(c*x^2+b*x+a)^(
5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+
b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4
*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1
/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x
^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2)))))-2/7*a/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*
x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/
8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))+(d*h^
2+2*e*g*h+f*g^2)*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(
5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4
*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3
/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+(2*d*g*h+e*g^2)*(1/5*(c*
x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c
-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+d*g^2*(1/8*(2*c*x+b)/c*(c*x^2+b*x+
a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c
-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1634 vs. $2(745) = 1490$.
time = 2.19, size = 3271, normalized size = 4.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas"
)
```

```
[Out] [1/6881280*(105*(32*(24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d + (7*b^6*c^2
 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*f)*g^2 - 96*(8*(b^5*c^3 - 8
*a*b^3*c^4 + 16*a^2*b*c^5)*d + (3*b^7*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*f)*g*h + (32*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*
a^3*c^5)*d + 3*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3
+ 256*a^4*c^4)*f)*h^2 - 16*(24*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*g^2 -
 4*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*g*h + 3*(3*b^7
*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*h^2)*e)*sqrt(c)*log(-8*c
^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*
c) + 4*(215040*c^8*f*h^2*x^7 + 15360*(32*c^8*f*g*h + 17*b*c^7*f*h^2)*x^6 +
1280*(224*c^8*f*g^2 + 480*b*c^7*f*g*h + (224*c^8*d + 3*(b^2*c^6 + 84*a*c^7)
*f)*h^2)*x^5 + 128*(2912*b*c^7*f*g^2 + 96*(56*c^8*d + (b^2*c^6 + 64*a*c^7)*
f)*g*h + (2912*b*c^7*d - 3*(11*b^3*c^5 - 52*a*b*c^6)*f)*h^2)*x^4 + 16*(224*
(120*c^8*d + (3*b^2*c^6 + 140*a*c^7)*f)*g^2 + 96*(616*b*c^7*d - (9*b^3*c^5
- 44*a*b*c^6)*f)*g*h + (224*(3*b^2*c^6 + 140*a*c^7)*d + 3*(99*b^4*c^4 - 568
*a*b^2*c^5 + 560*a^2*c^6)*f)*h^2)*x^3 - 224*(120*(3*b^3*c^5 - 20*a*b*c^6)*d
 + (105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*f)*g^2 + 96*(56*(15*b^4*c
^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d + (315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*
a^2*b^2*c^4 - 2048*a^3*c^5)*f)*g*h - (224*(105*b^5*c^3 - 760*a*b^3*c^4 + 12
96*a^2*b*c^5)*d + 3*(3465*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 588
16*a^3*b*c^4)*f)*h^2 + 8*(224*(360*b*c^7*d - (7*b^3*c^5 - 36*a*b*c^6)*f)*g^
2 + 96*(56*(b^2*c^6 + 32*a*c^7)*d + (21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c
^6)*f)*g*h - (224*(7*b^3*c^5 - 36*a*b*c^6)*d + 3*(231*b^5*c^3 - 1560*a*b^3*
c^4 + 2416*a^2*b*c^5)*f)*h^2)*x^2 + 2*(224*(120*(b^2*c^6 + 20*a*c^7)*d + (3
5*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*f)*g^2 - 96*(56*(5*b^3*c^5 - 28*a*
b*c^6)*d + (105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*f)*g*h + (224*(35
*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*d + 3*(1155*b^6*c^2 - 8988*a*b^4*c^
3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*f)*h^2)*x + 16*(15360*c^8*h^2*x^6 + 1
280*(28*c^8*g*h + 15*b*c^7*h^2)*x^5 + 128*(168*c^8*g^2 + 364*b*c^7*g*h + 3*
(b^2*c^6 + 64*a*c^7)*h^2)*x^4 + 16*(1848*b*c^7*g^2 + 28*(3*b^2*c^6 + 140*a*
c^7)*g*h - 3*(9*b^3*c^5 - 44*a*b*c^6)*h^2)*x^3 + 168*(15*b^4*c^4 - 100*a*b^
2*c^5 + 128*a^2*c^6)*g^2 - 28*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5
)*g*h + 3*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*
h^2 + 8*(168*(b^2*c^6 + 32*a*c^7)*g^2 - 28*(7*b^3*c^5 - 36*a*b*c^6)*g*h + 3
*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*h^2)*x^2 - 2*(168*(5*b^3*c^5 -
```

```
28*a*b*c^6)*g^2 - 28*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*g*h + 3*(10
5*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*h^2)*x)*e)*sqrt(c*x^2 + b*x + a
))/c^7, -1/3440640*(105*(32*(24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d + (7
*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*f)*g^2 - 96*(8*(b^5
*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d + (3*b^7*c - 28*a*b^5*c^2 + 80*a^2*b^3
*c^3 - 64*a^3*b*c^4)*f)*g*h + (32*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c
^4 - 64*a^3*c^5)*d + 3*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*
b^2*c^3 + 256*a^4*c^4)*f)*h^2 - 16*(24*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^
5)*g^2 - 4*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*g*h +
3*(3*b^7*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*h^2)*e)*sqrt(-c)
*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a
*c)) - 2*(215040*c^8*f*h^2*x^7 + 15360*(32*c^8*f*g*h + 17*b*c^7*f*h^2)*x^6
+ 1280*(224*c^8*f*g^2 + 480*b*c^7*f*g*h + (224*c^8*d + 3*(b^2*c^6 + 84*a*c^
7)*f)*h^2)*x^5 + 128*(2912*b*c^7*f*g^2 + 96*(56*c^8*d + (b^2*c^6 + 64*a*c^7
)*f)*g*h + (2912*b*c^7*d - 3*(11*b^3*c^5 - 52*a*b*c^6)*f)*h^2)*x^4 + 16*(22
4*(120*c^8*d + (3*b^2*c^6 + 140*a*c^7)*f)*g^2 + 96*(616*b*c^7*d - (9*b^3*c^
5 - 44*a*b*c^6)*f)*g*h + (224*(3*b^2*c^6 + 140*a*c^7)*d + 3*(99*b^4*c^4 - 5
68*a*b^2*c^5 + 560*a^2*c^6)*f)*h^2)*x^3 - 224*(120*(3*b^3*c^5 - 20*a*b*c^6)
*d + (105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*f)*g^2 + 96*(56*(15*b^4
*c^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d + (315*b^6*c^2 - 2520*a*b^4*c^3 + 548
8*a^2*b^2*c^4 - 2048*a^3*c^5)*f)*g*h - (224*(105*b^5*c^3 - 760*a*b^3*c^4 +
1296*a^2*b*c^5)*d + 3*(3465*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 5
8816*a^3*b*c^4)*f)*h^2 + 8*(224*(360*b*c^7*d - (7*b^3*c^5 - 36*a*b*c^6)*f)*
g^2 + 96*(56*(b^2*c^6 + 32*a*c^7)*d + (21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2
*c^6)*f)*g*h - (224*(7*b^3*c^5 - 36*a*b*c^6)*d + 3*(231*b^5*c^3 - 1560*a*b^
3*c^4 + 2416*a^2*b*c^5)*f)*h^2)*x^2 + 2*(224*(120*(b^2*c^6 + 20*a*c^7)*d +
(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*f)*g^2 - 96*(56*(5*b^3*c^5 - 28*
a*b*c^6)*d + (105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*f)*g*h + (224*(
35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*d + 3*(1155*b^6*c^2 - 8988*a*b^4*
c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*f)*h^2)*x + 16*(15360*c^8*h^2*x^6 +
 1280*(28*c^8*g*h + 15*b*c^7*h^2)*x^5 + 128*(168*c^8*g^2 + 364*b*c^7*g*h +
3*(b^2*c^6 + 64*a*c^7)*h^2)*x^4 + 16*(1848*b*c^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{\frac{3}{2}} \left(d+ex+fx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((g + h*x)**2*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1852 vs. 2(745) = 1490.

time = 4.55, size = 1852, normalized size = 2.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/1720320*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*(14*c*f*h^2*x + (32*c^8
*f*g*h + 17*b*c^7*f*h^2 + 16*c^8*h^2*e)/c^7)*x + (224*c^8*f*g^2 + 480*b*c^7
*f*g*h + 224*c^8*d*h^2 + 3*b^2*c^6*f*h^2 + 252*a*c^7*f*h^2 + 448*c^8*g*h*e
+ 240*b*c^7*h^2*e)/c^7)*x + (2912*b*c^7*f*g^2 + 5376*c^8*d*g*h + 96*b^2*c^6
*f*g*h + 6144*a*c^7*f*g*h + 2912*b*c^7*d*h^2 - 33*b^3*c^5*f*h^2 + 156*a*b*c
^6*f*h^2 + 2688*c^8*g^2*e + 5824*b*c^7*g*h*e + 48*b^2*c^6*h^2*e + 3072*a*c^
7*h^2*e)/c^7)*x + (26880*c^8*d*g^2 + 672*b^2*c^6*f*g^2 + 31360*a*c^7*f*g^2
+ 59136*b*c^7*d*g*h - 864*b^3*c^5*f*g*h + 4224*a*b*c^6*f*g*h + 672*b^2*c^6*
d*h^2 + 31360*a*c^7*d*h^2 + 297*b^4*c^4*f*h^2 - 1704*a*b^2*c^5*f*h^2 + 1680
*a^2*c^6*f*h^2 + 29568*b*c^7*g^2*e + 1344*b^2*c^6*g*h*e + 62720*a*c^7*g*h*e
 - 432*b^3*c^5*h^2*e + 2112*a*b*c^6*h^2*e)/c^7)*x + (80640*b*c^7*d*g^2 - 15
68*b^3*c^5*f*g^2 + 8064*a*b*c^6*f*g^2 + 5376*b^2*c^6*d*g*h + 172032*a*c^7*d
*g*h + 2016*b^4*c^4*f*g*h - 11904*a*b^2*c^5*f*g*h + 12288*a^2*c^6*f*g*h - 1
568*b^3*c^5*d*h^2 + 8064*a*b*c^6*d*h^2 - 693*b^5*c^3*f*h^2 + 4680*a*b^3*c^4
*f*h^2 - 7248*a^2*b*c^5*f*h^2 + 2688*b^2*c^6*g^2*e + 86016*a*c^7*g^2*e - 31
36*b^3*c^5*g*h*e + 16128*a*b*c^6*g*h*e + 1008*b^4*c^4*h^2*e - 5952*a*b^2*c^
5*h^2*e + 6144*a^2*c^6*h^2*e)/c^7)*x + (26880*b^2*c^6*d*g^2 + 537600*a*c^7*
d*g^2 + 7840*b^4*c^4*f*g^2 - 48384*a*b^2*c^5*f*g^2 + 53760*a^2*c^6*f*g^2 -
26880*b^3*c^5*d*g*h + 150528*a*b*c^6*d*g*h - 10080*b^5*c^3*f*g*h + 69888*a*
b^3*c^4*f*g*h - 112128*a^2*b*c^5*f*g*h + 7840*b^4*c^4*d*h^2 - 48384*a*b^2*c
^5*d*h^2 + 53760*a^2*c^6*d*h^2 + 3465*b^6*c^2*f*h^2 - 26964*a*b^4*c^3*f*h^2
 + 56688*a^2*b^2*c^4*f*h^2 - 20160*a^3*c^5*f*h^2 - 13440*b^3*c^5*g^2*e + 75
264*a*b*c^6*g^2*e + 15680*b^4*c^4*g*h*e - 96768*a*b^2*c^5*g*h*e + 107520*a^
2*c^6*g*h*e - 5040*b^5*c^3*h^2*e + 34944*a*b^3*c^4*h^2*e - 56064*a^2*b*c^5*
h^2*e)/c^7)*x - (80640*b^3*c^5*d*g^2 - 537600*a*b*c^6*d*g^2 + 23520*b^5*c^3
*f*g^2 - 170240*a*b^3*c^4*f*g^2 + 290304*a^2*b*c^5*f*g^2 - 80640*b^4*c^4*d*
g*h + 537600*a*b^2*c^5*d*g*h - 688128*a^2*c^6*d*g*h - 30240*b^6*c^2*f*g*h +
 241920*a*b^4*c^3*f*g*h - 526848*a^2*b^2*c^4*f*g*h + 196608*a^3*c^5*f*g*h +
 23520*b^5*c^3*d*h^2 - 170240*a*b^3*c^4*d*h^2 + 290304*a^2*b*c^5*d*h^2 + 10
395*b^7*c*f*h^2 - 91980*a*b^5*c^2*f*h^2 + 244944*a^2*b^3*c^3*f*h^2 - 176448
*a^3*b*c^4*f*h^2 - 40320*b^4*c^4*g^2*e + 268800*a*b^2*c^5*g^2*e - 344064*a^
2*c^6*g^2*e + 47040*b^5*c^3*g*h*e - 340480*a*b^3*c^4*g*h*e + 580608*a^2*b*c
^5*g*h*e - 15120*b^6*c^2*h^2*e + 120960*a*b^4*c^3*h^2*e - 263424*a^2*b^2*c^
4*h^2*e + 98304*a^3*c^5*h^2*e)/c^7) - 1/32768*(768*b^4*c^4*d*g^2 - 6144*a*b
^2*c^5*d*g^2 + 12288*a^2*c^6*d*g^2 + 224*b^6*c^2*f*g^2 - 1920*a*b^4*c^3*f*g
^2 + 4608*a^2*b^2*c^4*f*g^2 - 2048*a^3*c^5*f*g^2 - 768*b^5*c^3*d*g*h + 6144
*a*b^3*c^4*d*g*h - 12288*a^2*b*c^5*d*g*h - 288*b^7*c*f*g*h + 2688*a*b^5*c^2
*f*g*h - 7680*a^2*b^3*c^3*f*g*h + 6144*a^3*b*c^4*f*g*h + 224*b^6*c^2*d*h^2
```

```
- 1920*a*b^4*c^3*d*h^2 + 4608*a^2*b^2*c^4*d*h^2 - 2048*a^3*c^5*d*h^2 + 99*b
^8*f*h^2 - 1008*a*b^6*c*f*h^2 + 3360*a^2*b^4*c^2*f*h^2 - 3840*a^3*b^2*c^3*f
*h^2 + 768*a^4*c^4*f*h^2 - 384*b^5*c^3*g^2*e + 3072*a*b^3*c^4*g^2*e - 6144*
a^2*b*c^5*g^2*e + 448*b^6*c^2*g*h*e - 3840*a*b^4*c^3*g*h*e + 9216*a^2*b^2*c
^4*g*h*e - 4096*a^3*c^5*g*h*e - 144*b^7*c*h^2*e + 1344*a*b^5*c^2*h^2*e - 38
40*a^2*b^3*c^3*h^2*e + 3072*a^3*b*c^4*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g+h\,x)^2\,\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.198 $\int (g+hx) \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx$

Optimal. Leaf size=418

$$-\frac{\left(b^2-4ac\right)\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh))\right)(b+2cx)\sqrt{a+}}{1024c^5}$$

```
[Out] 1/384*(48*c^3*d*g-9*b^3*f*h-8*c^2*(a*e*h+a*f*g+3*b*d*h+3*b*e*g)+2*b*c*(6*a*
f*h+7*b*(e*h+f*g)))*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^4+1/7*f*(h*x+g)^2*(c*x^
2+b*x+a)^(5/2)/c/h+1/840*(63*b^2*f*h^2-24*c^2*(5*f*g^2-7*h*(d*h+e*g))-2*c*h
*(24*a*f*h+49*b*(e*h+f*g))-10*c*h*(9*b*f*h-14*c*e*h+10*c*f*g)*x)*(c*x^2+b*x
+a)^(5/2)/c^3/h+1/2048*(-4*a*c+b^2)^2*(48*c^3*d*g-9*b^3*f*h-8*c^2*(a*e*h+a*
f*g+3*b*d*h+3*b*e*g)+2*b*c*(6*a*f*h+7*b*(e*h+f*g)))*arctanh(1/2*(2*c*x+b)/c
^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)-1/1024*(-4*a*c+b^2)*(48*c^3*d*g-9*b^3*
f*h-8*c^2*(a*e*h+a*f*g+3*b*d*h+3*b*e*g)+2*b*c*(6*a*f*h+7*b*(e*h+f*g)))*(2*c
*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

Rubi [A]
time = 0.36, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1667, 793, 626, 635, 212}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-8c^2(aeh+afg+3bdh+3beg)+2bc(6afh+7b(eh+fg))-9b^3fh+48c^3dg\right)}{384c^4}+\frac{\left(a+bx+cx^2\right)^{5/2}\left(-2ch(24afh+49b(eh+fg))+63b^2fh^2-10chx(9bfh-14ceh+10cfg)-24c^2\left(5fg^2-7h(dh+eg)\right)\right)}{840c^3h}+\frac{\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(aeh+afg+3bdh+3beg)+2bc(6afh+7b(eh+fg))-9b^3fh+48c^3dg\right)}{2048c^{11/2}}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-8c^2(aeh+afg+3bdh+3beg)+2bc(6afh+7b(eh+fg))-9b^3fh+48c^3dg\right)}{1024c^5}+\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -1/1024*((b^2 - 4*a*c)*(48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b*e*g + a*f*g + 3
*b*d*h + a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*(b + 2*c*x)*Sqrt[a + b
*x + c*x^2])/c^5 + ((48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b*e*g + a*f*g + 3*b*
d*h + a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*(b + 2*c*x)*(a + b*x + c*
x^2)^(3/2))/(384*c^4) + (f*(g + h*x)^2*(a + b*x + c*x^2)^(5/2))/(7*c*h) + (
(63*b^2*f*h^2 - 24*c^2*(5*f*g^2 - 7*h*(e*g + d*h)) - 2*c*h*(24*a*f*h + 49*b
*(f*g + e*h)) - 10*c*h*(10*c*f*g - 14*c*e*h + 9*b*f*h)*x)*(a + b*x + c*x^2)
^(5/2))/(840*c^3*h) + ((b^2 - 4*a*c)^2*(48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b
*e*g + a*f*g + 3*b*d*h + a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2048*c^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch} + \frac{\int (g+hx)\left(-\frac{1}{2}h(5bfg-\right.}{}$$

$$= \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch} + \frac{(63b^2fh^2-24c^2(5fg^2-}{}$$

$$= \frac{(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc}{384c^4}$$

$$= -\frac{\left(b^2-4ac\right)\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh\right.}{}$$

Mathematica [A]
time = 4.27, size = 598, normalized size = 1.43

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(945*b^6*f*h - 210*b^5*c*(7*f*g + 7*e*h +
3*f*h*x) + 28*b^4*c*(-270*a*f*h + c*(90*e*g + 90*d*h + 35*f*g*x + 35*e*h*x
+ 18*f*h*x^2)) - 16*b^3*c^2*(105*c*d*(3*g + h*x) - 7*a*(95*f*g + 95*e*h + 3
9*f*h*x) + c*x*(7*e*(15*g + 7*h*x) + f*x*(49*g + 27*h*x))) + 48*b^2*c^2*(34
3*a^2*f*h - 2*a*c*(175*d*h + 7*e*(25*g + 9*h*x) + f*x*(63*g + 31*h*x)) + 2*
c^2*x*(7*d*(5*g + 2*h*x) + x*(7*e*(2*g + h*x) + f*x*(7*g + 4*h*x)))) + 32*b
*c^3*(-3*a^2*(189*f*g + 189*e*h + 73*f*h*x) + 6*a*c*(7*d*(25*g + 7*h*x) + x
*(7*e*(7*g + 3*h*x) + f*x*(21*g + 11*h*x))) + 4*c^2*x^2*(21*d*(15*g + 11*h*
x) + x*(7*e*(33*g + 26*h*x) + 2*f*x*(91*g + 75*h*x)))) + 64*c^3*(-96*a^3*f*
h + 3*a^2*c*(112*d*h + 7*e*(16*g + 5*h*x) + f*x*(35*g + 16*h*x)) + 4*c^3*x^
3*(21*d*(5*g + 4*h*x) + 2*x*(7*e*(6*g + 5*h*x) + 5*f*x*(7*g + 6*h*x))) + 2*
a*c^2*x*(21*d*(25*g + 16*h*x) + x*(7*e*(48*g + 35*h*x) + f*x*(245*g + 192*h
*x))))) + 105*(b^2 - 4*a*c)^2*(-48*c^3*d*g + 9*b^3*f*h + 8*c^2*(3*b*e*g + a
*f*g + 3*b*d*h + a*e*h) - 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*Log[b + 2*c*x
- 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(215040*c^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(392) = 784$.
time = 0.14, size = 936, normalized size = 2.24

method	result

default	hf	$\frac{x^{2}\left(c\,x^{2}+bx+a\right)^{\frac{5}{2}}}{7c}-\frac{9b\left(\frac{x\left(c\,x^{2}+bx+a\right)^{\frac{5}{2}}}{6c}-\frac{7b\left(\frac{\left(c\,x^{2}+bx+a\right)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(2cx+b)\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^{2}\right)\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx}}{4c}\right.}{}\right.}{2c}\right.}{12c}\right.}{}$

risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] h*f*(1/7*x^2*(c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/
12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3
/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)
/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x
+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)
^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
))-2/7*a/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a
)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-
b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))+(e*h+f*g)*(1/6
*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(
2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b
*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1
/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/
4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2)))))+(d*h+e*g)*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*
(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c
*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2)))))+d*g*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(
1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 943 vs. $2(404) = 808$.
time = 0.82, size = 1889, normalized size = 4.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")

[Out] [-1/430080*(105*(2*(24*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d + (7*b^6*c -
60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*f)*g - 3*(8*(b^5*c^2 - 8*a*b^3
*c^3 + 16*a^2*b*c^4)*d + (3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^
3)*f)*h - 2*(12*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*g - (7*b^6*c - 60*a*
b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*h)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*
c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(15360
*c^7*f*h*x^6 + 1280*(14*c^7*f*g + 15*b*c^6*f*h)*x^5 + 128*(182*b*c^6*f*g +
3*(56*c^7*d + (b^2*c^5 + 64*a*c^6)*f)*h)*x^4 + 16*(14*(120*c^7*d + (3*b^2*c
^5 + 140*a*c^6)*f)*g + 3*(616*b*c^6*d - (9*b^3*c^4 - 44*a*b*c^5)*f)*h)*x^3
+ 8*(14*(360*b*c^6*d - (7*b^3*c^4 - 36*a*b*c^5)*f)*g + 3*(56*(b^2*c^5 + 32*
a*c^6)*d + (21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*f)*h)*x^2 - 14*(120*(
3*b^3*c^4 - 20*a*b*c^5)*d + (105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*c^4)*
f)*g + 3*(56*(15*b^4*c^3 - 100*a*b^2*c^4 + 128*a^2*c^5)*d + (315*b^6*c - 25
20*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4)*f)*h + 2*(14*(120*(b^2*c^5
+ 20*a*c^6)*d + (35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*f)*g - 3*(56*(5*
b^3*c^4 - 28*a*b*c^5)*d + (105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*f)
*h)*x + 14*(1280*c^7*h*x^5 + 128*(12*c^7*g + 13*b*c^6*h)*x^4 + 16*(132*b*c^
6*g + (3*b^2*c^5 + 140*a*c^6)*h)*x^3 + 8*(12*(b^2*c^5 + 32*a*c^6)*g - (7*b^
3*c^4 - 36*a*b*c^5)*h)*x^2 + 12*(15*b^4*c^3 - 100*a*b^2*c^4 + 128*a^2*c^5)*
g - (105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*c^4)*h - 2*(12*(5*b^3*c^4 - 2
8*a*b*c^5)*g - (35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*h)*x)*e)*sqrt(c*x
^2 + b*x + a))/c^6, -1/215040*(105*(2*(24*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c
^5)*d + (7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*f)*g - 3*(8
*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d + (3*b^7 - 28*a*b^5*c + 80*a^2*b^
3*c^2 - 64*a^3*b*c^3)*f)*h - 2*(12*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*g
 - (7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*h)*e)*sqrt(-c)*a
rctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c
)) - 2*(15360*c^7*f*h*x^6 + 1280*(14*c^7*f*g + 15*b*c^6*f*h)*x^5 + 128*(182
*b*c^6*f*g + 3*(56*c^7*d + (b^2*c^5 + 64*a*c^6)*f)*h)*x^4 + 16*(14*(120*c^7
*d + (3*b^2*c^5 + 140*a*c^6)*f)*g + 3*(616*b*c^6*d - (9*b^3*c^4 - 44*a*b*c^
5)*f)*h)*x^3 + 8*(14*(360*b*c^6*d - (7*b^3*c^4 - 36*a*b*c^5)*f)*g + 3*(56*(
b^2*c^5 + 32*a*c^6)*d + (21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*f)*h)*x^
2 - 14*(120*(3*b^3*c^4 - 20*a*b*c^5)*d + (105*b^5*c^2 - 760*a*b^3*c^3 + 129
6*a^2*b*c^4)*f)*g + 3*(56*(15*b^4*c^3 - 100*a*b^2*c^4 + 128*a^2*c^5)*d + (3
15*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4)*f)*h + 2*(14*(
120*(b^2*c^5 + 20*a*c^6)*d + (35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*f)*
g - 3*(56*(5*b^3*c^4 - 28*a*b*c^5)*d + (105*b^5*c^2 - 728*a*b^3*c^3 + 1168*
a^2*b*c^4)*f)*h)*x + 14*(1280*c^7*h*x^5 + 128*(12*c^7*g + 13*b*c^6*h)*x^4 +
 16*(132*b*c^6*g + (3*b^2*c^5 + 140*a*c^6)*h)*x^3 + 8*(12*(b^2*c^5 + 32*a*c
^6)*g - (7*b^3*c^4 - 36*a*b*c^5)*h)*x^2 + 12*(15*b^4*c^3 - 100*a*b^2*c^4 +
128*a^2*c^5)*g - (105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*c^4)*h - 2*(12*(
5*b^3*c^4 - 28*a*b*c^5)*g - (35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*h)*x
)*e)*sqrt(c*x^2 + b*x + a))/c^6]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (g+hx)\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
[Out] Integral((g + h*x)*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 955 vs. 2(404) = 808.
time = 4.93, size = 955, normalized size = 2.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
[Out] 1/107520*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*c*f*h*x + (14*c^7*f*g +
15*b*c^6*f*h + 14*c^7*h*e)/c^6)*x + (182*b*c^6*f*g + 168*c^7*d*h + 3*b^2*c^
5*f*h + 192*a*c^6*f*h + 168*c^7*g*e + 182*b*c^6*h*e)/c^6)*x + (1680*c^7*d*g
 + 42*b^2*c^5*f*g + 1960*a*c^6*f*g + 1848*b*c^6*d*h - 27*b^3*c^4*f*h + 132*
a*b*c^5*f*h + 1848*b*c^6*g*e + 42*b^2*c^5*h*e + 1960*a*c^6*h*e)/c^6)*x + (5
040*b*c^6*d*g - 98*b^3*c^4*f*g + 504*a*b*c^5*f*g + 168*b^2*c^5*d*h + 5376*a
*c^6*d*h + 63*b^4*c^3*f*h - 372*a*b^2*c^4*f*h + 384*a^2*c^5*f*h + 168*b^2*c
^5*g*e + 5376*a*c^6*g*e - 98*b^3*c^4*h*e + 504*a*b*c^5*h*e)/c^6)*x + (1680*
b^2*c^5*d*g + 33600*a*c^6*d*g + 490*b^4*c^3*f*g - 3024*a*b^2*c^4*f*g + 3360
*a^2*c^5*f*g - 840*b^3*c^4*d*h + 4704*a*b*c^5*d*h - 315*b^5*c^2*f*h + 2184*
a*b^3*c^3*f*h - 3504*a^2*b*c^4*f*h - 840*b^3*c^4*g*e + 4704*a*b*c^5*g*e + 4
90*b^4*c^3*h*e - 3024*a*b^2*c^4*h*e + 3360*a^2*c^5*h*e)/c^6)*x - (5040*b^3*
c^4*d*g - 33600*a*b*c^5*d*g + 1470*b^5*c^2*f*g - 10640*a*b^3*c^3*f*g + 1814
4*a^2*b*c^4*f*g - 2520*b^4*c^3*d*h + 16800*a*b^2*c^4*d*h - 21504*a^2*c^5*d*
h - 945*b^6*c*f*h + 7560*a*b^4*c^2*f*h - 16464*a^2*b^2*c^3*f*h + 6144*a^3*c
^4*f*h - 2520*b^4*c^3*g*e + 16800*a*b^2*c^4*g*e - 21504*a^2*c^5*g*e + 1470*
b^5*c^2*h*e - 10640*a*b^3*c^3*h*e + 18144*a^2*b*c^4*h*e)/c^6) - 1/2048*(48*
b^4*c^3*d*g - 384*a*b^2*c^4*d*g + 768*a^2*c^5*d*g + 14*b^6*c*f*g - 120*a*b^
4*c^2*f*g + 288*a^2*b^2*c^3*f*g - 128*a^3*c^4*f*g - 24*b^5*c^2*d*h + 192*a*
b^3*c^3*d*h - 384*a^2*b*c^4*d*h - 9*b^7*f*h + 84*a*b^5*c*f*h - 240*a^2*b^3*
c^2*f*h + 192*a^3*b*c^3*f*h - 24*b^5*c^2*g*e + 192*a*b^3*c^3*g*e - 384*a^2*
b*c^4*g*e + 14*b^6*c*h*e - 120*a*b^4*c^2*h*e + 288*a^2*b^2*c^3*h*e - 128*a^
3*c^4*h*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(
11/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (g+h\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
[Out] int((g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.199 $\int (a+bx+cx^2)^{3/2} (d+ex+fx^2) \, dx$

Optimal. Leaf size=236

$$-\frac{(b^2-4ac)\,(24c^2d+7b^2f-4c(3be+af))\,(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{(24c^2d-12bce+7b^2f-4acf)\,(b+2c}{192c^3}$$

```
[Out] 1/192*(-4*a*c*f+7*b^2*f-12*b*c*e+24*c^2*d)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^
3+1/60*(-7*b*f+12*c*e)*(c*x^2+b*x+a)^(5/2)/c^2+1/6*f*x*(c*x^2+b*x+a)^(5/2)/
c+1/1024*(-4*a*c+b^2)^2*(24*c^2*d+7*b^2*f-4*c*(a*f+3*b*e))*arctanh(1/2*(2*c
*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)-1/512*(-4*a*c+b^2)*(24*c^2*d+7*b
^2*f-4*c*(a*f+3*b*e))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^4
```

Rubi [A]
time = 0.13, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1675, 654, 626, 635, 212}

$$\frac{(b^2-4ac)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+3be)+7b^2f+24c^2d)}{1024c^{9/2}}-\frac{(b^2-4ac)\,(b+2cx)\sqrt{a+bx+cx^2}\,(-4c(af+3be)+7b^2f+24c^2d)}{512c^4}+\frac{(b+2cx)\,(a+bx+cx^2)^{3/2}\,(-4acf+7b^2f-12bce+24c^2d)}{192c^3}+\frac{(a+bx+cx^2)^{5/2}\,(12ce-7bf)}{60c^2}+\frac{fx(a+bx+cx^2)^{5/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -1/512*((b^2 - 4*a*c)*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*(b + 2*c*x)*
Sqrt[a + b*x + c*x^2])/c^4 + ((24*c^2*d - 12*b*c*e + 7*b^2*f - 4*a*c*f)*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3) + ((12*c*e - 7*b*f)*(a + b*x +
c*x^2)^(5/2))/(60*c^2) + (f*x*(a + b*x + c*x^2)^(5/2))/(6*c) + ((b^2 - 4*a*
c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \frac{fx(a+bx+cx^2)^{5/2}}{6c}+\frac{\int \left(6cd-af+\frac{1}{2}(12ce-7bf)x\right)\left(a+bx+\right.}{6c}$$

$$=\frac{(12ce-7bf)\left(a+bx+cx^2\right)^{5/2}}{60c^2}+\frac{fx(a+bx+cx^2)^{5/2}}{6c}+\frac{(2c(6c}{}$$

$$=\frac{(24c^2d-12bce+7b^2f-4acf)\,(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3}+\frac{(12}{}$$

$$=-\frac{(b^2-4ac)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+}}{512c^4}$$

$$=-\frac{(b^2-4ac)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+}}{512c^4}$$

$$=-\frac{(b^2-4ac)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+}}{512c^4}$$

Mathematica [A]

time = 0.15, size = 290, normalized size = 1.23

$$\frac{2\sqrt{c}\,\sqrt{a+x(b+cx)}\,(-105b^5f+10b^4c(18e+7fx)-8b^3c(45cd-95af+cx(15e+7fx))+48b^2c^2(-a(25e+9fx)+cx(5d+x(2e+fx)))+16bc^2(-81a^2f+6ac(25d+x(7e+3fx))+4c^2x^2(45d+x(33e+26fx)))+32c^3(3a^2(16e+5fx)+4c^2x^3(15d+2x(6e+5fx))+2acx(75d+x(48e+35fx))))-15(b^2-4ac)^2\,(24c^2d+7b^2f-4c(3be+af))\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)}{15360c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^5*f + 10*b^4*c*(18*e + 7*f*x) - 8*
b^3*c*(45*c*d - 95*a*f + c*x*(15*e + 7*f*x)) + 48*b^2*c^2*(-(a*(25*e + 9*f*
x)) + c*x*(5*d + x*(2*e + f*x))) + 16*b*c^2*(-81*a^2*f + 6*a*c*(25*d + x*(7
*e + 3*f*x)) + 4*c^2*x^2*(45*d + x*(33*e + 26*f*x))) + 32*c^3*(3*a^2*(16*e
+ 5*f*x) + 4*c^2*x^3*(15*d + 2*x*(6*e + 5*f*x)) + 2*a*c*x*(75*d + x*(48*e +
 35*f*x)))) - 15*(b^2 - 4*a*c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*L
og[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(15360*c^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(210) = 420$.
time = 0.00, size = 499, normalized size = 2.11

method	result
default	$f\left(\frac{x(cx^2+bx+a)^{\frac{5}{2}}}{6c}-\frac{7b\left(\frac{(cx^2+bx+a)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(2cx+b)(cx^2+bx+a)^{\frac{3}{2}}}{8c}+\frac{3(4ac-b^2)\left(\frac{(2cx+b)\sqrt{cx^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+c \cdots}{\sqrt{c} \cdots}\right.}{\cdots}\right.}{16c}\right.}{2c}\right.}{12c}\right.$

risch	$-\frac{(-1280c^5 f x^5-1664b c^4 f x^4-1536c^5 e x^4-2240a c^4 f x^3-48b^2 c^3 f x^3-2112b c^4 e x^3-1920c^5 d x^3-288ab c^3 f x^2-3072a c^4 e x^2+56b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*
(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c
*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)
/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+e*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1
/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x
^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2)))))+d*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*
(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.46, size = 849, normalized size = 3.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d + (7*b^6 - 60*a*b^
4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*f - 12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*x^5 + 1664*b*c^5*f*x^4 + 16*(120
*c^6*d + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(360*b*c^5*d - (7*b^3*c^3 - 36*
```

```
a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d - (105*b^5*c - 760*a*b^3*c
^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 + 20*a*c^5)*d + (35*b^4*c^2 - 216*
a*b^2*c^3 + 240*a^2*c^4)*f)*x + 12*(128*c^6*x^4 + 176*b*c^5*x^3 + 15*b^4*c^
2 - 100*a*b^2*c^3 + 128*a^2*c^4 + 8*(b^2*c^4 + 32*a*c^5)*x^2 - 2*(5*b^3*c^3
 - 28*a*b*c^4)*x)*e)*sqrt(c*x^2 + b*x + a))/c^5, -1/15360*(15*(24*(b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*d + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*
a^3*c^3)*f - 12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e)*sqrt(-c)*arctan(1/2
*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1
280*c^6*f*x^5 + 1664*b*c^5*f*x^4 + 16*(120*c^6*d + (3*b^2*c^4 + 140*a*c^5)*
f)*x^3 + 8*(360*b*c^5*d - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3
- 20*a*b*c^4)*d - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(
b^2*c^4 + 20*a*c^5)*d + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*f)*x + 1
2*(128*c^6*x^4 + 176*b*c^5*x^3 + 15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4 +
 8*(b^2*c^4 + 32*a*c^5)*x^2 - 2*(5*b^3*c^3 - 28*a*b*c^4)*x)*e)*sqrt(c*x^2 +
 b*x + a))/c^5]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

Giac [A]
time = 4.42, size = 417, normalized size = 1.77

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,cfx+\frac{13\,bc^5f+12\,c^6e}{c^5}\right)x+\frac{120\,c^6d+3\,b^2c^4f+140\,ac^5f+132\,bc^5e}{c^5}\right)x+\frac{360\,bc^5d-7\,b^3c^3f+36\,abc^4f+12\,b^2c^4e+384\,ac^5e}{c^5}\right)x+\frac{120\,b^2c^4d+2400\,ac^5d+35\,b^4c^2f-216\,ab^2c^3f+240\,a^2c^4f-60\,b^3c^3e+336\,abc^4e}{c^5}\right)x-\frac{360\,b^3c^3d-2400\,abc^4d+105\,b^5cf-760\,ab^3c^2f+1296\,a^2bc^3f-180\,b^4c^2e+1200\,ab^2c^3e-1536\,a^2c^4e}{c^5}\right)-\frac{\left(24\,b^4c^2d-192\,ab^2c^3d+384\,a^2c^4d+7\,b^6f-60\,ab^4cf+144\,a^2b^2c^2f-64\,a^3c^3f-12\,b^5ce+96\,ab^3c^2e-192\,a^2bc^3e\right)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{1024\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*f*x + (13*b*c^5*f + 12*c^6*e
)/c^5)*x + (120*c^6*d + 3*b^2*c^4*f + 140*a*c^5*f + 132*b*c^5*e)/c^5)*x + (
360*b*c^5*d - 7*b^3*c^3*f + 36*a*b*c^4*f + 12*b^2*c^4*e + 384*a*c^5*e)/c^5)
*x + (120*b^2*c^4*d + 2400*a*c^5*d + 35*b^4*c^2*f - 216*a*b^2*c^3*f + 240*a
^2*c^4*f - 60*b^3*c^3*e + 336*a*b*c^4*e)/c^5)*x - (360*b^3*c^3*d - 2400*a*b
*c^4*d + 105*b^5*c*f - 760*a*b^3*c^2*f + 1296*a^2*b*c^3*f - 180*b^4*c^2*e +
 1200*a*b^2*c^3*e - 1536*a^2*c^4*e)/c^5) - 1/1024*(24*b^4*c^2*d - 192*a*b^2
*c^3*d + 384*a^2*c^4*d + 7*b^6*f - 60*a*b^4*c*f + 144*a^2*b^2*c^2*f - 64*a^
3*c^3*f - 12*b^5*c*e + 96*a*b^3*c^2*e - 192*a^2*b*c^3*e)*log(abs(-2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
[Out] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.200 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{g+hx}\,dx$

Optimal. Leaf size=660

$$\frac{(3b^4fh^4 + 6b^2ch^3(bfg - beh - 2afh) + 128c^4g^2(fg^2 - h(eg - dh)) - 32c^3h(5bg - 4ah)\left(fg^2 - h(eg - dh)\right) -}{}$$

```
[Out] -1/48*(8*c*h*(b*f*g-2*c*d*h)-(-3*b*h+8*c*g)*(b*f*h-2*c*e*h+2*c*f*g)+6*c*h*(
b*f*h-2*c*e*h+2*c*f*g)*x)*(c*x^2+b*x+a)^(3/2)/c^2/h^3+1/5*f*(c*x^2+b*x+a)^(
5/2)/c/h-1/256*(4*c*h*(-b*h+2*c*g)*(8*c*h*(-2*a*h+b*g)*(b*f*g-2*c*d*h)-g*(-
4*a*c*h-3*b^2*h+8*b*c*g)*(b*f*h-2*c*e*h+2*c*f*g))-2*(4*c^2*g^2-1/2*b^2*h^2-
2*c*h*(-a*h+b*g))*(8*c*h*(-b*h+2*c*g)*(b*f*g-2*c*d*h)-(b*f*h-2*c*e*h+2*c*f*
g)*(16*c^2*g^2-3*b^2*h^2-4*c*h*(-3*a*h+2*b*g))))*arctanh(1/2*(2*c*x+b)/c^(1
/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)/h^6+(a*h^2-b*g*h+c*g^2)^(3/2)*(f*g^2-h*(-d
*h+e*g))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(
c*x^2+b*x+a)^(1/2))/h^6+1/128*(3*b^4*f*h^4+6*b^2*c*h^3*(-2*a*f*h-b*e*h+b*f*
g)+128*c^4*g^2*(f*g^2-h*(-d*h+e*g))-32*c^3*h*(-4*a*h+5*b*g)*(f*g^2-h*(-d*h+
e*g))-8*b*c^2*h^2*(3*a*h*(-e*h+f*g)-2*b*(d*h^2-e*g*h+f*g^2))+2*c*h*(8*c*h*(
-b*h+2*c*g)*(b*f*g-2*c*d*h)-(b*f*h-2*c*e*h+2*c*f*g)*(16*c^2*g^2-3*b^2*h^2-4
*c*h*(-3*a*h+2*b*g)))*x)*(c*x^2+b*x+a)^(1/2)/c^3/h^5
```

Rubi [A]
time = 1.07, antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 828, 857, 635, 212, 738}

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((3*b^4*f*h^4 + 6*b^2*c*h^3*(b*f*g - b*e*h - 2*a*f*h) - 32*c^3*h*(5*b*g - 4
*a*h)*(f*g^2 - h*(e*g - d*h)) + 128*c^4*(f*g^4 - g^2*h*(e*g - d*h)) - 8*b*c
^2*h^2*(3*a*h*(f*g - e*h) - 2*b*(f*g^2 - e*g*h + d*h^2)) + 2*c*h*(8*c*h*(2*
c*g - b*h)*(b*f*g - 2*c*d*h) - (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g^2 - 3*
b^2*h^2 - 4*c*h*(2*b*g - 3*a*h)))*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*h^5) -
 ((8*c*h*(b*f*g - 2*c*d*h) - (8*c*g - 3*b*h)*(2*c*f*g - 2*c*e*h + b*f*h) +
6*c*h*(2*c*f*g - 2*c*e*h + b*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(48*c^2*h^3)
+ (f*(a + b*x + c*x^2)^(5/2))/(5*c*h) - ((4*c*h*(2*c*g - b*h)*(8*c*h*(b*g -
 2*a*h)*(b*f*g - 2*c*d*h) - g*(8*b*c*g - 3*b^2*h - 4*a*c*h)*(2*c*f*g - 2*c*
e*h + b*f*h)) - 2*(4*c^2*g^2 - (b^2*h^2)/2 - 2*c*h*(b*g - a*h))*(8*c*h*(2*c
*g - b*h)*(b*f*g - 2*c*d*h) - (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g^2 - 3*b
^2*h^2 - 4*c*h*(2*b*g - 3*a*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
```

```
*x + c*x^2])])/(256*c^(7/2)*h^6) + ((c*g^2 - b*g*h + a*h^2)^(3/2)*(f*g^2 -
h*(e*g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*
h + a*h^2]*Sqrt[a + b*x + c*x^2])])/h^6
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{g+hx}\,dx = \frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch} + \frac{\int \frac{\left(-\frac{5}{2}h(bfg-2cdh)-\frac{5}{2}h(2cfg-2ceh+bfh)x\right)\left(a+bx+cx^2\right)^{3/}}{g+hx}}{5ch^2}$$

$$= -\frac{(8ch(bfg-2cdh)-(8cg-3bh)(2cfg-2ceh+bfh)+6ch(2cf}{48c^2h^3}$$

$$= \frac{(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h\right.}{}$$

$$= \frac{(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h\right.}{}$$

$$= \frac{(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h\right.}{}$$

$$= \frac{(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h\right.}{}$$

Mathematica [**A**]
time = 10.81, size = 752, normalized size = 1.14

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((2*h*Sqrt[a + x*(b + c*x)]*(45*b^4*f*h^4 - 30*b^2*c*h^3*(10*a*f*h + b*(-3*
f*g + 3*e*h + f*h*x)) + 12*c^2*h^2*(32*a^2*f*h^2 + 2*a*b*h*(-25*f*g + 25*e*
```

```
h + 7*f*h*x) + b^2*(5*h*(-4*e*g + 4*d*h + e*h*x) + f*(20*g^2 - 5*g*h*x + 2*
h^2*x^2))) + 32*c^4*(f*(60*g^4 - 30*g^3*h*x + 20*g^2*h^2*x^2 - 15*g*h^3*x^3
 + 12*h^4*x^4) + 5*h*(2*d*h*(6*g^2 - 3*g*h*x + 2*h^2*x^2) + e*(-12*g^3 + 6*
g^2*h*x - 4*g*h^2*x^2 + 3*h^3*x^3))) + 16*c^3*h*(a*h*(5*h*(-32*e*g + 32*d*h
 + 15*e*h*x) + f*(160*g^2 - 75*g*h*x + 48*h^2*x^2)) + b*(f*(-150*g^3 + 70*g
^2*h*x - 45*g*h^2*x^2 + 33*h^3*x^3) + 5*h*(2*d*h*(-15*g + 7*h*x) + e*(30*g^
2 - 14*g*h*x + 9*h^2*x^2))))))/c^3 + 3840*(c*g^2 + h*(-(b*g) + a*h))^(3/2)*
(f*g^2 + h*(-(e*g) + d*h))*Log[g + h*x] - (15*(3*b^5*f*h^5 - 6*b^3*c*h^4*(-
(b*f*g) + b*e*h + 4*a*f*h) - 384*c^4*g*h*(b*g - a*h)*(f*g^2 + h*(-(e*g) + d
*h)) + 256*c^5*(f*g^5 + g^3*h*(-(e*g) + d*h)) + 16*b*c^2*h^3*(3*a^2*f*h^2 +
 3*a*b*h*(-(f*g) + e*h) + b^2*(f*g^2 - e*g*h + d*h^2)) + 96*c^3*h^2*(a^2*h^
2*(f*g - e*h) + b^2*g*(f*g^2 - e*g*h + d*h^2) - 2*a*b*h*(f*g^2 - e*g*h + d*
h^2)))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(7/2) - 3840*(c*
g^2 + h*(-(b*g) + a*h))^(3/2)*(f*g^2 + h*(-(e*g) + d*h))*Log[-(b*g) + 2*a*h
 - 2*c*g*x + b*h*x + 2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)]
])/(3840*h^6)
```

Maple [A]

time = 0.22, size = 990, normalized size = 1.50

method	result
default	$\frac{fh\left(\frac{(cx^2+bx+a)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(2cx+b)(cx^2+bx+a)^{\frac{3}{2}}}{8c}+\frac{3(4ac-b^2)\left(\frac{(2cx+b)\sqrt{cx^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right.}{8c^{\frac{3}{2}}}\right.}{16c}\right.}{2c}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x,method=_RETURNVERBOSE)
```

```
[Out] 1/h^2*(f*h*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a
)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-
b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+e*h*(1/8*(2*c*x
+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)
```

```
^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
)-g*f*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b
)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2)))))+(d*h^2-e*g*h+f*g^2)/h^3*(1/3*((x+1/h*g)^2*c+(b*h-2*c*g)
/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)/h*(1/4*(2*c*(x+
1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3
/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h
*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)))+(a*h^2-b*g*h+c*g^2)/h^2*(((x+1/
h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*
c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g
)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h
^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g
)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/
h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re deta
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+cx^2)^{\frac{3}{2}}\,(d+ex+fx^2)}{g+hx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{g+h\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

3.201 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

Optimal. Leaf size=754

$$-\frac{(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3g(5fg^2-h(4eg-3dh))+16c^2h(4ah(2fg-eh)-b(19fg^2-\ldots}{64\ldots}$$

```
[Out] -1/24*(3*b*f*h^2*(-a*h+b*g)+8*c^2*g*(5*f*g^2-h*(-3*d*h+4*e*g))+c*h*(8*a*h*(
-e*h+2*f*g)-b*(43*f*g^2-8*h*(-3*d*h+4*e*g)))+6*c*h^2*(4*c*e*g+b*f*g-5*c*f*g
^2/h-4*c*d*h-a*f*h)*x)*(c*x^2+b*x+a)^(3/2)/c/h^3/(a*h^2-b*g*h+c*g^2)-(f*g^2
-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)+1/128*(3*b
^4*f*h^4+8*b^2*c*h^3*(-3*a*f*h-b*e*h+2*b*f*g)+128*c^4*g^2*(5*f*g^2-h*(-3*d*
h+4*e*g))+48*c^2*h^2*(a^2*f*h^2-2*a*b*h*(-e*h+2*f*g)+b^2*(d*h^2-2*e*g*h+3*f
*g^2))+192*c^3*h*(a*h*(d*h^2-2*e*g*h+3*f*g^2)-b*g*(2*d*h^2-3*e*g*h+4*f*g^2)
))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/h^6-1/2*(2*c*
g*(5*f*g^2-h*(-3*d*h+4*e*g))+h*(2*a*h*(-e*h+2*f*g)-b*(3*d*h^2-5*e*g*h+7*f*g
^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x
^2+b*x+a)^(1/2))*(a*h^2-b*g*h+c*g^2)^(1/2)/h^6-1/64*(3*b^3*f*h^3+4*b*c*h^2*
(-3*a*f*h-2*b*e*h+4*b*f*g)+64*c^3*g*(5*f*g^2-h*(-3*d*h+4*e*g))+16*c^2*h*(4*
a*h*(-e*h+2*f*g)-b*(9*d*h^2-14*e*g*h+19*f*g^2))+2*c*h*(3*b^2*f*h^2+4*c*h*(-
3*a*f*h-2*b*e*h+4*b*f*g)-16*c^2*(5*f*g^2-h*(-3*d*h+4*e*g)))*x)*(c*x^2+b*x+a
)^(1/2)/c^2/h^5
```

Rubi [A]
time = 1.45, antiderivative size = 750, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1664, 828, 857, 635, 212, 738}

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -1/64*((3*b^3*f*h^3 + 4*b*c*h^2*(4*b*f*g - 2*b*e*h - 3*a*f*h) + 64*c^3*(5*f
*g^3 - g*h*(4*e*g - 3*d*h)) - 16*c^2*h*(19*b*f*g^2 - b*h*(14*e*g - 9*d*h) -
 4*a*h*(2*f*g - e*h)) + 2*c*h*(3*b^2*f*h^2 + 4*c*h*(4*b*f*g - 2*b*e*h - 3*a
*f*h) - 16*c^2*(5*f*g^2 - h*(4*e*g - 3*d*h)))*x)*Sqrt[a + b*x + c*x^2])/(c^
2*h^5) - ((3*b*f*h*(b*g - a*h) + (8*c^2*(5*f*g^3 - g*h*(4*e*g - 3*d*h)))/h
- c*(43*b*f*g^2 - 8*b*h*(4*e*g - 3*d*h) - 8*a*h*(2*f*g - e*h)) + 6*c*h*(4*c
*e*g + b*f*g - (5*c*f*g^2)/h - 4*c*d*h - a*f*h)*x)*(a + b*x + c*x^2)^(3/2))
/(24*c*h^2*(c*g^2 - b*g*h + a*h^2)) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c
*x^2)^(5/2))/(h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) + ((3*b^4*f*h^4 + 8*b^2*
c*h^3*(2*b*f*g - b*e*h - 3*a*f*h) + 128*c^4*(5*f*g^4 - g^2*h*(4*e*g - 3*d*h
```

```
)) + 48*c^2*h^2*(a^2*f*h^2 - 2*a*b*h*(2*f*g - e*h) + b^2*(3*f*g^2 - 2*e*g*h
 + d*h^2)) + 192*c^3*h*(a*h*(3*f*g^2 - 2*e*g*h + d*h^2) - b*g*(4*f*g^2 - 3*
e*g*h + 2*d*h^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
(128*c^(5/2)*h^6) - (Sqrt[c*g^2 - b*g*h + a*h^2]*(2*c*(5*f*g^3 - g*h*(4*e*g
 - 3*d*h)) - h*(7*b*f*g^2 - b*h*(5*e*g - 3*d*h) - 2*a*h*(2*f*g - e*h)))*Arc
Tanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a
+ b*x + c*x^2])])/(2*h^6)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
```

```
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = -\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{h(cg^2-bgh+ah^2)(g+hx)} - \frac{\int \frac{\left(\frac{1}{2}\left(-2cdg+5beg+2afg-\frac{5bf\ldots}{h}\right.\right.}{\ldots}}{\ldots}$$

$$= -\frac{\left(3bfh(bg-ah)+\frac{8c^2(5fg^3-gh(4eg-3dh))}{h}-c(43bfg^2-8bh(4eg-3\ldots\right.}{24\ldots}$$

$$= -\frac{(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3(5fg^3-gh(4eg-3\ldots}{\ldots}$$

$$= -\frac{(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3(5fg^3-gh(4eg-3\ldots}{\ldots}$$

$$= -\frac{(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3(5fg^3-gh(4eg-3\ldots}{\ldots}$$

$$= -\frac{(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3(5fg^3-gh(4eg-3\ldots}{\ldots}$$

Mathematica [**A**]
time = 10.82, size = 722, normalized size = 0.96

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] ((2*h*Sqrt[a + x*(b + c*x)]*(-9*b^3*f*h^3*(g + h*x) + 6*b*c*h^2*(g + h*x)*(
10*a*f*h + b*(-8*f*g + 4*e*h + f*h*x)) - 16*c^3*(f*(60*g^4 + 30*g^3*h*x - 1
0*g^2*h^2*x^2 + 5*g*h^3*x^3 - 3*h^4*x^4) - 2*h*(3*d*h*(-6*g^2 - 3*g*h*x + h
^2*x^2) + 2*e*(12*g^3 + 6*g^2*h*x - 2*g*h^2*x^2 + h^3*x^3))) + 8*c^2*h*(a*h
*(8*h*(7*e*g - 3*d*h + 4*e*h*x) + f*(-88*g^2 - 49*g*h*x + 15*h^2*x^2)) + b*
(f*(114*g^3 + 62*g^2*h*x - 19*g*h^2*x^2 + 9*h^3*x^3) + 2*h*(3*d*h*(9*g + 5*
h*x) + e*(-42*g^2 - 23*g*h*x + 7*h^2*x^2))))))/(c^2*(g + h*x)) - 192*Sqrt[c
*g^2 + h*(-(b*g) + a*h)]*(2*c*(5*f*g^3 + g*h*(-4*e*g + 3*d*h)) + h*(-7*b*f*
g^2 + b*h*(5*e*g - 3*d*h) - 2*a*h*(-2*f*g + e*h)))*Log[g + h*x] + (3*(3*b^4
*f*h^4 - 8*b^2*c*h^3*(-2*b*f*g + b*e*h + 3*a*f*h) + 128*c^4*(5*f*g^4 + g^2*
h*(-4*e*g + 3*d*h)) + 48*c^2*h^2*(a^2*f*h^2 + 2*a*b*h*(-2*f*g + e*h) + b^2*
(3*f*g^2 - 2*e*g*h + d*h^2)) - 192*c^3*h*(-(a*h*(3*f*g^2 - 2*e*g*h + d*h^2)
) + b*g*(4*f*g^2 - 3*e*g*h + 2*d*h^2)))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a +
x*(b + c*x)]])/c^(5/2) + 192*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*(2*c*(5*f*g^3 +
 g*h*(-4*e*g + 3*d*h)) + h*(-7*b*f*g^2 + b*h*(5*e*g - 3*d*h) - 2*a*h*(-2*f*
g + e*h)))*Log[-(b*g) + 2*a*h - 2*c*g*x + b*h*x + 2*Sqrt[c*g^2 + h*(-(b*g)
+ a*h)]*Sqrt[a + x*(b + c*x)]])/(384*h^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1852 vs. $2(724) = 1448$.
time = 0.19, size = 1853, normalized size = 2.46

method	result	size
default	Expression too large to display	1853
risch	Expression too large to display	10250

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^2*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b
)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))))+1/h^3*(e*h-2*f*g)*(1/3*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1
/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)/h*(1/4*(2*c*(x+1/h*g)+
(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)
/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln(
(1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h
*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)))+(a*h^2-b*g*h+c*g^2)/h^2*(((x+1/h*g)^2*
c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*
ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+
1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+
```

```
1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/
h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))))+1/h^4*(d*h^2-e*g*h+f*g^2)
*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*
g)+(a*h^2-b*g*h+c*g^2)/h^2)^(5/2)+3/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(1/
3*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2
*(b*h-2*c*g)/h*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c
*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)
/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+
((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)))+(a*
h^2-b*g*h+c*g^2)/h^2*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c
*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/
2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c
^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2
-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+
1/h*g))))+4*c/(a*h^2-b*g*h+c*g^2)*h^2*(1/8*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*
((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+3/16*
(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c*(1/4*(2*c*(x+1/h*g)+(b*h-
2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)
^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*
(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re deta
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{\left(g+hx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**2,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

3.202 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3} dx$

Optimal. Leaf size=824

$$-\frac{(b^2fh^3(bg-ah)-8c^3g^2(10fg^2-3h(2eg-dh))-2c^2h(2ah(19fg^2-9egh+3dh^2)-3bg(22fg^2-12egh}{}$$

```
[Out] -1/12*(4*c*g*(6*e*g-10*f*g^2/h-3*d*h)-4*a*h*(-3*e*h+7*f*g)+b*(31*f*g^2-3*h*
(-d*h+5*e*g))+2*h*(3*c*e*g+2*b*f*g-5*c*f*g^2/h-3*c*d*h-2*a*f*h)*x)*(c*x^2+b
*x+a)^(3/2)/h^2/(a*h^2-b*g*h+c*g^2)/(h*x+g)-1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2
+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^2-1/16*(b^3*f*h^3+6*b*c*h^2*(-2
*a*f*h-b*e*h+3*b*f*g)+16*c^3*g*(10*f*g^2-3*h*(-d*h+2*e*g))+24*c^2*h*(a*h*(-
e*h+3*f*g)-b*(d*h^2-3*e*g*h+6*f*g^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2
+b*x+a)^(1/2))/c^(3/2)/h^6+1/8*(8*c^2*g^2*(10*f*g^2-3*h*(-d*h+2*e*g))+4*c*h
*(a*h*(3*d*h^2-9*e*g*h+19*f*g^2)-b*g*(6*d*h^2-15*e*g*h+28*f*g^2))+h^2*(8*a^
2*f*h^2-4*a*b*h*(-3*e*h+10*f*g)+b^2*(35*f*g^2-3*h*(-d*h+5*e*g))))*arctanh(1
/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2)
)/h^6/(a*h^2-b*g*h+c*g^2)^(1/2)-1/8*(b^2*f*h^3*(-a*h+b*g)-8*c^3*g^2*(10*f*g
^2-3*h*(-d*h+2*e*g))-2*c^2*h*(2*a*h*(3*d*h^2-9*e*g*h+19*f*g^2)-3*b*g*(5*d*h
^2-12*e*g*h+22*f*g^2))-c*h^2*(8*a^2*f*h^2-18*a*b*h*(-e*h+3*f*g)+b^2*(53*f*g
^2-6*h*(-d*h+4*e*g)))+2*c*h*(b*f*h^2*(-a*h+b*g)+2*c^2*g*(10*f*g^2-3*h*(-d*h
+2*e*g))+c*h*(2*a*h*(-3*e*h+7*f*g)-3*b*(d*h^2-3*e*g*h+6*f*g^2)))*x)*(c*x^2+
b*x+a)^(1/2)/c/h^5/(a*h^2-b*g*h+c*g^2)
```

Rubi [A]
time = 1.26, antiderivative size = 819, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1664, 826, 828, 857, 635, 212, 738}

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] -1/8*((b^2*f*h^2*(b*g - a*h) + 8*c^3*g^2*(6*e*g - (10*f*g^2)/h - 3*d*h) - 2
*c^2*(2*a*h*(19*f*g^2 - 9*e*g*h + 3*d*h^2) - 3*b*g*(22*f*g^2 - 12*e*g*h + 5
*d*h^2)) - c*h*(8*a^2*f*h^2 - 18*a*b*h*(3*f*g - e*h) + b^2*(53*f*g^2 - 6*h*
(4*e*g - d*h))) + 2*c*(b*f*h^2*(b*g - a*h) + 2*c^2*(10*f*g^3 - 3*g*h*(2*e*g
 - d*h)) + c*h*(2*a*h*(7*f*g - 3*e*h) - 3*b*(6*f*g^2 - 3*e*g*h + d*h^2)))*x
)*Sqrt[a + b*x + c*x^2])/(c*h^4*(c*g^2 - b*g*h + a*h^2)) - ((31*b*f*g^2 + 4
*c*g*(6*e*g - (10*f*g^2)/h - 3*d*h) - 3*b*h*(5*e*g - d*h) - 4*a*h*(7*f*g -
3*e*h) + 2*h*(3*c*e*g + 2*b*f*g - (5*c*f*g^2)/h - 3*c*d*h - 2*a*f*h)*x)*(a
+ b*x + c*x^2)^(3/2))/(12*h^2*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) - ((f*g^2
```

```
- h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(2*h*(c*g^2 - b*g*h + a*h^2)*(g +
 h*x)^2) - ((b^3*f*h^3 + 6*b*c*h^2*(3*b*f*g - b*e*h - 2*a*f*h) + 16*c^3*(10
*f*g^3 - 3*g*h*(2*e*g - d*h)) - 24*c^2*h*(6*b*f*g^2 - b*h*(3*e*g - d*h) - a
*h*(3*f*g - e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
(16*c^(3/2)*h^6) + ((8*c^2*(10*f*g^4 - 3*g^2*h*(2*e*g - d*h)) - 4*c*h*(28*b
*f*g^3 - 3*b*g*h*(5*e*g - 2*d*h) - a*h*(19*f*g^2 - 9*e*g*h + 3*d*h^2)) + h^
2*(8*a^2*f*h^2 - 4*a*b*h*(10*f*g - 3*e*h) + b^2*(35*f*g^2 - 3*h*(5*e*g - d*
h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2
]*Sqrt[a + b*x + c*x^2])])/(8*h^6*Sqrt[c*g^2 - b*g*h + a*h^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
```

```
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = -\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{2h(cg^2-bgh+ah^2)(g+hx)^2} - \int \frac{\left(\frac{1}{2}\left(-4cdg+5beg+4afg-5b\right.\right.}{}$$

$$= -\frac{\left(31bfg^2+4cg\left(6eg-\frac{10fg^2}{h}-3dh\right)-3bh(5eg-dh)-4ah(7f\right.}{12h^2(cg^2-}$$

$$= -\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2(2ah(19fg^2\right.}{}$$

$$= -\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2(2ah(19fg^2\right.}{}$$

$$= -\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2(2ah(19fg^2\right.}{}$$

$$= -\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2(2ah(19fg^2\right.}{}$$

Mathematica [A]
time = 11.06, size = 692, normalized size = 0.84

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] (2*h*Sqrt[a + x*(b + c*x)]*((3*b^2*f*h^2)/c + 24*c*(6*f*g^2 + h*(-3*e*g + d
*h)) + 2*h*(16*a*f*h + 15*b*(-3*f*g + e*h)) + 2*h*(7*b*f*h + 6*c*(-3*f*g +
e*h))*x + 8*c*f*h^2*x^2 - (12*(c*g^2 + h*(-(b*g) + a*h))*(f*g^2 + h*(-(e*g)
 + d*h)))/(g + h*x)^2 + (12*c*(9*f*g^3 + g*h*(-7*e*g + 5*d*h)) - 6*h*(13*b*
f*g^2 + b*h*(-9*e*g + 5*d*h) + 4*a*h*(-2*f*g + e*h)))/(g + h*x)) + (6*(8*c^
2*(10*f*g^4 + 3*g^2*h*(-2*e*g + d*h)) - 4*c*h*(28*b*f*g^3 + 3*b*g*h*(-5*e*g
 + 2*d*h) + a*h*(-19*f*g^2 + 9*e*g*h - 3*d*h^2)) + h^2*(8*a^2*f*h^2 + 4*a*b
*h*(-10*f*g + 3*e*h) + b^2*(35*f*g^2 + 3*h*(-5*e*g + d*h))))*Log[g + h*x])/
Sqrt[c*g^2 + h*(-(b*g) + a*h)] - (3*(b^3*f*h^3 - 6*b*c*h^2*(-3*b*f*g + b*e*
h + 2*a*f*h) + 16*c^3*(10*f*g^3 + 3*g*h*(-2*e*g + d*h)) - 24*c^2*h*(6*b*f*g
^2 + b*h*(-3*e*g + d*h) + a*h*(-3*f*g + e*h)))*Log[b + 2*c*x + 2*Sqrt[c]*Sq
rt[a + x*(b + c*x)]])/c^(3/2) - (6*(8*c^2*(10*f*g^4 + 3*g^2*h*(-2*e*g + d*h
)) - 4*c*h*(28*b*f*g^3 + 3*b*g*h*(-5*e*g + 2*d*h) + a*h*(-19*f*g^2 + 9*e*g*
```

```
h - 3*d*h^2)) + h^2*(8*a^2*f*h^2 + 4*a*b*h*(-10*f*g + 3*e*h) + b^2*(35*f*g^
2 + 3*h*(-5*e*g + d*h))))*Log[-(b*g) + 2*a*h - 2*c*g*x + b*h*x + 2*Sqrt[c*g
^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)]])/Sqrt[c*g^2 + h*(-(b*g) + a*h
)])/(48*h^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3599 vs. $2(792) = 1584$.
time = 0.19, size = 3600, normalized size = 4.37

method	result	size
default	Expression too large to display	3600
risch	Expression too large to display	19889

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

```
[Out] f/h^3*(1/3*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^
(3/2)+1/2*(b*h-2*c*g)/h*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c
+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g
*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))
/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1
/2)))+(a*h^2-b*g*h+c*g^2)/h^2*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^
2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*
g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)
^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(
(2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h
^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(
1/2))/(x+1/h*g))))+(e*h-2*f*g)/h^4*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((
x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(5/2)+3/2*(b*
h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(1/3*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+
(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)/h*(1/4*(2*c*(x+1/h*g)+(b*h-2
*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^
(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(
b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a
*h^2-b*g*h+c*g^2)/h^2)^(1/2)))+(a*h^2-b*g*h+c*g^2)/h^2*(((x+1/h*g)^2*c+(b*h
-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/
2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)
+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*
g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)
+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(
a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))))+4*c/(a*h^2-b*g*h+c*g^2)*h^2*(1/
8*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a
*h^2-b*g*h+c*g^2)/h^2)^(3/2)+3/16*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^
2/h^2)/c*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c*g)/h*
(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(
```

```
b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/
h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)))))+(d*h^2-
e*g*h+f*g^2)/h^5*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(
b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(5/2)+1/4*(b*h-2*c*g)*h/(a*
h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-
2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(5/2)+3/2*(b*h-2*c*g)*h/(a*h^2-
b*g*h+c*g^2)*(1/3*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2
)/h^2)^(3/2)+1/2*(b*h-2*c*g)/h*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h
*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*
h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+
1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)))+(a*h^2-b*g*h+c*g^2)/h^2*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g
)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*
(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^
2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1
/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c
*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)
/h^2)^(1/2))/(x+1/h*g))))+4*c/(a*h^2-b*g*h+c*g^2)*h^2*(1/8*(2*c*(x+1/h*g)+(
b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/
h^2)^(3/2)+3/16*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c*(1/4*(2*c
*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b
*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/
c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*
g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)))))+3/2*c/(a*h^2-b*g*h+c*g^2)
*h^2*(1/3*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(
3/2)+1/2*(b*h-2*c*g)/h*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+
(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*
h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/
c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/
2)))+(a*h^2-b*g*h+c*g^2)/h^2*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2
-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g
))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^
(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((
2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^
2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1
/2))/(x+1/h*g)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+cx^2)^{\frac{3}{2}}\,(d+ex+fx^2)}{(g+hx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**3,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.41Unable to divide
, perhaps due to rounding error%%%{1,[6,0,0,0,9,0,0,0]%%%}+%%%{%%{[-6,0]:[1
,0,%%%
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^2+b\,x+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

3.203 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx$

Optimal. Leaf size=833

$$-\frac{\left(8c^2g^2(10fg^2-h(4eg-dh))-2ch(3bg(18fg^2-6egh+dh^2)-2ah(23fg^2-8egh+2dh^2))+h^2(12a^2f\right.}{}$$

```
[Out] -1/12*(2*c*g*(4*e*g-10*f*g^2/h-d*h)-6*a*h*(-e*h+3*f*g)+b*(17*f*g^2-h*(d*h+5
*e*g))+2*h*(2*c*e*g+3*b*f*g-5*c*f*g^2/h-2*c*d*h-3*a*f*h)*x)*(c*x^2+b*x+a)^(
3/2)/h^2/(a*h^2-b*g*h+c*g^2)/(h*x+g)^2-1/3*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+
a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^3-1/16*(16*c^3*g^3*(10*f*g^2-h*(-d*h
+4*e*g))-b*h^3*(24*a^2*f*h^2-6*a*b*h*(-e*h+10*f*g)+b^2*(-d*h^2-5*e*g*h+35*f
*g^2))+6*c*h^2*(4*a^2*h^2*(-e*h+4*f*g)+b^2*g*(d*h^2-10*e*g*h+35*f*g^2)-2*a*
b*h*(d*h^2-7*e*g*h+25*f*g^2))-24*c^2*g*h*(b*g*(d*h^2-5*e*g*h+14*f*g^2)-a*h*
(d*h^2-4*e*g*h+11*f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*
g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/(a*h^2-b*g*h+c*g^2)^(3/2)+1/8*(3*
b^2*f*h^2-12*c*h*(-a*f*h-b*e*h+4*b*f*g)+8*c^2*(10*f*g^2-h*(-d*h+4*e*g)))*ar
ctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/c^(1/2)-1/8*(8*c^2*g^2
*(10*f*g^2-h*(-d*h+4*e*g))-2*c*h*(3*b*g*(d*h^2-6*e*g*h+18*f*g^2)-2*a*h*(2*d
*h^2-8*e*g*h+23*f*g^2))+h^2*(12*a^2*f*h^2-6*a*b*h*(-e*h+7*f*g)+b^2*(29*f*g^
2-h*(d*h+5*e*g)))+2*h*(3*b*f*h^2*(-a*h+b*g)+2*c^2*g*(10*f*g^2-h*(-d*h+4*e*g
))+c*h*(6*a*h*(-e*h+3*f*g)-b*(d*h^2-7*e*g*h+22*f*g^2)))*x)*(c*x^2+b*x+a)^(1
/2)/h^5/(a*h^2-b*g*h+c*g^2)/(h*x+g)
```

Rubi [A]
time = 1.33, antiderivative size = 829, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1664, 826, 857, 635, 212, 738}

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -1/8*((12*a^2*f*h^3 - 8*c^2*g^2*(4*e*g - (10*f*g^2)/h - d*h) - 6*a*b*h^2*(7
*f*g - e*h) + 4*a*c*h*(23*f*g^2 - 2*h*(4*e*g - d*h)) - 6*b*c*g*(18*f*g^2 -
h*(6*e*g - d*h)) + b^2*h*(29*f*g^2 - h*(5*e*g + d*h)) + 2*(3*b*f*h^2*(b*g -
 a*h) + 2*c^2*(10*f*g^3 - g*h*(4*e*g - d*h)) - c*h*(22*b*f*g^2 - b*h*(7*e*g
 - d*h) - 6*a*h*(3*f*g - e*h)))*x)*Sqrt[a + b*x + c*x^2])/(h^4*(c*g^2 - b*g
*h + a*h^2)*(g + h*x)) - ((17*b*f*g^2 + 2*c*g*(4*e*g - (10*f*g^2)/h - d*h)
- b*h*(5*e*g + d*h) - 6*a*h*(3*f*g - e*h) + 2*h*(2*c*e*g + 3*b*f*g - (5*c*f
*g^2)/h - 2*c*d*h - 3*a*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(12*h^2*(c*g^2 - b
*g*h + a*h^2)*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/
```

```
2))/(3*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) + ((3*b^2*f*h^2 - 12*c*h*(4*b
*f*g - b*e*h - a*f*h) + 8*c^2*(10*f*g^2 - h*(4*e*g - d*h)))*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*h^6) - ((16*c^3*(10*f*g
^5 - g^3*h*(4*e*g - d*h)) - b*h^3*(24*a^2*f*h^2 - 6*a*b*h*(10*f*g - e*h) +
b^2*(35*f*g^2 - 5*e*g*h - d*h^2)) + 6*c*h^2*(4*a^2*h^2*(4*f*g - e*h) + b^2*
g*(35*f*g^2 - 10*e*g*h + d*h^2) - 2*a*b*h*(25*f*g^2 - 7*e*g*h + d*h^2)) - 2
4*c^2*g*h*(b*g*(14*f*g^2 - 5*e*g*h + d*h^2) - a*h*(11*f*g^2 - 4*e*g*h + d*h
^2)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2
]*Sqrt[a + b*x + c*x^2])])/(16*h^6*(c*g^2 - b*g*h + a*h^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
```

```
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = -\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{3h(cg^2-bgh+ah^2)(g+hx)^3} - \int \frac{\left(\frac{1}{2}\left(-6cdg+5beg+6afg-5\ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(17bfg^2+2cg\left(4eg-\frac{10fg^2}{h}-dh\right)-bh(5eg+dh)-6ah(3fg-\ldots\right.}{12h^2(cg^2-b\ldots}$$

$$= -\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ac\ldots\right.}{\ldots}$$

$$= -\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ac\ldots\right.}{\ldots}$$

$$= -\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ac\ldots\right.}{\ldots}$$

$$= -\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ac\ldots\right.}{\ldots}$$

Mathematica [A]
time = 13.68, size = 879, normalized size = 1.06

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] (2*h*Sqrt[a + x*(b + c*x)]*(6*(5*b*f*h + 4*c*(-4*f*g + e*h)) + 12*c*f*h*x -
 (8*(c*g^2 + h*(-(b*g) + a*h))*(f*g^2 + h*(-(e*g) + d*h)))/(g + h*x)^3 + (2
*(26*c*f*g^3 + 2*c*g*h*(-10*e*g + 7*d*h) - 6*a*h^2*(-2*f*g + e*h) + b*h*(-1
9*f*g^2 + h*(13*e*g - 7*d*h))))/(g + h*x)^2 - (4*c^2*(47*f*g^4 + g^2*h*(-26
*e*g + 11*d*h)) + 3*h^2*(8*a^2*f*h^2 + 2*a*b*h*(-18*f*g + 5*e*h) + b^2*(29*
f*g^2 - 11*e*g*h + d*h^2)) + 2*c*h*(b*g*(-136*f*g^2 + 67*e*g*h - 22*d*h^2)
+ 2*a*h*(50*f*g^2 - 23*e*g*h + 8*d*h^2)))/((c*g^2 + h*(-(b*g) + a*h))*(g +
h*x))) - (3*(16*c^3*(10*f*g^5 + g^3*h*(-4*e*g + d*h)) + 6*c*h^2*(-4*a^2*h^2
*(-4*f*g + e*h) + b^2*g*(35*f*g^2 - 10*e*g*h + d*h^2) - 2*a*b*h*(25*f*g^2 -
 7*e*g*h + d*h^2)) - 24*c^2*g*h*(b*g*(14*f*g^2 - 5*e*g*h + d*h^2) - a*h*(11
*f*g^2 - 4*e*g*h + d*h^2)) + b*h^3*(-24*a^2*f*h^2 - 6*a*b*h*(-10*f*g + e*h)
 + b^2*(-35*f*g^2 + 5*e*g*h + d*h^2)))*Log[g + h*x])/(c*g^2 + h*(-(b*g) + a
*h))^(3/2) + (6*(3*b^2*f*h^2 + 12*c*h*(-4*b*f*g + b*e*h + a*f*h) + 8*c^2*(1
0*f*g^2 + h*(-4*e*g + d*h)))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)
]])/Sqrt[c] + (3*(16*c^3*(10*f*g^5 + g^3*h*(-4*e*g + d*h)) + 6*c*h^2*(-4*a^
2*h^2*(-4*f*g + e*h) + b^2*g*(35*f*g^2 - 10*e*g*h + d*h^2) - 2*a*b*h*(25*f*
g^2 - 7*e*g*h + d*h^2)) - 24*c^2*g*h*(b*g*(14*f*g^2 - 5*e*g*h + d*h^2) - a*
h*(11*f*g^2 - 4*e*g*h + d*h^2)) + b*h^3*(-24*a^2*f*h^2 - 6*a*b*h*(-10*f*g +
 e*h) + b^2*(-35*f*g^2 + 5*e*g*h + d*h^2)))*Log[-(b*g) + 2*a*h - 2*c*g*x +
b*h*x + 2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)]])/(c*g^2 + h
*(-(b*g) + a*h))^(3/2))/(48*h^6)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6059 vs. $2(801) = 1602$.
time = 0.16, size = 6060, normalized size = 7.27

method	result	size
default	Expression too large to display	6060
risch	Expression too large to display	33510

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{\left(g+hx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**4,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7319 vs. 2(815) = 1630.
time = 11.49, size = 7319, normalized size = 8.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*c*f*x/h^4 - (16*c^2*f*g*h^10 - 5*b*c*f*h^11 -
4*c^2*h^11*e)/(c*h^15)) - 1/8*(160*c^3*f*g^5 - 336*b*c^2*f*g^4*h + 16*c^3*d
*g^3*h^2 + 210*b^2*c*f*g^3*h^2 + 264*a*c^2*f*g^3*h^2 - 24*b*c^2*d*g^2*h^3 -
 35*b^3*f*g^2*h^3 - 300*a*b*c*f*g^2*h^3 + 6*b^2*c*d*g*h^4 + 24*a*c^2*d*g*h^
4 + 60*a*b^2*f*g*h^4 + 96*a^2*c*f*g*h^4 + b^3*d*h^5 - 12*a*b*c*d*h^5 - 24*a
^2*b*f*h^5 - 64*c^3*g^4*h*e + 120*b*c^2*g^3*h^2*e - 60*b^2*c*g^2*h^3*e - 96
*a*c^2*g^2*h^3*e + 5*b^3*g*h^4*e + 84*a*b*c*g*h^4*e - 6*a*b^2*h^5*e - 24*a^
2*c*h^5*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt
(-c*g^2 + b*g*h - a*h^2))/((c*g^2*h^6 - b*g*h^7 + a*h^8)*sqrt(-c*g^2 + b*g*
h - a*h^2)) - 1/24*(480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^(7/2)*f*g^5
```

h^2 - 912(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^(5/2)*f*g^4*h^3 + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^(7/2)*d*g^3*h^4 + 522*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c^(3/2)*f*g^3*h^4 + 552*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^(5/2)*f*g^3*h^4 - 216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^(5/2)*d*g^2*h^5 - 87*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*sqrt(c)*f*g^2*h^5 - 540*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^(3/2)*f*g^2*h^5 + 78*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c^(3/2)*d*g*h^6 + 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^(5/2)*d*g*h^6 + 108*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*sqrt(c)*f*g*h^6 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c^(3/2)*f*g*h^6 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*sqrt(c)*d*h^7 - 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^(3/2)*d*h^7 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*sqrt(c)*f*h^7 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^(7/2)*g^4*h^3*e + 504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^(5/2)*g^3*h^4*e - 252*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c^(3/2)*g^2*h^5*e - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^(5/2)*g^2*h^5*e + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*sqrt(c)*g*h^6*e + 228*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^(3/2)*g*h^6*e - 30*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*sqrt(c)*h^7*e - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c^(3/2)*h^7*e + 1680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^4*f*g^6*h - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^3*f*g^5*h^2 + 432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^4*d*g^4*h^3 + 1362*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^2*f*g^4*h^3 + 1464*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^3*f*g^4*h^3 - 504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^3*d*g^3*h^4 - 147*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c*f*g^3*h^4 - 876*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^2*f*g^3*h^4 + 54*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^2*d*g^2*h^5 + 216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^3*d*g^2*h^5 - 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c*f*g^2*h^5 - 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^2*f*g^2*h^5 + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c*d*g*h^6 + 84*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^2*d*g*h^6 + 216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*c*f*g*h^6 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c*d*h^7 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^2*d*h^7 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*c*f*h^7 - 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^4*g^5*h^2*e + 1464*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^3*g^4*h^3*e - 540*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^2*g^3*h^4*e - 672*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^3*g^3*h^4*e + 21*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c*g^2*h^5*e + 180*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^2*g^2*h^5*e + 90*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c*g*h^6*e + 168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^2*g*h^6*e - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*c*h^7*e + 1504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^(9/2)*f*g^7 - 1072*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^(7/2)*f*g^6*h + 352*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^(9/2)*d*g^5*h^2 - 1308*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^(5/2)*f*g^5*h^2 - 656*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^(7/2)*f*g^5*h^2 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^(7/

```
2)*d*g^4*h^3 + 1042*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c^(3/2)*f*g^4
*h^3 + 4056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^(5/2)*f*g^4*h^3 - 4
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^(5/2)*d*g^3*h^4 - 272*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^(7/2)*d*g^3*h^4 - 136*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^3*b^4*sqrt(c)*f*g^3*h^4 - 2712*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^3*a*b^2*c^(3/2)*f*g^3*h^4 - 2208*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*a^2*c^(5/2)*f*g^3*h^4 + 106*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^
3*c^(3/2)*d*g^2*h^5 + 840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^(5/2)
*d*g^2*h^5 + 328*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*sqrt(c)*f*g^2*
h^5 + 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c^(3/2)*f*g^2*h^5 +
8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*sqrt(c)*d*g*h^6 - 144*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c^(3/2)*d*g*...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

3.204 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx$

Optimal. Leaf size=1097

$$(64c^3g^4(5fg-eh)-16c^2g^2h(bg(41fg-7eh)-8ah(5fg-eh))+4ch^2(2b^2g^2(46fg-5eh)+16a^2h^2(5fg-$$

```
[Out] -1/96*(16*c^2*g^4*(-e*h+5*f*g)-h^2*(16*a^2*h^2*(-2*e*h+f*g)-b^2*g*(3*d*h^2+
5*e*g*h+35*f*g^2)+4*a*b*h*(3*d*h^2+7*e*g*h+7*f*g^2))-4*c*g*h*(b*g*(3*d*h^2-
5*e*g*h+31*f*g^2)-a*h*(9*d*h^2-5*e*g*h+25*f*g^2))+3*h*(8*c^2*g^2*(5*f*g^2-h
*(d*h+e*g))+h^2*(16*a^2*f*h^2-8*a*b*h*(-e*h+6*f*g)+b^2*(-3*d*h^2-5*e*g*h+29
*f*g^2))-4*c*h*(2*b*g*(-d*h^2-2*e*g*h+9*f*g^2)-a*h*(d*h^2-5*e*g*h+17*f*g^2)
))*x)*(c*x^2+b*x+a)^(3/2)/h^3/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^3-1/4*(f*g^2-h*
(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^4+1/128*(128*
c^4*g^5*(-e*h+5*f*g)-64*c^3*g^3*h*(b*g*(-5*e*h+28*f*g)-5*a*h*(-e*h+5*f*g))+
8*c*h^3*(24*a^3*f*h^3-12*a^2*b*h^2*(-e*h+10*f*g)-5*b^3*g^2*(-e*h+14*f*g)+3*
a*b^2*h*(-d*h^2-5*e*g*h+55*f*g^2))-48*c^2*h^2*(10*a*b*g^2*h*(-e*h+6*f*g)-5*
b^2*g^3*(-e*h+7*f*g)-a^2*h^2*(d*h^2-5*e*g*h+25*f*g^2))+b^2*h^4*(48*a^2*f*h^
2-8*a*b*h*(e*h+10*f*g)+b^2*(3*d*h^2+5*e*g*h+35*f*g^2)))*arctanh(1/2*(b*g-2*
a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/(a*h
^2-b*g*h+c*g^2)^(5/2)-1/2*(-3*b*f*h-2*c*e*h+10*c*f*g)*arctanh(1/2*(2*c*x+b)
/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/h^6+1/64*(64*c^3*g^4*(-e*h+5*f*g)-16*
c^2*g^2*h*(b*g*(-7*e*h+41*f*g)-8*a*h*(-e*h+5*f*g))+4*c*h^2*(2*b^2*g^2*(-5*e
*h+46*f*g)+16*a^2*h^2*(-e*h+5*f*g)-a*b*h*(-3*d*h^2-25*e*g*h+173*f*g^2))-b*h
^3*(48*a^2*f*h^2-8*a*b*h*(e*h+10*f*g)+b^2*(3*d*h^2+5*e*g*h+35*f*g^2))+2*c*h
*(16*c^2*g^3*(-e*h+5*f*g)-4*c*h*(6*b*g^2*(-e*h+6*f*g)-a*h*(35*f*g^2-h*(-3*d
*h+7*e*g)))+h^2*(48*a^2*f*h^2-8*a*b*h*(-e*h+14*f*g)+b^2*(61*f*g^2-h*(3*d*h+
5*e*g))))*x)*(c*x^2+b*x+a)^(1/2)/h^5/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)
```

Rubi [A]

time = 1.89, antiderivative size = 1096, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1664, 824, 826, 857, 635, 212, 738}

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (((64*c^3*g^4*(5*f*g - e*h))/h - 16*c^2*g^2*(b*g*(41*f*g - 7*e*h) - 8*a*h*(
5*f*g - e*h)) + 4*c*h*(2*b^2*g^2*(46*f*g - 5*e*h) + 16*a^2*h^2*(5*f*g - e*h
) - a*b*h*(173*f*g^2 - 25*e*g*h - 3*d*h^2)) - b*h^2*(48*a^2*f*h^2 - 8*a*b*h
*(10*f*g + e*h) + b^2*(35*f*g^2 + 5*e*g*h + 3*d*h^2)) + 2*c*(16*c^2*g^3*(5*
```

```
f*g - e*h) - 4*c*h*(6*b*g^2*(6*f*g - e*h) - a*h*(35*f*g^2 - h*(7*e*g - 3*d*
h))) + h^2*(48*a^2*f*h^2 - 8*a*b*h*(14*f*g - e*h) + b^2*(61*f*g^2 - h*(5*e*
g + 3*d*h))))*x)*Sqrt[a + b*x + c*x^2])/(64*h^4*(c*g^2 - b*g*h + a*h^2)^2*(
g + h*x)) - (((16*c^2*g^4*(5*f*g - e*h))/h - h*(16*a^2*h^2*(f*g - 2*e*h) -
b^2*g*(35*f*g^2 + 5*e*g*h + 3*d*h^2) + 4*a*b*h*(7*f*g^2 + 7*e*g*h + 3*d*h^2
)) - 4*c*g*(b*g*(31*f*g^2 - 5*e*g*h + 3*d*h^2) - a*h*(25*f*g^2 - 5*e*g*h +
9*d*h^2)) + 3*h*((40*c^2*f*g^4)/h + 16*a^2*f*h^3 - 8*c^2*g^2*(e*g + d*h) -
8*a*b*h^2*(6*f*g - e*h) + 4*a*c*h*(17*f*g^2 - h*(5*e*g - d*h)) - 8*b*c*g*(9
*f*g^2 - h*(2*e*g + d*h)) + b^2*h*(29*f*g^2 - h*(5*e*g + 3*d*h)))*x)*(a + b
*x + c*x^2)^(3/2))/(96*h^2*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^3) - ((f*g^2
 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(4*h*(c*g^2 - b*g*h + a*h^2)*(g
+ h*x)^4) - (Sqrt[c]*(10*c*f*g - 2*c*e*h - 3*b*f*h)*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*h^6) + ((128*c^4*g^5*(5*f*g - e*h) - 64
*c^3*g^3*h*(b*g*(28*f*g - 5*e*h) - 5*a*h*(5*f*g - e*h)) + 8*c*h^3*(24*a^3*f
*h^3 - 12*a^2*b*h^2*(10*f*g - e*h) - 5*b^3*g^2*(14*f*g - e*h) + 3*a*b^2*h*(
55*f*g^2 - 5*e*g*h - d*h^2)) - 48*c^2*h^2*(10*a*b*g^2*h*(6*f*g - e*h) - 5*b
^2*g^3*(7*f*g - e*h) - a^2*h^2*(25*f*g^2 - 5*e*g*h + d*h^2)) + b^2*h^4*(48*
a^2*f*h^2 - 8*a*b*h*(10*f*g + e*h) + b^2*(35*f*g^2 + 5*e*g*h + 3*d*h^2)))*A
rcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[
a + b*x + c*x^2])])/(128*h^6*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
```

```
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = -\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{4h(cg^2-bgh+ah^2)(g+hx)^4} - \frac{\int \frac{\left(\frac{1}{2}\left(-8cdg+5beg+8afg-5b\right.\right.}{}}{}$$

$$= -\frac{\left(\frac{16c^2g^4(5fg-eh)}{h} - h(16a^2h^2(fg-2eh) - b^2g(35fg^2+5egh+3d\right.}{}$$

$$= \frac{\left(\frac{64c^3g^4(5fg-eh)}{h} - 16c^2g^2(bg(41fg-7eh) - 8ah(5fg-eh)) + 4ch\right.}{}$$

$$= \frac{\left(\frac{64c^3g^4(5fg-eh)}{h} - 16c^2g^2(bg(41fg-7eh) - 8ah(5fg-eh)) + 4ch\right.}{}$$

$$= \frac{\left(\frac{64c^3g^4(5fg-eh)}{h} - 16c^2g^2(bg(41fg-7eh) - 8ah(5fg-eh)) + 4ch\right.}{}$$

$$= \frac{\left(\frac{64c^3g^4(5fg-eh)}{h} - 16c^2g^2(bg(41fg-7eh) - 8ah(5fg-eh)) + 4ch\right.}{}$$

Mathematica [A]
time = 16.57, size = 1835, normalized size = 1.67

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] ((a + x*(b + c*x))^(3/2)*((c*f)/h^5 - ((c*g^2 - b*g*h + a*h^2)*(f*g^2 - e*g
*h + d*h^2))/(4*h^5*(g + h*x)^4) + (34*c*f*g^3 - 26*c*e*g^2*h - 25*b*f*g^2*
h + 18*c*d*g*h^2 + 17*b*e*g*h^2 + 16*a*f*g*h^2 - 9*b*d*h^3 - 8*a*e*h^3)/(24
*h^5*(g + h*x)^3) + (-344*c^2*f*g^4 + 184*c^2*e*g^3*h + 504*b*c*f*g^3*h - 7
2*c^2*d*g^2*h^2 - 240*b*c*e*g^2*h^2 - 163*b^2*f*g^2*h^2 - 380*a*c*f*g^2*h^2
 + 72*b*c*d*g*h^3 + 59*b^2*e*g*h^3 + 172*a*c*e*g*h^3 + 208*a*b*f*g*h^3 - 3*
b^2*d*h^4 - 60*a*c*d*h^4 - 56*a*b*e*h^4 - 48*a^2*f*h^4)/(96*h^5*(c*g^2 - b*
g*h + a*h^2)*(g + h*x)^2) + (1232*c^3*f*g^5 - 400*c^3*e*g^4*h - 2680*b*c^2*
f*g^4*h + 48*c^3*d*g^3*h^2 + 776*b*c^2*e*g^3*h^2 + 1718*b^2*c*f*g^3*h^2 + 2
296*a*c^2*f*g^3*h^2 - 72*b*c^2*d*g^2*h^3 - 382*b^2*c*e*g^2*h^3 - 728*a*c^2*
e*g^2*h^3 - 279*b^3*f*g^2*h^3 - 2716*a*b*c*f*g^2*h^3 + 6*b^2*c*d*g*h^4 + 12
0*a*c^2*d*g*h^4 + 15*b^3*e*g*h^4 + 668*a*b*c*e*g*h^4 + 528*a*b^2*f*g*h^4 +
```

```
992*a^2*c*f*g*h^4 + 9*b^3*d*h^5 - 60*a*b*c*d*h^5 - 24*a*b^2*e*h^5 - 256*a^2
*c*e*h^5 - 240*a^2*b*f*h^5)/(192*h^5*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x))))
/(a + b*x + c*x^2) + ((640*c^4*f*g^6 - 128*c^4*e*g^5*h - 1792*b*c^3*f*g^5*h
 + 320*b*c^3*e*g^4*h^2 + 1680*b^2*c^2*f*g^4*h^2 + 1600*a*c^3*f*g^4*h^2 - 24
0*b^2*c^2*e*g^3*h^3 - 320*a*c^3*e*g^3*h^3 - 560*b^3*c*f*g^3*h^3 - 2880*a*b*
c^2*f*g^3*h^3 + 40*b^3*c*e*g^2*h^4 + 480*a*b*c^2*e*g^2*h^4 + 35*b^4*f*g^2*h
^4 + 1320*a*b^2*c*f*g^2*h^4 + 1200*a^2*c^2*f*g^2*h^4 + 5*b^4*e*g*h^5 - 120*
a*b^2*c*e*g*h^5 - 240*a^2*c^2*e*g*h^5 - 80*a*b^3*f*g*h^5 - 960*a^2*b*c*f*g*
h^5 + 3*b^4*d*h^6 - 24*a*b^2*c*d*h^6 + 48*a^2*c^2*d*h^6 - 8*a*b^3*e*h^6 + 9
6*a^2*b*c*e*h^6 + 48*a^2*b^2*f*h^6 + 192*a^3*c*f*h^6)*(a + x*(b + c*x))^(3/
2)*Log[g + h*x])/(128*h^6*(c*g^2 - b*g*h + a*h^2)^(5/2)*(a + b*x + c*x^2)^(
3/2)) + (((-5*c^4*f*g^5)/(h^6*(c*g^2 - b*g*h + a*h^2)^2) + (c^4*e*g^4)/(h^5
*(c*g^2 - b*g*h + a*h^2)^2) + (23*b*c^3*f*g^4)/(2*h^5*(c*g^2 - b*g*h + a*h^
2)^2) - (2*c^2*(b*c*e + 4*b^2*f + 5*a*c*f)*g^3)/(h^4*(c*g^2 - b*g*h + a*h^2
)^2) + (3*b^3*c*f*g^2)/(2*h^3*(c*g^2 - b*g*h + a*h^2)^2) + (c^2*(b^2*e + 2*
a*c*e + 13*a*b*f)*g^2)/(h^3*(c*g^2 - b*g*h + a*h^2)^2) + (-2*a*b*c^2*e*g -
3*a*b^2*c*f*g - 5*a^2*c^2*f*g)/(h^2*(c*g^2 - b*g*h + a*h^2)^2) + (a^2*c^2*e
)/(h*(c*g^2 - b*g*h + a*h^2)^2) + (3*a^2*b*c*f)/(2*h*(c*g^2 - b*g*h + a*h^2
)^2))*(a + x*(b + c*x))^(3/2)*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + b*x + c*x^
2]])/(Sqrt[c]*(a + b*x + c*x^2)^(3/2)) - ((640*c^4*f*g^6 - 128*c^4*e*g^5*h
- 1792*b*c^3*f*g^5*h + 320*b*c^3*e*g^4*h^2 + 1680*b^2*c^2*f*g^4*h^2 + 1600*
a*c^3*f*g^4*h^2 - 240*b^2*c^2*e*g^3*h^3 - 320*a*c^3*e*g^3*h^3 - 560*b^3*c*f
*g^3*h^3 - 2880*a*b*c^2*f*g^3*h^3 + 40*b^3*c*e*g^2*h^4 + 480*a*b*c^2*e*g^2*
h^4 + 35*b^4*f*g^2*h^4 + 1320*a*b^2*c*f*g^2*h^4 + 1200*a^2*c^2*f*g^2*h^4 +
5*b^4*e*g*h^5 - 120*a*b^2*c*e*g*h^5 - 240*a^2*c^2*e*g*h^5 - 80*a*b^3*f*g*h^
5 - 960*a^2*b*c*f*g*h^5 + 3*b^4*d*h^6 - 24*a*b^2*c*d*h^6 + 48*a^2*c^2*d*h^6
 - 8*a*b^3*e*h^6 + 96*a^2*b*c*e*h^6 + 48*a^2*b^2*f*h^6 + 192*a^3*c*f*h^6)*(
a + x*(b + c*x))^(3/2)*Log[-(b*g) + 2*a*h - 2*c*g*x + b*h*x + 2*Sqrt[c*g^2
- b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2]])/(128*h^6*(c*g^2 - b*g*h + a*h^2)^(
5/2)*(a + b*x + c*x^2)^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10037 vs. $2(1065) = 2130$.
time = 0.20, size = 10038, normalized size = 9.15

method	result	size
default	Expression too large to display	10038
risch	Expression too large to display	51442

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+cx^2)^{\frac{3}{2}}\,(d+ex+fx^2)}{(g+hx)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**5,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

3.205 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$

Optimal. Leaf size=1226

$$-\frac{\left(128c^4fg^7-32c^3fg^5h(11bg-10ah)+8c^2gh^2(38b^2fg^4+2a^2h^2(13fg^2+3dh^2)-abgh(65fg^2+3dh^2))-\right.}{}$$

```
[Out] -1/48*(16*c^2*f*g^5-2*c*g*h*(-6*a*d*h^3-10*a*f*g^2*h+3*b*d*g*h^2+13*b*f*g^3
)-h^2*(4*a^2*h^2*(-3*e*h+2*f*g)-b^2*g*(7*f*g^2+3*h*(d*h+e*g))+2*a*b*h*(f*g^
2+3*h*(d*h+2*e*g)))+h*(4*c^2*(-3*d*g^2*h^2+7*f*g^4)+2*c*g*h*(2*a*h*(-3*e*h+
14*f*g)-b*(-6*d*h^2-3*e*g*h+28*f*g^2))+h^2*(16*a^2*f*h^2-2*a*b*h*(-3*e*h+22
*f*g)+b^2*(25*f*g^2-3*h*(d*h+e*g))))*x)*(c*x^2+b*x+a)^(3/2)/h^3/(a*h^2-b*g*
h+c*g^2)^2/(h*x+g)^4-1/5*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-
b*g*h+c*g^2)/(h*x+g)^5+c^(3/2)*f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a
)^(1/2))/h^6-1/256*(256*c^5*f*g^7-896*c^4*f*g^5*h*(-a*h+b*g)+32*c^3*g*h^2*(
35*b^2*f*g^4-70*a*b*f*g^3*h+a^2*h^2*(-3*d*h^2+35*f*g^2))-16*c^2*h^3*(35*b^3
*f*g^4-6*a^3*h^3*(-e*h+6*f*g)+3*a^2*b*h^2*(-d*h^2-e*g*h+35*f*g^2)-3*a*b^2*g
*h*(d*h^2+35*f*g^2))+b^3*h^5*(16*a^2*f*h^2-2*a*b*h*(3*e*h+10*f*g)+b^2*(7*f*
g^2+3*h*(d*h+e*g)))-2*b*c*h^4*(96*a^3*f*h^3-24*a^2*b*h^2*(e*h+8*f*g)-b^3*(-
3*d*g*h^2+35*f*g^3)+4*a*b^2*h*(35*f*g^2+3*h*(d*h+e*g))))*arctanh(1/2*(b*g-2
*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/(a*
h^2-b*g*h+c*g^2)^(7/2)-1/128*(128*c^4*f*g^7-32*c^3*f*g^5*h*(-10*a*h+11*b*g)
+8*c^2*g*h^2*(38*b^2*f*g^4+2*a^2*h^2*(3*d*h^2+13*f*g^2)-a*b*g*h*(3*d*h^2+65
*f*g^2))-2*c*h^3*(8*a^3*h^3*(-3*e*h+2*f*g)-2*a*b^2*g^2*h*(3*e*h+34*f*g)+4*a
^2*b*h^2*(3*d*h^2+6*e*g*h+5*f*g^2)+b^3*(-3*d*g^2*h^2+35*f*g^4))-b*h^4*(-2*a
*h+b*g)*(16*a^2*f*h^2-2*a*b*h*(3*e*h+10*f*g)+b^2*(7*f*g^2+3*h*(d*h+e*g)))+h
*(128*c*f*(c*g^2-h*(-a*h+b*g))^3+(-b*h+2*c*g)*(32*c^3*f*g^5-8*c^2*g*h*(3*a*
d*h^3-11*a*f*g^2*h+10*b*f*g^3)+2*c*h^2*(4*a^2*h^2*(-3*e*h+10*f*g)-6*a*b*h*(
-d*h^2-e*g*h+11*f*g^2)+b^2*(3*d*g*h^2+29*f*g^3))-b*h^3*(16*a^2*f*h^2-2*a*b*
h*(3*e*h+10*f*g)+b^2*(7*f*g^2+3*h*(d*h+e*g)))))*x)*(c*x^2+b*x+a)^(1/2)/h^5/
(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^2
```

Rubi [A]
time = 2.46, antiderivative size = 1223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1664, 824, 857, 635, 212, 738}

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -1/128*(((128*c^4*f*g^7)/h - 32*c^3*f*g^5*(11*b*g - 10*a*h) + 8*c^2*g*h*(38
*b^2*f*g^4 + 2*a^2*h^2*(13*f*g^2 + 3*d*h^2) - a*b*g*h*(65*f*g^2 + 3*d*h^2))
```

```
 - b*h^3*(b*g - 2*a*h)*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*
g^2 + 3*h*(e*g + d*h))) - 2*c*h^2*(8*a^3*h^3*(2*f*g - 3*e*h) - 2*a*b^2*g^2*
h*(34*f*g + 3*e*h) + b^3*(35*f*g^4 - 3*d*g^2*h^2) + 4*a^2*b*h^2*(5*f*g^2 +
3*h*(2*e*g + d*h))) + (128*c*f*(c*g^2 - h*(b*g - a*h))^3 + (2*c*g - b*h)*(3
2*c^3*f*g^5 - 8*c^2*g*h*(10*b*f*g^3 - 11*a*f*g^2*h + 3*a*d*h^3) + 2*c*h^2*(
4*a^2*h^2*(10*f*g - 3*e*h) - 6*a*b*h*(11*f*g^2 - e*g*h - d*h^2) + b^2*(29*f
*g^3 + 3*d*g*h^2)) - b*h^3*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(
7*f*g^2 + 3*h*(e*g + d*h)))))*x)*Sqrt[a + b*x + c*x^2])/(h^4*(c*g^2 - b*g*h
 + a*h^2)^3*(g + h*x)^2) - ((16*c^2*f*g^5 - 2*c*g*h*(13*b*f*g^3 - 10*a*f*g^
2*h + 3*b*d*g*h^2 - 6*a*d*h^3) - h^2*(4*a^2*h^2*(2*f*g - 3*e*h) - b^2*g*(7*
f*g^2 + 3*h*(e*g + d*h)) + 2*a*b*h*(f*g^2 + 3*h*(2*e*g + d*h))) + h^2*(16*a
^2*f*h^3 + 4*a*c*g*h*(14*f*g - 3*e*h) + c^2*((28*f*g^4)/h - 12*d*g^2*h) + b
^2*h*(25*f*g^2 - 3*h*(e*g + d*h)) - b*(56*c*f*g^3 - 6*c*g*h*(e*g + 2*d*h) +
 2*a*h^2*(22*f*g - 3*e*h)))*x)*(a + b*x + c*x^2)^(3/2))/(48*h^3*(c*g^2 - b*
g*h + a*h^2)^2*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5
/2))/(5*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) + (c^(3/2)*f*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h^6 - ((256*c^5*f*g^7 - 896*c^4*f*
g^5*h*(b*g - a*h) + 32*c^3*g*h^2*(35*b^2*f*g^4 - 70*a*b*f*g^3*h + a^2*h^2*(
35*f*g^2 - 3*d*h^2)) - 16*c^2*h^3*(35*b^3*f*g^4 - 6*a^3*h^3*(6*f*g - e*h) +
 3*a^2*b*h^2*(35*f*g^2 - e*g*h - d*h^2) - 3*a*b^2*g*h*(35*f*g^2 + d*h^2)) +
 b^3*h^5*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*g^2 + 3*h*(e*g
 + d*h))) - 2*b*c*h^4*(96*a^3*f*h^3 - 24*a^2*b*h^2*(8*f*g + e*h) - b^3*(35*
f*g^3 - 3*d*g*h^2) + 4*a*b^2*h*(35*f*g^2 + 3*h*(e*g + d*h))))*ArcTanh[(b*g
- 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*
x^2])])/(256*h^6*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = -\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{5h(cg^2-bgh+ah^2)(g+hx)^5} - \frac{\int \left(-\frac{5}{2}\left(2cdg-beg-2afg+\frac{bfg^2}{h}\right.\right.}{5}$$

$$= -\frac{\left(16c^2fg^5 - 2cgh(13bfg^3 - 10afg^2h + 3bdgh^2 - 6adh^3) - h^2(4a\right.}{}$$

$$= -\frac{\left(\frac{128c^4fg^7}{h} - 32c^3fg^5(11bg-10ah) + 8c^2gh(38b^2fg^4 + 2a^2h^2(13f\right.}{}$$

$$= -\frac{\left(\frac{128c^4fg^7}{h} - 32c^3fg^5(11bg-10ah) + 8c^2gh(38b^2fg^4 + 2a^2h^2(13f\right.}{}$$

$$= -\frac{\left(\frac{128c^4fg^7}{h} - 32c^3fg^5(11bg-10ah) + 8c^2gh(38b^2fg^4 + 2a^2h^2(13f\right.}{}$$

$$= -\frac{\left(\frac{128c^4fg^7}{h} - 32c^3fg^5(11bg-10ah) + 8c^2gh(38b^2fg^4 + 2a^2h^2(13f\right.}{}$$

Mathematica [A]
time = 16.74, size = 2229, normalized size = 1.82

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] ((a + x*(b + c*x))^(3/2)*(-1/5*((c*g^2 - b*g*h + a*h^2)*(f*g^2 - e*g*h + d*
h^2))/(h^5*(g + h*x)^5) + (42*c*f*g^3 - 32*c*e*g^2*h - 31*b*f*g^2*h + 22*c*
d*g*h^2 + 21*b*e*g*h^2 + 20*a*f*g*h^2 - 11*b*d*h^3 - 10*a*e*h^3)/(40*h^5*(g
 + h*x)^4) + (-548*c^2*f*g^4 + 288*c^2*e*g^3*h + 808*b*c*f*g^3*h - 108*c^2*
d*g^2*h^2 - 378*b*c*e*g^2*h^2 - 263*b^2*f*g^2*h^2 - 616*a*c*f*g^2*h^2 + 108
*b*c*d*g*h^3 + 93*b^2*e*g*h^3 + 276*a*c*e*g*h^3 + 340*a*b*f*g*h^3 - 3*b^2*d
*h^4 - 96*a*c*d*h^4 - 90*a*b*e*h^4 - 80*a^2*f*h^4)/(240*h^5*(c*g^2 - b*g*h
+ a*h^2)*(g + h*x)^3) + (2608*c^3*f*g^5 - 768*c^3*e*g^4*h - 5752*b*c^2*f*g^
4*h + 48*c^3*d*g^3*h^2 + 1512*b*c^2*e*g^3*h^2 + 3734*b^2*c*f*g^3*h^2 + 5048
*a*c^2*f*g^3*h^2 - 72*b*c^2*d*g^2*h^3 - 744*b^2*c*e*g^2*h^3 - 1488*a*c^2*e*
g^2*h^3 - 605*b^3*f*g^2*h^3 - 6084*a*b*c*f*g^2*h^3 - 6*b^2*c*d*g*h^4 + 168*
a*c^2*d*g*h^4 + 15*b^3*e*g*h^4 + 1404*a*b*c*e*g*h^4 + 1180*a*b^2*f*g*h^4 +
```

```
2320*a^2*c*f*g*h^4 + 15*b^3*d*h^5 - 84*a*b*c*d*h^5 - 30*a*b^2*e*h^5 - 600*a
^2*c*e*h^5 - 560*a^2*b*f*h^5)/(960*h^5*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^
2) + (-4384*c^4*f*g^6 + 384*c^4*e*g^5*h + 12768*b*c^3*f*g^5*h + 96*c^4*d*g^
4*h^2 - 1008*b*c^3*e*g^4*h^2 - 12324*b^2*c^2*f*g^4*h^2 - 12528*a*c^3*f*g^4*
h^2 - 192*b*c^3*d*g^3*h^3 + 744*b^2*c^2*e*g^3*h^3 + 1248*a*c^3*e*g^3*h^3 +
4000*b^3*c*f*g^3*h^3 + 23808*a*b*c^2*f*g^3*h^3 + 36*b^2*c^2*d*g^2*h^4 + 432
*a*c^3*d*g^2*h^4 - 30*b^3*c*e*g^2*h^4 - 2088*a*b*c^2*e*g^2*h^4 - 105*b^4*f*
g^2*h^4 - 11100*a*b^2*c*f*g^2*h^4 - 11424*a^2*c^2*f*g^2*h^4 + 60*b^3*c*d*g*
h^5 - 432*a*b*c^2*d*g*h^5 - 45*b^4*e*g*h^5 + 360*a*b^2*c*e*g*h^5 + 1584*a^2
*c^2*e*g*h^5 + 300*a*b^3*f*g*h^5 + 9840*a^2*b*c*f*g*h^5 - 45*b^4*d*h^6 + 30
0*a*b^2*c*d*h^6 - 384*a^2*c^2*d*h^6 + 90*a*b^3*e*h^6 - 600*a^2*b*c*e*h^6 -
240*a^2*b^2*f*h^6 - 2560*a^3*c*f*h^6)/(1920*h^5*(c*g^2 - b*g*h + a*h^2)^3*(
g + h*x))))/(a + b*x + c*x^2) - ((256*c^5*f*g^7 - 896*b*c^4*f*g^6*h + 1120*
b^2*c^3*f*g^5*h^2 + 896*a*c^4*f*g^5*h^2 - 560*b^3*c^2*f*g^4*h^3 - 2240*a*b*
c^3*f*g^4*h^3 + 70*b^4*c*f*g^3*h^4 + 1680*a*b^2*c^2*f*g^3*h^4 + 1120*a^2*c^
3*f*g^3*h^4 + 7*b^5*f*g^2*h^5 - 280*a*b^3*c*f*g^2*h^5 - 1680*a^2*b*c^2*f*g^
2*h^5 - 6*b^4*c*d*g*h^6 + 48*a*b^2*c^2*d*g*h^6 - 96*a^2*c^3*d*g*h^6 + 3*b^5
*e*g*h^6 - 24*a*b^3*c*e*g*h^6 + 48*a^2*b*c^2*e*g*h^6 - 20*a*b^4*f*g*h^6 + 3
84*a^2*b^2*c*f*g*h^6 + 576*a^3*c^2*f*g*h^6 + 3*b^5*d*h^7 - 24*a*b^3*c*d*h^7
 + 48*a^2*b*c^2*d*h^7 - 6*a*b^4*e*h^7 + 48*a^2*b^2*c*e*h^7 - 96*a^3*c^2*e*h
^7 + 16*a^2*b^3*f*h^7 - 192*a^3*b*c*f*h^7)*(a + x*(b + c*x))^(3/2)*Log[g +
h*x])/(256*h^6*(c*g^2 - b*g*h + a*h^2)^(7/2)*(a + b*x + c*x^2)^(3/2)) + (((
c^5*f*g^6)/(h^6*(c*g^2 - b*g*h + a*h^2)^3) - (3*b*c^4*f*g^5)/(h^5*(c*g^2 -
b*g*h + a*h^2)^3) + (3*c^3*(b^2 + a*c)*f*g^4)/(h^4*(c*g^2 - b*g*h + a*h^2)^
3) - (b*c^2*(b^2 + 6*a*c)*f*g^3)/(h^3*(c*g^2 - b*g*h + a*h^2)^3) + (3*a*c^2
*(b^2 + a*c)*f*g^2)/(h^2*(c*g^2 - b*g*h + a*h^2)^3) + (a^2*c^2*f*(-3*b*g +
a*h))/(h*(c*g^2 - b*g*h + a*h^2)^3))*(a + x*(b + c*x))^(3/2)*Log[b + 2*c*x
+ 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(Sqrt[c]*(a + b*x + c*x^2)^(3/2)) + ((2
56*c^5*f*g^7 - 896*b*c^4*f*g^6*h + 1120*b^2*c^3*f*g^5*h^2 + 896*a*c^4*f*g^5
*h^2 - 560*b^3*c^2*f*g^4*h^3 - 2240*a*b*c^3*f*g^4*h^3 + 70*b^4*c*f*g^3*h^4
+ 1680*a*b^2*c^2*f*g^3*h^4 + 1120*a^2*c^3*f*g^3*h^4 + 7*b^5*f*g^2*h^5 - 280
*a*b^3*c*f*g^2*h^5 - 1680*a^2*b*c^2*f*g^2*h^5 - 6*b^4*c*d*g*h^6 + 48*a*b^2*
c^2*d*g*h^6 - 96*a^2*c^3*d*g*h^6 + 3*b^5*e*g*h^6 - 24*a*b^3*c*e*g*h^6 + 48*
a^2*b*c^2*e*g*h^6 - 20*a*b^4*f*g*h^6 + 384*a^2*b^2*c*f*g*h^6 + 576*a^3*c^2*
f*g*h^6 + 3*b^5*d*h^7 - 24*a*b^3*c*d*h^7 + 48*a^2*b*c^2*d*h^7 - 6*a*b^4*e*h
^7 + 48*a^2*b^2*c*e*h^7 - 96*a^3*c^2*e*h^7 + 16*a^2*b^3*f*h^7 - 192*a^3*b*c
*f*h^7)*(a + x*(b + c*x))^(3/2)*Log[-(b*g) + 2*a*h - 2*c*g*x + b*h*x + 2*Sq
rt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2]])/(256*h^6*(c*g^2 - b*g*h +
 a*h^2)^(7/2)*(a + b*x + c*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 13371 vs. $2(1196) = 2392$.

time = 0.14, size = 13372, normalized size = 10.91

method	result	size

default	Expression too large to display	13372

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx + cx^2)^{\frac{3}{2}} (d + ex + fx^2)}{(g + hx)^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**6,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**6, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

3.206 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx$

Optimal. Leaf size=657

$$-\frac{(b^2-4ac)\left(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)+b^2(7fg^2+5egh+7dh^2)-4c(3bg(eg+2dh)+a(fg^2\right.}{512\left(cg^2-bgh+ah^2\right)^4(g+hx)^2}$$

```
[Out] 1/192*(24*c^2*d*g^2+24*a^2*f*h^2-12*a*b*h*(e*h+2*f*g)+b^2*(7*d*h^2+5*e*g*h+
7*f*g^2)-4*c*(3*b*g*(2*d*h+e*g)+a*(d*h^2-7*e*g*h+f*g^2)))*(b*g-2*a*h+(-b*h+
2*c*g)*x)*(c*x^2+b*x+a)^(3/2)/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^4-1/6*(f*g^2-h*
(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^6+1/60*(2*c*g
*(5*f*g^2+h*(-7*d*h+e*g))+h*(12*a*h*(-e*h+2*f*g)-b*(-7*d*h^2-5*e*g*h+17*f*g
^2)))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^5+1/1024*(-4*a*c+
b^2)^2*(24*c^2*d*g^2+24*a^2*f*h^2-12*a*b*h*(e*h+2*f*g)+b^2*(7*d*h^2+5*e*g*h
+7*f*g^2)-4*c*(3*b*g*(2*d*h+e*g)+a*(d*h^2-7*e*g*h+f*g^2)))*arctanh(1/2*(b*g
-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^
2-b*g*h+c*g^2)^(9/2)-1/512*(-4*a*c+b^2)*(24*c^2*d*g^2+24*a^2*f*h^2-12*a*b*h
*(e*h+2*f*g)+b^2*(7*d*h^2+5*e*g*h+7*f*g^2)-4*c*(3*b*g*(2*d*h+e*g)+a*(d*h^2-
7*e*g*h+f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*
h+c*g^2)^4/(h*x+g)^2
```

Rubi [A]
time = 0.73, antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1664, 820, 734, 738, 212}

$$\frac{(a+bx+cx^2)^{3/2}(-2ah+x(2cg-bh)+bg)(24a^2fh^2-4c(-ah(7eg-dh)+afg^2+3bg(2dh+eg))-12abh(eh+2fg)+b^2(h(7dh+5eg)+7fg^2)+24c^2dg^2)}{192(g+hx)^4(ah^2-bgh+cg^2)^3}-\frac{(b^2-4ac)\sqrt{a+bx+cx^2}(-2ah+x(2cg-bh)+bg)(24a^2fh^2-4c(-ah(7eg-dh)+afg^2+3bg(2dh+eg))-12abh(eh+2fg)+b^2(h(7dh+5eg)+7fg^2)+24c^2dg^2)}{512(g+hx)^2(ah^2-bgh+cg^2)^4}+\frac{(b^2-4ac)^2\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)(24a^2fh^2-4c(-ah(7eg-dh)+afg^2+3bg(2dh+eg))-12abh(eh+2fg)+b^2(h(7dh+5eg)+7fg^2)+24c^2dg^2)}{1024(ah^2-bgh+cg^2)^{9/2}}-\frac{(a+bx+cx^2)^{5/2}(fg^2-h(eg-dh))}{6h(g+hx)^6(ah^2-bgh+cg^2)}+\frac{(a+bx+cx^2)^{5/2}(2c(gh(eg-7dh)+5fg^3)-h(-12ah(2fg-eh)-bh(7dh+5eg)+17bfg^2))}{60h(g+hx)^5(ah^2-bgh+cg^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] -1/512*((b^2 - 4*a*c)*(24*c^2*d*g^2 + 24*a^2*f*h^2 - 12*a*b*h*(2*f*g + e*h)
 - 4*c*(a*f*g^2 - a*h*(7*e*g - d*h) + 3*b*g*(e*g + 2*d*h)) + b^2*(7*f*g^2 +
 h*(5*e*g + 7*d*h)))*(b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])
/((c*g^2 - b*g*h + a*h^2)^4*(g + h*x)^2) + ((24*c^2*d*g^2 + 24*a^2*f*h^2 -
12*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*(7*e*g - d*h) + 3*b*g*(e*g + 2*
d*h)) + b^2*(7*f*g^2 + h*(5*e*g + 7*d*h)))*(b*g - 2*a*h + (2*c*g - b*h)*x)*
(a + b*x + c*x^2)^(3/2))/(192*(c*g^2 - b*g*h + a*h^2)^3*(g + h*x)^4) - ((f*
g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(6*h*(c*g^2 - b*g*h + a*h^2)*
(g + h*x)^6) + ((2*c*(5*f*g^3 + g*h*(e*g - 7*d*h)) - h*(17*b*f*g^2 - b*h*(5
*e*g + 7*d*h) - 12*a*h*(2*f*g - e*h)))*(a + b*x + c*x^2)^(5/2))/(60*h*(c*g^
2 - b*g*h + a*h^2)^2*(g + h*x)^5) + ((b^2 - 4*a*c)^2*(24*c^2*d*g^2 + 24*a^2
*f*h^2 - 12*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*(7*e*g - d*h) + 3*b*g*
```

```
(e*g + 2*d*h)) + b^2*(7*f*g^2 + h*(5*e*g + 7*d*h)))*ArcTanh[(b*g - 2*a*h +
(2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(1
024*(c*g^2 - b*g*h + a*h^2)^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = -\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{6h(cg^2-bgh+ah^2)(g+hx)^6} - \frac{\int \frac{\left(\frac{1}{2}\left(-12cdg+5beg+12afg-5\right.\right.}{}}{}$$

$$= -\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{6h(cg^2-bgh+ah^2)(g+hx)^6} + \frac{(2c(5fg^3+gh(eg-7dh}{}$$

$$= \frac{(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7eg-dh)}{192(}$$

$$= -\frac{(b^2-4ac)(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-a}{}$$

$$= -\frac{(b^2-4ac)(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-a}{}$$

$$= -\frac{(b^2-4ac)(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-a}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1548 vs. $2(657) = 1314$.
time = 16.80, size = 1548, normalized size = 2.36

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(-1280*(c*g^2 + h*(-(b*g) + a*h))*(f*g^2 + h*(-(e*g)
 + d*h)) + 128*(50*c*f*g^3 + 2*c*g*h*(-19*e*g + 13*d*h) - 12*a*h^2*(-2*f*g
+ e*h) + b*h*(-37*f*g^2 + h*(25*e*g - 13*d*h)))*(g + h*x) - (16*(8*c^2*(100
*f*g^4 + g^2*h*(-52*e*g + 19*d*h)) + 3*h^2*(40*a^2*f*h^2 + 4*a*b*h*(-42*f*g
 + 11*e*h) + b^2*(129*f*g^2 - 45*e*g*h + d*h^2)) - 4*c*h*(a*h*(-227*f*g^2 +
 101*e*g*h - 35*d*h^2) + b*g*(296*f*g^2 - 137*e*g*h + 38*d*h^2)))*(g + h*x)
^2)/(c*g^2 + h*(-(b*g) + a*h)) + (8*(16*c^3*(100*f*g^5 + g^3*h*(-28*e*g + d
*h)) - 24*c^2*g*h*(a*h*(-131*f*g^2 + 37*e*g*h - 3*d*h^2) + b*g*(148*f*g^2 -
 37*e*g*h + d*h^2)) - 6*c*h^2*(8*a^2*h^2*(-31*f*g + 8*e*h) + b^2*g*(-387*f*
```

```
g^2 + 73*e*g*h + d*h^2) + 2*a*b*h*(319*f*g^2 - 71*e*g*h + 3*d*h^2)) + b*h^3
*(-360*a^2*f*h^2 - 12*a*b*h*(-62*f*g + e*h) + b^2*(-377*f*g^2 + 5*e*g*h + 7
*d*h^2)))*(g + h*x)^3)/(c*g^2 + h*(-(b*g) + a*h))^2 - (2*(64*c^4*(50*f*g^6
- g^4*h*(2*e*g + d*h)) - 32*c^3*g^2*h*(b*g*(296*f*g^2 - 11*e*g*h - 4*d*h^2)
 + 3*a*h*(-99*f*g^2 + 5*e*g*h + 4*d*h^2)) + 5*b^2*h^4*(24*a^2*f*h^2 - 12*a*
b*h*(2*f*g + e*h) + b^2*(7*f*g^2 + 5*e*g*h + 7*d*h^2)) + 24*c^2*h^2*(b^2*g^
3*(387*f*g - 11*e*h) + 2*a^2*h^2*(193*f*g^2 - 19*e*g*h + 5*d*h^2) + 2*a*b*g
*h*(-386*f*g^2 + 19*e*g*h + 8*d*h^2)) - 4*c*h^3*(-600*a^3*f*h^3 + 12*a^2*b*
h^2*(174*f*g - 7*e*h) + 6*a*b^2*h*(-367*f*g^2 + 4*e*g*h + 9*d*h^2) + b^3*g*
(754*f*g^2 + 5*e*g*h + 16*d*h^2)))*(g + h*x)^4)/(c*g^2 + h*(-(b*g) + a*h))^
3 + ((128*c^5*(10*f*g^7 + g^5*h*(2*e*g + d*h)) + 15*b^3*h^5*(24*a^2*f*h^2 -
 12*a*b*h*(2*f*g + e*h) + b^2*(7*f*g^2 + 5*e*g*h + 7*d*h^2)) - 64*c^4*g^3*h
*(b*g*(74*f*g^2 + 13*e*g*h + 5*d*h^2) - a*h*(83*f*g^2 + 19*e*g*h + 14*d*h^2
)) + 48*c^3*g*h^2*(2*a^2*h^2*(89*f*g^2 + 29*e*g*h - 27*d*h^2) + b^2*g^2*(12
9*f*g^2 + 17*e*g*h + 2*d*h^2) - 2*a*b*g*h*(151*f*g^2 + 30*e*g*h + 14*d*h^2)
) - 10*b*c*h^4*(240*a^3*f*h^3 - 24*a^2*b*h^2*(11*f*g + 5*e*h) + 4*a*b^2*h*(
25*f*g^2 + 5*e*g*h + 19*d*h^2) + b^3*g*(-7*f*g^2 + 13*e*g*h + 29*d*h^2)) -
8*c^2*h^3*(24*a^3*h^3*(-41*f*g + 8*e*h) + 6*a^2*b*h^2*(325*f*g^2 + 31*e*g*h
 - 27*d*h^2) + b^3*g^2*(377*f*g^2 + 10*e*g*h - 22*d*h^2) - 6*a*b^2*g*h*(243
*f*g^2 + 23*e*g*h + 41*d*h^2)))*(g + h*x)^5)/(c*g^2 + h*(-(b*g) + a*h))^4))
/(7680*h^5*(g + h*x)^6) + ((b^2 - 4*a*c)^2*(24*c^2*d*g^2 + 24*a^2*f*h^2 - 1
2*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 + a*h*(-7*e*g + d*h) + 3*b*g*(e*g + 2*
d*h)) + b^2*(7*f*g^2 + h*(5*e*g + 7*d*h)))*Log[g + h*x])/(1024*(c*g^2 + h*(
-(b*g) + a*h))^(9/2)) - ((b^2 - 4*a*c)^2*(24*c^2*d*g^2 + 24*a^2*f*h^2 - 12*
a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 + a*h*(-7*e*g + d*h) + 3*b*g*(e*g + 2*d*
h)) + b^2*(7*f*g^2 + h*(5*e*g + 7*d*h)))*Log[-(b*g) + 2*a*h - 2*c*g*x + b*h
*x + 2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)]])/(1024*(c*g^2
+ h*(-(b*g) + a*h))^(9/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 20683 vs. $2(631) = 1262$.
time = 0.16, size = 20684, normalized size = 31.48

method	result	size
default	Expression too large to display	20684

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+cx^2)^{\frac{3}{2}}\,(d+ex+fx^2)}{(g+hx)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**7,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**7, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 48804 vs. 2(648) = 1296.
time = 8.05, size = 48804, normalized size = 74.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="giac")
```

```
[Out] 1/512*(24*b^4*c^2*d*g^2 - 192*a*b^2*c^3*d*g^2 + 384*a^2*c^4*d*g^2 + 7*b^6*f
*g^2 - 60*a*b^4*c*f*g^2 + 144*a^2*b^2*c^2*f*g^2 - 64*a^3*c^3*f*g^2 - 24*b^5
*c*d*g*h + 192*a*b^3*c^2*d*g*h - 384*a^2*b*c^3*d*g*h - 24*a*b^5*f*g*h + 192
*a^2*b^3*c*f*g*h - 384*a^3*b*c^2*f*g*h + 7*b^6*d*h^2 - 60*a*b^4*c*d*h^2 + 1
44*a^2*b^2*c^2*d*h^2 - 64*a^3*c^3*d*h^2 + 24*a^2*b^4*f*h^2 - 192*a^3*b^2*c*
```

f*h^2 + 384*a^4*c^2*f*h^2 - 12*b^5*c*g^2*e + 96*a*b^3*c^2*g^2*e - 192*a^2*b
*c^3*g^2*e + 5*b^6*g*h*e - 12*a*b^4*c*g*h*e - 144*a^2*b^2*c^2*g*h*e + 448*a
^3*c^3*g*h*e - 12*a*b^5*h^2*e + 96*a^2*b^3*c*h^2*e - 192*a^3*b*c^2*h^2*e)*a
rctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*
g*h - a*h^2))/((c^4*g^8 - 4*b*c^3*g^7*h + 6*b^2*c^2*g^6*h^2 + 4*a*c^3*g^6*h
^2 - 4*b^3*c*g^5*h^3 - 12*a*b*c^2*g^5*h^3 + b^4*g^4*h^4 + 12*a*b^2*c*g^4*h^
4 + 6*a^2*c^2*g^4*h^4 - 4*a*b^3*g^3*h^5 - 12*a^2*b*c*g^3*h^5 + 6*a^2*b^2*g^
2*h^6 + 4*a^3*c*g^2*h^6 - 4*a^3*b*g*h^7 + a^4*h^8)*sqrt(-c*g^2 + b*g*h - a*
h^2)) + 1/7680*(15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*c^6*f*g^8*h^5
- 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b*c^5*f*g^7*h^6 + 92160*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^11*b^2*c^4*f*g^6*h^7 + 61440*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^11*a*c^5*f*g^6*h^7 - 61440*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^11*b^3*c^3*f*g^5*h^8 - 184320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^11*a*b*c^4*f*g^5*h^8 + 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^4*c^
2*f*g^4*h^9 + 184320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^2*c^3*f*g^4
h^9 + 92160(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*c^4*f*g^4*h^9 - 614
40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^3*c^2*f*g^3*h^10 - 184320*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b*c^3*f*g^3*h^10 - 360*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^11*b^4*c^2*d*g^2*h^11 + 2880*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^11*a*b^2*c^3*d*g^2*h^11 - 5760*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^11*a^2*c^4*d*g^2*h^11 - 105*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^
6*f*g^2*h^11 + 900*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^4*c*f*g^2*h^1
1 + 90000*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b^2*c^2*f*g^2*h^11 + 6
2400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^3*c^3*f*g^2*h^11 + 360*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^11*b^5*c*d*g*h^12 - 2880*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^11*a*b^3*c^2*d*g*h^12 + 5760*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^11*a^2*b*c^3*d*g*h^12 + 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*
b^5*f*g*h^12 - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b^3*c*f*g*h^
12 - 55680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^3*b*c^2*f*g*h^12 - 105*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^6*d*h^13 + 900*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^11*a*b^4*c*d*h^13 - 2160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^11*a^2*b^2*c^2*d*h^13 + 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^3*c
^3*d*h^13 - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b^4*f*h^13 + 288
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^3*b^2*c*f*h^13 + 9600*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^11*a^4*c^2*f*h^13 + 180*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^11*b^5*c*g^2*h^11*e - 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^1
1*a*b^3*c^2*g^2*h^11*e + 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b*
c^3*g^2*h^11*e - 75*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^6*g*h^12*e + 1
80*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*b^4*c*g*h^12*e + 2160*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^11*a^2*b^2*c^2*g*h^12*e - 6720*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^11*a^3*c^3*g*h^12*e + 180*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^11*a*b^5*h^13*e - 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^2*b^3
*c*h^13*e + 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a^3*b*c^2*h^13*e +
76800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*c^(13/2)*f*g^9*h^4 - 276480*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b*c^(11/2)*f*g^8*h^5 + 337920*(sqrt(c)

```
*x - sqrt(c*x^2 + b*x + a))^10*b^2*c^(9/2)*f*g^7*h^6 + 307200*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^10*a*c^(11/2)*f*g^7*h^6 - 122880*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^10*b^3*c^(7/2)*f*g^6*h^7 - 798720*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^10*a*b*c^(9/2)*f*g^6*h^7 - 46080*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^10*b^4*c^(5/2)*f*g^5*h^8 + 552960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^10*a*b^2*c^(7/2)*f*g^5*h^8 + 460800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10
*a^2*c^(9/2)*f*g^5*h^8 + 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^5*c
^(3/2)*f*g^4*h^9 + 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^3*c^(5/
2)*f*g^4*h^9 - 737280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b*c^(7/2)*
f*g^4*h^9 - 3960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^4*c^(5/2)*d*g^3*h
^10 + 31680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^2*c^(7/2)*d*g^3*h^10
 - 63360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*c^(9/2)*d*g^3*h^10 - 11
55*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*b^6*sqrt(c)*f*g^3*h^10 - 112980*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^10*a*b^4*c^(3/2)*f*g^3*h^10 + 68400*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^10*a^2*b^2*c^(5/2)*f*g^3*h^10 + 317760*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^10*a^3*c^(7/2)*f*g^3*h^10 + 3960*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^10*b^5*c^(3/2)*d*g^2*...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^7} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x)
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7, x)
```

3.207 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx$

Optimal. Leaf size=1062

$$-\frac{\left(b^2-4ac\right)\left(48c^3dg^3-8c^2g(3bg(eg+3dh)+a(fg^2-8egh+3dh^2))-bh(24a^2fh^2-2abh(10fg+7eh)+\right.}{}$$

```
[Out] 1/384*(48*c^3*d*g^3-8*c^2*g*(3*b*g*(3*d*h+e*g)+a*(3*d*h^2-8*e*g*h+f*g^2))-b
*h*(24*a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))+2*c*
(4*a^2*h^2*(-e*h+8*f*g)-2*a*b*h*(-3*d*h^2+13*e*g*h+13*f*g^2)+b^2*g*(21*d*h^
2+10*e*g*h+7*f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(3/2)/(a*h^2
-b*g*h+c*g^2)^4/(h*x+g)^4-1/7*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a
*h^2-b*g*h+c*g^2)/(h*x+g)^7+1/84*(2*c*g*(5*f*g^2+h*(-9*d*h+2*e*g))+h*(14*a*
h*(-e*h+2*f*g)-b*(-9*d*h^2-5*e*g*h+19*f*g^2)))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2
-b*g*h+c*g^2)^2/(h*x+g)^6+1/840*(4*c^2*g^2*(5*f*g^2+h*(-51*d*h+2*e*g))-7*h^
2*(24*a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))-2*c*h
*(3*b*g*(-34*d*h^2-15*e*g*h+8*f*g^2)-2*a*h*(12*d*h^2-61*e*g*h+26*f*g^2)))*(
c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^5+1/2048*(-4*a*c+b^2)^2*
(48*c^3*d*g^3-8*c^2*g*(3*b*g*(3*d*h+e*g)+a*(3*d*h^2-8*e*g*h+f*g^2))-b*h*(24
*a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))+2*c*(4*a^2
*h^2*(-e*h+8*f*g)-2*a*b*h*(-3*d*h^2+13*e*g*h+13*f*g^2)+b^2*g*(21*d*h^2+10*e
*g*h+7*f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^
(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(11/2)-1/1024*(-4*a*c+b^2)*(
48*c^3*d*g^3-8*c^2*g*(3*b*g*(3*d*h+e*g)+a*(3*d*h^2-8*e*g*h+f*g^2))-b*h*(24*
a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))+2*c*(4*a^2*
h^2*(-e*h+8*f*g)-2*a*b*h*(-3*d*h^2+13*e*g*h+13*f*g^2)+b^2*g*(21*d*h^2+10*e*
g*h+7*f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+
c*g^2)^5/(h*x+g)^2
```

Rubi [A]
time = 1.74, antiderivative size = 1062, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1664, 848, 820, 734, 738, 212}

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -1/1024*((b^2 - 4*a*c)*(48*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - a*h*(8*e*g - 3*d*
h) + 3*b*g*(e*g + 3*d*h)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) +
b^2*(5*f*g^2 + h*(5*e*g + 9*d*h))) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h
*(13*f*g^2 + h*(13*e*g - 3*d*h)) + b^2*(7*f*g^3 + g*h*(10*e*g + 21*d*h)))*
```

```
(b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/((c*g^2 - b*g*h + a*
h^2)^5*(g + h*x)^2) + ((48*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - a*h*(8*e*g - 3*d*
h) + 3*b*g*(e*g + 3*d*h)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) +
b^2*(5*f*g^2 + h*(5*e*g + 9*d*h))) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h
*(13*f*g^2 + h*(13*e*g - 3*d*h)) + b^2*(7*f*g^3 + g*h*(10*e*g + 21*d*h))))*
(b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(384*(c*g^2 - b*g*
h + a*h^2)^4*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2
))/(7*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^7) + ((2*c*(5*f*g^3 + g*h*(2*e*g
- 9*d*h)) - h*(19*b*f*g^2 - b*h*(5*e*g + 9*d*h) - 14*a*h*(2*f*g - e*h)))*(a
 + b*x + c*x^2)^(5/2))/(84*h*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^6) + ((4*c
^2*(5*f*g^4 + g^2*h*(2*e*g - 51*d*h)) - 7*h^2*(24*a^2*f*h^2 - 2*a*b*h*(10*f
*g + 7*e*h) + b^2*(5*f*g^2 + 5*e*g*h + 9*d*h^2)) - 2*c*h*(3*b*g*(8*f*g^2 -
15*e*g*h - 34*d*h^2) - 2*a*h*(26*f*g^2 - 61*e*g*h + 12*d*h^2)))*(a + b*x +
c*x^2)^(5/2))/(840*h*(c*g^2 - b*g*h + a*h^2)^3*(g + h*x)^5) + ((b^2 - 4*a*c
)^2*(48*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - a*h*(8*e*g - 3*d*h) + 3*b*g*(e*g + 3
*d*h)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) + b^2*(5*f*g^2 + h*(5
*e*g + 9*d*h))) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h*(13*f*g^2 + h*(13*
e*g - 3*d*h)) + b^2*(7*f*g^3 + g*h*(10*e*g + 21*d*h))))*ArcTanh[(b*g - 2*a*
h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])]
)/(2048*(c*g^2 - b*g*h + a*h^2)^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
```

```
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = -\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{7h(cg^2-bgh+ah^2)(g+hx)^7} - \frac{\int \left(\frac{1}{2}\left(-14cdg+5beg+14afg-5\right.\right.}{}$$

$$= -\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{7h(cg^2-bgh+ah^2)(g+hx)^7} + \frac{(2c(5fg^3+gh(2eg-9d}{}$$

$$= -\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{7h(cg^2-bgh+ah^2)(g+hx)^7} + \frac{(2c(5fg^3+gh(2eg-9d}{}$$

$$= \frac{(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3dh))-bh(24a}{}$$

$$= -\frac{(b^2-4ac)(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3d}{}$$

$$= -\frac{(b^2-4ac)(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3d}{}$$

$$= -\frac{(b^2-4ac)(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3d}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 3059 vs. $2(1062) = 2124$.
time = 17.20, size = 3059, normalized size = 2.88

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] ((a + x*(b + c*x))^(3/2)*(-1/7*((c*g^2 - b*g*h + a*h^2)*(f*g^2 - e*g*h + d*
h^2))/(h^5*(g + h*x)^7) + (58*c*f*g^3 - 44*c*e*g^2*h - 43*b*f*g^2*h + 30*c*
d*g*h^2 + 29*b*e*g*h^2 + 28*a*f*g*h^2 - 15*b*d*h^3 - 14*a*e*h^3)/(84*h^5*(g
 + h*x)^6) + (-1100*c^2*f*g^4 + 568*c^2*e*g^3*h + 1632*b*c*f*g^3*h - 204*c^
2*d*g^2*h^2 - 750*b*c*e*g^2*h^2 - 535*b^2*f*g^2*h^2 - 1256*a*c*f*g^2*h^2 +
204*b*c*d*g*h^3 + 185*b^2*e*g*h^3 + 556*a*c*e*g*h^3 + 700*a*b*f*g*h^3 - 3*b
^2*d*h^4 - 192*a*c*d*h^4 - 182*a*b*e*h^4 - 168*a^2*f*h^4)/(840*h^5*(c*g^2 -
 b*g*h + a*h^2)*(g + h*x)^5) + (8000*c^3*f*g^5 - 2176*c^3*e*g^4*h - 17824*b
*c^2*f*g^4*h + 48*c^3*d*g^3*h^2 + 4328*b*c^2*e*g^3*h^2 + 11702*b^2*c*f*g^3*
```

h^2 + 15832*a*c^2*f*g^3*h^2 - 72*b*c^2*d*g^2*h^3 - 2140*b^2*c*e*g^2*h^3 - 4
352*a*c^2*e*g^2*h^3 - 1905*b^3*f*g^2*h^3 - 19396*a*b*c*f*g^2*h^3 - 30*b^2*c
*d*g*h^4 + 264*a*c^2*d*g*h^4 + 15*b^3*e*g*h^4 + 4220*a*b*c*e*g*h^4 + 3780*a
*b^2*f*g*h^4 + 7616*a^2*c*f*g*h^4 + 27*b^3*d*h^5 - 132*a*b*c*d*h^5 - 42*a*b
^2*e*h^5 - 1960*a^2*c*e*h^5 - 1848*a^2*b*f*h^5)/(6720*h^5*(c*g^2 - b*g*h +
a*h^2)^2*(g + h*x)^4) + (-6400*c^4*f*g^6 + 128*c^4*e*g^5*h + 19072*b*c^3*f*
g^5*h + 96*c^4*d*g^4*h^2 - 368*b*c^3*e*g^4*h^2 - 18852*b^2*c^2*f*g^4*h^2 -
19216*a*c^3*f*g^4*h^2 - 192*b*c^3*d*g^3*h^3 + 288*b^2*c^2*e*g^3*h^3 + 512*a
*c^3*e*g^3*h^3 + 6152*b^3*c*f*g^3*h^3 + 37920*a*b*c^2*f*g^3*h^3 - 36*b^2*c^
2*d*g^2*h^4 + 720*a*c^3*d*g^2*h^4 + 50*b^3*c*e*g^2*h^4 - 1128*a*b*c^2*e*g^2
*h^4 - 35*b^4*f*g^2*h^4 - 18276*a*b^2*c*f*g^2*h^4 - 19200*a^2*c^2*f*g^2*h^4
 + 132*b^3*c*d*g*h^5 - 720*a*b*c^2*d*g*h^5 - 35*b^4*e*g*h^5 + 48*a*b^2*c*e*
g*h^5 + 1392*a^2*c^2*e*g*h^5 + 140*a*b^3*f*g*h^5 + 17808*a^2*b*c*f*g*h^5 -
63*b^4*d*h^6 + 372*a*b^2*c*d*h^6 - 384*a^2*c^2*d*h^6 + 98*a*b^3*e*h^6 - 504
*a^2*b*c*e*h^6 - 168*a^2*b^2*f*h^6 - 5376*a^3*c*f*h^6)/(13440*h^5*(c*g^2 -
b*g*h + a*h^2)^3*(g + h*x)^3) + (1280*c^5*f*g^7 + 512*c^5*e*g^6*h - 4992*b*
c^4*f*g^6*h + 384*c^5*d*g^5*h^2 - 1728*b*c^4*e*g^5*h^2 + 6928*b^2*c^3*f*g^5
*h^2 + 5696*a*c^4*f*g^5*h^2 - 960*b*c^4*d*g^4*h^3 + 1696*b^2*c^3*e*g^4*h^3
+ 2816*a*c^4*e*g^4*h^3 - 3496*b^3*c^2*f*g^4*h^3 - 17056*a*b*c^3*f*g^4*h^3 +
 96*b^2*c^3*d*g^3*h^4 + 3456*a*c^4*d*g^3*h^4 + 80*b^3*c^2*e*g^3*h^4 - 7360*
a*b*c^3*e*g^3*h^4 - 210*b^4*c*f*g^3*h^4 + 15504*a*b^2*c^2*f*g^3*h^4 + 10464
*a^2*c^3*f*g^3*h^4 + 816*b^3*c^2*d*g^2*h^5 - 5184*a*b*c^3*d*g^2*h^5 - 420*b
^4*c*e*g^2*h^5 + 2304*a*b^2*c^2*e*g^2*h^5 + 9024*a^2*c^3*e*g^2*h^5 + 175*b^
5*f*g^2*h^5 - 280*a*b^3*c*f*g^2*h^5 - 24720*a^2*b*c^2*f*g^2*h^5 - 966*b^4*c
*d*g*h^6 + 6096*a*b^2*c^2*d*g*h^6 - 7008*a^2*c^3*d*g*h^6 + 175*b^5*e*g*h^6
+ 56*a*b^3*c*e*g*h^6 - 5520*a^2*b*c^2*e*g*h^6 - 700*a*b^4*f*g*h^6 + 3024*a^
2*b^2*c*f*g*h^6 + 16128*a^3*c^2*f*g*h^6 + 315*b^5*d*h^7 - 2184*a*b^3*c*d*h^
7 + 3504*a^2*b*c^2*d*h^7 - 490*a*b^4*e*h^7 + 3024*a^2*b^2*c*e*h^7 - 3360*a^
3*c^2*e*h^7 + 840*a^2*b^3*f*h^7 - 4704*a^3*b*c*f*h^7)/(53760*h^5*(c*g^2 - b
*g*h + a*h^2)^4*(g + h*x)^2) + (2560*c^6*f*g^8 + 1024*c^6*e*g^7*h - 11264*b
*c^5*f*g^7*h + 768*c^6*d*g^6*h^2 - 3968*b*c^5*e*g^6*h^2 + 18208*b^2*c^4*f*g
^6*h^2 + 13952*a*c^5*f*g^6*h^2 - 2304*b*c^5*d*g^5*h^3 + 4864*b^2*c^4*e*g^5*
h^3 + 6656*a*c^5*e*g^5*h^3 - 11744*b^3*c^3*f*g^5*h^3 - 48512*a*b*c^4*f*g^5*
h^3 + 960*b^2*c^4*d*g^4*h^4 + 7680*a*c^5*d*g^4*h^4 - 800*b^3*c^3*e*g^4*h^4
- 20480*a*b*c^4*e*g^4*h^4 + 700*b^4*c^2*f*g^4*h^4 + 54720*a*b^2*c^3*f*g^4*h
^4 + 32320*a^2*c^4*f*g^4*h^4 + 1920*b^3*c^3*d*g^3*h^5 - 15360*a*b*c^4*d*g^3
*h^5 - 1400*b^4*c^2*e*g^3*h^5 + 12480*a*b^2*c^3*e*g^3*h^5 + 23680*a^2*c^4*e
*g^3*h^5 + 1120*b^5*c*f*g^3*h^5 - 11200*a*b^3*c^2*f*g^3*h^5 - 88320*a^2*b*c
^3*f*g^3*h^5 - 5124*b^4*c^2*d*g^2*h^6 + 35232*a*b^2*c^3*d*g^2*h^6 - 47424*a
^2*c^4*d*g^2*h^6 + 1750*b^5*c*e*g^2*h^6 - 8176*a*b^3*c^2*e*g^2*h^6 - 11808*
a^2*b*c^3*e*g^2*h^6 - 525*b^6*f*g^2*h^6 - 560*a*b^4*c*f*g^2*h^6 + 27216*a^2
*b^2*c^2*f*g^2*h^6 + 59904*a^3*c^3*f*g^2*h^6 + 3780*b^5*c*d*g*h^7 - 27552*a
*b^3*c^2*d*g*h^7 + 47424*a^2*b*c^3*d*g*h^7 - 525*b^6*e*g*h^7 - 980*a*b^4*c*
e*g*h^7 + 25872*a^2*b^2*c^2*e*g*h^7 - 42432*a^3*c^3*e*g*h^7 + 2100*a*b^5*f*
g*h^7 - 8960*a^2*b^3*c*f*g*h^7 - 17472*a^3*b*c^2*f*g*h^7 - 945*b^6*d*h^8 +

```
7560*a*b^4*c*d*h^8 - 16464*a^2*b^2*c^2*d*h^8 + 6144*a^3*c^3*d*h^8 + 1470*a*
b^5*e*h^8 - 10640*a^2*b^3*c*e*h^8 + 18144*a^3*b*c^2*e*h^8 - 2520*a^2*b^4*f*
h^8 + 16800*a^3*b^2*c*f*h^8 - 21504*a^4*c^2*f*h^8)/(107520*h^5*(c*g^2 - b*g
*h + a*h^2)^5*(g + h*x))))/(a + b*x + c*x^2) - ((b^2 - 4*a*c)^2*(-48*c^3*d*
g^3 + 24*b*c^2*e*g^3 - 14*b^2*c*f*g^3 + 8*a*c^2*f*g^3 + 72*b*c^2*d*g^2*h -
20*b^2*c*e*g^2*h - 64*a*c^2*e*g^2*h + 5*b^3*f*g^2*h + 52*a*b*c*f*g^2*h - 42
*b^2*c*d*g*h^2 + 24*a*c^2*d*g*h^2 + 5*b^3*e*g*h^2 + 52*a*b*c*e*g*h^2 - 20*a
*b^2*f*g*h^2 - 64*a^2*c*f*g*h^2 + 9*b^3*d*h^3 - 12*a*b*c*d*h^3 - 14*a*b^2*e
*h^3 + 8*a^2*c*e*h^3 + 24*a^2*b*f*h^3)*(a + x*(b + c*x))^(3/2)*Log[g + h*x]
)/(2048*(c*g^2 - b*g*h + a*h^2)^(11/2)*(a + b*x + c*x^2)^(3/2)) + ((b^2 - 4
*a*c)^2*(-48*c^3*d*g^3 + 24*b*c^2*e*g^3 - 14*b^2*c*f*g^3 + 8*a*c^2*f*g^3 +
72*b*c^2*d*g^2*h - 20*b^2*c*e*g^2*h - 64*a*c^2*e*g^2*h + 5*b^3*f*g^2*h + 52
*a*b*c*f*g^2*h - 42*b^2*c*d*g*h^2 + 24*a*c^2*d*...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 31329 vs. $2(1032) = 2064$.
time = 0.14, size = 31330, normalized size = 29.50

method	result	size
default	Expression too large to display	31330

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**8,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**8, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 76083 vs. 2(1063) = 2126.
time = 76.54, size = 76083, normalized size = 71.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="giac")
```

```
[Out] 1/1024*(48*b^4*c^3*d*g^3 - 384*a*b^2*c^4*d*g^3 + 768*a^2*c^5*d*g^3 + 14*b^6
*c*f*g^3 - 120*a*b^4*c^2*f*g^3 + 288*a^2*b^2*c^3*f*g^3 - 128*a^3*c^4*f*g^3
- 72*b^5*c^2*d*g^2*h + 576*a*b^3*c^3*d*g^2*h - 1152*a^2*b*c^4*d*g^2*h - 5*b
^7*f*g^2*h - 12*a*b^5*c*f*g^2*h + 336*a^2*b^3*c^2*f*g^2*h - 832*a^3*b*c^3*f
*g^2*h + 42*b^6*c*d*g*h^2 - 360*a*b^4*c^2*d*g*h^2 + 864*a^2*b^2*c^3*d*g*h^2
 - 384*a^3*c^4*d*g*h^2 + 20*a*b^6*f*g*h^2 - 96*a^2*b^4*c*f*g*h^2 - 192*a^3*
b^2*c^2*f*g*h^2 + 1024*a^4*c^3*f*g*h^2 - 9*b^7*d*h^3 + 84*a*b^5*c*d*h^3 - 2
40*a^2*b^3*c^2*d*h^3 + 192*a^3*b*c^3*d*h^3 - 24*a^2*b^5*f*h^3 + 192*a^3*b^3
*c*f*h^3 - 384*a^4*b*c^2*f*h^3 - 24*b^5*c^2*g^3*e + 192*a*b^3*c^3*g^3*e - 3
84*a^2*b*c^4*g^3*e + 20*b^6*c*g^2*h*e - 96*a*b^4*c^2*g^2*h*e - 192*a^2*b^2*
c^3*g^2*h*e + 1024*a^3*c^4*g^2*h*e - 5*b^7*g*h^2*e - 12*a*b^5*c*g*h^2*e + 3
36*a^2*b^3*c^2*g*h^2*e - 832*a^3*b*c^3*g*h^2*e + 14*a*b^6*h^3*e - 120*a^2*b
^4*c*h^3*e + 288*a^3*b^2*c^2*h^3*e - 128*a^4*c^3*h^3*e)*arctan(-((sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/((c^
5*g^10 - 5*b*c^4*g^9*h + 10*b^2*c^3*g^8*h^2 + 5*a*c^4*g^8*h^2 - 10*b^3*c^2*
g^7*h^3 - 20*a*b*c^3*g^7*h^3 + 5*b^4*c*g^6*h^4 + 30*a*b^2*c^2*g^6*h^4 + 10*
a^2*c^3*g^6*h^4 - b^5*g^5*h^5 - 20*a*b^3*c*g^5*h^5 - 30*a^2*b*c^2*g^5*h^5 +
 5*a*b^4*g^4*h^6 + 30*a^2*b^2*c*g^4*h^6 + 10*a^3*c^2*g^4*h^6 - 10*a^2*b^3*g
^3*h^7 - 20*a^3*b*c*g^3*h^7 + 10*a^3*b^2*g^2*h^8 + 5*a^4*c*g^2*h^8 - 5*a^4*
b*g*h^9 + a^5*h^10)*sqrt(-c*g^2 + b*g*h - a*h^2)) - 1/107520*(5040*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^13*b^4*c^3*d*g^3*h^12 - 40320*(sqrt(c)*x - sqrt
```

(c*x^2 + b*x + a))^13*a*b^2*c^4*d*g^3*h^12 + 80640*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^13*a^2*c^5*d*g^3*h^12 + 1470*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^13*b^6*c*f*g^3*h^12 - 12600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a*b^4*
c^2*f*g^3*h^12 + 30240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^2*b^2*c^3*f
*g^3*h^12 - 13440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^3*c^4*f*g^3*h^12
 - 7560*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*b^5*c^2*d*g^2*h^13 + 60480*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a*b^3*c^3*d*g^2*h^13 - 120960*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^13*a^2*b*c^4*d*g^2*h^13 - 525*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^13*b^7*f*g^2*h^13 - 1260*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^13*a*b^5*c*f*g^2*h^13 + 35280*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*
a^2*b^3*c^2*f*g^2*h^13 - 87360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^3*b
*c^3*f*g^2*h^13 + 4410*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*b^6*c*d*g*h^1
4 - 37800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a*b^4*c^2*d*g*h^14 + 90720
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^2*b^2*c^3*d*g*h^14 - 40320*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^13*a^3*c^4*d*g*h^14 + 2100*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^13*a*b^6*f*g*h^14 - 10080*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^13*a^2*b^4*c*f*g*h^14 - 20160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a
^3*b^2*c^2*f*g*h^14 + 107520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^4*c^3
*f*g*h^14 - 945*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*b^7*d*h^15 + 8820*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a*b^5*c*d*h^15 - 25200*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^13*a^2*b^3*c^2*d*h^15 + 20160*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^13*a^3*b*c^3*d*h^15 - 2520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
13*a^2*b^5*f*h^15 + 20160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^3*b^3*c*
f*h^15 - 40320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^4*b*c^2*f*h^15 - 25
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*b^5*c^2*g^3*h^12*e + 20160*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^13*a*b^3*c^3*g^3*h^12*e - 40320*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^13*a^2*b*c^4*g^3*h^12*e + 2100*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^13*b^6*c*g^2*h^13*e - 10080*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^13*a*b^4*c^2*g^2*h^13*e - 20160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*
a^2*b^2*c^3*g^2*h^13*e + 107520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^3*
c^4*g^2*h^13*e - 525*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*b^7*g*h^14*e -
1260*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a*b^5*c*g*h^14*e + 35280*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^13*a^2*b^3*c^2*g*h^14*e - 87360*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^13*a^3*b*c^3*g*h^14*e + 1470*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^13*a*b^6*h^15*e - 12600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13
*a^2*b^4*c*h^15*e + 30240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^3*b^2*c^
2*h^15*e - 13440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^13*a^4*c^3*h^15*e - 21
5040*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*c^(15/2)*f*g^10*h^5 + 1075200*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^12*b*c^(13/2)*f*g^9*h^6 - 2150400*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^12*b^2*c^(11/2)*f*g^8*h^7 - 1075200*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^12*a*c^(13/2)*f*g^8*h^7 + 2150400*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^12*b^3*c^(9/2)*f*g^7*h^8 + 4300800*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^12*a*b*c^(11/2)*f*g^7*h^8 - 1075200*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^12*b^4*c^(7/2)*f*g^6*h^9 - 6451200*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^12*a*b^2*c^(9/2)*f*g^6*h^9 - 2150400*(sqrt(c)*x - sqrt(c*x^2 + b

```
*x + a))^12*a^2*c^(11/2)*f*g^6*h^9 + 215040*(sq...
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x)
[Out] \text{Hanged}
```

3.208 $\int (1+2x)^3 \sqrt{2-x+3x^2} \left(1+3x+4x^2\right) dx$

Optimal. Leaf size=143

$$\frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552}+\frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{2}{21}(1+2x)^4\left(2-\right.$$

```
[Out] 17/105*(1+2*x)^2*(3*x^2-x+2)^(3/2)+67/378*(1+2*x)^3*(3*x^2-x+2)^(3/2)+2/21*
(1+2*x)^4*(3*x^2-x+2)^(3/2)-1/68040*(75295+26982*x)*(3*x^2-x+2)^(3/2)+12403
9/93312*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+5393/15552*(1-6*x)*(3*x^2-x+
2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 846, 793, 626, 633, 221}

$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4+\frac{67}{378}\left(3x^2-x+2\right)^{3/2}(2x+1)^3+\frac{17}{105}\left(3x^2-x+2\right)^{3/2}(2x+1)^2-\frac{(26982x+75295)\left(3x^2-x+2\right)^{3/2}}{68040}+\frac{5393(1-6x)\sqrt{3x^2-x+2}}{15552}+\frac{124039\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{31104\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^3*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (5393*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/15552 + (17*(1 + 2*x)^2*(2 - x + 3*x^2
)^(3/2))/105 + (67*(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2))/378 + (2*(1 + 2*x)^4*
(2 - x + 3*x^2)^(3/2))/21 - ((75295 + 26982*x)*(2 - x + 3*x^2)^(3/2))/68040
 + (124039*ArcSinh[(1 - 6*x)/Sqrt[23]])/(31104*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)^3\sqrt{2-x+3x^2}\,(1+3x+4x^2)\,dx = \frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)^{3/2}+\frac{1}{84}\int (1+2x)^3(-32+268x$$

$$= \frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)$$

$$= \frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{67}{378}(1+2x)^3\left(2-x+3x^2\right.$$

$$= \frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{67}{378}(1+2x)^3\left(2-x+3x^2\right.$$

$$= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552}+\frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3}$$

$$= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552}+\frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3}$$

$$= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552}+\frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3}$$

Mathematica [A]
time = 0.43, size = 80, normalized size = 0.56

$$\frac{6\sqrt{2-x+3x^2}\,(-543069+1493894x+3280872x^2+5497776x^3+7491456x^4+6462720x^5+2488320x^6)+4341365\sqrt{3}\,\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{3265920}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^3*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(-543069 + 1493894*x + 3280872*x^2 + 5497776*x^3 + 7
491456*x^4 + 6462720*x^5 + 2488320*x^6) + 4341365*Sqrt[3]*Log[1 - 6*x + 2*S
qrt[6 - 3*x + 9*x^2]])/3265920
```

Maple [A]
time = 0.17, size = 115, normalized size = 0.80

method	result
risch	$\frac{(2488320x^6+6462720x^5+7491456x^4+5497776x^3+3280872x^2+1493894x-543069)\sqrt{3x^2-x+2}}{544320}-\frac{124039\sqrt{3}\ \mathrm{arcsinh}\left(\frac{6}{}\right.}{93312}$
trager	$\left(\frac{32}{7}x^6+\frac{748}{63}x^5+\frac{1858}{135}x^4+\frac{38179}{3780}x^3+\frac{19529}{3240}x^2+\frac{746947}{272160}x-\frac{60341}{60480}\right)\sqrt{3x^2-x+2}+\frac{124039\,\mathrm{RootOf}\left(\_Z^2-\right.}{}$

default	$\frac{32x^4(3x^2-x+2)^{\frac{3}{2}}}{21}+\frac{844x^3(3x^2-x+2)^{\frac{3}{2}}}{189}+\frac{1594x^2(3x^2-x+2)^{\frac{3}{2}}}{315}+\frac{7849x(3x^2-x+2)^{\frac{3}{2}}}{3780}-\frac{124039\sqrt{3}\ \mathrm{arcsinh}\left(\frac{6\sqrt{23}\,(x}{23}\right.}{93312}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 32/21*x^4*(3*x^2-x+2)^(3/2)+844/189*x^3*(3*x^2-x+2)^(3/2)+1594/315*x^2*(3*x
^2-x+2)^(3/2)+7849/3780*x*(3*x^2-x+2)^(3/2)-124039/93312*3^(1/2)*arcsinh(6/
23*23^(1/2)*(x-1/6))-5393/15552*(6*x-1)*(3*x^2-x+2)^(1/2)-45739/68040*(3*x^
2-x+2)^(3/2)
```

Maxima [A]
time = 0.50, size = 126, normalized size = 0.88

$$\frac{32}{21}\,{\left(3\,x^{2}-x+2\right)}^{\frac{3}{2}}x^{4}+\frac{844}{189}\,{\left(3\,x^{2}-x+2\right)}^{\frac{3}{2}}x^{3}+\frac{1594}{315}\,{\left(3\,x^{2}-x+2\right)}^{\frac{3}{2}}x^{2}+\frac{7849}{3780}\,{\left(3\,x^{2}-x+2\right)}^{\frac{3}{2}}x-\frac{45739}{68040}\,{\left(3\,x^{2}-x+2\right)}^{\frac{3}{2}}-\frac{5393}{2592}\,\sqrt{3\,x^{2}-x+2}\,x-\frac{124039}{93312}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,{\left(6\,x-1\right)}\right)+\frac{5393}{15552}\,\sqrt{3\,x^{2}-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/21*(3*x^2 - x + 2)^(3/2)*x^4 + 844/189*(3*x^2 - x + 2)^(3/2)*x^3 + 1594/
315*(3*x^2 - x + 2)^(3/2)*x^2 + 7849/3780*(3*x^2 - x + 2)^(3/2)*x - 45739/6
8040*(3*x^2 - x + 2)^(3/2) - 5393/2592*sqrt(3*x^2 - x + 2)*x - 124039/93312
*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 5393/15552*sqrt(3*x^2 - x + 2)
```

Fricas [A]
time = 0.35, size = 83, normalized size = 0.58

$$\frac{1}{544320}\,{\left(2488320\,x^{6}+6462720\,x^{5}+7491456\,x^{4}+5497776\,x^{3}+3280872\,x^{2}+1493894\,x-543069\right)}\sqrt{3\,x^{2}-x+2}+\frac{124039}{186624}\,\sqrt{3}\,\log\left(4\,\sqrt{3}\,\sqrt{3\,x^{2}-x+2}\,{\left(6\,x-1\right)}-72\,x^{2}+24\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/544320*(2488320*x^6 + 6462720*x^5 + 7491456*x^4 + 5497776*x^3 + 3280872*x
^2 + 1493894*x - 543069)*sqrt(3*x^2 - x + 2) + 124039/186624*sqrt(3)*log(4*
sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x+1\right)^{3}\sqrt{3x^{2}-x+2}\cdot\left(4x^{2}+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)**3*sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1), x)
```

Giac [A]
time = 3.95, size = 78, normalized size = 0.55

$$\frac{1}{544320}\left(2\left(12\left(6\left(8\left(30\left(72x+187\right)x+6503\right)x+38179\right)x+136703\right)x+746947\right)x-543069\right)\sqrt{3x^2-x+2}+\frac{124039}{93312}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/544320*(2*(12*(6*(8*(30*(72*x + 187)*x + 6503)*x + 38179)*x + 136703)*x +
 746947)*x - 543069)*sqrt(3*x^2 - x + 2) + 124039/93312*sqrt(3)*log(-2*sqrt
(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [B]
time = 5.54, size = 170, normalized size = 1.19

$$\frac{1594x^2\left(3x^2-x+2\right)^{3/2}}{315}+\frac{844x^3\left(3x^2-x+2\right)^{3/2}}{189}+\frac{32x^4\left(3x^2-x+2\right)^{3/2}}{21}-\frac{137057\sqrt{3}\ \ln\left(\sqrt{3x^2-x+2}+\frac{\sqrt{3}\left(3x-\frac{1}{2}\right)}{3}\right)}{136080}-\frac{5959\left(\frac{x}{2}-\frac{1}{12}\right)\sqrt{3x^2-x+2}}{1890}-\frac{45739\sqrt{3x^2-x+2}\left(72x^2-6x+45\right)}{1632960}+\frac{7849x\left(3x^2-x+2\right)^{3/2}}{3780}-\frac{1051997\sqrt{3}\ \ln\left(2\sqrt{3x^2-x+2}+\frac{\sqrt{3}\left(6x-1\right)}{3}\right)}{3265920}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] (1594*x^2*(3*x^2 - x + 2)^(3/2))/315 + (844*x^3*(3*x^2 - x + 2)^(3/2))/189
+ (32*x^4*(3*x^2 - x + 2)^(3/2))/21 - (137057*3^(1/2)*log((3*x^2 - x + 2)^(
1/2) + (3^(1/2)*(3*x - 1/2))/3))/136080 - (5959*(x/2 - 1/12)*(3*x^2 - x + 2
)^(1/2))/1890 - (45739*(3*x^2 - x + 2)^(1/2)*(72*x^2 - 6*x + 45))/1632960 +
 (7849*x*(3*x^2 - x + 2)^(3/2))/3780 - (1051997*3^(1/2)*log(2*(3*x^2 - x +
2)^(1/2) + (3^(1/2)*(6*x - 1))/3))/3265920
```

3.209 $\int (1+2x)^2 \sqrt{2-x+3x^2} \left(1+3x+4x^2\right) dx$

Optimal. Leaf size=118

$$\frac{235(1-6x)\sqrt{2-x+3x^2}}{1296}+\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{1}{810}(25+306x)\left(2-\right.$$

```
[Out] 1/5*(1+2*x)^2*(3*x^2-x+2)^(3/2)+1/9*(1+2*x)^3*(3*x^2-x+2)^(3/2)+1/810*(25+3
06*x)*(3*x^2-x+2)^(3/2)+5405/7776*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+23
5/1296*(1-6*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 846, 793, 626, 633, 221}

$$\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3+\frac{1}{5}\left(3x^2-x+2\right)^{3/2}(2x+1)^2+\frac{1}{810}(306x+25)\left(3x^2-x+2\right)^{3/2}+\frac{235(1-6x)\sqrt{3x^2-x+2}}{1296}+\frac{5405\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{2592\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^2*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (235*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/1296 + ((1 + 2*x)^2*(2 - x + 3*x^2)^(3/
2))/5 + ((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2))/9 + ((25 + 306*x)*(2 - x + 3*x^
2)^(3/2))/810 + (5405*ArcSinh[(1 - 6*x)/Sqrt[23]])/(2592*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{1}{72}\int (1+2x)^2(-12+216x$$

$$=\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}$$

$$=\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}$$

$$=\frac{235(1-6x)\sqrt{2-x+3x^2}}{1296}+\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}$$

$$=\frac{235(1-6x)\sqrt{2-x+3x^2}}{1296}+\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}$$

$$=\frac{235(1-6x)\sqrt{2-x+3x^2}}{1296}+\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}$$

Mathematica [A]
time = 0.36, size = 75, normalized size = 0.64

$$\frac{6\sqrt{2-x+3x^2}\,(5607+14638x+22344x^2+33552x^3+35712x^4+17280x^5)+27025\sqrt{3}\,\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{38880}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^2*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(5607 + 14638*x + 22344*x^2 + 33552*x^3 + 35712*x^4
+ 17280*x^5) + 27025*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/38880
```

Maple [A]
time = 0.11, size = 98, normalized size = 0.83

method	result
risch	$\frac{(17280x^5+35712x^4+33552x^3+22344x^2+14638x+5607)\sqrt{3x^2-x+2}}{6480}-\frac{5405\sqrt{3}\ \mathrm{arcsinh}\left(\frac{6\sqrt{23}\,(x-\frac{1}{6})}{23}\right)}{7776}$
trager	$\left(\frac{8}{3}x^5+\frac{248}{45}x^4+\frac{233}{45}x^3+\frac{931}{270}x^2+\frac{7319}{3240}x+\frac{623}{720}\right)\sqrt{3x^2-x+2}+\frac{5405\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z\right.\right.}{}$

default	$\frac{8x^3(3x^2-x+2)^{\frac{3}{2}}}{9}+\frac{32x^2(3x^2-x+2)^{\frac{3}{2}}}{15}+\frac{83x(3x^2-x+2)^{\frac{3}{2}}}{45}+\frac{277(3x^2-x+2)^{\frac{3}{2}}}{810}-\frac{235(6x-1)\sqrt{3x^2-x+2}}{1296}-\frac{5405\sqrt{3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/9*x^3*(3*x^2-x+2)^(3/2)+32/15*x^2*(3*x^2-x+2)^(3/2)+83/45*x*(3*x^2-x+2)^(
3/2)+277/810*(3*x^2-x+2)^(3/2)-235/1296*(6*x-1)*(3*x^2-x+2)^(1/2)-5405/7776
*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Maxima [A]
time = 0.49, size = 109, normalized size = 0.92

$$\frac{8}{9}\left(3x^2-x+2\right)^{\frac{3}{2}}x^3+\frac{32}{15}\left(3x^2-x+2\right)^{\frac{3}{2}}x^2+\frac{83}{45}\left(3x^2-x+2\right)^{\frac{3}{2}}x+\frac{277}{810}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{235}{216}\sqrt{3x^2-x+2}\,x-\frac{5405}{7776}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}\,(6x-1)\right)+\frac{235}{1296}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*(3*x^2 - x + 2)^(3/2)*x^3 + 32/15*(3*x^2 - x + 2)^(3/2)*x^2 + 83/45*(3*
x^2 - x + 2)^(3/2)*x + 277/810*(3*x^2 - x + 2)^(3/2) - 235/216*sqrt(3*x^2 -
 x + 2)*x - 5405/7776*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 235/1296*s
qrt(3*x^2 - x + 2)
```

Fricas [A]
time = 0.39, size = 78, normalized size = 0.66

$$\frac{1}{6480}\left(17280x^5+35712x^4+33552x^3+22344x^2+14638x+5607\right)\sqrt{3x^2-x+2}+\frac{5405}{15552}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6480*(17280*x^5 + 35712*x^4 + 33552*x^3 + 22344*x^2 + 14638*x + 5607)*sqr
t(3*x^2 - x + 2) + 5405/15552*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*
x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)^2\sqrt{3x^2-x+2}\cdot\left(4x^2+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)**2*sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1), x)
```

Giac [A]
time = 5.15, size = 73, normalized size = 0.62

$$\frac{1}{6480}\left(2\left(12\left(6\left(8\left(15x+31\right)x+233\right)x+931\right)x+7319\right)x+5607\right)\sqrt{3x^{2}-x+2}+\frac{5405}{7776}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^{2}-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6480*(2*(12*(6*(8*(15*x + 31)*x + 233)*x + 931)*x + 7319)*x + 5607)*sqrt(
3*x^2 - x + 2) + 5405/7776*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 -
 x + 2)) + 1)
```

Mupad [B]
time = 5.15, size = 153, normalized size = 1.30

$$\frac{32x^{2}\left(3x^{2}-x+2\right)^{3/2}}{15}+\frac{8x^{3}\left(3x^{2}-x+2\right)^{3/2}}{9}-\frac{2783\sqrt{3}\ln\left(\sqrt{3x^{2}-x+2}+\frac{\sqrt{3}\left(3x-\frac{1}{2}\right)}{3}\right)}{3240}-\frac{121\left(\frac{x}{2}-\frac{1}{12}\right)\sqrt{3x^{2}-x+2}}{45}+\frac{277\sqrt{3x^{2}-x+2}\left(72x^{2}-6x+45\right)}{19440}+\frac{83x\left(3x^{2}-x+2\right)^{3/2}}{45}+\frac{6371\sqrt{3}\ln\left(2\sqrt{3x^{2}-x+2}+\frac{\sqrt{3}\left(6x-1\right)}{3}\right)}{38880}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] (32*x^2*(3*x^2 - x + 2)^(3/2))/15 + (8*x^3*(3*x^2 - x + 2)^(3/2))/9 - (2783
*3^(1/2)*log((3*x^2 - x + 2)^(1/2) + (3^(1/2)*(3*x - 1/2))/3))/3240 - (121*
(x/2 - 1/12)*(3*x^2 - x + 2)^(1/2))/45 + (277*(3*x^2 - x + 2)^(1/2)*(72*x^2
 - 6*x + 45))/19440 + (83*x*(3*x^2 - x + 2)^(3/2))/45 + (6371*3^(1/2)*log(2
*(3*x^2 - x + 2)^(1/2) + (3^(1/2)*(6*x - 1))/3))/38880
```

3.210 $\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)dx$

Optimal. Leaf size=93

$$\frac{19(1-6x)\sqrt{2-x+3x^2}}{2592}+\frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{(745+738x)\left(2-x+3x^2\right)^{3/2}}{1620}+\frac{437\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{5184\sqrt{3}}$$

```
[Out] 2/15*(1+2*x)^2*(3*x^2-x+2)^(3/2)+1/1620*(745+738*x)*(3*x^2-x+2)^(3/2)+437/1
5552*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+19/2592*(1-6*x)*(3*x^2-x+2)^(1/
2)
```

Rubi [A]

time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1667, 793, 626, 633, 221}

$$\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2+\frac{(738x+745)\left(3x^2-x+2\right)^{3/2}}{1620}+\frac{19(1-6x)\sqrt{3x^2-x+2}}{2592}+\frac{437\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{5184\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (19*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/2592 + (2*(1 + 2*x)^2*(2 - x + 3*x^2)^(3
/2))/15 + ((745 + 738*x)*(2 - x + 3*x^2)^(3/2))/1620 + (437*ArcSinh[(1 - 6*
x)/Sqrt[23]])/(5184*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{60}\int (1+2x)(8+164x)\sqrt{2$$

$$= \frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{(745+738x)\left(2-x+3x^2\right)}{1620}$$

$$= \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592}+\frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+$$

$$= \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592}+\frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+$$

$$= \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592}+\frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+$$

Mathematica [A]
time = 0.27, size = 70, normalized size = 0.75

$$\frac{6\sqrt{2-x+3x^2}\left(15471+17374x+24072x^2+31536x^3+20736x^4\right)+2185\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{77760}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(15471 + 17374*x + 24072*x^2 + 31536*x^3 + 20736*x^4
) + 2185*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/77760
```

Maple [A]
time = 0.10, size = 81, normalized size = 0.87

method	result
risch	$\frac{(20736x^4+31536x^3+24072x^2+17374x+15471)\sqrt{3x^2-x+2}}{12960} - \frac{437\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\ (x-\frac{1}{6})}{23}\right)}{15552}$
trager	$\left(\frac{8}{5}x^4+\frac{73}{30}x^3+\frac{1003}{540}x^2+\frac{8687}{6480}x+\frac{191}{160}\right)\sqrt{3x^2-x+2} - \frac{437\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x}\right.}{15552}$
default	$\frac{8x^2(3x^2-x+2)^{\frac{3}{2}}}{15} + \frac{89x(3x^2-x+2)^{\frac{3}{2}}}{90} + \frac{961(3x^2-x+2)^{\frac{3}{2}}}{1620} - \frac{19(6x-1)\sqrt{3x^2-x+2}}{2592} - \frac{437\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\ (x-}{23}\right.}{15552}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/15*x^2*(3*x^2-x+2)^(3/2)+89/90*x*(3*x^2-x+2)^(3/2)+961/1620*(3*x^2-x+2)^(
3/2)-19/2592*(6*x-1)*(3*x^2-x+2)^(1/2)-437/15552*3^(1/2)*arcsinh(6/23*23^(1
/2)*(x-1/6))
```

Maxima [A]
time = 0.49, size = 92, normalized size = 0.99

$$\frac{8}{15}\,{\left(3\,x^{2}-x+2\right)}^{\frac{3}{2}}x^{2}+\frac{89}{90}\,{\left(3\,x^{2}-x+2\right)}^{\frac{3}{2}}x+\frac{961}{1620}\,{\left(3\,x^{2}-x+2\right)}^{\frac{3}{2}}-\frac{19}{432}\,\sqrt{3\,x^{2}-x+2}\,x-\frac{437}{15552}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(6\,x-1)\right)+\frac{19}{2592}\,\sqrt{3\,x^{2}-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/15*(3*x^2 - x + 2)^(3/2)*x^2 + 89/90*(3*x^2 - x + 2)^(3/2)*x + 961/1620*(
3*x^2 - x + 2)^(3/2) - 19/432*sqrt(3*x^2 - x + 2)*x - 437/15552*sqrt(3)*arc
sinh(1/23*sqrt(23)*(6*x - 1)) + 19/2592*sqrt(3*x^2 - x + 2)
```

Fricas [A]
time = 0.39, size = 73, normalized size = 0.78

$$\frac{1}{12960}\,{\left(20736\,x^{4}+31536\,x^{3}+24072\,x^{2}+17374\,x+15471\right)}\sqrt{3\,x^{2}-x+2}+\frac{437}{31104}\,\sqrt{3}\,\log\left(4\,\sqrt{3}\,\sqrt{3\,x^{2}-x+2}\,(6\,x-1)-72\,x^{2}+24\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12960*(20736*x^4 + 31536*x^3 + 24072*x^2 + 17374*x + 15471)*sqrt(3*x^2 -
x + 2) + 437/31104*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72
*x^2 + 24*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)\sqrt{3x^2-x+2}\cdot\left(4x^2+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)*sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1), x)
```

Giac [A]
time = 4.18, size = 68, normalized size = 0.73

$$\frac{1}{12960}\left(2\left(12\left(18\left(48x+73\right)x+1003\right)x+8687\right)x+15471\right)\sqrt{3x^2-x+2}+\frac{437}{15552}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12960*(2*(12*(18*(48*x + 73)*x + 1003)*x + 8687)*x + 15471)*sqrt(3*x^2 -
x + 2) + 437/15552*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
 + 1)
```

Mupad [B]
time = 4.88, size = 136, normalized size = 1.46

$$\frac{8x^2\left(3x^2-x+2\right)^{3/2}}{15}-\frac{253\sqrt{3}\ln\left(\sqrt{3x^2-x+2}+\frac{\sqrt{3}\left(3x-\frac{1}{2}\right)}{3}\right)}{810}-\frac{44\left(\frac{x}{2}-\frac{1}{12}\right)\sqrt{3x^2-x+2}}{45}+\frac{961\sqrt{3x^2-x+2}\left(72x^2-6x+45\right)}{38880}+\frac{89x\left(3x^2-x+2\right)^{3/2}}{90}+\frac{22103\sqrt{3}\ln\left(2\sqrt{3x^2-x+2}+\frac{\sqrt{3}\left(6x-1\right)}{3}\right)}{77760}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] (8*x^2*(3*x^2 - x + 2)^(3/2))/15 - (253*3^(1/2)*log((3*x^2 - x + 2)^(1/2) +
 (3^(1/2)*(3*x - 1/2))/3))/810 - (44*(x/2 - 1/12)*(3*x^2 - x + 2)^(1/2))/45
 + (961*(3*x^2 - x + 2)^(1/2)*(72*x^2 - 6*x + 45))/38880 + (89*x*(3*x^2 - x
 + 2)^(3/2))/90 + (22103*3^(1/2)*log(2*(3*x^2 - x + 2)^(1/2) + (3^(1/2)*(6*
x - 1))/3))/77760
```

3.211 $\int \frac{\sqrt{2-x+3x^2}\,(1+3x+4x^2)}{1+2x}\,dx$

Optimal. Leaf size=101

$$\frac{1}{72}(13+30x)\sqrt{2-x+3x^2}+\frac{2}{9}\left(2-x+3x^2\right)^{3/2}-\frac{43\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}}-\frac{1}{8}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

```
[Out] 2/9*(3*x^2-x+2)^(3/2)-43/432*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1/8*arc
tanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+1/72*(13+30*x)*(3*x^
2-x+2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1667, 828, 857, 633, 221, 738, 212}

$$\frac{2}{9}\left(3x^2-x+2\right)^{3/2}+\frac{1}{72}(30x+13)\sqrt{3x^2-x+2}-\frac{1}{8}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)-\frac{43\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] ((13 + 30*x)*Sqrt[2 - x + 3*x^2])/72 + (2*(2 - x + 3*x^2)^(3/2))/9 - (43*Ar
cSinh[(1 - 6*x)/Sqrt[23]])/(144*Sqrt[3]) - (Sqrt[13]*ArcTanh[(9 - 8*x)/(2*S
qrt[13]*Sqrt[2 - x + 3*x^2])])/8
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{2-x+3x^2}\,(1+3x+4x^2)}{1+2x}\,dx = \frac{2}{9}\left(2-x+3x^2\right)^{3/2} + \frac{1}{36}\int \frac{(48+60x)\sqrt{2-x+3x^2}}{1+2x}\,dx$$

$$= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2} + \frac{2}{9}\left(2-x+3x^2\right)^{3/2} - \frac{\int \frac{-3324-1032x}{(1+2x)\sqrt{2-x+}}}{1728}$$

$$= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2} + \frac{2}{9}\left(2-x+3x^2\right)^{3/2} + \frac{43}{144}\int \frac{1}{\sqrt{2-x+}}$$

$$= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2} + \frac{2}{9}\left(2-x+3x^2\right)^{3/2} - \frac{13}{4}\text{Subst}\left(\int \frac{}{52-}\right.$$

$$= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2} + \frac{2}{9}\left(2-x+3x^2\right)^{3/2} - \frac{43\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right.}{144\sqrt{3}}$$

Mathematica [A]
time = 0.25, size = 104, normalized size = 1.03

$$\frac{1}{432}\left(6\sqrt{2-x+3x^2}\,(45+14x+48x^2)+108\sqrt{13}\,\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}\,x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)-43\sqrt{3}\,\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(45 + 14*x + 48*x^2) + 108*Sqrt[13]*ArcTanh[(Sqrt[3]
 + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] - 43*Sqrt[3]*Log[1 - 6*x
+ 2*Sqrt[6 - 3*x + 9*x^2]])/432
```

Maple [A]
time = 0.17, size = 95, normalized size = 0.94

method	result
risch	$\frac{(48x^2+14x+45)\sqrt{3x^2-x+2}}{72}+\frac{43\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{432}-\frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+}}\right.}{8}$
default	$\frac{5(6x-1)\sqrt{3x^2-x+2}}{72}+\frac{43\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{432}+\frac{2(3x^2-x+2)^{\frac{3}{2}}}{9}+\frac{\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}{8}-$

trager	$\left(\frac{2}{3}x^2+\frac{7}{36}x+\frac{5}{8}\right)\sqrt{3x^2-x+2}+\frac{43\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}-\mathrm{RootOf}\left(\_\right.\right.}{432}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(2*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 5/72*(6*x-1)*(3*x^2-x+2)^(1/2)+43/432*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6)
)+2/9*(3*x^2-x+2)^(3/2)+1/8*(12*(x+1/2)^2-16*x+5)^(1/2)-1/8*13^(1/2)*arctan
h(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Maxima [A]
time = 0.51, size = 96, normalized size = 0.95

$$\frac{2}{9}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{5}{12}\sqrt{3x^2-x+2}\,x+\frac{43}{432}\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{1}{8}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2x+1|}-\frac{9\sqrt{23}}{23\,|2x+1|}\right)+\frac{13}{72}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="maxima")
```

```
[Out] 2/9*(3*x^2 - x + 2)^(3/2) + 5/12*sqrt(3*x^2 - x + 2)*x + 43/432*sqrt(3)*arc
sinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1/8*sqrt(13)*arcsinh(8/23*sqrt(23)*
x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 13/72*sqrt(3*x^2 - x + 2)
```

Fricas [A]
time = 0.36, size = 115, normalized size = 1.14

$$\frac{1}{72}\left(48x^2+14x+45\right)\sqrt{3x^2-x+2}+\frac{43}{864}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)+\frac{1}{16}\sqrt{13}\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}\,(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="fricas")
```

```
[Out] 1/72*(48*x^2 + 14*x + 45)*sqrt(3*x^2 - x + 2) + 43/864*sqrt(3)*log(-4*sqrt(
3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 1/16*sqrt(13)*log(
-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2
+ 4*x + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{3x^2-x+2}\cdot\left(4x^2+3x+1\right)}{2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x),x)
```

```
[Out] Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

Giac [A]
time = 4.41, size = 126, normalized size = 1.25

$$\frac{1}{72}\left(2\left(24x+7\right)x+45\right)\sqrt{3x^2-x+2}-\frac{43}{432}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3x^2-x+2}\right)+\frac{1}{8}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="giac")
```

```
[Out] 1/72*(2*(24*x + 7)*x + 45)*sqrt(3*x^2 - x + 2) - 43/432*sqrt(3)*log(-6*sqrt
(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 1/8*sqrt(13)*log(-1/2*abs(-4*sqr
t(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqr
t(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3x^2-x+2}\,\left(4x^2+3x+1\right)}{2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
```

```
[Out] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

3.212 $\int \frac{\sqrt{2-x+3x^2}\,(1+3x+4x^2)}{(1+2x)^2}\,dx$

Optimal. Leaf size=108

$$-\frac{1}{156}(67-96x)\sqrt{2-x+3x^2}-\frac{(2-x+3x^2)^{3/2}}{13(1+2x)}-\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}+\frac{17\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{8\sqrt{13}}$$

```
[Out] -1/13*(3*x^2-x+2)^(3/2)/(1+2*x)-11/18*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2
)+17/104*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-1/156*(6
7-96*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1664, 828, 857, 633, 221, 738, 212}

$$-\frac{(3x^2-x+2)^{3/2}}{13(2x+1)}-\frac{1}{156}(67-96x)\sqrt{3x^2-x+2}+\frac{17\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{8\sqrt{13}}-\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] -1/156*((67 - 96*x)*Sqrt[2 - x + 3*x^2]) - (2 - x + 3*x^2)^(3/2)/(13*(1 + 2
*x)) - (11*ArcSinh[(1 - 6*x)/Sqrt[23]])/(6*Sqrt[3]) + (17*ArcTanh[(9 - 8*x)
/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(8*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-x+3x^2}\,(1+3x+4x^2)}{(1+2x)^2}\,dx = -\frac{(2-x+3x^2)^{3/2}}{13(1+2x)} - \frac{1}{13}\int \frac{\left(-\frac{15}{2}-32x\right)\sqrt{2-x+3x^2}}{1+2x}\,dx$$

$$= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2} - \frac{(2-x+3x^2)^{3/2}}{13(1+2x)} + \frac{1}{624}\int \frac{-}{(1+2}$$

$$= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2} - \frac{(2-x+3x^2)^{3/2}}{13(1+2x)} + \frac{11}{6}\int \frac{1}{\sqrt{2-x}}$$

$$= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2} - \frac{(2-x+3x^2)^{3/2}}{13(1+2x)} + \frac{17}{4}\text{Subst}\left(\int \frac{}{5}\right.$$

$$= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2} - \frac{(2-x+3x^2)^{3/2}}{13(1+2x)} - \frac{11\sinh^{-1}\left(\frac{1-}{\sqrt{}}\right.}{6\sqrt{3}}$$

Mathematica [A]
time = 0.41, size = 110, normalized size = 1.02

$$\frac{\sqrt{2-x+3x^2}\,(-7-2x+12x^2)}{12+24x} - \frac{17\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}\,x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{4\sqrt{13}} - \frac{11\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] (Sqrt[2 - x + 3*x^2]*(-7 - 2*x + 12*x^2))/(12 + 24*x) - (17*ArcTanh[(Sqrt[3
] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(4*Sqrt[13]) - (11*Log[
1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(6*Sqrt[3])
```

Maple [A]
time = 0.17, size = 123, normalized size = 1.14

method	result
risch	$\frac{36x^4-18x^3+5x^2+3x-14}{12(2x+1)\sqrt{3x^2-x+2}} + \frac{11\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{18} + \frac{17\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right.}{104}$

trager	$\frac{(12x^2-2x-7)\sqrt{3x^2-x+2}}{24x+12}-\frac{11\,\mathrm{RootOf}(\_Z^2-3)\ln\left(-6\,\mathrm{RootOf}(\_Z^2-3)x+6\sqrt{3x^2-x+2}+\mathrm{RootOf}(\_Z^2-3)\right.}{18}$
default	$\frac{(6x-1)\sqrt{3x^2-x+2}}{12}+\frac{11\sqrt{3}\,\mathrm{arcsinh}\left(\frac{6\sqrt{23}\,(x-\frac{1}{6})}{23}\right)}{18}-\frac{17\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}{104}+\frac{17\sqrt{13}\,\mathrm{arctanh}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(2*x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(6*x-1)*(3*x^2-x+2)^(1/2)+11/18*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
-17/104*(12*(x+1/2)^2-16*x+5)^(1/2)+17/104*13^(1/2)*arctanh(2/13*(9/2-4*x)*
13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))-1/26/(x+1/2)*(3*(x+1/2)^2-4*x+5/4)^(3
/2)+1/52*(6*x-1)*(3*(x+1/2)^2-4*x+5/4)^(1/2)
```

Maxima [A]
time = 0.52, size = 103, normalized size = 0.95

$$\frac{1}{2}\sqrt{3x^2-x+2}\,x+\frac{11}{18}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{17}{104}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2x+1|}-\frac{9\sqrt{23}}{23\,|2x+1|}\right)-\frac{1}{3}\sqrt{3x^2-x+2}-\frac{\sqrt{3x^2-x+2}}{4(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 - x + 2)*x + 11/18*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sq
rt(23)) - 17/104*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(
23)/abs(2*x + 1)) - 1/3*sqrt(3*x^2 - x + 2) - 1/4*sqrt(3*x^2 - x + 2)/(2*x
+ 1)
```

Fricas [A]
time = 0.34, size = 133, normalized size = 1.23

$$\frac{572\sqrt{3}(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+153\sqrt{13}(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)-220x^2+196x-185}{4x^2+4x+1}\right)+156(12x^2-2x-7)\sqrt{3x^2-x+2}}{1872(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/1872*(572*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
- 72*x^2 + 24*x - 25) + 153*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2 -
 x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 156*(12*x^2
 - 2*x - 7)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3x^2-x+2}\cdot(4x^2+3x+1)}{(2x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x)**2,x)
```

```
[Out] Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(85) = 170.
time = 5.50, size = 380, normalized size = 3.52

$$\frac{17}{104}\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)\mathrm{sgn}\left(\frac{1}{2x+1}\right)-\frac{11}{18}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{2\sqrt{13}}{2x+1}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)}\right)\mathrm{sgn}\left(\frac{1}{2x+1}\right)-\frac{1}{8}\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)+\frac{67\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)^3\mathrm{sgn}\left(\frac{1}{2x+1}\right)-57\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)^2\mathrm{sgn}\left(\frac{1}{2x+1}\right)+129\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)\mathrm{sgn}\left(\frac{1}{2x+1}\right)+27\sqrt{13}\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)}{12\left(\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)^2-3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="giac")
```

```
[Out] 17/104*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqr
t(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 11/18*sqrt(3)*log(1/2*abs(-2*sqrt(
3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1))/(sqr
t(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)))*sgn(1
/(2*x + 1)) - 1/8*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1/(2*x + 1))
+ 1/12*(67*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^3
*sgn(1/(2*x + 1)) - 57*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) +
sqrt(13)/(2*x + 1))^2*sgn(1/(2*x + 1)) + 129*(sqrt(-8/(2*x + 1) + 13/(2*x +
 1)^2 + 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 27*sqrt(13)*sgn(1/(2*x
+ 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^2 -
3)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3\,x^2-x+2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
```

```
[Out] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

3.213 $\int \frac{\sqrt{2-x+3x^2}\,(1+3x+4x^2)}{(1+2x)^3}\,dx$

Optimal. Leaf size=115

$$\frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)}-\frac{(2-x+3x^2)^{3/2}}{26(1+2x)^2}+\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}}-\frac{803\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{208\sqrt{13}}$$

```
[Out] -1/26*(3*x^2-x+2)^(3/2)/(1+2*x)^2+11/24*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1
/2)-803/2704*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+11/1
04*(7+10*x)*(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1664, 826, 857, 633, 221, 738, 212}

$$-\frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2}+\frac{11(10x+7)\sqrt{3x^2-x+2}}{104(2x+1)}-\frac{803\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{208\sqrt{13}}+\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] (11*(7 + 10*x)*Sqrt[2 - x + 3*x^2])/(104*(1 + 2*x)) - (2 - x + 3*x^2)^(3/2)
/(26*(1 + 2*x)^2) + (11*ArcSinh[(1 - 6*x)/Sqrt[23]])/(8*Sqrt[3]) - (803*Arc
Tanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(208*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-x+3x^2}\,(1+3x+4x^2)}{(1+2x)^3}\,dx = -\frac{(2-x+3x^2)^{3/2}}{26(1+2x)^2} - \frac{1}{26}\int \frac{\left(-\frac{33}{2}-55x\right)\sqrt{2-x+3x^2}}{(1+2x)^2}\,dx$$

$$= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{(2-x+3x^2)^{3/2}}{26(1+2x)^2} + \frac{1}{208}\int \frac{517-}{(1+2x)\sqrt{2}}$$

$$= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{(2-x+3x^2)^{3/2}}{26(1+2x)^2} - \frac{11}{8}\int \frac{1}{\sqrt{2-x+3x^2}}$$

$$= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{(2-x+3x^2)^{3/2}}{26(1+2x)^2} - \frac{803}{104}\text{Subst}\left(\int \frac{1}{52-}\right.$$

$$= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{(2-x+3x^2)^{3/2}}{26(1+2x)^2} + \frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}}$$

Mathematica [A]

time = 0.44, size = 111, normalized size = 0.97

$$\frac{\frac{39\sqrt{2-x+3x^2}\,(69+268x+208x^2)}{(1+2x)^2} + 2409\sqrt{13}\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}\,x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right) + 1859\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{4056}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((39*Sqrt[2 - x + 3*x^2]*(69 + 268*x + 208*x^2))/(1 + 2*x)^2 + 2409*Sqrt[13
]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] + 1859*
Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/4056
```

Maple [A]

time = 0.16, size = 125, normalized size = 1.09

method	result
risch	$\frac{624x^4+596x^3+355x^2+467x+138}{104(2x+1)^2\sqrt{3x^2-x+2}} - \frac{11\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{24} - \frac{803\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+}}\right)}{2704}$
trager	$\frac{(208x^2+268x+69)\sqrt{3x^2-x+2}}{104(2x+1)^2} + \frac{11\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}+\mathrm{RootOf}\left(\_Z^2\right.\right.}{24}$

default	$\frac{803\sqrt{12\left(x+\frac{1}{2}\right)^{2}-16x+5}}{2704}-\frac{11\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{24}-\frac{803\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^{2}-}}\right.}{2704}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(2*x+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 803/2704*(12*(x+1/2)^2-16*x+5)^(1/2)-11/24*3^(1/2)*arcsinh(6/23*23^(1/2)*(x
-1/6))-803/2704*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x
+5)^(1/2))+11/338/(x+1/2)*(3*(x+1/2)^2-4*x+5/4)^(3/2)-11/676*(6*x-1)*(3*(x+
1/2)^2-4*x+5/4)^(1/2)-1/104/(x+1/2)^2*(3*(x+1/2)^2-4*x+5/4)^(3/2)
```

Maxima [A]
time = 0.51, size = 114, normalized size = 0.99

$$-\frac{11}{24}\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{803}{2704}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{55}{104}\sqrt{3\,x^{2}-x+2}-\frac{\left(3\,x^{2}-x+2\right)^{\frac{3}{2}}}{26\,\left(4\,x^{2}+4\,x+1\right)}+\frac{11\sqrt{3\,x^{2}-x+2}}{52\,\left(2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="maxima")
```

```
[Out] -11/24*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 803/2704*sqrt(13)
*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 55/10
4*sqrt(3*x^2 - x + 2) - 1/26*(3*x^2 - x + 2)^(3/2)/(4*x^2 + 4*x + 1) + 11/5
2*sqrt(3*x^2 - x + 2)/(2*x + 1)
```

Fricas [A]
time = 0.33, size = 149, normalized size = 1.30

$$\frac{3718\sqrt{3}\,\left(4\,x^{2}+4\,x+1\right)\log\left(4\sqrt{3}\sqrt{3\,x^{2}-x+2}\,\left(6\,x-1\right)-72\,x^{2}+24\,x-25\right)+2409\sqrt{13}\,\left(4\,x^{2}+4\,x+1\right)\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^{2}-x+2}\,\left(8\,x-9\right)+220\,x^{2}-196\,x+185}{4\,x^{2}+4\,x+1}\right)+156\,\left(208\,x^{2}+268\,x+69\right)\sqrt{3\,x^{2}-x+2}}{16224\,\left(4\,x^{2}+4\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="fricas")
```

```
[Out] 1/16224*(3718*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(
6*x - 1) - 72*x^2 + 24*x - 25) + 2409*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*sq
rt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x
+ 1)) + 156*(208*x^2 + 268*x + 69)*sqrt(3*x^2 - x + 2))/(4*x^2 + 4*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{3x^{2}-x+2}\cdot\left(4x^{2}+3x+1\right)}{\left(2x+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x)**3,x)
```

```
[Out] Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-389344,[6]%%%}+%%%{%%{[1168032,0]:[1,0,-3]%%},[5]%%%}+%%%{-5
84016,
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3\,x^2-x+2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
```

```
[Out] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)
```

3.214 $\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx$

Optimal. Leaf size=158

$$\frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976}+\frac{54593(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872}-\frac{11(283-5850x)\left(2-x+3x^2\right)^{5/2}}{58320}+\frac{913}{486}$$

```
[Out] 54593/559872*(1-6*x)*(3*x^2-x+2)^(3/2)-11/58320*(283-5850*x)*(3*x^2-x+2)^(5
/2)+913/486*x^2*(3*x^2-x+2)^(5/2)+77/81*x^3*(3*x^2-x+2)^(5/2)+2/27*(1+2*x)^
4*(3*x^2-x+2)^(5/2)+28879697/26873856*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2
)+1255639/4478976*(1-6*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 12, 793, 626, 633, 221}

$$\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4+\frac{913}{486}x^2\left(3x^2-x+2\right)^{5/2}-\frac{11(283-5850x)\left(3x^2-x+2\right)^{5/2}}{58320}+\frac{54593(1-6x)\left(3x^2-x+2\right)^{3/2}}{559872}+\frac{1255639(1-6x)\sqrt{3x^2-x+2}}{4478976}+\frac{77}{81}x^3\left(3x^2-x+2\right)^{5/2}+\frac{28879697\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8957952\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (1255639*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/4478976 + (54593*(1 - 6*x)*(2 - x +
 3*x^2)^(3/2))/559872 - (11*(283 - 5850*x)*(2 - x + 3*x^2)^(5/2))/58320 + (
913*x^2*(2 - x + 3*x^2)^(5/2))/486 + (77*x^3*(2 - x + 3*x^2)^(5/2))/81 + (2
*(1 + 2*x)^4*(2 - x + 3*x^2)^(5/2))/27 + (28879697*ArcSinh[(1 - 6*x)/Sqrt[2
3]])/(8957952*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx = \frac{2}{27}(1+2x)^4 \left(2-x+3x^2\right)^{5/2} + \frac{1}{108} \int 308x(1+2x)^3 \, (2$$

$$= \frac{2}{27}(1+2x)^4 \left(2-x+3x^2\right)^{5/2} + \frac{77}{27} \int x(1+2x)^3 \left(2-x\right.$$

$$= \frac{77}{81}x^3 \left(2-x+3x^2\right)^{5/2} + \frac{2}{27}(1+2x)^4 \left(2-x+3x^2\right)^{5/2} +$$

$$= \frac{913}{486}x^2 \left(2-x+3x^2\right)^{5/2} + \frac{77}{81}x^3 \left(2-x+3x^2\right)^{5/2} + \frac{2}{27}(1$$

$$= -\frac{11(283-5850x)\left(2-x+3x^2\right)^{5/2}}{58320} + \frac{913}{486}x^2 \left(2-x+3x\right.$$

$$= \frac{54593(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872} - \frac{11(283-5850x)\,(2-}{58320}$$

$$= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)\,(2-x}{559872}$$

$$= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)\,(2-x}{559872}$$

$$= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)\,(2-x}{559872}$$

Mathematica [**A**]
time = 0.64, size = 90, normalized size = 0.57

$$\frac{6\sqrt{2-x+3x^2}\left(12499587+84014278x+201289704x^2+421626672x^3+649452672x^4+711210240x^5+635765760x^6+510105600x^7+238878720x^8\right)+144398485\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{134369280}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(12499587 + 84014278*x + 201289704*x^2 + 421626672*x
^3 + 649452672*x^4 + 711210240*x^5 + 635765760*x^6 + 510105600*x^7 + 238878
720*x^8) + 144398485*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/134369
280
```

Maple [**A**]
time = 0.12, size = 134, normalized size = 0.85

method	result
risch	$\frac{(238878720x^8+510105600x^7+635765760x^6+711210240x^5+649452672x^4+421626672x^3+201289704x^2+84014278x+12499587)\sqrt{3x}}{22394880}$
trager	$\left(\frac{32}{3}x^8+\frac{205}{9}x^7+\frac{511}{18}x^6+\frac{20579}{648}x^5+\frac{563761}{19440}x^4+\frac{2927963}{155520}x^3+\frac{8387071}{933120}x^2+\frac{42007139}{11197440}x+\frac{1388843}{2488320}\right)\sqrt{3x^2-}$
default	$\frac{32x^4(3x^2-x+2)^{\frac{5}{2}}}{27}+\frac{1099x(3x^2-x+2)^{\frac{5}{2}}}{648}-\frac{28879697\sqrt{3}\,\mathrm{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{26873856}-\frac{1255639(6x-1)\sqrt{3x^2-x+2}}{4478976}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 32/27*x^4*(3*x^2-x+2)^(5/2)+1099/648*x*(3*x^2-x+2)^(5/2)-28879697/26873856*
3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-1255639/4478976*(6*x-1)*(3*x^2-x+2)^
(1/2)-54593/559872*(6*x-1)*(3*x^2-x+2)^(3/2)+1207/58320*(3*x^2-x+2)^(5/2)+1
777/486*x^2*(3*x^2-x+2)^(5/2)+269/81*x^3*(3*x^2-x+2)^(5/2)
```

Maxima [A]
time = 0.51, size = 155, normalized size = 0.98

$$\frac{32}{27}\left(3x^2-x+2\right)^{\frac{5}{2}}x^4+\frac{269}{81}\left(3x^2-x+2\right)^{\frac{5}{2}}x^3+\frac{1777}{486}\left(3x^2-x+2\right)^{\frac{5}{2}}x^2+\frac{1099}{648}\left(3x^2-x+2\right)^{\frac{5}{2}}x+\frac{1207}{58320}\left(3x^2-x+2\right)^{\frac{5}{2}}-\frac{54593}{93312}\left(3x^2-x+2\right)^{\frac{3}{2}}x+\frac{54593}{559872}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{1255639}{746496}\sqrt{3x^2-x+2}\,x-\frac{28879697}{26873856}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}\,(6x-1)\right)+\frac{1255639}{4478976}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 32/27*(3*x^2 - x + 2)^(5/2)*x^4 + 269/81*(3*x^2 - x + 2)^(5/2)*x^3 + 1777/4
86*(3*x^2 - x + 2)^(5/2)*x^2 + 1099/648*(3*x^2 - x + 2)^(5/2)*x + 1207/5832
0*(3*x^2 - x + 2)^(5/2) - 54593/93312*(3*x^2 - x + 2)^(3/2)*x + 54593/55987
2*(3*x^2 - x + 2)^(3/2) - 1255639/746496*sqrt(3*x^2 - x + 2)*x - 28879697/2
6873856*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 1255639/4478976*sqrt(3*x
^2 - x + 2)
```

Fricas [A]
time = 0.33, size = 93, normalized size = 0.59

$$\frac{1}{22394880}\left(238878720x^8+510105600x^7+635765760x^6+711210240x^5+649452672x^4+421626672x^3+201289704x^2+84014278x+12499587\right)\sqrt{3x^2-x+2}+\frac{28879697}{53747712}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/22394880*(238878720*x^8 + 510105600*x^7 + 635765760*x^6 + 711210240*x^5 +
 649452672*x^4 + 421626672*x^3 + 201289704*x^2 + 84014278*x + 12499587)*sqr
```

```
t(3*x^2 - x + 2) + 28879697/53747712*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x +
 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)^3 \left(3x^2-x+2\right)^{\frac{3}{2}} \cdot \left(4x^2+3x+1\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
[Out] Integral((2*x + 1)**3*(3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1), x)
```

Giac [A]
time = 3.68, size = 88, normalized size = 0.56

$$\frac{1}{22394880}\left(2\left(12\left(6\left(8\left(30\left(36\left(2\left(96x+205\right)x+511\right)x+20579\right)x+563761\right)x+2927963\right)x+8387071\right)x+42007139\right)x+12499587\right)\sqrt{3x^2-x+2}+\frac{28879697}{26873856}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
[Out] 1/22394880*(2*(12*(6*(8*(30*(36*(2*(96*x + 205)*x + 511)*x + 20579)*x + 563
761)*x + 2927963)*x + 8387071)*x + 42007139)*x + 12499587)*sqrt(3*x^2 - x +
 2) + 28879697/26873856*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x
+ 2)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (2\,x+1)^3\,\left(3\,x^2-x+2\right)^{3/2}\,\left(4\,x^2+3\,x+1\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
[Out] int((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

3.215 $\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx$

Optimal. Leaf size=141

$$\frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648}+\frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456}+\frac{8}{63}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{12}(1+2x)^3\left(2-x\right.$$

```
[Out] 91/3456*(1-6*x)*(3*x^2-x+2)^(3/2)+8/63*(1+2*x)^2*(3*x^2-x+2)^(5/2)+1/12*(1+
2*x)^3*(3*x^2-x+2)^(5/2)+13/2520*(29+50*x)*(3*x^2-x+2)^(5/2)+48139/165888*a
rcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2093/27648*(1-6*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 846, 793, 626, 633, 221}

$$\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3+\frac{8}{63}\left(3x^2-x+2\right)^{5/2}(2x+1)^2+\frac{13(50x+29)\left(3x^2-x+2\right)^{5/2}}{2520}+\frac{91(1-6x)\left(3x^2-x+2\right)^{3/2}}{3456}+\frac{2093(1-6x)\sqrt{3x^2-x+2}}{27648}+\frac{48139\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{55296\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (2093*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/27648 + (91*(1 - 6*x)*(2 - x + 3*x^2)^
(3/2))/3456 + (8*(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2))/63 + ((1 + 2*x)^3*(2 -
x + 3*x^2)^(5/2))/12 + (13*(29 + 50*x)*(2 - x + 3*x^2)^(5/2))/2520 + (48139
*ArcSinh[(1 - 6*x)/Sqrt[23]])/(55296*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx = \frac{1}{12}(1+2x)^3 \left(2-x+3x^2\right)^{5/2} + \frac{1}{96} \int (1+2x)^2(20+256x$$

$$= \frac{8}{63}(1+2x)^2 \left(2-x+3x^2\right)^{5/2} + \frac{1}{12}(1+2x)^3 \left(2-x+3x^2\right.$$

$$= \frac{8}{63}(1+2x)^2 \left(2-x+3x^2\right)^{5/2} + \frac{1}{12}(1+2x)^3 \left(2-x+3x^2\right.$$

$$= \frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456} + \frac{8}{63}(1+2x)^2 \left(2-x+3x^2\right)$$

$$= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648} + \frac{91(1-6x)\left(2-x+3x^2\right)^{3/}}{3456}$$

$$= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648} + \frac{91(1-6x)\left(2-x+3x^2\right)^{3/}}{3456}$$

$$= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648} + \frac{91(1-6x)\left(2-x+3x^2\right)^{3/}}{3456}$$

Mathematica [A]
time = 0.55, size = 85, normalized size = 0.60

$$\frac{6\sqrt{2-x+3x^2}\,(1517367+2735918x+5694024x^2+10119792x^3+12173952x^4+10656000x^5+9262080x^6+5806080x^7)+1684865\sqrt{3}\,\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{5806080}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(1517367 + 2735918*x + 5694024*x^2 + 10119792*x^3 +
12173952*x^4 + 10656000*x^5 + 9262080*x^6 + 5806080*x^7) + 1684865*Sqrt[3]*
Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/5806080
```

Maple [A]
time = 0.14, size = 117, normalized size = 0.83

method	result
risch	$\frac{(5806080x^7+9262080x^6+10656000x^5+12173952x^4+10119792x^3+5694024x^2+2735918x+1517367)\sqrt{3x^2-x+2}}{967680} - \frac{48139\sqrt{}}{}$
trager	$\left(6x^7+\frac{67}{7}x^6+\frac{925}{84}x^5+\frac{4529}{360}x^4+\frac{210829}{20160}x^3+\frac{33893}{5760}x^2+\frac{1367959}{483840}x+\frac{505789}{322560}\right)\sqrt{3x^2-x+2} - \frac{48139\,\mathrm{Root}}{}$

default	$\frac{319x(3x^2-x+2)^{\frac{5}{2}}}{252} - \frac{48139\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\ \left(x-\frac{1}{6}\right)}{23}\right)}{165888} - \frac{2093(6x-1)\sqrt{3x^2-x+2}}{27648} - \frac{91(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{3456} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 319/252*x*(3*x^2-x+2)^(5/2)-48139/165888*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1
/6))-2093/27648*(6*x-1)*(3*x^2-x+2)^(1/2)-91/3456*(6*x-1)*(3*x^2-x+2)^(3/2)
+907/2520*(3*x^2-x+2)^(5/2)+95/63*x^2*(3*x^2-x+2)^(5/2)+2/3*x^3*(3*x^2-x+2)
^(5/2)
```

Maxima [A]
time = 0.52, size = 138, normalized size = 0.98

$$\frac{2}{3}\,{\left(3\,x^{2}-x+2\right)}^{\frac{5}{2}}x^{3}+\frac{95}{63}\,{\left(3\,x^{2}-x+2\right)}^{\frac{5}{2}}x^{2}+\frac{319}{252}\,{\left(3\,x^{2}-x+2\right)}^{\frac{5}{2}}x+\frac{907}{2520}\,{\left(3\,x^{2}-x+2\right)}^{\frac{5}{2}}-\frac{91}{576}\,{\left(3\,x^{2}-x+2\right)}^{\frac{3}{2}}x+\frac{91}{3456}\,{\left(3\,x^{2}-x+2\right)}^{\frac{3}{2}}-\frac{2093}{4608}\,\sqrt{3\,x^{2}-x+2}\,x-\frac{48139}{165888}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,{\left(6\,x-1\right)}\right)+\frac{2093}{27648}\,\sqrt{3\,x^{2}-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 2/3*(3*x^2 - x + 2)^(5/2)*x^3 + 95/63*(3*x^2 - x + 2)^(5/2)*x^2 + 319/252*(
3*x^2 - x + 2)^(5/2)*x + 907/2520*(3*x^2 - x + 2)^(5/2) - 91/576*(3*x^2 - x
 + 2)^(3/2)*x + 91/3456*(3*x^2 - x + 2)^(3/2) - 2093/4608*sqrt(3*x^2 - x +
2)*x - 48139/165888*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 2093/27648*s
qrt(3*x^2 - x + 2)
```

Fricas [A]
time = 0.34, size = 88, normalized size = 0.62

$$\frac{1}{967680}\,{\left(5806080\,x^{7}+9262080\,x^{6}+10656000\,x^{5}+12173952\,x^{4}+10119792\,x^{3}+5694024\,x^{2}+2735918\,x+1517367\right)}\sqrt{3\,x^{2}-x+2}+\frac{48139}{331776}\,\sqrt{3}\,\log\left(4\,\sqrt{3}\,\sqrt{3\,x^{2}-x+2}\,{\left(6\,x-1\right)}-72\,x^{2}+24\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/967680*(5806080*x^7 + 9262080*x^6 + 10656000*x^5 + 12173952*x^4 + 1011979
2*x^3 + 5694024*x^2 + 2735918*x + 1517367)*sqrt(3*x^2 - x + 2) + 48139/3317
76*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x+1\right)^{2}\left(3x^{2}-x+2\right)^{\frac{3}{2}}\cdot\left(4x^{2}+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
[Out] Integral((2*x + 1)**2*(3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1), x)
```

Giac [A]
time = 4.53, size = 83, normalized size = 0.59

$$\frac{1}{967680}(2(12(2(8(30(12(42x+67)x+925)x+31703)x+210829)x+237251)x+1367959)x+1517367)\sqrt{3x^2-x+2}+\frac{48139}{165888}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
[Out] 1/967680*(2*(12*(2*(8*(30*(12*(42*x + 67)*x + 925)*x + 31703)*x + 210829)*x
 + 237251)*x + 1367959)*x + 1517367)*sqrt(3*x^2 - x + 2) + 48139/165888*sqr
t(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (2x+1)^2\left(3x^2-x+2\right)^{3/2}\left(4x^2+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
[Out] int((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

3.216 $\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx$

Optimal. Leaf size=116

$$-\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736}-\frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592}+\frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{378}(109+102x$$

```
[Out] -71/2592*(1-6*x)*(3*x^2-x+2)^(3/2)+2/21*(1+2*x)^2*(3*x^2-x+2)^(5/2)+1/378*(
109+102*x)*(3*x^2-x+2)^(5/2)-37559/124416*arcsinh(1/23*(1-6*x)*23^(1/2))*3^
(1/2)-1633/20736*(1-6*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1667, 793, 626, 633, 221}

$$\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}+\frac{1}{378}(102x+109)\left(3x^2-x+2\right)^{5/2}-\frac{71(1-6x)\left(3x^2-x+2\right)^{3/2}}{2592}-\frac{1633(1-6x)\sqrt{3x^2-x+2}}{20736}-\frac{37559\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{41472\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (-1633*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/20736 - (71*(1 - 6*x)*(2 - x + 3*x^2)
^(3/2))/2592 + (2*(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2))/21 + ((109 + 102*x)*(2
 - x + 3*x^2)^(5/2))/378 - (37559*ArcSinh[(1 - 6*x)/Sqrt[23]])/(41472*Sqrt[
3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx = \frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{84}\int (1+2x)(40+204x)$$

$$= \frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{378}(109+102x)\left(2-x+\right.$$

$$= -\frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592}+\frac{2}{21}(1+2x)^2\left(2-x+3x^2\right.$$

$$= -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736}-\frac{71(1-6x)\left(2-x+3x^2\right)}{2592}$$

$$= -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736}-\frac{71(1-6x)\left(2-x+3x^2\right)}{2592}$$

$$= -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736}-\frac{71(1-6x)\left(2-x+3x^2\right)}{2592}$$

Mathematica [A]

time = 0.39, size = 80, normalized size = 0.69

$$\frac{6\sqrt{2-x+3x^2}\,(203337+275410x+531384x^2+744336x^3+653184x^4+518400x^5+497664x^6)-262913\sqrt{3}\,\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{870912}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(203337 + 275410*x + 531384*x^2 + 744336*x^3 + 65318
4*x^4 + 518400*x^5 + 497664*x^6) - 262913*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 -
3*x + 9*x^2]])/870912
```

Maple [A]
time = 0.10, size = 100, normalized size = 0.86

method	result
risch	$\frac{(497664x^6+518400x^5+653184x^4+744336x^3+531384x^2+275410x+203337)\sqrt{3x^2-x+2}}{145152}+\frac{37559\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}}{23}\right.}{124416}$
trager	$\left(\frac{24}{7}x^6+\frac{25}{7}x^5+\frac{9}{2}x^4+\frac{1723}{336}x^3+\frac{3163}{864}x^2+\frac{137705}{72576}x+\frac{7531}{5376}\right)\sqrt{3x^2-x+2}-\frac{37559\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\right.}{}$
default	$\frac{8x^2(3x^2-x+2)^{\frac{5}{2}}}{21}+\frac{41x(3x^2-x+2)^{\frac{5}{2}}}{63}+\frac{145(3x^2-x+2)^{\frac{5}{2}}}{378}+\frac{71(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{2592}+\frac{1633(6x-1)\sqrt{3x^2-x+2}}{20736}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 8/21*x^2*(3*x^2-x+2)^(5/2)+41/63*x*(3*x^2-x+2)^(5/2)+145/378*(3*x^2-x+2)^(5
/2)+71/2592*(6*x-1)*(3*x^2-x+2)^(3/2)+1633/20736*(6*x-1)*(3*x^2-x+2)^(1/2)+
37559/124416*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Maxima [A]
time = 0.50, size = 121, normalized size = 1.04

$$\frac{8}{21}\,(3\,x^2-x+2)^{\frac{5}{2}}x^2+\frac{41}{63}\,(3\,x^2-x+2)^{\frac{5}{2}}x+\frac{145}{378}\,(3\,x^2-x+2)^{\frac{5}{2}}+\frac{71}{432}\,(3\,x^2-x+2)^{\frac{3}{2}}x-\frac{71}{2592}\,(3\,x^2-x+2)^{\frac{3}{2}}+\frac{1633}{3456}\,\sqrt{3\,x^2-x+2}\,x+\frac{37559}{124416}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(6\,x-1)\right)-\frac{1633}{20736}\,\sqrt{3\,x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 8/21*(3*x^2 - x + 2)^(5/2)*x^2 + 41/63*(3*x^2 - x + 2)^(5/2)*x + 145/378*(3
*x^2 - x + 2)^(5/2) + 71/432*(3*x^2 - x + 2)^(3/2)*x - 71/2592*(3*x^2 - x +
 2)^(3/2) + 1633/3456*sqrt(3*x^2 - x + 2)*x + 37559/124416*sqrt(3)*arcsinh(
1/23*sqrt(23)*(6*x - 1)) - 1633/20736*sqrt(3*x^2 - x + 2)
```

Fricas [A]
time = 0.39, size = 83, normalized size = 0.72

$$\frac{1}{145152}\left(497664\,x^{6}+518400\,x^{5}+653184\,x^{4}+744336\,x^{3}+531384\,x^{2}+275410\,x+203337\right)\sqrt{3\,x^{2}-x+2}+\frac{37559}{248832}\,\sqrt{3}\,\log\left(-4\,\sqrt{3}\,\sqrt{3\,x^{2}-x+2}\,(6\,x-1)-72\,x^{2}+24\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/145152*(497664*x^6 + 518400*x^5 + 653184*x^4 + 744336*x^3 + 531384*x^2 +
275410*x + 203337)*sqrt(3*x^2 - x + 2) + 37559/248832*sqrt(3)*log(-4*sqrt(3
)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)\left(3x^{2}-x+2\right)^{\frac{3}{2}}\cdot\left(4x^{2}+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
```

```
[Out] Integral((2*x + 1)*(3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1), x)
```

Giac [A]
time = 4.16, size = 78, normalized size = 0.67

$$\frac{1}{145152}\left(2\left(12\left(18\left(24\left(2\left(24\,x+25\right)x+63\right)x+1723\right)x+22141\right)x+137705\right)x+203337\right)\sqrt{3\,x^{2}-x+2}-\frac{37559}{124416}\,\sqrt{3}\,\log\left(-2\,\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^{2}-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/145152*(2*(12*(18*(24*(2*(24*x + 25)*x + 63)*x + 1723)*x + 22141)*x + 137
705)*x + 203337)*sqrt(3*x^2 - x + 2) - 37559/124416*sqrt(3)*log(-2*sqrt(3)*
(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (2\,x+1)\,\left(3\,x^{2}-x+2\right)^{3/2}\,\left(4\,x^{2}+3\,x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] int((2*x + 1)*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

3.217 $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx$

Optimal. Leaf size=124

$$\frac{(869+402x)\sqrt{2-x+3x^2}}{1152}+\frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2}+\frac{2}{15}\left(2-x+3x^2\right)^{5/2}+\frac{2203\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{2304\sqrt{3}}-$$

```
[Out] 1/144*(7+30*x)*(3*x^2-x+2)^(3/2)+2/15*(3*x^2-x+2)^(5/2)+2203/6912*arcsinh(1
/23*(1-6*x)*23^(1/2))*3^(1/2)-13/32*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+
2)^(1/2))*13^(1/2)+1/1152*(869+402*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1667, 828, 857, 633, 221, 738, 212}

$$\frac{2}{15}\left(3x^2-x+2\right)^{5/2}+\frac{1}{144}(30x+7)\left(3x^2-x+2\right)^{3/2}+\frac{(402x+869)\sqrt{3x^2-x+2}}{1152}-\frac{13}{32}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)+\frac{2203\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{2304\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] ((869 + 402*x)*Sqrt[2 - x + 3*x^2])/1152 + ((7 + 30*x)*(2 - x + 3*x^2)^(3/2
))/144 + (2*(2 - x + 3*x^2)^(5/2))/15 + (2203*ArcSinh[(1 - 6*x)/Sqrt[23]])/
(2304*Sqrt[3]) - (13*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*
x^2])])/32
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{2}{15}(2-x+3x^2)^{5/2} + \frac{1}{60}\int \frac{(80+100x)(2-x+3x^2)^{3/2}}{1+2x}\,dx$$

$$= \frac{1}{144}(7+30x)(2-x+3x^2)^{3/2} + \frac{2}{15}(2-x+3x^2)^{5/2} - \int \frac{(-13380-$$

$$= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)(2-x+3x^2)^{3/2} + \frac{2}{1}$$

$$= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)(2-x+3x^2)^{3/2} + \frac{2}{1}$$

$$= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)(2-x+3x^2)^{3/2} + \frac{2}{1}$$

$$= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)(2-x+3x^2)^{3/2} + \frac{2}{1}$$

Mathematica [A]
time = 0.38, size = 114, normalized size = 0.92

$$\frac{6\sqrt{2-x+3x^2}\,(7977+1058x+9624x^2-1008x^3+6912x^4)+28080\sqrt{13}\,\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}\,x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)+11015\sqrt{3}\,\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{34560}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(7977 + 1058*x + 9624*x^2 - 1008*x^3 + 6912*x^4) + 2
8080*Sqrt[13]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[
13]] + 11015*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/34560
```

Maple [A]
time = 0.16, size = 151, normalized size = 1.22

method	result
risch	$\frac{(6912x^4-1008x^3+9624x^2+1058x+7977)\sqrt{3x^2-x+2}}{5760} - \frac{2203\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,(x-\frac{1}{6})}{23}\right)}{6912} - \frac{13\sqrt{13}\,\operatorname{arctanh}\left(\right.}{}$

trager	$\left(\frac{6}{5}x^4-\frac{7}{40}x^3+\frac{401}{240}x^2+\frac{529}{2880}x+\frac{2659}{1920}\right)\sqrt{3x^2-x+2}+\frac{13\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x}}{}\right.}{32}$
default	$\frac{5(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{144}+\frac{115(6x-1)\sqrt{3x^2-x+2}}{1152}-\frac{2203\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,(x-\frac{1}{6})}{23}\right)}{6912}+\frac{2(3x^2-x+2)^{\frac{5}{2}}}{15}+\frac{\left(3(x+\frac{1}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(2*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 5/144*(6*x-1)*(3*x^2-x+2)^(3/2)+115/1152*(6*x-1)*(3*x^2-x+2)^(1/2)-2203/691
2*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+2/15*(3*x^2-x+2)^(5/2)+1/12*(3*(x+
1/2)^2-4*x+5/4)^(3/2)-1/24*(6*x-1)*(3*(x+1/2)^2-4*x+5/4)^(1/2)+13/32*(12*(x
+1/2)^2-16*x+5)^(1/2)-13/32*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x
+1/2)^2-16*x+5)^(1/2))
```

Maxima [A]
time = 0.52, size = 125, normalized size = 1.01

$$\frac{2}{15}\,(3\,x^2-x+2)^{\frac{5}{2}}+\frac{5}{24}\,(3\,x^2-x+2)^{\frac{3}{2}}x+\frac{7}{144}\,(3\,x^2-x+2)^{\frac{3}{2}}+\frac{67}{192}\,\sqrt{3\,x^2-x+2}\,x-\frac{2203}{6912}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\,\sqrt{23}\,x-\frac{1}{23}\,\sqrt{23}\right)+\frac{13}{32}\,\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\,\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\,\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{869}{1152}\,\sqrt{3\,x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="maxima")
```

```
[Out] 2/15*(3*x^2 - x + 2)^(5/2) + 5/24*(3*x^2 - x + 2)^(3/2)*x + 7/144*(3*x^2 -
x + 2)^(3/2) + 67/192*sqrt(3*x^2 - x + 2)*x - 2203/6912*sqrt(3)*arcsinh(6/2
3*sqrt(23)*x - 1/23*sqrt(23)) + 13/32*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(
2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 869/1152*sqrt(3*x^2 - x + 2)
```

Fricas [A]
time = 0.37, size = 125, normalized size = 1.01

$$\frac{1}{5760}\,(6912\,x^4-1008\,x^3+9624\,x^2+1058\,x+7977)\sqrt{3\,x^2-x+2}+\frac{2203}{13824}\,\sqrt{3}\,\log\left(4\,\sqrt{3}\,\sqrt{3\,x^2-x+2}\,(6\,x-1)-72\,x^2+24\,x-25\right)+\frac{13}{64}\,\sqrt{13}\,\log\left(-\frac{4\,\sqrt{13}\,\sqrt{3\,x^2-x+2}\,(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="fricas")
```

```
[Out] 1/5760*(6912*x^4 - 1008*x^3 + 9624*x^2 + 1058*x + 7977)*sqrt(3*x^2 - x + 2)
 + 2203/13824*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2
+ 24*x - 25) + 13/64*sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9
) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2-x+2)^{\frac{3}{2}}\cdot(4x^2+3x+1)}{2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x),x)
```

```
[Out] Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

Giac [A]
time = 4.21, size = 136, normalized size = 1.10

$$\frac{1}{5760}\left(2\left(12\left(6\left(48x-7\right)x+401\right)x+529\right)x+7977\right)\sqrt{3x^2-x+2}+\frac{2203}{6912}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3x^2-x+2}\right)+\frac{13}{32}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="giac")
```

```
[Out] 1/5760*(2*(12*(6*(48*x - 7)*x + 401)*x + 529)*x + 7977)*sqrt(3*x^2 - x + 2)
 + 2203/6912*sqrt(3)*log(-6*sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) +
13/32*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(
3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(3\,x^2-x+2\right)}^{3/2}\,\left(4\,x^2+3\,x+1\right)}{2\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
```

```
[Out] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

3.218 $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx$

Optimal. Leaf size=131

$$-\frac{1}{192}(349-294x)\sqrt{2-x+3x^2}-\frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2}-\frac{\left(2-x+3x^2\right)^{5/2}}{13(1+2x)}-\frac{2327\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{384\sqrt{3}}-$$

```
[Out] -1/104*(23-38*x)*(3*x^2-x+2)^(3/2)-1/13*(3*x^2-x+2)^(5/2)/(1+2*x)-2327/1152
*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+25/32*arctanh(1/26*(9-8*x)*13^(1/2)
/(3*x^2-x+2)^(1/2))*13^(1/2)-1/192*(349-294*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1664, 828, 857, 633, 221, 738, 212}

$$-\frac{(3x^2-x+2)^{5/2}}{13(2x+1)}-\frac{1}{104}(23-38x)\left(3x^2-x+2\right)^{3/2}-\frac{1}{192}(349-294x)\sqrt{3x^2-x+2}+\frac{25}{32}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)-\frac{2327\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{384\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] -1/192*((349 - 294*x)*Sqrt[2 - x + 3*x^2]) - ((23 - 38*x)*(2 - x + 3*x^2)^(
3/2))/104 - (2 - x + 3*x^2)^(5/2)/(13*(1 + 2*x)) - (2327*ArcSinh[(1 - 6*x)/
Sqrt[23]])/(384*Sqrt[3]) + (25*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[
2 - x + 3*x^2])])/32
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = -\frac{(2-x+3x^2)^{5/2}}{13(1+2x)} - \frac{1}{13}\int \frac{\left(-\frac{13}{2}-38x\right)(2-x+3x^2)^{3/2}}{1+2x}\,dx$$

$$= -\frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} - \frac{(2-x+3x^2)^{5/2}}{13(1+2x)} + \frac{\int \frac{(-78+7644x}{}}{}$$

$$= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2} - \frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/}$$

$$= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2} - \frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/}$$

$$= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2} - \frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/}$$

$$= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2} - \frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/}$$

Mathematica [A]
time = 0.45, size = 121, normalized size = 0.92

$$\frac{\frac{6\sqrt{2-x+3x^2}\,(-493-332x+564x^2-96x^3+288x^4)}{1+2x} - 1800\sqrt{13}\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}\,x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right) - 2327\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{1152}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
[Out] ((6*Sqrt[2 - x + 3*x^2]*(-493 - 332*x + 564*x^2 - 96*x^3 + 288*x^4))/(1 + 2
*x) - 1800*Sqrt[13]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])
/Sqrt[13]] - 2327*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/1152
```

Maple [A]
time = 0.17, size = 179, normalized size = 1.37

method	result
risch	$\frac{864x^6-576x^5+2364x^4-1752x^3-19x^2-171x-986}{192(2x+1)\sqrt{3x^2-x+2}} + \frac{2327\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{1152} + \frac{25\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}\right.}{13\sqrt{12\,(x+}}\right.}{32}$

trager	$\frac{(288x^4-96x^3+564x^2-332x-493)\sqrt{3x^2-x+2}}{384x+192}+\frac{25\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{-8\,\mathrm{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+}}{2x+1}\right.}{32}$
default	$\frac{(6x-1)\left(3x^2-x+2\right)^{\frac{3}{2}}}{24}+\frac{23(6x-1)\sqrt{3x^2-x+2}}{192}+\frac{2327\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{1152}-\frac{25\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}{156}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(2*x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(6*x-1)*(3*x^2-x+2)^(3/2)+23/192*(6*x-1)*(3*x^2-x+2)^(1/2)+2327/1152*3
^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-25/156*(3*(x+1/2)^2-4*x+5/4)^(3/2)+13
/96*(6*x-1)*(3*(x+1/2)^2-4*x+5/4)^(1/2)-25/32*(12*(x+1/2)^2-16*x+5)^(1/2)+2
5/32*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))-
1/26/(x+1/2)*(3*(x+1/2)^2-4*x+5/4)^(5/2)+1/52*(6*x-1)*(3*(x+1/2)^2-4*x+5/4)
^(3/2)
```

Maxima [A]
time = 0.51, size = 132, normalized size = 1.01

$$\frac{1}{4}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{1}{8}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{49}{32}\sqrt{3x^2-x+2}\,x+\frac{2327}{1152}\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{25}{32}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2x+1|}-\frac{9\sqrt{23}}{23\,|2x+1|}\right)-\frac{349}{192}\sqrt{3x^2-x+2}-\frac{\left(3x^2-x+2\right)^{\frac{3}{2}}}{4\,(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(3*x^2 - x + 2)^(3/2)*x - 1/8*(3*x^2 - x + 2)^(3/2) + 49/32*sqrt(3*x^2
- x + 2)*x + 2327/1152*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 2
5/32*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x
+ 1)) - 349/192*sqrt(3*x^2 - x + 2) - 1/4*(3*x^2 - x + 2)^(3/2)/(2*x + 1)
```

Fricas [A]
time = 0.40, size = 143, normalized size = 1.09

$$\frac{2327\sqrt{3}\,(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)+900\sqrt{13}\,(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}\,(8x-9)-220x^2+196x-185}{4x^2+4x+1}\right)+12\left(288x^4-96x^3+564x^2-332x-493\right)\sqrt{3x^2-x+2}}{2304\,(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2304*(2327*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
 - 72*x^2 + 24*x - 25) + 900*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2
- x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 12*(288*x^
4 - 96*x^3 + 564*x^2 - 332*x - 493)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^{2}-x+2\right)^{\frac{3}{2}}\cdot\left(4x^{2}+3x+1\right)}{\left(2x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x)**2,x)
```

```
[Out] Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 570 vs. 2(104) = 208.
time = 4.47, size = 570, normalized size = 4.35

$$\frac{25}{32}\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^{2}}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)-\frac{2327}{1152}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^{2}}+3}+\frac{2\sqrt{13}}{2x+1}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^{2}}+3}+\frac{\sqrt{13}}{2x+1}\right)}\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)-\frac{13}{32}\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^{2}}+3}\,\operatorname{sgn}\left(\frac{1}{2x+1}\right)+\frac{5929\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^{2}}+3}+\frac{\sqrt{13}}{2x+1}\right)^{7}\operatorname{sgn}\left(\frac{1}{2x+1}\right)-7272\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^{2}}+3}+\frac{\sqrt{13}}{2x+1}\right)^{6}\operatorname{sgn}\left(\frac{1}{2x+1}\right)+25101\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^{2}}+3}+\frac{\sqrt{13}}{2x+1}\right)^{5}\operatorname{sgn}\left(\frac{1}{2x+1}\right)-48\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^{2}}+3}+\frac{\sqrt{13}}{2x+1}\right)^{4}\operatorname{sgn}\left(\frac{1}{2x+1}\right)+112359\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^{2}}+3}+\frac{\sqrt{13}}{2x+1}\right)^{3}\operatorname{sgn}\left(\frac{1}{2x+1}\right)-69336\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^{2}}+3}+\frac{\sqrt{13}}{2x+1}\right)^{2}\operatorname{sgn}\left(\frac{1}{2x+1}\right)+71955\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^{2}}+3}+\frac{\sqrt{13}}{2x+1}\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)+24624\sqrt{13}\operatorname{sgn}\left(\frac{1}{2x+1}\right)}{192\left(\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^{2}}+3}+\frac{\sqrt{13}}{2x+1}\right)^{2}-3\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="giac")
```

```
[Out] 25/32*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt
(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 2327/1152*sqrt(3)*log(1/2*abs(-2*sq
rt(3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1))/(
sqrt(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)))*sg
n(1/(2*x + 1)) - 13/32*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1/(2*x +
 1)) + 1/192*(5929*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x
 + 1))^7*sgn(1/(2*x + 1)) - 7272*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)
^2 + 3) + sqrt(13)/(2*x + 1))^6*sgn(1/(2*x + 1)) + 25101*(sqrt(-8/(2*x + 1)
 + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^5*sgn(1/(2*x + 1)) - 48*sqrt(1
3)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^4*sgn(1/(
2*x + 1)) + 112359*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x
 + 1))^3*sgn(1/(2*x + 1)) - 69336*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1
)^2 + 3) + sqrt(13)/(2*x + 1))^2*sgn(1/(2*x + 1)) + 71955*(sqrt(-8/(2*x + 1
) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 24624*sqrt
(13)*sgn(1/(2*x + 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)
/(2*x + 1))^2 - 3)^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(3\,x^2-x+2\right)}^{3/2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
```

```
[Out] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

3.219 $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx$

Optimal. Leaf size=138

$$\frac{1}{624}(1858-771x)\sqrt{2-x+3x^2}+\frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)}-\frac{\left(2-x+3x^2\right)^{5/2}}{26(1+2x)^2}+\frac{1519\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{192\sqrt{3}}-$$

```
[Out] 1/312*(151+122*x)*(3*x^2-x+2)^(3/2)/(1+2*x)-1/26*(3*x^2-x+2)^(5/2)/(1+2*x)^
2+1519/576*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1153/832*arctanh(1/26*(9-
8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+1/624*(1858-771*x)*(3*x^2-x+2)^(1
/2)
```

Rubi [A]
time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1664, 826, 828, 857, 633, 221, 738, 212}

$$-\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}+\frac{(122x+151)\left(3x^2-x+2\right)^{3/2}}{312(2x+1)}+\frac{1}{624}(1858-771x)\sqrt{3x^2-x+2}-\frac{1153\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{64\sqrt{13}}+\frac{1519\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{192\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((1858 - 771*x)*Sqrt[2 - x + 3*x^2])/624 + ((151 + 122*x)*(2 - x + 3*x^2)^(
3/2))/(312*(1 + 2*x)) - (2 - x + 3*x^2)^(5/2)/(26*(1 + 2*x)^2) + (1519*ArcS
inh[(1 - 6*x)/Sqrt[23]])/(192*Sqrt[3]) - (1153*ArcTanh[(9 - 8*x)/(2*Sqrt[13
]*Sqrt[2 - x + 3*x^2])])/(64*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
```

```
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = -\frac{(2-x+3x^2)^{5/2}}{26(1+2x)^2} - \frac{1}{26}\int \frac{\left(-\frac{31}{2}-61x\right)(2-x+3x^2)^{3/2}}{(1+2x)^2}\,dx$$

$$= \frac{(151+122x)(2-x+3x^2)^{3/2}}{312(1+2x)} - \frac{(2-x+3x^2)^{5/2}}{26(1+2x)^2} + \frac{1}{208}\int \frac{(639 -$$

$$= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)(2-x+3x^2)^{3/2}}{312(1+2x)}$$

$$= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)(2-x+3x^2)^{3/2}}{312(1+2x)}$$

$$= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)(2-x+3x^2)^{3/2}}{312(1+2x)}$$

$$= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)(2-x+3x^2)^{3/2}}{312(1+2x)}$$

Mathematica [A]
time = 0.58, size = 121, normalized size = 0.88

$$\frac{\frac{156\sqrt{2-x+3x^2}\,(182+627x+390x^2-68x^3+96x^4)}{(1+2x)^2}+20754\sqrt{13}\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}\,x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)+19747\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{7488}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((156*Sqrt[2 - x + 3*x^2]*(182 + 627*x + 390*x^2 - 68*x^3 + 96*x^4))/(1 + 2
*x)^2 + 20754*Sqrt[13]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^
2])/Sqrt[13]] + 19747*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/7488
```

Maple [A]
time = 0.16, size = 162, normalized size = 1.17

method	result
risch	$\frac{288x^6-300x^5+1430x^4+1355x^3+699x^2+1072x+364}{48(2x+1)^2\sqrt{3x^2-x+2}}-\frac{1519\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\ \left(x-\frac{1}{6}\right)}{23}\right)}{576}-\frac{1153\sqrt{13}\ \operatorname{arctanh}\left(\frac{}{13\sqrt{12}}\right.}{832}$
trager	$\frac{(96x^4-68x^3+390x^2+627x+182)\sqrt{3x^2-x+2}}{48(2x+1)^2}+\frac{1153\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}}{2x+1}\right.}{832}$
default	$\frac{1153\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}{4056}-\frac{257(6x-1)\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}{1248}-\frac{1519\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\ \left(x-\frac{1}{6}\right)}{23}\right)}{576}+\frac{1153\sqrt{12}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(2*x+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1153/4056*(3*(x+1/2)^2-4*x+5/4)^(3/2)-257/1248*(6*x-1)*(3*(x+1/2)^2-4*x+5/4
)^(1/2)-1519/576*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+1153/832*(12*(x+1/2
)^2-16*x+5)^(1/2)-1153/832*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+
1/2)^2-16*x+5)^(1/2))+15/338/(x+1/2)*(3*(x+1/2)^2-4*x+5/4)^(5/2)-15/676*(6*
x-1)*(3*(x+1/2)^2-4*x+5/4)^(3/2)-1/104/(x+1/2)^2*(3*(x+1/2)^2-4*x+5/4)^(5/2
)
```

Maxima [A]

time = 0.52, size = 143, normalized size = 1.04

$$\frac{61}{312}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{\left(3x^2-x+2\right)^{\frac{5}{2}}}{26\left(4x^2+4x+1\right)}-\frac{257}{208}\sqrt{3x^2-x+2}\,x-\frac{1519}{576}\sqrt{3}\ \operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{1153}{832}\sqrt{13}\ \operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2x+1|}-\frac{9\sqrt{23}}{23\,|2x+1|}\right)+\frac{929}{312}\sqrt{3x^2-x+2}+\frac{15\left(3x^2-x+2\right)^{\frac{3}{2}}}{52\left(2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="maxima")
```

```
[Out] 61/312*(3*x^2 - x + 2)^(3/2) - 1/26*(3*x^2 - x + 2)^(5/2)/(4*x^2 + 4*x + 1)
 - 257/208*sqrt(3*x^2 - x + 2)*x - 1519/576*sqrt(3)*arcsinh(6/23*sqrt(23)*x
 - 1/23*sqrt(23)) + 1153/832*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1)
- 9/23*sqrt(23)/abs(2*x + 1)) + 929/312*sqrt(3*x^2 - x + 2) + 15/52*(3*x^2
- x + 2)^(3/2)/(2*x + 1)
```

Fricas [A]

time = 0.44, size = 159, normalized size = 1.15

$$\frac{19747\sqrt{3}\left(4x^2+4x+1\right)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}\left(6x-1\right)-72x^2+24x-25\right)+10377\sqrt{13}\left(4x^2+4x+1\right)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}\left(8x-9\right)+220x^2-196x+185}{4x^2+4x+1}\right)+312\left(96x^4-68x^3+390x^2+627x+182\right)\sqrt{3x^2-x+2}}{14976\left(4x^2+4x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="fricas")
```

```
[Out] 1/14976*(19747*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*
(6*x - 1) - 72*x^2 + 24*x - 25) + 10377*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*
sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*
x + 1)) + 312*(96*x^4 - 68*x^3 + 390*x^2 + 627*x + 182)*sqrt(3*x^2 - x + 2)
)/(4*x^2 + 4*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2 - x + 2)^{\frac{3}{2}} \cdot (4x^2 + 3x + 1)}{(2x + 1)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x)**3,x)
```

```
[Out] Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-18688512,[6]%%%}+%%%{%%{[56065536,0]:[1,0,-3]%%},[5]%%%}+%%%
{-2803
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(3\,x^2-x+2\right)}^{3/2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
```

```
[Out] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)
```

3.220 $\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx$

Optimal. Leaf size=189

$$\frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936}+\frac{117047(1-6x)\left(2-x+3x^2\right)^{3/2}}{1492992}+\frac{5089(1-6x)\left(2-x+3x^2\right)^{5/2}}{155520}-\frac{(26353}{}$$

```
[Out] 117047/1492992*(1-6*x)*(3*x^2-x+2)^(3/2)+5089/155520*(1-6*x)*(3*x^2-x+2)^(5
/2)-1/498960*(26353-21350*x)*(3*x^2-x+2)^(7/2)+133/1485*(1+2*x)^2*(3*x^2-x+
2)^(7/2)+29/330*(1+2*x)^3*(3*x^2-x+2)^(7/2)+2/33*(1+2*x)^4*(3*x^2-x+2)^(7/2
)+61917863/71663616*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2692081/11943936
*(1-6*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 846, 793, 626, 633, 221}

$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4+\frac{29}{330}\left(3x^2-x+2\right)^{7/2}(2x+1)^3+\frac{133\left(3x^2-x+2\right)^{7/2}(2x+1)^2}{1485}-\frac{(26353-21350x)\left(3x^2-x+2\right)^{7/2}}{498960}+\frac{5089(1-6x)\left(3x^2-x+2\right)^{5/2}}{155520}+\frac{117047(1-6x)\left(3x^2-x+2\right)^{3/2}}{1492992}+\frac{2692081(1-6x)\sqrt{3x^2-x+2}}{11943936}+\frac{61917863\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{23887872\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (2692081*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/11943936 + (117047*(1 - 6*x)*(2 - x
 + 3*x^2)^(3/2))/1492992 + (5089*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/155520 -
((26353 - 21350*x)*(2 - x + 3*x^2)^(7/2))/498960 + (133*(1 + 2*x)^2*(2 - x
+ 3*x^2)^(7/2))/1485 + (29*(1 + 2*x)^3*(2 - x + 3*x^2)^(7/2))/330 + (2*(1 +
 2*x)^4*(2 - x + 3*x^2)^(7/2))/33 + (61917863*ArcSinh[(1 - 6*x)/Sqrt[23]])/
(23887872*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right)\,dx = \frac{2}{33}(1+2x)^4 \left(2-x+3x^2\right)^{7/2} + \frac{1}{132}\int (1+2x)^3(32+348$$

$$= \frac{29}{330}(1+2x)^3 \left(2-x+3x^2\right)^{7/2} + \frac{2}{33}(1+2x)^4 \left(2-x+3x$$

$$= \frac{133(1+2x)^2 \left(2-x+3x^2\right)^{7/2}}{1485} + \frac{29}{330}(1+2x)^3 \left(2-x+3$$

$$= -\frac{(26353-21350x)\left(2-x+3x^2\right)^{7/2}}{498960} + \frac{133(1+2x)^2 (2-}{1485}$$

$$= \frac{5089(1-6x)\left(2-x+3x^2\right)^{5/2}}{155520} - \frac{(26353-21350x)(2-x}{498960}$$

$$= \frac{117047(1-6x)\left(2-x+3x^2\right)^{3/2}}{1492992} + \frac{5089(1-6x)(2-x+}{155520}$$

$$= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936} + \frac{117047(1-6x)(2-x}{1492992}$$

$$= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936} + \frac{117047(1-6x)(2-x}{1492992}$$

$$= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936} + \frac{117047(1-6x)(2-x}{1492992}$$

Mathematica [A]
time = 0.84, size = 100, normalized size = 0.53

$$\frac{6\sqrt{2-x+3x^2}\left(9173509857+26646633218x+72088585464x^2+161269204752x^3+263636134272x^4+347247744768x^5+415908006912x^6+419978151936x^7+308846297088x^8+207681159168x^9+120394874880x^{10}\right)+23838377255\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{27590492160}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(9173509857 + 26646633218*x + 72088585464*x^2 + 1612
69204752*x^3 + 263636134272*x^4 + 347247744768*x^5 + 415908006912*x^6 + 419
978151936*x^7 + 308846297088*x^8 + 207681159168*x^9 + 120394874880*x^10) +
23838377255*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/27590492160
```

Maple [A]
time = 0.12, size = 153, normalized size = 0.81

method	result
risch	$\frac{(120394874880x^{10}+207681159168x^9+308846297088x^8+419978151936x^7+415908006912x^6+347247744768x^5+263636134272x^4+1}{4598415360}$
trager	$\left(\frac{288}{11}x^{10}+\frac{2484}{55}x^9+\frac{3694}{55}x^8+\frac{120557}{1320}x^7+\frac{557147}{6160}x^6+\frac{50238389}{665280}x^5+\frac{32692973}{570240}x^4+\frac{1119925033}{31933440}x^3+\frac{42909872}{2737152}\right.$
default	$\frac{32x^4(3x^2-x+2)^{\frac{7}{2}}}{33}+\frac{436x^3(3x^2-x+2)^{\frac{7}{2}}}{165}+\frac{4258x^2(3x^2-x+2)^{\frac{7}{2}}}{1485}+\frac{10073x(3x^2-x+2)^{\frac{7}{2}}}{7128}-\frac{61917863\sqrt{3}\,\mathrm{arcsinh}\left(\frac{6\sqrt{23}}{2}\right.}{71663616}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x+1)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)

[Out] 32/33*x^4*(3*x^2-x+2)^(7/2)+436/165*x^3*(3*x^2-x+2)^(7/2)+4258/1485*x^2*(3*x^2-x+2)^(7/2)+10073/7128*x*(3*x^2-x+2)^(7/2)-61917863/71663616*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-2692081/11943936*(6*x-1)*(3*x^2-x+2)^(1/2)-5089/155520*(6*x-1)*(3*x^2-x+2)^(5/2)-117047/1492992*(6*x-1)*(3*x^2-x+2)^(3/2)+92423/498960*(3*x^2-x+2)^(7/2)

Maxima [A]
time = 0.50, size = 184, normalized size = 0.97

$$\frac{32}{33}\,{\left(3\,x^{2}-x+2\right)}^{\frac{7}{2}}x^{4}+\frac{436}{165}\,{\left(3\,x^{2}-x+2\right)}^{\frac{7}{2}}x^{3}+\frac{4258}{1485}\,{\left(3\,x^{2}-x+2\right)}^{\frac{7}{2}}x^{2}+\frac{10073}{7128}\,{\left(3\,x^{2}-x+2\right)}^{\frac{7}{2}}x+\frac{92423}{498960}\,{\left(3\,x^{2}-x+2\right)}^{\frac{7}{2}}-\frac{5089}{25920}\,{\left(3\,x^{2}-x+2\right)}^{\frac{5}{2}}x+\frac{5089}{155520}\,{\left(3\,x^{2}-x+2\right)}^{\frac{5}{2}}-\frac{117047}{248832}\,{\left(3\,x^{2}-x+2\right)}^{\frac{3}{2}}x+\frac{117047}{1492992}\,{\left(3\,x^{2}-x+2\right)}^{\frac{3}{2}}-\frac{2692081}{1990656}\,\sqrt{3\,x^{2}-x+2}\,x-\frac{61917863}{71663616}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(6\,x-1)\right)+\frac{2692081}{11943936}\,\sqrt{3\,x^{2}-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")

[Out] 32/33*(3*x^2 - x + 2)^(7/2)*x^4 + 436/165*(3*x^2 - x + 2)^(7/2)*x^3 + 4258/1485*(3*x^2 - x + 2)^(7/2)*x^2 + 10073/7128*(3*x^2 - x + 2)^(7/2)*x + 92423/498960*(3*x^2 - x + 2)^(7/2) - 5089/25920*(3*x^2 - x + 2)^(5/2)*x + 5089/155520*(3*x^2 - x + 2)^(5/2) - 117047/248832*(3*x^2 - x + 2)^(3/2)*x + 117047/1492992*(3*x^2 - x + 2)^(3/2) - 2692081/1990656*sqrt(3*x^2 - x + 2)*x - 61917863/71663616*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 2692081/11943936*sqrt(3*x^2 - x + 2)

Fricas [A]
time = 0.35, size = 103, normalized size = 0.54

$$\frac{1}{4598415360}\,\left(120394874880\,x^{10}+207681159168\,x^{9}+308846297088\,x^{8}+419978151936\,x^{7}+415908006912\,x^{6}+347247744768\,x^{5}+263636134272\,x^{4}+161269204752\,x^{3}+72088585464\,x^{2}+26646633218\,x+9173509857\right)\sqrt{3\,x^{2}-x+2}+\frac{61917863}{143327232}\,\sqrt{3}\,\log\left(4\,\sqrt{3}\,\sqrt{3\,x^{2}-x+2}\,(6\,x-1)-72\,x^{2}+24\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")

[Out] 1/4598415360*(120394874880*x^10 + 207681159168*x^9 + 308846297088*x^8 + 419978151936*x^7 + 415908006912*x^6 + 347247744768*x^5 + 263636134272*x^4 + 16

```
1269204752*x^3 + 72088585464*x^2 + 26646633218*x + 9173509857)*sqrt(3*x^2 -
 x + 2) + 61917863/143327232*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
 - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)^3 \left(3x^2-x+2\right)^{\frac{5}{2}} \cdot \left(4x^2+3x+1\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

```
[Out] Integral((2*x + 1)**3*(3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1), x)
```

Giac [A]
time = 4.57, size = 98, normalized size = 0.52

$$\frac{1}{4598415360}\left(2\left(12\left(6\left(8\left(6\left(36\left(14\left(48\left(18\left(40\,x+69\right)x+1847\right)x+120557\right)x+1671441\right)x+50238389\right)x+228850811\right)x+1119925033\right)x+3003691061\right)x+13323316609\right)x+9173509857\right)\sqrt{3\,x^2-x+2}+\frac{61917863}{71663616}\,\sqrt{3}\,\log\left(-2\,\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/4598415360*(2*(12*(6*(8*(6*(36*(14*(48*(18*(40*x + 69)*x + 1847)*x + 1205
57)*x + 1671441)*x + 50238389)*x + 228850811)*x + 1119925033)*x + 300369106
1)*x + 13323316609)*x + 9173509857)*sqrt(3*x^2 - x + 2) + 61917863/71663616
*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(2\,x+1\right)}^3\,{\left(3\,x^2-x+2\right)}^{5/2}\,\left(4\,x^2+3\,x+1\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] int((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

3.221 $\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx$

Optimal. Leaf size=164

$$-\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976}-\frac{6739(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872}-\frac{293(1-6x)\left(2-x+3x^2\right)^{5/2}}{58320}+\frac{37}{405}(1+2$$

[Out] -6739/559872*(1-6*x)*(3*x^2-x+2)^(3/2)-293/58320*(1-6*x)*(3*x^2-x+2)^(5/2)+37/405*(1+2*x)^2*(3*x^2-x+2)^(7/2)+1/15*(1+2*x)^3*(3*x^2-x+2)^(7/2)+1/17010*(2731+3430*x)*(3*x^2-x+2)^(7/2)-3564931/26873856*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-154997/4478976*(1-6*x)*(3*x^2-x+2)^(1/2)

Rubi [A]
time = 0.08, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 846, 793, 626, 633, 221}

$$\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}+\frac{37}{405}(2x+1)^2\left(3x^2-x+2\right)^{7/2}+\frac{(3430x+2731)\left(3x^2-x+2\right)^{7/2}}{17010}-\frac{293(1-6x)\left(3x^2-x+2\right)^{5/2}}{58320}-\frac{6739(1-6x)\left(3x^2-x+2\right)^{3/2}}{559872}-\frac{154997(1-6x)\sqrt{3x^2-x+2}}{4478976}-\frac{3564931\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8957952\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]

[Out] (-154997*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/4478976 - (6739*(1 - 6*x)*(2 - x + 3*x^2)^(3/2))/559872 - (293*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/58320 + (37*(1 + 2*x)^2*(2 - x + 3*x^2)^(7/2))/405 + ((1 + 2*x)^3*(2 - x + 3*x^2)^(7/2))/15 + ((2731 + 3430*x)*(2 - x + 3*x^2)^(7/2))/17010 - (3564931*ArcSinh[(1 - 6*x)/Sqrt[23]])/(8957952*Sqrt[3])

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right)\, dx = \frac{1}{15}(1+2x)^3 \left(2-x+3x^2\right)^{7/2} + \frac{1}{120}\int (1+2x)^2(52+29$$

$$= \frac{37}{405}(1+2x)^2 \left(2-x+3x^2\right)^{7/2} + \frac{1}{15}(1+2x)^3 \left(2-x+3$$

$$= \frac{37}{405}(1+2x)^2 \left(2-x+3x^2\right)^{7/2} + \frac{1}{15}(1+2x)^3 \left(2-x+3$$

$$= -\frac{293(1-6x)\left(2-x+3x^2\right)^{5/2}}{58320} + \frac{37}{405}(1+2x)^2 \left(2-x+\right.$$

$$= -\frac{6739(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872} - \frac{293(1-6x)\left(2-x+\right.}{58320}$$

$$= -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976} - \frac{6739(1-6x)\left(2-x\right.}{559872}$$

$$= -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976} - \frac{6739(1-6x)\left(2-x\right.}{559872}$$

$$= -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976} - \frac{6739(1-6x)\left(2-x\right.}{559872}$$

Mathematica [A]
time = 0.76, size = 95, normalized size = 0.58

$$\frac{6\sqrt{2-x+3x^2}\left(387182961+692659234x+1693765752x^2+3096104976x^3+4171579776x^4+4996802304x^5+5671627776x^6+4427716608x^7+2675441664x^8+2257403904x^9\right)-124772585\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{940584960}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(387182961 + 692659234*x + 1693765752*x^2 + 30961049
76*x^3 + 4171579776*x^4 + 4996802304*x^5 + 5671627776*x^6 + 4427716608*x^7
+ 2675441664*x^8 + 2257403904*x^9) - 124772585*Sqrt[3]*Log[1 - 6*x + 2*Sqrt
[6 - 3*x + 9*x^2]])/940584960
```

Maple [A]
time = 0.12, size = 136, normalized size = 0.83

method	result

risch	$\frac{(2257403904x^9+2675441664x^8+4427716608x^7+5671627776x^6+4996802304x^5+4171579776x^4+3096104976x^3+1693765752x^2+6926}{156764160}$
trager	$\left(\frac{72}{5}x^9+\frac{256}{15}x^8+\frac{1271}{45}x^7+\frac{22793}{630}x^6+\frac{722917}{22680}x^5+\frac{517309}{19440}x^4+\frac{21500729}{1088640}x^3+\frac{10081939}{933120}x^2+\frac{346329617}{78382080}x+\frac{430}{174}\right.$
default	$\frac{8x^3(3x^2-x+2)^{\frac{7}{2}}}{15}+\frac{472x^2(3x^2-x+2)^{\frac{7}{2}}}{405}+\frac{235x(3x^2-x+2)^{\frac{7}{2}}}{243}+\frac{3564931\sqrt{3}\,\mathrm{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{26873856}+\frac{154997(6x-1)\sqrt{}}{4478}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 8/15*x^3*(3*x^2-x+2)^(7/2)+472/405*x^2*(3*x^2-x+2)^(7/2)+235/243*x*(3*x^2-x
+2)^(7/2)+3564931/26873856*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+154997/44
78976*(6*x-1)*(3*x^2-x+2)^(1/2)+293/58320*(6*x-1)*(3*x^2-x+2)^(5/2)+6739/55
9872*(6*x-1)*(3*x^2-x+2)^(3/2)+5419/17010*(3*x^2-x+2)^(7/2)
```

Maxima [A]
time = 0.52, size = 167, normalized size = 1.02

$$\frac{8}{15}\left(3x^2-x+2\right)^{\frac{7}{2}}x^3+\frac{472}{405}\left(3x^2-x+2\right)^{\frac{7}{2}}x^2+\frac{235}{243}\left(3x^2-x+2\right)^{\frac{7}{2}}x+\frac{5419}{17010}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{293}{9720}\left(3x^2-x+2\right)^{\frac{5}{2}}x-\frac{293}{58320}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{6739}{93312}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{6739}{559872}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{154997}{746496}\sqrt{3x^2-x+2}\,x+\frac{3564931}{26873856}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}\,(6x-1)\right)-\frac{154997}{4478976}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 8/15*(3*x^2 - x + 2)^(7/2)*x^3 + 472/405*(3*x^2 - x + 2)^(7/2)*x^2 + 235/24
3*(3*x^2 - x + 2)^(7/2)*x + 5419/17010*(3*x^2 - x + 2)^(7/2) + 293/9720*(3*
x^2 - x + 2)^(5/2)*x - 293/58320*(3*x^2 - x + 2)^(5/2) + 6739/93312*(3*x^2
- x + 2)^(3/2)*x - 6739/559872*(3*x^2 - x + 2)^(3/2) + 154997/746496*sqrt(3
*x^2 - x + 2)*x + 3564931/26873856*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1))
 - 154997/4478976*sqrt(3*x^2 - x + 2)
```

Fricas [A]
time = 0.37, size = 98, normalized size = 0.60

$$\frac{1}{156764160}\left(2257403904x^9+2675441664x^8+4427716608x^7+5671627776x^6+4996802304x^5+4171579776x^4+3096104976x^3+1693765752x^2+692659234x+387182961\right)\sqrt{3x^2-x+2}+\frac{3564931}{53747712}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/156764160*(2257403904*x^9 + 2675441664*x^8 + 4427716608*x^7 + 5671627776*
x^6 + 4996802304*x^5 + 4171579776*x^4 + 3096104976*x^3 + 1693765752*x^2 + 6
92659234*x + 387182961)*sqrt(3*x^2 - x + 2) + 3564931/53747712*sqrt(3)*log(
-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)^2 \left(3x^2-x+2\right)^{\frac{5}{2}} \cdot \left(4x^2+3x+1\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

```
[Out] Integral((2*x + 1)**2*(3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1), x)
```

Giac [A]

time = 6.09, size = 93, normalized size = 0.57

$$\frac{1}{156764160}\,(2\,(12\,(6\,(8\,(6\,(36\,(14\,(24\,(27\,x+32)x+1271)x+22793)x+722917)x+3621163)x+21500729)x+70573573)x+346329617)x+387182961)\sqrt{3\,x^2-x+2}-\frac{3564931}{26873856}\,\sqrt{3}\,\log\left(-2\,\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/156764160*(2*(12*(6*(8*(6*(36*(14*(24*(27*x + 32)*x + 1271)*x + 22793)*x
+ 722917)*x + 3621163)*x + 21500729)*x + 70573573)*x + 346329617)*x + 38718
2961)*sqrt(3*x^2 - x + 2) - 3564931/26873856*sqrt(3)*log(-2*sqrt(3)*(sqrt(3
)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(2\,x+1\right)}^2\,{\left(3\,x^2-x+2\right)}^{5/2}\,\left(4\,x^2+3\,x+1\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] int((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

3.222 $\int (1+2x) \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx$

Optimal. Leaf size=139

$$-\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968}-\frac{51175(1-6x)\left(2-x+3x^2\right)^{3/2}}{746496}-\frac{445(1-6x)\left(2-x+3x^2\right)^{5/2}}{15552}+\frac{2}{27}(1+2x$$

```
[Out] -51175/746496*(1-6*x)*(3*x^2-x+2)^(3/2)-445/15552*(1-6*x)*(3*x^2-x+2)^(5/2)
+2/27*(1+2*x)^2*(3*x^2-x+2)^(7/2)+1/648*(137+122*x)*(3*x^2-x+2)^(7/2)-27071
575/35831808*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1177025/5971968*(1-6*x)
*(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1667, 793, 626, 633, 221}

$$\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}+\frac{1}{648}(122x+137)\left(3x^2-x+2\right)^{7/2}-\frac{445(1-6x)\left(3x^2-x+2\right)^{5/2}}{15552}-\frac{51175(1-6x)\left(3x^2-x+2\right)^{3/2}}{746496}-\frac{1177025(1-6x)\sqrt{3x^2-x+2}}{5971968}-\frac{27071575\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{11943936\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (-1177025*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/5971968 - (51175*(1 - 6*x)*(2 - x
+ 3*x^2)^(3/2))/746496 - (445*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/15552 + (2*(
1 + 2*x)^2*(2 - x + 3*x^2)^(7/2))/27 + ((137 + 122*x)*(2 - x + 3*x^2)^(7/2)
)/648 - (27071575*ArcSinh[(1 - 6*x)/Sqrt[23]])/(11943936*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx = \frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+\frac{1}{108}\int (1+2x)(72+244$$

$$= \frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+\frac{1}{648}(137+122x)\left(2-x\right.$$

$$= -\frac{445(1-6x)\left(2-x+3x^2\right)^{5/2}}{15552}+\frac{2}{27}(1+2x)^2\left(2-x+3\right.$$

$$= -\frac{51175(1-6x)\left(2-x+3x^2\right)^{3/2}}{746496}-\frac{445(1-6x)\left(2-x+\right.}{15552}$$

$$= -\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968}-\frac{51175(1-6x)\left(2-\right.}{746496}$$

$$= -\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968}-\frac{51175(1-6x)\left(2-\right.}{746496}$$

$$= -\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968}-\frac{51175(1-6x)\left(2-\right.}{746496}$$

Mathematica [A]
time = 0.63, size = 90, normalized size = 0.65

$$\frac{6\sqrt{2-x+3x^2}\left(10960335+19860062x+41031048x^2+58946544x^3+66969216x^4+80034048x^5+79377408x^6+30357504x^7+47775744x^8\right)-27071575\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{35831808}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(10960335 + 19860062*x + 41031048*x^2 + 58946544*x^3
 + 66969216*x^4 + 80034048*x^5 + 79377408*x^6 + 30357504*x^7 + 47775744*x^8
) - 27071575*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/35831808
```

Maple [A]
time = 0.10, size = 119, normalized size = 0.86

method	result
risch	$\frac{\left(47775744x^8+30357504x^7+79377408x^6+80034048x^5+66969216x^4+58946544x^3+41031048x^2+19860062x+10960335\right)\sqrt{3x^2-x}}{5971968}$
trager	$\left(8x^8+\frac{61}{12}x^7+\frac{319}{24}x^6+\frac{11579}{864}x^5+\frac{58133}{5184}x^4+\frac{409351}{41472}x^3+\frac{1709627}{248832}x^2+\frac{9930031}{2985984}x+\frac{1217815}{663552}\right)\sqrt{3x^2-x+}$
default	$\frac{8x^2(3x^2-x+2)^{\frac{7}{2}}}{27}+\frac{157x(3x^2-x+2)^{\frac{7}{2}}}{324}+\frac{185(3x^2-x+2)^{\frac{7}{2}}}{648}+\frac{445(6x-1)(3x^2-x+2)^{\frac{5}{2}}}{15552}+\frac{51175(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{746496}+1177$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 8/27*x^2*(3*x^2-x+2)^(7/2)+157/324*x*(3*x^2-x+2)^(7/2)+185/648*(3*x^2-x+2)^
(7/2)+445/15552*(6*x-1)*(3*x^2-x+2)^(5/2)+51175/746496*(6*x-1)*(3*x^2-x+2)^
(3/2)+1177025/5971968*(6*x-1)*(3*x^2-x+2)^(1/2)+27071575/35831808*3^(1/2)*a
rcsinh(6/23*23^(1/2)*(x-1/6))
```

Maxima [A]
time = 0.51, size = 150, normalized size = 1.08

$$\frac{8}{27}\,{\left(3\,x^2-x+2\right)}^{\frac{7}{2}}x^2+\frac{157}{324}\,{\left(3\,x^2-x+2\right)}^{\frac{7}{2}}x+\frac{185}{648}\,{\left(3\,x^2-x+2\right)}^{\frac{7}{2}}+\frac{445}{2592}\,{\left(3\,x^2-x+2\right)}^{\frac{5}{2}}x-\frac{445}{15552}\,{\left(3\,x^2-x+2\right)}^{\frac{5}{2}}+\frac{51175}{124416}\,{\left(3\,x^2-x+2\right)}^{\frac{3}{2}}x-\frac{51175}{746496}\,{\left(3\,x^2-x+2\right)}^{\frac{3}{2}}+\frac{1177025}{995328}\,\sqrt{3\,x^2-x+2}\,x+\frac{27071575}{35831808}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(6\,x-1)\right)-\frac{1177025}{5971968}\,\sqrt{3\,x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 8/27*(3*x^2 - x + 2)^(7/2)*x^2 + 157/324*(3*x^2 - x + 2)^(7/2)*x + 185/648*
(3*x^2 - x + 2)^(7/2) + 445/2592*(3*x^2 - x + 2)^(5/2)*x - 445/15552*(3*x^2
 - x + 2)^(5/2) + 51175/124416*(3*x^2 - x + 2)^(3/2)*x - 51175/746496*(3*x^
2 - x + 2)^(3/2) + 1177025/995328*sqrt(3*x^2 - x + 2)*x + 27071575/35831808
```

```
*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 1177025/5971968*sqrt(3*x^2 - x
+ 2)
```

Fricas [A]
time = 0.34, size = 93, normalized size = 0.67

$$\frac{1}{5971968}\left(47775744\,x^{8}+30357504\,x^{7}+79377408\,x^{6}+80034048\,x^{5}+66969216\,x^{4}+58946544\,x^{3}+41031048\,x^{2}+19860062\,x+10960335\right)\sqrt{3\,x^{2}-x+2}+\frac{27071575}{71663616}\,\sqrt{3}\,\log\left(-4\,\sqrt{3}\,\sqrt{3\,x^{2}-x+2}\,(6\,x-1)-72\,x^{2}+24\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
[Out] 1/5971968*(47775744*x^8 + 30357504*x^7 + 79377408*x^6 + 80034048*x^5 + 6696
9216*x^4 + 58946544*x^3 + 41031048*x^2 + 19860062*x + 10960335)*sqrt(3*x^2
- x + 2) + 27071575/71663616*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*
x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)\left(3x^{2}-x+2\right)^{\frac{5}{2}}\cdot\left(4x^{2}+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
[Out] Integral((2*x + 1)*(3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1), x)
```

Giac [A]
time = 3.57, size = 88, normalized size = 0.63

$$\frac{1}{5971968}\left(2\,(12\,(6\,(8\,(6\,(36\,(2\,(96\,x+61)x+319)x+11579)x+58133)x+409351)x+1709627)x+9930031)x+10960335\right)\sqrt{3\,x^{2}-x+2}-\frac{27071575}{35831808}\,\sqrt{3}\,\log\left(-2\,\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^{2}-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
[Out] 1/5971968*(2*(12*(6*(8*(6*(36*(2*(96*x + 61)*x + 319)*x + 11579)*x + 58133)
*x + 409351)*x + 1709627)*x + 9930031)*x + 10960335)*sqrt(3*x^2 - x + 2) -
27071575/35831808*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
+ 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (2\,x+1)\,\left(3\,x^{2}-x+2\right)^{5/2}\,\left(4\,x^{2}+3\,x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
[Out] int((2*x + 1)*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

3.223 $\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{1+2x}\,dx$

Optimal. Leaf size=147

$$\frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944}+\frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368}+\frac{(29+150x)\left(2-x+3x^2\right)^{5/2}}{1080}+\frac{2}{21}(2-$$

```
[Out] 1/10368*(2449+2154*x)*(3*x^2-x+2)^(3/2)+1/1080*(29+150*x)*(3*x^2-x+2)^(5/2)
+2/21*(3*x^2-x+2)^(7/2)+944521/497664*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2
)-169/128*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+1/82944
*(221999-17850*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1667, 828, 857, 633, 221, 738, 212}

$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}+\frac{(150x+29)\left(3x^2-x+2\right)^{5/2}}{1080}+\frac{(2154x+2449)\left(3x^2-x+2\right)^{3/2}}{10368}+\frac{(221999-17850x)\sqrt{3x^2-x+2}}{82944}-\frac{169}{128}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)+\frac{944521\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{165888\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] ((221999 - 17850*x)*Sqrt[2 - x + 3*x^2])/82944 + ((2449 + 2154*x)*(2 - x +
3*x^2)^(3/2))/10368 + ((29 + 150*x)*(2 - x + 3*x^2)^(5/2))/1080 + (2*(2 - x
 + 3*x^2)^(7/2))/21 + (944521*ArcSinh[(1 - 6*x)/Sqrt[23]])/(165888*Sqrt[3])
 - (169*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/128
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{2}{21}(2-x+3x^2)^{7/2} + \frac{1}{84}\int \frac{(112+140x)(2-x+3x^2)^{5/2}}{1+2x}\,dx$$

$$= \frac{(29+150x)(2-x+3x^2)^{5/2}}{1080} + \frac{2}{21}(2-x+3x^2)^{7/2} - \frac{\int \frac{(-29708-2010}{1}}{1}$$

$$= \frac{(2449+2154x)(2-x+3x^2)^{3/2}}{10368} + \frac{(29+150x)(2-x+3x^2)^{5/2}}{1080} +$$

$$= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944} + \frac{(2449+2154x)(2-x+3x^2)^{3}}{10368}$$

$$= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944} + \frac{(2449+2154x)(2-x+3x^2)^{3}}{10368}$$

$$= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944} + \frac{(2449+2154x)(2-x+3x^2)^{3}}{10368}$$

$$= \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944} + \frac{(2449+2154x)(2-x+3x^2)^{3}}{10368}$$

Mathematica [A]
time = 0.61, size = 124, normalized size = 0.84

$$\frac{6\sqrt{2-x+3x^2}\,(11665053-2120998x+12466776x^2-3646512x^3+15700608x^4-3836160x^5+7464960x^6)+45995040\sqrt{13}\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)+33058235\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{17418240}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(11665053 - 2120998*x + 12466776*x^2 - 3646512*x^3 +
 15700608*x^4 - 3836160*x^5 + 7464960*x^6) + 45995040*Sqrt[13]*ArcTanh[(Sqr
t[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] + 33058235*Sqrt[3]*Lo
g[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/17418240
```

Maple [A]
time = 0.16, size = 207, normalized size = 1.41

method	result

risch	$\frac{(7464960x^6-3836160x^5+15700608x^4-3646512x^3+12466776x^2-2120998x+11665053)\sqrt{3x^2-x+2}}{2903040} - \frac{944521\sqrt{3}\ \text{arcsi}}{49}$
trager	$\left(\frac{18}{7}x^6 - \frac{37}{28}x^5 + \frac{649}{120}x^4 - \frac{8441}{6720}x^3 + \frac{74207}{17280}x^2 - \frac{1060499}{1451520}x + \frac{144013}{35840}\right)\sqrt{3x^2-x+2} + \frac{169\,\text{RootOf}\left(\_Z^2-13\right.}{}$
default	$\frac{5(6x-1)(3x^2-x+2)^{\frac{5}{2}}}{216} + \frac{575(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{10368} + \frac{13225(6x-1)\sqrt{3x^2-x+2}}{82944} - \frac{944521\sqrt{3}\ \text{arcsinh}\left(\frac{6\sqrt{23}\,(x-\frac{1}{6}}{23}\right.}{497664}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(2*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 5/216*(6*x-1)*(3*x^2-x+2)^(5/2)+575/10368*(6*x-1)*(3*x^2-x+2)^(3/2)+13225/8
2944*(6*x-1)*(3*x^2-x+2)^(1/2)-944521/497664*3^(1/2)*arcsinh(6/23*23^(1/2)*
(x-1/6))+2/21*(3*x^2-x+2)^(7/2)+1/20*(3*(x+1/2)^2-4*x+5/4)^(5/2)-1/48*(6*x-
1)*(3*(x+1/2)^2-4*x+5/4)^(3/2)-25/128*(6*x-1)*(3*(x+1/2)^2-4*x+5/4)^(1/2)+1
3/48*(3*(x+1/2)^2-4*x+5/4)^(3/2)+169/128*(12*(x+1/2)^2-16*x+5)^(1/2)-169/12
8*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Maxima [A]
time = 0.51, size = 154, normalized size = 1.05

$$\frac{2}{21}\,(3\,x^2-x+2)^{\frac{7}{2}}+\frac{5}{36}\,(3\,x^2-x+2)^{\frac{5}{2}}x+\frac{29}{1080}\,(3\,x^2-x+2)^{\frac{5}{2}}+\frac{359}{1728}\,(3\,x^2-x+2)^{\frac{3}{2}}x+\frac{2449}{10368}\,(3\,x^2-x+2)^{\frac{3}{2}}-\frac{2975}{13824}\,\sqrt{3\,x^2-x+2}\,x-\frac{944521}{497664}\,\sqrt{3}\,\text{arsinh}\left(\frac{6}{23}\,\sqrt{23}\,x-\frac{1}{23}\,\sqrt{23}\right)+\frac{169}{128}\,\sqrt{13}\,\text{arsinh}\left(\frac{8\,\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\,\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{221999}{82944}\,\sqrt{3\,x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="maxima")
```

```
[Out] 2/21*(3*x^2 - x + 2)^(7/2) + 5/36*(3*x^2 - x + 2)^(5/2)*x + 29/1080*(3*x^2
- x + 2)^(5/2) + 359/1728*(3*x^2 - x + 2)^(3/2)*x + 2449/10368*(3*x^2 - x +
 2)^(3/2) - 2975/13824*sqrt(3*x^2 - x + 2)*x - 944521/497664*sqrt(3)*arcsin
h(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 169/128*sqrt(13)*arcsinh(8/23*sqrt(23)
*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 221999/82944*sqrt(3*x^2 - x
 + 2)
```

Fricas [A]
time = 0.41, size = 135, normalized size = 0.92

$$\frac{1}{2903040}\,(7464960\,x^6-3836160\,x^5+15700608\,x^4-3646512\,x^3+12466776\,x^2-2120998\,x+11665053)\sqrt{3\,x^2-x+2}+\frac{944521}{995328}\,\sqrt{3}\,\log\left(4\,\sqrt{3}\,\sqrt{3\,x^2-x+2}\,(6\,x-1)-72\,x^2+24\,x-25\right)+\frac{169}{256}\,\sqrt{13}\,\log\left(-\frac{4\,\sqrt{13}\,\sqrt{3\,x^2-x+2}\,(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="fricas")
```

```
[Out] 1/2903040*(7464960*x^6 - 3836160*x^5 + 15700608*x^4 - 3646512*x^3 + 1246677
6*x^2 - 2120998*x + 11665053)*sqrt(3*x^2 - x + 2) + 944521/995328*sqrt(3)*l
og(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 169/256*
sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x +
 185)/(4*x^2 + 4*x + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2 - x + 2)^{\frac{5}{2}} \cdot (4x^2 + 3x + 1)}{2x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x),x)
```

```
[Out] Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

Giac [A]
time = 4.59, size = 146, normalized size = 0.99

$$\frac{1}{2903040}\,(2\,(12\,(18\,(8\,(30\,(72\,x-37)x+4543)x-8441)x+519449)x-1060499)x+11665053)\sqrt{3\,x^2-x+2}+\frac{944521}{497664}\,\sqrt{3}\,\log\left(-6\,\sqrt{3}\,x+\sqrt{3}+6\,\sqrt{3\,x^2-x+2}\right)+\frac{169}{128}\,\sqrt{13}\,\log\left(-\frac{\left|-4\,\sqrt{3}\,x-2\,\sqrt{13}-2\,\sqrt{3}+4\,\sqrt{3\,x^2-x+2}\right|}{2\left(2\,\sqrt{3}\,x-\sqrt{13}+\sqrt{3}-2\,\sqrt{3\,x^2-x+2}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="giac")
```

```
[Out] 1/2903040*(2*(12*(18*(8*(30*(72*x - 37)*x + 4543)*x - 8441)*x + 519449)*x -
 1060499)*x + 11665053)*sqrt(3*x^2 - x + 2) + 944521/497664*sqrt(3)*log(-6*
sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 169/128*sqrt(13)*log(-1/2*ab
s(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 - x + 2))/(2*sqrt(3)
*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(3\,x^2-x+2\right)}^{5/2}\,\left(4\,x^2+3\,x+1\right)}{2\,x+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
```

```
[Out] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

3.224 $\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx$

Optimal. Leaf size=154

$$-\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912}-\frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2}-\frac{11(37-60x)\left(2-x+3x^2\right)^{5/2}}{2340}-\frac{(2-}{13}$$

```
[Out] -11/864*(67-78*x)*(3*x^2-x+2)^(3/2)-11/2340*(37-60*x)*(3*x^2-x+2)^(5/2)-1/1
3*(3*x^2-x+2)^(7/2)/(1+2*x)-315623/41472*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(
1/2)+429/128*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-11/6
912*(4727-3090*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1664, 828, 857, 633, 221, 738, 212}

$$-\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}-\frac{11(37-60x)\left(3x^2-x+2\right)^{5/2}}{2340}-\frac{11}{864}(67-78x)\left(3x^2-x+2\right)^{3/2}-\frac{11(4727-3090x)\sqrt{3x^2-x+2}}{6912}+\frac{429}{128}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)-\frac{315623\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{13824\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] (-11*(4727 - 3090*x)*Sqrt[2 - x + 3*x^2])/6912 - (11*(67 - 78*x)*(2 - x + 3
*x^2)^(3/2))/864 - (11*(37 - 60*x)*(2 - x + 3*x^2)^(5/2))/2340 - (2 - x + 3
*x^2)^(7/2)/(13*(1 + 2*x)) - (315623*ArcSinh[(1 - 6*x)/Sqrt[23]])/(13824*Sq
rt[3]) + (429*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])
/128
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = -\frac{(2-x+3x^2)^{7/2}}{13(1+2x)} - \frac{1}{13}\int \frac{\left(-\frac{11}{2}-44x\right)(2-x+3x^2)^{5/2}}{1+2x}\,dx$$

$$= -\frac{11(37-60x)(2-x+3x^2)^{5/2}}{2340} - \frac{(2-x+3x^2)^{7/2}}{13(1+2x)} + \int \frac{(-286+1487}{}$$

$$= -\frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{11(37-60x)(2-x+3x^2)^{5/2}}{2340}$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right.$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right.$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right.$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right.$$

Mathematica [A]
time = 0.64, size = 131, normalized size = 0.85

$$\frac{\frac{6\sqrt{2-x+3x^2}(-364257-322972x+310660x^2-115680x^3+251424x^4-65664x^5+103680x^6)}{1+2x} - 1389960\sqrt{13}\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right) - 1578115\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{207360}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] ((6*Sqrt[2 - x + 3*x^2]*(-364257 - 322972*x + 310660*x^2 - 115680*x^3 + 251
424*x^4 - 65664*x^5 + 103680*x^6))/(1 + 2*x) - 1389960*Sqrt[13]*ArcTanh[(Sq
rt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] - 1578115*Sqrt[3]*Lo
g[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/207360
```

Maple [A]
time = 0.17, size = 235, normalized size = 1.53

method	result

risch	$\frac{311040x^8-300672x^7+1027296x^6-729792x^5+1550508x^4-1510936x^3-148479x^2-281687x-728514}{34560(2x+1)\sqrt{3x^2-x+2}}+\frac{315623\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{2}}{}\right.}{41472}$
trager	$\frac{(103680x^6-65664x^5+251424x^4-115680x^3+310660x^2-322972x-364257)\sqrt{3x^2-x+2}}{69120x+34560}-\frac{315623\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\right.}{}$
default	$\frac{(6x-1)(3x^2-x+2)^{\frac{5}{2}}}{36}+\frac{115(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{1728}+\frac{2645(6x-1)\sqrt{3x^2-x+2}}{13824}+\frac{315623\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,(x-\frac{1}{6})}{23}\right)}{41472}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(2*x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*(6*x-1)*(3*x^2-x+2)^(5/2)+115/1728*(6*x-1)*(3*x^2-x+2)^(3/2)+2645/1382
4*(6*x-1)*(3*x^2-x+2)^(1/2)+315623/41472*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1
/6))-33/260*(3*(x+1/2)^2-4*x+5/4)^(5/2)+19/192*(6*x-1)*(3*(x+1/2)^2-4*x+5/4
)^(3/2)+965/1536*(6*x-1)*(3*(x+1/2)^2-4*x+5/4)^(1/2)-11/16*(3*(x+1/2)^2-4*x
+5/4)^(3/2)-429/128*(12*(x+1/2)^2-16*x+5)^(1/2)+429/128*13^(1/2)*arctanh(2/
13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))-1/26/(x+1/2)*(3*(x+1/2)^
2-4*x+5/4)^(7/2)+1/52*(6*x-1)*(3*(x+1/2)^2-4*x+5/4)^(5/2)
```

Maxima [A]

time = 0.51, size = 161, normalized size = 1.05

$$\frac{1}{6}\left(3x^2-x+2\right)^{\frac{5}{2}}x-\frac{7}{90}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{143}{144}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{737}{864}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{\left(3x^2-x+2\right)^{\frac{5}{2}}}{4(2x+1)}+\frac{5665}{1152}\sqrt{3x^2-x+2}\,x+\frac{315623}{41472}\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{429}{128}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23|2x+1|}-\frac{9\sqrt{23}}{23|2x+1|}\right)-\frac{51997}{6912}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*x^2 - x + 2)^(5/2)*x - 7/90*(3*x^2 - x + 2)^(5/2) + 143/144*(3*x^2 -
 x + 2)^(3/2)*x - 737/864*(3*x^2 - x + 2)^(3/2) - 1/4*(3*x^2 - x + 2)^(5/2)
/(2*x + 1) + 5665/1152*sqrt(3*x^2 - x + 2)*x + 315623/41472*sqrt(3)*arcsinh
(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 429/128*sqrt(13)*arcsinh(8/23*sqrt(23)*
x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) - 51997/6912*sqrt(3*x^2 - x +
2)
```

Fricas [A]

time = 0.37, size = 153, normalized size = 0.99

$$\frac{1578115\sqrt{3}(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+694980\sqrt{13}(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)-220x^2+196x-185}{4x^2+4x+1}\right)+12\left(103680x^6-65664x^5+251424x^4-115680x^3+310660x^2-322972x-364257\right)\sqrt{3x^2-x+2}}{414720(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="fricas")

[Out] 1/414720*(1578115*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 694980*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 12*(103680*x^6 - 65664*x^5 + 251424*x^4 - 115680*x^3 + 310660*x^2 - 322972*x - 364257)*sqrt(3*x^2 - x + 2))/(2*x + 1)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2 - x + 2\right)^{\frac{5}{2}} \cdot \left(4x^2 + 3x + 1\right)}{\left(2x + 1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x)**2,x)

[Out] Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 760 vs. 2(123) = 246.
time = 6.41, size = 760, normalized size = 4.94

$$\frac{429}{128}\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)-\frac{315623}{41472}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{2\sqrt{13}}{2x+1}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)}\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)-\frac{169}{128}\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}\operatorname{sgn}\left(\frac{1}{2x+1}\right)+\ldots$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="giac")

[Out] 429/128*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 315623/41472*sqrt(3)*log(1/2*abs(-2*sqrt(3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1))/(sqrt(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)))*sgn(1/(2*x + 1)) - 169/128*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1/(2*x + 1)) + 1/34560*(5154065*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^11*sgn(1/(2*x + 1)) - 7837020*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^10*sgn(1/(2*x + 1)) + 39468815*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^9*sgn(1/(2*x + 1)) - 14445540*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^8*sgn(1/(2*x + 1)) + 460893402*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^7*sgn(1/(2*x + 1)) - 343084680*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^6*sgn(1/(2*x + 1)) + 944150094*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^5*sgn(1/(2*x + 1)) - 22871160*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^4*sgn(1/(2*x + 1)) + 1397032245*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^3*sgn(1/(2*x + 1)) - 683367516*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*

```
x + 1))^2*sgn(1/(2*x + 1)) + 392684355*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2
+ 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 197538588*sqrt(13)*sgn(1/(2*x
 + 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^2 -
 3)^6
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^2-x+2\right)^{5/2}\,\left(4\,x^2+3\,x+1\right)}{\left(2\,x+1\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
[Out] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

3.225 $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx$

Optimal. Leaf size=161

$$\frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536}+\frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2}+\frac{(257+134x)\left(2-x+3x^2\right)^{5/2}}{520(1+2x)}-\frac{(2-}{2}$$

```
[Out] 1/832*(1227-838*x)*(3*x^2-x+2)^(3/2)+1/520*(257+134*x)*(3*x^2-x+2)^(5/2)/(1
+2*x)-1/26*(3*x^2-x+2)^(7/2)/(1+2*x)^2+118423/9216*arcsinh(1/23*(1-6*x)*23^
(1/2))*3^(1/2)-1631/256*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13
^(1/2)+1/1536*(21317-10470*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1664, 826, 828, 857, 633, 221, 738, 212}

$$-\frac{(3x^2-x+2)^{7/2}}{26(2x+1)^2}+\frac{(134x+257)(3x^2-x+2)^{5/2}}{520(2x+1)}+\frac{1}{832}(1227-838x)(3x^2-x+2)^{3/2}+\frac{(21317-10470x)\sqrt{3x^2-x+2}}{1536}-\frac{1631}{256}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)+\frac{118423\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3072\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((21317 - 10470*x)*Sqrt[2 - x + 3*x^2])/1536 + ((1227 - 838*x)*(2 - x + 3*x
^2)^(3/2))/832 + ((257 + 134*x)*(2 - x + 3*x^2)^(5/2))/(520*(1 + 2*x)) - (2
 - x + 3*x^2)^(7/2)/(26*(1 + 2*x)^2) + (118423*ArcSinh[(1 - 6*x)/Sqrt[23]])
/(3072*Sqrt[3]) - (1631*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x +
 3*x^2])])/256
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
```

```
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = -\frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2} - \frac{1}{26}\int \frac{\left(-\frac{29}{2}-67x\right)(2-x+3x^2)^{5/2}}{(1+2x)^2}\,dx$$

$$= \frac{(257+134x)(2-x+3x^2)^{5/2}}{520(1+2x)} - \frac{(2-x+3x^2)^{7/2}}{26(1+2x)^2} + \frac{1}{208}\int \frac{(793 \ldots$$

$$= \frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2} + \frac{(257+134x)(2-x+3x^2)^{5\ldots}}{520(1+2x)}$$

$$= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)\left(2-x+3x\ldots\right.$$

$$= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)\left(2-x+3x\ldots\right.$$

$$= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)\left(2-x+3x\ldots\right.$$

$$= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)\left(2-x+3x\ldots\right.$$

Mathematica [A]
time = 0.79, size = 131, normalized size = 0.81

$$\frac{\frac{6\sqrt{2-x+3x^2}\,(142057+464446x+256564x^2-76200x^3+83616x^4-22464x^5+27648x^6)}{(1+2x)^2} + 587160\sqrt{13}\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right) + 592115\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{46080}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((6*Sqrt[2 - x + 3*x^2]*(142057 + 464446*x + 256564*x^2 - 76200*x^3 + 83616
*x^4 - 22464*x^5 + 27648*x^6))/(1 + 2*x)^2 + 587160*Sqrt[13]*ArcTanh[(Sqrt[
```

```
3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] + 592115*Sqrt[3]*Log[1
- 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/46080
```

Maple [A]
time = 0.18, size = 199, normalized size = 1.24

method	result
risch	$\frac{82944x^8-95040x^7+328608x^6-357144x^5+1013124x^4+984374x^3+474853x^2+786835x+284114}{7680(2x+1)^2\sqrt{3x^2-x+2}} - \frac{118423\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\ (}{23}\right.}{9216}$
trager	$\frac{(27648x^6-22464x^5+83616x^4-76200x^3+256564x^2+464446x+142057)\sqrt{3x^2-x+2}}{7680(2x+1)^2} + \frac{118423\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{Ro}\right.}{}$
default	$\frac{1631\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}{256} - \frac{\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{7}{2}}}{104\left(x+\frac{1}{2}\right)^2} - \frac{1745(6x-1)\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}{1536} + \frac{19\left(3\left(x+\frac{1}{2}\right)^2-\right.}{338(x+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(2*x+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1631/256*(12*(x+1/2)^2-16*x+5)^(1/2)-1/104/(x+1/2)^2*(3*(x+1/2)^2-4*x+5/4)^
(7/2)-1745/1536*(6*x-1)*(3*(x+1/2)^2-4*x+5/4)^(1/2)+19/338/(x+1/2)*(3*(x+1/
2)^2-4*x+5/4)^(7/2)-19/676*(6*x-1)*(3*(x+1/2)^2-4*x+5/4)^(5/2)-419/2496*(6*
x-1)*(3*(x+1/2)^2-4*x+5/4)^(3/2)-118423/9216*3^(1/2)*arcsinh(6/23*23^(1/2)*
(x-1/6))-1631/256*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16
*x+5)^(1/2))+1631/1248*(3*(x+1/2)^2-4*x+5/4)^(3/2)+1631/6760*(3*(x+1/2)^2-4
*x+5/4)^(5/2)
```

Maxima [A]
time = 0.51, size = 172, normalized size = 1.07

$$\frac{67}{520}\left(3x^2-x+2\right)^{\frac{5}{2}} - \frac{\left(3x^2-x+2\right)^{\frac{7}{2}}}{26\left(4x^2+4x+1\right)} - \frac{419}{416}\left(3x^2-x+2\right)^{\frac{3}{2}}x + \frac{1227}{832}\left(3x^2-x+2\right)^{\frac{3}{2}} + \frac{19\left(3x^2-x+2\right)^{\frac{5}{2}}}{52\left(2x+1\right)} - \frac{1745}{256}\sqrt{3x^2-x+2}\,x - \frac{118423}{9216}\sqrt{3}\ \operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}\,x - \frac{1}{23}\sqrt{23}\right) + \frac{1631}{256}\sqrt{13}\ \operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\left|2x+1\right|} - \frac{9\sqrt{23}}{23\left|2x+1\right|}\right) + \frac{21317}{1536}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="maxima")
```

```
[Out] 67/520*(3*x^2 - x + 2)^(5/2) - 1/26*(3*x^2 - x + 2)^(7/2)/(4*x^2 + 4*x + 1)
 - 419/416*(3*x^2 - x + 2)^(3/2)*x + 1227/832*(3*x^2 - x + 2)^(3/2) + 19/52
*(3*x^2 - x + 2)^(5/2)/(2*x + 1) - 1745/256*sqrt(3*x^2 - x + 2)*x - 118423/
9216*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1631/256*sqrt(13)*a
rcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 21317/1
536*sqrt(3*x^2 - x + 2)
```

Fricas [A]
time = 0.34, size = 169, normalized size = 1.05

$$\frac{592115\,\sqrt{3}\,(4\,x^2+4\,x+1)\log\left(4\,\sqrt{3}\,\sqrt{3\,x^2-x+2}\,(6\,x-1)-72\,x^2+24\,x-25\right)+293580\,\sqrt{13}\,(4\,x^2+4\,x+1)\log\left(-\frac{4\,\sqrt{13}\,\sqrt{3\,x^2-x+2}\,(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)+12\,(27648\,x^6-22464\,x^5+83616\,x^4-76200\,x^3+256564\,x^2+464446\,x+142057)\sqrt{3\,x^2-x+2}}{92160\,(4\,x^2+4\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="fricas")
```

```
[Out] 1/92160*(592115*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)
*(6*x - 1) - 72*x^2 + 24*x - 25) + 293580*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(
4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 +
4*x + 1)) + 12*(27648*x^6 - 22464*x^5 + 83616*x^4 - 76200*x^3 + 256564*x^2
+ 464446*x + 142057)*sqrt(3*x^2 - x + 2))/(4*x^2 + 4*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2-x+2)^{\frac{5}{2}}\cdot(4x^2+3x+1)}{(2x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x)**3,x)
```

```
[Out] Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-299016192,[6]%%%}+%%%{%%{[897048576,0]:[1,0,-3]%%},[5]%%%}+%
%%{-44
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(3\,x^2-x+2\right)}^{5/2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
```

```
[Out] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)
```

3.226 $\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=693

$$\frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)(g+hx)^2\sqrt{a+bx+cx^2}}{240c^3h}-\frac{(9bfh\ \cdots}{}$$

```
[Out] 1/256*(256*c^5*d*g^3-63*b^5*f*h^3+70*b^3*c*h^2*(4*a*f*h+b*e*h+3*b*f*g)-80*b
*c^2*h*(3*a^2*f*h^2+3*a*b*h*(e*h+3*f*g)+b^2*(d*h^2+3*e*g*h+3*f*g^2))-128*c^
4*g*(b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+96*c^3*(a^2*h^2*(e*h+3*f*g)+b
^2*g*(f*g^2+3*h*(d*h+e*g))+2*a*b*h*(3*f*g^2+h*(d*h+3*e*g))))*arctanh(1/2*(2
*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/240*(63*b^2*f*h^2-2*c*h*(32
*a*f*h+35*b*e*h+24*b*f*g)-c^2*(12*f*g^2-20*h*(4*d*h+3*e*g)))*(h*x+g)^2*(c*x
^2+b*x+a)^(1/2)/c^3/h-1/40*(9*b*f*h+2*c*(-5*e*h+f*g))*(h*x+g)^3*(c*x^2+b*x+
a)^(1/2)/c^2/h+1/5*f*(h*x+g)^4*(c*x^2+b*x+a)^(1/2)/c/h+1/1920*(945*b^4*f*h^
4-64*c^4*g^2*(3*f*g^2-5*h*(16*d*h+3*e*g))-210*b^2*c*h^3*(14*a*f*h+5*b*(e*h+
3*f*g))+8*c^2*h^2*(128*a^2*f*h^2+275*a*b*h*(e*h+3*f*g)+3*b^2*(129*f*g^2+50*
h*(d*h+3*e*g)))-16*c^3*h*(16*a*h*(13*f*g^2+5*h*(d*h+3*e*g))+b*g*(39*f*g^2+5
*h*(54*d*h+47*e*g)))-2*c*h*(315*b^3*f*h^3-14*b*c*h^2*(46*a*f*h+25*b*e*h+39*
b*f*g)+16*c^3*g*(3*f*g^2-5*h*(10*d*h+3*e*g))+8*c^2*h*(a*h*(45*e*h+71*f*g)+b
*(50*d*h^2+80*e*g*h+21*f*g^2)))*x)*(c*x^2+b*x+a)^(1/2)/c^5/h
```

Rubi [A]
time = 1.20, antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1667, 846, 793, 635, 212}

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((63*b^2*f*h^2 - 2*c*h*(24*b*f*g + 35*b*e*h + 32*a*f*h) - c^2*(12*f*g^2 - 2
0*h*(3*e*g + 4*d*h)))*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(240*c^3*h) - ((9*
b*f*h + 2*c*(f*g - 5*e*h))*(g + h*x)^3*Sqrt[a + b*x + c*x^2])/(40*c^2*h) +
(f*(g + h*x)^4*Sqrt[a + b*x + c*x^2])/(5*c*h) + ((945*b^4*f*h^4 - 64*c^4*(3
*f*g^4 - 5*g^2*h*(3*e*g + 16*d*h)) - 210*b^2*c*h^3*(14*a*f*h + 5*b*(3*f*g +
 e*h)) + 8*c^2*h^2*(128*a^2*f*h^2 + 275*a*b*h*(3*f*g + e*h) + 3*b^2*(129*f*
g^2 + 50*h*(3*e*g + d*h))) - 16*c^3*h*(16*a*h*(13*f*g^2 + 5*h*(3*e*g + d*h)
) + b*g*(39*f*g^2 + 5*h*(47*e*g + 54*d*h))) - 2*c*h*(315*b^3*f*h^3 - 14*b*c
*h^2*(39*b*f*g + 25*b*e*h + 46*a*f*h) + 16*c^3*(3*f*g^3 - 5*g*h*(3*e*g + 10
*d*h)) + 8*c^2*h*(21*b*f*g^2 + 10*b*h*(8*e*g + 5*d*h) + a*h*(71*f*g + 45*e*
h)))*x)*Sqrt[a + b*x + c*x^2])/(1920*c^5*h) + ((256*c^5*d*g^3 - 63*b^5*f*h^
```

```
3 + 70*b^3*c*h^2*(3*b*f*g + b*e*h + 4*a*f*h) - 128*c^4*g*(a*f*g^2 + 3*a*h*(
e*g + d*h) + b*g*(e*g + 3*d*h)) - 80*b*c^2*h*(3*a^2*f*h^2 + 3*a*b*h*(3*f*g
+ e*h) + b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 96*c^3*(a^2*h^2*(3*f*g + e*h) +
 b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 2*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
```

```
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch} + \frac{\int \frac{(g+hx)^3\left(-\frac{1}{2}h(bfg-10cdh+8afh)-\frac{1}{2}h(2cfg-10ceh+9b\ldots}{\sqrt{a+bx+cx^2}}}{5ch^2}$$

$$= -\frac{(9bfh+2c(fg-5eh))(g+hx)^3\sqrt{a+bx+cx^2}}{40c^2h} + \frac{f(g+hx)^4\sqrt{a+bx\ldots}}{5ch}$$

$$= \frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)\ldots}{240c^3h}$$

$$= \frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)\ldots}{240c^3h}$$

$$= \frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)\ldots}{240c^3h}$$

$$= \frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)\ldots}{240c^3h}$$

Mathematica [**A**]
time = 2.64, size = 588, normalized size = 0.85

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(945*b^4*f*h^3 - 210*b^2*c*h^2*(5*b*e*h +
14*a*f*h + 3*b*f*(5*g + h*x)) + 32*c^4*(10*d*h*(18*g^2 + 9*g*h*x + 2*h^2*x^
2) + 15*e*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3) + 3*f*x*(10*g^3 + 20*
g^2*h*x + 15*g*h^2*x^2 + 4*h^3*x^3)) + 4*c^2*h*(256*a^2*f*h^2 + 2*a*b*h*(82
5*f*g + 275*e*h + 161*f*h*x) + b^2*(25*h*(36*e*g + 12*d*h + 7*e*h*x) + 3*f*
(300*g^2 + 175*g*h*x + 42*h^2*x^2))) - 16*c^3*(a*h*(5*h*(48*e*g + 16*d*h +
9*e*h*x) + f*(240*g^2 + 135*g*h*x + 32*h^2*x^2)) + b*(3*f*(30*g^3 + 50*g^2*
h*x + 35*g*h^2*x^2 + 9*h^3*x^3) + 5*h*(2*d*h*(27*g + 5*h*x) + e*(54*g^2 + 3
0*g*h*x + 7*h^2*x^2))))) + 15*(-256*c^5*d*g^3 + 63*b^5*f*h^3 - 70*b^3*c*h^2
*(3*b*f*g + b*e*h + 4*a*f*h) + 128*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + b*g
```

```
*(e*g + 3*d*h)) + 80*b*c^2*h*(3*a^2*f*h^2 + 3*a*b*h*(3*f*g + e*h) + b^2*(3*
f*g^2 + 3*e*g*h + d*h^2)) - 96*c^3*(a^2*h^2*(3*f*g + e*h) + b^2*g*(f*g^2 +
3*h*(e*g + d*h)) + 2*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*Log[b + 2*c*x - 2*
Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(3840*c^(11/2))
```

Maple [A]
time = 0.14, size = 1309, normalized size = 1.89

method	result	size
default	Expression too large to display	1309
risch	Expression too large to display	1387

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*h^3*(1/5*x^4/c*(c*x^2+b*x+a)^(1/2)-9/10*b/c*(1/4*x^3/c*(c*x^2+b*x+a)^(1/2
)-7/8*b/c*(1/3*x^2/c*(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/2*x/c*(c*x^2+b*x+a)^(1/
2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
)))-2/3*a/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))))-3/4*a/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*
x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-4/5*a/c*(1/3*x
^2/c*(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*
(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2/3*a/c*(1/c
*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2)))))+(e*h^3+3*f*g*h^2)*(1/4*x^3/c*(c*x^2+b*x+a)^(1/2)-7/8*b/c*(1/3*x^2/c*
(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^
2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1
/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2/3*a/c*(1/c*(c*x
^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))
-3/4*a/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*
b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+(d*h^3+3*e*g*h^2+3*f*g^2*h)*(1/3*x^
2/c*(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(
c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)
))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2/3*a/c*(1/c*
(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))))+(3*d*g*h^2+3*e*g^2*h+f*g^3)*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*
(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(3*d*g^2*h+e
*g^3)*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2)))+d*g^3*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.67, size = 1527, normalized size = 2.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/7680*(15*(32*(8*c^5*d + (3*b^2*c^3 - 4*a*c^4)*f)*g^3 - 48*(8*b*c^4*d + (
5*b^3*c^2 - 12*a*b*c^3)*f)*g^2*h + 6*(16*(3*b^2*c^3 - 4*a*c^4)*d + (35*b^4*
c - 120*a*b^2*c^2 + 48*a^2*c^3)*f)*g*h^2 - (16*(5*b^3*c^2 - 12*a*b*c^3)*d +
 (63*b^5 - 280*a*b^3*c + 240*a^2*b*c^2)*f)*h^3 - 2*(64*b*c^4*g^3 - 48*(3*b^
2*c^3 - 4*a*c^4)*g^2*h + 24*(5*b^3*c^2 - 12*a*b*c^3)*g*h^2 - (35*b^4*c - 12
0*a*b^2*c^2 + 48*a^2*c^3)*h^3)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 -
4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(384*c^5*f*h^3*x^4
 - 1440*b*c^4*f*g^3 + 240*(24*c^5*d + (15*b^2*c^3 - 16*a*c^4)*f)*g^2*h - 30
*(144*b*c^4*d + 5*(21*b^3*c^2 - 44*a*b*c^3)*f)*g*h^2 + (80*(15*b^2*c^3 - 16
*a*c^4)*d + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*f)*h^3 + 144*(10*c^
5*f*g*h^2 - 3*b*c^4*f*h^3)*x^3 + 8*(240*c^5*f*g^2*h - 210*b*c^4*f*g*h^2 + (
80*c^5*d + (63*b^2*c^3 - 64*a*c^4)*f)*h^3)*x^2 + 2*(480*c^5*f*g^3 - 1200*b*
c^4*f*g^2*h + 30*(48*c^5*d + (35*b^2*c^3 - 36*a*c^4)*f)*g*h^2 - (400*b*c^4*
d + 7*(45*b^3*c^2 - 92*a*b*c^3)*f)*h^3)*x + 10*(48*c^5*h^3*x^3 + 192*c^5*g^
3 - 432*b*c^4*g^2*h + 24*(15*b^2*c^3 - 16*a*c^4)*g*h^2 - 5*(21*b^3*c^2 - 44
*a*b*c^3)*h^3 + 8*(24*c^5*g*h^2 - 7*b*c^4*h^3)*x^2 + 2*(144*c^5*g^2*h - 120
*b*c^4*g*h^2 + (35*b^2*c^3 - 36*a*c^4)*h^3)*x)*e)*sqrt(c*x^2 + b*x + a))/c^
6, -1/3840*(15*(32*(8*c^5*d + (3*b^2*c^3 - 4*a*c^4)*f)*g^3 - 48*(8*b*c^4*d
+ (5*b^3*c^2 - 12*a*b*c^3)*f)*g^2*h + 6*(16*(3*b^2*c^3 - 4*a*c^4)*d + (35*b
^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f)*g*h^2 - (16*(5*b^3*c^2 - 12*a*b*c^3)*
d + (63*b^5 - 280*a*b^3*c + 240*a^2*b*c^2)*f)*h^3 - 2*(64*b*c^4*g^3 - 48*(3
*b^2*c^3 - 4*a*c^4)*g^2*h + 24*(5*b^3*c^2 - 12*a*b*c^3)*g*h^2 - (35*b^4*c -
```

```
 120*a*b^2*c^2 + 48*a^2*c^3)*h^3)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x +
 a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(384*c^5*f*h^3*x^4 -
1440*b*c^4*f*g^3 + 240*(24*c^5*d + (15*b^2*c^3 - 16*a*c^4)*f)*g^2*h - 30*(1
44*b*c^4*d + 5*(21*b^3*c^2 - 44*a*b*c^3)*f)*g*h^2 + (80*(15*b^2*c^3 - 16*a*
c^4)*d + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*f)*h^3 + 144*(10*c^5*f
*g*h^2 - 3*b*c^4*f*h^3)*x^3 + 8*(240*c^5*f*g^2*h - 210*b*c^4*f*g*h^2 + (80*
c^5*d + (63*b^2*c^3 - 64*a*c^4)*f)*h^3)*x^2 + 2*(480*c^5*f*g^3 - 1200*b*c^4
*f*g^2*h + 30*(48*c^5*d + (35*b^2*c^3 - 36*a*c^4)*f)*g*h^2 - (400*b*c^4*d +
 7*(45*b^3*c^2 - 92*a*b*c^3)*f)*h^3)*x + 10*(48*c^5*h^3*x^3 + 192*c^5*g^3 -
 432*b*c^4*g^2*h + 24*(15*b^2*c^3 - 16*a*c^4)*g*h^2 - 5*(21*b^3*c^2 - 44*a*
b*c^3)*h^3 + 8*(24*c^5*g*h^2 - 7*b*c^4*h^3)*x^2 + 2*(144*c^5*g^2*h - 120*b*
c^4*g*h^2 + (35*b^2*c^3 - 36*a*c^4)*h^3)*x)*e)*sqrt(c*x^2 + b*x + a))/c^6]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**3*(d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

Giac [**A**]
time = 5.52, size = 822, normalized size = 1.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*f*h^3*x/c + (30*c^4*f*g*h^2 - 9*b*
c^3*f*h^3 + 10*c^4*h^3*e)/c^5)*x + (240*c^4*f*g^2*h - 210*b*c^3*f*g*h^2 + 8
0*c^4*d*h^3 + 63*b^2*c^2*f*h^3 - 64*a*c^3*f*h^3 + 240*c^4*g*h^2*e - 70*b*c^
3*h^3*e)/c^5)*x + (480*c^4*f*g^3 - 1200*b*c^3*f*g^2*h + 1440*c^4*d*g*h^2 +
1050*b^2*c^2*f*g*h^2 - 1080*a*c^3*f*g*h^2 - 400*b*c^3*d*h^3 - 315*b^3*c*f*h
^3 + 644*a*b*c^2*f*h^3 + 1440*c^4*g^2*h*e - 1200*b*c^3*g*h^2*e + 350*b^2*c^
2*h^3*e - 360*a*c^3*h^3*e)/c^5)*x - (1440*b*c^3*f*g^3 - 5760*c^4*d*g^2*h -
3600*b^2*c^2*f*g^2*h + 3840*a*c^3*f*g^2*h + 4320*b*c^3*d*g*h^2 + 3150*b^3*c
*f*g*h^2 - 6600*a*b*c^2*f*g*h^2 - 1200*b^2*c^2*d*h^3 + 1280*a*c^3*d*h^3 - 9
45*b^4*f*h^3 + 2940*a*b^2*c*f*h^3 - 1024*a^2*c^2*f*h^3 - 1920*c^4*g^3*e + 4
320*b*c^3*g^2*h*e - 3600*b^2*c^2*g*h^2*e + 3840*a*c^3*g*h^2*e + 1050*b^3*c*
h^3*e - 2200*a*b*c^2*h^3*e)/c^5) - 1/256*(256*c^5*d*g^3 + 96*b^2*c^3*f*g^3
- 128*a*c^4*f*g^3 - 384*b*c^4*d*g^2*h - 240*b^3*c^2*f*g^2*h + 576*a*b*c^3*f
*g^2*h + 288*b^2*c^3*d*g*h^2 - 384*a*c^4*d*g*h^2 + 210*b^4*c*f*g*h^2 - 720*
```

```
a*b^2*c^2*f*g*h^2 + 288*a^2*c^3*f*g*h^2 - 80*b^3*c^2*d*h^3 + 192*a*b*c^3*d*
h^3 - 63*b^5*f*h^3 + 280*a*b^3*c*f*h^3 - 240*a^2*b*c^2*f*h^3 - 128*b*c^4*g^
3*e + 288*b^2*c^3*g^2*h*e - 384*a*c^4*g^2*h*e - 240*b^3*c^2*g*h^2*e + 576*a
*b*c^3*g*h^2*e + 70*b^4*c*h^3*e - 240*a*b^2*c^2*h^3*e + 96*a^2*c^3*h^3*e)*l
og(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g+h\,x)^3\,(f\,x^2+e\,x+d)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

3.227 $\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=420

$$-\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h}+\frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch}-\frac{(105b^3fh^3+32c^3g(fg^2-4$$

```
[Out] 1/128*(128*c^4*d*g^2+35*b^4*f*h^2-40*b^2*c*h*(3*a*f*h+b*e*h+2*b*f*g)-64*c^3
*(b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+48*c^2*(a^2*f*h^2+2*a*b*h*(e*h+2
*f*g)+b^2*(d*h^2+2*e*g*h+f*g^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+
a)^(1/2))/c^(9/2)-1/24*(7*b*f*h-8*c*e*h+2*c*f*g)*(h*x+g)^2*(c*x^2+b*x+a)^(1
/2)/c^2/h+1/4*f*(h*x+g)^3*(c*x^2+b*x+a)^(1/2)/c/h-1/192*(105*b^3*f*h^3+32*c
^3*g*(f*g^2-4*h*(3*d*h+e*g))-20*b*c*h^2*(11*a*f*h+6*b*(e*h+2*f*g))+8*c^2*h*
(16*a*h*(e*h+2*f*g)+b*(11*f*g^2+18*h*(d*h+2*e*g)))-2*c*h*(35*b^2*f*h^2-4*c*
h*(9*a*f*h+10*b*e*h+6*b*f*g)-8*c^2*(f*g^2-2*h*(3*d*h+2*e*g))))*x)*(c*x^2+b*x
+a)^(1/2)/c^4/h
```

Rubi [A]
time = 0.58, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1667, 846, 793, 635, 212}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2\left(h(dh+2eg)+fg^2\right)\right)-40b^2ch(3afh+beh+2bfg)-64c^3\left(ah(dh+2eg)+afg^2+bg(2dh+eg)\right)+35b^4fh^2+128c^4dg^2\right)}{128c^{9/2}}-\frac{\sqrt{a+bx+cx^2}\left(-2chx\left(-4ch(9afh+10beh+6bfg)+35b^2fh^2-8c^2\left(fg^2-2h(3dh+2eg)\right)\right)+8c^2h\left(16ah(eh+2fg)+18bh(dh+2eg)+11bfg^2\right)-20bch^2(11afh+6b(eh+2fg))+105b^3fh^3+32c^3\left(fg^3-4gh(3dh+eg)\right)\right)}{192c^4h}-\frac{(g+hx)^2\sqrt{a+bx+cx^2}\,(7bfh-8ceh+2cfg)}{24c^2h}+\frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] -1/24*((2*c*f*g - 8*c*e*h + 7*b*f*h)*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(c^
2*h) + (f*(g + h*x)^3*Sqrt[a + b*x + c*x^2])/(4*c*h) - ((105*b^3*f*h^3 + 32
*c^3*(f*g^3 - 4*g*h*(e*g + 3*d*h)) - 20*b*c*h^2*(11*a*f*h + 6*b*(2*f*g + e*
h)) + 8*c^2*h*(11*b*f*g^2 + 18*b*h*(2*e*g + d*h) + 16*a*h*(2*f*g + e*h)) -
2*c*h*(35*b^2*f*h^2 - 4*c*h*(6*b*f*g + 10*b*e*h + 9*a*f*h) - 8*c^2*(f*g^2 -
 2*h*(2*e*g + 3*d*h)))*x)*Sqrt[a + b*x + c*x^2])/(192*c^4*h) + ((128*c^4*d*
g^2 + 35*b^4*f*h^2 - 40*b^2*c*h*(2*b*f*g + b*e*h + 3*a*f*h) - 64*c^3*(a*f*g
^2 + a*h*(2*e*g + d*h) + b*g*(e*g + 2*d*h)) + 48*c^2*(a^2*f*h^2 + 2*a*b*h*(
2*f*g + e*h) + b^2*(f*g^2 + h*(2*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + b*x + c*x^2])])/(128*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} + \frac{\int \frac{(g+hx)^2\left(-\frac{1}{2}h(bfg-8cdh+6afh)-\frac{1}{2}h(2cfg-8ceh+7bf}{\sqrt{a+bx+cx^2}}}{4ch^2}$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx}}{4ch}$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx}}{4ch}$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx}}{4ch}$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx}}{4ch}$$

Mathematica [**A**]
time = 1.28, size = 341, normalized size = 0.81

$$\frac{2\sqrt{c}\,\sqrt{a+x(b+cx)}\,(-105b^3fh^2+10bch(22afh+b(24fg+12eh+7fhx))+16c^3(6dh(4g+hx)+4e(3g^2+3ghx+h^2x^2)+fx(6g^2+8ghx+3h^2x^2))-8c^2(2bh(18eg+9dh+5ehx)+ah(32fg+16eh+9fhx)+bf(18g^2+20ghx+7h^2x^2)))+3(-128c^4dg^2-35b^4fh^2+40b^2ch(2bfg+beh+3afh)+64c^3(afg^2+ah(2eg+dh)+bg(eg+2dh))-48c^2(a^2fh^2+2abh(2fg+eh)+b^2(fg^2+h(2eg+dh))))\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)}{384c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^3*f*h^2 + 10*b*c*h*(22*a*f*h + b*(
24*f*g + 12*e*h + 7*f*h*x)) + 16*c^3*(6*d*h*(4*g + h*x) + 4*e*(3*g^2 + 3*g*
h*x + h^2*x^2) + f*x*(6*g^2 + 8*g*h*x + 3*h^2*x^2)) - 8*c^2*(2*b*h*(18*e*g
+ 9*d*h + 5*e*h*x) + a*h*(32*f*g + 16*e*h + 9*f*h*x) + b*f*(18*g^2 + 20*g*h
*x + 7*h^2*x^2))) + 3*(-128*c^4*d*g^2 - 35*b^4*f*h^2 + 40*b^2*c*h*(2*b*f*g
+ b*e*h + 3*a*f*h) + 64*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + b*g*(e*g + 2*d*h
)) - 48*c^2*(a^2*f*h^2 + 2*a*b*h*(2*f*g + e*h) + b^2*(f*g^2 + h*(2*e*g + d*
h))))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c^(9/2))
```

Maple [**A**]
time = 0.13, size = 740, normalized size = 1.76

method	result

default	$f\,h^{2} \left(\frac{x^{3} \sqrt{c\,x^{2}+b x +a}}{4 c}-\frac{7 b \left(\frac{x^{2} \sqrt{c\,x^{2}+b x +a}}{3 c}-\frac{5 b \left(\frac{x \sqrt{c\,x^{2}+b x +a}}{2 c}-\frac{3 b \left(\frac{\sqrt{c\,x^{2}+b x +a}}{c}-\frac{b \ln \left(\frac{b}{2} \cdots\right.}{\cdots}\right.}{4 c} \cdots\right.}{\cdots}\right.}{\cdots}\right.$
risch	$\frac{(48 f\,h^{2} c^{3} x^{3}-56 b\,c^{2} f\,h^{2} x^{2}+64 c^{3} e\,h^{2} x^{2}+128 c^{3} f g h\,x^{2}-72 a\,c^{2} f\,h^{2} x +70 b^{2} c f\,h^{2} x -80 b\,c^{2} e\,h^{2} x -160 b\,c^{2} f g h x +96 c^{3} d\,h^{2} x +192 c^{3} e g h x + \cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*h^2*(1/4*x^3/c*(c*x^2+b*x+a)^(1/2)-7/8*b/c*(1/3*x^2/c*(c*x^2+b*x+a)^(1/2)
-5/6*b/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*
b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2/3*a/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2
*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-3/4*a/c*(1/2*x/c*(
c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))))+(e*h^2+2*f*g*h)*(1/3*x^2/c*(c*x^2+b*x+a)^(1/2)-5/6*b/c
```

```
*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2/3*a/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+(d*h^2+2*e*g*h+f*g^2)*(1/2
*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln(
(1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x+a)^(1/2)))+(2*d*g*h+e*g^2)*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^
(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+d*g^2*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.52, size = 905, normalized size = 2.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/768*(3*(16*(8*c^4*d + (3*b^2*c^2 - 4*a*c^3)*f)*g^2 - 16*(8*b*c^3*d + (5*
b^3*c - 12*a*b*c^2)*f)*g*h + (16*(3*b^2*c^2 - 4*a*c^3)*d + (35*b^4 - 120*a*
b^2*c + 48*a^2*c^2)*f)*h^2 - 8*(8*b*c^3*g^2 - 4*(3*b^2*c^2 - 4*a*c^3)*g*h +
 (5*b^3*c - 12*a*b*c^2)*h^2)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*h^2*x^3 -
144*b*c^3*f*g^2 + 16*(24*c^4*d + (15*b^2*c^2 - 16*a*c^3)*f)*g*h - (144*b*c^
3*d + 5*(21*b^3*c - 44*a*b*c^2)*f)*h^2 + 8*(16*c^4*f*g*h - 7*b*c^3*f*h^2)*x
^2 + 2*(48*c^4*f*g^2 - 80*b*c^3*f*g*h + (48*c^4*d + (35*b^2*c^2 - 36*a*c^3)
*f)*h^2)*x + 8*(8*c^4*h^2*x^2 + 24*c^4*g^2 - 36*b*c^3*g*h + (15*b^2*c^2 - 1
6*a*c^3)*h^2 + 2*(12*c^4*g*h - 5*b*c^3*h^2)*x)*e)*sqrt(c*x^2 + b*x + a))/c^
5, -1/384*(3*(16*(8*c^4*d + (3*b^2*c^2 - 4*a*c^3)*f)*g^2 - 16*(8*b*c^3*d +
(5*b^3*c - 12*a*b*c^2)*f)*g*h + (16*(3*b^2*c^2 - 4*a*c^3)*d + (35*b^4 - 120
*a*b^2*c + 48*a^2*c^2)*f)*h^2 - 8*(8*b*c^3*g^2 - 4*(3*b^2*c^2 - 4*a*c^3)*g*
```

```
h + (5*b^3*c - 12*a*b*c^2)*h^2)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a
)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(48*c^4*f*h^2*x^3 - 144
*b*c^3*f*g^2 + 16*(24*c^4*d + (15*b^2*c^2 - 16*a*c^3)*f)*g*h - (144*b*c^3*d
 + 5*(21*b^3*c - 44*a*b*c^2)*f)*h^2 + 8*(16*c^4*f*g*h - 7*b*c^3*f*h^2)*x^2
+ 2*(48*c^4*f*g^2 - 80*b*c^3*f*g*h + (48*c^4*d + (35*b^2*c^2 - 36*a*c^3)*f)
*h^2)*x + 8*(8*c^4*h^2*x^2 + 24*c^4*g^2 - 36*b*c^3*g*h + (15*b^2*c^2 - 16*a
*c^3)*h^2 + 2*(12*c^4*g*h - 5*b*c^3*h^2)*x)*e)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**2*(d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

Giac [A]
time = 5.04, size = 457, normalized size = 1.09

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6fh^2x}{c}+\frac{16c^3fgh-7bc^2fh^2+8c^3h^2e}{c^4}\right)x+\frac{48c^3fg^2-80bc^2fgh+48c^3dh^2+35b^2cfh^2-36ac^2fh^2+96c^3ghe-40bc^2h^2e}{c^4}\right)x-\frac{144bc^2fg^2-384c^3dgh-240b^2cfgh+256ac^2fgh+144bc^2dh^2+105b^3fh^2-220abcfh^2-192c^3g^2e+288bc^2ghe-120b^2ch^2e+128ac^2h^2e}{c^4}\right)-\frac{(128c^4dg^2+48b^2c^2fg^2-64ac^3fg^2-128bc^3dgh-80b^3cfgh+192abc^2fgh+48b^2c^2dh^2-64ac^3dh^2+35b^4fh^2-120ab^2cfh^2+48a^2c^2fh^2-64bc^3g^2e+96b^2c^2ghe-128ac^3ghe-40b^3ch^2e+96abc^2h^2e)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{128c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*h^2*x/c + (16*c^3*f*g*h - 7*b*c^2*f*
h^2 + 8*c^3*h^2*e)/c^4)*x + (48*c^3*f*g^2 - 80*b*c^2*f*g*h + 48*c^3*d*h^2 +
 35*b^2*c*f*h^2 - 36*a*c^2*f*h^2 + 96*c^3*g*h*e - 40*b*c^2*h^2*e)/c^4)*x -
(144*b*c^2*f*g^2 - 384*c^3*d*g*h - 240*b^2*c*f*g*h + 256*a*c^2*f*g*h + 144*
b*c^2*d*h^2 + 105*b^3*f*h^2 - 220*a*b*c*f*h^2 - 192*c^3*g^2*e + 288*b*c^2*g
*h*e - 120*b^2*c*h^2*e + 128*a*c^2*h^2*e)/c^4) - 1/128*(128*c^4*d*g^2 + 48*
b^2*c^2*f*g^2 - 64*a*c^3*f*g^2 - 128*b*c^3*d*g*h - 80*b^3*c*f*g*h + 192*a*b
*c^2*f*g*h + 48*b^2*c^2*d*h^2 - 64*a*c^3*d*h^2 + 35*b^4*f*h^2 - 120*a*b^2*c
*f*h^2 + 48*a^2*c^2*f*h^2 - 64*b*c^3*g^2*e + 96*b^2*c^2*g*h*e - 128*a*c^3*g
*h*e - 40*b^3*c*h^2*e + 96*a*b*c^2*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g+h\,x)^2\,(f\,x^2+e\,x+d)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

3.228 $\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=223

$$\frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch}+\frac{(15b^2fh^2-8c^2(fg^2-3h(eg+dh))-2ch(8afh+9b(fg+eh))-2ch(2cfg-}{24c^3h}$$

```
[Out] 1/16*(16*c^3*d*g-5*b^3*f*h-8*c^2*(a*e*h+a*f*g+b*d*h+b*e*g)+6*b*c*(2*a*f*h+b
*e*h+b*f*g))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1/3
*f*(h*x+g)^2*(c*x^2+b*x+a)^(1/2)/c/h+1/24*(15*b^2*f*h^2-8*c^2*(f*g^2-3*h*(d
*h+e*g))-2*c*h*(8*a*f*h+9*b*(e*h+f*g))-2*c*h*(5*b*f*h-6*c*e*h+2*c*f*g)*x)*(
c*x^2+b*x+a)^(1/2)/c^3/h
```

Rubi [A]
time = 0.17, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1667, 793, 635, 212}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(aeh+afg+bdh+beg)+6bc(2afh+beh+bfg)-5b^3fh+16c^3dg\right)}{16c^{7/2}}+\frac{\sqrt{a+bx+cx^2}\left(-2ch(8afh+9b(eh+fg))+15b^2fh^2-2chx(5bfh-6ceh+2cfg)-8c^2(fg^2-3h(dh+eg))\right)}{24c^3h}+\frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (f*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c*h) + ((15*b^2*f*h^2 - 8*c^2*(f*g
^2 - 3*h*(e*g + d*h)) - 2*c*h*(8*a*f*h + 9*b*(f*g + e*h)) - 2*c*h*(2*c*f*g
- 6*c*e*h + 5*b*f*h)*x)*Sqrt[a + b*x + c*x^2])/(24*c^3*h) + ((16*c^3*d*g -
5*b^3*f*h - 8*c^2*(b*e*g + a*f*g + b*d*h + a*e*h) + 6*b*c*(b*f*g + b*e*h +
2*a*f*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(7/
2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\int \frac{(g+hx)\left(-\frac{1}{2}h(bfg-6cdh+4afh)-\frac{1}{2}h(2cfg-6ceh+5bfh)x\cdots\right)}{\sqrt{a+bx+cx^2}}\cdots}{3ch^2}$$

$$= \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\left(15b^2fh^2-8c^2\left(fg^2-3h(eg+dh)\right)-2ch(8a\cdots\right)}{\cdots}$$

$$= \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\left(15b^2fh^2-8c^2\left(fg^2-3h(eg+dh)\right)-2ch(8a\cdots\right)}{\cdots}$$

$$= \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\left(15b^2fh^2-8c^2\left(fg^2-3h(eg+dh)\right)-2ch(8a\cdots\right)}{\cdots}$$

Mathematica [A]
time = 0.66, size = 178, normalized size = 0.80

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(15b^2fh+4c^2\left(6eg+6dh+3fgx+3ehx+2fhx^2\right)-2c(8afh+b(9fg+9eh+5fhx))\right)+3\left(-16c^3dg+5b^3fh+8c^2(beg+afg+bdh+aeh)-6bc(bfg+beh+2afh)\right)\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{48c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^2*f*h + 4*c^2*(6*e*g + 6*d*h + 3*f*g
*x + 3*e*h*x + 2*f*h*x^2) - 2*c*(8*a*f*h + b*(9*f*g + 9*e*h + 5*f*h*x))) +
3*(-16*c^3*d*g + 5*b^3*f*h + 8*c^2*(b*e*g + a*f*g + b*d*h + a*e*h) - 6*b*c*
(b*f*g + b*e*h + 2*a*f*h))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]
)/(48*c^(7/2))

Maple [A]
time = 0.13, size = 389, normalized size = 1.74

method	result
default	$fh\left(\frac{x^2\sqrt{c x^2+bx+a}}{3c}-\frac{5b\left(\frac{x\sqrt{c x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}\right.}{6c}\right.$
risch	$-\frac{\left(-8fh\,c^2x^2+10bcfhx-12c^2ehx-12c^2fgx+16acfh-15b^2fh+18bceh+18bcfg-24c^2dh-24c^2eg\right)\sqrt{c x^2+bx+a}}{24c^3}+\frac{3\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] f*h*(1/3*x^2/c*(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4
b/c(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2/
3*a/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))))+(e*h+f*g)*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+
b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2
*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(d*h+e*g)*(1/c*(c*x
^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+
d*g*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.43, size = 473, normalized size = 2.12

$$\left[-\frac{3\left(2\left(8c^{3}d+\left(3b^{2}c-4ac^{2}\right)f\right)g-\left(8bc^{2}d+\left(5b^{3}-12abc\right)f\right)h-2\left(4bc^{2}g-\left(3b^{2}c-4ac^{2}\right)h\right)e\right)\sqrt{c}\log\left(-8c^{2}x^{2}-8bcx-b^{2}+4\sqrt{cx^{2}+bx+a}\left(2cx+b\right)\sqrt{c}-4ac\right)-4\left(8c^{3}fhx^{2}-18bc^{2}fg+\left(24c^{3}d+\left(15b^{2}c-16ac^{2}\right)f\right)h+2\left(6c^{3}fg-5bc^{2}fh\right)x+6\left(2c^{3}hx+4c^{3}g-3bc^{2}h\right)e\right)\sqrt{cx^{2}+bx+a}}{96c^{4}}, -\frac{3\left(2\left(8c^{3}d+\left(3b^{2}c-4ac^{2}\right)f\right)g-\left(8bc^{2}d+\left(5b^{3}-12abc\right)f\right)h-2\left(4bc^{2}g-\left(3b^{2}c-4ac^{2}\right)h\right)e\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^{2}+bx+a}\left(2cx+b\right)\sqrt{-c}}{2\left(c^{2}x^{2}+bcx+ac\right)}\right)-2\left(8c^{3}fhx^{2}-18bc^{2}fg+\left(24c^{3}d+\left(15b^{2}c-16ac^{2}\right)f\right)h+2\left(6c^{3}fg-5bc^{2}fh\right)x+6\left(2c^{3}hx+4c^{3}g-3bc^{2}h\right)e\right)\sqrt{cx^{2}+bx+a}}{48c^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(2*(8*c^3*d + (3*b^2*c - 4*a*c^2)*f)*g - (8*b*c^2*d + (5*b^3 - 12
*a*b*c)*f)*h - 2*(4*b*c^2*g - (3*b^2*c - 4*a*c^2)*h)*e)*sqrt(c)*log(-8*c^2*
x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c)
- 4*(8*c^3*f*h*x^2 - 18*b*c^2*f*g + (24*c^3*d + (15*b^2*c - 16*a*c^2)*f)*h
+ 2*(6*c^3*f*g - 5*b*c^2*f*h)*x + 6*(2*c^3*h*x + 4*c^3*g - 3*b*c^2*h)*e)*sq
rt(c*x^2 + b*x + a))/c^4, -1/48*(3*(2*(8*c^3*d + (3*b^2*c - 4*a*c^2)*f)*g -
 (8*b*c^2*d + (5*b^3 - 12*a*b*c)*f)*h - 2*(4*b*c^2*g - (3*b^2*c - 4*a*c^2)*
h)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x
^2 + b*c*x + a*c)) - 2*(8*c^3*f*h*x^2 - 18*b*c^2*f*g + (24*c^3*d + (15*b^2*
c - 16*a*c^2)*f)*h + 2*(6*c^3*f*g - 5*b*c^2*f*h)*x + 6*(2*c^3*h*x + 4*c^3*g
 - 3*b*c^2*h)*e)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g+hx)\left(d+ex+fx^{2}\right)}{\sqrt{a+bx+cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)*(d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

Giac [A]
time = 5.54, size = 210, normalized size = 0.94

$$\frac{1}{24}\sqrt{cx^{2}+bx+a}\left(2\left(\frac{4fhx}{c}+\frac{6c^{2}fg-5bcfh+6c^{2}he}{c^{3}}\right)x-\frac{18bcfg-24c^{2}dh-15b^{2}fh+16acfh-24c^{2}ge+18bche}{c^{3}}\right)-\frac{\left(16c^{3}dg+6b^{2}cfg-8ac^{2}fg-8bc^{2}dh-5b^{3}fh+12abcfh-8bc^{2}ge+6b^{2}che-8ac^{2}he\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^{2}+bx+a}\right)\sqrt{c}-b\right|\right)}{16c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*(4*f*h*x/c + (6*c^2*f*g - 5*b*c*f*h + 6*c^2*h
*e)/c^3)*x - (18*b*c*f*g - 24*c^2*d*h - 15*b^2*f*h + 16*a*c*f*h - 24*c^2*g*
e + 18*b*c*h*e)/c^3) - 1/16*(16*c^3*d*g + 6*b^2*c*f*g - 8*a*c^2*f*g - 8*b*c
^2*d*h - 5*b^3*f*h + 12*a*b*c*f*h - 8*b*c^2*g*e + 6*b^2*c*h*e - 8*a*c^2*h*e
)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g+h\,x)\,\left(f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

3.229 $\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=116

$$\frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}+\frac{(8c^2d+3b^2f-4c(be+af))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}$$

```
[Out] 1/8*(8*c^2*d+3*b^2*f-4*c*(a*f+b*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*
x+a)^(1/2))/c^(5/2)+1/4*(-3*b*f+4*c*e)*(c*x^2+b*x+a)^(1/2)/c^2+1/2*f*x*(c*x
^2+b*x+a)^(1/2)/c
```

Rubi [A]
time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1675, 654, 635, 212}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+be)+3b^2f+8c^2d)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}\,(4ce-3bf)}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((4*c*e - 3*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (f*x*Sqrt[a + b*x + c*x^2
])/(2*c) + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\int \frac{2cd-af+\frac{1}{2}(4ce-3bf)x}{\sqrt{a+bx+cx^2}}\,dx}{2c}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3b\right.}{4c^2}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3b\right.}{}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{(8c^2d+3b^2f-4c(be+af))\,\mathrm{t}}{8c}$$

Mathematica [A]
time = 0.03, size = 99, normalized size = 0.85

$$\frac{2\sqrt{c}\,(4ce-3bf+2cfx)\sqrt{a+x(b+cx)}+(-8c^2d-3b^2f+4c(be+af))\log\left(c^2\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)}{8c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*(4*c*e - 3*b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)] + (-8*c^2*d - 3*
b^2*f + 4*c*(b*e + a*f))*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x
)])])/(8*c^(5/2))
```

Maple [A]
time = 0.10, size = 188, normalized size = 1.62

method	result

risch	$-\frac{(-2cfx+3bf-4ce)\sqrt{c\,x^2+bx+a}}{4c^2}-\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)af}{2c^{\frac{3}{2}}}+\frac{3\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)b^2f}{8c^{\frac{5}{2}}}$
default	$f\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx\right.}}{2c^{\frac{3}{2}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+e*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+d*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.39, size = 231, normalized size = 1.99

$$\left[-\frac{(8\,c^2d-4\,bce+(3\,b^2-4\,ac)f)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{c}-4\,ac\right)-4\,(2\,c^2fx-3\,bcf+4\,c^2e)\sqrt{cx^2+bx+a}}{16\,c^3},-\frac{(8\,c^2d-4\,bce+(3\,b^2-4\,ac)f)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)-2\,(2\,c^2fx-3\,bcf+4\,c^2e)\sqrt{cx^2+bx+a}}{8\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*
b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*c
```

```
^2*f*x - 3*b*c*f + 4*c^2*e)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((8*c^2*d - 4*
b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*f*x - 3*b*c*f + 4*c^2*e)
*sqrt(c*x^2 + b*x + a))/c^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2}{\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

Giac [A]
time = 4.34, size = 98, normalized size = 0.84

$$\frac{1}{4}\sqrt{cx^2 + bx + a}\left(\frac{2\,fx}{c} - \frac{3\,bf - 4\,ce}{c^2}\right) - \frac{(8\,c^2d + 3\,b^2f - 4\,acf - 4\,bce)\log\left(\left|-2\left(\sqrt{c}\,x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} - b\right|\right)}{8\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*f*x/c - (3*b*f - 4*c*e)/c^2) - 1/8*(8*c^2*d +
3*b^2*f - 4*a*c*f - 4*b*c*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*sqrt(c) - b))/c^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2 + e\,x + d}{\sqrt{c\,x^2 + b\,x + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2), x)
```

3.230 $\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=179

$$\frac{f\sqrt{a+bx+cx^2}}{ch}-\frac{(2cfg-2ceh+bfh)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}h^2}+\frac{(fg^2-h(eg-dh))\tanh^{-1}\left(\frac{}{2\sqrt{c}}\right.}{h^2\sqrt{cg^2-}}$$

```
[Out] -1/2*(b*f*h-2*c*e*h+2*c*f*g)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/c^(3/2)/h^2+(f*g^2-h*(-d*h+e*g))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x
)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^2/(a*h^2-b*g*h+c*g^2)^(1
/2)+f*(c*x^2+b*x+a)^(1/2)/c/h
```

Rubi [A]
time = 0.17, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1667, 857, 635, 212, 738}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bfh-2ceh+2cfg)}{2c^{3/2}h^2}+\frac{(fg^2-h(eg-dh))\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{h^2\sqrt{ah^2-bgh+cg^2}}+\frac{f\sqrt{a+bx+cx^2}}{ch}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (f*Sqrt[a + b*x + c*x^2])/(c*h) - ((2*c*f*g - 2*c*e*h + b*f*h)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*h^2) + ((f*g^2 - h*(
e*g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h +
 a*h^2]*Sqrt[a + b*x + c*x^2])])/(h^2*Sqrt[c*g^2 - b*g*h + a*h^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \frac{f\sqrt{a+bx+cx^2}}{ch} + \frac{\int \frac{-\frac{1}{2}h(bfg-2cdh)-\frac{1}{2}h(2cfg-2ceh+bfh)x}{(g+hx)\sqrt{a+bx+cx^2}}\,dx}{ch^2}$$

$$= \frac{f\sqrt{a+bx+cx^2}}{ch} - \frac{(2cfg-2ceh+bfh)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2ch^2} + \frac{(fg^2-$$

$$= \frac{f\sqrt{a+bx+cx^2}}{ch} - \frac{(2cfg-2ceh+bfh)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+}}\right.}{ch^2}$$

$$= \frac{f\sqrt{a+bx+cx^2}}{ch} - \frac{(2cfg-2ceh+bfh)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}h^2}$$

Mathematica [A]

time = 0.72, size = 186, normalized size = 1.04

$$\frac{\frac{2fh\sqrt{a+x(b+cx)}}{c} + \frac{4\sqrt{-cg^2+h(bg-ah)}\,(fg^2+h(-eg+dh))\tan^{-1}\left(\frac{\sqrt{c}\,(g+hx)-h\sqrt{a+x(b+cx)}}{\sqrt{-cg^2+h(bg-ah)}}\right)}{cg^2+h(-bg+ah)} + \frac{(2cfg-2ceh+bfh)\log\left(c\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{c^{3/2}}}{2h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((2*f*h*Sqrt[a + x*(b + c*x)])/c + (4*Sqrt[-(c*g^2) + h*(b*g - a*h)]*(f*g^2
 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + x*(b + c*x)])/S
qrt[-(c*g^2) + h*(b*g - a*h)]])/(c*g^2 + h*(-(b*g) + a*h)) + ((2*c*f*g - 2*
c*e*h + b*f*h)*Log[c*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2
))/(2*h^2)
```

Maple [A]
time = 0.15, size = 293, normalized size = 1.64

method	result
default	$\frac{fh\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)+\frac{eh\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}-\frac{gf\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+\cdots}\right)}{\sqrt{c}}}{h^2}$
risch	$\frac{f\sqrt{c\,x^2+bx+a}}{ch}-\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)bf}{2c^{\frac{3}{2}}h}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)e}{\sqrt{c}\,h}-\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2\cdots}\right)}{\sqrt{c}\,h}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/h^2*(f*h*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2)))+e*h*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)
-g*f*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2))-(d*h^2-e*g*h+f*g^
2)/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2
*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c
*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `assum
e?` for
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2+e\,x+d}{(g+h\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.231 $\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=241

$$-\frac{(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,h^2}-\frac{(2c(fg^3-dgh^2)+h(2ah(2fg-$$

```
[Out] -1/2*(2*c*(-d*g*h^2+f*g^3)+h*(2*a*h*(-e*h+2*f*g)-b*(-d*h^2-e*g*h+3*f*g^2)))
*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*
x+a)^(1/2))/h^2/(a*h^2-b*g*h+c*g^2)^(3/2)+f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(
c*x^2+b*x+a)^(1/2))/h^2/c^(1/2)-(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1/2)/h/
(a*h^2-b*g*h+c*g^2)/(h*x+g)
```

Rubi [A]
time = 0.22, antiderivative size = 239, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1664, 857, 635, 212, 738}

$$-\frac{\sqrt{a+bx+cx^2}\,(fg^2-h(eg-dh))}{h(g+hx)\left(ah^2-bgh+cg^2\right)}-\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(2c(fg^3-dgh^2)-h(-2ah(2fg-eh)-bh(dh+eg)+3bfg^2)\right)}{2h^2\left(ah^2-bgh+cg^2\right)^{3/2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,h^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(((f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(h*(c*g^2 - b*g*h + a*h^2
)*(g + h*x))) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
(Sqrt[c]*h^2) - ((2*c*(f*g^3 - d*g*h^2) - h*(3*b*f*g^2 - b*h*(e*g + d*h) -
2*a*h*(2*f*g - e*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2
 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*h^2*(c*g^2 - b*g*h + a*h^2)^(
3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} - \frac{\int \frac{\frac{1}{2}\left(-2cdg+beg+2afg-\frac{bfg^2}{h}+bdh-2aeh\right.}{(g+hx)\sqrt{a+bx+}}}{cg^2-bgh+a}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} + \frac{f\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{h^2} - \frac{(2c(f}{}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} + \frac{(2f)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b}{\sqrt{a+}}\right.}{h^2}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} + \frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right.}{\sqrt{c}\,h^2}$$

Mathematica [A]

time = 1.24, size = 236, normalized size = 0.98

$$-\frac{\frac{h(fg^2+h(-eg+dh))\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)}+\frac{\sqrt{-cg^2+h(bg-ah)}\,(2c(fg^3-dgh^2)+h(-3bfg^2+bh(eg+dh)-2ah(-2fg+eh)))\tan^{-1}\left(\frac{\sqrt{c}\,(g+hx)-h\sqrt{a+x(b+cx)}}{\sqrt{-cg^2+h(bg-ah)}}\right)}{(cg^2+h(-bg+ah))^2}+\frac{f\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)}{\sqrt{c}}}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(((h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(-(b*g)
 + a*h))*(g + h*x)) + (Sqrt[-(c*g^2) + h*(b*g - a*h)]*(2*c*(f*g^3 - d*g*h^2
) + h*(-3*b*f*g^2 + b*h*(e*g + d*h) - 2*a*h*(-2*f*g + e*h)))*ArcTan[(Sqrt[c
]*(g + h*x) - h*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*g^2) + h*(b*g - a*h)]])/(c*
g^2 + h*(-(b*g) + a*h))^2 + (f*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*
x)]])/Sqrt[c])/h^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(221) = 442$.
time = 0.13, size = 485, normalized size = 2.01

method	result
default	$\frac{f\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{h^2\sqrt{c}}-\frac{(eh-2gf)\ln\left(\frac{\frac{2a\,h^2-2bgh+2c\,g^2}{h^2}+\frac{(bh-2cg)\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a\,h^2-bgh+c\,g^2}{h^2}}\sqrt{\left(x+\frac{g}{h}\right)^2 c}}{x+\frac{g}{h}}\right.}{h^3\sqrt{\frac{a\,h^2-bgh+c\,g^2}{h^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f/h^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/h^3*(e*h-2*f*g)
/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/
h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*
(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))+1/h^4*(d*h^2-e*g*h+f*g
^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1
/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/
((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h
*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(
x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `assum
e?` for
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)**2*sqrt(a + b*x + c*x**2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^2\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(1/2)), x)
```

3.232 $\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=336

$$-\frac{(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{2h\,(cg^2-bgh+ah^2)\,(g+hx)^2}+\frac{(2cg(fg^2+h(eg-3dh))+h(4ah(2fg-eh)-b(5fg^2-egh-3dh$$

```
[Out] 1/8*(8*c^2*d*g^2+8*a^2*f*h^2-4*a*b*h*(e*h+2*f*g)+b^2*(3*d*h^2+e*g*h+3*f*g^2
)-4*c*(b*g*(2*d*h+e*g)+a*(d*h^2-3*e*g*h+f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b
*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*
g^2)^(5/2)-1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1/2)/h/(a*h^2-b*g*h+c*g^
2)/(h*x+g)^2+1/4*(2*c*g*(f*g^2+h*(-3*d*h+e*g))+h*(4*a*h*(-e*h+2*f*g)-b*(-3*
d*h^2-e*g*h+5*f*g^2)))*(c*x^2+b*x+a)^(1/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)
```

Rubi [A]
time = 0.39, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1664, 820, 738, 212}

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\,\sqrt{ah^2-bgh+cg^2}}\right)(8a^2fh^2-4c(-ah(3eg-dh)+afg^2+bg(2dh+eg))-4abh(eh+2fg)+b^2(h(3dh+eg)+3fg^2)+8c^2dg^2)}{8\,(ah^2-bgh+cg^2)^{5/2}}-\frac{\sqrt{a+bx+cx^2}\,(fg^2-h(eg-dh))}{2h(g+hx)^2\,(ah^2-bgh+cg^2)}+\frac{\sqrt{a+bx+cx^2}\,(2c(gh(eg-3dh)+fg^3)-h(-4ah(2fg-eh)-bh(3dh+eg)+5bfg^2))}{4h(g+hx)\,(ah^2-bgh+cg^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/2*((f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(h*(c*g^2 - b*g*h + a*
h^2)*(g + h*x)^2) + ((2*c*(f*g^3 + g*h*(e*g - 3*d*h)) - h*(5*b*f*g^2 - b*h*
(e*g + 3*d*h) - 4*a*h*(2*f*g - e*h)))*Sqrt[a + b*x + c*x^2])/(4*h*(c*g^2 -
b*g*h + a*h^2)^2*(g + h*x)) + ((8*c^2*d*g^2 + 8*a^2*f*h^2 - 4*a*b*h*(2*f*g
+ e*h) - 4*c*(a*f*g^2 - a*h*(3*e*g - d*h) + b*g*(e*g + 2*d*h)) + b^2*(3*f*g
^2 + h*(e*g + 3*d*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^
2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*g^2 - b*g*h + a*h^2)^(5/2
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} - \frac{\int \frac{\frac{1}{2}\left(-4cdg+beg+4afg-\frac{bfg^2}{h}+3bdh-4aeh\right.}{(g+hx)^2\sqrt{a+b}}}{2\left(cg^2-bg\right.}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{(2c(fg^3+gh(eg-3dh))-h(5b}{4h\,(c}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{(2c(fg^3+gh(eg-3dh))-h(5b}{4h\,(c}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{(2c(fg^3+gh(eg-3dh))-h(5b}{4h\,(c}$$

Mathematica [A]
time = 10.81, size = 415, normalized size = 1.24

$$\frac{-2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b+cx)}\left(2(cg^2+h(-bg+ah))(fg^2+h(-eg+dh))-(2c(fg^3+gh(eg-3dh))+h(-5bfg^2+bh(eg+3dh)-4ah(-2fg+eh)))(g+hx)\right)+h\left(8c^2dg^2+8a^2fh^2-4abh(2fg+eh)-4c(afg^2+ah(-3eg+dh)+bg(eg+2dh))+b^2(3fg^2+h(eg+3dh))\right)(g+hx)^2\log(g+hx)+h\left(-8c^2dg^2-8a^2fh^2+4abh(2fg+eh)+4c(afg^2+ah(-3eg+dh)+bg(eg+2dh))-b^2(3fg^2+h(eg+3dh))\right)(g+hx)^2\log\left(-bg+2ah-2cgx+bhx+2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b+cx)}\right)}{8h\left(cg^2+h(-bg+ah)\right)^{5/2}(g+hx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)]*(2*(c*g^2 + h*(-(b
*g) + a*h))*(f*g^2 + h*(-(e*g) + d*h)) - (2*c*(f*g^3 + g*h*(e*g - 3*d*h)) +
 h*(-5*b*f*g^2 + b*h*(e*g + 3*d*h) - 4*a*h*(-2*f*g + e*h)))*(g + h*x)) + h*
(8*c^2*d*g^2 + 8*a^2*f*h^2 - 4*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 + a*h*(-3
*e*g + d*h) + b*g*(e*g + 2*d*h)) + b^2*(3*f*g^2 + h*(e*g + 3*d*h)))*(g + h*
x)^2*Log[g + h*x] + h*(-8*c^2*d*g^2 - 8*a^2*f*h^2 + 4*a*b*h*(2*f*g + e*h) +
 4*c*(a*f*g^2 + a*h*(-3*e*g + d*h) + b*g*(e*g + 2*d*h)) - b^2*(3*f*g^2 + h*
(e*g + 3*d*h)))*(g + h*x)^2*Log[-(b*g) + 2*a*h - 2*c*g*x + b*h*x + 2*Sqrt[c
*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)]])/(8*h*(c*g^2 + h*(-(b*g) +
a*h))^(5/2)*(g + h*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. $2(318) = 636$.
time = 0.16, size = 1013, normalized size = 3.01

method	result
default	$-\frac{f \ln\left(\frac{\frac{2a h^2-2bgh+2c g^2}{h^2}+\frac{(bh-2cg)\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a h^2-bgh+c g^2}{h^2}}\,\sqrt{\left(x+\frac{g}{h}\right)^2 c+\frac{(bh-2cg)\left(x+\frac{g}{h}\right)}{h}+\frac{a h^2-bgh+c g^2}{h^2}}}{x+\frac{g}{h}}\right)}{h^3\sqrt{\frac{a h^2-bgh+c g^2}{h^2}}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -f/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2
*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c
*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))+(e*h-2*f*g)/h^4*
(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g
)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+
1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/
```

```
h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+(d*h^2-e*g*h+f*g^2)/h^5*(-
1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h
*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-
1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^
2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/
h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*
g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/2*c/(a*h^2-b*g*h+c*g^2)*h^
2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)
/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h
*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. $2(327) = 654$.
time = 20.11, size = 2170, normalized size = 6.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/16*(((8*c^2*d + (3*b^2 - 4*a*c)*f)*g^4 - 8*(b*c*d + a*b*f)*g^3*h + (8*a
^2*f + (3*b^2 - 4*a*c)*d)*g^2*h^2 + ((8*c^2*d + (3*b^2 - 4*a*c)*f)*g^2*h^2
- 8*(b*c*d + a*b*f)*g*h^3 + (8*a^2*f + (3*b^2 - 4*a*c)*d)*h^4)*x^2 + 2*((8*
c^2*d + (3*b^2 - 4*a*c)*f)*g^3*h - 8*(b*c*d + a*b*f)*g^2*h^2 + (8*a^2*f + (
3*b^2 - 4*a*c)*d)*g*h^3)*x - (4*b*c*g^4 + 4*a*b*g^2*h^2 - (b^2 + 12*a*c)*g^
3*h + (4*b*c*g^2*h^2 + 4*a*b*h^4 - (b^2 + 12*a*c)*g*h^3)*x^2 + 2*(4*b*c*g^3
*h + 4*a*b*g*h^3 - (b^2 + 12*a*c)*g^2*h^2)*x)*e)*sqrt(c*g^2 - b*g*h + a*h^2
)*log((8*a*b*g*h - 8*a^2*h^2 - (b^2 + 4*a*c)*g^2 - (8*c^2*g^2 - 8*b*c*g*h +
 (b^2 + 4*a*c)*h^2)*x^2 + 4*sqrt(c*g^2 - b*g*h + a*h^2)*sqrt(c*x^2 + b*x +
a)*(b*g - 2*a*h + (2*c*g - b*h)*x) - 2*(4*b*c*g^2 + 4*a*b*h^2 - (3*b^2 + 4*
```

```
a*c)*g*h)*x)/(h^2*x^2 + 2*g*h*x + g^2)) + 4*(3*b*c*f*g^5 - 7*a*b*d*g*h^4 +
2*a^2*d*h^5 + (8*c^2*d - 3*(b^2 + 2*a*c)*f)*g^4*h - (13*b*c*d - 9*a*b*f)*g^
3*h^2 - (6*a^2*f - 5*(b^2 + 2*a*c)*d)*g^2*h^3 - (2*c^2*f*g^5 - 7*b*c*f*g^4*
h + 3*a*b*d*h^5 - (6*c^2*d - 5*(b^2 + 2*a*c)*f)*g^3*h^2 + (9*b*c*d - 13*a*b
*f)*g^2*h^3 + (8*a^2*f - 3*(b^2 + 2*a*c)*d)*g*h^4)*x - (4*c^2*g^5 - 5*b*c*g
^4*h + a*b*g^2*h^3 - 2*a^2*g*h^4 + (b^2 + 2*a*c)*g^3*h^2 + (2*c^2*g^4*h - b
*c*g^3*h^2 + 5*a*b*g*h^4 - 4*a^2*h^5 - (b^2 + 2*a*c)*g^2*h^3)*x)*e)*sqrt(c*
x^2 + b*x + a))/(c^3*g^8 - 3*b*c^2*g^7*h - 3*a^2*b*g^3*h^5 + a^3*g^2*h^6 +
3*(b^2*c + a*c^2)*g^6*h^2 - (b^3 + 6*a*b*c)*g^5*h^3 + 3*(a*b^2 + a^2*c)*g^4
*h^4 + (c^3*g^6*h^2 - 3*b*c^2*g^5*h^3 - 3*a^2*b*g*h^7 + a^3*h^8 + 3*(b^2*c
+ a*c^2)*g^4*h^4 - (b^3 + 6*a*b*c)*g^3*h^5 + 3*(a*b^2 + a^2*c)*g^2*h^6)*x^2
 + 2*(c^3*g^7*h - 3*b*c^2*g^6*h^2 - 3*a^2*b*g^2*h^6 + a^3*g*h^7 + 3*(b^2*c
+ a*c^2)*g^5*h^3 - (b^3 + 6*a*b*c)*g^4*h^4 + 3*(a*b^2 + a^2*c)*g^3*h^5)*x),
 1/8*(((8*c^2*d + (3*b^2 - 4*a*c)*f)*g^4 - 8*(b*c*d + a*b*f)*g^3*h + (8*a^2
*f + (3*b^2 - 4*a*c)*d)*g^2*h^2 + ((8*c^2*d + (3*b^2 - 4*a*c)*f)*g^2*h^2 -
8*(b*c*d + a*b*f)*g*h^3 + (8*a^2*f + (3*b^2 - 4*a*c)*d)*h^4)*x^2 + 2*((8*c^
2*d + (3*b^2 - 4*a*c)*f)*g^3*h - 8*(b*c*d + a*b*f)*g^2*h^2 + (8*a^2*f + (3*
b^2 - 4*a*c)*d)*g*h^3)*x - (4*b*c*g^4 + 4*a*b*g^2*h^2 - (b^2 + 12*a*c)*g^3*
h + (4*b*c*g^2*h^2 + 4*a*b*h^4 - (b^2 + 12*a*c)*g*h^3)*x^2 + 2*(4*b*c*g^3*h
 + 4*a*b*g*h^3 - (b^2 + 12*a*c)*g^2*h^2)*x)*e)*sqrt(-c*g^2 + b*g*h - a*h^2)
*arctan(-1/2*sqrt(-c*g^2 + b*g*h - a*h^2)*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*
h + (2*c*g - b*h)*x)/(a*c*g^2 - a*b*g*h + a^2*h^2 + (c^2*g^2 - b*c*g*h + a*
c*h^2)*x^2 + (b*c*g^2 - b^2*g*h + a*b*h^2)*x)) - 2*(3*b*c*f*g^5 - 7*a*b*d*g
*h^4 + 2*a^2*d*h^5 + (8*c^2*d - 3*(b^2 + 2*a*c)*f)*g^4*h - (13*b*c*d - 9*a*
b*f)*g^3*h^2 - (6*a^2*f - 5*(b^2 + 2*a*c)*d)*g^2*h^3 - (2*c^2*f*g^5 - 7*b*c
*f*g^4*h + 3*a*b*d*h^5 - (6*c^2*d - 5*(b^2 + 2*a*c)*f)*g^3*h^2 + (9*b*c*d -
 13*a*b*f)*g^2*h^3 + (8*a^2*f - 3*(b^2 + 2*a*c)*d)*g*h^4)*x - (4*c^2*g^5 -
5*b*c*g^4*h + a*b*g^2*h^3 - 2*a^2*g*h^4 + (b^2 + 2*a*c)*g^3*h^2 + (2*c^2*g^
4*h - b*c*g^3*h^2 + 5*a*b*g*h^4 - 4*a^2*h^5 - (b^2 + 2*a*c)*g^2*h^3)*x)*e)*
sqrt(c*x^2 + b*x + a))/(c^3*g^8 - 3*b*c^2*g^7*h - 3*a^2*b*g^3*h^5 + a^3*g^2
*h^6 + 3*(b^2*c + a*c^2)*g^6*h^2 - (b^3 + 6*a*b*c)*g^5*h^3 + 3*(a*b^2 + a^2
*c)*g^4*h^4 + (c^3*g^6*h^2 - 3*b*c^2*g^5*h^3 - 3*a^2*b*g*h^7 + a^3*h^8 + 3*
(b^2*c + a*c^2)*g^4*h^4 - (b^3 + 6*a*b*c)*g^3*h^5 + 3*(a*b^2 + a^2*c)*g^2*h
^6)*x^2 + 2*(c^3*g^7*h - 3*b*c^2*g^6*h^2 - 3*a^2*b*g^2*h^6 + a^3*g*h^7 + 3*
(b^2*c + a*c^2)*g^5*h^3 - (b^3 + 6*a*b*c)*g^4*h^4 + 3*(a*b^2 + a^2*c)*g^3*h
^5)*x)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2}{(g + hx)^3 \sqrt{a + bx + cx^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+b*x+a)**(1/2),x)
```

[Out] Integral((d + e*x + f*x**2)/((g + h*x)**3*sqrt(a + b*x + c*x**2)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2307 vs. $2(327) = 654$.

time = 4.25, size = 2307, normalized size = 6.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")

[Out] 1/4*(8*c^2*d*g^2 + 3*b^2*f*g^2 - 4*a*c*f*g^2 - 8*b*c*d*g*h - 8*a*b*f*g*h + 3*b^2*d*h^2 - 4*a*c*d*h^2 + 8*a^2*f*h^2 - 4*b*c*g^2*e + b^2*g*h*e + 12*a*c*g*h*e - 4*a*b*h^2*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/((c^2*g^4 - 2*b*c*g^3*h + b^2*g^2*h^2 + 2*a*c*g^2*h^2 - 2*a*b*g*h^3 + a^2*h^4)*sqrt(-c*g^2 + b*g*h - a*h^2)) + 1/4*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*f*g^4*h - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*f*g^3*h^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^2*h^3 + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*d*g*h^4 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*f*g*h^4 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*h^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*h^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*g^2*h^3*e - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*g*h^4*e - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*g*h^4*e + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*h^5*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*f*g^5 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*f*g^4*h - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*g^3*h^2 - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*f*g^3*h^2 + 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*f*g^3*h^2 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*d*g^2*h^3 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*f*g^2*h^3 - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*g*h^4 + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*g*h^4 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*f*g*h^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*g^4*h*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*g^3*h^2*e + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*g^2*h^3*e - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*g^2*h^3*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*g*h^4*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*h^5*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*f*g^5 - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*f*g^4*h - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*f*g^4*h - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*d*g^3*h^2 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*f*g^3*h^2 + 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*f*g^3*h^2 + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d*g^2*h^3 + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*g^2*h^3 - 11*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))

```
*a*b^2*f*g^2*h^3 - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*f*g^2*h^3 -
 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d*g*h^4 - 28*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a*b*c*d*g*h^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*
b*f*g*h^4 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d*h^5 + 4*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a^2*c*d*h^5 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*b*c^2*g^4*h*e - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*g^3*h^2*e
+ (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*g^2*h^3*e - 16*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*a*b*c*g^2*h^3*e + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
*b^2*g*h^4*e + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*g*h^4*e - 4*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*h^5*e + 2*b^2*c^(3/2)*f*g^5 - 5*b^3*
sqrt(c)*f*g^4*h - 4*a*b*c^(3/2)*f*g^4*h - 6*b^2*c^(3/2)*d*g^3*h^2 + 21*a*b^
2*sqrt(c)*f*g^3*h^2 + 4*a^2*c^(3/2)*f*g^3*h^2 + 3*b^3*sqrt(c)*d*g^2*h^3 + 2
0*a*b*c^(3/2)*d*g^2*h^3 - 32*a^2*b*sqrt(c)*f*g^2*h^3 - 11*a*b^2*sqrt(c)*d*g
*h^4 - 12*a^2*c^(3/2)*d*g*h^4 + 16*a^3*sqrt(c)*f*g*h^4 + 8*a^2*b*sqrt(c)*d*
h^5 + 2*b^2*c^(3/2)*g^4*h*e + b^3*sqrt(c)*g^3*h^2*e - 8*a*b*c^(3/2)*g^3*h^2
*e - 5*a*b^2*sqrt(c)*g^2*h^3*e + 4*a^2*c^(3/2)*g^2*h^3*e + 12*a^2*b*sqrt(c)
*g*h^4*e - 8*a^3*sqrt(c)*h^5*e)/((c^2*g^4*h^2 - 2*b*c*g^3*h^3 + b^2*g^2*h^4
 + 2*a*c*g^2*h^4 - 2*a*b*g*h^5 + a^2*h^6)*((sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*g + b*g - a*h)^2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^3\,\sqrt{c\,x^2+b\,x+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(1/2)), x)
```

3.233 $\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=504

$$\frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-(2c^2d-bce+b^2f-2acf)\,x\right)(g+hx)^3}{c\,(b^2-4ac)\sqrt{a+bx+cx^2}}+\frac{(12c^2d-6bce+7b^2f-16acf)\,h(g+hx)^2}{3c^2\,(b^2-4ac)}$$

```
[Out] -1/16*(35*b^3*f*h^3-30*b*c*h^2*(2*a*f*h+b*e*h+3*b*f*g)-16*c^3*g*(f*g^2+3*h*
(d*h+e*g))+24*c^2*h*(a*h*(e*h+3*f*g)+b*(d*h^2+3*e*g*h+3*f*g^2)))*arctanh(1/
2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)+2*(c*(2*a*e-b*(d+a*f/c))-(
-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)*(h*x+g)^3/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/
2)+1/3*(-16*a*c*f+7*b^2*f-6*b*c*e+12*c^2*d)*h*(h*x+g)^2*(c*x^2+b*x+a)^(1/2)
/c^2/(-4*a*c+b^2)+1/24*h*(192*c^4*d*g^2+105*b^4*f*h^2-10*b^2*c*h*(46*a*f*h+
9*b*(e*h+3*f*g))-16*c^3*(3*b*g*(3*d*h+2*e*g)+4*a*(3*d*h^2+9*e*g*h+7*f*g^2))
+8*c^2*(32*a^2*f*h^2+39*a*b*h*(e*h+3*f*g)+b^2*(20*f*g^2+9*h*(d*h+3*e*g)))+2
*c*h*(48*c^3*d*g-35*b^3*f*h-8*c^2*(9*a*e*h+11*a*f*g+3*b*d*h+3*b*e*g)+2*b*c*
(58*a*f*h+15*b*e*h+17*b*f*g))*x)*(c*x^2+b*x+a)^(1/2)/c^4/(-4*a*c+b^2)
```

Rubi [A]

time = 0.67, antiderivative size = 502, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1658, 846, 793, 635, 212}

$$\frac{h\sqrt{a+bx+cx^2}\left(8c(32a^2fh^2+39abh(eh+3fg)+b^2(9h(dh+3eg)+20fg^2))+2hx(-8c^2(9aeh+11afg+3bdh+3beg)+2bc(58afh+15beh+17bfg)-35b^3fh+48c^3dg)-10b^2h(46afh+9b(eh+3fg))-16c^2(4a(3dh^2+9egh+7fg^2)+3bg(3dh+2eg))+\frac{105b^4fh^2}{c}+192c^3dg^2\right)}{24c^3\,(b^2-4ac)}-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(24c^2h(ah(eh+3fg)+bh(dh+3eg)+3bfg^2)-30bch^2(2afh+beh+3bfg)+35b^3fh^3-16c^3(3gh(dh+eg)+fg^3))}{16c^{9/2}}+\frac{2(g+hx)^3\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x(-2acf+b^2f-bce+2c^2d)\right)}{c\,(b^2-4ac)\sqrt{a+bx+cx^2}}+\frac{h(g+hx)^2\sqrt{a+bx+cx^2}\,(-16acf+7b^2f-6bce+12c^2d)}{3c^2\,(b^2-4ac)}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x)^3)/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + ((12*c^2*d - 6*b*c*e +
 7*b^2*f - 16*a*c*f)*h*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c^2*(b^2 - 4*a
*c)) + (h*(192*c^3*d*g^2 + (105*b^4*f*h^2)/c - 10*b^2*h*(46*a*f*h + 9*b*(3*
f*g + e*h)) - 16*c^2*(3*b*g*(2*e*g + 3*d*h) + 4*a*(7*f*g^2 + 9*e*g*h + 3*d*
h^2)) + 8*c*(32*a^2*f*h^2 + 39*a*b*h*(3*f*g + e*h) + b^2*(20*f*g^2 + 9*h*(3
*e*g + d*h))) + 2*h*(48*c^3*d*g - 35*b^3*f*h - 8*c^2*(3*b*e*g + 11*a*f*g +
3*b*d*h + 9*a*e*h) + 2*b*c*(17*b*f*g + 15*b*e*h + 58*a*f*h))*x)*Sqrt[a + b*
x + c*x^2])/(24*c^3*(b^2 - 4*a*c)) - ((35*b^3*f*h^3 - 30*b*c*h^2*(3*b*f*g +
 b*e*h + 2*a*f*h) - 16*c^3*(f*g^3 + 3*g*h*(e*g + d*h)) + 24*c^2*h*(3*b*f*g^
2 + b*h*(3*e*g + d*h) + a*h*(3*f*g + e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(16*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int \cdots}{\cdots}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{(12c^2d \cdots}{\cdots}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{(12c^2d \cdots}{\cdots}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{(12c^2d \cdots}{\cdots}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{(12c^2d \cdots}{\cdots}$$

Mathematica [A]
time = 4.08, size = 713, normalized size = 1.41

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[c]*(105*b^5*f*h^3*x + 5*b^4*h^2*(21*a*f*h + c*x*(-54*f*g - 18*e*h
+ 7*f*h*x)) - 2*b^3*c*h*(5*a*h*(27*f*g + 9*e*h + 53*f*h*x) + c*x*(3*h*(-36*
e*g - 12*d*h + 5*e*h*x) + f*(-108*g^2 + 45*g*h*x + 7*h^2*x^2))) - 16*c^2*(-
16*a^3*f*h^3 + 6*c^3*d*g^3*x + a*c^2*(6*d*h*(-3*g^2 - 3*g*h*x + h^2*x^2) -
3*e*(2*g^3 + 6*g^2*h*x - 6*g*h^2*x^2 - h^3*x^3) + f*x*(-6*g^3 + 18*g^2*h*x
+ 9*g*h^2*x^2 + 2*h^3*x^3)) + a^2*c*h*(f*(36*g^2 + 27*g*h*x - 8*h^2*x^2) +
3*h*(4*d*h + 3*e*(4*g + h*x)))) - 8*b*c^2*(-6*c^2*g^2*(-(d*g) + e*g*x + 3*d
*h*x) - a^2*h^2*(117*f*g + 39*e*h + 61*f*h*x) + a*c*(f*(6*g^3 + 90*g^2*h*x
- 45*g*h^2*x^2 - 7*h^3*x^3) + 3*h*(2*d*h*(3*g + 5*h*x) + e*(6*g^2 + 30*g*h*
x - 5*h^2*x^2)))) + 4*b^2*c*(-115*a^2*f*h^3 + a*c*h*(3*h*(18*e*g + 6*d*h +
31*e*h*x) + f*(54*g^2 + 279*g*h*x - 43*h^2*x^2)) + c^2*x*(f*(-12*g^3 + 18*g
^2*h*x + 9*g*h^2*x^2 + 2*h^3*x^3) + 3*h*(2*d*h*(-6*g + h*x) + e*(-12*g^2 +
6*g*h*x + h^2*x^2))))) - 3*(b^2 - 4*a*c)*(35*b^3*f*h^3 - 30*b*c*h^2*(3*b*f*
g + b*e*h + 2*a*f*h) - 16*c^3*g*(f*g^2 + 3*h*(e*g + d*h)) + 24*c^2*h*(3*b*f
*g^2 + b*h*(3*e*g + d*h) + a*h*(3*f*g + e*h)))*Sqrt[a + x*(b + c*x)]*Log[b
```

```
+ 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(48*c^(9/2)*(-b^2 + 4*a*c)*Sqrt
[a + x*(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1365 vs. $2(484) = 968$.
time = 0.16, size = 1366, normalized size = 2.71

method	result	size
default	Expression too large to display	1366
risch	Expression too large to display	2575

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*h^3*(1/3*x^4/c/(c*x^2+b*x+a)^(1/2)-7/6*b/c*(1/2*x^3/c/(c*x^2+b*x+a)^(1/2)
-5/4*b/c*(x^2/c/(c*x^2+b*x+a)^(1/2)-3/2*b/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b
/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))
+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2*a/c*(-1/c/(c*x^2+
b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))-3/2*a/c*(-x/c/
(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-
b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))))-4/3*a/c*(x^2/c/(c*x^2+b*x+a)^(1/2)-3/2*b/c*(-x/c/(c*x^2+b*x+a)^(1/2)
-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2*a/c*(-1/c/(
c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))))+(e*h^3+
3*f*g*h^2)*(1/2*x^3/c/(c*x^2+b*x+a)^(1/2)-5/4*b/c*(x^2/c/(c*x^2+b*x+a)^(1/2
)-3/2*b/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(
2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+
(c*x^2+b*x+a)^(1/2)))-2*a/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-
b^2)/(c*x^2+b*x+a)^(1/2)))-3/2*a/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/
(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+(d*h^3+3*e*g*h^2+3*f*g^2*h)
*(x^2/c/(c*x^2+b*x+a)^(1/2)-3/2*b/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c
/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2*a/c*(-1/c/(c*x^2+b*x+a)^(
1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))+(3*d*g*h^2+3*e*g^2*h+f
*g^3)*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*
x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2)))+(3*d*g^2*h+e*g^3)*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)
/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+2*d*g^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+
a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1530 vs. $2(497) = 994$.
time = 7.40, size = 3063, normalized size = 6.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/96*(3*(16*(a*b^2*c^3 - 4*a^2*c^4)*f*g^3 - 72*(a*b^3*c^2 - 4*a^2*b*c^3)*
f*g^2*h + 6*(8*(a*b^2*c^3 - 4*a^2*c^4)*d + 3*(5*a*b^4*c - 24*a^2*b^2*c^2 +
16*a^3*c^3)*f)*g*h^2 - (24*(a*b^3*c^2 - 4*a^2*b*c^3)*d + 5*(7*a*b^5 - 40*a^
2*b^3*c + 48*a^3*b*c^2)*f)*h^3 + (16*(b^2*c^4 - 4*a*c^5)*f*g^3 - 72*(b^3*c^
3 - 4*a*b*c^4)*f*g^2*h + 6*(8*(b^2*c^4 - 4*a*c^5)*d + 3*(5*b^4*c^2 - 24*a*b
^2*c^3 + 16*a^2*c^4)*f)*g*h^2 - (24*(b^3*c^3 - 4*a*b*c^4)*d + 5*(7*b^5*c -
40*a*b^3*c^2 + 48*a^2*b*c^3)*f)*h^3)*x^2 + (16*(b^3*c^3 - 4*a*b*c^4)*f*g^3
- 72*(b^4*c^2 - 4*a*b^2*c^3)*f*g^2*h + 6*(8*(b^3*c^3 - 4*a*b*c^4)*d + 3*(5*
b^5*c - 24*a*b^3*c^2 + 16*a^2*b*c^3)*f)*g*h^2 - (24*(b^4*c^2 - 4*a*b^2*c^3)
*d + 5*(7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*f)*h^3)*x + 6*(8*(a*b^2*c^3 -
4*a^2*c^4)*g^2*h - 12*(a*b^3*c^2 - 4*a^2*b*c^3)*g*h^2 + (5*a*b^4*c - 24*a^2
*b^2*c^2 + 16*a^3*c^3)*h^3 + (8*(b^2*c^4 - 4*a*c^5)*g^2*h - 12*(b^3*c^3 - 4
*a*b*c^4)*g*h^2 + (5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*h^3)*x^2 + (8*(b^
3*c^3 - 4*a*b*c^4)*g^2*h - 12*(b^4*c^2 - 4*a*b^2*c^3)*g*h^2 + (5*b^5*c - 24
*a*b^3*c^2 + 16*a^2*b*c^3)*h^3)*x)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^
2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(8*(b^2*c^4 -
4*a*c^5)*f*h^3*x^4 - 48*(b*c^5*d + a*b*c^4*f)*g^3 + 72*(4*a*c^5*d + (3*a*b^
2*c^3 - 8*a^2*c^4)*f)*g^2*h - 18*(8*a*b*c^4*d + (15*a*b^3*c^2 - 52*a^2*b*c^
3)*f)*g*h^2 + (24*(3*a*b^2*c^3 - 8*a^2*c^4)*d + (105*a*b^4*c - 460*a^2*b^2*
c^2 + 256*a^3*c^3)*f)*h^3 + 2*(18*(b^2*c^4 - 4*a*c^5)*f*g*h^2 - 7*(b^3*c^3
- 4*a*b*c^4)*f*h^3)*x^3 + (72*(b^2*c^4 - 4*a*c^5)*f*g^2*h - 90*(b^3*c^3 - 4
*a*b*c^4)*f*g*h^2 + (24*(b^2*c^4 - 4*a*c^5)*d + (35*b^4*c^2 - 172*a*b^2*c^3
 + 128*a^2*c^4)*f)*h^3)*x^2 - (48*(2*c^6*d + (b^2*c^4 - 2*a*c^5)*f)*g^3 - 7
2*(2*b*c^5*d + (3*b^3*c^3 - 10*a*b*c^4)*f)*g^2*h + 18*(8*(b^2*c^4 - 2*a*c^5
)*d + (15*b^4*c^2 - 62*a*b^2*c^3 + 24*a^2*c^4)*f)*g*h^2 - (24*(3*b^3*c^3 -
10*a*b*c^4)*d + (105*b^5*c - 530*a*b^3*c^2 + 488*a^2*b*c^3)*f)*h^3)*x + 6*(
```

```
16*a*c^5*g^3 - 24*a*b*c^4*g^2*h + 2*(b^2*c^4 - 4*a*c^5)*h^3*x^3 + 12*(3*a*b
^2*c^3 - 8*a^2*c^4)*g*h^2 - (15*a*b^3*c^2 - 52*a^2*b*c^3)*h^3 + (12*(b^2*c^
4 - 4*a*c^5)*g*h^2 - 5*(b^3*c^3 - 4*a*b*c^4)*h^3)*x^2 + (8*b*c^5*g^3 - 24*(
b^2*c^4 - 2*a*c^5)*g^2*h + 12*(3*b^3*c^3 - 10*a*b*c^4)*g*h^2 - (15*b^4*c^2
- 62*a*b^2*c^3 + 24*a^2*c^4)*h^3)*x)*e)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^5 -
 4*a^2*c^6 + (b^2*c^6 - 4*a*c^7)*x^2 + (b^3*c^5 - 4*a*b*c^6)*x), -1/48*(3*(
16*(a*b^2*c^3 - 4*a^2*c^4)*f*g^3 - 72*(a*b^3*c^2 - 4*a^2*b*c^3)*f*g^2*h + 6
*(8*(a*b^2*c^3 - 4*a^2*c^4)*d + 3*(5*a*b^4*c - 24*a^2*b^2*c^2 + 16*a^3*c^3)
*f)*g*h^2 - (24*(a*b^3*c^2 - 4*a^2*b*c^3)*d + 5*(7*a*b^5 - 40*a^2*b^3*c + 4
8*a^3*b*c^2)*f)*h^3 + (16*(b^2*c^4 - 4*a*c^5)*f*g^3 - 72*(b^3*c^3 - 4*a*b*c
^4)*f*g^2*h + 6*(8*(b^2*c^4 - 4*a*c^5)*d + 3*(5*b^4*c^2 - 24*a*b^2*c^3 + 16
*a^2*c^4)*f)*g*h^2 - (24*(b^3*c^3 - 4*a*b*c^4)*d + 5*(7*b^5*c - 40*a*b^3*c^
2 + 48*a^2*b*c^3)*f)*h^3)*x^2 + (16*(b^3*c^3 - 4*a*b*c^4)*f*g^3 - 72*(b^4*c
^2 - 4*a*b^2*c^3)*f*g^2*h + 6*(8*(b^3*c^3 - 4*a*b*c^4)*d + 3*(5*b^5*c - 24*
a*b^3*c^2 + 16*a^2*b*c^3)*f)*g*h^2 - (24*(b^4*c^2 - 4*a*b^2*c^3)*d + 5*(7*b
^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*f)*h^3)*x + 6*(8*(a*b^2*c^3 - 4*a^2*c^4)*
g^2*h - 12*(a*b^3*c^2 - 4*a^2*b*c^3)*g*h^2 + (5*a*b^4*c - 24*a^2*b^2*c^2 +
16*a^3*c^3)*h^3 + (8*(b^2*c^4 - 4*a*c^5)*g^2*h - 12*(b^3*c^3 - 4*a*b*c^4)*g
*h^2 + (5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*h^3)*x^2 + (8*(b^3*c^3 - 4*a
*b*c^4)*g^2*h - 12*(b^4*c^2 - 4*a*b^2*c^3)*g*h^2 + (5*b^5*c - 24*a*b^3*c^2
+ 16*a^2*b*c^3)*h^3)*x)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(8*(b^2*c^4 - 4*a*c^5)*f*h^3*x^
4 - 48*(b*c^5*d + a*b*c^4*f)*g^3 + 72*(4*a*c^5*d + (3*a*b^2*c^3 - 8*a^2*c^4
)*f)*g^2*h - 18*(8*a*b*c^4*d + (15*a*b^3*c^2 - 52*a^2*b*c^3)*f)*g*h^2 + (24
*(3*a*b^2*c^3 - 8*a^2*c^4)*d + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3
)*f)*h^3 + 2*(18*(b^2*c^4 - 4*a*c^5)*f*g*h^2 - 7*(b^3*c^3 - 4*a*b*c^4)*f*h^
3)*x^3 + (72*(b^2*c^4 - 4*a*c^5)*f*g^2*h - 90*(b^3*c^3 - 4*a*b*c^4)*f*g*h^2
 + (24*(b^2*c^4 - 4*a*c^5)*d + (35*b^4*c^2 - 172*a*b^2*c^3 + 128*a^2*c^4)*f
)*h^3)*x^2 - (48*(2*c^6*d + (b^2*c^4 - 2*a*c^5)*f)*g^3 - 72*(2*b*c^5*d + (3
*b^3*c^3 - 10*a*b*c^4)*f)*g^2*h + 18*(8*(b^2*c^4 - 2*a*c^5)*d + (15*b^4*c^2
 - 62*a*b^2*c^3 + 24*a^2*c^4)*f)*g*h^2 - (24*(3*b^3*c^3 - 10*a*b*c^4)*d + (
105*b^5*c - 530*a*b^3*c^2 + 488*a^2*b*c^3)*f)*h^3)*x + 6*(16*a*c^5*g^3 - 24
*a*b*c^4*g^2*h + 2*(b^2*c^4 - 4*a*c^5)*h^3*x^3 + 12*(3*a*b^2*c^3 - 8*a^2*c^
4)*g*h^2 - (15*a*b^3*c^2 - 52*a^2*b*c^3)*h^3 + (12*(b^2*c^4 - 4*a*c^5)*g*h^
2 - 5*(b^3*c^3 - 4*a*b*c^4)*h^3)*x^2 + (8*b*c^5*g^3 - 24*(b^2*c^4 - 2*a*c^5
)*g^2*h + 12*(3*b^3*c^3 - 10*a*b*c^4)*g*h^2 - (15*b^4*c^2 - 62*a*b^2*c^3 +
24*a^2*c^4)*h^3)*x)*e)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^5 - 4*a^2*c^6 + (b^2
*c^6 - 4*a*c^7)*x^2 + (b^3*c^5 - 4*a*b*c^6)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g + hx)^3 (d + ex + fx^2)}{(a + bx + cx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**3*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1054 vs. $2(497) = 994$.
time = 3.21, size = 1054, normalized size = 2.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*(((2*(4*(b^2*c^3*f*h^3 - 4*a*c^4*f*h^3)*x/(b^2*c^4 - 4*a*c^5) + (18*b^
2*c^3*f*g*h^2 - 72*a*c^4*f*g*h^2 - 7*b^3*c^2*f*h^3 + 28*a*b*c^3*f*h^3 + 6*b
^2*c^3*h^3*e - 24*a*c^4*h^3*e)/(b^2*c^4 - 4*a*c^5))*x + (72*b^2*c^3*f*g^2*h
 - 288*a*c^4*f*g^2*h - 90*b^3*c^2*f*g*h^2 + 360*a*b*c^3*f*g*h^2 + 24*b^2*c^
3*d*h^3 - 96*a*c^4*d*h^3 + 35*b^4*c*f*h^3 - 172*a*b^2*c^2*f*h^3 + 128*a^2*c
^3*f*h^3 + 72*b^2*c^3*g*h^2*e - 288*a*c^4*g*h^2*e - 30*b^3*c^2*h^3*e + 120*
a*b*c^3*h^3*e)/(b^2*c^4 - 4*a*c^5))*x - (96*c^5*d*g^3 + 48*b^2*c^3*f*g^3 -
96*a*c^4*f*g^3 - 144*b*c^4*d*g^2*h - 216*b^3*c^2*f*g^2*h + 720*a*b*c^3*f*g^
2*h + 144*b^2*c^3*d*g*h^2 - 288*a*c^4*d*g*h^2 + 270*b^4*c*f*g*h^2 - 1116*a*
b^2*c^2*f*g*h^2 + 432*a^2*c^3*f*g*h^2 - 72*b^3*c^2*d*h^3 + 240*a*b*c^3*d*h^
3 - 105*b^5*f*h^3 + 530*a*b^3*c*f*h^3 - 488*a^2*b*c^2*f*h^3 - 48*b*c^4*g^3*
e + 144*b^2*c^3*g^2*h*e - 288*a*c^4*g^2*h*e - 216*b^3*c^2*g*h^2*e + 720*a*b
*c^3*g*h^2*e + 90*b^4*c*h^3*e - 372*a*b^2*c^2*h^3*e + 144*a^2*c^3*h^3*e)/(b
^2*c^4 - 4*a*c^5))*x - (48*b*c^4*d*g^3 + 48*a*b*c^3*f*g^3 - 288*a*c^4*d*g^2
*h - 216*a*b^2*c^2*f*g^2*h + 576*a^2*c^3*f*g^2*h + 144*a*b*c^3*d*g*h^2 + 27
0*a*b^3*c*f*g*h^2 - 936*a^2*b*c^2*f*g*h^2 - 72*a*b^2*c^2*d*h^3 + 192*a^2*c^
3*d*h^3 - 105*a*b^4*f*h^3 + 460*a^2*b^2*c*f*h^3 - 256*a^3*c^2*f*h^3 - 96*a*
c^4*g^3*e + 144*a*b*c^3*g^2*h*e - 216*a*b^2*c^2*g*h^2*e + 576*a^2*c^3*g*h^2
*e + 90*a*b^3*c*h^3*e - 312*a^2*b*c^2*h^3*e)/(b^2*c^4 - 4*a*c^5))/sqrt(c*x^
2 + b*x + a) - 1/16*(16*c^3*f*g^3 - 72*b*c^2*f*g^2*h + 48*c^3*d*g*h^2 + 90*
b^2*c*f*g*h^2 - 72*a*c^2*f*g*h^2 - 24*b*c^2*d*h^3 - 35*b^3*f*h^3 + 60*a*b*c
*f*h^3 + 48*c^3*g^2*h*e - 72*b*c^2*g*h^2*e + 30*b^2*c*h^3*e - 24*a*c^2*h^3*
e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g+h\,x\right)}^3\,\left(f\,x^2+e\,x+d\right)}{{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

3.234 $\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=289

$$\frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{h\left(32c^3dg-15b^3fh-8c^2(2beg+8afg+\right.}{}$$

```
[Out] 1/8*(15*b^2*f*h^2-12*c*h*(a*f*h+b*e*h+2*b*f*g)+8*c^2*(f*g^2+h*(d*h+2*e*g)))
*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+2*(c*(2*a*e-b*(
d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)*(h*x+g)^2/c/(-4*a*c+b^2)/(c*x^2
+b*x+a)^(1/2)+1/4*h*(32*c^3*d*g-15*b^3*f*h-8*c^2*(4*a*e*h+8*a*f*g+b*d*h+2*b
*e*g)+4*b*c*(13*a*f*h+3*b*e*h+6*b*f*g)+2*c*(-12*a*c*f+5*b^2*f-4*b*c*e+8*c^2
*d)*h*x)*(c*x^2+b*x+a)^(1/2)/c^3/(-4*a*c+b^2)
```

Rubi [A]

time = 0.23, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1658, 793, 635, 212}

$$\frac{2(g+hx)^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-12ch(afh+beh+2bfg)+15b^2fh^2+8c^2\left(h(dh+2eg)+fg^2\right)\right)}{8c^{7/2}}+\frac{h\sqrt{a+bx+cx^2}\left(2hx\left(-12acf+5b^2f-4bce+8c^2d\right)-8c(4aeh+8afg+bdh+2beg)+4b(13afh+3beh+6bfg)-\frac{15b^3fh}{c}+32c^2dg\right)}{4c^2\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x)^2)/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (h*(32*c^2*d*g - (15*b
^3*f*h)/c - 8*c*(2*b*e*g + 8*a*f*g + b*d*h + 4*a*e*h) + 4*b*(6*b*f*g + 3*b*
e*h + 13*a*f*h) + 2*(8*c^2*d - 4*b*c*e + 5*b^2*f - 12*a*c*f)*h*x)*Sqrt[a +
b*x + c*x^2])/(4*c^2*(b^2 - 4*a*c)) + ((15*b^2*f*h^2 - 12*c*h*(2*b*f*g + b*
e*h + a*f*h) + 8*c^2*(f*g^2 + h*(2*e*g + d*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\,(b^2-4ac)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{(g+\cdots}{}}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\,(b^2-4ac)\sqrt{a+bx+cx^2}} + \frac{h\left(32c^2\cdots\right.}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\,(b^2-4ac)\sqrt{a+bx+cx^2}} + \frac{h\left(32c^2\cdots\right.}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\,(b^2-4ac)\sqrt{a+bx+cx^2}} + \frac{h\left(32c^2\cdots\right.}{}$$

Mathematica [A]
time = 2.31, size = 390, normalized size = 1.35

$$\frac{-\frac{2\sqrt{c}\left(15b^4fh^2x+b^3h(15afh+cx(-24fg-12eh+5fhx))+4bc\left(-13a^2fh^2+2c^2g(-egx+d(g-2hx))+ac\left(2h(2eg+dh+5ehx)+f\left(2g^2+20ghx-5h^2x^2\right)\right)\right)-2b^2c\left(ah(12fg+6eh+31fhx)+cx\left(2h(-4eg-2dh+ehx)+f\left(-4g^2+4ghx+h^2x^2\right)\right)\right)+8c^2\left(2c^2dg^2x+a^2h(8fg+4eh+3fhx)+ac\left(-2dh(2g+hx)-2e\left(g^2+2ghx-h^2x^2\right)+fx\left(-2g^2+4ghx+h^2x^2\right)\right)\right)\right)}{(b^2-4ac)\sqrt{a+x(b+cx)}}+(-15b^2fh^2+12ch(2bfg+beh+afh)-8c^2(fg^2+h(2eg+dh)))\log\left(c^3\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{8c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((-2*Sqrt[c]*(15*b^4*f*h^2*x + b^3*h*(15*a*f*h + c*x*(-24*f*g - 12*e*h + 5*
f*h*x)) + 4*b*c*(-13*a^2*f*h^2 + 2*c^2*g*(-(e*g*x) + d*(g - 2*h*x)) + a*c*(
2*h*(2*e*g + d*h + 5*e*h*x) + f*(2*g^2 + 20*g*h*x - 5*h^2*x^2))) - 2*b^2*c*
(a*h*(12*f*g + 6*e*h + 31*f*h*x) + c*x*(2*h*(-4*e*g - 2*d*h + e*h*x) + f*(-
4*g^2 + 4*g*h*x + h^2*x^2))) + 8*c^2*(2*c^2*d*g^2*x + a^2*h*(8*f*g + 4*e*h
+ 3*f*h*x) + a*c*(-2*d*h*(2*g + h*x) - 2*e*(g^2 + 2*g*h*x - h^2*x^2) + f*x*
(-2*g^2 + 4*g*h*x + h^2*x^2)))))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) + (-
15*b^2*f*h^2 + 12*c*h*(2*b*f*g + b*e*h + a*f*h) - 8*c^2*(f*g^2 + h*(2*e*g +
 d*h)))*Log[c^3*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(8*c^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(273) = 546$.
time = 0.16, size = 775, normalized size = 2.68

method	result
default	$f h^2 \left(\frac{x^3}{2c\sqrt{c x^2+bx+a}} - \frac{5b\left(\frac{x^2}{c\sqrt{c x^2+bx+a}} - \frac{3b\left(-\frac{x}{c\sqrt{c x^2+bx+a}} - \frac{b\left(-\frac{1}{c\sqrt{c x^2+bx+a}} - \frac{}{c(4c}\right)}{2c}\right.}{}\right.}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*h^2*(1/2*x^3/c/(c*x^2+b*x+a)^(1/2)-5/4*b/c*(x^2/c/(c*x^2+b*x+a)^(1/2)-3/2
*b/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x
+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^
2+b*x+a)^(1/2)))-2*a/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/
(c*x^2+b*x+a)^(1/2)))-3/2*a/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^
```

```
2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln(
(1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+(e*h^2+2*f*g*h)*(x^2/c/(c*x^2+b*
x+a)^(1/2)-3/2*b/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1
/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2*a/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b
)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))+(d*h^2+2*e*g*h+f*g^2)*(-x/c/(c*x^2+b*x+
a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(2*d*
g*h+e*g^2)*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2))+2*d*g^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 914 vs. $2(279) = 558$.
time = 6.18, size = 1831, normalized size = 6.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/16*((8*(a*b^2*c^2 - 4*a^2*c^3)*f*g^2 - 24*(a*b^3*c - 4*a^2*b*c^2)*f*g*h
 + (8*(a*b^2*c^2 - 4*a^2*c^3)*d + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*f
)*h^2 + (8*(b^2*c^3 - 4*a*c^4)*f*g^2 - 24*(b^3*c^2 - 4*a*b*c^3)*f*g*h + (8*
(b^2*c^3 - 4*a*c^4)*d + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*h^2)*x^2
 + (8*(b^3*c^2 - 4*a*b*c^3)*f*g^2 - 24*(b^4*c - 4*a*b^2*c^2)*f*g*h + (8*(b^
3*c^2 - 4*a*b*c^3)*d + 3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*f)*h^2)*x + 4*
(4*(a*b^2*c^2 - 4*a^2*c^3)*g*h - 3*(a*b^3*c - 4*a^2*b*c^2)*h^2 + (4*(b^2*c^
3 - 4*a*c^4)*g*h - 3*(b^3*c^2 - 4*a*b*c^3)*h^2)*x^2 + (4*(b^3*c^2 - 4*a*b*c
^3)*g*h - 3*(b^4*c - 4*a*b^2*c^2)*h^2)*x)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c
*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*(b^2
*c^3 - 4*a*c^4)*f*h^2*x^3 - 8*(b*c^4*d + a*b*c^3*f)*g^2 + 8*(4*a*c^4*d + (3
```

```
*a*b^2*c^2 - 8*a^2*c^3)*f)*g*h - (8*a*b*c^3*d + (15*a*b^3*c - 52*a^2*b*c^2)
*f)*h^2 + (8*(b^2*c^3 - 4*a*c^4)*f*g*h - 5*(b^3*c^2 - 4*a*b*c^3)*f*h^2)*x^2
 - (8*(2*c^5*d + (b^2*c^3 - 2*a*c^4)*f)*g^2 - 8*(2*b*c^4*d + (3*b^3*c^2 - 1
0*a*b*c^3)*f)*g*h + (8*(b^2*c^3 - 2*a*c^4)*d + (15*b^4*c - 62*a*b^2*c^2 + 2
4*a^2*c^3)*f)*h^2)*x + 4*(4*a*c^4*g^2 - 4*a*b*c^3*g*h + (b^2*c^3 - 4*a*c^4)
*h^2*x^2 + (3*a*b^2*c^2 - 8*a^2*c^3)*h^2 + (2*b*c^4*g^2 - 4*(b^2*c^3 - 2*a*
c^4)*g*h + (3*b^3*c^2 - 10*a*b*c^3)*h^2)*x)*e)*sqrt(c*x^2 + b*x + a))/(a*b^
2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x), -1/
8*((8*(a*b^2*c^2 - 4*a^2*c^3)*f*g^2 - 24*(a*b^3*c - 4*a^2*b*c^2)*f*g*h + (8
*(a*b^2*c^2 - 4*a^2*c^3)*d + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*f)*h^2
 + (8*(b^2*c^3 - 4*a*c^4)*f*g^2 - 24*(b^3*c^2 - 4*a*b*c^3)*f*g*h + (8*(b^2*
c^3 - 4*a*c^4)*d + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*h^2)*x^2 + (8
*(b^3*c^2 - 4*a*b*c^3)*f*g^2 - 24*(b^4*c - 4*a*b^2*c^2)*f*g*h + (8*(b^3*c^2
 - 4*a*b*c^3)*d + 3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*f)*h^2)*x + 4*(4*(a
*b^2*c^2 - 4*a^2*c^3)*g*h - 3*(a*b^3*c - 4*a^2*b*c^2)*h^2 + (4*(b^2*c^3 - 4
*a*c^4)*g*h - 3*(b^3*c^2 - 4*a*b*c^3)*h^2)*x^2 + (4*(b^3*c^2 - 4*a*b*c^3)*g
*h - 3*(b^4*c - 4*a*b^2*c^2)*h^2)*x)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*
x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*(b^2*c^3 - 4*a*
c^4)*f*h^2*x^3 - 8*(b*c^4*d + a*b*c^3*f)*g^2 + 8*(4*a*c^4*d + (3*a*b^2*c^2
- 8*a^2*c^3)*f)*g*h - (8*a*b*c^3*d + (15*a*b^3*c - 52*a^2*b*c^2)*f)*h^2 + (
8*(b^2*c^3 - 4*a*c^4)*f*g*h - 5*(b^3*c^2 - 4*a*b*c^3)*f*h^2)*x^2 - (8*(2*c^
5*d + (b^2*c^3 - 2*a*c^4)*f)*g^2 - 8*(2*b*c^4*d + (3*b^3*c^2 - 10*a*b*c^3)*
f)*g*h + (8*(b^2*c^3 - 2*a*c^4)*d + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*
f)*h^2)*x + 4*(4*a*c^4*g^2 - 4*a*b*c^3*g*h + (b^2*c^3 - 4*a*c^4)*h^2*x^2 +
(3*a*b^2*c^2 - 8*a^2*c^3)*h^2 + (2*b*c^4*g^2 - 4*(b^2*c^3 - 2*a*c^4)*g*h +
(3*b^3*c^2 - 10*a*b*c^3)*h^2)*x)*e)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a
^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**2*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(279) = 558.
time = 4.29, size = 580, normalized size = 2.01

$$\frac{\left(\left(\left(\frac{2\,(b^2c^2fh^2-4\,ac^3fh^2)x}{b^2c^3-4\,ac^4}+\frac{8\,b^2c^2fgh-32\,ac^3fgh-5\,b^3cfh^2+20\,abc^2fh^2+4\,b^2c^2h^2e-16\,ac^3h^2e}{b^2c^3-4\,ac^4}\right)x-\frac{16\,c^4dg^2+8\,b^2c^2fg^2-16\,ac^3fg^2-16\,bc^3dgh-24\,b^3cfgh+80\,abc^2fgh+8\,b^2c^2dh^2-16\,ac^3dh^2+15\,b^4fh^2-62\,ab^2cfh^2+24\,a^2c^2fh^2-8\,bc^3g^2e+16\,b^2c^2ghe-32\,ac^3ghe-12\,b^3ch^2e+40\,abc^2h^2e}{b^2c^3-4\,ac^4}\right)x-\frac{8\,bc^3dg^2+8\,abc^2fg^2-32\,ac^3dgh-24\,ab^2cfgh+64\,a^2c^2fgh+8\,abc^2dh^2+15\,ab^3fh^2-52\,a^2bcfh^2-16\,ac^3g^2e+16\,abc^2ghe-12\,ab^2ch^2e+32\,a^2c^2h^2e}{b^2c^3-4\,ac^4}\right)}{4\,\sqrt{cx^2+bx+a}}-\frac{(8\,c^2fg^2-24\,bcfgh+8\,c^2dh^2+15\,b^2fh^2-12\,acfh^2+16\,c^2ghe-12\,bch^2e)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{8\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(((2*(b^2*c^2*f*h^2 - 4*a*c^3*f*h^2)*x/(b^2*c^3 - 4*a*c^4) + (8*b^2*c^2
*f*g*h - 32*a*c^3*f*g*h - 5*b^3*c*f*h^2 + 20*a*b*c^2*f*h^2 + 4*b^2*c^2*h^2*
e - 16*a*c^3*h^2*e)/(b^2*c^3 - 4*a*c^4))*x - (16*c^4*d*g^2 + 8*b^2*c^2*f*g^
2 - 16*a*c^3*f*g^2 - 16*b*c^3*d*g*h - 24*b^3*c*f*g*h + 80*a*b*c^2*f*g*h + 8
*b^2*c^2*d*h^2 - 16*a*c^3*d*h^2 + 15*b^4*f*h^2 - 62*a*b^2*c*f*h^2 + 24*a^2*
c^2*f*h^2 - 8*b*c^3*g^2*e + 16*b^2*c^2*g*h*e - 32*a*c^3*g*h*e - 12*b^3*c*h^
2*e + 40*a*b*c^2*h^2*e)/(b^2*c^3 - 4*a*c^4))*x - (8*b*c^3*d*g^2 + 8*a*b*c^2
*f*g^2 - 32*a*c^3*d*g*h - 24*a*b^2*c*f*g*h + 64*a^2*c^2*f*g*h + 8*a*b*c^2*d
*h^2 + 15*a*b^3*f*h^2 - 52*a^2*b*c*f*h^2 - 16*a*c^3*g^2*e + 16*a*b*c^2*g*h*
e - 12*a*b^2*c*h^2*e + 32*a^2*c^2*h^2*e)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^2 +
b*x + a) - 1/8*(8*c^2*f*g^2 - 24*b*c*f*g*h + 8*c^2*d*h^2 + 15*b^2*f*h^2 - 1
2*a*c*f*h^2 + 16*c^2*g*h*e - 12*b*c*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g+hx)^2\,(f\,x^2+e\,x+d)}{(c\,x^2+b\,x+a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

3.235 $\int \frac{(g+hx)(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=186

$$\frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{\left(4c^2d-2bce+3b^2f-8acf\right)h\sqrt{a+bx+}}{c^2\left(b^2-4ac\right)}$$

```
[Out] -1/2*(3*b*f*h-2*c*(e*h+f*g))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/c^(5/2)+2*(c*(2*a*e-b*(d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)*(h*
x+g)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)+(4*c^2*d+3*b^2*f-2*c*(4*a*f+b*e))*h
*(c*x^2+b*x+a)^(1/2)/c^2/(-4*a*c+b^2)
```

Rubi [A]
time = 0.14, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1658, 654, 635, 212}

$$\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{h\sqrt{a+bx+cx^2}\left(-8acf+3b^2f-2bce+4c^2d\right)}{c^2\left(b^2-4ac\right)}-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(3bfh-2c(eh+fg))}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x))/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + ((4*c^2*d - 2*b*c*e + 3*
b^2*f - 8*a*c*f)*h*Sqrt[a + b*x + c*x^2])/(c^2*(b^2 - 4*a*c)) - ((3*b*f*h -
 2*c*(f*g + e*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
2*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{-\frac{b^2fg\cdots}{}}{}}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(4c^2d+3\cdots\right.}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(4c^2d+3\cdots\right.}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(4c^2d+3\cdots\right.}{}$$

Mathematica [A]
time = 0.92, size = 197, normalized size = 1.06

$$\frac{-3b^3fhx+2bc(aeh-cegx+cd(g-hx)+af(g+5hx))+b^2(-3afh+cx(2fg+2eh-fhx))+4c(2a^2fh+c^2dgx-ac(dh+fx(g-hx)+e(g+hx)))}{c^2\left(-b^2+4ac\right)\sqrt{a+x(b+cx)}}+\frac{(3bfh-2c(fg+eh))\log\left(c^2\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

[Out] (-3*b^3*f*h*x + 2*b*c*(a*e*h - c*e*g*x + c*d*(g - h*x) + a*f*(g + 5*h*x)) + b^2*(-3*a*f*h + c*x*(2*f*g + 2*e*h - f*h*x)) + 4*c*(2*a^2*f*h + c^2*d*g*x - a*c*(d*h + f*x*(g - h*x) + e*(g + h*x))))/(c^2*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)]) + ((3*b*f*h - 2*c*(f*g + e*h))*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(2*c^(5/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(173) = 346$.
time = 0.14, size = 411, normalized size = 2.21

method	result
default	$fh\left(\frac{x^2}{c\sqrt{cx^2+bx+a}} - \frac{3b\left(-\frac{x}{c\sqrt{cx^2+bx+a}} - \frac{b\left(-\frac{1}{c\sqrt{cx^2+bx+a}} - \frac{b(2cx+b)}{c(4ac-b^2)\sqrt{cx^2+bx+a}}\right)}{2c} + \right.}{2c}\right.$
risch	$\frac{4cdgx}{(4ac-b^2)\sqrt{cx^2+bx+a}} - \frac{b^2dh}{c(4ac-b^2)\sqrt{cx^2+bx+a}} - \frac{b^2eg}{c(4ac-b^2)\sqrt{cx^2+bx+a}} + \frac{3xbfh}{2c^2\sqrt{cx^2+bx+}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] f*h*(x^2/c/(c*x^2+b*x+a)^(1/2)-3/2*b/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2*a/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))+(e*h+f*g)*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(d*h+e*g)*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+2*d*g*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(175) = 350$.
time = 4.10, size = 919, normalized size = 4.94

$$\left[\frac{\left(2\,{\left(a b^{2} c - 4\,a^{2} c^{2}\right)} f g - 3\,{\left(a b^{3} - 4\,a^{2} b c\right)} f h + {\left(2\,{\left(b^{2} c^{2} - 4\,a c^{3}\right)} f g - 3\,{\left(b^{3} c - 4\,a b c^{2}\right)} f h\right)} x^{2} + {\left(2\,{\left(b^{3} c - 4\,a b c^{2}\right)} f g - 3\,{\left(b^{4} - 4\,a b^{2} c\right)} f h\right)} x + 2\,{\left({\left(b^{2} c^{2} - 4\,a c^{3}\right)} h x^{2} + {\left(b^{3} c - 4\,a b c^{2}\right)} h x + {\left(a b^{2} c - 4\,a^{2} c^{2}\right)} h\right)} e\right)} \sqrt{c} \log\left(-8\,c^{2} x^{2} - 8\,b c x - b^{2} - 4\,\sqrt{c x^{2} + b x + a} {\left(2\,c x + b\right)} \sqrt{c} - 4\,a c\right) + 4\,{\left({\left(b^{2} c^{2} - 4\,a c^{3}\right)} f h x^{2} - 2\,{\left(b c^{3} d + a b c^{2} f\right)} g + {\left(4\,a c^{3} d + {\left(3\,a b^{2} c - 8\,a^{2} c^{2}\right)} f\right)} h - {\left(2\,{\left(2\,c^{4} d + {\left(b^{2} c^{2} - 2\,a c^{3}\right)} f\right)} g - {\left(2\,b c^{3} d + {\left(3\,b^{3} c - 10\,a b c^{2}\right)} f\right)} h\right)} x + 2\,{\left(2\,a c^{3} g - a b c^{2} h + {\left(b c^{3} g - {\left(b^{2} c^{2} - 2\,a c^{3}\right)} h\right)} x\right)} e\right)} \sqrt{c x^{2} + b x + a}}{4\,{\left(a b^{2} c^{3} - 4\,a^{2} c^{4} + {\left(b^{2} c^{4} - 4\,a c^{5}\right)} x^{2} + {\left(b^{3} c^{3} - 4\,a b c^{4}\right)} x\right)}}, -\frac{{\left(2\,{\left(a b^{2} c - 4\,a^{2} c^{2}\right)} f g - 3\,{\left(a b^{3} - 4\,a^{2} b c\right)} f h + {\left(2\,{\left(b^{2} c^{2} - 4\,a c^{3}\right)} f g - 3\,{\left(b^{3} c - 4\,a b c^{2}\right)} f h\right)} x^{2} + {\left(2\,{\left(b^{3} c - 4\,a b c^{2}\right)} f g - 3\,{\left(b^{4} - 4\,a b^{2} c\right)} f h\right)} x + 2\,{\left({\left(b^{2} c^{2} - 4\,a c^{3}\right)} h x^{2} + {\left(b^{3} c - 4\,a b c^{2}\right)} h x + {\left(a b^{2} c - 4\,a^{2} c^{2}\right)} h\right)} e\right)} \sqrt{-c} \arctan\left(\frac{\sqrt{c x^{2} + b x + a} {\left(2\,c x + b\right)} \sqrt{-c}}{2\,{\left(c^{2} x^{2} + b c x + a c\right)}}\right) - 2\,{\left({\left(b^{2} c^{2} - 4\,a c^{3}\right)} f h x^{2} - 2\,{\left(b c^{3} d + a b c^{2} f\right)} g + {\left(4\,a c^{3} d + {\left(3\,a b^{2} c - 8\,a^{2} c^{2}\right)} f\right)} h - {\left(2\,{\left(2\,c^{4} d + {\left(b^{2} c^{2} - 2\,a c^{3}\right)} f\right)} g - {\left(2\,b c^{3} d + {\left(3\,b^{3} c - 10\,a b c^{2}\right)} f\right)} h\right)} x + 2\,{\left(2\,a c^{3} g - a b c^{2} h + {\left(b c^{3} g - {\left(b^{2} c^{2} - 2\,a c^{3}\right)} h\right)} x\right)} e\right)} \sqrt{c x^{2} + b x + a}}{2\,{\left(a b^{2} c^{3} - 4\,a^{2} c^{4} + {\left(b^{2} c^{4} - 4\,a c^{5}\right)} x^{2} + {\left(b^{3} c^{3} - 4\,a b c^{4}\right)} x\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*(a*b^2*c - 4*a^2*c^2)*f*g - 3*(a*b^3 - 4*a^2*b*c)*f*h + (2*(b^2*c^
2 - 4*a*c^3)*f*g - 3*(b^3*c - 4*a*b*c^2)*f*h)*x^2 + (2*(b^3*c - 4*a*b*c^2)*
f*g - 3*(b^4 - 4*a*b^2*c)*f*h)*x + 2*((b^2*c^2 - 4*a*c^3)*h*x^2 + (b^3*c -
4*a*b*c^2)*h*x + (a*b^2*c - 4*a^2*c^2)*h)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c
*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*((b^2*c
^2 - 4*a*c^3)*f*h*x^2 - 2*(b*c^3*d + a*b*c^2*f)*g + (4*a*c^3*d + (3*a*b^2*c
 - 8*a^2*c^2)*f)*h - (2*(2*c^4*d + (b^2*c^2 - 2*a*c^3)*f)*g - (2*b*c^3*d +
(3*b^3*c - 10*a*b*c^2)*f)*h)*x + 2*(2*a*c^3*g - a*b*c^2*h + (b*c^3*g - (b^2
*c^2 - 2*a*c^3)*h)*x)*e)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^3 - 4*a^2*c^4 + (b
^2*c^4 - 4*a*c^5)*x^2 + (b^3*c^3 - 4*a*b*c^4)*x), -1/2*((2*(a*b^2*c - 4*a^2
*c^2)*f*g - 3*(a*b^3 - 4*a^2*b*c)*f*h + (2*(b^2*c^2 - 4*a*c^3)*f*g - 3*(b^3
*c - 4*a*b*c^2)*f*h)*x^2 + (2*(b^3*c - 4*a*b*c^2)*f*g - 3*(b^4 - 4*a*b^2*c)
*f*h)*x + 2*((b^2*c^2 - 4*a*c^3)*h*x^2 + (b^3*c - 4*a*b*c^2)*h*x + (a*b^2*c
 - 4*a^2*c^2)*h)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*s
qrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*((b^2*c^2 - 4*a*c^3)*f*h*x^2 - 2*(b*c^
3*d + a*b*c^2*f)*g + (4*a*c^3*d + (3*a*b^2*c - 8*a^2*c^2)*f)*h - (2*(2*c^4*
d + (b^2*c^2 - 2*a*c^3)*f)*g - (2*b*c^3*d + (3*b^3*c - 10*a*b*c^2)*f)*h)*x
+ 2*(2*a*c^3*g - a*b*c^2*h + (b*c^3*g - (b^2*c^2 - 2*a*c^3)*h)*x)*e)*sqrt(c
*x^2 + b*x + a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^2 + (b^3*c^
3 - 4*a*b*c^4)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g + hx\right)\left(d + ex + fx^{2}\right)}{\left(a + bx + cx^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Giac [A]
time = 4.52, size = 271, normalized size = 1.46

$$\frac{\left(\frac{(b^2cfh-4ac^2fh)x}{b^2c^2-4ac^3} - \frac{4c^3dg+2b^2cfg-4ac^2fg-2bc^2dh-3b^3fh+10abcfh-2bc^2ge+2b^2che-4ac^2he}{b^2c^2-4ac^3}\right)x - \frac{2bc^2dg+2abcfg-4ac^2dh-3ab^2fh+8a^2cfh-4ac^2ge+2abche}{b^2c^2-4ac^3}}{\sqrt{cx^2+bx+a}} - \frac{(2cfg-3bfh+2che)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{2c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] (((b^2*c*f*h - 4*a*c^2*f*h)*x/(b^2*c^2 - 4*a*c^3) - (4*c^3*d*g + 2*b^2*c*f*
g - 4*a*c^2*f*g - 2*b*c^2*d*h - 3*b^3*f*h + 10*a*b*c*f*h - 2*b*c^2*g*e + 2*
b^2*c*h*e - 4*a*c^2*h*e)/(b^2*c^2 - 4*a*c^3))*x - (2*b*c^2*d*g + 2*a*b*c*f*
g - 4*a*c^2*d*h - 3*a*b^2*f*h + 8*a^2*c*f*h - 4*a*c^2*g*e + 2*a*b*c*h*e)/(b
^2*c^2 - 4*a*c^3))/sqrt(c*x^2 + b*x + a) - 1/2*(2*c*f*g - 3*b*f*h + 2*c*h*e
)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g+h\,x)\,\left(f\,x^2+e\,x+d\right)}{{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

3.236 $\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

```
[Out] f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)+2*(c*(2*a*e-b*
(d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(
1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1674, 12, 635, 212}

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(
c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-(2c^2d-bce+b^2f-2acf)\,x\right)}{c\,(b^2-4ac)\sqrt{a+bx+cx^2}} - \frac{2\int -\frac{(b^2-4ac)f}{2c\sqrt{a+bx+cx^2}}}{b^2-4ac}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-(2c^2d-bce+b^2f-2acf)\,x\right)}{c\,(b^2-4ac)\sqrt{a+bx+cx^2}} + \frac{f\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{c}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-(2c^2d-bce+b^2f-2acf)\,x\right)}{c\,(b^2-4ac)\sqrt{a+bx+cx^2}} + \frac{(2f)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,\right.}{c}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-(2c^2d-bce+b^2f-2acf)\,x\right)}{c\,(b^2-4ac)\sqrt{a+bx+cx^2}} + \frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+b}}\right.}{c^{3/2}}$$

Mathematica [**A**]
time = 0.10, size = 114, normalized size = 1.03

$$\frac{\frac{2\sqrt{c}\,(abf+2c^2dx+b^2fx+bc(d-ex)-2ac(e+fx))}{\sqrt{a+x(b+cx)}}+(b^2-4ac)\,f\log\left(c\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)}{c^{3/2}\,(-b^2+4ac)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
[Out] ((2*Sqrt[c]*(a*b*f + 2*c^2*d*x + b^2*f*x + b*c*(d - e*x) - 2*a*c*(e + f*x))
)/Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*f*Log[c*(b + 2*c*x - 2*Sqrt[c]*Sqrt
[a + x*(b + c*x)])])/(c^(3/2)*(-b^2 + 4*a*c))
```

Maple [**A**]
time = 0.00, size = 201, normalized size = 1.81

method	result
default	$f\left(-\frac{x}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx-}\right.}{c^{\frac{3}{2}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)
/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2)))+e*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+
b*x+a)^(1/2))+2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(101) = 202$.
time = 0.62, size = 435, normalized size = 3.92

$$\left[\frac{\left((b^{2}c-4\,ac^{2})fx^{2}+(b^{3}-4\,abc)fx+(ab^{2}-4\,a^{2}c)f\right)\sqrt{c}\,\log\left(-8\,c^{2}x^{2}-8\,bcx-b^{2}-4\,\sqrt{cx^{2}+bx+a}\,(2\,cx+b)\sqrt{c}-4\,ac\right)-4\left(bc^{2}d+abcf+(2\,c^{3}d+(b^{2}c-2\,ac^{2})f)x-(bc^{2}x+2\,ac^{2})e\right)\sqrt{cx^{2}+bx+a}}{2\left(ab^{2}c^{2}-4\,a^{2}c^{3}+(b^{2}c^{3}-4\,ac^{4})x^{2}+(b^{3}c^{2}-4\,abc^{3})x\right)},\ -\frac{\left((b^{2}c-4\,ac^{2})fx^{2}+(b^{3}-4\,abc)fx+(ab^{2}-4\,a^{2}c)f\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{2}+bx+a}\,(2\,cx+b)\sqrt{-c}}{2\,(c^{2}x^{2}+bcx+ac)}\right)+2\left(bc^{2}d+abcf+(2\,c^{3}d+(b^{2}c-2\,ac^{2})f)x-(bc^{2}x+2\,ac^{2})e\right)\sqrt{cx^{2}+bx+a}}{ab^{2}c^{2}-4\,a^{2}c^{3}+(b^{2}c^{3}-4\,ac^{4})x^{2}+(b^{3}c^{2}-4\,abc^{3})x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)
*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x +
b)*sqrt(c) - 4*a*c) - 4*(b*c^2*d + a*b*c*f + (2*c^3*d + (b^2*c - 2*a*c^2)*f
)*x - (b*c^2*x + 2*a*c^2)*e)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3
+ (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -(((b^2*c - 4*a*c^2)*
f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)*sqrt(-c)*arctan(1/2*sqrt
(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(b*c^2*
d + a*b*c*f + (2*c^3*d + (b^2*c - 2*a*c^2)*f)*x - (b*c^2*x + 2*a*c^2)*e)*sq
```

```
rt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^
3*c^2 - 4*a*b*c^3)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Giac [A]
time = 4.36, size = 122, normalized size = 1.10

$$-\frac{2\left(\frac{(2\,c^2d+b^2f-2\,acf-bce)x}{b^2c-4\,ac^2}+\frac{bcd+abf-2\,ace}{b^2c-4\,ac^2}\right)}{\sqrt{cx^2+bx+a}}-\frac{f\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^2*d + b^2*f - 2*a*c*f - b*c*e)*x/(b^2*c - 4*a*c^2) + (b*c*d + a*b*
f - 2*a*c*e)/(b^2*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - f*log(abs(-2*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2)
```

Mupad [B]
time = 4.54, size = 143, normalized size = 1.29

$$\frac{f\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{c^{3/2}}-\frac{e\,(4\,a+2\,b\,x)}{(4\,a\,c-b^2)\,\sqrt{c\,x^2+b\,x+a}}+\frac{d\,\left(\frac{b}{2}+c\,x\right)}{\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}+\frac{f\,\left(\frac{a\,b}{2}-x\,\left(a\,c-\frac{b^2}{2}\right)\right)}{c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] (f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) - (e*(4*a +
2*b*x))/((4*a*c - b^2)*(a + b*x + c*x^2)^(1/2)) + (d*(b/2 + c*x))/((a*c - b
^2/4)*(a + b*x + c*x^2)^(1/2)) + (f*((a*b)/2 - x*(a*c - b^2/2)))/(c*(a*c -
b^2/4)*(a + b*x + c*x^2)^(1/2))
```

3.237 $\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=225

$$\frac{2(b^2dh - b(cdg + afg + aeh) + 2a(ceg - cdh + afh) - (2c^2dg + bf(bg - ah) - c(beg + 2afg + bdh - 2aeh)}{(b^2 - 4ac)\left(cg^2 - bgh + ah^2\right)\sqrt{a + bx + cx^2}}$$

```
[Out] (f*g^2-h*(-d*h+e*g))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*
g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(3/2)+2*(b^2*d*h-b*(a*e
*h+a*f*g+c*d*g)+2*a*(a*f*h-c*d*h+c*e*g)-(2*c^2*d*g+b*f*(-a*h+b*g)-c*(-2*a*e
*h+2*a*f*g+b*d*h+b*e*g))*x)/(-4*a*c+b^2)/(a*h^2-b*g*h+c*g^2)/(c*x^2+b*x+a)^
(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1660, 12, 738, 212}

$$\frac{2(-x(-c(-2aeh+2afg+bdh+beg)+bf(bg-ah)+2c^2dg)-b(aeh+afg+cdg)+2a(afh-cdh+ceg)+b^2dh)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(ah^2-bgh+cg^2)}+\frac{(fg^2-h(eg-dh))\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{(ah^2-bgh+cg^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^2*d*h - b*(c*d*g + a*f*g + a*e*h) + 2*a*(c*e*g - c*d*h + a*f*h) - (2*
c^2*d*g + b*f*(b*g - a*h) - c*(b*e*g + 2*a*f*g + b*d*h - 2*a*e*h))*x))/((b^
2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)*Sqrt[a + b*x + c*x^2]) + ((f*g^2 - h*(e*
g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a
*h^2]*Sqrt[a + b*x + c*x^2])])/(c*g^2 - b*g*h + a*h^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-(2c^2dg+bf(bg}{\left(b^2-4ac\right)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx}}$$

$$= \frac{2(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-(2c^2dg+bf(bg}{\left(b^2-4ac\right)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx}}$$

$$= \frac{2(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-(2c^2dg+bf(bg}{\left(b^2-4ac\right)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx}}$$

$$= \frac{2(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-(2c^2dg+bf(bg}{\left(b^2-4ac\right)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx}}$$

Mathematica [A]
time = 1.08, size = 235, normalized size = 1.04

$$2\left(\frac{-2a^2fh+2c^2dgx+b^2(-dh+fgx)+2ac(-eg+dh-fgx+ehx)+bc(-egx+d(g-hx))+ab(eh+f(g-hx))}{\left(b^2-4ac\right)\left(-cg^2+h(bg-ah)\right)\sqrt{a+x(b+cx)}}+\frac{\sqrt{-cg^2+h(bg-ah)}\,\left(fg^2+h(-eg+dh)\right)\tan^{-1}\left(\frac{\sqrt{c}\,(g+hx)-h\sqrt{a+x(b+cx)}}{\sqrt{-cg^2+h(bg-ah)}}\right)}{\left(cg^2+h(-bg+ah)\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] 2*((-2*a^2*f*h + 2*c^2*d*g*x + b^2*(-(d*h) + f*g*x) + 2*a*c*(-(e*g) + d*h -
 f*g*x + e*h*x) + b*c*(-(e*g*x) + d*(g - h*x)) + a*b*(e*h + f*(g - h*x)))/(
```

```
(b^2 - 4*a*c)*(-(c*g^2) + h*(b*g - a*h))*Sqrt[a + x*(b + c*x)]) + (Sqrt[-(c
*g^2) + h*(b*g - a*h)]*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[c]*(g + h*x)
 - h*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*g^2) + h*(b*g - a*h)]])/(c*g^2 + h*(-(
b*g) + a*h))^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(215) = 430$.

time = 0.13, size = 547, normalized size = 2.43

method	result
default	$\frac{fh\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}\right)+\frac{2eh(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{2gf(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}}{h^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/h^2*(f*h*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2))+2*e*h*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-2*g*f*(2*c*x+b)/(4
*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+(d*h^2-e*g*h+f*g^2)/h^3*(1/(a*h^2-b*g*h+c*g^
2)*h^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/(4*c*(a*h
^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*
g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2)*h^2/((a*h^2-b*g*h+c
*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2
-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `assum
e?` for
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(220) = 440$.
time = 11.94, size = 1961, normalized size = 8.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(((a*b^2 - 4*a^2*c)*f*g^2 + (a*b^2 - 4*a^2*c)*d*h^2 + ((b^2*c - 4*a*c
^2)*f*g^2 + (b^2*c - 4*a*c^2)*d*h^2)*x^2 + ((b^3 - 4*a*b*c)*f*g^2 + (b^3 -
4*a*b*c)*d*h^2)*x - ((b^2*c - 4*a*c^2)*g*h*x^2 + (b^3 - 4*a*b*c)*g*h*x + (a
*b^2 - 4*a^2*c)*g*h)*e)*sqrt(c*g^2 - b*g*h + a*h^2)*log((8*a*b*g*h - 8*a^2*
h^2 - (b^2 + 4*a*c)*g^2 - (8*c^2*g^2 - 8*b*c*g*h + (b^2 + 4*a*c)*h^2)*x^2 +
 4*sqrt(c*g^2 - b*g*h + a*h^2)*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g
- b*h)*x) - 2*(4*b*c*g^2 + 4*a*b*h^2 - (3*b^2 + 4*a*c)*g*h)*x)/(h^2*x^2 + 2
*g*h*x + g^2)) + 4*((b*c^2*d + a*b*c*f)*g^3 - (2*(b^2*c - a*c^2)*d + (a*b^2
 + 2*a^2*c)*f)*g^2*h + (3*a^2*b*f + (b^3 - a*b*c)*d)*g*h^2 - (2*a^3*f + (a*
b^2 - 2*a^2*c)*d)*h^3 + ((2*c^3*d + (b^2*c - 2*a*c^2)*f)*g^3 - (3*b*c^2*d +
 (b^3 - a*b*c)*f)*g^2*h + ((b^2*c + 2*a*c^2)*d + 2*(a*b^2 - a^2*c)*f)*g*h^2
 - (a*b*c*d + a^2*b*f)*h^3)*x - (2*a*c^2*g^3 - 3*a*b*c*g^2*h - a^2*b*h^3 +
(a*b^2 + 2*a^2*c)*g*h^2 + (b*c^2*g^3 + 3*a*b*c*g*h^2 - 2*a^2*c*h^3 - (b^2*c
 + 2*a*c^2)*g^2*h)*x)*e)*sqrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*g^
4 - 2*(a*b^3*c - 4*a^2*b*c^2)*g^3*h + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*g^2
*h^2 - 2*(a^2*b^3 - 4*a^3*b*c)*g*h^3 + (a^3*b^2 - 4*a^4*c)*h^4 + ((b^2*c^3
- 4*a*c^4)*g^4 - 2*(b^3*c^2 - 4*a*b*c^3)*g^3*h + (b^4*c - 2*a*b^2*c^2 - 8*a
^2*c^3)*g^2*h^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*g*h^3 + (a^2*b^2*c - 4*a^3*c^2)
*h^4)*x^2 + ((b^3*c^2 - 4*a*b*c^3)*g^4 - 2*(b^4*c - 4*a*b^2*c^2)*g^3*h + (b
^5 - 2*a*b^3*c - 8*a^2*b*c^2)*g^2*h^2 - 2*(a*b^4 - 4*a^2*b^2*c)*g*h^3 + (a^
2*b^3 - 4*a^3*b*c)*h^4)*x), (((a*b^2 - 4*a^2*c)*f*g^2 + (a*b^2 - 4*a^2*c)*d
*h^2 + ((b^2*c - 4*a*c^2)*f*g^2 + (b^2*c - 4*a*c^2)*d*h^2)*x^2 + ((b^3 - 4*
a*b*c)*f*g^2 + (b^3 - 4*a*b*c)*d*h^2)*x - ((b^2*c - 4*a*c^2)*g*h*x^2 + (b^3
 - 4*a*b*c)*g*h*x + (a*b^2 - 4*a^2*c)*g*h)*e)*sqrt(-c*g^2 + b*g*h - a*h^2)*
arctan(-1/2*sqrt(-c*g^2 + b*g*h - a*h^2)*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h
 + (2*c*g - b*h)*x)/(a*c*g^2 - a*b*g*h + a^2*h^2 + (c^2*g^2 - b*c*g*h + a*c
*h^2)*x^2 + (b*c*g^2 - b^2*g*h + a*b*h^2)*x)) - 2*((b*c^2*d + a*b*c*f)*g^3
- (2*(b^2*c - a*c^2)*d + (a*b^2 + 2*a^2*c)*f)*g^2*h + (3*a^2*b*f + (b^3 - a
*b*c)*d)*g*h^2 - (2*a^3*f + (a*b^2 - 2*a^2*c)*d)*h^3 + ((2*c^3*d + (b^2*c -
 2*a*c^2)*f)*g^3 - (3*b*c^2*d + (b^3 - a*b*c)*f)*g^2*h + ((b^2*c + 2*a*c^2)
*d + 2*(a*b^2 - a^2*c)*f)*g*h^2 - (a*b*c*d + a^2*b*f)*h^3)*x - (2*a*c^2*g^3
 - 3*a*b*c*g^2*h - a^2*b*h^3 + (a*b^2 + 2*a^2*c)*g*h^2 + (b*c^2*g^3 + 3*a*b
*c*g*h^2 - 2*a^2*c*h^3 - (b^2*c + 2*a*c^2)*g^2*h)*x)*e)*sqrt(c*x^2 + b*x +
a))/((a*b^2*c^2 - 4*a^2*c^3)*g^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*g^3*h + (a*b^4
 - 2*a^2*b^2*c - 8*a^3*c^2)*g^2*h^2 - 2*(a^2*b^3 - 4*a^3*b*c)*g*h^3 + (a^3*
b^2 - 4*a^4*c)*h^4 + ((b^2*c^3 - 4*a*c^4)*g^4 - 2*(b^3*c^2 - 4*a*b*c^3)*g^3
```

```
*h + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*g^2*h^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*
g*h^3 + (a^2*b^2*c - 4*a^3*c^2)*h^4)*x^2 + ((b^3*c^2 - 4*a*b*c^3)*g^4 - 2*(
b^4*c - 4*a*b^2*c^2)*g^3*h + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*g^2*h^2 - 2*(a
*b^4 - 4*a^2*b^2*c)*g*h^3 + (a^2*b^3 - 4*a^3*b*c)*h^4)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 719 vs. 2(220) = 440.
time = 3.91, size = 719, normalized size = 3.20

$$-\frac{2\left(\frac{\left(2c^3dg^3+b^2cfg^3-2ac^2fg^3-3bc^2dg^2h-b^3fg^2h+abcfg^2h+b^2cdgh^2+2ac^2dgh^2+2ab^2fgh^2-2a^2cfgh^2-abcdh^3-a^2bfh^3-bc^2g^3e+b^2cg^2he+2ac^2g^2he-3abcgh^2e+2a^2ch^3e\right)x}{b^2c^2g^4-4ac^3g^4-2b^3cg^3h+8abc^2g^3h+b^4g^2h^2-2ab^2cg^2h^2-8a^2c^2g^2h^2-2ab^3gh^3+8a^2bcgh^3+a^2b^2h^4-4a^3ch^4}+\frac{bc^2dg^3+abcfg^3-2b^2cdg^2h+2ac^2dg^2h-ab^2fg^2h-2a^2cfg^2h+b^3dgh^2-abcdgh^2+3a^2bfgh^2-ab^2dh^3+2a^2cdh^3-2a^3fh^3-2ac^2g^3e+3abcg^2he-ab^2gh^2e-2a^2cgh^2e+a^2bh^3e}{b^2c^2g^4-4ac^3g^4-2b^3cg^3h+8abc^2g^3h+b^4g^2h^2-2ab^2cg^2h^2-8a^2c^2g^2h^2-2ab^3gh^3+8a^2bcgh^3+a^2b^2h^4-4a^3ch^4}\right)}{\sqrt{cx^2+bx+a}}+\frac{2\left(fg^2+dh^2-ghe\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)h+\sqrt{c}g}{\sqrt{-cg^2+bgh-ah^2}}\right)}{\left(cg^2-bgh+ah^2\right)\sqrt{-cg^2+bgh-ah^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d*g^3 + b^2*c*f*g^3 - 2*a*c^2*f*g^3 - 3*b*c^2*d*g^2*h - b^3*f*g^
2*h + a*b*c*f*g^2*h + b^2*c*d*g*h^2 + 2*a*c^2*d*g*h^2 + 2*a*b^2*f*g*h^2 - 2
*a^2*c*f*g*h^2 - a*b*c*d*h^3 - a^2*b*f*h^3 - b*c^2*g^3*e + b^2*c*g^2*h*e +
2*a*c^2*g^2*h*e - 3*a*b*c*g*h^2*e + 2*a^2*c*h^3*e)*x/(b^2*c^2*g^4 - 4*a*c^3
*g^4 - 2*b^3*c*g^3*h + 8*a*b*c^2*g^3*h + b^4*g^2*h^2 - 2*a*b^2*c*g^2*h^2 -
8*a^2*c^2*g^2*h^2 - 2*a*b^3*g*h^3 + 8*a^2*b*c*g*h^3 + a^2*b^2*h^4 - 4*a^3*c
*h^4) + (b*c^2*d*g^3 + a*b*c*f*g^3 - 2*b^2*c*d*g^2*h + 2*a*c^2*d*g^2*h - a*
b^2*f*g^2*h - 2*a^2*c*f*g^2*h + b^3*d*g*h^2 - a*b*c*d*g*h^2 + 3*a^2*b*f*g*h
^2 - a*b^2*d*h^3 + 2*a^2*c*d*h^3 - 2*a^3*f*h^3 - 2*a*c^2*g^3*e + 3*a*b*c*g^
2*h*e - a*b^2*g*h^2*e - 2*a^2*c*g*h^2*e + a^2*b*h^3*e)/(b^2*c^2*g^4 - 4*a*c
^3*g^4 - 2*b^3*c*g^3*h + 8*a*b*c^2*g^3*h + b^4*g^2*h^2 - 2*a*b^2*c*g^2*h^2
- 8*a^2*c^2*g^2*h^2 - 2*a*b^3*g*h^3 + 8*a^2*b*c*g*h^3 + a^2*b^2*h^4 - 4*a^3
*c*h^4))/sqrt(c*x^2 + b*x + a) + 2*(f*g^2 + d*h^2 - g*h*e)*arctan(-((sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/(
(c*g^2 - b*g*h + a*h^2)*sqrt(-c*g^2 + b*g*h - a*h^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{\left(g+h\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.238 $\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=421

$$-\frac{2(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b(c^2dg^2+a^2fh^2+ac(fg^2+2egh-3d}{(b^2-4ac)\left(cg^2-bgh+\right.}$$

```
[Out] 1/2*(2*c*g*(f*g^2-h*(-3*d*h+2*e*g))-h*(2*a*h*(-e*h+2*f*g)-b*(-3*d*h^2+e*g*h
+f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/
(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(5/2)-2*(b^3*d*h^2-b^2*h*(a*e*h+2*
c*d*g)-2*a*c*(c*g*(-2*d*h+e*g)+a*h*(-e*h+2*f*g))+b*(c^2*d*g^2+a^2*f*h^2+a*c
*(-3*d*h^2+2*e*g*h+f*g^2))+c*(2*c^2*d*g^2+2*a^2*f*h^2-a*b*h*(e*h+2*f*g)+b^2
*(d*h^2+f*g^2)-c*(b*g*(2*d*h+e*g)+2*a*(d*h^2-2*e*g*h+f*g^2)))*x)/(-4*a*c+b^
2)/(a*h^2-b*g*h+c*g^2)^2/(c*x^2+b*x+a)^(1/2)-h*(f*g^2-h*(-d*h+e*g))*(c*x^2+
b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)
```

Rubi [A]
time = 0.61, antiderivative size = 418, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1660, 820, 738, 212}

$$-\frac{2(cx(2a^2fh^2-c(2a(dh^2-2egh+fg^2)+bg(2dh+eg))-abh(eh+2fg)+b^2(dh^2+fg^2)+2c^2dg^2)+b(a^2fh^2+ac(-3dh^2+2egh+fg^2)+c^2dg^2)-b^2h(aeh+2cdg)-2ac(ah(2fg-eh)+cg(eg-2dh))+b^3dh^2)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left(ah^2-bgh+cg^2\right)^2}-\frac{h\sqrt{a+bx+cx^2}\,(fg^2-h(eg-dh))}{(g+hx)\left(ah^2-bgh+cg^2\right)^2}+\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)(h(-2ah(2fg-eh)+bh(eg-3dh)+bfg^2)+2c(fg^3-gh(2eg-3dh)))}{2\left(ah^2-bgh+cg^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b^3*d*h^2 - b^2*h*(2*c*d*g + a*e*h) - 2*a*c*(c*g*(e*g - 2*d*h) + a*h*(
2*f*g - e*h)) + b*(c^2*d*g^2 + a^2*f*h^2 + a*c*(f*g^2 + 2*e*g*h - 3*d*h^2))
 + c*(2*c^2*d*g^2 + 2*a^2*f*h^2 - a*b*h*(2*f*g + e*h) + b^2*(f*g^2 + d*h^2)
 - c*(b*g*(e*g + 2*d*h) + 2*a*(f*g^2 - 2*e*g*h + d*h^2)))*x))/((b^2 - 4*a*c
)*(c*g^2 - b*g*h + a*h^2)^2*Sqrt[a + b*x + c*x^2]) - (h*(f*g^2 - h*(e*g - d
*h))*Sqrt[a + b*x + c*x^2])/((c*g^2 - b*g*h + a*h^2)^2*(g + h*x)) + ((2*c*(
f*g^3 - g*h*(2*e*g - 3*d*h)) + h*(b*f*g^2 + b*h*(e*g - 3*d*h) - 2*a*h*(2*f*
g - e*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h +
a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b(c}{}$$

$$= -\frac{2(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b(c}{}$$

$$= -\frac{2(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b(c}{}$$

$$= -\frac{2(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b(c}{}$$

Mathematica [**A**]
time = 11.93, size = 474, normalized size = 1.13

$$\frac{1}{2}\left(-\frac{2h(fg^2+h(-eg+dh))\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))^2(g+hx)}-\frac{4(b^3dh^2+b^2(-aeh^2+cfg^2x+cdh(-2g+hx))+2c(c^2dg^2x+a^2h(-2fg+eh+fhx)-ac(fg^2x+eg(g-2hx)+dh(-2g+hx)))+b(a^2fh^2+c^2g(-egx+d(g-2hx))+ac(fg(g-2hx)-h(-2eg+3dh+ehx))))}{(b^2-4ac)(cg^2+h(-bg+ah))^2\sqrt{a+x(b+cx)}}+\frac{(2c(fg^3+gh(-2eg+3dh))+h(bfg^2+bh(eg-3dh)+2ah(-2fg+eh)))\log(g+hx)}{(cg^2+h(-bg+ah))^{5/2}}-\frac{(2c(fg^3+gh(-2eg+3dh))+h(bfg^2+bh(eg-3dh)+2ah(-2fg+eh)))\log\left(-bg+2ah-2cgx+bhx+2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b+cx)}\right)}{(cg^2+h(-bg+ah))^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] ((-2*h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(-(b*g
) + a*h))^2*(g + h*x)) - (4*(b^3*d*h^2 + b^2*(-(a*e*h^2) + c*f*g^2*x + c*d*
h*(-2*g + h*x)) + 2*c*(c^2*d*g^2*x + a^2*h*(-2*f*g + e*h + f*h*x) - a*c*(f*
g^2*x + e*g*(g - 2*h*x) + d*h*(-2*g + h*x))) + b*(a^2*f*h^2 + c^2*g*(-(e*g*
x) + d*(g - 2*h*x)) + a*c*(f*g*(g - 2*h*x) - h*(-2*e*g + 3*d*h + e*h*x)))))
/((b^2 - 4*a*c)*(c*g^2 + h*(-(b*g) + a*h))^2*Sqrt[a + x*(b + c*x)]) + ((2*c
*(f*g^3 + g*h*(-2*e*g + 3*d*h)) + h*(b*f*g^2 + b*h*(e*g - 3*d*h) + 2*a*h*(-
2*f*g + e*h)))*Log[g + h*x])/(c*g^2 + h*(-(b*g) + a*h))^(5/2) - ((2*c*(f*g^
3 + g*h*(-2*e*g + 3*d*h)) + h*(b*f*g^2 + b*h*(e*g - 3*d*h) + 2*a*h*(-2*f*g
+ e*h)))*Log[-(b*g) + 2*a*h - 2*c*g*x + b*h*x + 2*Sqrt[c*g^2 + h*(-(b*g) +
a*h)]*Sqrt[a + x*(b + c*x)]])/(c*g^2 + h*(-(b*g) + a*h))^(5/2))/2
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. $2(407) = 814$.
time = 0.16, size = 1115, normalized size = 2.65

method	result

default	$\frac{2f(2cx+b)}{h^2(4ac-b^2)\sqrt{c\,x^2+bx+a}}+\frac{(eh-2gf)\left(\frac{h^2}{\left(a\,h^2-bgh+c\,g^2\right)\sqrt{\left(x+\frac{g}{h}\right)^2 c+\frac{(bh-2cg)\left(x+\frac{g}{h}\right)}{h}+\frac{a\,h^2-bgh+c\,g^2}{h^2}}}-\frac{}{(a}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f/h^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/h^3*(e*h-2*f*g)*(1/(a*h
^2-b*g*h+c*g^2)*h^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g
^2)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+1/h*g)+(b*h-2*c*g)
/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h-2*c
*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2)*h^2/((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(
x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+
1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/h^4*(d*h^2-e*g*h+f*g^2
)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h
*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(1
/(a*h^2-b*g*h+c*g^2)*h^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*
h+c*g^2)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+1/h*g)+(b*h-2
*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*
h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2)*h
^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g
)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/
h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))-4*c/(a*h^2-b*g*h+c*
g^2)*h^2*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*
c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)
^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `assum
e?` for
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2639 vs. $2(418) = 836$.
time = 33.81, size = 5320, normalized size = 12.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*((2*(a*b^2*c - 4*a^2*c^2)*f*g^4 + (a*b^3 - 4*a^2*b*c)*f*g^3*h - 3*(a*b
^3 - 4*a^2*b*c)*d*g*h^3 + 2*(3*(a*b^2*c - 4*a^2*c^2)*d - 2*(a^2*b^2 - 4*a^3
*c)*f)*g^2*h^2 + (2*(b^2*c^2 - 4*a*c^3)*f*g^3*h + (b^3*c - 4*a*b*c^2)*f*g^2
*h^2 - 3*(b^3*c - 4*a*b*c^2)*d*h^4 + 2*(3*(b^2*c^2 - 4*a*c^3)*d - 2*(a*b^2*
c - 4*a^2*c^2)*f)*g*h^3)*x^3 + (2*(b^2*c^2 - 4*a*c^3)*f*g^4 + 3*(b^3*c - 4*
a*b*c^2)*f*g^3*h - 3*(b^4 - 4*a*b^2*c)*d*h^4 + (6*(b^2*c^2 - 4*a*c^3)*d + (
b^4 - 8*a*b^2*c + 16*a^2*c^2)*f)*g^2*h^2 + (3*(b^3*c - 4*a*b*c^2)*d - 4*(a*
b^3 - 4*a^2*b*c)*f)*g*h^3)*x^2 + (2*(b^3*c - 4*a*b*c^2)*f*g^4 + (b^4 - 2*a*
b^2*c - 8*a^2*c^2)*f*g^3*h - 3*(a*b^3 - 4*a^2*b*c)*d*h^4 + 3*(2*(b^3*c - 4*
a*b*c^2)*d - (a*b^3 - 4*a^2*b*c)*f)*g^2*h^2 - (3*(b^4 - 6*a*b^2*c + 8*a^2*c
^2)*d + 4*(a^2*b^2 - 4*a^3*c)*f)*g*h^3)*x - (4*(a*b^2*c - 4*a^2*c^2)*g^3*h
- (a*b^3 - 4*a^2*b*c)*g^2*h^2 - 2*(a^2*b^2 - 4*a^3*c)*g*h^3 + (4*(b^2*c^2 -
 4*a*c^3)*g^2*h^2 - (b^3*c - 4*a*b*c^2)*g*h^3 - 2*(a*b^2*c - 4*a^2*c^2)*h^4
)*x^3 + (4*(b^2*c^2 - 4*a*c^3)*g^3*h + 3*(b^3*c - 4*a*b*c^2)*g^2*h^2 - (b^4
 - 2*a*b^2*c - 8*a^2*c^2)*g*h^3 - 2*(a*b^3 - 4*a^2*b*c)*h^4)*x^2 + (4*(b^3*
c - 4*a*b*c^2)*g^3*h - (b^4 - 8*a*b^2*c + 16*a^2*c^2)*g^2*h^2 - 3*(a*b^3 -
4*a^2*b*c)*g*h^3 - 2*(a^2*b^2 - 4*a^3*c)*h^4)*x)*e)*sqrt(c*g^2 - b*g*h + a*
h^2)*log((8*a*b*g*h - 8*a^2*h^2 - (b^2 + 4*a*c)*g^2 - (8*c^2*g^2 - 8*b*c*g*
h + (b^2 + 4*a*c)*h^2)*x^2 - 4*sqrt(c*g^2 - b*g*h + a*h^2)*sqrt(c*x^2 + b*x
 + a)*(b*g - 2*a*h + (2*c*g - b*h)*x) - 2*(4*b*c*g^2 + 4*a*b*h^2 - (3*b^2 +
 4*a*c)*g*h)*x)/(h^2*x^2 + 2*g*h*x + g^2)) - 4*((a^2*b^2 - 4*a^3*c)*d*h^5 +
 2*(b*c^3*d + a*b*c^2*f)*g^5 - (2*(3*b^2*c^2 - 4*a*c^3)*d + (a*b^2*c + 12*a
^2*c^2)*f)*g^4*h + (6*(b^3*c - 2*a*b*c^2)*d - (a*b^3 - 16*a^2*b*c)*f)*g^3*h
^2 - ((2*b^4 - 3*a*b^2*c - 4*a^2*c^2)*d + (a^2*b^2 + 12*a^3*c)*f)*g^2*h^3 +
 (2*a^3*b*f + (a*b^3 - 2*a^2*b*c)*d)*g*h^4 + ((4*c^4*d + (3*b^2*c^2 - 8*a*c
^3)*f)*g^4*h - (8*b*c^3*d + (3*b^3*c - 4*a*b*c^2)*f)*g^3*h^2 + ((7*b^2*c^2
- 4*a*c^3)*d + (7*a*b^2*c - 4*a^2*c^2)*f)*g^2*h^3 - (8*a^2*b*c*f + (3*b^3*c
 - 4*a*b*c^2)*d)*g*h^4 + (4*a^3*c*f + (3*a*b^2*c - 8*a^2*c^2)*d)*h^5)*x^2 +
```

```
 (2*(2*c^4*d + (b^2*c^2 - 2*a*c^3)*f)*g^5 - (6*b*c^3*d + (b^3*c + 2*a*b*c^2
)*f)*g^4*h + (8*a*c^3*d - (b^4 - 8*a*b^2*c + 8*a^2*c^2)*f)*g^3*h^2 + (a*b^3
*f + (5*b^3*c - 16*a*b*c^2)*d)*g^2*h^3 - ((3*b^4 - 8*a*b^2*c - 4*a^2*c^2)*d
 + 2*(a^2*b^2 + 2*a^3*c)*f)*g*h^4 + (2*a^3*b*f + (3*a*b^3 - 10*a^2*b*c)*d)*
h^5)*x - (4*a*c^3*g^5 - 8*a*b*c^2*g^4*h + (7*a*b^2*c - 4*a^2*c^2)*g^3*h^2 -
 (3*a*b^3 - 4*a^2*b*c)*g^2*h^3 + (3*a^2*b^2 - 8*a^3*c)*g*h^4 + (2*b*c^3*g^4
*h + 2*a^2*b*c*h^5 - (b^2*c^2 + 12*a*c^3)*g^3*h^2 - (b^3*c - 16*a*b*c^2)*g^
2*h^3 - (a*b^2*c + 12*a^2*c^2)*g*h^4)*x^2 + (2*b*c^3*g^5 + b^3*c*g^3*h^2 -
2*(b^2*c^2 + 2*a*c^3)*g^4*h - (b^4 - 8*a*b^2*c + 8*a^2*c^2)*g^2*h^3 - (a*b^
3 + 2*a^2*b*c)*g*h^4 + 2*(a^2*b^2 - 2*a^3*c)*h^5)*x)*e)*sqrt(c*x^2 + b*x +
a))/((a*b^2*c^3 - 4*a^2*c^4)*g^7 - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*g^6*h + 3*(a
*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*g^5*h^2 - (a*b^5 + 2*a^2*b^3*c - 24*a^3
*b*c^2)*g^4*h^3 + 3*(a^2*b^4 - 3*a^3*b^2*c - 4*a^4*c^2)*g^3*h^4 - 3*(a^3*b^
3 - 4*a^4*b*c)*g^2*h^5 + (a^4*b^2 - 4*a^5*c)*g*h^6 + ((b^2*c^4 - 4*a*c^5)*g
^6*h - 3*(b^3*c^3 - 4*a*b*c^4)*g^5*h^2 + 3*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c
^4)*g^4*h^3 - (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*g^3*h^4 + 3*(a*b^4*c - 3
*a^2*b^2*c^2 - 4*a^3*c^3)*g^2*h^5 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*g*h^6 + (a^
3*b^2*c - 4*a^4*c^2)*h^7)*x^3 + ((b^2*c^4 - 4*a*c^5)*g^7 - 2*(b^3*c^3 - 4*a
*b*c^4)*g^6*h + 3*(a*b^2*c^3 - 4*a^2*c^4)*g^5*h^2 + (2*b^5*c - 11*a*b^3*c^2
 + 12*a^2*b*c^3)*g^4*h^3 - (b^6 - a*b^4*c - 15*a^2*b^2*c^2 + 12*a^3*c^3)*g^
3*h^4 + 3*(a*b^5 - 4*a^2*b^3*c)*g^2*h^5 - (3*a^2*b^4 - 13*a^3*b^2*c + 4*a^4
*c^2)*g*h^6 + (a^3*b^3 - 4*a^4*b*c)*h^7)*x^2 + ((b^3*c^3 - 4*a*b*c^4)*g^7 -
 (3*b^4*c^2 - 13*a*b^2*c^3 + 4*a^2*c^4)*g^6*h + 3*(b^5*c - 4*a*b^3*c^2)*g^5
*h^2 - (b^6 - a*b^4*c - 15*a^2*b^2*c^2 + 12*a^3*c^3)*g^4*h^3 + (2*a*b^5 - 1
1*a^2*b^3*c + 12*a^3*b*c^2)*g^3*h^4 + 3*(a^3*b^2*c - 4*a^4*c^2)*g^2*h^5 - 2
*(a^3*b^3 - 4*a^4*b*c)*g*h^6 + (a^4*b^2 - 4*a^5*c)*h^7)*x), 1/2*((2*(a*b^2*
c - 4*a^2*c^2)*f*g^4 + (a*b^3 - 4*a^2*b*c)*f*g^3*h - 3*(a*b^3 - 4*a^2*b*c)*
d*g*h^3 + 2*(3*(a*b^2*c - 4*a^2*c^2)*d - 2*(a^2*b^2 - 4*a^3*c)*f)*g^2*h^2 +
 (2*(b^2*c^2 - 4*a*c^3)*f*g^3*h + (b^3*c - 4*a*b*c^2)*f*g^2*h^2 - 3*(b^3*c
- 4*a*b*c^2)*d*h^4 + 2*(3*(b^2*c^2 - 4*a*c^3)*d - 2*(a*b^2*c - 4*a^2*c^2)*f
)*g*h^3)*x^3 + (2*(b^2*c^2 - 4*a*c^3)*f*g^4 + 3*(b^3*c - 4*a*b*c^2)*f*g^3*h
 - 3*(b^4 - 4*a*b^2*c)*d*h^4 + (6*(b^2*c^2 - 4*a*c^3)*d + (b^4 - 8*a*b^2*c
+ 16*a^2*c^2)*f)*g^2*h^2 + (3*(b^3*c - 4*a*b*c^2)*d - 4*(a*b^3 - 4*a^2*b*c)
*f)*g*h^3)*x^2 + (2*(b^3*c - 4*a*b*c^2)*f*g^4 + (b^4 - 2*a*b^2*c - 8*a^2*c^
2)*f*g^3*h - 3*(a*b^3 - 4*a^2*b*c)*d*h^4 + 3*(2*(b^3*c - 4*a*b*c^2)*d - (a*
b^3 - 4*a^2*b*c)*f)*g^2*h^2 - (3*(b^4 - 6*a*b^2*c + 8*a^2*c^2)*d + 4*(a^2*b
^2 - 4*a^3*c)*f)*g*h^3)*x - (4*(a*b^2*c - 4*a^2*c^2)*g^3*h - (a*b^3 - 4*a^2
*b*c)*g^2*h^2 - 2*(a^2*b^2 - 4*a^3*c)*g*h^3 + (4*(b^2*c^2 - 4*a*c^3)*g^2*h^
2 - (b^3*c - 4*a*b*c^2)*g*h^3 - 2*(a*b^2*c - 4*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+b*x+a)**(3/2),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^2\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)), x)
```

3.239 $\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=713

$$\frac{2(b^4dh^3 - b^3h^2(3cdg + aeh) + b^2h(3c^2dg^2 + a^2fh^2 + ach(3eg - 4dh)) - bc(c^2dg^3 + 3a^2h^2(fg - eh) + acg(\ldots}{\ldots}$$

```
[Out] 1/8*(8*c^2*g^2*(6*d*h^2-3*e*g*h+f*g^2)+h^2*(8*a^2*f*h^2+4*a*b*h*(-3*e*h+2*f
*g)-b^2*(f*g^2+3*h*(-5*d*h+e*g)))-4*c*h*(a*h*(3*d*h^2-9*e*g*h+11*f*g^2)-b*g
*(2*f*g^2+3*h*(-4*d*h+e*g))))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2
-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(7/2)+2*(b^4*d
*h^3-b^3*h^2*(a*e*h+3*c*d*g)+b^2*h*(3*c^2*d*g^2+a^2*f*h^2+a*c*h*(-4*d*h+3*e
*g))-b*c*(c^2*d*g^3+3*a^2*h^2*(-e*h+f*g)+a*c*g*(-9*d*h^2+3*e*g*h+f*g^2))-2*
a*c*(a^2*f*h^3-c^2*g^2*(-3*d*h+e*g)-a*c*h*(d*h^2-3*e*g*h+3*f*g^2))-c*(2*c^3
*d*g^3-b*(a^2*f-a*b*e+b^2*d)*h^3-c^2*g*(b*g*(3*d*h+e*g)+2*a*(3*d*h^2-3*e*g*
h+f*g^2))+c*(2*a^2*h^2*(-e*h+3*f*g)-3*a*b*h*(-d*h^2+e*g*h+f*g^2)+b^2*(3*d*g
*h^2+f*g^3)))*x)/(-4*a*c+b^2)/(a*h^2-b*g*h+c*g^2)^3/(c*x^2+b*x+a)^(1/2)-1/2
*h*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^2
-1/4*h*(2*c*g*(3*f*g^2-h*(-7*d*h+5*e*g))-h*(4*a*h*(-e*h+2*f*g)-b*(-7*d*h^2+
3*e*g*h+f*g^2)))*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)
```

Rubi [A]
time = 2.12, antiderivative size = 707, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1660, 1664, 820, 738, 212}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^4*d*h^3 - b^3*h^2*(3*c*d*g + a*e*h) + b^2*h*(3*c^2*d*g^2 + a^2*f*h^2
+ a*c*h*(3*e*g - 4*d*h)) - b*c*(c^2*d*g^3 + 3*a^2*h^2*(f*g - e*h) + a*c*g*(
f*g^2 + 3*e*g*h - 9*d*h^2)) - 2*a*c*(a^2*f*h^3 - c^2*g^2*(e*g - 3*d*h) - a*
c*h*(3*f*g^2 - 3*e*g*h + d*h^2)) - c*(2*c^3*d*g^3 - b*(b^2*d - a*b*e + a^2*
f)*h^3 - c^2*g*(2*a*f*g^2 - 6*a*h*(e*g - d*h) + b*g*(e*g + 3*d*h)) + c*(2*a
^2*h^2*(3*f*g - e*h) + b^2*(f*g^3 + 3*d*g*h^2) - 3*a*b*h*(f*g^2 + h*(e*g -
d*h))))*x))/((b^2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)^3*Sqrt[a + b*x + c*x^2])
 - (h*(f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(2*(c*g^2 - b*g*h + a*
h^2)^2*(g + h*x)^2) - (h*(6*c*f*g^3 - 2*c*g*h*(5*e*g - 7*d*h) - 4*a*h^2*(2*
f*g - e*h) + b*h*(f*g^2 + h*(3*e*g - 7*d*h)))*Sqrt[a + b*x + c*x^2])/(4*(c*
g^2 - b*g*h + a*h^2)^3*(g + h*x)) + ((8*c^2*g^2*(f*g^2 - 3*e*g*h + 6*d*h^2)
 + 4*c*h*(2*b*f*g^3 + 3*b*g*h*(e*g - 4*d*h) - a*h*(11*f*g^2 - 9*e*g*h + 3*d
```

```
*h^2)) + h^2*(8*a^2*f*h^2 + 4*a*b*h*(2*f*g - 3*e*h) - b^2*(f*g^2 + 3*h*(e*g
 - 5*d*h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h
+ a*h^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
```

```
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2(b^4dh^3 - b^3h^2(3cdg+aeh) + b^2h(3c^2dg^2 + a^2fh^2 + ach(3eg-4dh))}{}$$

$$= \frac{2(b^4dh^3 - b^3h^2(3cdg+aeh) + b^2h(3c^2dg^2 + a^2fh^2 + ach(3eg-4dh))}{}$$

$$= \frac{2(b^4dh^3 - b^3h^2(3cdg+aeh) + b^2h(3c^2dg^2 + a^2fh^2 + ach(3eg-4dh))}{}$$

$$= \frac{2(b^4dh^3 - b^3h^2(3cdg+aeh) + b^2h(3c^2dg^2 + a^2fh^2 + ach(3eg-4dh))}{}$$

$$= \frac{2(b^4dh^3 - b^3h^2(3cdg+aeh) + b^2h(3c^2dg^2 + a^2fh^2 + ach(3eg-4dh))}{}$$

Mathematica [A]
time = 14.93, size = 775, normalized size = 1.09

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] ((-2*Sqrt[a + x*(b + c*x)]*((2*h*(c*g^2 + h*(-(b*g) + a*h))*(f*g^2 + h*(-(e
*g) + d*h)))/(g + h*x)^2 + (h*(6*c*f*g^3 + 2*c*g*h*(-5*e*g + 7*d*h) + 4*a*h
^2*(-2*f*g + e*h) + b*h*(f*g^2 + h*(3*e*g - 7*d*h))))/(g + h*x) + (8*(-(b^4
*d*h^3) + b^3*h^2*(a*e*h + c*d*(3*g - h*x)) + b^2*(-(a^2*f*h^3) + a*c*h^2*(
-3*e*g + 4*d*h + e*h*x) + c^2*(f*g^3*x - 3*d*g*h*(g - h*x))) + 2*c*(a^3*f*h
^3 + c^3*d*g^3*x - a*c^2*g*(f*g^2*x + e*g*(g - 3*h*x) + 3*d*h*(-g + h*x)) -
 a^2*c*h*(3*f*g*(g - h*x) + h*(-3*e*g + d*h + e*h*x))) + b*c*(-(a^2*h^2*(-3
*f*g + 3*e*h + f*h*x)) + c^2*g^2*(-(e*g*x) + d*(g - 3*h*x)) + a*c*(f*g^2*(g
 - 3*h*x) + 3*h*(e*g*(g - h*x) + d*h*(-3*g + h*x))))))/((b^2 - 4*a*c)*(a +
```

```
x*(b + c*x)))))/(c*g^2 + h*(-(b*g) + a*h))^3 + ((8*c^2*g^2*(f*g^2 - 3*e*g*h
 + 6*d*h^2) + 4*c*h*(2*b*f*g^3 + 3*b*g*h*(e*g - 4*d*h) + a*h*(-11*f*g^2 + 9
*e*g*h - 3*d*h^2)) + h^2*(8*a^2*f*h^2 + 4*a*b*h*(2*f*g - 3*e*h) - b^2*(f*g^
2 + 3*h*(e*g - 5*d*h))))*Log[g + h*x])/(c*g^2 + h*(-(b*g) + a*h))^(7/2) - (
(8*c^2*g^2*(f*g^2 - 3*e*g*h + 6*d*h^2) + 4*c*h*(2*b*f*g^3 + 3*b*g*h*(e*g -
4*d*h) + a*h*(-11*f*g^2 + 9*e*g*h - 3*d*h^2)) + h^2*(8*a^2*f*h^2 + 4*a*b*h*
(2*f*g - 3*e*h) - b^2*(f*g^2 + 3*h*(e*g - 5*d*h))))*Log[-(b*g) + 2*a*h - 2*
c*g*x + b*h*x + 2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)]])/(c
*g^2 + h*(-(b*g) + a*h))^(7/2))/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2267 vs. $2(693) = 1386$.
time = 0.14, size = 2268, normalized size = 3.18

method	result	size
default	Expression too large to display	2268

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] f/h^3*(1/(a*h^2-b*g*h+c*g^2)*h^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+1/h*g
)+(b*h-2*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)
^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+
c*g^2)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b
*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h
-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+(e*h-2*f*g)
/h^4*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+
1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)
*(1/(a*h^2-b*g*h+c*g^2)*h^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b
*g*h+c*g^2)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+1/h*g)+(b*
h-2*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)^2*c+
(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2
)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*
c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*
g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))-4*c/(a*h^2-b*g*h
+c*g^2)*h^2*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h
-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h
^2)^(1/2))+(d*h^2-e*g*h+f*g^2)/h^5*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^
2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-5/4
*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)/((
x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/2*(b*
h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(1/(a*h^2-b*g*h+c*g^2)*h^2/((x+1/h*g)^2*c+(b
*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b
*g*h+c*g^2)*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h
```

```
-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h
^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(
a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^
(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
)/(x+1/h*g)))-4*c/(a*h^2-b*g*h+c*g^2)*h^2*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/(4*
c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(
x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))-3/2*c/(a*h^2-b*g*h+c*g^2)*h^2*(1/(
a*h^2-b*g*h+c*g^2)*h^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+
c*g^2)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+1/h*g)+(b*h-2*c
*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h-
2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2)*h^2
/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/
h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*
(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5370 vs. 2(714) = 1428.
time = 140.89, size = 10782, normalized size = 15.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/16*((8*(a*b^2*c^2 - 4*a^2*c^3)*f*g^6 + 8*(a*b^3*c - 4*a^2*b*c^2)*f*g^5*
h + (48*(a*b^2*c^2 - 4*a^2*c^3)*d - (a*b^4 + 40*a^2*b^2*c - 176*a^3*c^2)*f)
*g^4*h^2 - 8*(6*(a*b^3*c - 4*a^2*b*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*f)*g^3*h^
3 + (3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*d + 8*(a^3*b^2 - 4*a^4*c)*f)*g
^2*h^4 + (8*(b^2*c^3 - 4*a*c^4)*f*g^4*h^2 + 8*(b^3*c^2 - 4*a*b*c^3)*f*g^3*h
^3 + (48*(b^2*c^3 - 4*a*c^4)*d - (b^4*c + 40*a*b^2*c^2 - 176*a^2*c^3)*f)*g^
2*h^4 - 8*(6*(b^3*c^2 - 4*a*b*c^3)*d - (a*b^3*c - 4*a^2*b*c^2)*f)*g*h^5 + (
```

3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*d + 8*(a^2*b^2*c - 4*a^3*c^2)*f)*h^
6)*x^4 + (16*(b^2*c^3 - 4*a*c^4)*f*g^5*h + 24*(b^3*c^2 - 4*a*b*c^3)*f*g^4*h
^2 + 2*(48*(b^2*c^3 - 4*a*c^4)*d + (3*b^4*c - 56*a*b^2*c^2 + 176*a^2*c^3)*f
)*g^3*h^3 - (48*(b^3*c^2 - 4*a*b*c^3)*d + (b^5 + 24*a*b^3*c - 112*a^2*b*c^2
)*f)*g^2*h^4 - 2*(3*(3*b^4*c - 8*a*b^2*c^2 - 16*a^2*c^3)*d - 4*(a*b^4 - 2*a
^2*b^2*c - 8*a^3*c^2)*f)*g*h^5 + (3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*d +
8*(a^2*b^3 - 4*a^3*b*c)*f)*h^6)*x^3 + (8*(b^2*c^3 - 4*a*c^4)*f*g^6 + 24*(b
^3*c^2 - 4*a*b*c^3)*f*g^5*h + 3*(16*(b^2*c^3 - 4*a*c^4)*d + (5*b^4*c - 32*a
*b^2*c^2 + 48*a^2*c^3)*f)*g^4*h^2 + 2*(24*(b^3*c^2 - 4*a*b*c^3)*d - (b^5 +
32*a*b^3*c - 144*a^2*b*c^2)*f)*g^3*h^3 - 3*(3*(9*b^4*c - 40*a*b^2*c^2 + 16*
a^2*c^3)*d - (5*a*b^4 - 32*a^2*b^2*c + 48*a^3*c^2)*f)*g^2*h^4 + 6*((5*b^5 -
32*a*b^3*c + 48*a^2*b*c^2)*d + 4*(a^2*b^3 - 4*a^3*b*c)*f)*g*h^5 + (3*(5*a*
b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*d + 8*(a^3*b^2 - 4*a^4*c)*f)*h^6)*x^2 + (8
*(b^3*c^2 - 4*a*b*c^3)*f*g^6 + 8*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*f*g^5*h
+ (48*(b^3*c^2 - 4*a*b*c^3)*d - (b^5 + 24*a*b^3*c - 112*a^2*b*c^2)*f)*g^4*h
^2 - 2*(24*(b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d - (3*a*b^4 - 56*a^2*b^2*c +
176*a^3*c^2)*f)*g^3*h^3 + 3*((5*b^5 - 56*a*b^3*c + 144*a^2*b*c^2)*d + 8*(a^
2*b^3 - 4*a^3*b*c)*f)*g^2*h^4 + 2*(3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*
d + 8*(a^3*b^2 - 4*a^4*c)*f)*g*h^5)*x - 3*(8*(a*b^2*c^2 - 4*a^2*c^3)*g^5*h
- 4*(a*b^3*c - 4*a^2*b*c^2)*g^4*h^2 + (a*b^4 - 16*a^2*b^2*c + 48*a^3*c^2)*g
^3*h^3 + 4*(a^2*b^3 - 4*a^3*b*c)*g^2*h^4 + (8*(b^2*c^3 - 4*a*c^4)*g^3*h^3 -
4*(b^3*c^2 - 4*a*b*c^3)*g^2*h^4 + (b^4*c - 16*a*b^2*c^2 + 48*a^2*c^3)*g*h^
5 + 4*(a*b^3*c - 4*a^2*b*c^2)*h^6)*x^4 + (16*(b^2*c^3 - 4*a*c^4)*g^4*h^2 -
2*(b^4*c + 8*a*b^2*c^2 - 48*a^2*c^3)*g^2*h^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*
c^2)*g*h^5 + 4*(a*b^4 - 4*a^2*b^2*c)*h^6)*x^3 + (8*(b^2*c^3 - 4*a*c^4)*g^5*
h + 12*(b^3*c^2 - 4*a*b*c^3)*g^4*h^2 - (7*b^4*c - 24*a*b^2*c^2 - 16*a^2*c^3
)*g^3*h^3 + 2*(b^5 - 16*a*b^3*c + 48*a^2*b*c^2)*g^2*h^4 + 3*(3*a*b^4 - 16*a
^2*b^2*c + 16*a^3*c^2)*g*h^5 + 4*(a^2*b^3 - 4*a^3*b*c)*h^6)*x^2 + (8*(b^3*c
^2 - 4*a*b*c^3)*g^5*h - 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*g^4*h^2 + (b^5
- 24*a*b^3*c + 80*a^2*b*c^2)*g^3*h^3 + 6*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
)*g^2*h^4 + 8*(a^2*b^3 - 4*a^3*b*c)*g*h^5)*x)*e)*sqrt(c*g^2 - b*g*h + a*h^2
)*log((8*a*b*g*h - 8*a^2*h^2 - (b^2 + 4*a*c)*g^2 - (8*c^2*g^2 - 8*b*c*g*h +
(b^2 + 4*a*c)*h^2)*x^2 + 4*sqrt(c*g^2 - b*g*h + a*h^2)*sqrt(c*x^2 + b*x +
a)*(b*g - 2*a*h + (2*c*g - b*h)*x) - 2*(4*b*c*g^2 + 4*a*b*h^2 - (3*b^2 + 4*
a*c)*g*h)*x)/(h^2*x^2 + 2*g*h*x + g^2)) - 4*(11*(a^2*b^3 - 4*a^3*b*c)*d*g*h
^6 - 2*(a^3*b^2 - 4*a^4*c)*d*h^7 - 8*(b*c^4*d + a*b*c^3*f)*g^7 + 16*(5*a^2*
c^3*f + (2*b^2*c^3 - 3*a*c^4)*d)*g^6*h - (16*(3*b^3*c^2 - 7*a*b*c^3)*d - (9
*a*b^3*c - 116*a^2*b*c^2)*f)*g^5*h^2 + (32*(b^4*c - 3*a*b^2*c^2 + a^2*c^3)*
d - (a*b^4 - 34*a^2*b^2*c - 40*a^3*c^2)*f)*g^4*h^3 - ((8*b^5 - 33*a*b^3*c +
44*a^2*b*c^2)*d + (13*a^2*b^3 - 12*a^3*b*c)*f)*g^3*h^4 - ((a*b^4 + 14*a^2*
b^2*c - 88*a^3*c^2)*d - 2*(7*a^3*b^2 - 20*a^4*c)*f)*g^2*h^5 - (2*(8*c^5*d +
(7*b^2*c^3 - 20*a*c^4)*f)*g^5*h^2 - (40*b*c^4*d + (13*b^3*c^2 - 12*a*b*c^3
)*f)*g^4*h^3 + (2*(31*b^2*c^3 - 44*a*c^4)*d - (b^4*c - 34*a*b^2*c^2 - 40*a^
2*c^3)*f)*g^3*h^4 - ((53*b^3*c^2 - 132*a*b*c^3)*d - (9*a*b^3*c - 116*a^2*b*
c^2)*f)*g^2*h^5 + (80*a^3*c^2*f + (15*b^4*c - 14*a*b^2*c^2 - 104*a^2*c^3)*d

```
)*g*h^6 - (8*a^3*b*c*f + (15*a*b^3*c - 52*a^2*b*c^2)*d)*h^7)*x^3 - (8*(4*c^
5*d + (3*b^2*c^3 - 8*a*c^4)*f)*g^6*h - (72*b*c^4*d + (19*b^3*c^2 - 4*a*b*c^
3)*f)*g^5*h^2 + 2*(40*(b^2*c^3 - a*c^4)*d - (2*b^4*c - 37*a*b^2*c^2 - 4*a^2
*c^3)*f)*g^4*h^3 - ((27*b^3*c^2 - 28*a*b*c^3)*d + (b^5 - 7*a*b^3*c + 92*a^2
*b*c^2)*f)*g^3*h^4 - (2*(14*b^4*c - 73*a*b^2*c^2 + 68*a^2*c^3)*d - (9*a*b^4
 - 58*a^2*b^2*c + 88*a^3*c^2)*f)*g^2*h^5 + (24*a^3*b*c*f + (15*b^5 - 49*a*b
^3*c - 20*a^2*b*c^2)*d)*g*h^6 - ((15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*d +
 8*(a^3*b^2 - 2*a^4*c)*f)*h^7)*x^2 + (5*(a^2*b^3 - 4*a^3*b*c)*d*h^7 - 8*(2*
c^5*d + (b^2*c^3 - 2*a*c^4)*f)*g^7 + 24*(b*c^4*d + a*b*c^3*f)*g^6*h + (16*(
b^2*c^3 - 4*a*c^4)*d + (9*b^4*c - 58*a*b^2*c^2 + 88*a^2*c^3)*f)*g^5*h^2 - (
80*(b^3*c^2 - 3*a*b*c^3)*d + (b^5 - 7*a*b^3*c + 92*a^2*b*c^2)*f)*g^4*h^3 +
((81*b^4*c - 274*a*b^2*c^2 + 40*a^2*c^3)*d - 2*(2*a*b^4 - 37*a^2*b^2*c - 4*
a^3*c^2)*f)*g^3*h^4 - ((25*b^5 - 63*a*b^3*c - 76*a^2*b*c^2)*d + (19*a^2*b^3
 - 4*a^3*b*c)*f)*g^2*h^5 + 2*((10*a*b^4 - 47*a^2*b^2*c + 44*a^3*c^2)*d + 4*
(3*a^3*b^2 - 8*a^4*c)*f)*g*h^6)*x + (16*a*c^4*g...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5637 vs. $2(714) = 1428$.
time = 5.83, size = 5637, normalized size = 7.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^7*d*g^9 + b^2*c^5*f*g^9 - 2*a*c^6*f*g^9 - 9*b*c^6*d*g^8*h - 3*b^3*
c^4*f*g^8*h + 3*a*b*c^5*f*g^8*h + 18*b^2*c^5*d*g^7*h^2 + 3*b^4*c^3*f*g^7*h^
2 + 6*a*b^2*c^4*f*g^7*h^2 - 21*b^3*c^4*d*g^6*h^3 - b^5*c^2*f*g^6*h^3 - 13*a
*b^3*c^3*f*g^6*h^3 - 16*a^2*b*c^4*f*g^6*h^3 + 15*b^4*c^3*d*g^5*h^4 + 6*a*b^
2*c^4*d*g^5*h^4 - 12*a^2*c^5*d*g^5*h^4 + 6*a*b^4*c^2*f*g^5*h^4 + 36*a^2*b^2
*c^3*f*g^5*h^4 + 12*a^3*c^4*f*g^5*h^4 - 6*b^5*c^2*d*g^4*h^5 - 15*a*b^3*c^3*
d*g^4*h^5 + 30*a^2*b*c^4*d*g^4*h^5 - 21*a^2*b^3*c^2*f*g^4*h^5 - 42*a^3*b*c^
3*f*g^4*h^5 + b^6*c*d*g^3*h^6 + 12*a*b^4*c^2*d*g^3*h^6 - 18*a^2*b^2*c^3*d*g
^3*h^6 - 16*a^3*c^4*d*g^3*h^6 + a^2*b^4*c*f*g^3*h^6 + 34*a^3*b^2*c^2*f*g^3*
h^6 + 16*a^4*c^3*f*g^3*h^6 - 3*a*b^5*c*d*g^2*h^7 - 3*a^2*b^3*c^2*d*g^2*h^7
+ 24*a^3*b*c^3*d*g^2*h^7 - 3*a^3*b^3*c*f*g^2*h^7 - 24*a^4*b*c^2*f*g^2*h^7 +
```

3*a^2*b^4*c*d*g*h^8 - 6*a^3*b^2*c^2*d*g*h^8 - 6*a^4*c^3*d*g*h^8 + 3*a^4*b^2*c*f*g*h^8 + 6*a^5*c^2*f*g*h^8 - a^3*b^3*c*d*h^9 + 3*a^4*b*c^2*d*h^9 - a^5*b*c*f*h^9 - b*c^6*g^9*e + 3*b^2*c^5*g^8*h*e + 6*a*c^6*g^8*h*e - 3*b^3*c^4*g^7*h^2*e - 24*a*b*c^5*g^7*h^2*e + b^4*c^3*g^6*h^3*e + 34*a*b^2*c^4*g^6*h^3*e + 16*a^2*c^5*g^6*h^3*e - 21*a*b^3*c^3*g^5*h^4*e - 42*a^2*b*c^4*g^5*h^4*e + 6*a*b^4*c^2*g^4*h^5*e + 36*a^2*b^2*c^3*g^4*h^5*e + 12*a^3*c^4*g^4*h^5*e - a*b^5*c*g^3*h^6*e - 13*a^2*b^3*c^2*g^3*h^6*e - 16*a^3*b*c^3*g^3*h^6*e + 3*a^2*b^4*c*g^2*h^7*e + 6*a^3*b^2*c^2*g^2*h^7*e - 3*a^3*b^3*c*g*h^8*e + 3*a^4*b*c^2*g*h^8*e + a^4*b^2*c*h^9*e - 2*a^5*c^2*h^9*e)*x/(b^2*c^6*g^12 - 4*a*c^7*g^12 - 6*b^3*c^5*g^11*h + 24*a*b*c^6*g^11*h + 15*b^4*c^4*g^10*h^2 - 54*a*b^2*c^5*g^10*h^2 - 24*a^2*c^6*g^10*h^2 - 20*b^5*c^3*g^9*h^3 + 50*a*b^3*c^4*g^9*h^3 + 120*a^2*b*c^5*g^9*h^3 + 15*b^6*c^2*g^8*h^4 - 225*a^2*b^2*c^4*g^8*h^4 - 60*a^3*c^5*g^8*h^4 - 6*b^7*c*g^7*h^5 - 36*a*b^5*c^2*g^7*h^5 + 180*a^2*b^3*c^3*g^7*h^5 + 240*a^3*b*c^4*g^7*h^5 + b^8*g^6*h^6 + 26*a*b^6*c*g^6*h^6 - 30*a^2*b^4*c^2*g^6*h^6 - 340*a^3*b^2*c^3*g^6*h^6 - 80*a^4*c^4*g^6*h^6 - 6*a*b^7*g^5*h^7 - 36*a^2*b^5*c*g^5*h^7 + 180*a^3*b^3*c^2*g^5*h^7 + 240*a^4*b*c^3*g^5*h^7 + 15*a^2*b^6*g^4*h^8 - 225*a^4*b^2*c^2*g^4*h^8 - 60*a^5*c^3*g^4*h^8 - 20*a^3*b^5*g^3*h^9 + 50*a^4*b^3*c*g^3*h^9 + 120*a^5*b*c^2*g^3*h^9 + 15*a^4*b^4*g^2*h^10 - 54*a^5*b^2*c*g^2*h^10 - 24*a^6*c^2*g^2*h^10 - 6*a^5*b^3*g*h^11 + 24*a^6*b*c*g*h^11 + a^6*b^2*h^12 - 4*a^7*c*h^12) + (b*c^6*d*g^9 + a*b*c^5*f*g^9 - 6*b^2*c^5*d*g^8*h + 6*a*c^6*d*g^8*h - 3*a*b^2*c^4*f*g^8*h - 6*a^2*c^5*f*g^8*h + 15*b^3*c^4*d*g^7*h^2 - 24*a*b*c^5*d*g^7*h^2 + 3*a*b^3*c^3*f*g^7*h^2 + 24*a^2*b*c^4*f*g^7*h^2 - 20*b^4*c^3*d*g^6*h^3 + 34*a*b^2*c^4*d*g^6*h^3 + 16*a^2*c^5*d*g^6*h^3 - a*b^4*c^2*f*g^6*h^3 - 34*a^2*b^2*c^3*f*g^6*h^3 - 16*a^3*c^4*f*g^6*h^3 + 15*b^5*c^2*d*g^5*h^4 - 15*a*b^3*c^3*d*g^5*h^4 - 54*a^2*b*c^4*d*g^5*h^4 + 21*a^2*b^3*c^2*f*g^5*h^4 + 42*a^3*b*c^3*f*g^5*h^4 - 6*b^6*c*d*g^4*h^5 - 9*a*b^4*c^2*d*g^4*h^5 + 66*a^2*b^2*c^3*d*g^4*h^5 + 12*a^3*c^4*d*g^4*h^5 - 6*a^2*b^4*c*f*g^4*h^5 - 36*a^3*b^2*c^2*f*g^4*h^5 - 12*a^4*c^3*f*g^4*h^5 + b^7*d*g^3*h^6 + 11*a*b^5*c*d*g^3*h^6 - 31*a^2*b^3*c^2*d*g^3*h^6 - 32*a^3*b*c^3*d*g^3*h^6 + a^2*b^5*f*g^3*h^6 + 13*a^3*b^3*c*f*g^3*h^6 + 16*a^4*b*c^2*f*g^3*h^6 - 3*a*b^6*d*g^2*h^7 + 30*a^3*b^2*c^2*d*g^2*h^7 - 3*a^3*b^4*f*g^2*h^7 - 6*a^4*b^2*c*f*g^2*h^7 + 3*a^2*b^5*d*g*h^8 - 9*a^3*b^3*c*d*g*h^8 - 3*a^4*b*c^2*d*g*h^8 + 3*a^4*b^3*f*g*h^8 - 3*a^5*b*c*f*g*h^8 - a^3*b^4*d*h^9 + 4*a^4*b^2*c*d*h^9 - 2*a^5*c^2*d*h^9 - a^5*b^2*f*h^9 + 2*a^6*c*f*h^9 - 2*a*c^6*g^9*e + 9*a*b*c^5*g^8*h*e - 18*a*b^2*c^4*g^7*h^2*e + 21*a*b^3*c^3*g^6*h^3*e - 15*a*b^4*c^2*g^5*h^4*e - 6*a^2*b^2*c^3*g^5*h^4*e + 12*a^3*c^4*g^5*h^4*e + 6*a*b^5*c*g^4*h^5*e + 15*a^2*b^3*c^2*g^4*h^5*e - 30*a^3*b*c^3*g^4*h^5*e - a*b^6*g^3*h^6*e - 12*a^2*b^4*c*g^3*h^6*e + 18*a^3*b^2*c^2*g^3*h^6*e + 16*a^4*c^3*g^3*h^6*e + 3*a^2*b^5*g^2*h^7*e + 3*a^3*b^3*c*g^2*h^7*e - 24*a^4*b*c^2*g^2*h^7*e - 3*a^3*b^4*g*h^8*e + 6*a^4*b^2*c*g*h^8*e + 6*a^5*c^2*g*h^8*e + a^4*b^3*h^9*e - 3*a^5*b*c*h^9*e)/(b^2*c^6*g^12 - 4*a*c^7*g^12 - 6*b^3*c^5*g^11*h + 24*a*b*c^6*g^11*h + 15*b^4*c^4*g^10*h^2 - 54*a*b^2*c^5*g^10*h^2 - 24*a^2*c^6*g^10*h^2 - 20*b^5*c^3*g^9*h^3 + 50*a*b^3*c^4*g^9*h^3 + 120*a^2*b*c^5*g^9*h^3 + 15*b^6*c^2*g^8*h^4 - 225*a^2*b^2*c^4*g^8*h^4 - 60*a^3*c^5*g^8*h^4 - 6*b^7*c*g^7*h^5 - 36*a*b^5*c^2

```
*g^7*h^5 + 180*a^2*b^3*c^3*g^7*h^5 + 240*a^3*b*c^4*g^7*h^5 + b^8*g^6*h^6 +
26*a*b^6*c*g^6*h^6 - 30*a^2*b^4*c^2*g^6*h^6 - 340*a^3*b^2*c^3*g^6*h^6 - 80*
a^4*c^4*g^6*h^6 - 6*a*b^7*g^5*h^7 - 36*a^2*b^5*c*g^5*h^7 + 180*a^3*b^3*c^2*
g^5*h^7 + 240*a^4*b*c^3*g^5*h^7 + 15*a^2*b^6*g^4*h^8 - 225*a^4*b^2*c^2*g^4*
h^8 - 60*a^5*c^3*g^4*h^8 - 20*a^3*b^5*g^3*h^9 + 50*a^4*b^3*c*g^3*h^9 + 120*
a^5*b*c^2*g^3*h^9 + 15*a^4*b^4*g^2*h^10 - 54*a^5*b^2*c*g^2*h^10 - 24*a^6*c^
2*g^2*h^10 - 6*a^5*b^3*g*h^11 + 24*a^6*b*c*g*h^11 + a^6*b^2*h^12 - 4*a^7*c*
h^12))/sqrt(c*x^2 + b*x + a) + 1/4*(8*c^2*f*g^4 + 8*b*c*f*g^3*h + 48*c^2*d*
g^2*h^2 - b^2*f*g^2*h^2 - 44*a*c*f*g^2*h^2 - 48*b*c*d*g*h^3 + 8*a*b*f*g*h^3
 + 15*b^2*d*h^4 - 12*a*c*d*h^4 + 8*a^2*f*h^4 - 24*c^2*g^3*h*e + 12*b*c*g^2*
h^2*e - 3*b^2*g*h^3*e + 36*a*c*g*h^3*e - 12*a*b...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^3\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)), x)
```

3.240 $\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

Optimal. Leaf size=120

$$\frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2}+\frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2}+\frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2}-\frac{(24897+6298x)\sqrt{2-x}}{3240}$$

[Out] 9211/3888*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+44/135*(1+2*x)^2*(3*x^2-x+2)^(1/2)+19/60*(1+2*x)^3*(3*x^2-x+2)^(1/2)+2/15*(1+2*x)^4*(3*x^2-x+2)^(1/2)-1/3240*(24897+6298*x)*(3*x^2-x+2)^(1/2)

Rubi [A]
time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1667, 846, 793, 633, 221}

$$\frac{2}{15}\sqrt{3x^2-x+2}\,(2x+1)^4+\frac{19}{60}\sqrt{3x^2-x+2}\,(2x+1)^3+\frac{44}{135}\sqrt{3x^2-x+2}\,(2x+1)^2-\frac{(6298x+24897)\sqrt{3x^2-x+2}}{3240}+\frac{9211\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{1296\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]

[Out] (44*(1 + 2*x)^2*Sqrt[2 - x + 3*x^2])/135 + (19*(1 + 2*x)^3*Sqrt[2 - x + 3*x^2])/60 + (2*(1 + 2*x)^4*Sqrt[2 - x + 3*x^2])/15 - ((24897 + 6298*x)*Sqrt[2 - x + 3*x^2])/3240 + (9211*ArcSinh[(1 - 6*x)/Sqrt[23]])/(1296*Sqrt[3])

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} + \frac{1}{60}\int \frac{(1+2x)^3(-64+228x)}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} + \frac{1}{720}\int \frac{(1+2x}{\sqrt{}}$$

$$= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2} + \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{}$$

$$= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2} + \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{}$$

$$= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2} + \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{}$$

$$= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2} + \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{}$$

Mathematica [A]
time = 0.37, size = 70, normalized size = 0.58

$$\frac{6\sqrt{2-x+3x^2}\,(-22383+7538x+26904x^2+22032x^3+6912x^4)+46055\sqrt{3}\,\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{19440}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
[Out] (6*Sqrt[2 - x + 3*x^2]*(-22383 + 7538*x + 26904*x^2 + 22032*x^3 + 6912*x^4)
 + 46055*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/19440
```

Maple [A]
time = 0.11, size = 96, normalized size = 0.80

method	result
risch	$\frac{(6912x^4+22032x^3+26904x^2+7538x-22383)\sqrt{3x^2-x+2}}{3240} - \frac{9211\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,(x-\frac{1}{6})}{23}\right)}{3888}$
trager	$\left(\frac{32}{15}x^4+\frac{34}{5}x^3+\frac{1121}{135}x^2+\frac{3769}{1620}x-\frac{829}{120}\right)\sqrt{3x^2-x+2} - \frac{9211\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3}\right.}{3888}$

default	$\frac{32x^4\sqrt{3x^2-x+2}}{15}+\frac{34x^3\sqrt{3x^2-x+2}}{5}+\frac{1121x^2\sqrt{3x^2-x+2}}{135}+\frac{3769x\sqrt{3x^2-x+2}}{1620}-\frac{829\sqrt{3x}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 32/15*x^4*(3*x^2-x+2)^(1/2)+34/5*x^3*(3*x^2-x+2)^(1/2)+1121/135*x^2*(3*x^2-
x+2)^(1/2)+3769/1620*x*(3*x^2-x+2)^(1/2)-829/120*(3*x^2-x+2)^(1/2)-9211/388
8*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Maxima [A]
time = 0.50, size = 97, normalized size = 0.81

$$\frac{32}{15}\sqrt{3x^2-x+2}\,x^4+\frac{34}{5}\sqrt{3x^2-x+2}\,x^3+\frac{1121}{135}\sqrt{3x^2-x+2}\,x^2+\frac{3769}{1620}\sqrt{3x^2-x+2}\,x-\frac{9211}{3888}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}\,(6x-1)\right)-\frac{829}{120}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/15*sqrt(3*x^2 - x + 2)*x^4 + 34/5*sqrt(3*x^2 - x + 2)*x^3 + 1121/135*sqr
t(3*x^2 - x + 2)*x^2 + 3769/1620*sqrt(3*x^2 - x + 2)*x - 9211/3888*sqrt(3)*
arcsinh(1/23*sqrt(23)*(6*x - 1)) - 829/120*sqrt(3*x^2 - x + 2)
```

Fricas [A]
time = 0.35, size = 73, normalized size = 0.61

$$\frac{1}{3240}\left(6912x^4+22032x^3+26904x^2+7538x-22383\right)\sqrt{3x^2-x+2}+\frac{9211}{7776}\sqrt{3}\,\log\left(4\sqrt{3}\,\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3240*(6912*x^4 + 22032*x^3 + 26904*x^2 + 7538*x - 22383)*sqrt(3*x^2 - x +
 2) + 9211/7776*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^
2 + 24*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(2x+1)^3\cdot(4x^2+3x+1)}{\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/sqrt(3*x**2 - x + 2), x)
```

Giac [A]
time = 5.51, size = 68, normalized size = 0.57

$$\frac{1}{3240}\left(2\left(12\left(18\left(16x+51\right)x+1121\right)x+3769\right)x-22383\right)\sqrt{3x^2-x+2}+\frac{9211}{3888}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3240*(2*(12*(18*(16*x + 51)*x + 1121)*x + 3769)*x - 22383)*sqrt(3*x^2 - x
 + 2) + 9211/3888*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
+ 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
```

```
[Out] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

3.241 $\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

Optimal. Leaf size=95

$$-\frac{143}{324}(3-2x)\sqrt{2-x+3x^2}+\frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2}+\frac{4147\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{648\sqrt{3}}$$

```
[Out] 4147/1944*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-143/324*(3-2*x)*(3*x^2-x+2
)^(1/2)+11/27*(1+2*x)^2*(3*x^2-x+2)^(1/2)+1/6*(1+2*x)^3*(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1667, 846, 793, 633, 221}

$$\frac{1}{6}\sqrt{3x^2-x+2}\,(2x+1)^3+\frac{11}{27}\sqrt{3x^2-x+2}\,(2x+1)^2-\frac{143}{324}(3-2x)\sqrt{3x^2-x+2}+\frac{4147\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{648\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (-143*(3 - 2*x)*Sqrt[2 - x + 3*x^2])/324 + (11*(1 + 2*x)^2*Sqrt[2 - x + 3*x
^2])/27 + ((1 + 2*x)^3*Sqrt[2 - x + 3*x^2])/6 + (4147*ArcSinh[(1 - 6*x)/Sqr
t[23]])/(648*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2}+\frac{1}{48}\int \frac{(1+2x)^2(-44+176x)}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2}+\frac{1}{432}\int \frac{(1+2x)}{\sqrt{2}}$$

$$= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2}+\frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{}$$

$$= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2}+\frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{}$$

$$= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2}+\frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{}$$

Mathematica [A]
time = 0.30, size = 65, normalized size = 0.68

$$\frac{6\sqrt{2-x+3x^2}\,(-243+1138x+1176x^2+432x^3)+4147\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{1944}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(-243 + 1138*x + 1176*x^2 + 432*x^3) + 4147*Sqrt[3]*
Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/1944
```

Maple [A]
time = 0.11, size = 79, normalized size = 0.83

method	result
risch	$\frac{(432x^3+1176x^2+1138x-243)\sqrt{3x^2-x+2}}{324}-\frac{4147\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,(x-\frac{1}{6})}{23}\right)}{1944}$
trager	$\left(\frac{4}{3}x^3+\frac{98}{27}x^2+\frac{569}{162}x-\frac{3}{4}\right)\sqrt{3x^2-x+2}+\frac{4147\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+}\right.}{1944}$
default	$\frac{4x^3\sqrt{3x^2-x+2}}{3}+\frac{98x^2\sqrt{3x^2-x+2}}{27}+\frac{569x\sqrt{3x^2-x+2}}{162}-\frac{3\sqrt{3x^2-x+2}}{4}-\frac{4147\sqrt{3}\,\operatorname{arcsin}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*x^3*(3*x^2-x+2)^(1/2)+98/27*x^2*(3*x^2-x+2)^(1/2)+569/162*x*(3*x^2-x+2)
^(1/2)-3/4*(3*x^2-x+2)^(1/2)-4147/1944*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6
))
```

Maxima [A]
time = 0.50, size = 80, normalized size = 0.84

$$\frac{4}{3}\sqrt{3\,x^2-x+2}\,x^3+\frac{98}{27}\sqrt{3\,x^2-x+2}\,x^2+\frac{569}{162}\sqrt{3\,x^2-x+2}\,x-\frac{4147}{1944}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(6\,x-1)\right)-\frac{3}{4}\sqrt{3\,x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3*x^2 - x + 2)*x^3 + 98/27*sqrt(3*x^2 - x + 2)*x^2 + 569/162*sqrt(
3*x^2 - x + 2)*x - 4147/1944*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 3/4
*sqrt(3*x^2 - x + 2)
```

Fricas [A]
time = 0.36, size = 68, normalized size = 0.72

$$\frac{1}{324}\left(432\,x^{3}+1176\,x^{2}+1138\,x-243\right)\sqrt{3\,x^{2}-x+2}+\frac{4147}{3888}\,\sqrt{3}\,\log\left(4\,\sqrt{3}\,\sqrt{3\,x^{2}-x+2}\,(6\,x-1)-72\,x^{2}+24\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/324*(432*x^3 + 1176*x^2 + 1138*x - 243)*sqrt(3*x^2 - x + 2) + 4147/3888*s
qrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(2x+1)^{2}\cdot(4x^{2}+3x+1)}{\sqrt{3x^{2}-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/sqrt(3*x**2 - x + 2), x)
```

Giac [A]
time = 8.71, size = 63, normalized size = 0.66

$$\frac{1}{324}\left(2\left(12\left(18\,x+49\right)x+569\right)x-243\right)\sqrt{3\,x^{2}-x+2}+\frac{4147}{1944}\,\sqrt{3}\,\log\left(-2\,\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^{2}-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/324*(2*(12*(18*x + 49)*x + 569)*x - 243)*sqrt(3*x^2 - x + 2) + 4147/1944*
sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(2\,x+1)^{2}\,(4\,x^{2}+3\,x+1)}{\sqrt{3\,x^{2}-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
```

```
[Out] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

3.242 $\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

Optimal. Leaf size=70

$$\frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{54}(69+62x)\sqrt{2-x+3x^2}+\frac{251\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}$$

```
[Out] 251/324*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2/9*(1+2*x)^2*(3*x^2-x+2)^(1
/2)+1/54*(69+62*x)*(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {1667, 793, 633, 221}

$$\frac{2}{9}\sqrt{3x^2-x+2}\,(2x+1)^2+\frac{1}{54}(62x+69)\sqrt{3x^2-x+2}+\frac{251\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (2*(1 + 2*x)^2*Sqrt[2 - x + 3*x^2])/9 + ((69 + 62*x)*Sqrt[2 - x + 3*x^2])/5
4 + (251*ArcSinh[(1 - 6*x)/Sqrt[23]])/(108*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{36}\int \frac{(1+2x)(-24+124x)}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{54}(69+62x)\sqrt{2-x+3x^2}-\frac{251}{108}\int \frac{1}{\sqrt{2-x}\cdots}$$

$$= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{54}(69+62x)\sqrt{2-x+3x^2}-\frac{251\text{Subst}\left(\int \cdots\right)}{\cdots}$$

$$= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{54}(69+62x)\sqrt{2-x+3x^2}+\frac{251\sinh^{-1}\left(\frac{1-\cdots}{\sqrt{\cdots}}\right)}{108\sqrt{3}}$$

Mathematica [A]
time = 0.19, size = 60, normalized size = 0.86

$$\frac{1}{324}\left(6\sqrt{2-x+3x^2}\left(81+110x+48x^2\right)+251\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(81 + 110*x + 48*x^2) + 251*Sqrt[3]*Log[1 - 6*x + 2*
Sqrt[6 - 3*x + 9*x^2]])/324
```

Maple [A]
time = 0.11, size = 62, normalized size = 0.89

method	result
risch	$\frac{(48x^2+110x+81)\sqrt{3x^2-x+2}}{54}-\frac{251\sqrt{3}\,\mathrm{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{324}$
default	$\frac{8x^2\sqrt{3x^2-x+2}}{9}+\frac{55x\sqrt{3x^2-x+2}}{27}+\frac{3\sqrt{3x^2-x+2}}{2}-\frac{251\sqrt{3}\,\mathrm{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{324}$
trager	$\left(\frac{8}{9}x^2+\frac{55}{27}x+\frac{3}{2}\right)\sqrt{3x^2-x+2}+\frac{251\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}+\mathrm{RootOf}\right.}{324}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/9*x^2*(3*x^2-x+2)^(1/2)+55/27*x*(3*x^2-x+2)^(1/2)+3/2*(3*x^2-x+2)^(1/2)-2
51/324*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Maxima [A]
time = 0.49, size = 63, normalized size = 0.90

$$\frac{8}{9}\sqrt{3x^2-x+2}\,x^2+\frac{55}{27}\sqrt{3x^2-x+2}\,x-\frac{251}{324}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}\,(6x-1)\right)+\frac{3}{2}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*sqrt(3*x^2 - x + 2)*x^2 + 55/27*sqrt(3*x^2 - x + 2)*x - 251/324*sqrt(3)
*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 3/2*sqrt(3*x^2 - x + 2)
```

Fricas [A]
time = 0.39, size = 63, normalized size = 0.90

$$\frac{1}{54}\left(48x^2+110x+81\right)\sqrt{3x^2-x+2}+\frac{251}{648}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(48*x^2 + 110*x + 81)*sqrt(3*x^2 - x + 2) + 251/648*sqrt(3)*log(4*sqrt
(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(2x+1)\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)*(4*x**2 + 3*x + 1)/sqrt(3*x**2 - x + 2), x)
```

Giac [A]
time = 7.06, size = 58, normalized size = 0.83

$$\frac{1}{54}\left(2\left(24\,x+55\right)x+81\right)\sqrt{3\,x^2-x+2}+\frac{251}{324}\,\sqrt{3}\,\log\left(-2\,\sqrt{3}\,\left(\sqrt{3}\,x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/54*(2*(24*x + 55)*x + 81)*sqrt(3*x^2 - x + 2) + 251/324*sqrt(3)*log(-2*sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x+1\right)\,\left(4\,x^2+3\,x+1\right)}{\sqrt{3\,x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
```

```
[Out] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

3.243 $\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx$

Optimal. Leaf size=78

$$\frac{2}{3}\sqrt{2-x+3x^2}-\frac{5\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2\sqrt{13}}$$

```
[Out] -5/18*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1/26*arctanh(1/26*(9-8*x)*13^(
1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+2/3*(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1667, 857, 633, 221, 738, 212}

$$\frac{2}{3}\sqrt{3x^2-x+2}-\frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}-\frac{5\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] (2*Sqrt[2 - x + 3*x^2])/3 - (5*ArcSinh[(1 - 6*x)/Sqrt[23]])/(6*Sqrt[3]) - A
rcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])]/(2*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx &= \frac{2}{3}\sqrt{2-x+3x^2}+\frac{1}{12}\int \frac{16+20x}{(1+2x)\sqrt{2-x+3x^2}}\,dx \\
&= \frac{2}{3}\sqrt{2-x+3x^2}+\frac{1}{2}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx+\frac{5}{6}\int \frac{1}{\sqrt{2-x+3x^2}}\,d \\
&= \frac{2}{3}\sqrt{2-x+3x^2}+\frac{5\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{6\sqrt{69}}-\mathrm{Subst}\left(\int \frac{1}{52-\cdots}\right. \\
&= \frac{2}{3}\sqrt{2-x+3x^2}-\frac{5\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2\sqrt{13}}
\end{aligned}$$

Mathematica [A]

time = 0.23, size = 93, normalized size = 1.19

$$\frac{2}{3}\sqrt{2-x+3x^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}\,x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{\sqrt{13}}-\frac{5\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] (2*Sqrt[2 - x + 3*x^2])/3 + ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x +
 3*x^2])/Sqrt[13]]/Sqrt[13] - (5*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(6
*Sqrt[3])
```

Maple [A]
time = 0.16, size = 60, normalized size = 0.77

method	result
default	$\frac{5\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{18}+\frac{2\sqrt{3x^2-x+2}}{3}-\frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{26}$
risch	$\frac{5\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{18}+\frac{2\sqrt{3x^2-x+2}}{3}-\frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{26}$
trager	$\frac{2\sqrt{3x^2-x+2}}{3}+\frac{\operatorname{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\operatorname{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}-9\operatorname{RootOf}\left(\_Z^2-13\right)}{2x+1}\right)}{26}-$ 5 Root

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/18*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+2/3*(3*x^2-x+2)^(1/2)-1/26*13^(
1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Maxima [A]
time = 0.52, size = 67, normalized size = 0.86

$$\frac{5}{18}\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{1}{26}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{2}{3}\sqrt{3\,x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/18*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1/26*sqrt(13)*arcsi
nh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 2/3*sqrt(3*
x^2 - x + 2)
```

Fricas [A]
time = 0.34, size = 105, normalized size = 1.35

$$\frac{5}{36}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+\frac{1}{52}\sqrt{13}\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+\frac{2}{3}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
[Out] 5/36*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x -
 25) + 1/52*sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x
^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 2/3*sqrt(3*x^2 - x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(1/2),x)
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*sqrt(3*x**2 - x + 2)), x)
```

Giac [A]
time = 8.00, size = 116, normalized size = 1.49

$$-\frac{5}{18}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3x^2-x+2}\right)+\frac{1}{26}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{2}{3}\sqrt{3x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
[Out] -5/18*sqrt(3)*log(-6*sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 1/26*sq
rt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 -
x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))) + 2/3*s
qrt(3*x^2 - x + 2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{(2\,x+1)\,\sqrt{3\,x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(1/2)),x)
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(1/2)), x)
```

3.244 $\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx$

Optimal. Leaf size=83

$$-\frac{\sqrt{2-x+3x^2}}{13(1+2x)}-\frac{\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}}+\frac{9\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{26\sqrt{13}}$$

```
[Out] -1/3*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+9/338*arctanh(1/26*(9-8*x)*13^(
1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-1/13*(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1664, 857, 633, 221, 738, 212}

$$-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}+\frac{9\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{26\sqrt{13}}-\frac{\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] -1/13*Sqrt[2 - x + 3*x^2]/(1 + 2*x) - ArcSinh[(1 - 6*x)/Sqrt[23]]/Sqrt[3] +
 (9*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(26*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx &= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)}-\frac{1}{13}\int \frac{-\frac{17}{2}-26x}{(1+2x)\sqrt{2-x+3x^2}}\,dx \\
&= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)}-\frac{9}{26}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx+\int \frac{1}{\sqrt{2-x+3x^2}}\,d \\
&\qquad \mathrm{Subst}\Bigg(\\
&= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)}+\frac{9}{13}\mathrm{Subst}\left(\int \frac{1}{52-x^2}\,dx, x, \frac{9-8x}{\sqrt{2-x+3x^2}}\right)+ \\
&= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)}-\frac{\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}}+\frac{9\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{26\sqrt{13}}
\end{aligned}$$

Mathematica [A]

time = 0.29, size = 99, normalized size = 1.19

$$-\frac{\sqrt{2-x+3x^2}}{13+26x}-\frac{9\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}\,x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{13\sqrt{13}}-\frac{\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] -(Sqrt[2 - x + 3*x^2]/(13 + 26*x)) - (9*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*
Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(13*Sqrt[13]) - Log[1 - 6*x + 2*Sqrt[6 - 3*
x + 9*x^2]]/Sqrt[3]
```

Maple [A]
time = 0.16, size = 67, normalized size = 0.81

method	result
default	$\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{3}+\frac{9\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{338}-\frac{\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+}}{26\left(x+\frac{1}{2}\right)}$
risch	$-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}+\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{3}+\frac{9\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{338}$
trager	$-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}+\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(6\operatorname{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}-\operatorname{RootOf}\left(\_Z^2-3\right)\right)}{3}-\frac{9\operatorname{RootO}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^2/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+9/338*13^(1/2)*arctanh(2/13*(9/2
-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))-1/26/(x+1/2)*(3*(x+1/2)^2-4*x+5
/4)^(1/2)
```

Maxima [A]
time = 0.50, size = 74, normalized size = 0.89

$$\frac{1}{3}\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{9}{338}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)-\frac{\sqrt{3\,x^2-x+2}}{13\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 9/338*sqrt(13)*arcsi
nh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) - 1/13*sqrt(3
*x^2 - x + 2)/(2*x + 1)
```

Fricas [A]
time = 0.37, size = 123, normalized size = 1.48

$$\frac{338\sqrt{3}\,(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)+27\sqrt{13}\,(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}\,(8x-9)-220x^2+196x-185}{4x^2+4x+1}\right)-156\sqrt{3x^2-x+2}}{2028\,(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2028*(338*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
- 72*x^2 + 24*x - 25) + 27*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2 -
x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) - 156*sqrt(3*x
^2 - x + 2))/(2*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)^2\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*sqrt(3*x**2 - x + 2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(66) = 132.
time = 3.20, size = 191, normalized size = 2.30

$$\frac{9\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)}{338\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)}-\frac{\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{2\sqrt{13}}{2x+1}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)}\right)}{3\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)}-\frac{\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}}{26\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 9/338*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt
(13)/(2*x + 1)) - 4)/sgn(1/(2*x + 1)) - 1/3*sqrt(3)*log(1/2*abs(-2*sqrt(3)
+ 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1))/(sqrt(3
) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)))/sgn(1/(2
*x + 1)) - 1/26*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)/sgn(1/(2*x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^2\,\sqrt{3\,x^2-x+2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)),x)
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)), x)
```

3.245 $\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx$

Optimal. Leaf size=89

$$-\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}+\frac{7\sqrt{2-x+3x^2}}{169(1+2x)}-\frac{581\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{676\sqrt{13}}$$

```
[Out] -581/8788*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-1/26*(3
*x^2-x+2)^(1/2)/(1+2*x)^2+7/169*(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1664, 820, 738, 212}

$$\frac{7\sqrt{3x^2-x+2}}{169(2x+1)}-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}-\frac{581\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{676\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] -1/26*Sqrt[2 - x + 3*x^2]/(1 + 2*x)^2 + (7*Sqrt[2 - x + 3*x^2])/(169*(1 + 2
*x)) - (581*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(676*Sqrt[
13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
```

```
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx = -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2} - \frac{1}{26}\int \frac{-\frac{35}{2}-49x}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx$$

$$= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2} + \frac{7\sqrt{2-x+3x^2}}{169(1+2x)} + \frac{581}{676}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx$$

$$= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2} + \frac{7\sqrt{2-x+3x^2}}{169(1+2x)} - \frac{581}{338}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx, x, \frac{9 \ldots}{\sqrt{2-\ldots}}\right)$$

$$= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2} + \frac{7\sqrt{2-x+3x^2}}{169(1+2x)} - \frac{581\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{676\sqrt{13}}$$

Mathematica [A]
time = 0.33, size = 77, normalized size = 0.87

$$\frac{\frac{13(1+28x)\sqrt{2-x+3x^2}}{(1+2x)^2} + 581\sqrt{13}\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{4394}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] ((13*(1 + 28*x)*Sqrt[2 - x + 3*x^2])/(1 + 2*x)^2 + 581*Sqrt[13]*ArcTanh[(Sq
rt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/4394
```

Maple [A]
time = 0.13, size = 74, normalized size = 0.83

method	result
risch	$\frac{84x^3-25x^2+55x+2}{338(2x+1)^2\sqrt{3x^2-x+2}}-\frac{581\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{8788}$
default	$-\frac{581\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{8788}+\frac{7\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}{338\left(x+\frac{1}{2}\right)}-\frac{\sqrt{3\left(x+\frac{1}{2}\right)^2-4x-}}{104\left(x+\frac{1}{2}\right)^2}$
trager	$\frac{(28x+1)\sqrt{3x^2-x+2}}{338(2x+1)^2}-\frac{581\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{-8\,\mathrm{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}+9\,\mathrm{RootOf}\left(\_Z^2-13\right)}{2x+1}\right)}{8788}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^3/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -581/8788*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1
/2))+7/338/(x+1/2)*(3*(x+1/2)^2-4*x+5/4)^(1/2)-1/104/(x+1/2)^2*(3*(x+1/2)^2
-4*x+5/4)^(1/2)
```

Maxima [A]
time = 0.51, size = 82, normalized size = 0.92

$$\frac{581}{8788}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)-\frac{\sqrt{3\,x^2-x+2}}{26\,(4\,x^2+4\,x+1)}+\frac{7\sqrt{3\,x^2-x+2}}{169\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 581/8788*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(
2*x + 1)) - 1/26*sqrt(3*x^2 - x + 2)/(4*x^2 + 4*x + 1) + 7/169*sqrt(3*x^2 -
 x + 2)/(2*x + 1)
```

Fricas [A]
time = 0.34, size = 96, normalized size = 1.08

$$\frac{581\sqrt{13}\,(4\,x^2+4\,x+1)\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}\,(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)+52\sqrt{3\,x^2-x+2}\,(28\,x+1)}{17576\,(4\,x^2+4\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/17576*(581*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2
)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 52*sqrt(3*x^2 - x
 + 2)*(28*x + 1))/(4*x^2 + 4*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)^3 \sqrt{3x^2 - x + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*sqrt(3*x**2 - x + 2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(71) = 142.
time = 4.71, size = 204, normalized size = 2.29

$$\frac{581}{8788} \sqrt{13} \log \left(-\frac{\left|-4 \sqrt{3} x - 2 \sqrt{13} - 2 \sqrt{3} + 4 \sqrt{3 x^2 - x + 2}\right|}{2 \left(2 \sqrt{3} x - \sqrt{13} + \sqrt{3} - 2 \sqrt{3 x^2 - x + 2}\right)}\right) + \frac{190 \left(\sqrt{3} x - \sqrt{3 x^2 - x + 2}\right)^3 - 53 \sqrt{3} \left(\sqrt{3} x - \sqrt{3 x^2 - x + 2}\right)^2 - 489 \sqrt{3} x + 289 \sqrt{3} + 489 \sqrt{3 x^2 - x + 2}}{338 \left(2 \left(\sqrt{3} x - \sqrt{3 x^2 - x + 2}\right)^2 + 2 \sqrt{3} \left(\sqrt{3} x - \sqrt{3 x^2 - x + 2}\right) - 5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 581/8788*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sq
rt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2
))) + 1/338*(190*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 - 53*sqrt(3)*(sqrt(3)*
x - sqrt(3*x^2 - x + 2))^2 - 489*sqrt(3)*x + 289*sqrt(3) + 489*sqrt(3*x^2 -
 x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3)*(sqrt(3)*x - sq
rt(3*x^2 - x + 2)) - 5)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2 + 3\,x + 1}{{\left(2\,x + 1\right)}^3\,\sqrt{3\,x^2 - x + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)), x)
```

3.246 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}}+\frac{746}{81}\sqrt{2-x+3x^2}+\frac{412}{81}x\sqrt{2-x+3x^2}+\frac{32}{27}x^2\sqrt{2-x+3x^2}+\frac{353\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{81\sqrt{3}}$$

```
[Out] 353/243*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2/1863*(12839-3871*x)/(3*x^2
-x+2)^(1/2)+746/81*(3*x^2-x+2)^(1/2)+412/81*x*(3*x^2-x+2)^(1/2)+32/27*x^2*(
3*x^2-x+2)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1674, 1675, 654, 633, 221}

$$\frac{32}{27}\sqrt{3x^2-x+2}\,x^2+\frac{412}{81}\sqrt{3x^2-x+2}\,x+\frac{746}{81}\sqrt{3x^2-x+2}+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}+\frac{353\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{81\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(12839 - 3871*x))/(1863*Sqrt[2 - x + 3*x^2]) + (746*Sqrt[2 - x + 3*x^2])
/81 + (412*x*Sqrt[2 - x + 3*x^2])/81 + (32*x^2*Sqrt[2 - x + 3*x^2])/27 + (3
53*ArcSinh[(1 - 6*x)/Sqrt[23]])/(81*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}}+\frac{2}{23}\int \frac{\frac{1127}{81}+\frac{7682x}{27}+\frac{2852x^2}{9}+\frac{368x^3}{3}}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}}+\frac{32}{27}x^2\sqrt{2-x+3x^2}+\frac{2}{207}\int \frac{\frac{1127}{9}+2070x+\frac{9476x}{3}}{\sqrt{2-x+3x^2}}$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}}+\frac{412}{81}x\sqrt{2-x+3x^2}+\frac{32}{27}x^2\sqrt{2-x+3x^2}+\frac{1}{621}$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}}+\frac{746}{81}\sqrt{2-x+3x^2}+\frac{412}{81}x\sqrt{2-x+3x^2}+\frac{32}{27}x^2$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}}+\frac{746}{81}\sqrt{2-x+3x^2}+\frac{412}{81}x\sqrt{2-x+3x^2}+\frac{32}{27}x^2$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}}+\frac{746}{81}\sqrt{2-x+3x^2}+\frac{412}{81}x\sqrt{2-x+3x^2}+\frac{32}{27}x^2$$

Mathematica [A]

time = 0.49, size = 70, normalized size = 0.68

$$\frac{\frac{6(29997-2974x+23207x^2+13110x^3+3312x^4)}{\sqrt{2-x+3x^2}}+8119\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{5589}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] ((6*(29997 - 2974*x + 23207*x^2 + 13110*x^3 + 3312*x^4))/Sqrt[2 - x + 3*x^2
] + 8119*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/5589
```

Maple [A]
time = 0.11, size = 115, normalized size = 1.12

method	result
risch	$\frac{\frac{32}{9}x^4+\frac{380}{27}x^3+\frac{2018}{81}x^2-\frac{5948}{1863}x+\frac{2222}{69}}{\sqrt{3x^2-x+2}}-\frac{353\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{243}$
trager	$\frac{\frac{32}{9}x^4+\frac{380}{27}x^3+\frac{2018}{81}x^2-\frac{5948}{1863}x+\frac{2222}{69}}{\sqrt{3x^2-x+2}}-\frac{353\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}-\mathrm{RootOf}\left(\_Z^2-3\right.\right.}{243}$
default	$\frac{557}{18\sqrt{3x^2-x+2}}-\frac{353\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{243}+\frac{2018x^2}{81\sqrt{3x^2-x+2}}+\frac{353x}{81\sqrt{3x^2-x+2}}+\frac{3}{9\sqrt{3x^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 557/18/(3*x^2-x+2)^(1/2)-353/243*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+201
8/81*x^2/(3*x^2-x+2)^(1/2)+353/81*x/(3*x^2-x+2)^(1/2)+32/9*x^4/(3*x^2-x+2)^
(1/2)+380/27*x^3/(3*x^2-x+2)^(1/2)-521/414*(6*x-1)/(3*x^2-x+2)^(1/2)
```

Maxima [A]
time = 0.51, size = 97, normalized size = 0.94

$$\frac{32\,x^4}{9\,\sqrt{3\,x^2-x+2}}+\frac{380\,x^3}{27\,\sqrt{3\,x^2-x+2}}+\frac{2018\,x^2}{81\,\sqrt{3\,x^2-x+2}}-\frac{353}{243}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(6\,x-1)\right)-\frac{5948\,x}{1863\,\sqrt{3\,x^2-x+2}}+\frac{2222}{69\,\sqrt{3\,x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 32/9*x^4/sqrt(3*x^2 - x + 2) + 380/27*x^3/sqrt(3*x^2 - x + 2) + 2018/81*x^2
/sqrt(3*x^2 - x + 2) - 353/243*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 5
948/1863*x/sqrt(3*x^2 - x + 2) + 2222/69/sqrt(3*x^2 - x + 2)
```

Fricas [A]
time = 0.37, size = 97, normalized size = 0.94

$$\frac{8119\sqrt{3}\,(3\,x^2-x+2)\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}\,(6\,x-1)-72\,x^2+24\,x-25\right)+12\,(3312\,x^4+13110\,x^3+23207\,x^2-2974\,x+29997)\sqrt{3\,x^2-x+2}}{11178\,(3\,x^2-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/11178*(8119*sqrt(3)*(3*x^2 - x + 2)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*
x - 1) - 72*x^2 + 24*x - 25) + 12*(3312*x^4 + 13110*x^3 + 23207*x^2 - 2974*
x + 29997)*sqrt(3*x^2 - x + 2))/(3*x^2 - x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(2x+1)^3\cdot(4x^2+3x+1)}{(3x^2-x+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

Giac [A]
time = 4.12, size = 67, normalized size = 0.65

$$\frac{353}{243}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^2-x+2}\right)+1\right)+\frac{2\,((23\,(6\,(24\,x+95)x+1009)x-2974)x+29997)}{1863\sqrt{3\,x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 353/243*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/1
863*((23*(6*(24*x + 95)*x + 1009)*x - 2974)*x + 29997)/sqrt(3*x^2 - x + 2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(2\,x+1)^3\,(4\,x^2+3\,x+1)}{(3\,x^2-x+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
```

```
[Out] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

3.247 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{112}{27}\sqrt{2-x+3x^2}+\frac{8}{9}x\sqrt{2-x+3x^2}-\frac{64\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

```
[Out] -64/27*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2/621*(1249-2273*x)/(3*x^2-x+
2)^(1/2)+112/27*(3*x^2-x+2)^(1/2)+8/9*x*(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1674, 1675, 654, 633, 221}

$$\frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}+\frac{8}{9}x\sqrt{3x^2-x+2}+\frac{112}{27}\sqrt{3x^2-x+2}-\frac{64\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(1249 - 2273*x))/(621*Sqrt[2 - x + 3*x^2]) + (112*Sqrt[2 - x + 3*x^2])/2
7 + (8*x*Sqrt[2 - x + 3*x^2])/9 - (64*ArcSinh[(1 - 6*x)/Sqrt[23]])/(9*Sqrt[
3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{2}{23}\int \frac{\frac{2116}{27}+\frac{1150x}{9}+\frac{184x^2}{3}}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{8}{9}x\sqrt{2-x+3x^2}+\frac{1}{69}\int \frac{\frac{3128}{9}+\frac{2576x}{3}}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{112}{27}\sqrt{2-x+3x^2}+\frac{8}{9}x\sqrt{2-x+3x^2}+\frac{64}{9}\int \frac{}{\sqrt{2}}$$

$$64\text{Subst}$$

$$= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{112}{27}\sqrt{2-x+3x^2}+\frac{8}{9}x\sqrt{2-x+3x^2}+ \text{———}$$

$$= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}}+\frac{112}{27}\sqrt{2-x+3x^2}+\frac{8}{9}x\sqrt{2-x+3x^2}-\frac{64\sinh^{-1}}{9}$$

Mathematica [A]
time = 0.44, size = 65, normalized size = 0.79

$$\frac{2(1275-1003x+1196x^2+276x^3)}{207\sqrt{2-x+3x^2}}-\frac{64\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(1275 - 1003*x + 1196*x^2 + 276*x^3))/(207*Sqrt[2 - x + 3*x^2]) - (64*Lo
g[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(9*Sqrt[3])
```

Maple [A]
time = 0.11, size = 98, normalized size = 1.20

method	result
risch	$\frac{\frac{8}{3}x^{3}+\frac{104}{9}x^{2}-\frac{2006}{207}x+\frac{850}{69}}{\sqrt{3x^{2}-x+2}}+\frac{64\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{27}$
trager	$\frac{\frac{8}{3}x^{3}+\frac{104}{9}x^{2}-\frac{2006}{207}x+\frac{850}{69}}{\sqrt{3x^{2}-x+2}}-\frac{64\,\mathrm{RootOf}\left(\_Z^{2}-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^{2}-3\right)x+6\sqrt{3x^{2}-x+2}+\mathrm{RootOf}\left(\_Z^{2}-3\right)\right)}{27}$
default	$\frac{8x^{3}}{3\sqrt{3x^{2}-x+2}}+\frac{104x^{2}}{9\sqrt{3x^{2}-x+2}}-\frac{64x}{9\sqrt{3x^{2}-x+2}}+\frac{107}{9\sqrt{3x^{2}-x+2}}-\frac{89(6x-1)}{207\sqrt{3x^{2}-x+2}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/3*x^3/(3*x^2-x+2)^(1/2)+104/9*x^2/(3*x^2-x+2)^(1/2)-64/9*x/(3*x^2-x+2)^(1
/2)+107/9/(3*x^2-x+2)^(1/2)-89/207*(6*x-1)/(3*x^2-x+2)^(1/2)+64/27*3^(1/2)*
arcsinh(6/23*23^(1/2)*(x-1/6))
```

Maxima [A]
time = 0.51, size = 80, normalized size = 0.98

$$\frac{8\,x^{3}}{3\,\sqrt{3\,x^{2}-x+2}}+\frac{104\,x^{2}}{9\,\sqrt{3\,x^{2}-x+2}}+\frac{64}{27}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(6\,x-1)\right)-\frac{2006\,x}{207\,\sqrt{3\,x^{2}-x+2}}+\frac{850}{69\,\sqrt{3\,x^{2}-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^3/sqrt(3*x^2 - x + 2) + 104/9*x^2/sqrt(3*x^2 - x + 2) + 64/27*sqrt(3)
*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 2006/207*x/sqrt(3*x^2 - x + 2) + 850/69
/sqrt(3*x^2 - x + 2)
```

Fricas [A]
time = 0.36, size = 92, normalized size = 1.12

$$\frac{2\left(368\sqrt{3}\,(3\,x^{2}-x+2)\log\left(-4\sqrt{3}\,\sqrt{3\,x^{2}-x+2}\,(6\,x-1)-72\,x^{2}+24\,x-25\right)+3\,(276\,x^{3}+1196\,x^{2}-1003\,x+1275)\sqrt{3\,x^{2}-x+2}\right)}{621\,(3\,x^{2}-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/621*(368*sqrt(3)*(3*x^2 - x + 2)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
- 1) - 72*x^2 + 24*x - 25) + 3*(276*x^3 + 1196*x^2 - 1003*x + 1275)*sqrt(3*
x^2 - x + 2))/(3*x^2 - x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^2 \cdot (4x^2+3x+1)}{(3x^2-x+2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

Giac [A]
time = 3.97, size = 62, normalized size = 0.76

$$-\frac{64}{27}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2-x+2}\right)+1\right) + \frac{2\left(\left(92\left(3x+13\right)x-1003\right)x+1275\right)}{207\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -64/27*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/20
7*((92*(3*x + 13)*x - 1003)*x + 1275)/sqrt(3*x^2 - x + 2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+1)^2\,(4x^2+3x+1)}{(3x^2-x+2)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
```

```
[Out] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

3.248 $\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{8}{9}\sqrt{2-x+3x^2}-\frac{14\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}$$

```
[Out] -14/9*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-2/207*(73+367*x)/(3*x^2-x+2)^(
1/2)+8/9*(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1674, 654, 633, 221}

$$-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}}+\frac{8}{9}\sqrt{3x^2-x+2}-\frac{14\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (-2*(73 + 367*x))/(207*Sqrt[2 - x + 3*x^2]) + (8*Sqrt[2 - x + 3*x^2])/9 - (
14*ArcSinh[(1 - 6*x)/Sqrt[23]])/(3*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx &= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{2}{23}\int \frac{\frac{437}{9}+\frac{92x}{3}}{\sqrt{2-x+3x^2}}\,dx\\ &= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{8}{9}\sqrt{2-x+3x^2}+\frac{14}{3}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx\\ &= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{8}{9}\sqrt{2-x+3x^2}+\frac{14\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-\right.}{3\sqrt{69}}\\ &= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{8}{9}\sqrt{2-x+3x^2}-\frac{14\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 60, normalized size = 0.95

$$\frac{2(37-153x+92x^2)}{69\sqrt{2-x+3x^2}}-\frac{14\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(37 - 153*x + 92*x^2))/(69*Sqrt[2 - x + 3*x^2]) - (14*Log[1 - 6*x + 2*Sq
rt[6 - 3*x + 9*x^2]])/(3*Sqrt[3])
```

Maple [A]
time = 0.12, size = 81, normalized size = 1.29

method	result

risch	$\frac{\frac{8}{3}x^2-\frac{102}{23}x+\frac{74}{69}}{\sqrt{3x^2-x+2}}+\frac{14\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{9}$
trager	$\frac{\frac{8}{3}x^2-\frac{102}{23}x+\frac{74}{69}}{\sqrt{3x^2-x+2}}-\frac{14\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}+\mathrm{RootOf}\left(\_Z^2-3\right)\right)}{9}$
default	$\frac{8x^2}{3\sqrt{3x^2-x+2}}-\frac{14x}{3\sqrt{3x^2-x+2}}+\frac{10}{9\sqrt{3x^2-x+2}}+\frac{\frac{16x}{69}-\frac{8}{207}}{\sqrt{3x^2-x+2}}+\frac{14\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,(x-}{23}\right.}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/3*x^2/(3*x^2-x+2)^(1/2)-14/3*x/(3*x^2-x+2)^(1/2)+10/9/(3*x^2-x+2)^(1/2)+8
/207*(6*x-1)/(3*x^2-x+2)^(1/2)+14/9*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Maxima [A]
time = 0.50, size = 63, normalized size = 1.00

$$\frac{8\,x^2}{3\,\sqrt{3\,x^2-x+2}}+\frac{14}{9}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(6\,x-1)\right)-\frac{102\,x}{23\,\sqrt{3\,x^2-x+2}}+\frac{74}{69\,\sqrt{3\,x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^2/sqrt(3*x^2 - x + 2) + 14/9*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1))
 - 102/23*x/sqrt(3*x^2 - x + 2) + 74/69/sqrt(3*x^2 - x + 2)
```

Fricas [A]
time = 0.35, size = 87, normalized size = 1.38

$$\frac{161\,\sqrt{3}\,(3\,x^2-x+2)\log\left(-4\,\sqrt{3}\,\sqrt{3\,x^2-x+2}\,(6\,x-1)-72\,x^2+24\,x-25\right)+6\,(92\,x^2-153\,x+37)\sqrt{3\,x^2-x+2}}{207\,(3\,x^2-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/207*(161*sqrt(3)*(3*x^2 - x + 2)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
- 1) - 72*x^2 + 24*x - 25) + 6*(92*x^2 - 153*x + 37)*sqrt(3*x^2 - x + 2))/(
3*x^2 - x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(2x+1)\,(4x^2+3x+1)}{(3x^2-x+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

Giac [A]
time = 2.79, size = 57, normalized size = 0.90

$$-\frac{14}{9}\sqrt{3}\,\log\left(-2\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^2-x+2}\right)+1\right)+\frac{2\left((92\,x-153)x+37\right)}{69\sqrt{3\,x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -14/9*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/69*
((92*x - 153)*x + 37)/sqrt(3*x^2 - x + 2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(2\,x+1)\,\left(4\,x^2+3\,x+1\right)}{\left(3\,x^2-x+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
```

```
[Out] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

3.249 $\int \frac{1+3x+4x^2}{(1+2x)(2-x+3x^2)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}-\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\,\sqrt{2-x+3x^2}}\right)}{13\sqrt{13}}$$

```
[Out] -2/169*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-2/299*(101
-77*x)/(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1660, 12, 738, 212}

$$-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}}-\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\,\sqrt{3x^2-x+2}}\right)}{13\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (-2*(101 - 77*x))/(299*Sqrt[2 - x + 3*x^2]) - (2*ArcTanh[(9 - 8*x)/(2*Sqrt[
13]*Sqrt[2 - x + 3*x^2])])/(13*Sqrt[13])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx &= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}+\frac{2}{23}\int \frac{23}{13(1+2x)\sqrt{2-x+3x^2}}\,dx \\
&= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}+\frac{2}{13}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx \\
&= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}-\frac{4}{13}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right) \\
&= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}-\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{13\sqrt{13}}
\end{aligned}$$

Mathematica [A]
time = 0.37, size = 70, normalized size = 1.13

$$\frac{2(-101+77x)}{299\sqrt{2-x+3x^2}}+\frac{4\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}\,x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{13\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (2*(-101 + 77*x))/(299*Sqrt[2 - x + 3*x^2]) + (4*ArcTanh[(Sqrt[3] + 2*Sqrt[
3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(13*Sqrt[13])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(48) = 96$.
time = 0.14, size = 102, normalized size = 1.65

method	result

risch	$\frac{-\frac{202}{299}+\frac{154x}{299}}{\sqrt{3x^2-x+2}}-\frac{2\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{169}$
trager	$\frac{-\frac{202}{299}+\frac{154x}{299}}{\sqrt{3x^2-x+2}}+\frac{2\operatorname{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\operatorname{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}-9\operatorname{RootOf}\left(\_Z^2-13\right)}{2x+1}\right)}{169}$
default	$\frac{\frac{10x}{23}-\frac{5}{69}}{\sqrt{3x^2-x+2}}-\frac{2}{3\sqrt{3x^2-x+2}}+\frac{1}{13\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}+\frac{\frac{24x}{299}-\frac{4}{299}}{\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}-\frac{2\sqrt{13}\ \cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/69*(6*x-1)/(3*x^2-x+2)^(1/2)-2/3/(3*x^2-x+2)^(1/2)+1/13/(3*(x+1/2)^2-4*x+
5/4)^(1/2)+4/299*(6*x-1)/(3*(x+1/2)^2-4*x+5/4)^(1/2)-2/169*13^(1/2)*arctanh
(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

Maxima [A]
time = 0.51, size = 64, normalized size = 1.03

$$\frac{2}{169}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{154\,x}{299\sqrt{3\,x^2-x+2}}-\frac{202}{299\sqrt{3\,x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/169*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x
 + 1)) + 154/299*x/sqrt(3*x^2 - x + 2) - 202/299/sqrt(3*x^2 - x + 2)
```

Fricas [A]
time = 0.38, size = 96, normalized size = 1.55

$$\frac{23\sqrt{13}\,(3\,x^2-x+2)\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}\,(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)+26\sqrt{3\,x^2-x+2}\,(77\,x-101)}{3887\,(3\,x^2-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3887*(23*sqrt(13)*(3*x^2 - x + 2)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8
*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 26*sqrt(3*x^2 - x + 2
)*(77*x - 101))/(3*x^2 - x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)\left(3x^2 - x + 2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 - x + 2)**(3/2)), x)
```

Giac [A]
time = 5.32, size = 91, normalized size = 1.47

$$\frac{2}{169}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x - 2\sqrt{13} - 2\sqrt{3} + 4\sqrt{3x^2 - x + 2}\right|}{2\left(2\sqrt{3}x - \sqrt{13} + \sqrt{3} - 2\sqrt{3x^2 - x + 2}\right)}\right) + \frac{2(77x - 101)}{299\sqrt{3x^2 - x + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/169*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(
3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
 + 2/299*(77*x - 101)/sqrt(3*x^2 - x + 2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{4\,x^2 + 3\,x + 1}{(2\,x + 1)\,\left(3\,x^2 - x + 2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(3/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(3/2)), x)
```

3.250 $\int \frac{1+3x+4x^2}{(1+2x)^2(2-x+3x^2)^{3/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}+\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{169\sqrt{13}}$$

```
[Out] 2/2197*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-2/3887*(19
7-837*x)/(3*x^2-x+2)^(1/2)-4/169*(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 820, 738, 212}

$$-\frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{169(2x+1)}+\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (-2*(197 - 837*x))/(3887*Sqrt[2 - x + 3*x^2]) - (4*Sqrt[2 - x + 3*x^2])/(16
9*(1 + 2*x)) + (2*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(169
*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
```

```
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx &= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}+\frac{2}{23}\int \frac{\frac{184}{169}-\frac{230x}{169}}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx\\ &= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}-\frac{2}{169}\int \frac{1}{(1+2x)\sqrt{2-x+3}\end{aligned}$$

$$= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}+\frac{4}{169}\mathrm{Subst}\left(\int \frac{1}{52-x^2}\,dx,x\right.$$

$$= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}+\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+}}\right.}{169\sqrt{13}}$$

Mathematica [A]
time = 0.40, size = 90, normalized size = 1.03

$$\frac{2\sqrt{2-x+3x^2}\left(-289+489x+1536x^2\right)}{3887\left(2+3x+x^2+6x^3\right)}-\frac{4\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (2*Sqrt[2 - x + 3*x^2]*(-289 + 489*x + 1536*x^2))/(3887*(2 + 3*x + x^2 + 6*
x^3)) - (4*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]
])/(169*Sqrt[13])
```

Maple [A]
time = 0.13, size = 109, normalized size = 1.25

method	result
risch	$\frac{\frac{3072}{3887}x^2+\frac{978}{3887}x-\frac{578}{3887}}{(2x+1)\sqrt{3x^2-x+2}}+\frac{2\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{2197}$
trager	$\frac{2(1536x^2+489x-289)\sqrt{3x^2-x+2}}{3887(6x^3+x^2+3x+2)}-\frac{2\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}-9\,\mathrm{RootOf}\left(\_Z\right.}{2x+1}\right.}{2197}$
default	$\frac{\frac{12x}{23}-\frac{2}{23}}{\sqrt{3x^2-x+2}}-\frac{1}{169\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}-\frac{82(6x-1)}{3887\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}+\frac{2\sqrt{13}\,\operatorname{arctanh}\left(\frac{}{13\sqrt{1}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^2/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/23*(6*x-1)/(3*x^2-x+2)^(1/2)-1/169/(3*(x+1/2)^2-4*x+5/4)^(1/2)-82/3887*(6
*x-1)/(3*(x+1/2)^2-4*x+5/4)^(1/2)+2/2197*13^(1/2)*arctanh(2/13*(9/2-4*x)*13
^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))-1/26/(x+1/2)/(3*(x+1/2)^2-4*x+5/4)^(1/2
)
```

Maxima [A]
time = 0.52, size = 96, normalized size = 1.10

$$-\frac{2}{2197}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2x+1|}-\frac{9\sqrt{23}}{23\,|2x+1|}\right)+\frac{1536\,x}{3887\sqrt{3x^2-x+2}}-\frac{279}{3887\sqrt{3x^2-x+2}}-\frac{1}{13\left(2\sqrt{3x^2-x+2}\,x+\sqrt{3x^2-x+2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/2197*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2
*x + 1)) + 1536/3887*x/sqrt(3*x^2 - x + 2) - 279/3887/sqrt(3*x^2 - x + 2) -
 1/13/(2*sqrt(3*x^2 - x + 2)*x + sqrt(3*x^2 - x + 2))
```

Fricas [A]
time = 0.37, size = 106, normalized size = 1.22

$$\frac{23\sqrt{13}\left(6x^3+x^2+3x+2\right)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}\,(8x-9)-220x^2+196x-185}{4x^2+4x+1}\right)+26\left(1536x^2+489x-289\right)\sqrt{3x^2-x+2}}{50531\left(6x^3+x^2+3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/50531*(23*sqrt(13)*(6*x^3 + x^2 + 3*x + 2)*log((4*sqrt(13)*sqrt(3*x^2 - x
 + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 26*(1536*x^2
+ 489*x - 289)*sqrt(3*x^2 - x + 2))/(6*x^3 + x^2 + 3*x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)^2\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 - x + 2)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(69) = 138.
time = 3.86, size = 168, normalized size = 1.93

$$-\frac{2}{50531}\sqrt{13}\left(256\sqrt{13}\sqrt{3}+23\log\left(\sqrt{13}\sqrt{3}-4\right)\right)\mathrm{sgn}\left(\frac{1}{2x+1}\right)-\frac{2\left(\frac{\frac{1047}{\mathrm{sgn}\left(\frac{1}{2x+1}\right)}+\frac{299}{(2x+1)\mathrm{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}-\frac{768}{\mathrm{sgn}\left(\frac{1}{2x+1}\right)}\right)}{3887\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}}+\frac{2\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)}{2197\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -2/50531*sqrt(13)*(256*sqrt(13)*sqrt(3) + 23*log(sqrt(13)*sqrt(3) - 4))*sgn
(1/(2*x + 1)) - 2/3887*((1047/sgn(1/(2*x + 1)) + 299/((2*x + 1)*sgn(1/(2*x
+ 1))))/(2*x + 1) - 768/sgn(1/(2*x + 1)))/sqrt(-8/(2*x + 1) + 13/(2*x + 1)^
2 + 3) + 2/2197*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 +
 3) + sqrt(13)/(2*x + 1)) - 4)/sgn(1/(2*x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^2\,{\left(3\,x^2-x+2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)), x)
```

3.251 $\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{3/2}}\,dx$

Optimal. Leaf size=112

$$\frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}-\frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2}-\frac{4\sqrt{2-x+3x^2}}{2197(1+2x)}-\frac{487\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2197\sqrt{13}}$$

```
[Out] -487/28561*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+2/5053
1*(2363+3693*x)/(3*x^2-x+2)^(1/2)-2/169*(3*x^2-x+2)^(1/2)/(1+2*x)^2-4/2197*
(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A]
time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1660, 1664, 820, 738, 212}

$$\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{2197(2x+1)}-\frac{2\sqrt{3x^2-x+2}}{169(2x+1)^2}-\frac{487\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (2*(2363 + 3693*x))/(50531*Sqrt[2 - x + 3*x^2]) - (2*Sqrt[2 - x + 3*x^2])/(
169*(1 + 2*x)^2) - (4*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)) - (487*ArcTanh[
(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2197*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
```

```
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}+\frac{2}{23}\int \frac{\frac{8349}{2197}+\frac{20838x}{2197}+\frac{23828x^2}{2197}}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}-\frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2}-\frac{1}{299}\int \frac{-\frac{11615}{169}-\frac{22034x}{169}}{(1+2x)^2\sqrt{2-x+3x}}$$

$$= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}-\frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2}-\frac{4\sqrt{2-x+3x^2}}{2197(1+2x)}+\frac{487\int \frac{}{(1+2}}{}$$

$$= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}-\frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2}-\frac{4\sqrt{2-x+3x^2}}{2197(1+2x)}-\frac{974\text{Subst}(}{}$$

$$= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}-\frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2}-\frac{4\sqrt{2-x+3x^2}}{2197(1+2x)}-\frac{487\tanh^{-}}{}$$

Mathematica [A]
time = 0.48, size = 87, normalized size = 0.78

$$\frac{2(1673+13306x+23281x^2+14496x^3)}{50531(1+2x)^2\sqrt{2-x+3x^2}}+\frac{974\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{2197\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (2*(1673 + 13306*x + 23281*x^2 + 14496*x^3))/(50531*(1 + 2*x)^2*Sqrt[2 - x
+ 3*x^2]) + (974*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sq
rt[13]])/(2197*Sqrt[13])
```

Maple [A]
time = 0.12, size = 111, normalized size = 0.99

method	result
risch	$\frac{\frac{28992}{50531}x^3+\frac{46562}{50531}x^2+\frac{26612}{50531}x+\frac{3346}{50531}}{(2x+1)^2\sqrt{3x^2-x+2}}-\frac{487\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{28561}$
trager	$\frac{\frac{28992}{50531}x^3+\frac{46562}{50531}x^2+\frac{26612}{50531}x+\frac{3346}{50531}}{(2x+1)^2\sqrt{3x^2-x+2}}-\frac{487\,\text{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{-8\,\text{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}+9\,\text{RootOf}\left(\_Z^2-\right.}{2x+1}\right.}{28561}$

default	$\frac{487}{4394\sqrt{3\left(x+\frac{1}{2}\right)^{2}-4x+\frac{5}{4}}}+\frac{\frac{7248x}{50531}-\frac{1208}{50531}}{\sqrt{3\left(x+\frac{1}{2}\right)^{2}-4x+\frac{5}{4}}}-\frac{487\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^{2}-16x\cdots}}\right.}{28561}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^3/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 487/4394/(3*(x+1/2)^2-4*x+5/4)^(1/2)+1208/50531*(6*x-1)/(3*(x+1/2)^2-4*x+5/
4)^(1/2)-487/28561*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-1
6*x+5)^(1/2))+3/338/(x+1/2)/(3*(x+1/2)^2-4*x+5/4)^(1/2)-1/104/(x+1/2)^2/(3*
(x+1/2)^2-4*x+5/4)^(1/2)
```

Maxima [A]
time = 0.52, size = 145, normalized size = 1.29

$$\frac{487}{28561}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{7248\,x}{50531\sqrt{3\,x^2-x+2}}+\frac{8785}{101062\sqrt{3\,x^2-x+2}}-\frac{1}{26\left(4\sqrt{3\,x^2-x+2}\,x^2+4\sqrt{3\,x^2-x+2}\,x+\sqrt{3\,x^2-x+2}\right)}+\frac{3}{169\left(2\sqrt{3\,x^2-x+2}\,x+\sqrt{3\,x^2-x+2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 487/28561*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs
(2*x + 1)) + 7248/50531*x/sqrt(3*x^2 - x + 2) + 8785/101062/sqrt(3*x^2 - x
+ 2) - 1/26/(4*sqrt(3*x^2 - x + 2)*x^2 + 4*sqrt(3*x^2 - x + 2)*x + sqrt(3*x
^2 - x + 2)) + 3/169/(2*sqrt(3*x^2 - x + 2)*x + sqrt(3*x^2 - x + 2))
```

Fricas [A]
time = 0.39, size = 126, normalized size = 1.12

$$\frac{11201\sqrt{13}\,(12\,x^4+8\,x^3+7\,x^2+7\,x+2)\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}\,(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)+52\,(14496\,x^3+23281\,x^2+13306\,x+1673)\sqrt{3\,x^2-x+2}}{1313806\,(12\,x^4+8\,x^3+7\,x^2+7\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1313806*(11201*sqrt(13)*(12*x^4 + 8*x^3 + 7*x^2 + 7*x + 2)*log(-(4*sqrt(1
3)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)
) + 52*(14496*x^3 + 23281*x^2 + 13306*x + 1673)*sqrt(3*x^2 - x + 2))/(12*x^
4 + 8*x^3 + 7*x^2 + 7*x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)^3\,(3x^2-x+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*(3*x**2 - x + 2)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(90) = 180.
time = 4.72, size = 223, normalized size = 1.99

$$\frac{487}{28561}\,\sqrt{13}\,\log\left(-\frac{\left|-4\,\sqrt{3}\,x-2\,\sqrt{13}-2\,\sqrt{3}+4\,\sqrt{3\,x^{2}-x+2}\right|}{2\,\left(2\,\sqrt{3}\,x-\sqrt{13}+\sqrt{3}-2\,\sqrt{3\,x^{2}-x+2}\right)}\right)+\frac{2\,(3693\,x+2363)}{50531\,\sqrt{3\,x^{2}-x+2}}+\frac{2\,\left(62\,\left(\sqrt{3}\,x-\sqrt{3\,x^{2}-x+2}\right)^{3}-37\,\sqrt{3}\,\left(\sqrt{3}\,x-\sqrt{3\,x^{2}-x+2}\right)^{2}+263\,\sqrt{3}\,x-71\,\sqrt{3}-263\,\sqrt{3\,x^{2}-x+2}\right)}{2197\,\left(2\,\left(\sqrt{3}\,x-\sqrt{3\,x^{2}-x+2}\right)^{2}+2\,\sqrt{3}\,\left(\sqrt{3}\,x-\sqrt{3\,x^{2}-x+2}\right)-5\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 487/28561*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*s
qrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x +
2))) + 2/50531*(3693*x + 2363)/sqrt(3*x^2 - x + 2) + 2/2197*(62*(sqrt(3)*x
- sqrt(3*x^2 - x + 2))^3 - 37*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 +
 263*sqrt(3)*x - 71*sqrt(3) - 263*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt
(3*x^2 - x + 2))^2 + 2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^3\,{\left(3\,x^2-x+2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)), x)
```

3.252 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx$

Optimal. Leaf size=86

$$\frac{2(12839-3871x)}{5589(2-x+3x^2)^{3/2}}-\frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}}+\frac{32}{27}\sqrt{2-x+3x^2}-\frac{296\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{27\sqrt{3}}$$

```
[Out] 2/5589*(12839-3871*x)/(3*x^2-x+2)^(3/2)-296/81*arcsinh(1/23*(1-6*x)*23^(1/2
))*3^(1/2)-28/128547*(35809+42240*x)/(3*x^2-x+2)^(1/2)+32/27*(3*x^2-x+2)^(1
/2)
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1674, 654, 633, 221}

$$\frac{2(12839-3871x)}{5589(3x^2-x+2)^{3/2}}+\frac{32}{27}\sqrt{3x^2-x+2}-\frac{28(42240x+35809)}{128547\sqrt{3x^2-x+2}}-\frac{296\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{27\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(12839 - 3871*x))/(5589*(2 - x + 3*x^2)^(3/2)) - (28*(35809 + 42240*x))/
(128547*Sqrt[2 - x + 3*x^2]) + (32*Sqrt[2 - x + 3*x^2])/27 - (296*ArcSinh[(
1 - 6*x)/Sqrt[23]])/(27*Sqrt[3])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}} + \frac{2}{69}\int \frac{-\frac{4361}{81}+\frac{7682x}{9}+\frac{2852x^2}{3}+368x^3}{\left(2-x+3x^2\right)^{3/2}}\,dx$$

$$= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}} - \frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}} + \frac{4\int \frac{\frac{37030}{9}+\frac{4232x}{3}}{\sqrt{2-x+3x^2}}\,dx}{1587}$$

$$= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}} - \frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}} + \frac{32}{27}\sqrt{2-x+3x^2} + \frac{296}{27}$$

$$296$$

$$= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}} - \frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}} + \frac{32}{27}\sqrt{2-x+3x^2} + \frac{}{}$$

$$= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}} - \frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}} + \frac{32}{27}\sqrt{2-x+3x^2} - \frac{296}{}$$

Mathematica [A]
time = 0.68, size = 70, normalized size = 0.81

$$\frac{2\left(-44739-119459x+8630x^2-247904x^3+76176x^4\right)}{14283\left(2-x+3x^2\right)^{3/2}} - \frac{296\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{27\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(-44739 - 119459*x + 8630*x^2 - 247904*x^3 + 76176*x^4))/(14283*(2 - x +
 3*x^2)^(3/2)) - (296*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(27*Sqrt[3])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(69) = 138$.
time = 0.13, size = 163, normalized size = 1.90

method	result
risch	$\frac{\frac{32}{3}x^4-\frac{495808}{14283}x^3+\frac{17260}{14283}x^2-\frac{238918}{14283}x-\frac{3314}{529}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{296\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\,(x-\frac{1}{6})}{23}\right)}{81}$
trager	$\frac{\frac{32}{3}x^4-\frac{495808}{14283}x^3+\frac{17260}{14283}x^2-\frac{238918}{14283}x-\frac{3314}{529}}{(3x^2-x+2)^{\frac{3}{2}}}-\frac{296\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}+\mathrm{RootOf}\right.}{81}$
default	$-\frac{148}{81\sqrt{3x^2-x+2}}-\frac{1727}{1458(3x^2-x+2)^{\frac{3}{2}}}+\frac{296\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\,(x-\frac{1}{6})}{23}\right)}{81}-\frac{296x}{27\sqrt{3x^2-x+2}}+\frac{\frac{130528x}{42849}}{\sqrt{3x^2-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -148/81/(3*x^2-x+2)^(1/2)-1727/1458/(3*x^2-x+2)^(3/2)+296/81*3^(1/2)*arcsin
h(6/23*23^(1/2)*(x-1/6))-296/27*x/(3*x^2-x+2)^(1/2)+65264/128547*(6*x-1)/(3
*x^2-x+2)^(1/2)+13763/33534*(6*x-1)/(3*x^2-x+2)^(3/2)+8/27*x^2/(3*x^2-x+2)^
(3/2)-461/81*x/(3*x^2-x+2)^(3/2)+32/3*x^4/(3*x^2-x+2)^(3/2)-296/27*x^3/(3*x
^2-x+2)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(69) = 138$.
time = 0.51, size = 202, normalized size = 2.35

$$\frac{32\,x^4}{3\,(3\,x^2-x+2)^{\frac{3}{2}}}+\frac{296}{42849}\,x\left(\frac{426\,x}{\sqrt{3\,x^2-x+2}}-\frac{4761\,x^2}{(3\,x^2-x+2)^{\frac{3}{2}}}-\frac{71}{\sqrt{3\,x^2-x+2}}+\frac{805\,x}{(3\,x^2-x+2)^{\frac{3}{2}}}-\frac{2162}{(3\,x^2-x+2)^{\frac{3}{2}}}\right)+\frac{296}{81}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(6\,x-1)\right)-\frac{42032}{42849}\sqrt{3\,x^2-x+2}-\frac{47072\,x}{42849\sqrt{3\,x^2-x+2}}+\frac{52\,x^2}{9\,(3\,x^2-x+2)^{\frac{3}{2}}}-\frac{23104}{14283\sqrt{3\,x^2-x+2}}-\frac{7742\,x}{1863\,(3\,x^2-x+2)^{\frac{3}{2}}}+\frac{1666}{1863\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/3*x^4/(3*x^2 - x + 2)^(3/2) + 296/42849*x*(426*x/sqrt(3*x^2 - x + 2) - 4
761*x^2/(3*x^2 - x + 2)^(3/2) - 71/sqrt(3*x^2 - x + 2) + 805*x/(3*x^2 - x +
 2)^(3/2) - 2162/(3*x^2 - x + 2)^(3/2)) + 296/81*sqrt(3)*arcsinh(1/23*sqrt(
23)*(6*x - 1)) - 42032/42849*sqrt(3*x^2 - x + 2) - 47072/42849*x/sqrt(3*x^2
 - x + 2) + 52/9*x^2/(3*x^2 - x + 2)^(3/2) - 23104/14283/sqrt(3*x^2 - x + 2
) - 7742/1863*x/(3*x^2 - x + 2)^(3/2) + 1666/1863/(3*x^2 - x + 2)^(3/2)
```

Fricas [A]
time = 0.38, size = 117, normalized size = 1.36

$$\frac{2\left(39146\sqrt{3}\,(9\,x^4-6\,x^3+13\,x^2-4\,x+4)\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}\,(6\,x-1)-72\,x^2+24\,x-25\right)+3\,(76176\,x^4-247904\,x^3+8630\,x^2-119459\,x-44739)\sqrt{3\,x^2-x+2}\right)}{42849\,(9\,x^4-6\,x^3+13\,x^2-4\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/42849*(39146*sqrt(3)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-4*sqrt(3)*sq
rt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 3*(76176*x^4 - 247904*x
^3 + 8630*x^2 - 119459*x - 44739)*sqrt(3*x^2 - x + 2))/(9*x^4 - 6*x^3 + 13*
x^2 - 4*x + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^3 \cdot (4x^2+3x+1)}{(3x^2-x+2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

Giac [A]
time = 2.62, size = 67, normalized size = 0.78

$$-\frac{296}{81}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)+\frac{2\left((2\left(8\left(4761x-15494\right)x+4315\right)x-119459)x-44739\right)}{14283\left(3x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -296/81*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/1
4283*((2*(8*(4761*x - 15494)*x + 4315)*x - 119459)*x - 44739)/(3*x^2 - x +
2)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+1)^3\,(4x^2+3x+1)}{(3x^2-x+2)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
```

```
[Out] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2), x)
```

3.253 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx$

Optimal. Leaf size=68

$$\frac{2(1249-2273x)}{1863\,(2-x+3x^2)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}}-\frac{16\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

```
[Out] 2/1863*(1249-2273*x)/(3*x^2-x+2)^(3/2)-16/27*arcsinh(1/23*(1-6*x)*23^(1/2))
*3^(1/2)-8/42849*(23257-1473*x)/(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1674, 12, 633, 221}

$$\frac{2(1249-2273x)}{1863\,(3x^2-x+2)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{3x^2-x+2}}-\frac{16\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(1249 - 2273*x))/(1863*(2 - x + 3*x^2)^(3/2)) - (8*(23257 - 1473*x))/(42
849*Sqrt[2 - x + 3*x^2]) - (16*ArcSinh[(1 - 6*x)/Sqrt[23]])/(9*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx &= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{1802}{27}+\frac{1150x}{3}+184x^2}{\left(2-x+3x^2\right)^{3/2}}\,dx\\ &= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}}+\frac{4\int \frac{2116}{3\sqrt{2-x+3x^2}}\,dx}{1587}\\ &= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}}+\frac{16}{9}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx\\ &= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}}+\frac{16\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx\right.}{9\sqrt{69}}\\ &= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}}-\frac{16\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.58, size = 65, normalized size = 0.96

$$\frac{2(-17481+5837x-31664x^2+1964x^3)}{4761\left(2-x+3x^2\right)^{3/2}}-\frac{16\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
[Out] (2*(-17481 + 5837*x - 31664*x^2 + 1964*x^3))/(4761*(2 - x + 3*x^2)^(3/2)) -
 (16*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(9*Sqrt[3])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(55) = 110$.
time = 0.17, size = 146, normalized size = 2.15

method	result
risch	$\frac{\frac{3928}{4761}x^3-\frac{63328}{4761}x^2+\frac{11674}{4761}x-\frac{11654}{1587}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{16\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{27}$
trager	$\frac{\frac{3928}{4761}x^3-\frac{63328}{4761}x^2+\frac{11674}{4761}x-\frac{11654}{1587}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{16\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}-\mathrm{RootOf}\left(\_Z^2-3\right)\right)}{27}$
default	$-\frac{16x^3}{9(3x^2-x+2)^{\frac{3}{2}}}-\frac{92x^2}{9(3x^2-x+2)^{\frac{3}{2}}}-\frac{67x}{27(3x^2-x+2)^{\frac{3}{2}}}-\frac{2653}{486(3x^2-x+2)^{\frac{3}{2}}}+\frac{\frac{4585x}{1863}-\frac{4585}{11178}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{\frac{37784x}{14283}-\frac{18892}{42849}}{\sqrt{3x^2-x+2}}-\frac{}{9\sqrt{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/9*x^3/(3*x^2-x+2)^(3/2)-92/9*x^2/(3*x^2-x+2)^(3/2)-67/27*x/(3*x^2-x+2)^
(3/2)-2653/486/(3*x^2-x+2)^(3/2)+4585/11178*(6*x-1)/(3*x^2-x+2)^(3/2)+18892
/42849*(6*x-1)/(3*x^2-x+2)^(1/2)-16/9*x/(3*x^2-x+2)^(1/2)-8/27/(3*x^2-x+2)^
(1/2)+16/27*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(55) = 110$.
time = 0.53, size = 185, normalized size = 2.72

$$\frac{16}{14283}\,x\left(\frac{426\,x}{\sqrt{3\,x^2-x+2}}-\frac{4761\,x^2}{(3\,x^2-x+2)^{\frac{3}{2}}}-\frac{71}{\sqrt{3\,x^2-x+2}}+\frac{805\,x}{(3\,x^2-x+2)^{\frac{3}{2}}}-\frac{2162}{(3\,x^2-x+2)^{\frac{3}{2}}}\right)+\frac{16}{27}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(6\,x-1)\right)-\frac{2272}{14283}\sqrt{3\,x^2-x+2}+\frac{28184\,x}{14283\,\sqrt{3\,x^2-x+2}}-\frac{28\,x^2}{3\,(3\,x^2-x+2)^{\frac{3}{2}}}-\frac{2956}{4761\,\sqrt{3\,x^2-x+2}}-\frac{106\,x}{621\,(3\,x^2-x+2)^{\frac{3}{2}}}-\frac{3394}{621\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 16/14283*x*(426*x/sqrt(3*x^2 - x + 2) - 4761*x^2/(3*x^2 - x + 2)^(3/2) - 71
/sqrt(3*x^2 - x + 2) + 805*x/(3*x^2 - x + 2)^(3/2) - 2162/(3*x^2 - x + 2)^(
3/2)) + 16/27*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 2272/14283*sqrt(3*
x^2 - x + 2) + 28184/14283*x/sqrt(3*x^2 - x + 2) - 28/3*x^2/(3*x^2 - x + 2)
^(3/2) - 2956/4761/sqrt(3*x^2 - x + 2) - 106/621*x/(3*x^2 - x + 2)^(3/2) -
3394/621/(3*x^2 - x + 2)^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(55) = 110$.
time = 0.40, size = 112, normalized size = 1.65

$$\frac{2\left(2116\sqrt{3}\,(9\,x^4-6\,x^3+13\,x^2-4\,x+4)\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}\,(6\,x-1)-72\,x^2+24\,x-25\right)+3\,(1964\,x^3-31664\,x^2+5837\,x-17481)\sqrt{3\,x^2-x+2}\right)}{14283\,(9\,x^4-6\,x^3+13\,x^2-4\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/14283*(2116*sqrt(3)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-4*sqrt(3)*sqr
t(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 3*(1964*x^3 - 31664*x^2
+ 5837*x - 17481)*sqrt(3*x^2 - x + 2))/(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^2 \cdot (4x^2+3x+1)}{(3x^2-x+2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

Giac [A]
time = 3.18, size = 62, normalized size = 0.91

$$-\frac{16}{27}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^2-x+2}\right)+1\right)+\frac{2\left((4\,(491\,x-7916)x+5837)x-17481\right)}{4761\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -16/27*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/47
61*((4*(491*x - 7916)*x + 5837)*x - 17481)/(3*x^2 - x + 2)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{(2\,x+1)}^2\,(4\,x^2+3\,x+1)}{{(3\,x^2-x+2)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
```

```
[Out] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2), x)
```

3.254 $\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{2(73+367x)}{621\left(2-x+3x^2\right)^{3/2}}-\frac{4(3889-4290x)}{14283\sqrt{2-x+3x^2}}$$

```
[Out] -2/621*(73+367*x)/(3*x^2-x+2)^(3/2)-4/14283*(3889-4290*x)/(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1674, 650}

$$-\frac{4(3889-4290x)}{14283\sqrt{3x^2-x+2}}-\frac{2(367x+73)}{621\left(3x^2-x+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (-2*(73 + 367*x))/(621*(2 - x + 3*x^2)^(3/2)) - (4*(3889 - 4290*x))/(14283*
Sqrt[2 - x + 3*x^2])
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = -\frac{2(73+367x)}{621(2-x+3x^2)^{3/2}} + \frac{2}{69}\int \frac{\frac{577}{9}+92x}{(2-x+3x^2)^{3/2}}\,dx$$
$$= -\frac{2(73+367x)}{621(2-x+3x^2)^{3/2}} - \frac{4(3889-4290x)}{14283\sqrt{2-x+3x^2}}$$

Mathematica [A]
time = 0.41, size = 33, normalized size = 0.70

$$\frac{2(-1915+1833x-3546x^2+2860x^3)}{1587(2-x+3x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(-1915 + 1833*x - 3546*x^2 + 2860*x^3))/(1587*(2 - x + 3*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(39) = 78.
time = 0.18, size = 86, normalized size = 1.83

method	result	size
gosper	$\frac{\frac{5720}{1587}x^3-\frac{2364}{529}x^2+\frac{1222}{529}x-\frac{3830}{1587}}{(3x^2-x+2)^{\frac{3}{2}}}$	30
trager	$\frac{\frac{5720}{1587}x^3-\frac{2364}{529}x^2+\frac{1222}{529}x-\frac{3830}{1587}}{(3x^2-x+2)^{\frac{3}{2}}}$	30
risch	$\frac{\frac{5720}{1587}x^3-\frac{2364}{529}x^2+\frac{1222}{529}x-\frac{3830}{1587}}{(3x^2-x+2)^{\frac{3}{2}}}$	30
default	$-\frac{8x^2}{3(3x^2-x+2)^{\frac{3}{2}}}-\frac{13x}{9(3x^2-x+2)^{\frac{3}{2}}}-\frac{295}{162(3x^2-x+2)^{\frac{3}{2}}}+\frac{\frac{715x}{621}-\frac{715}{3726}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{\frac{5720x}{4761}-\frac{2860}{14283}}{\sqrt{3x^2-x+2}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -8/3*x^2/(3*x^2-x+2)^(3/2)-13/9*x/(3*x^2-x+2)^(3/2)-295/162/(3*x^2-x+2)^(3/
2)+715/3726*(6*x-1)/(3*x^2-x+2)^(3/2)+2860/14283*(6*x-1)/(3*x^2-x+2)^(1/2)
```

Maxima [A]
time = 0.31, size = 76, normalized size = 1.62

$$\frac{5720\,x}{4761\sqrt{3\,x^2-x+2}} - \frac{8\,x^2}{3\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{2860}{14283\sqrt{3\,x^2-x+2}} - \frac{182\,x}{621\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{1250}{621\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5720/4761*x/sqrt(3*x^2 - x + 2) - 8/3*x^2/(3*x^2 - x + 2)^(3/2) - 2860/1428
3/sqrt(3*x^2 - x + 2) - 182/621*x/(3*x^2 - x + 2)^(3/2) - 1250/621/(3*x^2 -
 x + 2)^(3/2)
```

Fricas [A]
time = 0.36, size = 51, normalized size = 1.09

$$\frac{2\left(2860\,x^3-3546\,x^2+1833\,x-1915\right)\sqrt{3\,x^2-x+2}}{1587\left(9\,x^4-6\,x^3+13\,x^2-4\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/1587*(2860*x^3 - 3546*x^2 + 1833*x - 1915)*sqrt(3*x^2 - x + 2)/(9*x^4 - 6
*x^3 + 13*x^2 - 4*x + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

Giac [A]
time = 3.92, size = 28, normalized size = 0.60

$$\frac{2\left(\left(2\left(1430\,x-1773\right)x+1833\right)x-1915\right)}{1587\left(3\,x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/1587*((2*(1430*x - 1773)*x + 1833)*x - 1915)/(3*x^2 - x + 2)^(3/2)
```

Mupad [B]
time = 4.20, size = 49, normalized size = 1.04

$$-\frac{442\,x-5720\,x\left(3\,x^2-x+2\right)+15556\,x^2+11490}{\sqrt{3\,x^2-x+2}\,\left(14283\,x^2-4761\,x+9522\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
```

```
[Out] -(442*x - 5720*x*(3*x^2 - x + 2) + 15556*x^2 + 11490)/((3*x^2 - x + 2)^(1/2
)*(14283*x^2 - 4761*x + 9522))
```

3.255 $\int \frac{1+3x+4x^2}{(1+2x)(2-x+3x^2)^{5/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}}-\frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}}-\frac{8\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{169\sqrt{13}}$$

```
[Out] -2/897*(101-77*x)/(3*x^2-x+2)^(3/2)-8/2197*arctanh(1/26*(9-8*x)*13^(1/2)/(3
*x^2-x+2)^(1/2))*13^(1/2)-4/268203*(691-13668*x)/(3*x^2-x+2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1660, 836, 12, 738, 212}

$$-\frac{4(691-13668x)}{268203\sqrt{3x^2-x+2}}-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}-\frac{8\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (-2*(101 - 77*x))/(897*(2 - x + 3*x^2)^(3/2)) - (4*(691 - 13668*x))/(268203
*Sqrt[2 - x + 3*x^2]) - (8*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2
])])/(169*Sqrt[13])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{223}{13}+\frac{308x}{13}}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx$$

$$= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} + \frac{4\int \frac{3174}{13(1+2x)\sqrt{2-x+3x}}}{20631}$$

$$= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} + \frac{8}{169}\int \frac{1}{(1+2x)\sqrt{2-x}}$$

$$= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} - \frac{16}{169}\text{Subst}\left(\int \frac{1}{52-x^2}\right.$$

$$= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} - \frac{8\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-}}\right.}{169\sqrt{13}}$$

Mathematica [A]
time = 0.50, size = 80, normalized size = 0.94

$$\frac{2(-32963+79077x-31482x^2+82008x^3)}{268203\,(2-x+3x^2)^{3/2}}+\frac{16\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}\,x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (2*(-32963 + 79077*x - 31482*x^2 + 82008*x^3))/(268203*(2 - x + 3*x^2)^(3/2
)) + (16*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])
/(169*Sqrt[13])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(67) = 134.
time = 0.12, size = 158, normalized size = 1.86

method	result
trager	$\frac{\frac{54672}{89401}x^3-\frac{20988}{89401}x^2+\frac{52718}{89401}x-\frac{65926}{268203}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}-9\,\mathrm{RootOf}\left(\_Z^2-1\right.}{2x+1}\right.}{2197}$
default	$\frac{\frac{10x}{69}-\frac{5}{207}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{\frac{80x}{529}-\frac{40}{1587}}{\sqrt{3x^2-x+2}}-\frac{2}{9(3x^2-x+2)^{\frac{3}{2}}}+\frac{1}{39\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{\frac{8x}{299}-\frac{4}{897}}{\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{4704}{8940}\ldots\sqrt{3\left(x+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/207*(6*x-1)/(3*x^2-x+2)^(3/2)+40/1587*(6*x-1)/(3*x^2-x+2)^(1/2)-2/9/(3*x^
2-x+2)^(3/2)+1/39/(3*(x+1/2)^2-4*x+5/4)^(3/2)+4/897*(6*x-1)/(3*(x+1/2)^2-4*
x+5/4)^(3/2)+784/89401*(6*x-1)/(3*(x+1/2)^2-4*x+5/4)^(1/2)+4/169/(3*(x+1/2)
^2-4*x+5/4)^(1/2)-8/2197*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/
2)^2-16*x+5)^(1/2))
```

Maxima [A]
time = 0.52, size = 93, normalized size = 1.09

$$\frac{8}{2197}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{18224\,x}{89401\sqrt{3\,x^2-x+2}}-\frac{2764}{268203\sqrt{3\,x^2-x+2}}+\frac{154\,x}{897\,(3\,x^2-x+2)^{\frac{3}{2}}}-\frac{202}{897\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/2197*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*
x + 1)) + 18224/89401*x/sqrt(3*x^2 - x + 2) - 2764/268203/sqrt(3*x^2 - x +
2) + 154/897*x/(3*x^2 - x + 2)^(3/2) - 202/897/(3*x^2 - x + 2)^(3/2)
```

Fricas [A]
time = 0.35, size = 126, normalized size = 1.48

$$\frac{2\left(3174\sqrt{13}\left(9x^4-6x^3+13x^2-4x+4\right)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+13\left(82008x^3-31482x^2+79077x-32963\right)\sqrt{3x^2-x+2}\right)}{3486639\left(9x^4-6x^3+13x^2-4x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3486639*(3174*sqrt(13)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-(4*sqrt(13
)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1))
 + 13*(82008*x^3 - 31482*x^2 + 79077*x - 32963)*sqrt(3*x^2 - x + 2))/(9*x^4
 - 6*x^3 + 13*x^2 - 4*x + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{4x^2+3x+1}{(2x+1)\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 - x + 2)**(5/2)), x)
```

Giac [A]
time = 3.99, size = 101, normalized size = 1.19

$$\frac{8}{2197}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{2\left(3\left(6\left(4556x-1749\right)x+26359\right)x-32963\right)}{268203\left(3x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 8/2197*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt
(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))
) + 2/268203*(3*(6*(4556*x - 1749)*x + 26359)*x - 32963)/(3*x^2 - x + 2)^(3
/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{4\,x^2+3\,x+1}{\left(2\,x+1\right)\,{\left(3\,x^2-x+2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(5/2)),x)
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(5/2)), x)
```

3.256 $\int \frac{1+3x+4x^2}{(1+2x)^2(2-x+3x^2)^{5/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{2(197-837x)}{11661\,(2-x+3x^2)^{3/2}}-\frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}}-\frac{16\sqrt{2-x+3x^2}}{2197(1+2x)}-\frac{56\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\,\sqrt{2-x+3x^2}}\right)}{2197\sqrt{13}}$$

```
[Out] -2/11661*(197-837*x)/(3*x^2-x+2)^(3/2)-56/28561*arctanh(1/26*(9-8*x)*13^(1/
2)/(3*x^2-x+2)^(1/2))*13^(1/2)-24/1162213*(841-6633*x)/(3*x^2-x+2)^(1/2)-16
/2197*(3*x^2-x+2)^(1/2)/(1+2*x)
```

Rubi [A]
time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1660, 820, 738, 212}

$$-\frac{24(841-6633x)}{1162213\sqrt{3x^2-x+2}}-\frac{16\sqrt{3x^2-x+2}}{2197(2x+1)}-\frac{2(197-837x)}{11661\,(3x^2-x+2)^{3/2}}-\frac{56\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\,\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (-2*(197 - 837*x))/(11661*(2 - x + 3*x^2)^(3/2)) - (24*(841 - 6633*x))/(116
2213*Sqrt[2 - x + 3*x^2]) - (16*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)) - (56
*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2197*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
```

```
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{2226}{169}+\frac{462x}{13}+\frac{6696x^2}{169}}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}}-\frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}}+\frac{4\int \frac{\frac{50784}{2197}+\frac{190}{21}}{(1+2x)^2\sqrt{2-x}}}{1587}$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}}-\frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}}-\frac{16\sqrt{2-x+3x^2}}{2197(1+2x)}$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}}-\frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}}-\frac{16\sqrt{2-x+3x^2}}{2197(1+2x)}$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}}-\frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}}-\frac{16\sqrt{2-x+3x^2}}{2197(1+2x)}$$

Mathematica [A]
time = 0.57, size = 92, normalized size = 0.84

$$\frac{2(-170239+569989x+1021566x^2+133308x^3+1318464x^4)}{3486639(1+2x)\left(2-x+3x^2\right)^{3/2}}+\frac{112\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{2197\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (2*(-170239 + 569989*x + 1021566*x^2 + 133308*x^3 + 1318464*x^4))/(3486639*
(1 + 2*x)*(2 - x + 3*x^2)^(3/2)) + (112*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*
Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(2197*Sqrt[13])
```

Maple [A]
time = 0.12, size = 165, normalized size = 1.50

method	result
risch	$\frac{\frac{878976}{1162213}x^4+\frac{52388}{89401}x^2+\frac{1139978}{3486639}x-\frac{340478}{3486639}+\frac{168}{2197}x^3}{(3x^2-x+2)^{\frac{3}{2}}(2x+1)}-\frac{56\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{28561}$
trager	$\frac{\frac{878976}{1162213}x^4+\frac{52388}{89401}x^2+\frac{1139978}{3486639}x-\frac{340478}{3486639}+\frac{168}{2197}x^3}{(3x^2-x+2)^{\frac{3}{2}}(2x+1)}-\frac{56\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{-8\,\mathrm{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}+9\,\mathrm{R}}{2x+1}\right.}{28561}$
default	$\frac{\frac{4x}{23}-\frac{2}{69}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{\frac{96x}{529}-\frac{16}{529}}{\sqrt{3x^2-x+2}}+\frac{7}{507\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}-\frac{128(6x-1)}{11661\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}-\frac{10736(6x-1)}{1162213\sqrt{3\left(x+\frac{1}{2}\right)^2-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^2/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/69*(6*x-1)/(3*x^2-x+2)^(3/2)+16/529*(6*x-1)/(3*x^2-x+2)^(1/2)+7/507/(3*(x
+1/2)^2-4*x+5/4)^(3/2)-128/11661*(6*x-1)/(3*(x+1/2)^2-4*x+5/4)^(3/2)-10736/
1162213*(6*x-1)/(3*(x+1/2)^2-4*x+5/4)^(1/2)+28/2197/(3*(x+1/2)^2-4*x+5/4)^(
1/2)-56/28561*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5
)^(1/2))-1/26/(x+1/2)/(3*(x+1/2)^2-4*x+5/4)^(3/2)
```

Maxima [A]
time = 0.52, size = 125, normalized size = 1.14

$$\frac{56}{28561}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{146496\,x}{1162213\,\sqrt{3\,x^2-x+2}}-\frac{9604}{1162213\,\sqrt{3\,x^2-x+2}}+\frac{420\,x}{3887\,(3\,x^2-x+2)^{\frac{3}{2}}}-\frac{1}{13\left(2\,(3\,x^2-x+2)^{\frac{3}{2}}x+(3\,x^2-x+2)^{\frac{3}{2}}\right)}-\frac{49}{11661\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 56/28561*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(
2*x + 1)) + 146496/1162213*x/sqrt(3*x^2 - x + 2) - 9604/1162213/sqrt(3*x^2
```

```
- x + 2) + 420/3887*x/(3*x^2 - x + 2)^(3/2) - 1/13/(2*(3*x^2 - x + 2)^(3/2)
*x + (3*x^2 - x + 2)^(3/2)) - 49/11661/(3*x^2 - x + 2)^(3/2)
```

Fricas [A]
time = 0.34, size = 141, normalized size = 1.28

$$\frac{2\left(22218\sqrt{13}\,(18x^5-3x^4+20x^3+5x^2+4x+4)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}\,(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+13\,(1318464x^4+133308x^3+1021566x^2+569989x-170239)\sqrt{3x^2-x+2}\right)}{45326307\,(18x^5-3x^4+20x^3+5x^2+4x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45326307*(22218*sqrt(13)*(18*x^5 - 3*x^4 + 20*x^3 + 5*x^2 + 4*x + 4)*log(
-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2
+ 4*x + 1)) + 13*(1318464*x^4 + 133308*x^3 + 1021566*x^2 + 569989*x - 17023
9)*sqrt(3*x^2 - x + 2))/(18*x^5 - 3*x^4 + 20*x^3 + 5*x^2 + 4*x + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)^2\,(3x^2-x+2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 - x + 2)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(88) = 176.
time = 3.64, size = 233, normalized size = 2.12

$$-\frac{56}{15108769}\sqrt{13}\left(872\sqrt{13}\sqrt{3}-529\log\left(\sqrt{13}\sqrt{3}-4\right)\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)-\frac{56\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)}{28561\operatorname{sgn}\left(\frac{1}{2x+1}\right)}+\frac{8\left(\frac{\frac{\frac{13\left(\frac{77756}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}+\frac{20631}{(2x+1)\operatorname{sgn}\left(\frac{1}{2x+1}\right)}\right)}{2x+1}-\frac{1399650}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}+\frac{625905}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}-\frac{164808}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}\right)}{3486639\left(\frac{8}{2x+1}-\frac{13}{(2x+1)^2}-3\right)\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -56/15108769*sqrt(13)*(872*sqrt(13)*sqrt(3) - 529*log(sqrt(13)*sqrt(3) - 4)
)*sgn(1/(2*x + 1)) - 56/28561*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13
/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)) - 4)/sgn(1/(2*x + 1)) + 8/3486639*(
((13*(77756/sgn(1/(2*x + 1)) + 20631/((2*x + 1)*sgn(1/(2*x + 1))))/(2*x + 1
) - 1399650/sgn(1/(2*x + 1)))/(2*x + 1) + 625905/sgn(1/(2*x + 1)))/(2*x + 1
) - 164808/sgn(1/(2*x + 1)))/((8/(2*x + 1) - 13/(2*x + 1)^2 - 3)*sqrt(-8/(2
*x + 1) + 13/(2*x + 1)^2 + 3))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^2\,{\left(3\,x^2-x+2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)),x)
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)), x)
```

3.257 $\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$\frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}}+\frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}}-\frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2}-\frac{144\sqrt{2-x+3x^2}}{28561(1+2x)}-\frac{2084\tanh^{-1}\left(\right.}{28}$$

```
[Out] 2/151593*(2363+3693*x)/(3*x^2-x+2)^(3/2)-2084/371293*arctanh(1/26*(9-8*x)*1
3^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+12/15108769*(25771+103526*x)/(3*x^2-x+2
)^(1/2)-8/2197*(3*x^2-x+2)^(1/2)/(1+2*x)^2-144/28561*(3*x^2-x+2)^(1/2)/(1+2
*x)
```

Rubi [A]

time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1660, 1664, 820, 738, 212}

$$\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}-\frac{144\sqrt{3x^2-x+2}}{28561(2x+1)}-\frac{8\sqrt{3x^2-x+2}}{2197(2x+1)^2}+\frac{12(103526x+25771)}{15108769\sqrt{3x^2-x+2}}-\frac{2084\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{28561\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (2*(2363 + 3693*x))/(151593*(2 - x + 3*x^2)^(3/2)) + (12*(25771 + 103526*x)
)/(15108769*Sqrt[2 - x + 3*x^2]) - (8*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)^
2) - (144*Sqrt[2 - x + 3*x^2])/(28561*(1 + 2*x)) - (2084*ArcTanh[(9 - 8*x)/
(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(28561*Sqrt[13])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
```

```
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{32433}{2197}+\frac{106830x}{2197}+\frac{160116x^2}{2197}+\frac{59088x^3}{2197}}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} + \frac{4\int \frac{\frac{1434648}{28561}+\frac{3345396x}{28561}}{(1+2x)^3\sqrt{2-}}}{1587}$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} - \frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2}$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} - \frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2}$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} - \frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2}$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} - \frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2}$$

Mathematica [A]
time = 0.64, size = 105, normalized size = 0.78

$$\frac{2\sqrt{2-x+3x^2}\left(847141+10777477x+21890266x^2+19381992x^3+20074356x^4+20304864x^5\right)}{45326307\left(2+3x+x^2+6x^3\right)^2} + \frac{4168\tanh^{-1}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{28561\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)),x]
[Out] (2*Sqrt[2 - x + 3*x^2]*(847141 + 10777477*x + 21890266*x^2 + 19381992*x^3 +
 20074356*x^4 + 20304864*x^5))/(45326307*(2 + 3*x + x^2 + 6*x^3)^2) + (4168
*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(28561*
Sqrt[13])
```

Maple [A]
time = 0.12, size = 148, normalized size = 1.10

method	result
risch	$\frac{\frac{13536576}{15108769}x^5+\frac{13382904}{15108769}x^4+\frac{12921328}{15108769}x^3+\frac{43780532}{45326307}x^2+\frac{21554954}{45326307}x+\frac{1694282}{45326307}}{(2x+1)^2(3x^2-x+2)^{\frac{3}{2}}} - \frac{2084\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x}}\right.}{371293}$

trager	$\frac{2(20304864x^5+20074356x^4+19381992x^3+21890266x^2+10777477x+847141)\sqrt{3x^2-x+2}}{45326307(6x^3+x^2+3x+2)^2}+\frac{2084\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\ldots\right.}{}$
default	$\frac{521}{13182\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{\frac{1772x}{50531}-\frac{886}{151593}}{\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{\frac{1128048x}{15108769}-\frac{188008}{15108769}}{\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}+\frac{1042}{28561\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^3/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 521/13182/(3*(x+1/2)^2-4*x+5/4)^(3/2)+886/151593*(6*x-1)/(3*(x+1/2)^2-4*x+5
/4)^(3/2)+188008/15108769*(6*x-1)/(3*(x+1/2)^2-4*x+5/4)^(1/2)+1042/28561/(3
*(x+1/2)^2-4*x+5/4)^(1/2)-2084/371293*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1
/2)/(12*(x+1/2)^2-16*x+5)^(1/2))-1/338/(x+1/2)/(3*(x+1/2)^2-4*x+5/4)^(3/2)-
1/104/(x+1/2)^2/(3*(x+1/2)^2-4*x+5/4)^(3/2)
```

Maxima [A]
time = 0.52, size = 174, normalized size = 1.29

$$\frac{2084}{371293}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{1128048\,x}{15108769\sqrt{3\,x^2-x+2}}+\frac{363210}{15108769\sqrt{3\,x^2-x+2}}+\frac{1772\,x}{50531\,(3\,x^2-x+2)^{\frac{3}{2}}}-\frac{1}{26\left(4\,(3\,x^2-x+2)^{\frac{3}{2}}x^2+4\,(3\,x^2-x+2)^{\frac{3}{2}}x+(3\,x^2-x+2)^{\frac{3}{2}}\right)}-\frac{1}{169\left(2\,(3\,x^2-x+2)^{\frac{3}{2}}x+(3\,x^2-x+2)^{\frac{3}{2}}\right)}+\frac{10211}{303186\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2084/371293*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/a
bs(2*x + 1)) + 1128048/15108769*x/sqrt(3*x^2 - x + 2) + 363210/15108769/sqr
t(3*x^2 - x + 2) + 1772/50531*x/(3*x^2 - x + 2)^(3/2) - 1/26/(4*(3*x^2 - x
+ 2)^(3/2)*x^2 + 4*(3*x^2 - x + 2)^(3/2)*x + (3*x^2 - x + 2)^(3/2)) - 1/169
/(2*(3*x^2 - x + 2)^(3/2)*x + (3*x^2 - x + 2)^(3/2)) + 10211/303186/(3*x^2
- x + 2)^(3/2)
```

Fricas [A]
time = 0.41, size = 156, normalized size = 1.16

$$\frac{2\left(826827\sqrt{13}\,(36\,x^6+12\,x^5+37\,x^4+30\,x^3+13\,x^2+12\,x+4)\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}\,(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)+13\,(20304864\,x^5+20074356\,x^4+19381992\,x^3+21890266\,x^2+10777477\,x+847141)\sqrt{3\,x^2-x+2}\right)}{589241991\,(36\,x^6+12\,x^5+37\,x^4+30\,x^3+13\,x^2+12\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/589241991*(826827*sqrt(13)*(36*x^6 + 12*x^5 + 37*x^4 + 30*x^3 + 13*x^2 +
12*x + 4)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x
+ 185)/(4*x^2 + 4*x + 1)) + 13*(20304864*x^5 + 20074356*x^4 + 19381992*x^3
```

```
+ 21890266*x^2 + 10777477*x + 847141)*sqrt(3*x^2 - x + 2))/(36*x^6 + 12*x^5
 + 37*x^4 + 30*x^3 + 13*x^2 + 12*x + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)^3 (3x^2-x+2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*(3*x**2 - x + 2)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(109) = 218.
time = 3.49, size = 233, normalized size = 1.73

$$\frac{2084}{371293}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{2\left(3\left(6\left(310578x-26213\right)x+1455755\right)x+1634293\right)}{45326307\left(3x^2-x+2\right)^{\frac{3}{2}}}-\frac{8\left(66\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^3+21\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^2-1015\sqrt{3}x+431\sqrt{3}+1015\sqrt{3x^2-x+2}\right)}{28561\left(2\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^2+2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)-5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 2084/371293*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4
*sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x
+ 2))) + 2/45326307*(3*(6*(310578*x - 26213)*x + 1455755)*x + 1634293)/(3*x
^2 - x + 2)^(3/2) - 8/28561*(66*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 + 21*sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 - 1015*sqrt(3)*x + 431*sqrt(3) +
1015*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3
)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^3\,{\left(3\,x^2-x+2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)), x)
```

3.258 $\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=208

$$-\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}+\frac{\left(6bceh^2-3b^2fh^2+4c^2(fg^2-h(eg+2dh))\right)(b+2cx)}{3ch^2(2cg-bh)^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}+\frac{\ldots}{3h^3(2cg-b\ldots}$$

```
[Out] -f/c/h^3/(-g*(-b*h+c*g)+b*h^2*x+c*h^2*x^2)^(1/2)+1/3*(6*b*c*e*h^2-3*b^2*f*h
^2+4*c^2*(f*g^2-h*(2*d*h+e*g)))*(2*c*x+b)/c/h^2/(-b*h+2*c*g)^3/(-g*(-b*h+c*
g)+b*h^2*x+c*h^2*x^2)^(1/2)+2/3*(d*h^2-e*g*h+f*g^2)/h^3/(-b*h+2*c*g)/(h*x+g
)/(-g*(-b*h+c*g)+b*h^2*x+c*h^2*x^2)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.064$, Rules used = {1652, 806, 627}

$$\frac{(b+2cx)\left(-3b^2fh^2+6bceh^2+4c^2(fg^2-h(2dh+eg))\right)}{3ch^2(2cg-bh)^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}+\frac{2(dh^2-egh+fg^2)}{3h^3(g+hx)(2cg-bh)\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}-\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*(-(c*g^2) + b*g*h + b*h^2*x + c*h^2*x^2)^(
3/2)),x]
```

```
[Out] -(f/(c*h^3*Sqrt[-(g*(c*g - b*h)) + b*h^2*x + c*h^2*x^2])) + ((6*b*c*e*h^2 -
 3*b^2*f*h^2 + 4*c^2*(f*g^2 - h*(e*g + 2*d*h)))*(b + 2*c*x))/(3*c*h^2*(2*c*
g - b*h)^3*Sqrt[-(g*(c*g - b*h)) + b*h^2*x + c*h^2*x^2]) + (2*(f*g^2 - e*g*
h + d*h^2))/(3*h^3*(2*c*g - b*h)*(g + h*x)*Sqrt[-(g*(c*g - b*h)) + b*h^2*x
+ c*h^2*x^2])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 1652

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d - b
*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = -\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} - \frac{\int \frac{\frac{1}{2}h^3(bfg-2cdh)+\frac{1}{2}}{(g+hx)(-cg^2+b}}{}$$

$$= -\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} + \frac{}{3h^3(2cg-bh)(g}$$

$$= -\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} + \frac{(6bceh^2-3b^2fh}{3ch^2(2cg-b}$$

Mathematica [**A**]
time = 0.30, size = 219, normalized size = 1.05

$$\frac{2b^2h^2(-h(2eg+dh+3ehx)+f(8g^2+12ghx+3h^2x^2))+8c^2(fg^2(2g^2+2ghx-h^2x^2)+h(eg(g^2+ghx+h^2x^2)+dh(-g^2+2ghx+2h^2x^2)))-4bch(2fg^2(4g+5hx)+h(-2dh(2g+hx)+e(g^2+2ghx+3h^2x^2)))}{3h^3(-2cg+bh)^3(g+hx)\sqrt{(g+hx)(-cg+bh+chx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*(-(c*g^2) + b*g*h + b*h^2*x + c*h^2*
x^2)^(3/2)),x]
```

```
[Out] (2*b^2*h^2*(-(h*(2*e*g + d*h + 3*e*h*x)) + f*(8*g^2 + 12*g*h*x + 3*h^2*x^2)
) + 8*c^2*(f*g^2*(2*g^2 + 2*g*h*x - h^2*x^2) + h*(e*g*(g^2 + g*h*x + h^2*x^
2) + d*h*(-g^2 + 2*g*h*x + 2*h^2*x^2))) - 4*b*c*h*(2*f*g^2*(4*g + 5*h*x) +
h*(-2*d*h*(2*g + h*x) + e*(g^2 + 2*g*h*x + 3*h^2*x^2))))/(3*h^3*(-2*c*g + b
*h)^3*(g + h*x)*Sqrt[(g + h*x)*(-(c*g) + b*h + c*h*x)])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(198) = 396$.
time = 0.15, size = 424, normalized size = 2.04

method	result
gosper	$-\frac{2(chx+bh-cg)\left(-3b^2f\,h^4x^2+6bce\,h^4x^2-8c^2d\,h^4x^2-4c^2eg\,h^3x^2+4c^2f\,g^2h^2x^2+3b^2e\,h^4x-12b^2fg\,h^3x-4bcd\,h^4x+4bceg\,h^3x+20bcf}{3(b^3h^3-6b^2cg\,h^2+12b\,c^2g^2h-}$
trager	$-\frac{2\left(-3b^2f\,h^4x^2+6bce\,h^4x^2-8c^2d\,h^4x^2-4c^2eg\,h^3x^2+4c^2f\,g^2h^2x^2+3b^2e\,h^4x-12b^2fg\,h^3x-4bcd\,h^4x+4bceg\,h^3x+20bcf\,g^2h^2x-8c^2dg}{3h^3(b^2h^2-}$
default	$\frac{fh\left(-\frac{1}{c\,h^2\sqrt{c\,x^2h^2+bx\,h^2+bgh-c\,g^2}}-\frac{b\left(2c\,h^2x+b\,h^2\right)}{c\left(4c\,h^2\left(bgh-c\,g^2\right)-b^2h^4\right)\sqrt{c\,x^2h^2+bx\,h^2+bgh-c\,g^2}}\right)+\frac{}{\left(4c\,h^2\left(bg\right.\right.}}{h^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/h^2*(f*h*(-1/c/h^2/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(1/2)-b/c*(2*c*h^2*x+b
*h^2)/(4*c*h^2*(b*g*h-c*g^2)-b^2*h^4)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(1/2)
)+2*e*h*(2*c*h^2*x+b*h^2)/(4*c*h^2*(b*g*h-c*g^2)-b^2*h^4)/(c*h^2*x^2+b*h^2*
x+b*g*h-c*g^2)^(1/2)-2*g*f*(2*c*h^2*x+b*h^2)/(4*c*h^2*(b*g*h-c*g^2)-b^2*h^4
)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(1/2))+(d*h^2-e*g*h+f*g^2)/h^3*(-2/3/(b*h
^2-2*c*g*h)/(x+1/h*g)/(c*h^2*(x+1/h*g)^2+(b*h^2-2*c*g*h)*(x+1/h*g))^(1/2)+8
/3*c*h^2/(b*h^2-2*c*g*h)^3*(2*c*h^2*(x+1/h*g)+b*h^2-2*c*g*h)/(c*h^2*(x+1/h*
g)^2+(b*h^2-2*c*g*h)*(x+1/h*g))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(201) = 402$.
time = 12.96, size = 490, normalized size = 2.36

$$\frac{2\,{\left(8\,c^{2} f g^{4} - 16\,b c f g^{3} h + 8\,b c d g h^{3} - b^{2} d h^{4} - 4\,{\left(c^{2} d - 2\,b^{2} f\right)} g^{2} h^{2} - {\left(4\,c^{2} f g^{2} h^{2} - {\left(8\,c^{2} d + 3\,b^{2} f\right)} h^{4}\right)} x^{2} + 4\,{\left(2\,c^{2} f g^{3} h - 5\,b c f g^{2} h^{2} + b c d h^{4} + {\left(2\,c^{2} d + 3\,b^{2} f\right)} g h^{3}\right)} x + {\left(4\,c^{2} g^{3} h - 2\,b c g^{2} h^{2} - 2\,b^{2} g h^{3} + 2\,{\left(2\,c^{2} g h^{3} - 3\,b c h^{4}\right)} x^{2} + {\left(4\,c^{2} g^{2} h^{2} - 4\,b c g h^{3} - 3\,b^{2} h^{4}\right)} x\right)} e\right)} \sqrt{c h^{2} x^{2} + b h^{2} x - c g^{2} + b g h}}{3\,{\left(8\,c^{4} g^{6} h^{3} - 20\,b c^{3} g^{5} h^{4} + 18\,b^{2} c^{2} g^{4} h^{5} - 7\,b^{3} c g^{3} h^{6} + b^{4} g^{2} h^{7} - {\left(8\,c^{4} g^{3} h^{6} - 12\,b c^{3} g^{2} h^{7} + 6\,b^{2} c^{2} g h^{8} - b^{3} c h^{9}\right)} x^{3} - {\left(8\,c^{4} g^{4} h^{5} - 4\,b c^{3} g^{3} h^{6} - 6\,b^{2} c^{2} g^{2} h^{7} + 5\,b^{3} c g h^{8} - b^{4} h^{9}\right)} x^{2} + {\left(8\,c^{4} g^{5} h^{4} - 28\,b c^{3} g^{4} h^{5} + 30\,b^{2} c^{2} g^{3} h^{6} - 13\,b^{3} c g^{2} h^{7} + 2\,b^{4} g h^{8}\right)} x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, al
gorithm="fricas")
```

```
[Out] 2/3*(8*c^2*f*g^4 - 16*b*c*f*g^3*h + 8*b*c*d*g*h^3 - b^2*d*h^4 - 4*(c^2*d -
2*b^2*f)*g^2*h^2 - (4*c^2*f*g^2*h^2 - (8*c^2*d + 3*b^2*f)*h^4)*x^2 + 4*(2*c
^2*f*g^3*h - 5*b*c*f*g^2*h^2 + b*c*d*h^4 + (2*c^2*d + 3*b^2*f)*g*h^3)*x + (
4*c^2*g^3*h - 2*b*c*g^2*h^2 - 2*b^2*g*h^3 + 2*(2*c^2*g*h^3 - 3*b*c*h^4)*x^2
 + (4*c^2*g^2*h^2 - 4*b*c*g*h^3 - 3*b^2*h^4)*x)*e)*sqrt(c*h^2*x^2 + b*h^2*x
 - c*g^2 + b*g*h)/(8*c^4*g^6*h^3 - 20*b*c^3*g^5*h^4 + 18*b^2*c^2*g^4*h^5 -
7*b^3*c*g^3*h^6 + b^4*g^2*h^7 - (8*c^4*g^3*h^6 - 12*b*c^3*g^2*h^7 + 6*b^2*c
^2*g*h^8 - b^3*c*h^9)*x^3 - (8*c^4*g^4*h^5 - 4*b*c^3*g^3*h^6 - 6*b^2*c^2*g^
2*h^7 + 5*b^3*c*g*h^8 - b^4*h^9)*x^2 + (8*c^4*g^5*h^4 - 28*b*c^3*g^4*h^5 +
30*b^2*c^2*g^3*h^6 - 13*b^3*c*g^2*h^7 + 2*b^4*g*h^8)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2}{\left(\left(g + hx\right)\left(bh - cg + chx\right)\right)^{\frac{3}{2}}\left(g + hx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*h**2*x**2+b*h**2*x+b*g*h-c*g**2)**(3/2)
,x)
```

```
[Out] Integral((d + e*x + f*x**2)/(((g + h*x)*(b*h - c*g + c*h*x))**(3/2)*(g + h*
x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, al
gorithm="giac")
```

```
[Out] integrate((f*x^2 + x*e + d)/((c*h^2*x^2 + b*h^2*x - c*g^2 + b*g*h)^(3/2)*(h
*x + g)), x)
```

Mupad [B]
time = 5.75, size = 1089, normalized size = 5.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(3/2
)),x)
```

```
[Out] (16*c^2*f*g^4*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 2*b^2*d*h^4*(b*
h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 8*c^2*d*g^2*h^2*(b*h^2*x - c*g^2
 + c*h^2*x^2 + b*g*h)^(1/2) + 16*b^2*f*g^2*h^2*(b*h^2*x - c*g^2 + c*h^2*x^2
 + b*g*h)^(1/2) + 16*c^2*d*h^4*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1
/2) + 6*b^2*f*h^4*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 4*b^2*e
*g*h^3*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 8*c^2*e*g^3*h*(b*h^2*x
 - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 6*b^2*e*h^4*x*(b*h^2*x - c*g^2 + c*h^
2*x^2 + b*g*h)^(1/2) + 8*b*c*d*h^4*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^
(1/2) - 8*c^2*f*g^2*h^2*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 4
*b*c*e*g^2*h^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 12*b*c*e*h^4*x
^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 16*c^2*d*g*h^3*x*(b*h^2*x
- c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 24*b^2*f*g*h^3*x*(b*h^2*x - c*g^2 + c*
h^2*x^2 + b*g*h)^(1/2) + 16*c^2*f*g^3*h*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*
g*h)^(1/2) + 8*c^2*e*g^2*h^2*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2)
+ 8*c^2*e*g*h^3*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 16*b*c*d*
g*h^3*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 32*b*c*f*g^3*h*(b*h^2*x
 - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 8*b*c*e*g*h^3*x*(b*h^2*x - c*g^2 + c*
h^2*x^2 + b*g*h)^(1/2) - 40*b*c*f*g^2*h^2*x*(b*h^2*x - c*g^2 + c*h^2*x^2 +
b*g*h)^(1/2))/(3*b^4*g^2*h^7 + 24*c^4*g^6*h^3 + 3*b^4*h^9*x^2 - 60*b*c^3*g^
5*h^4 - 21*b^3*c*g^3*h^6 + 3*b^3*c*h^9*x^3 + 24*c^4*g^5*h^4*x + 54*b^2*c^2*
g^4*h^5 - 24*c^4*g^4*h^5*x^2 - 24*c^4*g^3*h^6*x^3 + 6*b^4*g*h^8*x + 18*b^2*
c^2*g^2*h^7*x^2 - 84*b*c^3*g^4*h^5*x - 39*b^3*c*g^2*h^7*x - 15*b^3*c*g*h^8*
x^2 + 90*b^2*c^2*g^3*h^6*x + 12*b*c^3*g^3*h^6*x^2 + 36*b*c^3*g^2*h^7*x^3 -
18*b^2*c^2*g*h^8*x^3)
```

3.259 $\int \sqrt{d+ex}\,\sqrt{a+bx+cx^2}\,(A+Bx+Cx^2)\,dx$

Optimal. Leaf size=906

$$\frac{2\sqrt{d+ex}\,(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3d(8Cd^2-3e(4Bd-7Ae))+3c^2e(ae(Cd-5Be)-b(\cdots}{3\cdots}$$

```
[Out] 2/9*C*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(3/2)/c/e-2/21*(-3*B*c*e+2*C*b*e+2*C*c*d)
*(c*x^2+b*x+a)^(3/2)*(e*x+d)^(1/2)/c^2/e+2/315*(8*b^3*C*e^3-3*b*c*e^2*(4*B*
b*e-C*a*e+C*b*d)+c^3*d*(8*C*d^2-3*e*(-7*A*e+4*B*d))+3*c^2*e*(a*e*(-5*B*e+C*
d)-b*(-7*A*e^2-2*B*d*e+C*d^2))+3*c*e*(8*b^2*C*e^2-c*e*(12*B*b*e+7*C*a*e+C*b
*d)-c^2*(2*C*d^2-3*e*(7*A*e+B*d)))*x)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^3
/e^3+1/315*(2*(4*c^2*d^2-b^2*e^2-3/2*c*e*(-2*a*e+b*d))*(8*b^2*C*e^2-c*e*(12
*B*b*e+7*C*a*e+C*b*d)-c^2*(2*C*d^2-3*e*(7*A*e+B*d)))-5*c*e*(-b*e+2*c*d)*(6*
b^2*C*d*e+c*e*(21*A*c*d-3*B*a*e-5*C*a*d)+b*(2*a*C*e^2-c*d*(9*B*e+C*d))))*El
lipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)
,(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*
(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^4/
e^4/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-
2/315*(a*e^2-b*d*e+c*d^2)*(8*b^3*C*e^3-3*c^2*e^2*(-7*A*b*e-10*B*a*e+B*b*d+2
*C*a*d)+3*b*c*e^2*(-4*B*b*e-9*C*a*e+C*b*d)-2*c^3*d*(8*C*d^2-3*e*(-7*A*e+4*B
*d)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*
2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)
/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^4/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a
)^(1/2)
```

Rubi [A]
time = 1.60, antiderivative size = 905, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {1667, 846, 828, 857, 732, 435, 430}

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(8*b^3*C*e^3 - 3*b*c*e^2*(b*C*d + 4*b*B*e - a*C*e) + c^3*(
8*C*d^3 - 3*d*e*(4*B*d - 7*A*e)) - 3*c^2*e*(b*C*d^2 - b*e*(2*B*d + 7*A*e) -
```

```
 a*e*(C*d - 5*B*e)) + 3*c*e*(8*b^2*C*e^2 - c*e*(b*C*d + 12*b*B*e + 7*a*C*e)
 - c^2*(2*C*d^2 - 3*e*(B*d + 7*A*e)))*x)*Sqrt[a + b*x + c*x^2])/(315*c^3*e^
3) - (2*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2)
)/(21*c^2*e) + (2*C*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(9*c*e) - (Sqr
t[2]*Sqrt[b^2 - 4*a*c]*(5*c*e*(2*c*d - b*e)*(6*b^2*C*d*e + 2*a*b*C*e^2 - b*
c*d*(C*d + 9*B*e) + c*e*(21*A*c*d - 5*a*C*d - 3*a*B*e)) - 2*(4*c^2*d^2 - b^
2*e^2 - (3*c*e*(b*d - 2*a*e))/2)*(8*b^2*C*e^2 - c*e*(b*C*d + 12*b*B*e + 7*a
*C*e) - c^2*(2*C*d^2 - 3*e*(B*d + 7*A*e))))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e)])/(315*c^4*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*
d^2 - b*d*e + a*e^2)*(8*b^3*C*e^3 - 3*c^2*e^2*(b*B*d + 2*a*C*d - 7*A*b*e -
10*a*B*e) + 3*b*c*e^2*(b*C*d - 4*b*B*e - 9*a*C*e) - 2*c^3*(8*C*d^3 - 3*d*e*
(4*B*d - 7*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*S
qrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315*c^4*e^4*Sqrt[d + e*x]*Sqrt[a +
 b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
```

```
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \sqrt{d+ex}\,\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx = \frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce} + \frac{2\int \sqrt{d+ex}\,\left(-\frac{3}{2}e(bC\right.}{}$$

$$= -\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}\,\left(a+bx+cx^2\right)^{3/2}}{21c^2e} +$$

$$= \frac{2\sqrt{d+ex}\,\left(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3(8C\right.}{}$$

$$= \frac{2\sqrt{d+ex}\,\left(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3(8C\right.}{}$$

$$= \frac{2\sqrt{d+ex}\,\left(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3(8C\right.}{}$$

$$= \frac{2\sqrt{d+ex}\,\left(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3(8C\right.}{}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 33.40, size = 15669, normalized size = 17.29

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2),x]
```

```
[Out] Result too large to show
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 19954 vs. 2(834) = 1668.
time = 0.22, size = 19955, normalized size = 22.03

method	result	size
elliptic	Expression too large to display	1736
risch	Expression too large to display	7113
default	Expression too large to display	19955

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(x*e + d), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 991, normalized size = 1.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/945*((16*C*c^5*d^5 - 8*(2*C*b*c^4 + 3*B*c^5)*d^4*e - (5*C*b^2*c^3 - 42*A*
c^5 - 3*(10*C*a + 9*B*b)*c^4)*d^3*e^2 - (5*C*b^3*c^2 + 3*(22*B*a + 21*A*b)*
c^4 - 3*(7*C*a*b + 4*B*b^2)*c^3)*d^2*e^3 - (16*C*b^4*c - 378*A*a*c^4 + 3*(2
2*C*a^2 + 41*B*a*b + 21*A*b^2)*c^3 - 3*(28*C*a*b^2 + 9*B*b^3)*c^2)*d*e^4 +
(16*C*b^5 - 9*(10*B*a^2 + 21*A*a*b)*c^3 + 3*(41*C*a^2*b + 41*B*a*b^2 + 14*A
*b^3)*c^2 - 24*(4*C*a*b^3 + B*b^4)*c)*e^5)*sqrt(c)*e^(1/2)*weierstrassPInve
rse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^
3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-
3)/c^3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c) + 3*(16*C*c^5*d^4*e - 8*(C*b*c^
4 + 3*B*c^5)*d^3*e^2 - 3*(2*C*b^2*c^3 - 14*A*c^5 - (6*C*a + 5*B*b)*c^4)*d^2
*e^3 - (8*C*b^3*c^2 + 6*(8*B*a + 7*A*b)*c^4 - 15*(2*C*a*b + B*b^2)*c^3)*d*e
^4 + (16*C*b^4*c - 126*A*a*c^4 + 3*(14*C*a^2 + 29*B*a*b + 14*A*b^2)*c^3 - 2
4*(3*C*a*b^2 + B*b^3)*c^2)*e^5)*sqrt(c)*e^(1/2)*weierstrassZeta(4/3*(c^2*d^
```

```
2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2
*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, weierst
rassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*
(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*
e^3)*e^(-3)/c^3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c)) + 3*(8*C*c^5*d^3*e^2
+ (35*C*c^5*x^3 + 8*C*b^3*c^2 + 3*(10*B*a + 7*A*b)*c^4 - 3*(9*C*a*b + 4*B*b
^2)*c^3 + 5*(C*b*c^4 + 9*B*c^5)*x^2 - (6*C*b^2*c^3 - 63*A*c^5 - (14*C*a + 9
*B*b)*c^4)*x)*e^5 + (5*C*c^5*d*x^2 + (2*C*b*c^4 + 9*B*c^5)*d*x - (3*C*b^2*c
^3 - 21*A*c^5 - 2*(4*C*a + 3*B*b)*c^4)*d)*e^4 - 3*(2*C*c^5*d^2*x + (C*b*c^4
 + 4*B*c^5)*d^2)*e^3)*sqrt(c*x^2 + b*x + a)*sqrt(x*e + d))*e^(-5)/c^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex}\,(A+Bx+Cx^2)\,\sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(A + B*x + C*x**2)*sqrt(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d+e\,x}\,\left(C\,x^2+B\,x+A\right)\,\sqrt{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)*(A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x)^(1/2)*(A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2), x)
```

3.260 $\int \frac{\sqrt{a+bx+cx^2}\,(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=668

$$-\frac{2\sqrt{d+ex}\,(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe)+3ce(6cCd-7Bce+4bCe)x)}{105c^2e^3}$$

```
[Out] 2/7*C*(c*x^2+b*x+a)^(3/2)*(e*x+d)^(1/2)/c/e-2/105*(5*c*e*(-7*A*c*e+C*a*e+3*
C*b*d)-(-b*e+4*c*d)*(-7*B*c*e+4*C*b*e+6*C*c*d)+3*c*e*(-7*B*c*e+4*C*b*e+6*C*
c*d)*x)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/e^3+1/105*(5*c*e*(-b*e+2*c*d)
*(-7*A*c*e+C*a*e+3*C*b*d)-(-7*B*c*e+4*C*b*e+6*C*c*d)*(8*c^2*d^2-2*b^2*e^2-3
*c*e*(-2*a*e+b*d)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-
4*a*c+b^2))^(1/2)/c^3/e^4/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2))))^(1/2)+2/105*(a*e^2-b*d*e+c*d^2)*(4*b^2*C*e^2+c*e*(-7*B*b*e-
10*C*a*e+8*C*b*d)+c^2*(48*C*d^2-14*e*(-5*A*e+4*B*d)))*EllipticF(1/2*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)
^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*
(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(
1/2))))^(1/2)/c^3/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 0.72, antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1667, 828, 857, 732, 435, 430}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/Sqrt[d + e*x],x]

[Out] (-2*Sqrt[d + e*x]*(5*c*e*(3*b*C*d - 7*A*c*e + a*C*e) - (4*c*d - b*e)*(6*c*C
*d - 7*B*c*e + 4*b*C*e) + 3*c*e*(6*c*C*d - 7*B*c*e + 4*b*C*e)*x)*Sqrt[a + b
*x + c*x^2])/(105*c^2*e^3) + (2*C*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2))/(7
*c*e) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(5*c*e*(2*c*d - b*e)*(3*b*C*d - 7*A*c*e
+ a*C*e) - (6*c*C*d - 7*B*c*e + 4*b*C*e)*(8*c^2*d^2 - 2*b^2*e^2 - 3*c*e*(b*
```

```
d - 2*a*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ell
ipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[
2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^
3*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
+ c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(4*b^2*C*e
^2 + c*e*(8*b*C*d - 7*b*B*e - 10*a*C*e) + c^2*(48*C*d^2 - 14*e*(4*B*d - 5*A
*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^3*e^4*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2
])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\,(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \frac{2C\sqrt{d+ex}\,(a+bx+cx^2)^{3/2}}{7ce} + \frac{2\int \frac{(-\frac{1}{2}e(3bCd-7Ace+aCe)-\frac{1}{2}e(6cCd-7B}{\sqrt{d+e}}}{7ce^2}$$

$$= -\frac{2\sqrt{d+ex}\,(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce}{105c^2e^3}$$

$$= -\frac{2\sqrt{d+ex}\,(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce}{105c^2e^3}$$

$$= -\frac{2\sqrt{d+ex}\,(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce}{105c^2e^3}$$

$$= -\frac{2\sqrt{d+ex}\,(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce}{105c^2e^3}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 33.15, size = 9965, normalized size = 14.92

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/Sqrt[d + e*x],x]
```

```
[Out] Result too large to show
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 12760 vs. 2(604) = 1208.
time = 0.15, size = 12761, normalized size = 19.10

method	result

elliptic	$\frac{\sqrt{(c\,x^2+bx+a)\,(ex+d)}\left(\frac{2C\,x^2\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}{7e}+\frac{2\left(cB+bC-\frac{2C(3eb+3cd)}{7e}\right)x\,\cdots}{\cdots}\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/sqrt(x*e + d), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 737, normalized size = 1.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/315*((48*C*c^4*d^4 - 8*(5*C*b*c^3 + 7*B*c^4)*d^3*e - (10*C*b^2*c^2 - 70*A
*c^4 - (62*C*a + 49*B*b)*c^3)*d^2*e^2 - (5*C*b^3*c + 14*(6*B*a + 5*A*b)*c^3
 - 2*(11*C*a*b + 7*B*b^2)*c^2)*d*e^3 - (8*C*b^4 - 210*A*a*c^3 + (30*C*a^2 +
 63*B*a*b + 35*A*b^2)*c^2 - (41*C*a*b^2 + 14*B*b^3)*c)*e^4)*sqrt(c)*e^(1/2)
*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2
, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9
*a*b*c)*e^3)*e^(-3)/c^3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c) + 3*(48*C*c^4*
d^3*e - 8*(2*C*b*c^3 + 7*B*c^4)*d^2*e^2 - (9*C*b^2*c^2 - 70*A*c^4 - (26*C*a
 + 21*B*b)*c^3)*d*e^3 - (8*C*b^3*c + 7*(6*B*a + 5*A*b)*c^3 - (29*C*a*b + 14
*B*b^2)*c^2)*e^4)*sqrt(c)*e^(1/2)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e +
(b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c
- 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, weierstrassPInverse(4
/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3
*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^
3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c)) + 3*(24*C*c^4*d^2*e^2 + (15*C*c^4*x
^2 - 4*C*b^2*c^2 + 35*A*c^4 + (10*C*a + 7*B*b)*c^3 + 3*(C*b*c^3 + 7*B*c^4)*
x)*e^4 - (18*C*c^4*d*x + (5*C*b*c^3 + 28*B*c^4)*d)*e^3)*sqrt(c*x^2 + b*x +
a)*sqrt(x*e + d))*e^(-5)/c^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{\sqrt{d+ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/sqrt(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/sqrt(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(C\,x^2+B\,x+A\right)\,\sqrt{c\,x^2+b\,x+a}}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(1/2),x)
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(1/2), x)
```

3.261 $\int \frac{\sqrt{a+bx+cx^2}\,(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=749

$$-\frac{2\sqrt{d+ex}\,\big(bCe^2(bd-ae)+c^2d(24Cd^2-5e(4Bd-3Ae))+ce(ae(9Cd-5Be)-5b(5Cd^2-4Bde+3Ae}{15ce^3\left(cd^2-bde+ae^2\right)}$$

```
[Out] -2*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(
1/2)-2/15*(b*C*e^2*(-a*e+b*d)+c^2*d*(24*C*d^2-5*e*(-3*A*e+4*B*d))+c*e*(a*e*
(-5*B*e+9*C*d)-5*b*(3*A*e^2-4*B*d*e+5*C*d^2))+3*c*e^2*(5*B*c*d+b*C*d-6*c*C*
d^2/e-5*A*c*e-a*C*e)*x)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e^3/(a*e^2-b*d*
e+c*d^2)-1/15*(2*b^2*C*e^2+c*e*(-5*B*b*e-6*C*a*e+8*C*b*d)-c^2*(48*C*d^2-10*
e*(-3*A*e+4*B*d)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/
2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4
*a*c+b^2))^(1/2)/c^2/e^4/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2))))^(1/2)+2/15*(b*C*e^2*(-a*e+b*d)-2*c^2*d*(24*C*d^2-5*e*(-3*A*e
+4*B*d))-c*e*(2*a*e*(-5*B*e+9*C*d)-b*(32*C*d^2-5*e*(-3*A*e+5*B*d))))*Ellipt
icF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2
*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*
a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b
+(-4*a*c+b^2)^(1/2))))^(1/2)/c^2/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 0.90, antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1664, 828, 857, 732, 435, 430}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(b*C*e^2*(b*d - a*e) + c^2*(24*C*d^3 - 5*d*e*(4*B*d - 3*A
*e)) + c*e*(a*e*(9*C*d - 5*B*e) - 5*b*(5*C*d^2 - 4*B*d*e + 3*A*e^2)) + 3*c*
e^2*(5*B*c*d + b*C*d - (6*c*C*d^2)/e - 5*A*c*e - a*C*e)*x)*Sqrt[a + b*x + c
*x^2])/(15*c*e^3*(c*d^2 - b*d*e + a*e^2)) - (2*(C*d^2 - e*(B*d - A*e))*(a +
```

```
 b*x + c*x^2)^(3/2))/(e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (Sqrt[2]*S
qrt[b^2 - 4*a*c]*(2*b^2*C*e^2 + c*e*(8*b*C*d - 5*b*B*e - 6*a*C*e) - c^2*(48
*C*d^2 - 10*e*(4*B*d - 3*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/
(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[
b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e)])/(15*c^2*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(b*C*e^2*(b*d -
a*e) - 2*c^2*d*(24*C*d^2 - 5*e*(4*B*d - 3*A*e)) + c*e*(32*b*C*d^2 - 5*b*e*(
5*B*d - 3*A*e) - 2*a*e*(9*C*d - 5*B*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elliptic
F[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
(-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c^2*e^4*
Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
```

```
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\,(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\,(a+bx+cx^2)^{3/2}}{e\,(cd^2-bde+ae^2)\,\sqrt{d+ex}} - \frac{2\int \frac{\left(-\frac{3bCd^2-be(3Bd-2Ae}{2}\right.}{}}{}$$

$$= -\frac{2\sqrt{d+ex}\,\left(bCe^2(bd-ae)+c^2(24Cd^3-5de(4Bd-3Ae))+c\right.}{}$$

$$= -\frac{2\sqrt{d+ex}\,\left(bCe^2(bd-ae)+c^2(24Cd^3-5de(4Bd-3Ae))+c\right.}{}$$

$$= -\frac{2\sqrt{d+ex}\,\left(bCe^2(bd-ae)+c^2(24Cd^3-5de(4Bd-3Ae))+c\right.}{}$$

$$= -\frac{2\sqrt{d+ex}\,\left(bCe^2(bd-ae)+c^2(24Cd^3-5de(4Bd-3Ae))+c\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 32.90, size = 13240, normalized size = 17.68

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8220 vs. 2(687) = 1374.
time = 0.18, size = 8221, normalized size = 10.98

method	result
elliptic	$\sqrt{(c x^2+bx+a)(ex+d)}\left(-\frac{2\left(ce\, x^2+bex+ae\right)\left(A\, e^2-Bde+C\, d^2\right)}{e^4\sqrt{\left(x+\frac{d}{e}\right)\left(ce\, x^2+bex+ae\right)}}+\frac{2Cx\sqrt{ce\, x^3+be\, x^2+cd\, x^2+aex+}}{5e^2}\right.$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(x*e + d)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 790, normalized size = 1.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="fri
cas")
```

```
[Out] -2/45*((48*C*c^3*d^4 - (2*C*b^3 + 15*(2*B*a + A*b)*c^2 - (9*C*a*b + 5*B*b^2
)*c)*x*e^4 - ((7*C*b^2*c - 30*A*c^3 - (42*C*a + 25*B*b)*c^2)*d*x + (2*C*b^3
 + 15*(2*B*a + A*b)*c^2 - (9*C*a*b + 5*B*b^2)*c)*d)*e^3 - (8*(4*C*b*c^2 + 5
*B*c^3)*d^2*x + (7*C*b^2*c - 30*A*c^3 - (42*C*a + 25*B*b)*c^2)*d^2)*e^2 + 8
*(6*C*c^3*d^3*x - (4*C*b*c^2 + 5*B*c^3)*d^3)*e)*sqrt(c)*e^(1/2)*weierstrass
PInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c
^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)
*e^(-3)/c^3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c) + 3*(48*C*c^3*d^3*e - (2*C
*b^2*c - 30*A*c^3 - (6*C*a + 5*B*b)*c^2)*x*e^4 - (8*(C*b*c^2 + 5*B*c^3)*d*x
 + (2*C*b^2*c - 30*A*c^3 - (6*C*a + 5*B*b)*c^2)*d)*e^3 + 8*(6*C*c^3*d^2*x -
 (C*b*c^2 + 5*B*c^3)*d^2)*e^2)*sqrt(c)*e^(1/2)*weierstrassZeta(4/3*(c^2*d^2
 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*
e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, weierstr
assPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(
2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e
^3)*e^(-3)/c^3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c)) + 3*(24*C*c^3*d^2*e^2
- (3*C*c^3*x^2 - 15*A*c^3 + (C*b*c^2 + 5*B*c^3)*x)*e^4 + (6*C*c^3*d*x - (C*
b*c^2 + 20*B*c^3)*d)*e^3)*sqrt(c*x^2 + b*x + a)*sqrt(x*e + d))/(c^3*x*e^6 +
 c^3*d*e^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx + Cx^2)\sqrt{a + bx + cx^2}}{(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(3/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(x*e + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(C\,x^2 + B\,x + A)\,\sqrt{c\,x^2 + b\,x + a}}{(d + e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(3/2),x)
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(3/2), x)
```

3.262 $\int \frac{\sqrt{a+bx+cx^2}\,(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=712

$$-\frac{2\left(e(bd-ae)(7Cd-3Be)-cd(8Cd^2-e(4Bd-Ae))+e^2\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-aCe\right)x\right)\sqrt{a+}}{3e^3\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

[Out] -2/3*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(3/2)-2/3*(e*(-a*e+b*d)*(-3*B*e+7*C*d)-c*d*(8*C*d^2-e*(-A*e+4*B*d))+e^2*(B
*c*d+b*C*d-2*c*C*d^2/e-A*c*e-a*C*e)*x)*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e
+c*d^2)/(e*x+d)^(1/2)+1/3*(2*(4*c*d-1/2*b*e)*(B*c*d+b*C*d-2*c*C*d^2/e-A*c*e
-a*C*e)+6*c*(b*d*(-B*e+C*d)+e*(A*c*d+B*a*e-C*a*d)))*EllipticE(1/2*((b+2*c*x
+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(
1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e
x+d)^(1/2)(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/e^3/(a*e^2-b*d*e+c*d^2)
/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3
(e(-3*B*b*e-2*C*a*e+8*C*b*d)-2*c*(8*C*d^2-e*(-A*e+4*B*d)))*EllipticF(1/2*
((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a
*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)
^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2))))^(1/2)/c/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)

Rubi [A]
time = 0.77, antiderivative size = 711, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1664, 826, 857, 732, 435, 430}

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,(e(-2aCe-3bBe+8bCd)-2c(8Cd^2-e(4Bd-Ae)))\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{3ce^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}+\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(2\left(4cd-\frac{be}{2}\right)\left(-aCe-Ace+bCd+Bcd-\frac{2cCd^2}{e}\right)+6c(e(aBe-aCd+Acd)+bd(Cd-Be))\right)E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{3ce^3\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2(a+bx+cx^2)^{3/2}(Cd^2-e(Bd-Ae))}{3e(d+ex)^{3/2}(ae^2-bde+cd^2)}+\frac{2\sqrt{a+bx+cx^2}\left(-ex\left(-aCe-Ace+bCd+Bcd-\frac{2cCd^2}{e}\right)-(bd-ae)(7Cd-3Be)-cd(4Bd-Ae)+\frac{8cCd^3}{e}\right)}{3e^2\sqrt{d+ex}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(5/2),x]

[Out] (2*((8*c*C*d^3)/e - c*d*(4*B*d - A*e) - (b*d - a*e)*(7*C*d - 3*B*e) - e*(B*
c*d + b*C*d - (2*c*C*d^2)/e - A*c*e - a*C*e)*x)*Sqrt[a + b*x + c*x^2])/(3*e
^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (2*(C*d^2 - e*(B*d - A*e))*(a +
 b*x + c*x^2)^(3/2))/(3*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) + (Sqrt[
2]*Sqrt[b^2 - 4*a*c]*(2*(4*c*d - (b*e)/2)*(B*c*d + b*C*d - (2*c*C*d^2)/e -

```
A*c*e - a*C*e) + 6*c*(b*d*(C*d - B*e) + e*(A*c*d - a*C*d + a*B*e)))*Sqrt[d
+ e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^3*(c*d^2 - b*d*e + a
*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
+ c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e*(8*b*C*d - 3*b*B*e - 2*a*C*e) -
 2*c*(8*C*d^2 - e*(4*B*d - A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSi
n[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqr
t[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^4*Sqrt[d + e
*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
```

```
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\,(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\,(a+bx+cx^2)^{3/2}}{3e\,(cd^2-bde+ae^2)\,(d+ex)^{3/2}} - \frac{2\int \frac{\left(-\frac{3(bd(Cd-Be)+e(Acd-\cdots}{2e}\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{2\left(\frac{8cCd^3}{e} - cd(4Bd-Ae) - (bd-ae)(7Cd-3Be) - e\left(Bcd+bC\cdots\right.\right.}{3e^2\,(cd^2-bde+ae^2)\,\sqrt{d+\cdots}}$$

$$= \frac{2\left(\frac{8cCd^3}{e} - cd(4Bd-Ae) - (bd-ae)(7Cd-3Be) - e\left(Bcd+bC\cdots\right.\right.}{3e^2\,(cd^2-bde+ae^2)\,\sqrt{d+\cdots}}$$

$$= \frac{2\left(\frac{8cCd^3}{e} - cd(4Bd-Ae) - (bd-ae)(7Cd-3Be) - e\left(Bcd+bC\cdots\right.\right.}{3e^2\,(cd^2-bde+ae^2)\,\sqrt{d+\cdots}}$$

$$= \frac{2\left(\frac{8cCd^3}{e} - cd(4Bd-Ae) - (bd-ae)(7Cd-3Be) - e\left(Bcd+bC\cdots\right.\right.}{3e^2\,(cd^2-bde+ae^2)\,\sqrt{d+\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 32.97, size = 8456, normalized size = 11.88

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 21037 vs. 2(646) = 1292.
time = 0.18, size = 21038, normalized size = 29.55

method	result
elliptic	$\frac{\sqrt{(c\,x^2+bx+a)\,(ex+d)}\left(-\frac{2\left(A\,e^2-Bde+C\,d^2\right)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}{3e^5\left(x+\frac{d}{e}\right)^2}-\frac{2\left(ce\,x^2+bex\cdots\right.}{\cdots}\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(x*e + d)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 1353, normalized size = 1.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="fri
cas")
```

```
[Out] 2/9*((16*C*c^3*d^6 - (C*a*b^2 - 6*A*a*c^2 - (6*C*a^2 + 3*B*a*b - A*b^2)*c)*
x^2*e^6 + ((C*b^3 - 2*(6*B*a + A*b)*c^2 - 2*(7*C*a*b + B*b^2)*c)*d*x^2 - 2*
(C*a*b^2 - 6*A*a*c^2 - (6*C*a^2 + 3*B*a*b - A*b^2)*c)*d*x)*e^5 + ((6*C*b^2*
c + 2*A*c^3 + (26*C*a + 11*B*b)*c^2)*d^2*x^2 + 2*(C*b^3 - 2*(6*B*a + A*b)*c
^2 - 2*(7*C*a*b + B*b^2)*c)*d^2*x - (C*a*b^2 - 6*A*a*c^2 - (6*C*a^2 + 3*B*a
*b - A*b^2)*c)*d^2)*e^4 - (8*(3*C*b*c^2 + B*c^3)*d^3*x^2 - 2*(6*C*b^2*c + 2
*A*c^3 + (26*C*a + 11*B*b)*c^2)*d^3*x - (C*b^3 - 2*(6*B*a + A*b)*c^2 - 2*(7
*C*a*b + B*b^2)*c)*d^3)*e^3 + (16*C*c^3*d^4*x^2 - 16*(3*C*b*c^2 + B*c^3)*d^
4*x + (6*C*b^2*c + 2*A*c^3 + (26*C*a + 11*B*b)*c^2)*d^4)*e^2 + 8*(4*C*c^3*d
^5*x - (3*C*b*c^2 + B*c^3)*d^5)*e)*sqrt(c)*e^(1/2)*weierstrassPInverse(4/3*
(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*
c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3,
1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c) + 3*(16*C*c^3*d^5*e - (C*a*b*c + (6*B*a
 + A*b)*c^2)*x^2*e^6 + ((C*b^2*c + 2*A*c^3 + 7*(2*C*a + B*b)*c^2)*d*x^2 - 2
*(C*a*b*c + (6*B*a + A*b)*c^2)*d*x)*e^5 - (8*(2*C*b*c^2 + B*c^3)*d^2*x^2 -
2*(C*b^2*c + 2*A*c^3 + 7*(2*C*a + B*b)*c^2)*d^2*x + (C*a*b*c + (6*B*a + A*b
)*c^2)*d^2)*e^4 + (16*C*c^3*d^3*x^2 - 16*(2*C*b*c^2 + B*c^3)*d^3*x + (C*b^2
*c + 2*A*c^3 + 7*(2*C*a + B*b)*c^2)*d^3)*e^3 + 8*(4*C*c^3*d^4*x - (2*C*b*c^
2 + B*c^3)*d^4)*e^2)*sqrt(c)*e^(1/2)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e
 + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2
*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, weierstrassPInvers
e(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3
- 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)
/c^3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c)) + 3*(8*C*c^3*d^4*e^2 + (C*a*c^2*
x^2 - A*a*c^2 - (3*B*a + A*b)*c^2*x)*e^6 - (C*b*c^2*d*x^2 + 2*B*a*c^2*d - 2
*(A*c^3 + 2*(2*C*a + B*b)*c^2)*d*x)*e^5 + (C*c^3*d^2*x^2 - (9*C*b*c^2 + 5*B
*c^3)*d^2*x + (A*c^3 + 3*(2*C*a + B*b)*c^2)*d^2)*e^4 + (10*C*c^3*d^3*x - (7
*C*b*c^2 + 4*B*c^3)*d^3)*e^3)*sqrt(c*x^2 + b*x + a)*sqrt(x*e + d))/(c^3*d^4
*e^5 + a*c^2*x^2*e^9 - (b*c^2*d*x^2 - 2*a*c^2*d*x)*e^8 + (c^3*d^2*x^2 - 2*b
*c^2*d^2*x + a*c^2*d^2)*e^7 + (2*c^3*d^3*x - b*c^2*d^3)*e^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx + Cx^2)\sqrt{a + bx + cx^2}}{(d + ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(5/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(x*e + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(C\,x^2+B\,x+A\right)\,\sqrt{c\,x^2+b\,x+a}}{{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(5/2),x)
```

```
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(5/2), x)
```

3.263 $\int \frac{\sqrt{a+bx+cx^2}\,(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx$

Optimal. Leaf size=992

$$-\frac{2(c^2d^3(24Cd^2-e(4Bd+Ae))+e^2(15b^2Cd^3+5a^2e^2(Cd+Be)-abe(22Cd^2+3Bde+2Ae^2))-cde(bd(4}{}$$

```
[Out] -2/5*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(5/2)-2/15*(c^2*d^3*(24*C*d^2-e*(A*e+4*B*d))+e^2*(15*b^2*C*d^3+5*a^2*e^2*(
B*e+C*d)-a*b*e*(2*A*e^2+3*B*d*e+22*C*d^2))-c*d*e*(b*d*(A*e^2-6*B*d*e+41*C*d
^2)-a*e*(7*A*e^2-7*B*d*e+37*C*d^2))+e*(5*c^2*d^2*(6*C*d^2-e*(A*e+B*d))+e^2*
(15*a^2*C*e^2-5*a*b*e*(-B*e+8*C*d)+b^2*(-2*A*e^2-3*B*d*e+23*C*d^2))-c*e*(5*
b*d*(-A*e^2-2*B*d*e+11*C*d^2)-a*e*(3*A*e^2-13*B*d*e+53*C*d^2)))*x)*(c*x^2+b
*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(3/2)+1/15*(2*c^2*d^2*(24*C*d
^2-e*(A*e+4*B*d))+e^2*(30*a^2*C*e^2-5*a*b*e*(-B*e+14*C*d)+b^2*(-2*A*e^2-3*B
*d*e+38*C*d^2))-c*e*(b*d*(-2*A*e^2-13*B*d*e+88*C*d^2)-2*a*e*(3*A*e^2-8*B*d*
e+43*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))
^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+
b^2))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d
-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/15*(15*b*C*e^2*(-a*e+b*d)+2*c^2*d*(24*C
*d^2-e*(A*e+4*B*d))+c*e*(10*a*e*(-B*e+5*C*d)-b*(-A*e^2-9*B*d*e+64*C*d^2)))*
EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^4/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(1/2)/
(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 1.17, antiderivative size = 989, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1664, 824, 857, 732, 435, 430}

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(c^2*(24*C*d^5 - d^3*e*(4*B*d + A*e)) + e^2*(15*b^2*C*d^3 + 5*a^2*e^2*(
C*d + B*e) - a*b*e*(22*C*d^2 + 3*B*d*e + 2*A*e^2)) - c*d*e*(b*d*(41*C*d^2 -
 6*B*d*e + A*e^2) - a*e*(37*C*d^2 - 7*B*d*e + 7*A*e^2)) + e^2*((30*c^2*C*d^
4)/e + 15*a^2*C*e^3 - 5*c^2*d^2*(B*d + A*e) - 5*a*b*e^2*(8*C*d - B*e) + a*c
*e*(53*C*d^2 - e*(13*B*d - 3*A*e)) - 5*b*c*d*(11*C*d^2 - e*(2*B*d + A*e)) +
 b^2*e*(23*C*d^2 - e*(3*B*d + 2*A*e)))*x)*Sqrt[a + b*x + c*x^2])/(15*e^3*(c
*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(3/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a +
b*x + c*x^2)^(3/2))/(5*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(5/2)) + (Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(c^2*(48*C*d^4 - 2*d^2*e*(4*B*d + A*e)) + e^2*(30*a^2*C
*e^2 - 5*a*b*e*(14*C*d - B*e) + b^2*(38*C*d^2 - 3*B*d*e - 2*A*e^2)) - c*e*(
b*d*(88*C*d^2 - 13*B*d*e - 2*A*e^2) - 2*a*e*(43*C*d^2 - 8*B*d*e + 3*A*e^2))
)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*e^4*(c*d^2 - b
*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqr
t[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(15*b*C*e^2*(b*d - a*e)
+ c^2*(48*C*d^3 - 2*d*e*(4*B*d + A*e)) - c*e*(64*b*C*d^2 - b*e*(9*B*d + A*e
) - 10*a*e*(5*C*d - B*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c*e^4*(c*d^2 - b*d*e +
 a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\,(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\,(a+bx+cx^2)^{3/2}}{5e\,(cd^2-bde+ae^2)\,(d+ex)^{5/2}} - \frac{2\int \frac{\left(-\frac{3bCd^2-be(3Bd+2Ae}{\cdots}\right.}{\cdots}}{\cdots}$$

$$= -\frac{2\left(c^2(24Cd^5-d^3e(4Bd+Ae))+e^2(15b^2Cd^3+5a^2e^2(Cd+Be\cdots\right.}{\cdots}$$

$$= -\frac{2\left(c^2(24Cd^5-d^3e(4Bd+Ae))+e^2(15b^2Cd^3+5a^2e^2(Cd+Be\cdots\right.}{\cdots}$$

$$= -\frac{2\left(c^2(24Cd^5-d^3e(4Bd+Ae))+e^2(15b^2Cd^3+5a^2e^2(Cd+Be\cdots\right.}{\cdots}$$

$$= -\frac{2\left(c^2(24Cd^5-d^3e(4Bd+Ae))+e^2(15b^2Cd^3+5a^2e^2(Cd+Be\cdots\right.}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.45, size = 12997, normalized size = 13.10

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(7/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 48426 vs. 2(928) = 1856.
time = 0.21, size = 48427, normalized size = 48.82

method	result	size
elliptic	Expression too large to display	1766
default	Expression too large to display	48427

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(x*e + d)^(7/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 2405, normalized size = 2.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="fri
cas")
```

```
[Out] -2/45*((48*C*c^3*d^8 - (15*C*a^2*b - 5*B*a*b^2 + 2*A*b^3 + 3*(10*B*a^2 - 3*
A*a*b)*c)*x^3*e^8 + ((20*C*a*b^2 - 3*B*b^3 - 18*A*a*c^2 + (90*C*a^2 + 31*B*
a*b + 3*A*b^2)*c)*d*x^3 - 3*(15*C*a^2*b - 5*B*a*b^2 + 2*A*b^3 + 3*(10*B*a^2
 - 3*A*a*b)*c)*d*x^2)*e^7 - ((7*C*b^3 + (22*B*a - 3*A*b)*c^2 + (161*C*a*b +
 8*B*b^2)*c)*d^2*x^3 - 3*(20*C*a*b^2 - 3*B*b^3 - 18*A*a*c^2 + (90*C*a^2 + 3
1*B*a*b + 3*A*b^2)*c)*d^2*x^2 + 3*(15*C*a^2*b - 5*B*a*b^2 + 2*A*b^3 + 3*(10
*B*a^2 - 3*A*a*b)*c)*d^2*x)*e^6 + ((73*C*b^2*c - 2*A*c^3 + (122*C*a + 17*B*
b)*c^2)*d^3*x^3 - 3*(7*C*b^3 + (22*B*a - 3*A*b)*c^2 + (161*C*a*b + 8*B*b^2)
*c)*d^3*x^2 + 3*(20*C*a*b^2 - 3*B*b^3 - 18*A*a*c^2 + (90*C*a^2 + 31*B*a*b +
 3*A*b^2)*c)*d^3*x - (15*C*a^2*b - 5*B*a*b^2 + 2*A*b^3 + 3*(10*B*a^2 - 3*A*
a*b)*c)*d^3)*e^5 - (8*(14*C*b*c^2 + B*c^3)*d^4*x^3 - 3*(73*C*b^2*c - 2*A*c^
3 + (122*C*a + 17*B*b)*c^2)*d^4*x^2 + 3*(7*C*b^3 + (22*B*a - 3*A*b)*c^2 + (
161*C*a*b + 8*B*b^2)*c)*d^4*x - (20*C*a*b^2 - 3*B*b^3 - 18*A*a*c^2 + (90*C*
a^2 + 31*B*a*b + 3*A*b^2)*c)*d^4)*e^4 + (48*C*c^3*d^5*x^3 - 24*(14*C*b*c^2
```

```
+ B*c^3)*d^5*x^2 + 3*(73*C*b^2*c - 2*A*c^3 + (122*C*a + 17*B*b)*c^2)*d^5*x
- (7*C*b^3 + (22*B*a - 3*A*b)*c^2 + (161*C*a*b + 8*B*b^2)*c)*d^5)*e^3 + (14
4*C*c^3*d^6*x^2 - 24*(14*C*b*c^2 + B*c^3)*d^6*x + (73*C*b^2*c - 2*A*c^3 + (
122*C*a + 17*B*b)*c^2)*d^6)*e^2 + 8*(18*C*c^3*d^7*x - (14*C*b*c^2 + B*c^3)*
d^7)*e)*sqrt(c)*e^(1/2)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 -
 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*
c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, 1/3*(c*d + (3*c*x + b)*e)*e
^(-1)/c) + 3*(48*C*c^3*d^7*e + (6*A*a*c^2 + (30*C*a^2 + 5*B*a*b - 2*A*b^2)*
c)*x^3*e^8 - ((2*(8*B*a - A*b)*c^2 + (70*C*a*b + 3*B*b^2)*c)*d*x^3 - 3*(6*A
*a*c^2 + (30*C*a^2 + 5*B*a*b - 2*A*b^2)*c)*d*x^2)*e^7 + ((38*C*b^2*c - 2*A*
c^3 + (86*C*a + 13*B*b)*c^2)*d^2*x^3 - 3*(2*(8*B*a - A*b)*c^2 + (70*C*a*b +
 3*B*b^2)*c)*d^2*x^2 + 3*(6*A*a*c^2 + (30*C*a^2 + 5*B*a*b - 2*A*b^2)*c)*d^2
*x)*e^6 - (8*(11*C*b*c^2 + B*c^3)*d^3*x^3 - 3*(38*C*b^2*c - 2*A*c^3 + (86*C
*a + 13*B*b)*c^2)*d^3*x^2 + 3*(2*(8*B*a - A*b)*c^2 + (70*C*a*b + 3*B*b^2)*c
)*d^3*x - (6*A*a*c^2 + (30*C*a^2 + 5*B*a*b - 2*A*b^2)*c)*d^3)*e^5 + (48*C*c
^3*d^4*x^3 - 24*(11*C*b*c^2 + B*c^3)*d^4*x^2 + 3*(38*C*b^2*c - 2*A*c^3 + (8
6*C*a + 13*B*b)*c^2)*d^4*x - (2*(8*B*a - A*b)*c^2 + (70*C*a*b + 3*B*b^2)*c)
*d^4)*e^4 + (144*C*c^3*d^5*x^2 - 24*(11*C*b*c^2 + B*c^3)*d^5*x + (38*C*b^2*
c - 2*A*c^3 + (86*C*a + 13*B*b)*c^2)*d^5)*e^3 + 8*(18*C*c^3*d^6*x - (11*C*b
*c^2 + B*c^3)*d^6)*e^2)*sqrt(c)*e^(1/2)*weierstrassZeta(4/3*(c^2*d^2 - b*c*
d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(
b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, weierstrassPInv
erse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d
^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(
-3)/c^3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c)) + 3*(24*C*c^3*d^6*e^2 + (3*A*
a^2*c + (5*B*a^2 + A*a*b)*c*x + (6*A*a*c^2 + (15*C*a^2 + 5*B*a*b - 2*A*b^2)
*c)*x^2)*e^8 - ((2*(8*B*a - A*b)*c^2 + (40*C*a*b + 3*B*b^2)*c)*d*x^2 - (2*B
*a^2 - 5*A*a*b)*c*d - (10*A*a*c^2 + (20*C*a^2 - B*a*b - 5*A*b^2)*c)*d*x)*e^
7 + ((23*C*b^2*c - 2*A*c^3 + (56*C*a + 13*B*b)*c^2)*d^2*x^2 - (59*C*a*b*c +
 (20*B*a - 7*A*b)*c^2)*d^2*x + 2*(4*C*a^2*c + 5*A*a*c^2)*d^2)*e^6 - (2*(29*
C*b*c^2 + 4*B*c^3)*d^3*x^2 - (35*C*b^2*c - 6*A*c^3 + (90*C*a + 13*B*b)*c^2)
*d^3*x + (25*C*a*b*c + (10*B*a + A*b)*c^2)*d^3)*e^5 + (33*C*c^3*d^4*x^2 - 3
*(31*C*b*c^2 + 3*B*c^3)*d^4*x + (15*C*b^2*c - A*c^3 + 2*(20*C*a + 3*B*b)*c^
2)*d^4)*e^4 + (54*C*c^3*d^5*x - (41*C*b*c^2 + 4*B*c^3)*d^5)*e^3)*sqrt(c*x^2
 + b*x + a)*sqrt(x*e + d))/(c^3*d^7*e^5 + a^2*c*x^3*e^12 - (2*a*b*c*d*x^3 -
 3*a^2*c*d*x^2)*e^11 - (6*a*b*c*d^2*x^2 - 3*a^2*c*d^2*x - (b^2*c + 2*a*c^2)
*d^2*x^3)*e^10 - (2*b*c^2*d^3*x^3 + 6*a*b*c*d^3*x - a^2*c*d^3 - 3*(b^2*c +
2*a*c^2)*d^3*x^2)*e^9 + (c^3*d^4*x^3 - 6*b*c^2*d^4*x^2 - 2*a*b*c*d^4 + 3*(b
^2*c + 2*a*c^2)*d^4*x)*e^8 + (3*c^3*d^5*x^2 - 6*b*c^2*d^5*x + (b^2*c + 2*a*
c^2)*d^5)*e^7 + (3*c^3*d^6*x - 2*b*c^2*d^6)*e^6)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx + Cx^2)\sqrt{a + bx + cx^2}}{(d + ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(7/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(x*e + d)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(C\,x^2 + B\,x + A)\,\sqrt{c\,x^2 + b\,x + a}}{(d + e\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(7/2),x)
```

```
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(7/2), x)
```

3.264 $\int \frac{\sqrt{a+bx+cx^2}\,(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx$

Optimal. Leaf size=1363

$$\frac{2(2c^3d^3(24Cd^2+e(4Bd+3Ae))-be^3(35a^2Ce^2-14abe(3Cd+Be)+b^2(15Cd^2+6Bde+8Ae^2))+c^2de(}{}$$

```
[Out] -2/7*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(7/2)-2/105*(c^2*d^3*(24*C*d^2+e*(3*A*e+4*B*d))-e^2*(7*a^2*e^2*(-3*B*e+C*d
)-b^2*d*(8*A*e^2+6*B*d*e+15*C*d^2)+a*b*e*(12*A*e^2+23*B*d*e+12*C*d^2))-c*d*
e*(b*d*(15*A*e^2+6*B*d*e+43*C*d^2)-a*e*(19*A*e^2+9*B*d*e+33*C*d^2))+e*(7*c^
2*d^2*(6*C*d^2+e*(-3*A*e+B*d))+e^2*(35*a^2*C*e^2-7*a*b*e*(-B*e+12*C*d)+b^2*
(-4*A*e^2-3*B*d*e+45*C*d^2))+c*e*(a*e*(-5*A*e^2-9*B*d*e+93*C*d^2)-b*(-21*A*
d*e^2+91*C*d^3)))*x)*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^
(5/2)+2/105*(2*c^3*d^3*(24*C*d^2+e*(3*A*e+4*B*d))-b*e^3*(35*a^2*C*e^2-14*a*
b*e*(B*e+3*C*d)+b^2*(8*A*e^2+6*B*d*e+15*C*d^2))+c^2*d*e*(2*a*e*(69*C*d^2+e*
(-29*A*e+15*B*d))-b*d*(128*C*d^2+e*(9*A*e+19*B*d)))+c*e^2*(14*a^2*e^2*(-3*B
*e+11*C*d)-a*b*e*(237*C*d^2+e*(-29*A*e+B*d))+b^2*d*(103*C*d^2+e*(19*A*e+9*B
*d))))*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)^(1/2)-1/105*(2
*c^3*d^3*(24*C*d^2+e*(3*A*e+4*B*d))-b*e^3*(35*a^2*C*e^2-14*a*b*e*(B*e+3*C*d
)+b^2*(8*A*e^2+6*B*d*e+15*C*d^2))+c^2*d*e*(2*a*e*(69*C*d^2+e*(-29*A*e+15*B*
d))-b*d*(128*C*d^2+e*(9*A*e+19*B*d)))+c*e^2*(14*a^2*e^2*(-3*B*e+11*C*d)-a*b
*e*(237*C*d^2+e*(-29*A*e+B*d))+b^2*d*(103*C*d^2+e*(19*A*e+9*B*d))))*Ellipti
cE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*
e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a
*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^4/(a*e^
2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))^(1/2)+2/105*(2*c^2*d^2*(24*C*d^2+e*(3*A*e+4*B*d))+c*e*(2*a*e*(51*C*d
^2+e*(-5*A*e+12*B*d))-b*d*(104*C*d^2+3*e*(2*A*e+5*B*d)))+e^2*(70*a^2*C*e^2-
7*a*b*e*(B*e+18*C*d)+b^2*(60*C*d^2+e*(4*A*e+3*B*d))))*EllipticF(1/2*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)
^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*
(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(
1/2))))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]

time = 2.49, antiderivative size = 1363, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {1664, 824, 848, 857, 732, 435, 430}

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(9/2),x]
```

```
[Out] (2*(c^3*(48*C*d^5 + 2*d^3*e*(4*B*d + 3*A*e)) - b*e^3*(35*a^2*C*e^2 - 14*a*b
*e*(3*C*d + B*e) + b^2*(15*C*d^2 + 6*B*d*e + 8*A*e^2)) + c^2*d*e*(2*a*e*(69
*C*d^2 + e*(15*B*d - 29*A*e)) - b*(128*C*d^3 + d*e*(19*B*d + 9*A*e))) + c*e
^2*(14*a^2*e^2*(11*C*d - 3*B*e) - a*b*e*(237*C*d^2 + e*(B*d - 29*A*e)) + b^
2*(103*C*d^3 + d*e*(9*B*d + 19*A*e))))*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d
^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x]) - (2*(c^2*(24*C*d^5 + d^3*e*(4*B*d + 3
*A*e)) - e^2*(7*a^2*e^2*(C*d - 3*B*e) - b^2*d*(15*C*d^2 + 6*B*d*e + 8*A*e^2
) + a*b*e*(12*C*d^2 + 23*B*d*e + 12*A*e^2)) - c*d*e*(b*d*(43*C*d^2 + 6*B*d*
e + 15*A*e^2) - a*e*(33*C*d^2 + 9*B*d*e + 19*A*e^2)) + e*(7*c^2*(6*C*d^4 +
d^2*e*(B*d - 3*A*e)) + e^2*(35*a^2*C*e^2 - 7*a*b*e*(12*C*d - B*e) + b^2*(45
*C*d^2 - 3*B*d*e - 4*A*e^2)) + c*e*(a*e*(93*C*d^2 - 9*B*d*e - 5*A*e^2) - b*
(91*C*d^3 - 21*A*d*e^2)))*x)*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d^2 - b*d*e
 + a*e^2)^2*(d + e*x)^(5/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)
^(3/2))/(7*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(7/2)) - (Sqrt[2]*Sqrt[b^2 -
 4*a*c]*(2*c^3*(24*C*d^5 + d^3*e*(4*B*d + 3*A*e)) - b*e^3*(35*a^2*C*e^2 - 1
4*a*b*e*(3*C*d + B*e) + b^2*(15*C*d^2 + 6*B*d*e + 8*A*e^2)) + c^2*d*e*(2*a*
e*(69*C*d^2 + e*(15*B*d - 29*A*e)) - b*(128*C*d^3 + d*e*(19*B*d + 9*A*e)))
+ c*e^2*(14*a^2*e^2*(11*C*d - 3*B*e) - a*b*e*(237*C*d^2 + e*(B*d - 29*A*e))
 + b^2*(103*C*d^3 + d*e*(9*B*d + 19*A*e))))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e)])/(105*e^4*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[(c*(d + e*x
))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]
*Sqrt[b^2 - 4*a*c]*(c^2*(48*C*d^4 + 2*d^2*e*(4*B*d + 3*A*e)) + c*e*(2*a*e*(
51*C*d^2 + e*(12*B*d - 5*A*e)) - b*(104*C*d^3 + 3*d*e*(5*B*d + 2*A*e))) + e
^2*(70*a^2*C*e^2 - 7*a*b*e*(18*C*d + B*e) + b^2*(60*C*d^2 + e*(3*B*d + 4*A*
e))))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(105*e^4*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x
]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\,(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\,(a+bx+cx^2)^{3/2}}{7e\,(cd^2-bde+ae^2)\,(d+ex)^{7/2}} - \frac{2\int \frac{\left(-\frac{3bCd^2-be(3Bd+4Ae}{2}\right.}{}}{}$$

$$= -\frac{2(c^2(24Cd^5+d^3e(4Bd+3Ae)) - e^2(7a^2e^2(Cd-3Be) - b^2d(1}{}$$

$$= \frac{2(c^3(48Cd^5+2d^3e(4Bd+3Ae)) - be^3(35a^2Ce^2 - 14abe(3Cd+}{}$$

$$= \frac{2(c^3(48Cd^5+2d^3e(4Bd+3Ae)) - be^3(35a^2Ce^2 - 14abe(3Cd+}{}$$

$$= \frac{2(c^3(48Cd^5+2d^3e(4Bd+3Ae)) - be^3(35a^2Ce^2 - 14abe(3Cd+}{}$$

$$= \frac{2(c^3(48Cd^5+2d^3e(4Bd+3Ae)) - be^3(35a^2Ce^2 - 14abe(3Cd+}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 34.05, size = 19853, normalized size = 14.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(9/2),x]
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 88789 vs. 2(1293) = 2586.
time = 0.29, size = 88790, normalized size = 65.14

method	result	size
elliptic	Expression too large to display	2484
default	Expression too large to display	88790

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(x*e + d)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 4494, normalized size = 3.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="fri
cas")
```

```
[Out] 2/315*((48*C*c^4*d^10 - (35*C*a^2*b^2 - 14*B*a*b^3 + 8*A*b^4 + 30*A*a^2*c^2
 - (210*C*a^3 - 63*B*a^2*b + 41*A*a*b^2)*c)*x^4*e^10 + ((42*C*a*b^3 - 6*B*b
^4 + 52*(3*B*a^2 - 2*A*a*b)*c^2 - (364*C*a^2*b - B*a*b^2 - 23*A*b^3)*c)*d*x
^4 - 4*(35*C*a^2*b^2 - 14*B*a*b^3 + 8*A*b^4 + 30*A*a^2*c^2 - (210*C*a^3 - 6
3*B*a^2*b + 41*A*a*b^2)*c)*d*x^3)*e^9 - ((15*C*b^4 - 104*A*a*c^3 - (208*C*a
^2 - 106*B*a*b - 17*A*b^2)*c^2 - 3*(79*C*a*b^2 + 4*B*b^3)*c)*d^2*x^4 - 4*(4
2*C*a*b^3 - 6*B*b^4 + 52*(3*B*a^2 - 2*A*a*b)*c^2 - (364*C*a^2*b - B*a*b^2 -
 23*A*b^3)*c)*d^2*x^3 + 6*(35*C*a^2*b^2 - 14*B*a*b^3 + 8*A*b^4 + 30*A*a^2*c
^2 - (210*C*a^3 - 63*B*a^2*b + 41*A*a*b^2)*c)*d^2*x^2)*e^8 - ((47*C*b^3*c -
 12*(3*B*a - A*b)*c^3 + (384*C*a*b - 17*B*b^2)*c^2)*d^3*x^4 + 4*(15*C*b^4 -
 104*A*a*c^3 - (208*C*a^2 - 106*B*a*b - 17*A*b^2)*c^2 - 3*(79*C*a*b^2 + 4*B
```

*b^3)*c)*d^3*x^3 - 6*(42*C*a*b^3 - 6*B*b^4 + 52*(3*B*a^2 - 2*A*a*b)*c^2 - (364*C*a^2*b - B*a*b^2 - 23*A*b^3)*c)*d^3*x^2 + 4*(35*C*a^2*b^2 - 14*B*a*b^3 + 8*A*b^4 + 30*A*a^2*c^2 - (210*C*a^3 - 63*B*a^2*b + 41*A*a*b^2)*c)*d^3*x)*e^7 + ((158*C*b^2*c^2 + 6*A*c^4 + (174*C*a - 23*B*b)*c^3)*d^4*x^4 - 4*(47*C*b^3*c - 12*(3*B*a - A*b)*c^3 + (384*C*a*b - 17*B*b^2)*c^2)*d^4*x^3 - 6*(15*C*b^4 - 104*A*a*c^3 - (208*C*a^2 - 106*B*a*b - 17*A*b^2)*c^2 - 3*(79*C*a*b^2 + 4*B*b^3)*c)*d^4*x^2 + 4*(42*C*a*b^3 - 6*B*b^4 + 52*(3*B*a^2 - 2*A*a*b)*c^2 - (364*C*a^2*b - B*a*b^2 - 23*A*b^3)*c)*d^4*x - (35*C*a^2*b^2 - 14*B*a*b^3 + 8*A*b^4 + 30*A*a^2*c^2 - (210*C*a^3 - 63*B*a^2*b + 41*A*a*b^2)*c)*d^4)*e^6 - (8*(19*C*b*c^3 - B*c^4)*d^5*x^4 - 4*(158*C*b^2*c^2 + 6*A*c^4 + (174*C*a - 23*B*b)*c^3)*d^5*x^3 + 6*(47*C*b^3*c - 12*(3*B*a - A*b)*c^3 + (384*C*a*b - 17*B*b^2)*c^2)*d^5*x^2 + 4*(15*C*b^4 - 104*A*a*c^3 - (208*C*a^2 - 106*B*a*b - 17*A*b^2)*c^2 - 3*(79*C*a*b^2 + 4*B*b^3)*c)*d^5*x - (42*C*a*b^3 - 6*B*b^4 + 52*(3*B*a^2 - 2*A*a*b)*c^2 - (364*C*a^2*b - B*a*b^2 - 23*A*b^3)*c)*d^5)*e^5 + (48*C*c^4*d^6*x^4 - 32*(19*C*b*c^3 - B*c^4)*d^6*x^3 + 6*(158*C*b^2*c^2 + 6*A*c^4 + (174*C*a - 23*B*b)*c^3)*d^6*x^2 - 4*(47*C*b^3*c - 12*(3*B*a - A*b)*c^3 + (384*C*a*b - 17*B*b^2)*c^2)*d^6*x - (15*C*b^4 - 104*A*a*c^3 - (208*C*a^2 - 106*B*a*b - 17*A*b^2)*c^2 - 3*(79*C*a*b^2 + 4*B*b^3)*c)*d^6)*e^4 + (192*C*c^4*d^7*x^3 - 48*(19*C*b*c^3 - B*c^4)*d^7*x^2 + 4*(158*C*b^2*c^2 + 6*A*c^4 + (174*C*a - 23*B*b)*c^3)*d^7*x - (47*C*b^3*c - 12*(3*B*a - A*b)*c^3 + (384*C*a*b - 17*B*b^2)*c^2)*d^7)*e^3 + (288*C*c^4*d^8*x^2 - 32*(19*C*b*c^3 - B*c^4)*d^8*x + (158*C*b^2*c^2 + 6*A*c^4 + (174*C*a - 23*B*b)*c^3)*d^8)*e^2 + 8*(24*C*c^4*d^9*x - (19*C*b*c^3 - B*c^4)*d^9)*e)*sqrt(c)*e^(1/2)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c) + 3*(48*C*c^4*d^9*e - ((42*B*a^2 - 29*A*a*b)*c^2 + (35*C*a^2*b - 14*B*a*b^2 + 8*A*b^3)*c)*x^4*e^10 - ((58*A*a*c^3 - (154*C*a^2 - B*a*b + 19*A*b^2)*c^2 - 6*(7*C*a*b^2 - B*b^3)*c)*d*x^4 + 4*((42*B*a^2 - 29*A*a*b)*c^2 + (35*C*a^2*b - 14*B*a*b^2 + 8*A*b^3)*c)*d*x^3)*e^9 - (3*(5*C*b^3*c - (10*B*a - 3*A*b)*c^3 + (79*C*a*b - 3*B*b^2)*c^2)*d^2*x^4 + 4*(58*A*a*c^3 - (154*C*a^2 - B*a*b + 19*A*b^2)*c^2 - 6*(7*C*a*b^2 - B*b^3)*c)*d^2*x^3 + 6*((42*B*a^2 - 29*A*a*b)*c^2 + (35*C*a^2*b - 14*B*a*b^2 + 8*A*b^3)*c)*d^2*x^2)*e^8 + ((103*C*b^2*c^2 + 6*A*c^4 + (138*C*a - 19*B*b)*c^3)*d^3*x^4 - 12*(5*C*b^3*c - (10*B*a - 3*A*b)*c^3 + (79*C*a*b - 3*B*b^2)*c^2)*d^3*x^3 - 6*(58*A*a*c^3 - (154*C*a^2 - B*a*b + 19*A*b^2)*c^2 - 6*(7*C*a*b^2 - B*b^3)*c)*d^3*x^2 - 4*((42*B*a^2 - 29*A*a*b)*c^2 + (35*C*a^2*b - 14*B*a*b^2 + 8*A*b^3)*c)*d^3*x)*e^7 - (8*(16*C*b*c^3 - B*c^4)*d^4*x^4 - 4*(103*C*b^2*c^2 + 6*A*c^4 + (138*C*a - 19*B*b)*c^3)*d^4*x^3 + 18*(5*C*b^3*c - (10*B*a - 3*A*b)*c^3 + (79*C*a*b - 3*B*b^2)*c^2)*d^4*x^2 + 4*(58*A*a*c^3 - (154*C*a^2 - B*a*b + 19*A*b^2)*c^2 - 6*(7*C*a*b^2 - B*b^3)*c)*d^4*x + ((42*B*a^2 - 29*A*a*b)*c^2 + (35*C*a^2*b - 14*B*a*b^2 + 8*A*b^3)*c)*d^4)*e^6 + (48*C*c^4*d^5*x^4 - 32*(16*C*b*c^3 - B*c^4)*d^5*x^3 + 6*(103*C*b^2*c^2 + 6*A*c^4 + (138*C*a - 19*B*b)*c^3)*d^5*x^2 - 12*(5*C*b^3*c - (10*B*a - 3*A*b)*c^3 + (79*C*a*b - 3*B*b^2)*c^2)*d^5*x - (58*A*a*c^3 - (154*C*a^2 - B*a*b + 19*A*b^2)*c^2 - 6*(7*C*a*b^2 - B*b^3)*c)*d

```
^5)*e^5 + (192*C*c^4*d^6*x^3 - 48*(16*C*b*c^3 - B*c^4)*d^6*x^2 + 4*(103*C*b
^2*c^2 + 6*A*c^4 + (138*C*a - 19*B*b)*c^3)*d^6*x - 3*(5*C*b^3*c - (10*B*a -
 3*A*b)*c^3 + (79*C*a*b - 3*B*b^2)*c^2)*d^6)*e^4 + (288*C*c^4*d^7*x^2 - 32*
(16*C*b*c^3 - B*c^4)*d^7*x + (103*C*b^2*c^2 + 6*A*c^4 + (138*C*a - 19*B*b)*
c^3)*d^7)*e^3 + 8*(24*C*c^4*d^8*x - (16*C*b*c^3 - B*c^4)*d^8)*e^2)*sqrt(c)*
e^(1/2)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/
c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3
- 9*a*b*c)*e^3)*e^(-3)/c^3, weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b
^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c -
6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, 1/3*(c*d + (3*c*x + b)*
e)*e^(-1)/c)) + 3*(24*C*c^4*d^8*e^2 - (15*A*a^3...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + Bx + Cx^2)\sqrt{a + bx + cx^2}}{(d + ex)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(9/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(9/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(x*e + d)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(C\,x^2 + B\,x + A)\,\sqrt{c\,x^2 + b\,x + a}}{(d + e\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(9/2),x)
```

```
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(9/2), x)
```

3.265 $\int \frac{\sqrt{a+bx+cx^2}\,(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx$

Optimal. Leaf size=1904

$$\frac{2(2c^3d^3(8Cd^2+e(4Bd+5Ae))+3c^2de(2ae(9Cd^2+7Bde-9Ae^2)-bd(16Cd^2+7Bde+5Ae^2))+3ce^2(2}{}$$

```
[Out] -2/9*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(9/2)+2/315*(2*c^3*d^3*(8*C*d^2+e*(5*A*e+4*B*d))+3*c^2*d*e*(2*a*e*(-9*A*e^
2+7*B*d*e+9*C*d^2)-b*d*(5*A*e^2+7*B*d*e+16*C*d^2))+3*c*e^2*(2*a^2*e^2*(-5*B
*e+17*C*d)-a*b*e*(-9*A*e^2+5*B*d*e+41*C*d^2)+b^2*d*(7*A*e^2+3*B*d*e+15*C*d^
2))-b*e^3*(21*a^2*C*e^2-6*a*b*e*(2*B*e+3*C*d)+b^2*(8*A*e^2+4*B*d*e+5*C*d^2)
))*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)^(3/2)-2/105*(c^2*d
^3*(8*C*d^2+e*(5*A*e+4*B*d))-e^2*(3*a^2*e^2*(-5*B*e+3*C*d)-a*b*e*(-10*A*e^2
-17*B*d*e+2*C*d^2)-b^2*d*(8*A*e^2+4*B*d*e+5*C*d^2))-c*d*e*(3*b*d*(5*A*e^2+2
*B*d*e+5*C*d^2)-a*e*(13*A*e^2+11*B*d*e+7*C*d^2))+e*(3*c^2*d^2*(6*C*d^2+e*(-
5*A*e+3*B*d))+c*e*(a*e*(-7*A*e^2+B*d*e+47*C*d^2)-3*b*d*(-5*A*e^2+2*B*d*e+15
*C*d^2))+e^2*(21*a^2*C*e^2-3*a*b*e*(-B*e+16*C*d)+b^2*(25*C*d^2-e*(2*A*e+B*d
))))*x)*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(7/2)+2/315*(
2*c^4*d^4*(8*C*d^2+e*(5*A*e+4*B*d))+2*b^2*e^4*(21*a^2*C*e^2-6*a*b*e*(2*B*e+
3*C*d)+b^2*(8*A*e^2+4*B*d*e+5*C*d^2))-6*c^2*e^2*(a*b*d*e*(-34*A*e^2-5*B*d*e
+30*C*d^2)-a^2*e^2*(7*A*e^2-36*B*d*e+30*C*d^2)-b^2*d^2*(11*A*e^2+3*B*d*e+11
*C*d^2))-c*e^3*(126*a^3*C*e^3-3*a^2*b*e^2*(29*B*e+12*C*d)-6*a*b^2*e*(-12*A*
e^2+7*B*d*e+5*C*d^2)+b^3*d*(56*A*e^2+25*B*d*e+20*C*d^2))+c^3*d^2*e*(6*a*e*(
-34*A*e^2+8*B*d*e+11*C*d^2)-b*d*(56*C*d^2+5*e*(4*A*e+5*B*d))))*(c*x^2+b*x+a
)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^4/(e*x+d)^(1/2)-1/315*(2*c^4*d^4*(8*C*d^2+e
*(5*A*e+4*B*d))+2*b^2*e^4*(21*a^2*C*e^2-6*a*b*e*(2*B*e+3*C*d)+b^2*(8*A*e^2+
4*B*d*e+5*C*d^2))-6*c^2*e^2*(a*b*d*e*(-34*A*e^2-5*B*d*e+30*C*d^2)-a^2*e^2*(
7*A*e^2-36*B*d*e+30*C*d^2)-b^2*d^2*(11*A*e^2+3*B*d*e+11*C*d^2))-c*e^3*(126*
a^3*C*e^3-3*a^2*b*e^2*(29*B*e+12*C*d)-6*a*b^2*e*(-12*A*e^2+7*B*d*e+5*C*d^2)
+b^3*d*(56*A*e^2+25*B*d*e+20*C*d^2))+c^3*d^2*e*(6*a*e*(-34*A*e^2+8*B*d*e+11
*C*d^2)-b*d*(56*C*d^2+5*e*(4*A*e+5*B*d))))*EllipticE(1/2*((b+2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c
*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/
2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b
*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/315*(2*c^3
*d^3*(8*C*d^2+e*(5*A*e+4*B*d))+3*c^2*d*e*(2*a*e*(-9*A*e^2+7*B*d*e+9*C*d^2)-
```

```
b*d*(5*A*e^2+7*B*d*e+16*C*d^2))+3*c*e^2*(2*a^2*e^2*(-5*B*e+17*C*d)-a*b*e*(-
9*A*e^2+5*B*d*e+41*C*d^2)+b^2*d*(7*A*e^2+3*B*d*e+15*C*d^2))-b*e^3*(21*a^2*C
*e^2-6*a*b*e*(2*B*e+3*C*d)+b^2*(8*A*e^2+4*B*d*e+5*C*d^2)))*EllipticF(1/2*((
b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c
+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(
1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b
^2)^(1/2))))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1
/2)
```

Rubi [A]

time = 3.75, antiderivative size = 1904, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {1664, 824, 848, 857, 732, 435, 430}

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(11/2),x]
```

```
[Out] (2*(2*c^3*(8*C*d^5 + d^3*e*(4*B*d + 5*A*e)) + 3*c^2*d*e*(2*a*e*(9*C*d^2 + 7
*B*d*e - 9*A*e^2) - b*d*(16*C*d^2 + 7*B*d*e + 5*A*e^2)) + 3*c*e^2*(2*a^2*e^
2*(17*C*d - 5*B*e) - a*b*e*(41*C*d^2 + 5*B*d*e - 9*A*e^2) + b^2*d*(15*C*d^2
 + 3*B*d*e + 7*A*e^2)) - b*e^3*(21*a^2*C*e^2 - 6*a*b*e*(3*C*d + 2*B*e) + b^
2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)))*Sqrt[a + b*x + c*x^2])/(315*e^3*(c*d^2 -
b*d*e + a*e^2)^3*(d + e*x)^(3/2)) + (2*(2*c^4*(8*C*d^6 + d^4*e*(4*B*d + 5*A
*e)) - c^3*d^2*e*(56*b*C*d^3 + 5*b*d*e*(5*B*d + 4*A*e) - 6*a*e*(11*C*d^2 +
8*B*d*e - 34*A*e^2)) + 2*b^2*e^4*(21*a^2*C*e^2 - 6*a*b*e*(3*C*d + 2*B*e) +
b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)) - 6*c^2*e^2*(a*b*d*e*(30*C*d^2 - 5*B*d*e
 - 34*A*e^2) - a^2*e^2*(30*C*d^2 - 36*B*d*e + 7*A*e^2) - b^2*d^2*(11*C*d^2
+ 3*B*d*e + 11*A*e^2)) - c*e^3*(126*a^3*C*e^3 - 3*a^2*b*e^2*(12*C*d + 29*B*
e) - 6*a*b^2*e*(5*C*d^2 + 7*B*d*e - 12*A*e^2) + b^3*d*(20*C*d^2 + 25*B*d*e
+ 56*A*e^2)))*Sqrt[a + b*x + c*x^2])/(315*e^3*(c*d^2 - b*d*e + a*e^2)^4*Sqr
t[d + e*x]) - (2*(c^2*(8*C*d^5 + d^3*e*(4*B*d + 5*A*e)) - e^2*(3*a^2*e^2*(3
*C*d - 5*B*e) - a*b*e*(2*C*d^2 - 17*B*d*e - 10*A*e^2) - b^2*d*(5*C*d^2 + 4*
B*d*e + 8*A*e^2)) - c*d*e*(3*b*d*(5*C*d^2 + 2*B*d*e + 5*A*e^2) - a*e*(7*C*d
^2 + 11*B*d*e + 13*A*e^2)) + e^2*((3*c^2*(6*C*d^4 + d^2*e*(3*B*d - 5*A*e)))
/e + c*(a*e*(47*C*d^2 + e*(B*d - 7*A*e)) - 3*b*(15*C*d^3 + d*e*(2*B*d - 5*A
*e))) + e*(21*a^2*C*e^2 - 3*a*b*e*(16*C*d - B*e) + b^2*(25*C*d^2 - e*(B*d +
 2*A*e))))*x)*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d^2 - b*d*e + a*e^2)^2*(d
+ e*x)^(7/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)^(3/2))/(9*e*(c
*d^2 - b*d*e + a*e^2)*(d + e*x)^(9/2)) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c^4*
(8*C*d^6 + d^4*e*(4*B*d + 5*A*e)) - c^3*d^2*e*(56*b*C*d^3 + 5*b*d*e*(5*B*d
+ 4*A*e) - 6*a*e*(11*C*d^2 + 8*B*d*e - 34*A*e^2)) + 2*b^2*e^4*(21*a^2*C*e^2
 - 6*a*b*e*(3*C*d + 2*B*e) + b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)) - 6*c^2*e^2
*(a*b*d*e*(30*C*d^2 - 5*B*d*e - 34*A*e^2) - a^2*e^2*(30*C*d^2 - 36*B*d*e +
```

```
7*A*e^2) - b^2*d^2*(11*C*d^2 + 3*B*d*e + 11*A*e^2)) - c*e^3*(126*a^3*C*e^3
- 3*a^2*b*e^2*(12*C*d + 29*B*e) - 6*a*b^2*e*(5*C*d^2 + 7*B*d*e - 12*A*e^2)
+ b^3*d*(20*C*d^2 + 25*B*d*e + 56*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)])/(315*e^4*(c*d^2 - b*d*e + a*e^2)^4*Sqrt[(c*(d + e*x)
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*
Sqrt[b^2 - 4*a*c]*(2*c^3*(8*C*d^5 + d^3*e*(4*B*d + 5*A*e)) + 3*c^2*d*e*(2*a
*e*(9*C*d^2 + 7*B*d*e - 9*A*e^2) - b*d*(16*C*d^2 + 7*B*d*e + 5*A*e^2)) + 3*
c*e^2*(2*a^2*e^2*(17*C*d - 5*B*e) - a*b*e*(41*C*d^2 + 5*B*d*e - 9*A*e^2) +
b^2*d*(15*C*d^2 + 3*B*d*e + 7*A*e^2)) - b*e^3*(21*a^2*C*e^2 - 6*a*b*e*(3*C*
d + 2*B*e) + b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)))*Sqrt[(c*(d + e*x))/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*
EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sq
rt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315
*e^4*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
```

```
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\,(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\,(a+bx+cx^2)^{3/2}}{9e\,(cd^2-bde+ae^2)\,(d+ex)^{9/2}} - \frac{2\int \Big(-\frac{3\big(bCd^2-be(Bd+2Ae}{}}{}$$

$$= -\frac{2\Big(c^2(8Cd^5+d^3e(4Bd+5Ae)) - e^2(3a^2e^2(3Cd-5Be) - abe(2}{}$$

$$= \frac{2(2c^3(8Cd^5+d^3e(4Bd+5Ae)) + 3c^2de(2ae(9Cd^2+7Bde-9Ae}{}$$

$$= \frac{2(2c^3(8Cd^5+d^3e(4Bd+5Ae)) + 3c^2de(2ae(9Cd^2+7Bde-9Ae}{}$$

$$= \frac{2(2c^3(8Cd^5+d^3e(4Bd+5Ae)) + 3c^2de(2ae(9Cd^2+7Bde-9Ae}{}$$

$$= \frac{2(2c^3(8Cd^5+d^3e(4Bd+5Ae)) + 3c^2de(2ae(9Cd^2+7Bde-9Ae}{}$$

$$= \frac{2(2c^3(8Cd^5+d^3e(4Bd+5Ae)) + 3c^2de(2ae(9Cd^2+7Bde-9Ae}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 35.24, size = 29140, normalized size = 15.30

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(11/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 153622 vs. $2(1828) = 3656$.
time = 0.47, size = 153623, normalized size = 80.68

method	result	size
elliptic	Expression too large to display	3498
default	Expression too large to display	153623

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x,method=_RETURNVERBOS
E)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(x*e + d)^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.77, size = 7746, normalized size = 4.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="fr
icas")
```

```
[Out] 2/945*((16*C*c^5*d^12 + (42*C*a^2*b^3 - 24*B*a*b^4 + 16*A*b^5 - 3*(30*B*a^3
 - 41*A*a^2*b)*c^2 - 3*(63*C*a^3*b - 41*B*a^2*b^2 + 32*A*a*b^3)*c)*x^5*e^12
 - ((36*C*a*b^4 - 8*B*b^5 + 246*A*a^2*c^3 - 3*(186*C*a^3 - 115*B*a^2*b + 11
0*A*a*b^2)*c^2 - (69*C*a^2*b^2 + 42*B*a*b^3 - 64*A*b^4)*c)*d*x^5 - 5*(42*C*
a^2*b^3 - 24*B*a*b^4 + 16*A*b^5 - 3*(30*B*a^3 - 41*A*a^2*b)*c^2 - 3*(63*C*a
^3*b - 41*B*a^2*b^2 + 32*A*a*b^3)*c)*d*x^4)*e^11 + ((10*C*b^5 + 18*(26*B*a^
2 - 23*A*a*b)*c^3 - (630*C*a^2*b - 54*B*a*b^2 - 91*A*b^3)*c^2 + (33*C*a*b^3
 - 29*B*b^4)*c)*d^2*x^5 - 5*(36*C*a*b^4 - 8*B*b^5 + 246*A*a^2*c^3 - 3*(186*
```

C*a^3 - 115*B*a^2*b + 110*A*a*b^2)*c^2 - (69*C*a^2*b^2 + 42*B*a*b^3 - 64*A*
b^4)*c)*d^2*x^4 + 10*(42*C*a^2*b^3 - 24*B*a*b^4 + 16*A*b^5 - 3*(30*B*a^3 -
41*A*a^2*b)*c^2 - 3*(63*C*a^3*b - 41*B*a^2*b^2 + 32*A*a*b^3)*c)*d^2*x^3)*e^
10 - ((25*C*b^4*c - 276*A*a*c^4 - 2*(54*C*a^2 - 123*B*a*b - 22*A*b^2)*c^3 -
(318*C*a*b^2 + 29*B*b^3)*c^2)*d^3*x^5 - 5*(10*C*b^5 + 18*(26*B*a^2 - 23*A*
a*b)*c^3 - (630*C*a^2*b - 54*B*a*b^2 - 91*A*b^3)*c^2 + (33*C*a*b^3 - 29*B*b
^4)*c)*d^3*x^4 + 10*(36*C*a*b^4 - 8*B*b^5 + 246*A*a^2*c^3 - 3*(186*C*a^3 -
115*B*a^2*b + 110*A*a*b^2)*c^2 - (69*C*a^2*b^2 + 42*B*a*b^3 - 64*A*b^4)*c)*
d^3*x^3 - 10*(42*C*a^2*b^3 - 24*B*a*b^4 + 16*A*b^5 - 3*(30*B*a^3 - 41*A*a^2
*b)*c^2 - 3*(63*C*a^3*b - 41*B*a^2*b^2 + 32*A*a*b^3)*c)*d^3*x^2)*e^9 - ((44
*C*b^3*c^2 - (54*B*a - 25*A*b)*c^4 + (249*C*a*b - 29*B*b^2)*c^3)*d^4*x^5 +
5*(25*C*b^4*c - 276*A*a*c^4 - 2*(54*C*a^2 - 123*B*a*b - 22*A*b^2)*c^3 - (31
8*C*a*b^2 + 29*B*b^3)*c^2)*d^4*x^4 - 10*(10*C*b^5 + 18*(26*B*a^2 - 23*A*a*b
)*c^3 - (630*C*a^2*b - 54*B*a*b^2 - 91*A*b^3)*c^2 + (33*C*a*b^3 - 29*B*b^4)
*c)*d^4*x^3 + 10*(36*C*a*b^4 - 8*B*b^5 + 246*A*a^2*c^3 - 3*(186*C*a^3 - 115
*B*a^2*b + 110*A*a*b^2)*c^2 - (69*C*a^2*b^2 + 42*B*a*b^3 - 64*A*b^4)*c)*d^4
x^2 - 5(42*C*a^2*b^3 - 24*B*a*b^4 + 16*A*b^5 - 3*(30*B*a^3 - 41*A*a^2*b)*
c^2 - 3*(63*C*a^3*b - 41*B*a^2*b^2 + 32*A*a*b^3)*c)*d^4*x)*e^8 + ((91*C*b^2
*c^3 + 10*A*c^5 + (78*C*a - 29*B*b)*c^4)*d^5*x^5 - 5*(44*C*b^3*c^2 - (54*B*
a - 25*A*b)*c^4 + (249*C*a*b - 29*B*b^2)*c^3)*d^5*x^4 - 10*(25*C*b^4*c - 27
6*A*a*c^4 - 2*(54*C*a^2 - 123*B*a*b - 22*A*b^2)*c^3 - (318*C*a*b^2 + 29*B*b
^3)*c^2)*d^5*x^3 + 10*(10*C*b^5 + 18*(26*B*a^2 - 23*A*a*b)*c^3 - (630*C*a^2
*b - 54*B*a*b^2 - 91*A*b^3)*c^2 + (33*C*a*b^3 - 29*B*b^4)*c)*d^5*x^2 - 5*(3
6*C*a*b^4 - 8*B*b^5 + 246*A*a^2*c^3 - 3*(186*C*a^3 - 115*B*a^2*b + 110*A*a*
b^2)*c^2 - (69*C*a^2*b^2 + 42*B*a*b^3 - 64*A*b^4)*c)*d^5*x + (42*C*a^2*b^3
- 24*B*a*b^4 + 16*A*b^5 - 3*(30*B*a^3 - 41*A*a^2*b)*c^2 - 3*(63*C*a^3*b - 4
1*B*a^2*b^2 + 32*A*a*b^3)*c)*d^5)*e^7 - (8*(8*C*b*c^4 - B*c^5)*d^6*x^5 - 5*
(91*C*b^2*c^3 + 10*A*c^5 + (78*C*a - 29*B*b)*c^4)*d^6*x^4 + 10*(44*C*b^3*c^
2 - (54*B*a - 25*A*b)*c^4 + (249*C*a*b - 29*B*b^2)*c^3)*d^6*x^3 + 10*(25*C*
b^4*c - 276*A*a*c^4 - 2*(54*C*a^2 - 123*B*a*b - 22*A*b^2)*c^3 - (318*C*a*b^
2 + 29*B*b^3)*c^2)*d^6*x^2 - 5*(10*C*b^5 + 18*(26*B*a^2 - 23*A*a*b)*c^3 - (
630*C*a^2*b - 54*B*a*b^2 - 91*A*b^3)*c^2 + (33*C*a*b^3 - 29*B*b^4)*c)*d^6*x
+ (36*C*a*b^4 - 8*B*b^5 + 246*A*a^2*c^3 - 3*(186*C*a^3 - 115*B*a^2*b + 110
*A*a*b^2)*c^2 - (69*C*a^2*b^2 + 42*B*a*b^3 - 64*A*b^4)*c)*d^6)*e^6 + (16*C*
c^5*d^7*x^5 - 40*(8*C*b*c^4 - B*c^5)*d^7*x^4 + 10*(91*C*b^2*c^3 + 10*A*c^5
+ (78*C*a - 29*B*b)*c^4)*d^7*x^3 - 10*(44*C*b^3*c^2 - (54*B*a - 25*A*b)*c^4
+ (249*C*a*b - 29*B*b^2)*c^3)*d^7*x^2 - 5*(25*C*b^4*c - 276*A*a*c^4 - 2*(5
4*C*a^2 - 123*B*a*b - 22*A*b^2)*c^3 - (318*C*a*b^2 + 29*B*b^3)*c^2)*d^7*x +
(10*C*b^5 + 18*(26*B*a^2 - 23*A*a*b)*c^3 - (630*C*a^2*b - 54*B*a*b^2 - 91*
A*b^3)*c^2 + (33*C*a*b^3 - 29*B*b^4)*c)*d^7)*e^5 + (80*C*c^5*d^8*x^4 - 80*(
8*C*b*c^4 - B*c^5)*d^8*x^3 + 10*(91*C*b^2*c^3 + 10*A*c^5 + (78*C*a - 29*B*b
)*c^4)*d^8*x^2 - 5*(44*C*b^3*c^2 - (54*B*a - 25*A*b)*c^4 + (249*C*a*b - 29*
B*b^2)*c^3)*d^8*x - (25*C*b^4*c - 276*A*a*c^4 - 2*(54*C*a^2 - 123*B*a*b - 2
2*A*b^2)*c^3 - (318*C*a*b^2 + 29*B*b^3)*c^2)*d^8)*e^4 + (160*C*c^5*d^9*x^3
- 80*(8*C*b*c^4 - B*c^5)*d^9*x^2 + 5*(91*C*b^2*c^3 + 10*A*c^5 + (78*C*a - 2

```
9*B*b)*c^4)*d^9*x - (44*C*b^3*c^2 - (54*B*a - 25*A*b)*c^4 + (249*C*a*b - 29
*B*b^2)*c^3)*d^9)*e^3 + (160*C*c^5*d^10*x^2 - 40*(8*C*b*c^4 - B*c^5)*d^10*x
 + (91*C*b^2*c^3 + 10*A*c^5 + (78*C*a - 29*B*b)*c^4)*d^10)*e^2 + 8*(10*C*c^
5*d^11*x - (8*C*b*c^4 - B*c^5)*d^11)*e)*sqrt(c)*e^(1/2)*weierstrassPInverse
(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 -
 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/
c^3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c) + 3*(16*C*c^5*d^11*e + (42*A*a^2*c
^3 - 3*(42*C*a^3 - 29*B*a^2*b + 24*A*a*b^2)*c^2 + 2*(21*C*a^2*b^2 - 12*B*a*
b^3 + 8*A*b^4)*c)*x^5*e^12 - (2*(6*(18*B*a^2 - 17*A*a*b)*c^3 - (18*C*a^2*b
+ 21*B*a*b^2 - 28*A*b^3)*c^2 + 2*(9*C*a*b^3 - 2*B*b^4)*c)*d*x^5 - 5*(42*A*a
^2*c^3 - 3*(42*C*a^3 - 29*B*a^2*b + 24*A*a*b^2)*c^2 + 2*(21*C*a^2*b^2 - 12*
B*a*b^3 + 8*A*b^4)*c)*d*x^4)*e^11 + ((10*C*b^4*c - 204*A*a*c^4 + 6*(30*C*a^
2 + 5*B*a*b + 11*A*b^2)*c^3 + 5*(6*C*a*b^2 - 5*B*b^3)*c^2)*d^2*x^5 - 10*(6*
(18*B*a^2 - 17*A*a*b)*c^3 - (18*C*a^2*b + 21*B*...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(x*e + d)^(11/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(C\,x^2+B\,x+A\right)\,\sqrt{c\,x^2+b\,x+a}}{{\left(d+e\,x\right)}^{11/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(11/2),x)
```

```
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(11/2), x)
```

3.266 $\int \frac{(d+ex)^{3/2}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=724

$$\frac{2(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2(6Cd^2-7e(3Bd+5Ae)))\sqrt{d+ex}\sqrt{a+bx+cx^2}}{105c^3e}-\frac{2(2cC$$

```
[Out] -2/35*(-7*B*c*e+6*C*b*e+2*C*c*d)*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/e+2/
7*C*(e*x+d)^(5/2)*(c*x^2+b*x+a)^(1/2)/c/e+2/105*(24*b^2*C*e^2-c*e*(28*B*b*e
+25*C*a*e+15*C*b*d)-c^2*(6*C*d^2-7*e*(5*A*e+3*B*d)))*(e*x+d)^(1/2)*(c*x^2+b
*x+a)^(1/2)/c^3/e-1/105*(48*b^3*C*e^3-8*b*c*e^2*(7*B*b*e+13*C*a*e+9*C*b*d)+
c^3*d*(6*C*d^2-7*e*(20*A*e+3*B*d))+c^2*e*(a*e*(63*B*e+82*C*d)+b*(70*A*e^2+9
1*B*d*e+12*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-
4*a*c+b^2))^(1/2)/c^4/e^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2))))^(1/2)-2/105*(a*e^2-b*d*e+c*d^2)*(24*b^2*C*e^2-c*e*(28*B*b*e
+25*C*a*e+15*C*b*d)-c^2*(6*C*d^2-7*e*(5*A*e+3*B*d)))*EllipticF(1/2*((b+2*c*
x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^
(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(
-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))^(1/2)/c^4/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 1.06, antiderivative size = 724, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1667, 846, 857, 732, 435, 430}

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ae^2-bde+cd^2)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(-ce(25aCe+28bBe+15bCd)-(c^2(6Cd^2-7e(5Ae+3Bd)))+24b^2Ce^2)F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{105c^4e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(c^2e(ae(63Be+82Cd)+b(70Ae^2+91Bde+12Cd^2))-8bce^2(13aCe+7bBe+9bCd)+c^3(6Cd^3-7de(20Ae+3Bd))+48b^3Ce^3)E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{105c^4e^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(-ce(25aCe+28bBe+15bCd)-(c^2(6Cd^2-7e(5Ae+3Bd)))+24b^2Ce^2)}{105c^3e}-\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+2cCd)}{35c^2e}+\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*(24*b^2*C*e^2 - c*e*(15*b*C*d + 28*b*B*e + 25*a*C*e) - c^2*(6*C*d^2 - 7*
e*(3*B*d + 5*A*e)))*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(105*c^3*e) - (2*(
2*c*C*d - 7*B*c*e + 6*b*C*e)*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(35*c^2
*e) + (2*C*(d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(7*c*e) - (Sqrt[2]*Sqrt[b
```

```
^2 - 4*a*c]*(48*b^3*C*e^3 - 8*b*c*e^2*(9*b*C*d + 7*b*B*e + 13*a*C*e) + c^3*
(6*C*d^3 - 7*d*e*(3*B*d + 20*A*e)) + c^2*e*(a*e*(82*C*d + 63*B*e) + b*(12*C
*d^2 + 91*B*d*e + 70*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)])/(105*c^4*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e +
a*e^2)*(24*b^2*C*e^2 - c*e*(15*b*C*d + 28*b*B*e + 25*a*C*e) - c^2*(6*C*d^2
- 7*e*(3*B*d + 5*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*
a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^4*e^2*Sqrt[d + e*x]*Sq
rt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce} + \frac{2\int \frac{(d+ex)^{3/2}\left(-\frac{1}{2}e(bCd-7Ace+5aCe)-\frac{1}{2}e(2cCd}{\sqrt{a+bx+cx^2}}}{7ce^2}$$

$$= -\frac{2(2cCd-7Bce+6bCe)(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{35c^2e} + \frac{2C(d+ex)^{5/2}}{7}$$

$$= \frac{2(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2(6Cd^2-7e(3Bd+5Ae}{105c^3e}$$

$$= \frac{2(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2(6Cd^2-7e(3Bd+5Ae}{105c^3e}$$

$$= \frac{2(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2(6Cd^2-7e(3Bd+5Ae}{105c^3e}$$

$$= \frac{2(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2(6Cd^2-7e(3Bd+5Ae}{105c^3e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 31.56, size = 1314, normalized size = 1.81

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[d + e*x]*(a + b*x + c*x^2)*((2*(3*c^2*C*d^2 + 42*B*c^2*d*e - 33*b*c*C
*d*e - 28*b*B*c*e^2 + 35*A*c^2*e^2 + 24*b^2*C*e^2 - 25*a*c*C*e^2))/(105*c^3
*e) + (2*(8*c*C*d + 7*B*c*e - 6*b*C*e)*x)/(35*c^2) + (2*C*e*x^2)/(7*c)))/Sq
rt[a + x*(b + c*x)] + (2*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]*(-((48*b^3*C
```

```
*e^3 - 8*b*c*e^2*(9*b*C*d + 7*b*B*e + 13*a*C*e) + c^3*(6*C*d^3 - 7*d*e*(3*B
*d + 20*A*e)) + c^2*e*(a*e*(82*C*d + 63*B*e) + b*(12*C*d^2 + 91*B*d*e + 70*
A*e^2)))*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x
)))/(d + e*x))) + ((I/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d -
b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) +
 a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])*(48*b^3*C*e^3 - 8*b*c*e^2*(9*b*C*d + 7*b*B*e +
13*a*C*e) + c^3*(6*C*d^3 - 7*d*e*(3*B*d + 20*A*e)) + c^2*e*(a*e*(82*C*d + 6
3*B*e) + b*(12*C*d^2 + 91*B*d*e + 70*A*e^2)))*EllipticE[I*ArcSinh[(Sqrt[2]*
Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqr
t[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt
[(b^2 - 4*a*c)*e^2]))] - (-48*b^4*C*e^4 + 8*b^3*e^3*(15*c*C*d + 7*B*c*e + 6
*C*Sqrt[(b^2 - 4*a*c)*e^2]) - b^2*c*e^2*(78*c*C*d^2 - 152*a*C*e^2 + 7*c*e*(
21*B*d + 10*A*e) + 8*Sqrt[(b^2 - 4*a*c)*e^2]*(9*C*d + 7*B*e)) + c^2*(-50*a^
2*C*e^4 - 3*c*d^2*Sqrt[(b^2 - 4*a*c)*e^2]*(-2*C*d + 7*B*e) - 70*A*c*e^2*(3*
c*d^2 - a*e^2 + 2*d*Sqrt[(b^2 - 4*a*c)*e^2]) + a*e^2*(6*c*d*(17*C*d + 28*B*
e) + Sqrt[(b^2 - 4*a*c)*e^2]*(82*C*d + 63*B*e))) + b*(7*B*c^2*e^2*(15*c*d^2
 - 17*a*e^2 + 13*d*Sqrt[(b^2 - 4*a*c)*e^2]) + 2*c*e*(6*c*C*d^2*Sqrt[(b^2 -
4*a*c)*e^2] + 35*A*c*e^2*(3*c*d + Sqrt[(b^2 - 4*a*c)*e^2]) - a*C*e^2*(135*c
*d + 52*Sqrt[(b^2 - 4*a*c)*e^2]))))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^
2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]
], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*
a*c)*e^2]))]))/(Sqrt[2]*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqr
t[(b^2 - 4*a*c)*e^2])]*Sqrt[d + e*x])))/(105*c^4*e^3*Sqrt[a + x*(b + c*x)]*
Sqrt[((d + e*x)^2*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)
/(d + e*x)))/(d + e*x)))/e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 14083 vs. $2(654) = 1308$.
time = 0.18, size = 14084, normalized size = 19.45

method	result

elliptic	$\frac{\sqrt{(c\,x^2+bx+a)\,(ex+d)}\left(\frac{2eC\,x^2\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}{7c}+\frac{2\left(B\,e^2+2Cde-\frac{2eC(3eb+3cd)}{7c}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(x*e + d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 737, normalized size = 1.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/315*((6*C*c^4*d^4 + 3*(3*C*b*c^3 - 7*B*c^4)*d^3*e + (39*C*b^2*c^2 + 175*A
*c^4 - (71*C*a + 56*B*b)*c^3)*d^2*e^2 - (96*C*b^3*c + 7*(27*B*a + 25*A*b)*c
^3 - (260*C*a*b + 119*B*b^2)*c^2)*d*e^3 + (48*C*b^4 - 105*A*a*c^3 + (75*C*a
^2 + 147*B*a*b + 70*A*b^2)*c^2 - 8*(22*C*a*b^2 + 7*B*b^3)*c)*e^4)*sqrt(c)*e
^(1/2)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-
2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b
^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c) + 3*(6*C
*c^4*d^3*e + 3*(4*C*b*c^3 - 7*B*c^4)*d^2*e^2 - (72*C*b^2*c^2 + 140*A*c^4 -
(82*C*a + 91*B*b)*c^3)*d*e^3 + (48*C*b^3*c + 7*(9*B*a + 10*A*b)*c^3 - 8*(13
*C*a*b + 7*B*b^2)*c^2)*e^4)*sqrt(c)*e^(1/2)*weierstrassZeta(4/3*(c^2*d^2 -
b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e -
 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, weierstrass
PInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c
^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)
*e^(-3)/c^3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c)) + 3*(3*C*c^4*d^2*e^2 + (1
5*C*c^4*x^2 + 24*C*b^2*c^2 + 35*A*c^4 - (25*C*a + 28*B*b)*c^3 - 3*(6*C*b*c^
3 - 7*B*c^4)*x)*e^4 + 3*(8*C*c^4*d*x - (11*C*b*c^3 - 14*B*c^4)*d)*e^3)*sqrt
(c*x^2 + b*x + a)*sqrt(x*e + d))*e^(-3)/c^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(C*x**2+B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(A + B*x + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*(x*e + d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^{3/2}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(3/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((d + e*x)^(3/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

3.267 $\int \frac{\sqrt{d+ex}\,(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=557

$$-\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}{15c^2e}+\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}+\frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(8b^2C}{}$$

```
[Out] 2/5*C*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2)/c/e-2/15*(-5*B*c*e+4*C*b*e+2*C*c*d)
*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/e+1/15*(8*b^2*C*e^2-c*e*(10*B*b*e+9*
C*a*e+3*C*b*d)-c^2*(2*C*d^2-5*e*(3*A*e+B*d)))*EllipticE(1/2*((b+2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(
2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^
(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^3/e^2/(c*x^2+b*x+a)^(1/2)/(c*
(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/15*(-5*B*c*e+4*C*b*e+2*C*
c*d)*(a*e^2-b*d*e+c*d^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^
2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2
))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^3/e^2/(e*x+d)
^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]

time = 0.53, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$,
Rules used = {1667, 846, 857, 732, 435, 430}

$$\frac{\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{d+ex}\,\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\left(-ce(9aCe+10bBe+3bCd)-(c^2(2Cd^2-5e(3Ae+Bd)))+8b^2Ce^2\right)E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{15c^3e^2\sqrt{a+bx+cx^2}\,\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,(ae^2-bde+cd^2)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,(4bCe-5Bce+2cCd)F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{15c^3e^2\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}-\frac{2\sqrt{d+ex}\,\sqrt{a+bx+cx^2}\,(4bCe-5Bce+2cCd)}{15c^2e}+\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-2*(2*c*C*d - 5*B*c*e + 4*b*C*e)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(15*
c^2*e) + (2*C*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c*e) + (Sqrt[2]*Sqr
t[b^2 - 4*a*c]*(8*b^2*C*e^2 - c*e*(3*b*C*d + 10*b*B*e + 9*a*C*e) - c^2*(2*C
*d^2 - 5*e*(B*d + 3*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)])/(15*c^3*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
```

```
Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d - 5*B*c*e +
4*b*C*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c^3*e^2*Sqrt[d +
 e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
```

```
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce} + \frac{2\int \frac{\sqrt{d+ex}\,(-\frac{1}{2}e(bCd-5Ace+3aCe)-\frac{1}{2}e(2cCd}{\sqrt{a+bx+cx^2}}}{5ce^2}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a}}{5ce}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a}}{5ce}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a}}{5ce}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a}}{5ce}$$

Mathematica [C] Result contains complex when optimal does not.
time = 29.25, size = 992, normalized size = 1.78

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (((2*(c*C*d + 5*B*c*e - 4*b*C*e))/(15*c^2*e) + (2*C*x)/(5*c))*Sqrt[d + e*x]
*(a + b*x + c*x^2))/Sqrt[a + x*(b + c*x)] - (2*(d + e*x)^(3/2)*Sqrt[a + b*x
 + c*x^2]*((-8*b^2*C*e^2 + c*e*(3*b*C*d + 10*b*B*e + 9*a*C*e) + c^2*(2*C*d^
2 - 5*e*(B*d + 3*A*e)))*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) +
 (a*e)/(d + e*x)))/(d + e*x)) + ((I/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e
)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2
```

```
 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]
*((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(8*b^2*C*e^2 - c*e*(3*b*C*d + 10*
b*B*e + 9*a*C*e) + c^2*(-2*C*d^2 + 5*e*(B*d + 3*A*e)))*EllipticE[I*ArcSinh[
(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^
2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b
*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (8*b^3*C*e^3 - b^2*e^2*(11*c*C*d + 10*B*c
*e + 8*C*Sqrt[(b^2 - 4*a*c)*e^2]) + c*(c*d*Sqrt[(b^2 - 4*a*c)*e^2]*(2*C*d -
 5*B*e) - 15*A*c*e^2*(2*c*d + Sqrt[(b^2 - 4*a*c)*e^2]) + a*e^2*(14*c*C*d +
10*B*c*e + 9*C*Sqrt[(b^2 - 4*a*c)*e^2])) + b*c*e*(15*A*c*e^2 - 17*a*C*e^2 +
 3*C*d*Sqrt[(b^2 - 4*a*c)*e^2] + 5*B*(3*c*d*e + 2*e*Sqrt[(b^2 - 4*a*c)*e^2]
)))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 -
 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*Sqrt[(c*
d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[d +
e*x])))/(15*c^3*e^3*Sqrt[a + x*(b + c*x)]*Sqrt[((d + e*x)^2*(c*(-1 + d/(d +
 e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8160 vs. $2(493) = 986$.
time = 0.18, size = 8161, normalized size = 14.65

method	result
elliptic	$\frac{\sqrt{(c\,x^2+bx+a)\,(ex+d)}\left(\frac{2Cx\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}{5c}+\frac{2\left(Be+Cd-\frac{2(2eb+2cd)C}{5c}\right)\sqrt{c\cdots}}{\cdots}\cdots\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(x*e + d)/sqrt(c*x^2 + b*x + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 554, normalized size = 0.99

$$\frac{2\left(\left(2Cc^3d^3+(2Cbc^2-5Bc^3)d^2e+(7Cb^2c+30Ac^3-2(6Ca+5Bb)c^2)de^2-(8Cb^3+15(Ba+Ab)c^2-(21Cab+10Bb^2)c)e^3\right)\sqrt{c}\,e^{\frac{1}{2}}\,\mathrm{weierstrassPInverse}\left(\frac{4(c^2d^2-bcde+(b^2-3ac)e^2)e^{(-2)}}{3c^2},-\frac{4(2c^3d^3-3bc^2d^2e-3(b^2c-6ac^2)de^2+(2b^3-9abc)e^3)e^{(-3)}}{27c^3},\frac{(cd+(3cx+b)e)e^{(-1)}}{3c}\right)+3\left(2Cc^3d^2e+(3Cbc^2-5Bc^3)de^2-(8Cb^2c+15Ac^3-(9Ca+10Bb)c^2)e^3\right)\sqrt{c}\,e^{\frac{1}{2}}\,\mathrm{weierstrassZeta}\left(\frac{4(c^2d^2-bcde+(b^2-3ac)e^2)e^{(-2)}}{3c^2},-\frac{4(2c^3d^3-3bc^2d^2e-3(b^2c-6ac^2)de^2+(2b^3-9abc)e^3)e^{(-3)}}{27c^3},\mathrm{weierstrassPInverse}\left(\frac{4(c^2d^2-bcde+(b^2-3ac)e^2)e^{(-2)}}{3c^2},-\frac{4(2c^3d^3-3bc^2d^2e-3(b^2c-6ac^2)de^2+(2b^3-9abc)e^3)e^{(-3)}}{27c^3},\frac{(cd+(3cx+b)e)e^{(-1)}}{3c}\right)\right)+3\left(Cc^3de^2+(3Cc^3x-4Cbc^2+5Bc^3)e^3\right)\sqrt{cx^2+bx+a}\sqrt{ex+d}\right)e^{(-3)}}{45c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/45*((2*C*c^3*d^3 + (2*C*b*c^2 - 5*B*c^3)*d^2*e + (7*C*b^2*c + 30*A*c^3 -
2*(6*C*a + 5*B*b)*c^2)*d*e^2 - (8*C*b^3 + 15*(B*a + A*b)*c^2 - (21*C*a*b +
10*B*b^2)*c)*e^3)*sqrt(c)*e^(1/2)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*
e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^
2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, 1/3*(c*d + (3*c*x
 + b)*e)*e^(-1)/c) + 3*(2*C*c^3*d^2*e + (3*C*b*c^2 - 5*B*c^3)*d*e^2 - (8*C*
b^2*c + 15*A*c^3 - (9*C*a + 10*B*b)*c^2)*e^3)*sqrt(c)*e^(1/2)*weierstrassZe
ta(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3
 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3
)/c^3, weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-
2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b
^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c)) + 3*(C*
c^3*d*e^2 + (3*C*c^3*x - 4*C*b*c^2 + 5*B*c^3)*e^3)*sqrt(c*x^2 + b*x + a)*sq
rt(x*e + d))*e^(-3)/c^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(C*x**2+B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(A + B*x + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(x*e + d)/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int(((d + e*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

3.268 $\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=471

$$\frac{2C\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}{3ce}-\frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(2cCd-3Bce+2bCe)\sqrt{d+ex}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,E\left(\sin^{-}\right.}{3c^2e^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$

```
[Out] 2/3*C*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e-1/3*(-3*B*c*e+2*C*b*e+2*C*c*d)*
EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^
2/e^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2
)+2/3*(C*e*(-a*e+b*d)+c*(2*C*d^2-3*e*(-A*e+B*d)))*EllipticF(1/2*((b+2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/
2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*
(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)
)))^(1/2)/c^2/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1667, 857, 732, 435, 430}

$$\frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,(Ce(bd-ae)-3ce(Bd-Ae)+2cCd^2)\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{3c^2e^2\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}-\frac{\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{d+ex}\,\sqrt{-\frac{c\,(a+bx+cx^2)}{b^2-4ac}}\,(2bCe-3Bce+2cCd)\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{3c^2e^2\sqrt{a+bx+cx^2}\,\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{2C\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*C*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) - (Sqrt[2]*Sqrt[b^2 - 4*a
*c]*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)])/(3*c^2*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 + C*
e*(b*d - a*e) - 3*c*e*(B*d - A*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
```

```
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c^2*e^2*Sqrt[d
+ e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}\,dx = \frac{2C\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}{3ce} + \frac{2\int \frac{-\frac{1}{2}e(bCd-3Ace+aCe)-\frac{1}{2}e(2cCd-3Bce+2bCe)x}{\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}\,d}{3ce^2}$$

$$= \frac{2C\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}{3ce} - \frac{(2cCd-3Bce+2bCe)\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}}{3ce^2}$$

$$= \frac{2C\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}{3ce} - \frac{\left(\sqrt{2}\,\sqrt{b^2-4ac}\,(2cCd-3Bce+2bCe)\sqrt{\right.}{}$$

$$= \frac{2C\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}{3ce} - \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(2cCd-3Bce+2bCe)\sqrt{d}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 28.57, size = 980, normalized size = 2.08

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(4*c*C*e^2*(a + x*(b + c*x)) + ((d + e*x)*((-4*e^2*(2*c*C*d
- 3*B*c*e + 2*b*C*e)*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(
b^2 - 4*a*c)*e^2])]*(a + x*(b + c*x)))/(d + e*x)^2 + (I*Sqrt[2]*(2*c*C*d -
3*B*c*e + 2*b*C*e)*(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*a*e^2 +
```

```
 d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*
(d - e*x))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*a*e
^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x +
b*e*(-d + e*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Ellip
ticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(
b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]])/Sqrt[d + e*x] + (I*Sqrt[2]*
(2*b^2*C*e^2 - b*e*(3*B*c*e + 2*C*Sqrt[(b^2 - 4*a*c)*e^2]) + c*(6*A*c*e^2 -
 2*a*C*e^2 + Sqrt[(b^2 - 4*a*c)*e^2]*(-2*C*d + 3*B*e)))*Sqrt[(-2*a*e^2 + d*
Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d
- e*x))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*a*e^2
+ d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e
*(-d + e*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Elliptic
F[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2]
)/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]])/Sqrt[d + e*x]))/Sqrt[(c*d^2 +
e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]))/(6*c^2*e^3*Sq
rt[a + x*(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4250 vs. $2(413) = 826$.
time = 0.18, size = 4251, normalized size = 9.03 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/3/c^2*(-2*C*a*c*d*e^2-2*C*c^2*e^3*x^3-2*C*b*c*d*e^2*x-C*(-4*a*c+b^2)^(1/
2)*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*((-b-2*c*x+(
-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d))^(1/2)*((b+2*c*x+(-4*
a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*EllipticF(2^(1/2)
*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2
)+e*b-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d))^(1/2))*a*e^3+6*B*2^(1/2)*(-(
e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1
/2))*e/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2)
)*e/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(
e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/(
e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d))^(1/2))*c^2*d^2*e+6*B*2^(1/2)*(-(e*x+d)*c/(
e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*
(-4*a*c+b^2)^(1/2)-e*b+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4
*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+
b^2)^(1/2)+e*b-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/(e*(-4*a*c+
b^2)^(1/2)-e*b+2*c*d))^(1/2))*a*c*e^3+3*C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^
2)^(1/2)+e*b-2*c*d))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)
^(1/2)-e*b+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1
/2)+e*b-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+e
```

```
*b-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-e
*b+2*c*d))^(1/2))*a*b*e^3-3*C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+e*b
-2*c*d))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-e*b+2
*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*
d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(
1/2),(-(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d))^(
1/2))*b^2*d*e^2+3*A*(-4*a*c+b^2)^(1/2)*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^
(1/2)+e*b-2*c*d))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1
/2)-e*b+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)
+e*b-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+e*b-
2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-e*b+
2*c*d))^(1/2))*c*e^3+3*A*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*
d))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d)
)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(
1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2),
(-(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d))^(1/2))
*b*c*e^3-6*A*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*((
-b-2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d))^(1/2)*((b+
2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*Ellipti
cF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2),(-(e*(-4*a*c
+b^2)^(1/2)+e*b-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d))^(1/2))*c^2*d*e^2-6
*B*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*((-b-2*c*x+(
-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d))^(1/2)*((b+2*c*x+(-4*
a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*EllipticF(2^(1/2)
*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2
)+e*b-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d))^(1/2))*a*c*e^3-4*C*2^(1/2)*(
-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^
(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/
2))*e/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c
/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)
/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d))^(1/2))*a*b*e^3+4*C*2^(1/2)*(-(e*x+d)*c/(
e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*
(-4*a*c+b^2)^(1/2)-e*b+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4
*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+
b^2)^(1/2)+e*b-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/(e*(-4*a*c+
b^2)^(1/2)-e*b+2*c*d))^(1/2))*b^2*d*e^2+2*C*(-4*a*c+b^2)^(1/2)*2^(1/2)*(-(e
*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/
2))*e/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))
*e/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e
*(-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/(e
*(-4*a*c+b^2)^(1/2)-e*b+2*c*d))^(1/2))*c*d^2*e+6*C*2^(1/2)*(-(e*x+d)*c/(e*(
-4*a*c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4
*a*c+b^2)^(1/2)-e*b+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*
c+b^2)^(1/2)+e*b-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2
)^(1/2)+e*b-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/(e*(-4*a*c+b^2
```

```
)^(1/2)-e*b+2*c*d))^(1/2))*a*c*d*e^2-6*B*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2
)^(1/2)+e*b-2*c*d))^(1/2)*((-b-2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^
(1/2)-e*b+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(x*e + d)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 441, normalized size = 0.94

$$\frac{2\left(3\sqrt{cx^2+bx+a}\,\sqrt{xe+d}\,Cc^2e^2+\left(2\,Cc^2d^2+(Cbc-3\,Bc^2)de+(2\,Cb^2+9\,Ac^2-3\,(Ca+Bb)c)e^2\right)\sqrt{c}\,e^{\frac{1}{2}}\mathrm{weierstrassPInverse}\left(\frac{4\,(c^2d^2-bcde+(b^2-3\,ac)e^2)e^{(-2)}}{3\,c^2},-\frac{4\,(2\,c^3d^3-3\,bc^2d^2e-3\,(b^2c-6\,ac^2)de^2+(2\,b^3-9\,abc)e^3)e^{(-3)}}{27\,c^3},\frac{(cd+(3\,cx+b)e)e^{(-1)}}{3\,c}\right)+3\,(2\,Cc^2de+(2\,Cbc-3\,Bc^2)e^2)\sqrt{c}\,e^{\frac{1}{2}}\mathrm{weierstrassZeta}\left(\frac{4\,(c^2d^2-bcde+(b^2-3\,ac)e^2)e^{(-2)}}{3\,c^2},-\frac{4\,(2\,c^3d^3-3\,bc^2d^2e-3\,(b^2c-6\,ac^2)de^2+(2\,b^3-9\,abc)e^3)e^{(-3)}}{27\,c^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(c^2d^2-bcde+(b^2-3\,ac)e^2)e^{(-2)}}{3\,c^2},-\frac{4\,(2\,c^3d^3-3\,bc^2d^2e-3\,(b^2c-6\,ac^2)de^2+(2\,b^3-9\,abc)e^3)e^{(-3)}}{27\,c^3},\frac{(cd+(3\,cx+b)e)e^{(-1)}}{3\,c}\right)\right)\right)e^{(-3)}}{9\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/9*(3*sqrt(c*x^2 + b*x + a)*sqrt(x*e + d)*C*c^2*e^2 + (2*C*c^2*d^2 + (C*b*
c - 3*B*c^2)*d*e + (2*C*b^2 + 9*A*c^2 - 3*(C*a + B*b)*c)*e^2)*sqrt(c)*e^(1/
2)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c
^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 -
 9*a*b*c)*e^3)*e^(-3)/c^3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c) + 3*(2*C*c^2
*d*e + (2*C*b*c - 3*B*c^2)*e^2)*sqrt(c)*e^(1/2)*weierstrassZeta(4/3*(c^2*d^
2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2
*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, weierst
rassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*
(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*
e^3)*e^(-3)/c^3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c)))*e^(-3)/c^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\,x^2+B\,x+A}{\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x + C*x^2)/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.269 $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=508

$$-\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}-\frac{\sqrt{2}\sqrt{b^2-4ac}\,(Ce(bd-ae)-c(2Cd^2-e(Bd-Ae)))\sqrt{d+ex}}{ce^2\left(cd^2-bde+ae^2\right)}$$

```
[Out] -2*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(
1/2)-(C*e*(-a*e+b*d)-c*(2*C*d^2-e*(-A*e+B*d)))*EllipticE(1/2*((b+2*c*x+(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)
^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/e^2/(a*e^2-b*d*e+c*d^2)/(c*x
^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2*(-B*e+
2*C*d)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))
*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d
)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)
^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1664, 857, 732, 435, 430}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-Ce(bd-ae)-ce(Bd-Ae)+2cCd^2\right)E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce^2\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(e*(c*d^2 - b*d*e + a*e^
2)*Sqrt[d + e*x]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 - C*e*(b*d - a*e)
 - c*e*(B*d - A*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c
))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]
]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/
(c*e^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*C*d -
B*e)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a +
```

```
b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{2\int \frac{-\frac{bd(Cd-Be)+e(Acd-aCd+aBe)}{2e}+}{\sqrt{d+ex}\sqrt{}}}{cd^2-b}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{(2Cd-Be)\int \frac{1}{\sqrt{d+ex}\sqrt{a}}}{e^2}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\left(Bcd+bC\right.\right.}{}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{\sqrt{2}\sqrt{b^2-4ac}\left(Bcd+bCd\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 24.69, size = 772, normalized size = 1.52

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*(-(e^2*(C*d^2 + e*(-(B*d) + A*e))*(a + x*(b + c*x))) + (e^2*(2*c*C*d^2 +
 C*e*(-(b*d) + a*e) + c*e*(-(B*d) + A*e))*(a + x*(b + c*x)))/c - ((I/2)*(d
+ e*x)^(3/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(
b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2
*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^2
 - 4*a*c)*e^2])*(2*c*C*d^2 + C*e*(-(b*d) + a*e) + c*e*(-(B*d) + A*e))*Ellip
ticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(
```

```
b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (-(b^2*C*d*e^2) + 2*a*c*C*
d*e^2 - 2*a*B*c*e^3 - 2*c*C*d^2*Sqrt[(b^2 - 4*a*c)*e^2] + B*c*d*e*Sqrt[(b^2
 - 4*a*c)*e^2] - a*C*e^2*Sqrt[(b^2 - 4*a*c)*e^2] - A*c*e^2*(2*c*d + Sqrt[(b
^2 - 4*a*c)*e^2]) + b*(B*c*d*e^2 + A*c*e^3 + a*C*e^3 + C*d*e*Sqrt[(b^2 - 4*
a*c)*e^2]))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c
*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqr
t[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*
c*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]
)))/(e^3*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6052 vs. $2(456) = 912$.

time = 0.17, size = 6053, normalized size = 11.92

method	result
elliptic	$\frac{\sqrt{(c x^2+bx+a)(ex+d)}\left(-\frac{2\left(ce\,x^2+bex+ae\right)\left(A\,e^2-Bde+C\,d^2\right)}{e^2\left(a\,e^2-deb+c\,d^2\right)\sqrt{\left(x+\frac{d}{e}\right)\left(ce\,x^2+bex+ae\right)}}+\frac{2\left(\frac{Be-Cd}{e^2}-\frac{(eb-cd)\left(A\,e^2-Bde+\right.}{e^2\left(a\,e^2-deb+c\,d\right.}\right.}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(x*e + d)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 687, normalized size = 1.35

$$-\frac{2\left((2Cc^2d^4+(Cab-(3Ba-Ab)c)xe^4-((Cb^2+2Ac^2-2(2Ca+Bb)c)dx-(Cab-(3Ba-Ab)c)d)e^3-((2Cbc+Bc^2)d^2x+(Cb^2+2Ac^2-2(2Ca+Bb)c)d^2)e^2+(2Cc^2d^3x-(2Cbc+Bc^2)d^3)e)\sqrt{c}\,e^{\frac{1}{2}}\,\mathrm{weierstrassPInverse}\left(\frac{4(c^2d^2-bcde+(b^2-3ac)e^2)e^{(-2)}}{3c^2},-\frac{4(2c^3d^3-3bc^2d^2e-3(b^2c-6ac^2)de^2+(2b^3-9abc)e^3)e^{(-3)}}{27c^3},\frac{(cd+(3cx+b)e)e^{(-1)}}{3c}\right)+3(2Cc^2d^3e+(Cac+Ac^2)xe^4-((Cbc+Bc^2)dx-(Cac+Ac^2)d)e^3+(2Cc^2d^2x-(Cbc+Bc^2)d^2)e^2)\sqrt{c}\,e^{\frac{1}{2}}\,\mathrm{weierstrassZeta}\left(\frac{4(c^2d^2-bcde+(b^2-3ac)e^2)e^{(-2)}}{3c^2},-\frac{4(2c^3d^3-3bc^2d^2e-3(b^2c-6ac^2)de^2+(2b^3-9abc)e^3)e^{(-3)}}{27c^3},\mathrm{weierstrassPInverse}\left(\frac{4(c^2d^2-bcde+(b^2-3ac)e^2)e^{(-2)}}{3c^2},-\frac{4(2c^3d^3-3bc^2d^2e-3(b^2c-6ac^2)de^2+(2b^3-9abc)e^3)e^{(-3)}}{27c^3},\frac{(cd+(3cx+b)e)e^{(-1)}}{3c}\right)\right)+3(Cc^2d^2e^2-Bc^2de^3+Ac^2e^4)\sqrt{cx^2+bx+a}\sqrt{xe+d}\right)}{3(c^3d^3e^3+ac^2xe^6-(bc^2dx-ac^2d)e^5+(c^3d^2x-bc^2d^2)e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -2/3*((2*C*c^2*d^4 + (C*a*b - (3*B*a - A*b)*c)*x*e^4 - ((C*b^2 + 2*A*c^2 -
2*(2*C*a + B*b)*c)*d*x - (C*a*b - (3*B*a - A*b)*c)*d)*e^3 - ((2*C*b*c + B*c
^2)*d^2*x + (C*b^2 + 2*A*c^2 - 2*(2*C*a + B*b)*c)*d^2)*e^2 + (2*C*c^2*d^3*x
 - (2*C*b*c + B*c^2)*d^3)*e)*sqrt(c)*e^(1/2)*weierstrassPInverse(4/3*(c^2*d
^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^
2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, 1/3*(c
*d + (3*c*x + b)*e)*e^(-1)/c) + 3*(2*C*c^2*d^3*e + (C*a*c + A*c^2)*x*e^4 -
((C*b*c + B*c^2)*d*x - (C*a*c + A*c^2)*d)*e^3 + (2*C*c^2*d^2*x - (C*b*c + B
*c^2)*d^2)*e^2)*sqrt(c)*e^(1/2)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b
^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c -
6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, weierstrassPInverse(4/3
*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b
*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3,
 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c)) + 3*(C*c^2*d^2*e^2 - B*c^2*d*e^3 + A*
c^2*e^4)*sqrt(c*x^2 + b*x + a)*sqrt(x*e + d))/(c^3*d^3*e^3 + a*c^2*x*e^6 -
(b*c^2*d*x - a*c^2*d)*e^5 + (c^3*d^2*x - b*c^2*d^2)*e^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(d + ex)^{\frac{3}{2}} \sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((d + e*x)**(3/2)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(x*e + d)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\,x^2+B\,x+A}{{\left(d+e\,x\right)}^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x + C*x^2)/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.270 $\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=684

$$-\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{3e\,(cd^2-bde+ae^2)\,(d+ex)^{3/2}}+\frac{2(cd(2Cd^2+e(Bd-4Ae))+e(3ae(2Cd-Be)-b(4Cd^2-Bde}{3e\,(cd^2-bde+ae^2)^2\,\sqrt{d+ex}}$$

[Out] -2/3*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(3/2)+2/3*(c*d*(2*C*d^2+e*(-4*A*e+B*d))+e*(3*a*e*(-B*e+2*C*d)-b*(-2*A*e^2-B*d*e+4*C*d^2)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(1/2)-1/3*(c*d*(2*C*d^2+e*(-4*A*e+B*d))+e*(3*a*e*(-B*e+2*C*d)-b*(-2*A*e^2-B*d*e+4*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3*(3*C*e*(-a*e+b*d)-c*(2*C*d^2+e*(-A*e+B*d)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)

Rubi [A]

time = 0.71, antiderivative size = 680, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1664, 848, 857, 732, 435, 430}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,(-3Ce(bd-ae)+ce(Bd-Ae)+2cCd^2)\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{3ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)}-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,(3ae^2(2Cd-Be)-be(4Cd^2-e(2Ae+Bd))+cde(Bd-4Ae)+2cCd^3)\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{3e^2\sqrt{a+bx+cx^2}\,(ae^2-bde+cd^2)^2\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{a+bx+cx^2}\,(Cd^2-e(Bd-Ae))}{3e(d+ex)^{3/2}(ae^2-bde+cd^2)}+\frac{2\sqrt{a+bx+cx^2}\,(3ae^2(2Cd-Be)-be(4Cd^2-e(2Ae+Bd))+cde(Bd-4Ae)+2cCd^3)}{3e\sqrt{d+ex}\,(ae^2-bde+cd^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x + C*x^2)/((d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]

[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(3*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) + (2*(2*c*C*d^3 + c*d*e*(B*d - 4*A*e) + 3*a*e^2*(2*C*d - B*e) - b*e*(4*C*d^2 - e*(B*d + 2*A*e)))*Sqrt[a + b*x + c*x^2])/(3*e*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^3 + c*d*e*(B*d - 4*A*e) + 3*a*e^2*(2*C*d - B*e) - b*e*(4*C*d^2 - e*(B*d +

```
2*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
 (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*e^2*(c*d
^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 - 3*C*e
*(b*d - a*e) + c*e*(B*d - A*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[
b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^2*(c*d^2 - b*d
*e + a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{3e\,(cd^2-bde+ae^2)\,(d+ex)^{3/2}} - \frac{2\int \frac{-\frac{bCd^2-be(Bd+2Ae)+3e(Acd-aCd+\cdots}{2e}}{(d+ex)^{3/2}\cdots}}{3\,(cd^2\cdots}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{3e\,(cd^2-bde+ae^2)\,(d+ex)^{3/2}} + \frac{2(2cCd^3+cde(Bd-4Ae)+\cdots}{\cdots}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{3e\,(cd^2-bde+ae^2)\,(d+ex)^{3/2}} + \frac{2(2cCd^3+cde(Bd-4Ae)+\cdots}{\cdots}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{3e\,(cd^2-bde+ae^2)\,(d+ex)^{3/2}} + \frac{2(2cCd^3+cde(Bd-4Ae)+\cdots}{\cdots}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{3e\,(cd^2-bde+ae^2)\,(d+ex)^{3/2}} + \frac{2(2cCd^3+cde(Bd-4Ae)+\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 30.00, size = 1194, normalized size = 1.75

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(a + b*x + c*x^2)*((-2*(C*d^2 - B*d*e + A*e^2))/(3*e*(c*d^2
- b*d*e + a*e^2)*(d + e*x)^2) - (2*(-2*c*C*d^3 - B*c*d^2*e + 4*b*C*d^2*e -
b*B*d*e^2 + 4*A*c*d*e^2 - 6*a*C*d*e^2 - 2*A*b*e^3 + 3*a*B*e^3))/(3*e*(c*d^2
 - b*d*e + a*e^2)^2*(d + e*x))))/Sqrt[a + x*(b + c*x)] + (2*(d + e*x)^(3/2)
*Sqrt[a + b*x + c*x^2]*(-((2*c*C*d^3 + c*d*e*(B*d - 4*A*e) - 3*a*e^2*(-2*C*
```

```
d + B*e) + b*e*(-4*C*d^2 + e*(B*d + 2*A*e)))*(c*(-1 + d/(d + e*x))^2 + (e*(
b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x))) + ((I/2)*Sqrt[1 - (2*(c
*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x
))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*
a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(c*d*(2*C*d
^2 + e*(B*d - 4*A*e)) + e*(-4*b*C*d^2 + b*e*(B*d + 2*A*e) - 3*a*e*(-2*C*d +
 B*e)))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d +
 b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] - (2*a*c*C*d^2*
e^2 - 8*a*B*c*d*e^3 - 6*a^2*C*e^4 + 2*c*C*d^3*Sqrt[(b^2 - 4*a*c)*e^2] + B*c
*d^2*e*Sqrt[(b^2 - 4*a*c)*e^2] + 6*a*C*d*e^2*Sqrt[(b^2 - 4*a*c)*e^2] - 3*a*
B*e^3*Sqrt[(b^2 - 4*a*c)*e^2] + 2*A*c*e^2*(-3*c*d^2 + a*e^2 - 2*d*Sqrt[(b^2
 - 4*a*c)*e^2]) - b^2*e^2*(2*C*d^2 + e*(B*d + 2*A*e)) + b*e*(2*A*e^2*(3*c*d
 + Sqrt[(b^2 - 4*a*c)*e^2]) + 2*C*d*(3*a*e^2 - 2*d*Sqrt[(b^2 - 4*a*c)*e^2])
 + B*e*(3*c*d^2 + 3*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2])))*EllipticF[I*ArcSin
h[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*
e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d -
 b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*Sqrt[(c*d^2 + e*(-(b*d) + a*e)
)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[d + e*x])))/(3*e^3*(c*d^2
- b*d*e + a*e^2)^2*Sqrt[a + x*(b + c*x)]*Sqrt[((d + e*x)^2*(c*(-1 + d/(d +
e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 20480 vs. $2(620) = 1240$.

time = 0.16, size = 20481, normalized size = 29.94

method	result
elliptic	$\frac{\sqrt{(c\,x^2+bx+a)\,(ex+d)}\left(-\frac{2\left(A\,e^2-Bde+C\,d^2\right)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}{3e^3\left(a\,e^2-deb+c\,d^2\right)\left(x+\frac{d}{e}\right)^2}+\frac{2\left(ce\,x^2+bex\cdots\right.}{3e\cdots}\right.}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(x*e + d)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 1270, normalized size = 1.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/9*((2*C*c^2*d^6 + (9*C*a^2 - 3*B*a*b + 2*A*b^2 - 3*A*a*c)*x^2*e^6 - ((12*
C*a*b - B*b^2 - (9*B*a - 5*A*b)*c)*d*x^2 - 2*(9*C*a^2 - 3*B*a*b + 2*A*b^2 -
 3*A*a*c)*d*x)*e^5 + ((5*C*b^2 + 5*A*c^2 + (3*C*a - 4*B*b)*c)*d^2*x^2 - 2*(
12*C*a*b - B*b^2 - (9*B*a - 5*A*b)*c)*d^2*x + (9*C*a^2 - 3*B*a*b + 2*A*b^2
- 3*A*a*c)*d^2)*e^4 - ((5*C*b*c - B*c^2)*d^3*x^2 - 2*(5*C*b^2 + 5*A*c^2 + (
3*C*a - 4*B*b)*c)*d^3*x + (12*C*a*b - B*b^2 - (9*B*a - 5*A*b)*c)*d^3)*e^3 +
 (2*C*c^2*d^4*x^2 - 2*(5*C*b*c - B*c^2)*d^4*x + (5*C*b^2 + 5*A*c^2 + (3*C*a
 - 4*B*b)*c)*d^4)*e^2 + (4*C*c^2*d^5*x - (5*C*b*c - B*c^2)*d^5)*e)*sqrt(c)*
e^(1/2)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(
-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*
b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c) + 3*(2*
C*c^2*d^5*e - (3*B*a - 2*A*b)*c*x^2*e^6 - (2*(3*B*a - 2*A*b)*c*d*x + (4*A*c
^2 - (6*C*a + B*b)*c)*d*x^2)*e^5 - ((4*C*b*c - B*c^2)*d^2*x^2 + (3*B*a - 2*
A*b)*c*d^2 + 2*(4*A*c^2 - (6*C*a + B*b)*c)*d^2*x)*e^4 + (2*C*c^2*d^3*x^2 -
2*(4*C*b*c - B*c^2)*d^3*x - (4*A*c^2 - (6*C*a + B*b)*c)*d^3)*e^3 + (4*C*c^2
*d^4*x - (4*C*b*c - B*c^2)*d^4)*e^2)*sqrt(c)*e^(1/2)*weierstrassZeta(4/3*(c
^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^
2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, we
ierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -
4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*
b*c)*e^3)*e^(-3)/c^3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c)) + 3*(C*c^2*d^4*e
```

```
^2 - (A*a*c + (3*B*a - 2*A*b)*c*x)*e^6 - ((2*B*a - 3*A*b)*c*d + (4*A*c^2 -
(6*C*a + B*b)*c)*d*x)*e^5 - ((4*C*b*c - B*c^2)*d^2*x - 5*(C*a*c - A*c^2)*d^
2)*e^4 + (2*C*c^2*d^3*x - (3*C*b*c - 2*B*c^2)*d^3)*e^3)*sqrt(c*x^2 + b*x +
a)*sqrt(x*e + d))/(c^3*d^6*e^3 + a^2*c*x^2*e^9 - 2*(a*b*c*d*x^2 - a^2*c*d*x
)*e^8 - (4*a*b*c*d^2*x - a^2*c*d^2 - (b^2*c + 2*a*c^2)*d^2*x^2)*e^7 - 2*(b*
c^2*d^3*x^2 + a*b*c*d^3 - (b^2*c + 2*a*c^2)*d^3*x)*e^6 + (c^3*d^4*x^2 - 4*b
*c^2*d^4*x + (b^2*c + 2*a*c^2)*d^4)*e^5 + 2*(c^3*d^5*x - b*c^2*d^5)*e^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{\frac{5}{2}}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((A + B*x + C*x**2)/((d + e*x)**(5/2)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(x*e + d)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\,x^2+B\,x+A}{{\left(d+e\,x\right)}^{5/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d + e*x)^(5/2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((A + B*x + C*x^2)/((d + e*x)^(5/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.271 $\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=944

$$-\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{5e\,(cd^2-bde+ae^2)\,(d+ex)^{5/2}}+\frac{2(cd(2Cd^2+e(3Bd-8Ae))+e(5ae(2Cd-Be)-b(6Cd^2-Bde}{15e\,(cd^2-bde+ae^2)^2\,(d+ex)^{3/2}}$$

```
[Out] -2/5*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(5/2)+2/15*(c*d*(2*C*d^2+e*(-8*A*e+3*B*d))+e*(5*a*e*(-B*e+2*C*d)-b*(-4*A*e
^2-B*d*e+6*C*d^2)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(3/
2)+2/15*(c^2*d^2*(2*C*d^2+e*(-23*A*e+3*B*d))-e^2*(15*a^2*C*e^2-10*a*b*e*(B*
e+C*d)+b^2*(8*A*e^2+2*B*d*e+3*C*d^2))-c*e*(b*d*(-23*A*e^2-7*B*d*e+7*C*d^2)-
a*e*(9*A*e^2-29*B*d*e+19*C*d^2)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)
^3/(e*x+d)^(1/2)-1/15*(c^2*d^2*(2*C*d^2+e*(-23*A*e+3*B*d))-e^2*(15*a^2*C*e^
2-10*a*b*e*(B*e+C*d)+b^2*(8*A*e^2+2*B*d*e+3*C*d^2))-c*e*(b*d*(-23*A*e^2-7*B
*d*e+7*C*d^2)-a*e*(9*A*e^2-29*B*d*e+19*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)
/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d
)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)^3/(c*
x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/15*(c
*d*(2*C*d^2+e*(-8*A*e+3*B*d))+e*(5*a*e*(-B*e+2*C*d)-b*(-4*A*e^2-B*d*e+6*C*d
^2)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*
2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)
/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^
(1/2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 1.34, antiderivative size = 942, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1664, 848, 857, 732, 435, 430}

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(5*e*(c*d^2 - b*d*e + a*
e^2)*(d + e*x)^(5/2)) + (2*(2*c*C*d^3 + c*d*e*(3*B*d - 8*A*e) + 5*a*e^2*(2*
```

```
C*d - B*e) - b*e*(6*C*d^2 - e*(B*d + 4*A*e)))*Sqrt[a + b*x + c*x^2])/(15*e*
(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(3/2)) + (2*(c^2*(2*C*d^4 + d^2*e*(3*B*
d - 23*A*e)) - e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2*
B*d*e + 8*A*e^2)) - c*e*(b*d*(7*C*d^2 - 7*B*d*e - 23*A*e^2) - a*e*(19*C*d^2
 - 29*B*d*e + 9*A*e^2)))*Sqrt[a + b*x + c*x^2])/(15*e*(c*d^2 - b*d*e + a*e^
2)^3*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*(2*C*d^4 + d^2*e*(3*B
*d - 23*A*e)) - e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2
*B*d*e + 8*A*e^2)) - c*e*(b*d*(7*C*d^2 - 7*B*d*e - 23*A*e^2) - a*e*(19*C*d^
2 - 29*B*d*e + 9*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)])/(15*e^2*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(2*c*C*d^3 + c*d*e*(3*B*d - 8*A*e) + 5*a*e^2*(2*C*d - B*e) - b*e*(6*C*d^2 -
 e*(B*d + 4*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqr
t[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*
e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*e^2*(c*d^2 - b*d*e + a*e^2)^2*
Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
```

```
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} - \frac{2\int \frac{-\frac{bCd^2-be(Bd+4Ae)+5e(Acd-aCd}{2e}}{(d+ex)^5}}{5\,(c}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2(2cCd^3+cde(3Bd-8Ae)}{}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2(2cCd^3+cde(3Bd-8Ae)}{}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2(2cCd^3+cde(3Bd-8Ae)}{}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2(2cCd^3+cde(3Bd-8Ae)}{}$$

$$= -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2(2cCd^3+cde(3Bd-8Ae)}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 32.92, size = 1746, normalized size = 1.85

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(a + b*x + c*x^2)*((-2*(C*d^2 - B*d*e + A*e^2))/(5*e*(c*d^2
- b*d*e + a*e^2)*(d + e*x)^3) - (2*(-2*c*C*d^3 - 3*B*c*d^2*e + 6*b*C*d^2*e
```

```
- b*B*d*e^2 + 8*A*c*d*e^2 - 10*a*C*d*e^2 - 4*A*b*e^3 + 5*a*B*e^3))/(15*e*(c
*d^2 - b*d*e + a*e^2)^2*(d + e*x)^2) - (2*(-2*c^2*C*d^4 - 3*B*c^2*d^3*e + 7
*b*c*C*d^3*e - 7*b*B*c*d^2*e^2 + 23*A*c^2*d^2*e^2 + 3*b^2*C*d^2*e^2 - 19*a*
c*C*d^2*e^2 + 2*b^2*B*d*e^3 - 23*A*b*c*d*e^3 + 29*a*B*c*d*e^3 - 10*a*b*C*d*
e^3 + 8*A*b^2*e^4 - 10*a*b*B*e^4 - 9*a*A*c*e^4 + 15*a^2*C*e^4))/(15*e*(c*d^
2 - b*d*e + a*e^2)^3*(d + e*x))))/Sqrt[a + x*(b + c*x)] + (2*(d + e*x)^(3/2
)*Sqrt[a + b*x + c*x^2]*(-((c^2*(2*C*d^4 + d^2*e*(3*B*d - 23*A*e)) - e^2*(1
5*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2*B*d*e + 8*A*e^2)) + c
*e*(a*e*(19*C*d^2 - 29*B*d*e + 9*A*e^2) + b*d*(-7*C*d^2 + 7*B*d*e + 23*A*e^
2)))*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/
(d + e*x))) - ((I/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e
+ Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e
)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + S
qrt[(b^2 - 4*a*c)*e^2])*(c^2*(-2*C*d^4 + d^2*e*(-3*B*d + 23*A*e)) + e^2*(15
*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2*B*d*e + 8*A*e^2)) - c*
e*(a*e*(19*C*d^2 - 29*B*d*e + 9*A*e^2) + b*d*(-7*C*d^2 + 7*B*d*e + 23*A*e^2
)))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 -
 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (-30*A*c^3*d^3*e^
2 + 14*a*c^2*C*d^3*e^2 - 54*a*B*c^2*d^2*e^3 + 34*a*A*c^2*d*e^4 - 50*a^2*c*C
*d*e^4 + 10*a^2*B*c*e^5 + 2*c^2*C*d^4*Sqrt[(b^2 - 4*a*c)*e^2] + 3*B*c^2*d^3
*e*Sqrt[(b^2 - 4*a*c)*e^2] - 23*A*c^2*d^2*e^2*Sqrt[(b^2 - 4*a*c)*e^2] + 19*
a*c*C*d^2*e^2*Sqrt[(b^2 - 4*a*c)*e^2] - 29*a*B*c*d*e^3*Sqrt[(b^2 - 4*a*c)*e
^2] + 9*a*A*c*e^4*Sqrt[(b^2 - 4*a*c)*e^2] - 15*a^2*C*e^4*Sqrt[(b^2 - 4*a*c)
*e^2] + b^3*e^3*(3*C*d^2 + 2*e*(B*d + 4*A*e)) - b^2*e^2*(11*c*C*d^3 + c*d*e
*(9*B*d + 31*A*e) + 10*a*e^2*(C*d + B*e) + Sqrt[(b^2 - 4*a*c)*e^2]*(3*C*d^2
 + 2*e*(B*d + 4*A*e))) + b*(A*c*e^3*(45*c*d^2 - 17*a*e^2 + 23*d*Sqrt[(b^2 -
 4*a*c)*e^2]) + C*e*(15*a^2*e^4 - 7*c*d^3*Sqrt[(b^2 - 4*a*c)*e^2] + a*d*e^2
*(33*c*d + 10*Sqrt[(b^2 - 4*a*c)*e^2])) + B*e^2*(15*c^2*d^3 + 10*a*e^2*Sqrt
[(b^2 - 4*a*c)*e^2] + c*d*(37*a*e^2 + 7*d*Sqrt[(b^2 - 4*a*c)*e^2]))))*Ellip
ticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(
b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*Sqrt[(c*d^2 + e*(
-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[d + e*x])))/(
15*e^3*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[a + x*(b + c*x)]*Sqrt[((d + e*x)^2*(c
*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*
x)))/e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 46696 vs. $2(874) = 1748$.

time = 0.21, size = 46697, normalized size = 49.47

method	result	size
elliptic	Expression too large to display	1757
default	Expression too large to display	46697

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(x*e + d)^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 2611, normalized size = 2.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2/45*((2*C*c^3*d^8 - (15*C*a^2*b - 10*B*a*b^2 + 8*A*b^3 + 3*(5*B*a^2 - 7*A*
a*b)*c)*x^3*e^8 + ((10*C*a*b^2 - 2*B*b^3 - 42*A*a*c^2 + (60*C*a^2 - 31*B*a*
b + 27*A*b^2)*c)*d*x^3 - 3*(15*C*a^2*b - 10*B*a*b^2 + 8*A*b^3 + 3*(5*B*a^2
- 7*A*a*b)*c)*d*x^2)*e^7 - ((3*C*b^3 - (52*B*a - 33*A*b)*c^2 + (49*C*a*b -
8*B*b^2)*c)*d^2*x^3 - 3*(10*C*a*b^2 - 2*B*b^3 - 42*A*a*c^2 + (60*C*a^2 - 31
*B*a*b + 27*A*b^2)*c)*d^2*x^2 + 3*(15*C*a^2*b - 10*B*a*b^2 + 8*A*b^3 + 3*(5
*B*a^2 - 7*A*a*b)*c)*d^2*x)*e^6 + ((17*C*b^2*c + 22*A*c^3 - (2*C*a + 17*B*b
)*c^2)*d^3*x^3 - 3*(3*C*b^3 - (52*B*a - 33*A*b)*c^2 + (49*C*a*b - 8*B*b^2)*
c)*d^3*x^2 + 3*(10*C*a*b^2 - 2*B*b^3 - 42*A*a*c^2 + (60*C*a^2 - 31*B*a*b +
27*A*b^2)*c)*d^3*x - (15*C*a^2*b - 10*B*a*b^2 + 8*A*b^3 + 3*(5*B*a^2 - 7*A*
a*b)*c)*d^3)*e^5 - ((8*C*b*c^2 - 3*B*c^3)*d^4*x^3 - 3*(17*C*b^2*c + 22*A*c^
3 - (2*C*a + 17*B*b)*c^2)*d^4*x^2 + 3*(3*C*b^3 - (52*B*a - 33*A*b)*c^2 + (4
9*C*a*b - 8*B*b^2)*c)*d^4*x - (10*C*a*b^2 - 2*B*b^3 - 42*A*a*c^2 + (60*C*a^
2 - 31*B*a*b + 27*A*b^2)*c)*d^4)*e^4 + (2*C*c^3*d^5*x^3 - 3*(8*C*b*c^2 - 3*
B*c^3)*d^5*x^2 + 3*(17*C*b^2*c + 22*A*c^3 - (2*C*a + 17*B*b)*c^2)*d^5*x - (
3*C*b^3 - (52*B*a - 33*A*b)*c^2 + (49*C*a*b - 8*B*b^2)*c)*d^5)*e^3 + (6*C*c
^3*d^6*x^2 - 3*(8*C*b*c^2 - 3*B*c^3)*d^6*x + (17*C*b^2*c + 22*A*c^3 - (2*C*
```

a + 17*B*b)*c^2)*d^6)*e^2 + (6*C*c^3*d^7*x - (8*C*b*c^2 - 3*B*c^3)*d^7)*e)*
sqrt(c)*e^(1/2)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*
e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e
^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, 1/3*(c*d + (3*c*x + b)*e)*e^(-1)/c)
 + 3*(2*C*c^3*d^7*e + (9*A*a*c^2 - (15*C*a^2 - 10*B*a*b + 8*A*b^2)*c)*x^3*e
^8 - (((29*B*a - 23*A*b)*c^2 - 2*(5*C*a*b - B*b^2)*c)*d*x^3 - 3*(9*A*a*c^2
- (15*C*a^2 - 10*B*a*b + 8*A*b^2)*c)*d*x^2)*e^7 - ((3*C*b^2*c + 23*A*c^3 -
(19*C*a + 7*B*b)*c^2)*d^2*x^3 + 3*((29*B*a - 23*A*b)*c^2 - 2*(5*C*a*b - B*b
^2)*c)*d^2*x^2 - 3*(9*A*a*c^2 - (15*C*a^2 - 10*B*a*b + 8*A*b^2)*c)*d^2*x)*e
^6 - ((7*C*b*c^2 - 3*B*c^3)*d^3*x^3 + 3*(3*C*b^2*c + 23*A*c^3 - (19*C*a + 7
*B*b)*c^2)*d^3*x^2 + 3*((29*B*a - 23*A*b)*c^2 - 2*(5*C*a*b - B*b^2)*c)*d^3*
x - (9*A*a*c^2 - (15*C*a^2 - 10*B*a*b + 8*A*b^2)*c)*d^3)*e^5 + (2*C*c^3*d^4
x^3 - 3(7*C*b*c^2 - 3*B*c^3)*d^4*x^2 - 3*(3*C*b^2*c + 23*A*c^3 - (19*C*a
+ 7*B*b)*c^2)*d^4*x - ((29*B*a - 23*A*b)*c^2 - 2*(5*C*a*b - B*b^2)*c)*d^4)*
e^4 + (6*C*c^3*d^5*x^2 - 3*(7*C*b*c^2 - 3*B*c^3)*d^5*x - (3*C*b^2*c + 23*A*
c^3 - (19*C*a + 7*B*b)*c^2)*d^5)*e^3 + (6*C*c^3*d^6*x - (7*C*b*c^2 - 3*B*c^
3)*d^6)*e^2)*sqrt(c)*e^(1/2)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2
- 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a
*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, weierstrassPInverse(4/3*(c
^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/c^2, -4/27*(2*c^3*d^3 - 3*b*c^
2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/c^3, 1/
3*(c*d + (3*c*x + b)*e)*e^(-1)/c)) + 3*(C*c^3*d^6*e^2 - (3*A*a^2*c + (5*B*a
^2 - 4*A*a*b)*c*x - (9*A*a*c^2 - (15*C*a^2 - 10*B*a*b + 8*A*b^2)*c)*x^2)*e^
8 - (((29*B*a - 23*A*b)*c^2 - 2*(5*C*a*b - B*b^2)*c)*d*x^2 + 2*(B*a^2 - 5*A
*a*b)*c*d - 2*(5*A*a*c^2 - (10*C*a^2 - 13*B*a*b + 10*A*b^2)*c)*d*x)*e^7 - (
(3*C*b^2*c + 23*A*c^3 - (19*C*a + 7*B*b)*c^2)*d^2*x^2 + (2*(30*B*a - 29*A*b
)*c^2 - (4*C*a*b - 5*B*b^2)*c)*d^2*x + (5*A*a*c^2 + (8*C*a^2 - 10*B*a*b + 1
5*A*b^2)*c)*d^2)*e^6 - ((25*B*a - 41*A*b)*c^2*d^3 + (7*C*b*c^2 - 3*B*c^3)*d
^3*x^2 + 2*(27*A*c^3 - (25*C*a + 6*B*b)*c^2)*d^3*x)*e^5 + (2*C*c^3*d^4*x^2
- (22*C*b*c^2 - 9*B*c^3)*d^4*x - (34*A*c^3 - (25*C*a - B*b)*c^2)*d^4)*e^4 +
 3*(2*C*c^3*d^5*x - 3*(C*b*c^2 - B*c^3)*d^5)*e^3)*sqrt(c*x^2 + b*x + a)*sqr
t(x*e + d))/(c^4*d^9*e^3 + a^3*c*x^3*e^12 - 3*(a^2*b*c*d*x^3 - a^3*c*d*x^2)
e^11 - 3(3*a^2*b*c*d^2*x^2 - a^3*c*d^2*x - (a*b^2*c + a^2*c^2)*d^2*x^3)*e
^10 - (9*a^2*b*c*d^3*x - a^3*c*d^3 + (b^3*c + 6*a*b*c^2)*d^3*x^3 - 9*(a*b^2
*c + a^2*c^2)*d^3*x^2)*e^9 - 3*(a^2*b*c*d^4 - (b^2*c^2 + a*c^3)*d^4*x^3 + (
b^3*c + 6*a*b*c^2)*d^4*x^2 - 3*(a*b^2*c + a^2*c^2)*d^4*x)*e^8 - 3*(b*c^3*d^
5*x^3 - 3*(b^2*c^2 + a*c^3)*d^5*x^2 + (b^3*c + 6*a*b*c^2)*d^5*x - (a*b^2*c
+ a^2*c^2)*d^5)*e^7 + (c^4*d^6*x^3 - 9*b*c^3*d^6*x^2 + 9*(b^2*c^2 + a*c^3)*
d^6*x - (b^3*c + 6*a*b*c^2)*d^6)*e^6 + 3*(c^4*d^7*x^2 - 3*b*c^3*d^7*x + (b^
2*c^2 + a*c^3)*d^7)*e^5 + 3*(c^4*d^8*x - b*c^3*d^8)*e^4)

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{\frac{7}{2}}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(7/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((A + B*x + C*x**2)/((d + e*x)**(7/2)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(x*e + d)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\,x^2+B\,x+A}{{\left(d+e\,x\right)}^{7/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d + e*x)^(7/2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((A + B*x + C*x^2)/((d + e*x)^(7/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.272 $\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=510

$$\frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)}+\frac{(fh(bg-ah)(1+m)+c(2fg^2(1+p)-h(eg-dh)(3+m+2p)))\,(g+h}{}$$

```
[Out] f*(h*x+g)^(1+m)*(c*x^2+b*x+a)^(1+p)/c/h/(3+m+2*p)+(f*h*(-a*h+b*g)*(1+m)+c*(
2*f*g^2*(1+p)-h*(-d*h+e*g)*(3+m+2*p)))*(h*x+g)^(1+m)*(c*x^2+b*x+a)^p*Appell
F1(1+m,-p,-p,2+m,2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(
2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))/c/h^3/(1+m)/(3+m+2*p)/((1-2*c*(h*x+g)/(2*c
*g-h*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(
1/2))))^p)-(b*f*h*(2+m+p)+c*(2*f*g*(1+p)-e*h*(3+m+2*p)))*(h*x+g)^(2+m)*(c*x
^2+b*x+a)^p*AppellF1(2+m,-p,-p,3+m,2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/
2))),2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))/c/h^3/(2+m)/(3+m+2*p)/((
1-2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(h*x+g)/(2*c*g-h
*(b+(-4*a*c+b^2)^(1/2))))^p)
```

Rubi [A]

time = 0.51, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1667, 857, 773, 138}

$$\frac{(g+hx)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}\right)\left(fh(m+1)(bg-ah)-ch(m+2p+3)(eg-dh)+2cfg^2(p+1)\right)}{ch^3(m+1)(m+2p+3)}-\frac{(g+hx)^{m+2}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}(bfh(m+p+2)-ceh(m+2p+3)+2cfg(p+1))F_1\left(m+2;-p,-p;m+3;\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}\right)}{ch^3(m+2)(m+2p+3)}+\frac{f(g+hx)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ch(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^(1 + p))/(c*h*(3 + m + 2*p)) + ((f*h
*(b*g - a*h)*(1 + m) + 2*c*f*g^2*(1 + p) - c*h*(e*g - d*h)*(3 + m + 2*p))*(
g + h*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*(g
 + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b +
 Sqrt[b^2 - 4*a*c])*h)])/(c*h^3*(1 + m)*(3 + m + 2*p)*(1 - (2*c*(g + h*x))/
(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(2*c*g - (b + S
qrt[b^2 - 4*a*c])*h))^p) - ((2*c*f*g*(1 + p) + b*f*h*(2 + m + p) - c*e*h*(3
 + m + 2*p))*(g + h*x)^(2 + m)*(a + b*x + c*x^2)^p*AppellF1[2 + m, -p, -p,
3 + m, (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))
/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h)])/(c*h^3*(2 + m)*(3 + m + 2*p)*(1 - (2
*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(
2*c*g - (b + Sqrt[b^2 - 4*a*c])*h))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
```

```
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx = \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\int (g+hx)^m(-h(afh(\cdots}{\cdots}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)} - \frac{(2cfg(1+p)+bfh(2\cdots}{\cdots}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)} - \frac{\left((2cfg(1+p)+bfh\cdots\right.}{\cdots}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{(fh(bg-ah)(1+m)\cdots}{\cdots}$$

Mathematica [F]
time = 1.77, size = 0, normalized size = 0.00

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (hx+g)^m \left(c\,x^2+bx+a\right)^p \left(f\,x^2+ex+d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
[Out] int((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((f*x^2 + x*e + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
[Out] integral((f*x^2 + x*e + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**m*(c*x**2+b*x+a)**p*(f*x**2+e*x+d),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
[Out] integrate((f*x^2 + x*e + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g+h\,x)^m\,\left(c\,x^2+b\,x+a\right)^p\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x)
```

3.273 $\int (g+hx)^m \sqrt{a+bx+cx^2}\,(d+ex+fx^2)\,dx$

Optimal. Leaf size=496

$$\frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)}+\frac{(fh(bg-ah)(1+m)+c(3fg^2-h(eg-dh)(4+m)))\,(g+hx)^{1+m}\sqrt{a+b}}{ch^3(1+m)(4+m)\sqrt{1-\frac{2c(g+}{2cg-\left(b-\sqrt{b}\right.}}}$$

```
[Out] f*(h*x+g)^(1+m)*(c*x^2+b*x+a)^(3/2)/c/h/(4+m)+(f*h*(-a*h+b*g)*(1+m)+c*(3*f*
g^2-h*(-d*h+e*g)*(4+m)))*(h*x+g)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,2*c*(h*x+
g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1
/2))))*(c*x^2+b*x+a)^(1/2)/c/h^3/(1+m)/(4+m)/(1-2*c*(h*x+g)/(2*c*g-h*(b-(-4
*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))^(
1/2)-1/2*(b*f*h*(5+2*m)+c*(6*f*g-2*e*h*(4+m)))*(h*x+g)^(2+m)*AppellF1(2+m,-
1/2,-1/2,3+m,2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(2*c*
g-h*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2)/c/h^3/(2+m)/(4+m)/(1-2*c*(
h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(h*x+g)/(2*c*g-h*(b+(
-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1667, 857, 773, 138}

$$\frac{\sqrt{a+bx+cx^2}\,(g+hx)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}\right)(fh(m+1)(bg-ah)-ch(m+4)(eg-dh)+3cfg^2)}{ch^3(m+1)(m+4)\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{\sqrt{a+bx+cx^2}\,(g+hx)^{m+2}(bfh(2m+5)-2ceh(m+4)+6cfg)F_1\left(m+2;-\frac{1}{2},-\frac{1}{2};m+3;\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}\right)}{2ch^3(m+2)(m+4)\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{f\left(a+bx+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^(3/2))/(c*h*(4 + m)) + ((3*c*f*g^2 +
 f*h*(b*g - a*h)*(1 + m) - c*h*(e*g - d*h)*(4 + m))*(g + h*x)^(1 + m)*Sqrt[
a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (2*c*(g + h*x))/(2*c*g
- (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*
c])*h)])/(c*h^3*(1 + m)*(4 + m)*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b - Sqrt
[b^2 - 4*a*c])*h)]*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c]
)*h)]) - ((6*c*f*g - 2*c*e*h*(4 + m) + b*f*h*(5 + 2*m))*(g + h*x)^(2 + m)*S
qrt[a + b*x + c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (2*c*(g + h*x))/(2*
c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 -
4*a*c])*h)])/(2*c*h^3*(2 + m)*(4 + m)*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b
- Sqrt[b^2 - 4*a*c])*h)]*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 -
4*a*c])*h)])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\int (g+hx)^m\left(-\frac{1}{2}h(3bfg\right.}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)} + \frac{(3cfg^2+fh(bg-ah)(}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\left((3cfg^2+fh(bg-ah)\right.}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)} + \frac{(3cfg^2+fh(bg-ah)(}{}$$

Mathematica [**F**]
time = 1.45, size = 0, normalized size = 0.00

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2), x]
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int (hx+g)^m\left(f\,x^2+ex+d\right)\sqrt{c\,x^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
[Out] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(f*x^2 + x*e + d)*(h*x + g)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(f*x^2 + x*e + d)*(h*x + g)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (g + hx)^m \sqrt{a + bx + cx^2} \left(d + ex + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**m*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**m*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(f*x^2 + x*e + d)*(h*x + g)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g+h\,x\right)}^m\,\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^m*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^m*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.274 $\int (g+hx)^{-3-2p} \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx$

Optimal. Leaf size=590

$$-\frac{\left(fg^2-h(eg-dh)\right)(g+hx)^{-2(1+p)}\left(a+bx+cx^2\right)^{1+p}}{2h\left(cg^2-bgh+ah^2\right)(1+p)}-\frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4\right.}\right.}}{}$$

```
[Out] -1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1+p)/h/(a*h^2-b*g*h+c*g^2)/(1+p)/(
(h*x+g)^(2+2*p))-1/2*(2*c*(-d*g*h^2+f*g^3)+h*(2*a*h*(-e*h+2*f*g)-b*(-d*h^2-
e*g*h+3*f*g^2)))*(h*x+g)^(-1-2*p)*(c*x^2+b*x+a)^p*hypergeom([-p, -1-2*p],[-
2*p],-4*c*(h*x+g)*(-4*a*c+b^2)^(1/2)/(b+2*c*x-(-4*a*c+b^2)^(1/2))/(2*c*g-h*
(b+(-4*a*c+b^2)^(1/2))))*(b+2*c*x-(-4*a*c+b^2)^(1/2))/h^2/(a*h^2-b*g*h+c*g^
2)/(1+2*p)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2)))/(((2*c*g-h*(b-(-4*a*c+b^2)^(1/2
)))*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+2*c*x-(-4*a*c+b^2)^(1/2))/(2*c*g-h*(b+(
-4*a*c+b^2)^(1/2))))^p)-1/2*f*(c*x^2+b*x+a)^p*AppellF1(-2*p,-p,-p,1-2*p,2*c
*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b
^2)^(1/2))))/h^3/p/((h*x+g)^(2*p))/((1-2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)
^(1/2))))^p)/((1-2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))^p)
```

Rubi [A]
time = 0.47, antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1669, 773, 138, 820, 740}

$$-\frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}\right)}{2h^3p}-\frac{\left(-\sqrt{b^2-4ac}+b+2cx\right)(g+hx)^{-2p-1}\left(a+bx+cx^2\right)^p\left(\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)\left(2cg-h\left(b-\sqrt{b^2-4ac}\right)\right)}{\left(-\sqrt{b^2-4ac}+b+2cx\right)\left(2cg-h\left(\sqrt{b^2-4ac}+b\right)\right)}\right)^{-p}\left(2c\left(fg^3-dgh^2\right)-h\left(-2ah(2fg-eh)-bh(dh+eg)+3bfg^2\right)\right){}_2F_1\left(-2p-1,-p;-2p;-\frac{4c\sqrt{b^2-4ac}(g+hx)}{\left(2cg-\left(b+\sqrt{b^2-4ac}\right)h\right)\left(b+2cx-\sqrt{b^2-4ac}\right)}\right)}{2h^2(2p+1)\left(2cg-h\left(b-\sqrt{b^2-4ac}\right)\right)\left(ah^2-bgh+cg^2\right)}-\frac{(g+hx)^{-2(p+1)}\left(a+bx+cx^2\right)^{p+1}\left(fg^2-h(eg-dh)\right)}{2h(p+1)\left(ah^2-bgh+cg^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] -1/2*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(1 + p))/(h*(c*g^2 - b*g*h
+ a*h^2)*(1 + p)*(g + h*x)^(2*(1 + p))) - (f*(a + b*x + c*x^2)^p*AppellF1[-
2*p, -p, -p, 1 - 2*p, (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h),
(2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h)])/(2*h^3*p*(g + h*x)^(2
*p)*(1 - (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(
g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h))^p) - ((2*c*(f*g^3 - d*g*h^2)
 - h*(3*b*f*g^2 - b*h*(e*g + d*h) - 2*a*h*(2*f*g - e*h)))*(b - Sqrt[b^2 - 4
*a*c] + 2*c*x)*(g + h*x)^(-1 - 2*p)*(a + b*x + c*x^2)^p*Hypergeometric2F1[-
1 - 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(g + h*x))/((2*c*g - (b + Sqrt[b
^2 - 4*a*c])*h)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))])/(2*h^2*(2*c*g - (b - Sqr
t[b^2 - 4*a*c])*h)*(c*g^2 - b*g*h + a*h^2)*(1 + 2*p)*(((2*c*g - (b - Sqrt[b
^2 - 4*a*c])*h)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*g - (b + Sqrt[b^2 -
4*a*c])*h)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(b - Rt[b^2 - 4*a*c, 2] + 2*c*x))*(d + e*x)^(m + 1)*((a +
b*x + c*x^2)^p/((m + 1)*(2*c*d - b*e + e*Rt[b^2 - 4*a*c, 2])*((2*c*d - b*e
+ e*Rt[b^2 - 4*a*c, 2])*((b + Rt[b^2 - 4*a*c, 2] + 2*c*x)/((2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2*c*x))))^p))*Hypergeometric
2F1[m + 1, -p, m + 2, -4*c*Rt[b^2 - 4*a*c, 2]*((d + e*x)/((2*c*d - b*e - e*
Rt[b^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2*c*x)))], x] /; FreeQ[{a, b,
 c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1669

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> With[{q = Expon[Pq, x]}, Dist[Coeff[Pq, x, q]/e^q, Int[(d
 + e*x)^(m + q)*(a + b*x + c*x^2)^p, x], x] + Dist[1/e^q, Int[(d + e*x)^m*(
a + b*x + c*x^2)^p*ExpandToSum[e^q*Pq - Coeff[Pq, x, q]*(d + e*x)^q, x], x]
, x]] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c
```

```
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ[m, 0] && RationalQ[a, b, c
, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx=\frac{\int (g+hx)^{-3-2p}\left(-fg^2+dh^2-h(2fg-eh)x\right)(a+bx$$

$$h^2$$

$$=-\frac{\left(fg^2-h(eg-dh)\right)(g+hx)^{-2(1+p)}\left(a+bx+cx^2\right)^{1+}}{2h\left(cg^2-bgh+ah^2\right)(1+p)}$$

$$=-\frac{\left(fg^2-h(eg-dh)\right)(g+hx)^{-2(1+p)}\left(a+bx+cx^2\right)^{1+}}{2h\left(cg^2-bgh+ah^2\right)(1+p)}$$

Mathematica [F]
time = 3.14, size = 0, normalized size = 0.00

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x]
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (hx+g)^{-3-2p}\left(c\,x^2+bx+a\right)^p\left(f\,x^2+ex+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
[Out] int((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="maxi
ma")
```

```
[Out] integrate((f*x^2 + x*e + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="fric
as")
```

```
[Out] integral((f*x^2 + x*e + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**(-3-2*p)*(c*x**2+b*x+a)**p*(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="giac
")
```

```
[Out] integrate((f*x^2 + x*e + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,x^2+b\,x+a)^p\,(f\,x^2+e\,x+d)}{(g+h\,x)^{2\,p+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3),x)
```

```
[Out] int(((a + b*x + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3), x)
```

3.275 $\int (d+fx^2)^p (2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2$

Optimal. Leaf size=41

$$\frac{bf(3+2p)\left(d+fx^2\right)^{1+p}}{1+p}+2cfx\left(d+fx^2\right)^{1+p}$$

```
[Out] b*f*(3+2*p)*(f*x^2+d)^(1+p)/(1+p)+2*c*f*x*(f*x^2+d)^(1+p)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1829, 12, 267}

$$\frac{bf(2p+3)\left(d+fx^2\right)^{p+1}}{p+1}+2cfx\left(d+fx^2\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + f*x^2)^p*(2*c*d*f + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),x
]
```

```
[Out] (b*f*(3 + 2*p)*(d + f*x^2)^(1 + p))/(1 + p) + 2*c*f*x*(d + f*x^2)^(1 + p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int (d+fx^2)^p (2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2)\,dx = 2cfx(d+fx^2)^{1+p} + \frac{\int 2bf^3(3+2p)^2x(d+f}{f(3+2p)}$$
$$= 2cfx(d+fx^2)^{1+p} + (2bf^2(3+2p))\int x(d+$$
$$= \frac{bf(3+2p)(d+fx^2)^{1+p}}{1+p} + 2cfx(d+fx^2)^{1+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.28, size = 119, normalized size = 2.90

$$\frac{f(d+fx^2)^p\left(1+\frac{fx^2}{d}\right)^{-p}\left(6cd(1+p)x\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{fx^2}{d}\right)+(3+2p)\left(3b(d+fx^2)\left(1+\frac{fx^2}{d}\right)^p+2cf(1+p)x^3\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{fx^2}{d}\right)\right)\right)}{3(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + f*x^2)^p*(2*c*d*f + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*
x^2),x]
```

```
[Out] (f*(d + f*x^2)^p*(6*c*d*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((f*x^2)
/d)] + (3 + 2*p)*(3*b*(d + f*x^2)*(1 + (f*x^2)/d)^p + 2*c*f*(1 + p)*x^3*Hyp
ergeometric2F1[3/2, -p, 5/2, -((f*x^2)/d)])))/(3*(1 + p)*(1 + (f*x^2)/d)^p)
```

Maple [A]
time = 0.12, size = 36, normalized size = 0.88

method	result	size
gosper	$\frac{f\left(f\,x^2+d\right)^{1+p}(2pcx+2pb+2cx+3b)}{1+p}$	36
risch	$\frac{f\left(2cfp\,x^3+2bfp\,x^2+2cf\,x^3+3bf\,x^2+2cdpx+2bdp+2cdx+3bd\right)\left(f\,x^2+d\right)^p}{1+p}$	68
norman	$\frac{bdf(3+2p)\mathrm{e}^{p\ln\left(f\,x^2+d\right)}}{1+p}+\frac{b\,f^2(3+2p)x^2\mathrm{e}^{p\ln\left(f\,x^2+d\right)}}{1+p}+2c\,f^2x^3\mathrm{e}^{p\ln\left(f\,x^2+d\right)}+2cdfx\,\mathrm{e}^{p\ln\left(f\,x^2+d\right)}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x,method=_R
ETURNVERBOSE)
```

```
[Out] f*(f*x^2+d)^(1+p)*(2*c*p*x+2*b*p+2*c*x+3*b)/(1+p)
```

Maxima [A]
time = 0.32, size = 59, normalized size = 1.44

$$\frac{(2\,cf^2(p+1)x^3+bf^2(2\,p+3)x^2+2\,cdf(p+1)x+bdf(2\,p+3))(fx^2+d)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, al
gorithm="maxima")
```

```
[Out] (2*c*f^2*(p + 1)*x^3 + b*f^2*(2*p + 3)*x^2 + 2*c*d*f*(p + 1)*x + b*d*f*(2*p
 + 3))*(f*x^2 + d)^p/(p + 1)
```

Fricas [A]
time = 0.35, size = 75, normalized size = 1.83

$$\frac{(2\,bdfp + 2\,(cf^2p + cf^2)x^3 + 3\,bdf + (2\,bf^2p + 3\,bf^2)x^2 + 2\,(cdfp + cdf)x)(fx^2 + d)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, al
gorithm="fricas")
```

```
[Out] (2*b*d*f*p + 2*(c*f^2*p + c*f^2)*x^3 + 3*b*d*f + (2*b*f^2*p + 3*b*f^2)*x^2
+ 2*(c*d*f*p + c*d*f)*x)*(f*x^2 + d)^p/(p + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(37) = 74.
time = 3.36, size = 211, normalized size = 5.15

$$\begin{cases} \frac{2bdfp(d+fx^2)^p}{p+1} + \frac{3bdf(d+fx^2)^p}{p+1} + \frac{2bf^2px^2(d+fx^2)^p}{p+1} + \frac{3bf^2x^2(d+fx^2)^p}{p+1} + \frac{2cdfpx(d+fx^2)^p}{p+1} + \frac{2cdfx(d+fx^2)^p}{p+1} + \frac{2cf^2px^3(d+fx^2)^p}{p+1} + \frac{2cf^2x^3(d+fx^2)^p}{p+1} & \text{for}\ p \neq -1 \\ bf\log\left(x - \sqrt{-\frac{d}{f}}\right) + bf\log\left(x + \sqrt{-\frac{d}{f}}\right) + 2cfx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+d)**p*(2*c*d*f+2*b*f**2*(3+2*p)*x+2*c*f**2*(3+2*p)*x**2),
x)
```

```
[Out] Piecewise((2*b*d*f*p*(d + f*x**2)**p/(p + 1) + 3*b*d*f*(d + f*x**2)**p/(p +
 1) + 2*b*f**2*p*x**2*(d + f*x**2)**p/(p + 1) + 3*b*f**2*x**2*(d + f*x**2)*
*p/(p + 1) + 2*c*d*f*p*x*(d + f*x**2)**p/(p + 1) + 2*c*d*f*x*(d + f*x**2)**
p/(p + 1) + 2*c*f**2*p*x**3*(d + f*x**2)**p/(p + 1) + 2*c*f**2*x**3*(d + f*
x**2)**p/(p + 1), Ne(p, -1)), (b*f*log(x - sqrt(-d/f)) + b*f*log(x + sqrt(-
d/f)) + 2*c*f*x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(41) = 82.
time = 4.14, size = 141, normalized size = 3.44

$$\frac{2\,(fx^2 + d)^p cf^2px^3 + 2\,(fx^2 + d)^p bf^2px^2 + 2\,(fx^2 + d)^p cf^2x^3 + 2\,(fx^2 + d)^p cdfpx + 3\,(fx^2 + d)^p bf^2x^2 + 2\,(fx^2 + d)^p bdfp + 2\,(fx^2 + d)^p cdfx + 3\,(fx^2 + d)^p bdf}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, al
gorithm="giac")
```

```
[Out] (2*(f*x^2 + d)^p*c*f^2*p*x^3 + 2*(f*x^2 + d)^p*b*f^2*p*x^2 + 2*(f*x^2 + d)^
p*c*f^2*x^3 + 2*(f*x^2 + d)^p*c*d*f*p*x + 3*(f*x^2 + d)^p*b*f^2*x^2 + 2*(f*
x^2 + d)^p*b*d*f*p + 2*(f*x^2 + d)^p*c*d*f*x + 3*(f*x^2 + d)^p*b*d*f)/(p +
1)
```

Mupad [B]
time = 4.25, size = 58, normalized size = 1.41

$$\left(f\,x^2+d\right)^p\,\left(2\,c\,f^2\,x^3+2\,c\,d\,f\,x+\frac{b\,f^2\,x^2\,\left(2\,p+3\right)}{p+1}+\frac{b\,d\,f\,\left(2\,p+3\right)}{p+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*x^2)^p*(2*c*d*f + 2*b*f^2*x*(2*p + 3) + 2*c*f^2*x^2*(2*p + 3)),x
)
```

```
[Out] (d + f*x^2)^p*(2*c*f^2*x^3 + 2*c*d*f*x + (b*f^2*x^2*(2*p + 3))/(p + 1) + (b
*d*f*(2*p + 3))/(p + 1))
```

3.276 $\int (d+ex+fx^2)^p(-2ce^2+2cdf-ce^2p+2cf^2(3+2$

Optimal. Leaf size=46

$$-\frac{ce(2+p)\left(d+ex+fx^2\right)^{1+p}}{1+p}+2cfx\left(d+ex+fx^2\right)^{1+p}$$

[Out] -c*e*(2+p)*(f*x^2+e*x+d)^(1+p)/(1+p)+2*c*f*x*(f*x^2+e*x+d)^(1+p)

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1675, 643}

$$2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{ce(p+2)\left(d+ex+fx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f - c*e^2*p + 2*c*f^2*(3 + 2*p)*x^2),x]

[Out] -((c*e*(2 + p)*(d + e*x + f*x^2)^(1 + p))/(1 + p)) + 2*c*f*x*(d + e*x + f*x^2)^(1 + p)

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(d+ex+fx^2\right)^p\left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right)\,dx=2cfx\left(d+ex+fx^2\right)^{1+p}+\frac{\int\left(-ce^2f(2+p\right.}{}$$

$$=-\frac{ce(2+p)\left(d+ex+fx^2\right)^{1+p}}{1+p}+2cfx\left(d+\right.$$

Mathematica [A]
time = 0.34, size = 34, normalized size = 0.74

$$\frac{c(-e(2+p)+2f(1+p)x)(d+x(e+fx))^{1+p}}{1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f - c*e^2*p + 2*c*f^2*(3 +
2*p)*x^2),x]
```

```
[Out] (c*(-(e*(2 + p)) + 2*f*(1 + p)*x)*(d + x*(e + f*x))^(1 + p))/(1 + p)
```

Maple [A]
time = 0.20, size = 39, normalized size = 0.85

method	result
gosper	$-\frac{c\left(f\,x^{2}+e x+d\right)^{1+p}\left(-2 f p x+e p-2 f x+2 e\right)}{1+p}$
risch	$-\frac{c\left(-2 p\,f^{2} x^{3}-e f p\,x^{2}-2 f^{2} x^{3}-2 d f p x+e^{2} p x+d e p-2 d f x+2 e^{2} x+2 d e\right)\left(f\,x^{2}+e x+d\right)^{p}}{1+p}$
norman	$\frac{c\left(2 d f p-e^{2} p+2 d f-2 e^{2}\right) x\,{\mathrm e}^{p\ln\left(f\,x^{2}+e x+d\right)}}{1+p}+\frac{c e f p\,x^{2} {\mathrm e}^{p\ln\left(f\,x^{2}+e x+d\right)}}{1+p}+2 c\,f^{2} x^{3} {\mathrm e}^{p\ln\left(f\,x^{2}+e x+d\right)}-\frac{c d e\left(2+p\right) {\mathrm e}^{p\ln\left(f\,x^{2}+e x+\right.}}{1+p}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,method
=_RETURNVERBOSE)
```

```
[Out] -c*(f*x^2+e*x+d)^(1+p)*(-2*f*p*x+e*p-2*f*x+2*e)/(1+p)
```

Maxima [A]
time = 0.34, size = 69, normalized size = 1.50

$$\frac{(2\,cf^2(p+1)x^3+cfpx^2e-cd(p+2)e+(2\,cdf(p+1)-c(p+2)e^2)x)(fx^2+xe+d)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,
 algorithm="maxima")
```

```
[Out] (2*c*f^2*(p + 1)*x^3 + c*f*p*x^2*e - c*d*(p + 2)*e + (2*c*d*f*(p + 1) - c*(
p + 2)*e^2)*x)*(f*x^2 + x*e + d)^p/(p + 1)
```

Fricas [A]
time = 0.37, size = 82, normalized size = 1.78

$$\frac{(2\,(cf^2p+cf^2)x^3-(cp+2\,c)xe^2+2\,(cdfp+cdf)x+(cfpx^2-cdp-2\,cd)e)(fx^2+xe+d)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,
 algorithm="fricas")
```

```
[Out] (2*(c*f^2*p + c*f^2)*x^3 - (c*p + 2*c)*x*e^2 + 2*(c*d*f*p + c*d*f)*x + (c*f
*p*x^2 - c*d*p - 2*c*d)*e)*(f*x^2 + x*e + d)^p/(p + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(42) = 84$.
time = 58.71, size = 280, normalized size = 6.09

$$\begin{cases} -\frac{cdep(d+ex+fx^2)^p}{p+1} - \frac{2cde(d+ex+fx^2)^p}{p+1} + \frac{2cdfpx(d+ex+fx^2)^p}{p+1} + \frac{2cdfx(d+ex+fx^2)^p}{p+1} - \frac{ce^2px(d+ex+fx^2)^p}{p+1} - \frac{2ce^2x(d+ex+fx^2)^p}{p+1} + \frac{cefpx^2(d+ex+fx^2)^p}{p+1} + \frac{2cf^2px^3(d+ex+fx^2)^p}{p+1} + \frac{2cf^2x^3(d+ex+fx^2)^p}{p+1} & \text{for } p \neq -1 \\ -ce\log\left(\frac{e}{2f} + x - \frac{\sqrt{-4df+e^2}}{2f}\right) - ce\log\left(\frac{e}{2f} + x + \frac{\sqrt{-4df+e^2}}{2f}\right) + 2cfx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**p*(-2*c*e**2+2*c*d*f-c*e**2*p+2*c*f**2*(3+2*p)*x*
*2),x)
```

```
[Out] Piecewise((-c*d*e*p*(d + e*x + f*x**2)**p/(p + 1) - 2*c*d*e*(d + e*x + f*x*
*2)**p/(p + 1) + 2*c*d*f*p*x*(d + e*x + f*x**2)**p/(p + 1) + 2*c*d*f*x*(d +
 e*x + f*x**2)**p/(p + 1) - c*e**2*p*x*(d + e*x + f*x**2)**p/(p + 1) - 2*c*
e**2*x*(d + e*x + f*x**2)**p/(p + 1) + c*e*f*p*x**2*(d + e*x + f*x**2)**p/(
p + 1) + 2*c*f**2*p*x**3*(d + e*x + f*x**2)**p/(p + 1) + 2*c*f**2*x**3*(d +
 e*x + f*x**2)**p/(p + 1), Ne(p, -1)), (-c*e*log(e/(2*f) + x - sqrt(-4*d*f
+ e**2)/(2*f)) - c*e*log(e/(2*f) + x + sqrt(-4*d*f + e**2)/(2*f)) + 2*c*f*x
, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(49) = 98$.
time = 3.18, size = 191, normalized size = 4.15

$$\frac{2\,(fx^2+xe+d)^p cf^2px^3 + 2\,(fx^2+xe+d)^p cf^2x^3 + (fx^2+xe+d)^p cfpx^2e + 2\,(fx^2+xe+d)^p cdfpx + 2\,(fx^2+xe+d)^p cdfx - (fx^2+xe+d)^p cpxe^2 - (fx^2+xe+d)^p cdpe - 2\,(fx^2+xe+d)^p cxe^2 - 2\,(fx^2+xe+d)^p cde}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,
 algorithm="giac")
```

```
[Out] (2*(f*x^2 + x*e + d)^p*c*f^2*p*x^3 + 2*(f*x^2 + x*e + d)^p*c*f^2*x^3 + (f*x
^2 + x*e + d)^p*c*f*p*x^2*e + 2*(f*x^2 + x*e + d)^p*c*d*f*p*x + 2*(f*x^2 +
x*e + d)^p*c*d*f*x - (f*x^2 + x*e + d)^p*c*p*x*e^2 - (f*x^2 + x*e + d)^p*c*
d*p*e - 2*(f*x^2 + x*e + d)^p*c*x*e^2 - 2*(f*x^2 + x*e + d)^p*c*d*e)/(p + 1
)
```

Mupad [B]
time = 4.39, size = 78, normalized size = 1.70

$$(f\,x^2+e\,x+d)^p\,\left(2\,c\,f^2\,x^3+\frac{c\,x\,\left(2\,d\,f-e^2\,p-2\,e^2+2\,d\,f\,p\right)}{p+1}-\frac{c\,d\,e\,\left(p+2\right)}{p+1}+\frac{c\,e\,f\,p\,x^2}{p+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(d + e*x + f*x^2)^p*(2*c*e^2 - 2*c*d*f + c*e^2*p - 2*c*f^2*x^2*(2*p +
3)),x)
```

```
[Out] (d + e*x + f*x^2)^p*(2*c*f^2*x^3 + (c*x*(2*d*f - e^2*p - 2*e^2 + 2*d*f*p))/
(p + 1) - (c*d*e*(p + 2))/(p + 1) + (c*e*f*p*x^2)/(p + 1))
```

3.277 $\int (d+ex+fx^2)^p (-2ce^2+2cdf+3bef-ce^2p+2b$

Optimal. Leaf size=57

$$-\frac{(ce(2+p)-bf(3+2p))(d+ex+fx^2)^{1+p}}{1+p}+2cfx(d+ex+fx^2)^{1+p}$$

```
[Out] -(c*e*(2+p)-b*f*(3+2*p))*(f*x^2+e*x+d)^(1+p)/(1+p)+2*c*f*x*(f*x^2+e*x+d)^(1
+p)
```

Rubi [A]
time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 69, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1675, 643}

$$2cfx(d+ex+fx^2)^{p+1}-\frac{(ce(p+2)-bf(2p+3))(d+ex+fx^2)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f + 3*b*e*f - c*e^2*p + 2*b*e*f*p
 + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),x]
```

```
[Out] -(((c*e*(2 + p) - b*f*(3 + 2*p))*(d + e*x + f*x^2)^(1 + p))/(1 + p)) + 2*c*
f*x*(d + e*x + f*x^2)^(1 + p)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int (d+ex+fx^2)^p (-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2)\,dx = 2cfx(d$$

$$= -\frac{(ce(2$$

Mathematica [A]
time = 0.58, size = 43, normalized size = 0.75

$$\frac{(-ce(2+p)+bf(3+2p)+2cf(1+p)x)(d+x(e+fx))^{1+p}}{1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f + 3*b*e*f - c*e^2*p + 2*b
*e*f*p + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),x]
```

```
[Out] ((-(c*e*(2 + p)) + b*f*(3 + 2*p) + 2*c*f*(1 + p)*x)*(d + x*(e + f*x))^(1 +
p))/(1 + p)
```

Maple [A]
time = 0.16, size = 51, normalized size = 0.89

method	result
gosper	$\frac{(f\,x^2+ex+d)^{1+p}(2cfxp+2bfp-cep+2cfx+3bf-2ce)}{1+p}$
risch	$\frac{(2pc\,f^2x^3+2b\,f^2p\,x^2+cefp\,x^2+2c\,f^2x^3+2befpx+3b\,f^2x^2+2cdfpx-c\,e^2px+2bdfp+3befx-cdep+2cdfx-2c\,e^2x+3bdf-2cde)(f\,x^2+}{1+p}$
norman	$\frac{d(2bfp-cep+3bf-2ce)\mathrm{e}^{p\ln(f\,x^2+ex+d)}}{1+p}+\frac{(2befp+2cdfp-c\,e^2p+3bef+2cdf-2c\,e^2)x\,\mathrm{e}^{p\ln(f\,x^2+ex+d)}}{1+p}+\frac{f(2bfp+cep+3bf)x^2\mathrm{e}}{1+p}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f^2*(3+
2*p)*x+2*c*f^2*(3+2*p)*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (f*x^2+e*x+d)^(1+p)*(2*c*f*p*x+2*b*f*p-c*e*p+2*c*f*x+3*b*f-2*c*e)/(1+p)
```

Maxima [A]
time = 0.33, size = 100, normalized size = 1.75

$$\frac{(2\,cf^2(p+1)x^3+bdf(2\,p+3)-cd(p+2)e+(bf^2(2\,p+3)+cfpe)x^2+(2\,cdf(p+1)+bf(2\,p+3)e-c(p+2)e^2)x)(fx^2+xe+d)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f
^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="maxima")
```

```
[Out] (2*c*f^2*(p + 1)*x^3 + b*d*f*(2*p + 3) - c*d*(p + 2)*e + (b*f^2*(2*p + 3) +
 c*f*p*e)*x^2 + (2*c*d*f*(p + 1) + b*f*(2*p + 3)*e - c*(p + 2)*e^2)*x)*(f*x
^2 + x*e + d)^p/(p + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(59) = 118$.
time = 0.40, size = 123, normalized size = 2.16

$$\frac{(2\,bdfp+2\,(cf^2p+cf^2)x^3+3\,bdf+(2\,bf^2p+3\,bf^2)x^2-(cp+2\,c)xe^2+2\,(cdfp+cdf)x+(cfpx^2-cdp-2\,cd+(2\,bfp+3\,bf)x)e)(fx^2+xe+d)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f
^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="fricas")
```

```
[Out] (2*b*d*f*p + 2*(c*f^2*p + c*f^2)*x^3 + 3*b*d*f + (2*b*f^2*p + 3*b*f^2)*x^2
- (c*p + 2*c)*x*e^2 + 2*(c*d*f*p + c*d*f)*x + (c*f*p*x^2 - c*d*p - 2*c*d +
(2*b*f*p + 3*b*f)*x)*e)*(f*x^2 + x*e + d)^p/(p + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(51) = 102$.
time = 64.38, size = 483, normalized size = 8.47

$$\begin{cases} \frac{2bdfp(d+ex+fx^2)^p}{p+1} + \frac{3bdf(d+ex+fx^2)^p}{p+1} + \frac{2befpx(d+ex+fx^2)^p}{p+1} + \frac{3befx(d+ex+fx^2)^p}{p+1} + \frac{2bf^2px^2(d+ex+fx^2)^p}{p+1} + \frac{3bf^2x^2(d+ex+fx^2)^p}{p+1} - \frac{cdep(d+ex+fx^2)^p}{p+1} - \frac{2cde(d+ex+fx^2)^p}{p+1} + \frac{2cdfpx(d+ex+fx^2)^p}{p+1} + \frac{2cdfx(d+ex+fx^2)^p}{p+1} - \frac{ce^2px(d+ex+fx^2)^p}{p+1} - \frac{2ce^2x(d+ex+fx^2)^p}{p+1} + \frac{cefpx^2(d+ex+fx^2)^p}{p+1} + \frac{2cf^2px^3(d+ex+fx^2)^p}{p+1} + \frac{2cf^2x^3(d+ex+fx^2)^p}{p+1} & \text{for } p \neq -1 \\ bf\log\left(\frac{e}{2f} + x - \frac{\sqrt{-4df+e^2}}{2f}\right) + bf\log\left(\frac{e}{2f} + x + \frac{\sqrt{-4df+e^2}}{2f}\right) - ce\log\left(\frac{e}{2f} + x - \frac{\sqrt{-4df+e^2}}{2f}\right) - ce\log\left(\frac{e}{2f} + x + \frac{\sqrt{-4df+e^2}}{2f}\right) + 2cfx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**p*(-2*c*e**2+2*c*d*f+3*b*e*f-c*e**2*p+2*b*e*f*p+2
*b*f**2*(3+2*p)*x+2*c*f**2*(3+2*p)*x**2),x)
```

```
[Out] Piecewise((2*b*d*f*p*(d + e*x + f*x**2)**p/(p + 1) + 3*b*d*f*(d + e*x + f*x
**2)**p/(p + 1) + 2*b*e*f*p*x*(d + e*x + f*x**2)**p/(p + 1) + 3*b*e*f*x*(d
+ e*x + f*x**2)**p/(p + 1) + 2*b*f**2*p*x**2*(d + e*x + f*x**2)**p/(p + 1)
+ 3*b*f**2*x**2*(d + e*x + f*x**2)**p/(p + 1) - c*d*e*p*(d + e*x + f*x**2)*
*p/(p + 1) - 2*c*d*e*(d + e*x + f*x**2)**p/(p + 1) + 2*c*d*f*p*x*(d + e*x +
 f*x**2)**p/(p + 1) + 2*c*d*f*x*(d + e*x + f*x**2)**p/(p + 1) - c*e**2*p*x*
(d + e*x + f*x**2)**p/(p + 1) - 2*c*e**2*x*(d + e*x + f*x**2)**p/(p + 1) +
c*e*f*p*x**2*(d + e*x + f*x**2)**p/(p + 1) + 2*c*f**2*p*x**3*(d + e*x + f*x
**2)**p/(p + 1) + 2*c*f**2*x**3*(d + e*x + f*x**2)**p/(p + 1), Ne(p, -1)),
(b*f*log(e/(2*f) + x - sqrt(-4*d*f + e**2)/(2*f)) + b*f*log(e/(2*f) + x + s
qrt(-4*d*f + e**2)/(2*f)) - c*e*log(e/(2*f) + x - sqrt(-4*d*f + e**2)/(2*f)
) - c*e*log(e/(2*f) + x + sqrt(-4*d*f + e**2)/(2*f)) + 2*c*f*x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(59) = 118$.
time = 3.93, size = 314, normalized size = 5.51

$$\frac{2(fx^2+xe+d)^p cf^2px^3 + 2(fx^2+xe+d)^p bf^2px^2 + 2(fx^2+xe+d)^p cf^2x^3 + (fx^2+xe+d)^p cfpx^2e + 2(fx^2+xe+d)^p cdfpx + 3(fx^2+xe+d)^p bf^2x^2 + 2(fx^2+xe+d)^p bfpxe + 2(fx^2+xe+d)^p bdfp + 2(fx^2+xe+d)^p cdfx - (fx^2+xe+d)^p cpxe^2 - (fx^2+xe+d)^p cdpe + 3(fx^2+xe+d)^p bfxe + 3(fx^2+xe+d)^p bdf - 2(fx^2+xe+d)^p cxe^2 - 2(fx^2+xe+d)^p cde}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f
^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="giac")
```

```
[Out] (2*(f*x^2 + x*e + d)^p*c*f^2*p*x^3 + 2*(f*x^2 + x*e + d)^p*b*f^2*p*x^2 + 2*
(f*x^2 + x*e + d)^p*c*f^2*x^3 + (f*x^2 + x*e + d)^p*c*f*p*x^2*e + 2*(f*x^2
+ x*e + d)^p*c*d*f*p*x + 3*(f*x^2 + x*e + d)^p*b*f^2*x^2 + 2*(f*x^2 + x*e +
 d)^p*b*f*p*x*e + 2*(f*x^2 + x*e + d)^p*b*d*f*p + 2*(f*x^2 + x*e + d)^p*c*d
```

```
*f*x - (f*x^2 + x*e + d)^p*c*p*x*e^2 - (f*x^2 + x*e + d)^p*c*d*p*e + 3*(f*x
^2 + x*e + d)^p*b*f*x*e + 3*(f*x^2 + x*e + d)^p*b*d*f - 2*(f*x^2 + x*e + d)
^p*c*x*e^2 - 2*(f*x^2 + x*e + d)^p*c*d*e)/(p + 1)
```

Mupad [B]
time = 4.46, size = 120, normalized size = 2.11

$$(f\,x^2+e\,x+d)^p\,\left(\frac{x^2\,(3\,b\,f^2+2\,b\,f^2\,p+c\,e\,f\,p)}{p+1}+2\,c\,f^2\,x^3+\frac{d\,(3\,b\,f-2\,c\,e+2\,b\,f\,p-c\,e\,p)}{p+1}+\frac{x\,(3\,b\,e\,f-2\,c\,e^2+2\,c\,d\,f-c\,e^2\,p+2\,b\,e\,f\,p+2\,c\,d\,f\,p)}{p+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^p*(3*b*e*f - 2*c*e^2 + 2*c*d*f - c*e^2*p + 2*b*f^2*x*
(2*p + 3) + 2*c*f^2*x^2*(2*p + 3) + 2*b*e*f*p),x)
```

```
[Out] (d + e*x + f*x^2)^p*((x^2*(3*b*f^2 + 2*b*f^2*p + c*e*f*p))/(p + 1) + 2*c*f^
2*x^3 + (d*(3*b*f - 2*c*e + 2*b*f*p - c*e*p))/(p + 1) + (x*(3*b*e*f - 2*c*e
^2 + 2*c*d*f - c*e^2*p + 2*b*e*f*p + 2*c*d*f*p))/(p + 1))
```

3.278 $\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+1\right.\right.$

Optimal. Leaf size=20

$$(d+ex)^5 \left(a+bx+cx^2\right)^6$$

```
[Out] (e*x+d)^5*(c*x^2+b*x+a)^6
```

Rubi [A]
time = 0.28, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 75, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = {1638, 1604}

$$(d+ex)^5 \left(a+bx+cx^2\right)^6$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*x + c*x^2)^5*(d*(6*b*d + 5*a*e) + (12*c*d^2 + 17*b*d
*e + 5*a*e^2)*x + e*(29*c*d + 11*b*e)*x^2 + 17*c*e^2*x^3),x]
```

```
[Out] (d + e*x)^5*(a + b*x + c*x^2)^6
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rule 1638

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^(m + 1)*PolynomialQuotient[Pq, d + e*x, x]*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[PolynomialRemainder[Pq, d + e*x, x], 0]
```

Rubi steps

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x+e(29cd+11be)x^2+17ce^2x^3\right)\,dx =$$

$$=$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 167 vs. $2(20) = 40$.
time = 0.30, size = 167, normalized size = 8.35

$$x\left(6a^5(b+cx)(d+ex)^5+15a^4x(b+cx)^2(d+ex)^5+20a^3x^2(b+cx)^3(d+ex)^5+15a^2x^3(b+cx)^4(d+ex)^5+6ax^4(b+cx)^5(d+ex)^5+x^5(b+cx)^6(d+ex)^5+a^6e\left(5d^4+10d^3ex+10d^2e^2x^2+5de^3x^3+e^4x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*x + c*x^2)^5*(d*(6*b*d + 5*a*e) + (12*c*d^2 +
17*b*d*e + 5*a*e^2)*x + e*(29*c*d + 11*b*e)*x^2 + 17*c*e^2*x^3),x]
```

```
[Out] x*(6*a^5*(b + c*x)*(d + e*x)^5 + 15*a^4*x*(b + c*x)^2*(d + e*x)^5 + 20*a^3*
x^2*(b + c*x)^3*(d + e*x)^5 + 15*a^2*x^3*(b + c*x)^4*(d + e*x)^5 + 6*a*x^4*
(b + c*x)^5*(d + e*x)^5 + x^5*(b + c*x)^6*(d + e*x)^5 + a^6*e*(5*d^4 + 10*d
^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8418 vs. $2(20) = 40$.
time = 0.51, size = 8419, normalized size = 420.95

method	result	size
norman	Expression too large to display	2052
gosper	Expression too large to display	2460
risch	Expression too large to display	2468
default	Expression too large to display	8419

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c*d^2)*
x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1875 vs. $2(21) = 42$.
time = 0.32, size = 1875, normalized size = 93.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c
*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="maxima")
```

```
[Out] c^6*x^17*e^5 + (5*c^6*d*e^4 + 6*b*c^5*e^5)*x^16 + (10*c^6*d^2*e^3 + 30*b*c^
5*d*e^4 + 15*b^2*c^4*e^5 + 6*a*c^5*e^5)*x^15 + 5*(2*c^6*d^3*e^2 + 12*b*c^5*
d^2*e^3 + 4*b^3*c^3*e^5 + 6*a*b*c^4*e^5 + 3*(5*b^2*c^4*e^4 + 2*a*c^5*e^4)*d
)*x^14 + 5*(c^6*d^4*e + 12*b*c^5*d^3*e^2 + 3*b^4*c^2*e^5 + 12*a*b^2*c^3*e^5
```

```
 + 3*a^2*c^4*e^5 + 6*(5*b^2*c^4*e^3 + 2*a*c^5*e^3)*d^2 + 10*(2*b^3*c^3*e^4
+ 3*a*b*c^4*e^4)*d)*x^13 + (c^6*d^5 + 30*b*c^5*d^4*e + 6*b^5*c*e^5 + 60*a*b
^3*c^2*e^5 + 60*a^2*b*c^3*e^5 + 30*(5*b^2*c^4*e^2 + 2*a*c^5*e^2)*d^3 + 100*
(2*b^3*c^3*e^3 + 3*a*b*c^4*e^3)*d^2 + 75*(b^4*c^2*e^4 + 4*a*b^2*c^3*e^4 + a
^2*c^4*e^4)*d)*x^12 + (6*b*c^5*d^5 + b^6*e^5 + 30*a*b^4*c*e^5 + 90*a^2*b^2*
c^2*e^5 + 20*a^3*c^3*e^5 + 15*(5*b^2*c^4*e + 2*a*c^5*e)*d^4 + 100*(2*b^3*c^
3*e^2 + 3*a*b*c^4*e^2)*d^3 + 150*(b^4*c^2*e^3 + 4*a*b^2*c^3*e^3 + a^2*c^4*e
^3)*d^2 + 30*(b^5*c*e^4 + 10*a*b^3*c^2*e^4 + 10*a^2*b*c^3*e^4)*d)*x^11 + (6
*a*b^5*e^5 + 60*a^2*b^3*c*e^5 + 60*a^3*b*c^2*e^5 + 3*(5*b^2*c^4 + 2*a*c^5)*
d^5 + 50*(2*b^3*c^3*e + 3*a*b*c^4*e)*d^4 + 150*(b^4*c^2*e^2 + 4*a*b^2*c^3*e
^2 + a^2*c^4*e^2)*d^3 + 60*(b^5*c*e^3 + 10*a*b^3*c^2*e^3 + 10*a^2*b*c^3*e^3
)*d^2 + 5*(b^6*e^4 + 30*a*b^4*c*e^4 + 90*a^2*b^2*c^2*e^4 + 20*a^3*c^3*e^4)*
d)*x^10 + 5*(3*a^2*b^4*e^5 + 12*a^3*b^2*c*e^5 + 3*a^4*c^2*e^5 + 2*(2*b^3*c^
3 + 3*a*b*c^4)*d^5 + 15*(b^4*c^2*e + 4*a*b^2*c^3*e + a^2*c^4*e)*d^4 + 12*(b
^5*c*e^2 + 10*a*b^3*c^2*e^2 + 10*a^2*b*c^3*e^2)*d^3 + 2*(b^6*e^3 + 30*a*b^4
*c*e^3 + 90*a^2*b^2*c^2*e^3 + 20*a^3*c^3*e^3)*d^2 + 6*(a*b^5*e^4 + 10*a^2*b
^3*c*e^4 + 10*a^3*b*c^2*e^4)*d)*x^9 + 5*(4*a^3*b^3*e^5 + 6*a^4*b*c*e^5 + 3*
(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^5 + 6*(b^5*c*e + 10*a*b^3*c^2*e + 10*a^
2*b*c^3*e)*d^4 + 2*(b^6*e^2 + 30*a*b^4*c*e^2 + 90*a^2*b^2*c^2*e^2 + 20*a^3*
c^3*e^2)*d^3 + 12*(a*b^5*e^3 + 10*a^2*b^3*c*e^3 + 10*a^3*b*c^2*e^3)*d^2 + 1
5*(a^2*b^4*e^4 + 4*a^3*b^2*c*e^4 + a^4*c^2*e^4)*d)*x^8 + (15*a^4*b^2*e^5 +
6*a^5*c*e^5 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^5 + 5*(b^6*e + 30*a
*b^4*c*e + 90*a^2*b^2*c^2*e + 20*a^3*c^3*e)*d^4 + 60*(a*b^5*e^2 + 10*a^2*b^
3*c*e^2 + 10*a^3*b*c^2*e^2)*d^3 + 150*(a^2*b^4*e^3 + 4*a^3*b^2*c*e^3 + a^4*
c^2*e^3)*d^2 + 50*(2*a^3*b^3*e^4 + 3*a^4*b*c*e^4)*d)*x^7 + (6*a^5*b*e^5 + (
b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d^5 + 30*(a*b^5*e + 10*a^2*
b^3*c*e + 10*a^3*b*c^2*e)*d^4 + 150*(a^2*b^4*e^2 + 4*a^3*b^2*c*e^2 + a^4*c^
2*e^2)*d^3 + 100*(2*a^3*b^3*e^3 + 3*a^4*b*c*e^3)*d^2 + 15*(5*a^4*b^2*e^4 +
2*a^5*c*e^4)*d)*x^6 + (30*a^5*b*d*e^4 + a^6*e^5 + 6*(a*b^5 + 10*a^2*b^3*c +
 10*a^3*b*c^2)*d^5 + 75*(a^2*b^4*e + 4*a^3*b^2*c*e + a^4*c^2*e)*d^4 + 100*(
2*a^3*b^3*e^2 + 3*a^4*b*c*e^2)*d^3 + 30*(5*a^4*b^2*e^3 + 2*a^5*c*e^3)*d^2)*
x^5 + 5*(12*a^5*b*d^2*e^3 + a^6*d*e^4 + 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)
*d^5 + 10*(2*a^3*b^3*e + 3*a^4*b*c*e)*d^4 + 6*(5*a^4*b^2*e^2 + 2*a^5*c*e^2)
*d^3)*x^4 + 5*(12*a^5*b*d^3*e^2 + 2*a^6*d^2*e^3 + 2*(2*a^3*b^3 + 3*a^4*b*c)
*d^5 + 3*(5*a^4*b^2*e + 2*a^5*c*e)*d^4)*x^3 + (30*a^5*b*d^4*e + 10*a^6*d^3*
e^2 + 3*(5*a^4*b^2 + 2*a^5*c)*d^5)*x^2 + (6*a^5*b*d^5 + 5*a^6*d^4*e)*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1764 vs. $2(21) = 42$.

time = 0.41, size = 1764, normalized size = 88.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c
*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="fricas")
```

```
[Out] c^6*d^5*x^12 + 6*b*c^5*d^5*x^11 + 3*(5*b^2*c^4 + 2*a*c^5)*d^5*x^10 + 10*(2*
b^3*c^3 + 3*a*b*c^4)*d^5*x^9 + 15*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^5*x^8
 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^5*x^7 + 6*a^5*b*d^5*x + (b^6 +
 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d^5*x^6 + 6*(a*b^5 + 10*a^2*b^3*
c + 10*a^3*b*c^2)*d^5*x^5 + 15*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^5*x^4 +
10*(2*a^3*b^3 + 3*a^4*b*c)*d^5*x^3 + 3*(5*a^4*b^2 + 2*a^5*c)*d^5*x^2 + (c^6
*x^17 + 6*b*c^5*x^16 + 3*(5*b^2*c^4 + 2*a*c^5)*x^15 + 10*(2*b^3*c^3 + 3*a*b
*c^4)*x^14 + 15*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*x^13 + 6*(b^5*c + 10*a*b^
3*c^2 + 10*a^2*b*c^3)*x^12 + 6*a^5*b*x^6 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c
^2 + 20*a^3*c^3)*x^11 + a^6*x^5 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*x
^10 + 15*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*x^9 + 10*(2*a^3*b^3 + 3*a^4*b*c)
*x^8 + 3*(5*a^4*b^2 + 2*a^5*c)*x^7)*e^5 + 5*(c^6*d*x^16 + 6*b*c^5*d*x^15 +
3*(5*b^2*c^4 + 2*a*c^5)*d*x^14 + 10*(2*b^3*c^3 + 3*a*b*c^4)*d*x^13 + 15*(b^
4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d*x^12 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*
c^3)*d*x^11 + 6*a^5*b*d*x^5 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c
^3)*d*x^10 + a^6*d*x^4 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d*x^9 + 15
*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d*x^8 + 10*(2*a^3*b^3 + 3*a^4*b*c)*d*x^7
 + 3*(5*a^4*b^2 + 2*a^5*c)*d*x^6)*e^4 + 10*(c^6*d^2*x^15 + 6*b*c^5*d^2*x^14
 + 3*(5*b^2*c^4 + 2*a*c^5)*d^2*x^13 + 10*(2*b^3*c^3 + 3*a*b*c^4)*d^2*x^12 +
 15*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^2*x^11 + 6*(b^5*c + 10*a*b^3*c^2 +
10*a^2*b*c^3)*d^2*x^10 + 6*a^5*b*d^2*x^4 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c
^2 + 20*a^3*c^3)*d^2*x^9 + a^6*d^2*x^3 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b
*c^2)*d^2*x^8 + 15*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^2*x^7 + 10*(2*a^3*b^
3 + 3*a^4*b*c)*d^2*x^6 + 3*(5*a^4*b^2 + 2*a^5*c)*d^2*x^5)*e^3 + 10*(c^6*d^3
*x^14 + 6*b*c^5*d^3*x^13 + 3*(5*b^2*c^4 + 2*a*c^5)*d^3*x^12 + 10*(2*b^3*c^3
 + 3*a*b*c^4)*d^3*x^11 + 15*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^3*x^10 + 6*
(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^3*x^9 + 6*a^5*b*d^3*x^3 + (b^6 + 30
*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d^3*x^8 + a^6*d^3*x^2 + 6*(a*b^5 +
10*a^2*b^3*c + 10*a^3*b*c^2)*d^3*x^7 + 15*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)
*d^3*x^6 + 10*(2*a^3*b^3 + 3*a^4*b*c)*d^3*x^5 + 3*(5*a^4*b^2 + 2*a^5*c)*d^3
*x^4)*e^2 + 5*(c^6*d^4*x^13 + 6*b*c^5*d^4*x^12 + 3*(5*b^2*c^4 + 2*a*c^5)*d^
4*x^11 + 10*(2*b^3*c^3 + 3*a*b*c^4)*d^4*x^10 + 15*(b^4*c^2 + 4*a*b^2*c^3 +
a^2*c^4)*d^4*x^9 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^4*x^8 + 6*a^5*
b*d^4*x^2 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d^4*x^7 + a^6*
d^4*x + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d^4*x^6 + 15*(a^2*b^4 + 4*a
^3*b^2*c + a^4*c^2)*d^4*x^5 + 10*(2*a^3*b^3 + 3*a^4*b*c)*d^4*x^4 + 3*(5*a^4
*b^2 + 2*a^5*c)*d^4*x^3)*e
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2281 vs. $2(17) = 34$.
time = 0.17, size = 2281, normalized size = 114.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**5*(d*(5*a*e+6*b*d)+(5*a*e**2+17*b*d*e+
12*c*d**2)*x+e*(11*b*e+29*c*d)*x**2+17*c*e**2*x**3),x)

[Out] c**6*e**5*x**17 + x**16*(6*b*c**5*e**5 + 5*c**6*d*e**4) + x**15*(6*a*c**5*e
**5 + 15*b**2*c**4*e**5 + 30*b*c**5*d*e**4 + 10*c**6*d**2*e**3) + x**14*(30
*a*b*c**4*e**5 + 30*a*c**5*d*e**4 + 20*b**3*c**3*e**5 + 75*b**2*c**4*d*e**4
 + 60*b*c**5*d**2*e**3 + 10*c**6*d**3*e**2) + x**13*(15*a**2*c**4*e**5 + 60
*a*b**2*c**3*e**5 + 150*a*b*c**4*d*e**4 + 60*a*c**5*d**2*e**3 + 15*b**4*c**
2*e**5 + 100*b**3*c**3*d*e**4 + 150*b**2*c**4*d**2*e**3 + 60*b*c**5*d**3*e*
*2 + 5*c**6*d**4*e) + x**12*(60*a**2*b*c**3*e**5 + 75*a**2*c**4*d*e**4 + 60
*a*b**3*c**2*e**5 + 300*a*b**2*c**3*d*e**4 + 300*a*b*c**4*d**2*e**3 + 60*a*
c**5*d**3*e**2 + 6*b**5*c*e**5 + 75*b**4*c**2*d*e**4 + 200*b**3*c**3*d**2*e
**3 + 150*b**2*c**4*d**3*e**2 + 30*b*c**5*d**4*e + c**6*d**5) + x**11*(20*a
**3*c**3*e**5 + 90*a**2*b**2*c**2*e**5 + 300*a**2*b*c**3*d*e**4 + 150*a**2*
c**4*d**2*e**3 + 30*a*b**4*c*e**5 + 300*a*b**3*c**2*d*e**4 + 600*a*b**2*c**
3*d**2*e**3 + 300*a*b*c**4*d**3*e**2 + 30*a*c**5*d**4*e + b**6*e**5 + 30*b*
*5*c*d*e**4 + 150*b**4*c**2*d**2*e**3 + 200*b**3*c**3*d**3*e**2 + 75*b**2*c
**4*d**4*e + 6*b*c**5*d**5) + x**10*(60*a**3*b*c**2*e**5 + 100*a**3*c**3*d*
e**4 + 60*a**2*b**3*c*e**5 + 450*a**2*b**2*c**2*d*e**4 + 600*a**2*b*c**3*d*
*2*e**3 + 150*a**2*c**4*d**3*e**2 + 6*a*b**5*e**5 + 150*a*b**4*c*d*e**4 + 6
00*a*b**3*c**2*d**2*e**3 + 600*a*b**2*c**3*d**3*e**2 + 150*a*b*c**4*d**4*e
 + 6*a*c**5*d**5 + 5*b**6*d*e**4 + 60*b**5*c*d**2*e**3 + 150*b**4*c**2*d**3*
e**2 + 100*b**3*c**3*d**4*e + 15*b**2*c**4*d**5) + x**9*(15*a**4*c**2*e**5
 + 60*a**3*b**2*c*e**5 + 300*a**3*b*c**2*d*e**4 + 200*a**3*c**3*d**2*e**3 +
15*a**2*b**4*e**5 + 300*a**2*b**3*c*d*e**4 + 900*a**2*b**2*c**2*d**2*e**3 +
 600*a**2*b*c**3*d**3*e**2 + 75*a**2*c**4*d**4*e + 30*a*b**5*d*e**4 + 300*a
*b**4*c*d**2*e**3 + 600*a*b**3*c**2*d**3*e**2 + 300*a*b**2*c**3*d**4*e + 30
*a*b*c**4*d**5 + 10*b**6*d**2*e**3 + 60*b**5*c*d**3*e**2 + 75*b**4*c**2*d**
4*e + 20*b**3*c**3*d**5) + x**8*(30*a**4*b*c*e**5 + 75*a**4*c**2*d*e**4 + 2
0*a**3*b**3*e**5 + 300*a**3*b**2*c*d*e**4 + 600*a**3*b*c**2*d**2*e**3 + 200
*a**3*c**3*d**3*e**2 + 75*a**2*b**4*d*e**4 + 600*a**2*b**3*c*d**2*e**3 + 90
0*a**2*b**2*c**2*d**3*e**2 + 300*a**2*b*c**3*d**4*e + 15*a**2*c**4*d**5 + 6
0*a*b**5*d**2*e**3 + 300*a*b**4*c*d**3*e**2 + 300*a*b**3*c**2*d**4*e + 60*a
*b**2*c**3*d**5 + 10*b**6*d**3*e**2 + 30*b**5*c*d**4*e + 15*b**4*c**2*d**5)
 + x**7*(6*a**5*c*e**5 + 15*a**4*b**2*e**5 + 150*a**4*b*c*d*e**4 + 150*a**4
*c**2*d**2*e**3 + 100*a**3*b**3*d*e**4 + 600*a**3*b**2*c*d**2*e**3 + 600*a*
*3*b*c**2*d**3*e**2 + 100*a**3*c**3*d**4*e + 150*a**2*b**4*d**2*e**3 + 600*
a**2*b**3*c*d**3*e**2 + 450*a**2*b**2*c**2*d**4*e + 60*a**2*b*c**3*d**5 + 6
0*a*b**5*d**3*e**2 + 150*a*b**4*c*d**4*e + 60*a*b**3*c**2*d**5 + 5*b**6*d**
4*e + 6*b**5*c*d**5) + x**6*(6*a**5*b*e**5 + 30*a**5*c*d*e**4 + 75*a**4*b**
2*d*e**4 + 300*a**4*b*c*d**2*e**3 + 150*a**4*c**2*d**3*e**2 + 200*a**3*b**3
*d**2*e**3 + 600*a**3*b**2*c*d**3*e**2 + 300*a**3*b*c**2*d**4*e + 20*a**3*c
**3*d**5 + 150*a**2*b**4*d**3*e**2 + 300*a**2*b**3*c*d**4*e + 90*a**2*b**2*
c**2*d**5 + 30*a*b**5*d**4*e + 30*a*b**4*c*d**5 + b**6*d**5) + x**5*(a**6*e
**5 + 30*a**5*b*d*e**4 + 60*a**5*c*d**2*e**3 + 150*a**4*b**2*d**2*e**3 + 30
```

```
0*a**4*b*c*d**3*e**2 + 75*a**4*c**2*d**4*e + 200*a**3*b**3*d**3*e**2 + 300*
a**3*b**2*c*d**4*e + 60*a**3*b*c**2*d**5 + 75*a**2*b**4*d**4*e + 60*a**2*b*
*3*c*d**5 + 6*a*b**5*d**5) + x**4*(5*a**6*d*e**4 + 60*a**5*b*d**2*e**3 + 60
*a**5*c*d**3*e**2 + 150*a**4*b**2*d**3*e**2 + 150*a**4*b*c*d**4*e + 15*a**4
*c**2*d**5 + 100*a**3*b**3*d**4*e + 60*a**3*b**2*c*d**5 + 15*a**2*b**4*d**5
) + x**3*(10*a**6*d**2*e**3 + 60*a**5*b*d**3*e**2 + 30*a**5*c*d**4*e + 75*a
**4*b**2*d**4*e + 30*a**4*b*c*d**5 + 20*a**3*b**3*d**5) + x**2*(10*a**6*d**
3*e**2 + 30*a**5*b*d**4*e + 6*a**5*c*d**5 + 15*a**4*b**2*d**5) + x*(5*a**6*
d**4*e + 6*a**5*b*d**5)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2383 vs. $2(21) = 42$.
time = 5.70, size = 2383, normalized size = 119.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c
*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="giac")
```

```
[Out] c^6*x^17*e^5 + 5*c^6*d*x^16*e^4 + 10*c^6*d^2*x^15*e^3 + 10*c^6*d^3*x^14*e^2
 + 5*c^6*d^4*x^13*e + c^6*d^5*x^12 + 6*b*c^5*x^16*e^5 + 30*b*c^5*d*x^15*e^4
 + 60*b*c^5*d^2*x^14*e^3 + 60*b*c^5*d^3*x^13*e^2 + 30*b*c^5*d^4*x^12*e + 6*
b*c^5*d^5*x^11 + 15*b^2*c^4*x^15*e^5 + 6*a*c^5*x^15*e^5 + 75*b^2*c^4*d*x^14
*e^4 + 30*a*c^5*d*x^14*e^4 + 150*b^2*c^4*d^2*x^13*e^3 + 60*a*c^5*d^2*x^13*e
^3 + 150*b^2*c^4*d^3*x^12*e^2 + 60*a*c^5*d^3*x^12*e^2 + 75*b^2*c^4*d^4*x^11
*e + 30*a*c^5*d^4*x^11*e + 15*b^2*c^4*d^5*x^10 + 6*a*c^5*d^5*x^10 + 20*b^3*
c^3*x^14*e^5 + 30*a*b*c^4*x^14*e^5 + 100*b^3*c^3*d*x^13*e^4 + 150*a*b*c^4*d
*x^13*e^4 + 200*b^3*c^3*d^2*x^12*e^3 + 300*a*b*c^4*d^2*x^12*e^3 + 200*b^3*c
^3*d^3*x^11*e^2 + 300*a*b*c^4*d^3*x^11*e^2 + 100*b^3*c^3*d^4*x^10*e + 150*a
*b*c^4*d^4*x^10*e + 20*b^3*c^3*d^5*x^9 + 30*a*b*c^4*d^5*x^9 + 15*b^4*c^2*x^
13*e^5 + 60*a*b^2*c^3*x^13*e^5 + 15*a^2*c^4*x^13*e^5 + 75*b^4*c^2*d*x^12*e^
4 + 300*a*b^2*c^3*d*x^12*e^4 + 75*a^2*c^4*d*x^12*e^4 + 150*b^4*c^2*d^2*x^11
*e^3 + 600*a*b^2*c^3*d^2*x^11*e^3 + 150*a^2*c^4*d^2*x^11*e^3 + 150*b^4*c^2*
d^3*x^10*e^2 + 600*a*b^2*c^3*d^3*x^10*e^2 + 150*a^2*c^4*d^3*x^10*e^2 + 75*b
^4*c^2*d^4*x^9*e + 300*a*b^2*c^3*d^4*x^9*e + 75*a^2*c^4*d^4*x^9*e + 15*b^4*
c^2*d^5*x^8 + 60*a*b^2*c^3*d^5*x^8 + 15*a^2*c^4*d^5*x^8 + 6*b^5*c*x^12*e^5
+ 60*a*b^3*c^2*x^12*e^5 + 60*a^2*b*c^3*x^12*e^5 + 30*b^5*c*d*x^11*e^4 + 300
*a*b^3*c^2*d*x^11*e^4 + 300*a^2*b*c^3*d*x^11*e^4 + 60*b^5*c*d^2*x^10*e^3 +
600*a*b^3*c^2*d^2*x^10*e^3 + 600*a^2*b*c^3*d^2*x^10*e^3 + 60*b^5*c*d^3*x^9*
e^2 + 600*a*b^3*c^2*d^3*x^9*e^2 + 600*a^2*b*c^3*d^3*x^9*e^2 + 30*b^5*c*d^4*
x^8*e + 300*a*b^3*c^2*d^4*x^8*e + 300*a^2*b*c^3*d^4*x^8*e + 6*b^5*c*d^5*x^7
 + 60*a*b^3*c^2*d^5*x^7 + 60*a^2*b*c^3*d^5*x^7 + b^6*x^11*e^5 + 30*a*b^4*c*
x^11*e^5 + 90*a^2*b^2*c^2*x^11*e^5 + 20*a^3*c^3*x^11*e^5 + 5*b^6*d*x^10*e^4
 + 150*a*b^4*c*d*x^10*e^4 + 450*a^2*b^2*c^2*d*x^10*e^4 + 100*a^3*c^3*d*x^10
```

```
*e^4 + 10*b^6*d^2*x^9*e^3 + 300*a*b^4*c*d^2*x^9*e^3 + 900*a^2*b^2*c^2*d^2*x
^9*e^3 + 200*a^3*c^3*d^2*x^9*e^3 + 10*b^6*d^3*x^8*e^2 + 300*a*b^4*c*d^3*x^8
*e^2 + 900*a^2*b^2*c^2*d^3*x^8*e^2 + 200*a^3*c^3*d^3*x^8*e^2 + 5*b^6*d^4*x^
7*e + 150*a*b^4*c*d^4*x^7*e + 450*a^2*b^2*c^2*d^4*x^7*e + 100*a^3*c^3*d^4*x
^7*e + b^6*d^5*x^6 + 30*a*b^4*c*d^5*x^6 + 90*a^2*b^2*c^2*d^5*x^6 + 20*a^3*c
^3*d^5*x^6 + 6*a*b^5*x^10*e^5 + 60*a^2*b^3*c*x^10*e^5 + 60*a^3*b*c^2*x^10*e
^5 + 30*a*b^5*d*x^9*e^4 + 300*a^2*b^3*c*d*x^9*e^4 + 300*a^3*b*c^2*d*x^9*e^4
 + 60*a*b^5*d^2*x^8*e^3 + 600*a^2*b^3*c*d^2*x^8*e^3 + 600*a^3*b*c^2*d^2*x^8
*e^3 + 60*a*b^5*d^3*x^7*e^2 + 600*a^2*b^3*c*d^3*x^7*e^2 + 600*a^3*b*c^2*d^3
*x^7*e^2 + 30*a*b^5*d^4*x^6*e + 300*a^2*b^3*c*d^4*x^6*e + 300*a^3*b*c^2*d^4
*x^6*e + 6*a*b^5*d^5*x^5 + 60*a^2*b^3*c*d^5*x^5 + 60*a^3*b*c^2*d^5*x^5 + 15
*a^2*b^4*x^9*e^5 + 60*a^3*b^2*c*x^9*e^5 + 15*a^4*c^2*x^9*e^5 + 75*a^2*b^4*d
*x^8*e^4 + 300*a^3*b^2*c*d*x^8*e^4 + 75*a^4*c^2*d*x^8*e^4 + 150*a^2*b^4*d^2
*x^7*e^3 + 600*a^3*b^2*c*d^2*x^7*e^3 + 150*a^4*c^2*d^2*x^7*e^3 + 150*a^2*b^
4*d^3*x^6*e^2 + 600*a^3*b^2*c*d^3*x^6*e^2 + 150*a^4*c^2*d^3*x^6*e^2 + 75*a^
2*b^4*d^4*x^5*e + 300*a^3*b^2*c*d^4*x^5*e + 75*a^4*c^2*d^4*x^5*e + 15*a^2*b
^4*d^5*x^4 + 60*a^3*b^2*c*d^5*x^4 + 15*a^4*c^2*d^5*x^4 + 20*a^3*b^3*x^8*e^5
 + 30*a^4*b*c*x^8*e^5 + 100*a^3*b^3*d*x^7*e^4 + 150*a^4*b*c*d*x^7*e^4 + 200
*a^3*b^3*d^2*x^6*e^3 + 300*a^4*b*c*d^2*x^6*e^3 + 200*a^3*b^3*d^3*x^5*e^2 +
300*a^4*b*c*d^3*x^5*e^2 + 100*a^3*b^3*d^4*x^4*e + 150*a^4*b*c*d^4*x^4*e + 2
0*a^3*b^3*d^5*x^3 + 30*a^4*b*c*d^5*x^3 + 15*a^4*b^2*x^7*e^5 + 6*a^5*c*x^7*e
^5 + 75*a^4*b^2*d*x^6*e^4 + 30*a^5*c*d*x^6*e^4 + 150*a^4*b^2*d^2*x^5*e^3 +
60*a^5*c*d^2*x^5*e^3 + 150*a^4*b^2*d^3*x^4*e^2 + 60*a^5*c*d^3*x^4*e^2 + 75*
a^4*b^2*d^4*x^3*e + 30*a^5*c*d^4*x^3*e + 15*a^4*b^2*d^5*x^2 + 6*a^5*c*d^5*x
^2 + 6*a^5*b*x^6*e^5 + 30*a^5*b*d*x^5*e^4 + 60*a^5*b*d^2*x^4*e^3 + 60*a^5*b
*d^3*x^3*e^2 + 30*a^5*b*d^4*x^2*e + 6*a^5*b*d^5*x + a^6*x^5*e^5 + 5*a^6*d*x
^4*e^4 + 10*a^6*d^2*x^3*e^3 + 10*a^6*d^3*x^2*e^2 + 5*a^6*d^4*x*e
```

Mupad [B]
time = 4.87, size = 2026, normalized size = 101.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(a + b*x + c*x^2)^5*(d*(5*a*e + 6*b*d) + x*(5*a*e^2 + 12*c*
d^2 + 17*b*d*e) + e*x^2*(11*b*e + 29*c*d) + 17*c*e^2*x^3),x)
```

```
[Out] x^6*(b^6*d^5 + 6*a^5*b*e^5 + 20*a^3*c^3*d^5 + 75*a^4*b^2*d*e^4 + 90*a^2*b^2
*c^2*d^5 + 150*a^2*b^4*d^3*e^2 + 200*a^3*b^3*d^2*e^3 + 150*a^4*c^2*d^3*e^2
+ 30*a*b^4*c*d^5 + 30*a*b^5*d^4*e + 30*a^5*c*d*e^4 + 300*a^2*b^3*c*d^4*e +
300*a^3*b*c^2*d^4*e + 300*a^4*b*c*d^2*e^3 + 600*a^3*b^2*c*d^3*e^2) + x^11*(
b^6*e^5 + 6*b*c^5*d^5 + 20*a^3*c^3*e^5 + 75*b^2*c^4*d^4*e + 90*a^2*b^2*c^2*
e^5 + 150*a^2*c^4*d^2*e^3 + 200*b^3*c^3*d^3*e^2 + 150*b^4*c^2*d^2*e^3 + 30*
a*b^4*c*e^5 + 30*a*c^5*d^4*e + 30*b^5*c*d*e^4 + 300*a*b*c^4*d^3*e^2 + 300*a
*b^3*c^2*d*e^4 + 300*a^2*b*c^3*d*e^4 + 600*a*b^2*c^3*d^2*e^3) + x^5*(a^6*e^
```

5 + 6*a*b^5*d^5 + 60*a^2*b^3*c*d^5 + 60*a^3*b*c^2*d^5 + 75*a^2*b^4*d^4*e + 75*a^4*c^2*d^4*e + 60*a^5*c*d^2*e^3 + 200*a^3*b^3*d^3*e^2 + 150*a^4*b^2*d^2*e^3 + 30*a^5*b*d*e^4 + 300*a^3*b^2*c*d^4*e + 300*a^4*b*c*d^3*e^2) + x^3*(20*a^3*b^3*d^5 + 10*a^6*d^2*e^3 + 75*a^4*b^2*d^4*e + 60*a^5*b*d^3*e^2 + 30*a^4*b*c*d^5 + 30*a^5*c*d^4*e) + x^12*(c^6*d^5 + 6*b^5*c*e^5 + 60*a*b^3*c^2*e^5 + 60*a^2*b*c^3*e^5 + 60*a*c^5*d^3*e^2 + 75*a^2*c^4*d*e^4 + 75*b^4*c^2*d*e^4 + 150*b^2*c^4*d^3*e^2 + 200*b^3*c^3*d^2*e^3 + 30*b*c^5*d^4*e + 300*a*b*c^4*d^2*e^3 + 300*a*b^2*c^3*d*e^4) + x^7*(6*a^5*c*e^5 + 6*b^5*c*d^5 + 5*b^6*d^4*e + 15*a^4*b^2*e^5 + 60*a*b^3*c^2*d^5 + 60*a^2*b*c^3*d^5 + 60*a*b^5*d^3*e^2 + 100*a^3*b^3*d*e^4 + 100*a^3*c^3*d^4*e + 150*a^2*b^4*d^2*e^3 + 150*a^4*c^2*d^2*e^3 + 150*a*b^4*c*d^4*e + 150*a^4*b*c*d*e^4 + 450*a^2*b^2*c^2*d^4*e + 600*a^2*b^3*c*d^3*e^2 + 600*a^3*b*c^2*d^3*e^2 + 600*a^3*b^2*c*d^2*e^3) + x^10*(6*a*b^5*e^5 + 6*a*c^5*d^5 + 5*b^6*d*e^4 + 15*b^2*c^4*d^5 + 60*a^2*b^3*c*e^5 + 60*a^3*b*c^2*e^5 + 100*a^3*c^3*d*e^4 + 100*b^3*c^3*d^4*e + 60*b^5*c*d^2*e^3 + 150*a^2*c^4*d^3*e^2 + 150*b^4*c^2*d^3*e^2 + 150*a*b*c^4*d^4*e + 150*a*b^4*c*d*e^4 + 600*a*b^2*c^3*d^3*e^2 + 600*a*b^3*c^2*d^2*e^3 + 600*a^2*b*c^3*d^2*e^3 + 450*a^2*b^2*c^2*d*e^4) + x^8*(15*a^2*c^4*d^5 + 20*a^3*b^3*e^5 + 15*b^4*c^2*d^5 + 10*b^6*d^3*e^2 + 60*a*b^2*c^3*d^5 + 60*a*b^5*d^2*e^3 + 75*a^2*b^4*d*e^4 + 75*a^4*c^2*d*e^4 + 200*a^3*c^3*d^3*e^2 + 30*a^4*b*c*e^5 + 30*b^5*c*d^4*e + 900*a^2*b^2*c^2*d^3*e^2 + 300*a*b^3*c^2*d^4*e + 300*a*b^4*c*d^3*e^2 + 300*a^2*b*c^3*d^4*e + 300*a^3*b^2*c*d*e^4 + 600*a^2*b^3*c*d^2*e^3 + 600*a^3*b*c^2*d^2*e^3) + x^9*(15*a^2*b^4*e^5 + 15*a^4*c^2*e^5 + 20*b^3*c^3*d^5 + 10*b^6*d^2*e^3 + 60*a^3*b^2*c*e^5 + 75*a^2*c^4*d^4*e + 75*b^4*c^2*d^4*e + 60*b^5*c*d^3*e^2 + 200*a^3*c^3*d^2*e^3 + 30*a*b*c^4*d^5 + 30*a*b^5*d*e^4 + 900*a^2*b^2*c^2*d^2*e^3 + 300*a*b^2*c^3*d^4*e + 300*a*b^4*c*d^2*e^3 + 300*a^2*b^3*c*d*e^4 + 300*a^3*b*c^2*d*e^4 + 600*a*b^3*c^2*d^3*e^2 + 600*a^2*b*c^3*d^3*e^2) + x^4*(5*a^6*d*e^4 + 15*a^2*b^4*d^5 + 15*a^4*c^2*d^5 + 60*a^3*b^2*c*d^5 + 100*a^3*b^3*d^4*e + 60*a^5*b*d^2*e^3 + 60*a^5*c*d^3*e^2 + 150*a^4*b^2*d^3*e^2 + 150*a^4*b*c*d^4*e) + x^13*(5*c^6*d^4*e + 15*a^2*c^4*e^5 + 15*b^4*c^2*e^5 + 60*a*b^2*c^3*e^5 + 60*a*c^5*d^2*e^3 + 60*b*c^5*d^3*e^2 + 100*b^3*c^3*d*e^4 + 150*b^2*c^4*d^2*e^3 + 150*a*b*c^4*d*e^4) + c^6*e^5*x^17 + a^5*d^4*x*(5*a*e + 6*b*d) + 5*c^3*e^2*x^14*(4*b^3*e^3 + 2*c^3*d^3 + 6*a*b*c*e^3 + 6*a*c^2*d*e^2 + 12*b*c^2*d^2*e + 15*b^2*c*d*e^2) + c^5*e^4*x^16*(6*b*e + 5*c*d) + a^4*d^3*x^2*(10*a^2*e^2 + 15*b^2*d^2 + 6*a*c*d^2 + 30*a*b*d*e) + c^4*e^3*x^15*(15*b^2*e^2 + 10*c^2*d^2 + 6*a*c*e^2 + 30*b*c*d*e)

3.279 $\int \frac{x^2+x^3}{-2+x+x^2} \, dx$

Optimal. Leaf size=26

$$\frac{x^2}{2} + \frac{2}{3}\log(1-x) + \frac{4}{3}\log(2+x)$$

```
[Out] 1/2*x^2+2/3*ln(1-x)+4/3*ln(2+x)
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1607, 814, 646, 31}

$$\frac{x^2}{2} + \frac{2}{3}\log(1-x) + \frac{4}{3}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2 + x^3)/(-2 + x + x^2),x]
```

```
[Out] x^2/2 + (2*Log[1 - x])/3 + (4*Log[2 + x])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2+x^3}{-2+x+x^2}\,dx &= \int \frac{x^2(1+x)}{-2+x+x^2}\,dx\\
&= \int \left(x+\frac{2x}{-2+x+x^2}\right)dx\\
&= \frac{x^2}{2}+2\int \frac{x}{-2+x+x^2}\,dx\\
&= \frac{x^2}{2}+\frac{2}{3}\int \frac{1}{-1+x}\,dx+\frac{4}{3}\int \frac{1}{2+x}\,dx\\
&= \frac{x^2}{2}+\frac{2}{3}\log(1-x)+\frac{4}{3}\log(2+x)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 1.00

$$\frac{x^2}{2}+\frac{2}{3}\log(1-x)+\frac{4}{3}\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2 + x^3)/(-2 + x + x^2),x]
[Out] x^2/2 + (2*Log[1 - x])/3 + (4*Log[2 + x])/3
```

Maple [A]
time = 0.12, size = 19, normalized size = 0.73

method	result	size
default	$\frac{x^2}{2}+\frac{4\ln(x+2)}{3}+\frac{2\ln(-1+x)}{3}$	19
norman	$\frac{x^2}{2}+\frac{4\ln(x+2)}{3}+\frac{2\ln(-1+x)}{3}$	19
risch	$\frac{x^2}{2}+\frac{4\ln(x+2)}{3}+\frac{2\ln(-1+x)}{3}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2)/(x^2+x-2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+4/3*ln(x+2)+2/3*ln(-1+x)
```

Maxima [A]
time = 0.30, size = 18, normalized size = 0.69

$$\frac{1}{2}\,x^2+\frac{4}{3}\,\log\left(x+2\right)+\frac{2}{3}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)/(x^2+x-2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 4/3*log(x + 2) + 2/3*log(x - 1)
```

Fricas [A]
time = 0.37, size = 18, normalized size = 0.69

$$\frac{1}{2}x^2 + \frac{4}{3}\log(x+2) + \frac{2}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)/(x^2+x-2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 4/3*log(x + 2) + 2/3*log(x - 1)
```

Sympy [A]
time = 0.03, size = 20, normalized size = 0.77

$$\frac{x^2}{2} + \frac{2\log(x-1)}{3} + \frac{4\log(x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2)/(x**2+x-2),x)
```

```
[Out] x**2/2 + 2*log(x - 1)/3 + 4*log(x + 2)/3
```

Giac [A]
time = 5.89, size = 20, normalized size = 0.77

$$\frac{1}{2}x^2 + \frac{4}{3}\log(|x+2|) + \frac{2}{3}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)/(x^2+x-2),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 4/3*log(abs(x + 2)) + 2/3*log(abs(x - 1))
```

Mupad [B]
time = 0.05, size = 18, normalized size = 0.69

$$\frac{2\ln(x-1)}{3} + \frac{4\ln(x+2)}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3)/(x + x^2 - 2),x)
```

```
[Out] (2*log(x - 1))/3 + (4*log(x + 2))/3 + x^2/2
```

3.280 $\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=346

$$\frac{(80c^2e-70bcf+63b^2g-64acg)\,x^2\sqrt{a+bx+cx^2}}{240c^3}+\frac{(10cf-9bg)x^3\sqrt{a+bx+cx^2}}{40c^2}+\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

```
[Out] 1/256*(70*b^4*c*f+48*b^2*c^2*(-5*a*f+2*c*d)-32*a*c^3*(-3*a*f+4*c*d)-63*b^5*
g-40*b^3*c*(-7*a*g+2*c*e)+48*a*b*c^2*(-5*a*g+4*c*e))*arctanh(1/2*(2*c*x+b)/
c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/240*(-64*a*c*g+63*b^2*g-70*b*c*f+80
*c^2*e)*x^2*(c*x^2+b*x+a)^(1/2)/c^3+1/40*(-9*b*g+10*c*f)*x^3*(c*x^2+b*x+a)^
(1/2)/c^2+1/5*g*x^4*(c*x^2+b*x+a)^(1/2)/c-1/1920*(1050*b^3*c*f+40*b*c^2*(-5
5*a*f+36*c*d)-945*b^4*g-60*b^2*c*(-49*a*g+20*c*e)+256*a*c^2*(-4*a*g+5*c*e)-
2*c*(480*c^3*d-40*c^2*(9*a*f+10*b*e)-315*b^3*g+14*b*c*(46*a*g+25*b*f))*x)*(
c*x^2+b*x+a)^(1/2)/c^5
```

Rubi [A]

time = 0.47, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1667, 846, 793, 635, 212}

$$\frac{x^2\sqrt{a+bx+cx^2}\,(-64acg+63b^2g-70bcf+80c^2e)}{240c^3}-\frac{\sqrt{a+bx+cx^2}\,(-2cx(-40c^2(9af+10be)+14bc(46ag+25bf)-315b^3g+480c^3d)-60b^2c(20ce-49ag)+40bc^2(36cd-55af)+256ac^2(5ce-4ag)-945b^4g+1050b^3cf)}{1920c^5}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-40b^3c(2ce-7ag)+48b^2c^2(2cd-5af)+48abc^2(4ce-5ag)-32ac^3(4cd-3af)-63b^5g+70b^4cf)}{256c^{11/2}}+\frac{x^3\sqrt{a+bx+cx^2}\,(10cf-9bg)}{40c^2}+\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((80*c^2*e - 70*b*c*f + 63*b^2*g - 64*a*c*g)*x^2*Sqrt[a + b*x + c*x^2])/(24
0*c^3) + ((10*c*f - 9*b*g)*x^3*Sqrt[a + b*x + c*x^2])/(40*c^2) + (g*x^4*Sqr
t[a + b*x + c*x^2])/(5*c) - ((1050*b^3*c*f + 40*b*c^2*(36*c*d - 55*a*f) - 9
45*b^4*g - 60*b^2*c*(20*c*e - 49*a*g) + 256*a*c^2*(5*c*e - 4*a*g) - 2*c*(48
0*c^3*d - 40*c^2*(10*b*e + 9*a*f) - 315*b^3*g + 14*b*c*(25*b*f + 46*a*g))*x
)*Sqrt[a + b*x + c*x^2])/(1920*c^5) + ((70*b^4*c*f + 48*b^2*c^2*(2*c*d - 5*
a*f) - 32*a*c^3*(4*c*d - 3*a*f) - 63*b^5*g - 40*b^3*c*(2*c*e - 7*a*g) + 48*
a*b*c^2*(4*c*e - 5*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(256*c^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \frac{gx^4\sqrt{a+bx+cx^2}}{5c} + \frac{\int \frac{x^2\left(5cd+(5ce-4ag)x+\frac{1}{2}(10cf-9bg)x^2\right)}{\sqrt{a+bx+cx^2}}\,dx}{5c}$$

$$= \frac{(10cf-9bg)x^3\sqrt{a+bx+cx^2}}{40c^2} + \frac{gx^4\sqrt{a+bx+cx^2}}{5c} + \frac{\int \frac{x^2\left(\frac{1}{2}(40c^2d-30acf\cdots}{\cdots}}{\cdots}$$

$$= \frac{\left(80c^2e-70bcf+63b^2g-64acg\right)x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf-9bg)x^3\sqrt{a\cdots}}{40c^2}$$

$$= \frac{\left(80c^2e-70bcf+63b^2g-64acg\right)x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf-9bg)x^3\sqrt{a\cdots}}{40c^2}$$

$$= \frac{\left(80c^2e-70bcf+63b^2g-64acg\right)x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf-9bg)x^3\sqrt{a\cdots}}{40c^2}$$

$$= \frac{\left(80c^2e-70bcf+63b^2g-64acg\right)x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf-9bg)x^3\sqrt{a\cdots}}{40c^2}$$

Mathematica [A]
time = 1.09, size = 282, normalized size = 0.82

$$\frac{2\sqrt{c}\,\sqrt{a+x(b+cx)}\,\left(945b^4g-210b^3c(5f+3gx)+4b^2c(300ce-735ag+7cx(25f+18gx))-8bc^2(-a(275f+161gx)+2c(90d+x(50e+35fx+27gx^2)))+16c^2(64a^2g-ac(80e+x(45f+32gx))+2c^2x(30d+x(20e+3x(5f+4gx))))\right)+15\left(-70b^4cf-48b^2c^2(2cd-5af)+32ac^3(4cd-3af)+63b^5g+40b^3c(2ce-7ag)+48abc^2(-4ce+5ag)\right)\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)}{3840c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(945*b^4*g - 210*b^3*c*(5*f + 3*g*x) + 4*b
^2*c*(300*c*e - 735*a*g + 7*c*x*(25*f + 18*g*x)) - 8*b*c^2*(-(a*(275*f + 16
1*g*x)) + 2*c*(90*d + x*(50*e + 35*f*x + 27*g*x^2))) + 16*c^2*(64*a^2*g - a
*c*(80*e + x*(45*f + 32*g*x)) + 2*c^2*x*(30*d + x*(20*e + 3*x*(5*f + 4*g*x)
)))) + 15*(-70*b^4*c*f - 48*b^2*c^2*(2*c*d - 5*a*f) + 32*a*c^3*(4*c*d - 3*a
*f) + 63*b^5*g + 40*b^3*c*(2*c*e - 7*a*g) + 48*a*b*c^2*(-4*c*e + 5*a*g))*Lo
g[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(3840*c^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1160 vs. $2(320) = 640$.
time = 0.16, size = 1161, normalized size = 3.36 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*(1/5*x^4/c*(c*x^2+b*x+a)^(1/2)-9/10*b/c*(1/4*x^3/c*(c*x^2+b*x+a)^(1/2)-7/
8*b/c*(1/3*x^2/c*(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3
```

```
/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-
2/3*a/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^
2+b*x+a)^(1/2))))-3/4*a/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+
b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2
*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-4/5*a/c*(1/3*x^2/c
*(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x
^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-
1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2/3*a/c*(1/c*(c*
x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
))+f*(1/4*x^3/c*(c*x^2+b*x+a)^(1/2)-7/8*b/c*(1/3*x^2/c*(c*x^2+b*x+a)^(1/2)-
5/6*b/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b
/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2
*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2/3*a/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*
b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-3/4*a/c*(1/2*x/c*(c
*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))))+e*(1/3*x^2/c*(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/2*x/c*(c*x^
2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2)))-2/3*a/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*
x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+d*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1
/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for mo
re deta
```

Fricas [**A**]
time = 0.45, size = 711, normalized size = 2.05

$$\left[\frac{15\left(32\left(3b^2c^2-4ac^3\right)d+2\left(35b^3c-120ab^2c^2+48a^2c^3\right)f-\left(63b^5-280ab^3c+240a^2bc^2\right)g-16\left(5b^2c^2-12abc^3\right)e\right)\sqrt{c}\log\left(-8c^2x^2-8bcx-b^2-4\sqrt{cx^2+bx+a}\left(2cx+b\right)\sqrt{c}-4ac\right)+4\left(384c^5gx^4-1440bc^4d+48\left(10c^5f-9bc^4g\right)x^3-8\left(70bc^4f-\left(63b^2c^3-64ac^4\right)g\right)x^2-50\left(21b^2c^3-44abc^4\right)f+\left(945b^4c-2940ab^2c^2+1024a^2c^3\right)g+2\left(480c^5d+10\left(35b^2c^3-36ac^4\right)f-7\left(45b^3c^2-92abc^3\right)g\right)x+80\left(8c^4x^2-10bc^3x+15b^2c^2-16ac^3\right)e\right)\sqrt{cx^2+bx+a}}{7680c^6}, -\frac{15\left(32\left(3b^2c^2-4ac^3\right)d+2\left(35b^3c-120ab^2c^2+48a^2c^3\right)f-\left(63b^5-280ab^3c+240a^2bc^2\right)g-16\left(5b^2c^2-12abc^3\right)e\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}\left(2cx+b\right)\sqrt{-c}}{2\left(c^2x^2+bcx+ac\right)}\right)-2\left(384c^5gx^4-1440bc^4d+48\left(10c^5f-9bc^4g\right)x^3-8\left(70bc^4f-\left(63b^2c^3-64ac^4\right)g\right)x^2-50\left(21b^2c^3-44abc^4\right)f+\left(945b^4c-2940ab^2c^2+1024a^2c^3\right)g+2\left(480c^5d+10\left(35b^2c^3-36ac^4\right)f-7\left(45b^3c^2-92abc^3\right)g\right)x+80\left(8c^4x^2-10bc^3x+15b^2c^2-16ac^3\right)e\right)\sqrt{cx^2+bx+a}}{3840c^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/7680*(15*(32*(3*b^2*c^3 - 4*a*c^4)*d + 2*(35*b^4*c - 120*a*b^2*c^2 + 48*
a^2*c^3)*f - (63*b^5 - 280*a*b^3*c + 240*a^2*b*c^2)*g - 16*(5*b^3*c^2 - 12*
a*b*c^3)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a
)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(384*c^5*g*x^4 - 1440*b*c^4*d + 48*(10*c
^5*f - 9*b*c^4*g)*x^3 - 8*(70*b*c^4*f - (63*b^2*c^3 - 64*a*c^4)*g)*x^2 - 50
*(21*b^3*c^2 - 44*a*b*c^3)*f + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*
g + 2*(480*c^5*d + 10*(35*b^2*c^3 - 36*a*c^4)*f - 7*(45*b^3*c^2 - 92*a*b*c^
3)*g)*x + 80*(8*c^5*x^2 - 10*b*c^4*x + 15*b^2*c^3 - 16*a*c^4)*e)*sqrt(c*x^2
 + b*x + a))/c^6, -1/3840*(15*(32*(3*b^2*c^3 - 4*a*c^4)*d + 2*(35*b^4*c - 1
20*a*b^2*c^2 + 48*a^2*c^3)*f - (63*b^5 - 280*a*b^3*c + 240*a^2*b*c^2)*g - 1
6*(5*b^3*c^2 - 12*a*b*c^3)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*
c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(384*c^5*g*x^4 - 1440*b*c^4*
d + 48*(10*c^5*f - 9*b*c^4*g)*x^3 - 8*(70*b*c^4*f - (63*b^2*c^3 - 64*a*c^4)
*g)*x^2 - 50*(21*b^3*c^2 - 44*a*b*c^3)*f + (945*b^4*c - 2940*a*b^2*c^2 + 10
24*a^2*c^3)*g + 2*(480*c^5*d + 10*(35*b^2*c^3 - 36*a*c^4)*f - 7*(45*b^3*c^2
 - 92*a*b*c^3)*g)*x + 80*(8*c^5*x^2 - 10*b*c^4*x + 15*b^2*c^3 - 16*a*c^4)*e
)*sqrt(c*x^2 + b*x + a))/c^6]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(x**2*(d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x + c*x**2), x)
```

Giac [A]
time = 4.88, size = 330, normalized size = 0.95

$$\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(\frac{8\,gx}{c}+\frac{10\,c^4f-9\,bc^3g}{c^5}\right)x-\frac{70\,bc^3f-63\,b^2c^2g+64\,ac^3g-80\,c^4e}{c^5}\right)x+\frac{480\,c^4d+350\,b^2c^2f-360\,ac^3f-315\,b^3cg+644\,abc^2g-400\,bc^3e}{c^5}\right)x-\frac{1440\,bc^3d+1050\,b^3cf-2200\,abc^2f-945\,b^4g+2940\,ab^2cg-1024\,a^2c^2g-1200\,b^2c^2e+1280\,ac^3e}{c^5}\right)-\frac{(96\,b^2c^3d-128\,ac^4d+70\,b^4cf-240\,ab^2c^2f+96\,a^2c^3f-63\,b^5g+280\,ab^3cg-240\,a^2bc^2g-80\,b^3c^2e+192\,abc^3e)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{256\,c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*g*x/c + (10*c^4*f - 9*b*c^3*g)/c^5
)*x - (70*b*c^3*f - 63*b^2*c^2*g + 64*a*c^3*g - 80*c^4*e)/c^5)*x + (480*c^4
*d + 350*b^2*c^2*f - 360*a*c^3*f - 315*b^3*c*g + 644*a*b*c^2*g - 400*b*c^3*
e)/c^5)*x - (1440*b*c^3*d + 1050*b^3*c*f - 2200*a*b*c^2*f - 945*b^4*g + 294
0*a*b^2*c*g - 1024*a^2*c^2*g - 1200*b^2*c^2*e + 1280*a*c^3*e)/c^5) - 1/256*
(96*b^2*c^3*d - 128*a*c^4*d + 70*b^4*c*f - 240*a*b^2*c^2*f + 96*a^2*c^3*f -
```

```
 63*b^5*g + 280*a*b^3*c*g - 240*a^2*b*c^2*g - 80*b^3*c^2*e + 192*a*b*c^3*e)
*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(g\,x^3+f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2),x)
[Out] int((x^2*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2), x)
```

3.281 $\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=245

$$\frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2}+\frac{gx^3\sqrt{a+bx+cx^2}}{4c}+\frac{(192c^3d-16c^2(9be+8af)-105b^3g+20bc(6bf+11$$

```
[Out] -1/128*(40*b^3*c*f+32*b*c^2*(-3*a*f+2*c*d)-35*b^4*g-24*b^2*c*(-5*a*g+2*c*e)
+16*a*c^2*(-3*a*g+4*c*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
)/c^(9/2)+1/24*(-7*b*g+8*c*f)*x^2*(c*x^2+b*x+a)^(1/2)/c^2+1/4*g*x^3*(c*x^2+
b*x+a)^(1/2)/c+1/192*(192*c^3*d-16*c^2*(8*a*f+9*b*e)-105*b^3*g+20*b*c*(11*a
*g+6*b*f)+2*c*(-36*a*c*g+35*b^2*g-40*b*c*f+48*c^2*e)*x)*(c*x^2+b*x+a)^(1/2)
/c^4
```

Rubi [A]
time = 0.25, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1667, 793, 635, 212}

$$\frac{\sqrt{a+bx+cx^2}\,(2cx(-36acg+35b^2g-40bcf+48c^2e)-16c^2(8af+9be)+20bc(11ag+6bf)-105b^3g+192c^3d)}{192c^4}-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-24b^2c(2ce-5ag)+32bc^2(2cd-3af)+16ac^2(4ce-3ag)-35b^4g+40b^3cf)}{128c^{9/2}}+\frac{x^2\sqrt{a+bx+cx^2}\,(8cf-7bg)}{24c^2}+\frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((8*c*f - 7*b*g)*x^2*Sqrt[a + b*x + c*x^2])/(24*c^2) + (g*x^3*Sqrt[a + b*x
+ c*x^2])/(4*c) + ((192*c^3*d - 16*c^2*(9*b*e + 8*a*f) - 105*b^3*g + 20*b*c
*(6*b*f + 11*a*g) + 2*c*(48*c^2*e - 40*b*c*f + 35*b^2*g - 36*a*c*g)*x)*Sqrt
[a + b*x + c*x^2])/(192*c^4) - ((40*b^3*c*f + 32*b*c^2*(2*c*d - 3*a*f) - 35
*b^4*g - 24*b^2*c*(2*c*e - 5*a*g) + 16*a*c^2*(4*c*e - 3*a*g))*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{x\left(4cd+(4ce-3ag)x+\frac{1}{2}(8cf-7bg)x^2\right)}{\sqrt{a+bx+cx^2}}\,dx}{4c}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{x\left(12c^2d-8acf+7abg+\frac{1}{4}(\ldots\right.}{\sqrt{a+\ldots}}}{\ldots}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{(192c^3d-16c^2(9be+\ldots}{\ldots}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{(192c^3d-16c^2(9be+\ldots}{\ldots}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{(192c^3d-16c^2(9be+\ldots}{\ldots}$$

Mathematica [A]
time = 0.73, size = 199, normalized size = 0.81

$$\frac{2\sqrt{c}\,\sqrt{a+x(b+cx)}\,\left(-105b^3g+10bc(12bf+22ag+7bgx)-8c^2(18be+16af+10bfx+9agx+7bgx^2)+16c^3(12d+x(6e+4fx+3gx^2))\right)+3\left(40b^3cf+32bc^2(2cd-3af)-35b^4g-24b^2c(2ce-5ag)+16ac^2(4ce-3ag)\right)\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)}{384c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^3*g + 10*b*c*(12*b*f + 22*a*g + 7*
b*g*x) - 8*c^2*(18*b*e + 16*a*f + 10*b*f*x + 9*a*g*x + 7*b*g*x^2) + 16*c^3*
(12*d + x*(6*e + 4*f*x + 3*g*x^2))) + 3*(40*b^3*c*f + 32*b*c^2*(2*c*d - 3*a
*f) - 35*b^4*g - 24*b^2*c*(2*c*e - 5*a*g) + 16*a*c^2*(4*c*e - 3*a*g))*Log[b
 + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(223) = 446$.

time = 0.15, size = 669, normalized size = 2.73

method	result
risch	$\frac{\left(48g\,c^3x^3-56b\,c^2g\,x^2+64c^3f\,x^2-72a\,c^2gx+70b^2cgx-80b\,c^2fx+96c^3ex+220abcg-128a\,c^2f-105b^3g+120b^2cf-144b\,c^2e+192c^3d\right)\sqrt{\ }}{192c^4}$
default	$g\left(\frac{x^3\sqrt{c\,x^2+bx+a}}{4c}-\frac{7b\left(\frac{x^2\sqrt{c\,x^2+bx+a}}{3c}-\frac{5b\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+c}{\sqrt{c}}\right)}{[illegible]}\right)}{4c}\right)}{}\right)}{}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*(1/4*x^3/c*(c*x^2+b*x+a)^(1/2)-7/8*b/c*(1/3*x^2/c*(c*x^2+b*x+a)^(1/2)-5/6
*b/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^
(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2/3*a/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c
^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-3/4*a/c*(1/2*x/c*(c*x^
2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))))+f*(1/3*x^2/c*(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/2*x/c*(c*x^2+b
*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2)))-2/3*a/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))))+e*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*
(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+d*(1/c*(c*x^
2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.41, size = 511, normalized size = 2.09

$$\left[\frac{3\,(64\,bc^3d+8\,(5\,b^3c-12\,abc^2)f-(35\,b^4-120\,ab^2c+48\,a^2c^2)g-16\,(3\,b^2c^2-4\,ac^3)e)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{c}-4\,ac\right)+4\,(48\,c^4gx^3+192\,c^4d+8\,(8\,c^4f-7\,bc^3g)x^2+8\,(15\,b^2c^2-16\,ac^3)f-5\,(21\,b^3c-44\,abc^2)g-2\,(40\,bc^3f-(35\,b^2c^2-36\,ac^3)g)x+48\,(2\,c^4x-3\,bc^3)e)\sqrt{cx^2+bx+a}}{768\,c^5}, \frac{3\,(64\,bc^3d+8\,(5\,b^3c-12\,abc^2)f-(35\,b^4-120\,ab^2c+48\,a^2c^2)g-16\,(3\,b^2c^2-4\,ac^3)e)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)+2\,(48\,c^4gx^3+192\,c^4d+8\,(8\,c^4f-7\,bc^3g)x^2+8\,(15\,b^2c^2-16\,ac^3)f-5\,(21\,b^3c-44\,abc^2)g-2\,(40\,bc^3f-(35\,b^2c^2-36\,ac^3)g)x+48\,(2\,c^4x-3\,bc^3)e)\sqrt{cx^2+bx+a}}{384\,c^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(64*b*c^3*d + 8*(5*b^3*c - 12*a*b*c^2)*f - (35*b^4 - 120*a*b^2*c
+ 48*a^2*c^2)*g - 16*(3*b^2*c^2 - 4*a*c^3)*e)*sqrt(c)*log(-8*c^2*x^2 - 8*b*
c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^
4*g*x^3 + 192*c^4*d + 8*(8*c^4*f - 7*b*c^3*g)*x^2 + 8*(15*b^2*c^2 - 16*a*c^
3)*f - 5*(21*b^3*c - 44*a*b*c^2)*g - 2*(40*b*c^3*f - (35*b^2*c^2 - 36*a*c^3
)*g)*x + 48*(2*c^4*x - 3*b*c^3)*e)*sqrt(c*x^2 + b*x + a))/c^5, 1/384*(3*(64
*b*c^3*d + 8*(5*b^3*c - 12*a*b*c^2)*f - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)
```

```
*g - 16*(3*b^2*c^2 - 4*a*c^3)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*
(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*g*x^3 + 192*c^4*d
 + 8*(8*c^4*f - 7*b*c^3*g)*x^2 + 8*(15*b^2*c^2 - 16*a*c^3)*f - 5*(21*b^3*c
- 44*a*b*c^2)*g - 2*(40*b*c^3*f - (35*b^2*c^2 - 36*a*c^3)*g)*x + 48*(2*c^4*
x - 3*b*c^3)*e)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(x*(d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x + c*x**2), x)
```

Giac [A]
time = 3.91, size = 228, normalized size = 0.93

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6\,gx}{c}+\frac{8\,c^3f-7\,bc^2g}{c^4}\right)x-\frac{40\,bc^2f-35\,b^2cg+36\,ac^2g-48\,c^3e}{c^4}\right)x+\frac{192\,c^3d+120\,b^2cf-128\,ac^2f-105\,b^3g+220\,abcg-144\,bc^2e}{c^4}\right)+\frac{(64\,bc^3d+40\,b^3cf-96\,abc^2f-35\,b^4g+120\,ab^2cg-48\,a^2c^2g-48\,b^2c^2e+64\,ac^3e)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{128\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*g*x/c + (8*c^3*f - 7*b*c^2*g)/c^4)*x -
 (40*b*c^2*f - 35*b^2*c*g + 36*a*c^2*g - 48*c^3*e)/c^4)*x + (192*c^3*d + 12
0*b^2*c*f - 128*a*c^2*f - 105*b^3*g + 220*a*b*c*g - 144*b*c^2*e)/c^4) + 1/1
28*(64*b*c^3*d + 40*b^3*c*f - 96*a*b*c^2*f - 35*b^4*g + 120*a*b^2*c*g - 48*
a^2*c^2*g - 48*b^2*c^2*e + 64*a*c^3*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*sqrt(c) - b))/c^(9/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(g\,x^3+f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((x*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2), x)
```

3.282 $\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}} \, dx$

Optimal. Leaf size=177

$$\frac{(24c^2e-18bcf+15b^2g-16acg)\sqrt{a+bx+cx^2}}{24c^3}+\frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2}+\frac{gx^2\sqrt{a+bx+cx^2}}{3c}+\frac{(16c}{\quad}$$

```
[Out] 1/16*(16*c^3*d-8*c^2*(a*f+b*e)-5*b^3*g+6*b*c*(2*a*g+b*f))*arctanh(1/2*(2*c*
x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1/24*(-16*a*c*g+15*b^2*g-18*b*c*f
+24*c^2*e)*(c*x^2+b*x+a)^(1/2)/c^3+1/12*(-5*b*g+6*c*f)*x*(c*x^2+b*x+a)^(1/2
)/c^2+1/3*g*x^2*(c*x^2+b*x+a)^(1/2)/c
```

Rubi [A]
time = 0.14, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1675, 654, 635, 212}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d)}{16c^{7/2}}+\frac{\sqrt{a+bx+cx^2}(-16acg+15b^2g-18bcf+24c^2e)}{24c^3}+\frac{x\sqrt{a+bx+cx^2}(6cf-5bg)}{12c^2}+\frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((24*c^2*e - 18*b*c*f + 15*b^2*g - 16*a*c*g)*Sqrt[a + b*x + c*x^2])/(24*c^3
) + ((6*c*f - 5*b*g)*x*Sqrt[a + b*x + c*x^2])/(12*c^2) + (g*x^2*Sqrt[a + b*
x + c*x^2])/(3*c) + ((16*c^3*d - 8*c^2*(b*e + a*f) - 5*b^3*g + 6*b*c*(b*f +
 2*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(7/2
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx = \frac{gx^2\sqrt{a+bx+cx^2}}{3c} + \frac{\int \frac{3cd+(3ce-2ag)x+\frac{1}{2}(6cf-5bg)x^2}{\sqrt{a+bx+cx^2}}\,dx}{3c}$$

$$= \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + \frac{gx^2\sqrt{a+bx+cx^2}}{3c} + \frac{\int \frac{\frac{1}{2}(12c^2d-6acf+5abg)+\frac{1}{4}(24}{\sqrt{a+bx}}}{6}$$

$$= \frac{(24c^2e-18bcf+15b^2g-16acg)\sqrt{a+bx+cx^2}}{24c^3} + \frac{(6cf-5bg)x\sqrt{a+bx+c}}{12c^2}$$

$$= \frac{(24c^2e-18bcf+15b^2g-16acg)\sqrt{a+bx+cx^2}}{24c^3} + \frac{(6cf-5bg)x\sqrt{a+bx+c}}{12c^2}$$

$$= \frac{(24c^2e-18bcf+15b^2g-16acg)\sqrt{a+bx+cx^2}}{24c^3} + \frac{(6cf-5bg)x\sqrt{a+bx+c}}{12c^2}$$

Mathematica [A]
time = 0.55, size = 139, normalized size = 0.79

$$\frac{2\sqrt{c}\,\sqrt{a+x(b+cx)}\,(15b^2g-2c(9bf+8ag+5bgx)+4c^2(6e+x(3f+2gx)))+3(-16c^3d+8c^2(be+af)+5b^3g-6bc(bf+2ag))\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)}{48c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^2*g - 2*c*(9*b*f + 8*a*g + 5*b*g*x)
+ 4*c^2*(6*e + x*(3*f + 2*g*x))) + 3*(-16*c^3*d + 8*c^2*(b*e + a*f) + 5*b^3
*g - 6*b*c*(b*f + 2*a*g))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])
/(48*c^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(155) = 310$.
time = 0.15, size = 375, normalized size = 2.12

method	result
risch	$-\frac{(-8g\,c^2x^2+10bcgx-12c^2fx+16acg-15b^2g+18bcf-24c^2e)\sqrt{c\,x^2+bx+a}}{24c^3}+\frac{3\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)abg}{4c^{\frac{5}{2}}}-$
default	$g\left(\frac{x^2\sqrt{c\,x^2+bx+a}}{3c}-\frac{5b\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a}{}\right.}{6c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*(1/3*x^2/c*(c*x^2+b*x+a)^(1/2)-5/6*b/c*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b
/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2/3*
a/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))))+f*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/
2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)
*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+e*(1/c*(c*x^2+b*x+a)^(1/2)-1/
2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+d*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.44, size = 345, normalized size = 1.95

$$\left[-\frac{3\,{\left(16\,c^{3} d - 8\,b c^{2} e + 2\,{\left(3\,b^{2} c - 4\,a c^{2}\right)} f - {\left(5\,b^{3} - 12\,a b c\right)} g\right)} \sqrt{c} \log\left(-8\,c^{2} x^{2} - 8\,b c x - b^{2} + 4\,\sqrt{c x^{2} + b x + a} {\left(2\,c x + b\right)} \sqrt{c} - 4\,a c\right) - 4\,{\left(8\,c^{3} g x^{2} - 18\,b c^{2} f + 24\,c^{3} e + {\left(15\,b^{2} c - 16\,a c^{2}\right)} g + 2\,{\left(6\,c^{3} f - 5\,b c^{2} g\right)} x\right)} \sqrt{c x^{2} + b x + a}}{96\,c^{4}}, -\frac{3\,{\left(16\,c^{3} d - 8\,b c^{2} e + 2\,{\left(3\,b^{2} c - 4\,a c^{2}\right)} f - {\left(5\,b^{3} - 12\,a b c\right)} g\right)} \sqrt{-c} \arctan\left(\frac{\sqrt{c x^{2} + b x + a} {\left(2\,c x + b\right)} \sqrt{-c}}{2\,{\left(c^{2} x^{2} + b c x + a c\right)}}\right) - 2\,{\left(8\,c^{3} g x^{2} - 18\,b c^{2} f + 24\,c^{3} e + {\left(15\,b^{2} c - 16\,a c^{2}\right)} g + 2\,{\left(6\,c^{3} f - 5\,b c^{2} g\right)} x\right)} \sqrt{c x^{2} + b x + a}}{48\,c^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(16*c^3*d - 8*b*c^2*e + 2*(3*b^2*c - 4*a*c^2)*f - (5*b^3 - 12*a*b
*c)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*
c*x + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*g*x^2 - 18*b*c^2*f + 24*c^3*e + (15*b^
2*c - 16*a*c^2)*g + 2*(6*c^3*f - 5*b*c^2*g)*x)*sqrt(c*x^2 + b*x + a))/c^4,
-1/48*(3*(16*c^3*d - 8*b*c^2*e + 2*(3*b^2*c - 4*a*c^2)*f - (5*b^3 - 12*a*b*
c)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x
^2 + b*c*x + a*c)) - 2*(8*c^3*g*x^2 - 18*b*c^2*f + 24*c^3*e + (15*b^2*c - 1
6*a*c^2)*g + 2*(6*c^3*f - 5*b*c^2*g)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^{2} + gx^{3}}{\sqrt{a + bx + cx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x + c*x**2), x)
```

Giac [A]
time = 3.85, size = 149, normalized size = 0.84

$$\frac{1}{24}\,\sqrt{c x^{2} + b x + a} {\left(2\,{\left(\frac{4\,g x}{c} + \frac{6\,c^{2} f - 5\,b c g}{c^{3}}\right)} x - \frac{18\,b c f - 15\,b^{2} g + 16\,a c g - 24\,c^{2} e}{c^{3}}\right)} - \frac{{\left(16\,c^{3} d + 6\,b^{2} c f - 8\,a c^{2} f - 5\,b^{3} g + 12\,a b c g - 8\,b c^{2} e\right)} \log\left({\left| -2\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)} \sqrt{c} - b \right|}\right)}{16\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*(4*g*x/c + (6*c^2*f - 5*b*c*g)/c^3)*x - (18*b
*c*f - 15*b^2*g + 16*a*c*g - 24*c^2*e)/c^3) - 1/16*(16*c^3*d + 6*b^2*c*f -
8*a*c^2*f - 5*b^3*g + 12*a*b*c*g - 8*b*c^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(a + b*x + c*x^2)^(1/2),x)
[Out] int((d + e*x + f*x^2 + g*x^3)/(a + b*x + c*x^2)^(1/2), x)
```

3.283 $\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=155

$$\frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2}+\frac{gx\sqrt{a+bx+cx^2}}{2c}-\frac{d\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}+\frac{(8c^2e+3b^2g-4c(bf$$

[Out] 1/8*(8*c^2*e+3*b^2*g-4*c*(a*g+b*f))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)-d*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(1/2)+1/4*(-3*b*g+4*c*f)*(c*x^2+b*x+a)^(1/2)/c^2+1/2*g*x*(c*x^2+b*x+a)^(1/2)/c

Rubi [A]
time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1667, 857, 635, 212, 738}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(ag+bf)+3b^2g+8c^2e)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{4c^2}-\frac{d\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*x^2 + g*x^3)/(x*Sqrt[a + b*x + c*x^2]),x]

[Out] ((4*c*f - 3*b*g)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (g*x*Sqrt[a + b*x + c*x^2])/(2*c) - (d*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a] + ((8*c^2*e + 3*b^2*g - 4*c*(b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \frac{gx\sqrt{a+bx+cx^2}}{2c}+\frac{\int \frac{2cd+(2ce-ag)x+\frac{1}{2}(4cf-3bg)x^2}{x\sqrt{a+bx+cx^2}}\,dx}{2c}$$

$$= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2}+\frac{gx\sqrt{a+bx+cx^2}}{2c}+\frac{\int \frac{2c^2d+\frac{1}{4}(8c^2e+3b^2g-4c(bf+ag))x}{x\sqrt{a+bx+cx^2}}}{2c^2}$$

$$= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2}+\frac{gx\sqrt{a+bx+cx^2}}{2c}+d\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx+$$

$$= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2}+\frac{gx\sqrt{a+bx+cx^2}}{2c}-(2d)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,\right.$$

$$= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2}+\frac{gx\sqrt{a+bx+cx^2}}{2c}-\frac{d\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+}}\right.}{\sqrt{a}}$$

Mathematica [A]

time = 0.64, size = 135, normalized size = 0.87

$$\frac{1}{8}\left(\frac{2(4cf-3bg+2cgx)\sqrt{a+x(b+cx)}}{c^2}+\frac{16d\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{(-8c^2e-3b^2g+4c(bf+ag))\log\left(c^2\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)}{c^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((2*(4*c*f - 3*b*g + 2*c*g*x)*Sqrt[a + x*(b + c*x)])/c^2 + (16*d*ArcTanh[(S
qrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a] + ((-8*c^2*e - 3*b^2*g
+ 4*c*(b*f + a*g))*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/
c^(5/2))/8
```

Maple [A]
time = 0.15, size = 223, normalized size = 1.44

method	result
default	$g\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+b}\right.}{2c^{\frac{3}{2}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+f*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))/c^(1/2)-d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/
x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for mo
re deta

Fricas [A]
time = 1.80, size = 739, normalized size = 4.77

$$\left[\frac{8\sqrt{a}c^3d\log\left(-\frac{8abx+(b^2+4ac)x^2-4\sqrt{cx^2+bx+a}(bx+2a)\sqrt{a}+8a^2}{x^2}\right)-(4abcf-8ac^2e-(3ab^2-4a^2c)g)\sqrt{c}\log\left(-8c^2x^2-8bcx-b^2-4\sqrt{cx^2+bx+a}(2cx+b)\sqrt{c}-4ac\right)+4(2ac^2gx+4ac^2f-3abcg)\sqrt{cx^2+bx+a}}{16ac^3}, \frac{4\sqrt{a}c^3d\log\left(-\frac{8abx+(b^2+4ac)x^2-4\sqrt{cx^2+bx+a}(bx+2a)\sqrt{a}+8a^2}{x^2}\right)+(4abcf-8ac^2e-(3ab^2-4a^2c)g)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2cx+b)\sqrt{-c}}{2(c^2x^2+bcx+ac)}\right)+2(2ac^2gx+4ac^2f-3abcg)\sqrt{cx^2+bx+a}}{8ac^3}, \frac{16\sqrt{-a}c^3d\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2a)\sqrt{-a}}{2(acx^2+abx+a^2)}\right)-(4abcf-8ac^2e-(3ab^2-4a^2c)g)\sqrt{c}\log\left(-8c^2x^2-8bcx-b^2-4\sqrt{cx^2+bx+a}(2cx+b)\sqrt{c}-4ac\right)+4(2ac^2gx+4ac^2f-3abcg)\sqrt{cx^2+bx+a}}{16ac^3}, \frac{8\sqrt{-a}c^3d\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2a)\sqrt{-a}}{2(acx^2+abx+a^2)}\right)+(4abcf-8ac^2e-(3ab^2-4a^2c)g)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2cx+b)\sqrt{-c}}{2(c^2x^2+bcx+ac)}\right)+2(2ac^2gx+4ac^2f-3abcg)\sqrt{cx^2+bx+a}}{8ac^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*sqrt(a)*c^3*d*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b
*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - (4*a*b*c*f - 8*a*c^2*e - (3*a*b
^2 - 4*a^2*c)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*
x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*
g)*sqrt(c*x^2 + b*x + a))/(a*c^3), 1/8*(4*sqrt(a)*c^3*d*log(-(8*a*b*x + (b^
2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2)
+ (4*a*b*c*f - 8*a*c^2*e - (3*a*b^2 - 4*a^2*c)*g)*sqrt(-c)*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(2*a*c^2
*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2 + b*x + a))/(a*c^3), 1/16*(16*sqrt
(-a)*c^3*d*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 +
 a*b*x + a^2)) - (4*a*b*c*f - 8*a*c^2*e - (3*a*b^2 - 4*a^2*c)*g)*sqrt(c)*lo
g(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c)
- 4*a*c) + 4*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2 + b*x + a))/(
a*c^3), 1/8*(8*sqrt(-a)*c^3*d*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*
sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + (4*a*b*c*f - 8*a*c^2*e - (3*a*b^2 - 4*a
^2*c)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^
2*x^2 + b*c*x + a*c)) + 2*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2
+ b*x + a))/(a*c^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{x\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2 + g*x^3)/(x*(a + b*x + c*x^2)^(1/2)), x)
```

3.284 $\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=139

$$\frac{g\sqrt{a+bx+cx^2}}{c}-\frac{d\sqrt{a+bx+cx^2}}{ax}+\frac{(bd-2ae)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}+\frac{(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}$$

```
[Out] 1/2*(-2*a*e+b*d)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)
+1/2*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2
)+g*(c*x^2+b*x+a)^(1/2)/c-d*(c*x^2+b*x+a)^(1/2)/a/x
```

Rubi [A]
time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1664, 1667, 857, 635, 212, 738}

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}+\frac{(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}-\frac{d\sqrt{a+bx+cx^2}}{ax}+\frac{g\sqrt{a+bx+cx^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g*Sqrt[a + b*x + c*x^2])/c - (d*Sqrt[a + b*x + c*x^2])/(a*x) + ((b*d - 2*a
*e)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)) + (
(2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^
(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{ax} - \frac{\int \frac{\frac{1}{2}(bd-2ae)-afx-agx^2}{x\sqrt{a+bx+cx^2}}\,dx}{a}$$

$$= \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} - \frac{\int \frac{\frac{1}{2}c(bd-2ae)-\frac{1}{2}a(2cf-bg)x}{x\sqrt{a+bx+cx^2}}\,dx}{ac}$$

$$= \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} - \frac{(bd-2ae)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2a} + \frac{(2c$$

$$= \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} + \frac{(bd-2ae)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a}{\sqrt{a+b}}\right.}{a}$$

$$= \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} + \frac{(bd-2ae)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx}}\right.}{2a^{3/2}}$$

Mathematica [A]
time = 0.75, size = 122, normalized size = 0.88

$$\frac{(-cd+agx)\sqrt{a+x(b+cx)}}{acx} + \frac{(bd-2ae)\tanh^{-1}\left(\frac{-\sqrt{c}\,x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{(-2cf+bg)\log\left(c\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((-(c*d) + a*g*x)*Sqrt[a + x*(b + c*x)])/(a*c*x) + ((b*d - 2*a*e)*ArcTanh[(
-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/a^(3/2) + ((-2*c*f + b*g)*L
og[c*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(2*c^(3/2))
```

Maple [A]
time = 0.16, size = 175, normalized size = 1.26

method	result
risch	$-\frac{d\sqrt{c\,x^2+bx+a}}{ax} + \frac{g\sqrt{c\,x^2+bx+a}}{c} - \frac{gb\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}} + \frac{f\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+\right.}{\sqrt{c}}$
default	$g\left(\frac{\sqrt{c\,x^2+bx+a}}{c} - \frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right) + \frac{f\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}} + d\left(-\frac{\sqrt{c\,x^2}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2)))+f*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+d*(-1/a/x*
(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2)
)/x))-e/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 1.46, size = 711, normalized size = 5.12

$$\left[-\frac{(2a^2cf - a^2bg)\sqrt{c}\,x\log\left(-8c^2x^2 - 8bcx - b^2 + 4\sqrt{cx^2+bx+a}(2cx+b)\sqrt{c} - 4ac\right) + (bc^2dx - 2ac^2xe)\sqrt{a}\log\left(-\frac{8abx + (b^2+4ac)x^2 - 4\sqrt{cx^2+bx+a}(bx+2a)\sqrt{a} + 8a^2}{x^2}\right) - 4(a^2cgx - ac^2d)\sqrt{cx^2+bx+a}}{4a^2c^2x}, -\frac{2(2a^2cf - a^2bg)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^2+bx+a}(2cx+b)\sqrt{-c}}{2(c^2x^2+bcx+ac)}\right) + (bc^2dx - 2ac^2xe)\sqrt{a}\log\left(-\frac{8abx + (b^2+4ac)x^2 - 4\sqrt{cx^2+bx+a}(bx+2a)\sqrt{a} + 8a^2}{x^2}\right) - 4(a^2cgx - ac^2d)\sqrt{cx^2+bx+a}}{4a^2c^2x}, -\frac{(2a^2cf - a^2bg)\sqrt{c}\,x\log\left(-8c^2x^2 - 8bcx - b^2 + 4\sqrt{cx^2+bx+a}(2cx+b)\sqrt{c} - 4ac\right) + 2(bc^2dx - 2ac^2xe)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2a)\sqrt{-a}}{2(acx^2+abx+a^2)}\right) - 4(a^2cgx - ac^2d)\sqrt{cx^2+bx+a}}{4a^2c^2x}, -\frac{(2a^2cf - a^2bg)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^2+bx+a}(2cx+b)\sqrt{-c}}{2(c^2x^2+bcx+ac)}\right) + (bc^2dx - 2ac^2xe)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2a)\sqrt{-a}}{2(acx^2+abx+a^2)}\right) - 2(a^2cgx - ac^2d)\sqrt{cx^2+bx+a}}{2a^2c^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/4*((2*a^2*c*f - a^2*b*g)*sqrt(c)*x*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*s
qrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + (b*c^2*d*x - 2*a*c^2*x*
e)*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x
 + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^2 + b*x +
a))/(a^2*c^2*x), -1/4*(2*(2*a^2*c*f - a^2*b*g)*sqrt(-c)*x*arctan(1/2*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + (b*c^2*d*x
- 2*a*c^2*x*e)*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b
*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a^2*c*g*x - a*c^2*d)*sqrt(c*
x^2 + b*x + a))/(a^2*c^2*x), -1/4*((2*a^2*c*f - a^2*b*g)*sqrt(c)*x*log(-8*c
^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*
c) + 2*(b*c^2*d*x - 2*a*c^2*x*e)*sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*
(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 4*(a^2*c*g*x - a*c^2*d)*sqr
t(c*x^2 + b*x + a))/(a^2*c^2*x), -1/2*((2*a^2*c*f - a^2*b*g)*sqrt(-c)*x*arc
tan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c))
 + (b*c^2*d*x - 2*a*c^2*x*e)*sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x
 + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(a^2*c*g*x - a*c^2*d)*sqrt(c*
x^2 + b*x + a))/(a^2*c^2*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2 + gx^3}{x^2\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**2*sqrt(a + b*x + c*x**2)), x)
```

Giac [A]
time = 3.79, size = 171, normalized size = 1.23

$$\frac{\sqrt{cx^2 + bx + a}\, g}{c} - \frac{(bd - 2\,ae)\arctan\left(-\frac{\sqrt{c}\,x - \sqrt{cx^2 + bx + a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a} - \frac{(2\,cf - bg)\log\left(\left|2\left(\sqrt{c}\,x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} + b\right|\right)}{2\,c^{\frac{3}{2}}} + \frac{\left(\sqrt{c}\,x - \sqrt{cx^2 + bx + a}\right)bd + 2\,a\sqrt{c}\,d}{\left(\left(\sqrt{c}\,x - \sqrt{cx^2 + bx + a}\right)^2 - a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x^2 + b*x + a)*g/c - (b*d - 2*a*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))/sqrt(-a))/(sqrt(-a)*a) - 1/2*(2*c*f - b*g)*log(abs(2*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(3/2) + ((sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*b*d + 2*a*sqrt(c)*d)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a
)*a)
```

Mupad [B]
time = 4.46, size = 166, normalized size = 1.19

$$\frac{g\,\sqrt{c\,x^2 + b\,x + a}}{c} - \frac{e\,\ln\left(\frac{b}{2} + \frac{a}{x} + \frac{\sqrt{a}\,\sqrt{c\,x^2 + b\,x + a}}{x}\right)}{\sqrt{a}} + \frac{f\,\ln\left(\frac{\frac{b}{2} + c\,x}{\sqrt{c}} + \sqrt{c\,x^2 + b\,x + a}\right)}{\sqrt{c}} - \frac{b\,g\,\ln\left(\frac{\frac{b}{2} + c\,x}{\sqrt{c}} + \sqrt{c\,x^2 + b\,x + a}\right)}{2\,c^{3/2}} - \frac{d\,\sqrt{c\,x^2 + b\,x + a}}{a\,x} + \frac{b\,d\,\mathrm{atanh}\left(\frac{a + \frac{b\,x}{2}}{\sqrt{a}\,\sqrt{c\,x^2 + b\,x + a}}\right)}{2\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^2*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] (g*(a + b*x + c*x^2)^(1/2))/c - (e*log(b/2 + a/x + (a^(1/2)*(a + b*x + c*x^
2)^(1/2))/x))/a^(1/2) + (f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2
)))/c^(1/2) - (b*g*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/(2*c
^(3/2)) - (d*(a + b*x + c*x^2)^(1/2))/(a*x) + (b*d*atanh((a + (b*x)/2)/(a^(
1/2)*(a + b*x + c*x^2)^(1/2))))/(2*a^(3/2))
```

3.285 $\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=159

$$-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}+\frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x}-\frac{(3b^2d-4acd-4abe+8a^2f)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+c}}\right.}{8a^{5/2}}$$

[Out] -1/8*(8*a^2*f-4*a*b*e-4*a*c*d+3*b^2*d)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5/2)+g*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/2*d*(c*x^2+b*x+a)^(1/2)/a/x^2+1/4*(-4*a*e+3*b*d)*(c*x^2+b*x+a)^(1/2)/a^2/x

Rubi [A]
time = 0.14, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1664, 857, 635, 212, 738}

$$\frac{\sqrt{a+bx+cx^2}\,(3bd-4ae)}{4a^2x}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)(8a^2f-4abe-4acd+3b^2d)}{8a^{5/2}}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}+\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^3*Sqrt[a + b*x + c*x^2]),x]

[Out] -1/2*(d*Sqrt[a + b*x + c*x^2])/(a*x^2) + ((3*b*d - 4*a*e)*Sqrt[a + b*x + c*x^2])/(4*a^2*x) - ((3*b^2*d - 4*a*c*d - 4*a*b*e + 8*a^2*f)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(5/2)) + (g*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c]

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} - \frac{\int \frac{\frac{1}{2}(3bd-4ae)+(cd-2af)x-2agx^2}{x^2\sqrt{a+bx+cx^2}}\,dx}{2a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} + \frac{\int \frac{\frac{1}{4}\left(3b^2d-4abe-4a(cd-2af)\right)+2a^2}{x\sqrt{a+bx+cx^2}}}{2a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} + \frac{(3b^2d-4acd-4abe+8a^2}{8a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} - \frac{(3b^2d-4acd-4abe+8a^2}{}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} - \frac{(3b^2d-4acd-4abe+8a^2}{}$$

Mathematica [A]

time = 1.21, size = 170, normalized size = 1.07

$$\frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}\,(3bdx-2a(d+2ex))}{x^2}+(3b^2d+8a^2f)\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)+4a(cd+be)\tanh^{-1}\left(\frac{-\sqrt{c}\,x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)-\frac{4a^{5/2}g\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{\sqrt{c}}}{4a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(3*b*d*x - 2*a*(d + 2*e*x)))/x^2 + (3*b^2*d
 + 8*a^2*f)*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 4*a*(c*d
 + b*e)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]] - (4*a^(5/2
)*g*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/Sqrt[c])/(4*a^(5/2))
```

Maple [A]
time = 0.16, size = 245, normalized size = 1.54

method	result
risch	$-\frac{\sqrt{c\,x^{2}+bx+a}\,(4aex-3bdx+2ad)}{4a^{2}x^{2}}+\frac{g\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}-\frac{f\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{\sqrt{a}}$
default	$\frac{g\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}+d\left(-\frac{\sqrt{c\,x^{2}+bx+a}}{2a\,x^{2}}-\frac{3b\left(-\frac{\sqrt{c\,x^{2}+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+d*(-1/2/a/x^2*(c*x^2+
b*x+a)^(1/2)-3/4*b/a*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+
2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))+1/2*c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x
^2+b*x+a)^(1/2))/x))+e*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*
x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))-f/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2
+b*x+a)^(1/2))/x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 1.99, size = 803, normalized size = 5.05

$$\left[\frac{8a^3\sqrt{c}gx^2\log\left(-8c^2x^2-8bcx-b^2-4\sqrt{cx^2+bx+a}(2cx+b)\sqrt{c}-4ac\right)+\left(4abcx^2e-\left(8a^2cf+\left(3b^2c-4ac^2\right)d\right)x^2\right)\sqrt{a}\log\left(-\frac{8abx+\left(b^2+4ac\right)x^2+4\sqrt{cx^2+bx+a}(bx+2a)\sqrt{a}+8a^2}{x^2}\right)+4\left(3abcdx-4a^2cxe-2a^2cd\right)\sqrt{cx^2+bx+a}}{16a^3cx^2}, -\frac{16a^3\sqrt{-c}gx^2\arctan\left(\frac{\sqrt{cx^2+bx+a}(2cx+b)\sqrt{-c}}{2(c^2x^2+bcx+ac)}\right)-\left(4abcx^2e-\left(8a^2cf+\left(3b^2c-4ac^2\right)d\right)x^2\right)\sqrt{a}\log\left(-\frac{8abx+\left(b^2+4ac\right)x^2+4\sqrt{cx^2+bx+a}(bx+2a)\sqrt{a}+8a^2}{x^2}\right)-4\left(3abcdx-4a^2cxe-2a^2cd\right)\sqrt{cx^2+bx+a}}{16a^3cx^2}, \frac{4a^3\sqrt{c}gx^2\log\left(-8c^2x^2-8bcx-b^2-4\sqrt{cx^2+bx+a}(2cx+b)\sqrt{c}-4ac\right)-\left(4abcx^2e-\left(8a^2cf+\left(3b^2c-4ac^2\right)d\right)x^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2a)\sqrt{-a}}{2(acx^2+abx+a^2)}\right)+2\left(3abcdx-4a^2cxe-2a^2cd\right)\sqrt{cx^2+bx+a}}{8a^3cx^2}, -\frac{8a^3\sqrt{-c}gx^2\arctan\left(\frac{\sqrt{cx^2+bx+a}(2cx+b)\sqrt{-c}}{2(c^2x^2+bcx+ac)}\right)+\left(4abcx^2e-\left(8a^2cf+\left(3b^2c-4ac^2\right)d\right)x^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2a)\sqrt{-a}}{2(acx^2+abx+a^2)}\right)-2\left(3abcdx-4a^2cxe-2a^2cd\right)\sqrt{cx^2+bx+a}}{8a^3cx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*(8*a^3*sqrt(c)*g*x^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 +
b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + (4*a*b*c*x^2*e - (8*a^2*c*f + (3*b^
2*c - 4*a*c^2)*d)*x^2)*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c
*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(3*a*b*c*d*x - 4*a^2*
c*x*e - 2*a^2*c*d)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^2), -1/16*(16*a^3*sqrt(-
c)*g*x^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b
*c*x + a*c)) - (4*a*b*c*x^2*e - (8*a^2*c*f + (3*b^2*c - 4*a*c^2)*d)*x^2)*sq
rt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*
a)*sqrt(a) + 8*a^2)/x^2) - 4*(3*a*b*c*d*x - 4*a^2*c*x*e - 2*a^2*c*d)*sqrt(c
*x^2 + b*x + a))/(a^3*c*x^2), 1/8*(4*a^3*sqrt(c)*g*x^2*log(-8*c^2*x^2 - 8*b
*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (4*a*b*
c*x^2*e - (8*a^2*c*f + (3*b^2*c - 4*a*c^2)*d)*x^2)*sqrt(-a)*arctan(1/2*sqrt
(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(3*a*b*
c*d*x - 4*a^2*c*x*e - 2*a^2*c*d)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^2), -1/8*(
8*a^3*sqrt(-c)*g*x^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/
(c^2*x^2 + b*c*x + a*c)) + (4*a*b*c*x^2*e - (8*a^2*c*f + (3*b^2*c - 4*a*c^2
)*d)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a
*c*x^2 + a*b*x + a^2)) - 2*(3*a*b*c*d*x - 4*a^2*c*x*e - 2*a^2*c*d)*sqrt(c*x
^2 + b*x + a))/(a^3*c*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2 + gx^3}{x^3\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**3*sqrt(a + b*x + c*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(135) = 270.
time = 3.96, size = 352, normalized size = 2.21

$$-\frac{g\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)c-b\sqrt{c}\right|\right)}{\sqrt{c}}+\frac{\left(3b^2d-4acd+8a^2f-4abe\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}\,a^2}-\frac{3\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3b^2d-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3acd-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3abe-8\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2a^2\sqrt{c}e-5\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)ab^2d-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)a^2cd+4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)a^2be-8a^2b\sqrt{c}d+8a^3\sqrt{c}e}{4\left(\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2-a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -g*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*c - b*sqrt(c)))/sqrt(c) +
 1/4*(3*b^2*d - 4*a*c*d + 8*a^2*f - 4*a*b*e)*arctan(-(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^2) - 1/4*(3*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*b^2*d - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d - 4*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*e - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*a^2*sqrt(c)*e - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d - 4*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^2*b*e - 8*a^2*b*sqrt(c)*d + 8*a^3*sqrt(c)*e)/(((sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^2 - a)^2*a^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^3*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2 + g*x^3)/(x^3*(a + b*x + c*x^2)^(1/2)), x)
```

3.286 $\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=186

$$-\frac{d\sqrt{a+bx+cx^2}}{3ax^3}+\frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2}-\frac{(15b^2d-16acd-18abe+24a^2f)\sqrt{a+bx+cx^2}}{24a^3x}+\frac{(5b^3d}{}$$

```
[Out] 1/16*(5*b^3*d-6*a*b^2*e-4*a*b*(-2*a*f+3*c*d)+8*a^2*(-2*a*g+c*e))*arctanh(1/
2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(7/2)-1/3*d*(c*x^2+b*x+a)^(1/2)/
a/x^3+1/12*(-6*a*e+5*b*d)*(c*x^2+b*x+a)^(1/2)/a^2/x^2-1/24*(24*a^2*f-18*a*b
*e-16*a*c*d+15*b^2*d)*(c*x^2+b*x+a)^(1/2)/a^3/x
```

Rubi [A]
time = 0.19, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1664, 820, 738, 212}

$$\frac{\sqrt{a+bx+cx^2}\,(5bd-6ae)}{12a^2x^2}+\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(8a^2(ce-2ag)-6ab^2e-4ab(3cd-2af)+5b^3d\right)}{16a^{7/2}}-\frac{\sqrt{a+bx+cx^2}\,(24a^2f-18abe-16acd+15b^2d)}{24a^3x}-\frac{d\sqrt{a+bx+cx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^4*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/3*(d*Sqrt[a + b*x + c*x^2])/(a*x^3) + ((5*b*d - 6*a*e)*Sqrt[a + b*x + c*
x^2])/(12*a^2*x^2) - ((15*b^2*d - 16*a*c*d - 18*a*b*e + 24*a^2*f)*Sqrt[a +
b*x + c*x^2])/(24*a^3*x) + ((5*b^3*d - 6*a*b^2*e - 4*a*b*(3*c*d - 2*a*f) +
8*a^2*(c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]
)/(16*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
```

```
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx &= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3}-\frac{\int \frac{\frac{1}{2}(5bd-6ae)+(2cd-3af)x-3agx^2}{x^3\sqrt{a+bx+cx^2}}\,dx}{3a}\\ &= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3}+\frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2}+\frac{\int \frac{\frac{1}{4}(15b^2d-16acd-18abe+24a^2f}{x^2\sqrt{a+bx}}}{6a^2}\\ &= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3}+\frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2}-\frac{(15b^2d-16acd-18abe}{24a}\\ &= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3}+\frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2}-\frac{(15b^2d-16acd-18abe}{24a}\\ &= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3}+\frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2}-\frac{(15b^2d-16acd-18abe}{24a}\end{aligned}$$

Mathematica [A]

time = 0.97, size = 178, normalized size = 0.96

$$\frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}\left(-15b^2dx^2+2ax(5bd+8cdx+9bex)-4a^2(2d+3x(e+2fx))\right)}{x^3}+3(-5b^3d+16a^3g)\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)+6a(-6bcd-3b^2e+4ace+4abf)\tanh^{-1}\left(\frac{-\sqrt{c}\,x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{24a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^4*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(-15*b^2*d*x^2 + 2*a*x*(5*b*d + 8*c*d*x + 9
*b*e*x) - 4*a^2*(2*d + 3*x*(e + 2*f*x))))/x^3 + 3*(-5*b^3*d + 16*a^3*g)*Arc
Tanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 6*a*(-6*b*c*d - 3*b^2*e
 + 4*a*c*e + 4*a*b*f)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a
]])/(24*a^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(164) = 328$.
time = 0.16, size = 424, normalized size = 2.28

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,\left(24a^2 f\,x^2-18abe\,x^2-16acd\,x^2+15b^2 d\,x^2+12a^2 ex-10abdx+8d\,a^2\right)}{24a^3x^3}-\frac{g\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx}}{x}\right.}{\sqrt{a}}$
default	$e\left(-\frac{\sqrt{c\,x^2+bx+a}}{2a\,x^2}-\frac{3b\left(-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}+\frac{c\ln\left(\frac{2a+bx+2\sqrt{a}}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/2/a/x^2*(c*x^2+b*x+a)^(1/2)-3/4*b/a*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b
/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))+1/2*c/a^(3/2)*ln((2
*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))+f*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2
*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))+d*(-1/3/a/x^3*(c*
x^2+b*x+a)^(1/2)-5/6*b/a*(-1/2/a/x^2*(c*x^2+b*x+a)^(1/2)-3/4*b/a*(-1/a/x*(c
*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/
x))+1/2*c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))-2/3*c/a*(-
1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)
^(1/2))/x)))-g/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 2.10, size = 385, normalized size = 2.07

$$\left[-\frac{3\,{\left(2\,{\left(3\,a b^{2} - 4\,a^{2} c\right)} x^{3} e - {\left(8\,a^{2} b f - 16\,a^{3} g + {\left(5\,b^{3} - 12\,a b c\right)} d\right)} x^{3}\right)} \sqrt{a} \log\left(-\frac{8\,a b x + {\left(b^{2} + 4\,a c\right)} x^{2} + 4\,\sqrt{c x^{2} + b x + a} {\left(b x + 2\,a\right)} \sqrt{a} + 8\,a^{2}}{x^{2}}\right) - 4\,{\left(10\,a^{2} b d x - 8\,a^{3} d - {\left(24\,a^{3} f + {\left(15\,a b^{2} - 16\,a^{2} c\right)} d\right)} x^{2} + 6\,{\left(3\,a^{2} b x^{2} - 2\,a^{3} x\right)} e\right)} \sqrt{c x^{2} + b x + a}}{96\,a^{4} x^{3}}, \frac{3\,{\left(2\,{\left(3\,a b^{2} - 4\,a^{2} c\right)} x^{3} e - {\left(8\,a^{2} b f - 16\,a^{3} g + {\left(5\,b^{3} - 12\,a b c\right)} d\right)} x^{3}\right)} \sqrt{-a} \arctan\left(\frac{\sqrt{c x^{2} + b x + a} {\left(b x + 2\,a\right)} \sqrt{-a}}{2\,{\left(a c x^{2} + a b x + a^{2}\right)}}\right) + 2\,{\left(10\,a^{2} b d x - 8\,a^{3} d - {\left(24\,a^{3} f + {\left(15\,a b^{2} - 16\,a^{2} c\right)} d\right)} x^{2} + 6\,{\left(3\,a^{2} b x^{2} - 2\,a^{3} x\right)} e\right)} \sqrt{c x^{2} + b x + a}}{48\,a^{4} x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/96*(3*(2*(3*a*b^2 - 4*a^2*c)*x^3*e - (8*a^2*b*f - 16*a^3*g + (5*b^3 - 1
2*a*b*c)*d)*x^3)*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 +
 b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(10*a^2*b*d*x - 8*a^3*d - (
24*a^3*f + (15*a*b^2 - 16*a^2*c)*d)*x^2 + 6*(3*a^2*b*x^2 - 2*a^3*x)*e)*sqrt
(c*x^2 + b*x + a))/(a^4*x^3), 1/48*(3*(2*(3*a*b^2 - 4*a^2*c)*x^3*e - (8*a^2
*b*f - 16*a^3*g + (5*b^3 - 12*a*b*c)*d)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^2
 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(10*a^2*b*d*x
 - 8*a^3*d - (24*a^3*f + (15*a*b^2 - 16*a^2*c)*d)*x^2 + 6*(3*a^2*b*x^2 - 2*
a^3*x)*e)*sqrt(c*x^2 + b*x + a))/(a^4*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2 + gx^3}{x^4\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**4/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**4*sqrt(a + b*x + c*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 689 vs. 2(169) = 338.
time = 3.68, size = 689, normalized size = 3.70

$$-\frac{{\left(5\,b^{3} d - 12\,a b c d + 8\,a^{2} b f - 16\,a^{3} g - 6\,a b^{2} e + 8\,a^{2} c e\right)} \arctan\left(-\frac{\sqrt{c} x - \sqrt{c x^{2} + b x + a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a} a^{3}} + \frac{15\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)}^{5} b^{3} d - 36\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)}^{5} a b c d + 24\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)}^{5} a^{2} b f - 18\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)}^{5} a b^{2} e + 24\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)}^{5} a^{2} c e + 48\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)}^{4} a^{3} \sqrt{c} f - 40\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)}^{3} a b^{3} d + 96\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)}^{3} a^{2} b c d - 48\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)}^{3} a^{3} b f + 48\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)}^{3} a^{2} b^{2} e + 96\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)}^{2} a^{3} c^{\frac{3}{2}} d - 96\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)}^{2} a^{4} \sqrt{c} f + 48\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)}^{2} a^{3} b \sqrt{c} e + 33\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)} a^{2} b^{3} d + 36\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)} a^{3} b c d + 24\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)} a^{4} b f - 30\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)} a^{3} b^{2} e - 24\,{\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)} a^{4} c e + 48\,a^{3} b^{2} \sqrt{c} d - 32\,a^{4} c^{\frac{3}{2}} d + 48\,a^{5} \sqrt{c} f - 48\,a^{4} b \sqrt{c} e}{24\,{\left({\left(\sqrt{c} x - \sqrt{c x^{2} + b x + a}\right)}^{2} - a\right)}^{3} a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(5*b^3*d - 12*a*b*c*d + 8*a^2*b*f - 16*a^3*g - 6*a*b^2*e + 8*a^2*c*e)*
arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^3) + 1/24
*(15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*d - 36*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*a*b*c*d + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*f
 - 18*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*e + 24*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^5*a^2*c*e + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*
sqrt(c)*f - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*d + 96*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*d - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*a^3*b*f + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*e + 96*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^(3/2)*d - 96*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a^4*sqrt(c)*f + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
a^3*b*sqrt(c)*e + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*d + 36*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c*d + 24*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^4*b*f - 30*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*e - 24*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*c*e + 48*a^3*b^2*sqrt(c)*d - 32*a^4*c
^(3/2)*d + 48*a^5*sqrt(c)*f - 48*a^4*b*sqrt(c)*e)/(((sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2 - a)^3*a^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^4\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^4*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2 + g*x^3)/(x^4*(a + b*x + c*x^2)^(1/2)), x)
```

3.287 $\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=270

$$-\frac{d\sqrt{a+bx+cx^2}}{4ax^4}+\frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3}-\frac{(35b^2d-36acd-40abe+48a^2f)\sqrt{a+bx+cx^2}}{96a^3x^2}+\frac{(105\cdots}{}$$

```
[Out] -1/128*(35*b^4*d-40*a*b^3*e+16*a^2*c*(-4*a*f+3*c*d)-24*a*b^2*(-2*a*f+5*c*d)
+32*a^2*b*(-2*a*g+3*c*e))*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2)
)/a^(9/2)-1/4*d*(c*x^2+b*x+a)^(1/2)/a/x^4+1/24*(-8*a*e+7*b*d)*(c*x^2+b*x+a)
^(1/2)/a^2/x^3-1/96*(48*a^2*f-40*a*b*e-36*a*c*d+35*b^2*d)*(c*x^2+b*x+a)^(1/
2)/a^3/x^2+1/192*(105*b^3*d-120*a*b^2*e-4*a*b*(-36*a*f+55*c*d)+64*a^2*(-3*a
*g+2*c*e))*(c*x^2+b*x+a)^(1/2)/a^4/x
```

Rubi [A]
time = 0.28, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1664, 848, 820, 738, 212}

$$\frac{\sqrt{a+bx+cx^2}(7bd-8ae)}{24a^2x^3}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)(32a^2b(3ce-2ag)+16a^2c(3cd-4af)-40ab^3e-24ab^2(5cd-2af)+35b^4d)}{128a^{9/2}}+\frac{\sqrt{a+bx+cx^2}(64a^2(2ce-3ag)-120ab^2e-4ab(55cd-36af)+105b^3d)}{192a^4x}-\frac{\sqrt{a+bx+cx^2}(48a^2f-40abe-36acd+35b^2d)}{96a^3x^2}-\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^5*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/4*(d*Sqrt[a + b*x + c*x^2])/(a*x^4) + ((7*b*d - 8*a*e)*Sqrt[a + b*x + c*
x^2])/(24*a^2*x^3) - ((35*b^2*d - 36*a*c*d - 40*a*b*e + 48*a^2*f)*Sqrt[a +
b*x + c*x^2])/(96*a^3*x^2) + ((105*b^3*d - 120*a*b^2*e - 4*a*b*(55*c*d - 36
*a*f) + 64*a^2*(2*c*e - 3*a*g))*Sqrt[a + b*x + c*x^2])/(192*a^4*x) - ((35*b
^4*d - 40*a*b^3*e + 16*a^2*c*(3*c*d - 4*a*f) - 24*a*b^2*(5*c*d - 2*a*f) + 3
2*a^2*b*(3*c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^
2])])/(128*a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} - \frac{\int \frac{\frac{1}{2}(7bd-8ae)+(3cd-4af)x-4agx^2}{x^4\sqrt{a+bx+cx^2}}\,dx}{4a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} + \frac{\int \frac{\frac{1}{4}(35b^2d-40abe-12a(3cd-4af}{x^3\sqrt{a+bx}}}{12a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{(35b^2d-36acd-40abe-}{96a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{(35b^2d-36acd-40abe-}{96a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{(35b^2d-36acd-40abe-}{96a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{(35b^2d-36acd-40abe-}{96a}$$

Mathematica [A]
time = 1.62, size = 237, normalized size = 0.88

$$\frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}(105b^3dx^3-10abx^2(7bd+22cdx+12bex)+8a^2x(7bd+cx(9d+16ex)+2bx(5e+9fx))-16a^3(3d+4ex+6x^2(f+2gx)))}{x^4}+105b^4d\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)+24a(5b^3e+3b^2(5cd-2af)+2ac(-3cd+4af)+4ab(-3ce+2ag))\tanh^{-1}\left(\frac{-\sqrt{c}\,x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{192a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^5*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(105*b^3*d*x^3 - 10*a*b*x^2*(7*b*d + 22*c*d
*x + 12*b*e*x) + 8*a^2*x*(7*b*d + c*x*(9*d + 16*e*x) + 2*b*x*(5*e + 9*f*x))
 - 16*a^3*(3*d + 4*e*x + 6*x^2*(f + 2*g*x))))/x^4 + 105*b^4*d*ArcTanh[(Sqrt
[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 24*a*(5*b^3*e + 3*b^2*(5*c*d - 2*
a*f) + 2*a*c*(-3*c*d + 4*a*f) + 4*a*b*(-3*c*e + 2*a*g))*ArcTanh[(-(Sqrt[c]*
x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(192*a^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 748 vs. $2(244) = 488$.
time = 0.16, size = 749, normalized size = 2.77

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(192a^3g\,x^3-144a^2bf\,x^3-128a^2ce\,x^3+120a\,b^2e\,x^3+220abcd\,x^3-105b^3d\,x^3+96a^3f\,x^2-80a^2be\,x^2-72a^2cd\,x}{192a^4x^4}$

default	$d\left(-\frac{\sqrt{c\,x^2+bx+a}}{4a\,x^4}-\frac{7b\left(-\frac{\sqrt{c\,x^2+bx+a}}{3a\,x^3}-\frac{5b\left(-\frac{\sqrt{c\,x^2+bx+a}}{2a\,x^2}-\frac{3b\left(-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+b}{}\right.}{}\right.}{4a}\right.}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] d*(-1/4/a/x^4*(c*x^2+b*x+a)^(1/2)-7/8*b/a*(-1/3/a/x^3*(c*x^2+b*x+a)^(1/2)-5/6*b/a*(-1/2/a/x^2*(c*x^2+b*x+a)^(1/2)-3/4*b/a*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))+1/2*c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))-2/3*c/a*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)))-3/4*c/a*(-1/2/a/x^2*(c*x^2+b*x+a)^(1/2)-3/4*b/a*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))+1/2*c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)))+f*(-1/2/a/x^2*(c*x^2+b*x+a)^(1/2)-3/4*b/a*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*

```
(c*x^2+b*x+a)^(1/2))/x))+1/2*c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^
(1/2))/x))+g*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2
)*(c*x^2+b*x+a)^(1/2))/x))+e*(-1/3/a/x^3*(c*x^2+b*x+a)^(1/2)-5/6*b/a*(-1/2/
a/x^2*(c*x^2+b*x+a)^(1/2)-3/4*b/a*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)
*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))+1/2*c/a^(3/2)*ln((2*a+b*x+2
*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))-2/3*c/a*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b
/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 10.12, size = 545, normalized size = 2.02

$$\left[-\frac{3\left(8\left(5ab^3-12a^2bc\right)x^4e+\left(64a^3bg-\left(35b^4-120ab^2c+48a^2c^2\right)d-16\left(3a^2b^2-4a^3c\right)f\right)x^4\right)\sqrt{a}\log\left(-\frac{8abx+\left(b^2+4ac\right)x^2-4\sqrt{cx^2+bx+a}\left(bx+2a\right)\sqrt{a}+8a^2}{x^2}\right)-4\left(56a^3bdx-48a^4d+\left(144a^3bf-192a^4g+5\left(21ab^3-44a^2bc\right)d\right)x^3-2\left(48a^4f+\left(35a^2b^2-36a^3c\right)d\right)x^2+8\left(10a^3bx^2-8a^4x-\left(15a^2b^2-16a^3c\right)x^3\right)e\right)\sqrt{cx^2+bx+a}}{768a^5x^4}, -\frac{3\left(8\left(5ab^3-12a^2bc\right)x^4e+\left(64a^3bg-\left(35b^4-120ab^2c+48a^2c^2\right)d-16\left(3a^2b^2-4a^3c\right)f\right)x^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^2+bx+a}\left(bx+2a\right)\sqrt{-a}}{2\left(acx^2+abx+a^2\right)}\right)-2\left(56a^3bdx-48a^4d+\left(144a^3bf-192a^4g+5\left(21ab^3-44a^2bc\right)d\right)x^3-2\left(48a^4f+\left(35a^2b^2-36a^3c\right)d\right)x^2+8\left(10a^3bx^2-8a^4x-\left(15a^2b^2-16a^3c\right)x^3\right)e\right)\sqrt{cx^2+bx+a}}{384a^5x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/768*(3*(8*(5*a*b^3 - 12*a^2*b*c)*x^4*e + (64*a^3*b*g - (35*b^4 - 120*a*
b^2*c + 48*a^2*c^2)*d - 16*(3*a^2*b^2 - 4*a^3*c)*f)*x^4)*sqrt(a)*log(-(8*a*
b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a
^2)/x^2) - 4*(56*a^3*b*d*x - 48*a^4*d + (144*a^3*b*f - 192*a^4*g + 5*(21*a*
b^3 - 44*a^2*b*c)*d)*x^3 - 2*(48*a^4*f + (35*a^2*b^2 - 36*a^3*c)*d)*x^2 + 8
*(10*a^3*b*x^2 - 8*a^4*x - (15*a^2*b^2 - 16*a^3*c)*x^3)*e)*sqrt(c*x^2 + b*x
 + a))/(a^5*x^4), -1/384*(3*(8*(5*a*b^3 - 12*a^2*b*c)*x^4*e + (64*a^3*b*g -
 (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*d - 16*(3*a^2*b^2 - 4*a^3*c)*f)*x^4)*s
qrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*
b*x + a^2)) - 2*(56*a^3*b*d*x - 48*a^4*d + (144*a^3*b*f - 192*a^4*g + 5*(21
*a*b^3 - 44*a^2*b*c)*d)*x^3 - 2*(48*a^4*f + (35*a^2*b^2 - 36*a^3*c)*d)*x^2
+ 8*(10*a^3*b*x^2 - 8*a^4*x - (15*a^2*b^2 - 16*a^3*c)*x^3)*e)*sqrt(c*x^2 +
b*x + a))/(a^5*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2 + gx^3}{x^5\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x**3+f*x**2+e*x+d)/x**5/(c*x**2+b*x+a)**(1/2),x)

[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**5*sqrt(a + b*x + c*x**2)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1448 vs. 2(250) = 500.
time = 4.31, size = 1448, normalized size = 5.36

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")

[Out] 1/64*(35*b^4*d - 120*a*b^2*c*d + 48*a^2*c^2*d + 48*a^2*b^2*f - 64*a^3*c*f - 64*a^3*b*g - 40*a*b^3*e + 96*a^2*b*c*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^4) - 1/192*(105*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*b^4*d - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^2*c*d + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*c^2*d + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^2*f - 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*c*f - 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*g - 120*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^3*e + 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b*c*e - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*sqrt(c)*g - 385*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^4*d + 1320*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^2*c*d - 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*c^2*d - 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^2*f + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*c*f + 576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b*g + 440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^3*e - 1056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b*c*e - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*b*sqrt(c)*f + 1152*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*sqrt(c)*g - 768*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*c^(3/2)*e + 511*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^4*d - 1752*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^2*c*d - 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*c^2*d + 624*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^2*f + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*c*f - 576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b*g - 584*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^3*e + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b*c*e - 2048*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b*c^(3/2)*d + 768*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b*sqrt(c)*f - 1152*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2

```
*a^6*sqrt(c)*g - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^2*sqrt(c)*
e + 1024*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*c^(3/2)*e - 279*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*a^3*b^4*d - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^4*b^2*c*d + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*c^2*d - 240*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^2*f - 192*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*a^6*c*f + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b*g + 26
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^3*e + 288*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a^5*b*c*e - 384*a^4*b^3*sqrt(c)*d + 512*a^5*b*c^(3/2)*d - 38
4*a^6*b*sqrt(c)*f + 384*a^7*sqrt(c)*g + 384*a^5*b^2*sqrt(c)*e - 256*a^6*c^(
3/2)*e)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^4*a^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^5\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^5*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2 + g*x^3)/(x^5*(a + b*x + c*x^2)^(1/2)), x)
```

3.288 $\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=371

$$-\frac{d\sqrt{a+bx+cx^2}}{5ax^5}+\frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4}-\frac{(63b^2d-64acd-70abe+80a^2f)\sqrt{a+bx+cx^2}}{240a^3x^3}+\frac{(315}{}$$

```
[Out] 1/256*(63*b^5*d-70*a*b^4*e+48*a^2*b*c*(-4*a*f+5*c*d)-40*a*b^3*(-2*a*f+7*c*d
)-32*a^3*c*(-4*a*g+3*c*e)+48*a^2*b^2*(-2*a*g+5*c*e))*arctanh(1/2*(b*x+2*a)/
a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(11/2)-1/5*d*(c*x^2+b*x+a)^(1/2)/a/x^5+1/40*
(-10*a*e+9*b*d)*(c*x^2+b*x+a)^(1/2)/a^2/x^4-1/240*(80*a^2*f-70*a*b*e-64*a*c
*d+63*b^2*d)*(c*x^2+b*x+a)^(1/2)/a^3/x^3+1/960*(315*b^3*d-350*a*b^2*e-4*a*b
*(-100*a*f+161*c*d)+120*a^2*(-4*a*g+3*c*e))*(c*x^2+b*x+a)^(1/2)/a^4/x^2-1/1
920*(945*b^4*d-1050*a*b^3*e-60*a*b^2*(-20*a*f+49*c*d)+256*a^2*c*(-5*a*f+4*c
*d)+40*a^2*b*(-36*a*g+55*c*e))*(c*x^2+b*x+a)^(1/2)/a^5/x
```

Rubi [A]
time = 0.48, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1664, 848, 820, 738, 212}

$$\frac{\sqrt{a+bx+cx^2}(9bd-10ae)}{40a^2x^4}-\frac{\sqrt{a+bx+cx^2}\left(40a^2b(55ce-36ag)+256a^2c(4cd-5af)-1050ab^3e-60ab^2(49cd-20af)+945b^4d\right)}{1920a^5x}+\frac{\sqrt{a+bx+cx^2}\left(120a^2(3ce-4ag)-350ab^2e-4ab(161cd-100af)+315b^3d\right)}{960a^4x^2}-\frac{\sqrt{a+bx+cx^2}\left(80a^2f-70abe-64acd+63b^2d\right)}{240a^3x^3}+\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(-32a^3c(3ce-4ag)+48a^2b^2(5ce-2ag)+48a^2bc(5cd-4af)-70ab^4e-40ab^3(7cd-2af)+63b^5d\right)}{256a^{11/2}}-\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^6*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/5*(d*Sqrt[a + b*x + c*x^2])/(a*x^5) + ((9*b*d - 10*a*e)*Sqrt[a + b*x + c
*x^2])/(40*a^2*x^4) - ((63*b^2*d - 64*a*c*d - 70*a*b*e + 80*a^2*f)*Sqrt[a +
 b*x + c*x^2])/(240*a^3*x^3) + ((315*b^3*d - 350*a*b^2*e - 4*a*b*(161*c*d -
 100*a*f) + 120*a^2*(3*c*e - 4*a*g))*Sqrt[a + b*x + c*x^2])/(960*a^4*x^2) -
 ((945*b^4*d - 1050*a*b^3*e - 60*a*b^2*(49*c*d - 20*a*f) + 256*a^2*c*(4*c*d
 - 5*a*f) + 40*a^2*b*(55*c*e - 36*a*g))*Sqrt[a + b*x + c*x^2])/(1920*a^5*x)
 + ((63*b^5*d - 70*a*b^4*e + 48*a^2*b*c*(5*c*d - 4*a*f) - 40*a*b^3*(7*c*d -
 2*a*f) - 32*a^3*c*(3*c*e - 4*a*g) + 48*a^2*b^2*(5*c*e - 2*a*g))*ArcTanh[(2
*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(256*a^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} - \frac{\int \frac{\frac{1}{2}(9bd-10ae)+(4cd-5af)x-5agx^2}{x^5\sqrt{a+bx+cx^2}}\,dx}{5a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} + \frac{\int \frac{\frac{1}{4}(63b^2d-64acd-70abe+80a^2f}{x^4\sqrt{a+bx}}}{20a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{(63b^2d-64acd-70abe+}{240a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{(63b^2d-64acd-70abe+}{240a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{(63b^2d-64acd-70abe+}{240a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{(63b^2d-64acd-70abe+}{240a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{(63b^2d-64acd-70abe+}{240a}$$

Mathematica [A]
time = 2.18, size = 328, normalized size = 0.88

$$\frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}\left(-945b^4dx^4+210ab^2x^3(3bd+14cdx+5bex)-32a^4\left(12d+5x\left(3e+4fx+6gx^2\right)\right)-4a^2x^2\left(256c^2dx^2+2bcx(161d+275ex)+b^2(126d+25x(7e+12fx))\right)+16a^3x(cx(32d+5x(9e+16fx))+b(27d+5x(7e+2x(5f+9gx))))\right)}{x^5}-15(63b^5d+128a^4cg)\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)-30a(35b^4e+48a^2c^2e+20b^3(7cd-2af)+24abc(-5cd+4af)+24ab^2(-5ce+2ag))\tanh^{-1}\left(\frac{-\sqrt{c}\,x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{1920a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^6*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(-945*b^4*d*x^4 + 210*a*b^2*x^3*(3*b*d + 14
*c*d*x + 5*b*e*x) - 32*a^4*(12*d + 5*x*(3*e + 4*f*x + 6*g*x^2)) - 4*a^2*x^2
*(256*c^2*d*x^2 + 2*b*c*x*(161*d + 275*e*x) + b^2*(126*d + 25*x*(7*e + 12*f
*x))) + 16*a^3*x*(c*x*(32*d + 5*x*(9*e + 16*f*x)) + b*(27*d + 5*x*(7*e + 2*
x*(5*f + 9*g*x))))))/x^5 - 15*(63*b^5*d + 128*a^4*c*g)*ArcTanh[(Sqrt[c]*x -
 Sqrt[a + x*(b + c*x)])/Sqrt[a]] - 30*a*(35*b^4*e + 48*a^2*c^2*e + 20*b^3*(
7*c*d - 2*a*f) + 24*a*b*c*(-5*c*d + 4*a*f) + 24*a*b^2*(-5*c*e + 2*a*g))*Arc
Tanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(1920*a^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1290 vs. $2(341) = 682$.
time = 0.16, size = 1291, normalized size = 3.48

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(-1440a^3bg\,x^4-1280a^3cf\,x^4+1200a^2b^2f\,x^4+2200a^2bce\,x^4+1024a^2c^2d\,x^4-1050a\,b^3e\,x^4-2940a\,b^2cd\,x^4+\cdots}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] d*(-1/5/a/x^5*(c*x^2+b*x+a)^(1/2)-9/10*b/a*(-1/4/a/x^4*(c*x^2+b*x+a)^(1/2)-
7/8*b/a*(-1/3/a/x^3*(c*x^2+b*x+a)^(1/2)-5/6*b/a*(-1/2/a/x^2*(c*x^2+b*x+a)^(
1/2)-3/4*b/a*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2
)*(c*x^2+b*x+a)^(1/2))/x))+1/2*c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a
)^(1/2))/x))-2/3*c/a*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+
2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)))-3/4*c/a*(-1/2/a/x^2*(c*x^2+b*x+a)^(1/2)
-3/4*b/a*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c
*x^2+b*x+a)^(1/2))/x))+1/2*c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1
/2))/x)))-4/5*c/a*(-1/3/a/x^3*(c*x^2+b*x+a)^(1/2)-5/6*b/a*(-1/2/a/x^2*(c*x^
2+b*x+a)^(1/2)-3/4*b/a*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*
x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))+1/2*c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c
*x^2+b*x+a)^(1/2))/x))-2/3*c/a*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln
((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))))+e*(-1/4/a/x^4*(c*x^2+b*x+a)^
(1/2)-7/8*b/a*(-1/3/a/x^3*(c*x^2+b*x+a)^(1/2)-5/6*b/a*(-1/2/a/x^2*(c*x^2+b*
x+a)^(1/2)-3/4*b/a*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+2*
a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))+1/2*c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2
+b*x+a)^(1/2))/x))-2/3*c/a*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*
a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)))-3/4*c/a*(-1/2/a/x^2*(c*x^2+b*x+a)
^(1/2)-3/4*b/a*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+2*a^(1
/2)*(c*x^2+b*x+a)^(1/2))/x))+1/2*c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x
+a)^(1/2))/x)))+g*(-1/2/a/x^2*(c*x^2+b*x+a)^(1/2)-3/4*b/a*(-1/a/x*(c*x^2+b*
x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))+1/2
*c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))+f*(-1/3/a/x^3*(c*
x^2+b*x+a)^(1/2)-5/6*b/a*(-1/2/a/x^2*(c*x^2+b*x+a)^(1/2)-3/4*b/a*(-1/a/x*(c
*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/
x))+1/2*c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))-2/3*c/a*(-
1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)
^(1/2))/x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 14.22, size = 753, normalized size = 2.03

$$\left[\frac{15\,(2\,(35\,ab^4-120\,a^2b^2c+48\,a^3c^2)x^5e-((63\,b^5-280\,ab^3c+240\,a^2bc^2)d+16\,(5\,a^2b^3-12\,a^3bc)f-32\,(3\,a^3b^2-4\,a^4c)g)x^5)\sqrt{a}\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{cx^2+bx+a}\,(bx+2\,a)\sqrt{a}+8\,a^2}{x^2}\right)+4\,(432\,a^4bdx-384\,a^5d+(1440\,a^4bg-(945\,ab^4-2940\,a^2b^2c+1024\,a^3c^2)d-80\,(15\,a^3b^2-16\,a^4c)f)x^4+2\,(400\,a^4bf-480\,a^5g+7\,(45\,a^2b^3-92\,a^3bc)d)x^3-8\,(80\,a^5f+(63\,a^3b^2-64\,a^4c)d)x^2+10\,(56\,a^4bx^2-48\,a^5x+5\,(21\,a^2b^3-44\,a^3bc)x^4-2\,(35\,a^3b^2-36\,a^4c)x^3)e)\sqrt{cx^2+bx+a}}{7680\,a^6x^5},\ \frac{15\,(2\,(35\,ab^4-120\,a^2b^2c+48\,a^3c^2)x^5e-((63\,b^5-280\,ab^3c+240\,a^2bc^2)d+16\,(5\,a^2b^3-12\,a^3bc)f-32\,(3\,a^3b^2-4\,a^4c)g)x^5)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^2+bx+a}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^2+abx+a^2)}\right)+2\,(432\,a^4bdx-384\,a^5d+(1440\,a^4bg-(945\,ab^4-2940\,a^2b^2c+1024\,a^3c^2)d-80\,(15\,a^3b^2-16\,a^4c)f)x^4+2\,(400\,a^4bf-480\,a^5g+7\,(45\,a^2b^3-92\,a^3bc)d)x^3-8\,(80\,a^5f+(63\,a^3b^2-64\,a^4c)d)x^2+10\,(56\,a^4bx^2-48\,a^5x+5\,(21\,a^2b^3-44\,a^3bc)x^4-2\,(35\,a^3b^2-36\,a^4c)x^3)e)\sqrt{cx^2+bx+a}}{3840\,a^6x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/7680*(15*(2*(35*a*b^4 - 120*a^2*b^2*c + 48*a^3*c^2)*x^5*e - ((63*b^5 - 2
80*a*b^3*c + 240*a^2*b*c^2)*d + 16*(5*a^2*b^3 - 12*a^3*b*c)*f - 32*(3*a^3*b
^2 - 4*a^4*c)*g)*x^5)*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*
x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(432*a^4*b*d*x - 384*a
^5*d + (1440*a^4*b*g - (945*a*b^4 - 2940*a^2*b^2*c + 1024*a^3*c^2)*d - 80*(
15*a^3*b^2 - 16*a^4*c)*f)*x^4 + 2*(400*a^4*b*f - 480*a^5*g + 7*(45*a^2*b^3
- 92*a^3*b*c)*d)*x^3 - 8*(80*a^5*f + (63*a^3*b^2 - 64*a^4*c)*d)*x^2 + 10*(5
6*a^4*b*x^2 - 48*a^5*x + 5*(21*a^2*b^3 - 44*a^3*b*c)*x^4 - 2*(35*a^3*b^2 -
36*a^4*c)*x^3)*e)*sqrt(c*x^2 + b*x + a))/(a^6*x^5), 1/3840*(15*(2*(35*a*b^4
 - 120*a^2*b^2*c + 48*a^3*c^2)*x^5*e - ((63*b^5 - 280*a*b^3*c + 240*a^2*b*c
^2)*d + 16*(5*a^2*b^3 - 12*a^3*b*c)*f - 32*(3*a^3*b^2 - 4*a^4*c)*g)*x^5)*sq
rt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b
*x + a^2)) + 2*(432*a^4*b*d*x - 384*a^5*d + (1440*a^4*b*g - (945*a*b^4 - 29
40*a^2*b^2*c + 1024*a^3*c^2)*d - 80*(15*a^3*b^2 - 16*a^4*c)*f)*x^4 + 2*(400
*a^4*b*f - 480*a^5*g + 7*(45*a^2*b^3 - 92*a^3*b*c)*d)*x^3 - 8*(80*a^5*f + (
63*a^3*b^2 - 64*a^4*c)*d)*x^2 + 10*(56*a^4*b*x^2 - 48*a^5*x + 5*(21*a^2*b^3
 - 44*a^3*b*c)*x^4 - 2*(35*a^3*b^2 - 36*a^4*c)*x^3)*e)*sqrt(c*x^2 + b*x + a
))/(a^6*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**6/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**6*sqrt(a + b*x + c*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2177 vs. $2(350) = 700$.

time = 6.68, size = 2177, normalized size = 5.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/128*(63*b^5*d - 280*a*b^3*c*d + 240*a^2*b*c^2*d + 80*a^2*b^3*f - 192*a^3
*b*c*f - 96*a^3*b^2*g + 128*a^4*c*g - 70*a*b^4*e + 240*a^2*b^2*c*e - 96*a^3
*c^2*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^5
) + 1/1920*(945*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*d - 4200*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c*d + 3600*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^9*a^2*b*c^2*d + 1200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^3*
f - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b*c*f - 1440*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^9*a^3*b^2*g + 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^9*a^4*c*g - 1050*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^4*e + 3600*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^2*c*e - 1440*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^9*a^3*c^2*e - 4410*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*
a*b^5*d + 19600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c*d - 16800*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^2*d - 5600*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^7*a^3*b^3*f + 13440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*
a^4*b*c*f + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b^2*g - 3840*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*c*g + 4900*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^7*a^2*b^4*e - 16800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^
2*c*e + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*c^2*e + 7680*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^6*a^5*c^(3/2)*f + 3840*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^6*a^5*b*sqrt(c)*g + 8064*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5
*a^2*b^5*d - 35840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^3*c*d + 3072
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b*c^2*d + 10240*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^5*a^4*b^3*f - 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^5*a^5*b*c*f - 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^2*g - 896
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^4*e + 30720*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^5*a^4*b^2*c*e + 20480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^4*a^5*c^(5/2)*d + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^2*sqrt
(c)*f - 17920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*c^(3/2)*f - 11520*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b*sqrt(c)*g + 20480*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^4*a^5*b*c^(3/2)*e - 7110*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*a^3*b^5*d + 31600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c
*d + 16800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b*c^2*d - 8480*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^3*f + 1920*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*a^6*b*c*f + 8640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b^2*g
 + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7*c*g + 7900*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^3*a^4*b^4*e - 13920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
```

```
))^3*a^5*b^2*c*e - 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*c^2*e + 3
8400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^2*c^(3/2)*d - 10240*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*c^(5/2)*d - 7680*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*a^6*b^2*sqrt(c)*f + 12800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*a^7*c^(3/2)*f + 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b*sqrt
(c)*g + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^3*sqrt(c)*e - 2560
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b*c^(3/2)*e + 2895*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*a^4*b^5*d + 4200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*a^5*b^3*c*d - 3600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b*c^2*d + 2640
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^3*f + 2880*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a^7*b*c*f - 2400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b^2
*g - 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8*c*g - 2790*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^5*b^4*e - 3600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
a^6*b^2*c*e + 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*c^2*e + 3840*a^5
*b^4*sqrt(c)*d - 7680*a^6*b^2*c^(3/2)*d + 2048*a^7*c^(5/2)*d + 3840*a^7*b^2
*sqrt(c)*f - 2560*a^8*c^(3/2)*f - 3840*a^8*b*sqrt(c)*g - 3840*a^6*b^3*sqrt(
c)*e + 5120*a^7*b*c^(3/2)*e)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^5
*a^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^6\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^6*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2 + g*x^3)/(x^6*(a + b*x + c*x^2)^(1/2)), x)
```

3.289 $\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right)$

Optimal. Leaf size=258

$$\frac{\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^4}{4e^7}-\frac{\left(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4\right.}{5e^7}$$

```
[Out] 1/4*(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^4/e^7
-1/5*(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5)*(e*x+d)^5/e^7+
1/6*(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+21*e^4)*(e*x+d)^6/e^7-2/7*(200*
d^3+85*d^2*e+34*d*e^2+2*e^3)*(e*x+d)^7/e^7+1/8*(300*d^2+85*d*e+17*e^2)*(e*x
+d)^8/e^7-1/9*(120*d+17*e)*(e*x+d)^9/e^7+2*(e*x+d)^10/e^7
```

Rubi [A]
time = 0.16, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1642}

$$\frac{\left(300d^2+85de+17e^2\right)(d+ex)^8}{8e^7}-\frac{2\left(200d^3+85d^2e+34de^2+2e^3\right)(d+ex)^7}{7e^7}+\frac{\left(300d^4+170d^3e+102d^2e^2+12de^3+21e^4\right)(d+ex)^6}{6e^7}+\frac{\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^4}{4e^7}-\frac{\left(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5\right)(d+ex)^5}{5e^7}+\frac{2(d+ex)^{10}}{e^7}-\frac{(120d+17e)(d+ex)^9}{9e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d +
 e*x)^4)/(4*e^7) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^
4 - 7*e^5)*(d + e*x)^5)/(5*e^7) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*
d*e^3 + 21*e^4)*(d + e*x)^6)/(6*e^7) - (2*(200*d^3 + 85*d^2*e + 34*d*e^2 +
2*e^3)*(d + e*x)^7)/(7*e^7) + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^8)/(8*
e^7) - ((120*d + 17*e)*(d + e*x)^9)/(9*e^7) + (2*(d + e*x)^10)/e^7
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \int \left(\frac{\left(20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4\right.}{e^6}\right.$$

$$= \frac{\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3e+3d^2e^2-de^3+\right.}{4e^7}$$

Mathematica [A]
time = 0.03, size = 212, normalized size = 0.82

$$6d^3x+\frac{1}{2}d^2(7d+18e)x^2+d\left(7d^2+7de+6e^2\right)x^3+\frac{1}{4}\left(-4d^3+63d^2e+21de^2+6e^3\right)x^4+\frac{1}{5}\left(17d^3-12d^2e+63de^2+7e^3\right)x^5+\frac{1}{6}\left(-17d^3+51d^2e-12de^2+21e^3\right)x^6+\frac{1}{7}\left(20d^3-51d^2e+51de^2-4e^3\right)x^7+\frac{1}{8}e\left(60d^2-51de+17e^2\right)x^8+\frac{1}{9}(60d-17e)e^2x^9+2e^3x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d^3*x + (d^2*(7*d + 18*e)*x^2)/2 + d*(7*d^2 + 7*d*e + 6*e^2)*x^3 + ((-4*d
^3 + 63*d^2*e + 21*d*e^2 + 6*e^3)*x^4)/4 + ((17*d^3 - 12*d^2*e + 63*d*e^2 +
 7*e^3)*x^5)/5 + ((-17*d^3 + 51*d^2*e - 12*d*e^2 + 21*e^3)*x^6)/6 + ((20*d^
3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7)/7 + (e*(60*d^2 - 51*d*e + 17*e^2)*x^8
)/8 + ((60*d - 17*e)*e^2*x^9)/9 + 2*e^3*x^10
```

Maple [A]
time = 0.12, size = 208, normalized size = 0.81

method	result
norman	$2e^3x^{10}+\left(\frac{20}{3}d\,e^2-\frac{17}{9}e^3\right)x^9+\left(\frac{15}{2}d^2e-\frac{51}{8}d\,e^2+\frac{17}{8}e^3\right)x^8+\left(\frac{20}{7}d^3-\frac{51}{7}d^2e+\frac{51}{7}d\,e^2-\frac{4}{7}e^3\right)x^7+\left(-\right.$
default	$2e^3x^{10}+\frac{(60d\,e^2-17e^3)x^9}{9}+\frac{(60d^2e-51d\,e^2+17e^3)x^8}{8}+\frac{(20d^3-51d^2e+51d\,e^2-4e^3)x^7}{7}+\frac{(-17d^3+51d^2e-12d\,e^2+21e^3)x^6}{6}$
gosper	$-\frac{51}{7}x^7d^2e+\frac{51}{7}x^7d\,e^2+\frac{17}{2}x^6d^2e-2x^6d\,e^2-\frac{12}{5}x^5d^2e+\frac{63}{5}x^5d\,e^2+\frac{63}{4}x^4d^2e+\frac{21}{4}x^4d\,e^2+7d^2e\,x^3+$
risch	$-\frac{51}{7}x^7d^2e+\frac{51}{7}x^7d\,e^2+\frac{17}{2}x^6d^2e-2x^6d\,e^2-\frac{12}{5}x^5d^2e+\frac{63}{5}x^5d\,e^2+\frac{63}{4}x^4d^2e+\frac{21}{4}x^4d\,e^2+7d^2e\,x^3+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE
)
```

```
[Out] 2*e^3*x^10+1/9*(60*d*e^2-17*e^3)*x^9+1/8*(60*d^2*e-51*d*e^2+17*e^3)*x^8+1/7
*(20*d^3-51*d^2*e+51*d*e^2-4*e^3)*x^7+1/6*(-17*d^3+51*d^2*e-12*d*e^2+21*e^3
)*x^6+1/5*(17*d^3-12*d^2*e+63*d*e^2+7*e^3)*x^5+1/4*(-4*d^3+63*d^2*e+21*d*e^
2+6*e^3)*x^4+1/3*(21*d^3+21*d^2*e+18*d*e^2)*x^3+1/2*(7*d^3+18*d^2*e)*x^2+6*
d^3*x
```

Maxima [A]
time = 0.34, size = 199, normalized size = 0.77

$$2\,x^{10}e^3+\frac{1}{9}\,(60\,de^2-17\,e^3)x^9+\frac{1}{8}\,(60\,d^2e-51\,de^2+17\,e^3)x^8+\frac{1}{7}\,(20\,d^3-51\,d^2e+51\,de^2-4\,e^3)x^7-\frac{1}{6}\,(17\,d^3-51\,d^2e+12\,de^2-21\,e^3)x^6+\frac{1}{5}\,(17\,d^3-12\,d^2e+63\,de^2+7\,e^3)x^5-\frac{1}{4}\,(4\,d^3-63\,d^2e-21\,de^2-6\,e^3)x^4+6\,d^3x+(7\,d^3+7\,d^2e+6\,de^2)x^3+\frac{1}{2}\,(7\,d^3+18\,d^2e)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 2*x^10*e^3 + 1/9*(60*d*e^2 - 17*e^3)*x^9 + 1/8*(60*d^2*e - 51*d*e^2 + 17*e^
3)*x^8 + 1/7*(20*d^3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7 - 1/6*(17*d^3 - 51*
```

```
d^2*e + 12*d*e^2 - 21*e^3)*x^6 + 1/5*(17*d^3 - 12*d^2*e + 63*d*e^2 + 7*e^3)
*x^5 - 1/4*(4*d^3 - 63*d^2*e - 21*d*e^2 - 6*e^3)*x^4 + 6*d^3*x + (7*d^3 + 7
*d^2*e + 6*d*e^2)*x^3 + 1/2*(7*d^3 + 18*d^2*e)*x^2
```

Fricas [A]
time = 0.33, size = 203, normalized size = 0.79

$$\frac{20}{7}d^3x^7-\frac{17}{6}d^3x^6+\frac{17}{5}d^3x^5-d^3x^4+7d^3x^3+\frac{7}{2}d^3x^2+6d^3x+\frac{1}{2520}\left(5040x^{10}-4760x^9+5355x^8-1440x^7+8820x^6+3528x^5+3780x^4\right)e^3+\frac{1}{840}\left(5600dx^9-5355dx^8+6120dx^7-1680dx^6+10584dx^5+4410dx^4+5040dx^3\right)e^2+\frac{1}{140}\left(1050d^2x^8-1020d^2x^7+1190d^2x^6-336d^2x^5+2205d^2x^4+980d^2x^3+1260d^2x^2\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 20/7*d^3*x^7 - 17/6*d^3*x^6 + 17/5*d^3*x^5 - d^3*x^4 + 7*d^3*x^3 + 7/2*d^3*
x^2 + 6*d^3*x + 1/2520*(5040*x^10 - 4760*x^9 + 5355*x^8 - 1440*x^7 + 8820*x
^6 + 3528*x^5 + 3780*x^4)*e^3 + 1/840*(5600*d*x^9 - 5355*d*x^8 + 6120*d*x^7
 - 1680*d*x^6 + 10584*d*x^5 + 4410*d*x^4 + 5040*d*x^3)*e^2 + 1/140*(1050*d^
2*x^8 - 1020*d^2*x^7 + 1190*d^2*x^6 - 336*d^2*x^5 + 2205*d^2*x^4 + 980*d^2*
x^3 + 1260*d^2*x^2)*e
```

Sympy [A]
time = 0.03, size = 230, normalized size = 0.89

$$6d^3x+2e^3x^{10}+x^9\cdot\left(\frac{20de^2}{3}-\frac{17e^3}{9}\right)+x^8\cdot\left(\frac{15d^2e}{2}-\frac{51de^2}{8}+\frac{17e^3}{8}\right)+x^7\cdot\left(\frac{20d^3}{7}-\frac{51d^2e}{7}+\frac{51de^2}{7}-\frac{4e^3}{7}\right)+x^6\left(-\frac{17d^3}{6}+\frac{17d^2e}{2}-2de^2+\frac{7e^3}{2}\right)+x^5\cdot\left(\frac{17d^3}{5}-\frac{12d^2e}{5}+\frac{63de^2}{5}+\frac{7e^3}{5}\right)+x^4\left(-d^3+\frac{63d^2e}{4}+\frac{21de^2}{4}+\frac{3e^3}{2}\right)+x^3\cdot\left(7d^3+7d^2e+6de^2\right)+x^2\cdot\left(\frac{7d^3}{2}+9d^2e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 6*d**3*x + 2*e**3*x**10 + x**9*(20*d*e**2/3 - 17*e**3/9) + x**8*(15*d**2*e/
2 - 51*d*e**2/8 + 17*e**3/8) + x**7*(20*d**3/7 - 51*d**2*e/7 + 51*d*e**2/7
- 4*e**3/7) + x**6*(-17*d**3/6 + 17*d**2*e/2 - 2*d*e**2 + 7*e**3/2) + x**5*
(17*d**3/5 - 12*d**2*e/5 + 63*d*e**2/5 + 7*e**3/5) + x**4*(-d**3 + 63*d**2*
e/4 + 21*d*e**2/4 + 3*e**3/2) + x**3*(7*d**3 + 7*d**2*e + 6*d*e**2) + x**2*
(7*d**3/2 + 9*d**2*e)
```

Giac [A]
time = 4.69, size = 230, normalized size = 0.89

$$2x^{10}e^3+\frac{20}{3}dx^9e^2+\frac{15}{2}d^2x^8e+\frac{20}{7}d^3x^7-\frac{17}{9}x^9e^3-\frac{51}{8}dx^8e^2-\frac{51}{7}d^2x^7e-\frac{17}{6}d^3x^6+\frac{17}{8}x^8e^3+\frac{51}{7}dx^7e^2+\frac{17}{2}d^2x^6e+\frac{17}{5}d^3x^5-\frac{4}{7}x^7e^3-2dx^6e^2-\frac{12}{5}d^2x^5e-d^3x^4+\frac{7}{2}x^6e^3+\frac{63}{5}dx^5e^2+\frac{63}{4}d^2x^4e+7d^3x^3+\frac{7}{5}x^5e^3+\frac{21}{4}dx^4e^2+7d^2x^3e+\frac{7}{2}d^3x^2+\frac{3}{2}x^4e^3+6dx^3e^2+9d^2x^2e+6d^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 2*x^10*e^3 + 20/3*d*x^9*e^2 + 15/2*d^2*x^8*e + 20/7*d^3*x^7 - 17/9*x^9*e^3
- 51/8*d*x^8*e^2 - 51/7*d^2*x^7*e - 17/6*d^3*x^6 + 17/8*x^8*e^3 + 51/7*d*x^
```

```
7*e^2 + 17/2*d^2*x^6*e + 17/5*d^3*x^5 - 4/7*x^7*e^3 - 2*d*x^6*e^2 - 12/5*d^
2*x^5*e - d^3*x^4 + 7/2*x^6*e^3 + 63/5*d*x^5*e^2 + 63/4*d^2*x^4*e + 7*d^3*x
^3 + 7/5*x^5*e^3 + 21/4*d*x^4*e^2 + 7*d^2*x^3*e + 7/2*d^3*x^2 + 3/2*x^4*e^3
 + 6*d*x^3*e^2 + 9*d^2*x^2*e + 6*d^3*x
```

Mupad [B]
time = 4.20, size = 196, normalized size = 0.76

$$6\,d^3\,x+x^8\,\left(\frac{15\,d^2\,e}{2}-\frac{51\,d\,e^2}{8}+\frac{17\,e^3}{8}\right)-x^6\,\left(\frac{17\,d^3}{6}-\frac{17\,d^2\,e}{2}+2\,d\,e^2-\frac{7\,e^3}{2}\right)+x^4\,\left(-d^3+\frac{63\,d^2\,e}{4}+\frac{21\,d\,e^2}{4}+\frac{3\,e^3}{2}\right)+x^5\,\left(\frac{17\,d^3}{5}-\frac{12\,d^2\,e}{5}+\frac{63\,d\,e^2}{5}+\frac{7\,e^3}{5}\right)+x^7\,\left(\frac{20\,d^3}{7}-\frac{51\,d^2\,e}{7}+\frac{51\,d\,e^2}{7}-\frac{4\,e^3}{7}\right)+2\,e^3\,x^{10}+d\,x^3\,\left(7\,d^2+7\,d\,e+6\,e^2\right)+\frac{d^2\,x^2\,\left(7\,d+18\,e\right)}{2}+\frac{e^2\,x^9\,\left(60\,d-17\,e\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] 6*d^3*x + x^8*((15*d^2*e)/2 - (51*d*e^2)/8 + (17*e^3)/8) - x^6*(2*d*e^2 - (
17*d^2*e)/2 + (17*d^3)/6 - (7*e^3)/2) + x^4*((21*d*e^2)/4 + (63*d^2*e)/4 -
d^3 + (3*e^3)/2) + x^5*((63*d*e^2)/5 - (12*d^2*e)/5 + (17*d^3)/5 + (7*e^3)/
5) + x^7*((51*d*e^2)/7 - (51*d^2*e)/7 + (20*d^3)/7 - (4*e^3)/7) + 2*e^3*x^1
0 + d*x^3*(7*d*e + 7*d^2 + 6*e^2) + (d^2*x^2*(7*d + 18*e))/2 + (e^2*x^9*(60
*d - 17*e))/9
```

3.290 $\int (d+ex)^2 (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4)$

Optimal. Leaf size=157

$$6d^2x+\frac{1}{2}d(7d+12e)x^2+\frac{1}{3}\left(21d^2+14de+6e^2\right)x^3-\frac{1}{4}\left(4d^2-42de-7e^2\right)x^4+\frac{1}{5}\left(17d^2-8de+21e^2\right)x^5-\frac{1}{6}(17$$

```
[Out] 6*d^2*x+1/2*d*(7*d+12*e)*x^2+1/3*(21*d^2+14*d*e+6*e^2)*x^3-1/4*(4*d^2-42*d*
e-7*e^2)*x^4+1/5*(17*d^2-8*d*e+21*e^2)*x^5-1/6*(17*d^2-34*d*e+4*e^2)*x^6+1/
7*(20*d^2-34*d*e+17*e^2)*x^7+1/8*(40*d-17*e)*e*x^8+20/9*e^2*x^9
```

Rubi [A]
time = 0.11, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1642}

$$\frac{1}{7}x^7\left(20d^2-34de+17e^2\right)-\frac{1}{6}x^6\left(17d^2-34de+4e^2\right)+\frac{1}{5}x^5\left(17d^2-8de+21e^2\right)-\frac{1}{4}x^4\left(4d^2-42de-7e^2\right)+\frac{1}{3}x^3\left(21d^2+14de+6e^2\right)+6d^2x+\frac{1}{8}ex^8(40d-17e)+\frac{1}{2}dx^2(7d+12e)+\frac{20e^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d^2*x + (d*(7*d + 12*e)*x^2)/2 + ((21*d^2 + 14*d*e + 6*e^2)*x^3)/3 - ((4*
d^2 - 42*d*e - 7*e^2)*x^4)/4 + ((17*d^2 - 8*d*e + 21*e^2)*x^5)/5 - ((17*d^2
 - 34*d*e + 4*e^2)*x^6)/6 + ((20*d^2 - 34*d*e + 17*e^2)*x^7)/7 + ((40*d - 1
7*e)*e*x^8)/8 + (20*e^2*x^9)/9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^2 \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx = \int \left(6d^2+d(7d+12e)x+\left(21d^2+14de+6e^2\right)\right.$$

$$= 6d^2x+\frac{1}{2}d(7d+12e)x^2+\frac{1}{3}\left(21d^2+14de+6e\right.$$

Mathematica [A]
time = 0.02, size = 136, normalized size = 0.87

$$\frac{e^2x^3\left(5040+4410x+10584x^2-1680x^3+6120x^4-5355x^5+5600x^6\right)}{2520}+d^2\left(6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}\right)+de\left(6x^2+\frac{14x^3}{3}+\frac{21x^4}{2}-\frac{8x^5}{5}+\frac{17x^6}{3}-\frac{34x^7}{7}+5x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] (e^2*x^3*(5040 + 4410*x + 10584*x^2 - 1680*x^3 + 6120*x^4 - 5355*x^5 + 5600
*x^6))/2520 + d^2*(6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6
+ (20*x^7)/7) + d*e*(6*x^2 + (14*x^3)/3 + (21*x^4)/2 - (8*x^5)/5 + (17*x^6)
/3 - (34*x^7)/7 + 5*x^8)
```

Maple [A]
time = 0.12, size = 146, normalized size = 0.93

method	result
norman	$\frac{20e^2x^9}{9}+\left(5de-\frac{17}{8}e^2\right)x^8+\left(\frac{20}{7}d^2-\frac{34}{7}de+\frac{17}{7}e^2\right)x^7+\left(-\frac{17}{6}d^2+\frac{17}{3}de-\frac{2}{3}e^2\right)x^6+\left(\frac{17}{5}d^2-\frac{8}{5}de+\right.$
default	$\frac{20e^2x^9}{9}+\frac{(40de-17e^2)x^8}{8}+\frac{(20d^2-34de+17e^2)x^7}{7}+\frac{(-17d^2+34de-4e^2)x^6}{6}+\frac{(17d^2-8de+21e^2)x^5}{5}+\frac{(-4d^2+42de+7e^2)x^4}{4}$
gosper	$\frac{20}{9}e^2x^9+5x^8de-\frac{17}{8}x^8e^2+\frac{20}{7}x^7d^2-\frac{34}{7}x^7de+\frac{17}{7}x^7e^2-\frac{17}{6}x^6d^2+\frac{17}{3}x^6de-\frac{2}{3}x^6e^2+\frac{17}{5}x^5d^2-\frac{8}{5}x^5$
risch	$\frac{20}{9}e^2x^9+5x^8de-\frac{17}{8}x^8e^2+\frac{20}{7}x^7d^2-\frac{34}{7}x^7de+\frac{17}{7}x^7e^2-\frac{17}{6}x^6d^2+\frac{17}{3}x^6de-\frac{2}{3}x^6e^2+\frac{17}{5}x^5d^2-\frac{8}{5}x^5$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE
)
```

```
[Out] 20/9*e^2*x^9+1/8*(40*d*e-17*e^2)*x^8+1/7*(20*d^2-34*d*e+17*e^2)*x^7+1/6*(-1
7*d^2+34*d*e-4*e^2)*x^6+1/5*(17*d^2-8*d*e+21*e^2)*x^5+1/4*(-4*d^2+42*d*e+7*
e^2)*x^4+1/3*(21*d^2+14*d*e+6*e^2)*x^3+1/2*(7*d^2+12*d*e)*x^2+6*d^2*x
```

Maxima [A]
time = 0.30, size = 145, normalized size = 0.92

$$\frac{20}{9}x^9e^2+\frac{1}{8}(40\,de-17\,e^2)x^8+\frac{1}{7}(20\,d^2-34\,de+17\,e^2)x^7-\frac{1}{6}(17\,d^2-34\,de+4\,e^2)x^6+\frac{1}{5}(17\,d^2-8\,de+21\,e^2)x^5-\frac{1}{4}(4\,d^2-42\,de-7\,e^2)x^4+\frac{1}{3}(21\,d^2+14\,de+6\,e^2)x^3+6\,d^2x+\frac{1}{2}(7\,d^2+12\,de)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 20/9*x^9*e^2 + 1/8*(40*d*e - 17*e^2)*x^8 + 1/7*(20*d^2 - 34*d*e + 17*e^2)*x
^7 - 1/6*(17*d^2 - 34*d*e + 4*e^2)*x^6 + 1/5*(17*d^2 - 8*d*e + 21*e^2)*x^5
- 1/4*(4*d^2 - 42*d*e - 7*e^2)*x^4 + 1/3*(21*d^2 + 14*d*e + 6*e^2)*x^3 + 6*
d^2*x + 1/2*(7*d^2 + 12*d*e)*x^2
```

Fricas [A]
time = 0.38, size = 142, normalized size = 0.90

$$\frac{20}{7}\,d^2x^7-\frac{17}{6}\,d^2x^6+\frac{17}{5}\,d^2x^5-d^2x^4+7\,d^2x^3+\frac{7}{2}\,d^2x^2+6\,d^2x+\frac{1}{2520}\left(5600\,x^9-5355\,x^8+6120\,x^7-1680\,x^6+10584\,x^5+4410\,x^4+5040\,x^3\right)e^2+\frac{1}{210}\left(1050\,dx^8-1020\,dx^7+1190\,dx^6-336\,dx^5+2205\,dx^4+980\,dx^3+1260\,dx^2\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 20/7*d^2*x^7 - 17/6*d^2*x^6 + 17/5*d^2*x^5 - d^2*x^4 + 7*d^2*x^3 + 7/2*d^2*
x^2 + 6*d^2*x + 1/2520*(5600*x^9 - 5355*x^8 + 6120*x^7 - 1680*x^6 + 10584*x
^5 + 4410*x^4 + 5040*x^3)*e^2 + 1/210*(1050*d*x^8 - 1020*d*x^7 + 1190*d*x^6
 - 336*d*x^5 + 2205*d*x^4 + 980*d*x^3 + 1260*d*x^2)*e
```

Sympy [A]
time = 0.02, size = 158, normalized size = 1.01

$$6d^2x + \frac{20e^2x^9}{9} + x^8 \cdot \left(5de - \frac{17e^2}{8}\right) + x^7 \cdot \left(\frac{20d^2}{7} - \frac{34de}{7} + \frac{17e^2}{7}\right) + x^6\left(-\frac{17d^2}{6} + \frac{17de}{3} - \frac{2e^2}{3}\right) + x^5 \cdot \left(\frac{17d^2}{5} - \frac{8de}{5} + \frac{21e^2}{5}\right) + x^4\left(-d^2 + \frac{21de}{2} + \frac{7e^2}{4}\right) + x^3 \cdot \left(7d^2 + \frac{14de}{3} + 2e^2\right) + x^2 \cdot \left(\frac{7d^2}{2} + 6de\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 6*d**2*x + 20*e**2*x**9/9 + x**8*(5*d*e - 17*e**2/8) + x**7*(20*d**2/7 - 34
*d*e/7 + 17*e**2/7) + x**6*(-17*d**2/6 + 17*d*e/3 - 2*e**2/3) + x**5*(17*d*
*2/5 - 8*d*e/5 + 21*e**2/5) + x**4*(-d**2 + 21*d*e/2 + 7*e**2/4) + x**3*(7*
d**2 + 14*d*e/3 + 2*e**2) + x**2*(7*d**2/2 + 6*d*e)
```

Giac [A]
time = 3.91, size = 160, normalized size = 1.02

$$\frac{20}{9}x^9e^2 + 5\,dx^8e + \frac{20}{7}d^2x^7 - \frac{17}{8}x^8e^2 - \frac{34}{7}dx^7e - \frac{17}{6}d^2x^6 + \frac{17}{7}x^7e^2 + \frac{17}{3}dx^6e + \frac{17}{5}d^2x^5 - \frac{2}{3}x^6e^2 - \frac{8}{5}dx^5e - d^2x^4 + \frac{21}{5}x^5e^2 + \frac{21}{2}dx^4e + 7\,d^2x^3 + \frac{7}{4}x^4e^2 + \frac{14}{3}dx^3e + \frac{7}{2}d^2x^2 + 2\,x^3e^2 + 6\,dx^2e + 6\,d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 20/9*x^9*e^2 + 5*d*x^8*e + 20/7*d^2*x^7 - 17/8*x^8*e^2 - 34/7*d*x^7*e - 17/
6*d^2*x^6 + 17/7*x^7*e^2 + 17/3*d*x^6*e + 17/5*d^2*x^5 - 2/3*x^6*e^2 - 8/5*
d*x^5*e - d^2*x^4 + 21/5*x^5*e^2 + 21/2*d*x^4*e + 7*d^2*x^3 + 7/4*x^4*e^2 +
 14/3*d*x^3*e + 7/2*d^2*x^2 + 2*x^3*e^2 + 6*d*x^2*e + 6*d^2*x
```

Mupad [B]
time = 4.11, size = 137, normalized size = 0.87

$$x^3\left(7\,d^2 + \frac{14\,d\,e}{3} + 2\,e^2\right) + x^4\left(-d^2 + \frac{21\,d\,e}{2} + \frac{7\,e^2}{4}\right) - x^6\left(\frac{17\,d^2}{6} - \frac{17\,d\,e}{3} + \frac{2\,e^2}{3}\right) + x^5\left(\frac{17\,d^2}{5} - \frac{8\,d\,e}{5} + \frac{21\,e^2}{5}\right) + x^7\left(\frac{20\,d^2}{7} - \frac{34\,d\,e}{7} + \frac{17\,e^2}{7}\right) + 6\,d^2\,x + \frac{20\,e^2\,x^9}{9} + \frac{d\,x^2\,(7\,d + 12\,e)}{2} + \frac{e\,x^8\,(40\,d - 17\,e)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] x^3*((14*d*e)/3 + 7*d^2 + 2*e^2) + x^4*((21*d*e)/2 - d^2 + (7*e^2)/4) - x^6
*((17*d^2)/6 - (17*d*e)/3 + (2*e^2)/3) + x^5*((17*d^2)/5 - (8*d*e)/5 + (21*
e^2)/5) + x^7*((20*d^2)/7 - (34*d*e)/7 + (17*e^2)/7) + 6*d^2*x + (20*e^2*x^
9)/9 + (d*x^2*(7*d + 12*e))/2 + (e*x^8*(40*d - 17*e))/8
```

3.291 $\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right) dx$

Optimal. Leaf size=93

$$6dx+\frac{1}{2}(7d+6e)x^2+\frac{7}{3}(3d+e)x^3-\frac{1}{4}(4d-21e)x^4+\frac{1}{5}(17d-4e)x^5-\frac{17}{6}(d-e)x^6+\frac{1}{7}(20d-17e)x^7+\frac{5ex^8}{2}$$

```
[Out] 6*d*x+1/2*(7*d+6*e)*x^2+7/3*(3*d+e)*x^3-1/4*(4*d-21*e)*x^4+1/5*(17*d-4*e)*x^5-17/6*(d-e)*x^6+1/7*(20*d-17*e)*x^7+5/2*e*x^8
```

Rubi [A]
time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1642}

$$\frac{1}{7}x^7(20d-17e)-\frac{17}{6}x^6(d-e)+\frac{1}{5}x^5(17d-4e)-\frac{1}{4}x^4(4d-21e)+\frac{7}{3}x^3(3d+e)+\frac{1}{2}x^2(7d+6e)+6dx+\frac{5ex^8}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d*x + ((7*d + 6*e)*x^2)/2 + (7*(3*d + e)*x^3)/3 - ((4*d - 21*e)*x^4)/4 + ((17*d - 4*e)*x^5)/5 - (17*(d - e)*x^6)/6 + ((20*d - 17*e)*x^7)/7 + (5*e*x^8)/2
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right) dx=\int \left(6d+(7d+6e)x+7(3d+e)x^2-(4d-21e)\right.$$
$$=6dx+\frac{1}{2}(7d+6e)x^2+\frac{7}{3}(3d+e)x^3-\frac{1}{4}(4d-21$$

Mathematica [A]
time = 0.01, size = 93, normalized size = 1.00

$$6dx+\frac{1}{2}(7d+6e)x^2+\frac{7}{3}(3d+e)x^3+\frac{1}{4}(-4d+21e)x^4+\frac{1}{5}(17d-4e)x^5-\frac{17}{6}(d-e)x^6+\frac{1}{7}(20d-17e)x^7+\frac{5ex^8}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]

[Out] 6*d*x + ((7*d + 6*e)*x^2)/2 + (7*(3*d + e)*x^3)/3 + ((-4*d + 21*e)*x^4)/4 + ((17*d - 4*e)*x^5)/5 - (17*(d - e)*x^6)/6 + ((20*d - 17*e)*x^7)/7 + (5*e*x^8)/2

Maple [A]
time = 0.05, size = 84, normalized size = 0.90

method	result
norman	$\frac{5e\,x^8}{2}+\left(\frac{20d}{7}-\frac{17e}{7}\right)x^7+\left(-\frac{17d}{6}+\frac{17e}{6}\right)x^6+\left(\frac{17d}{5}-\frac{4e}{5}\right)x^5+\left(-d+\frac{21e}{4}\right)x^4+\left(7d+\frac{7e}{3}\right)x^3+\left(\frac{7d}{2}+\right.$
gosper	$\frac{5}{2}e\,x^8+\frac{20}{7}x^7d-\frac{17}{7}x^7e-\frac{17}{6}x^6d+\frac{17}{6}e\,x^6+\frac{17}{5}d\,x^5-\frac{4}{5}e\,x^5-d\,x^4+\frac{21}{4}e\,x^4+7d\,x^3+\frac{7}{3}e\,x^3+\frac{7}{2}d\,x$
default	$\frac{5e\,x^8}{2}+\frac{(20d-17e)x^7}{7}+\frac{(-17d+17e)x^6}{6}+\frac{(17d-4e)x^5}{5}+\frac{(-4d+21e)x^4}{4}+\frac{(21d+7e)x^3}{3}+\frac{(7d+6e)x^2}{2}+6dx$
risch	$\frac{5}{2}e\,x^8+\frac{20}{7}x^7d-\frac{17}{7}x^7e-\frac{17}{6}x^6d+\frac{17}{6}e\,x^6+\frac{17}{5}d\,x^5-\frac{4}{5}e\,x^5-d\,x^4+\frac{21}{4}e\,x^4+7d\,x^3+\frac{7}{3}e\,x^3+\frac{7}{2}d\,x$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)

[Out] 5/2*e*x^8+1/7*(20*d-17*e)*x^7+1/6*(-17*d+17*e)*x^6+1/5*(17*d-4*e)*x^5+1/4*(-4*d+21*e)*x^4+1/3*(21*d+7*e)*x^3+1/2*(7*d+6*e)*x^2+6*d*x

Maxima [A]
time = 0.31, size = 86, normalized size = 0.92

$$\frac{5}{2}x^8e+\frac{1}{7}(20\,d-17\,e)x^7-\frac{17}{6}(d-e)x^6+\frac{1}{5}(17\,d-4\,e)x^5-\frac{1}{4}(4\,d-21\,e)x^4+\frac{7}{3}(3\,d+e)x^3+\frac{1}{2}(7\,d+6\,e)x^2+6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxima")

[Out] 5/2*x^8*e + 1/7*(20*d - 17*e)*x^7 - 17/6*(d - e)*x^6 + 1/5*(17*d - 4*e)*x^5 - 1/4*(4*d - 21*e)*x^4 + 7/3*(3*d + e)*x^3 + 1/2*(7*d + 6*e)*x^2 + 6*d*x

Fricas [A]
time = 0.34, size = 81, normalized size = 0.87

$$\frac{20}{7}\,dx^7-\frac{17}{6}\,dx^6+\frac{17}{5}\,dx^5-dx^4+7\,dx^3+\frac{7}{2}\,dx^2+6\,dx+\frac{1}{420}\left(1050\,x^8-1020\,x^7+1190\,x^6-336\,x^5+2205\,x^4+980\,x^3+1260\,x^2\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fricas")

[Out] 20/7*d*x^7 - 17/6*d*x^6 + 17/5*d*x^5 - d*x^4 + 7*d*x^3 + 7/2*d*x^2 + 6*d*x + 1/420*(1050*x^8 - 1020*x^7 + 1190*x^6 - 336*x^5 + 2205*x^4 + 980*x^3 + 1260*x^2)*e

Sympy [A]
time = 0.01, size = 87, normalized size = 0.94

$$6dx+\frac{5ex^8}{2}+x^7\cdot\left(\frac{20d}{7}-\frac{17e}{7}\right)+x^6\left(-\frac{17d}{6}+\frac{17e}{6}\right)+x^5\cdot\left(\frac{17d}{5}-\frac{4e}{5}\right)+x^4\left(-d+\frac{21e}{4}\right)+x^3\cdot\left(7d+\frac{7e}{3}\right)+x^2\cdot\left(\frac{7d}{2}+3e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 6*d*x + 5*e*x**8/2 + x**7*(20*d/7 - 17*e/7) + x**6*(-17*d/6 + 17*e/6) + x**
5*(17*d/5 - 4*e/5) + x**4*(-d + 21*e/4) + x**3*(7*d + 7*e/3) + x**2*(7*d/2
+ 3*e)
```

Giac [A]
time = 4.41, size = 90, normalized size = 0.97

$$\frac{5}{2}x^8e+\frac{20}{7}dx^7-\frac{17}{7}x^7e-\frac{17}{6}dx^6+\frac{17}{6}x^6e+\frac{17}{5}dx^5-\frac{4}{5}x^5e-dx^4+\frac{21}{4}x^4e+7dx^3+\frac{7}{3}x^3e+\frac{7}{2}dx^2+3x^2e+6dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac"
)
```

```
[Out] 5/2*x^8*e + 20/7*d*x^7 - 17/7*x^7*e - 17/6*d*x^6 + 17/6*x^6*e + 17/5*d*x^5
- 4/5*x^5*e - d*x^4 + 21/4*x^4*e + 7*d*x^3 + 7/3*x^3*e + 7/2*d*x^2 + 3*x^2*
e + 6*d*x
```

Mupad [B]
time = 0.05, size = 77, normalized size = 0.83

$$\frac{5\,e\,x^8}{2}+\left(\frac{20\,d}{7}-\frac{17\,e}{7}\right)\,x^7+\left(\frac{17\,e}{6}-\frac{17\,d}{6}\right)\,x^6+\left(\frac{17\,d}{5}-\frac{4\,e}{5}\right)\,x^5+\left(\frac{21\,e}{4}-d\right)\,x^4+\left(7\,d+\frac{7\,e}{3}\right)\,x^3+\left(\frac{7\,d}{2}+3\,e\right)\,x^2+6\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] x^2*((7*d)/2 + 3*e) + x^3*(7*d + (7*e)/3) + x^5*((17*d)/5 - (4*e)/5) - x^6*
((17*d)/6 - (17*e)/6) + x^7*((20*d)/7 - (17*e)/7) + 6*d*x + (5*e*x^8)/2 - x
^4*(d - (21*e)/4)
```

3.292 $\int (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)\,dx$

Optimal. Leaf size=42

$$6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}$$

```
[Out] 6*x+7/2*x^2+7*x^3-x^4+17/5*x^5-17/6*x^6+20/7*x^7
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1671}

$$\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx &= \int \left(6+7x+21x^2-4x^3+17x^4-17x^5+20x^6\right)\,dx\\ &= 6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 42, normalized size = 1.00

$$6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

Maple [A]
time = 0.03, size = 35, normalized size = 0.83

method	result	size
gosper	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35
default	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35
norman	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35
risch	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
[Out] 6*x+7/2*x^2+7*x^3-x^4+17/5*x^5-17/6*x^6+20/7*x^7
```

Maxima [A]
time = 0.30, size = 34, normalized size = 0.81

$$\frac{20}{7}\,x^7-\frac{17}{6}\,x^6+\frac{17}{5}\,x^5-x^4+7\,x^3+\frac{7}{2}\,x^2+6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxima")
[Out] 20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

Fricas [A]
time = 0.34, size = 34, normalized size = 0.81

$$\frac{20}{7}\,x^7-\frac{17}{6}\,x^6+\frac{17}{5}\,x^5-x^4+7\,x^3+\frac{7}{2}\,x^2+6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fricas")
[Out] 20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

Sympy [A]
time = 0.01, size = 37, normalized size = 0.88

$$\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 20*x**7/7 - 17*x**6/6 + 17*x**5/5 - x**4 + 7*x**3 + 7*x**2/2 + 6*x
```

Giac [A]
time = 4.02, size = 34, normalized size = 0.81

$$\frac{20}{7}x^7 - \frac{17}{6}x^6 + \frac{17}{5}x^5 - x^4 + 7x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac")
[Out] 20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

Mupad [B]
time = 0.03, size = 34, normalized size = 0.81

$$\frac{20x^7}{7} - \frac{17x^6}{6} + \frac{17x^5}{5} - x^4 + 7x^3 + \frac{7x^2}{2} + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
[Out] 6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

3.293 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$

Optimal. Leaf size=228

$$-\frac{(20d^5+17d^4e+17d^3e^2+4d^2e^3+21de^4-7e^5)\,x}{e^6}+\frac{(20d^4+17d^3e+17d^2e^2+4de^3+21e^4)\,x^2}{2e^5}-\frac{(20d^3+17d$$

```
[Out] -(20*d^5+17*d^4*e+17*d^3*e^2+4*d^2*e^3+21*d*e^4-7*e^5)*x/e^6+1/2*(20*d^4+17
*d^3*e+17*d^2*e^2+4*d*e^3+21*e^4)*x^2/e^5-1/3*(20*d^3+17*d^2*e+17*d*e^2+4*e
^3)*x^3/e^4+1/4*(20*d^2+17*d*e+17*e^2)*x^4/e^3-1/5*(20*d+17*e)*x^5/e^2+10/3
*x^6/e+(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/
e^7
```

Rubi [A]
time = 0.12, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1642}

$$\frac{x^4(20d^2+17de+17e^2)}{4e^3}-\frac{x^3(20d^3+17d^2e+17de^2+4e^3)}{3e^4}+\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^7}+\frac{x^2(20d^4+17d^3e+17d^2e^2+4de^3+21e^4)}{2e^5}-\frac{x(20d^5+17d^4e+17d^3e^2+4d^2e^3+21de^4-7e^5)}{e^6}-\frac{x^5(20d+17e)}{5e^2}+\frac{10x^6}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

```
[Out] -(((20*d^5 + 17*d^4*e + 17*d^3*e^2 + 4*d^2*e^3 + 21*d*e^4 - 7*e^5)*x)/e^6)
+ ((20*d^4 + 17*d^3*e + 17*d^2*e^2 + 4*d*e^3 + 21*e^4)*x^2)/(2*e^5) - ((20*
d^3 + 17*d^2*e + 17*d*e^2 + 4*e^3)*x^3)/(3*e^4) + ((20*d^2 + 17*d*e + 17*e^
2)*x^4)/(4*e^3) - ((20*d + 17*e)*x^5)/(5*e^2) + (10*x^6)/(3*e) + ((5*d^2 -
2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/
e^7
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx = \int \left(\frac{-20d^5-17d^4e-17d^3e^2-4d^2e^3-21de^4+7e^5}{e^6}+\frac{(2$$

$$= -\frac{(20d^5+17d^4e+17d^3e^2+4d^2e^3+21de^4-7e^5)\,x}{e^6}+\frac{(20$$

Mathematica [A]
time = 0.04, size = 179, normalized size = 0.79

$$\frac{ex(-1200d^5+60d^4e(-17+10x)-10d^3e^2(102-51x+40x^2)+10d^2e^3(-24+51x-34x^2+30x^3)-5de^4(252-24x+68x^2-51x^3+48x^4)+e^5(420+630x-80x^2+255x^3-204x^4+200x^5))+60(20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5+6e^6)\log(d+ex)}{60e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

```
[Out] (e*x*(-1200*d^5 + 60*d^4*e*(-17 + 10*x) - 10*d^3*e^2*(102 - 51*x + 40*x^2)
+ 10*d^2*e^3*(-24 + 51*x - 34*x^2 + 30*x^3) - 5*d*e^4*(252 - 24*x + 68*x^2
- 51*x^3 + 48*x^4) + e^5*(420 + 630*x - 80*x^2 + 255*x^3 - 204*x^4 + 200*x^
5)) + 60*(20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5
 + 6*e^6)*Log[d + e*x])/(60*e^7)
```

Maple [A]
time = 0.12, size = 249, normalized size = 1.09

method	result
norman	$\frac{10x^6}{3e}-\frac{(20d+17e)x^5}{5e^2}+\frac{(20d^2+17de+17e^2)x^4}{4e^3}-\frac{(20d^3+17d^2e+17de^2+4e^3)x^3}{3e^4}+\frac{(20d^4+17d^3e+17d^2e^2+4de^3+21e^4)x^2}{2e^5}-$
default	$\frac{(20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5+6e^6)\ln(ex+d)}{e^7}-\frac{-\frac{10}{3}x^6e^5+4x^5e^4d+\frac{17}{5}x^5e^5-5d^2e^3x^4-\frac{17}{4}x^4e^4d-\frac{17}{4}x^4e^5+\frac{20}{3}d^3e}{}$
risch	$\frac{17x^4}{4e}-\frac{4x^3}{3e}+\frac{7x}{e}+\frac{6\ln(ex+d)}{e}-\frac{17x^5}{5e}+\frac{20\ln(ex+d)d^6}{e^7}+\frac{17\ln(ex+d)d^5}{e^6}+\frac{17\ln(ex+d)d^4}{e^5}+\frac{4\ln(ex+d)d^3}{e^4}+\frac{21\ln(e}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] (20*d^6+17*d^5*e+17*d^4*e^2+4*d^3*e^3+21*d^2*e^4-7*d*e^5+6*e^6)/e^7*ln(e*x+
d)-1/e^6*(-10/3*x^6*e^5+4*x^5*e^4*d+17/5*x^5*e^5-5*d^2*e^3*x^4-17/4*x^4*e^4
*d-17/4*x^4*e^5+20/3*d^3*e^2*x^3+17/3*d^2*e^3*x^3+17/3*d*e^4*x^3+4/3*e^5*x^
3-10*d^4*e*x^2-17/2*d^3*e^2*x^2-17/2*d^2*e^3*x^2-2*d*e^4*x^2-21/2*e^5*x^2+2
0*d^5*x+17*d^4*e*x+17*d^3*e^2*x+4*d^2*e^3*x+21*d*e^4*x-7*e^5*x)
```

Maxima [A]
time = 0.31, size = 207, normalized size = 0.91

$$(20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5+6e^6)e^{(-7)}\log(xe+d)+\frac{1}{60}(200x^6e^5-12(20de^4+17e^5)x^5+15(20d^2e^3+17de^4+17e^5)x^4-20(20d^3e^2+17d^2e^3+17de^4+4e^5)x^3+30(20d^4e+17d^3e^2+17d^2e^3+4de^4+21e^5)x^2-60(20d^5+17d^4e+17d^3e^2+4d^2e^3+21de^4-7e^5)x)e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="maxim
a")
```

```
[Out] (20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)
*e^(-7)*log(x*e + d) + 1/60*(200*x^6*e^5 - 12*(20*d*e^4 + 17*e^5)*x^5 + 15*
(20*d^2*e^3 + 17*d*e^4 + 17*e^5)*x^4 - 20*(20*d^3*e^2 + 17*d^2*e^3 + 17*d*e
```

```
^4 + 4*e^5)*x^3 + 30*(20*d^4*e + 17*d^3*e^2 + 17*d^2*e^3 + 4*d*e^4 + 21*e^5
)*x^2 - 60*(20*d^5 + 17*d^4*e + 17*d^3*e^2 + 4*d^2*e^3 + 21*d*e^4 - 7*e^5)*
x)*e^(-6)
```

Fricas [A]
time = 0.33, size = 212, normalized size = 0.93

$$-\frac{1}{60}\left(1200\,d^5xe-\left(200\,x^6-204\,x^5+255\,x^4-80\,x^3+630\,x^2+420\,x\right)e^6+5\left(48\,dx^5-51\,dx^4+68\,dx^3-24\,dx^2+252\,dx\right)e^5-10\left(30\,d^2x^4-34\,d^2x^3+51\,d^2x^2-24\,d^2x\right)e^4+10\left(40\,d^3x^3-51\,d^3x^2+102\,d^3x\right)e^3-60\left(10\,d^4x^2-17\,d^4x\right)e^2-60\left(20\,d^6+17\,d^5e+17\,d^4e^2+4\,d^3e^3+21\,d^2e^4-7\,de^5+6\,e^6\right)\log\left(xe+d\right)\right)e^{(-7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="frica
s")
```

```
[Out] -1/60*(1200*d^5*x*e - (200*x^6 - 204*x^5 + 255*x^4 - 80*x^3 + 630*x^2 + 420
*x)*e^6 + 5*(48*d*x^5 - 51*d*x^4 + 68*d*x^3 - 24*d*x^2 + 252*d*x)*e^5 - 10*
(30*d^2*x^4 - 34*d^2*x^3 + 51*d^2*x^2 - 24*d^2*x)*e^4 + 10*(40*d^3*x^3 - 51
*d^3*x^2 + 102*d^3*x)*e^3 - 60*(10*d^4*x^2 - 17*d^4*x)*e^2 - 60*(20*d^6 + 1
7*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)*log(x*e +
d))*e^(-7)
```

Sympy [A]
time = 0.22, size = 235, normalized size = 1.03

$$x^5\left(-\frac{4d}{e^2}-\frac{17}{5e}\right)+x^4\cdot\left(\frac{5d^2}{e^3}+\frac{17d}{4e^2}+\frac{17}{4e}\right)+x^3\left(-\frac{20d^3}{3e^4}-\frac{17d^2}{3e^3}-\frac{17d}{3e^2}-\frac{4}{3e}\right)+x^2\cdot\left(\frac{10d^4}{e^5}+\frac{17d^3}{2e^4}+\frac{17d^2}{2e^3}+\frac{2d}{e^2}+\frac{21}{2e}\right)+x\left(-\frac{20d^5}{e^6}-\frac{17d^4}{e^5}-\frac{17d^3}{e^4}-\frac{4d^2}{e^3}-\frac{21d}{e^2}+\frac{7}{e}\right)+\frac{10x^6}{3e}+\frac{(5d^2-2de+3e^2)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log\left(d+ex\right)}{e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d),x)
```

```
[Out] x**5*(-4*d/e**2 - 17/(5*e)) + x**4*(5*d**2/e**3 + 17*d/(4*e**2) + 17/(4*e))
 + x**3*(-20*d**3/(3*e**4) - 17*d**2/(3*e**3) - 17*d/(3*e**2) - 4/(3*e)) +
x**2*(10*d**4/e**5 + 17*d**3/(2*e**4) + 17*d**2/(2*e**3) + 2*d/e**2 + 21/(2
*e)) + x*(-20*d**5/e**6 - 17*d**4/e**5 - 17*d**3/e**4 - 4*d**2/e**3 - 21*d/
e**2 + 7/e) + 10*x**6/(3*e) + (5*d**2 - 2*d*e + 3*e**2)*(4*d**4 + 5*d**3*e
+ 3*d**2*e**2 - d*e**3 + 2*e**4)*log(d + e*x)/e**7
```

Giac [A]
time = 3.73, size = 228, normalized size = 1.00

$$\left(20\,d^6+17\,d^5e+17\,d^4e^2+4\,d^3e^3+21\,d^2e^4-7\,de^5+6\,e^6\right)e^{(-7)}\log\left(|xe+d|\right)+\frac{1}{60}\left(200\,x^6e^5-240\,dx^5e^4+300\,d^2x^4e^3-400\,d^3x^3e^2+600\,d^4x^2e-1200\,d^5x-204\,x^5e^5+255\,dx^4e^4-340\,d^2x^3e^3+510\,d^3x^2e^2-1020\,d^4xe+255\,x^4e^5-340\,dx^3e^4+510\,d^2x^2e^3-1020\,d^3xe^2-80\,x^3e^5+120\,dx^2e^4-240\,d^2xe^3+630\,x^2e^5-1260\,dxe^4+420\,xe^5\right)e^{(-6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="giac"
)
```

```
[Out] (20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)
*e^(-7)*log(abs(x*e + d)) + 1/60*(200*x^6*e^5 - 240*d*x^5*e^4 + 300*d^2*x^4
```

```
*e^3 - 400*d^3*x^3*e^2 + 600*d^4*x^2*e - 1200*d^5*x - 204*x^5*e^5 + 255*d*x
^4*e^4 - 340*d^2*x^3*e^3 + 510*d^3*x^2*e^2 - 1020*d^4*x*e + 255*x^4*e^5 - 3
40*d*x^3*e^4 + 510*d^2*x^2*e^3 - 1020*d^3*x*e^2 - 80*x^3*e^5 + 120*d*x^2*e^
4 - 240*d^2*x*e^3 + 630*x^2*e^5 - 1260*d*x*e^4 + 420*x*e^5)*e^(-6)
```

Mupad [B]
time = 4.14, size = 260, normalized size = 1.14

$$x\left(\frac{7}{e}-\frac{d\left(\frac{21}{e}+\frac{d\left(\frac{4}{e}+\frac{d\left(\frac{17}{e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)-x^5\left(\frac{4\,d}{e^2}+\frac{17}{5\,e}\right)+x^4\left(\frac{17}{4\,e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{4\,e}\right)-x^3\left(\frac{4}{3\,e}+\frac{d\left(\frac{17}{e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{e}\right)}{3\,e}\right)+x^2\left(\frac{21}{2\,e}+\frac{d\left(\frac{4}{e}+\frac{d\left(\frac{17}{e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{e}\right)}{e}\right)}{2\,e}\right)+\frac{10\,x^6}{3\,e}+\frac{\ln\left(d+e\,x\right)\,\left(20\,d^6+17\,d^5\,e+17\,d^4\,e^2+4\,d^3\,e^3+21\,d^2\,e^4-7\,d\,e^5+6\,e^6\right)}{e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x),x)
```

```
[Out] x*(7/e - (d*(21/e + (d*(4/e + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/e))/e)
)/e) - x^5*((4*d)/e^2 + 17/(5*e)) + x^4*(17/(4*e) + (d*((20*d)/e^2 + 17/e))
/(4*e)) - x^3*(4/(3*e) + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/(3*e)) + x^
2*(21/(2*e) + (d*(4/e + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/e))/(2*e)) +
 (10*x^6)/(3*e) + (log(d + e*x)*(17*d^5*e - 7*d*e^5 + 20*d^6 + 6*e^6 + 21*d
^2*e^4 + 4*d^3*e^3 + 17*d^4*e^2))/e^7
```

3.294 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$

Optimal. Leaf size=228

$$\frac{(100d^4+68d^3e+51d^2e^2+8de^3+21e^4)\,x}{e^6}-\frac{(80d^3+51d^2e+34de^2+4e^3)\,x^2}{2e^5}+\frac{(60d^2+34de+17e^2)\,x^3}{3e^4}-\frac{(40$$

```
[Out] (100*d^4+68*d^3*e+51*d^2*e^2+8*d*e^3+21*e^4)*x/e^6-1/2*(80*d^3+51*d^2*e+34*
d*e^2+4*e^3)*x^2/e^5+1/3*(60*d^2+34*d*e+17*e^2)*x^3/e^4-1/4*(40*d+17*e)*x^4
/e^3+4*x^5/e^2-(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/e^
7/(e*x+d)-(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5)*ln(e*x+d)
/e^7
```

Rubi [A]
time = 0.13, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = $\{1642\}$

$$\frac{x^3(60d^2+34de+17e^2)}{3e^4}-\frac{x^2(80d^3+51d^2e+34de^2+4e^3)}{2e^5}-\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^7(d+ex)}+\frac{x(100d^4+68d^3e+51d^2e^2+8de^3+21e^4)}{e^6}-\frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)\log(d+ex)}{e^7}-\frac{x^4(40d+17e)}{4e^3}+\frac{4x^5}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x]
```

```
[Out] ((100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x)/e^6 - ((80*d^3 + 5
1*d^2*e + 34*d*e^2 + 4*e^3)*x^2)/(2*e^5) + ((60*d^2 + 34*d*e + 17*e^2)*x^3)
/(3*e^4) - ((40*d + 17*e)*x^4)/(4*e^3) + (4*x^5)/e^2 - ((5*d^2 - 2*d*e + 3*
e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e^7*(d + e*x)) - ((120
*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*Log[d + e*x])
/e^7
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx = \int \left(\frac{100d^4+68d^3e+51d^2e^2+8de^3+21e^4}{e^6}-\frac{(80d^3+51d\right.$$

$$= \frac{(100d^4+68d^3e+51d^2e^2+8de^3+21e^4)\,x}{e^6}-\frac{(80d^3+51d$$

Mathematica [A]

time = 0.06, size = 223, normalized size = 0.98

$$\frac{12e(100d^4+68d^3e+51d^2e^2+8de^3+21e^4)\,x-6e^2(80d^3+51d^2e+34de^2+4e^3)\,x^2+4e^3(60d^2+34de+17e^2)\,x^3-3e^4(40d+17e)x^4+48e^5x^5-\frac{12(20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5+6e^6)}{d+ex}-12(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)\log(d+ex)}{12e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x
]
```

```
[Out] (12*e*(100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x - 6*e^2*(80*d^
3 + 51*d^2*e + 34*d*e^2 + 4*e^3)*x^2 + 4*e^3*(60*d^2 + 34*d*e + 17*e^2)*x^3
 - 3*e^4*(40*d + 17*e)*x^4 + 48*e^5*x^5 - (12*(20*d^6 + 17*d^5*e + 17*d^4*e
^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6))/(d + e*x) - 12*(120*d^5 + 8
5*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*Log[d + e*x])/(12*e^7
)
```

Maple [A]

time = 0.11, size = 240, normalized size = 1.05

method	result
norman	$\frac{\frac{\left(120d^6+85d^5e+68d^4e^2+12d^3e^3+42d^2e^4-7d\,e^5+6e^6\right)x}{e^6d}+\frac{4x^6}{e}-\frac{(24d+17e)x^5}{4e^2}+\frac{\left(120d^2+85de+68e^2\right)x^4}{12e^3}-\frac{\left(120d^3+85d^2e+68d\,e^2+12e^3\right)x^3}{6e^4}+\frac{(12}{}}{ex+d}$
default	$\frac{\left(-120d^5-85d^4e-68d^3e^2-12d^2e^3-42d\,e^4+7e^5\right)\ln(ex+d)}{e^7}-\frac{20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7d\,e^5+6e^6}{e^7(ex+d)}+\frac{4e^4x^5-10d\,e^3x}{}$
risch	$\frac{17x^3}{3e^2}-\frac{2x^2}{e^2}-\frac{6}{e(ex+d)}+\frac{7\ln(ex+d)}{e^2}-\frac{17x^4}{4e^2}-\frac{120\ln(ex+d)d^5}{e^7}-\frac{85\ln(ex+d)d^4}{e^6}-\frac{68\ln(ex+d)d^3}{e^5}-\frac{12\ln(ex+d)d^2}{e^4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] (-120*d^5-85*d^4*e-68*d^3*e^2-12*d^2*e^3-42*d*e^4+7*e^5)/e^7*ln(e*x+d)-(20*
d^6+17*d^5*e+17*d^4*e^2+4*d^3*e^3+21*d^2*e^4-7*d*e^5+6*e^6)/e^7/(e*x+d)+1/e
^6*(4*e^4*x^5-10*d*e^3*x^4-17/4*x^4*e^4+20*d^2*e^2*x^3+34/3*d*e^3*x^3+17/3*
e^4*x^3-40*d^3*e*x^2-51/2*d^2*e^2*x^2-17*d*e^3*x^2-2*e^4*x^2+100*d^4*x+68*d
^3*e*x+51*d^2*e^2*x+8*d*e^3*x+21*e^4*x)
```

Maxima [A]

time = 0.35, size = 214, normalized size = 0.94

$$-\left(120\,d^5+85\,d^4e+68\,d^3e^2+12\,d^2e^3+42\,de^4-7\,e^5\right)e^{(-7)}\log\left(xe+d\right)+\frac{1}{12}\left(48\,x^5e^4-3\left(40\,de^3+17\,e^4\right)x^4+4\left(60\,d^2e^2+34\,de^3+17\,e^4\right)x^3-6\left(80\,d^3e+51\,d^2e^2+34\,de^3+4\,e^4\right)x^2+12\left(100\,d^4+68\,d^3e+51\,d^2e^2+8\,de^3+21\,e^4\right)x\right)e^{(-6)}-\frac{20\,d^6+17\,d^5e+17\,d^4e^2+4\,d^3e^3+21\,d^2e^4-7\,de^5+6\,e^6}{xe^8+de^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="max
ima")
```

```
[Out] -(120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*e^(-7)*l
og(x*e + d) + 1/12*(48*x^5*e^4 - 3*(40*d*e^3 + 17*e^4)*x^4 + 4*(60*d^2*e^2
+ 34*d*e^3 + 17*e^4)*x^3 - 6*(80*d^3*e + 51*d^2*e^2 + 34*d*e^3 + 4*e^4)*x^2
 + 12*(100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x)*e^(-6) - (20*
d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)/(x*
e^8 + d*e^7)
```

Fricas [A]
time = 0.35, size = 293, normalized size = 1.29

$$-\frac{240\,d^{6}-(48\,x^{6}-51\,x^{5}+68\,x^{4}-24\,x^{3}+252\,x^{2}-72)e^{6}+(72\,dx^{5}-85\,dx^{4}+136\,dx^{3}-72\,dx^{2}-252\,dx-84\,d)e^{5}-2\,(60\,d^{2}x^{4}-85\,d^{2}x^{3}+204\,d^{2}x^{2}+48\,d^{2}x-126\,d^{2})e^{4}+6\,(40\,d^{3}x^{3}-85\,d^{3}x^{2}-102\,d^{3}x+8\,d^{3})e^{3}-12\,(60\,d^{4}x^{2}+68\,d^{4}x-17\,d^{4})e^{2}-12\,(100\,d^{5}x-17\,d^{5})e+12\,(120\,d^{6}-7\,xe^{6}+7\,(6\,dx-d)e^{5}+6\,(2\,d^{2}x+7\,d^{2})e^{4}+4\,(17\,d^{3}x+3\,d^{3})e^{3}+17\,(5\,d^{4}x+4\,d^{4})e^{2}+5\,(24\,d^{5}x+17\,d^{5})e)\log(xe+d)}{12\,(xe^{8}+de^{7})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="fri
cas")
```

```
[Out] -1/12*(240*d^6 - (48*x^6 - 51*x^5 + 68*x^4 - 24*x^3 + 252*x^2 - 72)*e^6 + (
72*d*x^5 - 85*d*x^4 + 136*d*x^3 - 72*d*x^2 - 252*d*x - 84*d)*e^5 - 2*(60*d^
2*x^4 - 85*d^2*x^3 + 204*d^2*x^2 + 48*d^2*x - 126*d^2)*e^4 + 6*(40*d^3*x^3
- 85*d^3*x^2 - 102*d^3*x + 8*d^3)*e^3 - 12*(60*d^4*x^2 + 68*d^4*x - 17*d^4)
*e^2 - 12*(100*d^5*x - 17*d^5)*e + 12*(120*d^6 - 7*x*e^6 + 7*(6*d*x - d)*e^
5 + 6*(2*d^2*x + 7*d^2)*e^4 + 4*(17*d^3*x + 3*d^3)*e^3 + 17*(5*d^4*x + 4*d^
4)*e^2 + 5*(24*d^5*x + 17*d^5)*e)*log(x*e + d))/(x*e^8 + d*e^7)
```

Sympy [A]
time = 0.45, size = 238, normalized size = 1.04

$$x^{4}\left(-\frac{10d}{e^{3}}-\frac{17}{4e^{2}}\right)+x^{3}\cdot\left(\frac{20d^{2}}{e^{4}}+\frac{34d}{3e^{3}}+\frac{17}{3e^{2}}\right)+x^{2}\left(-\frac{40d^{3}}{e^{5}}-\frac{51d^{2}}{2e^{4}}-\frac{17d}{e^{3}}-\frac{2}{e^{2}}\right)+x\left(\frac{100d^{4}}{e^{6}}+\frac{68d^{3}}{e^{5}}+\frac{51d^{2}}{e^{4}}+\frac{8d}{e^{3}}+\frac{21}{e^{2}}\right)+\frac{-20d^{6}-17d^{5}e-17d^{4}e^{2}-4d^{3}e^{3}-21d^{2}e^{4}+7de^{5}-6e^{6}}{de^{7}+e^{8}x}+\frac{4x^{5}}{e^{2}}-\frac{(120d^{5}+85d^{4}e+68d^{3}e^{2}+12d^{2}e^{3}+42de^{4}-7e^{5})\log(d+ex)}{e^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2,x)
```

```
[Out] x**4*(-10*d/e**3 - 17/(4*e**2)) + x**3*(20*d**2/e**4 + 34*d/(3*e**3) + 17/(
3*e**2)) + x**2*(-40*d**3/e**5 - 51*d**2/(2*e**4) - 17*d/e**3 - 2/e**2) + x
*(100*d**4/e**6 + 68*d**3/e**5 + 51*d**2/e**4 + 8*d/e**3 + 21/e**2) + (-20*
d**6 - 17*d**5*e - 17*d**4*e**2 - 4*d**3*e**3 - 21*d**2*e**4 + 7*d*e**5 - 6
*e**6)/(d*e**7 + e**8*x) + 4*x**5/e**2 - (120*d**5 + 85*d**4*e + 68*d**3*e*
*2 + 12*d**2*e**3 + 42*d*e**4 - 7*e**5)*log(d + e*x)/e**7
```

Giac [A]
time = 3.96, size = 308, normalized size = 1.35

$$-\frac{1}{12}\,(xe+d)^{5}\left(\frac{3\,(120\,de+17\,e^{2})e^{(-1)}}{xe+d}-\frac{4\,(300\,d^{2}e^{2}+85\,de^{3}+17\,e^{4})e^{(-2)}}{(xe+d)^{2}}+\frac{12\,(200\,d^{3}e^{3}+85\,d^{2}e^{4}+34\,de^{5}+2\,e^{6})e^{(-3)}}{(xe+d)^{3}}-\frac{12\,(300\,d^{4}e^{4}+170\,d^{3}e^{5}+102\,d^{2}e^{6}+12\,de^{7}+21\,e^{8})e^{(-4)}}{(xe+d)^{4}}-48\right)e^{(-7)}+(120\,d^{5}+85\,d^{4}e+68\,d^{3}e^{2}+12\,d^{2}e^{3}+42\,de^{4}-7\,e^{5})e^{(-7)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^{2}}\right)-\left(\frac{20\,d^{6}e^{5}}{xe+d}+\frac{17\,d^{5}e^{6}}{xe+d}+\frac{17\,d^{4}e^{7}}{xe+d}+\frac{4\,d^{3}e^{8}}{xe+d}+\frac{21\,d^{2}e^{9}}{xe+d}-\frac{7\,de^{10}}{xe+d}+\frac{6\,e^{11}}{xe+d}\right)e^{(-12)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="gia
c")
```

```
[Out] -1/12*(x*e + d)^5*(3*(120*d*e + 17*e^2)*e^(-1)/(x*e + d) - 4*(300*d^2*e^2 +
 85*d*e^3 + 17*e^4)*e^(-2)/(x*e + d)^2 + 12*(200*d^3*e^3 + 85*d^2*e^4 + 34*
d*e^5 + 2*e^6)*e^(-3)/(x*e + d)^3 - 12*(300*d^4*e^4 + 170*d^3*e^5 + 102*d^2
*e^6 + 12*d*e^7 + 21*e^8)*e^(-4)/(x*e + d)^4 - 48)*e^(-7) + (120*d^5 + 85*d
^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*e^(-7)*log(abs(x*e + d)*
e^(-1)/(x*e + d)^2) - (20*d^6*e^5/(x*e + d) + 17*d^5*e^6/(x*e + d) + 17*d^4
*e^7/(x*e + d) + 4*d^3*e^8/(x*e + d) + 21*d^2*e^9/(x*e + d) - 7*d*e^10/(x*e
 + d) + 6*e^11/(x*e + d))*e^(-12)
```

Mupad [B]
time = 4.18, size = 363, normalized size = 1.59

$$x^3\left(\frac{17}{3\,e^2}-\frac{20\,d^2}{3\,e^4}+\frac{2\,d\,\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{3\,e}\right)-x^2\left(\frac{2}{e^2}+\frac{d\,\left(\frac{17}{e^2}-\frac{20\,d^2}{e^4}+\frac{2\,d\,\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{e}\right)}{e}-\frac{d^2\,\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{2\,e^2}\right)-x^4\left(\frac{10\,d}{e^3}+\frac{17}{4\,e^2}\right)+x\left(\frac{21}{e^2}+\frac{2\,d\,\left(\frac{4}{e^2}+\frac{2\,d\,\left(\frac{17}{e^2}-\frac{20\,d^2}{e^4}+\frac{2\,d\,\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{e}\right)}{e}-\frac{d^2\,\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{e^2}\right)}{e}-\frac{d^2\,\left(\frac{17}{e^2}-\frac{20\,d^2}{e^4}+\frac{2\,d\,\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{e}\right)}{e^2}\right)+\frac{4\,x^5}{e^2}-\frac{\ln\left(d+e\,x\right)\,\left(120\,d^5+85\,d^4\,e+68\,d^3\,e^2+12\,d^2\,e^3+42\,d\,e^4-7\,e^5\right)}{e^7}-\frac{20\,d^6+17\,d^5\,e+17\,d^4\,e^2+4\,d^3\,e^3+21\,d^2\,e^4-7\,d\,e^5+6\,e^6}{e\,\left(x\,e^7+d\,e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^2,x)
```

```
[Out] x^3*(17/(3*e^2) - (20*d^2)/(3*e^4) + (2*d*((40*d)/e^3 + 17/e^2))/(3*e)) - x
^2*(2/e^2 + (d*(17/e^2 - (20*d^2)/e^4 + (2*d*((40*d)/e^3 + 17/e^2))/e))/e -
 (d^2*((40*d)/e^3 + 17/e^2))/(2*e^2)) - x^4*((10*d)/e^3 + 17/(4*e^2)) + x*(
21/e^2 + (2*d*(4/e^2 + (2*d*(17/e^2 - (20*d^2)/e^4 + (2*d*((40*d)/e^3 + 17/
e^2))/e))/e - (d^2*((40*d)/e^3 + 17/e^2))/e^2))/e - (d^2*(17/e^2 - (20*d^2)
/e^4 + (2*d*((40*d)/e^3 + 17/e^2))/e))/e^2) + (4*x^5)/e^2 - (log(d + e*x)*(
42*d*e^4 + 85*d^4*e + 120*d^5 - 7*e^5 + 12*d^2*e^3 + 68*d^3*e^2))/e^7 - (17
*d^5*e - 7*d*e^5 + 20*d^6 + 6*e^6 + 21*d^2*e^4 + 4*d^3*e^3 + 17*d^4*e^2)/(e
*(d*e^6 + e^7*x))
```

3.295 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3} \, dx$

Optimal. Leaf size=231

$$-\frac{(200d^3+102d^2e+51de^2+4e^3)\,x}{e^6}+\frac{(120d^2+51de+17e^2)\,x^2}{2e^5}-\frac{(60d+17e)x^3}{3e^4}+\frac{5x^4}{e^3}-\frac{(5d^2-2de+3e^2)\,(4d$$

```
[Out] -(200*d^3+102*d^2*e+51*d*e^2+4*e^3)*x/e^6+1/2*(120*d^2+51*d*e+17*e^2)*x^2/e
^5-1/3*(60*d+17*e)*x^3/e^4+5*x^4/e^3-1/2*(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e
+3*d^2*e^2-d*e^3+2*e^4)/e^7/(e*x+d)^2+(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e
^3+42*d*e^4-7*e^5)/e^7/(e*x+d)+(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+21*e
^4)*ln(e*x+d)/e^7
```

Rubi [A]
time = 0.13, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = $\{1642\}$

$$\frac{x^2(120d^2+51de+17e^2)}{2e^5}-\frac{x(200d^3+102d^2e+51de^2+4e^3)}{e^6}-\frac{(5d^2-2de+3e^2)\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{2e^7(d+ex)^2}+\frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)\log(d+ex)}{e^7}+\frac{120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5}{e^7(d+ex)}-\frac{x^3(60d+17e)}{3e^4}+\frac{5x^4}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x]
```

```
[Out] -(((200*d^3 + 102*d^2*e + 51*d*e^2 + 4*e^3)*x)/e^6) + ((120*d^2 + 51*d*e +
17*e^2)*x^2)/(2*e^5) - ((60*d + 17*e)*x^3)/(3*e^4) + (5*x^4)/e^3 - ((5*d^2
- 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(2*e^7*(d +
 e*x)^2) + (120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5
)/(e^7*(d + e*x)) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4
)*Log[d + e*x])/e^7
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3} \, dx = \int \left(\frac{-200d^3-102d^2e-51de^2-4e^3}{e^6}+\frac{(120d^2+51de+}{e^5}\right.$$

$$= -\frac{(200d^3+102d^2e+51de^2+4e^3)\,x}{e^6}+\frac{(120d^2+51de+1}{2e^5}$$

Mathematica [A]
time = 0.04, size = 204, normalized size = 0.88

$$\frac{660d^6+d^5e(459-480x)-51d^4e^2(-7+2x+40x^2)-3d^3e^3(-20-34x+357x^2+200x^3)+d^2e^4(189+48x-561x^2-340x^3+150x^4)-de^5(21-252x+48x^2+204x^3-85x^4+60x^5)+e^6(-18-42x-24x^3+51x^4-34x^5+30x^6)+6(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)(d+ex)^2\log(d+ex)}{6e^7(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x
]
```

```
[Out] (660*d^6 + d^5*e*(459 - 480*x) - 51*d^4*e^2*(-7 + 2*x + 40*x^2) - 3*d^3*e^3
*(-20 - 34*x + 357*x^2 + 200*x^3) + d^2*e^4*(189 + 48*x - 561*x^2 - 340*x^3
 + 150*x^4) - d*e^5*(21 - 252*x + 48*x^2 + 204*x^3 - 85*x^4 + 60*x^5) + e^6
*(-18 - 42*x - 24*x^3 + 51*x^4 - 34*x^5 + 30*x^6) + 6*(300*d^4 + 170*d^3*e
+ 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*(d + e*x)^2*Log[d + e*x])/(6*e^7*(d + e*
x)^2)
```

Maple [A]
time = 0.10, size = 236, normalized size = 1.02

method	result
norman	$\frac{\frac{\left(600d^5+340d^4e+204d^3e^2+24d^2e^3+42d\,e^4-7e^5\right)x}{e^6}+\frac{5x^6}{e}+\frac{900d^6+510d^5e+306d^4e^2+36d^3e^3+63d^2e^4-7d\,e^5-6e^6}{2e^7}-\frac{(30d+17e)x^5}{3e^2}+\frac{\left(150d^2+85de\right.}{6e^3}}{(ex+d)^2}$
default	$\frac{\left(300d^4+170d^3e+102d^2e^2+12d\,e^3+21e^4\right)\ln(ex+d)}{e^7}-\frac{20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7d\,e^5+6e^6}{2e^7(ex+d)^2}-\frac{-120d^5-85d^4e-68d^3}{e^7(}$
risch	$\frac{5x^4}{e^3}-\frac{20d\,x^3}{e^4}-\frac{17x^3}{3e^3}+\frac{60d^2x^2}{e^5}+\frac{51d\,x^2}{2e^4}+\frac{17x^2}{2e^3}-\frac{200d^3x}{e^6}-\frac{102d^2x}{e^5}-\frac{51dx}{e^4}-\frac{4x}{e^3}+\frac{(120d^5+85d^4e+68d^3e^2+12d^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] (300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+21*e^4)*ln(e*x+d)/e^7-1/2*(20*d^6+1
7*d^5*e+17*d^4*e^2+4*d^3*e^3+21*d^2*e^4-7*d*e^5+6*e^6)/e^7/(e*x+d)^2-(-120*
d^5-85*d^4*e-68*d^3*e^2-12*d^2*e^3-42*d*e^4+7*e^5)/e^7/(e*x+d)-1/e^6*(-5*e^
3*x^4+20*d*e^2*x^3+17/3*e^3*x^3-60*d^2*e*x^2-51/2*d*e^2*x^2-17/2*e^3*x^2+20
0*d^3*x+102*d^2*e*x+51*d*e^2*x+4*e^3*x)
```

Maxima [A]
time = 0.29, size = 221, normalized size = 0.96

$$\left(300\,d^4+170\,d^3e+102\,d^2e^2+12\,de^3+21\,e^4\right)e^{(-7)}\log\left(xe+d\right)+\frac{1}{6}\left(30\,x^4e^3-2\left(60\,de^2+17\,e^3\right)x^3+3\left(120\,d^2e+51\,de^2+17\,e^3\right)x^2-6\left(200\,d^3+102\,d^2e+51\,de^2+4\,e^3\right)x\right)e^{(-6)}+\frac{220\,d^6+153\,d^5e+119\,d^4e^2+20\,d^3e^3+63\,d^2e^4+2\left(120\,d^5e+85\,d^4e^2+68\,d^3e^3+12\,d^2e^4+42\,de^5-7\,e^6\right)x-7\,de^5-6\,e^6}{2\left(x^2e^9+2\,dxe^8+d^2e^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="max
ima")
```

```
[Out] (300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*e^(-7)*log(x*e + d)
 + 1/6*(30*x^4*e^3 - 2*(60*d*e^2 + 17*e^3)*x^3 + 3*(120*d^2*e + 51*d*e^2 +
17*e^3)*x^2 - 6*(200*d^3 + 102*d^2*e + 51*d*e^2 + 4*e^3)*x)*e^(-6) + 1/2*(2
20*d^6 + 153*d^5*e + 119*d^4*e^2 + 20*d^3*e^3 + 63*d^2*e^4 + 2*(120*d^5*e +
 85*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e^5 - 7*e^6)*x - 7*d*e^5 - 6*e
^6)/(x^2*e^9 + 2*d*x*e^8 + d^2*e^7)
```

Fricas [A]
time = 0.37, size = 329, normalized size = 1.42

$$\frac{660\,d^6+(30\,x^6-34\,x^5+51\,x^4-24\,x^3-42\,x-18)e^6-(60\,dx^5-85\,dx^4+204\,dx^3+48\,dx^2-252\,dx+21\,d)e^5+(150\,d^2x^4-340\,d^2x^3-561\,d^2x^2+48\,d^2x+189\,d^2)e^4-3\,(200\,d^3x^3+357\,d^3x^2-34\,d^3x-20\,d^3)e^3-51\,(40\,d^4x^2+2\,d^4x-7\,d^4)e^2-3\,(160\,d^5x-153\,d^5)e+6\,(300\,d^6+21\,x^2e^6+6\,(2\,dx^2+7\,dx)e^5+3\,(34\,d^2x^2+8\,d^2x+7\,d^2)e^4+2\,(85\,d^3x^2+102\,d^3x+6\,d^3)e^3+2\,(150\,d^4x^2+170\,d^4x+51\,d^4)e^2+10\,(60\,d^5x+17\,d^5)e)\log(xe+d)}{6\,(x^2e^9+2\,dxe^8+d^2e^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="fri
cas")
```

```
[Out] 1/6*(660*d^6 + (30*x^6 - 34*x^5 + 51*x^4 - 24*x^3 - 42*x - 18)*e^6 - (60*d*
x^5 - 85*d*x^4 + 204*d*x^3 + 48*d*x^2 - 252*d*x + 21*d)*e^5 + (150*d^2*x^4
- 340*d^2*x^3 - 561*d^2*x^2 + 48*d^2*x + 189*d^2)*e^4 - 3*(200*d^3*x^3 + 35
7*d^3*x^2 - 34*d^3*x - 20*d^3)*e^3 - 51*(40*d^4*x^2 + 2*d^4*x - 7*d^4)*e^2
- 3*(160*d^5*x - 153*d^5)*e + 6*(300*d^6 + 21*x^2*e^6 + 6*(2*d*x^2 + 7*d*x)
*e^5 + 3*(34*d^2*x^2 + 8*d^2*x + 7*d^2)*e^4 + 2*(85*d^3*x^2 + 102*d^3*x + 6
*d^3)*e^3 + 2*(150*d^4*x^2 + 170*d^4*x + 51*d^4)*e^2 + 10*(60*d^5*x + 17*d^
5)*e)*log(x*e + d))/(x^2*e^9 + 2*d*x*e^8 + d^2*e^7)
```

Sympy [A]
time = 0.89, size = 248, normalized size = 1.07

$$x^3\left(-\frac{20d}{e^4}-\frac{17}{3e^3}\right)+x^2\cdot\left(\frac{60d^2}{e^5}+\frac{51d}{2e^4}+\frac{17}{2e^3}\right)+x\left(-\frac{200d^3}{e^6}-\frac{102d^2}{e^5}-\frac{51d}{e^4}-\frac{4}{e^3}\right)+\frac{220d^6+153d^5e+119d^4e^2+20d^3e^3+63d^2e^4-7de^5-6e^6+x(240d^5e+170d^4e^2+136d^3e^3+24d^2e^4+84de^5-14e^6)}{2d^2e^7+4de^8x+2e^9x^2}+\frac{5x^4}{e^3}+\frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)\log(d+ex)}{e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3,x)
```

```
[Out] x**3*(-20*d/e**4 - 17/(3*e**3)) + x**2*(60*d**2/e**5 + 51*d/(2*e**4) + 17/(
2*e**3)) + x*(-200*d**3/e**6 - 102*d**2/e**5 - 51*d/e**4 - 4/e**3) + (220*d
**6 + 153*d**5*e + 119*d**4*e**2 + 20*d**3*e**3 + 63*d**2*e**4 - 7*d*e**5 -
 6*e**6 + x*(240*d**5*e + 170*d**4*e**2 + 136*d**3*e**3 + 24*d**2*e**4 + 84
*d*e**5 - 14*e**6))/(2*d**2*e**7 + 4*d*e**8*x + 2*e**9*x**2) + 5*x**4/e**3
+ (300*d**4 + 170*d**3*e + 102*d**2*e**2 + 12*d*e**3 + 21*e**4)*log(d + e*x
)/e**7
```

Giac [A]
time = 3.97, size = 216, normalized size = 0.94

$$(300\,d^4+170\,d^3e+102\,d^2e^2+12\,de^3+21\,e^4)e^{(-7)}\log(|xe+d|)+\frac{1}{6}\,(30\,x^4e^9-120\,dx^3e^8+360\,d^2x^2e^7-1200\,d^3xe^6-34\,x^3e^9+153\,dx^2e^8-612\,d^2xe^7+51\,x^2e^9-306\,dxe^8-24\,xe^9)e^{(-12)}+\frac{(220\,d^6+153\,d^5e+119\,d^4e^2+20\,d^3e^3+63\,d^2e^4+2\,(120\,d^5e+85\,d^4e^2+68\,d^3e^3+12\,d^2e^4+42\,de^5-7\,e^6)x-7\,de^5-6\,e^6)e^{(-7)}}{2\,(xe+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="gia
c")
```

```
[Out] (300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*e^(-7)*log(abs(x*e
+ d)) + 1/6*(30*x^4*e^9 - 120*d*x^3*e^8 + 360*d^2*x^2*e^7 - 1200*d^3*x*e^6
- 34*x^3*e^9 + 153*d*x^2*e^8 - 612*d^2*x*e^7 + 51*x^2*e^9 - 306*d*x*e^8 - 2
4*x*e^9)*e^(-12) + 1/2*(220*d^6 + 153*d^5*e + 119*d^4*e^2 + 20*d^3*e^3 + 63
*d^2*e^4 + 2*(120*d^5*e + 85*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e^5 -
 7*e^6)*x - 7*d*e^5 - 6*e^6)*e^(-7)/(x*e + d)^2
```

Mupad [B]

time = 0.09, size = 297, normalized size = 1.29

$$x^2\,\left(\frac{17}{2\,e^3}-\frac{30\,d^2}{e^5}+\frac{3\,d\,\left(\frac{60\,d}{e^4}+\frac{17}{e^3}\right)}{2\,e}\right)-x^3\,\left(\frac{20\,d}{e^4}+\frac{17}{3\,e^3}\right)+\frac{x\,\left(120\,d^5+85\,d^4\,e+68\,d^3\,e^2+12\,d^2\,e^3+42\,d\,e^4-7\,e^5\right)+\frac{220\,d^6+153\,d^5\,e+119\,d^4\,e^2+20\,d^3\,e^3+63\,d^2\,e^4-7\,d\,e^5-6\,e^6}{2\,e}}{d^2\,e^6+2\,d\,e^7\,x+e^8\,x^2}-x\,\left(\frac{4}{e^3}+\frac{20\,d^3}{e^6}+\frac{3\,d\,\left(\frac{17}{e^3}-\frac{60\,d^2}{e^5}+\frac{3\,d\,\left(\frac{60\,d}{e^4}+\frac{17}{e^3}\right)}{e}\right)}{e}-\frac{3\,d^2\,\left(\frac{60\,d}{e^4}+\frac{17}{e^3}\right)}{e^2}\right)+\frac{5\,x^4}{e^3}+\frac{\ln\left(d+e\,x\right)\,\left(300\,d^4+170\,d^3\,e+102\,d^2\,e^2+12\,d\,e^3+21\,e^4\right)}{e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^3,x)
```

```
[Out] x^2*(17/(2*e^3) - (30*d^2)/e^5 + (3*d*((60*d)/e^4 + 17/e^3))/(2*e)) - x^3*(
(20*d)/e^4 + 17/(3*e^3)) + (x*(42*d*e^4 + 85*d^4*e + 120*d^5 - 7*e^5 + 12*d
^2*e^3 + 68*d^3*e^2) + (153*d^5*e - 7*d*e^5 + 220*d^6 - 6*e^6 + 63*d^2*e^4
+ 20*d^3*e^3 + 119*d^4*e^2)/(2*e))/(d^2*e^6 + e^8*x^2 + 2*d*e^7*x) - x*(4/e
^3 + (20*d^3)/e^6 + (3*d*(17/e^3 - (60*d^2)/e^5 + (3*d*((60*d)/e^4 + 17/e^3
))/e))/e - (3*d^2*((60*d)/e^4 + 17/e^3))/e^2) + (5*x^4)/e^3 + (log(d + e*x)
*(12*d*e^3 + 170*d^3*e + 300*d^4 + 21*e^4 + 102*d^2*e^2))/e^7
```

3.296 $\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)$

Optimal. Leaf size=391

$$\frac{\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^4}{4e^9}-\frac{\left(5d^2-2de+3e^2\right)\left(160d^5+127d^4e+88d^3e^2\right.}{5e^9}$$

```
[Out] 1/4*(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^4/e
^9-1/5*(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*d^3*e^2-4*d^2*e^3+64*d*e^4
-11*e^5)*(e*x+d)^5/e^9+1/6*(2800*d^6+945*d^5*e+1665*d^4*e^2+370*d^3*e^3+888
*d^2*e^4-195*d*e^5+107*e^6)*(e*x+d)^6/e^9-1/7*(5600*d^5+1575*d^4*e+2220*d^3
*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*(e*x+d)^7/e^9+1/8*(7000*d^4+1575*d^3*e+1
665*d^2*e^2+185*d*e^3+148*e^4)*(e*x+d)^8/e^9-1/9*(5600*d^3+945*d^2*e+666*d*
e^2+37*e^3)*(e*x+d)^9/e^9+1/10*(2800*d^2+315*d*e+111*e^2)*(e*x+d)^10/e^9-5/
11*(160*d+9*e)*(e*x+d)^11/e^9+25/3*(e*x+d)^12/e^9
```

Rubi [A]
time = 0.25, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}=0.026$, Rules used = {1642}

$$\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{10}}{10e^9}-\frac{\left(5600d^3+945d^2e+666de^2+37e^3\right)(d+ex)^9}{9e^9}+\frac{\left(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4\right)(d+ex)^8}{8e^9}+\frac{\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^4}{4e^9}-\frac{\left(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5\right)(d+ex)^7}{7e^9}-\frac{\left(5d^2-2de+3e^2\right)\left(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5\right)(d+ex)^5}{5e^9}+\frac{\left(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6\right)(d+ex)^6}{6e^9}+\frac{25(d+ex)^{12}}{3e^9}-\frac{5(160d+9e)(d+ex)^{11}}{11e^9}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
 + e*x)^4)/(4*e^9) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d^3
*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x)^5)/(5*e^9) + ((2800*d^6 + 9
45*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*
(d + e*x)^6)/(6*e^9) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3
 + 592*d*e^4 - 65*e^5)*(d + e*x)^7)/(7*e^9) + ((7000*d^4 + 1575*d^3*e + 166
5*d^2*e^2 + 185*d*e^3 + 148*e^4)*(d + e*x)^8)/(8*e^9) - ((5600*d^3 + 945*d^
2*e + 666*d*e^2 + 37*e^3)*(d + e*x)^9)/(9*e^9) + ((2800*d^2 + 315*d*e + 111
*e^2)*(d + e*x)^10)/(10*e^9) - (5*(160*d + 9*e)*(d + e*x)^11)/(11*e^9) + (2
5*(d + e*x)^12)/(3*e^9)
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\, dx = \int \left(\frac{\left(5d^2-2de+3e^2\right)^2 \left(4d^4+5d^3e+3d^2e^2-\right.}{e^8}\right.$$

$$= \frac{\left(5d^2-2de+3e^2\right)^2 \left(4d^4+5d^3e+3d^2e^2-de^3\right.}{4e^9}$$

Mathematica [A]
time = 0.03, size = 277, normalized size = 0.71

$$18d^3x+\frac{3}{2}d^2(11d+18e)x^2+\frac{1}{3}d\left(107d^2+99de+54e^2\right)x^3+\frac{1}{4}\left(65d^3+321d^2e+99de^2+18e^3\right)x^4+\frac{1}{5}\left(148d^3+195d^2e+321de^2+33e^3\right)x^5+\frac{1}{6}\left(-37d^3+444d^2e+195de^2+107e^3\right)x^6+\frac{1}{7}\left(111d^3-111d^2e+444de^2+65e^3\right)x^7+\frac{1}{8}\left(-45d^3+333d^2e-111de^2+148e^3\right)x^8+\frac{1}{9}\left(100d^3-135d^2e+333de^2-37e^3\right)x^9+\frac{3}{10}e\left(100d^2-45de+37e^2\right)x^{10}+\frac{15}{11}(20d-3e)e^2x^{11}+\frac{25e^3x^{12}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] 18*d^3*x + (3*d^2*(11*d + 18*e)*x^2)/2 + (d*(107*d^2 + 99*d*e + 54*e^2)*x^3
)/3 + ((65*d^3 + 321*d^2*e + 99*d*e^2 + 18*e^3)*x^4)/4 + ((148*d^3 + 195*d^
2*e + 321*d*e^2 + 33*e^3)*x^5)/5 + ((-37*d^3 + 444*d^2*e + 195*d*e^2 + 107*
e^3)*x^6)/6 + ((111*d^3 - 111*d^2*e + 444*d*e^2 + 65*e^3)*x^7)/7 + ((-45*d^
3 + 333*d^2*e - 111*d*e^2 + 148*e^3)*x^8)/8 + ((100*d^3 - 135*d^2*e + 333*d
*e^2 - 37*e^3)*x^9)/9 + (3*e*(100*d^2 - 45*d*e + 37*e^2)*x^10)/10 + (15*(20
*d - 3*e)*e^2*x^11)/11 + (25*e^3*x^12)/3
```

Maple [A]
time = 0.13, size = 264, normalized size = 0.68

method	result
norman	$\frac{25e^3x^{12}}{3}+\left(\frac{300}{11}d\,e^2-\frac{45}{11}e^3\right)x^{11}+\left(30d^2e-\frac{27}{2}d\,e^2+\frac{111}{10}e^3\right)x^{10}+\left(\frac{100}{9}d^3-15d^2e+37d\,e^2-\frac{37}{9}e^3\right)$
default	$\frac{25e^3x^{12}}{3}+\frac{\left(300d\,e^2-45e^3\right)x^{11}}{11}+\frac{\left(300d^2e-135d\,e^2+111e^3\right)x^{10}}{10}+\frac{\left(100d^3-135d^2e+333d\,e^2-37e^3\right)x^9}{9}+\frac{\left(-45d^3+333d^2e-11\right.}{8}$
gosper	$-\frac{111}{7}x^7d^2e+\frac{444}{7}x^7d\,e^2+74x^6d^2e+\frac{65}{2}x^6d\,e^2+39x^5d^2e+\frac{321}{5}x^5d\,e^2+\frac{321}{4}x^4d^2e+\frac{99}{4}x^4d\,e^2+33d$
risch	$-\frac{111}{7}x^7d^2e+\frac{444}{7}x^7d\,e^2+74x^6d^2e+\frac{65}{2}x^6d\,e^2+39x^5d^2e+\frac{321}{5}x^5d\,e^2+\frac{321}{4}x^4d^2e+\frac{99}{4}x^4d\,e^2+33d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBO
SE)
```

```
[Out] 25/3*e^3*x^12+1/11*(300*d*e^2-45*e^3)*x^11+1/10*(300*d^2*e-135*d*e^2+111*e^
3)*x^10+1/9*(100*d^3-135*d^2*e+333*d*e^2-37*e^3)*x^9+1/8*(-45*d^3+333*d^2*e
-111*d*e^2+148*e^3)*x^8+1/7*(111*d^3-111*d^2*e+444*d*e^2+65*e^3)*x^7+1/6*(-
37*d^3+444*d^2*e+195*d*e^2+107*e^3)*x^6+1/5*(148*d^3+195*d^2*e+321*d*e^2+33
```

```
*e^3)*x^5+1/4*(65*d^3+321*d^2*e+99*d*e^2+18*e^3)*x^4+1/3*(107*d^3+99*d^2*e+
54*d*e^2)*x^3+1/2*(33*d^3+54*d^2*e)*x^2+18*d^3*x
```

Maxima [A]
time = 0.30, size = 254, normalized size = 0.65

$$\frac{25}{3}\,x^{12}e^{3}+\frac{15}{11}\,\left(20\,de^{2}-3\,e^{3}\right)x^{11}+\frac{3}{10}\,\left(100\,d^{2}e-45\,de^{2}+37\,e^{3}\right)x^{10}+\frac{1}{9}\,\left(100\,d^{3}-135\,d^{2}e+333\,de^{2}-37\,e^{3}\right)x^{9}-\frac{1}{8}\,\left(45\,d^{3}-333\,d^{2}e+111\,de^{2}-148\,e^{3}\right)x^{8}+\frac{1}{7}\,\left(111\,d^{3}-111\,d^{2}e+444\,de^{2}+65\,e^{3}\right)x^{7}-\frac{1}{6}\,\left(37\,d^{3}-444\,d^{2}e-195\,de^{2}-107\,e^{3}\right)x^{6}+\frac{1}{5}\,\left(148\,d^{3}+195\,d^{2}e+321\,de^{2}+33\,e^{3}\right)x^{5}+\frac{1}{4}\,\left(65\,d^{3}+321\,d^{2}e+99\,de^{2}+18\,e^{3}\right)x^{4}+18\,d^{3}x+\frac{1}{3}\,\left(107\,d^{3}+99\,d^{2}e+54\,de^{2}\right)x^{3}+\frac{3}{2}\,\left(11\,d^{3}+18\,d^{2}e\right)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 25/3*x^12*e^3 + 15/11*(20*d*e^2 - 3*e^3)*x^11 + 3/10*(100*d^2*e - 45*d*e^2
+ 37*e^3)*x^10 + 1/9*(100*d^3 - 135*d^2*e + 333*d*e^2 - 37*e^3)*x^9 - 1/8*(
45*d^3 - 333*d^2*e + 111*d*e^2 - 148*e^3)*x^8 + 1/7*(111*d^3 - 111*d^2*e +
444*d*e^2 + 65*e^3)*x^7 - 1/6*(37*d^3 - 444*d^2*e - 195*d*e^2 - 107*e^3)*x^
6 + 1/5*(148*d^3 + 195*d^2*e + 321*d*e^2 + 33*e^3)*x^5 + 1/4*(65*d^3 + 321*
d^2*e + 99*d*e^2 + 18*e^3)*x^4 + 18*d^3*x + 1/3*(107*d^3 + 99*d^2*e + 54*d*
e^2)*x^3 + 3/2*(11*d^3 + 18*d^2*e)*x^2
```

Fricas [A]
time = 0.38, size = 257, normalized size = 0.66

$$\frac{100}{9}\,d^{3}x^{9}-\frac{45}{8}\,d^{3}x^{8}+\frac{111}{7}\,d^{3}x^{7}-\frac{37}{6}\,d^{3}x^{6}+\frac{148}{5}\,d^{3}x^{5}+\frac{65}{4}\,d^{3}x^{4}+\frac{107}{3}\,d^{3}x^{3}+\frac{33}{2}\,d^{3}x^{2}+18\,d^{3}x+\frac{1}{6930}\,\left(57750\,x^{12}-28350\,x^{11}+76923\,x^{10}-28490\,x^{9}+128205\,x^{8}+64350\,x^{7}+123585\,x^{6}+45738\,x^{5}+31185\,x^{4}\right)e^{3}+\frac{1}{3080}\,\left(84000\,dx^{11}-41580\,dx^{10}+113960\,dx^{9}-42735\,dx^{8}+195360\,dx^{7}+100100\,dx^{6}+197736\,dx^{5}+76230\,dx^{4}+55440\,dx^{3}\right)e^{2}+\frac{1}{56}\,\left(1680\,d^{2}x^{10}-840\,d^{2}x^{9}+2331\,d^{2}x^{8}-888\,d^{2}x^{7}+4144\,d^{2}x^{6}+2184\,d^{2}x^{5}+4494\,d^{2}x^{4}+1848\,d^{2}x^{3}+1512\,d^{2}x^{2}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] 100/9*d^3*x^9 - 45/8*d^3*x^8 + 111/7*d^3*x^7 - 37/6*d^3*x^6 + 148/5*d^3*x^5
 + 65/4*d^3*x^4 + 107/3*d^3*x^3 + 33/2*d^3*x^2 + 18*d^3*x + 1/6930*(57750*x
^12 - 28350*x^11 + 76923*x^10 - 28490*x^9 + 128205*x^8 + 64350*x^7 + 123585
*x^6 + 45738*x^5 + 31185*x^4)*e^3 + 1/3080*(84000*d*x^11 - 41580*d*x^10 + 1
13960*d*x^9 - 42735*d*x^8 + 195360*d*x^7 + 100100*d*x^6 + 197736*d*x^5 + 76
230*d*x^4 + 55440*d*x^3)*e^2 + 1/56*(1680*d^2*x^10 - 840*d^2*x^9 + 2331*d^2
*x^8 - 888*d^2*x^7 + 4144*d^2*x^6 + 2184*d^2*x^5 + 4494*d^2*x^4 + 1848*d^2*
x^3 + 1512*d^2*x^2)*e
```

Sympy [A]
time = 0.03, size = 298, normalized size = 0.76

$$18d^{3}x+\frac{25e^{3}x^{12}}{3}+x^{11}\cdot\left(\frac{300de^{2}}{11}-\frac{45e^{3}}{11}\right)+x^{10}\cdot\left(30d^{2}e-\frac{27de^{2}}{2}+\frac{111e^{3}}{10}\right)+x^{9}\cdot\left(\frac{100d^{3}}{9}-15d^{2}e+37de^{2}-\frac{37e^{3}}{9}\right)+x^{8}\left(-\frac{45d^{3}}{8}+\frac{333d^{2}e}{8}-\frac{111de^{2}}{8}+\frac{37e^{3}}{2}\right)+x^{7}\cdot\left(\frac{111d^{3}}{7}-\frac{111d^{2}e}{7}+\frac{444de^{2}}{7}+\frac{65e^{3}}{7}\right)+x^{6}\left(-\frac{37d^{3}}{6}+74d^{2}e+\frac{65de^{2}}{2}+\frac{107e^{3}}{6}\right)+x^{5}\cdot\left(\frac{148d^{3}}{5}+39d^{2}e+\frac{321de^{2}}{5}+\frac{33e^{3}}{5}\right)+x^{4}\cdot\left(\frac{65d^{3}}{4}+\frac{321d^{2}e}{4}+\frac{99de^{2}}{4}+\frac{9e^{3}}{2}\right)+x^{3}\cdot\left(\frac{107d^{3}}{3}+33d^{2}e+18de^{2}\right)+x^{2}\cdot\left(\frac{33d^{3}}{2}+27d^{2}e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 18*d**3*x + 25*e**3*x**12/3 + x**11*(300*d*e**2/11 - 45*e**3/11) + x**10*(3
0*d**2*e - 27*d*e**2/2 + 111*e**3/10) + x**9*(100*d**3/9 - 15*d**2*e + 37*d
*e**2 - 37*e**3/9) + x**8*(-45*d**3/8 + 333*d**2*e/8 - 111*d*e**2/8 + 37*e*
*3/2) + x**7*(111*d**3/7 - 111*d**2*e/7 + 444*d*e**2/7 + 65*e**3/7) + x**6*
(-37*d**3/6 + 74*d**2*e + 65*d*e**2/2 + 107*e**3/6) + x**5*(148*d**3/5 + 39
*d**2*e + 321*d*e**2/5 + 33*e**3/5) + x**4*(65*d**3/4 + 321*d**2*e/4 + 99*d
*e**2/4 + 9*e**3/2) + x**3*(107*d**3/3 + 33*d**2*e + 18*d*e**2) + x**2*(33*
d**3/2 + 27*d**2*e)
```

Giac [**A**]
time = 4.12, size = 296, normalized size = 0.76

$$\frac{25}{3}\,x^{12}e^{3}+\frac{300}{11}\,dx^{11}e^{2}+30\,d^{2}x^{10}e+\frac{100}{9}\,d^{3}x^{9}-\frac{45}{11}\,x^{11}e^{3}-\frac{27}{2}\,dx^{10}e^{2}-15\,d^{2}x^{9}e-\frac{45}{8}\,d^{3}x^{8}+\frac{111}{10}\,x^{10}e^{3}+37\,dx^{9}e^{2}+\frac{333}{8}\,d^{2}x^{8}e+\frac{111}{7}\,d^{3}x^{7}-\frac{37}{9}\,x^{9}e^{3}-\frac{111}{8}\,dx^{8}e^{2}-\frac{111}{7}\,d^{2}x^{7}e-\frac{37}{6}\,d^{3}x^{6}+\frac{37}{2}\,x^{8}e^{3}+\frac{444}{7}\,dx^{7}e^{2}+74\,d^{2}x^{6}e+\frac{148}{5}\,d^{3}x^{5}+\frac{65}{7}\,x^{7}e^{3}+\frac{65}{2}\,dx^{6}e^{2}+39\,d^{2}x^{5}e+\frac{65}{4}\,d^{3}x^{4}+\frac{107}{6}\,x^{6}e^{3}+\frac{321}{5}\,dx^{5}e^{2}+\frac{321}{4}\,d^{2}x^{4}e+\frac{107}{3}\,d^{3}x^{3}+\frac{33}{5}\,x^{5}e^{3}+\frac{99}{4}\,dx^{4}e^{2}+33\,d^{2}x^{3}e+\frac{33}{2}\,d^{3}x^{2}+\frac{9}{2}\,x^{4}e^{3}+18\,dx^{3}e^{2}+27\,d^{2}x^{2}e+18\,d^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] 25/3*x^12*e^3 + 300/11*d*x^11*e^2 + 30*d^2*x^10*e + 100/9*d^3*x^9 - 45/11*x
^11*e^3 - 27/2*d*x^10*e^2 - 15*d^2*x^9*e - 45/8*d^3*x^8 + 111/10*x^10*e^3 +
 37*d*x^9*e^2 + 333/8*d^2*x^8*e + 111/7*d^3*x^7 - 37/9*x^9*e^3 - 111/8*d*x^
8*e^2 - 111/7*d^2*x^7*e - 37/6*d^3*x^6 + 37/2*x^8*e^3 + 444/7*d*x^7*e^2 + 7
4*d^2*x^6*e + 148/5*d^3*x^5 + 65/7*x^7*e^3 + 65/2*d*x^6*e^2 + 39*d^2*x^5*e
+ 65/4*d^3*x^4 + 107/6*x^6*e^3 + 321/5*d*x^5*e^2 + 321/4*d^2*x^4*e + 107/3*
d^3*x^3 + 33/5*x^5*e^3 + 99/4*d*x^4*e^2 + 33*d^2*x^3*e + 33/2*d^3*x^2 + 9/2
*x^4*e^3 + 18*d*x^3*e^2 + 27*d^2*x^2*e + 18*d^3*x
```

Mupad [**B**]
time = 4.25, size = 251, normalized size = 0.64

$$18\,d^3\,x+x^3\,\left(\frac{107\,d^3}{3}+33\,d^2\,e+18\,d\,e^2\right)+x^9\,\left(\frac{100\,d^3}{9}-15\,d^2\,e+37\,d\,e^2-\frac{37\,e^3}{9}\right)+x^6\,\left(-\frac{37\,d^3}{6}+74\,d^2\,e+\frac{65\,d\,e^2}{2}+\frac{107\,e^3}{6}\right)+x^4\,\left(\frac{65\,d^3}{4}+\frac{321\,d^2\,e}{4}+\frac{99\,d\,e^2}{4}+\frac{9\,e^3}{2}\right)-x^8\,\left(\frac{45\,d^3}{8}-\frac{333\,d^2\,e}{8}+\frac{111\,d\,e^2}{8}-\frac{37\,e^3}{2}\right)+x^5\,\left(\frac{148\,d^3}{5}+39\,d^2\,e+\frac{321\,d\,e^2}{5}+\frac{33\,e^3}{5}\right)+x^7\,\left(\frac{111\,d^3}{7}-\frac{111\,d^2\,e}{7}+\frac{444\,d\,e^2}{7}+\frac{65\,e^3}{7}\right)+\frac{25\,e^3\,x^{12}}{3}+\frac{3\,e\,x^{10}\,\left(100\,d^2-45\,d\,e+37\,e^2\right)}{10}+\frac{3\,d^2\,x^2\,\left(11\,d+18\,e\right)}{2}+\frac{15\,e^2\,x^{11}\,\left(20\,d-3\,e\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] 18*d^3*x + x^3*(18*d*e^2 + 33*d^2*e + (107*d^3)/3) + x^9*(37*d*e^2 - 15*d^2
*e + (100*d^3)/9 - (37*e^3)/9) + x^6*((65*d*e^2)/2 + 74*d^2*e - (37*d^3)/6
+ (107*e^3)/6) + x^4*((99*d*e^2)/4 + (321*d^2*e)/4 + (65*d^3)/4 + (9*e^3)/2
) - x^8*((111*d*e^2)/8 - (333*d^2*e)/8 + (45*d^3)/8 - (37*e^3)/2) + x^5*((3
21*d*e^2)/5 + 39*d^2*e + (148*d^3)/5 + (33*e^3)/5) + x^7*((444*d*e^2)/7 - (
111*d^2*e)/7 + (111*d^3)/7 + (65*e^3)/7) + (25*e^3*x^12)/3 + (3*e*x^10*(100
*d^2 - 45*d*e + 37*e^2))/10 + (3*d^2*x^2*(11*d + 18*e))/2 + (15*e^2*x^11*(2
0*d - 3*e))/11
```

3.297 $\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)$

Optimal. Leaf size=201

$$18d^2x+\frac{3}{2}d(11d+12e)x^2+\frac{1}{3}\left(107d^2+66de+18e^2\right)x^3+\frac{1}{4}\left(65d^2+214de+33e^2\right)x^4+\frac{1}{5}\left(148d^2+130de+107e^2\right.$$

```
[Out] 18*d^2*x+3/2*d*(11*d+12*e)*x^2+1/3*(107*d^2+66*d*e+18*e^2)*x^3+1/4*(65*d^2+
214*d*e+33*e^2)*x^4+1/5*(148*d^2+130*d*e+107*e^2)*x^5-1/6*(37*d^2-296*d*e-6
5*e^2)*x^6+37/7*(3*d^2-2*d*e+4*e^2)*x^7-1/8*(45*d^2-222*d*e+37*e^2)*x^8+1/9
*(100*d^2-90*d*e+111*e^2)*x^9+1/2*(40*d-9*e)*e*x^10+100/11*e^2*x^11
```

Rubi [A]
time = 0.16, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = $\{1642\}$

$$\frac{1}{9}x^9\left(100d^2-90de+111e^2\right)-\frac{1}{8}x^8\left(45d^2-222de+37e^2\right)+\frac{37}{7}x^7\left(3d^2-2de+4e^2\right)-\frac{1}{6}x^6\left(37d^2-296de-65e^2\right)+\frac{1}{5}x^5\left(148d^2+130de+107e^2\right)+\frac{1}{4}x^4\left(65d^2+214de+33e^2\right)+\frac{1}{3}x^3\left(107d^2+66de+18e^2\right)+18d^2x+\frac{1}{2}ex^{10}(40d-9e)+\frac{3}{2}dx^2(11d+12e)+\frac{100e^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*d^2*x + (3*d*(11*d + 12*e)*x^2)/2 + ((107*d^2 + 66*d*e + 18*e^2)*x^3)/3
+ ((65*d^2 + 214*d*e + 33*e^2)*x^4)/4 + ((148*d^2 + 130*d*e + 107*e^2)*x^5)
/5 - ((37*d^2 - 296*d*e - 65*e^2)*x^6)/6 + (37*(3*d^2 - 2*d*e + 4*e^2)*x^7)
/7 - ((45*d^2 - 222*d*e + 37*e^2)*x^8)/8 + ((100*d^2 - 90*d*e + 111*e^2)*x^
9)/9 + ((40*d - 9*e)*e*x^10)/2 + (100*e^2*x^11)/11
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx &= \int \left(18d^2+3d(11d+12e)x+\left(107d^2+66de+\right.\right.\\ &= 18d^2x+\frac{3}{2}d(11d+12e)x^2+\frac{1}{3}\left(107d^2+66de+\right.\end{aligned}$$

Mathematica [A]
time = 0.02, size = 201, normalized size = 1.00

$$18d^2x+\frac{3}{2}d(11d+12e)x^2+\frac{1}{3}\left(107d^2+66de+18e^2\right)x^3+\frac{1}{4}\left(65d^2+214de+33e^2\right)x^4+\frac{1}{5}\left(148d^2+130de+107e^2\right)x^5+\frac{1}{6}\left(-37d^2+296de+65e^2\right)x^6+\frac{37}{7}\left(3d^2-2de+4e^2\right)x^7+\frac{1}{8}\left(-45d^2+222de-37e^2\right)x^8+\frac{1}{9}\left(100d^2-90de+111e^2\right)x^9+\frac{1}{2}(40d-9e)ex^{10}+\frac{100e^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] 18*d^2*x + (3*d*(11*d + 12*e)*x^2)/2 + ((107*d^2 + 66*d*e + 18*e^2)*x^3)/3
+ ((65*d^2 + 214*d*e + 33*e^2)*x^4)/4 + ((148*d^2 + 130*d*e + 107*e^2)*x^5)
/5 + ((-37*d^2 + 296*d*e + 65*e^2)*x^6)/6 + (37*(3*d^2 - 2*d*e + 4*e^2)*x^7
)/7 + ((-45*d^2 + 222*d*e - 37*e^2)*x^8)/8 + ((100*d^2 - 90*d*e + 111*e^2)*
x^9)/9 + ((40*d - 9*e)*e*x^10)/2 + (100*e^2*x^11)/11
```

Maple [A]
time = 0.12, size = 186, normalized size = 0.93

method	result
norman	$\frac{100e^2x^{11}}{11} + \left(20de - \frac{9}{2}e^2\right)x^{10} + \left(\frac{100}{9}d^2 - 10de + \frac{37}{3}e^2\right)x^9 + \left(-\frac{45}{8}d^2 + \frac{111}{4}de - \frac{37}{8}e^2\right)x^8 + \left(\frac{111}{7}d^2 -\right.$
default	$\frac{100e^2x^{11}}{11} + \frac{(200de-45e^2)x^{10}}{10} + \frac{(100d^2-90de+111e^2)x^9}{9} + \frac{(-45d^2+222de-37e^2)x^8}{8} + \frac{(111d^2-74de+148e^2)x^7}{7} + \frac{(-37d}{}$
gosper	$\frac{65}{4}x^4d^2 + \frac{33}{4}x^4e^2 + \frac{107}{3}x^3d^2 - \frac{37}{6}x^6d^2 + \frac{65}{6}x^6e^2 + \frac{148}{5}x^5d^2 + \frac{111}{7}x^7d^2 + \frac{148}{7}x^7e^2 - \frac{37}{8}x^8e^2 + \frac{33}{2}d^2x^2$
risch	$\frac{65}{4}x^4d^2 + \frac{33}{4}x^4e^2 + \frac{107}{3}x^3d^2 - \frac{37}{6}x^6d^2 + \frac{65}{6}x^6e^2 + \frac{148}{5}x^5d^2 + \frac{111}{7}x^7d^2 + \frac{148}{7}x^7e^2 - \frac{37}{8}x^8e^2 + \frac{33}{2}d^2x^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBO
SE)
```

```
[Out] 100/11*e^2*x^11+1/10*(200*d*e-45*e^2)*x^10+1/9*(100*d^2-90*d*e+111*e^2)*x^9
+1/8*(-45*d^2+222*d*e-37*e^2)*x^8+1/7*(111*d^2-74*d*e+148*e^2)*x^7+1/6*(-37
*d^2+296*d*e+65*e^2)*x^6+1/5*(148*d^2+130*d*e+107*e^2)*x^5+1/4*(65*d^2+214*
d*e+33*e^2)*x^4+1/3*(107*d^2+66*d*e+18*e^2)*x^3+1/2*(33*d^2+36*d*e)*x^2+18*
d^2*x
```

Maxima [A]
time = 0.30, size = 185, normalized size = 0.92

$$\frac{100}{11}x^{11}e^2 + \frac{1}{2}(40\,de - 9\,e^2)x^{10} + \frac{1}{9}(100\,d^2 - 90\,de + 111\,e^2)x^9 - \frac{1}{8}(45\,d^2 - 222\,de + 37\,e^2)x^8 + \frac{37}{7}(3\,d^2 - 2\,de + 4\,e^2)x^7 - \frac{1}{6}(37\,d^2 - 296\,de - 65\,e^2)x^6 + \frac{1}{5}(148\,d^2 + 130\,de + 107\,e^2)x^5 + \frac{1}{4}(65\,d^2 + 214\,de + 33\,e^2)x^4 + \frac{1}{3}(107\,d^2 + 66\,de + 18\,e^2)x^3 + 18\,d^2x + \frac{3}{2}(11\,d^2 + 12\,de)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 100/11*x^11*e^2 + 1/2*(40*d*e - 9*e^2)*x^10 + 1/9*(100*d^2 - 90*d*e + 111*e
^2)*x^9 - 1/8*(45*d^2 - 222*d*e + 37*e^2)*x^8 + 37/7*(3*d^2 - 2*d*e + 4*e^2
)*x^7 - 1/6*(37*d^2 - 296*d*e - 65*e^2)*x^6 + 1/5*(148*d^2 + 130*d*e + 107*
e^2)*x^5 + 1/4*(65*d^2 + 214*d*e + 33*e^2)*x^4 + 1/3*(107*d^2 + 66*d*e + 18
*e^2)*x^3 + 18*d^2*x + 3/2*(11*d^2 + 12*d*e)*x^2
```

Fricas [A]
time = 0.36, size = 180, normalized size = 0.90

$$\frac{100}{9}d^2x^9 - \frac{45}{8}d^2x^8 + \frac{111}{7}d^2x^7 - \frac{37}{6}d^2x^6 + \frac{148}{5}d^2x^5 + \frac{65}{4}d^2x^4 + \frac{107}{3}d^2x^3 + \frac{33}{2}d^2x^2 + 18\,d^2x + \frac{1}{9240}\left(84000\,x^{11} - 41580\,x^{10} + 113960\,x^9 - 42735\,x^8 + 195360\,x^7 + 100100\,x^6 + 197736\,x^5 + 76230\,x^4 + 55440\,x^3\right)e^2 + \frac{1}{84}\left(1680\,dx^{10} - 840\,dx^9 + 2331\,dx^8 - 888\,dx^7 + 4144\,dx^6 + 2184\,dx^5 + 4494\,dx^4 + 1848\,dx^3 + 1512\,dx^2\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] 100/9*d^2*x^9 - 45/8*d^2*x^8 + 111/7*d^2*x^7 - 37/6*d^2*x^6 + 148/5*d^2*x^5
 + 65/4*d^2*x^4 + 107/3*d^2*x^3 + 33/2*d^2*x^2 + 18*d^2*x + 1/9240*(84000*x
^11 - 41580*x^10 + 113960*x^9 - 42735*x^8 + 195360*x^7 + 100100*x^6 + 19773
6*x^5 + 76230*x^4 + 55440*x^3)*e^2 + 1/84*(1680*d*x^10 - 840*d*x^9 + 2331*d
*x^8 - 888*d*x^7 + 4144*d*x^6 + 2184*d*x^5 + 4494*d*x^4 + 1848*d*x^3 + 1512
*d*x^2)*e
```

Sympy [A]
time = 0.02, size = 206, normalized size = 1.02

$$18d^2x + \frac{100e^2x^{11}}{11} + x^{10}\cdot\left(20de - \frac{9e^2}{2}\right) + x^9\cdot\left(\frac{100d^2}{9} - 10de + \frac{37e^2}{3}\right) + x^8\left(-\frac{45d^2}{8} + \frac{111de}{4} - \frac{37e^2}{8}\right) + x^7\cdot\left(\frac{111d^2}{7} - \frac{74de}{7} + \frac{148e^2}{7}\right) + x^6\left(-\frac{37d^2}{6} + \frac{148de}{3} + \frac{65e^2}{6}\right) + x^5\cdot\left(\frac{148d^2}{5} + 26de + \frac{107e^2}{5}\right) + x^4\cdot\left(\frac{65d^2}{4} + \frac{107de}{2} + \frac{33e^2}{4}\right) + x^3\cdot\left(\frac{107d^2}{3} + 22de + 6e^2\right) + x^2\cdot\left(\frac{33d^2}{2} + 18de\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 18*d**2*x + 100*e**2*x**11/11 + x**10*(20*d*e - 9*e**2/2) + x**9*(100*d**2/
9 - 10*d*e + 37*e**2/3) + x**8*(-45*d**2/8 + 111*d*e/4 - 37*e**2/8) + x**7*
(111*d**2/7 - 74*d*e/7 + 148*e**2/7) + x**6*(-37*d**2/6 + 148*d*e/3 + 65*e*
*2/6) + x**5*(148*d**2/5 + 26*d*e + 107*e**2/5) + x**4*(65*d**2/4 + 107*d*e
/2 + 33*e**2/4) + x**3*(107*d**2/3 + 22*d*e + 6*e**2) + x**2*(33*d**2/2 + 1
8*d*e)
```

Giac [A]
time = 3.19, size = 206, normalized size = 1.02

$$\frac{100}{11}x^{11}e^2 + 20\,dx^{10}e + \frac{100}{9}d^2x^9 - \frac{9}{2}x^{10}e^2 - 10\,dx^9e - \frac{45}{8}d^2x^8 + \frac{37}{3}x^9e^2 + \frac{111}{4}dx^8e + \frac{111}{7}d^2x^7 - \frac{37}{8}x^8e^2 - \frac{74}{7}dx^7e - \frac{37}{6}d^2x^6 + \frac{148}{7}x^7e^2 + \frac{148}{3}dx^6e + \frac{148}{5}d^2x^5 + \frac{65}{6}x^6e^2 + 26\,dx^5e + \frac{65}{4}d^2x^4 + \frac{107}{5}x^5e^2 + \frac{107}{2}dx^4e + \frac{107}{3}d^2x^3 + \frac{33}{4}x^4e^2 + 22\,dx^3e + \frac{33}{2}d^2x^2 + 6\,x^3e^2 + 18\,dx^2e + 18\,d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] 100/11*x^11*e^2 + 20*d*x^10*e + 100/9*d^2*x^9 - 9/2*x^10*e^2 - 10*d*x^9*e -
 45/8*d^2*x^8 + 37/3*x^9*e^2 + 111/4*d*x^8*e + 111/7*d^2*x^7 - 37/8*x^8*e^2
 - 74/7*d*x^7*e - 37/6*d^2*x^6 + 148/7*x^7*e^2 + 148/3*d*x^6*e + 148/5*d^2*
x^5 + 65/6*x^6*e^2 + 26*d*x^5*e + 65/4*d^2*x^4 + 107/5*x^5*e^2 + 107/2*d*x^
4*e + 107/3*d^2*x^3 + 33/4*x^4*e^2 + 22*d*x^3*e + 33/2*d^2*x^2 + 6*x^3*e^2
+ 18*d*x^2*e + 18*d^2*x
```

Mupad [B]
time = 0.11, size = 175, normalized size = 0.87

$$x^3\left(\frac{107\,d^2}{3}+22\,d\,e+6\,e^2\right)+x^9\left(\frac{100\,d^2}{9}-10\,d\,e+\frac{37\,e^2}{3}\right)+x^4\left(\frac{65\,d^2}{4}+\frac{107\,d\,e}{2}+\frac{33\,e^2}{4}\right)-x^8\left(\frac{45\,d^2}{8}-\frac{111\,d\,e}{4}+\frac{37\,e^2}{8}\right)+x^6\left(-\frac{37\,d^2}{6}+\frac{148\,d\,e}{3}+\frac{65\,e^2}{6}\right)+x^5\left(\frac{148\,d^2}{5}+26\,d\,e+\frac{107\,e^2}{5}\right)+x^7\left(\frac{111\,d^2}{7}-\frac{74\,d\,e}{7}+\frac{148\,e^2}{7}\right)+18\,d^2\,x+\frac{100\,e^2\,x^{11}}{11}+\frac{3\,d\,x^2\,(11\,d+12\,e)}{2}+\frac{e\,x^{10}\,(40\,d-9\,e)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] x^3*(22*d*e + (107*d^2)/3 + 6*e^2) + x^9*((100*d^2)/9 - 10*d*e + (37*e^2)/3
) + x^4*((107*d*e)/2 + (65*d^2)/4 + (33*e^2)/4) - x^8*((45*d^2)/8 - (111*d*
e)/4 + (37*e^2)/8) + x^6*((148*d*e)/3 - (37*d^2)/6 + (65*e^2)/6) + x^5*(26*
d*e + (148*d^2)/5 + (107*e^2)/5) + x^7*((111*d^2)/7 - (74*d*e)/7 + (148*e^2
)/7) + 18*d^2*x + (100*e^2*x^11)/11 + (3*d*x^2*(11*d + 12*e))/2 + (e*x^10*(
40*d - 9*e))/2
```

3.298 $\int (d+ex) \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) d$

Optimal. Leaf size=121

$$18dx+\frac{3}{2}(11d+6e)x^2+\frac{1}{3}(107d+33e)x^3+\frac{1}{4}(65d+107e)x^4+\frac{1}{5}(148d+65e)x^5-\frac{37}{6}(d-4e)x^6+\frac{37}{7}(3d-e)x^7-\frac{3}{8}(15d$$

```
[Out] 18*d*x+3/2*(11*d+6*e)*x^2+1/3*(107*d+33*e)*x^3+1/4*(65*d+107*e)*x^4+1/5*(14
8*d+65*e)*x^5-37/6*(d-4*e)*x^6+37/7*(3*d-e)*x^7-3/8*(15*d-37*e)*x^8+5/9*(20
*d-9*e)*x^9+10*e*x^10
```

Rubi [A]
time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1642}

$$\frac{5}{9}x^9(20d-9e)-\frac{3}{8}x^8(15d-37e)+\frac{37}{7}x^7(3d-e)-\frac{37}{6}x^6(d-4e)+\frac{1}{5}x^5(148d+65e)+\frac{1}{4}x^4(65d+107e)+\frac{1}{3}x^3(107d+33e)+\frac{3}{2}x^2(11d+6e)+18dx+10ex^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*d*x + (3*(11*d + 6*e)*x^2)/2 + ((107*d + 33*e)*x^3)/3 + ((65*d + 107*e)*
x^4)/4 + ((148*d + 65*e)*x^5)/5 - (37*(d - 4*e)*x^6)/6 + (37*(3*d - e)*x^7)
/7 - (3*(15*d - 37*e)*x^8)/8 + (5*(20*d - 9*e)*x^9)/9 + 10*e*x^10
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx &= \int \left(18d+3(11d+6e)x+(107d+33e)x^2+(65\right.\\ &= 18dx+\frac{3}{2}(11d+6e)x^2+\frac{1}{3}(107d+33e)x^3+\frac{1}{4}(\end{aligned}$$

Mathematica [A]
time = 0.01, size = 121, normalized size = 1.00

$$18dx+\frac{3}{2}(11d+6e)x^2+\frac{1}{3}(107d+33e)x^3+\frac{1}{4}(65d+107e)x^4+\frac{1}{5}(148d+65e)x^5-\frac{37}{6}(d-4e)x^6+\frac{37}{7}(3d-e)x^7-\frac{3}{8}(15d-37e)x^8+\frac{5}{9}(20d-9e)x^9+10ex^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*d*x + (3*(11*d + 6*e)*x^2)/2 + ((107*d + 33*e)*x^3)/3 + ((65*d + 107*e)*
x^4)/4 + ((148*d + 65*e)*x^5)/5 - (37*(d - 4*e)*x^6)/6 + (37*(3*d - e)*x^7)
/7 - (3*(15*d - 37*e)*x^8)/8 + (5*(20*d - 9*e)*x^9)/9 + 10*e*x^10
```

Maple [A]
time = 0.11, size = 108, normalized size = 0.89

method	result
norman	$10e\,x^{10}+\left(\frac{100d}{9}-5e\right)x^9+\left(-\frac{45d}{8}+\frac{111e}{8}\right)x^8+\left(\frac{111d}{7}-\frac{37e}{7}\right)x^7+\left(-\frac{37d}{6}+\frac{74e}{3}\right)x^6+\left(\frac{148d}{5}+13e\right)$
gosper	$10e\,x^{10}+\frac{100}{9}x^9d-5x^9e-\frac{45}{8}x^8d+\frac{111}{8}e\,x^8+\frac{111}{7}x^7d-\frac{37}{7}x^7e-\frac{37}{6}x^6d+\frac{74}{3}e\,x^6+\frac{148}{5}d\,x^5+13e\,x$
default	$10e\,x^{10}+\frac{(100d-45e)x^9}{9}+\frac{(-45d+111e)x^8}{8}+\frac{(111d-37e)x^7}{7}+\frac{(-37d+148e)x^6}{6}+\frac{(148d+65e)x^5}{5}+\frac{(65d+107e)x^4}{4}+$
risch	$10e\,x^{10}+\frac{100}{9}x^9d-5x^9e-\frac{45}{8}x^8d+\frac{111}{8}e\,x^8+\frac{111}{7}x^7d-\frac{37}{7}x^7e-\frac{37}{6}x^6d+\frac{74}{3}e\,x^6+\frac{148}{5}d\,x^5+13e\,x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE
)
```

```
[Out] 10*e*x^10+1/9*(100*d-45*e)*x^9+1/8*(-45*d+111*e)*x^8+1/7*(111*d-37*e)*x^7+1
/6*(-37*d+148*e)*x^6+1/5*(148*d+65*e)*x^5+1/4*(65*d+107*e)*x^4+1/3*(107*d+3
3*e)*x^3+1/2*(33*d+18*e)*x^2+18*d*x
```

Maxima [A]
time = 0.31, size = 114, normalized size = 0.94

$$10\,x^{10}e+\frac{5}{9}\,(20\,d-9\,e)x^9-\frac{3}{8}\,(15\,d-37\,e)x^8+\frac{37}{7}\,(3\,d-e)x^7-\frac{37}{6}\,(d-4\,e)x^6+\frac{1}{5}\,(148\,d+65\,e)x^5+\frac{1}{4}\,(65\,d+107\,e)x^4+\frac{1}{3}\,(107\,d+33\,e)x^3+\frac{3}{2}\,(11\,d+6\,e)x^2+18\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 10*x^10*e + 5/9*(20*d - 9*e)*x^9 - 3/8*(15*d - 37*e)*x^8 + 37/7*(3*d - e)*x
^7 - 37/6*(d - 4*e)*x^6 + 1/5*(148*d + 65*e)*x^5 + 1/4*(65*d + 107*e)*x^4 +
 1/3*(107*d + 33*e)*x^3 + 3/2*(11*d + 6*e)*x^2 + 18*d*x
```

Fricas [A]
time = 0.40, size = 103, normalized size = 0.85

$$\frac{100}{9}\,dx^9-\frac{45}{8}\,dx^8+\frac{111}{7}\,dx^7-\frac{37}{6}\,dx^6+\frac{148}{5}\,dx^5+\frac{65}{4}\,dx^4+\frac{107}{3}\,dx^3+\frac{33}{2}\,dx^2+18\,dx+\frac{1}{168}\,(1680\,x^{10}-840\,x^9+2331\,x^8-888\,x^7+4144\,x^6+2184\,x^5+4494\,x^4+1848\,x^3+1512\,x^2)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 100/9*d*x^9 - 45/8*d*x^8 + 111/7*d*x^7 - 37/6*d*x^6 + 148/5*d*x^5 + 65/4*d*
x^4 + 107/3*d*x^3 + 33/2*d*x^2 + 18*d*x + 1/168*(1680*x^10 - 840*x^9 + 2331
*x^8 - 888*x^7 + 4144*x^6 + 2184*x^5 + 4494*x^4 + 1848*x^3 + 1512*x^2)*e
```

Sympy [A]
time = 0.02, size = 112, normalized size = 0.93

$$18dx + 10ex^{10} + x^9 \cdot \left(\frac{100d}{9} - 5e\right) + x^8\left(-\frac{45d}{8} + \frac{111e}{8}\right) + x^7 \cdot \left(\frac{111d}{7} - \frac{37e}{7}\right) + x^6\left(-\frac{37d}{6} + \frac{74e}{3}\right) + x^5 \cdot \left(\frac{148d}{5} + 13e\right) + x^4 \cdot \left(\frac{65d}{4} + \frac{107e}{4}\right) + x^3 \cdot \left(\frac{107d}{3} + 11e\right) + x^2 \cdot \left(\frac{33d}{2} + 9e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 18*d*x + 10*e*x**10 + x**9*(100*d/9 - 5*e) + x**8*(-45*d/8 + 111*e/8) + x**
7*(111*d/7 - 37*e/7) + x**6*(-37*d/6 + 74*e/3) + x**5*(148*d/5 + 13*e) + x*
*4*(65*d/4 + 107*e/4) + x**3*(107*d/3 + 11*e) + x**2*(33*d/2 + 9*e)
```

Giac [A]
time = 3.96, size = 116, normalized size = 0.96

$$10\,x^{10}e + \frac{100}{9}\,dx^9 - 5\,x^9e - \frac{45}{8}\,dx^8 + \frac{111}{8}\,x^8e + \frac{111}{7}\,dx^7 - \frac{37}{7}\,x^7e - \frac{37}{6}\,dx^6 + \frac{74}{3}\,x^6e + \frac{148}{5}\,dx^5 + 13\,x^5e + \frac{65}{4}\,dx^4 + \frac{107}{4}\,x^4e + \frac{107}{3}\,dx^3 + 11\,x^3e + \frac{33}{2}\,dx^2 + 9\,x^2e + 18\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 10*x^10*e + 100/9*d*x^9 - 5*x^9*e - 45/8*d*x^8 + 111/8*x^8*e + 111/7*d*x^7
- 37/7*x^7*e - 37/6*d*x^6 + 74/3*x^6*e + 148/5*d*x^5 + 13*x^5*e + 65/4*d*x^
4 + 107/4*x^4*e + 107/3*d*x^3 + 11*x^3*e + 33/2*d*x^2 + 9*x^2*e + 18*d*x
```

Mupad [B]
time = 4.17, size = 101, normalized size = 0.83

$$10\,e\,x^{10} + \left(\frac{100\,d}{9} - 5\,e\right)\,x^9 + \left(\frac{111\,e}{8} - \frac{45\,d}{8}\right)\,x^8 + \left(\frac{111\,d}{7} - \frac{37\,e}{7}\right)\,x^7 + \left(\frac{74\,e}{3} - \frac{37\,d}{6}\right)\,x^6 + \left(\frac{148\,d}{5} + 13\,e\right)\,x^5 + \left(\frac{65\,d}{4} + \frac{107\,e}{4}\right)\,x^4 + \left(\frac{107\,d}{3} + 11\,e\right)\,x^3 + \left(\frac{33\,d}{2} + 9\,e\right)\,x^2 + 18\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] x^2*((33*d)/2 + 9*e) + x^9*((100*d)/9 - 5*e) + x^3*((107*d)/3 + 11*e) - x^6
*((37*d)/6 - (74*e)/3) + x^7*((111*d)/7 - (37*e)/7) + x^5*((148*d)/5 + 13*e
) - x^8*((45*d)/8 - (111*e)/8) + x^4*((65*d)/4 + (107*e)/4) + 18*d*x + 10*e
*x^10
```

3.299 $\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)\,dx$

Optimal. Leaf size=60

$$18x+\frac{33x^2}{2}+\frac{107x^3}{3}+\frac{65x^4}{4}+\frac{148x^5}{5}-\frac{37x^6}{6}+\frac{111x^7}{7}-\frac{45x^8}{8}+\frac{100x^9}{9}$$

```
[Out] 18*x+33/2*x^2+107/3*x^3+65/4*x^4+148/5*x^5-37/6*x^6+111/7*x^7-45/8*x^8+100/
9*x^9
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {1671}

$$\frac{100x^9}{9}-\frac{45x^8}{8}+\frac{111x^7}{7}-\frac{37x^6}{6}+\frac{148x^5}{5}+\frac{65x^4}{4}+\frac{107x^3}{3}+\frac{33x^2}{2}+18x$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 + (
111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)\,dx = \int (18+33x+107x^2+65x^3+148x^4-37x^5+111x^6$$
$$= 18x+\frac{33x^2}{2}+\frac{107x^3}{3}+\frac{65x^4}{4}+\frac{148x^5}{5}-\frac{37x^6}{6}+\frac{111x^7}{7}$$

Mathematica [A]
time = 0.00, size = 60, normalized size = 1.00

$$18x+\frac{33x^2}{2}+\frac{107x^3}{3}+\frac{65x^4}{4}+\frac{148x^5}{5}-\frac{37x^6}{6}+\frac{111x^7}{7}-\frac{45x^8}{8}+\frac{100x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 + (
111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

Maple [A]
time = 0.10, size = 45, normalized size = 0.75

method	result	size
gosper	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45
default	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45
norman	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45
risch	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 18*x+33/2*x^2+107/3*x^3+65/4*x^4+148/5*x^5-37/6*x^6+111/7*x^7-45/8*x^8+100/
9*x^9
```

Maxima [A]
time = 0.32, size = 44, normalized size = 0.73

$$\frac{100}{9}\,x^9-\frac{45}{8}\,x^8+\frac{111}{7}\,x^7-\frac{37}{6}\,x^6+\frac{148}{5}\,x^5+\frac{65}{4}\,x^4+\frac{107}{3}\,x^3+\frac{33}{2}\,x^2+18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3*
x^3 + 33/2*x^2 + 18*x
```

Fricas [A]
time = 0.37, size = 44, normalized size = 0.73

$$\frac{100}{9}\,x^9-\frac{45}{8}\,x^8+\frac{111}{7}\,x^7-\frac{37}{6}\,x^6+\frac{148}{5}\,x^5+\frac{65}{4}\,x^4+\frac{107}{3}\,x^3+\frac{33}{2}\,x^2+18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3*
x^3 + 33/2*x^2 + 18*x
```

Sympy [A]
time = 0.01, size = 56, normalized size = 0.93

$$\frac{100x^9}{9}-\frac{45x^8}{8}+\frac{111x^7}{7}-\frac{37x^6}{6}+\frac{148x^5}{5}+\frac{65x^4}{4}+\frac{107x^3}{3}+\frac{33x^2}{2}+18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 100*x**9/9 - 45*x**8/8 + 111*x**7/7 - 37*x**6/6 + 148*x**5/5 + 65*x**4/4 +
107*x**3/3 + 33*x**2/2 + 18*x
```

Giac [A]
time = 3.88, size = 44, normalized size = 0.73

$$\frac{100}{9}\,x^{9}-\frac{45}{8}\,x^{8}+\frac{111}{7}\,x^{7}-\frac{37}{6}\,x^{6}+\frac{148}{5}\,x^{5}+\frac{65}{4}\,x^{4}+\frac{107}{3}\,x^{3}+\frac{33}{2}\,x^{2}+18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3*
x^3 + 33/2*x^2 + 18*x
```

Mupad [B]
time = 0.03, size = 44, normalized size = 0.73

$$\frac{100\,x^9}{9}-\frac{45\,x^8}{8}+\frac{111\,x^7}{7}-\frac{37\,x^6}{6}+\frac{148\,x^5}{5}+\frac{65\,x^4}{4}+\frac{107\,x^3}{3}+\frac{33\,x^2}{2}+18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] 18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 + (
111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

3.300 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$

Optimal. Leaf size=352

$$-\frac{(100d^7+45d^6e+111d^5e^2+37d^4e^3+148d^3e^4-65d^2e^5+107de^6-33e^7)\,x}{e^8}+\frac{(100d^6+45d^5e+111d^4e^2+3}{}$$

```
[Out] -(100*d^7+45*d^6*e+111*d^5*e^2+37*d^4*e^3+148*d^3*e^4-65*d^2*e^5+107*d*e^6-
33*e^7)*x/e^8+1/2*(100*d^6+45*d^5*e+111*d^4*e^2+37*d^3*e^3+148*d^2*e^4-65*d
*e^5+107*e^6)*x^2/e^7-1/3*(100*d^5+45*d^4*e+111*d^3*e^2+37*d^2*e^3+148*d*e^
4-65*e^5)*x^3/e^6+1/4*(100*d^4+45*d^3*e+111*d^2*e^2+37*d*e^3+148*e^4)*x^4/e
^5-1/5*(100*d^3+45*d^2*e+111*d*e^2+37*e^3)*x^5/e^4+1/6*(100*d^2+45*d*e+111*
e^2)*x^6/e^3-5/7*(20*d+9*e)*x^7/e^2+25/2*x^8/e+(5*d^2-2*d*e+3*e^2)^2*(4*d^4
+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/e^9
```

Rubi [A]
time = 0.21, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1642}

$$\frac{x^6(100d^2+45de+111e^2)}{6e^3}-\frac{x^5(100d^3+45d^2e+111de^2+37e^3)}{5e^4}+\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^9}+\frac{x^4(100d^4+45d^3e+111d^2e^2+37de^3+148e^4)}{4e^5}-\frac{x^3(100d^5+45d^4e+111d^3e^2+37d^2e^3+148de^4-65e^5)}{3e^6}+\frac{x^2(100d^6+45d^5e+111d^4e^2+37d^3e^3+148d^2e^4-65de^5+107e^6)}{2e^7}-\frac{x(100d^7+45d^6e+111d^5e^2+37d^4e^3+148d^3e^4-65d^2e^5+107de^6-33e^7)}{e^8}-\frac{5x^7(20d+9e)}{7e^2}+\frac{25x^8}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

```
[Out] -(((100*d^7 + 45*d^6*e + 111*d^5*e^2 + 37*d^4*e^3 + 148*d^3*e^4 - 65*d^2*e^
5 + 107*d*e^6 - 33*e^7)*x)/e^8) + ((100*d^6 + 45*d^5*e + 111*d^4*e^2 + 37*d
^3*e^3 + 148*d^2*e^4 - 65*d*e^5 + 107*e^6)*x^2)/(2*e^7) - ((100*d^5 + 45*d^
4*e + 111*d^3*e^2 + 37*d^2*e^3 + 148*d*e^4 - 65*e^5)*x^3)/(3*e^6) + ((100*d
^4 + 45*d^3*e + 111*d^2*e^2 + 37*d*e^3 + 148*e^4)*x^4)/(4*e^5) - ((100*d^3
+ 45*d^2*e + 111*d*e^2 + 37*e^3)*x^5)/(5*e^4) + ((100*d^2 + 45*d*e + 111*e^
2)*x^6)/(6*e^3) - (5*(20*d + 9*e)*x^7)/(7*e^2) + (25*x^8)/(2*e) + ((5*d^2 -
 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x
])/e^9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx = \int \left(\frac{-100d^7-45d^6e-111d^5e^2-37d^4e^3-148d^3e^4+6}{e^8}\right.$$

$$= -\frac{(100d^7+45d^6e+111d^5e^2+37d^4e^3+148d^3e^4-65d^2}{e^8}$$

Mathematica [A]
time = 0.08, size = 262, normalized size = 0.74

$$\frac{x(-42000d^7+2100d^6e(-9+10x)-70d^5e^2(666-135x+200x^2)+210d^4e^3(-74+111x-30x^2+50x^3)-105d^3e^4(592-74x+148x^2-45x^3+80x^4)+35d^2e^5(780+888x-148x^2+333x^3-108x^4+200x^5)-de^6(44940+13650x+20720x^2-3885x^3+9324x^4-3150x^5+6000x^6)+2e^7(6930+11235x+4550x^2+7770x^3-1554x^4+3885x^5-1350x^6+2625x^7))}{420e^8}+\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x
]
```

```
[Out] (x*(-42000*d^7 + 2100*d^6*e*(-9 + 10*x) - 70*d^5*e^2*(666 - 135*x + 200*x^2
) + 210*d^4*e^3*(-74 + 111*x - 30*x^2 + 50*x^3) - 105*d^3*e^4*(592 - 74*x +
 148*x^2 - 45*x^3 + 80*x^4) + 35*d^2*e^5*(780 + 888*x - 148*x^2 + 333*x^3 -
 108*x^4 + 200*x^5) - d*e^6*(44940 + 13650*x + 20720*x^2 - 3885*x^3 + 9324*
x^4 - 3150*x^5 + 6000*x^6) + 2*e^7*(6930 + 11235*x + 4550*x^2 + 7770*x^3 -
1554*x^4 + 3885*x^5 - 1350*x^6 + 2625*x^7)))/(420*e^8) + ((5*d^2 - 2*d*e +
3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/e^9
```

Maple [A]
time = 0.11, size = 416, normalized size = 1.18

method	result
norman	$\frac{25x^8}{2e}-\frac{5(20d+9e)x^7}{7e^2}+\frac{(100d^2+45de+111e^2)x^6}{6e^3}-\frac{(100d^3+45d^2e+111d\,e^2+37e^3)x^5}{5e^4}+\frac{(100d^4+45d^3e+111d^2e^2+37d\,e^3+14}{4e^5}$
default	$\frac{(100d^8+45d^7e+111d^6e^2+37d^5e^3+148d^4e^4-65d^3e^5+107d^2e^6-33d\,e^7+18e^8)\ln(ex+d)}{e^9}-\frac{-50d^6e\,x^2+\frac{100}{3}d^5e^2x^3-25d^4e^3x^4+20d}{}$
risch	$\frac{37x^4}{e}+\frac{65x^3}{3e}+\frac{33x}{e}+\frac{18\ln(ex+d)}{e}-\frac{45x^7}{7e}-\frac{37x^5}{5e}+\frac{25x^8}{2e}+\frac{111\ln(ex+d)d^6}{e^7}+\frac{37\ln(ex+d)d^5}{e^6}+\frac{148\ln(ex+d)d^4}{e^5}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x,method=_RETURNVERBOSE
)
```

```
[Out] (100*d^8+45*d^7*e+111*d^6*e^2+37*d^5*e^3+148*d^4*e^4-65*d^3*e^5+107*d^2*e^6
-33*d*e^7+18*e^8)/e^9*ln(e*x+d)-1/e^8*(-50*d^6*e*x^2+100/3*d^5*e^2*x^3-25*d
^4*e^3*x^4+20*d^3*e^4*x^5+100/7*x^7*e^6*d-50/3*d^2*e^5*x^6-74*d^2*e^5*x^2+6
5/2*d*e^6*x^2-45/4*d^3*e^4*x^4-111/4*d^2*e^5*x^4-37/4*d*e^6*x^4+15*d^4*e^3*
x^3+37*d^3*e^4*x^3+37/3*d^2*e^5*x^3+148/3*d*e^6*x^3-45/2*d^5*e^2*x^2-111/2*
```

```
d^4*e^3*x^2-37/2*d^3*e^4*x^2+9*d^2*e^5*x^5+111/5*d*e^6*x^5-15/2*x^6*e^6*d+4
5/7*x^7*e^7+45*d^6*e*x+111*d^5*e^2*x+37*d^4*e^3*x+148*d^3*e^4*x-65*d^2*e^5*
x+107*d*e^6*x-25/2*x^8*e^7+37/5*e^7*x^5-37*e^7*x^4-65/3*e^7*x^3-107/2*e^7*x
^2-37/2*x^6*e^7+100*d^7*x-33*e^7*x)
```

Maxima [A]
time = 0.32, size = 328, normalized size = 0.93

$$(100\,d^8+45\,d^7e+111\,d^6e^2+37\,d^5e^3+148\,d^4e^4-65\,d^3e^5+107\,d^2e^6-33\,de^7+18\,e^8)e^{(-9)}\log(xe+d)+\frac{1}{420}\,(5250\,x^8e^7-300\,(20\,de^6+9\,e^7)x^7+70\,(100\,d^2e^5+45\,de^6+111\,e^7)x^6-84\,(100\,d^3e^4+45\,d^2e^5+111\,de^6+37\,e^7)x^5+105\,(100\,d^4e^3+45\,d^3e^4+111\,d^2e^5+37\,de^6+148\,e^7)x^4-140\,(100\,d^5e^2+45\,d^4e^3+111\,d^3e^4+37\,d^2e^5+148\,de^6-65\,e^7)x^3+210\,(100\,d^6e+45\,d^5e^2+111\,d^4e^3+37\,d^3e^4+148\,d^2e^5-65\,de^6+107\,e^7)x^2-420\,(100\,d^7+45\,d^6e+111\,d^5e^2+37\,d^4e^3+148\,d^3e^4-65\,d^2e^5+107\,de^6-33\,e^7)x)e^{(-8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] (100*d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 +
 107*d^2*e^6 - 33*d*e^7 + 18*e^8)*e^(-9)*log(x*e + d) + 1/420*(5250*x^8*e^7
 - 300*(20*d*e^6 + 9*e^7)*x^7 + 70*(100*d^2*e^5 + 45*d*e^6 + 111*e^7)*x^6 -
 84*(100*d^3*e^4 + 45*d^2*e^5 + 111*d*e^6 + 37*e^7)*x^5 + 105*(100*d^4*e^3
+ 45*d^3*e^4 + 111*d^2*e^5 + 37*d*e^6 + 148*e^7)*x^4 - 140*(100*d^5*e^2 + 4
5*d^4*e^3 + 111*d^3*e^4 + 37*d^2*e^5 + 148*d*e^6 - 65*e^7)*x^3 + 210*(100*d
^6*e + 45*d^5*e^2 + 111*d^4*e^3 + 37*d^3*e^4 + 148*d^2*e^5 - 65*d*e^6 + 107
*e^7)*x^2 - 420*(100*d^7 + 45*d^6*e + 111*d^5*e^2 + 37*d^4*e^3 + 148*d^3*e^
4 - 65*d^2*e^5 + 107*d*e^6 - 33*e^7)*x)*e^(-8)
```

Fricas [A]
time = 0.37, size = 341, normalized size = 0.97

$$-\frac{1}{420}\,(42000\,d^7xe-2\,(2625\,x^8-1350\,x^7+3885\,x^6-1554\,x^5+7770\,x^4+4550\,x^3+11235\,x^2+6930\,x)e^8+(6000\,dx^7-3150\,dx^6+9324\,dx^5-3885\,dx^4+20720\,dx^3+13650\,dx^2+44940\,dx)e^7-35\,(200\,d^2x^6-108\,d^2x^5+333\,d^2x^4-148\,d^2x^3+888\,d^2x^2+780\,d^2x)e^6+105\,(80\,d^3x^5-45\,d^3x^4+148\,d^3x^3-74\,d^3x^2+592\,d^3x)e^5-210\,(50\,d^4x^4-30\,d^4x^3+111\,d^4x^2-74\,d^4x)e^4+70\,(200\,d^5x^3-135\,d^5x^2+666\,d^5x)e^3-2100\,(10\,d^6x^2-9\,d^6x)e^2-420\,(100\,d^8+45\,d^7e+111\,d^6e^2+37\,d^5e^3+148\,d^4e^4-65\,d^3e^5+107\,d^2e^6-33\,de^7+18\,e^8)\log(xe+d))e^{(-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] -1/420*(42000*d^7*x*e - 2*(2625*x^8 - 1350*x^7 + 3885*x^6 - 1554*x^5 + 7770
*x^4 + 4550*x^3 + 11235*x^2 + 6930*x)*e^8 + (6000*d*x^7 - 3150*d*x^6 + 9324
*d*x^5 - 3885*d*x^4 + 20720*d*x^3 + 13650*d*x^2 + 44940*d*x)*e^7 - 35*(200*
d^2*x^6 - 108*d^2*x^5 + 333*d^2*x^4 - 148*d^2*x^3 + 888*d^2*x^2 + 780*d^2*x
)*e^6 + 105*(80*d^3*x^5 - 45*d^3*x^4 + 148*d^3*x^3 - 74*d^3*x^2 + 592*d^3*x
)*e^5 - 210*(50*d^4*x^4 - 30*d^4*x^3 + 111*d^4*x^2 - 74*d^4*x)*e^4 + 70*(20
0*d^5*x^3 - 135*d^5*x^2 + 666*d^5*x)*e^3 - 2100*(10*d^6*x^2 - 9*d^6*x)*e^2
- 420*(100*d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3
*e^5 + 107*d^2*e^6 - 33*d*e^7 + 18*e^8)*log(x*e + d))*e^(-9)
```

Sympy [A]
time = 0.34, size = 372, normalized size = 1.06

$$x^7\left(-\frac{100d}{7e^2}-\frac{45}{7e}\right)+x^6\cdot\left(\frac{50d^2}{3e^3}+\frac{15d}{2e^2}+\frac{37}{2e}\right)+x^5\left(-\frac{20d^3}{e^4}-\frac{9d^2}{e^3}-\frac{111d}{5e^2}-\frac{37}{5e}\right)+x^4\cdot\left(\frac{25d^4}{e^5}+\frac{45d^3}{4e^4}+\frac{111d^2}{4e^3}+\frac{37d}{4e^2}+\frac{37}{e}\right)+x^3\left(-\frac{100d^5}{3e^6}-\frac{15d^4}{e^5}-\frac{37d^3}{e^4}-\frac{37d^2}{3e^3}-\frac{148d}{3e^2}+\frac{65}{3e}\right)+x^2\cdot\left(\frac{50d^6}{e^7}+\frac{45d^5}{2e^6}+\frac{111d^4}{2e^5}+\frac{37d^3}{2e^4}+\frac{74d^2}{e^3}-\frac{65d}{2e^2}+\frac{107}{2e}\right)+x\left(-\frac{100d^7}{e^8}-\frac{45d^6}{e^7}-\frac{111d^5}{e^6}-\frac{37d^4}{e^5}-\frac{148d^3}{e^4}+\frac{65d^2}{e^3}-\frac{107d}{e^2}+\frac{33}{e}\right)+\frac{25x^8}{2e}+\frac{(5d^2-2de+3e^2)^2\cdot(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d),x)
```

```
[Out] x**7*(-100*d/(7*e**2) - 45/(7*e)) + x**6*(50*d**2/(3*e**3) + 15*d/(2*e**2)
+ 37/(2*e)) + x**5*(-20*d**3/e**4 - 9*d**2/e**3 - 111*d/(5*e**2) - 37/(5*e)
) + x**4*(25*d**4/e**5 + 45*d**3/(4*e**4) + 111*d**2/(4*e**3) + 37*d/(4*e**
2) + 37/e) + x**3*(-100*d**5/(3*e**6) - 15*d**4/e**5 - 37*d**3/e**4 - 37*d*
*2/(3*e**3) - 148*d/(3*e**2) + 65/(3*e)) + x**2*(50*d**6/e**7 + 45*d**5/(2*
e**6) + 111*d**4/(2*e**5) + 37*d**3/(2*e**4) + 74*d**2/e**3 - 65*d/(2*e**2)
 + 107/(2*e)) + x*(-100*d**7/e**8 - 45*d**6/e**7 - 111*d**5/e**6 - 37*d**4/
e**5 - 148*d**3/e**4 + 65*d**2/e**3 - 107*d/e**2 + 33/e) + 25*x**8/(2*e) +
(5*d**2 - 2*d*e + 3*e**2)**2*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*
e**4)*log(d + e*x)/e**9
```

Giac [A]
time = 3.38, size = 378, normalized size = 1.07

$$\left(100\,d^8+45\,d^7e+111\,d^6e^2+37\,d^5e^3+148\,d^4e^4-65\,d^3e^5+107\,d^2e^6-33\,de^7+18\,e^8\right)e^{(-9)}\log\left(|xe+d|\right)+\frac{1}{420}\left(5250\,x^8e^7-6000\,dx^7e^6+7000\,d^2x^6e^5-8400\,d^3x^5e^4+10500\,d^4x^4e^3-14000\,d^5x^3e^2+21000\,d^6x^2e-42000\,d^7x-2700\,x^7e^7+3150\,dx^6e^6-3780\,d^2x^5e^5+4725\,d^3x^4e^4-6300\,d^4x^3e^3+9450\,d^5x^2e^2-18900\,d^6xe+7770\,x^6e^7-9324\,dx^5e^6+11655\,d^2x^4e^5-15540\,d^3x^3e^4+23310\,d^4x^2e^3-46620\,d^5xe^2-3108\,x^5e^7+3885\,dx^4e^6-5180\,d^2x^3e^5+7770\,d^3x^2e^4-15540\,d^4xe^3+15540\,x^4e^7-20720\,dx^3e^6+31080\,d^2x^2e^5-62160\,d^3xe^4+9100\,x^3e^7-13650\,dx^2e^6+27300\,d^2xe^5+22470\,x^2e^7-44940\,dxe^6+13860\,xe^7\right)e^{(-8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] (100*d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 +
 107*d^2*e^6 - 33*d*e^7 + 18*e^8)*e^(-9)*log(abs(x*e + d)) + 1/420*(5250*x^
8*e^7 - 6000*d*x^7*e^6 + 7000*d^2*x^6*e^5 - 8400*d^3*x^5*e^4 + 10500*d^4*x^
4*e^3 - 14000*d^5*x^3*e^2 + 21000*d^6*x^2*e - 42000*d^7*x - 2700*x^7*e^7 +
3150*d*x^6*e^6 - 3780*d^2*x^5*e^5 + 4725*d^3*x^4*e^4 - 6300*d^4*x^3*e^3 + 9
450*d^5*x^2*e^2 - 18900*d^6*x*e + 7770*x^6*e^7 - 9324*d*x^5*e^6 + 11655*d^2
*x^4*e^5 - 15540*d^3*x^3*e^4 + 23310*d^4*x^2*e^3 - 46620*d^5*x*e^2 - 3108*x
^5*e^7 + 3885*d*x^4*e^6 - 5180*d^2*x^3*e^5 + 7770*d^3*x^2*e^4 - 15540*d^4*x
*e^3 + 15540*x^4*e^7 - 20720*d*x^3*e^6 + 31080*d^2*x^2*e^5 - 62160*d^3*x*e^
4 + 9100*x^3*e^7 - 13650*d*x^2*e^6 + 27300*d^2*x*e^5 + 22470*x^2*e^7 - 4494
0*d*x*e^6 + 13860*x*e^7)*e^(-8)
```

Mupad [B]

time = 0.08, size = 434, normalized size = 1.23

$$x\left(\frac{33}{e}-\frac{d\left(\frac{107}{e}-\frac{d\left(\frac{65}{e}-\frac{d\left(\frac{148}{e}+\frac{d\left(\frac{37}{e}+\frac{d\left(\frac{111}{e}+\frac{d\left(\frac{100d}{e^2}+\frac{45}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)-x^7\left(\frac{100d}{7e^2}+\frac{45}{7e}\right)+x^6\left(\frac{37}{2e}+\frac{d\left(\frac{100d}{e^2}+\frac{45}{e}\right)}{6e}\right)-x^5\left(\frac{37}{5e}+\frac{d\left(\frac{111}{e}+\frac{d\left(\frac{100d}{e^2}+\frac{45}{e}\right)}{e}\right)}{5e}\right)+x^4\left(\frac{37}{e}+\frac{d\left(\frac{37}{e}+\frac{d\left(\frac{111}{e}+\frac{d\left(\frac{100d}{e^2}+\frac{45}{e}\right)}{e}\right)}{e}\right)}{4e}\right)+x^3\left(\frac{65}{3e}-\frac{d\left(\frac{148}{e}+\frac{d\left(\frac{37}{e}+\frac{d\left(\frac{111}{e}+\frac{d\left(\frac{100d}{e^2}+\frac{45}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{3e}\right)+x^2\left(\frac{107}{2e}-\frac{d\left(\frac{65}{e}-\frac{d\left(\frac{148}{e}+\frac{d\left(\frac{37}{e}+\frac{d\left(\frac{111}{e}+\frac{d\left(\frac{100d}{e^2}+\frac{45}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{2e}\right)+\frac{25x^8}{2e}+\frac{\ln(d+ex)\,(100d^8+45d^7e+111d^6e^2+37d^5e^3+148d^4e^4-65d^3e^5+107d^2e^6-33de^7+18e^8)}{e^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x),x)
```

```
[Out] x*(33/e - (d*(107/e - (d*(65/e - (d*(148/e + (d*(37/e + (d*(111/e + (d*((10
0*d)/e^2 + 45/e))/e))/e))/e))/e))/e))/e) - x^7*((100*d)/(7*e^2) + 45/(7*e))
 + x^6*(37/(2*e) + (d*((100*d)/e^2 + 45/e))/(6*e)) - x^5*(37/(5*e) + (d*(11
1/e + (d*((100*d)/e^2 + 45/e))/e))/(5*e)) + x^4*(37/e + (d*(37/e + (d*(111/
e + (d*((100*d)/e^2 + 45/e))/e))/e))/(4*e)) + x^3*(65/(3*e) - (d*(148/e + (
d*(37/e + (d*(111/e + (d*((100*d)/e^2 + 45/e))/e))/e))/e))/(3*e)) + x^2*(10
7/(2*e) - (d*(65/e - (d*(148/e + (d*(37/e + (d*(111/e + (d*((100*d)/e^2 + 4
5/e))/e))/e))/e))/e))/(2*e)) + (25*x^8)/(2*e) + (log(d + e*x)*(45*d^7*e - 3
3*d*e^7 + 100*d^8 + 18*e^8 + 107*d^2*e^6 - 65*d^3*e^5 + 148*d^4*e^4 + 37*d^
5*e^3 + 111*d^6*e^2))/e^9
```

3.301 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$

Optimal. Leaf size=353

$$\frac{(700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5+107e^6)\,x}{e^8}-\frac{(600d^5+225d^4e+444d^3e^2+111d^2}{2e^7}$$

```
[Out] (700*d^6+270*d^5*e+555*d^4*e^2+148*d^3*e^3+444*d^2*e^4-130*d*e^5+107*e^6)*x
/e^8-1/2*(600*d^5+225*d^4*e+444*d^3*e^2+111*d^2*e^3+296*d*e^4-65*e^5)*x^2/e
^7+1/3*(500*d^4+180*d^3*e+333*d^2*e^2+74*d*e^3+148*e^4)*x^3/e^6-1/4*(400*d^
3+135*d^2*e+222*d*e^2+37*e^3)*x^4/e^5+3/5*(100*d^2+30*d*e+37*e^2)*x^5/e^4-5
/6*(40*d+9*e)*x^6/e^3+100/7*x^7/e^2-(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*
d^2*e^2-d*e^3+2*e^4)/e^9/(e*x+d)-(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*
d^3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)*ln(e*x+d)/e^9
```

Rubi [A]

time = 0.21, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1642}

$$\frac{3x^5(100d^2+30de+37e^2)}{5e^4}-\frac{x^4(400d^3+135d^2e+222de^2+37e^3)}{4e^5}-\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^9(d+ex)}+\frac{x^3(500d^4+180d^3e+333d^2e^2+74de^3+148e^4)}{3e^6}-\frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)\log(d+ex)}{e^9}-\frac{x^2(600d^5+225d^4e+444d^3e^2+111d^2e^3+296de^4-65e^5)}{2e^7}+\frac{x(700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5+107e^6)}{e^8}-\frac{5x^6(40d+9e)}{6e^3}+\frac{100x^7}{7e^2}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x]
```

```
[Out] ((700*d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130*d*e^5
 + 107*e^6)*x)/e^8 - ((600*d^5 + 225*d^4*e + 444*d^3*e^2 + 111*d^2*e^3 + 29
6*d*e^4 - 65*e^5)*x^2)/(2*e^7) + ((500*d^4 + 180*d^3*e + 333*d^2*e^2 + 74*d
*e^3 + 148*e^4)*x^3)/(3*e^6) - ((400*d^3 + 135*d^2*e + 222*d*e^2 + 37*e^3)*
x^4)/(4*e^5) + (3*(100*d^2 + 30*d*e + 37*e^2)*x^5)/(5*e^4) - (5*(40*d + 9*e
)*x^6)/(6*e^3) + (100*x^7)/(7*e^2) - ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*
d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e^9*(d + e*x)) - ((5*d^2 - 2*d*e + 3*e
^2)*(160*d^5 + 127*d^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*Log[
d + e*x])/e^9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx = \int \left(\frac{700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-13}{e^8} \right.$$

$$= \frac{(700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de}{e^8}$$

Mathematica [A]
time = 0.09, size = 342, normalized size = 0.97

$$\frac{420e(700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5+107e^6)x-210e^2(600d^5+225d^4e+444d^3e^2+111d^2e^3+296de^4-65e^5)x^2+140e^3(500d^4+180d^3e+333d^2e^2+74de^3+148e^4)x^3-105e^4(400d^3+135d^2e+222de^2+37e^3)x^4+252e^5(100d^2+30de+37e^2)x^5-350e^6(40d+9e)x^6+6000e^7x^7-\frac{420(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{d+ex}-420(800d^7+315d^6e+666d^5e^2+185d^4e^3+592d^3e^4-195d^2e^5+214de^6-33e^7)\log(d+ex)}{420e^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2
,x]
```

```
[Out] (420*e*(700*d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130
*d*e^5 + 107*e^6)*x - 210*e^2*(600*d^5 + 225*d^4*e + 444*d^3*e^2 + 111*d^2*
e^3 + 296*d*e^4 - 65*e^5)*x^2 + 140*e^3*(500*d^4 + 180*d^3*e + 333*d^2*e^2
+ 74*d*e^3 + 148*e^4)*x^3 - 105*e^4*(400*d^3 + 135*d^2*e + 222*d*e^2 + 37*e
^3)*x^4 + 252*e^5*(100*d^2 + 30*d*e + 37*e^2)*x^5 - 350*e^6*(40*d + 9*e)*x^
6 + 6000*e^7*x^7 - (420*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*
e^2 - d*e^3 + 2*e^4))/(d + e*x) - 420*(800*d^7 + 315*d^6*e + 666*d^5*e^2 +
185*d^4*e^3 + 592*d^3*e^4 - 195*d^2*e^5 + 214*d*e^6 - 33*e^7)*Log[d + e*x])
/(420*e^9)
```

Maple [A]
time = 0.10, size = 401, normalized size = 1.14

method	result
norman	$\frac{\left(800d^8+315d^7e+666d^6e^2+185d^5e^3+592d^4e^4-195d^3e^5+214d^2e^6-33d\,e^7+18e^8\right)x}{e^8 d}+\frac{100x^8}{7e}-\frac{5(160d+63e)x^7}{42e^2}+\frac{\left(800d^2+315de+666e^2\right)x^6}{30e^3}-\frac{\left(800d^3\right.}{}$
default	$\frac{\left(-800d^7-315d^6e-666d^5e^2-185d^4e^3-592d^3e^4+195d^2e^5-214d\,e^6+33e^7\right)\ln(ex+d)}{e^9}-\frac{100d^8+45d^7e+111d^6e^2+37d^5e^3+148d^4e^4-6}{e^9(ex+d)}$
risch	$\frac{148x^3}{3e^2}+\frac{65x^2}{2e^2}-\frac{18}{e(ex+d)}+\frac{33\ln(ex+d)}{e^2}-\frac{15x^6}{2e^2}-\frac{37x^4}{4e^2}+\frac{100x^7}{7e^2}-\frac{666\ln(ex+d)d^5}{e^7}-\frac{185\ln(ex+d)d^4}{e^6}-\frac{592\ln(ex+d)}{e^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x,method=_RETURNVERBO
SE)
```

```
[Out] (-800*d^7-315*d^6*e-666*d^5*e^2-185*d^4*e^3-592*d^3*e^4+195*d^2*e^5-214*d*e
^6+33*e^7)/e^9*ln(e*x+d)-(100*d^8+45*d^7*e+111*d^6*e^2+37*d^5*e^3+148*d^4*e
```

```
^4-65*d^3*e^5+107*d^2*e^6-33*d*e^7+18*e^8)/e^9/(e*x+d)+1/e^8*(18*x^5*e^5*d-
15/2*x^6*e^6-111/2*d^2*e^4*x^2+500/3*d^4*e^2*x^3+60*d^3*e^3*x^3+111*d^2*e^4
*x^3+74/3*d*e^5*x^3-300*d^5*e*x^2-225/2*d^4*e^2*x^2-222*d^3*e^3*x^2+700*d^6
*x+100/7*x^7*e^6-37/4*e^6*x^4+111/5*x^5*e^6+148/3*e^6*x^3+65/2*e^6*x^2+107*
e^6*x-100*d^3*e^3*x^4+60*d^2*e^4*x^5-100/3*x^6*e^5*d-135/4*d^2*e^4*x^4-111/
2*d*e^5*x^4-148*d*e^5*x^2+270*d^5*e*x+555*d^4*e^2*x+148*d^3*e^3*x+444*d^2*e
^4*x-130*d*e^5*x)
```

Maxima [**A**]
time = 0.36, size = 335, normalized size = 0.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="m
axima")
```

```
[Out] -(800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e^4 - 195*d^2*e
^5 + 214*d*e^6 - 33*e^7)*e^(-9)*log(x*e + d) + 1/420*(6000*x^7*e^6 - 350*(4
0*d*e^5 + 9*e^6)*x^6 + 252*(100*d^2*e^4 + 30*d*e^5 + 37*e^6)*x^5 - 105*(400
*d^3*e^3 + 135*d^2*e^4 + 222*d*e^5 + 37*e^6)*x^4 + 140*(500*d^4*e^2 + 180*d
^3*e^3 + 333*d^2*e^4 + 74*d*e^5 + 148*e^6)*x^3 - 210*(600*d^5*e + 225*d^4*e
^2 + 444*d^3*e^3 + 111*d^2*e^4 + 296*d*e^5 - 65*e^6)*x^2 + 420*(700*d^6 + 2
70*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130*d*e^5 + 107*e^6)*x
)*e^(-8) - (100*d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 6
5*d^3*e^5 + 107*d^2*e^6 - 33*d*e^7 + 18*e^8)/(x*e^10 + d*e^9)
```

Fricas [**A**]
time = 0.38, size = 449, normalized size = 1.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="f
ricas")
```

```
[Out] -1/420*(42000*d^8 - (6000*x^8 - 3150*x^7 + 9324*x^6 - 3885*x^5 + 20720*x^4
+ 13650*x^3 + 44940*x^2 - 7560)*e^8 + (8000*d*x^7 - 4410*d*x^6 + 13986*d*x^
5 - 6475*d*x^4 + 41440*d*x^3 + 40950*d*x^2 - 44940*d*x - 13860*d)*e^7 - 35*
(320*d^2*x^6 - 189*d^2*x^5 + 666*d^2*x^4 - 370*d^2*x^3 + 3552*d^2*x^2 - 156
0*d^2*x - 1284*d^2)*e^6 + 105*(160*d^3*x^5 - 105*d^3*x^4 + 444*d^3*x^3 - 37
0*d^3*x^2 - 1776*d^3*x - 260*d^3)*e^5 - 70*(400*d^4*x^4 - 315*d^4*x^3 + 199
8*d^4*x^2 + 888*d^4*x - 888*d^4)*e^4 + 70*(800*d^5*x^3 - 945*d^5*x^2 - 3330
*d^5*x + 222*d^5)*e^3 - 420*(400*d^6*x^2 + 270*d^6*x - 111*d^6)*e^2 - 2100*
(140*d^7*x - 9*d^7)*e + 420*(800*d^8 - 33*x*e^8 + (214*d*x - 33*d)*e^7 - (1
95*d^2*x - 214*d^2)*e^6 + (592*d^3*x - 195*d^3)*e^5 + 37*(5*d^4*x + 16*d^4)
```

```
*e^4 + 37*(18*d^5*x + 5*d^5)*e^3 + 9*(35*d^6*x + 74*d^6)*e^2 + 5*(160*d^7*x
 + 63*d^7)*e)*log(x*e + d))/(x*e^10 + d*e^9)
```

Sympy [A]
time = 0.69, size = 393, normalized size = 1.11

$$x^{6}\left(-\frac{100d}{3e^{3}}-\frac{15}{2e^{2}}\right)+x^{5}\cdot\left(\frac{60d^{2}}{e^{4}}+\frac{18d}{e^{3}}+\frac{111}{5e^{2}}\right)+x^{4}\left(-\frac{100d^{3}}{e^{5}}-\frac{135d^{2}}{4e^{4}}-\frac{111d}{2e^{3}}-\frac{37}{4e^{2}}\right)+x^{3}\cdot\left(\frac{500d^{4}}{3e^{6}}+\frac{60d^{3}}{e^{5}}+\frac{111d^{2}}{e^{4}}+\frac{74d}{3e^{3}}+\frac{148}{3e^{2}}\right)+x^{2}\left(-\frac{300d^{5}}{e^{7}}-\frac{225d^{4}}{2e^{6}}-\frac{222d^{3}}{e^{5}}-\frac{111d^{2}}{2e^{4}}-\frac{148d}{e^{3}}+\frac{65}{2e^{2}}\right)+x\left(\frac{700d^{6}}{e^{8}}+\frac{270d^{5}}{e^{7}}+\frac{555d^{4}}{e^{6}}+\frac{148d^{3}}{e^{5}}+\frac{444d^{2}}{e^{4}}-\frac{130d}{e^{3}}+\frac{107}{e^{2}}\right)+\frac{-100d^{8}-45d^{7}e-111d^{6}e^{2}-37d^{5}e^{3}-148d^{4}e^{4}+65d^{3}e^{5}-107d^{2}e^{6}+33de^{7}-18e^{8}}{de^{9}+e^{10}x}+\frac{100x^{7}}{7e^{2}}-\frac{(5d^{2}-2de+3e^{2})(160d^{5}+127d^{4}e+88d^{3}e^{2}-4d^{2}e^{3}+64de^{4}-11e^{5})\log(d+ex)}{e^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2,x)
```

```
[Out] x**6*(-100*d/(3*e**3) - 15/(2*e**2)) + x**5*(60*d**2/e**4 + 18*d/e**3 + 111
/(5*e**2)) + x**4*(-100*d**3/e**5 - 135*d**2/(4*e**4) - 111*d/(2*e**3) - 37
/(4*e**2)) + x**3*(500*d**4/(3*e**6) + 60*d**3/e**5 + 111*d**2/e**4 + 74*d/
(3*e**3) + 148/(3*e**2)) + x**2*(-300*d**5/e**7 - 225*d**4/(2*e**6) - 222*d
**3/e**5 - 111*d**2/(2*e**4) - 148*d/e**3 + 65/(2*e**2)) + x*(700*d**6/e**8
 + 270*d**5/e**7 + 555*d**4/e**6 + 148*d**3/e**5 + 444*d**2/e**4 - 130*d/e*
*3 + 107/e**2) + (-100*d**8 - 45*d**7*e - 111*d**6*e**2 - 37*d**5*e**3 - 14
8*d**4*e**4 + 65*d**3*e**5 - 107*d**2*e**6 + 33*d*e**7 - 18*e**8)/(d*e**9 +
 e**10*x) + 100*x**7/(7*e**2) - (5*d**2 - 2*d*e + 3*e**2)*(160*d**5 + 127*d
**4*e + 88*d**3*e**2 - 4*d**2*e**3 + 64*d*e**4 - 11*e**5)*log(d + e*x)/e**9
```

Giac [A]
time = 3.45, size = 459, normalized size = 1.30

$$-\frac{1}{420}(xe+d)^{7}\left(\frac{350(160de+9e^{2})e^{(-1)}}{xe+d}-\frac{84(2800d^{2}e^{2}+315de^{3}+111e^{4})e^{(-2)}}{(xe+d)^{2}}+\frac{105(5600d^{3}e^{3}+945d^{2}e^{4}+666de^{5}+37e^{6})e^{(-3)}}{(xe+d)^{3}}-\frac{140(7000d^{4}e^{4}+1575d^{3}e^{5}+1665d^{2}e^{6}+185de^{7}+148e^{8})e^{(-4)}}{(xe+d)^{4}}+\frac{210(5600d^{5}e^{5}+1575d^{4}e^{6}+2220d^{3}e^{7}+370d^{2}e^{8}+592de^{9}-65e^{10})e^{(-5)}}{(xe+d)^{5}}-\frac{420(2800d^{6}e^{6}+945d^{5}e^{7}+1665d^{4}e^{8}+370d^{3}e^{9}+888d^{2}e^{10}-195de^{11}+107e^{12})e^{(-6)}}{(xe+d)^{6}}-6000\right)e^{(-9)}+(800d^{7}+315d^{6}e+666d^{5}e^{2}+185d^{4}e^{3}+592d^{3}e^{4}-195d^{2}e^{5}+214de^{6}-33e^{7})e^{(-9)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^{2}}\right)-\left(\frac{100d^{8}e^{7}}{xe+d}+\frac{45d^{7}e^{8}}{xe+d}+\frac{111d^{6}e^{9}}{xe+d}+\frac{37d^{5}e^{10}}{xe+d}+\frac{148d^{4}e^{11}}{xe+d}-\frac{65d^{3}e^{12}}{xe+d}+\frac{107d^{2}e^{13}}{xe+d}-\frac{33de^{14}}{xe+d}+\frac{18e^{15}}{xe+d}\right)e^{(-16)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="g
iac")
```

```
[Out] -1/420*(x*e + d)^7*(350*(160*d*e + 9*e^2)*e^(-1)/(x*e + d) - 84*(2800*d^2*e
^2 + 315*d*e^3 + 111*e^4)*e^(-2)/(x*e + d)^2 + 105*(5600*d^3*e^3 + 945*d^2*
e^4 + 666*d*e^5 + 37*e^6)*e^(-3)/(x*e + d)^3 - 140*(7000*d^4*e^4 + 1575*d^3
*e^5 + 1665*d^2*e^6 + 185*d*e^7 + 148*e^8)*e^(-4)/(x*e + d)^4 + 210*(5600*d
^5*e^5 + 1575*d^4*e^6 + 2220*d^3*e^7 + 370*d^2*e^8 + 592*d*e^9 - 65*e^10)*e
^(-5)/(x*e + d)^5 - 420*(2800*d^6*e^6 + 945*d^5*e^7 + 1665*d^4*e^8 + 370*d^
3*e^9 + 888*d^2*e^10 - 195*d*e^11 + 107*e^12)*e^(-6)/(x*e + d)^6 - 6000)*e^
(-9) + (800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e^4 - 195
*d^2*e^5 + 214*d*e^6 - 33*e^7)*e^(-9)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)
- (100*d^8*e^7/(x*e + d) + 45*d^7*e^8/(x*e + d) + 111*d^6*e^9/(x*e + d) + 3
7*d^5*e^10/(x*e + d) + 148*d^4*e^11/(x*e + d) - 65*d^3*e^12/(x*e + d) + 107
*d^2*e^13/(x*e + d) - 33*d*e^14/(x*e + d) + 18*e^15/(x*e + d))*e^(-16)
```

Mupad [B]
time = 4.22, size = 939, normalized size = 2.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^2,x)
```

```
[Out] x^2*(65/(2*e^2) - (d*(148/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^
4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))
/e - (d^2*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^2))
/e + (d^2*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/
e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/(2*e^2)) + x^3*(148/(3*e^2
) + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e
^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/(3*e) - (d^2*(111/e^2 - (10
0*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/(3*e^2)) - x^4*(37/(4*e^2) +
(d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/(2*e) - (d^2
*((200*d)/e^3 + 45/e^2))/(4*e^2)) + x^5*(111/(5*e^2) - (20*d^2)/e^4 + (2*d*
((200*d)/e^3 + 45/e^2))/(5*e)) - x^6*((100*d)/(3*e^3) + 15/(2*e^2)) - x*((2
*d*(65/e^2 - (2*d*(148/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 +
 (2*d*((200*d)/e^3 + 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/e
- (d^2*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^2))/e
+ (d^2*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2
))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/e^2))/e - 107/e^2 + (d^2*(148
/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 4
5/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/e - (d^2*(111/e^2 - (100
*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^2))/e^2) + (100*x^7)/(7*e^2)
 - (45*d^7*e - 33*d*e^7 + 100*d^8 + 18*e^8 + 107*d^2*e^6 - 65*d^3*e^5 + 148
*d^4*e^4 + 37*d^5*e^3 + 111*d^6*e^2)/(e*(d*e^8 + e^9*x)) - (log(d + e*x)*(2
14*d*e^6 + 315*d^6*e + 800*d^7 - 33*e^7 - 195*d^2*e^5 + 592*d^3*e^4 + 185*d
^4*e^3 + 666*d^5*e^2))/e^9
```

3.302 $\int \frac{(3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)}{(d+ex)^3} \, dx$

Optimal. Leaf size=354

$$-\frac{(2100d^5+675d^4e+1110d^3e^2+222d^2e^3+444de^4-65e^5)\,x}{e^8}+\frac{(1500d^4+450d^3e+666d^2e^2+111de^3+148e^4)}{2e^7}$$

```
[Out] -(2100*d^5+675*d^4*e+1110*d^3*e^2+222*d^2*e^3+444*d*e^4-65*e^5)*x/e^8+1/2*(
1500*d^4+450*d^3*e+666*d^2*e^2+111*d*e^3+148*e^4)*x^2/e^7-1/3*(1000*d^3+270
*d^2*e+333*d*e^2+37*e^3)*x^3/e^6+3/4*(200*d^2+45*d*e+37*e^2)*x^4/e^5-3*(20*
d+3*e)*x^5/e^4+50/3*x^6/e^3-1/2*(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^2*
e^2-d*e^3+2*e^4)/e^9/(e*x+d)^2+(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*d^
3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)/e^9/(e*x+d)+(2800*d^6+945*d^5*e+1665*d^4*e
^2+370*d^3*e^3+888*d^2*e^4-195*d*e^5+107*e^6)*ln(e*x+d)/e^9
```

Rubi [A]
time = 0.22, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1642}

$$\frac{3x^4(200d^2+45de+37e^2)}{4e^5}-\frac{x^3(1000d^3+270d^2e+333de^2+37e^3)}{3e^6}-\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{2e^9(d+ex)^2}+\frac{x^2(1500d^4+450d^3e+666d^2e^2+111de^3+148e^4)}{2e^7}+\frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)}{e^9(d+ex)}-\frac{x(2100d^5+675d^4e+1110d^3e^2+222d^2e^3+444de^4-65e^5)}{e^8}+\frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)\log(d+ex)}{e^9}-\frac{3x^5(20d+3e)}{e^4}+\frac{50x^6}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x]
```

```
[Out] -(((2100*d^5 + 675*d^4*e + 1110*d^3*e^2 + 222*d^2*e^3 + 444*d*e^4 - 65*e^5)
*x)/e^8) + ((1500*d^4 + 450*d^3*e + 666*d^2*e^2 + 111*d*e^3 + 148*e^4)*x^2)
/(2*e^7) - ((1000*d^3 + 270*d^2*e + 333*d*e^2 + 37*e^3)*x^3)/(3*e^6) + (3*(
200*d^2 + 45*d*e + 37*e^2)*x^4)/(4*e^5) - (3*(20*d + 3*e)*x^5)/e^4 + (50*x^
6)/(3*e^3) - ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^
3 + 2*e^4))/(2*e^9*(d + e*x)^2) + ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d
^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5))/(e^9*(d + e*x)) + ((280
0*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 +
107*e^6)*Log[d + e*x])/e^9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx = \int \left(\frac{-2100d^5-675d^4e-1110d^3e^2-222d^2e^3-444de^4}{e^8} \right.$$

$$= -\frac{(2100d^5+675d^4e+1110d^3e^2+222d^2e^3+444de^4-6}{e^8}$$

Mathematica [A]

time = 0.07, size = 311, normalized size = 0.88

$$\frac{9000d^8-390d^7e(-9+40x)-18d^6e^2(-407+240x+2300x^2)-2d^5e^3(-999+2664x+6750x^2+5600x^3)+4d^4e^4(1554-111x-5661x^2-945x^3+700x^4)-d^3e^5(1950-1776x+4662x^2+6660x^3-945x^4+1120x^5)+d^2e^6(1926-1560x-9768x^2-1480x^3+1665x^4-378x^5+560x^6)+de^7(-198+2568x+1560x^2-3552x^3+370x^4-666x^5+189x^6-320x^7)+e^8(-108-396x+780x^3+888x^4-148x^5+333x^6-108x^7+200x^8)+12(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)(d+ex)^2\log(d+ex)}{12e^9(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3
,x]
```

```
[Out] (9000*d^8 - 390*d^7*e*(-9 + 40*x) - 18*d^6*e^2*(-407 + 240*x + 2300*x^2) -
2*d^5*e^3*(-999 + 2664*x + 6750*x^2 + 5600*x^3) + 4*d^4*e^4*(1554 - 111*x -
 5661*x^2 - 945*x^3 + 700*x^4) - d^3*e^5*(1950 - 1776*x + 4662*x^2 + 6660*x
^3 - 945*x^4 + 1120*x^5) + d^2*e^6*(1926 - 1560*x - 9768*x^2 - 1480*x^3 + 1
665*x^4 - 378*x^5 + 560*x^6) + d*e^7*(-198 + 2568*x + 1560*x^2 - 3552*x^3 +
 370*x^4 - 666*x^5 + 189*x^6 - 320*x^7) + e^8*(-108 - 396*x + 780*x^3 + 888
*x^4 - 148*x^5 + 333*x^6 - 108*x^7 + 200*x^8) + 12*(2800*d^6 + 945*d^5*e +
1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*(d + e*x)^2
*Log[d + e*x])/(12*e^9*(d + e*x)^2)
```

Maple [A]

time = 0.11, size = 391, normalized size = 1.10

method	result
norman	$\frac{\left(5600d^7+1890d^6e+3330d^5e^2+740d^4e^3+1776d^3e^4-390d^2e^5+214d\,e^6-33e^7\right)x}{e^8}+\frac{50x^8}{3e}+\frac{8400d^8+2835d^7e+4995d^6e^2+1110d^5e^3+2664d^4e^4-585d^3}{2e^9}$
default	$\frac{\left(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195d\,e^5+107e^6\right)\ln(ex+d)}{e^9}-\frac{100d^8+45d^7e+111d^6e^2+37d^5e^3+148d^4e^4-65d^3e^5}{2e^9(ex+d)^2}$
risch	$\frac{107\ln(ex+d)}{e^3}+\frac{74x^2}{e^3}+\frac{65x}{e^3}-\frac{9x^5}{e^3}-\frac{37x^3}{3e^3}+\frac{50x^6}{3e^3}+\frac{(800d^7+315d^6e+666d^5e^2+185d^4e^3+592d^3e^4-195d^2e^5+214d\,e^6-33}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x,method=_RETURNVERBO
SE)
```

```
[Out] (2800*d^6+945*d^5*e+1665*d^4*e^2+370*d^3*e^3+888*d^2*e^4-195*d*e^5+107*e^6)
*ln(e*x+d)/e^9-1/2*(100*d^8+45*d^7*e+111*d^6*e^2+37*d^5*e^3+148*d^4*e^4-65*
```

```
d^3*e^5+107*d^2*e^6-33*d*e^7+18*e^8)/e^9/(e*x+d)^2-(-800*d^7-315*d^6*e-666*
d^5*e^2-185*d^4*e^3-592*d^3*e^4+195*d^2*e^5-214*d*e^6+33*e^7)/e^9/(e*x+d)-1
/e^8*(-50/3*x^6*e^5+60*x^5*e^4*d+9*x^5*e^5-150*d^2*e^3*x^4-135/4*x^4*e^4*d-
111/4*x^4*e^5+1000/3*d^3*e^2*x^3+90*d^2*e^3*x^3+111*d*e^4*x^3+37/3*e^5*x^3-
750*d^4*e*x^2-225*d^3*e^2*x^2-333*d^2*e^3*x^2-111/2*d*e^4*x^2-74*e^5*x^2+21
00*d^5*x+675*d^4*e*x+1110*d^3*e^2*x+222*d^2*e^3*x+444*d*e^4*x-65*e^5*x)
```

Maxima [A]
time = 0.30, size = 342, normalized size = 0.97

$$\left(2800\,d^{6}+945\,d^{5}e+1665\,d^{4}e^{2}+370\,d^{3}e^{3}+888\,d^{2}e^{4}-195\,de^{5}+107\,e^{6}\right)e^{(-9)}\log\left(xe+d\right)+\frac{1}{12}\left(200\,x^{6}e^{5}-36\left(20\,de^{4}+3\,e^{5}\right)x^{5}+9\left(200\,d^{2}e^{3}+45\,de^{4}+37\,e^{5}\right)x^{4}-4\left(1000\,d^{3}e^{2}+270\,d^{2}e^{3}+333\,de^{4}+37\,e^{5}\right)x^{3}+6\left(1500\,d^{4}e+450\,d^{3}e^{2}+666\,d^{2}e^{3}+111\,de^{4}+148\,e^{5}\right)x^{2}-12\left(2100\,d^{5}+675\,d^{4}e+1110\,d^{3}e^{2}+222\,d^{2}e^{3}+444\,de^{4}-65\,e^{5}\right)x\right)e^{(-8)}+\frac{1500\,d^{8}+585\,d^{7}e+1221\,d^{6}e^{2}+333\,d^{5}e^{3}+1036\,d^{4}e^{4}-325\,d^{3}e^{5}+321\,d^{2}e^{6}+2\left(800\,d^{7}e+315\,d^{6}e^{2}+666\,d^{5}e^{3}+185\,d^{4}e^{4}+592\,d^{3}e^{5}-195\,d^{2}e^{6}+214\,de^{7}-33\,e^{8}\right)x-33\,de^{7}-18\,e^{8}}{2\left(x^{2}e^{11}+2\,dxe^{10}+d^{2}e^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="m
axima")
```

```
[Out] (2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^
5 + 107*e^6)*e^(-9)*log(x*e + d) + 1/12*(200*x^6*e^5 - 36*(20*d*e^4 + 3*e^5
)*x^5 + 9*(200*d^2*e^3 + 45*d*e^4 + 37*e^5)*x^4 - 4*(1000*d^3*e^2 + 270*d^2
*e^3 + 333*d*e^4 + 37*e^5)*x^3 + 6*(1500*d^4*e + 450*d^3*e^2 + 666*d^2*e^3
+ 111*d*e^4 + 148*e^5)*x^2 - 12*(2100*d^5 + 675*d^4*e + 1110*d^3*e^2 + 222*
d^2*e^3 + 444*d*e^4 - 65*e^5)*x)*e^(-8) + 1/2*(1500*d^8 + 585*d^7*e + 1221*
d^6*e^2 + 333*d^5*e^3 + 1036*d^4*e^4 - 325*d^3*e^5 + 321*d^2*e^6 + 2*(800*d
^7*e + 315*d^6*e^2 + 666*d^5*e^3 + 185*d^4*e^4 + 592*d^3*e^5 - 195*d^2*e^6
+ 214*d*e^7 - 33*e^8)*x - 33*d*e^7 - 18*e^8)/(x^2*e^11 + 2*d*x*e^10 + d^2*e
^9)
```

Fricas [A]
time = 0.38, size = 501, normalized size = 1.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="f
ricas")
```

```
[Out] 1/12*(9000*d^8 + (200*x^8 - 108*x^7 + 333*x^6 - 148*x^5 + 888*x^4 + 780*x^3
 - 396*x - 108)*e^8 - (320*d*x^7 - 189*d*x^6 + 666*d*x^5 - 370*d*x^4 + 3552
*d*x^3 - 1560*d*x^2 - 2568*d*x + 198*d)*e^7 + (560*d^2*x^6 - 378*d^2*x^5 +
1665*d^2*x^4 - 1480*d^2*x^3 - 9768*d^2*x^2 - 1560*d^2*x + 1926*d^2)*e^6 - (
1120*d^3*x^5 - 945*d^3*x^4 + 6660*d^3*x^3 + 4662*d^3*x^2 - 1776*d^3*x + 195
0*d^3)*e^5 + 4*(700*d^4*x^4 - 945*d^4*x^3 - 5661*d^4*x^2 - 111*d^4*x + 1554
*d^4)*e^4 - 2*(5600*d^5*x^3 + 6750*d^5*x^2 + 2664*d^5*x - 999*d^5)*e^3 - 18
*(2300*d^6*x^2 + 240*d^6*x - 407*d^6)*e^2 - 390*(40*d^7*x - 9*d^7)*e + 12*(
2800*d^8 + 107*x^2*e^8 - (195*d*x^2 - 214*d*x)*e^7 + (888*d^2*x^2 - 390*d^2
*x + 107*d^2)*e^6 + (370*d^3*x^2 + 1776*d^3*x - 195*d^3)*e^5 + 37*(45*d^4*x
```

```
^2 + 20*d^4*x + 24*d^4)*e^4 + 5*(189*d^5*x^2 + 666*d^5*x + 74*d^5)*e^3 + 5*
(560*d^6*x^2 + 378*d^6*x + 333*d^6)*e^2 + 35*(160*d^7*x + 27*d^7)*e)*log(x*
e + d))/(x^2*e^11 + 2*d*x*e^10 + d^2*e^9)
```

Sympy [A]
time = 1.50, size = 394, normalized size = 1.11

$$x^5\left(-\frac{60d}{e^4}-\frac{9}{e^3}\right)+x^4\cdot\left(\frac{150d^2}{e^5}+\frac{135d}{4e^4}+\frac{111}{4e^3}\right)+x^3\left(-\frac{1000d^3}{3e^6}-\frac{90d^2}{e^5}-\frac{111d}{e^4}-\frac{37}{3e^3}\right)+x^2\cdot\left(\frac{750d^4}{e^7}+\frac{225d^3}{e^6}+\frac{333d^2}{e^5}+\frac{111d}{2e^4}+\frac{74}{e^3}\right)+x\left(-\frac{2100d^5}{e^8}-\frac{675d^4}{e^7}-\frac{1110d^3}{e^6}-\frac{222d^2}{e^5}-\frac{444d}{e^4}+\frac{65}{e^3}\right)+\frac{1500d^8+585d^7e+1221d^6e^2+333d^5e^3+1036d^4e^4-325d^3e^5+321d^2e^6-33de^7-18e^8+x(1600d^7e+630d^6e^2+1332d^5e^3+370d^4e^4+1184d^3e^5-390d^2e^6+428de^7-66e^8)}{2d^2e^9+4de^{10}x+2e^{11}x^2}+\frac{50x^6}{3e^3}+\frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)\log(d+ex)}{e^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3,x)
[Out] x**5*(-60*d/e**4 - 9/e**3) + x**4*(150*d**2/e**5 + 135*d/(4*e**4) + 111/(4*
e**3)) + x**3*(-1000*d**3/(3*e**6) - 90*d**2/e**5 - 111*d/e**4 - 37/(3*e**3
)) + x**2*(750*d**4/e**7 + 225*d**3/e**6 + 333*d**2/e**5 + 111*d/(2*e**4) +
 74/e**3) + x*(-2100*d**5/e**8 - 675*d**4/e**7 - 1110*d**3/e**6 - 222*d**2/
e**5 - 444*d/e**4 + 65/e**3) + (1500*d**8 + 585*d**7*e + 1221*d**6*e**2 + 3
33*d**5*e**3 + 1036*d**4*e**4 - 325*d**3*e**5 + 321*d**2*e**6 - 33*d*e**7 -
 18*e**8 + x*(1600*d**7*e + 630*d**6*e**2 + 1332*d**5*e**3 + 370*d**4*e**4
+ 1184*d**3*e**5 - 390*d**2*e**6 + 428*d*e**7 - 66*e**8))/(2*d**2*e**9 + 4*
d*e**10*x + 2*e**11*x**2) + 50*x**6/(3*e**3) + (2800*d**6 + 945*d**5*e + 16
65*d**4*e**2 + 370*d**3*e**3 + 888*d**2*e**4 - 195*d*e**5 + 107*e**6)*log(d
 + e*x)/e**9
```

Giac [A]
time = 4.82, size = 354, normalized size = 1.00

$$(2800\,d^6+945\,d^5e+1665\,d^4e^2+370\,d^3e^3+888\,d^2e^4-195\,de^5+107\,e^6)e^{(-9)}\log\left(|xe+d|\right)+\frac{1}{12}\left(200\,x^6e^{15}-720\,dx^5e^{14}+1800\,d^2x^4e^{13}-4000\,d^3x^3e^{12}+9000\,d^4x^2e^{11}-25200\,d^5xe^{10}-108\,x^5e^{15}+405\,dx^4e^{14}-1080\,d^2x^3e^{13}+2700\,d^3x^2e^{12}-8100\,d^4xe^{11}+333\,x^4e^{15}-1332\,dx^3e^{14}+3996\,d^2x^2e^{13}-13320\,d^3xe^{12}-148\,x^3e^{15}+666\,dx^2e^{14}-2664\,d^2xe^{13}+888\,x^2e^{15}-5328\,dxe^{14}+780\,xe^{15}\right)e^{(-18)}+\frac{(1500\,d^8+585\,d^7e+1221\,d^6e^2+333\,d^5e^3+1036\,d^4e^4-325\,d^3e^5+321\,d^2e^6+2\,(800\,d^7e+315\,d^6e^2+666\,d^5e^3+185\,d^4e^4+592\,d^3e^5-195\,d^2e^6+214\,de^7-33\,e^8)x-33\,de^7-18\,e^8)e^{(-9)}}{2\,(xe+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="g
iac")
[Out] (2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^
5 + 107*e^6)*e^(-9)*log(abs(x*e + d)) + 1/12*(200*x^6*e^15 - 720*d*x^5*e^14
 + 1800*d^2*x^4*e^13 - 4000*d^3*x^3*e^12 + 9000*d^4*x^2*e^11 - 25200*d^5*x*
e^10 - 108*x^5*e^15 + 405*d*x^4*e^14 - 1080*d^2*x^3*e^13 + 2700*d^3*x^2*e^1
2 - 8100*d^4*x*e^11 + 333*x^4*e^15 - 1332*d*x^3*e^14 + 3996*d^2*x^2*e^13 -
13320*d^3*x*e^12 - 148*x^3*e^15 + 666*d*x^2*e^14 - 2664*d^2*x*e^13 + 888*x^
2*e^15 - 5328*d*x*e^14 + 780*x*e^15)*e^(-18) + 1/2*(1500*d^8 + 585*d^7*e +
1221*d^6*e^2 + 333*d^5*e^3 + 1036*d^4*e^4 - 325*d^3*e^5 + 321*d^2*e^6 + 2*(
800*d^7*e + 315*d^6*e^2 + 666*d^5*e^3 + 185*d^4*e^4 + 592*d^3*e^5 - 195*d^2
*e^6 + 214*d*e^7 - 33*e^8)*x - 33*d*e^7 - 18*e^8)*e^(-9)/(x*e + d)^2
```

Mupad [B]
time = 0.13, size = 771, normalized size = 2.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^3,x)
```

```
[Out] x^4*(111/(4*e^3) - (75*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/(4*e)) - x^3
*(37/(3*e^3) + (100*d^3)/(3*e^6) + (d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300
*d)/e^4 + 45/e^3))/e))/e - (d^2*((300*d)/e^4 + 45/e^3))/e^2) - x^5*((60*d)/
e^4 + 9/e^3) + x*(65/e^3 - (3*d*(148/e^3 + (3*d*(37/e^3 + (100*d^3)/e^6 + (
3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/e - (3*d^2*
((300*d)/e^4 + 45/e^3))/e^2))/e - (3*d^2*(111/e^3 - (300*d^2)/e^5 + (3*d*((
300*d)/e^4 + 45/e^3))/e))/e^2 + (d^3*((300*d)/e^4 + 45/e^3))/e^3))/e + (3*d
^2*(37/e^3 + (100*d^3)/e^6 + (3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/
e^4 + 45/e^3))/e))/e - (3*d^2*((300*d)/e^4 + 45/e^3))/e^2))/e^2 - (d^3*(111
/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/e^3) + (x*(214*d*e^
6 + 315*d^6*e + 800*d^7 - 33*e^7 - 195*d^2*e^5 + 592*d^3*e^4 + 185*d^4*e^3
+ 666*d^5*e^2) + (585*d^7*e - 33*d*e^7 + 1500*d^8 - 18*e^8 + 321*d^2*e^6 -
325*d^3*e^5 + 1036*d^4*e^4 + 333*d^5*e^3 + 1221*d^6*e^2)/(2*e))/(d^2*e^8 +
e^10*x^2 + 2*d*e^9*x) + (50*x^6)/(3*e^3) + x^2*(74/e^3 + (3*d*(37/e^3 + (10
0*d^3)/e^6 + (3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e
))/e - (3*d^2*((300*d)/e^4 + 45/e^3))/e^2))/(2*e) - (3*d^2*(111/e^3 - (300*
d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/(2*e^2) + (d^3*((300*d)/e^4 + 4
5/e^3))/(2*e^3)) + (log(d + e*x)*(945*d^5*e - 195*d*e^5 + 2800*d^6 + 107*e^
6 + 888*d^2*e^4 + 370*d^3*e^3 + 1665*d^4*e^2))/e^9
```

3.303 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx$

Optimal. Leaf size=360

$$\frac{2(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)\,x}{e^8}-\frac{(2000d^3+450d^2e+444de^2+37e^3)\,x^2}{2e^7}+\frac{(1000d^2+180de}{3e^6}$$

```
[Out] 2*(1750*d^4+450*d^3*e+555*d^2*e^2+74*d*e^3+74*e^4)*x/e^8-1/2*(2000*d^3+450*
d^2*e+444*d*e^2+37*e^3)*x^2/e^7+1/3*(1000*d^2+180*d*e+111*e^2)*x^3/e^6-5/4*
(80*d+9*e)*x^4/e^5+20*x^5/e^4-1/3*(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^
2*e^2-d*e^3+2*e^4)/e^9/(e*x+d)^3+1/2*(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e
+88*d^3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)/e^9/(e*x+d)^2+(-2800*d^6-945*d^5*e-1
665*d^4*e^2-370*d^3*e^3-888*d^2*e^4+195*d*e^5-107*e^6)/e^9/(e*x+d)-(5600*d^
5+1575*d^4*e+2220*d^3*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*ln(e*x+d)/e^9
```

Rubi [A]

time = 0.21, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = $\{1642\}$

$$\frac{x^3(1000d^2+180de+111e^2)}{3e^6}-\frac{x^2(2000d^3+450d^2e+444de^2+37e^3)}{2e^7}-\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{3e^9(d+ex)^3}+\frac{2x(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)}{e^8}+\frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)}{2e^9(d+ex)^2}-\frac{(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5)\log(d+ex)}{e^9}-\frac{2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6}{e^9(d+ex)}-\frac{5x^4(80d+9e)}{4e^5}+\frac{20x^5}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^4,x]
```

```
[Out] (2*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4)*x)/e^8 - ((2000
*d^3 + 450*d^2*e + 444*d*e^2 + 37*e^3)*x^2)/(2*e^7) + ((1000*d^2 + 180*d*e
+ 111*e^2)*x^3)/(3*e^6) - (5*(80*d + 9*e)*x^4)/(4*e^5) + (20*x^5)/e^4 - ((5
*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(3*e
^9*(d + e*x)^3) + ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d^3*e^
2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5))/(2*e^9*(d + e*x)^2) - (2800*d^6 + 945*d
^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)/(e^9
*(d + e*x)) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*
e^4 - 65*e^5)*Log[d + e*x])/e^9
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx = \int \left(\frac{2(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)}{e^8} - \frac{(2\cdots}{\cdots}\right.$$

$$= \frac{2(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)\,x}{e^8} - \frac{(2000\cdots}{\cdots}$$

Mathematica [A]
time = 0.08, size = 344, normalized size = 0.96

$$\frac{24e(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)x-6e^2(2000d^3+450d^2e+444de^2+37e^3)x^2+4e^3(1000d^2+180de+111e^2)x^3-15e^4(80d+9e)x^4+240e^5x^5-\frac{4(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{(d+ex)^3}+\frac{6(800d^7+315d^6e+666d^5e^2+185d^4e^3+592d^3e^4-195d^2e^5+214de^6-33e^7)}{(d+ex)^2}-\frac{12(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)}{d+ex}-12(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5)\log(d+ex)}{12e^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^4
,x]
```

```
[Out] (24*e*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4)*x - 6*e^2*(2
000*d^3 + 450*d^2*e + 444*d*e^2 + 37*e^3)*x^2 + 4*e^3*(1000*d^2 + 180*d*e +
 111*e^2)*x^3 - 15*e^4*(80*d + 9*e)*x^4 + 240*e^5*x^5 - (4*(5*d^2 - 2*d*e +
 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(d + e*x)^3 + (6*(
800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e^4 - 195*d^2*e^5
 + 214*d*e^6 - 33*e^7))/(d + e*x)^2 - (12*(2800*d^6 + 945*d^5*e + 1665*d^4*
e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6))/(d + e*x) - 12*(560
0*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*Log[d
 + e*x])/(12*e^9)
```

Maple [A]
time = 0.11, size = 382, normalized size = 1.06

method	result
norman	$\frac{\frac{20x^8}{e}-\frac{61600d^8+17325d^7e+24420d^6e^2+4070d^5e^3+6512d^4e^4-715d^3e^5+214d^2e^6+33d\,e^7+36e^8}{6e^9}-\frac{5(32d+9e)x^7}{4e^2}+\frac{(1120d^2+315de+444e^2)x^6}{12e^3}-\frac{(112\cdots}{\cdots}}{\cdots}$
default	$\frac{(-5600d^5-1575d^4e-2220d^3e^2-370d^2e^3-592d\,e^4+65e^5)\ln(ex+d)}{e^9}-\frac{-800d^7-315d^6e-666d^5e^2-185d^4e^3-592d^3e^4+195d^2e^5-214\cdots}{2e^9(ex+d)^2}$
risch	$\frac{20x^5}{e^4}-\frac{100d\,x^4}{e^5}-\frac{45x^4}{4e^4}+\frac{1000d^2x^3}{3e^6}+\frac{60d\,x^3}{e^5}+\frac{37x^3}{e^4}-\frac{1000d^3x^2}{e^7}-\frac{225d^2x^2}{e^6}-\frac{222d\,x^2}{e^5}-\frac{37x^2}{2e^4}+\frac{3500d^4x}{e^8}+\frac{900d^3\cdots}{e^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x,method=_RETURNVERBO
SE)
```

```
[Out] (-5600*d^5-1575*d^4*e-2220*d^3*e^2-370*d^2*e^3-592*d*e^4+65*e^5)/e^9*ln(e*x
+d)-1/2*(-800*d^7-315*d^6*e-666*d^5*e^2-185*d^4*e^3-592*d^3*e^4+195*d^2*e^5
```

-214*d*e^6+33*e^7)/e^9/(e*x+d)^2-1/e^9*(2800*d^6+945*d^5*e+1665*d^4*e^2+370*d^3*e^3+888*d^2*e^4-195*d*e^5+107*e^6)/(e*x+d)-1/3*(100*d^8+45*d^7*e+111*d^6*e^2+37*d^5*e^3+148*d^4*e^4-65*d^3*e^5+107*d^2*e^6-33*d*e^7+18*e^8)/e^9/(e*x+d)^3+1/e^8*(20*e^4*x^5-100*d*e^3*x^4-45/4*x^4*e^4+1000/3*d^2*e^2*x^3+60*d*e^3*x^3+37*e^4*x^3-1000*d^3*e*x^2-225*d^2*e^2*x^2-222*d*e^3*x^2-37/2*e^4*x^2+3500*d^4*x+900*d^3*e*x+1110*d^2*e^2*x+148*d*e^3*x+148*e^4*x)

Maxima [A]
time = 0.32, size = 353, normalized size = 0.98

$$-\left(5600\,d^5 + 1575\,d^4e + 2220\,d^3e^2 + 370\,d^2e^3 + 592\,de^4 - 65\,e^5\right)e^{(-9)}\log\left(ex + d\right) + \frac{1}{12}\left(240\,x^5e^4 - 15\left(80\,de^3 + 9\,e^4\right)x^4 + 4\left(1000\,d^2e^2 + 180\,de^3 + 111\,e^4\right)x^3 - 6\left(2000\,d^3e + 450\,d^2e^2 + 444\,de^3 + 37\,e^4\right)x^2 + 24\left(1750\,d^4 + 450\,d^3e + 555\,d^2e^2 + 74\,de^3 + 74\,e^4\right)x\right)e^{(-8)} - \frac{14600\,d^8 + 4815\,d^7e + 8214\,d^6e^2 + 1739\,d^5e^3 + 3848\,d^4e^4 - 715\,d^3e^5 + 6\left(2800\,d^6e^2 + 945\,d^5e^3 + 1665\,d^4e^4 + 370\,d^3e^5 + 888\,d^2e^6 - 195\,de^7 + 107\,e^8\right)x^2 + 214\,d^2e^6 + 3\left(10400\,d^7e + 3465\,d^6e^2 + 5994\,d^5e^3 + 1295\,d^4e^4 + 2960\,d^3e^5 - 585\,d^2e^6 + 214\,de^7 + 33\,e^8\right)x + 33\,de^7 + 36\,e^8}{6\left(x^3e^{12} + 3\,dx^2e^{11} + 3\,d^2xe^{10} + d^3e^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="m
axima")
```

```
[Out] -(5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*
e^(-9)*log(x*e + d) + 1/12*(240*x^5*e^4 - 15*(80*d*e^3 + 9*e^4)*x^4 + 4*(10
00*d^2*e^2 + 180*d*e^3 + 111*e^4)*x^3 - 6*(2000*d^3*e + 450*d^2*e^2 + 444*d
*e^3 + 37*e^4)*x^2 + 24*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74
*e^4)*x)*e^(-8) - 1/6*(14600*d^8 + 4815*d^7*e + 8214*d^6*e^2 + 1739*d^5*e^3
 + 3848*d^4*e^4 - 715*d^3*e^5 + 6*(2800*d^6*e^2 + 945*d^5*e^3 + 1665*d^4*e^
4 + 370*d^3*e^5 + 888*d^2*e^6 - 195*d*e^7 + 107*e^8)*x^2 + 214*d^2*e^6 + 3*
(10400*d^7*e + 3465*d^6*e^2 + 5994*d^5*e^3 + 1295*d^4*e^4 + 2960*d^3*e^5 -
585*d^2*e^6 + 214*d*e^7 + 33*e^8)*x + 33*d*e^7 + 36*e^8)/(x^3*e^12 + 3*d*x^
2*e^11 + 3*d^2*x*e^10 + d^3*e^9)
```

Fricas [A]
time = 0.38, size = 539, normalized size = 1.50

$$-\frac{29200\,d^8 - 3\left(80\,x^8 - 45\,x^7 + 148\,x^6 - 74\,x^5 + 592\,x^4 - 428\,x^2 - 66\,x - 24\right)e^8 + 3\left(160\,dx^7 - 105\,dx^6 + 444\,dx^5 - 370\,dx^4 - 1776\,dx^3 - 780\,dx^2 + 428\,dx + 22\,d\right)e^7 - \left(1120\,d^2x^6 - 945\,d^2x^5 + 6660\,d^2x^4 + 4662\,d^2x^3 - 5328\,d^2x^2 + 3510\,d^2x \ldots\right.}{\ldots}$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="f
ricas")
```

```
[Out] -1/12*(29200*d^8 - 3*(80*x^8 - 45*x^7 + 148*x^6 - 74*x^5 + 592*x^4 - 428*x^
2 - 66*x - 24)*e^8 + 3*(160*d*x^7 - 105*d*x^6 + 444*d*x^5 - 370*d*x^4 - 177
6*d*x^3 - 780*d*x^2 + 428*d*x + 22*d)*e^7 - (1120*d^2*x^6 - 945*d^2*x^5 + 6
660*d^2*x^4 + 4662*d^2*x^3 - 5328*d^2*x^2 + 3510*d^2*x - 428*d^2)*e^6 + (33
60*d^3*x^5 - 4725*d^3*x^4 - 32412*d^3*x^3 - 666*d^3*x^2 + 15984*d^3*x - 143
0*d^3)*e^5 - 2*(8400*d^4*x^4 + 12510*d^4*x^3 + 8658*d^4*x^2 - 2997*d^4*x -
3848*d^4)*e^4 - 2*(47000*d^5*x^3 + 9180*d^5*x^2 - 11322*d^5*x - 1739*d^5)*e
^3 - 6*(13400*d^6*x^2 - 1665*d^6*x - 2738*d^6)*e^2 + 30*(680*d^7*x + 321*d^
7)*e + 12*(5600*d^8 - 65*x^3*e^8 + (592*d*x^3 - 195*d*x^2)*e^7 + (370*d^2*x
^3 + 1776*d^2*x^2 - 195*d^2*x)*e^6 + (2220*d^3*x^3 + 1110*d^3*x^2 + 1776*d^
```

```
3*x - 65*d^3)*e^5 + (1575*d^4*x^3 + 6660*d^4*x^2 + 1110*d^4*x + 592*d^4)*e^
4 + 5*(1120*d^5*x^3 + 945*d^5*x^2 + 1332*d^5*x + 74*d^5)*e^3 + 15*(1120*d^6
*x^2 + 315*d^6*x + 148*d^6)*e^2 + 525*(32*d^7*x + 3*d^7)*e)*log(x*e + d))/(
x^3*e^12 + 3*d*x^2*e^11 + 3*d^2*x*e^10 + d^3*e^9)
```

Sympy [A]
time = 2.73, size = 401, normalized size = 1.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**4,x)
```

```
[Out] x**4*(-100*d/e**5 - 45/(4*e**4)) + x**3*(1000*d**2/(3*e**6) + 60*d/e**5 + 3
7/e**4) + x**2*(-1000*d**3/e**7 - 225*d**2/e**6 - 222*d/e**5 - 37/(2*e**4))
 + x*(3500*d**4/e**8 + 900*d**3/e**7 + 1110*d**2/e**6 + 148*d/e**5 + 148/e*
*4) + (-14600*d**8 - 4815*d**7*e - 8214*d**6*e**2 - 1739*d**5*e**3 - 3848*d
**4*e**4 + 715*d**3*e**5 - 214*d**2*e**6 - 33*d*e**7 - 36*e**8 + x**2*(-168
00*d**6*e**2 - 5670*d**5*e**3 - 9990*d**4*e**4 - 2220*d**3*e**5 - 5328*d**2
*e**6 + 1170*d*e**7 - 642*e**8) + x*(-31200*d**7*e - 10395*d**6*e**2 - 1798
2*d**5*e**3 - 3885*d**4*e**4 - 8880*d**3*e**5 + 1755*d**2*e**6 - 642*d*e**7
 - 99*e**8))/(6*d**3*e**9 + 18*d**2*e**10*x + 18*d*e**11*x**2 + 6*e**12*x**
3) + 20*x**5/e**4 - (5600*d**5 + 1575*d**4*e + 2220*d**3*e**2 + 370*d**2*e*
*3 + 592*d*e**4 - 65*e**5)*log(d + e*x)/e**9
```

Giac [A]
time = 3.85, size = 345, normalized size = 0.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="g
iac")
```

```
[Out] -(5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*
e^(-9)*log(abs(x*e + d)) + 1/12*(240*x^5*e^16 - 1200*d*x^4*e^15 + 4000*d^2*
x^3*e^14 - 12000*d^3*x^2*e^13 + 42000*d^4*x*e^12 - 135*x^4*e^16 + 720*d*x^3
*e^15 - 2700*d^2*x^2*e^14 + 10800*d^3*x*e^13 + 444*x^3*e^16 - 2664*d*x^2*e^
15 + 13320*d^2*x*e^14 - 222*x^2*e^16 + 1776*d*x*e^15 + 1776*x*e^16)*e^(-20)
 - 1/6*(14600*d^8 + 4815*d^7*e + 8214*d^6*e^2 + 1739*d^5*e^3 + 3848*d^4*e^4
 - 715*d^3*e^5 + 6*(2800*d^6*e^2 + 945*d^5*e^3 + 1665*d^4*e^4 + 370*d^3*e^5
 + 888*d^2*e^6 - 195*d*e^7 + 107*e^8)*x^2 + 214*d^2*e^6 + 3*(10400*d^7*e +
3465*d^6*e^2 + 5994*d^5*e^3 + 1295*d^4*e^4 + 2960*d^3*e^5 - 585*d^2*e^6 + 2
14*d*e^7 + 33*e^8)*x + 33*d*e^7 + 36*e^8)*e^(-9)/(x*e + d)^3
```

Mupad [B]
time = 4.28, size = 560, normalized size = 1.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^4,x)
```

```
[Out] x^3*(37/e^4 - (200*d^2)/e^6 + (4*d*((400*d)/e^5 + 45/e^4))/(3*e)) - x^2*(37
/(2*e^4) + (200*d^3)/e^7 + (2*d*(111/e^4 - (600*d^2)/e^6 + (4*d*((400*d)/e^
5 + 45/e^4))/e))/e - (3*d^2*((400*d)/e^5 + 45/e^4))/e^2) - (x*(107*d*e^6 +
(3465*d^6*e)/2 + 5200*d^7 + (33*e^7)/2 - (585*d^2*e^5)/2 + 1480*d^3*e^4 + (
1295*d^4*e^3)/2 + 2997*d^5*e^2) + (33*d*e^7 + 4815*d^7*e + 14600*d^8 + 36*e
^8 + 214*d^2*e^6 - 715*d^3*e^5 + 3848*d^4*e^4 + 1739*d^5*e^3 + 8214*d^6*e^2
)/(6*e) + x^2*(2800*d^6*e - 195*d*e^6 + 107*e^7 + 888*d^2*e^5 + 370*d^3*e^4
 + 1665*d^4*e^3 + 945*d^5*e^2))/(d^3*e^8 + e^11*x^3 + 3*d^2*e^9*x + 3*d*e^1
0*x^2) - x^4*((100*d)/e^5 + 45/(4*e^4)) + x*(148/e^4 - (100*d^4)/e^8 + (4*d
*(37/e^4 + (400*d^3)/e^7 + (4*d*(111/e^4 - (600*d^2)/e^6 + (4*d*((400*d)/e^
5 + 45/e^4))/e))/e - (6*d^2*((400*d)/e^5 + 45/e^4))/e^2))/e - (6*d^2*(111/e
^4 - (600*d^2)/e^6 + (4*d*((400*d)/e^5 + 45/e^4))/e))/e^2 + (4*d^3*((400*d)
/e^5 + 45/e^4))/e^3) + (20*x^5)/e^4 - (log(d + e*x)*(592*d*e^4 + 1575*d^4*e
 + 5600*d^5 - 65*e^5 + 370*d^2*e^3 + 2220*d^3*e^2))/e^9
```

3.304 $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

Optimal. Leaf size=221

$$\frac{(10125d^3+34350d^2e-13215de^2-5108e^3)\,x}{15625}-\frac{(4125d^3-6075d^2e-6870de^2+881e^3)\,x^2}{6250}+\frac{(500d^3-2475d^2$$

```
[Out] 1/15625*(10125*d^3+34350*d^2*e-13215*d*e^2-5108*e^3)*x-1/6250*(4125*d^3-607
5*d^2*e-6870*d*e^2+881*e^3)*x^2+1/1875*(500*d^3-2475*d^2*e+1215*d*e^2+458*e
^3)*x^3+3/500*e*(100*d^2-165*d*e+27*e^2)*x^4+3/125*(20*d-11*e)*e^2*x^5+2/15
*e^3*x^6+1/156250*(57250*d^3-66075*d^2*e-76620*d*e^2+23431*e^3)*ln(5*x^2+2*
x+3)-1/1093750*(52875*d^3+449175*d^2*e-274845*d*e^2-53189*e^3)*arctan(1/14*
(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1642, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)(52875d^3+449175d^2e-274845de^2-53189e^3)}{78125\sqrt{14}}+\frac{3}{500}ex^4(100d^2-165de+27e^2)+\frac{x^3(500d^3-2475d^2e+1215de^2+458e^3)}{1875}-\frac{x^2(4125d^3-6075d^2e-6870de^2+881e^3)}{6250}+\frac{(57250d^3-66075d^2e-76620de^2+23431e^3)\log(5x^2+2x+3)}{156250}+\frac{x(10125d^3+34350d^2e-13215de^2-5108e^3)}{15625}+\frac{3}{125}e^2x^5(20d-11e)+\frac{2e^3x^6}{15}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] ((10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 5108*e^3)*x)/15625 - ((4125*d^3 -
 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2)/6250 + ((500*d^3 - 2475*d^2*e + 12
15*d*e^2 + 458*e^3)*x^3)/1875 + (3*e*(100*d^2 - 165*d*e + 27*e^2)*x^4)/500
+ (3*(20*d - 11*e)*e^2*x^5)/125 + (2*e^3*x^6)/15 - ((52875*d^3 + 449175*d^2
*e - 274845*d*e^2 - 53189*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(78125*Sqrt[14])
 + ((57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*Log[3 + 2*x + 5*x^2
])/156250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \int \left(\frac{10125d^3+34350d^2e-13215de^2-5108e^3}{15625} - \frac{(4125d^3-6\right.$$

$$= \frac{\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)x}{15625} - \frac{(4125d^3-60}{}$$

$$= \frac{\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)x}{15625} - \frac{(4125d^3-60}{}$$

$$= \frac{\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)x}{15625} - \frac{(4125d^3-60}{}$$

$$= \frac{\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)x}{15625} - \frac{(4125d^3-60}{}$$

Mathematica [A]
time = 0.08, size = 178, normalized size = 0.81

$$\frac{35x(250d^3(486-495x+200x^2)+450d^2e(916+405x-550x^2+250x^3)+45de^2(-3524+4580x+2700x^2-4125x^3+2000x^4)+e^3(-61296-26430x+45800x^2+30375x^3-49500x^4+25000x^5))-6\sqrt{14}\,(52875d^3+449175d^2e-274845de^2-53189e^3)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)+42(57250d^3-66075d^2e-76620de^2+23431e^3)\log\left(3+2x+5x^2\right)}{6562500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x
]
```

```
[Out] (35*x*(250*d^3*(486 - 495*x + 200*x^2) + 450*d^2*e*(916 + 405*x - 550*x^2 +
 250*x^3) + 45*d*e^2*(-3524 + 4580*x + 2700*x^2 - 4125*x^3 + 2000*x^4) + e^
```

```
3*(-61296 - 26430*x + 45800*x^2 + 30375*x^3 - 49500*x^4 + 25000*x^5)) - 6*S
qrt[14]*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*ArcTan[(1 + 5
*x)/Sqrt[14]] + 42*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*Log[
3 + 2*x + 5*x^2])/6562500
```

Maple [A]

time = 0.26, size = 222, normalized size = 1.00

method	result
default	$\frac{2e^3x^6}{15}+\frac{12x^5de^2}{25}-\frac{33x^5e^3}{125}+\frac{3x^4d^2e}{5}-\frac{99x^4de^2}{100}+\frac{81e^3x^4}{500}+\frac{4d^3x^3}{15}-\frac{33d^2ex^3}{25}+\frac{81de^2x^3}{125}+\frac{458e^3x^3}{1875}-\frac{33x^2d^3}{50}+$
risch	$\frac{687de^2x^2}{625}+\frac{1374d^2ex}{625}+\frac{12x^5de^2}{25}+\frac{3x^4d^2e}{5}-\frac{99x^4de^2}{100}-\frac{33d^2ex^3}{25}-\frac{2643de^2x}{3125}+\frac{81d^3x}{125}-\frac{2643d^2e\ln(350x^2+140x+2}{6250}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x,method=_RETURNVERBOSE
)
```

```
[Out] 2/15*e^3*x^6+12/25*x^5*d*e^2-33/125*x^5*e^3+3/5*x^4*d^2*e-99/100*x^4*d*e^2+
81/500*e^3*x^4+4/15*d^3*x^3-33/25*d^2*e*x^3+81/125*d*e^2*x^3+458/1875*e^3*x
^3-33/50*x^2*d^3+243/250*d^2*e*x^2+687/625*d*e^2*x^2-881/6250*e^3*x^2+81/12
5*d^3*x+1374/625*d^2*e*x-2643/3125*d*e^2*x-5108/15625*e^3*x+1/156250*(57250
*d^3-66075*d^2*e-76620*d*e^2+23431*e^3)*ln(5*x^2+2*x+3)+1/218750*(-10575*d^
3-89835*d^2*e+54969*d*e^2+53189/5*e^3)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/
2))
```

Maxima [A]

time = 0.54, size = 197, normalized size = 0.89

$$\frac{2}{15}x^6e^3+\frac{3}{125}\left(20\,de^2-11\,e^3\right)x^5+\frac{3}{500}\left(100\,d^2e-165\,de^2+27\,e^3\right)x^4+\frac{1}{1875}\left(500\,d^3-2475\,d^2e+1215\,de^2+458\,e^3\right)x^3-\frac{1}{6250}\left(4125\,d^3-6075\,d^2e-6870\,de^2+881\,e^3\right)x^2-\frac{1}{1093750}\sqrt{14}\left(52875\,d^3+449175\,d^2e-274845\,de^2-53189\,e^3\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)+\frac{1}{15625}\left(10125\,d^3+34350\,d^2e-13215\,de^2-5108\,e^3\right)x+\frac{1}{156250}\left(57250\,d^3-66075\,d^2e-76620\,de^2+23431\,e^3\right)\log\left(5\,x^2+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] 2/15*x^6*e^3 + 3/125*(20*d*e^2 - 11*e^3)*x^5 + 3/500*(100*d^2*e - 165*d*e^2
 + 27*e^3)*x^4 + 1/1875*(500*d^3 - 2475*d^2*e + 1215*d*e^2 + 458*e^3)*x^3 -
 1/6250*(4125*d^3 - 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2 - 1/1093750*sqrt
(14)*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*arctan(1/14*sqrt
(14)*(5*x + 1)) + 1/15625*(10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 5108*e^3
)*x + 1/156250*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*log(5*x^
2 + 2*x + 3)
```

Fricas [A]

time = 0.37, size = 197, normalized size = 0.89

$$\frac{4}{15}\,d^3x^3-\frac{33}{50}\,d^3x^2+\frac{81}{125}\,d^3x-\frac{1}{1093750}\sqrt{14}\left(52875\,d^3+449175\,d^2e-274845\,de^2-53189\,e^3\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)+\frac{1}{187500}\left(25000\,x^6-49500\,x^5+30375\,x^4+45800\,x^3-26430\,x^2-61296\,x\right)e^3+\frac{3}{12500}\left(2000\,dx^5-4125\,dx^4+2700\,dx^3+4580\,dx^2-3524\,dx\right)e^2+\frac{3}{1250}\left(250\,d^2x^4-550\,d^2x^3+405\,d^2x^2+916\,d^2x\right)e+\frac{1}{156250}\left(57250\,d^3-66075\,d^2e-76620\,de^2+23431\,e^3\right)\log\left(5\,x^2+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] 4/15*d^3*x^3 - 33/50*d^3*x^2 + 81/125*d^3*x - 1/1093750*sqrt(14)*(52875*d^3
 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*arctan(1/14*sqrt(14)*(5*x + 1))
 + 1/187500*(25000*x^6 - 49500*x^5 + 30375*x^4 + 45800*x^3 - 26430*x^2 - 61
296*x)*e^3 + 3/12500*(2000*d*x^5 - 4125*d*x^4 + 2700*d*x^3 + 4580*d*x^2 - 3
524*d*x)*e^2 + 3/1250*(250*d^2*x^4 - 550*d^2*x^3 + 405*d^2*x^2 + 916*d^2*x)
*e + 1/156250*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*log(5*x^2
 + 2*x + 3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.74, size = 450, normalized size = 2.04

$$\frac{2e^3x^6}{15} + x^5\cdot\left(\frac{12de^2}{25} - \frac{33e^3}{125}\right) + x^4\cdot\left(\frac{3d^2e}{5} - \frac{99de^2}{100} + \frac{81e^3}{500}\right) + x^3\cdot\left(\frac{4d^3}{15} - \frac{33d^2e}{25} + \frac{81de^2}{125} + \frac{458e^3}{1875}\right) + x^2\left(-\frac{33d^3}{50} + \frac{243d^2e}{250} + \frac{687de^2}{625} - \frac{881e^3}{6250}\right) + x\left(\frac{81d^3}{125} + \frac{1374d^2e}{625} - \frac{2643de^2}{3125} - \frac{5108e^3}{15625}\right) + \left(\frac{229d^3}{625} - \frac{2643d^2e}{6250} - \frac{7662de^2}{15625} + \frac{23431e^3}{156250} - \frac{\sqrt{14}\,i(52875d^3 + 449175d^2e - 274845de^2 - 53189e^3)}{2187500}\right)\log\left(x + \frac{10575d^3 + 89835d^2e - 54969de^2 - \frac{53189e^3}{5} + \frac{\sqrt{14}\,i(52875d^3+449175d^2e-274845de^2-53189e^3)}{5}}{52875d^3 + 449175d^2e - 274845de^2 - 53189e^3}\right) + \left(\frac{229d^3}{625} - \frac{2643d^2e}{6250} - \frac{7662de^2}{15625} + \frac{23431e^3}{156250} + \frac{\sqrt{14}\,i(52875d^3 + 449175d^2e - 274845de^2 - 53189e^3)}{2187500}\right)\log\left(x + \frac{10575d^3 + 89835d^2e - 54969de^2 - \frac{53189e^3}{5} - \frac{\sqrt{14}\,i(52875d^3+449175d^2e-274845de^2-53189e^3)}{5}}{52875d^3 + 449175d^2e - 274845de^2 - 53189e^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] 2*e**3*x**6/15 + x**5*(12*d*e**2/25 - 33*e**3/125) + x**4*(3*d**2*e/5 - 99*
d*e**2/100 + 81*e**3/500) + x**3*(4*d**3/15 - 33*d**2*e/25 + 81*d*e**2/125
+ 458*e**3/1875) + x**2*(-33*d**3/50 + 243*d**2*e/250 + 687*d*e**2/625 - 88
1*e**3/6250) + x*(81*d**3/125 + 1374*d**2*e/625 - 2643*d*e**2/3125 - 5108*e
**3/15625) + (229*d**3/625 - 2643*d**2*e/6250 - 7662*d*e**2/15625 + 23431*e
**3/156250 - sqrt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189
*e**3)/2187500)*log(x + (10575*d**3 + 89835*d**2*e - 54969*d*e**2 - 53189*e
**3/5 + sqrt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3
)/5)/(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)) + (229*d**3
/625 - 2643*d**2*e/6250 - 7662*d*e**2/15625 + 23431*e**3/156250 + sqrt(14)*
I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)/2187500)*log(x
+ (10575*d**3 + 89835*d**2*e - 54969*d*e**2 - 53189*e**3/5 - sqrt(14)*I*(52
875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)/5)/(52875*d**3 + 449
175*d**2*e - 274845*d*e**2 - 53189*e**3))
```

Giac [A]
time = 3.20, size = 212, normalized size = 0.96

$$\frac{2}{15}x^6e^3 + \frac{12}{25}dx^5e^2 + \frac{3}{5}d^2x^4e + \frac{4}{15}d^3x^3 - \frac{33}{125}x^5e^3 - \frac{99}{100}dx^4e^2 - \frac{33}{25}d^2x^3e - \frac{33}{50}d^3x^2 + \frac{81}{500}x^4e^3 + \frac{81}{125}dx^3e^2 + \frac{243}{250}d^2x^2e + \frac{81}{125}d^3x + \frac{458}{1875}x^3e^3 + \frac{687}{625}dx^2e^2 + \frac{1374}{625}d^2xe - \frac{881}{6250}x^2e^3 - \frac{2643}{3125}dxe^2 - \frac{1}{1093750}\sqrt{14}\left(52875\,d^3 + 449175\,d^2e - 274845\,de^2 - 53189\,e^3\right)\arctan\left(\frac{1}{14}\sqrt{14}\left(5\,x+1\right)\right) - \frac{5108}{15625}xe^3 + \frac{1}{156250}\left(57250\,d^3 - 66075\,d^2e - 76620\,de^2 + 23431\,e^3\right)\log\left(5\,x^2 + 2\,x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] 2/15*x^6*e^3 + 12/25*d*x^5*e^2 + 3/5*d^2*x^4*e + 4/15*d^3*x^3 - 33/125*x^5*
e^3 - 99/100*d*x^4*e^2 - 33/25*d^2*x^3*e - 33/50*d^3*x^2 + 81/500*x^4*e^3 +
```

```
 81/125*d*x^3*e^2 + 243/250*d^2*x^2*e + 81/125*d^3*x + 458/1875*x^3*e^3 + 6
87/625*d*x^2*e^2 + 1374/625*d^2*x*e - 881/6250*x^2*e^3 - 2643/3125*d*x*e^2
- 1/1093750*sqrt(14)*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*
arctan(1/14*sqrt(14)*(5*x + 1)) - 5108/15625*x*e^3 + 1/156250*(57250*d^3 -
66075*d^2*e - 76620*d*e^2 + 23431*e^3)*log(5*x^2 + 2*x + 3)
```

Mupad [B]
time = 4.18, size = 397, normalized size = 1.80

$$x^2\left(\frac{26e^2(12d-5e)}{625}-\frac{33e(4d^2-5de+e^2)}{250}-\frac{3de^2}{50}+\frac{3d^2e}{2}-\frac{33d^3}{50}+\frac{622e^3}{3125}\right)-x^3\left(\frac{11e^2(12d-5e)}{375}+\frac{2e(4d^2-5de+e^2)}{25}-\frac{3de^2}{5}+d^2e-\frac{4d^3}{15}-\frac{111e^3}{625}\right)+x^5\left(\frac{e^2(12d-5e)}{25}-\frac{8e^3}{125}\right)-\ln(5x^2+2x+3)\left(-\frac{229d^3}{625}+\frac{2643d^2e}{6250}+\frac{7662de^2}{15625}-\frac{23431e^3}{156250}\right)-x^4\left(\frac{e^2(12d-5e)}{50}-\frac{3e(4d^2-5de+e^2)}{20}+\frac{11e^3}{125}\right)+\frac{2e^3x^6}{15}+x\left(\frac{61e^2(12d-5e)}{3125}+\frac{3d(d^2+de+2e^2)}{5}+\frac{156e(4d^2-5de+e^2)}{625}-\frac{129de^2}{125}+\frac{3d^2e}{5}+\frac{6d^3}{125}-\frac{7483e^3}{15625}\right)+\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,(-52875d^3-449175d^2e+274845de^2+53189e^3)}{1093750}+\frac{\sqrt{14}\,x\,(-52875d^3-449175d^2e+274845de^2+53189e^3)}{218750}}{-\frac{423d^3}{625}-\frac{17967d^2e}{3125}+\frac{54969de^2}{15625}+\frac{53189e^3}{78125}}\right)(-52875d^3-449175d^2e+274845de^2+53189e^3)}{1093750}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

```
[Out] x^2*((26*e^2*(12*d - 5*e))/625 - (33*e*(4*d^2 - 5*d*e + e^2))/250 - (3*d*e^
2)/50 + (3*d^2*e)/2 - (33*d^3)/50 + (622*e^3)/3125) - x^3*((11*e^2*(12*d -
5*e))/375 + (2*e*(4*d^2 - 5*d*e + e^2))/25 - (3*d*e^2)/5 + d^2*e - (4*d^3)/
15 - (111*e^3)/625) + x^5*((e^2*(12*d - 5*e))/25 - (8*e^3)/125) - log(2*x +
 5*x^2 + 3)*((7662*d*e^2)/15625 + (2643*d^2*e)/6250 - (229*d^3)/625 - (2343
1*e^3)/156250) - x^4*((e^2*(12*d - 5*e))/50 - (3*e*(4*d^2 - 5*d*e + e^2))/2
0 + (11*e^3)/125) + (2*e^3*x^6)/15 + x*((61*e^2*(12*d - 5*e))/3125 + (3*d*(
d*e + d^2 + 2*e^2))/5 + (156*e*(4*d^2 - 5*d*e + e^2))/625 - (129*d*e^2)/125
 + (3*d^2*e)/5 + (6*d^3)/125 - (7483*e^3)/15625) + (14^(1/2)*atan(((14^(1/2
)*(274845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/1093750 + (14^(1/2
)*x*(274845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/218750)/((54969*
d*e^2)/15625 - (17967*d^2*e)/3125 - (423*d^3)/625 + (53189*e^3)/78125))*(27
4845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/1093750
```

3.305 $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

Optimal. Leaf size=156

$$\frac{(2025d^2+4580de-881e^2)\,x}{3125}-\frac{(825d^2-810de-458e^2)\,x^2}{1250}+\frac{1}{375}\left(100d^2-330de+81e^2\right)x^3+\frac{1}{100}(40d-33e$$

```
[Out] 1/3125*(2025*d^2+4580*d*e-881*e^2)*x-1/1250*(825*d^2-810*d*e-458*e^2)*x^2+1
/375*(100*d^2-330*d*e+81*e^2)*x^3+1/100*(40*d-33*e)*e*x^4+4/25*e^2*x^5+1/15
625*(5725*d^2-4405*d*e-2554*e^2)*ln(5*x^2+2*x+3)-1/218750*(10575*d^2+59890*
d*e-18323*e^2)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1642, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)(10575d^2+59890de-18323e^2)}{15625\sqrt{14}}+\frac{1}{375}x^3(100d^2-330de+81e^2)-\frac{x^2(825d^2-810de-458e^2)}{1250}+\frac{(5725d^2-4405de-2554e^2)\log(5x^2+2x+3)}{15625}+\frac{x(2025d^2+4580de-881e^2)}{3125}+\frac{1}{100}ex^4(40d-33e)+\frac{4e^2x^5}{25}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] ((2025*d^2 + 4580*d*e - 881*e^2)*x)/3125 - ((825*d^2 - 810*d*e - 458*e^2)*x
^2)/1250 + ((100*d^2 - 330*d*e + 81*e^2)*x^3)/375 + ((40*d - 33*e)*e*x^4)/1
00 + (4*e^2*x^5)/25 - ((10575*d^2 + 59890*d*e - 18323*e^2)*ArcTan[(1 + 5*x)
/Sqrt[14]])/(15625*Sqrt[14]) + ((5725*d^2 - 4405*d*e - 2554*e^2)*Log[3 + 2*
x + 5*x^2])/15625
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \int \left(\frac{2025d^2+4580de-881e^2}{3125} - \frac{1}{625}\left(825d^2-810de-458e^2\right)\right.$$

$$= \frac{\left(2025d^2+4580de-881e^2\right)x}{3125} - \frac{\left(825d^2-810de-458e^2\right)x^2}{1250} +$$

$$= \frac{\left(2025d^2+4580de-881e^2\right)x}{3125} - \frac{\left(825d^2-810de-458e^2\right)x^2}{1250} +$$

$$= \frac{\left(2025d^2+4580de-881e^2\right)x}{3125} - \frac{\left(825d^2-810de-458e^2\right)x^2}{1250} +$$

$$= \frac{\left(2025d^2+4580de-881e^2\right)x}{3125} - \frac{\left(825d^2-810de-458e^2\right)x^2}{1250} +$$

Mathematica [A]
time = 0.06, size = 130, normalized size = 0.83

$$\frac{35x(50d^2(486-495x+200x^2)+60de(916+405x-550x^2+250x^3)+3e^2(-3524+4580x+2700x^2-4125x^3+2000x^4))-6\sqrt{14}\left(10575d^2+59890de-18323e^2\right)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)+84(5725d^2-4405de-2554e^2)\log\left(3+2x+5x^2\right)}{1312500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x
]
```

```
[Out] (35*x*(50*d^2*(486 - 495*x + 200*x^2) + 60*d*e*(916 + 405*x - 550*x^2 + 250
*x^3) + 3*e^2*(-3524 + 4580*x + 2700*x^2 - 4125*x^3 + 2000*x^4)) - 6*Sqrt[1
```

```
4]*(10575*d^2 + 59890*d*e - 18323*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]] + 84*(572
5*d^2 - 4405*d*e - 2554*e^2)*Log[3 + 2*x + 5*x^2])/1312500
```

Maple [A]
time = 0.15, size = 147, normalized size = 0.94

method	result
default	$\frac{4e^{2}x^{5}}{25}+\frac{2x^{4}de}{5}-\frac{33x^{4}e^{2}}{100}+\frac{4x^{3}d^{2}}{15}-\frac{22x^{3}de}{25}+\frac{27e^{2}x^{3}}{125}-\frac{33d^{2}x^{2}}{50}+\frac{81de\,x^{2}}{125}+\frac{229e^{2}x^{2}}{625}+\frac{81d^{2}x}{125}+\frac{916dex}{625}-\frac{881e}{312}$
risch	$-\frac{33x^{4}e^{2}}{100}+\frac{4x^{3}d^{2}}{15}-\frac{33d^{2}x^{2}}{50}-\frac{22x^{3}de}{25}+\frac{2x^{4}de}{5}+\frac{229d^{2}\ln\left(350x^{2}+140x+210\right)}{625}-\frac{2554e^{2}\ln\left(350x^{2}+140x+210\right)}{15625}+\frac{916d}{625}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x,method=_RETURNVERBOSE
)
```

```
[Out] 4/25*e^2*x^5+2/5*x^4*d*e-33/100*x^4*e^2+4/15*x^3*d^2-22/25*x^3*d*e+27/125*e
^2*x^3-33/50*d^2*x^2+81/125*d*e*x^2+229/625*e^2*x^2+81/125*d^2*x+916/625*d*
e*x-881/3125*e^2*x+1/31250*(11450*d^2-8810*d*e-5108*e^2)*ln(5*x^2+2*x+3)+1/
43750*(-2115*d^2-11978*d*e+18323/5*e^2)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1
/2))
```

Maxima [A]
time = 0.56, size = 140, normalized size = 0.90

$$\frac{4}{25}\,x^{5}e^{2}+\frac{1}{100}\,{\left(40\,de-33\,e^{2}\right)}x^{4}+\frac{1}{375}\,{\left(100\,d^{2}-330\,de+81\,e^{2}\right)}x^{3}-\frac{1}{1250}\,{\left(825\,d^{2}-810\,de-458\,e^{2}\right)}x^{2}-\frac{1}{218750}\,\sqrt{14}\,{\left(10575\,d^{2}+59890\,de-18323\,e^{2}\right)}\arctan\left(\frac{1}{14}\,\sqrt{14}\,{\left(5\,x+1\right)}\right)+\frac{1}{3125}\,{\left(2025\,d^{2}+4580\,de-881\,e^{2}\right)}x+\frac{1}{15625}\,{\left(5725\,d^{2}-4405\,de-2554\,e^{2}\right)}\log\left(5\,x^{2}+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] 4/25*x^5*e^2 + 1/100*(40*d*e - 33*e^2)*x^4 + 1/375*(100*d^2 - 330*d*e + 81*
e^2)*x^3 - 1/1250*(825*d^2 - 810*d*e - 458*e^2)*x^2 - 1/218750*sqrt(14)*(10
575*d^2 + 59890*d*e - 18323*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/3125*(
2025*d^2 + 4580*d*e - 881*e^2)*x + 1/15625*(5725*d^2 - 4405*d*e - 2554*e^2)
*log(5*x^2 + 2*x + 3)
```

Fricas [A]
time = 0.39, size = 137, normalized size = 0.88

$$\frac{4}{15}\,d^{2}x^{3}-\frac{33}{50}\,d^{2}x^{2}+\frac{81}{125}\,d^{2}x-\frac{1}{218750}\,\sqrt{14}\,{\left(10575\,d^{2}+59890\,de-18323\,e^{2}\right)}\arctan\left(\frac{1}{14}\,\sqrt{14}\,{\left(5\,x+1\right)}\right)+\frac{1}{12500}\,{\left(2000\,x^{5}-4125\,x^{4}+2700\,x^{3}+4580\,x^{2}-3524\,x\right)}e^{2}+\frac{1}{625}\,{\left(250\,dx^{4}-550\,dx^{3}+405\,dx^{2}+916\,dx\right)}e+\frac{1}{15625}\,{\left(5725\,d^{2}-4405\,de-2554\,e^{2}\right)}\log\left(5\,x^{2}+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] 4/15*d^2*x^3 - 33/50*d^2*x^2 + 81/125*d^2*x - 1/218750*sqrt(14)*(10575*d^2
+ 59890*d*e - 18323*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/12500*(2000*x^
5 - 4125*x^4 + 2700*x^3 + 4580*x^2 - 3524*x)*e^2 + 1/625*(250*d*x^4 - 550*d
*x^3 + 405*d*x^2 + 916*d*x)*e + 1/15625*(5725*d^2 - 4405*d*e - 2554*e^2)*lo
g(5*x^2 + 2*x + 3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.52, size = 303, normalized size = 1.94

$$\frac{4e^2x^5}{25} + x^4 \cdot \left(\frac{2de}{5} - \frac{33e^2}{100}\right) + x^3 \cdot \left(\frac{4d^2}{15} - \frac{22de}{25} + \frac{27e^2}{125}\right) + x^2\left(-\frac{33d^2}{50} + \frac{81de}{125} + \frac{229e^2}{625}\right) + x\left(\frac{81d^2}{125} + \frac{916de}{625} - \frac{881e^2}{3125}\right) + \left(\frac{229d^2}{625} - \frac{881de}{3125} - \frac{2554e^2}{15625} - \frac{\sqrt{14}\,i(10575d^2 + 59890de - 18323e^2)}{437500}\right)\log\left(x + \frac{2115d^2 + 11978de - \frac{18323e^2}{5} + \frac{\sqrt{14}\,i(10575d^2+59890de-18323e^2)}{5}}{10575d^2 + 59890de - 18323e^2}\right) + \left(\frac{229d^2}{625} - \frac{881de}{3125} - \frac{2554e^2}{15625} + \frac{\sqrt{14}\,i(10575d^2 + 59890de - 18323e^2)}{437500}\right)\log\left(x + \frac{2115d^2 + 11978de - \frac{18323e^2}{5} - \frac{\sqrt{14}\,i(10575d^2+59890de-18323e^2)}{5}}{10575d^2 + 59890de - 18323e^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] 4*e**2*x**5/25 + x**4*(2*d*e/5 - 33*e**2/100) + x**3*(4*d**2/15 - 22*d*e/25
 + 27*e**2/125) + x**2*(-33*d**2/50 + 81*d*e/125 + 229*e**2/625) + x*(81*d*
*2/125 + 916*d*e/625 - 881*e**2/3125) + (229*d**2/625 - 881*d*e/3125 - 2554
*e**2/15625 - sqrt(14)*I*(10575*d**2 + 59890*d*e - 18323*e**2)/437500)*log(
x + (2115*d**2 + 11978*d*e - 18323*e**2/5 + sqrt(14)*I*(10575*d**2 + 59890*
d*e - 18323*e**2)/5)/(10575*d**2 + 59890*d*e - 18323*e**2)) + (229*d**2/625
 - 881*d*e/3125 - 2554*e**2/15625 + sqrt(14)*I*(10575*d**2 + 59890*d*e - 18
323*e**2)/437500)*log(x + (2115*d**2 + 11978*d*e - 18323*e**2/5 - sqrt(14)*
I*(10575*d**2 + 59890*d*e - 18323*e**2)/5)/(10575*d**2 + 59890*d*e - 18323*
e**2))
```

Giac [A]
time = 3.30, size = 145, normalized size = 0.93

$$\frac{4}{25}\,x^5e^2 + \frac{2}{5}\,dx^4e + \frac{4}{15}\,d^2x^3 - \frac{33}{100}\,x^4e^2 - \frac{22}{25}\,dx^3e - \frac{33}{50}\,d^2x^2 + \frac{27}{125}\,x^3e^2 + \frac{81}{125}\,dx^2e + \frac{81}{125}\,d^2x + \frac{229}{625}\,x^2e^2 + \frac{916}{625}\,dxe - \frac{1}{218750}\,\sqrt{14}\,(10575\,d^2 + 59890\,de - 18323\,e^2)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x + 1)\right) - \frac{881}{3125}\,xe^2 + \frac{1}{15625}\,(5725\,d^2 - 4405\,de - 2554\,e^2)\log\left(5\,x^2 + 2\,x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] 4/25*x^5*e^2 + 2/5*d*x^4*e + 4/15*d^2*x^3 - 33/100*x^4*e^2 - 22/25*d*x^3*e
- 33/50*d^2*x^2 + 27/125*x^3*e^2 + 81/125*d*x^2*e + 81/125*d^2*x + 229/625*
x^2*e^2 + 916/625*d*x*e - 1/218750*sqrt(14)*(10575*d^2 + 59890*d*e - 18323*
e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) - 881/3125*x*e^2 + 1/15625*(5725*d^2 -
 4405*d*e - 2554*e^2)*log(5*x^2 + 2*x + 3)
```

Mupad [B]
time = 0.10, size = 223, normalized size = 1.43

$$x\left(\frac{4\,d\,e}{5} + \frac{52\,e\,(8\,d - 5\,e)}{625} + \frac{81\,d^2}{125} + \frac{419\,e^2}{3125}\right) - \ln\left(5\,x^2 + 2\,x + 3\right)\left(-\frac{229\,d^2}{625} + \frac{881\,d\,e}{3125} + \frac{2554\,e^2}{15625}\right) + x^4\left(\frac{e\,(8\,d - 5\,e)}{20} - \frac{2\,e^2}{25}\right) - x^3\left(\frac{2\,d\,e}{3} + \frac{2\,e\,(8\,d - 5\,e)}{75} - \frac{4\,d^2}{15} - \frac{31\,e^2}{375}\right) + x^2\left(d\,e - \frac{11\,e\,(8\,d - 5\,e)}{250} - \frac{33\,d^2}{50} + \frac{183\,e^2}{1250}\right) + \frac{4\,e^2\,x^5}{25} - \frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,(10575\,d^2+59890\,d\,e-18323\,e^2)}{218750} + \frac{\sqrt{14}\,x\,(10575\,d^2+59890\,d\,e-18323\,e^2)}{43750}}{\frac{423\,d^2}{625} + \frac{11978\,d\,e}{3125} - \frac{18323\,e^2}{15625}}\right)(10575\,d^2 + 59890\,d\,e - 18323\,e^2)}{218750}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

```
[Out] x*((4*d*e)/5 + (52*e*(8*d - 5*e))/625 + (81*d^2)/125 + (419*e^2)/3125) - lo
g(2*x + 5*x^2 + 3)*((881*d*e)/3125 - (229*d^2)/625 + (2554*e^2)/15625) + x^
4*((e*(8*d - 5*e))/20 - (2*e^2)/25) - x^3*((2*d*e)/3 + (2*e*(8*d - 5*e))/75
 - (4*d^2)/15 - (31*e^2)/375) + x^2*(d*e - (11*e*(8*d - 5*e))/250 - (33*d^2
)/50 + (183*e^2)/1250) + (4*e^2*x^5)/25 - (14^(1/2)*atan(((14^(1/2)*(59890*
d*e + 10575*d^2 - 18323*e^2))/218750 + (14^(1/2)*x*(59890*d*e + 10575*d^2 -
 18323*e^2))/43750)/((11978*d*e)/3125 + (423*d^2)/625 - (18323*e^2)/15625))
*(59890*d*e + 10575*d^2 - 18323*e^2))/218750
```

3.306 $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

Optimal. Leaf size=99

$$\frac{1}{625}(405d+458e)x-\frac{3}{250}(55d-27e)x^2+\frac{1}{75}(20d-33e)x^3+\frac{ex^4}{5}-\frac{(2115d+5989e)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{3125\sqrt{14}}+\frac{(2290d-88$$

```
[Out] 1/625*(405*d+458*e)*x-3/250*(55*d-27*e)*x^2+1/75*(20*d-33*e)*x^3+1/5*e*x^4+
1/6250*(2290*d-881*e)*ln(5*x^2+2*x+3)-1/43750*(2115*d+5989*e)*arctan(1/14*(
1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {1642, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)(2115d+5989e)}{3125\sqrt{14}}+\frac{1}{75}x^3(20d-33e)-\frac{3}{250}x^2(55d-27e)+\frac{(2290d-881e)\log(5x^2+2x+3)}{6250}+\frac{1}{625}x(405d+458e)+\frac{ex^4}{5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] ((405*d + 458*e)*x)/625 - (3*(55*d - 27*e)*x^2)/250 + ((20*d - 33*e)*x^3)/7
5 + (e*x^4)/5 - ((2115*d + 5989*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(3125*Sqrt[1
4]) + ((2290*d - 881*e)*Log[3 + 2*x + 5*x^2])/6250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \int \left(\frac{1}{625}(405d+458e) - \frac{3}{125}(55d-27e)x + \frac{1}{25}(20d-33e)x^2\right.$$

$$= \frac{1}{625}(405d+458e)x - \frac{3}{250}(55d-27e)x^2 + \frac{1}{75}(20d-33e)x^3 +$$

$$= \frac{1}{625}(405d+458e)x - \frac{3}{250}(55d-27e)x^2 + \frac{1}{75}(20d-33e)x^3 +$$

$$= \frac{1}{625}(405d+458e)x - \frac{3}{250}(55d-27e)x^2 + \frac{1}{75}(20d-33e)x^3 +$$

$$= \frac{1}{625}(405d+458e)x - \frac{3}{250}(55d-27e)x^2 + \frac{1}{75}(20d-33e)x^3 +$$

Mathematica [A]
time = 0.04, size = 86, normalized size = 0.87

$$\frac{35x(5d(486-495x+200x^2)+3e(916+405x-550x^2+250x^3)) - 3\sqrt{14}\,(2115d+5989e)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right) + 21(2290d-881e)\log\left(3+2x+5x^2\right)}{131250}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
[Out] (35*x*(5*d*(486 - 495*x + 200*x^2) + 3*e*(916 + 405*x - 550*x^2 + 250*x^3))
 - 3*Sqrt[14]*(2115*d + 5989*e)*ArcTan[(1 + 5*x)/Sqrt[14]] + 21*(2290*d - 8
81*e)*Log[3 + 2*x + 5*x^2])/131250
```

Maple [A]
time = 0.13, size = 83, normalized size = 0.84

method	result
default	$\frac{e\,x^4}{5}+\frac{4d\,x^3}{15}-\frac{11e\,x^3}{25}-\frac{33d\,x^2}{50}+\frac{81e\,x^2}{250}+\frac{81dx}{125}+\frac{458ex}{625}+\frac{(2290d-881e)\ln(5x^2+2x+3)}{6250}+\frac{\left(-423d-\frac{5989e}{5}\right)\sqrt{14}\,\mathrm{arc}}{8750}$
risch	$\frac{e\,x^4}{5}+\frac{4d\,x^3}{15}-\frac{11e\,x^3}{25}-\frac{33d\,x^2}{50}+\frac{81e\,x^2}{250}+\frac{81dx}{125}+\frac{458ex}{625}+\frac{229d\ln(350x^2+140x+210)}{625}-\frac{881e\ln(350x^2+140x+210)}{6250}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*e*x^4+4/15*d*x^3-11/25*e*x^3-33/50*d*x^2+81/250*e*x^2+81/125*d*x+458/62
5*e*x+1/6250*(2290*d-881*e)*ln(5*x^2+2*x+3)+1/8750*(-423*d-5989/5*e)*14^(1/
2)*arctan(1/28*(10*x+2)*14^(1/2))
```

Maxima [A]
time = 0.53, size = 90, normalized size = 0.91

$$\frac{1}{5}\,x^4e+\frac{1}{75}\,(20\,d-33\,e)x^3-\frac{3}{250}\,(55\,d-27\,e)x^2-\frac{1}{43750}\,\sqrt{14}\,(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{1}{625}\,(405\,d+458\,e)x+\frac{1}{6250}\,(2290\,d-881\,e)\log\left(5\,x^2+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="maxim
a")
```

```
[Out] 1/5*x^4*e + 1/75*(20*d - 33*e)*x^3 - 3/250*(55*d - 27*e)*x^2 - 1/43750*sqrt
(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(405*d + 458
*e)*x + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

Fricas [A]
time = 0.38, size = 85, normalized size = 0.86

$$\frac{4}{15}\,dx^3-\frac{33}{50}\,dx^2-\frac{1}{43750}\,\sqrt{14}\,(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{81}{125}\,dx+\frac{1}{1250}\,\left(250\,x^4-550\,x^3+405\,x^2+916\,x\right)e+\frac{1}{6250}\,(2290\,d-881\,e)\log\left(5\,x^2+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="frica
s")
```

```
[Out] 4/15*d*x^3 - 33/50*d*x^2 - 1/43750*sqrt(14)*(2115*d + 5989*e)*arctan(1/14*s
qrt(14)*(5*x + 1)) + 81/125*d*x + 1/1250*(250*x^4 - 550*x^3 + 405*x^2 + 916
*x)*e + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.32, size = 163, normalized size = 1.65

$$\frac{ex^4}{5}+x^3\cdot\left(\frac{4d}{15}-\frac{11e}{25}\right)+x^2\left(-\frac{33d}{50}+\frac{81e}{250}\right)+x\left(\frac{81d}{125}+\frac{458e}{625}\right)+\left(\frac{229d}{625}-\frac{881e}{6250}-\frac{\sqrt{14}\,i(2115d+5989e)}{87500}\right)\log\left(x+\frac{423d+\frac{5989e}{5}+\frac{\sqrt{14}\,i(2115d+5989e)}{5}}{2115d+5989e}\right)+\left(\frac{229d}{625}-\frac{881e}{6250}+\frac{\sqrt{14}\,i(2115d+5989e)}{87500}\right)\log\left(x+\frac{423d+\frac{5989e}{5}-\frac{\sqrt{14}\,i(2115d+5989e)}{5}}{2115d+5989e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] e*x**4/5 + x**3*(4*d/15 - 11*e/25) + x**2*(-33*d/50 + 81*e/250) + x*(81*d/1
25 + 458*e/625) + (229*d/625 - 881*e/6250 - sqrt(14)*I*(2115*d + 5989*e)/87
500)*log(x + (423*d + 5989*e/5 + sqrt(14)*I*(2115*d + 5989*e)/5)/(2115*d +
5989*e)) + (229*d/625 - 881*e/6250 + sqrt(14)*I*(2115*d + 5989*e)/87500)*lo
g(x + (423*d + 5989*e/5 - sqrt(14)*I*(2115*d + 5989*e)/5)/(2115*d + 5989*e)
)
```

Giac [A]
time = 4.95, size = 88, normalized size = 0.89

$$\frac{1}{5}x^4e+\frac{4}{15}dx^3-\frac{11}{25}x^3e-\frac{33}{50}dx^2+\frac{81}{250}x^2e-\frac{1}{43750}\sqrt{14}\,(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)+\frac{81}{125}dx+\frac{458}{625}xe+\frac{1}{6250}(2290\,d-881\,e)\log\left(5\,x^2+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="giac"
)
```

```
[Out] 1/5*x^4*e + 4/15*d*x^3 - 11/25*x^3*e - 33/50*d*x^2 + 81/250*x^2*e - 1/43750
*sqrt(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 81/125*d*x +
458/625*x*e + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

Mupad [B]
time = 0.07, size = 107, normalized size = 1.08

$$x^3\left(\frac{4\,d}{15}-\frac{11\,e}{25}\right)-x^2\left(\frac{33\,d}{50}-\frac{81\,e}{250}\right)+\ln\left(5\,x^2+2\,x+3\right)\left(\frac{229\,d}{625}-\frac{881\,e}{6250}\right)+\frac{e\,x^4}{5}+x\left(\frac{81\,d}{125}+\frac{458\,e}{625}\right)-\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,(2115\,d+5989\,e)}{43750}+\frac{\sqrt{14}\,x\,(2115\,d+5989\,e)}{8750}}{\frac{423\,d}{625}+\frac{5989\,e}{3125}}\right)(2115\,d+5989\,e)}{43750}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

```
[Out] x^3*((4*d)/15 - (11*e)/25) - x^2*((33*d)/50 - (81*e)/250) + log(2*x + 5*x^2
 + 3)*((229*d)/625 - (881*e)/6250) + (e*x^4)/5 + x*((81*d)/125 + (458*e)/62
5) - (14^(1/2)*atan(((14^(1/2)*(2115*d + 5989*e))/43750 + (14^(1/2)*x*(2115
*d + 5989*e))/8750)/((423*d)/625 + (5989*e)/3125))*(2115*d + 5989*e))/43750
```

3.307 $\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2} \, dx$

Optimal. Leaf size=56

$$\frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}-\frac{423\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{625\sqrt{14}}+\frac{229}{625}\log\left(3+2x+5x^2\right)$$

```
[Out] 81/125*x-33/50*x^2+4/15*x^3+229/625*ln(5*x^2+2*x+3)-423/8750*arctan(1/14*(1
+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1671, 648, 632, 210, 642}

$$-\frac{423\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)}{625\sqrt{14}}+\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{229}{625}\log\left(5x^2+2x+3\right)+\frac{81x}{125}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2),x]
```

```
[Out] (81*x)/125 - (33*x^2)/50 + (4*x^3)/15 - (423*ArcTan[(1 + 5*x)/Sqrt[14]])/(6
25*Sqrt[14]) + (229*Log[3 + 2*x + 5*x^2])/625
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx &= \int \left(\frac{81}{125}-\frac{33x}{25}+\frac{4x^2}{5}+\frac{7+458x}{125\left(3+2x+5x^2\right)}\right)dx\\ &= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}+\frac{1}{125}\int \frac{7+458x}{3+2x+5x^2}\,dx\\ &= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}+\frac{229}{625}\int \frac{2+10x}{3+2x+5x^2}\,dx-\frac{423}{625}\int \frac{1}{3+2x+5x^2}\,dx\\ &= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}+\frac{229}{625}\log\left(3+2x+5x^2\right)+\frac{846}{625}\text{Subst}\left(\int \frac{1}{-56-x^2}\,d\right.\\ &= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}-\frac{423\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{625\sqrt{14}}+\frac{229}{625}\log\left(3+2x+5x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 0.89

$$\frac{35x(486-495x+200x^2)-1269\sqrt{14}\,\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)+9618\log\left(3+2x+5x^2\right)}{26250}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2),x]
```

```
[Out] (35*x*(486 - 495*x + 200*x^2) - 1269*Sqrt[14]*ArcTan[(1 + 5*x)/Sqrt[14]] +
9618*Log[3 + 2*x + 5*x^2])/26250
```

Maple [A]
time = 0.13, size = 44, normalized size = 0.79

method	result	size
default	$\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{81x}{125}+\frac{229\ln(5x^2+2x+3)}{625}-\frac{423\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{8750}$	44
risch	$\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{81x}{125}+\frac{229\ln(25x^2+10x+15)}{625}-\frac{423\arctan\left(\frac{(1+5x)\sqrt{14}}{14}\right)\sqrt{14}}{8750}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x,method=_RETURNVERBOSE)
```

```
[Out] 4/15*x^3-33/50*x^2+81/125*x+229/625*ln(5*x^2+2*x+3)-423/8750*14^(1/2)*arcta
n(1/28*(10*x+2)*14^(1/2))
```

Maxima [A]
time = 0.54, size = 43, normalized size = 0.77

$$\frac{4}{15}\,x^3-\frac{33}{50}\,x^2-\frac{423}{8750}\,\sqrt{14}\,\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{81}{125}\,x+\frac{229}{625}\,\log\left(5\,x^2+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="maxima")
```

```
[Out] 4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

Fricas [A]
time = 0.37, size = 43, normalized size = 0.77

$$\frac{4}{15}\,x^3-\frac{33}{50}\,x^2-\frac{423}{8750}\,\sqrt{14}\,\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{81}{125}\,x+\frac{229}{625}\,\log\left(5\,x^2+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fricas")
```

```
[Out] 4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

Sympy [A]
time = 0.04, size = 61, normalized size = 1.09

$$\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{81x}{125}+\frac{229\log\left(x^2+\frac{2x}{5}+\frac{3}{5}\right)}{625}-\frac{423\sqrt{14}\,\operatorname{atan}\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{8750}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] 4*x**3/15 - 33*x**2/50 + 81*x/125 + 229*log(x**2 + 2*x/5 + 3/5)/625 - 423*s
qrt(14)*atan(5*sqrt(14)*x/14 + sqrt(14)/14)/8750
```

Giac [A]
time = 4.70, size = 43, normalized size = 0.77

$$\frac{4}{15}\,x^{3}-\frac{33}{50}\,x^{2}-\frac{423}{8750}\,\sqrt{14}\,\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{81}{125}\,x+\frac{229}{625}\,\log\left(5\,x^{2}+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="giac")
```

```
[Out] 4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

Mupad [B]
time = 0.04, size = 45, normalized size = 0.80

$$\frac{81\,x}{125}+\frac{229\,\ln\left(5\,x^2+2\,x+3\right)}{625}-\frac{423\,\sqrt{14}\,\mathrm{atan}\left(\frac{5\,\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{8750}-\frac{33\,x^2}{50}+\frac{4\,x^3}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3),x)
```

```
[Out] (81*x)/125 + (229*log(2*x + 5*x^2 + 3))/625 - (423*14^(1/2)*atan((5*14^(1/2
)*x)/14 + 14^(1/2)/14))/8750 - (33*x^2)/50 + (4*x^3)/15
```

3.308 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx$

Optimal. Leaf size=168

$$-\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}-\frac{(423d-1367e)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{125\sqrt{14}\,(5d^2-2de+3e^2)}+\frac{(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^3\,(5d^2-2de+3e^2)}+\frac{(458\dots}{2\dots}$$

```
[Out] -1/25*(20*d+33*e)*x/e^2+2/5*x^2/e+(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(
e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)+1/250*(458*d-7*e)*ln(5*x^2+2*x+3)/(5*d^2-2*d
*e+3*e^2)-1/1750*(423*d-1367*e)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+
3*e^2)*14^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1642, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)(423d-1367e)}{125\sqrt{14}\,(5d^2-2de+3e^2)}+\frac{(458d-7e)\log(5x^2+2x+3)}{250\,(5d^2-2de+3e^2)}+\frac{(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^3\,(5d^2-2de+3e^2)}-\frac{x(20d+33e)}{25e^2}+\frac{2x^2}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)),x]
```

```
[Out] -1/25*((20*d + 33*e)*x)/e^2 + (2*x^2)/(5*e) - ((423*d - 1367*e)*ArcTan[(1 +
 5*x)/Sqrt[14]])/(125*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)) + ((4*d^4 + 5*d^3*e
 + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)) +
 ((458*d - 7*e)*Log[3 + 2*x + 5*x^2])/(250*(5*d^2 - 2*d*e + 3*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)}\,dx &= \int \left(\frac{-20d-33e}{25e^2}+\frac{4x}{5e}+\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2\left(5d^2-2de+3e^2\right)(d+ex)}+\frac{7d+27}{25\left(5d^2-2d\right.}\right.\\ &= -\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}+\\ &= -\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}-\\ &= -\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}+\\ &= -\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}-\frac{(423d-1367e)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{125\sqrt{14}\left(5d^2-2de+3e^2\right)}+\frac{\left(4d^4+5d^3e+\right.}{e}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 146, normalized size = 0.87

$$\frac{70e\left(5d^2-2de+3e^2\right)x(-20d+e(-33+10x))-\sqrt{14}\,(423d-1367e)e^3\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)+1750\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)+7(458d-7e)e^3\log\left(3+2x+5x^2\right)}{1750e^3\left(5d^2-2de+3e^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)),x]
```

```
[Out] (70*e*(5*d^2 - 2*d*e + 3*e^2)*x*(-20*d + e*(-33 + 10*x)) - Sqrt[14]*(423*d
- 1367*e)*e^3*ArcTan[(1 + 5*x)/Sqrt[14]] + 1750*(4*d^4 + 5*d^3*e + 3*d^2*e^
```

```
2 - d*e^3 + 2*e^4)*Log[d + e*x] + 7*(458*d - 7*e)*e^3*Log[3 + 2*x + 5*x^2])
/(1750*e^3*(5*d^2 - 2*d*e + 3*e^2))
```

Maple [A]
time = 0.24, size = 142, normalized size = 0.85

method	result
default	$-\frac{-10e\,x^{2}+20dx+33ex}{25e^{2}}+\frac{\left(4d^{4}+5d^{3}e+3d^{2}e^{2}-d\,e^{3}+2e^{4}\right)\ln\left(ex+d\right)}{e^{3}\left(5d^{2}-2de+3e^{2}\right)}+\frac{\frac{\left(458d-7e\right)\ln\left(5x^{2}+2x+3\right)}{10}+\frac{\left(-\frac{423d}{5}+\frac{1367e}{5}\right)\sqrt{14}\,\arctan\left(\frac{(10}{}\right.}{14}}{125d^{2}-50de+75e^{2}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/25/e^2*(-10*e*x^2+20*d*x+33*e*x)+(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*l
n(e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)+1/(125*d^2-50*d*e+75*e^2)*(1/10*(458*d-7*e
)*ln(5*x^2+2*x+3)+1/14*(-423/5*d+1367/5*e)*14^(1/2)*arctan(1/28*(10*x+2)*14
^(1/2)))
```

Maxima [A]
time = 0.58, size = 159, normalized size = 0.95

$$\frac{1}{25}\left(10\,x^{2}e-(20\,d+33\,e)x\right)e^{(-2)}-\frac{\sqrt{14}\,(423\,d-1367\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)}{1750\,(5\,d^{2}-2\,de+3\,e^{2})}+\frac{(458\,d-7\,e)\log\left(5\,x^{2}+2\,x+3\right)}{250\,(5\,d^{2}-2\,de+3\,e^{2})}+\frac{(4\,d^{4}+5\,d^{3}e+3\,d^{2}e^{2}-de^{3}+2\,e^{4})\log\left(xe+d\right)}{5\,d^{2}e^{3}-2\,de^{4}+3\,e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="maxim
a")
```

```
[Out] 1/25*(10*x^2*e - (20*d + 33*e)*x)*e^(-2) - 1/1750*sqrt(14)*(423*d - 1367*e)
*arctan(1/14*sqrt(14)*(5*x + 1))/(5*d^2 - 2*d*e + 3*e^2) + 1/250*(458*d - 7
*e)*log(5*x^2 + 2*x + 3)/(5*d^2 - 2*d*e + 3*e^2) + (4*d^4 + 5*d^3*e + 3*d^2
*e^2 - d*e^3 + 2*e^4)*log(x*e + d)/(5*d^2*e^3 - 2*d*e^4 + 3*e^5)
```

Fricas [A]
time = 0.43, size = 163, normalized size = 0.97

$$-\frac{7000\,d^{3}xe+\sqrt{14}\,(423\,de^{3}-1367\,e^{4})\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)-210\,(10\,x^{2}-33\,x)e^{4}+140\,(10\,dx^{2}-3\,dx)e^{3}-1750\,(2\,d^{2}x^{2}-5\,d^{2}x)e^{2}-7\,(458\,de^{3}-7\,e^{4})\log\left(5\,x^{2}+2\,x+3\right)-1750\,(4\,d^{4}+5\,d^{3}e+3\,d^{2}e^{2}-de^{3}+2\,e^{4})\log\left(xe+d\right)}{1750\,(5\,d^{2}e^{3}-2\,de^{4}+3\,e^{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="frica
s")
```

```
[Out] -1/1750*(7000*d^3*x*e + sqrt(14)*(423*d*e^3 - 1367*e^4)*arctan(1/14*sqrt(14
)*(5*x + 1)) - 210*(10*x^2 - 33*x)*e^4 + 140*(10*d*x^2 - 3*d*x)*e^3 - 1750*
```

```
(2*d^2*x^2 - 5*d^2*x)*e^2 - 7*(458*d*e^3 - 7*e^4)*log(5*x^2 + 2*x + 3) - 17
50*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(x*e + d))/(5*d^2*e^3 -
 2*d*e^4 + 3*e^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.62, size = 158, normalized size = 0.94

$$\frac{1}{25}\left(10\,x^2 e - 20\,dx - 33\,xe\right)e^{(-2)} - \frac{\sqrt{14}\,(423\,d - 1367\,e)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{1750\,(5\,d^2 - 2\,de + 3\,e^2)} + \frac{(458\,d - 7\,e)\log\left(5\,x^2 + 2\,x + 3\right)}{250\,(5\,d^2 - 2\,de + 3\,e^2)} + \frac{(4\,d^4 + 5\,d^3 e + 3\,d^2 e^2 - de^3 + 2\,e^4)\log\left(|xe + d|\right)}{5\,d^2 e^3 - 2\,de^4 + 3\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="giac"
)
```

```
[Out] 1/25*(10*x^2*e - 20*d*x - 33*x*e)*e^(-2) - 1/1750*sqrt(14)*(423*d - 1367*e)
*arctan(1/14*sqrt(14)*(5*x + 1))/(5*d^2 - 2*d*e + 3*e^2) + 1/250*(458*d - 7
*e)*log(5*x^2 + 2*x + 3)/(5*d^2 - 2*d*e + 3*e^2) + (4*d^4 + 5*d^3*e + 3*d^2
*e^2 - d*e^3 + 2*e^4)*log(abs(x*e + d))/(5*d^2*e^3 - 2*d*e^4 + 3*e^5)
```

Mupad [**B**]
time = 6.39, size = 713, normalized size = 4.24

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)),x)
```

```
[Out] (2*x^2)/(5*e) - log(d + e*x)*(((458*d)/125 - (7*e)/125)/(5*d^2 - 2*d*e + 3*
e^2) - (165*d*e + 100*d^2 + 81*e^2)/(125*e^3)) - x*((4*(5*d + 2*e))/(25*e^2
) + 1/e) - (log((1791*d*e^2 + 1053*d^2*e - 28*d^3 + 916*e^3)/(25*e^2) - (x*
(321*d*e^2 + 2318*d^2*e + 1832*d^3 - 2249*e^3))/(25*e^2) + ((d*((423*14^(1/
2))/3500 - 229i/125) - e*((1367*14^(1/2))/3500 - 7i/250))*((4751*d*e^3 + 43
50*d^3*e - 1000*d^4 + 874*e^4 + 8490*d^2*e^2)/(25*e^2) + (x*(8200*d*e^3 - 6
250*d^3*e - 5000*d^4 + 2917*e^4 + 1850*d^2*e^2))/(25*e^2) - (((750*e^5 - 14
500*d*e^4 + 1250*d^2*e^3)/(25*e^2) - (x*(2500*d*e^4 + 10250*e^5 - 6250*d^2*
e^3))/(25*e^2))*(d*((423*14^(1/2))/3500 - 229i/125) - e*((1367*14^(1/2))/35
00 - 7i/250)))/(d^2*5i - d*e*2i + e^2*3i)))/(d^2*5i - d*e*2i + e^2*3i))*(d*
```

((423*14^(1/2))/3500 - 229i/125) - e*((1367*14^(1/2))/3500 - 7i/250)))/(d^2
*5i - d*e*2i + e^2*3i) + (log((1791*d*e^2 + 1053*d^2*e - 28*d^3 + 916*e^3)/
(25*e^2) - (x*(321*d*e^2 + 2318*d^2*e + 1832*d^3 - 2249*e^3))/(25*e^2) - ((
d*((423*14^(1/2))/3500 + 229i/125) - e*((1367*14^(1/2))/3500 + 7i/250))*((4
751*d*e^3 + 4350*d^3*e - 1000*d^4 + 874*e^4 + 8490*d^2*e^2)/(25*e^2) + (x*(
8200*d*e^3 - 6250*d^3*e - 5000*d^4 + 2917*e^4 + 1850*d^2*e^2))/(25*e^2) + (
((750*e^5 - 14500*d*e^4 + 1250*d^2*e^3)/(25*e^2) - (x*(2500*d*e^4 + 10250*e
^5 - 6250*d^2*e^3))/(25*e^2))*(d*((423*14^(1/2))/3500 + 229i/125) - e*((136
7*14^(1/2))/3500 + 7i/250)))/(d^2*5i - d*e*2i + e^2*3i)))/(d^2*5i - d*e*2i
+ e^2*3i))*(d*((423*14^(1/2))/3500 + 229i/125) - e*((1367*14^(1/2))/3500 +
7i/250)))/(d^2*5i - d*e*2i + e^2*3i)

3.309 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx$

Optimal. Leaf size=233

$$\frac{4x}{5e^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3\left(5d^2-2de+3e^2\right)(d+ex)}-\frac{\left(423d^2-2734de+293e^2\right)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{25\sqrt{14}\left(5d^2-2de+3e^2\right)^2}-\frac{\left(40d^5+d^4e+28d^3e^2+\right.}{e^3\left(5d^2\right.}$$

```
[Out] 4/5*x/e^2+(-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e^3/(5*d^2-2*d*e+3*e^2)/(e
*x+d)-(40*d^5+d^4*e+28*d^3*e^2+44*d^2*e^3-2*d*e^4+e^5)*ln(e*x+d)/e^3/(5*d^2
-2*d*e+3*e^2)^2+1/25*(229*d^2-7*d*e-136*e^2)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3
*e^2)^2-1/350*(423*d^2-2734*d*e+293*e^2)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d
^2-2*d*e+3*e^2)^2*14^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1642, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)\left(423d^2-2734de+293e^2\right)}{25\sqrt{14}\left(5d^2-2de+3e^2\right)^2}+\frac{\left(229d^2-7de-136e^2\right)\log\left(5x^2+2x+3\right)}{25\left(5d^2-2de+3e^2\right)^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3\left(5d^2-2de+3e^2\right)(d+ex)}-\frac{\left(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)^2}+\frac{4x}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)),x]
```

```
[Out] (4*x)/(5*e^2) - (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e^3*(5*d^2 -
 2*d*e + 3*e^2)*(d + e*x)) - ((423*d^2 - 2734*d*e + 293*e^2)*ArcTan[(1 + 5*
x)/Sqrt[14]])/(25*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) - ((40*d^5 + d^4*e +
28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e
+ 3*e^2)^2) + ((229*d^2 - 7*d*e - 136*e^2)*Log[3 + 2*x + 5*x^2])/(25*(5*d^2
 - 2*d*e + 3*e^2)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx = \int \left(\frac{4}{5e^2}+\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2(5d^2-2de+3e^2)(d+ex)^2}+\frac{-40d^5-d^4e-28d^3e^2-44d\ldots}{e^2(5d^2-2de+3e^2)^2\ldots}\right.$$

$$= \frac{4x}{5e^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)}-\frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-\ldots}{e^3(5d^2-2de+\ldots}$$

$$= \frac{4x}{5e^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)}-\frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-\ldots}{e^3(5d^2-2de+\ldots}$$

$$= \frac{4x}{5e^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)}-\frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-\ldots}{e^3(5d^2-2de+\ldots}$$

$$= \frac{4x}{5e^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)}-\frac{(423d^2-2734de+293e^2)\tan^{-1}(\ldots}{25\sqrt{14}\,(5d^2-2de+3e^2)\ldots}$$

Mathematica [A]

time = 0.11, size = 233, normalized size = 1.00

$$\frac{4x}{5e^2}+\frac{-4d^4-5d^3e-3d^2e^2+de^3-2e^4}{e^3(5d^2-2de+3e^2)(d+ex)}+\frac{(-423d^2+2734de-293e^2)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{25\sqrt{14}\,(5d^2-2de+3e^2)^2}+\frac{(-40d^5-d^4e-28d^3e^2-44d^2e^3+2de^4-e^5)\log(d+ex)}{e^3(5d^2-2de+3e^2)^2}+\frac{(229d^2-7de-136e^2)\log(3+2x+5x^2)}{25(5d^2-2de+3e^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)),x
]
```

```
[Out] (4*x)/(5*e^2) + (-4*d^4 - 5*d^3*e - 3*d^2*e^2 + d*e^3 - 2*e^4)/(e^3*(5*d^2
- 2*d*e + 3*e^2)*(d + e*x)) + ((-423*d^2 + 2734*d*e - 293*e^2)*ArcTan[(1 +
5*x)/Sqrt[14]])/(25*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) + ((-40*d^5 - d^4*e
 - 28*d^3*e^2 - 44*d^2*e^3 + 2*d*e^4 - e^5)*Log[d + e*x])/(e^3*(5*d^2 - 2*d
*e + 3*e^2)^2) + ((229*d^2 - 7*d*e - 136*e^2)*Log[3 + 2*x + 5*x^2])/(25*(5*
d^2 - 2*d*e + 3*e^2)^2)
```

Maple [A]

time = 0.26, size = 213, normalized size = 0.91

method	result
default	$-\frac{4d^4+5d^3e+3d^2e^2-d\,e^3+2e^4}{e^3(5d^2-2de+3e^2)(ex+d)}+\frac{\left(-40d^5-d^4e-28d^3e^2-44d^2e^3+2d\,e^4-e^5\right)\ln(ex+d)}{e^3\left(5d^2-2de+3e^2\right)^2}+\frac{4x}{5e^2}+\frac{\left(458d^2-14de-272e^2\right)\ln\left(5x^2+2x\right.}{10}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/e^3*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2)/(e*x+d)+(-
40*d^5-d^4*e-28*d^3*e^2-44*d^2*e^3+2*d*e^4-e^5)/e^3/(5*d^2-2*d*e+3*e^2)^2*l
n(e*x+d)+4/5*x/e^2+1/5/(5*d^2-2*d*e+3*e^2)^2*(1/10*(458*d^2-14*d*e-272*e^2)
*ln(5*x^2+2*x+3)+1/14*(-423/5*d^2+2734/5*d*e-293/5*e^2)*14^(1/2)*arctan(1/2
8*(10*x+2)*14^(1/2)))
```

Maxima [A]

time = 0.54, size = 274, normalized size = 1.18

$$\frac{4}{5}\,xe^{(-2)}-\frac{\sqrt{14}\,(423\,d^2-2734\,de+293\,e^2)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)}{350\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)}+\frac{(229\,d^2-7\,de-136\,e^2)\log\left(5\,x^2+2\,x+3\right)}{25\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)}-\frac{(40\,d^5+d^4e+28\,d^3e^2+44\,d^2e^3-2\,de^4+e^5)\log\left(xe+d\right)}{25\,d^4e^3-20\,d^3e^4+34\,d^2e^5-12\,de^6+9\,e^7}-\frac{4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4}{5\,d^3e^3-2\,d^2e^4+(5\,d^2e^4-2\,de^5+3\,e^6)x+3\,de^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] 4/5*x*e^(-2) - 1/350*sqrt(14)*(423*d^2 - 2734*d*e + 293*e^2)*arctan(1/14*sq
rt(14)*(5*x + 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) + 1/2
5*(229*d^2 - 7*d*e - 136*e^2)*log(5*x^2 + 2*x + 3)/(25*d^4 - 20*d^3*e + 34*
d^2*e^2 - 12*d*e^3 + 9*e^4) - (40*d^5 + d^4*e + 28*d^3*e^2 + 44*d^2*e^3 - 2
*d*e^4 + e^5)*log(x*e + d)/(25*d^4*e^3 - 20*d^3*e^4 + 34*d^2*e^5 - 12*d*e^6
 + 9*e^7) - (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(5*d^3*e^3 - 2*d^
2*e^4 + (5*d^2*e^4 - 2*d*e^5 + 3*e^6)*x + 3*d*e^5)
```

Fricas [A]

time = 0.44, size = 398, normalized size = 1.71

$$-\frac{7000\,d^6+\sqrt{14}\,(423\,d^3e^3+293\,xe^6-(2734\,dx-293\,d)e^5+(423\,d^2x-2734\,d^2)e^4)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)-420\,(6\,x^2-5)e^6+70\,(48\,dx^2-36\,dx-35\,d)e^5-70\,(136\,d^2x^2-48\,d^2x-105\,d^2)e^4+280\,(20\,d^3x^2-34\,d^3x+5\,d^3)e^3-350\,(20\,d^4x^2-16\,d^4x-17\,d^4)e^2-350\,(20\,d^5x-17\,d^5)e-14\,(229\,d^3e^3-136\,xe^6-(7\,dx+136\,d)e^5+(229\,d^2x-7\,d^2)e^4)\log\left(5\,x^2+2\,x+3\right)+350\,(40\,d^6+xe^6-(2\,dx-d)e^5+2\,(22\,d^2x-d^2)e^4+4\,(7\,d^3x+11\,d^3)e^3+(d^4x+28\,d^4)e^2+(40\,d^5x+d^5)e)\log\left(xe+d\right)}{350\,(25\,d^5e^3+9\,xe^8-3\,(4\,dx-3\,d)e^7+2\,(17\,d^2x-6\,d^2)e^6-2\,(10\,d^3x-17\,d^3)e^5+5\,(5\,d^4x-4\,d^4)e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] -1/350*(7000*d^6 + sqrt(14)*(423*d^3*e^3 + 293*x*e^6 - (2734*d*x - 293*d)*e
^5 + (423*d^2*x - 2734*d^2)*e^4)*arctan(1/14*sqrt(14)*(5*x + 1)) - 420*(6*x
^2 - 5)*e^6 + 70*(48*d*x^2 - 36*d*x - 35*d)*e^5 - 70*(136*d^2*x^2 - 48*d^2*
x - 105*d^2)*e^4 + 280*(20*d^3*x^2 - 34*d^3*x + 5*d^3)*e^3 - 350*(20*d^4*x^
2 - 16*d^4*x - 17*d^4)*e^2 - 350*(20*d^5*x - 17*d^5)*e - 14*(229*d^3*e^3 -
136*x*e^6 - (7*d*x + 136*d)*e^5 + (229*d^2*x - 7*d^2)*e^4)*log(5*x^2 + 2*x
+ 3) + 350*(40*d^6 + x*e^6 - (2*d*x - d)*e^5 + 2*(22*d^2*x - d^2)*e^4 + 4*(
7*d^3*x + 11*d^3)*e^3 + (d^4*x + 28*d^4)*e^2 + (40*d^5*x + d^5)*e)*log(x*e
+ d))/(25*d^5*e^3 + 9*x*e^8 - 3*(4*d*x - 3*d)*e^7 + 2*(17*d^2*x - 6*d^2)*e^
6 - 2*(10*d^3*x - 17*d^3)*e^5 + 5*(5*d^4*x - 4*d^4)*e^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.43, size = 355, normalized size = 1.52

$$\frac{1}{25}\,(40\,d+33\,e)e^{(-3)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)-\frac{\sqrt{14}\,(423\,d^2e^2-2734\,de^3+293\,e^4)\arctan\left(\frac{1}{14}\sqrt{14}\left(5\,d-\frac{5\,d^2}{xe+d}+\frac{2\,de}{xe+d}-\frac{3\,e^2}{xe+d}-e\right)e^{(-1)}\right)e^{(-2)}}{350\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)}+\frac{4}{5}\,(xe+d)e^{(-3)}+\frac{(229\,d^2-7\,de-136\,e^2)\log\left(-\frac{10\,d}{xe+d}+\frac{5\,d^2}{(xe+d)^2}+\frac{2\,e}{xe+d}-\frac{2\,de}{(xe+d)^2}+\frac{3\,e^2}{(xe+d)^2}+5\right)}{25\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)}-\frac{\frac{4\,d^4e^3}{xe+d}+\frac{5\,d^3e^4}{xe+d}+\frac{3\,d^2e^5}{xe+d}-\frac{de^6}{xe+d}+\frac{2\,e^7}{xe+d}}{5\,d^2e^6-2\,de^7+3\,e^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] 1/25*(40*d + 33*e)*e^(-3)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) - 1/350*sqrt
(14)*(423*d^2*e^2 - 2734*d*e^3 + 293*e^4)*arctan(1/14*sqrt(14)*(5*d - 5*d^2
/(x*e + d) + 2*d*e/(x*e + d) - 3*e^2/(x*e + d) - e)*e^(-1))*e^(-2)/(25*d^4
- 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) + 4/5*(x*e + d)*e^(-3) + 1/25*(
229*d^2 - 7*d*e - 136*e^2)*log(-10*d/(x*e + d) + 5*d^2/(x*e + d)^2 + 2*e/(x
*e + d) - 2*d*e/(x*e + d)^2 + 3*e^2/(x*e + d)^2 + 5)/(25*d^4 - 20*d^3*e + 3
4*d^2*e^2 - 12*d*e^3 + 9*e^4) - (4*d^4*e^3/(x*e + d) + 5*d^3*e^4/(x*e + d)
+ 3*d^2*e^5/(x*e + d) - d*e^6/(x*e + d) + 2*e^7/(x*e + d))/(5*d^2*e^6 - 2*d
*e^7 + 3*e^8)
```

Mupad [B]
time = 4.67, size = 312, normalized size = 1.34

$$\frac{4\,x}{5\,e^2}-\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1{}\mathrm{i}}{5}\right)\,\left(\left(\frac{423\,\sqrt{14}}{700}-\frac{229}{25}{}\mathrm{i}\right)\,d^2+\left(-\frac{1367\,\sqrt{14}}{350}+\frac{7}{25}{}\mathrm{i}\right)\,d\,e+\left(\frac{293\,\sqrt{14}}{700}+\frac{136}{25}{}\mathrm{i}\right)\,e^2\right)}{d^4\,25{}\mathrm{i}-d^3\,e\,20{}\mathrm{i}+d^2\,e^2\,34{}\mathrm{i}-d\,e^3\,12{}\mathrm{i}+e^4\,9{}\mathrm{i}}+\frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1{}\mathrm{i}}{5}\right)\,\left(\left(\frac{423\,\sqrt{14}}{700}+\frac{229}{25}{}\mathrm{i}\right)\,d^2+\left(-\frac{1367\,\sqrt{14}}{350}-\frac{7}{25}{}\mathrm{i}\right)\,d\,e+\left(\frac{293\,\sqrt{14}}{700}-\frac{136}{25}{}\mathrm{i}\right)\,e^2\right)}{d^4\,25{}\mathrm{i}-d^3\,e\,20{}\mathrm{i}+d^2\,e^2\,34{}\mathrm{i}-d\,e^3\,12{}\mathrm{i}+e^4\,9{}\mathrm{i}}-\frac{5\,\left(4\,d^4+5\,d^3\,e+3\,d^2\,e^2-d\,e^3+2\,e^4\right)}{e\,\left(5\,x\,e^3+5\,d\,e^2\right)\,\left(5\,d^2-2\,d\,e+3\,e^2\right)}-\frac{\ln\left(d+e\,x\right)\,\left(40\,d^5+d^4\,e+28\,d^3\,e^2+44\,d^2\,e^3-2\,d\,e^4+e^5\right)}{e^3\,{\left(5\,d^2-2\,d\,e+3\,e^2\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)),x)
```

```
[Out] (4*x)/(5*e^2) - (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^2*((423*14^(1/2))/700 -
229i/25) + e^2*((293*14^(1/2))/700 + 136i/25) - d*e*((1367*14^(1/2))/350 -
7i/25)))/(d^4*25i - d^3*e*20i - d*e^3*12i + e^4*9i + d^2*e^2*34i) + (log(x
+ (14^(1/2)*1i)/5 + 1/5)*(d^2*((423*14^(1/2))/700 + 229i/25) + e^2*((293*14
^(1/2))/700 - 136i/25) - d*e*((1367*14^(1/2))/350 + 7i/25)))/(d^4*25i - d^3
*e*20i - d*e^3*12i + e^4*9i + d^2*e^2*34i) - (5*(5*d^3*e - d*e^3 + 4*d^4 +
2*e^4 + 3*d^2*e^2))/(e*(5*d*e^2 + 5*e^3*x)*(5*d^2 - 2*d*e + 3*e^2)) - (log(
d + e*x)*(d^4*e - 2*d*e^4 + 40*d^5 + e^5 + 44*d^2*e^3 + 28*d^3*e^2))/(e^3*(
5*d^2 - 2*d*e + 3*e^2)^2)
```

3.310 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)}\,dx$

Optimal. Leaf size=317

$$-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2}+\frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)}-\frac{(423d^3-4101d^2e+879de^2 \cdots}{5\sqrt{14}\,(5d^2-2 \cdots}$$

```
[Out] 1/2*(-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e^3/(5*d^2-2*d*e+3*e^2)/(e*x+d)^
2+(40*d^5+d^4*e+28*d^3*e^2+44*d^2*e^3-2*d*e^4+e^5)/e^3/(5*d^2-2*d*e+3*e^2)^
2/(e*x+d)+(100*d^6-120*d^5*e+228*d^4*e^2-242*d^3*e^3+141*d^2*e^4+120*d*e^5-
e^6)*ln(e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)^3+1/10*(458*d^3-21*d^2*e-816*d*e^2+1
13*e^3)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^3-1/70*(423*d^3-4101*d^2*e+879*
d*e^2+703*e^3)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1642, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)\left(423d^3-4101d^2e+879de^2+703e^3\right)}{5\sqrt{14}\left(5d^2-2de+3e^2\right)^3}+\frac{\left(458d^3-21d^2e-816de^2+113e^3\right)\log\left(5x^2+2x+3\right)}{10\left(5d^2-2de+3e^2\right)^3}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2}+\frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)}+\frac{\left(100d^6-120d^5e+228d^4e^2-242d^3e^3+141d^2e^4+120de^5-e^6\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)),x]
```

```
[Out] -1/2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e^3*(5*d^2 - 2*d*e + 3*
e^2)*(d + e*x)^2) + (40*d^5 + d^4*e + 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e
^5)/(e^3*(5*d^2 - 2*d*e + 3*e^2)^2*(d + e*x)) - ((423*d^3 - 4101*d^2*e + 87
9*d*e^2 + 703*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(5*Sqrt[14]*(5*d^2 - 2*d*e +
 3*e^2)^3) + ((100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d^3*e^3 + 141*d^2*e^
4 + 120*d*e^5 - e^6)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)^3) + ((458*
d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*Log[3 + 2*x + 5*x^2])/(10*(5*d^2 - 2*
d*e + 3*e^2)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\,(3+2x+5x^2)}\,dx = \int \left(\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2\,(5d^2-2de+3e^2)\,(d+ex)^3} + \frac{-40d^5-d^4e-28d^3e^2-44d^2e^3+}{e^2\,(5d^2-2de+3e^2)^2\,(d+e}\right.$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\,(5d^2-2de+3e^2)\,(d+ex)^2} + \frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de}{e^3\,(5d^2-2de+3e^2)^2\,(d+ex)}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\,(5d^2-2de+3e^2)\,(d+ex)^2} + \frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de}{e^3\,(5d^2-2de+3e^2)^2\,(d+ex)}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\,(5d^2-2de+3e^2)\,(d+ex)^2} + \frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de}{e^3\,(5d^2-2de+3e^2)^2\,(d+ex)}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\,(5d^2-2de+3e^2)\,(d+ex)^2} + \frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de}{e^3\,(5d^2-2de+3e^2)^2\,(d+ex)}$$

Mathematica [A]
time = 0.28, size = 278, normalized size = 0.88

$$-\frac{\frac{35(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^3(d+ex)^2}-\frac{70(5d^2-2de+3e^2)(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5)}{e^3(d+ex)}+\sqrt{14}\,(423d^3-4101d^2e+879de^2+703e^3)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)+\frac{70(-100d^6+120d^5e-228d^4e^2+242d^3e^3-141d^2e^4-120de^5+e^6)\log(d+ex)}{e^3}-7(458d^3-21d^2e-816de^2+113e^3)\log\left(3+2x+5x^2\right)}{70\left(5d^2-2de+3e^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)),x
]
```

```
[Out] -1/70*((35*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 +
 2*e^4))/(e^3*(d + e*x)^2) - (70*(5*d^2 - 2*d*e + 3*e^2)*(40*d^5 + d^4*e +
28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5))/(e^3*(d + e*x)) + Sqrt[14]*(423*d
^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + (70*(-1
00*d^6 + 120*d^5*e - 228*d^4*e^2 + 242*d^3*e^3 - 141*d^2*e^4 - 120*d*e^5 +
e^6)*Log[d + e*x])/e^3 - 7*(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*Log[3
 + 2*x + 5*x^2])/(5*d^2 - 2*d*e + 3*e^2)^3
```

Maple [A]
time = 0.25, size = 298, normalized size = 0.94

method	result
default	$-\frac{4d^4+5d^3e+3d^2e^2-d\,e^3+2e^4}{2e^3(5d^2-2de+3e^2)(ex+d)^2}-\frac{-40d^5-d^4e-28d^3e^2-44d^2e^3+2d\,e^4-e^5}{e^3(5d^2-2de+3e^2)^2(ex+d)}+\frac{(100d^6-120d^5e+228d^4e^2-242d^3e^3+141d^2e^4+120d}{e^3(5d^2-2de+3e^2)^3}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/e^3/(5*d^2-2*d*e+3*e^2)/(e*x+d)^
2-(-40*d^5-d^4*e-28*d^3*e^2-44*d^2*e^3+2*d*e^4-e^5)/e^3/(5*d^2-2*d*e+3*e^2)
^2/(e*x+d)+(100*d^6-120*d^5*e+228*d^4*e^2-242*d^3*e^3+141*d^2*e^4+120*d*e^5
-e^6)*ln(e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)^3+1/(5*d^2-2*d*e+3*e^2)^3*(1/10*(45
8*d^3-21*d^2*e-816*d*e^2+113*e^3)*ln(5*x^2+2*x+3)+1/14*(-423/5*d^3+4101/5*d
^2*e-879/5*d*e^2-703/5*e^3)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2)))
```

Maxima [A]
time = 0.54, size = 455, normalized size = 1.44

$$-\frac{\sqrt{14}\,(423\,d^3-4101\,d^2e+879\,de^2+703\,e^3)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{70\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}+\frac{(458\,d^3-21\,d^2e-816\,de^2+113\,e^3)\log\left(5\,x^2+2\,x+3\right)}{10\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}+\frac{(100\,d^6-120\,d^5e+228\,d^4e^2-242\,d^3e^3+141\,d^2e^4+120\,de^5-e^6)\log\left(xe+d\right)}{125\,d^6e^3-150\,d^5e^4+285\,d^4e^5-188\,d^3e^6+171\,d^2e^7-54\,de^8+27\,e^9}+\frac{60\,d^6-15\,d^5e+39\,d^4e^2+84\,d^3e^3-25\,d^2e^4+2\,(40\,d^5e+d^4e^2+28\,d^3e^3+44\,d^2e^4-2\,de^5+e^6)x+9\,de^5-6\,e^6}{2\,(25\,d^6e^3-20\,d^5e^4+34\,d^4e^5-12\,d^3e^6+(25\,d^4e^5-20\,d^3e^6+34\,d^2e^7-12\,de^8+9\,e^9)x^2+9\,d^2e^7+2\,(25\,d^5e^4-20\,d^4e^5+34\,d^3e^6-12\,d^2e^7+9\,de^8)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] -1/70*sqrt(14)*(423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*arctan(1/14*sqr
t(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2
*e^4 - 54*d*e^5 + 27*e^6) + 1/10*(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)
*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 17
1*d^2*e^4 - 54*d*e^5 + 27*e^6) + (100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d
^3*e^3 + 141*d^2*e^4 + 120*d*e^5 - e^6)*log(x*e + d)/(125*d^6*e^3 - 150*d^5
*e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8 + 27*e^9) + 1/2*(
60*d^6 - 15*d^5*e + 39*d^4*e^2 + 84*d^3*e^3 - 25*d^2*e^4 + 2*(40*d^5*e + d^
```

```
4*e^2 + 28*d^3*e^3 + 44*d^2*e^4 - 2*d*e^5 + e^6)*x + 9*d*e^5 - 6*e^6)/(25*d
^6*e^3 - 20*d^5*e^4 + 34*d^4*e^5 - 12*d^3*e^6 + (25*d^4*e^5 - 20*d^3*e^6 +
34*d^2*e^7 - 12*d*e^8 + 9*e^9)*x^2 + 9*d^2*e^7 + 2*(25*d^5*e^4 - 20*d^4*e^5
 + 34*d^3*e^6 - 12*d^2*e^7 + 9*d*e^8)*x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(297) = 594$.
time = 0.55, size = 675, normalized size = 2.13

$$\frac{10500\,d^{8} - \sqrt{14}\,\left(423\,d^{5}e^{3} + 703\,x^{2}e^{8} + (879\,dx^{2} + 1406\,dx)e^{7} - (4101\,d^{2}x^{2} - 1758\,d^{2}x - 703\,d^{2})e^{6} + 3\,(141\,d^{3}x^{2} - 2734\,d^{3}x + 293\,d^{3})e^{5} + 3\,(282\,d^{4}x - 1367\,d^{4})e^{4}\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x + 1)\right) + 210\,(x - 3)e^{8} - 35\,(16\,dx - 39\,d)e^{7} + 105\,(94\,d^{2}x - 41\,d^{2})e^{6} - 35\,(28\,d^{3}x - 347\,d^{3})e^{5} + 70\,(167\,d^{4}x - 88\,d^{4})e^{4} + 105\,(172\,d^{5}x + 99\,d^{5})e^{3} - 525\,(10\,d^{6}x - 27\,d^{6})e^{2} + 175\,(80\,d^{7}x - 39\,d^{7})e + 7\,\left(458\,d^{5}e^{3} + 113\,x^{2}e^{8} - 2\,(408\,dx^{2} - 113\,dx)e^{7} - (21\,d^{2}x^{2} + 1632\,d^{2}x - 113\,d^{2})e^{6} + 2\,(229\,d^{3}x^{2} - 21\,d^{3}x - 408\,d^{3})e^{5} + (916\,d^{4}x - 21\,d^{4})e^{4}\right)\log\left(5\,x^{2} + 2\,x + 3\right) + 70\,\left(100\,d^{8} - x^{2}e^{8} + 2\,(60\,dx^{2} - dx)e^{7} + (141\,d^{2}x^{2} + 240\,d^{2}x - d^{2})e^{6} - 2\,(121\,d^{3}x^{2} - 141\,d^{3}x - 60\,d^{3})e^{5} + (228\,d^{4}x^{2} - 484\,d^{4}x + 141\,d^{4})e^{4} - 2\,(60\,d^{5}x^{2} - 228\,d^{5}x + 121\,d^{5})e^{3} + 4\,(25\,d^{6}x^{2} - 60\,d^{6}x + 57\,d^{6})e^{2} + 40\,(5\,d^{7}x - 3\,d^{7})e\right)\log\left(xe + d\right)}{70\,\left(125\,d^{8}e^{3} + 27\,x^{2}e^{11} - 54\,(dx^{2} - dx)e^{10} + 9\,(19\,d^{2}x^{2} - 12\,d^{2}x + 3\,d^{2})e^{9} - 2\,(94\,d^{3}x^{2} - 171\,d^{3}x + 27\,d^{3})e^{8} + (285\,d^{4}x^{2} - 376\,d^{4}x + 171\,d^{4})e^{7} - 2\,(75\,d^{5}x^{2} - 285\,d^{5}x + 94\,d^{5})e^{6} + 5\,(25\,d^{6}x^{2} - 60\,d^{6}x + 57\,d^{6})e^{5} + 50\,(5\,d^{7}x - 3\,d^{7})e^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] 1/70*(10500*d^8 - sqrt(14)*(423*d^5*e^3 + 703*x^2*e^8 + (879*d*x^2 + 1406*d
*x)*e^7 - (4101*d^2*x^2 - 1758*d^2*x - 703*d^2)*e^6 + 3*(141*d^3*x^2 - 2734
*d^3*x + 293*d^3)*e^5 + 3*(282*d^4*x - 1367*d^4)*e^4)*arctan(1/14*sqrt(14)*
(5*x + 1)) + 210*(x - 3)*e^8 - 35*(16*d*x - 39*d)*e^7 + 105*(94*d^2*x - 41*
d^2)*e^6 - 35*(28*d^3*x - 347*d^3)*e^5 + 70*(167*d^4*x - 88*d^4)*e^4 + 105*
(172*d^5*x + 99*d^5)*e^3 - 525*(10*d^6*x - 27*d^6)*e^2 + 175*(80*d^7*x - 39
*d^7)*e + 7*(458*d^5*e^3 + 113*x^2*e^8 - 2*(408*d*x^2 - 113*d*x)*e^7 - (21*
d^2*x^2 + 1632*d^2*x - 113*d^2)*e^6 + 2*(229*d^3*x^2 - 21*d^3*x - 408*d^3)*
e^5 + (916*d^4*x - 21*d^4)*e^4)*log(5*x^2 + 2*x + 3) + 70*(100*d^8 - x^2*e^
8 + 2*(60*d*x^2 - d*x)*e^7 + (141*d^2*x^2 + 240*d^2*x - d^2)*e^6 - 2*(121*d
^3*x^2 - 141*d^3*x - 60*d^3)*e^5 + (228*d^4*x^2 - 484*d^4*x + 141*d^4)*e^4
- 2*(60*d^5*x^2 - 228*d^5*x + 121*d^5)*e^3 + 4*(25*d^6*x^2 - 60*d^6*x + 57*
d^6)*e^2 + 40*(5*d^7*x - 3*d^7)*e)*log(x*e + d))/(125*d^8*e^3 + 27*x^2*e^11
 - 54*(d*x^2 - d*x)*e^10 + 9*(19*d^2*x^2 - 12*d^2*x + 3*d^2)*e^9 - 2*(94*d^
3*x^2 - 171*d^3*x + 27*d^3)*e^8 + (285*d^4*x^2 - 376*d^4*x + 171*d^4)*e^7 -
 2*(75*d^5*x^2 - 285*d^5*x + 94*d^5)*e^6 + 5*(25*d^6*x^2 - 60*d^6*x + 57*d^
6)*e^5 + 50*(5*d^7*x - 3*d^7)*e^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.71, size = 406, normalized size = 1.28

$$-\frac{\sqrt{14}\,(423\,d^{3} - 4101\,d^{2}e + 879\,de^{2} + 703\,e^{3})\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x + 1)\right)}{70\,(125\,d^{6} - 150\,d^{5}e + 285\,d^{4}e^{2} - 188\,d^{3}e^{3} + 171\,d^{2}e^{4} - 54\,de^{5} + 27\,e^{6})} + \frac{(458\,d^{3} - 21\,d^{2}e - 816\,de^{2} + 113\,e^{3})\log\left(5\,x^{2} + 2\,x + 3\right)}{10\,(125\,d^{6} - 150\,d^{5}e + 285\,d^{4}e^{2} - 188\,d^{3}e^{3} + 171\,d^{2}e^{4} - 54\,de^{5} + 27\,e^{6})} + \frac{(100\,d^{6} - 120\,d^{5}e + 228\,d^{4}e^{2} - 242\,d^{3}e^{3} + 141\,d^{2}e^{4} + 120\,de^{5} - e^{6})\log\left(|xe + d|\right)}{125\,d^{6}e^{3} - 150\,d^{5}e^{4} + 285\,d^{4}e^{5} - 188\,d^{3}e^{6} + 171\,d^{2}e^{7} - 54\,de^{8} + 27\,e^{9}} + \frac{\left(2\,(200\,d^{7} - 75\,d^{6}e + 258\,d^{5}e^{2} + 167\,d^{4}e^{3} - 14\,d^{3}e^{4} + 141\,d^{2}e^{5} - 8\,de^{6} + 3\,e^{7})x + (300\,d^{8} - 195\,d^{7}e + 405\,d^{6}e^{2} + 297\,d^{5}e^{3} - 176\,d^{4}e^{4} + 347\,d^{3}e^{5} - 123\,d^{2}e^{6} + 39\,de^{7} - 18\,e^{8})e^{(-1)}\right)e^{(-2)}}{2\,(5\,d^{2} - 2\,de + 3\,e^{2})^{3}(xe + d)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] -1/70*sqrt(14)*(423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*arctan(1/14*sqr
t(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2
*e^4 - 54*d*e^5 + 27*e^6) + 1/10*(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)
*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 17
1*d^2*e^4 - 54*d*e^5 + 27*e^6) + (100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d
^3*e^3 + 141*d^2*e^4 + 120*d*e^5 - e^6)*log(abs(x*e + d))/(125*d^6*e^3 - 15
0*d^5*e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8 + 27*e^9) +
1/2*(2*(200*d^7 - 75*d^6*e + 258*d^5*e^2 + 167*d^4*e^3 - 14*d^3*e^4 + 141*d
^2*e^5 - 8*d*e^6 + 3*e^7)*x + (300*d^8 - 195*d^7*e + 405*d^6*e^2 + 297*d^5*
e^3 - 176*d^4*e^4 + 347*d^3*e^5 - 123*d^2*e^6 + 39*d*e^7 - 18*e^8)*e^(-1))*
e^(-2)/((5*d^2 - 2*d*e + 3*e^2)^3*(x*e + d)^2)
```

Mupad [B]

time = 4.76, size = 493, normalized size = 1.56

$$\frac{\frac{60\,d^6-15\,d^5\,e+39\,d^4\,e^2+84\,d^3\,e^3-25\,d^2\,e^4+9\,d\,e^5-6\,e^6}{2\,e^3\,\left(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4\right)}+\frac{x\,\left(40\,d^5+d^4\,e+28\,d^3\,e^2+44\,d^2\,e^3-2\,d\,e^4+e^5\right)}{e^2\,\left(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4\right)}}{d^2+2\,d\,e\,x+e^2\,x^2}-\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1{}\mathrm{i}}{5}\right)\,\left(\left(\frac{423\,\sqrt{14}}{140}-\frac{229}{5}{}\mathrm{i}\right)\,d^3+\left(-\frac{4101\,\sqrt{14}}{140}+\frac{21}{10}{}\mathrm{i}\right)\,d^2\,e+\left(\frac{879\,\sqrt{14}}{140}+\frac{408}{5}{}\mathrm{i}\right)\,d\,e^2+\left(\frac{703\,\sqrt{14}}{140}-\frac{113}{10}{}\mathrm{i}\right)\,e^3\right)}{d^6\,125{}\mathrm{i}-d^5\,e\,150{}\mathrm{i}+d^4\,e^2\,285{}\mathrm{i}-d^3\,e^3\,188{}\mathrm{i}+d^2\,e^4\,171{}\mathrm{i}-d\,e^5\,54{}\mathrm{i}+e^6\,27{}\mathrm{i}}+\frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1{}\mathrm{i}}{5}\right)\,\left(\left(\frac{423\,\sqrt{14}}{140}+\frac{229}{5}{}\mathrm{i}\right)\,d^3+\left(-\frac{4101\,\sqrt{14}}{140}-\frac{21}{10}{}\mathrm{i}\right)\,d^2\,e+\left(\frac{879\,\sqrt{14}}{140}-\frac{408}{5}{}\mathrm{i}\right)\,d\,e^2+\left(\frac{703\,\sqrt{14}}{140}+\frac{113}{10}{}\mathrm{i}\right)\,e^3\right)}{d^6\,125{}\mathrm{i}-d^5\,e\,150{}\mathrm{i}+d^4\,e^2\,285{}\mathrm{i}-d^3\,e^3\,188{}\mathrm{i}+d^2\,e^4\,171{}\mathrm{i}-d\,e^5\,54{}\mathrm{i}+e^6\,27{}\mathrm{i}}+\frac{\ln\left(d+e\,x\right)\,\left(100\,d^6-120\,d^5\,e+228\,d^4\,e^2-242\,d^3\,e^3+141\,d^2\,e^4+120\,d\,e^5-e^6\right)}{e^3\,{\left(5\,d^2-2\,d\,e+3\,e^2\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^3*(2*x + 5*x^2 + 3)),x)
```

```
[Out] ((9*d*e^5 - 15*d^5*e + 60*d^6 - 6*e^6 - 25*d^2*e^4 + 84*d^3*e^3 + 39*d^4*e^
2)/(2*e^3*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x*(d^4*e
- 2*d*e^4 + 40*d^5 + e^5 + 44*d^2*e^3 + 28*d^3*e^2))/(e^2*(25*d^4 - 20*d^3*
e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)))/(d^2 + e^2*x^2 + 2*d*e*x) - (log(x - (
14^(1/2)*1i)/5 + 1/5)*(d^3*((423*14^(1/2))/140 - 229i/5) + e^3*((703*14^(1/
2))/140 - 113i/10) + d*e^2*((879*14^(1/2))/140 + 408i/5) - d^2*e*((4101*14^
(1/2))/140 - 21i/10)))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e
^4*171i - d^3*e^3*188i + d^4*e^2*285i) + (log(x + (14^(1/2)*1i)/5 + 1/5)*(d
^3*((423*14^(1/2))/140 + 229i/5) + e^3*((703*14^(1/2))/140 + 113i/10) + d*e
^2*((879*14^(1/2))/140 - 408i/5) - d^2*e*((4101*14^(1/2))/140 + 21i/10)))/(
d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*188i +
 d^4*e^2*285i) + (log(d + e*x)*(120*d*e^5 - 120*d^5*e + 100*d^6 - e^6 + 141
*d^2*e^4 - 242*d^3*e^3 + 228*d^4*e^2))/(e^3*(5*d^2 - 2*d*e + 3*e^2)^3)
```

3.311 $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

Optimal. Leaf size=189

$$\frac{(2800d^3-17220d^2e+9921de^2+6053e^3)\,x}{17500}+\frac{e(840d^2-1722de+373e^2)\,x^2}{3500}+\frac{1}{375}(60d-41e)e^2x^3+\frac{e^3x^4}{25}-\frac{(1$$

```
[Out] 1/17500*(2800*d^3-17220*d^2*e+9921*d*e^2+6053*e^3)*x+1/3500*e*(840*d^2-1722
*d*e+373*e^2)*x^2+1/375*(60*d-41*e)*e^2*x^3+1/25*e^3*x^4-1/3500*(1367+423*x
)*(e*x+d)^3/(5*x^2+2*x+3)-1/6250*(1025*d^3-1545*d^2*e-2601*d*e^2+832*e^3)*l
n(5*x^2+2*x+3)+1/1225000*(32825*d^3+317565*d^2*e-221643*d*e^2-67499*e^3)*ar
ctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1658, 1642, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)(32825d^3+317565d^2e-221643de^2-67499e^3)}{87500\sqrt{14}}+\frac{ex^2(840d^2-1722de+373e^2)}{3500}-\frac{(1025d^3-1545d^2e-2601de^2+832e^3)\log(5x^2+2x+3)}{6250}+\frac{x(2800d^3-17220d^2e+9921de^2+6053e^3)}{17500}+\frac{1}{375}e^2x^3(60d-41e)-\frac{(423x+1367)(d+ex)^3}{3500(5x^2+2x+3)}+\frac{e^3x^4}{25}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] ((2800*d^3 - 17220*d^2*e + 9921*d*e^2 + 6053*e^3)*x)/17500 + (e*(840*d^2 -
1722*d*e + 373*e^2)*x^2)/3500 + ((60*d - 41*e)*e^2*x^3)/375 + (e^3*x^4)/25
- ((1367 + 423*x)*(d + e*x)^3)/(3500*(3 + 2*x + 5*x^2)) + ((32825*d^3 + 317
565*d^2*e - 221643*d*e^2 - 67499*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(87500*Sq
rt[14]) - ((1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)*Log[3 + 2*x + 5*x
^2])/6250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = -\frac{(1367+423x)(d+ex)^3}{3500\,(3+2x+5x^2)} + \frac{1}{56}\int \frac{(d+ex)^2 \left(\frac{6}{125}(615d+136\right.}{}$$

$$= -\frac{(1367+423x)(d+ex)^3}{3500\,(3+2x+5x^2)} + \frac{1}{56}\int \left(\frac{2}{625}(2800d^3-17220d^2e\right.$$

$$= \frac{(2800d^3-17220d^2e+9921de^2+6053e^3)\,x}{17500} + \frac{e(840d^2-1722}{35}$$

$$= \frac{(2800d^3-17220d^2e+9921de^2+6053e^3)\,x}{17500} + \frac{e(840d^2-1722}{35}$$

$$= \frac{(2800d^3-17220d^2e+9921de^2+6053e^3)\,x}{17500} + \frac{e(840d^2-1722}{35}$$

$$= \frac{(2800d^3-17220d^2e+9921de^2+6053e^3)\,x}{17500} + \frac{e(840d^2-1722}{35}$$

Mathematica [A]
time = 0.10, size = 209, normalized size = 1.11

$$\frac{5880(500d^3-3075d^2e+1545de^2+867e^3)\,x+14700e(300d^2-615de+103e^2)\,x^2+49000(60d-41e)e^2x^3+735000e^3x^4-\frac{42\left(e^3(54969-53189x)+125d^3(1367+423x)+75d^2e(-1269+5989x)-15de^2(17967+18323x)\right)}{3+2x+5x^2}+15\sqrt{14}\,(32825d^3+317565d^2e-221643de^2-67499e^3)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)+2940(-1025d^3+1545d^2e+2601de^2-832e^3)\log(3+2x+5x^2)}{18375000}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2
,x]
```

```
[Out] (5880*(500*d^3 - 3075*d^2*e + 1545*d*e^2 + 867*e^3)*x + 14700*e*(300*d^2 -
615*d*e + 103*e^2)*x^2 + 49000*(60*d - 41*e)*e^2*x^3 + 735000*e^3*x^4 - (42
*(e^3*(54969 - 53189*x) + 125*d^3*(1367 + 423*x) + 75*d^2*e*(-1269 + 5989*x
) - 15*d*e^2*(17967 + 18323*x)))/(3 + 2*x + 5*x^2) + 15*Sqrt[14]*(32825*d^3
 + 317565*d^2*e - 221643*d*e^2 - 67499*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + 29
40*(-1025*d^3 + 1545*d^2*e + 2601*d*e^2 - 832*e^3)*Log[3 + 2*x + 5*x^2])/18
375000
```

Maple [A]
time = 0.18, size = 214, normalized size = 1.13

method	result
default	$\frac{e^3x^4}{25}+\frac{4d\,e^2x^3}{25}-\frac{41e^3x^3}{375}+\frac{6d^2e\,x^2}{25}-\frac{123d\,e^2x^2}{250}+\frac{103e^3x^2}{1250}+\frac{4d^3x}{25}-\frac{123d^2ex}{125}+\frac{309d\,e^2x}{625}+\frac{867e^3x}{3125}-\frac{(\frac{2115}{28}d^3+}{}$

risch	$-\frac{123 d\,e^{2} x^{2}}{250}-\frac{123 d^{2} e x}{125}+\frac{309 d\,e^{2} x}{625}+\frac{\frac{\left(-\frac{2115}{28} d^{3}-\frac{17967}{28} d^{2} e+\frac{54969}{140} d\,e^{2}+\frac{53189}{700} e^{3}\right) x}{3125}-\frac{1367 d^{3}}{17500}+\frac{3807 d^{2} e}{87500}+\frac{53901 d\,e^{2}}{437500}-\frac{54969 e^{3}}{2187500}}{x^{2}+\frac{2}{5} x+\frac{3}{5}}+\frac{4d\ldots}{25\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x,method=_RETURNVERBO
SE)
```

```
[Out] 1/25*e^3*x^4+4/25*d*e^2*x^3-41/375*e^3*x^3+6/25*d^2*e*x^2-123/250*d*e^2*x^2
+103/1250*e^3*x^2+4/25*d^3*x-123/125*d^2*e*x+309/625*d*e^2*x+867/3125*e^3*x
-1/3125*((2115/28*d^3+17967/28*d^2*e-54969/140*d*e^2-53189/700*e^3)*x+6835/
28*d^3-3807/28*d^2*e-53901/140*d*e^2+54969/700*e^3)/(x^2+2/5*x+3/5)-1/17500
0*(28700*d^3-43260*d^2*e-72828*d*e^2+23296*e^3)*ln(5*x^2+2*x+3)-1/245000*(-
6565*d^3-63513*d^2*e+221643/5*d*e^2+67499/5*e^3)*14^(1/2)*arctan(1/28*(10*x
+2)*14^(1/2))
```

Maxima [A]
time = 0.54, size = 203, normalized size = 1.07

$$\frac{1}{25}\,x^{4}e^{3}+\frac{1}{375}\,(60\,de^{2}-41\,e^{3})x^{3}+\frac{1}{1250}\,(300\,d^{2}e-615\,de^{2}+103\,e^{3})x^{2}+\frac{1}{1225000}\,\sqrt{14}\,(32825\,d^{3}+317565\,d^{2}e-221643\,de^{2}-67499\,e^{3})\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{1}{3125}\,(500\,d^{3}-3075\,d^{2}e+1545\,de^{2}+867\,e^{3})x-\frac{1}{6250}\,(1025\,d^{3}-1545\,d^{2}e-2601\,de^{2}+832\,e^{3})\log\left(5\,x^{2}+2\,x+3\right)-\frac{170875\,d^{3}-95175\,d^{2}e+(52875\,d^{3}+449175\,d^{2}e-274845\,de^{2}-53189\,e^{3})x-269505\,de^{2}+54969\,e^{3}}{437500\,(5\,x^{2}+2\,x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 1/25*x^4*e^3 + 1/375*(60*d*e^2 - 41*e^3)*x^3 + 1/1250*(300*d^2*e - 615*d*e^
2 + 103*e^3)*x^2 + 1/1225000*sqrt(14)*(32825*d^3 + 317565*d^2*e - 221643*d*
e^2 - 67499*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/3125*(500*d^3 - 3075*d
^2*e + 1545*d*e^2 + 867*e^3)*x - 1/6250*(1025*d^3 - 1545*d^2*e - 2601*d*e^2
 + 832*e^3)*log(5*x^2 + 2*x + 3) - 1/437500*(170875*d^3 - 95175*d^2*e + (52
875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*x - 269505*d*e^2 + 54969
*e^3)/(5*x^2 + 2*x + 3)
```

Fricas [A]
time = 0.38, size = 333, normalized size = 1.76

$$\frac{14700000\,d^{3}x^{3}+5880000\,d^{3}x^{2}+6599250\,d^{3}x-7176750\,d^{3}+15\,\sqrt{14}\,(164125\,d^{3}x^{2}+65650\,d^{3}x+98475\,d^{3}-67499\,(5\,x^{2}+2\,x+3)e^{3}-221643\,(5\,dx^{2}+2\,dx+3\,d)e^{2}+317565\,(5\,d^{2}x^{2}+2\,d^{2}x+3\,d^{2})e)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+14\,(262500\,x^{6}-612500\,x^{5}+411250\,x^{4}+1606500\,x^{3}+1052730\,x^{2}+1251987\,x-164907)e^{3}+210\,(70000\,dx^{5}-187250\,dx^{4}+172200\,dx^{3}-42630\,dx^{2}+184749\,dx+53901\,d)e^{2}+3150\,(7000\,d^{2}x^{4}-25900\,d^{2}x^{3}-7280\,d^{2}x^{2}-23209\,d^{2}x+1269\,d^{2})e-2940\,(5125\,d^{3}x^{2}+2050\,d^{3}x+3075\,d^{3}+832\,(5\,x^{2}+2\,x+3)e^{3}-2601\,(5\,dx^{2}+2\,dx+3\,d)e^{2}-1545\,(5\,d^{2}x^{2}+2\,d^{2}x+3\,d^{2})e)\log\left(5\,x^{2}+2\,x+3\right)}{18375000\,(5\,x^{2}+2\,x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] 1/18375000*(14700000*d^3*x^3 + 5880000*d^3*x^2 + 6599250*d^3*x - 7176750*d^
3 + 15*sqrt(14)*(164125*d^3*x^2 + 65650*d^3*x + 98475*d^3 - 67499*(5*x^2 +
2*x + 3)*e^3 - 221643*(5*d*x^2 + 2*d*x + 3*d)*e^2 + 317565*(5*d^2*x^2 + 2*d
```

```
^2*x + 3*d^2)*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(262500*x^6 - 612500*
x^5 + 411250*x^4 + 1606500*x^3 + 1052730*x^2 + 1251987*x - 164907)*e^3 + 21
0*(70000*d*x^5 - 187250*d*x^4 + 172200*d*x^3 - 42630*d*x^2 + 184749*d*x + 5
3901*d)*e^2 + 3150*(7000*d^2*x^4 - 25900*d^2*x^3 - 7280*d^2*x^2 - 23209*d^2
*x + 1269*d^2)*e - 2940*(5125*d^3*x^2 + 2050*d^3*x + 3075*d^3 + 832*(5*x^2
+ 2*x + 3)*e^3 - 2601*(5*d*x^2 + 2*d*x + 3*d)*e^2 - 1545*(5*d^2*x^2 + 2*d^2
*x + 3*d^2)*e)*log(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.43, size = 444, normalized size = 2.35

$$\frac{e^3x^4}{25}+x^3\cdot\left(\frac{4de^2}{25}-\frac{41e^3}{375}\right)+x^2\cdot\left(\frac{6d^2e}{25}-\frac{123de^2}{250}+\frac{103e^3}{1250}\right)+x\left(\frac{4d^3}{25}-\frac{123d^2e}{125}+\frac{309de^2}{625}+\frac{867e^3}{3125}\right)+\left(-\frac{41d^3}{250}+\frac{309d^2e}{1250}+\frac{2601de^2}{6250}-\frac{416e^3}{3125}-\frac{\sqrt{14}\,i(32825d^3+317565d^2e-221643de^2-67499e^3)}{2450000}\right)\log\left(x+\frac{6565d^3+63513d^2e-\frac{221643de^2}{5}-\frac{67499e^3}{5}-\frac{\sqrt{14}\,i(32825d^3+317565d^2e-221643de^2-67499e^3)}{5}}{32825d^3+317565d^2e-221643de^2-67499e^3}\right)+\left(-\frac{41d^3}{250}+\frac{309d^2e}{1250}+\frac{2601de^2}{6250}-\frac{416e^3}{3125}+\frac{\sqrt{14}\,i(32825d^3+317565d^2e-221643de^2-67499e^3)}{2450000}\right)\log\left(x+\frac{6565d^3+63513d^2e-\frac{221643de^2}{5}-\frac{67499e^3}{5}+\frac{\sqrt{14}\,i(32825d^3+317565d^2e-221643de^2-67499e^3)}{5}}{32825d^3+317565d^2e-221643de^2-67499e^3}\right)+\frac{-170875d^3+95175d^2e+269505de^2-54969e^3+x(-52875d^3-449175d^2e+274845de^2+53189e^3)}{2187500x^2+875000x+1312500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] e**3*x**4/25 + x**3*(4*d*e**2/25 - 41*e**3/375) + x**2*(6*d**2*e/25 - 123*d
*e**2/250 + 103*e**3/1250) + x*(4*d**3/25 - 123*d**2*e/125 + 309*d*e**2/625
 + 867*e**3/3125) + (-41*d**3/250 + 309*d**2*e/1250 + 2601*d*e**2/6250 - 41
6*e**3/3125 - sqrt(14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 6749
9*e**3)/2450000)*log(x + (6565*d**3 + 63513*d**2*e - 221643*d*e**2/5 - 6749
9*e**3/5 - sqrt(14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e
**3)/5)/(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)) + (-41*d
**3/250 + 309*d**2*e/1250 + 2601*d*e**2/6250 - 416*e**3/3125 + sqrt(14)*I*(
32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)/2450000)*log(x + (
6565*d**3 + 63513*d**2*e - 221643*d*e**2/5 - 67499*e**3/5 + sqrt(14)*I*(328
25*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)/5)/(32825*d**3 + 3175
65*d**2*e - 221643*d*e**2 - 67499*e**3)) + (-170875*d**3 + 95175*d**2*e + 2
69505*d*e**2 - 54969*e**3 + x*(-52875*d**3 - 449175*d**2*e + 274845*d*e**2
+ 53189*e**3))/(2187500*x**2 + 875000*x + 1312500)
```

Giac [A]
time = 4.30, size = 206, normalized size = 1.09

$$\frac{1}{25}x^4e^3+\frac{4}{25}dx^3e^2+\frac{6}{25}d^2x^2e+\frac{4}{25}d^3x-\frac{41}{375}x^3e^3-\frac{123}{250}dx^2e^2-\frac{123}{125}d^2xe+\frac{103}{1250}x^2e^3+\frac{309}{625}dxe^2+\frac{1}{1225000}\sqrt{14}\left(32825\,d^3+317565\,d^2e-221643\,de^2-67499\,e^3\right)\arctan\left(\frac{1}{14}\sqrt{14}\left(5\,x+1\right)\right)+\frac{867}{3125}xe^3-\frac{1}{6250}\left(1025\,d^3-1545\,d^2e-2601\,de^2+832\,e^3\right)\log\left(5\,x^2+2\,x+3\right)-\frac{170875\,d^3-95175\,d^2e+\left(52875\,d^3+449175\,d^2e-274845\,de^2-53189\,e^3\right)x-269505\,de^2+54969\,e^3}{437500\left(5\,x^2+2\,x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 1/25*x^4*e^3 + 4/25*d*x^3*e^2 + 6/25*d^2*x^2*e + 4/25*d^3*x - 41/375*x^3*e^
3 - 123/250*d*x^2*e^2 - 123/125*d^2*x*e + 103/1250*x^2*e^3 + 309/625*d*x*e^
2 + 1/1225000*sqrt(14)*(32825*d^3 + 317565*d^2*e - 221643*d*e^2 - 67499*e^3
)*arctan(1/14*sqrt(14)*(5*x + 1)) + 867/3125*x*e^3 - 1/6250*(1025*d^3 - 154
5*d^2*e - 2601*d*e^2 + 832*e^3)*log(5*x^2 + 2*x + 3) - 1/437500*(170875*d^3
```

```
 - 95175*d^2*e + (52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*x -
269505*d*e^2 + 54969*e^3)/(5*x^2 + 2*x + 3)
```

Mupad [B]
time = 0.15, size = 333, normalized size = 1.76

$$\frac{\frac{53901\,d\,e^2}{28}+\frac{19035\,d^2\,e}{28}+x\,\left(-\frac{10575\,d^3}{28}-\frac{89835\,d^2\,e}{28}+\frac{54969\,d\,e^2}{28}+\frac{53189\,e^3}{140}\right)-\frac{34175\,d^3}{28}-\frac{54969\,e^3}{140}}{15625\,x^2+6250\,x+9375}+x^3\,\left(\frac{e^2\,\left(12\,d-5\,e\right)}{75}-\frac{16\,e^3}{375}\right)-x\,\left(\frac{18\,e^2\,\left(12\,d-5\,e\right)}{625}+\frac{12\,e\,\left(4\,d^2-5\,d\,e+e^2\right)}{125}-\frac{9\,d\,e^2}{25}+\frac{3\,d^2\,e}{5}-\frac{4\,d^3}{25}-\frac{717\,e^3}{3125}\right)+\ln\left(5\,x^2+2\,x+3\right)\,\left(-\frac{41\,d^3}{250}+\frac{309\,d^2\,e}{1250}+\frac{2601\,d\,e^2}{6250}-\frac{416\,e^3}{3125}\right)-x^2\,\left(\frac{2\,e^2\,\left(12\,d-5\,e\right)}{125}-\frac{3\,e\,\left(4\,d^2-5\,d\,e+e^2\right)}{50}+\frac{36\,e^3}{625}\right)+\frac{e^3\,x^4}{25}-\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,\left(-32825\,d^3-317565\,d^2\,e+221643\,d\,e^2+67499\,e^3\right)}{1225000}+\frac{\sqrt{14}\,x\,\left(-32825\,d^3-317565\,d^2\,e+221643\,d\,e^2+67499\,e^3\right)}{245000}}{-\frac{1313\,d^3}{3500}-\frac{63513\,d^2\,e}{17500}+\frac{221643\,d\,e^2}{87500}+\frac{67499\,e^3}{87500}}\right)\,\left(-32825\,d^3-317565\,d^2\,e+221643\,d\,e^2+67499\,e^3\right)}{1225000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

```
[Out] ((53901*d*e^2)/28 + (19035*d^2*e)/28 + x*((54969*d*e^2)/28 - (89835*d^2*e)/
28 - (10575*d^3)/28 + (53189*e^3)/140) - (34175*d^3)/28 - (54969*e^3)/140)/
(6250*x + 15625*x^2 + 9375) + x^3*((e^2*(12*d - 5*e))/75 - (16*e^3)/375) -
x*((18*e^2*(12*d - 5*e))/625 + (12*e*(4*d^2 - 5*d*e + e^2))/125 - (9*d*e^2)
/25 + (3*d^2*e)/5 - (4*d^3)/25 - (717*e^3)/3125) + log(2*x + 5*x^2 + 3)*((2
601*d*e^2)/6250 + (309*d^2*e)/1250 - (41*d^3)/250 - (416*e^3)/3125) - x^2*(
(2*e^2*(12*d - 5*e))/125 - (3*e*(4*d^2 - 5*d*e + e^2))/50 + (36*e^3)/625) +
 (e^3*x^4)/25 - (14^(1/2)*atan(((14^(1/2)*(221643*d*e^2 - 317565*d^2*e - 32
825*d^3 + 67499*e^3))/1225000 + (14^(1/2)*x*(221643*d*e^2 - 317565*d^2*e -
32825*d^3 + 67499*e^3))/245000)/((221643*d*e^2)/87500 - (63513*d^2*e)/17500
 - (1313*d^3)/3500 + (67499*e^3)/87500))*(221643*d*e^2 - 317565*d^2*e - 328
25*d^3 + 67499*e^3))/1225000
```

3.312 $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

Optimal. Leaf size=140

$$\frac{(2800d^2-11480de+3307e^2)\,x}{17500}+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x^3}{75}-\frac{(1367+423x)(d+ex)^2}{3500\,(3+2x+5x^2)}+\frac{(32825d^2+211710d}{8}$$

```
[Out] 1/17500*(2800*d^2-11480*d*e+3307*e^2)*x+1/250*(40*d-41*e)*e*x^2+4/75*e^2*x^
3-1/3500*(1367+423*x)*(e*x+d)^2/(5*x^2+2*x+3)-1/6250*(1025*d^2-1030*d*e-867
*e^2)*ln(5*x^2+2*x+3)+1/1225000*(32825*d^2+211710*d*e-73881*e^2)*arctan(1/1
4*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1658, 1642, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)(32825d^2+211710de-73881e^2)}{87500\sqrt{14}}-\frac{(1025d^2-1030de-867e^2)\log(5x^2+2x+3)}{6250}+\frac{x(2800d^2-11480de+3307e^2)}{17500}+\frac{1}{250}ex^2(40d-41e)-\frac{(423x+1367)(d+ex)^2}{3500(5x^2+2x+3)}+\frac{4e^2x^3}{75}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] ((2800*d^2 - 11480*d*e + 3307*e^2)*x)/17500 + ((40*d - 41*e)*e*x^2)/250 + (
4*e^2*x^3)/75 - ((1367 + 423*x)*(d + e*x)^2)/(3500*(3 + 2*x + 5*x^2)) + ((3
2825*d^2 + 211710*d*e - 73881*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]])/(87500*Sqrt[
14]) - ((1025*d^2 - 1030*d*e - 867*e^2)*Log[3 + 2*x + 5*x^2])/6250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx = -\frac{(1367+423x)(d+ex)^2}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int \frac{(d+ex)\left(\frac{2}{125}(1845d+273\right.}{}$$

$$= -\frac{(1367+423x)(d+ex)^2}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int \left(\frac{2}{625}\left(2800d^2-11480de+\right.\right.$$

$$= \frac{\left(2800d^2-11480de+3307e^2\right)x}{17500}+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x}{75}$$

$$= \frac{\left(2800d^2-11480de+3307e^2\right)x}{17500}+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x}{75}$$

$$= \frac{\left(2800d^2-11480de+3307e^2\right)x}{17500}+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x}{75}$$

$$= \frac{\left(2800d^2-11480de+3307e^2\right)x}{17500}+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x}{75}$$

Mathematica [A]
time = 0.08, size = 150, normalized size = 1.07

$$\frac{5880(100d^2-410de+103e^2)x+14700(40d-41e)ex^2+196000e^2x^3-\frac{42(25d^2(1367+423x)+10de(-1269+5989x)-e^2(17967+18323x))}{3+2x+5x^2}+3\sqrt{14}\,(32825d^2+211710de-73881e^2)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)+588(-1025d^2+1030de+867e^2)\log\left(3+2x+5x^2\right)}{3675000}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2
,x]
```

```
[Out] (5880*(100*d^2 - 410*d*e + 103*e^2)*x + 14700*(40*d - 41*e)*e*x^2 + 196000*
e^2*x^3 - (42*(25*d^2*(1367 + 423*x) + 10*d*e*(-1269 + 5989*x) - e^2*(17967
 + 18323*x)))/(3 + 2*x + 5*x^2) + 3*Sqrt[14]*(32825*d^2 + 211710*d*e - 7388
1*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]] + 588*(-1025*d^2 + 1030*d*e + 867*e^2)*Lo
g[3 + 2*x + 5*x^2])/3675000
```

Maple [A]
time = 0.15, size = 145, normalized size = 1.04

method	result
default	$\frac{4e^2x^3}{75}+\frac{4de\,x^2}{25}-\frac{41e^2x^2}{250}+\frac{4d^2x}{25}-\frac{82dex}{125}+\frac{103e^2x}{625}-\frac{\left(\frac{423}{28}d^2+\frac{5989}{70}de-\frac{18323}{700}e^2\right)x+\frac{1367d^2}{28}-\frac{1269de}{70}-\frac{17967e^2}{700}}{625\left(x^2+\frac{2}{5}x+\frac{3}{5}\right)}-\frac{(28700}{}$
risch	$-\frac{41d^2\ln\left(350x^2+140x+210\right)}{250}+\frac{867e^2\ln\left(350x^2+140x+210\right)}{6250}+\frac{\frac{\left(-\frac{423}{28}d^2-\frac{5989}{70}de+\frac{18323}{700}e^2\right)x}{625}-\frac{1367d^2}{17500}+\frac{1269de}{43750}+\frac{17967e^2}{437500}}{x^2+\frac{2}{5}x+\frac{3}{5}}-\frac{82d}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x,method=_RETURNVERBO
SE)
```

```
[Out] 4/75*e^2*x^3+4/25*d*e*x^2-41/250*e^2*x^2+4/25*d^2*x-82/125*d*e*x+103/625*e^
2*x-1/625*((423/28*d^2+5989/70*d*e-18323/700*e^2)*x+1367/28*d^2-1269/70*d*e
-17967/700*e^2)/(x^2+2/5*x+3/5)-1/175000*(28700*d^2-28840*d*e-24276*e^2)*ln
(5*x^2+2*x+3)-1/245000*(-6565*d^2-42342*d*e+73881/5*e^2)*14^(1/2)*arctan(1/
28*(10*x+2)*14^(1/2))
```

Maxima [A]
time = 0.52, size = 146, normalized size = 1.04

$$\frac{4}{75}\,x^{3}e^{2}+\frac{1}{250}\,\left(40\,de-41\,e^{2}\right)x^{2}+\frac{1}{1225000}\,\sqrt{14}\,\left(32825\,d^{2}+211710\,de-73881\,e^{2}\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,\left(5\,x+1\right)\right)+\frac{1}{625}\,\left(100\,d^{2}-410\,de+103\,e^{2}\right)x-\frac{1}{6250}\,\left(1025\,d^{2}-1030\,de-867\,e^{2}\right)\log\left(5\,x^{2}+2\,x+3\right)-\frac{34175\,d^{2}+\left(10575\,d^{2}+59890\,de-18323\,e^{2}\right)x-12690\,de-17967\,e^{2}}{87500\,\left(5\,x^{2}+2\,x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 4/75*x^3*e^2 + 1/250*(40*d*e - 41*e^2)*x^2 + 1/1225000*sqrt(14)*(32825*d^2
+ 211710*d*e - 73881*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(100*d^2
- 410*d*e + 103*e^2)*x - 1/6250*(1025*d^2 - 1030*d*e - 867*e^2)*log(5*x^2 +
 2*x + 3) - 1/87500*(34175*d^2 + (10575*d^2 + 59890*d*e - 18323*e^2)*x - 12
690*d*e - 17967*e^2)/(5*x^2 + 2*x + 3)
```

Fricas [A]
time = 0.36, size = 234, normalized size = 1.67

$$\frac{2940000\,d^{2}x^{3}+1176000\,d^{2}x^{2}+1319850\,d^{2}x+3\,\sqrt{14}\,\left(164125\,d^{2}x^{2}+65650\,d^{2}x+98475\,d^{2}-73881\,\left(5\,x^{2}+2\,x+3\right)e^{2}+211710\,\left(5\,dx^{2}+2\,dx+3\,d\right)e\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,\left(5\,x+1\right)\right)-1435350\,d^{2}+14\,\left(70000\,x^{5}-187250\,x^{4}+172200\,x^{3}-42630\,x^{2}+184749\,x+53901\right)e^{2}+420\,\left(7000\,dx^{4}-25900\,dx^{3}-7280\,dx^{2}-23209\,dx+1269\,d\right)e-588\,\left(5125\,d^{2}x^{2}+2050\,d^{2}x+3075\,d^{2}-867\,\left(5\,x^{2}+2\,x+3\right)e^{2}-1030\,\left(5\,dx^{2}+2\,dx+3\,d\right)e\right)\log\left(5\,x^{2}+2\,x+3\right)}{3675000\,\left(5\,x^{2}+2\,x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] 1/3675000*(2940000*d^2*x^3 + 1176000*d^2*x^2 + 1319850*d^2*x + 3*sqrt(14)*(
164125*d^2*x^2 + 65650*d^2*x + 98475*d^2 - 73881*(5*x^2 + 2*x + 3)*e^2 + 21
1710*(5*d*x^2 + 2*d*x + 3*d)*e)*arctan(1/14*sqrt(14)*(5*x + 1)) - 1435350*d
^2 + 14*(70000*x^5 - 187250*x^4 + 172200*x^3 - 42630*x^2 + 184749*x + 53901
)*e^2 + 420*(7000*d*x^4 - 25900*d*x^3 - 7280*d*x^2 - 23209*d*x + 1269*d)*e
- 588*(5125*d^2*x^2 + 2050*d^2*x + 3075*d^2 - 867*(5*x^2 + 2*x + 3)*e^2 - 1
030*(5*d*x^2 + 2*d*x + 3*d)*e)*log(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.21, size = 298, normalized size = 2.13

$$\frac{4e^{2}x^{3}}{75}+x^{2}\cdot\left(\frac{4de}{25}-\frac{41e^{2}}{250}\right)+x\left(\frac{4d^{2}}{25}-\frac{82de}{125}+\frac{103e^{2}}{625}\right)+\left(-\frac{41d^{2}}{250}+\frac{103de}{625}+\frac{867e^{2}}{6250}-\frac{\sqrt{14}\,i(32825d^{2}+211710de-73881e^{2})}{2450000}\right)\log\left(x+\frac{6565d^{2}+42342de-\frac{73881e^{2}}{5}-\frac{\sqrt{14}\,i(32825d^{2}+211710de-73881e^{2})}{5}}{32825d^{2}+211710de-73881e^{2}}\right)+\left(-\frac{41d^{2}}{250}+\frac{103de}{625}+\frac{867e^{2}}{6250}+\frac{\sqrt{14}\,i(32825d^{2}+211710de-73881e^{2})}{2450000}\right)\log\left(x+\frac{6565d^{2}+42342de-\frac{73881e^{2}}{5}+\frac{\sqrt{14}\,i(32825d^{2}+211710de-73881e^{2})}{5}}{32825d^{2}+211710de-73881e^{2}}\right)+\frac{-34175d^{2}+12690de+17967e^{2}+x(-10575d^{2}-59890de+18323e^{2})}{437500x^{2}+175000x+262500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] 4*e**2*x**3/75 + x**2*(4*d*e/25 - 41*e**2/250) + x*(4*d**2/25 - 82*d*e/125
+ 103*e**2/625) + (-41*d**2/250 + 103*d*e/625 + 867*e**2/6250 - sqrt(14)*I*
(32825*d**2 + 211710*d*e - 73881*e**2)/2450000)*log(x + (6565*d**2 + 42342*
d*e - 73881*e**2/5 - sqrt(14)*I*(32825*d**2 + 211710*d*e - 73881*e**2)/5)/(
32825*d**2 + 211710*d*e - 73881*e**2)) + (-41*d**2/250 + 103*d*e/625 + 867*
e**2/6250 + sqrt(14)*I*(32825*d**2 + 211710*d*e - 73881*e**2)/2450000)*log(
x + (6565*d**2 + 42342*d*e - 73881*e**2/5 + sqrt(14)*I*(32825*d**2 + 211710
*d*e - 73881*e**2)/5)/(32825*d**2 + 211710*d*e - 73881*e**2)) + (-34175*d**
2 + 12690*d*e + 17967*e**2 + x*(-10575*d**2 - 59890*d*e + 18323*e**2))/(437
500*x**2 + 175000*x + 262500)
```

Giac [A]
time = 2.63, size = 145, normalized size = 1.04

$$\frac{4}{75}\,x^3e^2+\frac{4}{25}\,dx^2e+\frac{4}{25}\,d^2x-\frac{41}{250}\,x^2e^2-\frac{82}{125}\,dxe+\frac{1}{1225000}\,\sqrt{14}\,(32825\,d^2+211710\,de-73881\,e^2)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{103}{625}\,xe^2-\frac{1}{6250}\,(1025\,d^2-1030\,de-867\,e^2)\log\left(5\,x^2+2\,x+3\right)-\frac{34175\,d^2+(10575\,d^2+59890\,de-18323\,e^2)x-12690\,de-17967\,e^2}{87500\,(5\,x^2+2\,x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 4/75*x^3*e^2 + 4/25*d*x^2*e + 4/25*d^2*x - 41/250*x^2*e^2 - 82/125*d*x*e +
1/1225000*sqrt(14)*(32825*d^2 + 211710*d*e - 73881*e^2)*arctan(1/14*sqrt(14
)*(5*x + 1)) + 103/625*x*e^2 - 1/6250*(1025*d^2 - 1030*d*e - 867*e^2)*log(5
*x^2 + 2*x + 3) - 1/87500*(34175*d^2 + (10575*d^2 + 59890*d*e - 18323*e^2)*
x - 12690*d*e - 17967*e^2)/(5*x^2 + 2*x + 3)
```

Mupad [B]
time = 0.11, size = 211, normalized size = 1.51

$$\ln\left(5\,x^2+2\,x+3\right)\,\left(-\frac{41\,d^2}{250}+\frac{103\,d\,e}{625}+\frac{867\,e^2}{6250}\right)-x\,\left(\frac{2\,d\,e}{5}+\frac{4\,e\,(8\,d-5\,e)}{125}-\frac{4\,d^2}{25}-\frac{3\,e^2}{625}\right)+x^2\,\left(\frac{e\,(8\,d-5\,e)}{50}-\frac{8\,e^2}{125}\right)+\frac{\frac{1269\,d\,e}{14}-x\,\left(\frac{2115\,d^2}{28}+\frac{5989\,d\,e}{14}-\frac{18323\,e^2}{140}\right)-\frac{6835\,d^2}{28}+\frac{17967\,e^2}{140}}{3125\,x^2+1250\,x+1875}+\frac{4\,e^2\,x^3}{75}+\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,\left(32825\,d^2+211710\,d\,e-73881\,e^2\right)}{1225000}+\frac{\sqrt{14}\,x\,\left(32825\,d^2+211710\,d\,e-73881\,e^2\right)}{245000}}{\frac{1313\,d^2}{3500}+\frac{21171\,d\,e}{8750}-\frac{73881\,e^2}{87500}}\right)\,(32825\,d^2+211710\,d\,e-73881\,e^2)}{1225000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

```
[Out] log(2*x + 5*x^2 + 3)*((103*d*e)/625 - (41*d^2)/250 + (867*e^2)/6250) - x*((
2*d*e)/5 + (4*e*(8*d - 5*e))/125 - (4*d^2)/25 - (3*e^2)/625) + x^2*((e*(8*d
 - 5*e))/50 - (8*e^2)/125) + ((1269*d*e)/14 - x*((5989*d*e)/14 + (2115*d^2)
/28 - (18323*e^2)/140) - (6835*d^2)/28 + (17967*e^2)/140)/(1250*x + 3125*x^
2 + 1875) + (4*e^2*x^3)/75 + (14^(1/2)*atan(((14^(1/2)*(211710*d*e + 32825*
d^2 - 73881*e^2))/1225000 + (14^(1/2)*x*(211710*d*e + 32825*d^2 - 73881*e^2
))/245000)/((21171*d*e)/8750 + (1313*d^2)/3500 - (73881*e^2)/87500))*(21171
0*d*e + 32825*d^2 - 73881*e^2))/1225000
```

3.313 $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

Optimal. Leaf size=97

$$\frac{1}{125}(20d-41e)x+\frac{2ex^2}{25}-\frac{(1367+423x)(d+ex)}{3500\,(3+2x+5x^2)}+\frac{(6565d+21171e)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{17500\sqrt{14}}-\frac{(205d-103e)\log\,(3+}{1250}$$

```
[Out] 1/125*(20*d-41*e)*x+2/25*e*x^2-1/3500*(1367+423*x)*(e*x+d)/(5*x^2+2*x+3)-1/
1250*(205*d-103*e)*ln(5*x^2+2*x+3)+1/245000*(6565*d+21171*e)*arctan(1/14*(1
+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1658, 1671, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)(6565d+21171e)}{17500\sqrt{14}}-\frac{(423x+1367)(d+ex)}{3500\,(5x^2+2x+3)}-\frac{(205d-103e)\log\,(5x^2+2x+3)}{1250}+\frac{1}{125}x(20d-41e)+\frac{2ex^2}{25}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] ((20*d - 41*e)*x)/125 + (2*e*x^2)/25 - ((1367 + 423*x)*(d + e*x))/(3500*(3
+ 2*x + 5*x^2)) + ((6565*d + 21171*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(17500*Sq
rt[14]) - ((205*d - 103*e)*Log[3 + 2*x + 5*x^2])/1250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx = -\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int \frac{\frac{2}{125}(1845d+1367e)-\frac{168}{125}(55d}{3+}$$

$$= -\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int \left(\frac{56}{125}(20d-41e)+\frac{224ex}{25}+\right.$$

$$= \frac{1}{125}(20d-41e)x+\frac{2ex^2}{25}-\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)}+\frac{\int \frac{165d+481}{}}{}$$

$$= \frac{1}{125}(20d-41e)x+\frac{2ex^2}{25}-\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)}+\frac{(-205d+}{}$$

$$= \frac{1}{125}(20d-41e)x+\frac{2ex^2}{25}-\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)}-\frac{(205d-1}{}$$

$$= \frac{1}{125}(20d-41e)x+\frac{2ex^2}{25}-\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)}+\frac{(6565d+}{}$$

Mathematica [A]

time = 0.05, size = 96, normalized size = 0.99

$$\frac{1960(20d-41e)x+19600ex^2-\frac{14(5d(1367+423x)+e(-1269+5989x))}{3+2x+5x^2}+\sqrt{14}\,(6565d+21171e)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)+196(-205d+103e)\log\left(3+2x+5x^2\right)}{245000}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x
]
```

```
[Out] (1960*(20*d - 41*e)*x + 19600*e*x^2 - (14*(5*d*(1367 + 423*x) + e*(-1269 +
5989*x)))/(3 + 2*x + 5*x^2) + Sqrt[14]*(6565*d + 21171*e)*ArcTan[(1 + 5*x)/
Sqrt[14]] + 196*(-205*d + 103*e)*Log[3 + 2*x + 5*x^2])/245000
```

Maple [A]

time = 0.13, size = 87, normalized size = 0.90

method	result
default	$\frac{2e\,x^2}{25}+\frac{4dx}{25}-\frac{41ex}{125}-\frac{\left(\frac{423d}{140}+\frac{5989e}{700}\right)x+\frac{1367d}{140}-\frac{1269e}{700}}{125\left(x^2+\frac{2}{5}x+\frac{3}{5}\right)}-\frac{(5740d-2884e)\ln\left(5x^2+2x+3\right)}{35000}-\frac{\left(-1313d-\frac{21171e}{5}\right)\sqrt{14}\ \arctan\left(\right.}{49000}$
risch	$\frac{2e\,x^2}{25}+\frac{4dx}{25}-\frac{41ex}{125}+\frac{\frac{\left(-\frac{423d}{140}-\frac{5989e}{700}\right)x}{125}-\frac{1367d}{17500}+\frac{1269e}{87500}}{x^2+\frac{2}{5}x+\frac{3}{5}}-\frac{41d\ln\left(350x^2+140x+210\right)}{250}+\frac{103e\ln\left(350x^2+140x+210\right)}{1250}+\frac{1313}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] 2/25*e*x^2+4/25*d*x-41/125*e*x-1/125*((423/140*d+5989/700*e)*x+1367/140*d-1
269/700*e)/(x^2+2/5*x+3/5)-1/35000*(5740*d-2884*e)*ln(5*x^2+2*x+3)-1/49000*
(-1313*d-21171/5*e)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

Maxima [A]
time = 0.54, size = 96, normalized size = 0.99

$$\frac{2}{25}\,x^{2}e+\frac{1}{245000}\,\sqrt{14}\,(6565\,d+21171\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{1}{125}\,(20\,d-41\,e)x-\frac{1}{1250}\,(205\,d-103\,e)\log\left(5\,x^{2}+2\,x+3\right)-\frac{(2115\,d+5989\,e)x+6835\,d-1269\,e}{17500\,(5\,x^{2}+2\,x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="max
ima")
```

```
[Out] 2/25*x^2*e + 1/245000*sqrt(14)*(6565*d + 21171*e)*arctan(1/14*sqrt(14)*(5*x
 + 1)) + 1/125*(20*d - 41*e)*x - 1/1250*(205*d - 103*e)*log(5*x^2 + 2*x + 3
) - 1/17500*((2115*d + 5989*e)*x + 6835*d - 1269*e)/(5*x^2 + 2*x + 3)
```

Fricas [A]
time = 0.37, size = 142, normalized size = 1.46

$$\frac{196000\,dx^{3}+78400\,dx^{2}+\sqrt{14}\,(32825\,dx^{2}+13130\,dx+21171\,(5\,x^{2}+2\,x+3)e+19695\,d)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+87990\,dx+14\,(7000\,x^{4}-25900\,x^{3}-7280\,x^{2}-23209\,x+1269)e-196\,(1025\,dx^{2}+410\,dx-103\,(5\,x^{2}+2\,x+3)e+615\,d)\log\left(5\,x^{2}+2\,x+3\right)-95690\,d}{245000\,(5\,x^{2}+2\,x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="fri
cas")
```

```
[Out] 1/245000*(196000*d*x^3 + 78400*d*x^2 + sqrt(14)*(32825*d*x^2 + 13130*d*x +
21171*(5*x^2 + 2*x + 3)*e + 19695*d)*arctan(1/14*sqrt(14)*(5*x + 1)) + 8799
0*d*x + 14*(7000*x^4 - 25900*x^3 - 7280*x^2 - 23209*x + 1269)*e - 196*(1025
*d*x^2 + 410*d*x - 103*(5*x^2 + 2*x + 3)*e + 615*d)*log(5*x^2 + 2*x + 3) -
95690*d)/(5*x^2 + 2*x + 3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.58, size = 165, normalized size = 1.70

$$\frac{2ex^{2}}{25}+x\left(\frac{4d}{25}-\frac{41e}{125}\right)+\frac{-6835d+1269e+x(-2115d-5989e)}{87500x^{2}+35000x+52500}+\left(-\frac{41d}{250}+\frac{103e}{1250}-\frac{\sqrt{14}\,i(6565d+21171e)}{490000}\right)\log\left(x+\frac{1313d+\frac{21171e}{5}-\frac{\sqrt{14}\,i(6565d+21171e)}{5}}{6565d+21171e}\right)+\left(-\frac{41d}{250}+\frac{103e}{1250}+\frac{\sqrt{14}\,i(6565d+21171e)}{490000}\right)\log\left(x+\frac{1313d+\frac{21171e}{5}+\frac{\sqrt{14}\,i(6565d+21171e)}{5}}{6565d+21171e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] 2*e*x**2/25 + x*(4*d/25 - 41*e/125) + (-6835*d + 1269*e + x*(-2115*d - 5989
*e))/(87500*x**2 + 35000*x + 52500) + (-41*d/250 + 103*e/1250 - sqrt(14)*I*
(6565*d + 21171*e)/490000)*log(x + (1313*d + 21171*e/5 - sqrt(14)*I*(6565*d
```

```
 + 21171*e)/5)/(6565*d + 21171*e)) + (-41*d/250 + 103*e/1250 + sqrt(14)*I*(
6565*d + 21171*e)/490000)*log(x + (1313*d + 21171*e/5 + sqrt(14)*I*(6565*d
+ 21171*e)/5)/(6565*d + 21171*e))
```

Giac [A]
time = 5.90, size = 94, normalized size = 0.97

$$\frac{2}{25}\,x^2e+\frac{1}{245000}\,\sqrt{14}\,(6565\,d+21171\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{4}{25}\,dx-\frac{41}{125}\,xe-\frac{1}{1250}\,(205\,d-103\,e)\log\left(5\,x^2+2\,x+3\right)-\frac{(2115\,d+5989\,e)x+6835\,d-1269\,e}{17500\,(5\,x^2+2\,x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="gia
c")
```

```
[Out] 2/25*x^2*e + 1/245000*sqrt(14)*(6565*d + 21171*e)*arctan(1/14*sqrt(14)*(5*x
 + 1)) + 4/25*d*x - 41/125*x*e - 1/1250*(205*d - 103*e)*log(5*x^2 + 2*x + 3
) - 1/17500*((2115*d + 5989*e)*x + 6835*d - 1269*e)/(5*x^2 + 2*x + 3)
```

Mupad [B]
time = 4.15, size = 115, normalized size = 1.19

$$\frac{2\,e\,x^2}{25}-\ln\left(5\,x^2+2\,x+3\right)\,\left(\frac{41\,d}{250}-\frac{103\,e}{1250}\right)+x\,\left(\frac{4\,d}{25}-\frac{41\,e}{125}\right)-\frac{\frac{1367\,d}{28}-\frac{1269\,e}{140}+x\,\left(\frac{423\,d}{28}+\frac{5989\,e}{140}\right)}{625\,x^2+250\,x+375}+\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,(6565\,d+21171\,e)}{245000}+\frac{\sqrt{14}\,x\,(6565\,d+21171\,e)}{49000}}{\frac{1313\,d}{3500}+\frac{21171\,e}{17500}}\right)\,(6565\,d+21171\,e)}{245000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

```
[Out] (2*e*x^2)/25 - log(2*x + 5*x^2 + 3)*((41*d)/250 - (103*e)/1250) + x*((4*d)/
25 - (41*e)/125) - ((1367*d)/28 - (1269*e)/140 + x*((423*d)/28 + (5989*e)/1
40))/(250*x + 625*x^2 + 375) + (14^(1/2)*atan(((14^(1/2)*(6565*d + 21171*e)
)/245000 + (14^(1/2)*x*(6565*d + 21171*e))/49000)/((1313*d)/3500 + (21171*e
)/17500))*(6565*d + 21171*e))/245000
```

3.314 $\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2}\,dx$

Optimal. Leaf size=63

$$\frac{4x}{25}-\frac{1367+423x}{3500(3+2x+5x^2)}+\frac{1313\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{3500\sqrt{14}}-\frac{41}{250}\log\left(3+2x+5x^2\right)$$

```
[Out] 4/25*x+1/3500*(-1367-423*x)/(5*x^2+2*x+3)-41/250*ln(5*x^2+2*x+3)+1313/49000
*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1674, 1671, 648, 632, 210, 642}

$$\frac{1313\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)}{3500\sqrt{14}}-\frac{423x+1367}{3500(5x^2+2x+3)}-\frac{41}{250}\log\left(5x^2+2x+3\right)+\frac{4x}{25}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] (4*x)/25 - (1367 + 423*x)/(3500*(3 + 2*x + 5*x^2)) + (1313*ArcTan[(1 + 5*x)
/Sqrt[14]])/(3500*Sqrt[14]) - (41*Log[3 + 2*x + 5*x^2])/250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^2}\,dx &= -\frac{1367+423x}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int \frac{\frac{738}{25}-\frac{1848x}{25}+\frac{224x^2}{5}}{3+2x+5x^2}\,dx\\
&= -\frac{1367+423x}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int \left(\frac{224}{25}+\frac{2(33-1148x)}{25\left(3+2x+5x^2\right)}\right)dx\\
&= \frac{4x}{25}-\frac{1367+423x}{3500\left(3+2x+5x^2\right)}+\frac{1}{700}\int \frac{33-1148x}{3+2x+5x^2}\,dx\\
&= \frac{4x}{25}-\frac{1367+423x}{3500\left(3+2x+5x^2\right)}-\frac{41}{250}\int \frac{2+10x}{3+2x+5x^2}\,dx+\frac{1313\int \frac{1}{3+2x+5x^2}\,dx}{3500}\\
&= \frac{4x}{25}-\frac{1367+423x}{3500\left(3+2x+5x^2\right)}-\frac{41}{250}\log\left(3+2x+5x^2\right)-\frac{1313\text{Subst}\left(\int \frac{1}{-56-x}\right.}{175}\\
&= \frac{4x}{25}-\frac{1367+423x}{3500\left(3+2x+5x^2\right)}+\frac{1313\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{3500\sqrt{14}}-\frac{41}{250}\log\left(3+2x+5x^2\right.
\end{aligned}
$$

Mathematica [A]

time = 0.03, size = 59, normalized size = 0.94

$$\frac{7840x-\frac{14(1367+423x)}{3+2x+5x^2}+1313\sqrt{14}\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)-8036\log\left(3+2x+5x^2\right)}{49000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] (7840*x - (14*(1367 + 423*x))/(3 + 2*x + 5*x^2) + 1313*Sqrt[14]*ArcTan[(1 +
 5*x)/Sqrt[14]] - 8036*Log[3 + 2*x + 5*x^2])/49000
```

Maple [A]
time = 0.13, size = 51, normalized size = 0.81

method	result	size
risch	$\frac{4x}{25}+\frac{-\frac{423x}{17500}-\frac{1367}{17500}}{x^2+\frac{2}{5}x+\frac{3}{5}}-\frac{41\ln(25x^2+10x+15)}{250}+\frac{1313\arctan\left(\frac{(1+5x)\sqrt{14}}{14}\right)\sqrt{14}}{49000}$	50
default	$\frac{4x}{25}-\frac{\frac{423x}{700}+\frac{1367}{700}}{25\left(x^2+\frac{2}{5}x+\frac{3}{5}\right)}-\frac{41\ln(5x^2+2x+3)}{250}+\frac{1313\sqrt{14}\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{49000}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/25*x-1/25*(423/700*x+1367/700)/(x^2+2/5*x+3/5)-41/250*ln(5*x^2+2*x+3)+131
3/49000*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

Maxima [A]
time = 0.53, size = 52, normalized size = 0.83

$$\frac{1313}{49000}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)+\frac{4}{25}x-\frac{423x+1367}{3500(5x^2+2x+3)}-\frac{41}{250}\log\left(5x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="maxima")
```

```
[Out] 1313/49000*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4/25*x - 1/3500*(423*
x + 1367)/(5*x^2 + 2*x + 3) - 41/250*log(5*x^2 + 2*x + 3)
```

Fricas [A]
time = 0.34, size = 78, normalized size = 1.24

$$\frac{39200x^3+1313\sqrt{14}(5x^2+2x+3)\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)+15680x^2-8036(5x^2+2x+3)\log\left(5x^2+2x+3\right)+17598x-19138}{49000(5x^2+2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/49000*(39200*x^3 + 1313*sqrt(14)*(5*x^2 + 2*x + 3)*arctan(1/14*sqrt(14)*(
5*x + 1)) + 15680*x^2 - 8036*(5*x^2 + 2*x + 3)*log(5*x^2 + 2*x + 3) + 17598
*x - 19138)/(5*x^2 + 2*x + 3)
```

Sympy [A]
time = 0.06, size = 65, normalized size = 1.03

$$\frac{4x}{25} + \frac{-423x - 1367}{17500x^2 + 7000x + 10500} - \frac{41\log\left(x^2 + \frac{2x}{5} + \frac{3}{5}\right)}{250} + \frac{1313\sqrt{14}\,\operatorname{atan}\left(\frac{5\sqrt{14}\,x}{14} + \frac{\sqrt{14}}{14}\right)}{49000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] 4*x/25 + (-423*x - 1367)/(17500*x**2 + 7000*x + 10500) - 41*log(x**2 + 2*x/
5 + 3/5)/250 + 1313*sqrt(14)*atan(5*sqrt(14)*x/14 + sqrt(14)/14)/49000
```

Giac [A]
time = 4.35, size = 52, normalized size = 0.83

$$\frac{1313}{49000}\sqrt{14}\,\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right) + \frac{4}{25}\,x - \frac{423\,x + 1367}{3500\,(5\,x^2 + 2\,x + 3)} - \frac{41}{250}\,\log\left(5\,x^2 + 2\,x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="giac")
```

```
[Out] 1313/49000*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4/25*x - 1/3500*(423*
x + 1367)/(5*x^2 + 2*x + 3) - 41/250*log(5*x^2 + 2*x + 3)
```

Mupad [B]
time = 4.15, size = 52, normalized size = 0.83

$$\frac{4\,x}{25} - \frac{41\,\ln\left(5\,x^2 + 2\,x + 3\right)}{250} - \frac{\frac{423\,x}{17500} + \frac{1367}{17500}}{x^2 + \frac{2\,x}{5} + \frac{3}{5}} + \frac{1313\,\sqrt{14}\,\operatorname{atan}\left(\frac{5\,\sqrt{14}\,x}{14} + \frac{\sqrt{14}}{14}\right)}{49000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3)^2,x)
```

```
[Out] (4*x)/25 - (41*log(2*x + 5*x^2 + 3))/250 - ((423*x)/17500 + 1367/17500)/((2
*x)/5 + x^2 + 3/5) + (1313*14^(1/2)*atan((5*14^(1/2)*x)/14 + 14^(1/2)/14))/
49000
```

3.315 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx$

Optimal. Leaf size=224

$$-\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{\left(6565d^3-26423d^2e+11089de^2-6623e^3\right)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{700\sqrt{14}\left(5d^2-2de+3e^2\right)^2}+\frac{(4d^4}{}$$

```
[Out] 1/700*(-1367*d+293*e-(423*d-1367*e)*x)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)+(4
*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/e/(5*d^2-2*d*e+3*e^2)^2-1/50*
(205*d^3-61*d^2*e+23*d*e^2+14*e^3)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^2+1/
9800*(6565*d^3-26423*d^2*e+11089*d*e^2-6623*e^3)*arctan(1/14*(1+5*x)*14^(1/
2))/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1660, 1642, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)\left(6565d^3-26423d^2e+11089de^2-6623e^3\right)}{700\sqrt{14}\left(5d^2-2de+3e^2\right)^2}-\frac{x(423d-1367e)+1367d-293e}{700\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)}-\frac{\left(205d^3-61d^2e+23de^2+14e^3\right)\log\left(5x^2+2x+3\right)}{50\left(5d^2-2de+3e^2\right)^2}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e\left(5d^2-2de+3e^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^2),x]
```

```
[Out] -1/700*(1367*d - 293*e + (423*d - 1367*e)*x)/((5*d^2 - 2*d*e + 3*e^2)*(3 +
2*x + 5*x^2)) + ((6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*ArcTan[(
1 + 5*x)/Sqrt[14]])/(700*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) + ((4*d^4 + 5*
d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/(e*(5*d^2 - 2*d*e + 3*e^2)
^2) - ((205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*Log[3 + 2*x + 5*x^2])/(50*(
5*d^2 - 2*d*e + 3*e^2)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx = -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{1}{56}\int \frac{\frac{2\left(369d^2-421de+280e^2\right)}{5\left(5d^2-2de+3e^2\right)}-\frac{2(92\cdots}{5\cdots}}{(d+ex)\left(3+\cdots\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{1}{56}\int \left(\frac{56\left(4d^4+5d^3e+3d^2e^2\cdots\right.}{\left(5d^2-2de+3e^2\right)^2}\cdots\right.$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e\cdots\right.}{e\left(5d^2-2de+3e^2\cdots\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e\cdots\right.}{e\left(5d^2-2de+3e^2\cdots\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e\cdots\right.}{e\left(5d^2-2de+3e^2\cdots\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}+\frac{\left(6565d^3-26423d^2e+11089de\cdots\right.}{700\sqrt{14}\left(5d^2-\cdots\right.}$$

Mathematica [A]

time = 0.11, size = 186, normalized size = 0.83

$$\frac{\frac{14\left(5d^2-2de+3e^2\right)\left(-d(1367+423x)+e(293+1367x)\right)}{3+2x+5x^2}+\sqrt{14}\left(6565d^3-26423d^2e+11089de^2-6623e^3\right)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)+\frac{9800\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e}-196\left(205d^3-61d^2e+23de^2+14e^3\right)\log\left(3+2x+5x^2\right)}{9800\left(5d^2-2de+3e^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^2),x
]
```

```
[Out] ((14*(5*d^2 - 2*d*e + 3*e^2)*(-(d*(1367 + 423*x)) + e*(293 + 1367*x)))/(3 +
 2*x + 5*x^2) + Sqrt[14]*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*
ArcTan[(1 + 5*x)/Sqrt[14]] + (9800*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2
*e^4)*Log[d + e*x])/e - 196*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*Log[3
+ 2*x + 5*x^2])/(9800*(5*d^2 - 2*d*e + 3*e^2)^2)
```

Maple [A]

time = 0.15, size = 214, normalized size = 0.96

method	result
default	$\frac{\left(4d^4+5d^3e+3d^2e^2-d\,e^3+2e^4\right)\ln(ex+d)}{e\left(5d^2-2de+3e^2\right)^2}-\frac{\frac{\left(\frac{423}{700}d^3-\frac{7681}{3500}d^2e+\frac{4003}{3500}d\,e^2-\frac{4101}{3500}e^3\right)x+\frac{1367d^3}{700}-\frac{4199d^2e}{3500}+\frac{4687d\,e^2}{3500}-\frac{879e^3}{3500}}{x^2+\frac{2}{5}x+\frac{3}{5}}+\frac{\left(5740d^3-17\cdots\right.}{\cdots}}{\cdots}$

risch	$\frac{-\frac{(423d-1367e)x}{3500\left(5d^2-2de+3e^2\right)}-\frac{1367d-293e}{3500\left(5d^2-2de+3e^2\right)}}{x^2+\frac{2}{5}x+\frac{3}{5}}+\frac{4\ln(ex+d)d^4}{e(25d^4-20d^3e+34d^2e^2-12d\,e^3+9e^4)}+\frac{5\ln(ex+d)d^3}{25d^4-20d^3e+34d^2e^2-12d\,e^3+9e^4}+\frac{}{25d^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] (4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/e/(5*d^2-2*d*e+3*e^2)^2-1/(
5*d^2-2*d*e+3*e^2)^2*(((423/700*d^3-7681/3500*d^2*e+4003/3500*d*e^2-4101/35
00*e^3)*x+1367/700*d^3-4199/3500*d^2*e+4687/3500*d*e^2-879/3500*e^3)/(x^2+2
/5*x+3/5)+1/1400*(5740*d^3-1708*d^2*e+644*d*e^2+392*e^3)*ln(5*x^2+2*x+3)+1/
1960*(-1313*d^3+26423/5*d^2*e-11089/5*d*e^2+6623/5*e^3)*14^(1/2)*arctan(1/2
8*(10*x+2)*14^(1/2)))
```

Maxima [A]
time = 0.54, size = 281, normalized size = 1.25

$$\frac{\sqrt{14}\,(6565\,d^3-26423\,d^2e+11089\,de^2-6623\,e^3)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{9800\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)}-\frac{(205\,d^3-61\,d^2e+23\,de^2+14\,e^3)\log\left(5\,x^2+2\,x+3\right)}{50\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)}+\frac{(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4)\log\left(xe+d\right)}{25\,d^4e-20\,d^3e^2+34\,d^2e^3-12\,de^4+9\,e^5}-\frac{(423\,d-1367\,e)x+1367\,d-293\,e}{700\,(5\,(5\,d^2-2\,de+3\,e^2)x^2+15\,d^2+2\,(5\,d^2-2\,de+3\,e^2)x-6\,de+9\,e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="max
ima")
```

```
[Out] 1/9800*sqrt(14)*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*arctan(1/
14*sqrt(14)*(5*x + 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)
- 1/50*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*log(5*x^2 + 2*x + 3)/(25*d^
4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) + (4*d^4 + 5*d^3*e + 3*d^2*e^
2 - d*e^3 + 2*e^4)*log(x*e + d)/(25*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 - 12*d*
e^4 + 9*e^5) - 1/700*((423*d - 1367*e)*x + 1367*d - 293*e)/(5*(5*d^2 - 2*d*
e + 3*e^2)*x^2 + 15*d^2 + 2*(5*d^2 - 2*d*e + 3*e^2)*x - 6*d*e + 9*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. 2(213) = 426.
time = 0.43, size = 462, normalized size = 2.06

$$-\frac{\sqrt{14}\,(6623\,(5\,x^2+2\,x+3)e^4-11089\,(5\,dx^2+2\,dx+3\,d)e^3+26423\,(5\,d^2x^2+2\,d^2x+3\,d^2)e^2-6565\,(5\,d^3x^2+2\,d^3x+3\,d^3)e)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)-42\,(1367\,x+293)e^4+14\,(4003\,dx+4687\,d)e^3-14\,(7681\,d^2x+4199\,d^2)e^2+70\,(423\,d^3x+1367\,d^3)e+196\,(14\,(5\,x^2+2\,x+3)e^4+23\,(5\,dx^2+2\,dx+3\,d)e^3-61\,(5\,d^2x^2+2\,d^2x+3\,d^2)e^2+205\,(5\,d^3x^2+2\,d^3x+3\,d^3)e)\log\left(5\,x^2+2\,x+3\right)-9800\,(20\,d^4x^2+8\,d^4x+12\,d^4+2\,(5\,x^2+2\,x+3)e^4-(5\,dx^2+2\,dx+3\,d)e^3+3\,(5\,d^2x^2+2\,d^2x+3\,d^2)e^2+5\,(5\,d^3x^2+2\,d^3x+3\,d^3)e)\log\left(xe+d\right)}{9800\,(9\,(5\,x^2+2\,x+3)e^5-12\,(5\,dx^2+2\,dx+3\,d)e^4+34\,(5\,d^2x^2+2\,d^2x+3\,d^2)e^3-20\,(5\,d^3x^2+2\,d^3x+3\,d^3)e^2+25\,(5\,d^4x^2+2\,d^4x+3\,d^4)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="fri
cas")
```

```
[Out] -1/9800*(sqrt(14)*(6623*(5*x^2 + 2*x + 3)*e^4 - 11089*(5*d*x^2 + 2*d*x + 3*
d)*e^3 + 26423*(5*d^2*x^2 + 2*d^2*x + 3*d^2)*e^2 - 6565*(5*d^3*x^2 + 2*d^3*
```

```
x + 3*d^3)*e)*arctan(1/14*sqrt(14)*(5*x + 1)) - 42*(1367*x + 293)*e^4 + 14*
(4003*d*x + 4687*d)*e^3 - 14*(7681*d^2*x + 4199*d^2)*e^2 + 70*(423*d^3*x +
1367*d^3)*e + 196*(14*(5*x^2 + 2*x + 3)*e^4 + 23*(5*d*x^2 + 2*d*x + 3*d)*e^
3 - 61*(5*d^2*x^2 + 2*d^2*x + 3*d^2)*e^2 + 205*(5*d^3*x^2 + 2*d^3*x + 3*d^3
)*e)*log(5*x^2 + 2*x + 3) - 9800*(20*d^4*x^2 + 8*d^4*x + 12*d^4 + 2*(5*x^2
+ 2*x + 3)*e^4 - (5*d*x^2 + 2*d*x + 3*d)*e^3 + 3*(5*d^2*x^2 + 2*d^2*x + 3*d
^2)*e^2 + 5*(5*d^3*x^2 + 2*d^3*x + 3*d^3)*e)*log(x*e + d))/(9*(5*x^2 + 2*x
+ 3)*e^5 - 12*(5*d*x^2 + 2*d*x + 3*d)*e^4 + 34*(5*d^2*x^2 + 2*d^2*x + 3*d^2
)*e^3 - 20*(5*d^3*x^2 + 2*d^3*x + 3*d^3)*e^2 + 25*(5*d^4*x^2 + 2*d^4*x + 3*
d^4)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.47, size = 284, normalized size = 1.27

$$\frac{\sqrt{14}\,(6565\,d^3-26423\,d^2e+11089\,de^2-6623\,e^3)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{9800\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)}-\frac{(205\,d^3-61\,d^2e+23\,de^2+14\,e^3)\log\left(5\,x^2+2\,x+3\right)}{50\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)}+\frac{(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4)\log\left(|xe+d|\right)}{25\,d^4e-20\,d^3e^2+34\,d^2e^3-12\,de^4+9\,e^5}-\frac{6835\,d^3-4199\,d^2e+(2115\,d^3-7681\,d^2e+4003\,de^2-4101\,e^3)x+4687\,de^2-879\,e^3}{700\,(5\,d^2-2\,de+3\,e^2)^2(5\,x^2+2\,x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="gia
c")
```

```
[Out] 1/9800*sqrt(14)*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*arctan(1/
14*sqrt(14)*(5*x + 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)
- 1/50*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*log(5*x^2 + 2*x + 3)/(25*d^
4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) + (4*d^4 + 5*d^3*e + 3*d^2*e^
2 - d*e^3 + 2*e^4)*log(abs(x*e + d))/(25*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 -
12*d*e^4 + 9*e^5) - 1/700*(6835*d^3 - 4199*d^2*e + (2115*d^3 - 7681*d^2*e +
 4003*d*e^2 - 4101*e^3)*x + 4687*d*e^2 - 879*e^3)/((5*d^2 - 2*d*e + 3*e^2)^
2*(5*x^2 + 2*x + 3))
```

Mupad [**B**]
time = 4.61, size = 330, normalized size = 1.47

$$\frac{\ln\left(d+e\,x\right)\,\left(4\,d^4+5\,d^3\,e+3\,d^2\,e^2-d\,e^3+2\,e^4\right)}{e\,{\left(5\,d^2-2\,d\,e+3\,e^2\right)}^2}+\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1{}\mathrm{i}}{5}\right)\,\left(\left(\frac{1313\,\sqrt{14}}{3920}-\frac{41}{10}{}\mathrm{i}\right)\,d^3+\left(-\frac{26423\,\sqrt{14}}{19600}+\frac{61}{50}{}\mathrm{i}\right)\,d^2\,e+\left(\frac{11089\,\sqrt{14}}{19600}-\frac{23}{50}{}\mathrm{i}\right)\,d\,e^2+\left(-\frac{6623\,\sqrt{14}}{19600}-\frac{7}{25}{}\mathrm{i}\right)\,e^3\right)}{d^4\,25{}\mathrm{i}-d^3\,e\,20{}\mathrm{i}+d^2\,e^2\,34{}\mathrm{i}-d\,e^3\,12{}\mathrm{i}+e^4\,9{}\mathrm{i}}-\frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1{}\mathrm{i}}{5}\right)\,\left(\left(\frac{1313\,\sqrt{14}}{3920}+\frac{41}{10}{}\mathrm{i}\right)\,d^3+\left(-\frac{26423\,\sqrt{14}}{19600}-\frac{61}{50}{}\mathrm{i}\right)\,d^2\,e+\left(\frac{11089\,\sqrt{14}}{19600}+\frac{23}{50}{}\mathrm{i}\right)\,d\,e^2+\left(-\frac{6623\,\sqrt{14}}{19600}+\frac{7}{25}{}\mathrm{i}\right)\,e^3\right)}{d^4\,25{}\mathrm{i}-d^3\,e\,20{}\mathrm{i}+d^2\,e^2\,34{}\mathrm{i}-d\,e^3\,12{}\mathrm{i}+e^4\,9{}\mathrm{i}}-\frac{\frac{1367\,d-293\,e}{700\,\left(5\,d^2-2\,d\,e+3\,e^2\right)}+\frac{x\,\left(423\,d-1367\,e\right)}{700\,\left(5\,d^2-2\,d\,e+3\,e^2\right)}}{5\,x^2+2\,x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)^2),x)
```

```
[Out] (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^3*((1313*14^(1/2))/3920 - 41i/10) - e^3*
((6623*14^(1/2))/19600 + 7i/25) + d*e^2*((11089*14^(1/2))/19600 - 23i/50) -
 d^2*e*((26423*14^(1/2))/19600 - 61i/50)))/(d^4*25i - d^3*e*20i - d*e^3*12i
 + e^4*9i + d^2*e^2*34i) - ((1367*d - 293*e)/(700*(5*d^2 - 2*d*e + 3*e^2))
+ (x*(423*d - 1367*e))/(700*(5*d^2 - 2*d*e + 3*e^2)))/(2*x + 5*x^2 + 3) - (
log(x + (14^(1/2)*1i)/5 + 1/5)*(d^3*((1313*14^(1/2))/3920 + 41i/10) - e^3*(
(6623*14^(1/2))/19600 - 7i/25) + d*e^2*((11089*14^(1/2))/19600 + 23i/50) -
d^2*e*((26423*14^(1/2))/19600 + 61i/50)))/(d^4*25i - d^3*e*20i - d*e^3*12i
+ e^4*9i + d^2*e^2*34i) + (log(d + e*x)*(5*d^3*e - d*e^3 + 4*d^4 + 2*e^4 +
3*d^2*e^2))/(e*(5*d^2 - 2*d*e + 3*e^2)^2)
```

3.316 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^2}\,dx$

Optimal. Leaf size=313

$$-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{140\left(5d^2-2de+3e^2\right)^2(3+2x+5x^2)}+\frac{(1313d^4-100}{}$$

```
[Out] (-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)+1/14
0*(-1367*d^2+586*d*e+703*e^2-(423*d^2-2734*d*e+293*e^2)*x)/(5*d^2-2*d*e+3*e
^2)^2/(5*x^2+2*x+3)+(41*d^4-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)*ln(e*x+d)/(5
*d^2-2*d*e+3*e^2)^3-1/2*(41*d^4-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)*ln(5*x^2
+2*x+3)/(5*d^2-2*d*e+3*e^2)^3+1/392*(1313*d^4-10044*d^3*e+4290*d^2*e^2+156*
d*e^3-271*e^4)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1660, 1642, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)\left(1313d^4-10044d^3e+4290d^2e^2+156de^3-271e^4\right)}{28\sqrt{14}\left(5d^2-2de+3e^2\right)^3}-\frac{x\left(423d^2-2734de+293e^2\right)+1367d^2-586de-703e^2}{140\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^2}-\frac{\left(41d^4-8d^3e-60d^2e^2+24de^3-5e^4\right)\log\left(5x^2+2x+3\right)}{2\left(5d^2-2de+3e^2\right)^3}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)}+\frac{\left(41d^4-8d^3e-60d^2e^2+24de^3-5e^4\right)\log(d+ex)}{\left(5d^2-2de+3e^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^2),x]
```

```
[Out] -((4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e*(5*d^2 - 2*d*e + 3*e^2)^
2*(d + e*x))) - (1367*d^2 - 586*d*e - 703*e^2 + (423*d^2 - 2734*d*e + 293*e
^2)*x)/(140*(5*d^2 - 2*d*e + 3*e^2)^2*(3 + 2*x + 5*x^2)) + ((1313*d^4 - 100
44*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4)*ArcTan[(1 + 5*x)/Sqrt[14]])/
(28*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^3) + ((41*d^4 - 8*d^3*e - 60*d^2*e^2 +
 24*d*e^3 - 5*e^4)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^3 - ((41*d^4 - 8*d
^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d
*e + 3*e^2)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^2}\,dx = -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)} + \frac{1}{56}\int \frac{2(369d^4 \cdots}{\cdots}$$

$$= -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)} + \frac{1}{56}\int \left(\frac{56(4 \cdots}{(\cdots}\right.$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)} - \frac{1367d^2-586de-703e^2+\left(423d^2- \cdots\right.}{140\left(5d^2-2de+3e^2\right)^2(3+ \cdots}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)} - \frac{1367d^2-586de-703e^2+\left(423d^2- \cdots\right.}{140\left(5d^2-2de+3e^2\right)^2(3+ \cdots}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)} - \frac{1367d^2-586de-703e^2+\left(423d^2- \cdots\right.}{140\left(5d^2-2de+3e^2\right)^2(3+ \cdots}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)} - \frac{1367d^2-586de-703e^2+\left(423d^2- \cdots\right.}{140\left(5d^2-2de+3e^2\right)^2(3+ \cdots}$$

Mathematica [A]

time = 0.17, size = 270, normalized size = 0.86

$$\frac{-\frac{1960\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{e(d+ex)} - \frac{14\left(5d^2-2de+3e^2\right)\left(e^2(-703+293x)+d^2(1367+423x)-2de(293+1367x)\right)}{3+2x+5x^2} + 5\sqrt{14}\left(1313d^4-10044d^3e+4290d^2e^2+156de^3-271e^4\right)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right) + 1960(41d^4-8d^3e-60d^2e^2+24de^3-5e^4)\log(d+ex) + 980(-41d^4+8d^3e+60d^2e^2-24de^3+5e^4)\log\left(3+2x+5x^2\right)}{1960\left(5d^2-2de+3e^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^2)
,x]
```

```
[Out] ((-1960*(5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^
4))/(e*(d + e*x)) - (14*(5*d^2 - 2*d*e + 3*e^2)*(e^2*(-703 + 293*x) + d^2*(
1367 + 423*x) - 2*d*e*(293 + 1367*x)))/(3 + 2*x + 5*x^2) + 5*Sqrt[14]*(1313
*d^4 - 10044*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4)*ArcTan[(1 + 5*x)/S
qrt[14]] + 1960*(41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*Log[d +
e*x] + 980*(-41*d^4 + 8*d^3*e + 60*d^2*e^2 - 24*d*e^3 + 5*e^4)*Log[3 + 2*x
+ 5*x^2])/(1960*(5*d^2 - 2*d*e + 3*e^2)^3)
```

Maple [A]

time = 0.15, size = 303, normalized size = 0.97

method	result
default	$-\frac{4d^4+5d^3e+3d^2e^2-d\,e^3+2e^4}{\left(5d^2-2de+3e^2\right)^2 e(ex+d)} + \frac{\left(41d^4-8d^3e-60d^2e^2+24d\,e^3-5e^4\right)\ln(ex+d)}{\left(5d^2-2de+3e^2\right)^3} - \frac{\left(\frac{423}{140}d^4-\frac{3629}{175}d^3e+\frac{4101}{350}d^2e^2-\frac{2197}{175}d\,e^3+\frac{879}{700}e^4\right) \cdots}{x^2+\frac{2}{5} \cdots}$

risch	$\frac{-\frac{\left(2800d^4+3500d^3e+2523d^2e^2-3434d\,e^3+1693e^4\right)x^2}{140e\left(25d^4-20d^3e+34d^2e^2-12d\,e^3+9e^4\right)}-\frac{\left(1120d^4+1823d^3e-527d^2e^2-573d\,e^3-143e^4\right)x}{140e\left(25d^4-20d^3e+34d^2e^2-12d\,e^3+9e^4\right)}-\frac{1680d^4+3467d^3e+674d^2e^2-1123d\,e^3+840}{140e\left(25d^4-20d^3e+34d^2e^2-12d\,e^3+9e^4\right)}}{(5x^2+2x+3)(ex+d)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x,method=_RETURNVERBO
SE)
```

```
[Out] -(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2)^2/e/(e*x+d)+(41*
d^4-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^3-1/(5
*d^2-2*d*e+3*e^2)^3*(((423/140*d^4-3629/175*d^3*e+4101/350*d^2*e^2-2197/175
*d*e^3+879/700*e^4)*x+1367/140*d^4-1416/175*d^3*e+879/350*d^2*e^2-88/175*d*
e^3-2109/700*e^4)/(x^2+2/5*x+3/5)+1/280*(5740*d^4-1120*d^3*e-8400*d^2*e^2+3
360*d*e^3-700*e^4)*ln(5*x^2+2*x+3)+1/392*(-1313*d^4+10044*d^3*e-4290*d^2*e^
2-156*d*e^3+271*e^4)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2)))
```

Maxima [A]
time = 0.58, size = 509, normalized size = 1.63

$$\frac{\sqrt{14}\,(1313\,d^4-10044\,d^3e+4290\,d^2e^2+156\,de^3-271\,e^4)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{392\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}-\frac{(41\,d^4-8\,d^3e-60\,d^2e^2+24\,de^3-5\,e^4)\log(5\,x^2+2\,x+3)}{2\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}+\frac{(41\,d^4-8\,d^3e-60\,d^2e^2+24\,de^3-5\,e^4)\log(xe+d)}{125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6}-\frac{1680\,d^4+3467\,d^3e+(2800\,d^4+3500\,d^3e+2523\,d^2e^2-3434\,de^3+1693\,e^4)x^2+674\,d^2e^2+(1120\,d^4+1823\,d^3e-527\,d^2e^2-573\,de^3-143\,e^4)x-1123\,de^3+840\,e^4}{140\,(75\,d^5e-60\,d^4e^2+5\,(25\,d^4e^2-20\,d^3e^3+34\,d^2e^4-12\,de^5+9\,e^6)x^3+102\,d^3e^3+(125\,d^5e-50\,d^4e^2+130\,d^3e^3+8\,d^2e^4+21\,de^5+18\,e^6)x^2-36\,d^2e^4+(50\,d^5e+35\,d^4e^2+8\,d^3e^3+78\,d^2e^4-18\,de^5+27\,e^6)x+27\,de^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 1/392*sqrt(14)*(1313*d^4 - 10044*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4
)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*
d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(41*d^4 - 8*d^3*e - 60*d^2
*e^2 + 24*d*e^3 - 5*e^4)*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d^5*e + 285*d^
4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + (41*d^4 - 8*d^3*e
- 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*log(x*e + d)/(125*d^6 - 150*d^5*e + 285*d^
4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/140*(1680*d^4 +
3467*d^3*e + (2800*d^4 + 3500*d^3*e + 2523*d^2*e^2 - 3434*d*e^3 + 1693*e^4)
*x^2 + 674*d^2*e^2 + (1120*d^4 + 1823*d^3*e - 527*d^2*e^2 - 573*d*e^3 - 143
*e^4)*x - 1123*d*e^3 + 840*e^4)/(75*d^5*e - 60*d^4*e^2 + 5*(25*d^4*e^2 - 20
*d^3*e^3 + 34*d^2*e^4 - 12*d*e^5 + 9*e^6)*x^3 + 102*d^3*e^3 + (125*d^5*e -
50*d^4*e^2 + 130*d^3*e^3 + 8*d^2*e^4 + 21*d*e^5 + 18*e^6)*x^2 - 36*d^2*e^4
+ (50*d^5*e + 35*d^4*e^2 + 8*d^3*e^3 + 78*d^2*e^4 - 18*d*e^5 + 27*e^6)*x +
27*d*e^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(297) = 594$.
time = 0.47, size = 889, normalized size = 2.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] -1/1960*(196000*d^6*x^2 + 78400*d^6*x + 117600*d^6 + 5*sqrt(14)*(271*(5*x^3
 + 2*x^2 + 3*x)*e^6 - (780*d*x^3 - 1043*d*x^2 - 74*d*x - 813*d)*e^5 - 78*(2
75*d^2*x^3 + 120*d^2*x^2 + 169*d^2*x + 6*d^2)*e^4 + 6*(8370*d^3*x^3 - 227*d
^3*x^2 + 3592*d^3*x - 2145*d^3)*e^3 - (6565*d^4*x^3 - 47594*d^4*x^2 - 16149
*d^4*x - 30132*d^4)*e^2 - 1313*(5*d^5*x^2 + 2*d^5*x + 3*d^5)*e)*arctan(1/14
*sqrt(14)*(5*x + 1)) + 42*(1693*x^2 - 143*x + 840)*e^6 - 14*(13688*d*x^2 +
1433*d*x + 5049*d)*e^5 + 28*(11451*d^2*x^2 - 575*d^2*x + 4234*d^2)*e^4 - 28
*(5858*d^3*x^2 - 1829*d^3*x - 1719*d^3)*e^3 + 14*(14015*d^4*x^2 - 2921*d^4*
x + 1476*d^4)*e^2 + 350*(476*d^5*x^2 + 275*d^5*x + 559*d^5)*e - 980*(5*(5*x
^3 + 2*x^2 + 3*x)*e^6 - (120*d*x^3 + 23*d*x^2 + 62*d*x - 15*d)*e^5 + 12*(25
*d^2*x^3 + 11*d^2*x - 6*d^2)*e^4 + 4*(10*d^3*x^3 + 79*d^3*x^2 + 36*d^3*x +
45*d^3)*e^3 - (205*d^4*x^3 + 42*d^4*x^2 + 107*d^4*x - 24*d^4)*e^2 - 41*(5*d
^5*x^2 + 2*d^5*x + 3*d^5)*e)*log(5*x^2 + 2*x + 3) + 1960*(5*(5*x^3 + 2*x^2
+ 3*x)*e^6 - (120*d*x^3 + 23*d*x^2 + 62*d*x - 15*d)*e^5 + 12*(25*d^2*x^3 +
11*d^2*x - 6*d^2)*e^4 + 4*(10*d^3*x^3 + 79*d^3*x^2 + 36*d^3*x + 45*d^3)*e^3
 - (205*d^4*x^3 + 42*d^4*x^2 + 107*d^4*x - 24*d^4)*e^2 - 41*(5*d^5*x^2 + 2*
d^5*x + 3*d^5)*e)*log(x*e + d))/(27*(5*x^3 + 2*x^2 + 3*x)*e^8 - 27*(10*d*x^
3 - d*x^2 + 4*d*x - 3*d)*e^7 + 9*(95*d^2*x^3 + 8*d^2*x^2 + 45*d^2*x - 18*d^
2)*e^6 - (940*d^3*x^3 - 479*d^3*x^2 + 222*d^3*x - 513*d^3)*e^5 + (1425*d^4*
x^3 - 370*d^4*x^2 + 479*d^4*x - 564*d^4)*e^4 - 15*(50*d^5*x^3 - 75*d^5*x^2
- 8*d^5*x - 57*d^5)*e^3 + 25*(25*d^6*x^3 - 20*d^6*x^2 + 3*d^6*x - 18*d^6)*e
^2 + 125*(5*d^7*x^2 + 2*d^7*x + 3*d^7)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.77, size = 571, normalized size = 1.82

$$\frac{\sqrt{14}\left(1313\,d^4e^2-10044\,d^3e^3+4290\,d^2e^4+156\,de^5-271\,e^6\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\left(5\,d-\frac{5\,d^2}{xe+d}+\frac{2\,de}{xe+d}-\frac{3\,e^2}{xe+d}-e\right)e^{(-1)}\right)e^{(-2)}}{392\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6\right)}-\frac{\left(41\,d^4-8\,d^3e-60\,d^2e^2+24\,de^3-5\,e^4\right)\log\left(-\frac{10\,d}{xe+d}+\frac{5\,d^2}{(xe+d)^2}+\frac{2\,e}{xe+d}-\frac{2\,de}{(xe+d)^2}+\frac{3\,e^2}{(xe+d)^2}+5\right)}{2\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6\right)}-\frac{\frac{4\,d^4e^3}{xe+d}+\frac{5\,d^3e^4}{xe+d}+\frac{3\,d^2e^5}{xe+d}-\frac{de^6}{xe+d}+\frac{2\,e^7}{xe+d}}{25\,d^4e^4-20\,d^3e^5+34\,d^2e^6-12\,de^7+9\,e^8}+\frac{\frac{423\,d^3e-4101\,d^2e^2+879\,de^3+703\,e^4}{5\,d^2-2\,de+3\,e^2}-\frac{\left(423\,d^4e^2-5468\,d^3e^3+1758\,d^2e^4+2812\,de^5-457\,e^6\right)e^{(-1)}}{\left(5\,d^2-2\,de+3\,e^2\right)(xe+d)}}{28\left(5\,d^2-2\,de+3\,e^2\right)^2\left(\frac{10\,d}{xe+d}-\frac{5\,d^2}{(xe+d)^2}-\frac{2\,e}{xe+d}+\frac{2\,de}{(xe+d)^2}-\frac{3\,e^2}{(xe+d)^2}-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 1/392*sqrt(14)*(1313*d^4*e^2 - 10044*d^3*e^3 + 4290*d^2*e^4 + 156*d*e^5 - 2
71*e^6)*arctan(1/14*sqrt(14)*(5*d - 5*d^2/(x*e + d) + 2*d*e/(x*e + d) - 3*e
^2/(x*e + d) - e)*e^(-1))*e^(-2)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d
^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(41*d^4 - 8*d^3*e - 60*d^2*
e^2 + 24*d*e^3 - 5*e^4)*log(-10*d/(x*e + d) + 5*d^2/(x*e + d)^2 + 2*e/(x*e
+ d) - 2*d*e/(x*e + d)^2 + 3*e^2/(x*e + d)^2 + 5)/(125*d^6 - 150*d^5*e + 28
5*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - (4*d^4*e^3/(x*
e + d) + 5*d^3*e^4/(x*e + d) + 3*d^2*e^5/(x*e + d) - d*e^6/(x*e + d) + 2*e^
7/(x*e + d))/(25*d^4*e^4 - 20*d^3*e^5 + 34*d^2*e^6 - 12*d*e^7 + 9*e^8) + 1/
28*((423*d^3*e - 4101*d^2*e^2 + 879*d*e^3 + 703*e^4)/(5*d^2 - 2*d*e + 3*e^2
) - (423*d^4*e^2 - 5468*d^3*e^3 + 1758*d^2*e^4 + 2812*d*e^5 - 457*e^6)*e^(-
1)/((5*d^2 - 2*d*e + 3*e^2)*(x*e + d)))/((5*d^2 - 2*d*e + 3*e^2)^2*(10*d/(x
*e + d) - 5*d^2/(x*e + d)^2 - 2*e/(x*e + d) + 2*d*e/(x*e + d)^2 - 3*e^2/(x*
e + d)^2 - 5))
```

Mupad [B]
time = 4.84, size = 601, normalized size = 1.92

$$\ln(d+e\,x)\left(\frac{41}{25\,(5\,d^2-2\,d\,e+3\,e^2)}-\frac{4\,e^3\,(423\,d-1367\,e)}{125\,(5\,d^2-2\,d\,e+3\,e^2)^3}+\frac{2\,e\,(310\,d-1323\,e)}{125\,(5\,d^2-2\,d\,e+3\,e^2)^2}\right)-\frac{\frac{1680\,d^4+3467\,d^3\,e+674\,d^2\,e^2-1123\,d\,e^3+840\,e^4}{140\,e\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}-\frac{x\,(-1120\,d^4-1823\,d^3\,e+527\,d^2\,e^2+573\,d\,e^3+143\,e^4)}{140\,e\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}+\frac{x^2\,(2800\,d^4+3500\,d^3\,e+2523\,d^2\,e^2-3434\,d\,e^3+1693\,e^4)}{140\,e\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}}{5\,e\,x^3+(5\,d+2\,e)\,x^2+(2\,d+3\,e)\,x+3\,d}+\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(\left(\frac{1313\,\sqrt{14}}{784}-\frac{41}{2}\mathrm{i}\right)d^4+\left(-\frac{2511\,\sqrt{14}}{196}+4\mathrm{i}\right)d^3\,e+\left(\frac{2145\,\sqrt{14}}{392}+30\mathrm{i}\right)d^2\,e^2+\left(\frac{39\,\sqrt{14}}{196}-12\mathrm{i}\right)d\,e^3+\left(-\frac{271\,\sqrt{14}}{784}+\frac{5}{2}\mathrm{i}\right)e^4\right)}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5\,54\mathrm{i}+e^6\,27\mathrm{i}}-\frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(\left(\frac{1313\,\sqrt{14}}{784}+\frac{41}{2}\mathrm{i}\right)d^4+\left(-\frac{2511\,\sqrt{14}}{196}-4\mathrm{i}\right)d^3\,e+\left(\frac{2145\,\sqrt{14}}{392}-30\mathrm{i}\right)d^2\,e^2+\left(\frac{39\,\sqrt{14}}{196}+12\mathrm{i}\right)d\,e^3+\left(-\frac{271\,\sqrt{14}}{784}-\frac{5}{2}\mathrm{i}\right)e^4\right)}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5\,54\mathrm{i}+e^6\,27\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)^2),x)
```

```
[Out] log(d + e*x)*(41/(25*(5*d^2 - 2*d*e + 3*e^2)) - (4*e^3*(423*d - 1367*e))/(1
25*(5*d^2 - 2*d*e + 3*e^2)^3) + (2*e*(310*d - 1323*e))/(125*(5*d^2 - 2*d*e
+ 3*e^2)^2)) - ((3467*d^3*e - 1123*d*e^3 + 1680*d^4 + 840*e^4 + 674*d^2*e^2
)/(140*e*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) - (x*(573*d*e
^3 - 1823*d^3*e - 1120*d^4 + 143*e^4 + 527*d^2*e^2))/(140*e*(25*d^4 - 20*d^
3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x^2*(3500*d^3*e - 3434*d*e^3 + 280
0*d^4 + 1693*e^4 + 2523*d^2*e^2))/(140*e*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*
e^4 + 34*d^2*e^2)))/(3*d + x^2*(5*d + 2*e) + 5*e*x^3 + x*(2*d + 3*e)) + (lo
g(x - (14^(1/2)*1i)/5 + 1/5)*(d^4*((1313*14^(1/2))/784 - 41i/2) - e^4*((271
*14^(1/2))/784 - 5i/2) + d^2*e^2*((2145*14^(1/2))/392 + 30i) + d*e^3*((39*1
4^(1/2))/196 - 12i) - d^3*e*((2511*14^(1/2))/196 - 4i)))/(d^6*125i - d^5*e*
150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*188i + d^4*e^2*285i) -
(log(x + (14^(1/2)*1i)/5 + 1/5)*(d^4*((1313*14^(1/2))/784 + 41i/2) - e^4*((
271*14^(1/2))/784 + 5i/2) + d^2*e^2*((2145*14^(1/2))/392 - 30i) + d*e^3*((3
9*14^(1/2))/196 + 12i) - d^3*e*((2511*14^(1/2))/196 + 4i)))/(d^6*125i - d^5
*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*188i + d^4*e^2*285i)
```

3.317 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)^2}\,dx$

Optimal. Leaf size=412

$$-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2}-\frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{1367d^3-879d^2e-2109de^2+45}{28\left(5d^2-2de\right.}$$

```
[Out] 1/2*(-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)^
2+(-41*d^4+8*d^3*e+60*d^2*e^2-24*d*e^3+5*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)
+1/28*(-1367*d^3+879*d^2*e+2109*d*e^2-457*e^3-(423*d^3-4101*d^2*e+879*d*e^2
+703*e^3)*x)/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)+(205*d^5-19*d^4*e-846*d^3*
e^2+396*d^2*e^3+57*d*e^4-21*e^5)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^4-1/2*(205*d
^5-19*d^4*e-846*d^3*e^2+396*d^2*e^3+57*d*e^4-21*e^5)*ln(5*x^2+2*x+3)/(5*d^2
-2*d*e+3*e^2)^4+1/392*(6565*d^5-74017*d^4*e+35022*d^3*e^2+42858*d^2*e^3-172
47*d*e^4+579*e^5)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^4*14^(1
/2)
```

Rubi [A]
time = 0.55, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1660, 1642, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)\left(6565d^5-74017d^4e+35022d^3e^2+42858d^2e^3-17247de^4+579e^5\right)}{28\sqrt{14}\left(5d^2-2de+3e^2\right)^4}-\frac{1367d^3-879d^2e+x\left(423d^3-4101d^2e+879de^2+703e^3\right)-2109de^2+457e^3}{28\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^3}-\frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2}-\frac{\left(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5\right)\log\left(5x^2+2x+3\right)}{2\left(5d^2-2de+3e^2\right)^4}+\frac{\left(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5\right)\log(d+ex)}{\left(5d^2-2de+3e^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)^2),x]
```

```
[Out] -1/2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e*(5*d^2 - 2*d*e + 3*e^
2)^2*(d + e*x)^2) - (41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)/((5*
d^2 - 2*d*e + 3*e^2)^3*(d + e*x)) - (1367*d^3 - 879*d^2*e - 2109*d*e^2 + 45
7*e^3 + (423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*x)/(28*(5*d^2 - 2*d*e
+ 3*e^2)^3*(3 + 2*x + 5*x^2)) + ((6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 +
42858*d^2*e^3 - 17247*d*e^4 + 579*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]])/(28*Sqrt
[14]*(5*d^2 - 2*d*e + 3*e^2)^4) + ((205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*
d^2*e^3 + 57*d*e^4 - 21*e^5)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^4 - ((20
5*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*Log[3 + 2
*x + 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^4)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)^2}\,dx = -\frac{1367d^3-879d^2e-2109de^2+457e^3+(423d^3-4101d^2e+879de^2+7}{28\left(5d^2-2de+3e^2\right)^3\left(3+2x+5x^2\right)}$$

$$= -\frac{1367d^3-879d^2e-2109de^2+457e^3+(423d^3-4101d^2e+879de^2+7}{28\left(5d^2-2de+3e^2\right)^3\left(3+2x+5x^2\right)}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2} - \frac{41d^4-8d^3e-60d^2e^2+24de^3-5e}{\left(5d^2-2de+3e^2\right)^3(d+ex)}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2} - \frac{41d^4-8d^3e-60d^2e^2+24de^3-5e}{\left(5d^2-2de+3e^2\right)^3(d+ex)}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2} - \frac{41d^4-8d^3e-60d^2e^2+24de^3-5e}{\left(5d^2-2de+3e^2\right)^3(d+ex)}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2} - \frac{41d^4-8d^3e-60d^2e^2+24de^3-5e}{\left(5d^2-2de+3e^2\right)^3(d+ex)}$$

Mathematica [A]
time = 0.25, size = 363, normalized size = 0.88

$$\frac{-\frac{196\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{e(d+ex)^2}+\frac{392\left(5d^2-2de+3e^2\right)\left(-41d^4+8d^3e+60d^2e^2-24de^3+5e^4\right)}{d+ex}-\frac{14\left(5d^2-2de+3e^2\right)\left(3de^2(-703+293x)+d^3(1367+423x)+e^3(457+703x)-3d^2e(293+1367x)\right)}{3+2x+5x^2}+\sqrt{14}\left(6565d^5-74017d^4e+35022d^3e^2+42858d^2e^3-17247de^4+579e^5\right)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)+392\left(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5\right)\log(d+ex)+196\left(-205d^5+19d^4e+846d^3e^2-396d^2e^3-57de^4+21e^5\right)\log\left(3+2x+5x^2\right)}{392\left(5d^2-2de+3e^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)^2)
,x]
```

```
[Out] ((-196*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e
^4))/(e*(d + e*x)^2) + (392*(5*d^2 - 2*d*e + 3*e^2)*(-41*d^4 + 8*d^3*e + 60
*d^2*e^2 - 24*d*e^3 + 5*e^4))/(d + e*x) - (14*(5*d^2 - 2*d*e + 3*e^2)*(3*d*
e^2*(-703 + 293*x) + d^3*(1367 + 423*x) + e^3*(457 + 703*x) - 3*d^2*e*(293
+ 1367*x)))/(3 + 2*x + 5*x^2) + Sqrt[14]*(6565*d^5 - 74017*d^4*e + 35022*d^
3*e^2 + 42858*d^2*e^3 - 17247*d*e^4 + 579*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]] +
 392*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*L
og[d + e*x] + 196*(-205*d^5 + 19*d^4*e + 846*d^3*e^2 - 396*d^2*e^3 - 57*d*e
^4 + 21*e^5)*Log[3 + 2*x + 5*x^2])/(392*(5*d^2 - 2*d*e + 3*e^2)^4)
```

Maple [A]
time = 0.23, size = 400, normalized size = 0.97

method	result

default	$-\frac{4d^4+5d^3e+3d^2e^2-d\,e^3+2e^4}{2(5d^2-2de+3e^2)^2e(ex+d)^2}-\frac{41d^4-8d^3e-60d^2e^2+24d\,e^3-5e^4}{(5d^2-2de+3e^2)^3(ex+d)}+\frac{(205d^5-19d^4e-846d^3e^2+396d^2e^3+57d\,e^4-21e^5)\ln(ex+d)}{(5d^2-2de+3e^2)^4}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x,method=_RETURNVERBO
SE)
```

```
[Out] -1/2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2)^2/e/(e*x+d)^
2-(41*d^4-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)+
(205*d^5-19*d^4*e-846*d^3*e^2+396*d^2*e^3+57*d*e^4-21*e^5)*ln(e*x+d)/(5*d^2
-2*d*e+3*e^2)^4-1/(5*d^2-2*d*e+3*e^2)^4*(((423/28*d^5-21351/140*d^4*e+6933/
70*d^3*e^2-5273/70*d^2*e^3+1231/140*d*e^4+2109/140*e^5)*x+1367/28*d^5-7129/
140*d^4*e-2343/70*d^3*e^2+1933/70*d^2*e^3-7241/140*d*e^4+1371/140*e^5)/(x^2
+2/5*x+3/5)+1/280*(28700*d^5-2660*d^4*e-118440*d^3*e^2+55440*d^2*e^3+7980*d
*e^4-2940*e^5)*ln(5*x^2+2*x+3)+1/392*(-6565*d^5+74017*d^4*e-35022*d^3*e^2-4
2858*d^2*e^3+17247*d*e^4-579*e^5)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2)))
```

Maxima [A]
time = 0.55, size = 775, normalized size = 1.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 1/392*sqrt(14)*(6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42858*d^2*e^3 - 17
247*d*e^4 + 579*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(625*d^8 - 1000*d^7*e
+ 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 -
 216*d*e^7 + 81*e^8) - 1/2*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3
+ 57*d*e^4 - 21*e^5)*log(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6*
e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7
+ 81*e^8) + (205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21
*e^5)*log(x*e + d)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 20
86*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - 1/28*(840*d
^6 + 5525*d^5*e - 837*d^4*e^2 + (5740*d^4*e^2 - 697*d^3*e^3 - 12501*d^2*e^4
 + 4239*d*e^5 + 3*e^6)*x^3 - 6981*d^3*e^3 + (1400*d^6 + 6930*d^5*e + 3212*d
^4*e^2 - 15403*d^3*e^3 + 2349*d^2*e^4 - 549*d*e^5 + 597*e^6)*x^2 + 3355*d^2
*e^4 + (560*d^6 + 3195*d^5*e + 2105*d^4*e^2 - 4799*d^3*e^3 - 6623*d^2*e^4 +
 2454*d*e^5 - 252*e^6)*x - 714*d*e^5 + 252*e^6)/(375*d^8*e - 450*d^7*e^2 +
855*d^6*e^3 - 564*d^5*e^4 + 5*(125*d^6*e^3 - 150*d^5*e^4 + 285*d^4*e^5 - 18
```

```
8*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8 + 27*e^9)*x^4 + 513*d^4*e^5 + 2*(625*d^7
*e^2 - 625*d^6*e^3 + 1275*d^5*e^4 - 655*d^4*e^5 + 667*d^3*e^6 - 99*d^2*e^7
+ 81*d*e^8 + 27*e^9)*x^3 - 162*d^3*e^6 + (625*d^8*e - 250*d^7*e^2 + 1200*d^
6*e^3 - 250*d^5*e^4 + 958*d^4*e^5 - 150*d^3*e^6 + 432*d^2*e^7 - 54*d*e^8 +
81*e^9)*x^2 + 81*d^2*e^7 + 2*(125*d^8*e + 225*d^7*e^2 - 165*d^6*e^3 + 667*d
^5*e^4 - 393*d^4*e^5 + 459*d^3*e^6 - 135*d^2*e^7 + 81*d*e^8)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1459 vs. $2(388) = 776$.

time = 0.63, size = 1459, normalized size = 3.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] -1/392*(98000*d^8*x^2 + 39200*d^8*x + 58800*d^8 - sqrt(14)*(579*(5*x^4 + 2*
x^3 + 3*x^2)*e^8 - 3*(28745*d*x^4 + 9568*d*x^3 + 16475*d*x^2 - 1158*d*x)*e^
7 + 3*(71430*d^2*x^4 - 28918*d^2*x^3 + 20827*d^2*x^2 - 34108*d^2*x + 579*d^
2)*e^6 + 3*(58370*d^3*x^4 + 166208*d^3*x^3 + 63421*d^3*x^2 + 74218*d^3*x -
17247*d^3)*e^5 - (370085*d^4*x^4 - 202186*d^4*x^3 - 132327*d^4*x^2 - 295848
*d^4*x - 128574*d^4)*e^4 + (32825*d^5*x^4 - 727040*d^5*x^3 - 101263*d^5*x^2
 - 374058*d^5*x + 105066*d^5)*e^3 + (65650*d^6*x^3 - 343825*d^6*x^2 - 10864
4*d^6*x - 222051*d^6)*e^2 + 6565*(5*d^7*x^2 + 2*d^7*x + 3*d^7)*e)*arctan(1/
14*sqrt(14)*(5*x + 1)) + 126*(x^3 + 199*x^2 - 84*x + 84)*e^8 + 42*(4237*d*x
^3 - 947*d*x^2 + 2622*d*x - 882*d)*e^7 - 42*(15322*d^2*x^3 - 3710*d^2*x^2 +
 8679*d^2*x - 4251*d^2)*e^6 + 14*(44106*d^3*x^3 - 53652*d^3*x^2 + 11119*d^3
*x - 31223*d^3)*e^5 - 14*(43891*d^4*x^3 - 52187*d^4*x^2 + 17202*d^4*x - 282
26*d^4)*e^4 - 14*(14965*d^5*x^3 + 62649*d^5*x^2 + 18620*d^5*x + 16656*d^5)*
e^3 + 70*(5740*d^6*x^3 + 1280*d^6*x^2 + 1163*d^6*x - 2543*d^6)*e^2 + 70*(63
70*d^7*x^2 + 2971*d^7*x + 5189*d^7)*e - 196*(21*(5*x^4 + 2*x^3 + 3*x^2)*e^8
 - 3*(95*d*x^4 - 32*d*x^3 + 29*d*x^2 - 42*d*x)*e^7 - 3*(660*d^2*x^4 + 454*d
^2*x^3 + 437*d^2*x^2 + 100*d^2*x - 21*d^2)*e^6 + 3*(1410*d^3*x^4 - 756*d^3*
x^3 + 223*d^3*x^2 - 830*d^3*x - 57*d^3)*e^5 + (95*d^4*x^4 + 8498*d^4*x^3 +
1461*d^4*x^2 + 4284*d^4*x - 1188*d^4)*e^4 - (1025*d^5*x^4 + 220*d^5*x^3 - 3
691*d^5*x^2 - 1806*d^5*x - 2538*d^5)*e^3 - (2050*d^6*x^3 + 725*d^6*x^2 + 11
92*d^6*x - 57*d^6)*e^2 - 205*(5*d^7*x^2 + 2*d^7*x + 3*d^7)*e)*log(5*x^2 + 2
*x + 3) + 392*(21*(5*x^4 + 2*x^3 + 3*x^2)*e^8 - 3*(95*d*x^4 - 32*d*x^3 + 29
*d*x^2 - 42*d*x)*e^7 - 3*(660*d^2*x^4 + 454*d^2*x^3 + 437*d^2*x^2 + 100*d^2
*x - 21*d^2)*e^6 + 3*(1410*d^3*x^4 - 756*d^3*x^3 + 223*d^3*x^2 - 830*d^3*x
- 57*d^3)*e^5 + (95*d^4*x^4 + 8498*d^4*x^3 + 1461*d^4*x^2 + 4284*d^4*x - 11
88*d^4)*e^4 - (1025*d^5*x^4 + 220*d^5*x^3 - 3691*d^5*x^2 - 1806*d^5*x - 253
8*d^5)*e^3 - (2050*d^6*x^3 + 725*d^6*x^2 + 1192*d^6*x - 57*d^6)*e^2 - 205*(
5*d^7*x^2 + 2*d^7*x + 3*d^7)*e)*log(x*e + d))/(81*(5*x^4 + 2*x^3 + 3*x^2)*e
```

```
^11 - 54*(20*d*x^4 - 7*d*x^3 + 6*d*x^2 - 9*d*x)*e^10 + 27*(140*d^2*x^4 - 24
*d^2*x^3 + 67*d^2*x^2 - 42*d^2*x + 9*d^2)*e^9 - 24*(245*d^3*x^4 - 217*d^3*x
^3 + 66*d^3*x^2 - 171*d^3*x + 27*d^3)*e^8 + 14*(745*d^4*x^4 - 542*d^4*x^3 +
 381*d^4*x^2 - 396*d^4*x + 162*d^4)*e^7 - 28*(350*d^5*x^4 - 605*d^5*x^3 + 1
22*d^5*x^2 - 363*d^5*x + 126*d^5)*e^6 + 14*(750*d^6*x^4 - 1100*d^6*x^3 + 63
5*d^6*x^2 - 542*d^6*x + 447*d^6)*e^5 - 40*(125*d^7*x^4 - 475*d^7*x^3 + 110*
d^7*x^2 - 217*d^7*x + 147*d^7)*e^4 + 25*(125*d^8*x^4 - 350*d^8*x^3 + 335*d^
8*x^2 - 72*d^8*x + 252*d^8)*e^3 + 250*(25*d^9*x^3 - 10*d^9*x^2 + 7*d^9*x -
12*d^9)*e^2 + 625*(5*d^10*x^2 + 2*d^10*x + 3*d^10)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3/(5*x**2+2*x+3)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.98, size = 595, normalized size = 1.44

$$\frac{\sqrt{14}\left(6565\,d^5-74017\,d^4e+35022\,d^3e^2+42858\,d^2e^3-17247\,de^4+579\,e^5\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{392\,\left(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8\right)}-\frac{\left(205\,d^5-19\,d^4e-846\,d^3e^2+396\,d^2e^3+57\,de^4-21\,e^5\right)\log\left(5\,x^2+2\,x+3\right)}{2\,\left(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8\right)}+\frac{\left(205\,d^5e-19\,d^4e^2-846\,d^3e^3+396\,d^2e^4+57\,de^5-21\,e^6\right)\log\left(|xe+d|\right)}{625\,d^8e-1000\,d^7e^2+2100\,d^6e^3-1960\,d^5e^4+2086\,d^4e^5-1176\,d^3e^6+756\,d^2e^7-216\,de^8+81\,e^9}-\frac{4200\,d^8+25945\,d^7e-12715\,d^6e^2-16656\,d^5e^3+28226\,d^4e^4+\left(28700\,d^6e^2-14965\,d^5e^3-43891\,d^4e^4+44106\,d^3e^5-45966\,d^2e^6+12711\,de^7+9\,e^8\right)x^3-31223\,d^3e^5+\left(7000\,d^8+31850\,d^7e+6400\,d^6e^2-62649\,d^5e^3+52187\,d^4e^4-53652\,d^3e^5+11130\,d^2e^6-2841\,de^7+1791\,e^8\right)x^2+12753\,d^2e^6+\left(2800\,d^8+14855\,d^7e+5815\,d^6e^2-18620\,d^5e^3-17202\,d^4e^4+11119\,d^3e^5-26037\,d^2e^6+7866\,de^7-756\,e^8\right)x-2646\,de^7+756\,e^8}{28\,\left(5\,d^2-2\,de+3\,e^2\right)^4\left(5\,x^2+2\,x+3\right)(xe+d)^2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 1/392*sqrt(14)*(6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42858*d^2*e^3 - 17
247*d*e^4 + 579*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(625*d^8 - 1000*d^7*e
+ 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 -
 216*d*e^7 + 81*e^8) - 1/2*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3
+ 57*d*e^4 - 21*e^5)*log(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6*
e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7
+ 81*e^8) + (205*d^5*e - 19*d^4*e^2 - 846*d^3*e^3 + 396*d^2*e^4 + 57*d*e^5
- 21*e^6)*log(abs(x*e + d))/(625*d^8*e - 1000*d^7*e^2 + 2100*d^6*e^3 - 1960
*d^5*e^4 + 2086*d^4*e^5 - 1176*d^3*e^6 + 756*d^2*e^7 - 216*d*e^8 + 81*e^9)
- 1/28*(4200*d^8 + 25945*d^7*e - 12715*d^6*e^2 - 16656*d^5*e^3 + 28226*d^4*
e^4 + (28700*d^6*e^2 - 14965*d^5*e^3 - 43891*d^4*e^4 + 44106*d^3*e^5 - 4596
6*d^2*e^6 + 12711*d*e^7 + 9*e^8)*x^3 - 31223*d^3*e^5 + (7000*d^8 + 31850*d^
7*e + 6400*d^6*e^2 - 62649*d^5*e^3 + 52187*d^4*e^4 - 53652*d^3*e^5 + 11130*
d^2*e^6 - 2841*d*e^7 + 1791*e^8)*x^2 + 12753*d^2*e^6 + (2800*d^8 + 14855*d^
7*e + 5815*d^6*e^2 - 18620*d^5*e^3 - 17202*d^4*e^4 + 11119*d^3*e^5 - 26037*
d^2*e^6 + 7866*d*e^7 - 756*e^8)*x - 2646*d*e^7 + 756*e^8)*e^(-1)/((5*d^2 -
2*d*e + 3*e^2)^4*(5*x^2 + 2*x + 3)*(x*e + d)^2)
```

Mupad [B]

time = 4.94, size = 887, normalized size = 2.15

$$\ln(d+e\,x)\left(\frac{\frac{41\,d}{5}+\frac{29\,e}{5}}{{\left(5\,d^2-2\,d\,e+3\,e^2\right)}^2}+\frac{168\,e^4\,\left(458\,d-7\,e\right)}{125\,{\left(5\,d^2-2\,d\,e+3\,e^2\right)}^4}-\frac{2\,e^2\,\left(12610\,d+1329\,e\right)}{125\,{\left(5\,d^2-2\,d\,e+3\,e^2\right)}^3}\right)-\frac{\frac{840\,d^6+5525\,d^5\,e-837\,d^4\,e^2-6981\,d^3\,e^3+3355\,d^2\,e^4-714\,d\,e^5+252\,e^6}{28\,e\,\left(125\,d^6-150\,d^5\,e+285\,d^4\,e^2-188\,d^3\,e^3+171\,d^2\,e^4-54\,d\,e^5+27\,e^6\right)}+\frac{x^3\,\left(5740\,d^4\,e-697\,d^3\,e^2-12501\,d^2\,e^3+4239\,d\,e^4+3\,e^5\right)}{28\,\left(125\,d^6-150\,d^5\,e+285\,d^4\,e^2-188\,d^3\,e^3+171\,d^2\,e^4-54\,d\,e^5+27\,e^6\right)}+\frac{x^2\,\left(1400\,d^6+6930\,d^5\,e+3212\,d^4\,e^2-15403\,d^3\,e^3+2349\,d^2\,e^4-549\,d\,e^5+597\,e^6\right)}{28\,e\,\left(125\,d^6-150\,d^5\,e+285\,d^4\,e^2-188\,d^3\,e^3+171\,d^2\,e^4-54\,d\,e^5+27\,e^6\right)}+\frac{x\,\left(560\,d^6+3195\,d^5\,e+2105\,d^4\,e^2-4799\,d^3\,e^3-6623\,d^2\,e^4+2454\,d\,e^5-252\,e^6\right)}{28\,e\,\left(125\,d^6-150\,d^5\,e+285\,d^4\,e^2-188\,d^3\,e^3+171\,d^2\,e^4-54\,d\,e^5+27\,e^6\right)}}{x^2\,\left(5\,d^2+4\,d\,e+3\,e^2\right)+x\,\left(2\,d^2+6\,e\,d\right)+3\,d^2+x^3\,\left(2\,e^2+10\,d\,e\right)+5\,e^2\,x^4}+\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1{}\mathrm{i}}{5}\right)\,\left(\left(\frac{6565\,\sqrt{14}}{784}-\frac{205}{2}{}\mathrm{i}\right)\,d^5+\left(-\frac{74017\,\sqrt{14}}{784}+\frac{19}{2}{}\mathrm{i}\right)\,d^4\,e+\left(\frac{17511\,\sqrt{14}}{392}+423{}\mathrm{i}\right)\,d^3\,e^2+\left(\frac{21429\,\sqrt{14}}{392}-198{}\mathrm{i}\right)\,d^2\,e^3+\left(-\frac{17247\,\sqrt{14}}{784}-\frac{57}{2}{}\mathrm{i}\right)\,d\,e^4+\left(\frac{579\,\sqrt{14}}{784}+\frac{21}{2}{}\mathrm{i}\right)\,e^5\right)}{d^8\,625{}\mathrm{i}-d^7\,e\,1000{}\mathrm{i}+d^6\,e^2\,2100{}\mathrm{i}-d^5\,e^3\,1960{}\mathrm{i}+d^4\,e^4\,2086{}\mathrm{i}-d^3\,e^5\,1176{}\mathrm{i}+d^2\,e^6\,756{}\mathrm{i}-d\,e^7\,216{}\mathrm{i}+e^8\,81{}\mathrm{i}}-\frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1{}\mathrm{i}}{5}\right)\,\left(\left(\frac{6565\,\sqrt{14}}{784}+\frac{205}{2}{}\mathrm{i}\right)\,d^5+\left(-\frac{74017\,\sqrt{14}}{784}-\frac{19}{2}{}\mathrm{i}\right)\,d^4\,e+\left(\frac{17511\,\sqrt{14}}{392}-423{}\mathrm{i}\right)\,d^3\,e^2+\left(\frac{21429\,\sqrt{14}}{392}+198{}\mathrm{i}\right)\,d^2\,e^3+\left(-\frac{17247\,\sqrt{14}}{784}+\frac{57}{2}{}\mathrm{i}\right)\,d\,e^4+\left(\frac{579\,\sqrt{14}}{784}-\frac{21}{2}{}\mathrm{i}\right)\,e^5\right)}{d^8\,625{}\mathrm{i}-d^7\,e\,1000{}\mathrm{i}+d^6\,e^2\,2100{}\mathrm{i}-d^5\,e^3\,1960{}\mathrm{i}+d^4\,e^4\,2086{}\mathrm{i}-d^3\,e^5\,1176{}\mathrm{i}+d^2\,e^6\,756{}\mathrm{i}-d\,e^7\,216{}\mathrm{i}+e^8\,81{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^3*(2*x + 5*x^2 + 3)^2),x)
```

```
[Out] log(d + e*x)*(((41*d)/5 + (29*e)/5)/(5*d^2 - 2*d*e + 3*e^2)^2 + (168*e^4*(4
58*d - 7*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^4) - (2*e^2*(12610*d + 1329*e))/(
125*(5*d^2 - 2*d*e + 3*e^2)^3)) - ((5525*d^5*e - 714*d*e^5 + 840*d^6 + 252*
e^6 + 3355*d^2*e^4 - 6981*d^3*e^3 - 837*d^4*e^2)/(28*e*(125*d^6 - 150*d^5*e
 - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x^3*(42
39*d*e^4 + 5740*d^4*e + 3*e^5 - 12501*d^2*e^3 - 697*d^3*e^2))/(28*(125*d^6
- 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2))
 + (x^2*(6930*d^5*e - 549*d*e^5 + 1400*d^6 + 597*e^6 + 2349*d^2*e^4 - 15403
*d^3*e^3 + 3212*d^4*e^2))/(28*e*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 +
171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x*(2454*d*e^5 + 3195*d^5*e + 5
60*d^6 - 252*e^6 - 6623*d^2*e^4 - 4799*d^3*e^3 + 2105*d^4*e^2))/(28*e*(125*
d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e
^2)))/(x^2*(4*d*e + 5*d^2 + 3*e^2) + x*(6*d*e + 2*d^2) + 3*d^2 + x^3*(10*d*
e + 2*e^2) + 5*e^2*x^4) + (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^5*((6565*14^(1
/2))/784 - 205i/2) + e^5*((579*14^(1/2))/784 + 21i/2) + d^3*e^2*((17511*14^
(1/2))/392 + 423i) + d^2*e^3*((21429*14^(1/2))/392 - 198i) - d*e^4*((17247*
14^(1/2))/784 + 57i/2) - d^4*e*((74017*14^(1/2))/784 - 19i/2)))/(d^8*625i -
 d^7*e*1000i - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*e^5*1176i + d^4*e^
4*2086i - d^5*e^3*1960i + d^6*e^2*2100i) - (log(x + (14^(1/2)*1i)/5 + 1/5)*
(d^5*((6565*14^(1/2))/784 + 205i/2) + e^5*((579*14^(1/2))/784 - 21i/2) + d^
3*e^2*((17511*14^(1/2))/392 - 423i) + d^2*e^3*((21429*14^(1/2))/392 + 198i)
 - d*e^4*((17247*14^(1/2))/784 - 57i/2) - d^4*e*((74017*14^(1/2))/784 + 19i
/2)))/(d^8*625i - d^7*e*1000i - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*e
^5*1176i + d^4*e^4*2086i - d^5*e^3*1960i + d^6*e^2*2100i)
```

3.318 $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$

Optimal. Leaf size=171

$$\frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000\,(3+2x+5x^2)^2}+\frac{(d+ex)^2(3(11449d-2105e)+(11015d+49177}{196000\,(3+2x+5x^2)}$$

```
[Out] 1/980000*(83065*d-126009*e)*e^2*x+2/125*e^3*x^2-1/7000*(1367+423*x)*(e*x+d)
^3/(5*x^2+2*x+3)^2+1/196000*(e*x+d)^2*(34347*d-6315*e+(11015*d+49177*e)*x)/
(5*x^2+2*x+3)+3/6250*e*(100*d^2-245*d*e+47*e^2)*ln(5*x^2+2*x+3)+3/68600000*
(353125*d^3-855175*d^2*e+74085*d*e^2+556349*e^3)*arctan(1/14*(1+5*x)*14^(1/
2))*14^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1658, 1642, 648, 632, 210, 642}

$$\frac{3\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)(353125d^3-855175d^2e+74085de^2+556349e^3)}{4900000\sqrt{14}}+\frac{3e(100d^2-245de+47e^2)\log(5x^2+2x+3)}{6250}+\frac{e^2x(83065d-126009e)}{980000}+\frac{(d+ex)^2(x(11015d+49177e)+3(11449d-2105e))}{196000\,(5x^2+2x+3)}-\frac{(423x+1367)(d+ex)^3}{7000\,(5x^2+2x+3)^2}+\frac{2e^3x^2}{125}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] ((83065*d - 126009*e)*e^2*x)/980000 + (2*e^3*x^2)/125 - ((1367 + 423*x)*(d
+ e*x)^3)/(7000*(3 + 2*x + 5*x^2)^2) + ((d + e*x)^2*(3*(11449*d - 2105*e) +
 (11015*d + 49177*e)*x))/(196000*(3 + 2*x + 5*x^2)) + (3*(353125*d^3 - 8551
75*d^2*e + 74085*d*e^2 + 556349*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(4900000*S
qrt[14]) + (3*e*(100*d^2 - 245*d*e + 47*e^2)*Log[3 + 2*x + 5*x^2])/6250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx = -\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2}+\frac{1}{112}\int \frac{(d+ex)^2\left(\frac{6}{125}(1089d+136\right.}{}$$

$$= -\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)^2(3(11449d-2105e)+(1}{196000\left(3+2x+5\right.}$$

$$= -\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)^2(3(11449d-2105e)+(1}{196000\left(3+2x+5\right.}$$

$$= \frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2}+$$

$$= \frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2}+$$

$$= \frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2}+$$

$$= \frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2}+$$

Mathematica [A]
time = 0.13, size = 209, normalized size = 1.22

$$\frac{548800(60d-49e)e^2x+5488000e^3x^2-\frac{392\left(e^3(54969-53189x)+125d^3(1367+423x)+75d^2e(-1269+5989x)-15de^2(17967+18323x)\right)}{\left(3+2x+5x^2\right)^2}+\frac{14\left(e^3(2639639-3109005x)+125d^3(34347+11015x)+75d^2e(-44399+181765x)-15de^2(809167+647195x)\right)}{3+2x+5x^2}+15\sqrt{14}\left(353125d^3-855175d^2e+74085de^2+556349e^3\right)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)+164640e\left(100d^2-245de+47e^2\right)\log\left(3+2x+5x^2\right)}{343000000}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3
,x]
```

```
[Out] (548800*(60*d - 49*e)*e^2*x + 5488000*e^3*x^2 - (392*(e^3*(54969 - 53189*x)
 + 125*d^3*(1367 + 423*x) + 75*d^2*e*(-1269 + 5989*x) - 15*d*e^2*(17967 + 1
8323*x)))/(3 + 2*x + 5*x^2)^2 + (14*(e^3*(2639639 - 3109005*x) + 125*d^3*(3
4347 + 11015*x) + 75*d^2*e*(-44399 + 181765*x) - 15*d*e^2*(809167 + 647195*
x)))/(3 + 2*x + 5*x^2) + 15*Sqrt[14]*(353125*d^3 - 855175*d^2*e + 74085*d*e
^2 + 556349*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + 164640*e*(100*d^2 - 245*d*e +
 47*e^2)*Log[3 + 2*x + 5*x^2])/343000000
```

Maple [A]
time = 0.14, size = 209, normalized size = 1.22

method	result

default	$\frac{2e^3x^2}{125}+\frac{12d\,e^2x}{125}-\frac{49e^3x}{625}+\frac{\left(\frac{11015}{1568}d^3+\frac{109059}{1568}d^2e-\frac{388317}{7840}d\,e^2-\frac{621801}{39200}e^3\right)x^3+\left(\frac{38753}{1568}d^3+\frac{84921}{7840}d^2e-\frac{640827}{7840}d\,e^2+\frac{1396037}{196000}e^3\right)x^2\cdots}{25(5x^2+2x\cdots}$
risch	$\frac{12d\,e^2x}{125}+\frac{\frac{\left(\frac{11015}{1568}d^3+\frac{109059}{1568}d^2e-\frac{388317}{7840}d\,e^2-\frac{621801}{39200}e^3\right)x^3}{25}+\frac{\left(\frac{38753}{1568}d^3+\frac{84921}{7840}d^2e-\frac{640827}{7840}d\,e^2+\frac{1396037}{196000}e^3\right)x^2}{25}+\frac{\left(\frac{17979}{1568}d^3+\frac{173283}{7840}d^2e-\cdots\right.}{\cdots}}{(5x^2+2x+3)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x,method=_RETURNVERBO
SE)
```

```
[Out] 2/125*e^3*x^2+12/125*d*e^2*x-49/625*e^3*x+1/25*((11015/1568*d^3+109059/1568
*d^2*e-388317/7840*d*e^2-621801/39200*e^3)*x^3+(38753/1568*d^3+84921/7840*d
^2*e-640827/7840*d*e^2+1396037/196000*e^3)*x^2+(17979/1568*d^3+173283/7840*
d^2*e-73125/1568*d*e^2-511689/196000*e^3)*x+12953/1568*d^3-58599/7840*d^2*e
-230931/7840*d*e^2+1275957/196000*e^3)/(5*x^2+2*x+3)^2+3/9800000*(156800*d^
2*e-384160*d*e^2+73696*e^3)*ln(5*x^2+2*x+3)+3/13720000*(70625*d^3-171035*d^
2*e+14817*d*e^2+556349/5*e^3)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

Maxima [A]
time = 0.61, size = 213, normalized size = 1.25

$$\frac{2}{125}x^2e^3+\frac{3}{68600000}\sqrt{14}\left(353125\,d^3-855175\,d^2e+74085\,de^2+556349\,e^3\right)\arctan\left(\frac{1}{14}\sqrt{14}\left(5x+1\right)\right)+\frac{1}{625}\left(60\,de^2-49\,e^3\right)x+\frac{3}{6250}\left(100\,d^2e-245\,de^2+47\,e^3\right)\log\left(5x^2+2x+3\right)+\frac{5\left(275375\,d^3+2726475\,d^2e-1941585\,de^2-621801\,e^3\right)x^3+1619125\,d^3+\left(4844125\,d^3+2123025\,d^2e-16020675\,de^2+1396037\,e^3\right)x^2-1464975\,d^2e+3\left(749125\,d^3+1444025\,d^2e-3046875\,de^2-170563\,e^3\right)x-5773275\,de^2+1275957\,e^3}{4900000\left(25x^4+20x^3+34x^2+12x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="m
axima")
```

```
[Out] 2/125*x^2*e^3 + 3/68600000*sqrt(14)*(353125*d^3 - 855175*d^2*e + 74085*d*e^
2 + 556349*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(60*d*e^2 - 49*e^3)
*x + 3/6250*(100*d^2*e - 245*d*e^2 + 47*e^3)*log(5*x^2 + 2*x + 3) + 1/49000
00*(5*(275375*d^3 + 2726475*d^2*e - 1941585*d*e^2 - 621801*e^3)*x^3 + 16191
25*d^3 + (4844125*d^3 + 2123025*d^2*e - 16020675*d*e^2 + 1396037*e^3)*x^2 -
 1464975*d^2*e + 3*(749125*d^3 + 1444025*d^2*e - 3046875*d*e^2 - 170563*e^3
)*x - 5773275*d*e^2 + 1275957*e^3)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(156) = 312$.
time = 0.37, size = 408, normalized size = 2.39

$$\frac{19276250\,d^3x^3+67817750\,d^3x^2+31463250\,d^3x+22667750\,d^3+3\sqrt{14}\left(8828125\,d^3x^4+7062500\,d^3x^3+12006250\,d^3x^2+4237500\,d^3x+3178125\,d^3+556349\left(25x^4+20x^3+34x^2+12x+9\right)e^3+74085\left(25\,dx^4+20\,dx^3+34\,dx^2+12\,dx+9\,d\right)e^2-855175\left(25\,d^2x^4+20\,d^2x^3+34\,d^2x^2+12\,d^2x+9\,d^2\right)e\right)\arctan\left(\frac{1}{14}\sqrt{14}\left(5x+1\right)\right)+14\left(1960000\,x^6-8036000\,x^5-5017600\,x^4-15229645\,x^3-2508283\,x^2-3969129\,x+1275957\right)e^3+1050\left(156800\,dx^5+125440\,dx^4+83809\,dx^3-138345\,dx^2-65427\,dx-76977\,d\right)e^2+1050\left(181765\,d^2x^3+28307\,d^2x^2+57761\,d^2x-19533\,d^2\right)e+32928\left(47\left(25x^4+20x^3+34x^2+12x+9\right)e^3-245\left(25\,dx^4+20\,dx^3+34\,dx^2+12\,dx+9\,d\right)e^2+100\left(25\,d^2x^4+20\,d^2x^3+34\,d^2x^2+12\,d^2x+9\,d^2\right)e\right)\log\left(5x^2+2x+3\right)}{68600000\left(25x^4+20x^3+34x^2+12x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="f
ricas")
```

```
[Out] 1/68600000*(19276250*d^3*x^3 + 67817750*d^3*x^2 + 31463250*d^3*x + 22667750
*d^3 + 3*sqrt(14)*(8828125*d^3*x^4 + 7062500*d^3*x^3 + 12006250*d^3*x^2 + 4
237500*d^3*x + 3178125*d^3 + 556349*(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)*e
^3 + 74085*(25*d*x^4 + 20*d*x^3 + 34*d*x^2 + 12*d*x + 9*d)*e^2 - 855175*(25
*d^2*x^4 + 20*d^2*x^3 + 34*d^2*x^2 + 12*d^2*x + 9*d^2)*e)*arctan(1/14*sqrt(
14)*(5*x + 1)) + 14*(1960000*x^6 - 8036000*x^5 - 5017600*x^4 - 15229645*x^3
 - 2508283*x^2 - 3969129*x + 1275957)*e^3 + 1050*(156800*d*x^5 + 125440*d*x
^4 + 83809*d*x^3 - 138345*d*x^2 - 65427*d*x - 76977*d)*e^2 + 1050*(181765*d
^2*x^3 + 28307*d^2*x^2 + 57761*d^2*x - 19533*d^2)*e + 32928*(47*(25*x^4 + 2
0*x^3 + 34*x^2 + 12*x + 9)*e^3 - 245*(25*d*x^4 + 20*d*x^3 + 34*d*x^2 + 12*d
*x + 9*d)*e^2 + 100*(25*d^2*x^4 + 20*d^2*x^3 + 34*d^2*x^2 + 12*d^2*x + 9*d^
2)*e)*log(5*x^2 + 2*x + 3))/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.21, size = 469, normalized size = 2.74

$$\frac{2e^3x^2}{125} + x\left(\frac{12de^2}{125} - \frac{49e^3}{625}\right) + \left(\frac{3e(100d^2 - 245de + 47e^2)}{6250} - \frac{3\sqrt{14}\,i(353125d^3 - 855175d^2e + 74085de^2 + 556349e^3)}{137200000}\right)\log\left(x + \frac{211875d^3 - 1830225d^2e + 3271395de^2 - 285237e^3 + \frac{65856e(100d^2-245de+47e^2)}{5} - \frac{3\sqrt{14}\,i(353125d^3-855175d^2e+74085de^2+556349e^3)}{5}}{1059375d^3 - 2565525d^2e + 222255de^2 + 1669047e^3}\right) + \left(\frac{3e(100d^2 - 245de + 47e^2)}{6250} + \frac{3\sqrt{14}\,i(353125d^3 - 855175d^2e + 74085de^2 + 556349e^3)}{137200000}\right)\log\left(x + \frac{211875d^3 - 1830225d^2e + 3271395de^2 - 285237e^3 + \frac{65856e(100d^2-245de+47e^2)}{5} + \frac{3\sqrt{14}\,i(353125d^3-855175d^2e+74085de^2+556349e^3)}{5}}{1059375d^3 - 2565525d^2e + 222255de^2 + 1669047e^3}\right) + \frac{1619125d^3 - 1464975d^2e - 5773275de^2 + 1275957e^3 + x^3\cdot(1376875d^3 + 13632375d^2e - 9707925de^2 - 3109005e^3) + x^2\cdot(4844125d^3 + 2123025d^2e - 16020675de^2 + 1396037e^3) + x(2247375d^3 + 4332075d^2e - 9140625de^2 - 511689e^3)}{122500000x^4 + 98000000x^3 + 166600000x^2 + 58800000x + 44100000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] 2*e**3*x**2/125 + x*(12*d*e**2/125 - 49*e**3/625) + (3*e*(100*d**2 - 245*d*
e + 47*e**2)/6250 - 3*sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**
2 + 556349*e**3)/137200000)*log(x + (211875*d**3 - 1830225*d**2*e + 3271395
*d*e**2 - 285237*e**3 + 65856*e*(100*d**2 - 245*d*e + 47*e**2)/5 - 3*sqrt(1
4)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**2 + 556349*e**3)/5)/(1059375
*d**3 - 2565525*d**2*e + 222255*d*e**2 + 1669047*e**3)) + (3*e*(100*d**2 -
245*d*e + 47*e**2)/6250 + 3*sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085
*d*e**2 + 556349*e**3)/137200000)*log(x + (211875*d**3 - 1830225*d**2*e + 3
271395*d*e**2 - 285237*e**3 + 65856*e*(100*d**2 - 245*d*e + 47*e**2)/5 + 3*
sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**2 + 556349*e**3)/5)/(1
059375*d**3 - 2565525*d**2*e + 222255*d*e**2 + 1669047*e**3)) + (1619125*d*
*3 - 1464975*d**2*e - 5773275*d*e**2 + 1275957*e**3 + x**3*(1376875*d**3 +
13632375*d**2*e - 9707925*d*e**2 - 3109005*e**3) + x**2*(4844125*d**3 + 212
3025*d**2*e - 16020675*d*e**2 + 1396037*e**3) + x*(2247375*d**3 + 4332075*d
**2*e - 9140625*d*e**2 - 511689*e**3))/(122500000*x**4 + 98000000*x**3 + 16
6600000*x**2 + 58800000*x + 44100000)
```

Giac [A]
time = 3.11, size = 201, normalized size = 1.18

$$\frac{2}{125}\,x^2e^3 + \frac{12}{125}\,dxe^2 + \frac{3}{68600000}\,\sqrt{14}\,(353125\,d^3 - 855175\,d^2e + 74085\,de^2 + 556349\,e^3)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x + 1)\right) - \frac{49}{625}\,xe^3 + \frac{3}{6250}\,(100\,d^2e - 245\,de^2 + 47\,e^3)\log\left(5\,x^2 + 2\,x + 3\right) + \frac{5\,(275375\,d^3 + 2726475\,d^2e - 1941585\,de^2 - 621801\,e^3)x^3 + 1619125\,d^3 + (4844125\,d^3 + 2123025\,d^2e - 16020675\,de^2 + 1396037\,e^3)x^2 - 1464975\,d^2e + 3\,(749125\,d^3 + 1444025\,d^2e - 3046875\,de^2 - 170563\,e^3)x - 5773275\,de^2 + 1275957\,e^3}{4900000\,(5\,x^2 + 2\,x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="g
iac")
```

```
[Out] 2/125*x^2*e^3 + 12/125*d*x*e^2 + 3/68600000*sqrt(14)*(353125*d^3 - 855175*d
^2*e + 74085*d*e^2 + 556349*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) - 49/625*x
*e^3 + 3/6250*(100*d^2*e - 245*d*e^2 + 47*e^3)*log(5*x^2 + 2*x + 3) + 1/490
0000*(5*(275375*d^3 + 2726475*d^2*e - 1941585*d*e^2 - 621801*e^3)*x^3 + 161
9125*d^3 + (4844125*d^3 + 2123025*d^2*e - 16020675*d*e^2 + 1396037*e^3)*x^2
 - 1464975*d^2*e + 3*(749125*d^3 + 1444025*d^2*e - 3046875*d*e^2 - 170563*e
^3)*x - 5773275*d*e^2 + 1275957*e^3)/(5*x^2 + 2*x + 3)^2
```

Mupad [B]
time = 0.15, size = 299, normalized size = 1.75

$$x\left(\frac{e^2\,(12\,d-5\,e)}{125}-\frac{24\,e^3}{625}\right)-\frac{\frac{1154655\,d\,e^2}{1568}+\frac{292995\,d^2\,e}{1568}+x\left(-\frac{449475\,d^3}{1568}-\frac{866415\,d^2\,e}{1568}+\frac{1828125\,d\,e^2}{1568}+\frac{511689\,e^3}{7840}\right)-\frac{323825\,d^3}{1568}-\frac{1275957\,e^3}{7840}+x^3\left(-\frac{275375\,d^3}{1568}-\frac{2726475\,d^2\,e}{1568}+\frac{1941585\,d\,e^2}{1568}+\frac{621801\,e^3}{1568}\right)-x^2\left(\frac{968825\,d^3}{1568}+\frac{424605\,d^2\,e}{1568}-\frac{3204135\,d\,e^2}{1568}+\frac{1396037\,e^3}{7840}\right)}{15625\,x^4+12500\,x^3+21250\,x^2+7500\,x+5625}+\ln\left(5\,x^2+2\,x+3\right)\left(\frac{6\,d^2\,e}{125}-\frac{147\,d\,e^2}{1250}+\frac{141\,e^3}{6250}\right)+\frac{2\,e^3\,x^2}{125}+\frac{3\,\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{3\,\sqrt{14}\,\left(353125\,d^3-855175\,d^2\,e+74085\,d\,e^2+556349\,e^3\right)}{68600000}+\frac{3\,\sqrt{14}\,x\,\left(353125\,d^3-855175\,d^2\,e+74085\,d\,e^2+556349\,e^3\right)}{13720000}}{\frac{339\,d^3}{1568}-\frac{102621\,d^2\,e}{196000}+\frac{44451\,d\,e^2}{980000}+\frac{1669047\,e^3}{4900000}}\right)\left(353125\,d^3-855175\,d^2\,e+74085\,d\,e^2+556349\,e^3\right)}{68600000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)
```

```
[Out] x*((e^2*(12*d - 5*e))/125 - (24*e^3)/625) - ((1154655*d*e^2)/1568 + (292995
*d^2*e)/1568 + x*((1828125*d*e^2)/1568 - (866415*d^2*e)/1568 - (449475*d^3)
/1568 + (511689*e^3)/7840) - (323825*d^3)/1568 - (1275957*e^3)/7840 + x^3*(
(1941585*d*e^2)/1568 - (2726475*d^2*e)/1568 - (275375*d^3)/1568 + (621801*e
^3)/1568) - x^2*((424605*d^2*e)/1568 - (3204135*d*e^2)/1568 + (968825*d^3)/
1568 + (1396037*e^3)/7840))/(7500*x + 21250*x^2 + 12500*x^3 + 15625*x^4 + 5
625) + log(2*x + 5*x^2 + 3)*((6*d^2*e)/125 - (147*d*e^2)/1250 + (141*e^3)/6
250) + (2*e^3*x^2)/125 + (3*14^(1/2)*atan(((3*14^(1/2)*(74085*d*e^2 - 85517
5*d^2*e + 353125*d^3 + 556349*e^3))/68600000 + (3*14^(1/2)*x*(74085*d*e^2 -
 855175*d^2*e + 353125*d^3 + 556349*e^3))/13720000)/((44451*d*e^2)/980000 -
 (102621*d^2*e)/196000 + (339*d^3)/1568 + (1669047*e^3)/4900000))*(74085*d*
e^2 - 855175*d^2*e + 353125*d^3 + 556349*e^3))/68600000
```

3.319 $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$

Optimal. Leaf size=134

$$\frac{4e^2x}{125}-\frac{(1367+423x)(d+ex)^2}{7000\,(3+2x+5x^2)^2}+\frac{(d+ex)(34347d-6413e+5(2203d+8553e)x)}{196000\,(3+2x+5x^2)}+\frac{(211875d^2-342070de+\dots}{98000\dots}$$

```
[Out] 4/125*e^2*x-1/7000*(1367+423*x)*(e*x+d)^2/(5*x^2+2*x+3)^2+1/196000*(e*x+d)*
(34347*d-6413*e+5*(2203*d+8553*e)*x)/(5*x^2+2*x+3)+1/1250*(40*d-49*e)*e*ln(
5*x^2+2*x+3)+1/13720000*(211875*d^2-342070*d*e+14817*e^2)*arctan(1/14*(1+5*
x)*14^(1/2))*14^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1658, 1671, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)(211875d^2-342070de+14817e^2)}{980000\sqrt{14}}+\frac{(d+ex)(5x(2203d+8553e)+34347d-6413e)}{196000\,(5x^2+2x+3)}-\frac{(423x+1367)(d+ex)^2}{7000\,(5x^2+2x+3)^2}+\frac{e(40d-49e)\log(5x^2+2x+3)}{1250}+\frac{4e^2x}{125}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] (4*e^2*x)/125 - ((1367 + 423*x)*(d + e*x)^2)/(7000*(3 + 2*x + 5*x^2)^2) + (
(d + e*x)*(34347*d - 6413*e + 5*(2203*d + 8553*e)*x))/(196000*(3 + 2*x + 5*
x^2)) + ((211875*d^2 - 342070*d*e + 14817*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]])/
(980000*Sqrt[14]) + ((40*d - 49*e)*e*Log[3 + 2*x + 5*x^2])/1250
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx = -\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{1}{112}\int \frac{(d+ex)\left(\frac{2}{125}(3267d+273\cdots\right.}{\cdots}$$

$$= -\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)(34347d-6413e+5(220\cdots}{196000\left(3+2x+5x^2\cdots\right.}$$

$$= -\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)(34347d-6413e+5(220\cdots}{196000\left(3+2x+5x^2\cdots\right.}$$

$$= \frac{4e^2x}{125}-\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)(34347d-6413e+\cdots}{196000\left(3+2x\cdots\right.}$$

$$= \frac{4e^2x}{125}-\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)(34347d-6413e+\cdots}{196000\left(3+2x\cdots\right.}$$

$$= \frac{4e^2x}{125}-\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)(34347d-6413e+\cdots}{196000\left(3+2x\cdots\right.}$$

$$= \frac{4e^2x}{125}-\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2}+\frac{(d+ex)(34347d-6413e+\cdots}{196000\left(3+2x\cdots\right.}$$

Mathematica [A]
time = 0.12, size = 146, normalized size = 1.09

$$\frac{5\sqrt{14}\left(211875d^2-342070de+14817e^2\right)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)+70\left(\frac{5\left(5d^2\left(12953+17979x+38753x^2+11015x^3\right)+2de\left(-19533+57761x+28307x^2+181765x^3\right)+e^2\left(-76977-65427x-138345x^2+83809x^3+125440x^4+156800x^5\right)\right)}{\left(3+2x+5x^2\right)^2}+784(40d-49e)e\log\left(3+2x+5x^2\right)\right)}{68600000}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3
,x]
```

```
[Out] (5*Sqrt[14]*(211875*d^2 - 342070*d*e + 14817*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]
] + 70*((5*(5*d^2*(12953 + 17979*x + 38753*x^2 + 11015*x^3) + 2*d*e*(-19533
 + 57761*x + 28307*x^2 + 181765*x^3) + e^2*(-76977 - 65427*x - 138345*x^2 +
 83809*x^3 + 125440*x^4 + 156800*x^5)))/(3 + 2*x + 5*x^2)^2 + 784*(40*d - 4
9*e)*e*Log[3 + 2*x + 5*x^2]))/68600000
```

Maple [A]
time = 0.14, size = 146, normalized size = 1.09

method	result

default	$\frac{4e^2x}{125}+\frac{\left(\frac{2203}{1568}d^2+\frac{36353}{3920}de-\frac{129439}{39200}e^2\right)x^3+\left(\frac{38753}{7840}d^2+\frac{28307}{19600}de-\frac{213609}{39200}e^2\right)x^2+\left(\frac{17979}{7840}d^2+\frac{57761}{19600}de-\frac{4875}{1568}e^2\right)x+\frac{12953d^2}{7840}-\frac{19533de}{19600}-7}{5(5x^2+2x+3)^2}$
risch	$\frac{4e^2x}{125}+\frac{\frac{\left(\frac{2203}{1568}d^2+\frac{36353}{3920}de-\frac{129439}{39200}e^2\right)x^3}{5}+\frac{\left(\frac{38753}{7840}d^2+\frac{28307}{19600}de-\frac{213609}{39200}e^2\right)x^2}{5}+\frac{\left(\frac{17979}{7840}d^2+\frac{57761}{19600}de-\frac{4875}{1568}e^2\right)x}{5}+\frac{12953d^2}{39200}-\frac{19533de}{98000}-\frac{76}{19}}{(5x^2+2x+3)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x,method=_RETURNVERBO
SE)
```

```
[Out] 4/125*e^2*x+1/5*((2203/1568*d^2+36353/3920*d*e-129439/39200*e^2)*x^3+(38753
/7840*d^2+28307/19600*d*e-213609/39200*e^2)*x^2+(17979/7840*d^2+57761/19600
*d*e-4875/1568*e^2)*x+12953/7840*d^2-19533/19600*d*e-76977/39200*e^2)/(5*x^
2+2*x+3)^2+1/1960000*(62720*d*e-76832*e^2)*ln(5*x^2+2*x+3)+1/2744000*(42375
*d^2-68414*d*e+14817/5*e^2)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

Maxima [A]
time = 0.58, size = 154, normalized size = 1.15

$$\frac{1}{13720000}\sqrt{14}\left(211875\,d^2-342070\,de+14817\,e^2\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)+\frac{4}{125}\,xe^2+\frac{1}{1250}\left(40\,de-49\,e^2\right)\log\left(5\,x^2+2\,x+3\right)+\frac{(55075\,d^2+363530\,de-129439\,e^2)x^3+(193765\,d^2+56614\,de-213609\,e^2)x^2+64765\,d^2+(89895\,d^2+115522\,de-121875\,e^2)x-39066\,de-76977\,e^2}{196000\,(25\,x^4+20\,x^3+34\,x^2+12\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="m
axima")
```

```
[Out] 1/13720000*sqrt(14)*(211875*d^2 - 342070*d*e + 14817*e^2)*arctan(1/14*sqrt(
14)*(5*x + 1)) + 4/125*x*e^2 + 1/1250*(40*d*e - 49*e^2)*log(5*x^2 + 2*x + 3
) + 1/196000*((55075*d^2 + 363530*d*e - 129439*e^2)*x^3 + (193765*d^2 + 566
14*d*e - 213609*e^2)*x^2 + 64765*d^2 + (89895*d^2 + 115522*d*e - 121875*e^2
)*x - 39066*d*e - 76977*e^2)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(126) = 252.
time = 0.37, size = 278, normalized size = 2.07

$$\frac{3855250\,d^2x^3+13563550\,d^2x^2+6292650\,d^2x+\sqrt{14}\left(5296875\,d^2x^4+4237500\,d^2x^3+7203750\,d^2x^2+2542500\,d^2x+1906875\,d^2+14817\,(25\,x^4+20\,x^3+34\,x^2+12\,x+9)e^2-342070\,(25\,dx^4+20\,dx^3+34\,dx^2+12\,dx+9\,d)e\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)+4533550\,d^2+70\,(156800\,x^5+125440\,x^4+83809\,x^3-138345\,x^2-65427\,x-76977)e^2+140\,(181765\,dx^3+28307\,dx^2+57761\,dx-19533\,d)e-10976\,(49\,(25\,x^4+20\,x^3+34\,x^2+12\,x+9)e^2-40\,(25\,dx^4+20\,dx^3+34\,dx^2+12\,dx+9\,d)e)\log\left(5\,x^2+2\,x+3\right)}{13720000\,(25\,x^4+20\,x^3+34\,x^2+12\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="f
ricas")
```

```
[Out] 1/13720000*(3855250*d^2*x^3 + 13563550*d^2*x^2 + 6292650*d^2*x + sqrt(14)*(
5296875*d^2*x^4 + 4237500*d^2*x^3 + 7203750*d^2*x^2 + 2542500*d^2*x + 19068
```

```
75*d^2 + 14817*(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)*e^2 - 342070*(25*d*x^4
 + 20*d*x^3 + 34*d*x^2 + 12*d*x + 9*d)*e)*arctan(1/14*sqrt(14)*(5*x + 1)) +
 4533550*d^2 + 70*(156800*x^5 + 125440*x^4 + 83809*x^3 - 138345*x^2 - 65427
*x - 76977)*e^2 + 140*(181765*d*x^3 + 28307*d*x^2 + 57761*d*x - 19533*d)*e
- 10976*(49*(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)*e^2 - 40*(25*d*x^4 + 20*d
*x^3 + 34*d*x^2 + 12*d*x + 9*d)*e)*log(5*x^2 + 2*x + 3))/(25*x^4 + 20*x^3 +
 34*x^2 + 12*x + 9)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.99, size = 304, normalized size = 2.27

$$\frac{4e^2x}{125}+\left(\frac{e(40d-49e)}{1250}-\frac{\sqrt{14}\,i(211875d^2-342070de+14817e^2)}{27440000}\right)\log\left(x+\frac{42375d^2-244030de+218093e^2+\frac{21952e(40d-49e)}{5}-\frac{\sqrt{14}\,i(211875d^2-342070de+14817e^2)}{5}}{211875d^2-342070de+14817e^2}\right)+\left(\frac{e(40d-49e)}{1250}+\frac{\sqrt{14}\,i(211875d^2-342070de+14817e^2)}{27440000}\right)\log\left(x+\frac{42375d^2-244030de+218093e^2+\frac{21952e(40d-49e)}{5}+\frac{\sqrt{14}\,i(211875d^2-342070de+14817e^2)}{5}}{211875d^2-342070de+14817e^2}\right)+\frac{64765d^2-39066de-76977e^2+x^3\cdot(55075d^2+363530de-129439e^2)+x^2\cdot(193765d^2+56614de-213609e^2)+x(89895d^2+115522de-121875e^2)}{4900000x^4+3920000x^3+6664000x^2+2352000x+1764000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] 4*e**2*x/125 + (e*(40*d - 49*e)/1250 - sqrt(14)*I*(211875*d**2 - 342070*d*e
 + 14817*e**2)/27440000)*log(x + (42375*d**2 - 244030*d*e + 218093*e**2 + 2
1952*e*(40*d - 49*e)/5 - sqrt(14)*I*(211875*d**2 - 342070*d*e + 14817*e**2)
/5)/(211875*d**2 - 342070*d*e + 14817*e**2)) + (e*(40*d - 49*e)/1250 + sqrt
(14)*I*(211875*d**2 - 342070*d*e + 14817*e**2)/27440000)*log(x + (42375*d**
2 - 244030*d*e + 218093*e**2 + 21952*e*(40*d - 49*e)/5 + sqrt(14)*I*(211875
*d**2 - 342070*d*e + 14817*e**2)/5)/(211875*d**2 - 342070*d*e + 14817*e**2)
) + (64765*d**2 - 39066*d*e - 76977*e**2 + x**3*(55075*d**2 + 363530*d*e -
129439*e**2) + x**2*(193765*d**2 + 56614*d*e - 213609*e**2) + x*(89895*d**2
 + 115522*d*e - 121875*e**2))/(4900000*x**4 + 3920000*x**3 + 6664000*x**2 +
 2352000*x + 1764000)
```

Giac [A]
time = 3.74, size = 144, normalized size = 1.07

$$\frac{1}{13720000}\sqrt{14}\,(211875\,d^2-342070\,de+14817\,e^2)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)+\frac{4}{125}\,xe^2+\frac{1}{1250}\,(40\,de-49\,e^2)\log\left(5\,x^2+2\,x+3\right)+\frac{(55075\,d^2+363530\,de-129439\,e^2)x^3+(193765\,d^2+56614\,de-213609\,e^2)x^2+64765\,d^2+(89895\,d^2+115522\,de-121875\,e^2)x-39066\,de-76977\,e^2}{196000\,(5\,x^2+2\,x+3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="g
iac")
```

```
[Out] 1/13720000*sqrt(14)*(211875*d^2 - 342070*d*e + 14817*e^2)*arctan(1/14*sqrt(
14)*(5*x + 1)) + 4/125*x*e^2 + 1/1250*(40*d*e - 49*e^2)*log(5*x^2 + 2*x + 3
) + 1/196000*((55075*d^2 + 363530*d*e - 129439*e^2)*x^3 + (193765*d^2 + 566
14*d*e - 213609*e^2)*x^2 + 64765*d^2 + (89895*d^2 + 115522*d*e - 121875*e^2
)*x - 39066*d*e - 76977*e^2)/(5*x^2 + 2*x + 3)^2
```

Mupad [B]
time = 4.21, size = 203, normalized size = 1.51

$$\frac{x^3\left(\frac{55075\,d^2}{1568}+\frac{181765\,d\,e}{784}-\frac{129439\,e^2}{1568}\right)+x^2\left(\frac{193765\,d^2}{1568}+\frac{28307\,d\,e}{784}-\frac{213609\,e^2}{1568}\right)-\frac{19533\,d\,e}{784}+x\left(\frac{89895\,d^2}{1568}+\frac{57761\,d\,e}{784}-\frac{121875\,e^2}{1568}\right)+\frac{64765\,d^2}{1568}-\frac{76977\,e^2}{1568}}{3125\,x^4+2500\,x^3+4250\,x^2+1500\,x+1125}+\frac{4\,e^2\,x}{125}+\ln\left(5\,x^2+2\,x+3\right)\left(\frac{4\,d\,e}{125}-\frac{49\,e^2}{1250}\right)+\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,\left(211875\,d^2-342070\,d\,e+14817\,e^2\right)}{13720000}+\frac{\sqrt{14}\,x\,\left(211875\,d^2-342070\,d\,e+14817\,e^2\right)}{2744000}}{\frac{339\,d^2}{1568}-\frac{34207\,d\,e}{98000}+\frac{14817\,e^2}{980000}}\right)\,\left(211875\,d^2-342070\,d\,e+14817\,e^2\right)}{13720000}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)

[Out] (x^3*((181765*d*e)/784 + (55075*d^2)/1568 - (129439*e^2)/1568) + x^2*((2830
7*d*e)/784 + (193765*d^2)/1568 - (213609*e^2)/1568) - (19533*d*e)/784 + x*(
(57761*d*e)/784 + (89895*d^2)/1568 - (121875*e^2)/1568) + (64765*d^2)/1568
- (76977*e^2)/1568)/(1500*x + 4250*x^2 + 2500*x^3 + 3125*x^4 + 1125) + (4*e
^2*x)/125 + log(2*x + 5*x^2 + 3)*((4*d*e)/125 - (49*e^2)/1250) + (14^(1/2)*
atan(((14^(1/2)*(211875*d^2 - 342070*d*e + 14817*e^2))/13720000 + (14^(1/2)
x(211875*d^2 - 342070*d*e + 14817*e^2))/2744000)/((339*d^2)/1568 - (34207
*d*e)/98000 + (14817*e^2)/980000))*(211875*d^2 - 342070*d*e + 14817*e^2))/1
3720000

3.320 $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$

Optimal. Leaf size=103

$$-\frac{(1367+423x)(d+ex)}{7000\,(3+2x+5x^2)^2}+\frac{34347d-6511e+(11015d+36353e)x}{196000\,(3+2x+5x^2)}+\frac{(42375d-34207e)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{196000\sqrt{14}}+\frac{2}{125}$$

```
[Out] -1/7000*(1367+423*x)*(e*x+d)/(5*x^2+2*x+3)^2+1/196000*(34347*d-6511*e+(1101
5*d+36353*e)*x)/(5*x^2+2*x+3)+2/125*e*ln(5*x^2+2*x+3)+1/2744000*(42375*d-34
207*e)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1658, 1674, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)(42375d-34207e)}{196000\sqrt{14}}-\frac{(423x+1367)(d+ex)}{7000\,(5x^2+2x+3)^2}+\frac{x(11015d+36353e)+34347d-6511e}{196000\,(5x^2+2x+3)}+\frac{2}{125}e\log\left(5x^2+2x+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] -1/7000*((1367 + 423*x)*(d + e*x))/(3 + 2*x + 5*x^2)^2 + (34347*d - 6511*e
+ (11015*d + 36353*e)*x)/(196000*(3 + 2*x + 5*x^2)) + ((42375*d - 34207*e)*
ArcTan[(1 + 5*x)/Sqrt[14]])/(196000*Sqrt[14]) + (2*e*Log[3 + 2*x + 5*x^2])/
125
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1658

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x + c
*x^2)^(p + 1)*((f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx = -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{1}{112}\int \frac{\frac{2}{125}(3267d+1367e)-\frac{12}{25}(308\ldots}{(3\ldots}$$

$$= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353\ldots}{196000\left(3+2x+5x^2\right)}$$

$$= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353\ldots}{196000\left(3+2x+5x^2\right)}$$

$$= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353\ldots}{196000\left(3+2x+5x^2\right)}$$

$$= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353\ldots}{196000\left(3+2x+5x^2\right)}$$

Mathematica [A]
time = 0.06, size = 107, normalized size = 1.04

$$\frac{-6835d+1269e-2115dx-5989ex}{35000\left(3+2x+5x^2\right)^2}+\frac{171735d-44399e+55075dx+181765ex}{980000\left(3+2x+5x^2\right)}+\frac{(42375d-34207e)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{196000\sqrt{14}}+\frac{2}{125}e\log\left(3+2x+5x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x
]
```

```
[Out] (-6835*d + 1269*e - 2115*d*x - 5989*e*x)/(35000*(3 + 2*x + 5*x^2)^2) + (171
735*d - 44399*e + 55075*d*x + 181765*e*x)/(980000*(3 + 2*x + 5*x^2)) + ((42
375*d - 34207*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(196000*Sqrt[14]) + (2*e*Log[3
 + 2*x + 5*x^2])/125
```

Maple [A]
time = 0.12, size = 91, normalized size = 0.88

method	result
default	$\frac{25\left(\frac{36353e}{980000}+\frac{2203d}{196000}\right)x^3+25\left(\frac{28307e}{4900000}+\frac{38753d}{980000}\right)x^2+25\left(\frac{57761e}{4900000}+\frac{17979d}{980000}\right)x+\frac{12953d}{39200}-\frac{19533e}{196000}}{\left(5x^2+2x+3\right)^2}+\frac{2e\ln\left(5x^2+2x+3\right)}{125}+\frac{\left(8475d-\frac{34207e}{5}\right)\ldots}{\ldots}$
risch	$\frac{25\left(\frac{36353e}{980000}+\frac{2203d}{196000}\right)x^3+25\left(\frac{28307e}{4900000}+\frac{38753d}{980000}\right)x^2+25\left(\frac{57761e}{4900000}+\frac{17979d}{980000}\right)x+\frac{12953d}{39200}-\frac{19533e}{196000}}{\left(5x^2+2x+3\right)^2}+\frac{2e\ln\left(350x^2+140x+210\right)}{125}+\frac{339\sqrt{14}\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] 25*((36353/980000*e+2203/196000*d)*x^3+(28307/4900000*e+38753/980000*d)*x^2
+(57761/4900000*e+17979/980000*d)*x+12953/980000*d-19533/4900000*e)/(5*x^2+
2*x+3)^2+2/125*e*ln(5*x^2+2*x+3)+1/548800*(8475*d-34207/5*e)*14^(1/2)*arcta
n(1/28*(10*x+2)*14^(1/2))
```

Maxima [A]
time = 0.54, size = 107, normalized size = 1.04

$$\frac{1}{2744000}\sqrt{14}\,(42375\,d-34207\,e)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)+\frac{2}{125}\,e\log\left(5\,x^2+2\,x+3\right)+\frac{5\,(11015\,d+36353\,e)x^3+(193765\,d+28307\,e)x^2+(89895\,d+57761\,e)x+64765\,d-19533\,e}{196000\,(25\,x^4+20\,x^3+34\,x^2+12\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="max
ima")
```

```
[Out] 1/2744000*sqrt(14)*(42375*d - 34207*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 2/
125*e*log(5*x^2 + 2*x + 3) + 1/196000*(5*(11015*d + 36353*e)*x^3 + (193765*
d + 28307*e)*x^2 + (89895*d + 57761*e)*x + 64765*d - 19533*e)/(25*x^4 + 20*
x^3 + 34*x^2 + 12*x + 9)
```

Fricas [A]
time = 0.40, size = 163, normalized size = 1.58

$$\frac{771050\,dx^3+2712710\,dx^2+43904\,(25\,x^4+20\,x^3+34\,x^2+12\,x+9)e\log\left(5\,x^2+2\,x+3\right)+\sqrt{14}\,(1059375\,dx^4+847500\,dx^3+1440750\,dx^2+508500\,dx-34207\,(25\,x^4+20\,x^3+34\,x^2+12\,x+9)e+381375\,d)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)+1258530\,dx+14\,(181765\,x^3+28307\,x^2+57761\,x-19533)e+906710\,d}{2744000\,(25\,x^4+20\,x^3+34\,x^2+12\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="fri
cas")
```

```
[Out] 1/2744000*(771050*d*x^3 + 2712710*d*x^2 + 43904*(25*x^4 + 20*x^3 + 34*x^2 +
 12*x + 9)*e*log(5*x^2 + 2*x + 3) + sqrt(14)*(1059375*d*x^4 + 847500*d*x^3
+ 1440750*d*x^2 + 508500*d*x - 34207*(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)*
e + 381375*d)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1258530*d*x + 14*(181765*x^
3 + 28307*x^2 + 57761*x - 19533)*e + 906710*d)/(25*x^4 + 20*x^3 + 34*x^2 +
12*x + 9)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.94, size = 163, normalized size = 1.58

$$\left(\frac{2e}{125}-\frac{\sqrt{14}\,i(42375d-34207e)}{5488000}\right)\log\left(x+\frac{8475d-\frac{34207e}{5}-\frac{\sqrt{14}\,i(42375d-34207e)}{5}}{42375d-34207e}\right)+\left(\frac{2e}{125}+\frac{\sqrt{14}\,i(42375d-34207e)}{5488000}\right)\log\left(x+\frac{8475d-\frac{34207e}{5}+\frac{\sqrt{14}\,i(42375d-34207e)}{5}}{42375d-34207e}\right)+\frac{64765d-19533e+x^3\cdot(55075d+181765e)+x^2\cdot(193765d+28307e)+x(89895d+57761e)}{4900000x^4+3920000x^3+6664000x^2+2352000x+1764000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] (2*e/125 - sqrt(14)*I*(42375*d - 34207*e)/5488000)*log(x + (8475*d - 34207*
e/5 - sqrt(14)*I*(42375*d - 34207*e)/5)/(42375*d - 34207*e)) + (2*e/125 + s
qrt(14)*I*(42375*d - 34207*e)/5488000)*log(x + (8475*d - 34207*e/5 + sqrt(1
4)*I*(42375*d - 34207*e)/5)/(42375*d - 34207*e)) + (64765*d - 19533*e + x**
3*(55075*d + 181765*e) + x**2*(193765*d + 28307*e) + x*(89895*d + 57761*e))
/(4900000*x**4 + 3920000*x**3 + 6664000*x**2 + 2352000*x + 1764000)
```

Giac [A]
time = 7.65, size = 97, normalized size = 0.94

$$\frac{1}{2744000}\sqrt{14}\,(42375\,d-34207\,e)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)+\frac{2}{125}\,e\log\left(5\,x^2+2\,x+3\right)+\frac{5\,(11015\,d+36353\,e)x^3+(193765\,d+28307\,e)x^2+(89895\,d+57761\,e)x+64765\,d-19533\,e}{196000\,(5\,x^2+2\,x+3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="gia
c")
```

```
[Out] 1/2744000*sqrt(14)*(42375*d - 34207*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 2/
125*e*log(5*x^2 + 2*x + 3) + 1/196000*(5*(11015*d + 36353*e)*x^3 + (193765*
d + 28307*e)*x^2 + (89895*d + 57761*e)*x + 64765*d - 19533*e)/(5*x^2 + 2*x
+ 3)^2
```

Mupad [B]
time = 0.12, size = 125, normalized size = 1.21

$$\frac{\left(\frac{2203\,d}{7840}+\frac{36353\,e}{39200}\right)\,x^3+\left(\frac{38753\,d}{39200}+\frac{28307\,e}{196000}\right)\,x^2+\left(\frac{17979\,d}{39200}+\frac{57761\,e}{196000}\right)\,x+\frac{12953\,d}{39200}-\frac{19533\,e}{196000}}{25\,x^4+20\,x^3+34\,x^2+12\,x+9}+\frac{2\,e\,\ln\left(5\,x^2+2\,x+3\right)}{125}+\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,(42375\,d-34207\,e)}{2744000}+\frac{\sqrt{14}\,x\,(42375\,d-34207\,e)}{548800}}{\frac{339\,d}{1568}-\frac{34207\,e}{196000}}\right)\,(42375\,d-34207\,e)}{2744000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)
```

```
[Out] ((12953*d)/39200 - (19533*e)/196000 + x^3*((2203*d)/7840 + (36353*e)/39200)
 + x^2*((38753*d)/39200 + (28307*e)/196000) + x*((17979*d)/39200 + (57761*e
)/196000))/(12*x + 34*x^2 + 20*x^3 + 25*x^4 + 9) + (2*e*log(2*x + 5*x^2 + 3
))/125 + (14^(1/2)*atan(((14^(1/2)*(42375*d - 34207*e))/2744000 + (14^(1/2)
*x*(42375*d - 34207*e))/548800)/((339*d)/1568 - (34207*e)/196000))*(42375*d
 - 34207*e))/2744000
```

3.321 $\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3} dx$

Optimal. Leaf size=64

$$-\frac{1367+423x}{7000\,(3+2x+5x^2)^2}+\frac{34347+11015x}{196000\,(3+2x+5x^2)}+\frac{339\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1568\sqrt{14}}$$

```
[Out] 1/7000*(-1367-423*x)/(5*x^2+2*x+3)^2+1/196000*(34347+11015*x)/(5*x^2+2*x+3)
+339/21952*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1674, 12, 632, 210}

$$\frac{339\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)}{1568\sqrt{14}}-\frac{423x+1367}{7000\,(5x^2+2x+3)^2}+\frac{11015x+34347}{196000\,(5x^2+2x+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] -1/7000*(1367 + 423*x)/(3 + 2*x + 5*x^2)^2 + (34347 + 11015*x)/(196000*(3 +
 2*x + 5*x^2)) + (339*ArcTan[(1 + 5*x)/Sqrt[14]])/(1568*Sqrt[14])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx &= -\frac{1367+423x}{7000\,(3+2x+5x^2)^2}+\frac{1}{112}\int \frac{\frac{6534}{125}-\frac{3696x}{25}+\frac{448x^2}{5}}{(3+2x+5x^2)^2}\,dx\\ &= -\frac{1367+423x}{7000\,(3+2x+5x^2)^2}+\frac{34347+11015x}{196000\,(3+2x+5x^2)}+\frac{\int \frac{1356}{3+2x+5x^2}\,dx}{6272}\\ &= -\frac{1367+423x}{7000\,(3+2x+5x^2)^2}+\frac{34347+11015x}{196000\,(3+2x+5x^2)}+\frac{339\int \frac{1}{3+2x+5x^2}\,dx}{1568}\\ &= -\frac{1367+423x}{7000\,(3+2x+5x^2)^2}+\frac{34347+11015x}{196000\,(3+2x+5x^2)}-\frac{339}{784}\text{Subst}\left(\int \frac{1}{-56-x\right.\\ &= -\frac{1367+423x}{7000\,(3+2x+5x^2)^2}+\frac{34347+11015x}{196000\,(3+2x+5x^2)}+\frac{339\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1568\sqrt{14}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.83

$$\frac{\frac{14(12953+17979x+38753x^2+11015x^3)}{(3+2x+5x^2)^2}+8475\sqrt{14}\,\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{548800}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] ((14*(12953 + 17979*x + 38753*x^2 + 11015*x^3))/(3 + 2*x + 5*x^2)^2 + 8475*
Sqrt[14]*ArcTan[(1 + 5*x)/Sqrt[14]])/548800
```

Maple [A]
time = 0.11, size = 47, normalized size = 0.73

method	result	size

default	$\frac{\frac{2203}{7840}x^3+\frac{38753}{39200}x^2+\frac{17979}{39200}x+\frac{12953}{39200}}{(5x^2+2x+3)^2}+\frac{339\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{21952}$	47
risch	$\frac{\frac{2203}{7840}x^3+\frac{38753}{39200}x^2+\frac{17979}{39200}x+\frac{12953}{39200}}{(5x^2+2x+3)^2}+\frac{339\arctan\left(\frac{(1+5x)\sqrt{14}}{14}\right)\sqrt{14}}{21952}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 25*(2203/196000*x^3+38753/980000*x^2+17979/980000*x+12953/980000)/(5*x^2+2*
x+3)^2+339/21952*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

Maxima [A]
time = 0.54, size = 56, normalized size = 0.88

$$\frac{339}{21952}\sqrt{14}\,\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)+\frac{11015\,x^3+38753\,x^2+17979\,x+12953}{39200\,(25\,x^4+20\,x^3+34\,x^2+12\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="maxima")
```

```
[Out] 339/21952*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/39200*(11015*x^3 + 3
8753*x^2 + 17979*x + 12953)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

Fricas [A]
time = 0.34, size = 75, normalized size = 1.17

$$\frac{154210\,x^3+8475\,\sqrt{14}\,(25\,x^4+20\,x^3+34\,x^2+12\,x+9)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)+542542\,x^2+251706\,x+181342}{548800\,(25\,x^4+20\,x^3+34\,x^2+12\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="fricas")
```

```
[Out] 1/548800*(154210*x^3 + 8475*sqrt(14)*(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)*
arctan(1/14*sqrt(14)*(5*x + 1)) + 542542*x^2 + 251706*x + 181342)/(25*x^4 +
 20*x^3 + 34*x^2 + 12*x + 9)
```

Sympy [A]
time = 0.07, size = 61, normalized size = 0.95

$$\frac{11015x^3+38753x^2+17979x+12953}{980000x^4+784000x^3+1332800x^2+470400x+352800}+\frac{339\sqrt{14}\,\operatorname{atan}\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{21952}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] (11015*x**3 + 38753*x**2 + 17979*x + 12953)/(980000*x**4 + 784000*x**3 + 13
32800*x**2 + 470400*x + 352800) + 339*sqrt(14)*atan(5*sqrt(14)*x/14 + sqrt(
14)/14)/21952
```

Giac [A]
time = 4.55, size = 46, normalized size = 0.72

$$\frac{339}{21952}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)+\frac{11015x^3+38753x^2+17979x+12953}{39200(5x^2+2x+3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="giac")
```

```
[Out] 339/21952*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/39200*(11015*x^3 + 3
8753*x^2 + 17979*x + 12953)/(5*x^2 + 2*x + 3)^2
```

Mupad [B]
time = 0.05, size = 55, normalized size = 0.86

$$\frac{339\sqrt{14}\,\mathrm{atan}\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{21952}+\frac{\frac{2203x^3}{196000}+\frac{38753x^2}{980000}+\frac{17979x}{980000}+\frac{12953}{980000}}{x^4+\frac{4x^3}{5}+\frac{34x^2}{25}+\frac{12x}{25}+\frac{9}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3)^3,x)
```

```
[Out] (339*14^(1/2)*atan((5*14^(1/2)*x)/14 + 14^(1/2)/14))/21952 + ((17979*x)/980
000 + (38753*x^2)/980000 + (2203*x^3)/196000 + 12953/980000)/((12*x)/25 + (
34*x^2)/25 + (4*x^3)/5 + x^4 + 9/25)
```

3.322 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^3}\,dx$

Optimal. Leaf size=329

$$-\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+36207de^2+1831e^3+25(2203d^3-9033}{39200\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right.}$$

```
[Out] 1/1400*(-1367*d+293*e-(423*d-1367*e)*x)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)^2
+1/39200*(171735*d^3-92989*d^2*e+36207*d*e^2+1831*e^3+25*(2203*d^3-9033*d^2
*e+3635*d*e^2-1829*e^3)*x)/(5*d^2-2*d*e+3*e^2)^2/(5*x^2+2*x+3)+e*(4*d^4+5*d
^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^3-1/2*e*(4*d^4+5*
d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^3+1/21952*
(42375*d^5-16643*d^4*e+58530*d^3*e^2-56058*d^2*e^3+31811*d*e^4-8623*e^5)*ar
ctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1660, 814, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)\left(42375d^5-16643d^4e+58530d^3e^2-56058d^2e^3+31811de^4-8623e^5\right)}{1568\sqrt{14}\left(5d^2-2de+3e^2\right)^3}-\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}+\frac{171735d^3-92989d^2e+25x\left(2203d^3-9033d^2e+3635de^2-1829e^3\right)+36207de^2+1831e^3}{39200\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^2}-\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log\left(5x^2+2x+3\right)}{2\left(5d^2-2de+3e^2\right)^3}+\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{\left(5d^2-2de+3e^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^3),x]
```

```
[Out] -1/1400*(1367*d - 293*e + (423*d - 1367*e)*x)/((5*d^2 - 2*d*e + 3*e^2)*(3 +
 2*x + 5*x^2)^2) + (171735*d^3 - 92989*d^2*e + 36207*d*e^2 + 1831*e^3 + 25*
(2203*d^3 - 9033*d^2*e + 3635*d*e^2 - 1829*e^3)*x)/(39200*(5*d^2 - 2*d*e +
3*e^2)^2*(3 + 2*x + 5*x^2)) + ((42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 5
6058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]])/(1568*Sq
rt[14]*(5*d^2 - 2*d*e + 3*e^2)^3) + (e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3
 + 2*e^4)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^3 - (e*(4*d^4 + 5*d^3*e + 3
*d^2*e^2 - d*e^3 + 2*e^4)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^
3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx = -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{1}{112}\int \frac{\frac{2\left(3267d^2-2843de+2800e^2\right)}{25\left(5d^2-2de+3e^2\right)}}{(d+ex)}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+3620}{392}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+3620}{392}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+3620}{392}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+3620}{392}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+3620}{392}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}+\frac{171735d^3-92989d^2e+3620}{392}$$

Mathematica [A]
time = 0.20, size = 282, normalized size = 0.86

$$\frac{\frac{392\left(5d^2-2de+3e^2\right)^2\left(-d(1367+423x)+e(293+1367x)\right)}{\left(3+2x+5x^2\right)^2}+\frac{14\left(5d^2-2de+3e^2\right)\left(e^3(1831-45725x)+5d^3(34347+11015x)+de^2(36207+90875x)-d^2e(92989+225825x)\right)}{3+2x+5x^2}+25\sqrt{14}\left(42375d^5-16643d^4e+58530d^3e^2-56058d^2e^3+31811de^4-8623e^5\right)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)+548800e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)-274400e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log\left(3+2x+5x^2\right)}{548800\left(5d^2-2de+3e^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^3),x
]
```

```
[Out] ((392*(5*d^2 - 2*d*e + 3*e^2)^2*(-(d*(1367 + 423*x)) + e*(293 + 1367*x)))/(
3 + 2*x + 5*x^2)^2 + (14*(5*d^2 - 2*d*e + 3*e^2)*(e^3*(1831 - 45725*x) + 5*
d^3*(34347 + 11015*x) + d*e^2*(36207 + 90875*x) - d^2*e*(92989 + 225825*x))
)/(3 + 2*x + 5*x^2) + 25*Sqrt[14]*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2
- 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]] + 5488
00*e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x] - 274400*e*
(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[3 + 2*x + 5*x^2])/(548800
*(5*d^2 - 2*d*e + 3*e^2)^3)
```

Maple [A]
time = 0.23, size = 359, normalized size = 1.09

method	result
default	$\frac{e\left(4d^4+5d^3e+3d^2e^2-d\,e^3+2e^4\right)\ln(ex+d)}{\left(5d^2-2de+3e^2\right)^3}+\frac{25\left(\frac{2203}{1568}d^5-\frac{49571}{7840}d^4e+\frac{4285}{784}d^3e^2-\frac{21757}{3920}d^2e^3+\frac{14563}{7840}d\,e^4-\frac{5487}{7840}e^5\right)x^3+25\left(\frac{38753}{7840}d^5-\frac{10433}{1568}d^4\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] e*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^3+1/(
5*d^2-2*d*e+3*e^2)^3*(25*((2203/1568*d^5-49571/7840*d^4*e+4285/784*d^3*e^2-
21757/3920*d^2*e^3+14563/7840*d*e^4-5487/7840*e^5)*x^3+(38753/7840*d^5-1043
3/1568*d^4*e+655359/98000*d^3*e^2-388683/98000*d^2*e^3+250589/196000*d*e^4-
49377/196000*e^5)*x^2+(17979/7840*d^5-33127/7840*d^4*e+380997/98000*d^3*e^2
-250449/98000*d^2*e^3+147247/196000*d*e^4-11211/196000*e^5)*x+12953/7840*d^
5-11637/7840*d^4*e+118119/98000*d^3*e^2-28843/98000*d^2*e^3-25611/196000*d*
e^4+18063/196000*e^5)/(5*x^2+2*x+3)^2+1/15680*(-31360*d^4*e-39200*d^3*e^2-2
3520*d^2*e^3+7840*d*e^4-15680*e^5)*ln(5*x^2+2*x+3)+1/21952*(42375*d^5-16643
*d^4*e+58530*d^3*e^2-56058*d^2*e^3+31811*d*e^4-8623*e^5)*14^(1/2)*arctan(1/
28*(10*x+2)*14^(1/2)))
```

Maxima [A]
time = 0.56, size = 537, normalized size = 1.63

$$\frac{\sqrt{14}\left(42375\,d^5-16643\,d^4e+58530\,d^3e^2-56058\,d^2e^3+31811\,de^4-8623\,e^5\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{21952\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}-\frac{(4\,d^4e+5\,d^3e^2+3\,d^2e^3-de^4+2\,e^5)\log\left(5\,x^2+2\,x+3\right)}{2\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}+\frac{(4\,d^4e+5\,d^3e^2+3\,d^2e^3-de^4+2\,e^5)\log\left(xe+d\right)}{125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6}+\frac{25\,(2203\,d^3-9033\,d^2e+3635\,de^2-1829\,e^3)x^3+64765\,d^3+(193765\,d^3-183319\,d^2e+72557\,de^2-16459\,e^3)x^2-32279\,d^2e+(89895\,d^3-129677\,d^2e+46591\,de^2-3737\,e^3)x-4523\,de^2+6021\,e^3}{7840\,(25\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)x^4+225\,d^4+20\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)x^3-180\,d^3e+34\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)x^2+306\,d^2e^2+12\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)x-108\,de^3+81\,e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="max
ima")
```

```
[Out] 1/21952*sqrt(14)*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 +
 31811*d*e^4 - 8623*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d^5
*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(4*
d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(5*x^2 + 2*x + 3)/(125*d^
6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6
) + (4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(x*e + d)/(125*d^6
 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)
 + 1/7840*(25*(2203*d^3 - 9033*d^2*e + 3635*d*e^2 - 1829*e^3)*x^3 + 64765*d
^3 + (193765*d^3 - 183319*d^2*e + 72557*d*e^2 - 16459*e^3)*x^2 - 32279*d^2*
e + (89895*d^3 - 129677*d^2*e + 46591*d*e^2 - 3737*e^3)*x - 4523*d*e^2 + 60
21*e^3)/(25*(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*x^4 + 225*d
^4 + 20*(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*x^3 - 180*d^3*e
```

```
 + 34*(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*x^2 + 306*d^2*e^2
 + 12*(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*x - 108*d*e^3 + 8
1*e^4)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. $2(314) = 628$.
time = 0.55, size = 1019, normalized size = 3.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="fri
cas")
```

```
[Out] 1/109760*(3855250*d^5*x^3 + 13563550*d^5*x^2 + 6292650*d^5*x + 4533550*d^5
+ 5*sqrt(14)*(1059375*d^5*x^4 + 847500*d^5*x^3 + 1440750*d^5*x^2 + 508500*d
^5*x + 381375*d^5 - 8623*(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)*e^5 + 31811*
(25*d*x^4 + 20*d*x^3 + 34*d*x^2 + 12*d*x + 9*d)*e^4 - 56058*(25*d^2*x^4 + 2
0*d^2*x^3 + 34*d^2*x^2 + 12*d^2*x + 9*d^2)*e^3 + 58530*(25*d^3*x^4 + 20*d^3
*x^3 + 34*d^3*x^2 + 12*d^3*x + 9*d^3)*e^2 - 16643*(25*d^4*x^4 + 20*d^4*x^3
+ 34*d^4*x^2 + 12*d^4*x + 9*d^4)*e)*arctan(1/14*sqrt(14)*(5*x + 1)) - 42*(4
5725*x^3 + 16459*x^2 + 3737*x - 6021)*e^5 + 14*(364075*d*x^3 + 250589*d*x^2
 + 147247*d*x - 25611*d)*e^4 - 28*(543925*d^2*x^3 + 388683*d^2*x^2 + 250449
*d^2*x + 28843*d^2)*e^3 + 28*(535625*d^3*x^3 + 655359*d^3*x^2 + 380997*d^3*
x + 118119*d^3)*e^2 - 350*(49571*d^4*x^3 + 52165*d^4*x^2 + 33127*d^4*x + 11
637*d^4)*e - 54880*(2*(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)*e^5 - (25*d*x^4
 + 20*d*x^3 + 34*d*x^2 + 12*d*x + 9*d)*e^4 + 3*(25*d^2*x^4 + 20*d^2*x^3 + 3
4*d^2*x^2 + 12*d^2*x + 9*d^2)*e^3 + 5*(25*d^3*x^4 + 20*d^3*x^3 + 34*d^3*x^2
 + 12*d^3*x + 9*d^3)*e^2 + 4*(25*d^4*x^4 + 20*d^4*x^3 + 34*d^4*x^2 + 12*d^4
*x + 9*d^4)*e)*log(5*x^2 + 2*x + 3) + 109760*(2*(25*x^4 + 20*x^3 + 34*x^2 +
 12*x + 9)*e^5 - (25*d*x^4 + 20*d*x^3 + 34*d*x^2 + 12*d*x + 9*d)*e^4 + 3*(2
5*d^2*x^4 + 20*d^2*x^3 + 34*d^2*x^2 + 12*d^2*x + 9*d^2)*e^3 + 5*(25*d^3*x^4
 + 20*d^3*x^3 + 34*d^3*x^2 + 12*d^3*x + 9*d^3)*e^2 + 4*(25*d^4*x^4 + 20*d^4
*x^3 + 34*d^4*x^2 + 12*d^4*x + 9*d^4)*e)*log(x*e + d))/(3125*d^6*x^4 + 2500
*d^6*x^3 + 4250*d^6*x^2 + 1500*d^6*x + 1125*d^6 + 27*(25*x^4 + 20*x^3 + 34*
x^2 + 12*x + 9)*e^6 - 54*(25*d*x^4 + 20*d*x^3 + 34*d*x^2 + 12*d*x + 9*d)*e^
5 + 171*(25*d^2*x^4 + 20*d^2*x^3 + 34*d^2*x^2 + 12*d^2*x + 9*d^2)*e^4 - 188
*(25*d^3*x^4 + 20*d^3*x^3 + 34*d^3*x^2 + 12*d^3*x + 9*d^3)*e^3 + 285*(25*d^
4*x^4 + 20*d^4*x^3 + 34*d^4*x^2 + 12*d^4*x + 9*d^4)*e^2 - 150*(25*d^5*x^4 +
 20*d^5*x^3 + 34*d^5*x^2 + 12*d^5*x + 9*d^5)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.08, size = 460, normalized size = 1.40

$$\frac{\sqrt{14}\,(42375\,d^5 - 16643\,d^4e + 58530\,d^3e^2 - 56058\,d^2e^3 + 31811\,de^4 - 8623\,e^5)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{21952\,(125\,d^6 - 150\,d^5e + 285\,d^4e^2 - 188\,d^3e^3 + 171\,d^2e^4 - 54\,de^5 + 27\,e^6)} - \frac{(4\,d^4e + 5\,d^3e^2 + 3\,d^2e^3 - de^4 + 2\,e^5)\log(5\,x^2 + 2\,x + 3)}{2\,(125\,d^6 - 150\,d^5e + 285\,d^4e^2 - 188\,d^3e^3 + 171\,d^2e^4 - 54\,de^5 + 27\,e^6)} + \frac{(4\,d^4e^2 + 5\,d^3e^3 + 3\,d^2e^4 - de^5 + 2\,e^6)\log(|xe+d|)}{125\,d^6e - 150\,d^5e^2 + 285\,d^4e^3 - 188\,d^3e^4 + 171\,d^2e^5 - 54\,de^6 + 27\,e^7} + \frac{323825\,d^5 - 290925\,d^4e + 25\,(11015\,d^5 - 49571\,d^4e + 42850\,d^3e^2 - 43514\,d^2e^3 + 14563\,de^4 - 5487\,e^5)x^3 + 236238\,d^3e^2 + (968825\,d^5 - 1304125\,d^4e + 1310718\,d^3e^2 - 777366\,d^2e^3 + 250589\,de^4 - 49377\,e^5)x^2 - 57686\,d^2e^3 + (449475\,d^5 - 828175\,d^4e + 761994\,d^3e^2 - 500898\,d^2e^3 + 147247\,de^4 - 11211\,e^5)x - 25611\,de^4 + 18063\,e^5}{7840\,(5\,d^2 - 2\,de + 3\,e^2)^3(5\,x^2 + 2\,x + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="gia
c")
```

```
[Out] 1/21952*sqrt(14)*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 +
 31811*d*e^4 - 8623*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d^5
*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(4*
d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(5*x^2 + 2*x + 3)/(125*d^
6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6
) + (4*d^4*e^2 + 5*d^3*e^3 + 3*d^2*e^4 - d*e^5 + 2*e^6)*log(abs(x*e + d))/(
125*d^6*e - 150*d^5*e^2 + 285*d^4*e^3 - 188*d^3*e^4 + 171*d^2*e^5 - 54*d*e^
6 + 27*e^7) + 1/7840*(323825*d^5 - 290925*d^4*e + 25*(11015*d^5 - 49571*d^4
*e + 42850*d^3*e^2 - 43514*d^2*e^3 + 14563*d*e^4 - 5487*e^5)*x^3 + 236238*d
^3*e^2 + (968825*d^5 - 1304125*d^4*e + 1310718*d^3*e^2 - 777366*d^2*e^3 + 2
50589*d*e^4 - 49377*e^5)*x^2 - 57686*d^2*e^3 + (449475*d^5 - 828175*d^4*e +
 761994*d^3*e^2 - 500898*d^2*e^3 + 147247*d*e^4 - 11211*e^5)*x - 25611*d*e^
4 + 18063*e^5)/((5*d^2 - 2*d*e + 3*e^2)^3*(5*x^2 + 2*x + 3)^2)
```

Mupad [B]
time = 4.79, size = 641, normalized size = 1.95

$$\frac{\frac{x\,(89895\,d^3-129677\,d^2\,e+46591\,d\,e^2-3737\,e^3)}{7840\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}-\frac{-64765\,d^3+32279\,d^2\,e+4523\,d\,e^2-6021\,e^3}{7840\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}+\frac{5\,x^3\,(2203\,d^3-9033\,d^2\,e+3635\,d\,e^2-1829\,e^3)}{1568\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}+\frac{x^2\,(193765\,d^3-183319\,d^2\,e+72557\,d\,e^2-16459\,e^3)}{7840\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}}{25\,x^4+20\,x^3+34\,x^2+12\,x+9}+\ln(d+e\,x)\left(\frac{4\,e}{25\,(5\,d^2-2\,d\,e+3\,e^2)}+\frac{e^2\,(205\,d+21\,e)}{125\,(5\,d^2-2\,d\,e+3\,e^2)^2}-\frac{e^4\,(458\,d-7\,e)}{125\,(5\,d^2-2\,d\,e+3\,e^2)^3}\right)-\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1i}{5}\right)\left(-\frac{42375\,\sqrt{14}\,d^5}{43904}+\left(\frac{16643\,\sqrt{14}}{43904}+2i\right)d^4\,e+\left(-\frac{29265\,\sqrt{14}}{21952}+\frac{5}{2}i\right)d^3\,e^2+\left(\frac{28029\,\sqrt{14}}{21952}+\frac{3}{2}i\right)d^2\,e^3+\left(-\frac{31811\,\sqrt{14}}{43904}-\frac{1}{2}i\right)d\,e^4+\left(\frac{8623\,\sqrt{14}}{43904}+1i\right)e^5\right)}{d^6\,125i-d^5\,e\,150i+d^4\,e^2\,285i-d^3\,e^3\,188i+d^2\,e^4\,171i-d\,e^5\,54i+e^6\,27i}+\frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1i}{5}\right)\left(-\frac{42375\,\sqrt{14}\,d^5}{43904}+\left(\frac{16643\,\sqrt{14}}{43904}-2i\right)d^4\,e+\left(-\frac{29265\,\sqrt{14}}{21952}-\frac{5}{2}i\right)d^3\,e^2+\left(\frac{28029\,\sqrt{14}}{21952}-\frac{3}{2}i\right)d^2\,e^3+\left(-\frac{31811\,\sqrt{14}}{43904}+\frac{1}{2}i\right)d\,e^4+\left(\frac{8623\,\sqrt{14}}{43904}-i\right)e^5\right)}{d^6\,125i-d^5\,e\,150i+d^4\,e^2\,285i-d^3\,e^3\,188i+d^2\,e^4\,171i-d\,e^5\,54i+e^6\,27i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)^3),x)
```

```
[Out] ((x*(46591*d*e^2 - 129677*d^2*e + 89895*d^3 - 3737*e^3))/(7840*(25*d^4 - 20
*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) - (4523*d*e^2 + 32279*d^2*e - 6476
5*d^3 - 6021*e^3)/(7840*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)
) + (5*x^3*(3635*d*e^2 - 9033*d^2*e + 2203*d^3 - 1829*e^3))/(1568*(25*d^4 -
 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x^2*(72557*d*e^2 - 183319*d^
2*e + 193765*d^3 - 16459*e^3))/(7840*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4
+ 34*d^2*e^2)))/(12*x + 34*x^2 + 20*x^3 + 25*x^4 + 9) + log(d + e*x)*((4*e)
/(25*(5*d^2 - 2*d*e + 3*e^2)) + (e^2*(205*d + 21*e))/(125*(5*d^2 - 2*d*e +
3*e^2)^2) - (e^4*(458*d - 7*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^3)) - (log(x -
 (14^(1/2)*1i)/5 + 1/5)*(e^5*((8623*14^(1/2))/43904 + 1i) - (42375*14^(1/2)
*d^5)/43904 + d^2*e^3*((28029*14^(1/2))/21952 + 3i/2) - d^3*e^2*((29265*14^
```

```
(1/2))/21952 - 5i/2) + d^4*e*((16643*14^(1/2))/43904 + 2i) - d*e^4*((31811*
14^(1/2))/43904 + 1i/2)))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^
2*e^4*171i - d^3*e^3*188i + d^4*e^2*285i) + (log(x + (14^(1/2)*1i)/5 + 1/5)
*(e^5*((8623*14^(1/2))/43904 - 1i) - (42375*14^(1/2)*d^5)/43904 + d^2*e^3*(
(28029*14^(1/2))/21952 - 3i/2) - d^3*e^2*((29265*14^(1/2))/21952 + 5i/2) +
d^4*e*((16643*14^(1/2))/43904 - 2i) - d*e^4*((31811*14^(1/2))/43904 - 1i/2)
))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*18
8i + d^4*e^2*285i)
```

3.323 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^3}\,dx$

Optimal. Leaf size=443

$$-\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{(5d^2-2de+3e^2)^3(d+ex)}-\frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{280(5d^2-2de+3e^2)^2(3+2x+5x^2)^2}+\frac{171735d^4-1}{}$$

```
[Out] -e*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)+1/28
0*(-1367*d^2+586*d*e+703*e^2-(423*d^2-2734*d*e+293*e^2)*x)/(5*d^2-2*d*e+3*e
^2)^2/(5*x^2+2*x+3)^2+1/7840*(171735*d^4-117284*d^3*e-200502*d^2*e^2+104428
*d*e^3-23189*e^4+5*(11015*d^4-85924*d^3*e+34698*d^2*e^2+10348*d*e^3-3589*e^
4)*x)/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)+e*(40*d^5+83*d^4*e+12*d^3*e^2-76*
d^2*e^3+46*d*e^4-9*e^5)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^4-1/2*e*(40*d^5+83*d^
4*e+12*d^3*e^2-76*d^2*e^3+46*d*e^4-9*e^5)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^
2)^4+1/21952*(211875*d^6+3070*d^5*e+209039*d^4*e^2-921444*d^3*e^3+380621*d^
2*e^4-49586*d*e^5-43695*e^6)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e
^2)^4*14^(1/2)
```

Rubi [A]
time = 0.72, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1660, 1642, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{5x+1}{\sqrt{14}}\right)(211875d^6+3070d^5e+209039d^4e^2-921444d^3e^3+380621d^2e^4-49586de^5-43695e^6)}{1568\sqrt{14}(5d^2-2de+3e^2)^4}-\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{280(5x^2+2x+3)^2(5d^2-2de+3e^2)^2}+\frac{171735d^4-117284d^3e-200502d^2e^2+5x(11015d^4-85924d^3e+34698d^2e^2+10348de^3-3589e^4)+104428de^3-23189e^4}{7840(5x^2+2x+3)(5d^2-2de+3e^2)^3}-\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{(5d^2-2de+3e^2)^3(d+ex)}-\frac{e(40d^5+83d^4e+12d^3e^2-76d^2e^3+46de^4-9e^5)\log(5x^2+2x+3)}{2(5d^2-2de+3e^2)^4}+\frac{e(40d^5+83d^4e+12d^3e^2-76d^2e^3+46de^4-9e^5)\log(d+ex)}{(5d^2-2de+3e^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^3),x]
```

```
[Out] -((e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/((5*d^2 - 2*d*e + 3*e^2
)^3*(d + e*x))) - (1367*d^2 - 586*d*e - 703*e^2 + (423*d^2 - 2734*d*e + 293
*e^2)*x)/(280*(5*d^2 - 2*d*e + 3*e^2)^2*(3 + 2*x + 5*x^2)^2) + (171735*d^4
- 117284*d^3*e - 200502*d^2*e^2 + 104428*d*e^3 - 23189*e^4 + 5*(11015*d^4 -
 85924*d^3*e + 34698*d^2*e^2 + 10348*d*e^3 - 3589*e^4)*x)/(7840*(5*d^2 - 2*
d*e + 3*e^2)^3*(3 + 2*x + 5*x^2)) + ((211875*d^6 + 3070*d^5*e + 209039*d^4*
e^2 - 921444*d^3*e^3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*ArcTan[(1
+ 5*x)/Sqrt[14]])/(1568*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^4) + (e*(40*d^5 +
83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 + 46*d*e^4 - 9*e^5)*Log[d + e*x])/(5*d^2
 - 2*d*e + 3*e^2)^4 - (e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 + 46*
d*e^4 - 9*e^5)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^4)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx = -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)^2} + \frac{1}{112}\int \frac{2(3267d}{}$$

$$= -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)^2} + \frac{171735d^4-1}{}$$

$$= -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)^2} + \frac{171735d^4-1}{}$$

$$= -\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{\left(5d^2-2de+3e^2\right)^3(d+ex)} - \frac{1367d^2-586de-703e^2+(423d^2}{280\left(5d^2-2de+3e^2\right)^2(3}$$

$$= -\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{\left(5d^2-2de+3e^2\right)^3(d+ex)} - \frac{1367d^2-586de-703e^2+(423d^2}{280\left(5d^2-2de+3e^2\right)^2(3}$$

$$= -\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{\left(5d^2-2de+3e^2\right)^3(d+ex)} - \frac{1367d^2-586de-703e^2+(423d^2}{280\left(5d^2-2de+3e^2\right)^2(3}$$

$$= -\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{\left(5d^2-2de+3e^2\right)^3(d+ex)} - \frac{1367d^2-586de-703e^2+(423d^2}{280\left(5d^2-2de+3e^2\right)^2(3}$$

Mathematica [A]
time = 0.34, size = 389, normalized size = 0.88

$$\frac{-\frac{109760e\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{d+ex}-\frac{392\left(5d^2-2de+3e^2\right)^2\left(e^2(-703+293x)+d^2(1367+423x)-2de(293+1367x)\right)}{\left(3+2x+5x^2\right)^2}+\frac{14\left(5d^2-2de+3e^2\right)\left(5d^4(34347+11015x)+4de^3(26107+12935x)-e^4(23189+17945x)+6d^2e^2(-33417+28915x)-4d^3e(29321+107405x)\right)}{3+2x+5x^2}+5\sqrt{14}\left(211875d^6+3070d^5e+209039d^4e^2-921444d^3e^3+380621d^2e^4-49586de^5-43695e^6\right)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)+109760e\left(40d^5+83d^4e+12d^3e^2-76d^2e^3+46de^4-9e^5\right)\log(d+ex)-54880e\left(40d^5+83d^4e+12d^3e^2-76d^2e^3+46de^4-9e^5\right)\log\left(3+2x+5x^2\right)}{109760\left(5d^2-2de+3e^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^3)
,x]
```

```
[Out] ((-109760*e*(5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 +
2*e^4))/(d + e*x) - (392*(5*d^2 - 2*d*e + 3*e^2)^2*(e^2*(-703 + 293*x) + d^
2*(1367 + 423*x) - 2*d*e*(293 + 1367*x)))/(3 + 2*x + 5*x^2)^2 + (14*(5*d^2
- 2*d*e + 3*e^2)*(5*d^4*(34347 + 11015*x) + 4*d*e^3*(26107 + 12935*x) - e^4
*(23189 + 17945*x) + 6*d^2*e^2*(-33417 + 28915*x) - 4*d^3*e*(29321 + 107405
*x)))/(3 + 2*x + 5*x^2) + 5*Sqrt[14]*(211875*d^6 + 3070*d^5*e + 209039*d^4*
e^2 - 921444*d^3*e^3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*ArcTan[(1
+ 5*x)/Sqrt[14]] + 109760*e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 +
46*d*e^4 - 9*e^5)*Log[d + e*x] - 54880*e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 -
76*d^2*e^3 + 46*d*e^4 - 9*e^5)*Log[3 + 2*x + 5*x^2]])/(109760*(5*d^2 - 2*d*e
 + 3*e^2)^4)
```

Maple [A]
time = 0.15, size = 473, normalized size = 1.07

method	result
default	$\frac{e\left(40d^{5}+83d^{4}e+12d^{3}e^{2}-76d^{2}e^{3}+46d\,e^{4}-9e^{5}\right)\ln(ex+d)}{\left(5d^{2}-2de+3e^{2}\right)^{4}}-\frac{e\left(4d^{4}+5d^{3}e+3d^{2}e^{2}-d\,e^{3}+2e^{4}\right)}{\left(5d^{2}-2de+3e^{2}\right)^{3}(ex+d)}+\frac{25\left(\frac{11015}{1568}d^{6}-\frac{45165}{784}d^{5}e+\frac{378383}{7840}d^{4}e^{2}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x,method=_RETURNVERBO
SE)
```

```
[Out] e*(40*d^5+83*d^4*e+12*d^3*e^2-76*d^2*e^3+46*d*e^4-9*e^5)*ln(e*x+d)/(5*d^2-2
*d*e+3*e^2)^4-e*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2)^3
/(e*x+d)+1/(5*d^2-2*d*e+3*e^2)^4*(25*((11015/1568*d^6-45165/784*d^5*e+37838
3/7840*d^4*e^2-68857/1960*d^3*e^3+65453/7840*d^2*e^4+19111/3920*d*e^5-10767
/7840*e^6)*x^3+(38753/1568*d^6-183319/3920*d^5*e+504029/39200*d^4*e^2+5109/
9800*d^3*e^3-795401/39200*d^2*e^4+218053/19600*d*e^5-91101/39200*e^6)*x^2+(
17979/1568*d^6-129677/3920*d^5*e+606287/39200*d^4*e^2-3993/9800*d^3*e^3-869
99/7840*d^2*e^4+208007/19600*d*e^5-14979/7840*e^6)*x+12953/1568*d^6-32279/3
920*d^5*e-379131/39200*d^4*e^2+116869/9800*d^3*e^3-530209/39200*d^2*e^4+198
09/3920*d*e^5-6309/39200*e^6)/(5*x^2+2*x+3)^2+1/15680*(-313600*d^5*e-650720
*d^4*e^2-94080*d^3*e^3+595840*d^2*e^4-360640*d*e^5+70560*e^6)*ln(5*x^2+2*x+
3)+1/21952*(211875*d^6+3070*d^5*e+209039*d^4*e^2-921444*d^3*e^3+380621*d^2*
e^4-49586*d*e^5-43695*e^6)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 843 vs. 2(421) = 842.
time = 0.56, size = 843, normalized size = 1.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="m
axima")
```

```
[Out] 1/21952*sqrt(14)*(211875*d^6 + 3070*d^5*e + 209039*d^4*e^2 - 921444*d^3*e^3
 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*arctan(1/14*sqrt(14)*(5*x + 1)
)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176
*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - 1/2*(40*d^5*e + 83*d^4*e^2 +
 12*d^3*e^3 - 76*d^2*e^4 + 46*d*e^5 - 9*e^6)*log(5*x^2 + 2*x + 3)/(625*d^8
- 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 +
756*d^2*e^6 - 216*d*e^7 + 81*e^8) + (40*d^5*e + 83*d^4*e^2 + 12*d^3*e^3 - 7
```

```
6*d^2*e^4 + 46*d*e^5 - 9*e^6)*log(x*e + d)/(625*d^8 - 1000*d^7*e + 2100*d^6
*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7
 + 81*e^8) + 1/1568*(64765*d^5 - 5*(20345*d^4*e + 125124*d^3*e^2 - 11178*d^
2*e^3 - 18188*d*e^4 + 19269*e^5)*x^4 - 95100*d^4*e + (55075*d^5 - 361295*d^
4*e - 272442*d^3*e^2 - 173446*d^2*e^3 + 138539*d*e^4 - 93087*e^5)*x^3 - 200
706*d^3*e^2 + (193765*d^5 - 412485*d^4*e - 621062*d^3*e^2 - 56850*d^2*e^3 +
 144973*d*e^4 - 131589*e^5)*x^2 + 22292*d^2*e^3 + 3*(29965*d^5 - 77965*d^4*
e - 51590*d^3*e^2 - 21522*d^2*e^3 + 19493*d*e^4 - 13245*e^5)*x + 12009*d*e^
4 - 28224*e^5)/(1125*d^7 - 1350*d^6*e + 25*(125*d^6*e - 150*d^5*e^2 + 285*d
^4*e^3 - 188*d^3*e^4 + 171*d^2*e^5 - 54*d*e^6 + 27*e^7)*x^5 + 2565*d^5*e^2
+ 5*(625*d^7 - 250*d^6*e + 825*d^5*e^2 + 200*d^4*e^3 + 103*d^3*e^4 + 414*d^
2*e^5 - 81*d*e^6 + 108*e^7)*x^4 - 1692*d^4*e^3 + 2*(1250*d^7 + 625*d^6*e +
300*d^5*e^2 + 2965*d^4*e^3 - 1486*d^3*e^4 + 2367*d^2*e^5 - 648*d*e^6 + 459*
e^7)*x^3 + 1539*d^3*e^4 + 2*(2125*d^7 - 1800*d^6*e + 3945*d^5*e^2 - 1486*d^
4*e^3 + 1779*d^3*e^4 + 108*d^2*e^5 + 135*d*e^6 + 162*e^7)*x^2 - 486*d^2*e^5
 + 3*(500*d^7 - 225*d^6*e + 690*d^5*e^2 + 103*d^4*e^3 + 120*d^3*e^4 + 297*d
^2*e^5 - 54*d*e^6 + 81*e^7)*x + 243*d*e^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. $2(421) = 842$.
time = 0.66, size = 1690, normalized size = 3.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="f
ricas")
```

```
[Out] 1/21952*(3855250*d^7*x^3 + 13563550*d^7*x^2 + 6292650*d^7*x + 4533550*d^7 +
 sqrt(14)*(5296875*d^7*x^4 + 4237500*d^7*x^3 + 7203750*d^7*x^2 + 2542500*d^
7*x + 1906875*d^7 - 43695*(25*x^5 + 20*x^4 + 34*x^3 + 12*x^2 + 9*x)*e^7 - (
1239650*d*x^5 + 2084095*d*x^4 + 2559824*d*x^3 + 2080662*d*x^2 + 970614*d*x
+ 393255*d)*e^6 + (9515525*d^2*x^5 + 6372770*d^2*x^4 + 11949394*d^2*x^3 + 2
881528*d^2*x^2 + 2830557*d^2*x - 446274*d^2)*e^5 - (23036100*d^3*x^5 + 8913
355*d^3*x^4 + 23716676*d^3*x^3 - 1883786*d^3*x^2 + 3725544*d^3*x - 3425589*
d^3)*e^4 + (5225975*d^4*x^5 - 18855320*d^4*x^4 - 11321554*d^4*x^3 - 2882062
8*d^4*x^2 - 9175977*d^4*x - 8292996*d^4)*e^3 + (76750*d^5*x^5 + 5287375*d^5
*x^4 + 4285160*d^5*x^3 + 7144166*d^5*x^2 + 2536098*d^5*x + 1881351*d^5)*e^2
 + 5*(1059375*d^6*x^5 + 862850*d^6*x^4 + 1453030*d^6*x^3 + 529376*d^6*x^2 +
 388743*d^6*x + 5526*d^6)*e)*arctan(1/14*sqrt(14)*(5*x + 1)) - 378*(10705*x
^4 + 10343*x^3 + 14621*x^2 + 4415*x + 3136)*e^7 + 42*(155170*d*x^4 + 200597
*d*x^3 + 232699*d*x^2 + 84969*d*x + 30825*d)*e^6 - 14*(495935*d^2*x^4 + 126
2851*d^2*x^3 + 1118441*d^2*x^2 + 509331*d^2*x + 98262*d^2)*e^5 - 14*(153394
0*d^3*x^4 - 222261*d^3*x^3 + 1024621*d^3*x^2 + 42783*d^3*x + 586657*d^3)*e^
4 + 14*(1225515*d^4*x^4 - 1406231*d^4*x^3 - 279581*d^4*x^2 - 714975*d^4*x +
```

```
 227572*d^4)*e^3 - 70*(584930*d^5*x^4 + 94879*d^5*x^3 + 339809*d^5*x^2 + 72
75*d^5*x + 123807*d^5)*e^2 - 70*(101725*d^6*x^4 + 383325*d^6*x^3 + 489991*d
^6*x^2 + 269853*d^6*x + 121006*d^6)*e + 10976*(9*(25*x^5 + 20*x^4 + 34*x^3
+ 12*x^2 + 9*x)*e^7 - (1150*d*x^5 + 695*d*x^4 + 1384*d*x^3 + 246*d*x^2 + 30
6*d*x - 81*d)*e^6 + 2*(950*d^2*x^5 + 185*d^2*x^4 + 832*d^2*x^3 - 326*d^2*x^
2 + 66*d^2*x - 207*d^2)*e^5 - 4*(75*d^3*x^5 - 415*d^3*x^4 - 278*d^3*x^3 - 6
10*d^3*x^2 - 201*d^3*x - 171*d^3)*e^4 - (2075*d^4*x^5 + 1960*d^4*x^4 + 3062
*d^4*x^3 + 1404*d^4*x^2 + 891*d^4*x + 108*d^4)*e^3 - (1000*d^5*x^5 + 2875*d
^5*x^4 + 3020*d^5*x^3 + 3302*d^5*x^2 + 1356*d^5*x + 747*d^5)*e^2 - 40*(25*d
^6*x^4 + 20*d^6*x^3 + 34*d^6*x^2 + 12*d^6*x + 9*d^6)*e)*log(5*x^2 + 2*x + 3
) - 21952*(9*(25*x^5 + 20*x^4 + 34*x^3 + 12*x^2 + 9*x)*e^7 - (1150*d*x^5 +
695*d*x^4 + 1384*d*x^3 + 246*d*x^2 + 306*d*x - 81*d)*e^6 + 2*(950*d^2*x^5 +
 185*d^2*x^4 + 832*d^2*x^3 - 326*d^2*x^2 + 66*d^2*x - 207*d^2)*e^5 - 4*(75*
d^3*x^5 - 415*d^3*x^4 - 278*d^3*x^3 - 610*d^3*x^2 - 201*d^3*x - 171*d^3)*e^
4 - (2075*d^4*x^5 + 1960*d^4*x^4 + 3062*d^4*x^3 + 1404*d^4*x^2 + 891*d^4*x
+ 108*d^4)*e^3 - (1000*d^5*x^5 + 2875*d^5*x^4 + 3020*d^5*x^3 + 3302*d^5*x^2
 + 1356*d^5*x + 747*d^5)*e^2 - 40*(25*d^6*x^4 + 20*d^6*x^3 + 34*d^6*x^2 + 1
2*d^6*x + 9*d^6)*e)*log(x*e + d))/(15625*d^9*x^4 + 12500*d^9*x^3 + 21250*d^
9*x^2 + 7500*d^9*x + 5625*d^9 + 81*(25*x^5 + 20*x^4 + 34*x^3 + 12*x^2 + 9*x
)*e^9 - 27*(200*d*x^5 + 85*d*x^4 + 212*d*x^3 - 6*d*x^2 + 36*d*x - 27*d)*e^8
 + 108*(175*d^2*x^5 + 90*d^2*x^4 + 198*d^2*x^3 + 16*d^2*x^2 + 39*d^2*x - 18
*d^2)*e^7 - 84*(350*d^3*x^5 + 55*d^3*x^4 + 296*d^3*x^3 - 138*d^3*x^2 + 18*d
^3*x - 81*d^3)*e^6 + 14*(3725*d^4*x^5 + 880*d^4*x^4 + 3386*d^4*x^3 - 1068*d
^4*x^2 + 333*d^4*x - 756*d^4)*e^5 - 14*(3500*d^5*x^5 - 925*d^5*x^4 + 1780*d
^5*x^3 - 3386*d^5*x^2 - 528*d^5*x - 1341*d^5)*e^4 + 140*(375*d^6*x^5 - 50*d
^6*x^4 + 230*d^6*x^3 - 296*d^6*x^2 - 33*d^6*x - 126*d^6)*e^3 - 100*(250*d^7
*x^5 - 325*d^7*x^4 - 80*d^7*x^3 - 594*d^7*x^2 - 162*d^7*x - 189*d^7)*e^2 +
125*(125*d^8*x^5 - 100*d^8*x^4 + 10*d^8*x^3 - 212*d^8*x^2 - 51*d^8*x - 72*d
^8)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.06, size = 762, normalized size = 1.72

$$\frac{\sqrt{14}\,(211875\,d^6e^2 + 3070\,d^5e^3 + 209039\,d^4e^4 - 921444\,d^3e^5 + 380621\,d^2e^6 - 49586\,de^7 - 43695\,e^8)\arctan\left(\frac{1}{14}\sqrt{14}\left(5\,d - \frac{5\,d^2}{xe+d} + \frac{2\,de}{xe+d} - \frac{3\,e^2}{xe+d} - e\right)e^{(-1)}\right)e^{(-2)}}{21952\,(625\,d^8 - 1000\,d^7e + 2100\,d^6e^2 - 1960\,d^5e^3 + 2086\,d^4e^4 - 1176\,d^3e^5 + 756\,d^2e^6 - 216\,de^7 + 81\,e^8)} - \frac{(40\,d^5e + 83\,d^4e^2 + 12\,d^3e^3 - 76\,d^2e^4 + 46\,de^5 - 9\,e^6)\log\left(-\frac{10\,d}{xe+d} + \frac{5\,d^2}{(xe+d)^2} + \frac{2\,e}{xe+d} - \frac{2\,de}{(xe+d)^2} + \frac{3\,e^2}{(xe+d)^2} + 5\right)}{2\,(625\,d^8 - 1000\,d^7e + 2100\,d^6e^2 - 1960\,d^5e^3 + 2086\,d^4e^4 - 1176\,d^3e^5 + 756\,d^2e^6 - 216\,de^7 + 81\,e^8)} - \frac{\frac{4\,d^4e^7}{xe+d} + \frac{5\,d^3e^8}{xe+d} + \frac{3\,d^2e^9}{xe+d} - \frac{de^{10}}{xe+d} + \frac{2\,e^{11}}{xe+d}}{125\,d^6e^6 - 150\,d^5e^7 + 285\,d^4e^8 - 188\,d^3e^9 + 171\,d^2e^{10} - 54\,de^{11} + 27\,e^{12}} + \frac{275375\,d^5e - 3006775\,d^4e^2 + 1394650\,d^3e^3 + 1835350\,d^2e^4 - 734925\,de^5 - \frac{5\,(165225\,d^6e^2 - 1997830\,d^5e^3 + 1218421\,d^4e^4 + 1520564\,d^3e^5 - 947049\,d^2e^6 + 93386\,de^7 + 7963\,e^8)e^{(-1)}}{xe+d} + \frac{(826125\,d^7e^3 - 10957975\,d^6e^4 + 8449735\,d^5e^5 + 8211175\,d^4e^6 - 7879025\,d^3e^7 + 2996315\,d^2e^8 - 443947\,de^9 - 67267\,e^{10})e^{(-2)}}{(xe+d)^2} - \frac{(275375\,d^8e^4 - 3975600\,d^7e^5 + 3752280\,d^6e^6 + 2119880\,d^5e^7 - 3655050\,d^4e^8 + 4008480\,d^3e^9 - 1453312\,d^2e^{10} - 197784\,de^{11} + 66483\,e^{12})e^{(-3)}}{(xe+d)^3} + 17525\,e^6}{1568\,(5\,d^2 - 2\,de + 3\,e^2)^4\left(\frac{10\,d}{xe+d} - \frac{5\,d^2}{(xe+d)^2} - \frac{2\,e}{xe+d} + \frac{2\,de}{(xe+d)^2} - \frac{3\,e^2}{(xe+d)^2} - 5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="g
iac")
```

```
[Out] 1/21952*sqrt(14)*(211875*d^6*e^2 + 3070*d^5*e^3 + 209039*d^4*e^4 - 921444*d
^3*e^5 + 380621*d^2*e^6 - 49586*d*e^7 - 43695*e^8)*arctan(1/14*sqrt(14)*(5*
d - 5*d^2/(x*e + d) + 2*d*e/(x*e + d) - 3*e^2/(x*e + d) - e)*e^(-1))*e^(-2)
/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*
d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - 1/2*(40*d^5*e + 83*d^4*e^2 +
12*d^3*e^3 - 76*d^2*e^4 + 46*d*e^5 - 9*e^6)*log(-10*d/(x*e + d) + 5*d^2/(x*
e + d)^2 + 2*e/(x*e + d) - 2*d*e/(x*e + d)^2 + 3*e^2/(x*e + d)^2 + 5)/(625*
d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^
5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - (4*d^4*e^7/(x*e + d) + 5*d^3*e^8/(x
*e + d) + 3*d^2*e^9/(x*e + d) - d*e^10/(x*e + d) + 2*e^11/(x*e + d))/(125*d
^6*e^6 - 150*d^5*e^7 + 285*d^4*e^8 - 188*d^3*e^9 + 171*d^2*e^10 - 54*d*e^11
 + 27*e^12) + 1/1568*(275375*d^5*e - 3006775*d^4*e^2 + 1394650*d^3*e^3 + 18
35350*d^2*e^4 - 734925*d*e^5 - 5*(165225*d^6*e^2 - 1997830*d^5*e^3 + 121842
1*d^4*e^4 + 1520564*d^3*e^5 - 947049*d^2*e^6 + 93386*d*e^7 + 7963*e^8)*e^(-
1)/(x*e + d) + (826125*d^7*e^3 - 10957975*d^6*e^4 + 8449735*d^5*e^5 + 82111
75*d^4*e^6 - 7879025*d^3*e^7 + 2996315*d^2*e^8 - 443947*d*e^9 - 67267*e^10)
*e^(-2)/(x*e + d)^2 - (275375*d^8*e^4 - 3975600*d^7*e^5 + 3752280*d^6*e^6 +
 2119880*d^5*e^7 - 3655050*d^4*e^8 + 4008480*d^3*e^9 - 1453312*d^2*e^10 - 1
97784*d*e^11 + 66483*e^12)*e^(-3)/(x*e + d)^3 + 17525*e^6)/((5*d^2 - 2*d*e
+ 3*e^2)^4*(10*d/(x*e + d) - 5*d^2/(x*e + d)^2 - 2*e/(x*e + d) + 2*d*e/(x*e
 + d)^2 - 3*e^2/(x*e + d)^2 - 5)^2)
```

Mupad [B]
time = 4.99, size = 965, normalized size = 2.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)^3),x)
```

```
[Out] log(d + e*x)*((2*e^3*(620*d - 2417*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^3) - (6
*e^5*(423*d - 1367*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^4) + (e*(8*d + 23*e))/(
5*(5*d^2 - 2*d*e + 3*e^2)^2)) - ((3*x*(77965*d^4*e - 19493*d*e^4 - 29965*d^
5 + 13245*e^5 + 21522*d^2*e^3 + 51590*d^3*e^2))/(1568*(125*d^6 - 150*d^5*e
- 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) - (12009*d*
e^4 - 95100*d^4*e + 64765*d^5 - 28224*e^5 + 22292*d^2*e^3 - 200706*d^3*e^2)
/(1568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3
 + 285*d^4*e^2)) + (5*x^4*(20345*d^4*e - 18188*d*e^4 + 19269*e^5 - 11178*d^
2*e^3 + 125124*d^3*e^2))/(1568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 1
71*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x^3*(361295*d^4*e - 138539*d*e^
4 - 55075*d^5 + 93087*e^5 + 173446*d^2*e^3 + 272442*d^3*e^2))/(1568*(125*d^
6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2
)) + (x^2*(412485*d^4*e - 144973*d*e^4 - 193765*d^5 + 131589*e^5 + 56850*d^
```

2*e^3 + 621062*d^3*e^2))/(1568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)))/(9*d + x^2*(34*d + 12*e) + x^4*(25*d + 20*e) + x^3*(20*d + 34*e) + 25*e*x^5 + x*(12*d + 9*e)) + (log(x - (14^(1/2)*1i)/5 + 1/5)*((211875*14^(1/2)*d^6)/43904 - e^6*((43695*14^(1/2))/43904 - 9i/2) - d^3*e^3*((230361*14^(1/2))/10976 + 6i) + d^4*e^2*((209039*14^(1/2))/43904 - 83i/2) + d^2*e^4*((380621*14^(1/2))/43904 + 38i) + d^5*e*((1535*14^(1/2))/21952 - 20i) - d*e^5*((24793*14^(1/2))/21952 + 23i)))/(d^8*625i - d^7*e*1000i - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*e^5*1176i + d^4*e^4*2086i - d^5*e^3*1960i + d^6*e^2*2100i) - (log(x + (14^(1/2)*1i)/5 + 1/5)*((211875*14^(1/2)*d^6)/43904 - e^6*((43695*14^(1/2))/43904 + 9i/2) - d^3*e^3*((230361*14^(1/2))/10976 - 6i) + d^4*e^2*((209039*14^(1/2))/43904 + 83i/2) + d^2*e^4*((380621*14^(1/2))/43904 - 38i) + d^5*e*((1535*14^(1/2))/21952 + 20i) - d*e^5*((24793*14^(1/2))/21952 - 23i)))/(d^8*625i - d^7*e*1000i - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*e^5*1176i + d^4*e^4*2086i - d^5*e^3*1960i + d^6*e^2*2100i)

3.324 $\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$

Optimal. Leaf size=143

$$-\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768}+\frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480}-\frac{823(5+2x)^3\left(3-x+2x^2\right)^{3/2}}{1344}+\frac{5}{112}(5+2$$

```
[Out] 11433/4480*(5+2*x)^2*(2*x^2-x+3)^(3/2)-823/1344*(5+2*x)^3*(2*x^2-x+3)^(3/2)
+5/112*(5+2*x)^4*(2*x^2-x+3)^(3/2)-1/71680*(1005757+295276*x)*(2*x^2-x+3)^(
3/2)-1183005/131072*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-51435/32768*(1-4
*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1667, 793, 626, 633, 221}

$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4-\frac{823\left(2x^2-x+3\right)^{3/2}(2x+5)^3}{1344}+\frac{11433\left(2x^2-x+3\right)^{3/2}(2x+5)^2}{4480}-\frac{(295276x+1005757)\left(2x^2-x+3\right)^{3/2}}{71680}-\frac{51435(1-4x)\sqrt{2x^2-x+3}}{32768}-\frac{1183005\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x)*Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-51435*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/32768 + (11433*(5 + 2*x)^2*(3 - x +
2*x^2)^(3/2))/4480 - (823*(5 + 2*x)^3*(3 - x + 2*x^2)^(3/2))/1344 + (5*(5 +
 2*x)^4*(3 - x + 2*x^2)^(3/2))/112 - ((1005757 + 295276*x)*(3 - x + 2*x^2)^
(3/2))/71680 - (1183005*ArcSinh[(1 - 4*x)/Sqrt[23]])/(65536*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx = \frac{5}{112}(5+2x)^4\left(3-x+2x^2\right)^{3/2}+\frac{1}{224}\int (5+2x)$$

$$= -\frac{823(5+2x)^3\left(3-x+2x^2\right)^{3/2}}{1344}+\frac{5}{112}(5+2x)^4$$

$$= \frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480}-\frac{823(5+2x)^3}{1}$$

$$= \frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480}-\frac{823(5+2x)^3}{1}$$

$$= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768}+\frac{11433(5+2x)}{4}$$

$$= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768}+\frac{11433(5+2x)}{4}$$

$$= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768}+\frac{11433(5+2x)}{4}$$

Mathematica [A]
time = 0.86, size = 80, normalized size = 0.56

$$\frac{4\sqrt{3-x+2x^2}\,(6231117+14742332x+11357024x^2+20304768x^3+1390592x^4+12984320x^5+4915200x^6)-124215525\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{13762560}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x)*Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(6231117 + 14742332*x + 11357024*x^2 + 20304768*x^3
+ 1390592*x^4 + 12984320*x^5 + 4915200*x^6) - 124215525*Sqrt[2]*Log[1 - 4*x
 + 2*Sqrt[6 - 2*x + 4*x^2]])/13762560
```

Maple [A]
time = 0.13, size = 115, normalized size = 0.80

method	result
risch	$\frac{(4915200x^6+12984320x^5+1390592x^4+20304768x^3+11357024x^2+14742332x+6231117)\sqrt{2x^2-x+3}}{3440640}+\frac{1183005\sqrt{2}\ \text{arcsi}}{13}$
trager	$\left(\frac{10}{7}x^6+\frac{317}{84}x^5+\frac{97}{240}x^4+\frac{52877}{8960}x^3+\frac{50701}{15360}x^2+\frac{3685583}{860160}x+\frac{2077039}{1146880}\right)\sqrt{2x^2-x+3}-\frac{1183005\,\text{RootOf}\left(\_Z\right.}{}$
default	$\frac{242329(2x^2-x+3)^{\frac{3}{2}}}{215040}+\frac{51435(4x-1)\sqrt{2x^2-x+3}}{32768}+\frac{1183005\sqrt{2}\ \text{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{131072}-\frac{5179x(2x^2-x+3)^{\frac{3}{2}}}{17920}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 242329/215040*(2*x^2-x+3)^(3/2)+51435/32768*(4*x-1)*(2*x^2-x+3)^(1/2)+11830
05/131072*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-5179/17920*x*(2*x^2-x+3)^(
3/2)+283/1120*x^2*(2*x^2-x+3)^(3/2)+377/168*x^3*(2*x^2-x+3)^(3/2)+5/7*x^4*(
2*x^2-x+3)^(3/2)
```

Maxima [A]
time = 0.53, size = 126, normalized size = 0.88

$$\frac{5}{7}\,(2\,x^2-x+3)^{\frac{3}{2}}x^4+\frac{377}{168}\,(2\,x^2-x+3)^{\frac{3}{2}}x^3+\frac{283}{1120}\,(2\,x^2-x+3)^{\frac{3}{2}}x^2-\frac{5179}{17920}\,(2\,x^2-x+3)^{\frac{3}{2}}x+\frac{242329}{215040}\,(2\,x^2-x+3)^{\frac{3}{2}}+\frac{51435}{8192}\,\sqrt{2\,x^2-x+3}\,x+\frac{1183005}{131072}\,\sqrt{2}\ \text{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{51435}{32768}\,\sqrt{2\,x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="max
ima")
```

```
[Out] 5/7*(2*x^2 - x + 3)^(3/2)*x^4 + 377/168*(2*x^2 - x + 3)^(3/2)*x^3 + 283/112
0*(2*x^2 - x + 3)^(3/2)*x^2 - 5179/17920*(2*x^2 - x + 3)^(3/2)*x + 242329/2
```

```
15040*(2*x^2 - x + 3)^(3/2) + 51435/8192*sqrt(2*x^2 - x + 3)*x + 1183005/13
1072*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 51435/32768*sqrt(2*x^2 - x
+ 3)
```

Fricas [A]
time = 0.35, size = 83, normalized size = 0.58

$$\frac{1}{3440640}\left(4915200\,x^{6}+12984320\,x^{5}+1390592\,x^{4}+20304768\,x^{3}+11357024\,x^{2}+14742332\,x+6231117\right)\sqrt{2\,x^{2}-x+3}+\frac{1183005}{262144}\,\sqrt{2}\,\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^{2}-x+3}\,(4\,x-1)-32\,x^{2}+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/3440640*(4915200*x^6 + 12984320*x^5 + 1390592*x^4 + 20304768*x^3 + 113570
24*x^2 + 14742332*x + 6231117)*sqrt(2*x^2 - x + 3) + 1183005/262144*sqrt(2)
*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+5)\sqrt{2x^2-x+3}\cdot\left(5x^4-x^3+3x^2+x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((2*x + 5)*sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2), x
)
```

Giac [A]
time = 4.08, size = 78, normalized size = 0.55

$$\frac{1}{3440640}\left(4\left(8\left(4\left(16\left(20\left(120\,x+317\right)x+679\right)x+158631\right)x+354907\right)x+3685583\right)x+6231117\right)\sqrt{2\,x^{2}-x+3}-\frac{1183005}{131072}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^{2}-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/3440640*(4*(8*(4*(16*(20*(120*x + 317)*x + 679)*x + 158631)*x + 354907)*x
 + 3685583)*x + 6231117)*sqrt(2*x^2 - x + 3) - 1183005/131072*sqrt(2)*log(-
2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [B]
time = 1.72, size = 170, normalized size = 1.19

$$\frac{283\,x^{2}\,{\left(2\,x^{2}-x+3\right)}^{3/2}}{1120}+\frac{377\,x^{3}\,{\left(2\,x^{2}-x+3\right)}^{3/2}}{168}+\frac{5\,x^{4}\,{\left(2\,x^{2}-x+3\right)}^{3/2}}{7}+\frac{4478951\,\sqrt{2}\,\ln\left(\sqrt{2\,x^{2}-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{573440}+\frac{194737\,\left(\frac{x}{2}-\frac{1}{8}\right)\,\sqrt{2\,x^{2}-x+3}}{17920}+\frac{242329\,\sqrt{2\,x^{2}-x+3}\,\left(32\,x^{2}-4\,x+45\right)}{3440640}-\frac{5179\,x\,{\left(2\,x^{2}-x+3\right)}^{3/2}}{17920}+\frac{5573567\,\sqrt{2}\,\ln\left(2\,\sqrt{2\,x^{2}-x+3}+\frac{\sqrt{2}\,\left(4\,x-1\right)}{2}\right)}{4587520}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5)*(2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

```
[Out] (283*x^2*(2*x^2 - x + 3)^(3/2))/1120 + (377*x^3*(2*x^2 - x + 3)^(3/2))/168
+ (5*x^4*(2*x^2 - x + 3)^(3/2))/7 + (4478951*2^(1/2)*log((2*x^2 - x + 3)^(1
/2) + (2^(1/2)*(2*x - 1/2))/2))/573440 + (194737*(x/2 - 1/8)*(2*x^2 - x + 3
)^(1/2))/17920 + (242329*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*x + 45))/3440640
 - (5179*x*(2*x^2 - x + 3)^(3/2))/17920 + (5573567*2^(1/2)*log(2*(2*x^2 - x
 + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/4587520
```

3.325 $\int \sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)\,dx$

Optimal. Leaf size=124

$$-\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384}+\frac{287(3-x+2x^2)^{3/2}}{5120}-\frac{71x(3-x+2x^2)^{3/2}}{1280}+\frac{7}{80}x^2(3-x+2x^2)^{3/2}+\frac{5}{12}x^3(3$$

```
[Out] 287/5120*(2*x^2-x+3)^(3/2)-71/1280*x*(2*x^2-x+3)^(3/2)+7/80*x^2*(2*x^2-x+3)
^(3/2)+5/12*x^3*(2*x^2-x+3)^(3/2)-106007/65536*arcsinh(1/23*(1-4*x)*23^(1/2
))*2^(1/2)-4609/16384*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1675, 654, 626, 633, 221}

$$\frac{7}{80}(2x^2-x+3)^{3/2}x^2-\frac{71(2x^2-x+3)^{3/2}x}{1280}+\frac{287(2x^2-x+3)^{3/2}}{5120}-\frac{4609(1-4x)\sqrt{2x^2-x+3}}{16384}+\frac{5}{12}(2x^2-x+3)^{3/2}x^3-\frac{106007\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32768\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-4609*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16384 + (287*(3 - x + 2*x^2)^(3/2))/5
120 - (71*x*(3 - x + 2*x^2)^(3/2))/1280 + (7*x^2*(3 - x + 2*x^2)^(3/2))/80
+ (5*x^3*(3 - x + 2*x^2)^(3/2))/12 - (106007*ArcSinh[(1 - 4*x)/Sqrt[23]])/(
32768*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)dx=\frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2}+\frac{1}{12}\int \sqrt{3-x+2x^2}\left(24+12x-\right.$$

$$=\frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2}+\frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2}+\frac{1}{120}\int\left(\right.$$

$$=-\frac{71x(3-x+2x^2)^{3/2}}{1280}+\frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2}+\frac{5}{12}x^3\left(3-\right.$$

$$=\frac{287(3-x+2x^2)^{3/2}}{5120}-\frac{71x(3-x+2x^2)^{3/2}}{1280}+\frac{7}{80}x^2\left(3-x\right.$$

$$=-\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384}+\frac{287(3-x+2x^2)^{3/2}}{5120}-\frac{71}{}$$

$$=-\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384}+\frac{287(3-x+2x^2)^{3/2}}{5120}-\frac{71}{}$$

$$=-\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384}+\frac{287(3-x+2x^2)^{3/2}}{5120}-\frac{71}{}$$

Mathematica [A]
time = 0.38, size = 75, normalized size = 0.60

$$\frac{4\sqrt{3-x+2x^2}\left(-27807+221868x+105696x^2+258432x^3-59392x^4+204800x^5\right)-1590105\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{983040}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-27807 + 221868*x + 105696*x^2 + 258432*x^3 - 59392
*x^4 + 204800*x^5) - 1590105*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]]
)/983040
```

Maple [A]
time = 0.11, size = 98, normalized size = 0.79

method	result
risch	$\frac{(204800x^5-59392x^4+258432x^3+105696x^2+221868x-27807)\sqrt{2x^2-x+3}}{245760}+\frac{106007\sqrt{2}\,\mathrm{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{65536}$
trager	$\left(\frac{5}{6}x^5-\frac{29}{120}x^4+\frac{673}{640}x^3+\frac{1101}{2560}x^2+\frac{18489}{20480}x-\frac{9269}{81920}\right)\sqrt{2x^2-x+3}+\frac{106007\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\mathrm{RootOf}\right.}{}$
default	$\frac{5x^3(2x^2-x+3)^{\frac{3}{2}}}{12}+\frac{7x^2(2x^2-x+3)^{\frac{3}{2}}}{80}-\frac{71x(2x^2-x+3)^{\frac{3}{2}}}{1280}+\frac{287(2x^2-x+3)^{\frac{3}{2}}}{5120}+\frac{4609(4x-1)\sqrt{2x^2-x+3}}{16384}+\frac{106007}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/12*x^3*(2*x^2-x+3)^(3/2)+7/80*x^2*(2*x^2-x+3)^(3/2)-71/1280*x*(2*x^2-x+3)
^(3/2)+287/5120*(2*x^2-x+3)^(3/2)+4609/16384*(4*x-1)*(2*x^2-x+3)^(1/2)+1060
07/65536*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [A]
time = 0.49, size = 109, normalized size = 0.88

$$\frac{5}{12}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{7}{80}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{71}{1280}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{287}{5120}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{4609}{4096}\sqrt{2x^2-x+3}\,x+\frac{106007}{65536}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{4609}{16384}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/12*(2*x^2 - x + 3)^(3/2)*x^3 + 7/80*(2*x^2 - x + 3)^(3/2)*x^2 - 71/1280*(
2*x^2 - x + 3)^(3/2)*x + 287/5120*(2*x^2 - x + 3)^(3/2) + 4609/4096*sqrt(2*
x^2 - x + 3)*x + 106007/65536*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 46
09/16384*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 0.38, size = 78, normalized size = 0.63

$$\frac{1}{245760}\left(204800x^5-59392x^4+258432x^3+105696x^2+221868x-27807\right)\sqrt{2x^2-x+3}+\frac{106007}{131072}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/245760*(204800*x^5 - 59392*x^4 + 258432*x^3 + 105696*x^2 + 221868*x - 278
07)*sqrt(2*x^2 - x + 3) + 106007/131072*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 -
 x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x^2 - x + 3} \cdot \left(5x^4 - x^3 + 3x^2 + x + 2\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2), x)
```

Giac [A]
time = 2.70, size = 73, normalized size = 0.59

$$\frac{1}{245760}\,(4\,(8\,(4\,(16\,(100\,x-29)x+2019)x+3303)x+55467)x-27807)\sqrt{2\,x^2-x+3}-\frac{106007}{65536}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\,\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/245760*(4*(8*(4*(16*(100*x - 29)*x + 2019)*x + 3303)*x + 55467)*x - 27807
)*sqrt(2*x^2 - x + 3) - 106007/65536*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sq
rt(2*x^2 - x + 3)) + 1)
```

Mupad [B]
time = 0.77, size = 153, normalized size = 1.23

$$\frac{7\,x^2\,{\left(2\,x^2-x+3\right)}^{3/2}}{80}+\frac{5\,x^3\,{\left(2\,x^2-x+3\right)}^{3/2}}{12}+\frac{63779\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{40960}+\frac{2773\,\left(\frac{x}{2}-\frac{1}{8}\right)\,\sqrt{2\,x^2-x+3}}{1280}+\frac{287\,\sqrt{2\,x^2-x+3}\,\left(32\,x^2-4\,x+45\right)}{81920}-\frac{71\,x\,{\left(2\,x^2-x+3\right)}^{3/2}}{1280}+\frac{19803\,\sqrt{2}\,\ln\left(2\,\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(4\,x-1\right)}{2}\right)}{327680}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

```
[Out] (7*x^2*(2*x^2 - x + 3)^(3/2))/80 + (5*x^3*(2*x^2 - x + 3)^(3/2))/12 + (6377
9*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/40960 + (27
73*(x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/1280 + (287*(2*x^2 - x + 3)^(1/2)*(32
*x^2 - 4*x + 45))/81920 - (71*x*(2*x^2 - x + 3)^(3/2))/1280 + (19803*2^(1/2
)*log(2*(2*x^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/327680
```

3.326 $\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$

Optimal. Leaf size=149

$$\frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096}+\frac{4535}{768}\left(3-x+2x^2\right)^{3/2}-\frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2}+\frac{1}{16}(5+2x)^2\left(3-\right.$$

```
[Out] 4535/768*(2*x^2-x+3)^(3/2)-127/128*(5+2*x)*(2*x^2-x+3)^(3/2)+1/16*(5+2*x)^2
*(2*x^2-x+3)^(3/2)+5627989/16384*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-110
01/32*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/4096*(489
587-80844*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1667, 828, 857, 633, 221, 738, 212}

$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2-\frac{127}{128}\left(2x^2-x+3\right)^{3/2}(2x+5)+\frac{4535}{768}\left(2x^2-x+3\right)^{3/2}+\frac{(489587-80844x)\sqrt{2x^2-x+3}}{4096}-\frac{11001\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{16\sqrt{2}}+\frac{5627989\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

```
[Out] ((489587 - 80844*x)*Sqrt[3 - x + 2*x^2])/4096 + (4535*(3 - x + 2*x^2)^(3/2)
)/768 - (127*(5 + 2*x)*(3 - x + 2*x^2)^(3/2))/128 + ((5 + 2*x)^2*(3 - x + 2
*x^2)^(3/2))/16 + (5627989*ArcSinh[(1 - 4*x)/Sqrt[23]])/(8192*Sqrt[2]) - (1
1001*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(16*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{1}{16}(5+2x)^2\left(3-x+2x^2\right)^{3/2} + \frac{1}{160}\int \frac{\sqrt{3-x+2x^2}\,(-8$$

$$= -\frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2} + \frac{1}{16}(5+2x)^2\left(3-x+2\right.$$

$$= \frac{4535}{768}\left(3-x+2x^2\right)^{3/2} - \frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2} +$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096} + \frac{4535}{768}\left(3-x+2x^2\right)^{3/}$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096} + \frac{4535}{768}\left(3-x+2x^2\right)^{3/}$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096} + \frac{4535}{768}\left(3-x+2x^2\right)^{3/}$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096} + \frac{4535}{768}\left(3-x+2x^2\right)^{3/}$$

Mathematica [A]
time = 0.43, size = 103, normalized size = 0.69

$$\frac{4\sqrt{3-x+2x^2}\,(1561161-300404x+79840x^2-21120x^3+6144x^4)+33795072\sqrt{2}\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)+16883967\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{49152}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
[Out] (4*Sqrt[3 - x + 2*x^2]*(1561161 - 300404*x + 79840*x^2 - 21120*x^3 + 6144*x
^4) + 33795072*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 16883
967*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/49152
```

Maple [A]
time = 0.02, size = 127, normalized size = 0.85

$$\frac{x^2(2x^2-x+3)^{\frac{3}{2}}}{4} - \frac{47x(2x^2-x+3)^{\frac{3}{2}}}{64} + \frac{1925(2x^2-x+3)^{\frac{3}{2}}}{768} - \frac{20211(4x-1)\sqrt{2x^2-x+3}}{4096} - \frac{5627989\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x)
```

```
[Out] 1/4*x^2*(2*x^2-x+3)^(3/2)-47/64*x*(2*x^2-x+3)^(3/2)+1925/768*(2*x^2-x+3)^(3
/2)-20211/4096*(4*x-1)*(2*x^2-x+3)^(1/2)-5627989/16384*2^(1/2)*arcsinh(4/23
*23^(1/2)*(x-1/4))+3667/32*(2*(x+5/2)^2-11*x-19/2)^(1/2)-11001/32*2^(1/2)*a
rctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Maxima [A]
time = 0.48, size = 128, normalized size = 0.86

$$\frac{1}{4}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}x^{2}-\frac{47}{64}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}x+\frac{1925}{768}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}-\frac{20211}{1024}\,\sqrt{2\,x^{2}-x+3}\,x-\frac{5627989}{16384}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\,\sqrt{23}\,x-\frac{1}{23}\,\sqrt{23}\right)+\frac{11001}{32}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\,\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\,\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{489587}{4096}\,\sqrt{2\,x^{2}-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="max
ima")
```

```
[Out] 1/4*(2*x^2 - x + 3)^(3/2)*x^2 - 47/64*(2*x^2 - x + 3)^(3/2)*x + 1925/768*(2
*x^2 - x + 3)^(3/2) - 20211/1024*sqrt(2*x^2 - x + 3)*x - 5627989/16384*sqrt
(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 11001/32*sqrt(2)*arcsinh(22/
23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 489587/4096*sqr
t(2*x^2 - x + 3)
```

Fricas [A]
time = 0.43, size = 125, normalized size = 0.84

$$\frac{1}{12288}\left(6144\,x^{4}-21120\,x^{3}+79840\,x^{2}-300404\,x+1561161\right)\sqrt{2\,x^{2}-x+3}+\frac{5627989}{32768}\,\sqrt{2}\,\log\left(4\,\sqrt{2}\,\sqrt{2\,x^{2}-x+3}\,(4\,x-1)-32\,x^{2}+16\,x-25\right)+\frac{11001}{64}\,\sqrt{2}\,\log\left(-\frac{24\,\sqrt{2}\,\sqrt{2\,x^{2}-x+3}\,(22\,x-17)+1060\,x^{2}-1036\,x+1153}{4\,x^{2}+20\,x+25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="fri
cas")
```

```
[Out] 1/12288*(6144*x^4 - 21120*x^3 + 79840*x^2 - 300404*x + 1561161)*sqrt(2*x^2
- x + 3) + 5627989/32768*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1
) - 32*x^2 + 16*x - 25) + 11001/64*sqrt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x
+ 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^{2}-x+3}\cdot\left(5x^{4}-x^{3}+3x^{2}+x+2\right)}{2x+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5), x
)
```

Giac [A]
time = 3.15, size = 129, normalized size = 0.87

$$\frac{1}{12288}\left(4\left(8\left(12\left(16x-55\right)x+2495\right)x-75101\right)x+1561161\right)\sqrt{2x^2-x+3}+\frac{5627989}{16384}\sqrt{2}\log\left(-4\sqrt{2}x+\sqrt{2}+4\sqrt{2x^2-x+3}\right)-\frac{11001}{32}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{11001}{32}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="gia
c")
```

```
[Out] 1/12288*(4*(8*(12*(16*x - 55)*x + 2495)*x - 75101)*x + 1561161)*sqrt(2*x^2
- x + 3) + 5627989/16384*sqrt(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2
- x + 3)) - 11001/32*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2
- x + 3))) + 11001/32*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*
x^2 - x + 3)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{2\,x+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5),x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5), x)
```

3.327 $\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx$

Optimal. Leaf size=149

$$-\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432}-\frac{541}{384}\left(3-x+2x^2\right)^{3/2}-\frac{3667(3-x+2x^2)^{3/2}}{576(5+2x)}+\frac{5}{64}(5+2x)\left(3-x+2x\right.$$

```
[Out] -541/384*(2*x^2-x+3)^(3/2)-3667/576*(2*x^2-x+3)^(3/2)/(5+2*x)+5/64*(5+2*x)*
(2*x^2-x+3)^(3/2)-2551847/8192*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+23920
1/768*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-1/18432*(19
96953-333380*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1664, 1667, 828, 857, 633, 221, 738, 212}

$$\frac{5}{64}(2x+5)\left(2x^2-x+3\right)^{3/2}-\frac{3667(2x^2-x+3)^{3/2}}{576(2x+5)}-\frac{541}{384}\left(2x^2-x+3\right)^{3/2}-\frac{(1996953-333380x)\sqrt{2x^2-x+3}}{18432}+\frac{239201\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{384\sqrt{2}}-\frac{2551847\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x]
```

```
[Out] -1/18432*((1996953 - 333380*x)*Sqrt[3 - x + 2*x^2]) - (541*(3 - x + 2*x^2)^
(3/2))/384 - (3667*(3 - x + 2*x^2)^(3/2))/(576*(5 + 2*x)) + (5*(5 + 2*x)*(3
 - x + 2*x^2)^(3/2))/64 - (2551847*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4096*Sqrt[
2]) + (239201*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(384*S
qrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
```

```
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = -\frac{3667(3-x+2x^2)^{3/2}}{576(5+2x)} - \frac{1}{72}\int \frac{\sqrt{3-x+2x^2}\,\left(\frac{19341}{16}-\frac{631\ldots}{2}\right.}{5+2x\ldots}$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{576(5+2x)} + \frac{5}{64}(5+2x)\left(3-x+2x^2\right)^{3/2} - \int\ldots$$

$$= -\frac{541}{384}\left(3-x+2x^2\right)^{3/2} - \frac{3667(3-x+2x^2)^{3/2}}{576(5+2x)} + \frac{5}{64}(5+2\ldots$$

$$= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}\left(3-x+2x^2\right)\ldots$$

$$= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}\left(3-x+2x^2\right)\ldots$$

$$= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}\left(3-x+2x^2\right)\ldots$$

$$= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}\left(3-x+2x^2\right)\ldots$$

Mathematica [A]
time = 0.49, size = 110, normalized size = 0.74

$$\frac{\frac{4\sqrt{3-x+2x^2}\,(-3539439-728410x+94936x^2-17344x^3+3840x^4)}{5+2x} - 15308864\sqrt{2}\,\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right) - 7655541\sqrt{2}\,\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{24576}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x
]
```

```
[Out] ((4*Sqrt[3 - x + 2*x^2]*(-3539439 - 728410*x + 94936*x^2 - 17344*x^3 + 3840
*x^4))/(5 + 2*x) - 15308864*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2
])/6] - 7655541*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/24576
```

Maple [A]
time = 0.01, size = 152, normalized size = 1.02

$$\frac{5x(2x^2-x+3)^{\frac{3}{2}}}{32}-\frac{391(2x^2-x+3)^{\frac{3}{2}}}{384}+\frac{6001(4x-1)\sqrt{2x^2-x+3}}{2048}+\frac{2551847\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\ (x-\frac{1}{4})}{23}\right)}{8192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x)
```

```
[Out] 5/32*x*(2*x^2-x+3)^(3/2)-391/384*(2*x^2-x+3)^(3/2)+6001/2048*(4*x-1)*(2*x^2
-x+3)^(1/2)+2551847/8192*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-239201/2304
*(2*(x+5/2)^2-11*x-19/2)^(1/2)+239201/768*2^(1/2)*arctanh(1/12*(17/2-11*x)*
2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-3667/1152/(x+5/2)*(2*(x+5/2)^2-11*x-
19/2)^(3/2)+3667/2304*(4*x-1)*(2*(x+5/2)^2-11*x-19/2)^(1/2)
```

Maxima [A]
time = 0.51, size = 132, normalized size = 0.89

$$\frac{5}{32}\,{\left(2\,x^{2}-x+3\right)}^{\frac{3}{2}}x-\frac{391}{384}\,{\left(2\,x^{2}-x+3\right)}^{\frac{3}{2}}+\frac{6001}{512}\,\sqrt{2\,x^{2}-x+3}\,x+\frac{2551847}{8192}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\,\sqrt{23}\,x-\frac{1}{23}\,\sqrt{23}\right)-\frac{239201}{768}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\,\sqrt{23}\,x}{23\,{\left|2\,x+5\right|}}-\frac{17\,\sqrt{23}}{23\,{\left|2\,x+5\right|}}\right)-\frac{182769}{2048}\,\sqrt{2\,x^{2}-x+3}-\frac{3667\,\sqrt{2\,x^{2}-x+3}}{32\,{\left(2\,x+5\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="m
axima")
```

```
[Out] 5/32*(2*x^2 - x + 3)^(3/2)*x - 391/384*(2*x^2 - x + 3)^(3/2) + 6001/512*sqr
t(2*x^2 - x + 3)*x + 2551847/8192*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sq
rt(23)) - 239201/768*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*
sqrt(23)/abs(2*x + 5)) - 182769/2048*sqrt(2*x^2 - x + 3) - 3667/32*sqrt(2*x
^2 - x + 3)/(2*x + 5)
```

Fricas [A]
time = 0.40, size = 143, normalized size = 0.96

$$\frac{7655541\,\sqrt{2}\,{\left(2\,x+5\right)}\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^{2}-x+3}\,{\left(4\,x-1\right)}-32\,x^{2}+16\,x-25\right)+7654432\,\sqrt{2}\,{\left(2\,x+5\right)}\log\left(\frac{24\,\sqrt{2}\,\sqrt{2\,x^{2}-x+3}\,{\left(22\,x-17\right)}-1060\,x^{2}+1036\,x-1153}{4\,x^{2}+20\,x+25}\right)+8\,{\left(3840\,x^{4}-17344\,x^{3}+94936\,x^{2}-728410\,x-3539439\right)}\sqrt{2\,x^{2}-x+3}}{49152\,{\left(2\,x+5\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="fricas")

[Out] 1/49152*(7655541*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 7654432*sqrt(2)*(2*x + 5)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 8*(3840*x^4 - 17344*x^3 + 94936*x^2 - 728410*x - 3539439)*sqrt(2*x^2 - x + 3))/(2*x + 5)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2-x+3}\cdot\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x+5\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**2,x)

[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**2, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(118) = 236.
time = 4.24, size = 531, normalized size = 3.56

$$\frac{1}{24576}\sqrt{2}\left(7654432\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)+7655541\log\left(\left|\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}+1\right|\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)-7655541\log\left(\left|\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}-1\right|\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)-1408128\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}\operatorname{sgn}\left(\frac{1}{2x+5}\right)+\frac{2\left(16367883\left(\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}\right)^7\operatorname{sgn}\left(\frac{1}{2x+5}\right)-34896384\left(\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}\right)^6\operatorname{sgn}\left(\frac{1}{2x+5}\right)-93395\left(\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}\right)^5\operatorname{sgn}\left(\frac{1}{2x+5}\right)+25574400\left(\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}\right)^4\operatorname{sgn}\left(\frac{1}{2x+5}\right)+19752365\left(\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}\right)^3\operatorname{sgn}\left(\frac{1}{2x+5}\right)-31921920\left(\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}\right)^2\operatorname{sgn}\left(\frac{1}{2x+5}\right)-2445813\left(\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)+7663104\operatorname{sgn}\left(\frac{1}{2x+5}\right)\right)}{\left(\left(\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}\right)^2-1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="giac")

[Out] 1/24576*sqrt(2)*(7654432*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 7655541*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 7655541*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*sgn(1/(2*x + 5)) - 1408128*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)*sgn(1/(2*x + 5)) + 2*(16367883*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^7*sgn(1/(2*x + 5)) - 34896384*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^6*sgn(1/(2*x + 5)) - 93395*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^5*sgn(1/(2*x + 5)) + 25574400*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^4*sgn(1/(2*x + 5)) + 19752365*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3*sgn(1/(2*x + 5)) - 31921920*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2*sgn(1/(2*x + 5)) - 2445813*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x + 5)) + 7663104*sgn(1/(2*x + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^4)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2,x)
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2, x)
```

3.328 $\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$

Optimal. Leaf size=151

$$\frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944}+\frac{5}{48}\left(3-x+2x^2\right)^{3/2}-\frac{3667\left(3-x+2x^2\right)^{3/2}}{1152(5+2x)^2}+\frac{357391\left(3-x+2x^2\right)^{3/2}}{82944(5+2x)}+$$

```
[Out] 5/48*(2*x^2-x+3)^(3/2)-3667/1152*(2*x^2-x+3)^(3/2)/(5+2*x)^2+357391/82944*(
2*x^2-x+3)^(3/2)/(5+2*x)+117315/1024*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
-12670805/110592*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+
5/82944*(661065-110099*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1664, 1667, 828, 857, 633, 221, 738, 212}

$$\frac{357391\left(2x^2-x+3\right)^{3/2}}{82944(2x+5)}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{1152(2x+5)^2}+\frac{5}{48}\left(2x^2-x+3\right)^{3/2}+\frac{5(661065-110099x)\sqrt{2x^2-x+3}}{82944}-\frac{12670805\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{55296\sqrt{2}}+\frac{117315\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{512\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x]
```

```
[Out] (5*(661065 - 110099*x)*Sqrt[3 - x + 2*x^2])/82944 + (5*(3 - x + 2*x^2)^(3/2
))/48 - (3667*(3 - x + 2*x^2)^(3/2))/(1152*(5 + 2*x)^2) + (357391*(3 - x +
2*x^2)^(3/2))/(82944*(5 + 2*x)) + (117315*ArcSinh[(1 - 4*x)/Sqrt[23]])/(512
*Sqrt[2]) - (12670805*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])]
)/(55296*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
```

```
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = -\frac{3667(3-x+2x^2)^{3/2}}{1152(5+2x)^2} - \frac{1}{144}\int \frac{\sqrt{3-x+2x^2}\,\left(\frac{27681}{16} - \frac{14}{}\right.}{(5+2}$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{1152(5+2x)^2} + \frac{357391(3-x+2x^2)^{3/2}}{82944(5+2x)} + \frac{\int \sqrt{3}}{}$$

$$= \frac{5}{48}\left(3-x+2x^2\right)^{3/2} - \frac{3667(3-x+2x^2)^{3/2}}{1152(5+2x)^2} + \frac{357391(3-}{82944(5}$$

$$= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48}\left(3-x+2x^2\right)^{3/2}$$

$$= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48}\left(3-x+2x^2\right)^{3/2}$$

$$= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48}\left(3-x+2x^2\right)^{3/2}$$

$$= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48}\left(3-x+2x^2\right)^{3/2}$$

Mathematica [A]
time = 0.59, size = 110, normalized size = 0.73

$$\frac{\frac{12\sqrt{3-x+2x^2}\,(4880551+2959330x+272520x^2-25632x^3+3840x^4)}{(5+2x)^2} + 12670805\sqrt{2}\,\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right) + 6335010\sqrt{2}\,\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{55296}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x
]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(4880551 + 2959330*x + 272520*x^2 - 25632*x^3 + 38
40*x^4))/(5 + 2*x)^2 + 12670805*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4
*x^2])/6] + 6335010*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/55296
```

Maple [A]
time = 0.01, size = 158, normalized size = 1.05

$$\frac{5(2x^2-x+3)^{\frac{3}{2}}}{48}-\frac{149(4x-1)\sqrt{2x^2-x+3}}{256}-\frac{117315\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\ (x-\frac{1}{4})}{23}\right)}{1024}-\frac{3667\left(2\left(x+\frac{5}{2}\right)^2-11\right.}{4608\left(x+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x)
```

```
[Out] 5/48*(2*x^2-x+3)^(3/2)-149/256*(4*x-1)*(2*x^2-x+3)^(1/2)-117315/1024*2^(1/2
)*arcsinh(4/23*23^(1/2)*(x-1/4))-3667/4608/(x+5/2)^2*(2*(x+5/2)^2-11*x-19/2
)^(3/2)+357391/165888/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(3/2)+12670805/331776
*(2*(x+5/2)^2-11*x-19/2)^(1/2)-12670805/110592*2^(1/2)*arctanh(1/12*(17/2-1
1*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-357391/331776*(4*x-1)*(2*(x+5/2
)^2-11*x-19/2)^(1/2)
```

Maxima [A]
time = 0.49, size = 143, normalized size = 0.95

$$\frac{5}{48}\,{\left(2\,x^{2}-x+3\right)}^{\frac{3}{2}}-\frac{149}{64}\,\sqrt{2\,x^{2}-x+3}\,x-\frac{117315}{1024}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\,\sqrt{23}\,x-\frac{1}{23}\,\sqrt{23}\right)+\frac{12670805}{110592}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\,\sqrt{23}\,x}{23\,{\left|2\,x+5\right|}}-\frac{17\,\sqrt{23}}{23\,{\left|2\,x+5\right|}}\right)+\frac{3877}{144}\,\sqrt{2\,x^{2}-x+3}-\frac{3667\,{\left(2\,x^{2}-x+3\right)}^{\frac{3}{2}}}{1152\,{\left(4\,x^{2}+20\,x+25\right)}}+\frac{357391\,\sqrt{2\,x^{2}-x+3}}{4608\,{\left(2\,x+5\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="m
axima")
```

```
[Out] 5/48*(2*x^2 - x + 3)^(3/2) - 149/64*sqrt(2*x^2 - x + 3)*x - 117315/1024*sqr
t(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 12670805/110592*sqrt(2)*arc
sinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 3877/14
4*sqrt(2*x^2 - x + 3) - 3667/1152*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25)
 + 357391/4608*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

Fricas [A]
time = 0.36, size = 159, normalized size = 1.05

$$\frac{12670020\,\sqrt{2}\,{\left(4\,x^{2}+20\,x+25\right)}\log\left(4\,\sqrt{2}\,\sqrt{2\,x^{2}-x+3}\,{\left(4\,x-1\right)}-32\,x^{2}+16\,x-25\right)+12670805\,\sqrt{2}\,{\left(4\,x^{2}+20\,x+25\right)}\log\left(-\frac{24\,\sqrt{2}\,\sqrt{2\,x^{2}-x+3}\,{\left(22\,x-17\right)}+1060\,x^{2}-1036\,x+1153}{4\,x^{2}+20\,x+25}\right)+48\,{\left(3840\,x^{4}-25632\,x^{3}+272520\,x^{2}+2959330\,x+4880551\right)}\sqrt{2\,x^{2}-x+3}}{221184\,{\left(4\,x^{2}+20\,x+25\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="f
ricas")
```

```
[Out] 1/221184*(12670020*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 12670805*sqrt(2)*(4*x^2 + 20*x + 25
)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 11
53)/(4*x^2 + 20*x + 25)) + 48*(3840*x^4 - 25632*x^3 + 272520*x^2 + 2959330*
x + 4880551)*sqrt(2*x^2 - x + 3))/(4*x^2 + 20*x + 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3} \cdot (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**3,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**3
, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(120) = 240.
time = 5.25, size = 258, normalized size = 1.71

$$\frac{1}{768}\,(4\,(40\,x-467)x+19695)\sqrt{2\,x^2-x+3}+\frac{117315}{1024}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)-\frac{12670805}{110592}\,\sqrt{2}\,\log\left(\left|-2\,\sqrt{2}\,x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)+\frac{12670805}{110592}\,\sqrt{2}\,\log\left(\left|-2\,\sqrt{2}\,x-11\,\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)+\frac{\sqrt{2}\left(10693526\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^3+79895946\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^2-124044603\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+80334011\right)}{9216\left(2\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^2+10\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)-11\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="g
iac")
```

```
[Out] 1/768*(4*(40*x - 467)*x + 19695)*sqrt(2*x^2 - x + 3) + 117315/1024*sqrt(2)*
log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 12670805/110592*sqr
t(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 12670805/11
0592*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) +
1/9216*sqrt(2)*(10693526*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 7989
5946*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 124044603*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3)) + 80334011)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10
*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2 - x + 3}\,\left(5\,x^4 - x^3 + 3\,x^2 + x + 2\right)}{{\left(2\,x + 5\right)}^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3,x)
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3, x)
```

3.329 $\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$

Optimal. Leaf size=158

$$-\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968}-\frac{3667(3-x+2x^2)^{3/2}}{1728(5+2x)^3}+\frac{158527(3-x+2x^2)^{3/2}}{82944(5+2x)^2}-\frac{6467659(3-x+}{5971968(5+}$$

```
[Out] -3667/1728*(2*x^2-x+3)^(3/2)/(5+2*x)^3+158527/82944*(2*x^2-x+3)^(3/2)/(5+2*
x)^2-6467659/5971968*(2*x^2-x+3)^(3/2)/(5+2*x)-10939/512*arcsinh(1/23*(1-4*
x)*23^(1/2))*2^(1/2)+170114729/7962624*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^
2-x+3)^(1/2))*2^(1/2)-1/5971968*(44378877-7400779*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1664, 828, 857, 633, 221, 738, 212}

$$-\frac{6467659(2x^2-x+3)^{3/2}}{5971968(2x+5)}+\frac{158527(2x^2-x+3)^{3/2}}{82944(2x+5)^2}-\frac{3667(2x^2-x+3)^{3/2}}{1728(2x+5)^3}-\frac{(44378877-7400779x)\sqrt{2x^2-x+3}}{5971968}+\frac{170114729\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{3981312\sqrt{2}}-\frac{10939\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x]
```

```
[Out] -1/5971968*((44378877 - 7400779*x)*Sqrt[3 - x + 2*x^2]) - (3667*(3 - x + 2*
x^2)^(3/2))/(1728*(5 + 2*x)^3) + (158527*(3 - x + 2*x^2)^(3/2))/(82944*(5 +
 2*x)^2) - (6467659*(3 - x + 2*x^2)^(3/2))/(5971968*(5 + 2*x)) - (10939*Arc
Sinh[(1 - 4*x)/Sqrt[23]])/(256*Sqrt[2]) + (170114729*ArcTanh[(17 - 22*x)/(1
2*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(3981312*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = -\frac{3667(3-x+2x^2)^{3/2}}{1728(5+2x)^3} - \frac{1}{216}\int \frac{\sqrt{3-x+2x^2}\,\left(\frac{36021}{16} - 39\right.}{(5+2}$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{1728(5+2x)^3} + \frac{158527(3-x+2x^2)^{3/2}}{82944(5+2x)^2} + \frac{\int \sqrt{3}}{}$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{1728(5+2x)^3} + \frac{158527(3-x+2x^2)^{3/2}}{82944(5+2x)^2} - \frac{64676}{59}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667(3-x+2}{1728(5+2x}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667(3-x+2}{1728(5+2x}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667(3-x+2}{1728(5+2x}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667(3-x+2}{1728(5+2x}$$

Mathematica [A]
time = 0.68, size = 110, normalized size = 0.70

$$\frac{\frac{12\sqrt{3-x+2x^2}\,(-327735797-329667508x-97682900x^2-5453568x^3+414720x^4)}{(5+2x)^3} - 170114729\sqrt{2}\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right) - 85061664\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{3981312}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x
]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(-327735797 - 329667508*x - 97682900*x^2 - 5453568
*x^3 + 414720*x^4))/(5 + 2*x)^3 - 170114729*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt
[6 - 2*x + 4*x^2])/6] - 85061664*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x
^2]])/3981312
```

Maple [A]

time = 0.01, size = 165, normalized size = 1.04

$$\frac{5(4x-1)\sqrt{2x^2-x+3}}{128}+\frac{10939\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{512}+\frac{158527\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{331776\left(x+\frac{5}{2}\right)^2}-\frac{6467659}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x)
```

```
[Out] 5/128*(4*x-1)*(2*x^2-x+3)^(1/2)+10939/512*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-
1/4))+158527/331776/(x+5/2)^2*(2*(x+5/2)^2-11*x-19/2)^(3/2)-6467659/1194393
6/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(3/2)-170114729/23887872*(2*(x+5/2)^2-11*
x-19/2)^(1/2)+170114729/7962624*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2
*(x+5/2)^2-11*x-19/2)^(1/2))+6467659/23887872*(4*x-1)*(2*(x+5/2)^2-11*x-19/
2)^(1/2)-3667/13824/(x+5/2)^3*(2*(x+5/2)^2-11*x-19/2)^(3/2)
```

Maxima [A]
time = 0.50, size = 160, normalized size = 1.01

$$\frac{5}{32}\sqrt{2x^2-x+3}\,x+\frac{10939}{512}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{170114729}{7962624}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+5|}\right)-\frac{693775}{165888}\sqrt{2x^2-x+3}-\frac{3667\,(2x^2-x+3)^{\frac{3}{2}}}{1728\,(8x^3+60x^2+150x+125)}+\frac{158527\,(2x^2-x+3)^{\frac{3}{2}}}{82944\,(4x^2+20x+25)}-\frac{6467659\sqrt{2x^2-x+3}}{331776\,(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="m
axima")
```

```
[Out] 5/32*sqrt(2*x^2 - x + 3)*x + 10939/512*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/
23*sqrt(23)) - 170114729/7962624*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x +
 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 693775/165888*sqrt(2*x^2 - x + 3) - 36
67/1728*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 158527/82944
*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 6467659/331776*sqrt(2*x^2 - x
+ 3)/(2*x + 5)
```

Fricas [A]
time = 0.38, size = 173, normalized size = 1.09

$$\frac{170123328\sqrt{2}\,(8x^3+60x^2+150x+125)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)+170114729\sqrt{2}\,(8x^3+60x^2+150x+125)\log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}\,(22x-17)-1060x^2+1036x-1153}{4x^2+20x+25}\right)+48\,(414720x^4-5453568x^3-97682900x^2-329667508x-327735797)\sqrt{2x^2-x+3}}{15925248\,(8x^3+60x^2+150x+125)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="f
ricas")
```

```
[Out] 1/15925248*(170123328*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2)
*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 170114729*sqrt(2)*(8
```

```
*x^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17
) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(414720*x^4 - 54535
68*x^3 - 97682900*x^2 - 329667508*x - 327735797)*sqrt(2*x^2 - x + 3))/(8*x^
3 + 60*x^2 + 150*x + 125)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**4,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**4
, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(127) = 254.
time = 4.75, size = 304, normalized size = 1.92

$$\frac{1}{128}\sqrt{2x^2-x+3}\,(20x-413) - \frac{10939}{512}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right) + \frac{170114729}{7962624}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) - \frac{170114729}{7962624}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) - \frac{\sqrt{2}\left(575810908\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^5+9206213116\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^4+9688786604\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^3-73157325092\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+49481952947\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-20269228621\right)}{663552\left(2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-11\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="g
iac")
```

```
[Out] 1/128*sqrt(2*x^2 - x + 3)*(20*x - 413) - 10939/512*sqrt(2)*log(-2*sqrt(2)*(
sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 170114729/7962624*sqrt(2)*log(abs(-
2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 170114729/7962624*sqrt(2)
*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/663552*sqr
t(2)*(575810908*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 9206213116*(s
qrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 9688786604*sqrt(2)*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^3 - 73157325092*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 4948195
2947*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 20269228621)/(2*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) -
11)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4, x)
```

3.330 $\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$

Optimal. Leaf size=165

$$\frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)}-\frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4}+\frac{593771(3-x+2x^2)^{3/2}}{497664(5+2x)^3}-\frac{9363383(3-x\ldots}{23887872(5+\ldots}$$

```
[Out] -3667/2304*(2*x^2-x+3)^(3/2)/(5+2*x)^4+593771/497664*(2*x^2-x+3)^(3/2)/(5+2
*x)^3-9363383/23887872*(2*x^2-x+3)^(3/2)/(5+2*x)^2+259/128*arcsinh(1/23*(1-
4*x)*23^(1/2))*2^(1/2)-4640586097/2293235712*arctanh(1/24*(17-22*x)*2^(1/2)
/(2*x^2-x+3)^(1/2))*2^(1/2)+7/95551488*(52836655+9616196*x)*(2*x^2-x+3)^(1/
2)/(5+2*x)
```

Rubi [A]
time = 0.14, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1664, 826, 857, 633, 221, 738, 212}

$$-\frac{9363383(2x^2-x+3)^{3/2}}{23887872(2x+5)^2}+\frac{593771(2x^2-x+3)^{3/2}}{497664(2x+5)^3}-\frac{3667(2x^2-x+3)^{3/2}}{2304(2x+5)^4}+\frac{7(9616196x+52836655)\sqrt{2x^2-x+3}}{95551488(2x+5)}-\frac{4640586097\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1146617856\sqrt{2}}+\frac{259\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x]
```

```
[Out] (7*(52836655 + 9616196*x)*Sqrt[3 - x + 2*x^2])/(95551488*(5 + 2*x)) - (3667
*(3 - x + 2*x^2)^(3/2))/(2304*(5 + 2*x)^4) + (593771*(3 - x + 2*x^2)^(3/2))
/(497664*(5 + 2*x)^3) - (9363383*(3 - x + 2*x^2)^(3/2))/(23887872*(5 + 2*x)
^2) + (259*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2]) - (4640586097*ArcTanh[
(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1146617856*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = -\frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} - \frac{1}{288}\int \frac{\sqrt{3-x+2x^2}\,\left(\frac{44361}{16} - \right.}{(5+}$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} + \frac{593771(3-x+2x^2)^{3/2}}{497664(5+2x)^3} + \int \frac{\sqrt{}}{}$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} + \frac{593771(3-x+2x^2)^{3/2}}{497664(5+2x)^3} - \frac{9363}{23}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667(3-x+2}{2304(5+2}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667(3-x+2}{2304(5+2}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667(3-x+2}{2304(5+2}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667(3-x+2}{2304(5+2}$$

Mathematica [A]
time = 0.69, size = 110, normalized size = 0.67

$$\frac{\frac{12\sqrt{3-x+2x^2}\,(44676885233+62847867486x+31323229164x^2+6105343976x^3+238878720x^4)}{(5+2x)^4} + 4640586097\sqrt{2}\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right) + 2320109568\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{1146617856}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x
]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(44676885233 + 62847867486*x + 31323229164*x^2 + 6
105343976*x^3 + 238878720*x^4))/(5 + 2*x)^4 + 4640586097*Sqrt[2]*ArcTanh[(5
 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 2320109568*Sqrt[2]*Log[1 - 4*x + 2*Sqr
t[6 - 2*x + 4*x^2]])/1146617856
```

Maple [A]

time = 0.01, size = 167, normalized size = 1.01

$$\frac{593771\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{3981312\left(x+\frac{5}{2}\right)^{3}}+\frac{201573155\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{3439853568\left(x+\frac{5}{2}\right)}-\frac{201573155(4x-1)\sqrt{2\left(x+\frac{5}{2}\right)^{2}}}{6879707136}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x)
```

```
[Out] 593771/3981312/(x+5/2)^3*(2*(x+5/2)^2-11*x-19/2)^(3/2)+201573155/3439853568
/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(3/2)-201573155/6879707136*(4*x-1)*(2*(x+5
/2)^2-11*x-19/2)^(1/2)-9363383/95551488/(x+5/2)^2*(2*(x+5/2)^2-11*x-19/2)^(
3/2)-3667/36864/(x+5/2)^4*(2*(x+5/2)^2-11*x-19/2)^(3/2)-4640586097/22932357
12*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))+
4640586097/6879707136*(2*(x+5/2)^2-11*x-19/2)^(1/2)-259/128*2^(1/2)*arcsinh
(4/23*23^(1/2)*(x-1/4))
```

Maxima [A]
time = 0.50, size = 181, normalized size = 1.10

$$-\frac{259}{128}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{4640586097}{2293235712}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{16828343}{47775744}\sqrt{2\,x^{2}-x+3}-\frac{3667\,(2\,x^{2}-x+3)^{\frac{3}{2}}}{2304\,(16\,x^{4}+160\,x^{3}+600\,x^{2}+1000\,x+625)}+\frac{593771\,(2\,x^{2}-x+3)^{\frac{3}{2}}}{497664\,(8\,x^{3}+60\,x^{2}+150\,x+125)}-\frac{9363383\,(2\,x^{2}-x+3)^{\frac{3}{2}}}{23887872\,(4\,x^{2}+20\,x+25)}+\frac{201573155\sqrt{2\,x^{2}-x+3}}{95551488\,(2\,x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="m
axima")
```

```
[Out] -259/128*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 4640586097/2293
235712*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2
*x + 5)) + 16828343/47775744*sqrt(2*x^2 - x + 3) - 3667/2304*(2*x^2 - x + 3
)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 593771/497664*(2*x^2
- x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 9363383/23887872*(2*x^2 - x
 + 3)^(3/2)/(4*x^2 + 20*x + 25) + 201573155/95551488*sqrt(2*x^2 - x + 3)/(2
*x + 5)
```

Fricas [A]
time = 0.40, size = 189, normalized size = 1.15

$$\frac{4640219136\sqrt{2}\,(16\,x^{4}+160\,x^{3}+600\,x^{2}+1000\,x+625)\log\left(4\sqrt{2}\sqrt{2\,x^{2}-x+3}\,(4\,x-1)-32\,x^{2}+16\,x-25\right)+4640586097\sqrt{2}\,(16\,x^{4}+160\,x^{3}+600\,x^{2}+1000\,x+625)\log\left(-\frac{24\sqrt{2}\sqrt{2\,x^{2}-x+3}\,(22\,x-17)+1060\,x^{2}-1036\,x+1153}{4\,x^{2}+20\,x+25}\right)+48\,(238878720\,x^{4}+6105343976\,x^{3}+31323229164\,x^{2}+62847867486\,x+44676885233)\sqrt{2\,x^{2}-x+3}}{4586471424\,(16\,x^{4}+160\,x^{3}+600\,x^{2}+1000\,x+625)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="f
ricas")
```

[Out] 1/4586471424*(4640219136*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625
)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 46405
86097*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*log(-(24*sqrt(2)*
sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x +
 25)) + 48*(238878720*x^4 + 6105343976*x^3 + 31323229164*x^2 + 62847867486*
x + 44676885233)*sqrt(2*x^2 - x + 3))/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x
+ 625)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3} \cdot (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**5,x)

[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**5
, x)

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(134) = 268.
time = 5.52, size = 327, normalized size = 1.98

$$-\frac{1}{2293235712}\sqrt{2}\left(4640586097\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)+4640219136\log\left(\left|\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}+1\right|\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)-4640219136\log\left(\left|\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}-1\right|\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)+12\left(\frac{24\left(\frac{144\left(\frac{792072\operatorname{sgn}\left(\frac{1}{2x+5}\right)}{2x+5}-835793\operatorname{sgn}\left(\frac{1}{2x+5}\right)\right)}{2x+5}+57384361\operatorname{sgn}\left(\frac{1}{2x+5}\right)\right)}{2x+5}-464569597\operatorname{sgn}\left(\frac{1}{2x+5}\right)\right)\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{179159040\left(11\left(\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)-12\operatorname{sgn}\left(\frac{1}{2x+5}\right)\right)}{\left(\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}\right)^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="g
iac")

[Out] -1/2293235712*sqrt(2)*(4640586097*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^
2 + 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 4640219136*log(abs(sqrt(-11/
(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 4640
219136*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))
*sgn(1/(2*x + 5)) + 12*(24*(144*(792072*sgn(1/(2*x + 5))/(2*x + 5) - 835793
*sgn(1/(2*x + 5)))/(2*x + 5) + 57384361*sgn(1/(2*x + 5)))/(2*x + 5) - 46456
9597*sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 179159040
(11(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x +
5)) - 12*sgn(1/(2*x + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(
2*x + 5))^2 - 1))

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2 - x + 3}\,\left(5\,x^4 - x^3 + 3\,x^2 + x + 2\right)}{{\left(2\,x + 5\right)}^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5,x)
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5, x)
```

3.331 $\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$

Optimal. Leaf size=165

$$-\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2}-\frac{3667(3-x+2x^2)^{3/2}}{2880(5+2x)^5}+\frac{711961(3-x+2x^2)^{3/2}}{829440(5+2x)^4}-\frac{38732321}{17915\cdots}$$

```
[Out] -3667/2880*(2*x^2-x+3)^(3/2)/(5+2*x)^5+711961/829440*(2*x^2-x+3)^(3/2)/(5+2
*x)^4-38732321/179159040*(2*x^2-x+3)^(3/2)/(5+2*x)^3-5/64*arcsinh(1/23*(1-4
*x)*23^(1/2))*2^(1/2)+12895597463/165112971264*arctanh(1/24*(17-22*x)*2^(1/
2)/(2*x^2-x+3)^(1/2))*2^(1/2)-1/6879707136*(4583087983+3174439702*x)*(2*x^2
-x+3)^(1/2)/(5+2*x)^2
```

Rubi [A]
time = 0.14, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1664, 824, 857, 633, 221, 738, 212}

$$-\frac{38732321(2x^2-x+3)^{3/2}}{179159040(2x+5)^3}+\frac{711961(2x^2-x+3)^{3/2}}{829440(2x+5)^4}-\frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5}-\frac{(3174439702x+4583087983)\sqrt{2x^2-x+3}}{6879707136(2x+5)^2}+\frac{12895597463\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{82556485632\sqrt{2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x]
```

```
[Out] -1/6879707136*((4583087983 + 3174439702*x)*Sqrt[3 - x + 2*x^2])/(5 + 2*x)^2
 - (3667*(3 - x + 2*x^2)^(3/2))/(2880*(5 + 2*x)^5) + (711961*(3 - x + 2*x^2
)^(3/2))/(829440*(5 + 2*x)^4) - (38732321*(3 - x + 2*x^2)^(3/2))/(179159040
*(5 + 2*x)^3) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32*Sqrt[2]) + (12895597463
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(82556485632*Sqrt[2
])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5} - \frac{1}{360}\int \frac{\sqrt{3-x+2x^2}\left(\frac{52701}{16} - \right.}{(5+}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5} + \frac{711961\left(3-x+2x^2\right)^{3/2}}{829440(5+2x)^4} + \int \frac{\sqrt{}}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5} + \frac{711961\left(3-x+2x^2\right)^{3/2}}{829440(5+2x)^4} - \frac{3873}{1}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667(3}{2880}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667(3}{2880}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667(3}{2880}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667(3}{2880}$$

Mathematica [A]
time = 0.79, size = 110, normalized size = 0.67

$$\frac{-\frac{12\sqrt{3-x+2x^2}\left(3110673952831+5608297138216x+3919478861832x^2+1285267446304x^3+186470433136x^4\right)}{(5+2x)^5} - 64477987315\sqrt{2}\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right) - 32248627200\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{412782428160}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x
]
```

```
[Out] ((-12*Sqrt[3 - x + 2*x^2]*(3110673952831 + 5608297138216*x + 3919478861832*
x^2 + 1285267446304*x^3 + 186470433136*x^4))/(5 + 2*x)^5 - 64477987315*Sqrt
[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 32248627200*Sqrt[2]*Log[
1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/412782428160
```

Maple [A]

time = 0.02, size = 188, normalized size = 1.14

$$-\frac{38732321\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{1433272320\left(x+\frac{5}{2}\right)^{3}}-\frac{562688629\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{247669456896\left(x+\frac{5}{2}\right)}+\frac{562688629(4x-1)\sqrt{2\left(x+\frac{5}{2}\right.}}{495338913792}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x)
```

```
[Out] -38732321/1433272320/(x+5/2)^3*(2*(x+5/2)^2-11*x-19/2)^(3/2)-562688629/2476
69456896/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(3/2)+562688629/495338913792*(4*x-
1)*(2*(x+5/2)^2-11*x-19/2)^(1/2)-3667/92160/(x+5/2)^5*(2*(x+5/2)^2-11*x-19/
2)^(3/2)+46569601/6879707136/(x+5/2)^2*(2*(x+5/2)^2-11*x-19/2)^(3/2)+711961
/13271040/(x+5/2)^4*(2*(x+5/2)^2-11*x-19/2)^(3/2)+12895597463/165112971264*
2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-128
95597463/495338913792*(2*(x+5/2)^2-11*x-19/2)^(1/2)+5/64*2^(1/2)*arcsinh(4/
23*23^(1/2)*(x-1/4))
```

Maxima [A]
time = 0.50, size = 222, normalized size = 1.35

$$\frac{5}{64}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{12895597463}{165112971264}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{46569601}{3439853568}\sqrt{2\,x^{2}-x+3}-\frac{3667\,(2\,x^{2}-x+3)^{\frac{3}{2}}}{2880\,(32\,x^{5}+400\,x^{4}+2000\,x^{3}+5000\,x^{2}+6250\,x+3125)}+\frac{711961\,(2\,x^{2}-x+3)^{\frac{3}{2}}}{829440\,(16\,x^{4}+160\,x^{3}+600\,x^{2}+1000\,x+625)}-\frac{38732321\,(2\,x^{2}-x+3)^{\frac{3}{2}}}{179159040\,(8\,x^{3}+60\,x^{2}+150\,x+125)}+\frac{46569601\,(2\,x^{2}-x+3)^{\frac{3}{2}}}{1719926784\,(4\,x^{2}+20\,x+25)}-\frac{562688629\sqrt{2\,x^{2}-x+3}}{6879707136\,(2\,x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="m
axima")
```

```
[Out] 5/64*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 12895597463/1651129
71264*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*
x + 5)) - 46569601/3439853568*sqrt(2*x^2 - x + 3) - 3667/2880*(2*x^2 - x +
3)^(3/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 711961/
829440*(2*x^2 - x + 3)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) -
38732321/179159040*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 4
6569601/1719926784*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 562688629/68
79707136*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

Fricas [A]
time = 0.36, size = 203, normalized size = 1.23

$$\frac{64497254400\sqrt{2}\,(32\,x^{5}+400\,x^{4}+2000\,x^{3}+5000\,x^{2}+6250\,x+3125)\log\left(-4\sqrt{2}\sqrt{2\,x^{2}-x+3}\,(4\,x-1)-32\,x^{2}+16\,x-25\right)+64477987315\sqrt{2}\,(32\,x^{5}+400\,x^{4}+2000\,x^{3}+5000\,x^{2}+6250\,x+3125)\log\left(\frac{24\sqrt{2}\sqrt{2\,x^{2}-x+3}\,(22\,x-17)-1060\,x^{2}+1036\,x-1153}{4\,x^{2}+20\,x+25}\right)-48\,(186470433136\,x^{4}+1285267446304\,x^{3}+3919478861832\,x^{2}+5608297138216\,x+3110673952831)\sqrt{2\,x^{2}-x+3}}{1651129712640\,(32\,x^{5}+400\,x^{4}+2000\,x^{3}+5000\,x^{2}+6250\,x+3125)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="f
ricas")
```

```
[Out] 1/1651129712640*(64497254400*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^
2 + 6250*x + 3125)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 +
16*x - 25) + 64477987315*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 +
6250*x + 3125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 +
 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(186470433136*x^4 + 1285267446304
*x^3 + 3919478861832*x^2 + 5608297138216*x + 3110673952831)*sqrt(2*x^2 - x
+ 3))/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2-x+3}\cdot\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x+5\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**6,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**6
, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(134) = 268.
time = 3.77, size = 387, normalized size = 2.35

$$-\frac{5}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{12895597463}{165112971264}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)-\frac{12895597463}{165112971264}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)-\frac{\sqrt{2}\left(4368922304720\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^9+124570969998480\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^8+637804348664160\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^7+1828845222532320\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^6-3763189300187016\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^5-10794416351958120\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^4+25049834283305880\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^3-34708488692384520\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+10654664764755165\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-2507056315485767\right)}{68797071360\left(2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-11\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="g
iac")
```

```
[Out] -5/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 12895
597463/165112971264*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 -
 x + 3))) - 12895597463/165112971264*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt
(2) + 2*sqrt(2*x^2 - x + 3))) - 1/68797071360*sqrt(2)*(4368922304720*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 124570969998480*(sqrt(2)*x - sqrt(2
*x^2 - x + 3))^8 + 637804348664160*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^7 + 1828845222532320*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 37631893001870
16*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 - 10794416351958120*(sqrt(2)
*x - sqrt(2*x^2 - x + 3))^4 + 25049834283305880*sqrt(2)*(sqrt(2)*x - sqrt(2
*x^2 - x + 3))^3 - 34708488692384520*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 +
10654664764755165*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 2507056315485
767)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(
2*x^2 - x + 3)) - 11)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6,x)
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6, x)
```

3.332 $\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$

Optimal. Leaf size=169

$$-\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2}-\frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6}+\frac{92239(3-x+2x^2)^{3/2}}{138240(5+2x)^5}-\frac{5703277(3-x+2x^2}{39813120(5+2x)}$$

```
[Out] -3667/3456*(2*x^2-x+3)^(3/2)/(5+2*x)^6+92239/138240*(2*x^2-x+3)^(3/2)/(5+2*
x)^5-5703277/39813120*(2*x^2-x+3)^(3/2)/(5+2*x)^4+87677717/8599633920*(2*x^
2-x+3)^(3/2)/(5+2*x)^3-26972675/7925422620672*arctanh(1/24*(17-22*x)*2^(1/2
)/(2*x^2-x+3)^(1/2))*2^(1/2)-1172725/330225942528*(17-22*x)*(2*x^2-x+3)^(1/
2)/(5+2*x)^2
```

Rubi [A]
time = 0.13, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1664, 820, 734, 738, 212}

$$\frac{87677717(2x^2-x+3)^{3/2}}{8599633920(2x+5)^3}-\frac{5703277(2x^2-x+3)^{3/2}}{39813120(2x+5)^4}+\frac{92239(2x^2-x+3)^{3/2}}{138240(2x+5)^5}-\frac{3667(2x^2-x+3)^{3/2}}{3456(2x+5)^6}-\frac{1172725(17-22x)\sqrt{2x^2-x+3}}{330225942528(2x+5)^2}-\frac{26972675\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{3962711310336\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x]
```

```
[Out] (-1172725*(17 - 22*x)*Sqrt[3 - x + 2*x^2])/(330225942528*(5 + 2*x)^2) - (36
67*(3 - x + 2*x^2)^(3/2))/(3456*(5 + 2*x)^6) + (92239*(3 - x + 2*x^2)^(3/2)
)/(138240*(5 + 2*x)^5) - (5703277*(3 - x + 2*x^2)^(3/2))/(39813120*(5 + 2*x
)^4) + (87677717*(3 - x + 2*x^2)^(3/2))/(8599633920*(5 + 2*x)^3) - (2697267
5*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(3962711310336*Sqr
t[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
```

```
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = -\frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6} - \frac{1}{432}\int \frac{\sqrt{3-x+2x^2}\,\left(\frac{61041}{16} - \right.}{(5+}$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6} + \frac{92239(3-x+2x^2)^{3/2}}{138240(5+2x)^5} + \frac{\int \sqrt{3}}{}$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6} + \frac{92239(3-x+2x^2)^{3/2}}{138240(5+2x)^5} - \frac{57032}{398}$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6} + \frac{92239(3-x+2x^2)^{3/2}}{138240(5+2x)^5} - \frac{57032}{398}$$

$$= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667(3-x+2x^2)}{3456(5+2x)^6}$$

$$= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667(3-x+2x^2)}{3456(5+2x)^6}$$

$$= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667(3-x+2x^2)}{3456(5+2x)^6}$$

Mathematica [A]
time = 0.77, size = 86, normalized size = 0.51

$$\frac{\frac{12\sqrt{3-x+2x^2}\,(-219337079305+27245373694x+158340720344x^2+397498825328x^3+12256250416x^4+271409942624x^5)}{(5+2x)^6}+134863375\sqrt{2}\,\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{19813556551680}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x
]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(-219337079305 + 27245373694*x + 158340720344*x^2
+ 397498825328*x^3 + 12256250416*x^4 + 271409942624*x^5))/(5 + 2*x)^6 + 134
863375*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6])/19813556551680
```

Maple [A]
time = 0.17, size = 195, normalized size = 1.15

method	result

risch	$\frac{542819885248x^7-246897441792x^6+1596971228112x^5-44048633392x^4+1088646503028x^3+9102628728x^2+301073200387x-658011}{1651129712640(5+2x)^6\sqrt{2x^2-x+3}}$
trager	$\frac{(271409942624x^5+12256250416x^4+397498825328x^3+158340720344x^2+27245373694x-219337079305)\sqrt{2x^2-x+3}}{1651129712640(5+2x)^6} - \frac{269}{}$
default	$\frac{92239\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{4423680\left(x+\frac{5}{2}\right)^5} - \frac{5703277\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{637009920\left(x+\frac{5}{2}\right)^4} + \frac{87677717\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{68797071360\left(x+\frac{5}{2}\right)^3} - \frac{1172725\left(2\left(x+\frac{5}{2}\right)^2-11x\right.}{330225942528\left(x+\frac{5}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x,method=_RETURNVERBO
SE)
```

```
[Out] 92239/4423680/(x+5/2)^5*(2*(x+5/2)^2-11*x-19/2)^(3/2)-5703277/637009920/(x+
5/2)^4*(2*(x+5/2)^2-11*x-19/2)^(3/2)+87677717/68797071360/(x+5/2)^3*(2*(x+5
/2)^2-11*x-19/2)^(3/2)-1172725/330225942528/(x+5/2)^2*(2*(x+5/2)^2-11*x-19/
2)^(3/2)-12899975/11888133931008/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(3/2)+1289
9975/23776267862016*(4*x-1)*(2*(x+5/2)^2-11*x-19/2)^(1/2)-3667/221184/(x+5/
2)^6*(2*(x+5/2)^2-11*x-19/2)^(3/2)-26972675/7925422620672*2^(1/2)*arctanh(1
/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))+26972675/23776267862
016*(2*(x+5/2)^2-11*x-19/2)^(1/2)
```

Maxima [A]

time = 0.51, size = 250, normalized size = 1.48

$$\frac{26972675}{7925422620672}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{1172725}{165112971264}\sqrt{2\,x^2-x+3}-\frac{3667\,(2\,x^2-x+3)^{\frac{3}{2}}}{3456\,(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3+37500\,x^2+37500\,x+15625)}+\frac{92239\,(2\,x^2-x+3)^{\frac{3}{2}}}{138240\,(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125)}-\frac{5703277\,(2\,x^2-x+3)^{\frac{3}{2}}}{39813120\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)}+\frac{87677717\,(2\,x^2-x+3)^{\frac{3}{2}}}{8599633920\,(8\,x^3+60\,x^2+150\,x+125)}-\frac{1172725\,(2\,x^2-x+3)^{\frac{3}{2}}}{82556485632\,(4\,x^2+20\,x+25)}-\frac{12899975\,\sqrt{2\,x^2-x+3}}{330225942528\,(2\,x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="m
axima")
```

```
[Out] 26972675/7925422620672*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/2
3*sqrt(23)/abs(2*x + 5)) + 1172725/165112971264*sqrt(2*x^2 - x + 3) - 3667/
3456*(2*x^2 - x + 3)^(3/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500
*x^2 + 37500*x + 15625) + 92239/138240*(2*x^2 - x + 3)^(3/2)/(32*x^5 + 400*
x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) - 5703277/39813120*(2*x^2 - x +
3)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 87677717/8599633920*
(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 1172725/82556485632*
(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 12899975/330225942528*sqrt(2*x^
2 - x + 3)/(2*x + 5)
```

Fricas [A]
time = 0.36, size = 156, normalized size = 0.92

$$\frac{134863375 \sqrt{2} \left(64 x^6+960 x^5+6000 x^4+20000 x^3+37500 x^2+37500 x+15625\right) \log \left(-\frac{24 \sqrt{2} \sqrt{2 x^2-x+3} (22 x-17)+1060 x^2-1036 x+1153}{4 x^2+20 x+25}\right)+48 \left(271409942624 x^5+12256250416 x^4+397498825328 x^3+158340720344 x^2+27245373694 x-219337079305\right) \sqrt{2 x^2-x+3}}{79254226206720 \left(64 x^6+960 x^5+6000 x^4+20000 x^3+37500 x^2+37500 x+15625\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="f
ricas")
```

```
[Out] 1/79254226206720*(134863375*sqrt(2)*(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^
3 + 37500*x^2 + 37500*x + 15625)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x
 - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)) + 48*(271409942624*
x^5 + 12256250416*x^4 + 397498825328*x^3 + 158340720344*x^2 + 27245373694*x
 - 219337079305)*sqrt(2*x^2 - x + 3))/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*
x^3 + 37500*x^2 + 37500*x + 15625)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2-x+3} \cdot \left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**7,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**7
, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(139) = 278.
time = 3.24, size = 405, normalized size = 2.40

$$-\frac{26972675}{7925422620672} \sqrt{2} \log \left(\left|-2 \sqrt{2} x+\sqrt{2}+2 \sqrt{2 x^2-x+3}\right|\right)+\frac{26972675}{7925422620672} \sqrt{2} \log \left(\left|-2 \sqrt{2} x-11 \sqrt{2}+2 \sqrt{2 x^2-x+3}\right|\right)+\frac{\sqrt{2} \left(16506981498400 \sqrt{2} \left(\sqrt{2} x-\sqrt{2 x^2-x+3}\right)^{11}+389429252643040 \left(\sqrt{2} x-\sqrt{2 x^2-x+3}\right)^{10}+2263923918689840 \sqrt{2} \left(\sqrt{2} x-\sqrt{2 x^2-x+3}\right)^{9}+11663651054548560 \left(\sqrt{2} x-\sqrt{2 x^2-x+3}\right)^{8}+902212326134736 \sqrt{2} \left(\sqrt{2} x-\sqrt{2 x^2-x+3}\right)^{7}-84192729519861840 \left(\sqrt{2} x-\sqrt{2 x^2-x+3}\right)^{6}-4317200555009448 \sqrt{2} \left(\sqrt{2} x-\sqrt{2 x^2-x+3}\right)^{5}+351543414066518760 \left(\sqrt{2} x-\sqrt{2 x^2-x+3}\right)^{4}-376787166452923830 \sqrt{2} \left(\sqrt{2} x-\sqrt{2 x^2-x+3}\right)^{3}+356306707647610982 \left(\sqrt{2} x-\sqrt{2 x^2-x+3}\right)^{2}-82348353128195465 \sqrt{2} \left(\sqrt{2} x-\sqrt{2 x^2-x+3}\right)+15499394004553969\right)}{3302259425280 \left(2 \left(\sqrt{2} x-\sqrt{2 x^2-x+3}\right)^{2}+10 \sqrt{2} \left(\sqrt{2} x-\sqrt{2 x^2-x+3}\right)-11\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="g
iac")
```

```
[Out] -26972675/7925422620672*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x
^2 - x + 3))) + 26972675/7925422620672*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sq
rt(2) + 2*sqrt(2*x^2 - x + 3))) + 1/3302259425280*sqrt(2)*(16506981498400*s
qrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^11 + 389429252643040*(sqrt(2)*x -
sqrt(2*x^2 - x + 3))^10 + 2263923918689840*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2
- x + 3))^9 + 11663651054548560*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 90221
2326134736*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 - 84192729519861840*
```

```
(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 4317200555009448*sqrt(2)*(sqrt(2)*x -
 sqrt(2*x^2 - x + 3))^5 + 351543414066518760*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^4 - 376787166452923830*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 35
6306707647610982*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 82348353128195465*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 15499394004553969)/(2*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 1
1)^6
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7,x)
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7, x)
```

3.333 $\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$

Optimal. Leaf size=194

$$-\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2}-\frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7}+\frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6}-\frac{1464037(3-x+2}{13934592(5+2}$$

```
[Out] -3667/4032*(2*x^2-x+3)^(3/2)/(5+2*x)^7+948341/1741824*(2*x^2-x+3)^(3/2)/(5+
2*x)^6-1464037/13934592*(2*x^2-x+3)^(3/2)/(5+2*x)^5+19414831/4013162496*(2*
x^2-x+3)^(3/2)/(5+2*x)^4+246159769/866843099136*(2*x^2-x+3)^(3/2)/(5+2*x)^3
-289071245/570630428688384*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2)
)*2^(1/2)-12568315/23776267862016*(17-22*x)*(2*x^2-x+3)^(1/2)/(5+2*x)^2
```

Rubi [A]
time = 0.14, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1664, 848, 820, 734, 738, 212}

$$\frac{246159769(2x^2-x+3)^{3/2}}{866843099136(2x+5)^3}+\frac{19414831(2x^2-x+3)^{3/2}}{4013162496(2x+5)^4}-\frac{1464037(2x^2-x+3)^{3/2}}{13934592(2x+5)^5}+\frac{948341(2x^2-x+3)^{3/2}}{1741824(2x+5)^6}-\frac{3667(2x^2-x+3)^{3/2}}{4032(2x+5)^7}-\frac{12568315(17-22x)\sqrt{2x^2-x+3}}{23776267862016(2x+5)^2}-\frac{289071245\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{285315214344192\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x]
```

```
[Out] (-12568315*(17 - 22*x)*Sqrt[3 - x + 2*x^2])/(23776267862016*(5 + 2*x)^2) -
(3667*(3 - x + 2*x^2)^(3/2))/(4032*(5 + 2*x)^7) + (948341*(3 - x + 2*x^2)^(
3/2))/(1741824*(5 + 2*x)^6) - (1464037*(3 - x + 2*x^2)^(3/2))/(13934592*(5
+ 2*x)^5) + (19414831*(3 - x + 2*x^2)^(3/2))/(4013162496*(5 + 2*x)^4) + (24
6159769*(3 - x + 2*x^2)^(3/2))/(866843099136*(5 + 2*x)^3) - (289071245*ArcT
anh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(285315214344192*Sqrt[2]
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
```

```
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx = -\frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} - \frac{1}{504}\int \frac{\sqrt{3-x+2x^2}\,\left(\frac{69381}{16} - \ldots\right.}{(5+\ldots}$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} + \frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6} + \int \frac{\sqrt{\ldots}}{\ldots}$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} + \frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6} - \frac{1464\ldots}{13\ldots}$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} + \frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6} - \frac{1464\ldots}{13\ldots}$$

$$= -\frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7} + \frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6} - \frac{1464\ldots}{13\ldots}$$

$$= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667(3-x+2x^2\ldots}{4032(5+2x)^7\ldots}$$

$$= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667(3-x+2x^2\ldots}{4032(5+2x)^7\ldots}$$

$$= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667(3-x+2x^2\ldots}{4032(5+2x)^7\ldots}$$

Mathematica [A]
time = 1.01, size = 91, normalized size = 0.47

$$\frac{\frac{12\sqrt{3-x+2x^2}\,(-20465234808721+590492177460x+14716683780036x^2+41058010262368x^3+4982916071952x^4+27976951397184x^5+1574342277056x^6)}{(5+2x)^7} + 2023498715\sqrt{2}\,\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{1997206500409344}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x
]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(-20465234808721 + 590492177460*x + 14716683780036
*x^2 + 41058010262368*x^3 + 4982916071952*x^4 + 27976951397184*x^5 + 157434
2277056*x^6))/(5 + 2*x)^7 + 2023498715*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 -
2*x + 4*x^2])/6])/1997206500409344
```

Maple [A]
time = 0.17, size = 216, normalized size = 1.11

method	result
risch	$\frac{3148684554112x^8+54379560517312x^7-13288092422112x^6+161063958644336x^5+3324105513560x^4+109638331361988x^3+26290895}{166433875034112(5+2x)^7\sqrt{2x^2-x+3}}$
trager	$\frac{(1574342277056x^6+27976951397184x^5+4982916071952x^4+41058010262368x^3+14716683780036x^2+590492177460x-204652348087}{166433875034112(5+2x)^7}$
default	$-\frac{3667\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{516096\left(x+\frac{5}{2}\right)^7}-\frac{1464037\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{445906944\left(x+\frac{5}{2}\right)^5}+\frac{19414831\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{64210599936\left(x+\frac{5}{2}\right)^4}+\frac{246159769\left(2\left(x+\frac{5}{2}\right)^2-\right.}{6934744793088(x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x,method=_RETURNVERBO
SE)
```

```
[Out] -3667/516096/(x+5/2)^7*(2*(x+5/2)^2-11*x-19/2)^(3/2)-1464037/445906944/(x+5
/2)^5*(2*(x+5/2)^2-11*x-19/2)^(3/2)+19414831/64210599936/(x+5/2)^4*(2*(x+5/
2)^2-11*x-19/2)^(3/2)+246159769/6934744793088/(x+5/2)^3*(2*(x+5/2)^2-11*x-1
9/2)^(3/2)-12568315/23776267862016/(x+5/2)^2*(2*(x+5/2)^2-11*x-19/2)^(3/2)-
138251465/855945643032576/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(3/2)+138251465/1
711891286065152*(4*x-1)*(2*(x+5/2)^2-11*x-19/2)^(1/2)+948341/111476736/(x+5
/2)^6*(2*(x+5/2)^2-11*x-19/2)^(3/2)-289071245/570630428688384*2^(1/2)*arcta
nh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))+289071245/171189
1286065152*(2*(x+5/2)^2-11*x-19/2)^(1/2)
```

Maxima [A]

time = 0.51, size = 301, normalized size = 1.55

$$\frac{289071245}{570630428688384}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{12568315}{11888133931008}\sqrt{2\,x^2-x+3}-\frac{3667\,(2\,x^2-x+3)^{\frac{3}{2}}}{4032\,(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x+78125)}+\frac{948341\,(2\,x^2-x+3)^{\frac{3}{2}}}{1741824\,(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3+37500\,x^2+37500\,x+15625)}-\frac{1464037\,(2\,x^2-x+3)^{\frac{3}{2}}}{13934592\,(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125)}+\frac{19414831\,(2\,x^2-x+3)^{\frac{3}{2}}}{4013162496\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)}+\frac{246159769\,(2\,x^2-x+3)^{\frac{3}{2}}}{866843099136\,(8\,x^3+60\,x^2+150\,x+125)}-\frac{12568315\,(2\,x^2-x+3)^{\frac{3}{2}}}{5944066965504\,(4\,x^2+20\,x+25)}-\frac{138251465\,\sqrt{2\,x^2-x+3}}{23776267862016\,(2\,x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="m
axima")
```

```
[Out] 289071245/570630428688384*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 1
7/23*sqrt(23)/abs(2*x + 5)) + 12568315/11888133931008*sqrt(2*x^2 - x + 3) -
 3667/4032*(2*x^2 - x + 3)^(3/2)/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^
4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125) + 948341/1741824*(2*x^2 - x
 + 3)^(3/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x
+ 15625) - 1464037/13934592*(2*x^2 - x + 3)^(3/2)/(32*x^5 + 400*x^4 + 2000*
```

```
x^3 + 5000*x^2 + 6250*x + 3125) + 19414831/4013162496*(2*x^2 - x + 3)^(3/2)
/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 246159769/866843099136*(2*x^
2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 12568315/5944066965504*(2
*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 138251465/23776267862016*sqrt(2*x
^2 - x + 3)/(2*x + 5)
```

Fricas [A]
time = 0.36, size = 171, normalized size = 0.88

$$\frac{2023498715\,\sqrt{2}\,(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x+78125)\log\left(-\frac{24\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(22\,x-17)+1060\,x^2-1036\,x+1153}{4\,x^2+20\,x+25}\right)+48\,(1574342277056\,x^6+27976951397184\,x^5+4982916071952\,x^4+41058010262368\,x^3+14716683780036\,x^2+590492177460\,x-20465234808721)\,\sqrt{2\,x^2-x+3}}{7988826001637376\,(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x+78125)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="f
ricas")
```

```
[Out] 1/7988826001637376*(2023498715*sqrt(2)*(128*x^7 + 2240*x^6 + 16800*x^5 + 70
000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125)*log(-(24*sqrt(2)*sqrt
(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)
) + 48*(1574342277056*x^6 + 27976951397184*x^5 + 4982916071952*x^4 + 410580
10262368*x^3 + 14716683780036*x^2 + 590492177460*x - 20465234808721)*sqrt(2
*x^2 - x + 3))/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 2
62500*x^2 + 218750*x + 78125)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3} \cdot (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**8,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**8
, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(160) = 320.
time = 3.09, size = 456, normalized size = 2.35

$$-\frac{289071245}{570630428688384}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}\,x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)+\frac{289071245}{570630428688384}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}\,x-11\,\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)-\frac{\sqrt{2}\left(129503917760\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^{13}-3320259746027840\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^{12}-23966708071916736\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^{11}-186055342532355520\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^{10}-274256644494948976\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^{9}+796135370176031760\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^{8}+2531523139171005408\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^{7}-4610393811900786336\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^{6}-7997126854300052364\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^{5}+30842713619423538868\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^{4}-21873571601855032556\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^{3}+16204706960604668100\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^{2}-3196254593191113265\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+5367990322161117911\right)}{332867750068224\left(2\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^{2}+10\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)-11\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="g
iac")
```

```
[Out] -289071245/570630428688384*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(
2*x^2 - x + 3))) + 289071245/570630428688384*sqrt(2)*log(abs(-2*sqrt(2)*x -
```

```
 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/332867750068224*sqrt(2)*(12950391
7760*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^13 - 3320259746027840*(sqrt(
2)*x - sqrt(2*x^2 - x + 3))^12 - 23966708071916736*sqrt(2)*(sqrt(2)*x - sqr
t(2*x^2 - x + 3))^11 - 186055342532355520*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
^10 - 274256644494948976*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 7961
35370176031760*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 2531523139171005408*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 - 4610393811900786336*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^6 - 7997126854300052364*sqrt(2)*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^5 + 30842713619423538868*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4
- 21873571601855032556*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 162047
06960604668100*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 3196254593191113265*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 536799032216117911)/(2*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) -
11)^7
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8,x)
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8, x)
```

3.334 $\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)$

Optimal. Leaf size=166

$$-\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{92727(1-4x)\left(3-x+2x^2\right)^{3/2}}{131072}+\frac{69415(5+2x)^2\left(3-x+2x^2\right)^{5/2}}{32256}-\frac{112}{}$$

```
[Out] -92727/131072*(1-4*x)*(2*x^2-x+3)^(3/2)+69415/32256*(5+2*x)^2*(2*x^2-x+3)^(
5/2)-1121/2304*(5+2*x)^3*(2*x^2-x+3)^(5/2)+5/144*(5+2*x)^4*(2*x^2-x+3)^(5/2
)-3/143360*(661397+215900*x)*(2*x^2-x+3)^(5/2)-147157749/8388608*arcsinh(1/
23*(1-4*x)*23^(1/2))*2^(1/2)-6398163/2097152*(1-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1667, 793, 626, 633, 221}

$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4-\frac{1121\left(2x^2-x+3\right)^{5/2}(2x+5)^3}{2304}+\frac{69415\left(2x^2-x+3\right)^{5/2}(2x+5)^2}{32256}-\frac{3(215900x+661397)\left(2x^2-x+3\right)^{5/2}}{143360}-\frac{92727(1-4x)\left(2x^2-x+3\right)^{3/2}}{131072}-\frac{6398163(1-4x)\sqrt{2x^2-x+3}}{2097152}-\frac{147157749\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4194304\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x)*(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-6398163*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2097152 - (92727*(1 - 4*x)*(3 - x
+ 2*x^2)^(3/2))/131072 + (69415*(5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))/32256 -
(1121*(5 + 2*x)^3*(3 - x + 2*x^2)^(5/2))/2304 + (5*(5 + 2*x)^4*(3 - x + 2*x
^2)^(5/2))/144 - (3*(661397 + 215900*x)*(3 - x + 2*x^2)^(5/2))/143360 - (14
7157749*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4194304*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx = \frac{5}{144}(5+2x)^4\left(3-x+2x^2\right)^{5/2}+\frac{1}{288}\int (5+2\ldots$$

$$= -\frac{1121(5+2x)^3\left(3-x+2x^2\right)^{5/2}}{2304}+\frac{5}{144}(5+2\ldots$$

$$= \frac{69415(5+2x)^2\left(3-x+2x^2\right)^{5/2}}{32256}-\frac{1121(5+2\ldots}{}$$

$$= \frac{69415(5+2x)^2\left(3-x+2x^2\right)^{5/2}}{32256}-\frac{1121(5+2\ldots}{}$$

$$= -\frac{92727(1-4x)\left(3-x+2x^2\right)^{3/2}}{131072}+\frac{69415(5+\ldots}{}$$

$$= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{92727(1-\ldots}{}$$

$$= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{92727(1-\ldots}{}$$

$$= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{92727(1-\ldots}{}$$

Mathematica [A]
time = 0.71, size = 90, normalized size = 0.54

$$\frac{4\sqrt{3-x+2x^2}\left(1592737263+12357760788x+4870637856x^2+12669290112x^3+379086848x^4+12117893120x^5+1033175040x^6+2926837760x^7+1468006400x^8\right)-46354690935\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{2642411520}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x)*(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(1592737263 + 12357760788*x + 4870637856*x^2 + 12669
290112*x^3 + 379086848*x^4 + 12117893120*x^5 + 1033175040*x^6 + 2926837760*
x^7 + 1468006400*x^8) - 46354690935*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x +
4*x^2]])/2642411520
```

Maple [A]
time = 0.14, size = 134, normalized size = 0.81

method	result

risch	$\frac{(1468006400x^8+2926837760x^7+1033175040x^6+12117893120x^5+379086848x^4+12669290112x^3+4870637856x^2+12357760788x+159}{660602880}$
trager	$\left(\frac{20}{9}x^8+\frac{319}{72}x^7+\frac{1051}{672}x^6+\frac{295847}{16128}x^5+\frac{26443}{46080}x^4+\frac{32992943}{1720320}x^3+\frac{2415991}{327680}x^2+\frac{343271133}{18350080}x+\frac{176970807}{73400320}\right)\sqrt{2x}$
default	$\frac{6398163(4x-1)\sqrt{2x^2-x+3}}{2097152}+\frac{147157749\sqrt{2}\ \text{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{8388608}+\frac{92727(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{131072}+\frac{5645x(2x^2-}{21504}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x,method=_RETURNVERBOSE
)
```

```
[Out] 6398163/2097152*(4*x-1)*(2*x^2-x+3)^(1/2)+147157749/8388608*2^(1/2)*arcsinh
(4/23*23^(1/2)*(x-1/4))+92727/131072*(4*x-1)*(2*x^2-x+3)^(3/2)+5645/21504*x
*(2*x^2-x+3)^(5/2)+2005/8064*x^2*(2*x^2-x+3)^(5/2)+479/288*x^3*(2*x^2-x+3)^
(5/2)+5/9*x^4*(2*x^2-x+3)^(5/2)+120809/143360*(2*x^2-x+3)^(5/2)
```

Maxima [A]
time = 0.49, size = 155, normalized size = 0.93

$$\frac{5}{9}(2x^2-x+3)^{\frac{5}{2}}x^4+\frac{479}{288}(2x^2-x+3)^{\frac{5}{2}}x^3+\frac{2005}{8064}(2x^2-x+3)^{\frac{5}{2}}x^2+\frac{5645}{21504}(2x^2-x+3)^{\frac{5}{2}}x+\frac{120809}{143360}(2x^2-x+3)^{\frac{5}{2}}+\frac{92727}{32768}(2x^2-x+3)^{\frac{3}{2}}x-\frac{92727}{131072}(2x^2-x+3)^{\frac{3}{2}}+\frac{6398163}{524288}\sqrt{2x^2-x+3}\,x+\frac{147157749}{8388608}\sqrt{2}\ \text{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{6398163}{2097152}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 5/9*(2*x^2 - x + 3)^(5/2)*x^4 + 479/288*(2*x^2 - x + 3)^(5/2)*x^3 + 2005/80
64*(2*x^2 - x + 3)^(5/2)*x^2 + 5645/21504*(2*x^2 - x + 3)^(5/2)*x + 120809/
143360*(2*x^2 - x + 3)^(5/2) + 92727/32768*(2*x^2 - x + 3)^(3/2)*x - 92727/
131072*(2*x^2 - x + 3)^(3/2) + 6398163/524288*sqrt(2*x^2 - x + 3)*x + 14715
7749/8388608*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 6398163/2097152*sqr
t(2*x^2 - x + 3)
```

Fricas [A]
time = 0.36, size = 93, normalized size = 0.56

$$\frac{1}{660602880}\left(1468006400x^8+2926837760x^7+1033175040x^6+12117893120x^5+379086848x^4+12669290112x^3+4870637856x^2+12357760788x+1592737263\right)\sqrt{2x^2-x+3}+\frac{147157749}{16777216}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 1/660602880*(1468006400*x^8 + 2926837760*x^7 + 1033175040*x^6 + 12117893120
*x^5 + 379086848*x^4 + 12669290112*x^3 + 4870637856*x^2 + 12357760788*x + 1
```

```
592737263)*sqrt(2*x^2 - x + 3) + 147157749/16777216*sqrt(2)*log(-4*sqrt(2)*
sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+5)\left(2x^2-x+3\right)^{\frac{3}{2}}\cdot\left(5x^4-x^3+3x^2+x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2),x)
```

```
[Out] Integral((2*x + 5)*(2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)
, x)
```

Giac [A]
time = 5.29, size = 88, normalized size = 0.53

$$\frac{1}{660602880}\left(4\left(8\left(4\left(16\left(20\left(8\left(28\left(160x+319\right)x+3153\right)x+295847\right)x+185101\right)x+98978829\right)x+152207433\right)x+3089440197\right)x+1592737263\right)\sqrt{2x^2-x+3}-\frac{147157749}{8388608}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 1/660602880*(4*(8*(4*(16*(20*(8*(28*(160*x + 319)*x + 3153)*x + 295847)*x +
 185101)*x + 98978829)*x + 152207433)*x + 3089440197)*x + 1592737263)*sqrt(
2*x^2 - x + 3) - 147157749/8388608*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (2x+5)\left(2x^2-x+3\right)^{3/2}\left(5x^4-x^3+3x^2+x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5)*(2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

```
[Out] int((2*x + 5)*(2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2), x)
```

3.335 $\int (3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)\, dx$

Optimal. Leaf size=147

$$-\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576}-\frac{8597(1-4x)\,(3-x+2x^2)^{3/2}}{65536}+\frac{1167(3-x+2x^2)^{5/2}}{14336}+\frac{125x(3-x+2x^2)}{3584}$$

```
[Out] -8597/65536*(1-4*x)*(2*x^2-x+3)^(3/2)+1167/14336*(2*x^2-x+3)^(5/2)+125/3584
*x*(2*x^2-x+3)^(5/2)+23/448*x^2*(2*x^2-x+3)^(5/2)+5/16*x^3*(2*x^2-x+3)^(5/2
)-13643439/4194304*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-593193/1048576*(1
-4*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$,
Rules used = {1675, 654, 626, 633, 221}

$$\frac{23}{448}\left(2x^2-x+3\right)^{5/2}x^2+\frac{125\left(2x^2-x+3\right)^{5/2}x}{3584}+\frac{1167\left(2x^2-x+3\right)^{5/2}}{14336}-\frac{8597(1-4x)\left(2x^2-x+3\right)^{3/2}}{65536}-\frac{593193(1-4x)\sqrt{2x^2-x+3}}{1048576}+\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3-\frac{13643439\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2097152\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-593193*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1048576 - (8597*(1 - 4*x)*(3 - x +
2*x^2)^(3/2))/65536 + (1167*(3 - x + 2*x^2)^(5/2))/14336 + (125*x*(3 - x +
2*x^2)^(5/2))/3584 + (23*x^2*(3 - x + 2*x^2)^(5/2))/448 + (5*x^3*(3 - x + 2
*x^2)^(5/2))/16 - (13643439*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2097152*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx = \frac{5}{16}x^3\left(3-x+2x^2\right)^{5/2}+\frac{1}{16}\int \left(3-x+2x^2\right)^{3/2}\left(32+\right.$$

$$=\frac{23}{448}x^2\left(3-x+2x^2\right)^{5/2}+\frac{5}{16}x^3\left(3-x+2x^2\right)^{5/2}+\frac{1}{224}$$

$$=\frac{125x\left(3-x+2x^2\right)^{5/2}}{3584}+\frac{23}{448}x^2\left(3-x+2x^2\right)^{5/2}+\frac{5}{16}x^3$$

$$=\frac{1167\left(3-x+2x^2\right)^{5/2}}{14336}+\frac{125x\left(3-x+2x^2\right)^{5/2}}{3584}+\frac{23}{448}x^2$$

$$=-\frac{8597(1-4x)\left(3-x+2x^2\right)^{3/2}}{65536}+\frac{1167\left(3-x+2x^2\right)^{5/}}{14336}$$

$$=-\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576}-\frac{8597(1-4x)\,(3-x}{65536}$$

$$=-\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576}-\frac{8597(1-4x)\,(3-x}{65536}$$

$$=-\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576}-\frac{8597(1-4x)\,(3-x}{65536}$$

Mathematica [A]

time = 0.57, size = 85, normalized size = 0.58

$$\frac{4\sqrt{3-x+2x^2}\left(-1663407+27845612x+3845856x^2+27023744x^3-7497728x^4+29335552x^5-7667712x^6+9175040x^7\right)-95504073\sqrt{2}\,\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{29360128}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-1663407 + 27845612*x + 3845856*x^2 + 27023744*x^3
- 7497728*x^4 + 29335552*x^5 - 7667712*x^6 + 9175040*x^7) - 95504073*Sqrt[2
]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/29360128
```

Maple [A]
time = 0.15, size = 117, normalized size = 0.80

method	result
risch	$\frac{(9175040x^7-7667712x^6+29335552x^5-7497728x^4+27023744x^3+3845856x^2+27845612x-1663407)\sqrt{2x^2-x+3}}{7340032}+\frac{1364343}{}$
trager	$\left(\frac{5}{4}x^7-\frac{117}{112}x^6+\frac{3581}{896}x^5-\frac{523}{512}x^4+\frac{211123}{57344}x^3+\frac{17169}{32768}x^2+\frac{6961403}{1835008}x-\frac{1663407}{7340032}\right)\sqrt{2x^2-x+3}-\frac{1364343}{}$
default	$\frac{593193(4x-1)\sqrt{2x^2-x+3}}{1048576}+\frac{13643439\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{4194304}+\frac{8597(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{65536}+\frac{125x(2x^2-x+3}{3584}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 593193/1048576*(4*x-1)*(2*x^2-x+3)^(1/2)+13643439/4194304*2^(1/2)*arcsinh(4
/23*23^(1/2)*(x-1/4))+8597/65536*(4*x-1)*(2*x^2-x+3)^(3/2)+125/3584*x*(2*x^
2-x+3)^(5/2)+23/448*x^2*(2*x^2-x+3)^(5/2)+5/16*x^3*(2*x^2-x+3)^(5/2)+1167/1
4336*(2*x^2-x+3)^(5/2)
```

Maxima [A]
time = 0.50, size = 138, normalized size = 0.94

$$\frac{5}{16}\left(2x^2-x+3\right)^{\frac{5}{2}}x^3+\frac{23}{448}\left(2x^2-x+3\right)^{\frac{5}{2}}x^2+\frac{125}{3584}\left(2x^2-x+3\right)^{\frac{5}{2}}x+\frac{1167}{14336}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{8597}{16384}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{8597}{65536}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{593193}{262144}\sqrt{2x^2-x+3}\,x+\frac{13643439}{4194304}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{593193}{1048576}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 5/16*(2*x^2 - x + 3)^(5/2)*x^3 + 23/448*(2*x^2 - x + 3)^(5/2)*x^2 + 125/358
4*(2*x^2 - x + 3)^(5/2)*x + 1167/14336*(2*x^2 - x + 3)^(5/2) + 8597/16384*(
2*x^2 - x + 3)^(3/2)*x - 8597/65536*(2*x^2 - x + 3)^(3/2) + 593193/262144*s
qrt(2*x^2 - x + 3)*x + 13643439/4194304*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x
- 1)) - 593193/1048576*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 0.35, size = 88, normalized size = 0.60

$$\frac{1}{7340032}\left(9175040\,x^{7}-7667712\,x^{6}+29335552\,x^{5}-7497728\,x^{4}+27023744\,x^{3}+3845856\,x^{2}+27845612\,x-1663407\right)\sqrt{2\,x^{2}-x+3}+\frac{13643439}{8388608}\,\sqrt{2}\,\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^{2}-x+3}\,(4\,x-1)-32\,x^{2}+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 1/7340032*(9175040*x^7 - 7667712*x^6 + 29335552*x^5 - 7497728*x^4 + 2702374
4*x^3 + 3845856*x^2 + 27845612*x - 1663407)*sqrt(2*x^2 - x + 3) + 13643439/
8388608*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*
x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^{2}-x+3\right)^{\frac{3}{2}}\cdot\left(5x^{4}-x^{3}+3x^{2}+x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2), x)
```

Giac [A]
time = 5.21, size = 83, normalized size = 0.56

$$\frac{1}{7340032}\left(4\left(8\left(4\left(16\left(4\left(8\left(140\,x-117\right)x+3581\right)x-3661\right)x+211123\right)x+120183\right)x+6961403\right)x-1663407\right)\sqrt{2\,x^{2}-x+3}-\frac{13643439}{4194304}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^{2}-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/7340032*(4*(8*(4*(16*(4*(8*(140*x - 117)*x + 3581)*x - 3661)*x + 211123)*
x + 120183)*x + 6961403)*x - 1663407)*sqrt(2*x^2 - x + 3) - 13643439/419430
4*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(2\,x^2-x+3\right)}^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

```
[Out] int((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2), x)
```

3.336 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$

Optimal. Leaf size=172

$$\frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536}+\frac{(500141-123060x)\left(3-x+2x^2\right)^{3/2}}{12288}+\frac{3505}{896}\left(3-x+2x^2\right)^{5/2}-\frac{31}{44}$$

```
[Out] 1/12288*(500141-123060*x)*(2*x^2-x+3)^(3/2)+3505/896*(2*x^2-x+3)^(5/2)-311/
448*(5+2*x)*(2*x^2-x+3)^(5/2)+5/112*(5+2*x)^2*(2*x^2-x+3)^(5/2)+1622009981/
262144*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-99009/16*arctanh(1/24*(17-22*
x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/65536*(141051019-23482924*x)*(2*x^2
-x+3)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1667, 828, 857, 633, 221, 738, 212}

$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}-\frac{311}{448}(2x+5)\left(2x^2-x+3\right)^{5/2}+\frac{3505}{896}\left(2x^2-x+3\right)^{5/2}+\frac{(500141-123060x)\left(2x^2-x+3\right)^{3/2}}{12288}+\frac{(141051019-23482924x)\sqrt{2x^2-x+3}}{65536}-\frac{99009\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{8\sqrt{2}}+\frac{1622009981\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{131072\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

```
[Out] ((141051019 - 23482924*x)*Sqrt[3 - x + 2*x^2])/65536 + ((500141 - 123060*x)
*(3 - x + 2*x^2)^(3/2))/12288 + (3505*(3 - x + 2*x^2)^(5/2))/896 - (311*(5
+ 2*x)*(3 - x + 2*x^2)^(5/2))/448 + (5*(5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))/1
12 + (1622009981*ArcSinh[(1 - 4*x)/Sqrt[23]])/(131072*Sqrt[2]) - (99009*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(8*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{5}{112}(5+2x)^2(3-x+2x^2)^{5/2} + \frac{1}{224}\int \frac{(3-x+2x^2)^{3/2}}{\cdots}$$

$$= -\frac{311}{448}(5+2x)(3-x+2x^2)^{5/2} + \frac{5}{112}(5+2x)^2(3-x+\cdots$$

$$= \frac{3505}{896}(3-x+2x^2)^{5/2} - \frac{311}{448}(5+2x)(3-x+2x^2)^{5/2} + \cdots$$

$$= \frac{(500141-123060x)(3-x+2x^2)^{3/2}}{12288} + \frac{3505}{896}(3-x+2x\cdots$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536} + \frac{(500141-12\cdots}{\cdots}$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536} + \frac{(500141-12\cdots}{\cdots}$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536} + \frac{(500141-12\cdots}{\cdots}$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536} + \frac{(500141-12\cdots}{\cdots}$$

Mathematica [A]
time = 0.67, size = 113, normalized size = 0.66

$$\frac{4\sqrt{3-x+2x^2}\,(3149403255-609499532x+159973408x^2-46476672x^3+14493696x^4-3710976x^5+983040x^6)+68130865152\sqrt{2}\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)+34062209601\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{5505024}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x
]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(3149403255 - 609499532*x + 159973408*x^2 - 46476672
*x^3 + 14493696*x^4 - 3710976*x^5 + 983040*x^6) + 68130865152*Sqrt[2]*ArcTa
nh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 34062209601*Sqrt[2]*Log[1 - 4*x +
 2*Sqrt[6 - 2*x + 4*x^2]])/5505024
```

Maple [A]

time = 0.01, size = 183, normalized size = 1.06

$$\frac{5x^2(2x^2-x+3)^{\frac{5}{2}}}{28}-\frac{111x(2x^2-x+3)^{\frac{5}{2}}}{224}+\frac{1395(2x^2-x+3)^{\frac{5}{2}}}{896}-\frac{10255(4x-1)\,(2x^2-x+3)^{\frac{3}{2}}}{4096}-\frac{707595(4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x)
```

```
[Out] 5/28*x^2*(2*x^2-x+3)^(5/2)-111/224*x*(2*x^2-x+3)^(5/2)+1395/896*(2*x^2-x+3)
^(5/2)-10255/4096*(4*x-1)*(2*x^2-x+3)^(3/2)-707595/65536*(4*x-1)*(2*x^2-x+3
)^(1/2)-1622009981/262144*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+3667/96*(2
*(x+5/2)^2-11*x-19/2)^(3/2)-40337/512*(4*x-1)*(2*(x+5/2)^2-11*x-19/2)^(1/2)
+33003/16*(2*(x+5/2)^2-11*x-19/2)^(1/2)-99009/16*2^(1/2)*arctanh(1/12*(17/2
-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Maxima [A]
time = 0.52, size = 157, normalized size = 0.91

$$\frac{5}{28}\left(2x^2-x+3\right)^{\frac{5}{2}}x^2-\frac{111}{224}\left(2x^2-x+3\right)^{\frac{5}{2}}x+\frac{1395}{896}\left(2x^2-x+3\right)^{\frac{5}{2}}-\frac{10255}{1024}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{500141}{12288}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{5870731}{16384}\sqrt{2x^2-x+3}\,x-\frac{1622009981}{262144}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{99009}{16}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+5|}\right)+\frac{141051019}{65536}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="max
ima")
```

```
[Out] 5/28*(2*x^2 - x + 3)^(5/2)*x^2 - 111/224*(2*x^2 - x + 3)^(5/2)*x + 1395/896
*(2*x^2 - x + 3)^(5/2) - 10255/1024*(2*x^2 - x + 3)^(3/2)*x + 500141/12288*
(2*x^2 - x + 3)^(3/2) - 5870731/16384*sqrt(2*x^2 - x + 3)*x - 1622009981/26
2144*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 99009/16*sqrt(2)*ar
csinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 141051
019/65536*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 0.38, size = 135, normalized size = 0.78

$$\frac{1}{1376256}\left(983040x^6-3710976x^5+14493696x^4-46476672x^3+159973408x^2-609499532x+3149403255\right)\sqrt{2x^2-x+3}+\frac{1622009981}{524288}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)+\frac{99009}{32}\sqrt{2}\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}\,(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="fri
cas")
```

```
[Out] 1/1376256*(983040*x^6 - 3710976*x^5 + 14493696*x^4 - 46476672*x^3 + 1599734
08*x^2 - 609499532*x + 3149403255)*sqrt(2*x^2 - x + 3) + 1622009981/524288*
```

```
sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) +
 99009/32*sqrt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x
^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}} \cdot (5x^4 - x^3 + 3x^2 + x + 2)}{2x + 5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
, x)
```

Giac [A]
time = 4.94, size = 139, normalized size = 0.81

$$\frac{1}{1376256}\,(4\,(8\,(12\,(16\,(4\,(40\,x-151)x+2359)x-121033)x+4999169)x-152374883)x+3149403255)\sqrt{2\,x^2-x+3}+\frac{1622009981}{262144}\,\sqrt{2}\,\log\left(-4\,\sqrt{2}\,x+\sqrt{2}+4\,\sqrt{2\,x^2-x+3}\right)-\frac{99009}{16}\,\sqrt{2}\,\log\left(\left|-2\,\sqrt{2}\,x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)+\frac{99009}{16}\,\sqrt{2}\,\log\left(\left|-2\,\sqrt{2}\,x-11\,\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="gia
c")
```

```
[Out] 1/1376256*(4*(8*(12*(16*(4*(40*x - 151)*x + 2359)*x - 121033)*x + 4999169)*
x - 152374883)*x + 3149403255)*sqrt(2*x^2 - x + 3) + 1622009981/262144*sqrt
(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 99009/16*sqrt(2)*
log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 99009/16*sqrt(2)
*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(2\,x^2-x+3\right)}^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{2\,x+5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5),x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5), x)
```

3.337 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx$

Optimal. Leaf size=172

$$-\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768}-\frac{(909513-226052x)\left(3-x+2x^2\right)^{3/2}}{18432}-\frac{839}{960}\left(3-x+2x^2\right)^{5/2}-\frac{3}{}$$

```
[Out] -1/18432*(909513-226052*x)*(2*x^2-x+3)^(3/2)-839/960*(2*x^2-x+3)^(5/2)-3667
/576*(2*x^2-x+3)^(5/2)/(5+2*x)+5/96*(5+2*x)*(2*x^2-x+3)^(5/2)-982669459/131
072*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+959625/128*arctanh(1/24*(17-22*x
)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-1/32768*(85448933-14243732*x)*(2*x^2-x
+3)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1664, 1667, 828, 857, 633, 221, 738, 212}

$$\frac{5}{96}(2x+5)\left(2x^2-x+3\right)^{5/2}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}-\frac{839}{960}\left(2x^2-x+3\right)^{5/2}-\frac{(909513-226052x)\left(2x^2-x+3\right)^{3/2}}{18432}-\frac{(85448933-14243732x)\sqrt{2x^2-x+3}}{32768}+\frac{959625\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{64\sqrt{2}}-\frac{982669459\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x]
```

```
[Out] -1/32768*((85448933 - 14243732*x)*Sqrt[3 - x + 2*x^2]) - ((909513 - 226052*
x)*(3 - x + 2*x^2)^(3/2))/18432 - (839*(3 - x + 2*x^2)^(5/2))/960 - (3667*(
3 - x + 2*x^2)^(5/2))/(576*(5 + 2*x)) + (5*(5 + 2*x)*(3 - x + 2*x^2)^(5/2))
/96 - (982669459*ArcSinh[(1 - 4*x)/Sqrt[23]])/(65536*Sqrt[2]) + (959625*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(64*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
```

```
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = -\frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)} - \frac{1}{72}\int \frac{(3-x+2x^2)^{3/2}\left(\frac{26675}{16}\right.}{5}$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)} + \frac{5}{96}(5+2x)\left(3-x+2x^2\right)^{5/2}$$

$$= -\frac{839}{960}\left(3-x+2x^2\right)^{5/2} - \frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)} + \frac{5}{96}(5$$

$$= -\frac{(909513-226052x)\left(3-x+2x^2\right)^{3/2}}{18432} - \frac{839}{960}(3-x+$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-}{}$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-}{}$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-}{}$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-}{}$$

Mathematica [A]
time = 0.72, size = 120, normalized size = 0.70

$$\frac{\frac{4\sqrt{3-x+2x^2}\left(-6814208295-1404323114x+182033816x^2-35369408x^3+8283904x^4-1798144x^5+409600x^6\right)}{5+2x} - 29479680000\sqrt{2}\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right) - 14740041885\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{1966080}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2
,x]
```

```
[Out] ((4*Sqrt[3 - x + 2*x^2]*(-6814208295 - 1404323114*x + 182033816*x^2 - 35369
408*x^3 + 8283904*x^4 - 1798144*x^5 + 409600*x^6))/(5 + 2*x) - 29479680000*
Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 14740041885*Sqrt[2]*
Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/1966080
```

Maple [A]
time = 0.02, size = 208, normalized size = 1.21

$$\frac{5x(2x^2-x+3)^{\frac{5}{2}}}{48}-\frac{589(2x^2-x+3)^{\frac{5}{2}}}{960}+\frac{9059(4x-1)\,(2x^2-x+3)^{\frac{3}{2}}}{6144}+\frac{208357(4x-1)\,\sqrt{2x^2-x+3}}{32768}+\frac{982\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x)
```

```
[Out] 5/48*x*(2*x^2-x+3)^(5/2)-589/960*(2*x^2-x+3)^(5/2)+9059/6144*(4*x-1)*(2*x^2
-x+3)^(3/2)+208357/32768*(4*x-1)*(2*x^2-x+3)^(1/2)+982669459/131072*2^(1/2)
*arcsinh(4/23*23^(1/2)*(x-1/4))-3667/1152/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(
5/2)-106625/2304*(2*(x+5/2)^2-11*x-19/2)^(3/2)+1637/16*(4*x-1)*(2*(x+5/2)^2
-11*x-19/2)^(1/2)-319875/128*(2*(x+5/2)^2-11*x-19/2)^(1/2)+959625/128*2^(1/
2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))+3667/230
4*(4*x-1)*(2*(x+5/2)^2-11*x-19/2)^(3/2)
```

Maxima [A]
time = 0.51, size = 161, normalized size = 0.94

$$\frac{5}{48}\,{\left(2\,x^{2}-x+3\right)}^{\frac{5}{2}}x-\frac{589}{960}\,{\left(2\,x^{2}-x+3\right)}^{\frac{5}{2}}+\frac{9059}{1536}\,{\left(2\,x^{2}-x+3\right)}^{\frac{3}{2}}x-\frac{185827}{6144}\,{\left(2\,x^{2}-x+3\right)}^{\frac{3}{2}}+\frac{3560933}{8192}\,\sqrt{2\,x^{2}-x+3}\,x+\frac{982669459}{131072}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\,\sqrt{23}\,x-\frac{1}{23}\,\sqrt{23}\right)-\frac{959625}{128}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\,\sqrt{23}\,x}{23\,{\left|2\,x+5\right|}}-\frac{17\,\sqrt{23}}{23\,{\left|2\,x+5\right|}}\right)-\frac{85448933}{32768}\,\sqrt{2\,x^{2}-x+3}-\frac{3667\,{\left(2\,x^{2}-x+3\right)}^{\frac{3}{2}}}{32\,{\left(2\,x+5\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="m
axima")
```

```
[Out] 5/48*(2*x^2 - x + 3)^(5/2)*x - 589/960*(2*x^2 - x + 3)^(5/2) + 9059/1536*(2
*x^2 - x + 3)^(3/2)*x - 185827/6144*(2*x^2 - x + 3)^(3/2) + 3560933/8192*sq
rt(2*x^2 - x + 3)*x + 982669459/131072*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/
23*sqrt(23)) - 959625/128*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 1
7/23*sqrt(23)/abs(2*x + 5)) - 85448933/32768*sqrt(2*x^2 - x + 3) - 3667/32*
(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Fricas [A]
time = 0.42, size = 153, normalized size = 0.89

$$\frac{14740041885 \sqrt{2} (2x+5) \log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1) - 32x^2+16x-25\right) + 14739840000\sqrt{2}(2x+5)\log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)-1060x^2+1036x-1153}{4x^2+20x+25}\right) + 8(409600x^6 - 1798144x^5+8283904x^4-35369408x^3+182033816x^2-1404323114x-6814208295)\sqrt{2x^2-x+3}}{3932160(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="f
ricas")
```

```
[Out] 1/3932160*(14740041885*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)
*(4*x - 1) - 32*x^2 + 16*x - 25) + 14739840000*sqrt(2)*(2*x + 5)*log((24*sq
rt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 +
20*x + 25)) + 8*(409600*x^6 - 1798144*x^5 + 8283904*x^4 - 35369408*x^3 + 18
2033816*x^2 - 1404323114*x - 6814208295)*sqrt(2*x^2 - x + 3))/(2*x + 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2 - x + 3\right)^{\frac{3}{2}} \cdot \left(5x^4 - x^3 + 3x^2 + x + 2\right)}{\left(2x + 5\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 707 vs. 2(137) = 274.
time = 4.18, size = 707, normalized size = 4.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="g
iac")
```

```
[Out] 1/1966080*sqrt(2)*(14739840000*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 14740041885*log(abs(sqrt(-11/(2
*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 147400
41885*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*
sgn(1/(2*x + 5)) - 2027704320*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)*sgn(
1/(2*x + 5)) + 2*(45496763235*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6
/(2*x + 5))^11*sgn(1/(2*x + 5)) - 126553743360*(sqrt(-11/(2*x + 5) + 36/(2*
x + 5)^2 + 1) + 6/(2*x + 5))^10*sgn(1/(2*x + 5)) + 44062768335*(sqrt(-11/(2
```

```
*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^9*sgn(1/(2*x + 5)) + 331789824
00*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^8*sgn(1/(2*x +
5)) + 294206421582*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))
^7*sgn(1/(2*x + 5)) - 463672074240*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1
) + 6/(2*x + 5))^6*sgn(1/(2*x + 5)) + 35099942478*(sqrt(-11/(2*x + 5) + 36/
(2*x + 5)^2 + 1) + 6/(2*x + 5))^5*sgn(1/(2*x + 5)) + 171324610560*(sqrt(-11
/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^4*sgn(1/(2*x + 5)) + 600592
81615*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3*sgn(1/(2*x
 + 5)) - 105051009024*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
5))^2*sgn(1/(2*x + 5)) - 5210329245*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
1) + 6/(2*x + 5))*sgn(1/(2*x + 5)) + 17058392064*sgn(1/(2*x + 5)))/((sqrt(-
11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^6)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{\left(2\,x+5\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2,x)
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2, x)
```

3.338 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$

Optimal. Leaf size=174

$$\frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576}+\frac{(2154633-534617x)\left(3-x+2x^2\right)^{3/2}}{82944}+\frac{1}{16}\left(3-x+2x^2\right)^{5/2}-\cdots$$

```
[Out] 1/82944*(2154633-534617*x)*(2*x^2-x+3)^(3/2)+1/16*(2*x^2-x+3)^(5/2)-3667/11
52*(2*x^2-x+3)^(5/2)/(5+2*x)^2+438065/82944*(2*x^2-x+3)^(5/2)/(5+2*x)+12934
2063/32768*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-8083915/2048*arctanh(1/24
*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/24576*(33741483-5623292*x)*
(2*x^2-x+3)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1664, 1667, 828, 857, 633, 221, 738, 212}

$$\frac{438065(2x^2-x+3)^{5/2}}{82944(2x+5)}-\frac{3667(2x^2-x+3)^{5/2}}{1152(2x+5)^2}+\frac{1}{16}\left(2x^2-x+3\right)^{5/2}+\frac{(2154633-534617x)\left(2x^2-x+3\right)^{3/2}}{82944}+\frac{(33741483-5623292x)\sqrt{2x^2-x+3}}{24576}-\frac{8083915\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1024\sqrt{2}}+\frac{129342063\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16384\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x]
```

```
[Out] ((33741483 - 5623292*x)*Sqrt[3 - x + 2*x^2])/24576 + ((2154633 - 534617*x)*
(3 - x + 2*x^2)^(3/2))/82944 + (3 - x + 2*x^2)^(5/2)/16 - (3667*(3 - x + 2*
x^2)^(5/2))/(1152*(5 + 2*x)^2) + (438065*(3 - x + 2*x^2)^(5/2))/(82944*(5 +
 2*x)) + (129342063*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16384*Sqrt[2]) - (8083915
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1024*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
```

```
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = -\frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2} - \frac{1}{144}\int \frac{(3-x+2x^2)^{3/2}\left(\frac{35015}{16}\right.}{(5}$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2} + \frac{438065(3-x+2x^2)^{5/2}}{82944(5+2x)} + \frac{\int}{}$$

$$= \frac{1}{16}(3-x+2x^2)^{5/2} - \frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2} + \frac{438065(3}{8294}$$

$$= \frac{(2154633-534617x)(3-x+2x^2)^{3/2}}{82944} + \frac{1}{16}(3-x+2x$$

$$= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633-53}{}$$

$$= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633-53}{}$$

$$= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633-53}{}$$

$$= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633-53}{}$$

Mathematica [A]

time = 0.82, size = 120, normalized size = 0.69

$$\frac{\frac{4\sqrt{3-x+2x^2}\,(298966737+181223072x+16667188x^2-1620944x^3+253312x^4-43520x^5+8192x^6)}{(5+2x)^2}+258685280\sqrt{2}\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)+129342063\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{32768}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3
,x]
```

```
[Out] ((4*Sqrt[3 - x + 2*x^2]*(298966737 + 181223072*x + 16667188*x^2 - 1620944*x
^3 + 253312*x^4 - 43520*x^5 + 8192*x^6))/(5 + 2*x)^2 + 258685280*Sqrt[2]*Ar
cTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 129342063*Sqrt[2]*Log[1 - 4*x
+ 2*Sqrt[6 - 2*x + 4*x^2]])/32768
```

Maple [A]
time = 0.02, size = 214, normalized size = 1.23

$$\frac{(2x^2-x+3)^{\frac{5}{2}}}{16}-\frac{3667\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{5}{2}}}{4608\left(x+\frac{5}{2}\right)^2}+\frac{438065\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{5}{2}}}{165888\left(x+\frac{5}{2}\right)}-\frac{438065(4x-1)\left(2(x+\right.}{33177}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x)
```

```
[Out] 1/16*(2*x^2-x+3)^(5/2)-3667/4608/(x+5/2)^2*(2*(x+5/2)^2-11*x-19/2)^(5/2)+43
8065/165888/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(5/2)-438065/331776*(4*x-1)*(2*
(x+5/2)^2-11*x-19/2)^(3/2)-343745/6144*(4*x-1)*(2*(x+5/2)^2-11*x-19/2)^(1/2
)-149/512*(4*x-1)*(2*x^2-x+3)^(3/2)-8083915/2048*2^(1/2)*arctanh(1/12*(17/2
-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))+8083915/6144*(2*(x+5/2)^2-11*
x-19/2)^(1/2)-129342063/32768*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+808391
5/331776*(2*(x+5/2)^2-11*x-19/2)^(3/2)-10281/8192*(4*x-1)*(2*x^2-x+3)^(1/2)
```

Maxima [A]
time = 0.54, size = 172, normalized size = 0.99

$$\frac{1}{16}\left(2x^2-x+3\right)^{\frac{5}{2}}-\frac{149}{128}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{46691}{4608}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2x^2-x+3\right)^{\frac{5}{2}}}{1152\left(4x^2+20x+25\right)}-\frac{1405823}{6144}\sqrt{2x^2-x+3}\,x-\frac{129342063}{32768}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{8083915}{2048}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+5|}\right)+\frac{11247161}{8192}\sqrt{2x^2-x+3}+\frac{438065\left(2x^2-x+3\right)^{\frac{3}{2}}}{4608\left(2x+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="m
axima")
```

```
[Out] 1/16*(2*x^2 - x + 3)^(5/2) - 149/128*(2*x^2 - x + 3)^(3/2)*x + 46691/4608*(
2*x^2 - x + 3)^(3/2) - 3667/1152*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25)
- 1405823/6144*sqrt(2*x^2 - x + 3)*x - 129342063/32768*sqrt(2)*arcsinh(4/23
*sqrt(23)*x - 1/23*sqrt(23)) + 8083915/2048*sqrt(2)*arcsinh(22/23*sqrt(23)*
x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 11247161/8192*sqrt(2*x^2 -
x + 3) + 438065/4608*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Fricas [A]
time = 0.37, size = 169, normalized size = 0.97

$$\frac{129342063\sqrt{2}\left(4x^2+20x+25\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}\left(4x-1\right)-32x^2+16x-25\right)+129342640\sqrt{2}\left(4x^2+20x+25\right)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}\left(22x-17\right)+1060x^2-1036x+1153}{4x^2+20x+25}\right)+8\left(8192x^6-43520x^5+253312x^4-1620944x^3+16667188x^2+181223072x+298966737\right)\sqrt{2x^2-x+3}}{65536\left(4x^2+20x+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="f
ricas")
```

```
[Out] 1/65536*(129342063*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 129342640*sqrt(2)*(4*x^2 + 20*x + 2
5)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1
153)/(4*x^2 + 20*x + 25)) + 8*(8192*x^6 - 43520*x^5 + 253312*x^4 - 1620944*
x^3 + 16667188*x^2 + 181223072*x + 298966737)*sqrt(2*x^2 - x + 3))/(4*x^2 +
 20*x + 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2-x+3\right)^{\frac{3}{2}}\cdot\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x+5\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**3, x)
```

Giac [A]
time = 3.89, size = 268, normalized size = 1.54

$$\frac{1}{8192}\left(4\left(8\left(4\left(16x-165\right)x+4879\right)x-263469\right)x+8460377\right)\sqrt{2x^2-x+3}+\frac{129342063}{32768}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)-\frac{8083915}{2048}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{8083915}{2048}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{\sqrt{2}\left(14243182\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^3+109906674\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2-170996871\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+110506087\right)}{512\left(2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-11\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="g
iac")
```

```
[Out] 1/8192*(4*(8*(4*(16*x - 165)*x + 4879)*x - 263469)*x + 8460377)*sqrt(2*x^2
- x + 3) + 129342063/32768*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 -
 x + 3)) + 1) - 8083915/2048*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqr
t(2*x^2 - x + 3))) + 8083915/2048*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2)
 + 2*sqrt(2*x^2 - x + 3))) + 1/512*sqrt(2)*(14243182*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3))^3 + 109906674*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 170
996871*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 110506087)/(2*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) -
11)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3,x)
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3, x)
```

3.339 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$

Optimal. Leaf size=181

$$-\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776}-\frac{(138006843-34265045x)\left(3-x+2x^2\right)^{3/2}}{17915904}-\frac{3667(3-x+2x}{1728(5+2x)}$$

```
[Out] -1/17915904*(138006843-34265045*x)*(2*x^2-x+3)^(3/2)-3667/1728*(2*x^2-x+3)^
(5/2)/(5+2*x)^3+556255/248832*(2*x^2-x+3)^(5/2)/(5+2*x)^2-32865365/17915904
*(2*x^2-x+3)^(5/2)/(5+2*x)-19176431/16384*arcsinh(1/23*(1-4*x)*23^(1/2))*2^
(1/2)+517762327/442368*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^
(1/2)-1/331776*(135068604-22512089*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1664, 828, 857, 633, 221, 738, 212}

$$-\frac{32865365(2x^2-x+3)^{5/2}}{17915904(2x+5)}+\frac{556255(2x^2-x+3)^{5/2}}{248832(2x+5)^2}-\frac{3667(2x^2-x+3)^{5/2}}{1728(2x+5)^3}-\frac{(138006843-34265045x)\left(2x^2-x+3\right)^{3/2}}{17915904}-\frac{(135068604-22512089x)\sqrt{2x^2-x+3}}{331776}+\frac{517762327\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{221184\sqrt{2}}-\frac{19176431\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x]
```

```
[Out] -1/331776*((135068604 - 22512089*x)*Sqrt[3 - x + 2*x^2]) - ((138006843 - 34
265045*x)*(3 - x + 2*x^2)^(3/2))/17915904 - (3667*(3 - x + 2*x^2)^(5/2))/(1
728*(5 + 2*x)^3) + (556255*(3 - x + 2*x^2)^(5/2))/(248832*(5 + 2*x)^2) - (3
2865365*(3 - x + 2*x^2)^(5/2))/(17915904*(5 + 2*x)) - (19176431*ArcSinh[(1
- 4*x)/Sqrt[23]])/(8192*Sqrt[2]) + (517762327*ArcTanh[(17 - 22*x)/(12*Sqrt[
2]*Sqrt[3 - x + 2*x^2])])/(221184*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = -\frac{3667(3-x+2x^2)^{5/2}}{1728(5+2x)^3} - \frac{1}{216}\int \frac{(3-x+2x^2)^{3/2}\left(\frac{43355}{16}\right.}{(5}$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{1728(5+2x)^3} + \frac{556255(3-x+2x^2)^{5/2}}{248832(5+2x)^2} + \int$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{1728(5+2x)^3} + \frac{556255(3-x+2x^2)^{5/2}}{248832(5+2x)^2} - 32$$

$$= -\frac{(138006843-34265045x)(3-x+2x^2)^{3/2}}{17915904} - \frac{3667(3}{1728}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(1380068}{}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(1380068}{}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(1380068}{}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(1380068}{}$$

Mathematica [A]
time = 0.86, size = 120, normalized size = 0.66

$$\frac{\frac{12\sqrt{3-x+2x^2}\,(-1994650739-2006873194x-594798908x^2-33595416x^3+2626848x^4-315648x^5+46080x^6)}{(5+2x)^3} - 1035524654\sqrt{2}\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right) - 517763637\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{442368}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4
,x]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(-1994650739 - 2006873194*x - 594798908*x^2 - 3359
5416*x^3 + 2626848*x^4 - 315648*x^5 + 46080*x^6))/(5 + 2*x)^3 - 1035524654*
Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 517763637*Sqrt[2]*Lo
g[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/442368
```

Maple [A]

time = 0.02, size = 221, normalized size = 1.22

$$-\frac{3667\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{5}{2}}}{13824\left(x+\frac{5}{2}\right)^{3}}+\frac{556255\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{5}{2}}}{995328\left(x+\frac{5}{2}\right)^{2}}-\frac{32865365\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{5}{2}}}{35831808\left(x+\frac{5}{2}\right)}+ 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x)
```

```
[Out] -3667/13824/(x+5/2)^3*(2*(x+5/2)^2-11*x-19/2)^(5/2)+556255/995328/(x+5/2)^2
*(2*(x+5/2)^2-11*x-19/2)^(5/2)-32865365/35831808/(x+5/2)*(2*(x+5/2)^2-11*x-
19/2)^(5/2)+32865365/71663616*(4*x-1)*(2*(x+5/2)^2-11*x-19/2)^(3/2)+2240030
9/1327104*(4*x-1)*(2*(x+5/2)^2-11*x-19/2)^(1/2)+5/256*(4*x-1)*(2*x^2-x+3)^(
3/2)+517762327/442368*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2
-11*x-19/2)^(1/2))-517762327/1327104*(2*(x+5/2)^2-11*x-19/2)^(1/2)+19176431
/16384*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-517762327/71663616*(2*(x+5/2)
^2-11*x-19/2)^(3/2)+345/4096*(4*x-1)*(2*x^2-x+3)^(1/2)
```

Maxima [A]
time = 0.52, size = 189, normalized size = 1.04

$$\frac{5}{64}\left(2x^{2}-x+3\right)^{\frac{3}{2}}x-\frac{1094743}{497664}\left(2x^{2}-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{1728\left(8x^{3}+60x^{2}+150x+125\right)}+\frac{556255\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{248832\left(4x^{2}+20x+25\right)}+\frac{22512089}{331776}\sqrt{2x^{2}-x+3}\,x+\frac{19176431}{16384}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{517762327}{442368}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\left|2x+5\right|}-\frac{17\sqrt{23}}{23\left|2x+5\right|}\right)-\frac{11255717}{27648}\sqrt{2x^{2}-x+3}-\frac{32865365\left(2x^{2}-x+3\right)^{\frac{3}{2}}}{995328\left(2x+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="m
axima")
```

```
[Out] 5/64*(2*x^2 - x + 3)^(3/2)*x - 1094743/497664*(2*x^2 - x + 3)^(3/2) - 3667/
1728*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 556255/248832*(
2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25) + 22512089/331776*sqrt(2*x^2 - x +
 3)*x + 19176431/16384*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 5
17762327/442368*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(
23)/abs(2*x + 5)) - 11255717/27648*sqrt(2*x^2 - x + 3) - 32865365/995328*(2
*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Fricas [A]
time = 0.39, size = 183, normalized size = 1.01

$$\frac{517763637\sqrt{2}\left(8x^{3}+60x^{2}+150x+125\right)\log\left(-4\sqrt{2}\sqrt{2x^{2}-x+3}\left(4x-1\right)-32x^{2}+16x-25\right)+517762327\sqrt{2}\left(8x^{3}+60x^{2}+150x+125\right)\log\left(\frac{24\sqrt{2}\sqrt{2x^{2}-x+3}\left(22x-17\right)-1060x^{2}+1036x-1153}{4x^{2}+20x+25}\right)+24\left(46080x^{6}-315648x^{5}+2626848x^{4}-33595416x^{3}-594798908x^{2}-2006873194x-1994650739\right)\sqrt{2x^{2}-x+3}}{884736\left(8x^{3}+60x^{2}+150x+125\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="f
ricas")
```

```
[Out] 1/884736*(517763637*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 517762327*sqrt(2)*(8*x
^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17)
- 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 24*(46080*x^6 - 315648*x
^5 + 2626848*x^4 - 33595416*x^3 - 594798908*x^2 - 2006873194*x - 1994650739
)*sqrt(2*x^2 - x + 3))/(8*x^3 + 60*x^2 + 150*x + 125)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}} \cdot (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(146) = 292.
time = 3.75, size = 314, normalized size = 1.73

$$\frac{1}{4096}\left(4\left(8\left(20x - 287\right)x + 23341\right)x - 1004633\right)\sqrt{2x^2 - x + 3} - \frac{19176431}{16384}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{517762327}{442368}\sqrt{2}\log\left(\left|-2\sqrt{2}x + \sqrt{2} + 2\sqrt{2x^2 - x + 3}\right|\right) - \frac{517762327}{442368}\sqrt{2}\log\left(\left|-2\sqrt{2}x - 11\sqrt{2} + 2\sqrt{2x^2 - x + 3}\right|\right) - \frac{\sqrt{2}\left(1092794276\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right)^5 + 18284336132\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right)^4 + 20314214356\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right)^3 - 151449344092\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right)^2 + 102529692109\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) - 41882448755\right)}{36864\left(2\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right)^2 + 10\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) - 11\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="g
iac")
```

```
[Out] 1/4096*(4*(8*(20*x - 287)*x + 23341)*x - 1004633)*sqrt(2*x^2 - x + 3) - 191
76431/16384*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) +
 517762327/442368*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x
 + 3))) - 517762327/442368*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sq
rt(2*x^2 - x + 3))) - 1/36864*sqrt(2)*(1092794276*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3))^5 + 18284336132*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 2031
4214356*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 151449344092*(sqrt(2)
*x - sqrt(2*x^2 - x + 3))^2 + 102529692109*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2
- x + 3)) - 41882448755)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(2\,x^2-x+3\right)}^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4,x)
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4, x)
```

3.340 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$

Optimal. Leaf size=188

$$\frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496}+\frac{(762984903+67865260x)\left(3-x+2x^2\right)^{3/2}}{95551488(5+2x)}-\frac{3667(3-x+2x}{2304(5+2x)}$$

```
[Out] 1/95551488*(762984903+67865260*x)*(2*x^2-x+3)^(3/2)/(5+2*x)-3667/2304*(2*x^
2-x+3)^(5/2)/(5+2*x)^4+224815/165888*(2*x^2-x+3)^(5/2)/(5+2*x)^3-14477995/2
3887872*(2*x^2-x+3)^(5/2)/(5+2*x)^2+432565/2048*arcsinh(1/23*(1-4*x)*23^(1/
2))*2^(1/2)-8969688643/42467328*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^
(1/2))*2^(1/2)+1/31850496*(2339916063-389975609*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1664, 826, 828, 857, 633, 221, 738, 212}

$$-\frac{14477995\left(2x^2-x+3\right)^{5/2}}{23887872(2x+5)^2}+\frac{224815\left(2x^2-x+3\right)^{5/2}}{165888(2x+5)^3}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}+\frac{(67865260x+762984903)\left(2x^2-x+3\right)^{3/2}}{95551488(2x+5)}+\frac{(2339916063-389975609x)\sqrt{2x^2-x+3}}{31850496}-\frac{8969688643\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{21233664\sqrt{2}}+\frac{432565\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x]
```

```
[Out] ((2339916063 - 389975609*x)*Sqrt[3 - x + 2*x^2])/31850496 + ((762984903 + 6
7865260*x)*(3 - x + 2*x^2)^(3/2))/(95551488*(5 + 2*x)) - (3667*(3 - x + 2*x
^2)^(5/2))/(2304*(5 + 2*x)^4) + (224815*(3 - x + 2*x^2)^(5/2))/(165888*(5 +
 2*x)^3) - (14477995*(3 - x + 2*x^2)^(5/2))/(23887872*(5 + 2*x)^2) + (43256
5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*Sqrt[2]) - (8969688643*ArcTanh[(17 - 2
2*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(21233664*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = -\frac{3667(3-x+2x^2)^{5/2}}{2304(5+2x)^4} - \frac{1}{288}\int \frac{(3-x+2x^2)^{3/2}\left(\frac{51695}{16}\right.}{(5}$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{2304(5+2x)^4} + \frac{224815(3-x+2x^2)^{5/2}}{165888(5+2x)^3} + \frac{\int}{}$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{2304(5+2x)^4} + \frac{224815(3-x+2x^2)^{5/2}}{165888(5+2x)^3} - \frac{14}{}$$

$$= \frac{(762984903+67865260x)(3-x+2x^2)^{3/2}}{95551488(5+2x)} - \frac{3667(3-}{2304(}$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(7629849}{}$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(7629849}{}$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(7629849}{}$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(7629849}{}$$

Mathematica [A]
time = 0.83, size = 120, normalized size = 0.64

$$\frac{\frac{12\sqrt{3-x+2x^2}\,(86386856771+121473790266x+60528581892x^2+11761910072x^3+468043776x^4-29270016x^5+2949120x^6)}{(5+2x)^4}+8969688643\sqrt{2}\,\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)+4484833920\sqrt{2}\,\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{21233664}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5
,x]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(86386856771 + 121473790266*x + 60528581892*x^2 +
11761910072*x^3 + 468043776*x^4 - 29270016*x^5 + 2949120*x^6))/(5 + 2*x)^4
+ 8969688643*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 4484833
920*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/21233664
```

Maple [A]
time = 0.02, size = 204, normalized size = 1.09

$$\frac{224815\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{5}{2}}}{1327104\left(x+\frac{5}{2}\right)^{3}}-\frac{14477995\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{5}{2}}}{95551488\left(x+\frac{5}{2}\right)^{2}}+\frac{593321753\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{5}{2}}}{3439853568\left(x+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x)
```

```
[Out] 224815/1327104/(x+5/2)^3*(2*(x+5/2)^2-11*x-19/2)^(5/2)-14477995/95551488/(x
+5/2)^2*(2*(x+5/2)^2-11*x-19/2)^(5/2)+593321753/3439853568/(x+5/2)*(2*(x+5/
2)^2-11*x-19/2)^(5/2)-593321753/6879707136*(4*x-1)*(2*(x+5/2)^2-11*x-19/2)^
(3/2)-389975609/127401984*(4*x-1)*(2*(x+5/2)^2-11*x-19/2)^(1/2)-3667/36864/
(x+5/2)^4*(2*(x+5/2)^2-11*x-19/2)^(5/2)-8969688643/42467328*2^(1/2)*arctanh
(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))+8969688643/1274019
84*(2*(x+5/2)^2-11*x-19/2)^(1/2)-432565/2048*2^(1/2)*arcsinh(4/23*23^(1/2)*
(x-1/4))+8969688643/6879707136*(2*(x+5/2)^2-11*x-19/2)^(3/2)
```

Maxima [A]
time = 0.52, size = 210, normalized size = 1.12

$$\frac{16966315}{47775744}\left(2x^{2}-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{2304\left(16x^{4}+160x^{3}+600x^{2}+1000x+625\right)}+\frac{224815\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{165888\left(8x^{3}+60x^{2}+150x+125\right)}-\frac{14477995\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{23887872\left(4x^{2}+20x+25\right)}-\frac{389975609}{31850496}\sqrt{2x^{2}-x+3}\,x-\frac{432565}{2048}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{8969688643}{42467328}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\left|2x+5\right|}-\frac{17\sqrt{23}}{23\left|2x+5\right|}\right)+\frac{779972021}{10616832}\sqrt{2x^{2}-x+3}+\frac{593321753\left(2x^{2}-x+3\right)^{\frac{3}{2}}}{95551488\left(2x+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="m
axima")
```

```
[Out] 16966315/47775744*(2*x^2 - x + 3)^(3/2) - 3667/2304*(2*x^2 - x + 3)^(5/2)/(
16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 224815/165888*(2*x^2 - x + 3)^
(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 14477995/23887872*(2*x^2 - x + 3)^(5
/2)/(4*x^2 + 20*x + 25) - 389975609/31850496*sqrt(2*x^2 - x + 3)*x - 432565
/2048*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 8969688643/4246732
```

```
8*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x +
5)) + 779972021/10616832*sqrt(2*x^2 - x + 3) + 593321753/95551488*(2*x^2 -
x + 3)^(3/2)/(2*x + 5)
```

Fricas [A]
time = 0.43, size = 199, normalized size = 1.06

$$\frac{8969667840\,\sqrt{2}\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)\log\left(4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+8969688643\,\sqrt{2}\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)\log\left(-\frac{24\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(22\,x-17)+1060\,x^2-1036\,x+1153}{4\,x^2+20\,x+25}\right)+48\,(2949120\,x^6-29270016\,x^5+468043776\,x^4+11761910072\,x^3+60528581892\,x^2+121473790266\,x+86386856771)\sqrt{2\,x^2-x+3}}{84934656\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="f
ricas")
```

```
[Out] 1/84934656*(8969667840*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*
log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8969688
643*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*log(-(24*sqrt(2)*sq
rt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 2
5)) + 48*(2949120*x^6 - 29270016*x^5 + 468043776*x^4 + 11761910072*x^3 + 60
528581892*x^2 + 121473790266*x + 86386856771)*sqrt(2*x^2 - x + 3))/(16*x^4
+ 160*x^3 + 600*x^2 + 1000*x + 625)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}} \cdot (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**5,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 503 vs. 2(153) = 306.
time = 4.19, size = 503, normalized size = 2.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="g
iac")
```

```
[Out] -1/42467328*sqrt(2)*(8969688643*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2
+ 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 8969667840*log(abs(sqrt(-11/(2
*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 896966
```

```
7840*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*s
gn(1/(2*x + 5)) + 12*(24*(1296*(29336*sgn(1/(2*x + 5))/(2*x + 5) - 42907*sg
n(1/(2*x + 5)))/(2*x + 5) + 39923563*sgn(1/(2*x + 5)))/(2*x + 5) - 54131203
9*sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 13824*(80624
1*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^5*sgn(1/(2*x + 5
)) - 1152288*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^4*sgn
(1/(2*x + 5)) - 957352*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
 5))^3*sgn(1/(2*x + 5)) + 1529280*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)
 + 6/(2*x + 5))^2*sgn(1/(2*x + 5)) + 394431*(sqrt(-11/(2*x + 5) + 36/(2*x +
 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x + 5)) - 620352*sgn(1/(2*x + 5)))/((sqr
t(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{\left(2\,x+5\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5,x)
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5, x)
```

3.341 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$

Optimal. Leaf size=195

$$-\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)}+\frac{(246012435+44773976x)\,(3-x+2x^2)^{3/2}}{95551488(5+2x)^2}-\frac{3667(3-x+}{2880(5+}$$

```
[Out] 1/95551488*(246012435+44773976*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^2-3667/2880*(2*
x^2-x+3)^(5/2)/(5+2*x)^5+158527/165888*(2*x^2-x+3)^(5/2)/(5+2*x)^4-3730507/
11943936*(2*x^2-x+3)^(5/2)/(5+2*x)^3-23775/1024*arcsinh(1/23*(1-4*x)*23^(1/
2))*2^(1/2)+70991525167/3057647616*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+
3)^(1/2))*2^(1/2)-1/127401984*(5658774871+1028823716*x)*(2*x^2-x+3)^(1/2)/(
5+2*x)
```

Rubi [A]
time = 0.16, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1664, 826, 857, 633, 221, 738, 212}

$$-\frac{3730507(2x^2-x+3)^{5/2}}{11943936(2x+5)^3}+\frac{158527(2x^2-x+3)^{5/2}}{165888(2x+5)^4}-\frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5}+\frac{(44773976x+246012435)\,(2x^2-x+3)^{3/2}}{95551488(2x+5)^2}-\frac{(1028823716x+5658774871)\sqrt{2x^2-x+3}}{127401984(2x+5)}+\frac{70991525167\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1528823808\sqrt{2}}-\frac{23775\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{512\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x]
```

```
[Out] -1/127401984*((5658774871 + 1028823716*x)*Sqrt[3 - x + 2*x^2])/(5 + 2*x) +
((246012435 + 44773976*x)*(3 - x + 2*x^2)^(3/2))/(95551488*(5 + 2*x)^2) - (
3667*(3 - x + 2*x^2)^(5/2))/(2880*(5 + 2*x)^5) + (158527*(3 - x + 2*x^2)^(5
/2))/(165888*(5 + 2*x)^4) - (3730507*(3 - x + 2*x^2)^(5/2))/(11943936*(5 +
2*x)^3) - (23775*ArcSinh[(1 - 4*x)/Sqrt[23]])/(512*Sqrt[2]) + (70991525167*
ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1528823808*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = -\frac{3667(3-x+2x^2)^{5/2}}{2880(5+2x)^5} - \frac{1}{360}\int \frac{(3-x+2x^2)^{3/2}\left(\frac{60035}{16}\right.}{(}$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{2880(5+2x)^5} + \frac{158527(3-x+2x^2)^{5/2}}{165888(5+2x)^4} + \frac{\int}{}$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{2880(5+2x)^5} + \frac{158527(3-x+2x^2)^{5/2}}{165888(5+2x)^4} - \frac{37}{}$$

$$= \frac{(246012435+44773976x)(3-x+2x^2)^{3/2}}{95551488(5+2x)^2} - \frac{3667(3-}{2880(}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(2460}{}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(2460}{}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(2460}{}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(2460}{}$$

Mathematica [A]
time = 0.90, size = 120, normalized size = 0.62

$$\frac{\frac{12\sqrt{3-x+2x^2}\,(-17093312738327-30872393829992x-21590439797064x^2-7117092892448x^3-1023534029552x^4-30496849920x^5+1592524800x^6)}{(5+2x)^5} - 354957625835\sqrt{2}\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right) - 177479424000\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{7644119040}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6
,x]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(-17093312738327 - 30872393829992*x - 215904397970
64*x^2 - 7117092892448*x^3 - 1023534029552*x^4 - 30496849920*x^5 + 15925248
00*x^6))/(5 + 2*x)^5 - 354957625835*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x
 + 4*x^2])/6] - 177479424000*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]]
)/7644119040
```

Maple [A]
time = 0.02, size = 225, normalized size = 1.15

$$-\frac{3667\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{5}{2}}}{92160\left(x+\frac{5}{2}\right)^{5}}-\frac{3730507\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{5}{2}}}{95551488\left(x+\frac{5}{2}\right)^{3}}+\frac{134077495\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{5}{2}}}{6879707136\left(x+\frac{5}{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x)
```

```
[Out] -3667/92160/(x+5/2)^5*(2*(x+5/2)^2-11*x-19/2)^(5/2)-3730507/95551488/(x+5/2
)^3*(2*(x+5/2)^2-11*x-19/2)^(5/2)+134077495/6879707136/(x+5/2)^2*(2*(x+5/2)
^2-11*x-19/2)^(5/2)-4698578717/247669456896/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)
^(5/2)+4698578717/495338913792*(4*x-1)*(2*(x+5/2)^2-11*x-19/2)^(3/2)+308671
5581/9172942848*(4*x-1)*(2*(x+5/2)^2-11*x-19/2)^(1/2)+158527/2654208/(x+5/2
)^4*(2*(x+5/2)^2-11*x-19/2)^(5/2)+70991525167/3057647616*2^(1/2)*arctanh(1/
12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-70991525167/917294284
8*(2*(x+5/2)^2-11*x-19/2)^(1/2)+23775/1024*2^(1/2)*arcsinh(4/23*23^(1/2)*(x
-1/4))-70991525167/495338913792*(2*(x+5/2)^2-11*x-19/2)^(3/2)
```

Maxima [A]
time = 0.57, size = 251, normalized size = 1.29

$$-\frac{134077495}{3439853568}\left(2x^{2}-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{2880\left(32x^{5}+400x^{4}+2000x^{3}+5000x^{2}+6250x+3125\right)}+\frac{158527\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{165888\left(16x^{4}+160x^{3}+600x^{2}+1000x+625\right)}-\frac{3730507\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{11943936\left(8x^{3}+60x^{2}+150x+125\right)}+\frac{134077495\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{1719926784\left(4x^{2}+20x+25\right)}+\frac{3086715581}{2293235712}\sqrt{2x^{2}-x+3}\,x+\frac{23775}{1024}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{70991525167}{3057647616}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\left|2x+5\right|}-\frac{17\sqrt{23}}{23\left|2x+5\right|}\right)-\frac{6173186729}{764411904}\sqrt{2x^{2}-x+3}-\frac{4698578717\left(2x^{2}-x+3\right)^{\frac{3}{2}}}{6879707136\left(2x+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="m
axima")
```

```
[Out] -134077495/3439853568*(2*x^2 - x + 3)^(3/2) - 3667/2880*(2*x^2 - x + 3)^(5/
2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 158527/165888
*(2*x^2 - x + 3)^(5/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) - 373050
7/11943936*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 134077495
/1719926784*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25) + 3086715581/22932357
12*sqrt(2*x^2 - x + 3)*x + 23775/1024*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/2
3*sqrt(23)) - 70991525167/3057647616*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2
*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 6173186729/764411904*sqrt(2*x^2 -
x + 3) - 4698578717/6879707136*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Fricas [A]
time = 0.42, size = 213, normalized size = 1.09

$$\frac{354958848000\sqrt{2}\left(32x^{5}+400x^{4}+2000x^{3}+5000x^{2}+6250x+3125\right)\log\left(-4\sqrt{2}\sqrt{2x^{2}-x+3}\left(4x-1\right)-32x^{2}+16x-25\right)+354957625835\sqrt{2}\left(32x^{5}+400x^{4}+2000x^{3}+5000x^{2}+6250x+3125\right)\log\left(\frac{24\sqrt{2}\sqrt{2x^{2}-x+3}\left(22x-17\right)-1060x^{2}+1036x-1153}{4x^{2}+20x+25}\right)+48\left(1592524800x^{6}-30496849920x^{5}-1023534029552x^{4}-7117092892448x^{3}-21590439797064x^{2}-30872393829992x-17093312738327\right)\sqrt{2x^{2}-x+3}}{30576476160\left(32x^{5}+400x^{4}+2000x^{3}+5000x^{2}+6250x+3125\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="f
ricas")
```

```
[Out] 1/30576476160*(354958848000*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2
 + 6250*x + 3125)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 1
6*x - 25) + 354957625835*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 +
6250*x + 3125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 +
 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(1592524800*x^6 - 30496849920*x^5
 - 1023534029552*x^4 - 7117092892448*x^3 - 21590439797064*x^2 - 30872393829
992*x - 17093312738327)*sqrt(2*x^2 - x + 3))/(32*x^5 + 400*x^4 + 2000*x^3 +
 5000*x^2 + 6250*x + 3125)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}} \cdot (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**6,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**6, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(160) = 320.
time = 4.05, size = 406, normalized size = 2.08

$$\frac{1}{256}\sqrt{2x^2-x+3}\,(20x-633) - \frac{23775}{1024}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right) + \frac{70991525167}{3057647616}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) - \frac{70991525167}{3057647616}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) - \frac{\sqrt{2}\left(8281387393360\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^9 + 275661428628240\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^8 + 1560382703345760\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^7 + 4938646760855520\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^6 - 9673562837036232\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^5 - 30647310393849000\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^4 + 70060241036847960\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^3 - 97730658088823880\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2 + 30180638363071845\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right) - 7096913381268319\right)}{1274019840\left(2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2 + 10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right) - 11\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="g
iac")
```

```
[Out] 1/256*sqrt(2*x^2 - x + 3)*(20*x - 633) - 23775/1024*sqrt(2)*log(-2*sqrt(2)*
(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 70991525167/3057647616*sqrt(2)*log
(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 70991525167/3057647
616*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1
/1274019840*sqrt(2)*(8281387393360*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^9 + 275661428628240*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 156038270334576
0*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 + 4938646760855520*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^6 - 9673562837036232*sqrt(2)*(sqrt(2)*x - sqrt(2*x^
2 - x + 3))^5 - 30647310393849000*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 700
60241036847960*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 97730658088823
```

```
880*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 30180638363071845*sqrt(2)*(sqrt(2
)*x - sqrt(2*x^2 - x + 3)) - 7096913381268319)/(2*(sqrt(2)*x - sqrt(2*x^2 -
 x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{(2x+5)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6,x)
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6, x)
```

3.342 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$

Optimal. Leaf size=195

$$\frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)}-\frac{(9802984711+6793718806x)\,(3-x+2x^2)^{3/2}}{13759414272(5+2x)^3}-\frac{3667(3-}{3456}$$

```
[Out] -1/13759414272*(9802984711+6793718806*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^3-3667/3
456*(2*x^2-x+3)^(5/2)/(5+2*x)^6+182165/248832*(2*x^2-x+3)^(5/2)/(5+2*x)^5-1
4087245/71663616*(2*x^2-x+3)^(5/2)/(5+2*x)^4+369/256*arcsinh(1/23*(1-4*x)*2
3^(1/2))*2^(1/2)-1903976002333/1320903770112*arctanh(1/24*(17-22*x)*2^(1/2)
/(2*x^2-x+3)^(1/2))*2^(1/2)+1/55037657088*(151764102421+27596573612*x)*(2*x
^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A]
time = 0.17, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1664, 824, 826, 857, 633, 221, 738, 212}

$$-\frac{14087245(2x^2-x+3)^{5/2}}{71663616(2x+5)^4}+\frac{182165(2x^2-x+3)^{5/2}}{248832(2x+5)^5}-\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}-\frac{(6793718806x+9802984711)\,(2x^2-x+3)^{3/2}}{13759414272(2x+5)^3}+\frac{(27596573612x+151764102421)\sqrt{2x^2-x+3}}{55037657088(2x+5)}-\frac{1903976002333\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{660451885056\sqrt{2}}+\frac{369\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x]
```

```
[Out] ((151764102421 + 27596573612*x)*Sqrt[3 - x + 2*x^2])/(55037657088*(5 + 2*x)
) - ((9802984711 + 6793718806*x)*(3 - x + 2*x^2)^(3/2))/(13759414272*(5 + 2
*x)^3) - (3667*(3 - x + 2*x^2)^(5/2))/(3456*(5 + 2*x)^6) + (182165*(3 - x +
 2*x^2)^(5/2))/(248832*(5 + 2*x)^5) - (14087245*(3 - x + 2*x^2)^(5/2))/(716
63616*(5 + 2*x)^4) + (369*ArcSinh[(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2]) - (190
3976002333*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(66045188
5056*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = -\frac{3667(3-x+2x^2)^{5/2}}{3456(5+2x)^6} - \frac{1}{432}\int \frac{(3-x+2x^2)^{3/2}\left(\frac{68375}{16}\right.}{(}$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{3456(5+2x)^6} + \frac{182165(3-x+2x^2)^{5/2}}{248832(5+2x)^5} + \frac{\int}{}$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{3456(5+2x)^6} + \frac{182165(3-x+2x^2)^{5/2}}{248832(5+2x)^5} - \frac{14}{}$$

$$= -\frac{(9802984711+6793718806x)(3-x+2x^2)^{3/2}}{13759414272(5+2x)^3} - \frac{3667}{3}$$

$$= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(980}{}$$

$$= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(980}{}$$

$$= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(980}{}$$

$$= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(980}{}$$

Mathematica [A]

time = 0.97, size = 120, normalized size = 0.62

$$\frac{\frac{12\sqrt{3-x+2x^2}\,(458411625354581+1011372787716826x+910256842473992x^2+422554114856528x^3+103803827945872x^4+11854023276320x^5+275188285440x^6)}{(5+2x)^6}+1903976002333\sqrt{2}\,\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)+951979474944\sqrt{2}\,\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{660451885056}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7
,x]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(458411625354581 + 1011372787716826*x + 9102568424
73992*x^2 + 422554114856528*x^3 + 103803827945872*x^4 + 11854023276320*x^5
+ 275188285440*x^6))/(5 + 2*x)^6 + 1903976002333*Sqrt[2]*ArcTanh[(5 + 2*x -
 Sqrt[6 - 2*x + 4*x^2])/6] + 951979474944*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 -
2*x + 4*x^2]])/660451885056
```

Maple [A]
time = 0.02, size = 246, normalized size = 1.26

$$-\frac{3667\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{5}{2}}}{221184\left(x+\frac{5}{2}\right)^6}+\frac{182165\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{5}{2}}}{7962624\left(x+\frac{5}{2}\right)^5}+\frac{149610673\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{5}{2}}}{41278242816\left(x+\frac{5}{2}\right)^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x)
```

```
[Out] -3667/221184/(x+5/2)^6*(2*(x+5/2)^2-11*x-19/2)^(5/2)+182165/7962624/(x+5/2)
^5*(2*(x+5/2)^2-11*x-19/2)^(5/2)+149610673/41278242816/(x+5/2)^3*(2*(x+5/2)
^2-11*x-19/2)^(5/2)-3607708597/2972033482752/(x+5/2)^2*(2*(x+5/2)^2-11*x-19
/2)^(5/2)+125860542215/106993205379072/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(5/2
)-125860542215/213986410758144*(4*x-1)*(2*(x+5/2)^2-11*x-19/2)^(3/2)-827726
68391/3962711310336*(4*x-1)*(2*(x+5/2)^2-11*x-19/2)^(1/2)-14087245/11466178
56/(x+5/2)^4*(2*(x+5/2)^2-11*x-19/2)^(5/2)-1903976002333/1320903770112*2^(1
/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))+1903976
002333/3962711310336*(2*(x+5/2)^2-11*x-19/2)^(1/2)-369/256*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))+1903976002333/213986410758144*(2*(x+5/2)^2-11*x-19/2
)^(3/2)
```

Maxima [A]
time = 0.53, size = 297, normalized size = 1.52

$$\frac{3607708597}{1486016741376}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2x^2-x+3\right)^{\frac{5}{2}}}{3456\left(64x^6+960x^5+6000x^4+20000x^3+37500x^2+37500x+15625\right)}+\frac{182165\left(2x^2-x+3\right)^{\frac{5}{2}}}{248832\left(32x^5+400x^4+2000x^3+5000x^2+6250x+3125\right)}-\frac{14087245\left(2x^2-x+3\right)^{\frac{5}{2}}}{71663616\left(16x^4+160x^3+600x^2+1000x+625\right)}+\frac{149610673\left(2x^2-x+3\right)^{\frac{5}{2}}}{5159780352\left(8x^3+60x^2+150x+125\right)}-\frac{3607708597\left(2x^2-x+3\right)^{\frac{5}{2}}}{743008370688\left(4x^2+20x+25\right)}-\frac{82772668391}{990677827584}\sqrt{2x^2-x+3}\,x-\frac{369}{256}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{1903976002333}{1320903770112}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+5|}\right)+\frac{165562389227}{330225942528}\sqrt{2x^2-x+3}+\frac{125860542215\left(2x^2-x+3\right)^{\frac{3}{2}}}{2972033482752\,(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="m
axima")
```

```
[Out] 3607708597/1486016741376*(2*x^2 - x + 3)^(3/2) - 3667/3456*(2*x^2 - x + 3)^
(5/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 1562
5) + 182165/248832*(2*x^2 - x + 3)^(5/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 500
0*x^2 + 6250*x + 3125) - 14087245/71663616*(2*x^2 - x + 3)^(5/2)/(16*x^4 +
160*x^3 + 600*x^2 + 1000*x + 625) + 149610673/5159780352*(2*x^2 - x + 3)^(5
/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 3607708597/743008370688*(2*x^2 - x + 3
)^(5/2)/(4*x^2 + 20*x + 25) - 82772668391/990677827584*sqrt(2*x^2 - x + 3)*
x - 369/256*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 190397600233
3/1320903770112*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(
23)/abs(2*x + 5)) + 165562389227/330225942528*sqrt(2*x^2 - x + 3) + 1258605
42215/2972033482752*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Fricas [A]
time = 0.39, size = 229, normalized size = 1.17

$$\frac{1903958949888 \sqrt{2} (64 x^6 + 960 x^5 + 6000 x^4 + 20000 x^3 + 37500 x^2 + 37500 x + 15625) \log\left(4 \sqrt{2} \sqrt{2 x^2 - x + 3} (4 x - 1) - 32 x^2 + 16 x - 25\right) + 1903976002333 \sqrt{2} (64 x^6 + 960 x^5 + 6000 x^4 + 20000 x^3 + 37500 x^2 + 37500 x + 15625) \log\left(-\frac{24 \sqrt{2} \sqrt{2 x^2 - x + 3} (22 x - 17) + 1060 x^2 - 1036 x + 1153}{4 x^2 + 20 x + 25}\right) + 48 (275188285440 x^6 + 11854023276320 x^5 + 103803827945872 x^4 + 422554114856528 x^3 + 910256842473992 x^2 + 1011372787716826 x + 458411625354581) \sqrt{2 x^2 - x + 3}}{2641807540224 (64 x^6 + 960 x^5 + 6000 x^4 + 20000 x^3 + 37500 x^2 + 37500 x + 15625)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="f
ricas")
```

```
[Out] 1/2641807540224*(1903958949888*sqrt(2)*(64*x^6 + 960*x^5 + 6000*x^4 + 20000
*x^3 + 37500*x^2 + 37500*x + 15625)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25) + 1903976002333*sqrt(2)*(64*x^6 + 960*x^5 + 6000
*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 15625)*log(-(24*sqrt(2)*sqrt(2*x^2
 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)) + 48
*(275188285440*x^6 + 11854023276320*x^5 + 103803827945872*x^4 + 42255411485
6528*x^3 + 910256842473992*x^2 + 1011372787716826*x + 458411625354581)*sqrt
(2*x^2 - x + 3))/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 375
00*x + 15625)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2 - x + 3)^{\frac{3}{2}} \cdot (5x^4 - x^3 + 3x^2 + x + 2)}{(2x + 5)^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**7,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**7, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(160) = 320.

time = 3.17, size = 452, normalized size = 2.32

$$\frac{369}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)-\frac{1903976002333}{1320903770112}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{1903976002333}{1320903770112}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{5}{64}\sqrt{2x^2-x+3}+\frac{\sqrt{2}\left(159278433934432\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{11}+6347903280912544\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{10}+48544526840833424\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{9}+305716670132783088\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{8}+88313821135911024\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{7}-2423668581998843376\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{6}-397211131697032056\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{5}+11708897232532299576\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{4}-12803484860728491138\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{3}+12593033197867577234\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{2}-3042533760672408875\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+5895262632497801 95\right)}{110075314176\left(2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{2}+10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-11\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="g
iac")
```

```
[Out] 369/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 190
3976002333/1320903770112*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*
x^2 - x + 3))) + 1903976002333/1320903770112*sqrt(2)*log(abs(-2*sqrt(2)*x -
 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 5/64*sqrt(2*x^2 - x + 3) + 1/110075
314176*sqrt(2)*(159278433934432*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^1
1 + 6347903280912544*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^10 + 485445268408334
24*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 305716670132783088*(sqrt(2
)*x - sqrt(2*x^2 - x + 3))^8 + 88313821135911024*sqrt(2)*(sqrt(2)*x - sqrt(
2*x^2 - x + 3))^7 - 2423668581998843376*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6
 - 397211131697032056*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 1170889
7232532299576*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 - 12803484860728491138*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 12593033197867577234*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 - 3042533760672408875*sqrt(2)*(sqrt(2)*x - sqrt(2
*x^2 - x + 3)) + 5895262632497801 95)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2
 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^6
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(2\,x^2-x+3\right)}^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^7} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7,x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7, x)
```

3.343 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$

Optimal. Leaf size=195

$$-\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2}-\frac{(463558457+411822458x)\left(3-x+2x^2\right)^{3/2}}{2293235712(5+2x)^4}-\frac{3667(3}{4032}$$

```
[Out] -1/2293235712*(463558457+411822458*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^4-3667/4032
*(2*x^2-x+3)^(5/2)/(5+2*x)^7+114335/193536*(2*x^2-x+3)^(5/2)/(5+2*x)^6-1930
441/13934592*(2*x^2-x+3)^(5/2)/(5+2*x)^5-5/128*arcsinh(1/23*(1-4*x)*23^(1/2
))*2^(1/2)+412760561351/10567230160896*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^
2-x+3)^(1/2))*2^(1/2)-1/440301256704*(146583836191+101679102454*x)*(2*x^2-x
+3)^(1/2)/(5+2*x)^2
```

Rubi [A]
time = 0.16, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1664, 824, 857, 633, 221, 738, 212}

$$-\frac{1930441\left(2x^2-x+3\right)^{5/2}}{13934592(2x+5)^5}+\frac{114335\left(2x^2-x+3\right)^{5/2}}{193536(2x+5)^6}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}-\frac{(411822458x+463558457)\left(2x^2-x+3\right)^{3/2}}{2293235712(2x+5)^4}-\frac{(101679102454x+146583836191)\sqrt{2x^2-x+3}}{440301256704(2x+5)^2}+\frac{412760561351\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{5283615080448\sqrt{2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x]
```

```
[Out] -1/440301256704*((146583836191 + 101679102454*x)*Sqrt[3 - x + 2*x^2])/(5 +
2*x)^2 - ((463558457 + 411822458*x)*(3 - x + 2*x^2)^(3/2))/(2293235712*(5 +
 2*x)^4) - (3667*(3 - x + 2*x^2)^(5/2))/(4032*(5 + 2*x)^7) + (114335*(3 - x
 + 2*x^2)^(5/2))/(193536*(5 + 2*x)^6) - (1930441*(3 - x + 2*x^2)^(5/2))/(13
934592*(5 + 2*x)^5) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2]) + (41276
0561351*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(52836150804
48*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^8}\,dx = -\frac{3667(3-x+2x^2)^{5/2}}{4032(5+2x)^7} - \frac{1}{504}\int \frac{(3-x+2x^2)^{3/2}\left(\frac{76715}{16}\right.}{(}$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{4032(5+2x)^7} + \frac{114335(3-x+2x^2)^{5/2}}{193536(5+2x)^6} + \int$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{4032(5+2x)^7} + \frac{114335(3-x+2x^2)^{5/2}}{193536(5+2x)^6} - 19$$

$$= -\frac{(463558457+411822458x)(3-x+2x^2)^{3/2}}{2293235712(5+2x)^4} - \frac{3667(3}{403}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(4}{}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(4}{}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(4}{}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(4}{}$$

<u>Mathematica</u> [A]
time = 1.21, size = 120, normalized size = 0.62

$$\frac{-\frac{12\sqrt{3-x+2x^2}\,(3479517268702637+9065154700300572x+9976065367498188x^2+5966329646300704x^3+2069947287085104x^4+402255822731712x^5+38463671680832x^6)}{(5+2x)^7} - 2889323929457\sqrt{2}\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right) - 1444738498560\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{36985305563136}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8
,x]
```

```
[Out] ((-12*Sqrt[3 - x + 2*x^2]*(3479517268702637 + 9065154700300572*x + 99760653
67498188*x^2 + 5966329646300704*x^3 + 2069947287085104*x^4 + 40225582273171
2*x^5 + 38463671680832*x^6))/(5 + 2*x)^7 - 2889323929457*Sqrt[2]*ArcTanh[(5
 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 1444738498560*Sqrt[2]*Log[1 - 4*x + 2*
Sqrt[6 - 2*x + 4*x^2]])/36985305563136
```

Maple [A]
time = 0.02, size = 267, normalized size = 1.37

$$\frac{114335\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{5}{2}}}{12386304\left(x+\frac{5}{2}\right)^{6}}-\frac{3667\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{5}{2}}}{516096\left(x+\frac{5}{2}\right)^{7}}-\frac{1930441\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{5}{2}}}{445906944\left(x+\frac{5}{2}\right)^{5}}-329$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x)
```

```
[Out] 114335/12386304/(x+5/2)^6*(2*(x+5/2)^2-11*x-19/2)^(5/2)-3667/516096/(x+5/2)
^7*(2*(x+5/2)^2-11*x-19/2)^(5/2)-1930441/445906944/(x+5/2)^5*(2*(x+5/2)^2-1
1*x-19/2)^(5/2)-32967491/330225942528/(x+5/2)^3*(2*(x+5/2)^2-11*x-19/2)^(5/
2)+769352975/23776267862016/(x+5/2)^2*(2*(x+5/2)^2-11*x-19/2)^(5/2)-2745215
7541/855945643032576/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(5/2)+27452157541/1711
891286065152*(4*x-1)*(2*(x+5/2)^2-11*x-19/2)^(3/2)+17957520133/317016904826
88*(4*x-1)*(2*(x+5/2)^2-11*x-19/2)^(1/2)+7861079/9172942848/(x+5/2)^4*(2*(x
+5/2)^2-11*x-19/2)^(5/2)+412760561351/10567230160896*2^(1/2)*arctanh(1/12*(
17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-412760561351/317016904826
88*(2*(x+5/2)^2-11*x-19/2)^(1/2)+5/128*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4
))-412760561351/1711891286065152*(2*(x+5/2)^2-11*x-19/2)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(160) = 320.
time = 0.53, size = 348, normalized size = 1.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="m
axima")
```

```
[Out] -769352975/11888133931008*(2*x^2 - x + 3)^(3/2) - 3667/4032*(2*x^2 - x + 3)
^(5/2)/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^
2 + 218750*x + 78125) + 114335/193536*(2*x^2 - x + 3)^(5/2)/(64*x^6 + 960*x
^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 15625) - 1930441/13934592
*(2*x^2 - x + 3)^(5/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3
125) + 7861079/573308928*(2*x^2 - x + 3)^(5/2)/(16*x^4 + 160*x^3 + 600*x^2
+ 1000*x + 625) - 32967491/41278242816*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^
2 + 150*x + 125) + 769352975/5944066965504*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 2
0*x + 25) + 17957520133/7925422620672*sqrt(2*x^2 - x + 3)*x + 5/128*sqrt(2)
*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 412760561351/10567230160896*sqr
```

```
t(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) -
 35893173457/2641807540224*sqrt(2*x^2 - x + 3) - 27452157541/23776267862016
*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Fricas [A]
time = 0.44, size = 243, normalized size = 1.25

$$\frac{2889476997120\,\sqrt{2}\,(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x+78125)\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+2889323929457\,\sqrt{2}\,(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x+78125)\log\left(\frac{24\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(22\,x-17)-1060\,x^2+1036\,x-1153}{4\,x^2+20\,x+25}\right)-48\,(38463671680832\,x^6+402255822731712\,x^5+2069947287085104\,x^4+5966329646300704\,x^3+9976065367498188\,x^2+9065154700300572\,x+3479517268702637)\sqrt{2\,x^2-x+3}}{147941222252544\,(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x+78125)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="f
ricas")
```

```
[Out] 1/147941222252544*(2889476997120*sqrt(2)*(128*x^7 + 2240*x^6 + 16800*x^5 +
70000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125)*log(-4*sqrt(2)*sqrt
(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 2889323929457*sqrt(2)*(12
8*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^2 + 218750
*x + 78125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 10
36*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(38463671680832*x^6 + 40225582273171
2*x^5 + 2069947287085104*x^4 + 5966329646300704*x^3 + 9976065367498188*x^2
+ 9065154700300572*x + 3479517268702637)*sqrt(2*x^2 - x + 3))/(128*x^7 + 22
40*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**8,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**8, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 489 vs. 2(160) = 320.
time = 5.37, size = 489, normalized size = 2.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="g
iac")
```

```
[Out] -5/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 4127
60561351/10567230160896*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x
^2 - x + 3))) - 412760561351/10567230160896*sqrt(2)*log(abs(-2*sqrt(2)*x -
11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/6164217593856*sqrt(2)*(11218973984
12224*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^13 + 48260296303776704*(sqr
t(2)*x - sqrt(2*x^2 - x + 3))^12 + 444673458321712704*sqrt(2)*(sqrt(2)*x -
sqrt(2*x^2 - x + 3))^11 + 3996455936659982656*(sqrt(2)*x - sqrt(2*x^2 - x +
 3))^10 + 6725227967167489360*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 -
 17132661028483948080*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 - 637130120947372
46112*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 + 106515880136064432096*(
sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 + 226947197958946260516*sqrt(2)*(sqrt(2)
*x - sqrt(2*x^2 - x + 3))^5 - 856601202771483308188*(sqrt(2)*x - sqrt(2*x^2
 - x + 3))^4 + 617998258357377713732*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^3 - 467121785339763351756*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 9229208
073556056227*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 15161716093827501
349)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(
2*x^2 - x + 3)) - 11)^7
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^8} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8,x)
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8, x)
```

3.344 $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=120

$$\frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2}-\frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2}+\frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2}-\frac{(19227+4676x)\sqrt{3-}}{2048}$$

```
[Out] -85429/8192*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+761/256*(5+2*x)^2*(2*x^2
-x+3)^(1/2)-105/128*(5+2*x)^3*(2*x^2-x+3)^(1/2)+1/16*(5+2*x)^4*(2*x^2-x+3)^
(1/2)-1/2048*(19227+4676*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1667, 793, 633, 221}

$$\frac{1}{16}\sqrt{2x^2-x+3}\,(2x+5)^4-\frac{105}{128}\sqrt{2x^2-x+3}\,(2x+5)^3+\frac{761}{256}\sqrt{2x^2-x+3}\,(2x+5)^2-\frac{(4676x+19227)\sqrt{2x^2-x+3}}{2048}-\frac{85429\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (761*(5 + 2*x)^2*Sqrt[3 - x + 2*x^2])/256 - (105*(5 + 2*x)^3*Sqrt[3 - x + 2
*x^2])/128 + ((5 + 2*x)^4*Sqrt[3 - x + 2*x^2])/16 - ((19227 + 4676*x)*Sqrt[
3 - x + 2*x^2])/2048 - (85429*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4096*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} + \frac{1}{160}\int \frac{(5+2x)\left(-5055-4390x-\right.}{\sqrt{3-x+2x\right.}}$$

$$= -\frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} + \int$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}$$

Mathematica [A]
time = 0.34, size = 70, normalized size = 0.58

$$\frac{4\sqrt{3-x+2x^2}\left(2973-6916x+352x^2+7040x^3+2048x^4\right)-85429\sqrt{2}\,\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{8192}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(2973 - 6916*x + 352*x^2 + 7040*x^3 + 2048*x^4) - 85
429*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/8192
```

Maple [A]
time = 0.15, size = 95, normalized size = 0.79

method	result
risch	$\frac{(2048x^4+7040x^3+352x^2-6916x+2973)\sqrt{2x^2-x+3}}{2048}+\frac{85429\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{8192}$
trager	$\left(x^4+\frac{55}{16}x^3+\frac{11}{64}x^2-\frac{1729}{512}x+\frac{2973}{2048}\right)\sqrt{2x^2-x+3}-\frac{85429\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-4\,\mathrm{RootOf}\left(\_Z^2-2\right)x+4\sqrt{}\right.}{8192}$
default	$x^4\sqrt{2x^2-x+3}+\frac{55x^3\sqrt{2x^2-x+3}}{16}+\frac{11x^2\sqrt{2x^2-x+3}}{64}-\frac{1729x\sqrt{2x^2-x+3}}{512}+\frac{2973\sqrt{2x^2}}{204}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] x^4*(2*x^2-x+3)^(1/2)+55/16*x^3*(2*x^2-x+3)^(1/2)+11/64*x^2*(2*x^2-x+3)^(1/
2)-1729/512*x*(2*x^2-x+3)^(1/2)+2973/2048*(2*x^2-x+3)^(1/2)+85429/8192*2^(1
/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [A]
time = 0.49, size = 96, normalized size = 0.80

$$\sqrt{2\,x^2-x+3}\,x^4+\frac{55}{16}\,\sqrt{2\,x^2-x+3}\,x^3+\frac{11}{64}\,\sqrt{2\,x^2-x+3}\,x^2-\frac{1729}{512}\,\sqrt{2\,x^2-x+3}\,x+\frac{85429}{8192}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{2973}{2048}\,\sqrt{2\,x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="max
ima")
```

```
[Out] sqrt(2*x^2 - x + 3)*x^4 + 55/16*sqrt(2*x^2 - x + 3)*x^3 + 11/64*sqrt(2*x^2
- x + 3)*x^2 - 1729/512*sqrt(2*x^2 - x + 3)*x + 85429/8192*sqrt(2)*arcsinh(
1/23*sqrt(23)*(4*x - 1)) + 2973/2048*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 0.37, size = 73, normalized size = 0.61

$$\frac{1}{2048}\,(2048\,x^4+7040\,x^3+352\,x^2-6916\,x+2973)\sqrt{2\,x^2-x+3}+\frac{85429}{16384}\,\sqrt{2}\,\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2048*(2048*x^4 + 7040*x^3 + 352*x^2 - 6916*x + 2973)*sqrt(2*x^2 - x + 3)
+ 85429/16384*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2
 + 16*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/sqrt(2*x**2 - x + 3), x
)
```

Giac [A]
time = 5.84, size = 68, normalized size = 0.57

$$\frac{1}{2048}\left(4\left(8\left(4\left(16x+55\right)x+11\right)x-1729\right)x+2973\right)\sqrt{2x^2-x+3}-\frac{85429}{8192}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/2048*(4*(8*(4*(16*x + 55)*x + 11)*x - 1729)*x + 2973)*sqrt(2*x^2 - x + 3)
 - 85429/8192*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(1/2), x)
```

3.345 $\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=101

$$-\frac{505\sqrt{3-x+2x^2}}{1024}-\frac{409}{768}x\sqrt{3-x+2x^2}+\frac{19}{96}x^2\sqrt{3-x+2x^2}+\frac{5}{8}x^3\sqrt{3-x+2x^2}-\frac{6863\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

```
[Out] -6863/4096*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-505/1024*(2*x^2-x+3)^(1/2
)-409/768*x*(2*x^2-x+3)^(1/2)+19/96*x^2*(2*x^2-x+3)^(1/2)+5/8*x^3*(2*x^2-x+
3)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1675, 654, 633, 221}

$$\frac{19}{96}\sqrt{2x^2-x+3}\,x^2-\frac{409}{768}\sqrt{2x^2-x+3}\,x-\frac{505\sqrt{2x^2-x+3}}{1024}+\frac{5}{8}\sqrt{2x^2-x+3}\,x^3-\frac{6863\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (-505*Sqrt[3 - x + 2*x^2])/1024 - (409*x*Sqrt[3 - x + 2*x^2])/768 + (19*x^2
*Sqrt[3 - x + 2*x^2])/96 + (5*x^3*Sqrt[3 - x + 2*x^2])/8 - (6863*ArcSinh[(1
 - 4*x)/Sqrt[23]])/(2048*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \frac{5}{8}x^3\sqrt{3-x+2x^2}+\frac{1}{8}\int \frac{16+8x-21x^2+\frac{19x^3}{2}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{19}{96}x^2\sqrt{3-x+2x^2}+\frac{5}{8}x^3\sqrt{3-x+2x^2}+\frac{1}{48}\int \frac{96-9x-\frac{409x^2}{4}}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{409}{768}x\sqrt{3-x+2x^2}+\frac{19}{96}x^2\sqrt{3-x+2x^2}+\frac{5}{8}x^3\sqrt{3-x+2x^2}+\frac{1}{192}\int$$

$$= -\frac{505\sqrt{3-x+2x^2}}{1024}-\frac{409}{768}x\sqrt{3-x+2x^2}+\frac{19}{96}x^2\sqrt{3-x+2x^2}+\frac{5}{8}x^3\sqrt{}$$

$$= -\frac{505\sqrt{3-x+2x^2}}{1024}-\frac{409}{768}x\sqrt{3-x+2x^2}+\frac{19}{96}x^2\sqrt{3-x+2x^2}+\frac{5}{8}x^3\sqrt{}$$

$$= -\frac{505\sqrt{3-x+2x^2}}{1024}-\frac{409}{768}x\sqrt{3-x+2x^2}+\frac{19}{96}x^2\sqrt{3-x+2x^2}+\frac{5}{8}x^3\sqrt{}$$

Mathematica [**A**]
time = 0.30, size = 65, normalized size = 0.64

$$\frac{4\sqrt{3-x+2x^2}\left(-1515-1636x+608x^2+1920x^3\right)-20589\sqrt{2}\,\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{12288}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-1515 - 1636*x + 608*x^2 + 1920*x^3) - 20589*Sqrt[2
]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/12288
```

Maple [A]
time = 0.15, size = 79, normalized size = 0.78

method	result
risch	$\frac{(1920x^3+608x^2-1636x-1515)\sqrt{2x^2-x+3}}{3072}+\frac{6863\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{4096}$
trager	$\left(\frac{5}{8}x^3+\frac{19}{96}x^2-\frac{409}{768}x-\frac{505}{1024}\right)\sqrt{2x^2-x+3}+\frac{6863\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\mathrm{RootOf}\left(\_Z^2-2\right)x-\mathrm{RootOf}\left(\_Z^2-\right.\right.}{4096}$
default	$\frac{5x^3\sqrt{2x^2-x+3}}{8}+\frac{19x^2\sqrt{2x^2-x+3}}{96}-\frac{409x\sqrt{2x^2-x+3}}{768}-\frac{505\sqrt{2x^2-x+3}}{1024}+\frac{6863\sqrt{2}\,\mathrm{arcs}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 5/8*x^3*(2*x^2-x+3)^(1/2)+19/96*x^2*(2*x^2-x+3)^(1/2)-409/768*x*(2*x^2-x+3)
^(1/2)-505/1024*(2*x^2-x+3)^(1/2)+6863/4096*2^(1/2)*arcsinh(4/23*23^(1/2)*(
x-1/4))
```

Maxima [A]
time = 0.49, size = 80, normalized size = 0.79

$$\frac{5}{8}\sqrt{2x^2-x+3}\,x^3+\frac{19}{96}\sqrt{2x^2-x+3}\,x^2-\frac{409}{768}\sqrt{2x^2-x+3}\,x+\frac{6863}{4096}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4x-1)\right)-\frac{505}{1024}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
[Out] 5/8*sqrt(2*x^2 - x + 3)*x^3 + 19/96*sqrt(2*x^2 - x + 3)*x^2 - 409/768*sqrt(
2*x^2 - x + 3)*x + 6863/4096*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 505
/1024*sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 0.35, size = 68, normalized size = 0.67

$$\frac{1}{3072}\left(1920x^3+608x^2-1636x-1515\right)\sqrt{2x^2-x+3}+\frac{6863}{8192}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
[Out] 1/3072*(1920*x^3 + 608*x^2 - 1636*x - 1515)*sqrt(2*x^2 - x + 3) + 6863/8192
*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{5x^4-x^3+3x^2+x+2}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/sqrt(2*x**2 - x + 3), x)
```

Giac [A]
time = 3.65, size = 63, normalized size = 0.62

$$\frac{1}{3072}\left(4\left(8\left(60\,x+19\right)x-409\right)x-1515\right)\sqrt{2\,x^2-x+3}-\frac{6863}{4096}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\,\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3072*(4*(8*(60*x + 19)*x - 409)*x - 1515)*sqrt(2*x^2 - x + 3) - 6863/4096
*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(1/2), x)
```

3.346 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=126

$$\frac{1669}{128}\sqrt{3-x+2x^2}-\frac{337}{192}(5+2x)\sqrt{3-x+2x^2}+\frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2}+\frac{9657\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{2}}-\frac{3667\,\mathrm{t}}{}$$

```
[Out] 9657/512*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-3667/192*arctanh(1/24*(17-2
2*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1669/128*(2*x^2-x+3)^(1/2)-337/192*
(5+2*x)*(2*x^2-x+3)^(1/2)+5/48*(5+2*x)^2*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1667, 857, 633, 221, 738, 212}

$$\frac{5}{48}\sqrt{2x^2-x+3}\,(2x+5)^2-\frac{337}{192}\sqrt{2x^2-x+3}\,(2x+5)+\frac{1669}{128}\sqrt{2x^2-x+3}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{96\sqrt{2}}+\frac{9657\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (1669*Sqrt[3 - x + 2*x^2])/128 - (337*(5 + 2*x)*Sqrt[3 - x + 2*x^2])/192 +
(5*(5 + 2*x)^2*Sqrt[3 - x + 2*x^2])/48 + (9657*ArcSinh[(1 - 4*x)/Sqrt[23]])
/(256*Sqrt[2]) - (3667*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])
])/(96*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx = \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} + \frac{1}{96}\int \frac{-2183-3054x-4092x^2-2696x^3}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} + \frac{\int \frac{24504+12873}{(5+2x)\sqrt{3-}}}{30}$$

$$= \frac{1669}{128}\sqrt{3-x+2x^2} - \frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+}$$

$$= \frac{1669}{128}\sqrt{3-x+2x^2} - \frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+}$$

$$= \frac{1669}{128}\sqrt{3-x+2x^2} - \frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+}$$

$$= \frac{1669}{128}\sqrt{3-x+2x^2} - \frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+}$$

Mathematica [A]
time = 0.35, size = 93, normalized size = 0.74

$$\frac{4\sqrt{3-x+2x^2}\,(2637-548x+160x^2)+58672\sqrt{2}\,\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)+28971\sqrt{2}\,\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{1536}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(2637 - 548*x + 160*x^2) + 58672*Sqrt[2]*ArcTanh[(5
+ 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 28971*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 -
2*x + 4*x^2]])/1536
```

Maple [A]
time = 0.01, size = 92, normalized size = 0.73

$$\frac{5x^2\sqrt{2x^2-x+3}}{12} - \frac{137x\sqrt{2x^2-x+3}}{96} + \frac{879\sqrt{2x^2-x+3}}{128} - \frac{9657\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{512} - \frac{3667\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/12*x^2*(2*x^2-x+3)^(1/2)-137/96*x*(2*x^2-x+3)^(1/2)+879/128*(2*x^2-x+3)^(
1/2)-9657/512*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-3667/192*2^(1/2)*arcta
nh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Maxima [A]
time = 0.50, size = 99, normalized size = 0.79

$$\frac{5}{12}\sqrt{2x^2-x+3}\,x^2-\frac{137}{96}\sqrt{2x^2-x+3}\,x-\frac{9657}{512}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{3667}{192}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+5|}\right)+\frac{879}{128}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="max
ima")
```

```
[Out] 5/12*sqrt(2*x^2 - x + 3)*x^2 - 137/96*sqrt(2*x^2 - x + 3)*x - 9657/512*sqrt
(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 3667/192*sqrt(2)*arcsinh(22/
23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 879/128*sqrt(2*
x^2 - x + 3)
```

Fricas [A]
time = 0.41, size = 115, normalized size = 0.91

$$\frac{1}{384}\left(160x^2-548x+2637\right)\sqrt{2x^2-x+3}+\frac{9657}{1024}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)+\frac{3667}{384}\sqrt{2}\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}\,(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/384*(160*x^2 - 548*x + 2637)*sqrt(2*x^2 - x + 3) + 9657/1024*sqrt(2)*log(
4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 3667/384*sq
rt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x
+ 1153)/(4*x^2 + 20*x + 25))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*sqrt(2*x**2 - x + 3)),
 x)
```

Giac [A]
time = 3.83, size = 119, normalized size = 0.94

$$\frac{1}{384}\left(4\left(40x-137\right)x+2637\right)\sqrt{2x^2-x+3}+\frac{9657}{512}\sqrt{2}\log\left(-4\sqrt{2}x+\sqrt{2}+4\sqrt{2x^2-x+3}\right)-\frac{3667}{192}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{3667}{192}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/384*(4*(40*x - 137)*x + 2637)*sqrt(2*x^2 - x + 3) + 9657/512*sqrt(2)*log(
-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 3667/192*sqrt(2)*log(abs(
-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 3667/192*sqrt(2)*log(abs
(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4-x^3+3x^2+x+2}{\left(2x+5\right)\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(1/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(1/2)), x)
```

3.347 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=126

$$-\frac{243}{64}\sqrt{3-x+2x^2}-\frac{3667\sqrt{3-x+2x^2}}{576(5+2x)}+\frac{5}{32}(5+2x)\sqrt{3-x+2x^2}-\frac{2943\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}+\frac{158527\tanh^{-}}{}$$

```
[Out] -2943/256*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+158527/13824*arctanh(1/24*
(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-243/64*(2*x^2-x+3)^(1/2)-3667/
576*(2*x^2-x+3)^(1/2)/(5+2*x)+5/32*(5+2*x)*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1664, 1667, 857, 633, 221, 738, 212}

$$\frac{5}{32}\sqrt{2x^2-x+3}\,(2x+5)-\frac{243}{64}\sqrt{2x^2-x+3}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}+\frac{158527\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6912\sqrt{2}}-\frac{2943\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (-243*Sqrt[3 - x + 2*x^2])/64 - (3667*Sqrt[3 - x + 2*x^2])/(576*(5 + 2*x))
+ (5*(5 + 2*x)*Sqrt[3 - x + 2*x^2])/32 - (2943*ArcSinh[(1 - 4*x)/Sqrt[23]])
/(128*Sqrt[2]) + (158527*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2
])])/(6912*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} - \frac{1}{72}\int \frac{\frac{12007}{16}-1323x+486x^2-180x^3}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} - \frac{\int \frac{30314-27216x+34992x^2}{(5+2x)\sqrt{3-x+2x^2}}}{2304}$$

$$= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} -$$

$$= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} +$$

$$= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} +$$

$$= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} -$$

Mathematica [A]
time = 0.51, size = 100, normalized size = 0.79

$$\frac{\frac{24\sqrt{3-x+2x^2}\,(-6176-1287x+180x^2)}{5+2x} - 158527\sqrt{2}\,\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right) - 79461\sqrt{2}\,\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{6912}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(-6176 - 1287*x + 180*x^2))/(5 + 2*x) - 158527*Sqr
t[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 79461*Sqrt[2]*Log[1 - 4
*x + 2*Sqrt[6 - 2*x + 4*x^2]])/6912
```

Maple [A]
time = 0.01, size = 96, normalized size = 0.76

$$\frac{5x\sqrt{2x^2-x+3}}{16} - \frac{193\sqrt{2x^2-x+3}}{64} + \frac{2943\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{256} - \frac{3667\sqrt{2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}}}{1152\left(x+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/16*x*(2*x^2-x+3)^(1/2)-193/64*(2*x^2-x+3)^(1/2)+2943/256*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))-3667/1152/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(1/2)+1585
27/13824*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(
1/2))
```

Maxima [**A**]
time = 0.50, size = 103, normalized size = 0.82

$$\frac{5}{16}\sqrt{2\,x^2-x+3}\,x+\frac{2943}{256}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{158527}{13824}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{193}{64}\sqrt{2\,x^2-x+3}-\frac{3667\sqrt{2\,x^2-x+3}}{576\,(2\,x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] 5/16*sqrt(2*x^2 - x + 3)*x + 2943/256*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/2
3*sqrt(23)) - 158527/13824*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) -
17/23*sqrt(23)/abs(2*x + 5)) - 193/64*sqrt(2*x^2 - x + 3) - 3667/576*sqrt(2
*x^2 - x + 3)/(2*x + 5)
```

Fricas [**A**]
time = 0.37, size = 133, normalized size = 1.06

$$\frac{158922\sqrt{2}\,(2\,x+5)\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+158527\sqrt{2}\,(2\,x+5)\log\left(\frac{24\sqrt{2}\sqrt{2\,x^2-x+3}\,(22\,x-17)-1060\,x^2+1036\,x-1153}{4\,x^2+20\,x+25}\right)+96\,(180\,x^2-1287\,x-6176)\sqrt{2\,x^2-x+3}}{27648\,(2\,x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/27648*(158922*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x -
 1) - 32*x^2 + 16*x - 25) + 158527*sqrt(2)*(2*x + 5)*log((24*sqrt(2)*sqrt(2
*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25))
+ 96*(180*x^2 - 1287*x - 6176)*sqrt(2*x^2 - x + 3))/(2*x + 5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)^2\sqrt{2x^2 - x + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(1/2),x)
```

[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*sqrt(2*x**2 - x + 3
)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(99) = 198.
time = 3.68, size = 339, normalized size = 2.69

$$\frac{1}{13824}\sqrt{2}\left(\frac{158527\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}+\frac{158922\log\left(\left|\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}+1\right|\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}-\frac{158922\log\left(\left|\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}-1\right|\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}-\frac{44004\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}+\frac{108\left(3393\left(\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}\right)^3-4896\left(\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}\right)^2-743\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}-\frac{4458}{2x+5}+2256\right)}{\left(\left(\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}\right)^2-1\right)^2\operatorname{sgn}\left(\frac{1}{2x+5}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")

[Out] 1/13824*sqrt(2)*(158527*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 7
2/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 158922*log(abs(sqrt(-11/(2*x + 5) + 36
/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))/sgn(1/(2*x + 5)) - 158922*log(abs(sqr
t(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))/sgn(1/(2*x + 5))
- 44004*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)/sgn(1/(2*x + 5)) + 108*(33
93*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3 - 4896*(sqrt(
-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 743*sqrt(-11/(2*x +
5) + 36/(2*x + 5)^2 + 1) - 4458/(2*x + 5) + 2256)/(((sqrt(-11/(2*x + 5) + 3
6/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^2*sgn(1/(2*x + 5))))

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^2\,\sqrt{2\,x^2-x+3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(1/2)),x)

[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(1/2)), x)

3.348 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=128

$$\frac{5}{16}\sqrt{3-x+2x^2}-\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2}+\frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)}+\frac{149\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}-\frac{1546507\tanh^{-1}\left(\frac{\ldots}{12\ldots}\right.}{33177\ldots}$$

```
[Out] 149/64*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1546507/663552*arctanh(1/24*(
17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+5/16*(2*x^2-x+3)^(1/2)-3667/115
2*(2*x^2-x+3)^(1/2)/(5+2*x)^2+92239/27648*(2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A]

time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1664, 1667, 857, 633, 221, 738, 212}

$$\frac{92239\sqrt{2x^2-x+3}}{27648(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}+\frac{5}{16}\sqrt{2x^2-x+3}-\frac{1546507\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{331776\sqrt{2}}+\frac{149\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (5*Sqrt[3 - x + 2*x^2])/16 - (3667*Sqrt[3 - x + 2*x^2])/(1152*(5 + 2*x)^2)
+ (92239*Sqrt[3 - x + 2*x^2])/(27648*(5 + 2*x)) + (149*ArcSinh[(1 - 4*x)/Sq
rt[23]])/(32*Sqrt[2]) - (1546507*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x
 + 2*x^2])])/(331776*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} - \frac{1}{144}\int \frac{\frac{20347}{16}-\frac{6917x}{4}+972x^2-360x^3}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} + \frac{\int \frac{\frac{647841}{16}-67392x+12960x^2}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{10368}$$

$$= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} + \frac{\int}{(5+2}$$

$$= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} - \frac{149}{32}\int$$

$$= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} - \frac{15465}{}$$

$$= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} + \frac{149\,\mathrm{si}}{}$$

Mathematica [A]
time = 0.49, size = 100, normalized size = 0.78

$$\frac{\frac{12\sqrt{3-x+2x^2}\,(589187+357278x+34560x^2)}{(5+2x)^2}+1546507\sqrt{2}\,\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)+772416\sqrt{2}\,\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{331776}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(589187 + 357278*x + 34560*x^2))/(5 + 2*x)^2 + 154
6507*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 772416*Sqrt[2]*
Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/331776
```

Maple [A]
time = 0.01, size = 102, normalized size = 0.80

$$\frac{5\sqrt{2x^2-x+3}}{16} - \frac{149\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{64} + \frac{92239\sqrt{2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}}}{55296\left(x+\frac{5}{2}\right)} - \frac{1546507\sqrt{2}\,\operatorname{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/16*(2*x^2-x+3)^(1/2)-149/64*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+92239/
55296/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(1/2)-1546507/663552*2^(1/2)*arctanh(
1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-3667/4608/(x+5/2)^2
*(2*(x+5/2)^2-11*x-19/2)^(1/2)
```

Maxima [A]
time = 0.49, size = 114, normalized size = 0.89

$$-\frac{149}{64}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{1546507}{663552}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{5}{16}\sqrt{2\,x^2-x+3}-\frac{3667\sqrt{2\,x^2-x+3}}{1152\,(4\,x^2+20\,x+25)}+\frac{92239\sqrt{2\,x^2-x+3}}{27648\,(2\,x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] -149/64*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 1546507/663552*s
qrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5))
 + 5/16*sqrt(2*x^2 - x + 3) - 3667/1152*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x +
 25) + 92239/27648*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

Fricas [A]
time = 0.38, size = 149, normalized size = 1.16

$$\frac{1544832\sqrt{2}\,(4\,x^2+20\,x+25)\log\left(4\sqrt{2}\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+1546507\sqrt{2}\,(4\,x^2+20\,x+25)\log\left(-\frac{24\sqrt{2}\sqrt{2\,x^2-x+3}\,(22\,x-17)+1060\,x^2-1036\,x+1153}{4\,x^2+20\,x+25}\right)+48\,(34560\,x^2+357278\,x+589187)\sqrt{2\,x^2-x+3}}{1327104\,(4\,x^2+20\,x+25)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/1327104*(1544832*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 1546507*sqrt(2)*(4*x^2 + 20*x + 25)
*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 115
3)/(4*x^2 + 20*x + 25)) + 48*(34560*x^2 + 357278*x + 589187)*sqrt(2*x^2 - x
 + 3))/(4*x^2 + 20*x + 25)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*sqrt(2*x**2 - x + 3
)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(101) = 202$.
time = 3.38, size = 248, normalized size = 1.94

$$\frac{149}{64}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)-\frac{1546507}{663552}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}\,x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)+\frac{1546507}{663552}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}\,x-11\,\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)+\frac{5}{16}\,\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\left(2381290\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^3+16628406\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^2-25697445\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+16720645\right)}{55296\left(2\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)^2+10\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)-11\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] 149/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 1546
507/663552*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
 + 1546507/663552*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2
- x + 3))) + 5/16*sqrt(2*x^2 - x + 3) + 1/55296*sqrt(2)*(2381290*sqrt(2)*(s
qrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 16628406*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^2 - 25697445*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 16720645)/(2*(
sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
 + 3)) - 11)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^3\,\sqrt{2\,x^2-x+3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(1/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(1/2)), x)
```

3.349 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=135

$$-\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3}+\frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2}-\frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}+\frac{22389491\tanh^{-1}\cdots}{7\cdots}$$

```
[Out] -5/32*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+22389491/143327232*arctanh(1/2
4*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-3667/1728*(2*x^2-x+3)^(1/2)/
(5+2*x)^3+394907/248832*(2*x^2-x+3)^(1/2)/(5+2*x)^2-3163415/5971968*(2*x^2-
x+3)^(1/2)/(5+2*x)
```

Rubi [A]
time = 0.12, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1664, 857, 633, 221, 738, 212}

$$-\frac{3163415\sqrt{2x^2-x+3}}{5971968(2x+5)}+\frac{394907\sqrt{2x^2-x+3}}{248832(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}+\frac{22389491\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{71663616\sqrt{2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (-3667*Sqrt[3 - x + 2*x^2])/(1728*(5 + 2*x)^3) + (394907*Sqrt[3 - x + 2*x^2
])/(248832*(5 + 2*x)^2) - (3163415*Sqrt[3 - x + 2*x^2])/(5971968*(5 + 2*x))
 - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16*Sqrt[2]) + (22389491*ArcTanh[(17 - 2
2*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(71663616*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} - \frac{1}{216}\int \frac{\frac{28687}{16}-\frac{4271x}{2}+1458x^2-540x^3}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} + \frac{\int \frac{\frac{1464275}{16}-\frac{413797x}{4}+38880x^2}{(5+2x)^2\sqrt{3-x+2x^2}}\,d}{31104}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} -$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} -$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} -$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3} + \frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)} -$$

Mathematica [A]
time = 0.56, size = 100, normalized size = 0.74

$$\frac{-\frac{12\sqrt{3-x+2x^2}\,(44369687+44312764x+12653660x^2)}{(5+2x)^3} - 22389491\sqrt{2}\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right) - 11197440\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{71663616}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] ((-12*Sqrt[3 - x + 2*x^2]*(44369687 + 44312764*x + 12653660*x^2))/(5 + 2*x)
^3 - 22389491*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 111974
40*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/71663616
```

Maple [A]
time = 0.02, size = 109, normalized size = 0.81

$$\frac{5\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{32} - \frac{3163415\sqrt{2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}}}{11943936\left(x+\frac{5}{2}\right)} + \frac{22389491\sqrt{2}\,\operatorname{arctanh}\left(\frac{(\frac{17}{2}-\cdots}{12\sqrt{2\left(x+\frac{5}{2}\right.}}\right.}{143327232}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/32*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-3163415/11943936/(x+5/2)*(2*(x+
5/2)^2-11*x-19/2)^(1/2)+22389491/143327232*2^(1/2)*arctanh(1/12*(17/2-11*x)
*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))+394907/995328/(x+5/2)^2*(2*(x+5/2)^
2-11*x-19/2)^(1/2)-3667/13824/(x+5/2)^3*(2*(x+5/2)^2-11*x-19/2)^(1/2)
```

Maxima [A]
time = 0.54, size = 131, normalized size = 0.97

$$\frac{5}{32}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{22389491}{143327232}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{3667\sqrt{2\,x^2-x+3}}{1728\,(8\,x^3+60\,x^2+150\,x+125)}+\frac{394907\sqrt{2\,x^2-x+3}}{248832\,(4\,x^2+20\,x+25)}-\frac{3163415\sqrt{2\,x^2-x+3}}{5971968\,(2\,x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] 5/32*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 22389491/143327232*
sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)
) - 3667/1728*sqrt(2*x^2 - x + 3)/(8*x^3 + 60*x^2 + 150*x + 125) + 394907/2
48832*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x + 25) - 3163415/5971968*sqrt(2*x^2
- x + 3)/(2*x + 5)
```

Fricas [A]
time = 0.38, size = 163, normalized size = 1.21

$$\frac{22394880\sqrt{2}\,(8\,x^3+60\,x^2+150\,x+125)\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+22389491\sqrt{2}\,(8\,x^3+60\,x^2+150\,x+125)\log\left(\frac{24\sqrt{2}\sqrt{2\,x^2-x+3}\,(22\,x-17)-1060\,x^2+1036\,x-1153}{4\,x^2+20\,x+25}\right)-48\,(12653660\,x^2+44312764\,x+44369687)\sqrt{2\,x^2-x+3}}{286654464\,(8\,x^3+60\,x^2+150\,x+125)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/286654464*(22394880*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2)
*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 22389491*sqrt(2)*(8*
x^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17)
 - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(12653660*x^2 + 4431
2764*x + 44369687)*sqrt(2*x^2 - x + 3))/(8*x^3 + 60*x^2 + 150*x + 125)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*sqrt(2*x**2 - x + 3
)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(108) = 216.
time = 3.84, size = 285, normalized size = 2.11

$$-\frac{5}{32}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{22389491}{143327232}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)-\frac{22389491}{143327232}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)-\frac{\sqrt{2}\left(215012404\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^5+3010410772\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^4+2740802468\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^3-21459328844\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+14434519361\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-5957650879\right)}{11943936\left(2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-11\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] -5/32*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 22389
491/143327232*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3
))) - 22389491/143327232*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt
(2*x^2 - x + 3))) - 1/11943936*sqrt(2)*(215012404*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3))^5 + 3010410772*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 27408
02468*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 21459328844*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^2 + 14434519361*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
+ 3)) - 5957650879)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sq
rt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(1/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(1/2)), x)
```

3.350 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx$

Optimal. Leaf size=139

$$-\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4}+\frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3}-\frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2}+\frac{26800085\sqrt{3-x+2x^2}}{1719926784(5+2x)}+\frac{20532}{}$$

[Out] 2053207/41278242816*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-3667/2304*(2*x^2-x+3)^(1/2)/(5+2*x)^4+513097/497664*(2*x^2-x+3)^(1/2)/(5+2*x)^3-16295969/71663616*(2*x^2-x+3)^(1/2)/(5+2*x)^2+26800085/1719926784*(2*x^2-x+3)^(1/2)/(5+2*x)

Rubi [A]
time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1664, 820, 738, 212}

$$\frac{26800085\sqrt{2x^2-x+3}}{1719926784(2x+5)}-\frac{16295969\sqrt{2x^2-x+3}}{71663616(2x+5)^2}+\frac{513097\sqrt{2x^2-x+3}}{497664(2x+5)^3}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}+\frac{2053207\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{20639121408\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^5*Sqrt[3 - x + 2*x^2]),x]

[Out] (-3667*Sqrt[3 - x + 2*x^2])/(2304*(5 + 2*x)^4) + (513097*Sqrt[3 - x + 2*x^2])/(497664*(5 + 2*x)^3) - (16295969*Sqrt[3 - x + 2*x^2])/(71663616*(5 + 2*x)^2) + (26800085*Sqrt[3 - x + 2*x^2])/(1719926784*(5 + 2*x)) + (2053207*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(20639121408*Sqrt[2])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
```

```
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} - \frac{1}{288}\int \frac{\frac{37027}{16}-\frac{10167x}{4}+1944x^2-720x^3}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} + \frac{\int \frac{\frac{2607829}{16}-\frac{295607x}{2}+77760x^2}{(5+2x)^3\sqrt{3-x+2x^2}}\,d}{62208}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2}$$

Mathematica [A]
time = 0.57, size = 76, normalized size = 0.55

$$\frac{\frac{12\sqrt{3-x+2x^2}\,(-298655447-255525906x+43592076x^2+214400680x^3)}{(5+2x)^4} - 2053207\sqrt{2}\,\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{20639121408}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^5*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(-298655447 - 255525906*x + 43592076*x^2 + 2144006
80*x^3))/(5 + 2*x)^4 - 2053207*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*
x^2])/6])/20639121408
```

Maple [A]
time = 0.02, size = 116, normalized size = 0.83

$$-\frac{3667\sqrt{2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}}}{36864\left(x+\frac{5}{2}\right)^{4}}+\frac{513097\sqrt{2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}}}{3981312\left(x+\frac{5}{2}\right)^{3}}-\frac{16295969\sqrt{2\left(x+\frac{5}{2}\right)^{2}-11x-}}{286654464\left(x+\frac{5}{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x)
```

```
[Out] -3667/36864/(x+5/2)^4*(2*(x+5/2)^2-11*x-19/2)^(1/2)+513097/3981312/(x+5/2)^
3*(2*(x+5/2)^2-11*x-19/2)^(1/2)-16295969/286654464/(x+5/2)^2*(2*(x+5/2)^2-1
1*x-19/2)^(1/2)+26800085/3439853568/(x+5/2)*(2*(x+5/2)^2-11*x-19/2)^(1/2)+2
053207/41278242816*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11
*x-19/2)^(1/2))
```

Maxima [A]
time = 0.55, size = 149, normalized size = 1.07

$$-\frac{2053207}{41278242816}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{3667\sqrt{2\,x^{2}-x+3}}{2304\,(16\,x^{4}+160\,x^{3}+600\,x^{2}+1000\,x+625)}+\frac{513097\sqrt{2\,x^{2}-x+3}}{497664\,(8\,x^{3}+60\,x^{2}+150\,x+125)}-\frac{16295969\sqrt{2\,x^{2}-x+3}}{71663616\,(4\,x^{2}+20\,x+25)}+\frac{26800085\sqrt{2\,x^{2}-x+3}}{1719926784\,(2\,x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] -2053207/41278242816*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*
sqrt(23)/abs(2*x + 5)) - 3667/2304*sqrt(2*x^2 - x + 3)/(16*x^4 + 160*x^3 +
600*x^2 + 1000*x + 625) + 513097/497664*sqrt(2*x^2 - x + 3)/(8*x^3 + 60*x^2
 + 150*x + 125) - 16295969/71663616*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x + 25)
 + 26800085/1719926784*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

Fricas [A]
time = 0.36, size = 125, normalized size = 0.90

$$\frac{2053207\sqrt{2}\,(16\,x^{4}+160\,x^{3}+600\,x^{2}+1000\,x+625)\log\left(\frac{24\sqrt{2}\sqrt{2\,x^{2}-x+3}\,(22\,x-17)-1060\,x^{2}+1036\,x-1153}{4\,x^{2}+20\,x+25}\right)+48\,(214400680\,x^{3}+43592076\,x^{2}-255525906\,x-298655447)\sqrt{2\,x^{2}-x+3}}{82556485632\,(16\,x^{4}+160\,x^{3}+600\,x^{2}+1000\,x+625)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/82556485632*(2053207*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*
log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)
/(4*x^2 + 20*x + 25)) + 48*(214400680*x^3 + 43592076*x^2 - 255525906*x - 29
8655447)*sqrt(2*x^2 - x + 3))/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)^5 \sqrt{2x^2 - x + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**5/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**5*sqrt(2*x**2 - x + 3
)), x)
```

Giac [A]
time = 4.02, size = 164, normalized size = 1.18

$$\frac{1}{41278242816}\sqrt{2}\left(12\left(\frac{24\left(\frac{144\left(\frac{513097}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}-\frac{792072}{(2x+5)\operatorname{sgn}\left(\frac{1}{2x+5}\right)}\right)}{2x+5}-\frac{16295969}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}\right)}{2x+5}+\frac{26800085}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}\right)\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{2053207\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}-321601020\operatorname{sgn}\left(\frac{1}{2x+5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/41278242816*sqrt(2)*(12*(24*(144*(513097/sgn(1/(2*x + 5)) - 792072/((2*x
+ 5)*sgn(1/(2*x + 5))))/(2*x + 5) - 16295969/sgn(1/(2*x + 5)))/(2*x + 5) +
26800085/sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 20532
07*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 72/(2*x + 5) - 11)/sgn
(1/(2*x + 5)) - 321601020*sgn(1/(2*x + 5)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4 - x^3 + 3\,x^2 + x + 2}{(2\,x+5)^5\,\sqrt{2\,x^2 - x + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^5*(2*x^2 - x + 3)^(1/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^5*(2*x^2 - x + 3)^(1/2)), x)
```

3.351 $\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}} \, dx$

Optimal. Leaf size=124

$$-\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}-\frac{13153}{512}\sqrt{3-x+2x^2}+\frac{2645}{128}x\sqrt{3-x+2x^2}+\frac{153}{16}x^2\sqrt{3-x+2x^2}+\frac{5}{4}x^3\sqrt{3-x+2x^2}$$

```
[Out] 144217/2048*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-4/23*(346-533*x)/(2*x^2-
x+3)^(1/2)-13153/512*(2*x^2-x+3)^(1/2)+2645/128*x*(2*x^2-x+3)^(1/2)+153/16*
x^2*(2*x^2-x+3)^(1/2)+5/4*x^3*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1674, 1675, 654, 633, 221}

$$\frac{153}{16}\sqrt{2x^2-x+3}\,x^2+\frac{2645}{128}\sqrt{2x^2-x+3}\,x-\frac{13153}{512}\sqrt{2x^2-x+3}-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}\,x^3+\frac{144217\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-4*(346 - 533*x))/(23*Sqrt[3 - x + 2*x^2]) - (13153*Sqrt[3 - x + 2*x^2])/5
12 + (2645*x*Sqrt[3 - x + 2*x^2])/128 + (153*x^2*Sqrt[3 - x + 2*x^2])/16 +
(5*x^3*Sqrt[3 - x + 2*x^2])/4 + (144217*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*
Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{-759-\frac{575x}{2}+805x^2+\frac{1219x^3}{2}+115}{\sqrt{3-x+2x^2}}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}+\frac{5}{4}x^3\sqrt{3-x+2x^2}+\frac{1}{92}\int \frac{-6072-2300}{\sqrt{3}}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}+\frac{153}{16}x^2\sqrt{3-x+2x^2}+\frac{5}{4}x^3\sqrt{3-x+2x^2}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}+\frac{2645}{128}x\sqrt{3-x+2x^2}+\frac{153}{16}x^2\sqrt{3-x+2}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}-\frac{13153}{512}\sqrt{3-x+2x^2}+\frac{2645}{128}x\sqrt{3-x+2}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}-\frac{13153}{512}\sqrt{3-x+2x^2}+\frac{2645}{128}x\sqrt{3-x+2}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}-\frac{13153}{512}\sqrt{3-x+2x^2}+\frac{2645}{128}x\sqrt{3-x+2}$$

Mathematica [**A**]
time = 0.52, size = 75, normalized size = 0.60

$$\frac{\frac{4(-1616165+2124123x-510554x^2+418232x^3+210496x^4+29440x^5)}{\sqrt{3-x+2x^2}}+3316991\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{47104}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2)
,x]
```

```
[Out] ((4*(-1616165 + 2124123*x - 510554*x^2 + 418232*x^3 + 210496*x^4 + 29440*x^
5))/Sqrt[3 - x + 2*x^2] + 3316991*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*
x^2]])/47104
```

Maple [**A**]
time = 0.01, size = 132, normalized size = 1.06

$$\frac{5x^5}{2\sqrt{2x^2-x+3}}+\frac{143x^4}{8\sqrt{2x^2-x+3}}+\frac{2273x^3}{64\sqrt{2x^2-x+3}}+\frac{\frac{931255x}{23552}-\frac{931255}{94208}}{\sqrt{2x^2-x+3}}-\frac{11099x^2}{256\sqrt{2x^2-x+3}}+\frac{144217}{1024\sqrt{2x^2-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 5/2*x^5/(2*x^2-x+3)^(1/2)+143/8*x^4/(2*x^2-x+3)^(1/2)+2273/64*x^3/(2*x^2-x+
3)^(1/2)+931255/94208*(4*x-1)/(2*x^2-x+3)^(1/2)-11099/256*x^2/(2*x^2-x+3)^(
1/2)+144217/1024*x/(2*x^2-x+3)^(1/2)-144217/2048*2^(1/2)*arcsinh(4/23*23^(1
/2)*(x-1/4))-521655/4096/(2*x^2-x+3)^(1/2)
```

Maxima [**A**]
time = 0.55, size = 114, normalized size = 0.92

$$\frac{5\,x^5}{2\,\sqrt{2\,x^2-x+3}}+\frac{143\,x^4}{8\,\sqrt{2\,x^2-x+3}}+\frac{2273\,x^3}{64\,\sqrt{2\,x^2-x+3}}-\frac{11099\,x^2}{256\,\sqrt{2\,x^2-x+3}}-\frac{144217}{2048}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{2124123\,x}{11776\,\sqrt{2\,x^2-x+3}}-\frac{1616165}{11776\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 5/2*x^5/sqrt(2*x^2 - x + 3) + 143/8*x^4/sqrt(2*x^2 - x + 3) + 2273/64*x^3/s
qrt(2*x^2 - x + 3) - 11099/256*x^2/sqrt(2*x^2 - x + 3) - 144217/2048*sqrt(2
)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 2124123/11776*x/sqrt(2*x^2 - x + 3) -
1616165/11776/sqrt(2*x^2 - x + 3)
```

Fricas [**A**]
time = 0.37, size = 102, normalized size = 0.82

$$\frac{3316991\,\sqrt{2}\,(2\,x^2-x+3)\log\left(4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+8\,(29440\,x^5+210496\,x^4+418232\,x^3-510554\,x^2+2124123\,x-1616165)\sqrt{2\,x^2-x+3}}{94208\,(2\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/94208*(3316991*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*
(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(29440*x^5 + 210496*x^4 + 418232*x^3 -
510554*x^2 + 2124123*x - 1616165)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)^2 \cdot (5x^4 - x^3 + 3x^2 + x + 2)}{(2x^2 - x + 3)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)**2*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((2*x + 5)**2*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3
/2), x)
```

Giac [A]
time = 3.83, size = 72, normalized size = 0.58

$$\frac{144217}{2048}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) + 1\right) + \frac{(46(4(8(20x + 143)x + 2273)x - 11099)x + 2124123)x - 1616165}{11776\sqrt{2x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] 144217/2048*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) +
 1/11776*((46*(4*(8*(20*x + 143)*x + 2273)*x - 11099)*x + 2124123)*x - 1616
165)/sqrt(2*x^2 - x + 3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+5)^2 (5x^4 - x^3 + 3x^2 + x + 2)}{(2x^2 - x + 3)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2), x)
```

3.352 $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{-53+373x}{23\sqrt{3-x+2x^2}}+\frac{33}{64}\sqrt{3-x+2x^2}+\frac{193}{48}x\sqrt{3-x+2x^2}+\frac{5}{6}x^2\sqrt{3-x+2x^2}+\frac{3111\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

```
[Out] 3111/256*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+1/23*(-53+373*x)/(2*x^2-x+3
)^(1/2)+33/64*(2*x^2-x+3)^(1/2)+193/48*x*(2*x^2-x+3)^(1/2)+5/6*x^2*(2*x^2-x
+3)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1674, 1675, 654, 633, 221}

$$\frac{5}{6}\sqrt{2x^2-x+3}\,x^2+\frac{193}{48}\sqrt{2x^2-x+3}\,x+\frac{33}{64}\sqrt{2x^2-x+3}-\frac{53-373x}{23\sqrt{2x^2-x+3}}+\frac{3111\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] -1/23*(53 - 373*x)/Sqrt[3 - x + 2*x^2] + (33*Sqrt[3 - x + 2*x^2])/64 + (193
*x*Sqrt[3 - x + 2*x^2])/48 + (5*x^2*Sqrt[3 - x + 2*x^2])/6 + (3111*ArcSinh[
(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = -\frac{53-373x}{23\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{-\frac{575}{4}+161x^2+\frac{115x^3}{2}}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}}+\frac{5}{6}x^2\sqrt{3-x+2x^2}+\frac{1}{69}\int \frac{-\frac{1725}{2}-345x-}{\sqrt{3-x+2}}$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}}+\frac{193}{48}x\sqrt{3-x+2x^2}+\frac{5}{6}x^2\sqrt{3-x+2x^2}$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}}+\frac{33}{64}\sqrt{3-x+2x^2}+\frac{193}{48}x\sqrt{3-x+2x^2}+$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}}+\frac{33}{64}\sqrt{3-x+2x^2}+\frac{193}{48}x\sqrt{3-x+2x^2}+$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}}+\frac{33}{64}\sqrt{3-x+2x^2}+\frac{193}{48}x\sqrt{3-x+2x^2}+$$

Mathematica [A]

time = 0.47, size = 70, normalized size = 0.68

$$\frac{-3345+122607x-2162x^2+31832x^3+7360x^4}{4416\sqrt{3-x+2x^2}}+\frac{3111\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x
]
```

```
[Out] (-3345 + 122607*x - 2162*x^2 + 31832*x^3 + 7360*x^4)/(4416*Sqrt[3 - x + 2*x
^2]) + (3111*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(128*Sqrt[2])
```

Maple [A]
time = 0.16, size = 115, normalized size = 1.12

method	result
risch	$\frac{7360x^4+31832x^3-2162x^2+122607x-3345}{4416\sqrt{2x^2-x+3}}-\frac{3111\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{256}$
trager	$\frac{7360x^4+31832x^3-2162x^2+122607x-3345}{4416\sqrt{2x^2-x+3}}-\frac{3111\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\mathrm{RootOf}\left(\_Z^2-2\right)x-\mathrm{RootOf}\left(\_Z^2-2\right)+4\sqrt{2x^2-}\right.}{256}$
default	$\frac{\frac{10185x}{2944}-\frac{10185}{11776}}{\sqrt{2x^2-x+3}}+\frac{55}{512\sqrt{2x^2-x+3}}+\frac{3111x}{128\sqrt{2x^2-x+3}}+\frac{5x^4}{3\sqrt{2x^2-x+3}}+\frac{173x^3}{24\sqrt{2x^2-x+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 10185/11776*(4*x-1)/(2*x^2-x+3)^(1/2)+55/512/(2*x^2-x+3)^(1/2)+3111/128*x/(
2*x^2-x+3)^(1/2)+5/3*x^4/(2*x^2-x+3)^(1/2)+173/24*x^3/(2*x^2-x+3)^(1/2)-47/
96*x^2/(2*x^2-x+3)^(1/2)-3111/256*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [A]
time = 0.50, size = 97, normalized size = 0.94

$$\frac{5\,x^4}{3\,\sqrt{2\,x^2-x+3}}+\frac{173\,x^3}{24\,\sqrt{2\,x^2-x+3}}-\frac{47\,x^2}{96\,\sqrt{2\,x^2-x+3}}-\frac{3111}{256}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{40869\,x}{1472\,\sqrt{2\,x^2-x+3}}-\frac{1115}{1472\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="max
ima")
```

```
[Out] 5/3*x^4/sqrt(2*x^2 - x + 3) + 173/24*x^3/sqrt(2*x^2 - x + 3) - 47/96*x^2/sq
rt(2*x^2 - x + 3) - 3111/256*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 408
69/1472*x/sqrt(2*x^2 - x + 3) - 1115/1472/sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 0.35, size = 97, normalized size = 0.94

$$\frac{214659\sqrt{2}\,(2\,x^2-x+3)\log\left(4\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+8\,(7360\,x^4+31832\,x^3-2162\,x^2+122607\,x-3345)\sqrt{2\,x^2-x+3}}{35328\,(2\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/35328*(214659*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(
4*x - 1) - 32*x^2 + 16*x - 25) + 8*(7360*x^4 + 31832*x^3 - 2162*x^2 + 12260
7*x - 3345)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(2x+5)\,(5x^4-x^3+3x^2+x+2)}{(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3/2)
, x)
```

Giac [A]
time = 4.16, size = 67, normalized size = 0.65

$$\frac{3111}{256}\sqrt{2}\,\log\left(-2\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)+\frac{(46\,(4\,(40\,x+173)x-47)x+122607)x-3345}{4416\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="gia
c")
```

```
[Out] 3111/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/
4416*((46*(4*(40*x + 173)*x - 47)*x + 122607)*x - 3345)/sqrt(2*x^2 - x + 3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(2\,x+5)\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x^2-x+3\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2),x)
[Out] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2), x)
```

3.353 $\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{27}{32}\sqrt{3-x+2x^2}+\frac{5}{8}x\sqrt{3-x+2x^2}+\frac{213\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

```
[Out] 213/128*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+1/92*(89+219*x)/(2*x^2-x+3)^
(1/2)+27/32*(2*x^2-x+3)^(1/2)+5/8*x*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1674, 1675, 654, 633, 221}

$$\frac{5}{8}\sqrt{2x^2-x+3}\,x+\frac{27}{32}\sqrt{2x^2-x+3}+\frac{219x+89}{92\sqrt{2x^2-x+3}}+\frac{213\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (89 + 219*x)/(92*Sqrt[3 - x + 2*x^2]) + (27*Sqrt[3 - x + 2*x^2])/32 + (5*x*
Sqrt[3 - x + 2*x^2])/8 + (213*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{3/2}}\,dx &= \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{-\frac{345}{16}+\frac{69x}{8}+\frac{115x^2}{4}}{\sqrt{3-x+2x^2}}\,dx\\
&= \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{5}{8}x\sqrt{3-x+2x^2}+\frac{1}{46}\int \frac{-\frac{345}{2}+\frac{621x}{8}}{\sqrt{3-x+2x^2}}\,dx\\
&= \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{27}{32}\sqrt{3-x+2x^2}+\frac{5}{8}x\sqrt{3-x+2x^2}-\frac{213}{64}\int \frac{}{\sqrt{3-}}\\
&= \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{27}{32}\sqrt{3-x+2x^2}+\frac{5}{8}x\sqrt{3-x+2x^2}-\frac{213\text{Subst}\left(\int\right.}{}\\
&= \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{27}{32}\sqrt{3-x+2x^2}+\frac{5}{8}x\sqrt{3-x+2x^2}+\frac{213\sinh^{-1}\left(\right.}{64\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.42, size = 65, normalized size = 0.79

$$\frac{2575+2511x+782x^2+920x^3}{736\sqrt{3-x+2x^2}}+\frac{213\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (2575 + 2511*x + 782*x^2 + 920*x^3)/(736*Sqrt[3 - x + 2*x^2]) + (213*Log[1
- 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(64*Sqrt[2])
```

Maple [A]
time = 0.13, size = 98, normalized size = 1.20

method	result
risch	$\frac{920x^3+782x^2+2511x+2575}{736\sqrt{2x^2-x+3}}-\frac{213\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{128}$
trager	$\frac{920x^3+782x^2+2511x+2575}{736\sqrt{2x^2-x+3}}-\frac{213\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(4\operatorname{RootOf}\left(\_Z^2-2\right)x-\operatorname{RootOf}\left(\_Z^2-2\right)+4\sqrt{2x^2-x+3}\right)}{128}$
default	$\frac{5x^3}{4\sqrt{2x^2-x+3}}+\frac{17x^2}{16\sqrt{2x^2-x+3}}+\frac{213x}{64\sqrt{2x^2-x+3}}+\frac{901}{256\sqrt{2x^2-x+3}}+\frac{\frac{123x}{1472}-\frac{123}{5888}}{\sqrt{2x^2-x+3}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/4*x^3/(2*x^2-x+3)^(1/2)+17/16*x^2/(2*x^2-x+3)^(1/2)+213/64*x/(2*x^2-x+3)^
(1/2)+901/256/(2*x^2-x+3)^(1/2)+123/5888*(4*x-1)/(2*x^2-x+3)^(1/2)-213/128*
2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [A]
time = 0.52, size = 80, normalized size = 0.98

$$\frac{5\,x^3}{4\,\sqrt{2\,x^2-x+3}}+\frac{17\,x^2}{16\,\sqrt{2\,x^2-x+3}}-\frac{213}{128}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{2511\,x}{736\,\sqrt{2\,x^2-x+3}}+\frac{2575}{736\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 5/4*x^3/sqrt(2*x^2 - x + 3) + 17/16*x^2/sqrt(2*x^2 - x + 3) - 213/128*sqrt(
2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 2511/736*x/sqrt(2*x^2 - x + 3) + 2575
/736/sqrt(2*x^2 - x + 3)
```

Fricas [A]
time = 0.35, size = 92, normalized size = 1.12

$$\frac{4899\,\sqrt{2}\,(2\,x^2-x+3)\log\left(4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+8\,(920\,x^3+782\,x^2+2511\,x+2575)\sqrt{2\,x^2-x+3}}{5888\,(2\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5888*(4899*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
 - 1) - 32*x^2 + 16*x - 25) + 8*(920*x^3 + 782*x^2 + 2511*x + 2575)*sqrt(2*
x^2 - x + 3))/(2*x^2 - x + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x^2 - x + 3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3/2), x)
```

Giac [A]
time = 4.08, size = 62, normalized size = 0.76

$$\frac{213}{128}\sqrt{2}\,\log\left(-2\sqrt{2}\left(\sqrt{2}\,x - \sqrt{2\,x^2 - x + 3}\right) + 1\right) + \frac{(46\,(20\,x + 17)x + 2511)x + 2575}{736\sqrt{2\,x^2 - x + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] 213/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/7
36*((46*(20*x + 17)*x + 2511)*x + 2575)/sqrt(2*x^2 - x + 3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4 - x^3 + 3\,x^2 + x + 2}{(2\,x^2 - x + 3)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(3/2), x)
```

3.354 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{1191+917x}{3312\sqrt{3-x+2x^2}}+\frac{5}{8}\sqrt{3-x+2x^2}+\frac{39\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1728\sqrt{2}}$$

```
[Out] 39/32*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-3667/3456*arctanh(1/24*(17-22*
x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/3312*(1191+917*x)/(2*x^2-x+3)^(1/2)
+5/8*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1660, 1667, 857, 633, 221, 738, 212}

$$\frac{917x+1191}{3312\sqrt{2x^2-x+3}}+\frac{5}{8}\sqrt{2x^2-x+3}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1728\sqrt{2}}+\frac{39\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(3/2)),x]
```

```
[Out] (1191 + 917*x)/(3312*Sqrt[3 - x + 2*x^2]) + (5*Sqrt[3 - x + 2*x^2])/8 + (39
*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16*Sqrt[2]) - (3667*ArcTanh[(17 - 22*x)/(12*
Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1728*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{-\frac{6739}{576}+\frac{69x}{8}+\frac{115x^2}{4}}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} + \frac{1}{92}\int \frac{\frac{3611}{72}-\frac{897x}{2}}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} - \frac{39}{16}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx + \frac{3667}{288}$$

$$= \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} - \frac{3667}{144}\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \right.$$

$$= \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} + \frac{39\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}} - \frac{3667\tanh}{}$$

Mathematica [A]
time = 0.48, size = 93, normalized size = 0.92

$$\frac{\frac{12(7401-1153x+4140x^2)}{\sqrt{3-x+2x^2}} + 84341\sqrt{2}\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right) + 48438\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{39744}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(3/2)),x
]
```

```
[Out] ((12*(7401 - 1153*x + 4140*x^2))/Sqrt[3 - x + 2*x^2] + 84341*Sqrt[2]*ArcTan
h[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 48438*Sqrt[2]*Log[1 - 4*x + 2*Sqrt
[6 - 2*x + 4*x^2]])/39744
```

Maple [A]
time = 0.01, size = 148, normalized size = 1.47

$$\frac{5x^2}{4\sqrt{2x^2-x+3}} + \frac{39x}{16\sqrt{2x^2-x+3}} - \frac{309}{64\sqrt{2x^2-x+3}} - \frac{5507(4x-1)}{1472\sqrt{2x^2-x+3}} - \frac{39\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 5/4*x^2/(2*x^2-x+3)^(1/2)+39/16*x/(2*x^2-x+3)^(1/2)-309/64/(2*x^2-x+3)^(1/2
)-5507/1472*(4*x-1)/(2*x^2-x+3)^(1/2)-39/32*2^(1/2)*arcsinh(4/23*23^(1/2)*(
x-1/4))+3667/576/(2*(x+5/2)^2-11*x-19/2)^(1/2)+40337/13248*(4*x-1)/(2*(x+5/
2)^2-11*x-19/2)^(1/2)-3667/3456*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2
*(x+5/2)^2-11*x-19/2)^(1/2))
```

Maxima [A]
time = 0.50, size = 99, normalized size = 0.98

$$\frac{5\,x^2}{4\,\sqrt{2\,x^2-x+3}}-\frac{39}{32}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\,\sqrt{23}\,x-\frac{1}{23}\,\sqrt{23}\right)+\frac{3667}{3456}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\,\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\,\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{1153\,x}{3312\,\sqrt{2\,x^2-x+3}}+\frac{2467}{1104\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="max
ima")
```

```
[Out] 5/4*x^2/sqrt(2*x^2 - x + 3) - 39/32*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*
sqrt(23)) + 3667/3456*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23
*sqrt(23)/abs(2*x + 5)) - 1153/3312*x/sqrt(2*x^2 - x + 3) + 2467/1104/sqrt(
2*x^2 - x + 3)
```

Fricas [A]
time = 0.44, size = 149, normalized size = 1.48

$$\frac{96876\,\sqrt{2}\,(2\,x^2-x+3)\log\left(4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+84341\,\sqrt{2}\,(2\,x^2-x+3)\log\left(-\frac{24\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(22\,x-17)+1060\,x^2-1036\,x+1153}{4\,x^2+20\,x+25}\right)+48\,(4140\,x^2-1153\,x+7401)\sqrt{2\,x^2-x+3}}{158976\,(2\,x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/158976*(96876*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(
4*x - 1) - 32*x^2 + 16*x - 25) + 84341*sqrt(2)*(2*x^2 - x + 3)*log(-(24*sqr
t(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 2
0*x + 25)) + 48*(4140*x^2 - 1153*x + 7401)*sqrt(2*x^2 - x + 3))/(2*x^2 - x
+ 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\,(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(3/2),x)
```

[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*(2*x**2 - x + 3)**(3/2
)), x)

Giac [A]
time = 4.30, size = 118, normalized size = 1.17

$$\frac{39}{32}\sqrt{2}\,\log\left(-4\sqrt{2}\,x+\sqrt{2}\,+4\sqrt{2x^2-x+3}\right)-\frac{3667}{3456}\sqrt{2}\,\log\left(\left|-2\sqrt{2}\,x+\sqrt{2}\,+2\sqrt{2x^2-x+3}\right|\right)+\frac{3667}{3456}\sqrt{2}\,\log\left(\left|-2\sqrt{2}\,x-11\sqrt{2}\,+2\sqrt{2x^2-x+3}\right|\right)+\frac{(4140x-1153)x+7401}{3312\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="gia
c")

[Out] 39/32*sqrt(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 3667/34
56*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 3667/
3456*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) +
1/3312*((4140*x - 1153)*x + 7401)/sqrt(2*x^2 - x + 3)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\,\left(2x^2-x+3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(3/2)),x)

[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(3/2)), x)

3.355 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}+\frac{25951\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{41472\sqrt{2}}$$

```
[Out] -5/16*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+25951/82944*arctanh(1/24*(17-2
2*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/119232*(9897+2203*x)/(2*x^2-x+3)^
(1/2)-3667/10368*(2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A]

time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1660, 1664, 857, 633, 221, 738, 212}

$$\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}-\frac{3667\sqrt{2x^2-x+3}}{10368(2x+5)}+\frac{25951\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{41472\sqrt{2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(3/2)),x]
```

```
[Out] (9897 + 2203*x)/(119232*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(
10368*(5 + 2*x)) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(8*Sqrt[2]) + (25951*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(41472*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{9897+2203x}{119232\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{-\frac{33649}{20736}+\frac{131215x}{10368}+\frac{115x^2}{4}}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}-\frac{1}{828}\int \frac{\frac{100073}{192}-10\ldots}{(5+2x)\sqrt{3-x\ldots}}$$

$$= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}+\frac{5}{8}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}+\frac{25951\text{Subst}\left(\int \frac{1}{288-x^2}\,d\ldots\right.}{345\ldots}$$

$$= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}+\frac{25\ldots}{\ldots}$$

Mathematica [A]
time = 0.55, size = 110, normalized size = 1.02

$$-\frac{\sqrt{3-x+2x^2}\left(51351-48653x+53290x^2\right)}{79488\left(15+x+8x^2+4x^3\right)}-\frac{25951\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{20736\sqrt{2}}-\frac{5\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(3/2))
,x]
```

```
[Out] -1/79488*(Sqrt[3 - x + 2*x^2]*(51351 - 48653*x + 53290*x^2))/(15 + x + 8*x^
2 + 4*x^3) - (25951*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6])/(20736*Sq
rt[2]) - (5*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(8*Sqrt[2])
```

Maple [A]
time = 0.01, size = 152, normalized size = 1.41

$$-\frac{5x}{8\sqrt{2x^2-x+3}}+\frac{99}{32\sqrt{2x^2-x+3}}+\frac{\frac{1529x}{184}-\frac{1529}{736}}{\sqrt{2x^2-x+3}}+\frac{5\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{16}-\frac{3\ldots}{1152\left(x+\frac{5}{2}\right)\sqrt{2\left(\ldots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x)
```

```
[Out] -5/8*x/(2*x^2-x+3)^(1/2)+99/32/(2*x^2-x+3)^(1/2)+1529/736*(4*x-1)/(2*x^2-x+
3)^(1/2)+5/16*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-3667/1152/(x+5/2)/(2*(
x+5/2)^2-11*x-19/2)^(1/2)-25951/13824/(2*(x+5/2)^2-11*x-19/2)^(1/2)-637493/
317952*(4*x-1)/(2*(x+5/2)^2-11*x-19/2)^(1/2)+25951/82944*2^(1/2)*arctanh(1/
12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Maxima [A]
time = 0.51, size = 116, normalized size = 1.07

$$\frac{5}{16}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{25951}{82944}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{26645\,x}{79488\sqrt{2\,x^2-x+3}}+\frac{30313}{26496\sqrt{2\,x^2-x+3}}-\frac{3667}{576\left(2\sqrt{2\,x^2-x+3}\,x+5\sqrt{2\,x^2-x+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 5/16*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 25951/82944*sqrt(2)
*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 266
45/79488*x/sqrt(2*x^2 - x + 3) + 30313/26496/sqrt(2*x^2 - x + 3) - 3667/576
/(2*sqrt(2*x^2 - x + 3)*x + 5*sqrt(2*x^2 - x + 3))
```

Fricas [A]
time = 0.38, size = 157, normalized size = 1.45

$$\frac{596160\sqrt{2}\,(4\,x^3+8\,x^2+x+15)\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+596873\sqrt{2}\,(4\,x^3+8\,x^2+x+15)\log\left(\frac{24\sqrt{2}\sqrt{2\,x^2-x+3}\,(22\,x-17)-1060\,x^2+1036\,x-1153}{4\,x^2+20\,x+25}\right)-48\,(53290\,x^2-48653\,x+51351)\sqrt{2\,x^2-x+3}}{3815424\,(4\,x^3+8\,x^2+x+15)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/3815424*(596160*sqrt(2)*(4*x^3 + 8*x^2 + x + 15)*log(-4*sqrt(2)*sqrt(2*x^
2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 596873*sqrt(2)*(4*x^3 + 8*x^2
+ x + 15)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036
*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(53290*x^2 - 48653*x + 51351)*sqrt(2*x
^2 - x + 3))/(4*x^3 + 8*x^2 + x + 15)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)^2\,(2x^2 - x + 3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*(2*x**2 - x + 3)**(
3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(85) = 170.
time = 2.99, size = 225, normalized size = 2.08

$$\frac{1}{1907712}\sqrt{2}\left(\frac{12\left(\frac{\frac{315103}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}-\frac{1012092}{(2x+5)\operatorname{sgn}\left(\frac{1}{2x+5}\right)}}{2x+5}-\frac{26645}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}\right)}{\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}}+\frac{596873\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}+\frac{596160\log\left(\left|\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}+1\right|\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}-\frac{596160\log\left(\left|\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}-1\right|\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] 1/1907712*sqrt(2)*(12*((315103/sgn(1/(2*x + 5)) - 1012092/((2*x + 5)*sgn(1/
(2*x + 5))))/(2*x + 5) - 26645/sgn(1/(2*x + 5)))/sqrt(-11/(2*x + 5) + 36/(2
*x + 5)^2 + 1) + 596873*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 7
2/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 596160*log(abs(sqrt(-11/(2*x + 5) + 36
/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))/sgn(1/(2*x + 5)) - 596160*log(abs(sqr
t(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))/sgn(1/(2*x + 5)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^2\,{\left(2\,x^2-x+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(3/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(3/2)), x)
```

3.356 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=112

$$\frac{65991-8779x}{4292352\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2}+\frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)}-\frac{52631\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{5971968\sqrt{2}}$$

```
[Out] -52631/11943936*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1
/4292352*(65991-8779*x)/(2*x^2-x+3)^(1/2)-3667/20736*(2*x^2-x+3)^(1/2)/(5+2
*x)^2+115369/1492992*(2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A]
time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1660, 1664, 820, 738, 212}

$$\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}+\frac{115369\sqrt{2x^2-x+3}}{1492992(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{20736(2x+5)^2}-\frac{52631\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{5971968\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(3/2)),x]
```

```
[Out] (65991 - 8779*x)/(4292352*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])
/(20736*(5 + 2*x)^2) + (115369*Sqrt[3 - x + 2*x^2])/(1492992*(5 + 2*x)) - (
52631*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(5971968*Sqrt[
2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
```

```
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{3/2}}\,dx = \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{\frac{5168261}{746496}+\frac{3637795x}{186624}+\frac{5620625x^2}{186624}}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2} - \frac{\int \frac{\frac{842237}{1296}-\frac{4102487x}{2592}}{(5+2x)^2\sqrt{3-x+2x^2}}}{1656}$$

$$= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2} + \frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)} -$$

$$= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2} + \frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)} -$$

$$= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2} + \frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)} -$$

Mathematica [A]
time = 0.49, size = 76, normalized size = 0.68

$$\frac{\frac{12(11594283+5842933x+3263288x^2+3444340x^3)}{(5+2x)^2\sqrt{3-x+2x^2}} + 1210513\sqrt{2}\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{137355264}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(3/2))
,x]
```

```
[Out] ((12*(11594283 + 5842933*x + 3263288*x^2 + 3444340*x^3))/((5 + 2*x)^2*Sqrt[
3 - x + 2*x^2]) + 1210513*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])
/6])/137355264
```

Maple [A]
time = 0.02, size = 144, normalized size = 1.29

$$-\frac{5}{16\sqrt{2x^2-x+3}} - \frac{149(4x-1)}{368\sqrt{2x^2-x+3}} + \frac{196043}{165888\left(x+\frac{5}{2}\right)\sqrt{2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}}} + \frac{52631}{1990656\sqrt{2\left(x+\frac{5}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x)
```

```
[Out] -5/16/(2*x^2-x+3)^(1/2)-149/368*(4*x-1)/(2*x^2-x+3)^(1/2)+196043/165888/(x+
5/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2)+52631/1990656/(2*(x+5/2)^2-11*x-19/2)^(1
/2)+19399069/45785088*(4*x-1)/(2*(x+5/2)^2-11*x-19/2)^(1/2)-52631/11943936*
2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-366
7/4608/(x+5/2)^2/(2*(x+5/2)^2-11*x-19/2)^(1/2)
```

Maxima [A]
time = 0.53, size = 149, normalized size = 1.33

$$\frac{52631}{11943936}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{861085\,x}{11446272\sqrt{2\,x^2-x+3}}-\frac{1163201}{3815424\sqrt{2\,x^2-x+3}}-\frac{3667}{1152\left(4\sqrt{2\,x^2-x+3}\,x^2+20\sqrt{2\,x^2-x+3}\,x+25\sqrt{2\,x^2-x+3}\right)}+\frac{196043}{82944\left(2\sqrt{2\,x^2-x+3}\,x+5\sqrt{2\,x^2-x+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 52631/11943936*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(2
3)/abs(2*x + 5)) + 861085/11446272*x/sqrt(2*x^2 - x + 3) - 1163201/3815424/
sqrt(2*x^2 - x + 3) - 3667/1152/(4*sqrt(2*x^2 - x + 3)*x^2 + 20*sqrt(2*x^2
- x + 3)*x + 25*sqrt(2*x^2 - x + 3)) + 196043/82944/(2*sqrt(2*x^2 - x + 3)*
x + 5*sqrt(2*x^2 - x + 3))
```

Fricas [A]
time = 0.35, size = 126, normalized size = 1.12

$$\frac{1210513\sqrt{2}\,(8\,x^4+36\,x^3+42\,x^2+35\,x+75)\log\left(-\frac{24\sqrt{2}\sqrt{2\,x^2-x+3}\,(22\,x-17)+1060\,x^2-1036\,x+1153}{4\,x^2+20\,x+25}\right)+48\,(3444340\,x^3+3263288\,x^2+5842933\,x+11594283)\sqrt{2\,x^2-x+3}}{549421056\,(8\,x^4+36\,x^3+42\,x^2+35\,x+75)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/549421056*(1210513*sqrt(2)*(8*x^4 + 36*x^3 + 42*x^2 + 35*x + 75)*log(-(24
*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2
 + 20*x + 25)) + 48*(3444340*x^3 + 3263288*x^2 + 5842933*x + 11594283)*sqrt
(2*x^2 - x + 3))/(8*x^4 + 36*x^3 + 42*x^2 + 35*x + 75)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\,(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*(2*x**2 - x + 3)**(
3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(90) = 180.
time = 3.35, size = 220, normalized size = 1.96

$$-\frac{52631}{11943936}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)+\frac{52631}{11943936}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x-11\,\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)-\frac{8779\,x-65991}{4292352\,\sqrt{2\,x^2-x+3}}+\frac{\sqrt{2}\left(3594214\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^3+19874490\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^2-30140067\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+19989859\right)}{2985984\left(2\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^2+10\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)-11\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] -52631/11943936*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x +
 3))) + 52631/11943936*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2
*x^2 - x + 3))) - 1/4292352*(8779*x - 65991)/sqrt(2*x^2 - x + 3) + 1/298598
4*sqrt(2)*(3594214*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 19874490*(
sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 30140067*sqrt(2)*(sqrt(2)*x - sqrt(2*x
^2 - x + 3)) + 19989859)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^3\,{\left(2\,x^2-x+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(3/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(3/2)), x)
```

3.357 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}+\frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2}+\frac{430799\sqrt{3-x+2x^2}}{107495424(5+2x)}-\frac{3505819}{}$$

```
[Out] -3505819/2579890176*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/
2)+1/154524672*(369609-175877*x)/(2*x^2-x+3)^(1/2)-3667/31104*(2*x^2-x+3)^(
1/2)/(5+2*x)^3+152885/4478976*(2*x^2-x+3)^(1/2)/(5+2*x)^2+430799/107495424*
(2*x^2-x+3)^(1/2)/(5+2*x)
```

Rubi [A]
time = 0.13, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1660, 1664, 820, 738, 212}

$$\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}+\frac{430799\sqrt{2x^2-x+3}}{107495424(2x+5)}+\frac{152885\sqrt{2x^2-x+3}}{4478976(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{31104(2x+5)^3}-\frac{3505819\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1289945088\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(3/2)),x]
```

```
[Out] (369609 - 175877*x)/(154524672*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*
x^2])/(31104*(5 + 2*x)^3) + (152885*Sqrt[3 - x + 2*x^2])/(4478976*(5 + 2*x)
^2) + (430799*Sqrt[3 - x + 2*x^2])/(107495424*(5 + 2*x)) - (3505819*ArcTanh
[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1289945088*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
```

```
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{\frac{348877271}{26873856}+\frac{119871055x}{4478976}+\frac{73960295x^2}{2239488}+ 13\ldots}{(5+2x)^4\sqrt{3-x+2x^2}}$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3} - \frac{\int \frac{\frac{79609325}{124416}-\frac{71248733x}{31104}-13\ldots}{(5+2x)^3\sqrt{3-x+\ldots}}}{2484}$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3} + \frac{152885\sqrt{3-x+2x\ldots}}{4478976(5+2x)^2\ldots}$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3} + \frac{152885\sqrt{3-x+2x\ldots}}{4478976(5+2x)^2\ldots}$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3} + \frac{152885\sqrt{3-x+2x\ldots}}{4478976(5+2x)^2\ldots}$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3} + \frac{152885\sqrt{3-x+2x\ldots}}{4478976(5+2x)^2\ldots}$$

Mathematica [A]
time = 0.56, size = 81, normalized size = 0.59

$$\frac{\frac{12\left(1873786587+1257975811x+441046842x^2+572739684x^3+56754760x^4\right)}{(5+2x)^3\sqrt{3-x+2x^2}}+80633837\sqrt{2}\,\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{29668737024}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(3/2))
,x]
```

```
[Out] ((12*(1873786587 + 1257975811*x + 441046842*x^2 + 572739684*x^3 + 56754760*
x^4))/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]) + 80633837*Sqrt[2]*ArcTanh[(5 + 2*x
 - Sqrt[6 - 2*x + 4*x^2])/6])/29668737024
```

Maple [A]
time = 0.02, size = 151, normalized size = 1.10

$$\frac{\frac{5x}{46}-\frac{5}{184}}{\sqrt{2x^2-x+3}} - \frac{3127169}{35831808\left(x+\frac{5}{2}\right)\sqrt{2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}}} + \frac{3505819}{429981696\sqrt{2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}}} - \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 5/184*(4*x-1)/(2*x^2-x+3)^(1/2)-3127169/35831808/(x+5/2)/(2*(x+5/2)^2-11*x-
19/2)^(1/2)+3505819/429981696/(2*(x+5/2)^2-11*x-19/2)^(1/2)-261644215/98895
79008*(4*x-1)/(2*(x+5/2)^2-11*x-19/2)^(1/2)-3505819/2579890176*2^(1/2)*arct
anh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))+314233/995328/(
x+5/2)^2/(2*(x+5/2)^2-11*x-19/2)^(1/2)-3667/13824/(x+5/2)^3/(2*(x+5/2)^2-11
*x-19/2)^(1/2)
```

Maxima [A]
time = 0.51, size = 217, normalized size = 1.58

$$\frac{3505819}{2579890176}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{7094345\,x}{2472394752\sqrt{2\,x^2-x+3}}+\frac{6128291}{824131584\sqrt{2\,x^2-x+3}}-\frac{3667}{1728\left(8\sqrt{2\,x^2-x+3}\,x^3+60\sqrt{2\,x^2-x+3}\,x^2+150\sqrt{2\,x^2-x+3}\,x+125\sqrt{2\,x^2-x+3}\right)}+\frac{314233}{248832\left(4\sqrt{2\,x^2-x+3}\,x^2+20\sqrt{2\,x^2-x+3}\,x+25\sqrt{2\,x^2-x+3}\right)}-\frac{3127169}{17915904\left(2\sqrt{2\,x^2-x+3}\,x+5\sqrt{2\,x^2-x+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 3505819/2579890176*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sq
rt(23)/abs(2*x + 5)) + 7094345/2472394752*x/sqrt(2*x^2 - x + 3) + 6128291/8
24131584/sqrt(2*x^2 - x + 3) - 3667/1728/(8*sqrt(2*x^2 - x + 3)*x^3 + 60*sq
rt(2*x^2 - x + 3)*x^2 + 150*sqrt(2*x^2 - x + 3)*x + 125*sqrt(2*x^2 - x + 3)
) + 314233/248832/(4*sqrt(2*x^2 - x + 3)*x^2 + 20*sqrt(2*x^2 - x + 3)*x + 2
5*sqrt(2*x^2 - x + 3)) - 3127169/17915904/(2*sqrt(2*x^2 - x + 3)*x + 5*sqrt
(2*x^2 - x + 3))
```

Fricas [A]
time = 0.35, size = 141, normalized size = 1.03

$$\frac{80633837\sqrt{2}\left(16\,x^5+112\,x^4+264\,x^3+280\,x^2+325\,x+375\right)\log\left(-\frac{24\sqrt{2}\sqrt{2\,x^2-x+3}\,(22\,x-17)+1060\,x^2-1036\,x+1153}{4\,x^2+20\,x+25}\right)+48\left(56754760\,x^4+572739684\,x^3+441046842\,x^2+1257975811\,x+1873786587\right)\sqrt{2\,x^2-x+3}}{118674948096\left(16\,x^5+112\,x^4+264\,x^3+280\,x^2+325\,x+375\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/118674948096*(80633837*sqrt(2)*(16*x^5 + 112*x^4 + 264*x^3 + 280*x^2 + 32
5*x + 375)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 10
36*x + 1153)/(4*x^2 + 20*x + 25)) + 48*(56754760*x^4 + 572739684*x^3 + 4410
46842*x^2 + 1257975811*x + 1873786587)*sqrt(2*x^2 - x + 3))/(16*x^5 + 112*x
^4 + 264*x^3 + 280*x^2 + 325*x + 375)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)^4\,(2x^2 - x + 3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*(2*x**2 - x + 3)**(
3/2)), x)
```

<u>**Giac**</u> **[B]** Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(111) = 222.
time = 4.03, size = 271, normalized size = 1.98

$$-\frac{3505819}{2579890176}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{3505819}{2579890176}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)-\frac{175877x-369609}{154524672\sqrt{2x^2-x+3}}-\frac{\sqrt{2}\left(10398764\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^5-303070900\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^4-529738052\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^3+3644644652\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2-2612608649\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1052284471\right)}{214990848\left(2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-11\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] -3505819/2579890176*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 -
 x + 3))) + 3505819/2579890176*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) +
2*sqrt(2*x^2 - x + 3))) - 1/154524672*(175877*x - 369609)/sqrt(2*x^2 - x +
3) - 1/214990848*sqrt(2)*(10398764*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^5 - 303070900*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 - 529738052*sqrt(2)*(sq
rt(2)*x - sqrt(2*x^2 - x + 3))^3 + 3644644652*(sqrt(2)*x - sqrt(2*x^2 - x +
 3))^2 - 2612608649*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1052284471)
/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^
2 - x + 3)) - 11)^3
```

<u>**Mupad**</u> **[F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^4\,{\left(2\,x^2-x+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(3/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(3/2)), x)
```

3.358 $\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{247}{16}\sqrt{3-x+2x^2}+\frac{5}{4}x\sqrt{3-x+2x^2}-\frac{1471\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

```
[Out] -4/69*(346-533*x)/(2*x^2-x+3)^(3/2)-1471/64*arcsinh(1/23*(1-4*x)*23^(1/2))*
2^(1/2)+4/1587*(18982-20383*x)/(2*x^2-x+3)^(1/2)+247/16*(2*x^2-x+3)^(1/2)+5
/4*x*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1674, 1675, 654, 633, 221}

$$\frac{4(18982-20383x)}{1587\sqrt{2x^2-x+3}}+\frac{5}{4}x\sqrt{2x^2-x+3}+\frac{247}{16}\sqrt{2x^2-x+3}-\frac{4(346-533x)}{69\left(2x^2-x+3\right)^{3/2}}-\frac{1471\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-4*(346 - 533*x))/(69*(3 - x + 2*x^2)^(3/2)) + (4*(18982 - 20383*x))/(1587
*Sqrt[3 - x + 2*x^2]) + (247*Sqrt[3 - x + 2*x^2])/16 + (5*x*Sqrt[3 - x + 2*
x^2])/4 - (1471*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-145-\frac{1725x}{2}+2415x^2+\frac{3657x^3}{2}}{\left(3-x+2x^2\right)^{3/2}}$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{4\int \frac{\frac{33327}{2}+\frac{46023x}{4}+}{\sqrt{3-x+}}}{1587}$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{5}{4}x\sqrt{3-x+2x}$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{247}{16}\sqrt{3-x+2}$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{247}{16}\sqrt{3-x+2}$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{247}{16}\sqrt{3-x+2}$$

Mathematica [A]
time = 0.55, size = 75, normalized size = 0.71

$$\frac{6663133-6410082x+8639625x^2-3764360x^3+1440996x^4+126960x^5}{25392\left(3-x+2x^2\right)^{3/2}}-\frac{1471\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2)
,x]
```

```
[Out] (6663133 - 6410082*x + 8639625*x^2 - 3764360*x^3 + 1440996*x^4 + 126960*x^5
)/(25392*(3 - x + 2*x^2)^(3/2)) - (1471*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^
2]])/(32*Sqrt[2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(84) = 168.
time = 0.01, size = 180, normalized size = 1.71

$$\frac{577397}{2048\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{1471x^3}{48\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{19073x^2}{64\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{162931(4x-1)}{50784\sqrt{2x^2-x+3}}-\frac{753223(4x-1)}{141312\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] 577397/2048/(2*x^2-x+3)^(3/2)-1471/48*x^3/(2*x^2-x+3)^(3/2)+19073/64*x^2/(2
*x^2-x+3)^(3/2)-162931/50784*(4*x-1)/(2*x^2-x+3)^(1/2)-753223/141312*(4*x-1
)/(2*x^2-x+3)^(3/2)+5*x^5/(2*x^2-x+3)^(3/2)+227/4*x^4/(2*x^2-x+3)^(3/2)-322
57/512*x/(2*x^2-x+3)^(3/2)-1471/32*x/(2*x^2-x+3)^(1/2)+1471/64*2^(1/2)*arcs
inh(4/23*23^(1/2)*(x-1/4))-1471/128/(2*x^2-x+3)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(84) = 168.
time = 0.50, size = 219, normalized size = 2.09

$$\frac{5x^5}{(2x^2-x+3)^{\frac{3}{2}}}+\frac{227x^4}{4(2x^2-x+3)^{\frac{3}{2}}}+\frac{1471}{50784}x\left(\frac{284x}{\sqrt{2x^2-x+3}}-\frac{3174x^2}{(2x^2-x+3)^{\frac{3}{2}}}-\frac{71}{\sqrt{2x^2-x+3}}+\frac{805x}{(2x^2-x+3)^{\frac{3}{2}}}-\frac{3243}{(2x^2-x+3)^{\frac{3}{2}}}\right)+\frac{1471}{64}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{104441}{25392}\sqrt{2x^2-x+3}-\frac{383581x}{12696\sqrt{2x^2-x+3}}+\frac{321x^2}{(2x^2-x+3)^{\frac{3}{2}}}-\frac{15965}{4232\sqrt{2x^2-x+3}}-\frac{4147x}{46(2x^2-x+3)^{\frac{3}{2}}}+\frac{42883}{138(2x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] 5*x^5/(2*x^2 - x + 3)^(3/2) + 227/4*x^4/(2*x^2 - x + 3)^(3/2) + 1471/50784*
x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/sqrt(2*x
^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3/2)) + 1
471/64*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 104441/25392*sqrt(2*x^2 -
```

```
x + 3) - 383581/12696*x/sqrt(2*x^2 - x + 3) + 321*x^2/(2*x^2 - x + 3)^(3/2
) - 15965/4232/sqrt(2*x^2 - x + 3) - 4147/46*x/(2*x^2 - x + 3)^(3/2) + 4288
3/138/(2*x^2 - x + 3)^(3/2)
```

Fricas [A]
time = 0.37, size = 122, normalized size = 1.16

$$\frac{2334477 \sqrt{2} \left(4 x^{4}-4 x^{3}+13 x^{2}-6 x+9\right) \log \left(-4 \sqrt{2} \sqrt{2 x^{2}-x+3} (4 x-1)-32 x^{2}+16 x-25\right)+8 \left(126960 x^{5}+1440996 x^{4}-3764360 x^{3}+8639625 x^{2}-6410082 x+6663133\right) \sqrt{2 x^{2}-x+3}}{203136 \left(4 x^{4}-4 x^{3}+13 x^{2}-6 x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/203136*(2334477*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)
*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(126960*x^5 + 1440
996*x^4 - 3764360*x^3 + 8639625*x^2 - 6410082*x + 6663133)*sqrt(2*x^2 - x +
 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)^2 \cdot (5x^4 - x^3 + 3x^2 + x + 2)}{(2x^2 - x + 3)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)**2*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((2*x + 5)**2*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5
/2), x)
```

Giac [A]
time = 5.71, size = 71, normalized size = 0.68

$$-\frac{1471}{64} \sqrt{2} \log \left(-2 \sqrt{2} \left(\sqrt{2} x-\sqrt{2 x^{2}-x+3}\right)+1\right)+\frac{((4 (1587 (20 x+227) x-941090) x+8639625) x-6410082) x+6663133}{25392 \left(2 x^{2}-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] -1471/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/
25392*(((4*(1587*(20*x + 227)*x - 941090)*x + 8639625)*x - 6410082)*x + 666
3133)/(2*x^2 - x + 3)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(2\,x+5\right)}^2\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x^2-x+3\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2),x)
[Out] int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2), x)
```

3.359 $\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=86

$$\frac{-53+373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{5}{4}\sqrt{3-x+2x^2}-\frac{71\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

```
[Out] 1/69*(-53+373*x)/(2*x^2-x+3)^(3/2)-71/16*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(
1/2)+1/3174*(6055-28981*x)/(2*x^2-x+3)^(1/2)+5/4*(2*x^2-x+3)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1674, 654, 633, 221}

$$\frac{6055-28981x}{3174\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}-\frac{71\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] -1/69*(53 - 373*x)/(3 - x + 2*x^2)^(3/2) + (6055 - 28981*x)/(3174*Sqrt[3 -
x + 2*x^2]) + (5*Sqrt[3 - x + 2*x^2])/4 - (71*ArcSinh[(1 - 4*x)/Sqrt[23]])/
(8*Sqrt[2])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{233}{4}+483x^2+\frac{345x^3}{2}}{\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{4\int \frac{\frac{52371}{16}+\frac{7935x}{8}}{\sqrt{3-x+2x^2}}}{1587}$$

$$= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{5}{4}\sqrt{3-x+2x^2}$$

$$= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{5}{4}\sqrt{3-x+2x^2}$$

$$= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{5}{4}\sqrt{3-x+2x^2}$$

Mathematica [A]
time = 0.68, size = 70, normalized size = 0.81

$$\frac{102869-199290x+185337x^2-147664x^3+31740x^4}{6348\left(3-x+2x^2\right)^{3/2}}-\frac{71\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x
]
```

```
[Out] (102869 - 199290*x + 185337*x^2 - 147664*x^3 + 31740*x^4)/(6348*(3 - x + 2*
x^2)^(3/2)) - (71*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(8*Sqrt[2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(69) = 138$.
time = 0.01, size = 163, normalized size = 1.90

$$\frac{11749}{512\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{71x^3}{12\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{401x^2}{16\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{\frac{643x}{3174}-\frac{643}{12696}}{\sqrt{2x^2-x+3}}-\frac{2327(4x-1)}{35328\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{}{(2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] 11749/512/(2*x^2-x+3)^(3/2)-71/12*x^3/(2*x^2-x+3)^(3/2)+401/16*x^2/(2*x^2-x
+3)^(3/2)+643/12696*(4*x-1)/(2*x^2-x+3)^(1/2)-2327/35328*(4*x-1)/(2*x^2-x+3
)^(3/2)+5*x^4/(2*x^2-x+3)^(3/2)-945/128*x/(2*x^2-x+3)^(3/2)-71/8*x/(2*x^2-x
+3)^(1/2)+71/16*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-71/32/(2*x^2-x+3)^(1
/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(69) = 138$.
time = 0.51, size = 202, normalized size = 2.35

$$\frac{5x^4}{\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{71}{12696}x\left(\frac{284x}{\sqrt{2x^2-x+3}}-\frac{3174x^2}{\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{2x^2-x+3}}+\frac{805x}{\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{3243}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\right)+\frac{71}{16}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{5041}{6348}\sqrt{2x^2-x+3}-\frac{10007x}{3174\sqrt{2x^2-x+3}}+\frac{59x^2}{2\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{2959}{2116\sqrt{2x^2-x+3}}-\frac{807x}{92\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{7603}{276\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="max
ima")
```

```
[Out] 5*x^4/(2*x^2 - x + 3)^(3/2) + 71/12696*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*
x^2/(2*x^2 - x + 3)^(3/2) - 71/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^
(3/2) - 3243/(2*x^2 - x + 3)^(3/2)) + 71/16*sqrt(2)*arcsinh(1/23*sqrt(23)*(
4*x - 1)) - 5041/6348*sqrt(2*x^2 - x + 3) - 10007/3174*x/sqrt(2*x^2 - x + 3
) + 59/2*x^2/(2*x^2 - x + 3)^(3/2) - 2959/2116/sqrt(2*x^2 - x + 3) - 807/92
*x/(2*x^2 - x + 3)^(3/2) + 7603/276/(2*x^2 - x + 3)^(3/2)
```

Fricas [A]
time = 0.36, size = 117, normalized size = 1.36

$$\frac{112677\sqrt{2}\left(4x^4-4x^3+13x^2-6x+9\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+8\left(31740x^4-147664x^3+185337x^2-199290x+102869\right)\sqrt{2x^2-x+3}}{50784\left(4x^4-4x^3+13x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/50784*(112677*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(31740*x^4 - 147664*
```

```
x^3 + 185337*x^2 - 199290*x + 102869)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 +
 13*x^2 - 6*x + 9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
[Out] Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5/2)
, x)
```

Giac [A]
time = 3.85, size = 66, normalized size = 0.77

$$-\frac{71}{16}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{((4(7935x-36916)x+185337)x-199290)x+102869}{6348\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="gia
c")
[Out] -71/16*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/63
48*(((4*(7935*x - 36916)*x + 185337)*x - 199290)*x + 102869)/(2*x^2 - x + 3
)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2),x)
[Out] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2), x)
```

3.360 $\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=68

$$\frac{89+219x}{276\,(3-x+2x^2)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

```
[Out] 1/276*(89+219*x)/(2*x^2-x+3)^(3/2)-5/8*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/
2)+1/2116*(-1465-2604*x)/(2*x^2-x+3)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1674, 12, 633, 221}

$$\frac{219x+89}{276\,(2x^2-x+3)^{3/2}}-\frac{2604x+1465}{2116\sqrt{2x^2-x+3}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (89 + 219*x)/(276*(3 - x + 2*x^2)^(3/2)) - (1465 + 2604*x)/(2116*Sqrt[3 - x
 + 2*x^2]) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx &= \frac{89+219x}{276\,(3-x+2x^2)^{3/2}}+\frac{2}{69}\int \frac{-\frac{159}{16}+\frac{207x}{8}+\frac{345x^2}{4}}{(3-x+2x^2)^{3/2}}\,dx \\
&= \frac{89+219x}{276\,(3-x+2x^2)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}+\frac{4\int \frac{7935}{16\sqrt{3-x+2x^2}}\,dx}{1587} \\
&= \frac{89+219x}{276\,(3-x+2x^2)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}+\frac{5}{4}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx \\
&= \frac{89+219x}{276\,(3-x+2x^2)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}+\frac{5\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-\right.}{4\sqrt{46}} \\
&= \frac{89+219x}{276\,(3-x+2x^2)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.50, size = 65, normalized size = 0.96

$$\frac{-5569-7002x-489x^2-7812x^3}{3174\,(3-x+2x^2)^{3/2}}-\frac{5\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-5569 - 7002*x - 489*x^2 - 7812*x^3)/(3174*(3 - x + 2*x^2)^(3/2)) - (5*Log
[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(4*Sqrt[2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(55) = 110$.

time = 0.01, size = 146, normalized size = 2.15

$$-\frac{5x^3}{6\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{x^2}{8\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{47x}{64\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{271}{768\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{\frac{2423x}{4416}-\frac{2423}{17664}}{\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{\frac{692x}{1587}-}{\sqrt{2x^2-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -5/6*x^3/(2*x^2-x+3)^(3/2)-1/8*x^2/(2*x^2-x+3)^(3/2)-47/64*x/(2*x^2-x+3)^(3
/2)-271/768/(2*x^2-x+3)^(3/2)+2423/17664*(4*x-1)/(2*x^2-x+3)^(3/2)+173/1587
*(4*x-1)/(2*x^2-x+3)^(1/2)-5/4*x/(2*x^2-x+3)^(1/2)-5/16/(2*x^2-x+3)^(1/2)+5
/8*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(55) = 110.
time = 0.50, size = 185, normalized size = 2.72

$$\frac{5}{6348}x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}}-\frac{3174\,x^2}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^2-x+3}}+\frac{805\,x}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{3243}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}\right)+\frac{5}{8}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}\left(4\,x-1\right)\right)-\frac{355}{3174}\sqrt{2\,x^2-x+3}-\frac{58\,x}{1587\sqrt{2\,x^2-x+3}}+\frac{x^2}{2\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{1897}{6348\sqrt{2\,x^2-x+3}}-\frac{95\,x}{276\left(2\,x^2-x+3\right)^{\frac{3}{2}}}+\frac{41}{276\left(2\,x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/6348*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/s
qrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3/
2)) + 5/8*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 355/3174*sqrt(2*x^2 -
x + 3) - 58/1587*x/sqrt(2*x^2 - x + 3) + 1/2*x^2/(2*x^2 - x + 3)^(3/2) - 18
97/6348/sqrt(2*x^2 - x + 3) - 95/276*x/(2*x^2 - x + 3)^(3/2) + 41/276/(2*x^
2 - x + 3)^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(55) = 110.
time = 0.35, size = 112, normalized size = 1.65

$$\frac{7935\sqrt{2}\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}\left(4\,x-1\right)-32\,x^2+16\,x-25\right)-8\left(7812\,x^3+489\,x^2+7002\,x+5569\right)\sqrt{2\,x^2-x+3}}{25392\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/25392*(7935*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*sqr
t(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) - 8*(7812*x^3 + 489*x^2 +
7002*x + 5569)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5/2), x)
```

Giac [A]
time = 4.87, size = 62, normalized size = 0.91

$$-\frac{5}{8}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)-\frac{3\,((2604\,x+163)x+2334)x+5569}{3174\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 1/3174
*(3*((2604*x + 163)*x + 2334)*x + 5569)/(2*x^2 - x + 3)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x^2-x+3)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(5/2), x)
```

3.361 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{31104\sqrt{2}}$$

```
[Out] 1/9936*(1191+917*x)/(2*x^2-x+3)^(3/2)-3667/62208*arctanh(1/24*(17-22*x)*2^(
1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/1371168*(-335337-146729*x)/(2*x^2-x+3)^(1
/2)
```

Rubi [A]
time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1660, 12, 738, 212}

$$\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}-\frac{146729x+335337}{1371168\sqrt{2x^2-x+3}}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{31104\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(5/2)),x]
```

```
[Out] (1191 + 917*x)/(9936*(3 - x + 2*x^2)^(3/2)) - (335337 + 146729*x)/(1371168*
Sqrt[3 - x + 2*x^2]) - (3667*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2
*x^2])])/(31104*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{1877}{576}+\frac{695x}{18}+\frac{345x^2}{4}}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}+\frac{4\int \frac{1939843}{6912(5+2x)\sqrt{3-x+}}}{1587}$$

$$= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}+\frac{3667\int \frac{1}{(5+2x)\sqrt{3-x+}}}{5184}$$

$$= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}-\frac{3667\text{Subst}\left(\int \frac{1}{288-x^2}\,dx\right.}{259}$$

$$= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}-\frac{3667\tanh^{-1}\left(\frac{1}{12\sqrt{2}\sqrt{}}\right.}{31104\sqrt{2}}$$

Mathematica [A]
time = 0.53, size = 69, normalized size = 0.81

$$\frac{-841653+21696x-523945x^2-293458x^3}{1371168\left(3-x+2x^2\right)^{3/2}}+\frac{3667\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{15552\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(5/2)),x
]
```

```
[Out] (-841653 + 21696*x - 523945*x^2 - 293458*x^3)/(1371168*(3 - x + 2*x^2)^(3/2
)) + (3667*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6])/(15552*Sqrt[2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(67) = 134$.
time = 0.01, size = 190, normalized size = 2.24

$$-\frac{5x^2}{4\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{59x}{32\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{1597}{384\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{3817(4x-1)}{2944\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{3817(4x-1)}{4232\sqrt{2x^2-x+3}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -5/4*x^2/(2*x^2-x+3)^(3/2)+59/32*x/(2*x^2-x+3)^(3/2)-1597/384/(2*x^2-x+3)^(
3/2)-3817/2944*(4*x-1)/(2*x^2-x+3)^(3/2)-3817/4232*(4*x-1)/(2*x^2-x+3)^(1/2
)+3667/1728/(2*(x+5/2)^2-11*x-19/2)^(3/2)+40337/39744*(4*x-1)/(2*(x+5/2)^2-
11*x-19/2)^(3/2)+4800103/5484672*(4*x-1)/(2*(x+5/2)^2-11*x-19/2)^(1/2)+3667
/10368/(2*(x+5/2)^2-11*x-19/2)^(1/2)-3667/62208*2^(1/2)*arctanh(1/12*(17/2-
11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Maxima [A]
time = 0.50, size = 110, normalized size = 1.29

$$\frac{3667}{62208}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{146729\,x}{1371168\sqrt{2\,x^2-x+3}}-\frac{5\,x^2}{4\,(2\,x^2-x+3)^{\frac{3}{2}}}+\frac{173881}{457056\sqrt{2\,x^2-x+3}}+\frac{7127\,x}{9936\,(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{5813}{3312\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="max
ima")
```

```
[Out] 3667/62208*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/a
bs(2*x + 5)) - 146729/1371168*x/sqrt(2*x^2 - x + 3) - 5/4*x^2/(2*x^2 - x +
3)^(3/2) + 173881/457056/sqrt(2*x^2 - x + 3) + 7127/9936*x/(2*x^2 - x + 3)^
(3/2) - 5813/3312/(2*x^2 - x + 3)^(3/2)
```

Fricas [A]
time = 0.39, size = 126, normalized size = 1.48

$$\frac{1939843\sqrt{2}\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\log\left(-\frac{24\sqrt{2}\sqrt{2\,x^2-x+3}\,(22\,x-17)+1060\,x^2-1036\,x+1153}{4\,x^2+20\,x+25}\right)-48\,(293458\,x^3+523945\,x^2-21696\,x+841653)\sqrt{2\,x^2-x+3}}{65816064\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/65816064*(1939843*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-(24*sqr
t(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 2
0*x + 25)) - 48*(293458*x^3 + 523945*x^2 - 21696*x + 841653)*sqrt(2*x^2 - x
 + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x + 5)\left(2x^2 - x + 3\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*(2*x**2 - x + 3)**(5/2
)), x)
```

Giac [A]
time = 5.49, size = 92, normalized size = 1.08

$$-\frac{3667}{62208}\,\sqrt{2}\,\log\left(\left|-2\,\sqrt{2}\,x + \sqrt{2} + 2\,\sqrt{2\,x^2 - x + 3}\right|\right) + \frac{3667}{62208}\,\sqrt{2}\,\log\left(\left|-2\,\sqrt{2}\,x - 11\,\sqrt{2} + 2\,\sqrt{2\,x^2 - x + 3}\right|\right) - \frac{((293458\,x + 523945)x - 21696)x + 841653}{1371168\,(2\,x^2 - x + 3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="gia
c")
```

```
[Out] -3667/62208*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))
) + 3667/62208*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x
 + 3))) - 1/1371168*(((293458*x + 523945)*x - 21696)*x + 841653)/(2*x^2 - x
 + 3)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4 - x^3 + 3\,x^2 + x + 2}{(2\,x + 5)\,\left(2\,x^2 - x + 3\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(5/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(5/2)), x)
```

3.362 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=110

$$\frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}-\frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)}-\frac{2821\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{2239488\sqrt{2}}$$

[Out] 1/357696*(9897+2203*x)/(2*x^2-x+3)^(3/2)-2821/4478976*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/24681024*(-1255878+62021*x)/(2*x^2-x+3)^(1/2)-3667/186624*(2*x^2-x+3)^(1/2)/(5+2*x)

Rubi [A]
time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1660, 820, 738, 212}

$$-\frac{1255878-62021x}{24681024\sqrt{2x^2-x+3}}-\frac{3667\sqrt{2x^2-x+3}}{186624(2x+5)}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}-\frac{2821\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{2239488\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(5/2)),x]

[Out] (9897 + 2203*x)/(357696*(3 - x + 2*x^2)^(3/2)) - (1255878 - 62021*x)/(24681024*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(186624*(5 + 2*x)) - (2821*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(2239488*Sqrt[2])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
```

```
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{119353}{20736}+\frac{481765x}{10368}+\frac{113983x^2}{1296}}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}-\frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}}+\frac{4\int \frac{\frac{10109719}{124416}-\frac{4961}{622}}{(5+2x)^2\sqrt{3-x}}}{1587}$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}-\frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)}$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}-\frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)}$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}-\frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)}$$

Mathematica [A]
time = 0.67, size = 81, normalized size = 0.74

$$\frac{-\frac{12\left(79153407-18840090x+63941915x^2+10350004x^3+6767036x^4\right)}{(5+2x)\left(3-x+2x^2\right)^{3/2}}+1492309\sqrt{2}\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{1184689152}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))
,x]
```

```
[Out] ((-12*(79153407 - 18840090*x + 63941915*x^2 + 10350004*x^3 + 6767036*x^4))/
((5 + 2*x)*(3 - x + 2*x^2)^(3/2)) + 1492309*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt
[6 - 2*x + 4*x^2])/6])/1184689152
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(88) = 176$.
time = 0.02, size = 194, normalized size = 1.76

$$-\frac{5x}{16\left(2x^{2}-x+3\right)^{\frac{3}{2}}}+\frac{203}{192\left(2x^{2}-x+3\right)^{\frac{3}{2}}}+\frac{\frac{3173x}{1104}-\frac{3173}{4416}}{\left(2x^{2}-x+3\right)^{\frac{3}{2}}}+\frac{\frac{3173x}{1587}-\frac{3173}{6348}}{\sqrt{2x^{2}-x+3}}-\frac{3667}{1152\left(x+\frac{5}{2}\right)\left(2\left(x+\frac{5}{2}\right)^{2}-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -5/16*x/(2*x^2-x+3)^(3/2)+203/192/(2*x^2-x+3)^(3/2)+3173/4416*(4*x-1)/(2*x^
2-x+3)^(3/2)+3173/6348*(4*x-1)/(2*x^2-x+3)^(1/2)-3667/1152/(x+5/2)/(2*(x+5/
2)^2-11*x-19/2)^(3/2)+2821/124416/(2*(x+5/2)^2-11*x-19/2)^(3/2)-2081161/286
1568*(4*x-1)/(2*(x+5/2)^2-11*x-19/2)^(3/2)-199077743/394896384*(4*x-1)/(2*(
x+5/2)^2-11*x-19/2)^(1/2)+2821/746496/(2*(x+5/2)^2-11*x-19/2)^(1/2)-2821/44
78976*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2
))
```

Maxima [A]
time = 0.51, size = 127, normalized size = 1.15

$$\frac{2821}{4478976}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|}-\frac{17\sqrt{23}}{23|2x+5|}\right)-\frac{1691759x}{98724096\sqrt{2x^{2}-x+3}}+\frac{265339}{32908032\sqrt{2x^{2}-x+3}}-\frac{248617x}{715392\left(2x^{2}-x+3\right)^{\frac{3}{2}}}-\frac{3667}{576\left(2\left(2x^{2}-x+3\right)^{\frac{3}{2}}x+5\left(2x^{2}-x+3\right)^{\frac{3}{2}}\right)}+\frac{259621}{238464\left(2x^{2}-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] 2821/4478976*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)
/abs(2*x + 5)) - 1691759/98724096*x/sqrt(2*x^2 - x + 3) + 265339/32908032/s
qrt(2*x^2 - x + 3) - 248617/715392*x/(2*x^2 - x + 3)^(3/2) - 3667/576/(2*(2
*x^2 - x + 3)^(3/2)*x + 5*(2*x^2 - x + 3)^(3/2)) + 259621/238464/(2*x^2 - x
 + 3)^(3/2)
```

Fricas [**A**]
time = 0.38, size = 141, normalized size = 1.28

$$\frac{1492309\,\sqrt{2}\,\left(8\,x^{5}+12\,x^{4}+6\,x^{3}+53\,x^{2}-12\,x+45\right)\log\left(-\frac{24\,\sqrt{2}\,\sqrt{2\,x^{2}-x+3}\,\left(22\,x-17\right)+1060\,x^{2}-1036\,x+1153}{4\,x^{2}+20\,x+25}\right)-48\,\left(6767036\,x^{4}+10350004\,x^{3}+63941915\,x^{2}-18840090\,x+79153407\right)\sqrt{2\,x^{2}-x+3}}{4738756608\,\left(8\,x^{5}+12\,x^{4}+6\,x^{3}+53\,x^{2}-12\,x+45\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/4738756608*(1492309*sqrt(2)*(8*x^5 + 12*x^4 + 6*x^3 + 53*x^2 - 12*x + 45)
*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 115
3)/(4*x^2 + 20*x + 25)) - 48*(6767036*x^4 + 10350004*x^3 + 63941915*x^2 - 1
8840090*x + 79153407)*sqrt(2*x^2 - x + 3))/(8*x^5 + 12*x^4 + 6*x^3 + 53*x^2
 - 12*x + 45)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^{4} - x^{3} + 3x^{2} + x + 2}{\left(2x + 5\right)^{2} \left(2x^{2} - x + 3\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*(2*x**2 - x + 3)**(
5/2)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(88) = 176.
time = 4.95, size = 206, normalized size = 1.87

$$-\frac{1}{2369378304}\,\sqrt{2}\left(\frac{1492309\,\log\left(12\,\sqrt{-\frac{11}{2\,x+5}+\frac{36}{\left(2\,x+5\right)^{2}}+1}+\frac{72}{2\,x+5}-11\right)}{\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)}+\frac{12\left(\frac{\frac{\frac{48\left(\frac{23642785}{\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)}-\frac{52375761}{\left(2\,x+5\right)\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)}\right)}{2\,x+5}-\frac{240080735}{\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)}}{2\,x+5}+\frac{28660178}{\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)}}{2\,x+5}-\frac{1691759}{\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)}\right)}{\left(\frac{11}{2\,x+5}-\frac{36}{\left(2\,x+5\right)^{2}}-1\right)\sqrt{-\frac{11}{2\,x+5}+\frac{36}{\left(2\,x+5\right)^{2}}+1}}-20301108\,\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] -1/2369378304*sqrt(2)*(1492309*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
 1) + 72/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 12*(((48*(23642785/sgn(1/(2*x +
```

```
 5)) - 52375761/((2*x + 5)*sgn(1/(2*x + 5))))/(2*x + 5) - 240080735/sgn(1/(
2*x + 5)))/(2*x + 5) + 28660178/sgn(1/(2*x + 5)))/(2*x + 5) - 1691759/sgn(1
/(2*x + 5)))/((11/(2*x + 5) - 36/(2*x + 5)^2 - 1)*sqrt(-11/(2*x + 5) + 36/(
2*x + 5)^2 + 1)) - 20301108*sgn(1/(2*x + 5)))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^2\,{\left(2\,x^2-x+3\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(5/2)),x)
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(5/2)), x)
```

3.363 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$\frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}}-\frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2}-\frac{45979\sqrt{3-x+2x^2}}{26873856(5+2x)}+\frac{774079}{}$$

[Out] 1/12877056*(65991-8779*x)/(2*x^2-x+3)^(3/2)+774079/644972544*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/592344576*(-4679797+2148263*x)/(2*x^2-x+3)^(1/2)-3667/373248*(2*x^2-x+3)^(1/2)/(5+2*x)^2-45979/26873856*(2*x^2-x+3)^(1/2)/(5+2*x)

Rubi [A]
time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1660, 1664, 820, 738, 212}

$$-\frac{4679797-2148263x}{592344576\sqrt{2x^2-x+3}}-\frac{45979\sqrt{2x^2-x+3}}{26873856(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{373248(2x+5)^2}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}+\frac{774079\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{322486272\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(5/2)),x]

[Out] (65991 - 8779*x)/(12877056*(3 - x + 2*x^2)^(3/2)) - (4679797 - 2148263*x)/(592344576*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(373248*(5 + 2*x)^2) - (45979*Sqrt[3 - x + 2*x^2])/(26873856*(5 + 2*x)) + (774079*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(322486272*Sqrt[2])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
```

```
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{5/2}}\,dx = \frac{65991-8779x}{12877056(3-x+2x^2)^{3/2}} + \frac{2}{69}\int \frac{\frac{11115283}{746496}+\frac{3198845x}{62208}+\frac{605005x^2}{6912}-\frac{8779x^3}{23328}}{(5+2x)^3(3-x+2x^2)^{3/2}}$$

$$= \frac{65991-8779x}{12877056(3-x+2x^2)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} + \frac{4\int \frac{-\frac{171639869}{2985984}-1}{(5+2x)^3\sqrt{}}}{}$$

$$= \frac{65991-8779x}{12877056(3-x+2x^2)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+}}{373248(5+2}$$

$$= \frac{65991-8779x}{12877056(3-x+2x^2)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+}}{373248(5+2}$$

$$= \frac{65991-8779x}{12877056(3-x+2x^2)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+}}{373248(5+2}$$

$$= \frac{65991-8779x}{12877056(3-x+2x^2)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+}}{373248(5+2}$$

Mathematica [A]
time = 0.63, size = 94, normalized size = 0.70

$$\frac{\frac{12\sqrt{3-x+2x^2}\left(-8953831359+2280511668x-5919924791x^2-1503926130x^3+107028732x^4+217883368x^5\right)}{(15+x+8x^2+4x^3)^2}-409487791\sqrt{2}\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{170595237888}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(5/2))
,x]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(-8953831359 + 2280511668*x - 5919924791*x^2 - 150
3926130*x^3 + 107028732*x^4 + 217883368*x^5))/(15 + x + 8*x^2 + 4*x^3)^2 -
409487791*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6])/17059523788
8
```

Maple [A]
time = 0.01, size = 200, normalized size = 1.48

$$-\frac{5}{48\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{149(4x-1)}{1104\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{149(4x-1)}{1587\sqrt{2x^2-x+3}}+\frac{115369}{165888\left(x+\frac{5}{2}\right)\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -5/48/(2*x^2-x+3)^(3/2)-149/1104*(4*x-1)/(2*x^2-x+3)^(3/2)-149/1587*(4*x-1)
/(2*x^2-x+3)^(1/2)+115369/165888/(x+5/2)/(2*(x+5/2)^2-11*x-19/2)^(3/2)-7740
79/17915904/(2*(x+5/2)^2-11*x-19/2)^(3/2)+57937675/412065792*(4*x-1)/(2*(x+
5/2)^2-11*x-19/2)^(3/2)+5366174813/56865079296*(4*x-1)/(2*(x+5/2)^2-11*x-19
/2)^(1/2)-774079/107495424/(2*(x+5/2)^2-11*x-19/2)^(1/2)+774079/644972544*2
^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-3667
/4608/(x+5/2)^2/(2*(x+5/2)^2-11*x-19/2)^(3/2)
```

Maxima [A]
time = 0.50, size = 178, normalized size = 1.32

$$-\frac{774079}{644972544}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{27235421\,x}{14216269824\sqrt{2\,x^2-x+3}}-\frac{36393601}{4738756608\sqrt{2\,x^2-x+3}}+\frac{2323723\,x}{103016448\,(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{3667}{1152\left(4\,(2\,x^2-x+3)^{\frac{3}{2}}x^2+20\,(2\,x^2-x+3)^{\frac{3}{2}}x+25\,(2\,x^2-x+3)^{\frac{3}{2}}\right)}+\frac{115369}{82944\left(2\,(2\,x^2-x+3)^{\frac{3}{2}}x+5\,(2\,x^2-x+3)^{\frac{3}{2}}\right)}-\frac{5254255}{34338816\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] -774079/644972544*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqr
t(23)/abs(2*x + 5)) + 27235421/14216269824*x/sqrt(2*x^2 - x + 3) - 36393601
/4738756608/sqrt(2*x^2 - x + 3) + 2323723/103016448*x/(2*x^2 - x + 3)^(3/2)
 - 3667/1152/(4*(2*x^2 - x + 3)^(3/2)*x^2 + 20*(2*x^2 - x + 3)^(3/2)*x + 25
*(2*x^2 - x + 3)^(3/2)) + 115369/82944/(2*(2*x^2 - x + 3)^(3/2)*x + 5*(2*x^
2 - x + 3)^(3/2)) - 5254255/34338816/(2*x^2 - x + 3)^(3/2)
```

Fricas [A]
time = 0.36, size = 155, normalized size = 1.15

$$\frac{409487791\sqrt{2}\,(16\,x^6+64\,x^5+72\,x^4+136\,x^3+241\,x^2+30\,x+225)\log\left(\frac{24\sqrt{2}\sqrt{2\,x^2-x+3}\,(22\,x-17)-1060\,x^2+1036\,x-1153}{4\,x^2+20\,x+25}\right)+48\,(217883368\,x^5+107028732\,x^4-1503926130\,x^3-5919924791\,x^2+2280511668\,x-8953831359)\sqrt{2\,x^2-x+3}}{682380951552\,(16\,x^6+64\,x^5+72\,x^4+136\,x^3+241\,x^2+30\,x+225)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/682380951552*(409487791*sqrt(2)*(16*x^6 + 64*x^5 + 72*x^4 + 136*x^3 + 241
*x^2 + 30*x + 225)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x
^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(217883368*x^5 + 107028732*x^
4 - 1503926130*x^3 - 5919924791*x^2 + 2280511668*x - 8953831359)*sqrt(2*x^2
 - x + 3))/(16*x^6 + 64*x^5 + 72*x^4 + 136*x^3 + 241*x^2 + 30*x + 225)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\,(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*(2*x**2 - x + 3)**(
5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(109) = 218.
time = 4.37, size = 228, normalized size = 1.69

$$\frac{774079}{644972544}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)-\frac{774079}{644972544}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{\sqrt{2}\left(44558\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^3-10136238\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+16812201\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-10182217\right)}{53747712\left(2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-11\right)^2}+\frac{((4296526x-11507857)x+10720752)x-11003805}{592344576\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] 774079/644972544*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x
+ 3))) - 774079/644972544*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqr
t(2*x^2 - x + 3))) + 1/53747712*sqrt(2)*(44558*sqrt(2)*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^3 - 10136238*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 16812201*s
qrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 10182217)/(2*(sqrt(2)*x - sqrt(2
*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2 + 1
/592344576*(((4296526*x - 11507857)*x + 10720752)*x - 11003805)/(2*x^2 - x
+ 3)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^3\,{\left(2\,x^2-x+3\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(5/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(5/2)), x)
```

3.364 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=160

$$\frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}}-\frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^3}-\frac{89137\sqrt{3-x+2x^2}}{80621568(5+2x)^2}+\frac{47}{19}$$

```
[Out] 1/463574016*(369609-175877*x)/(2*x^2-x+3)^(3/2)+4778789/15479341056*arctanh
(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/31986607104*(-27754539
+31190998*x)/(2*x^2-x+3)^(1/2)-3667/559872*(2*x^2-x+3)^(1/2)/(5+2*x)^3-8913
7/80621568*(2*x^2-x+3)^(1/2)/(5+2*x)^2+475357/1934917632*(2*x^2-x+3)^(1/2)/
(5+2*x)
```

Rubi [A]
time = 0.17, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1660, 1664, 820, 738, 212}

$$-\frac{27754539-31190998x}{31986607104\sqrt{2x^2-x+3}}+\frac{475357\sqrt{2x^2-x+3}}{1934917632(2x+5)}-\frac{89137\sqrt{2x^2-x+3}}{80621568(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{559872(2x+5)^3}+\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}+\frac{4778789\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{7739670528\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(5/2)),x]
```

```
[Out] (369609 - 175877*x)/(463574016*(3 - x + 2*x^2)^(3/2)) - (27754539 - 3119099
8*x)/(31986607104*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(559872
*(5 + 2*x)^3) - (89137*Sqrt[3 - x + 2*x^2])/(80621568*(5 + 2*x)^2) + (47535
7*Sqrt[3 - x + 2*x^2])/(1934917632*(5 + 2*x)) + (4778789*ArcTanh[(17 - 22*x
)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(7739670528*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{606939313}{26873856}+\frac{727085495x}{13436928}+\frac{1867054850x^2}{2239488} - \cdots}{(5+2x)^4\left(3-x+2\cdots\right.}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} + \frac{4\int \frac{-481173\cdots}{40310\cdots}}{\cdots}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-\cdots}}{559872(\cdots}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-\cdots}}{559872(\cdots}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-\cdots}}{559872(\cdots}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-\cdots}}{559872(\cdots}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-\cdots}}{559872(\cdots}$$

Mathematica [A]
time = 0.72, size = 91, normalized size = 0.57

$$\frac{\frac{12\left(-95241881529+73621973154x-6702882569x^2+27484986184x^3+46210466520x^4+34872810880x^5+6664404208x^6\right)}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}} - 2527979381\sqrt{2}\,\tanh^{-1}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{4094285709312}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(5/2))
,x]
```

```
[Out] ((12*(-95241881529 + 73621973154*x - 6702882569*x^2 + 27484986184*x^3 + 462
10466520*x^4 + 34872810880*x^5 + 6664404208*x^6))/((5 + 2*x)^3*(3 - x + 2*x
^2)^(3/2)) - 2527979381*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6
])/4094285709312
```

Maple [A]

time = 0.01, size = 207, normalized size = 1.29

$$-\frac{4778789}{2579890176\sqrt{2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}}}-\frac{3667}{13824\left(x+\frac{5}{2}\right)^{3}\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{3}{2}}}+\frac{25\cdots}{110592\left(x+\frac{5}{2}\right)^{2}\left(2\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -4778789/2579890176/(2*(x+5/2)^2-11*x-19/2)^(1/2)-3667/13824/(x+5/2)^3/(2*(
x+5/2)^2-11*x-19/2)^(3/2)+25951/110592/(x+5/2)^2/(2*(x+5/2)^2-11*x-19/2)^(3
/2)-72646615/9889579008*(4*x-1)/(2*(x+5/2)^2-11*x-19/2)^(3/2)-34861/3981312
/(x+5/2)/(2*(x+5/2)^2-11*x-19/2)^(3/2)-8183108657/1364761903104*(4*x-1)/(2*
(x+5/2)^2-11*x-19/2)^(1/2)+10/1587*(4*x-1)/(2*x^2-x+3)^(1/2)+5/552*(4*x-1)/
(2*x^2-x+3)^(3/2)+4778789/15479341056*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1
/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))-4778789/429981696/(2*(x+5/2)^2-11*x-19/2
)^(3/2)
```

Maxima [A]
time = 0.52, size = 246, normalized size = 1.54

$$-\frac{4778789}{15479341056}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|}-\frac{17\sqrt{23}}{23|2x+5|}\right)+\frac{416525263x}{341190475776\sqrt{2x^2-x+3}}-\frac{245375387}{113730158592\sqrt{2x^2-x+3}}+\frac{16932905x}{2472394752(2x^2-x+3)^{\frac{3}{2}}}-\frac{3667}{1728\left(8(2x^2-x+3)^{\frac{3}{2}}x^3+60(2x^2-x+3)^{\frac{3}{2}}x^2+150(2x^2-x+3)^{\frac{3}{2}}x+125(2x^2-x+3)^{\frac{3}{2}}\right)}+\frac{25951}{27648\left(4(2x^2-x+3)^{\frac{3}{2}}x^2+20(2x^2-x+3)^{\frac{3}{2}}x+25(2x^2-x+3)^{\frac{3}{2}}\right)}-\frac{34861}{1990656\left(2(2x^2-x+3)^{\frac{3}{2}}x+5(2x^2-x+3)^{\frac{3}{2}}\right)}-\frac{10570421}{824131584(2x^2-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] -4778789/15479341056*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*
sqrt(23)/abs(2*x + 5)) + 416525263/341190475776*x/sqrt(2*x^2 - x + 3) - 245
375387/113730158592/sqrt(2*x^2 - x + 3) + 16932905/2472394752*x/(2*x^2 - x
+ 3)^(3/2) - 3667/1728/(8*(2*x^2 - x + 3)^(3/2)*x^3 + 60*(2*x^2 - x + 3)^(3
/2)*x^2 + 150*(2*x^2 - x + 3)^(3/2)*x + 125*(2*x^2 - x + 3)^(3/2)) + 25951/
27648/(4*(2*x^2 - x + 3)^(3/2)*x^2 + 20*(2*x^2 - x + 3)^(3/2)*x + 25*(2*x^2
 - x + 3)^(3/2)) - 34861/1990656/(2*(2*x^2 - x + 3)^(3/2)*x + 5*(2*x^2 - x
+ 3)^(3/2)) - 10570421/824131584/(2*x^2 - x + 3)^(3/2)
```

Fricas [A]
time = 0.37, size = 170, normalized size = 1.06

$$\frac{2527979381\sqrt{2}\left(32x^7+208x^6+464x^5+632x^4+1162x^3+1265x^2+600x+1125\right)\log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)-1060x^2+1036x-1153}{4x^2+20x+25}\right)+48\left(6664404208x^6+34872810880x^5+46210466520x^4+27484986184x^3-6702882569x^2+73621973154x-95241881529\right)\sqrt{2x^2-x+3}}{16377142837248\left(32x^7+208x^6+464x^5+632x^4+1162x^3+1265x^2+600x+1125\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/16377142837248*(2527979381*sqrt(2)*(32*x^7 + 208*x^6 + 464*x^5 + 632*x^4
+ 1162*x^3 + 1265*x^2 + 600*x + 1125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(
22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(666440420
8*x^6 + 34872810880*x^5 + 46210466520*x^4 + 27484986184*x^3 - 6702882569*x^
2 + 73621973154*x - 95241881529)*sqrt(2*x^2 - x + 3))/(32*x^7 + 208*x^6 + 4
64*x^5 + 632*x^4 + 1162*x^3 + 1265*x^2 + 600*x + 1125)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)^4 (2x^2 - x + 3)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*(2*x**2 - x + 3)**(
5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(130) = 260.
time = 4.21, size = 279, normalized size = 1.74

$$\frac{4778789}{15479341056}\sqrt{2}\log\left(\left|-2\sqrt{2}x + \sqrt{2} + 2\sqrt{2x^2 - x + 3}\right|\right) - \frac{4778789}{15479341056}\sqrt{2}\log\left(\left|-2\sqrt{2}x - 11\sqrt{2} + 2\sqrt{2x^2 - x + 3}\right|\right) + \frac{((15595499x - 21675019)x + 27298005)x - 14440149}{7996651776\,(2x^2 - x + 3)^{\frac{3}{2}}} + \frac{\sqrt{2}\left(38030012\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right)^5 + 734231900\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right)^4 + 122834956\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right)^3 - 2154595396\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right)^2 + 1659431083\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) - 760577429\right)}{3869835264\left(2\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right)^2 + 10\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 - x + 3}\right) - 11\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] 4778789/15479341056*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 -
 x + 3))) - 4778789/15479341056*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) +
 2*sqrt(2*x^2 - x + 3))) + 1/7996651776*(((15595499*x - 21675019)*x + 27298
005)*x - 14440149)/(2*x^2 - x + 3)^(3/2) + 1/3869835264*sqrt(2)*(38030012*s
qrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 734231900*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^4 + 122834956*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 2
154595396*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 1659431083*sqrt(2)*(sqrt(2)
*x - sqrt(2*x^2 - x + 3)) - 760577429)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4 - x^3 + 3\,x^2 + x + 2}{(2\,x+5)^4\,(2\,x^2 - x + 3)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(5/2)),x)
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(5/2)), x)
```

3.365 $\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=354

$$\frac{2(ab^2ci+2ac^2(cg-ai)-ab^3j-bc(c^2f+ach-3a^2j)-(2c^4f-c^3(bg+2ah)+b^4j-b^2c(bi+4aj)+c^2(b}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] 2/3*(a*b^2*c*i+2*a*c^2*(-a*i+c*g)-a*b^3*j-b*c*(-3*a^2*j+a*c*h+c^2*f)-(2*c^4
*f-c^3*(2*a*h+b*g)+b^4*j-b^2*c*(4*a*j+b*i)+c^2*(2*a^2*j+3*a*b*i+b^2*h))*x)/
c^3/(-4*a*c+b^2)/(c*x^2+b*x+a)^(3/2)+j*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2
+b*x+a)^(1/2))/c^(5/2)-2/3*(b^4*c*i+24*a^2*c^3*i+2*b^2*c^2*(-3*a*i+2*c*g)-b
^5*j-b^3*c*(-10*a*j+c*h)-4*b*c^2*(8*a^2*j+a*c*h+2*c^2*f)-c*(16*c^4*f-c^3*(-
8*a*h+8*b*g)-4*b^4*j+b^2*c*(28*a*j+b*i)+2*c^2*(-16*a^2*j-6*a*b*i+b^2*h))*x)
/c^3/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1674, 12, 635, 212}

$$\frac{2(-x(c^2(2a^2j+3abi+b^2h)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f)-bc(-3a^2j+ach+c^2f)-ab^3j+ab^2ci+2ac^2(cg-ai))}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{2(-cx(2c^2(-16a^2j-6abi+b^2h)+b^2c(28aj+bi)-c^3(8bg-8ah)-4b^4j+16c^4f)-4bc^2(8a^2j+ach+2c^2f)+24a^2c^3i-b^3c(ch-10aj)+2b^2c^2(2cg-3ai)+b^5(-j)+b^4ci)}{3c^3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}+\frac{j\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(a*b^2*c*i + 2*a*c^2*(c*g - a*i) - a*b^3*j - b*c*(c^2*f + a*c*h - 3*a^2*
j) - (2*c^4*f - c^3*(b*g + 2*a*h) + b^4*j - b^2*c*(b*i + 4*a*j) + c^2*(b^2*
h + 3*a*b*i + 2*a^2*j))*x))/(3*c^3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) -
 (2*(b^4*c*i + 24*a^2*c^3*i + 2*b^2*c^2*(2*c*g - 3*a*i) - b^5*j - b^3*c*(c*
h - 10*a*j) - 4*b*c^2*(2*c^2*f + a*c*h + 8*a^2*j) - c*(16*c^4*f - c^3*(8*b*
g - 8*a*h) - 4*b^4*j + b^2*c*(b*i + 28*a*j) + 2*c^2*(b^2*h - 6*a*b*i - 16*a
^2*j))*x))/(3*c^3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2]) + (j*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{f+gx+hx^2+365x^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx = -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a(365b^2c+2c^3g-bc^2h-b^3j)}{c^3}\right)-(365b^3c-bc^2(1095\cdots\right.}{3c^3(b^2-4ac)(a+bx\cdots}$$

$$= -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a(365b^2c+2c^3g-bc^2h-b^3j)}{c^3}\right)-(365b^3c-bc^2(1095\cdots\right.}{3c^3(b^2-4ac)(a+bx\cdots}$$

$$= -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a(365b^2c+2c^3g-bc^2h-b^3j)}{c^3}\right)-(365b^3c-bc^2(1095\cdots\right.}{3c^3(b^2-4ac)(a+bx\cdots}$$

$$= -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a(365b^2c+2c^3g-bc^2h-b^3j)}{c^3}\right)-(365b^3c-bc^2(1095\cdots\right.}{3c^3(b^2-4ac)(a+bx\cdots}$$

$$= -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a(365b^2c+2c^3g-bc^2h-b^3j)}{c^3}\right)-(365b^3c-bc^2(1095\cdots\right.}{3c^3(b^2-4ac)(a+bx\cdots}$$

Mathematica [A]
time = 1.92, size = 315, normalized size = 0.89

$$\frac{2(-3b^5jx^2-2b^4jx(3a+2cx^2)+b^3(-3a^2j+18acjx^2+c^2(-f-3gx+3hx^2+ix^3))+2b^2c(21a^2jx+c^2x(3f-6gx+hx^2)-ac(g-6hx+3ix^2-14jx^3))-8c^2(-2c^3fx^3+a^3(2i+3jx)-ac^2x(3f+hx^2)+a^2c(g+3ix^2+4jx^3))+4bc(5a^3j-2c^3x^2(-3f+gx)+2a^2c(h-3ix)+3ac^2(f-x(g-hx+ix^2)))))}{3c^2(b^2-4ac)^2(a+x(b+cx))^{3/2}}-\frac{j\log\left(c^2\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(-3*b^5*j*x^2 - 2*b^4*j*x*(3*a + 2*c*x^2) + b^3*(-3*a^2*j + 18*a*c*j*x^2
 + c^2*(-f - 3*g*x + 3*h*x^2 + i*x^3)) + 2*b^2*c*(21*a^2*j*x + c^2*x*(3*f -
 6*g*x + h*x^2) - a*c*(g - 6*h*x + 3*i*x^2 - 14*j*x^3)) - 8*c^2*(-2*c^3*f*x
^3 + a^3*(2*i + 3*j*x) - a*c^2*x*(3*f + h*x^2) + a^2*c*(g + 3*i*x^2 + 4*j*x
^3)) + 4*b*c*(5*a^3*j - 2*c^3*x^2*(-3*f + g*x) + 2*a^2*c*(h - 3*i*x) + 3*a*
c^2*(f - x*(g - h*x + i*x^2)))))/(3*c^2*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(
3/2)) - (j*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1405 vs. $2(338) = 676$.
time = 0.01, size = 1406, normalized size = 3.97

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 1/2*j/c^3*b/(c*x^2+b*x+a)^(1/2)+1/12*h*b/c^2/(c*x^2+b*x+a)^(3/2)-2/3*i*a/c^
2/(c*x^2+b*x+a)^(3/2)-i*x^2/c/(c*x^2+b*x+a)^(3/2)+1/24*i*b^2/c^3/(c*x^2+b*x
+a)^(3/2)-1/3*j*x^3/c/(c*x^2+b*x+a)^(3/2)-1/48*j*b^3/c^4/(c*x^2+b*x+a)^(3/2
)-j/c^2*x/(c*x^2+b*x+a)^(1/2)-8/3*g*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+2
/3*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b-1/2*h*x/c/(c*x^2+b*x+a)^(3/2)+1/6*h*
b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+1/3*h*a/c/(4*a*c-b^2)/(c*x^2+b*x+a)
^(3/2)*b+16/3*h*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-16/3*g*b*c/(4*a*c-b
^2)^2/(c*x^2+b*x+a)^(1/2)*x+j/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+1/1
2*i*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+2/3*i*b^3/c/(4*a*c-b^2)^2/(c*
x^2+b*x+a)^(1/2)*x-1/2*i*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-8*i*b*a/
(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-4*i*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)
^(1/2)-1/3*j*b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/4*j*b^3/c^3*a/(4
*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+2*j*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/
2)-1/24*j*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-1/4*i*b/c^2*x/(c*x^2+b*
x+a)^(3/2)+1/24*i*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+1/2*j*b^2/c^2*a/(
4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+4*j*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1
/2)*x-i*b/c*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-1/3*g/c/(c*x^2+b*x+a)^(3/2)
+j/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/3*i*b^4/c^2/(4*a*c
-b^2)^2/(c*x^2+b*x+a)^(1/2)+1/12*h*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+
4/3*h*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+2/3*h*b^3/c/(4*a*c-b^2)^2/(c*
x^2+b*x+a)^(1/2)+2/3*h*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+8/3*h*a/(4*a*c-b
^2)^2/(c*x^2+b*x+a)^(1/2)*b-2/3*g*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-1/3*g
*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+4/3*f/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2
)*x*c+32/3*f*c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+16/3*f*c/(4*a*c-b^2)^2
/(c*x^2+b*x+a)^(1/2)*b+1/2*j*b/c^2*x^2/(c*x^2+b*x+a)^(3/2)+1/8*j*b^2/c^3*x/
(c*x^2+b*x+a)^(3/2)-1/48*j*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-1/6*j*b^
5/c^3/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+1/3*j*b/c^3*a/(c*x^2+b*x+a)^(3/2)+1
/2*j/c^3*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 685 vs. 2(337) = 674.
time = 24.43, size = 1373, normalized size = 3.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/6*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 + 2*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3)*j*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*j*x^2 + 2*(a*b^5 -
 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*j)*
sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b
)*sqrt(c) - 4*a*c) + 4*(8*a^2*b*c^3*h - 16*a^3*c^3*i + (16*c^6*f - 8*b*c^5*
g + 2*(b^2*c^4 + 4*a*c^5)*h + (b^3*c^3 - 12*a*b*c^4)*i - 4*(b^4*c^2 - 7*a*b
^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f - 4*b^2*c^4*g + (b^3*c^3 + 4*a*b*
c^4)*h - 2*(a*b^2*c^3 + 4*a^2*c^4)*i - (b^5*c - 6*a*b^3*c^2)*j)*x^2 - (b^3*
c^3 - 12*a*b*c^4)*f - 2*(a*b^2*c^3 + 4*a^2*c^4)*g - (3*a^2*b^3*c - 20*a^3*b
*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i + 2*(b^2*c^4 + 4*a*c^5)*f - (b^3
*c^3 + 4*a*b*c^4)*g - 2*(a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*j)*x)*sqrt(c*
x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^3 +
(b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 1
6*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 + 2*(b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*j
*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2)*j)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt
(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(8*a^2*b*c^3*h - 16*a^3*c^3*i + (16*c^6*f
 - 8*b*c^5*g + 2*(b^2*c^4 + 4*a*c^5)*h + (b^3*c^3 - 12*a*b*c^4)*i - 4*(b^4*
c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f - 4*b^2*c^4*g + (b^3*c
```

```
^3 + 4*a*b*c^4)*h - 2*(a*b^2*c^3 + 4*a^2*c^4)*i - (b^5*c - 6*a*b^3*c^2)*j)*
x^2 - (b^3*c^3 - 12*a*b*c^4)*f - 2*(a*b^2*c^3 + 4*a^2*c^4)*g - (3*a^2*b^3*c
 - 20*a^3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i + 2*(b^2*c^4 + 4*a*c^
5)*f - (b^3*c^3 + 4*a*b*c^4)*g - 2*(a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*j)
*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4
*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*
c^6)*x^3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*
b^3*c^4 + 16*a^3*b*c^5)*x)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx + hx^2 + ix^3 + jx^4}{(a + bx + cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**4+i*x**3+h*x**2+g*x+f)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral((f + g*x + h*x**2 + i*x**3 + j*x**4)/(a + b*x + c*x**2)**(5/2), x)
```

Giac [A]

time = 4.55, size = 460, normalized size = 1.30

$$\frac{2\left(\left(\left(\frac{(16\,c^5f-8\,bc^4g+2\,b^2c^3h+8\,ac^4h-4\,b^4cj+28\,ab^2c^2j-32\,a^2c^3j+i\,b^3c^2-12i\,abc^3)x}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}+\frac{3\,(8\,bc^4f-4\,b^2c^3g+b^3c^2h+4\,abc^3h-b^5j+6\,ab^3cj-2i\,ab^2c^2-8i\,a^2c^3)}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}\right)x+\frac{3\,(2\,b^2c^3f+8\,ac^4f-b^3c^2g-4\,abc^3g+4\,ab^2c^2h-2\,ab^4j+14\,a^2b^2cj-8\,a^3c^2j-8i\,a^2bc^2)}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}\right)x-\frac{b^3c^2f-12\,abc^3f+2\,ab^2c^2g+8\,a^2c^3g-8\,a^2bc^2h+3\,a^2b^3j-20\,a^3bcj+16i\,a^3c^2}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}\right)}{3\,(cx^2+bx+a)^{\frac{3}{2}}}-\frac{j\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac"
)
```

```
[Out] 2/3*((((16*c^5*f - 8*b*c^4*g + 2*b^2*c^3*h + 8*a*c^4*h - 4*b^4*c*j + 28*a*b
^2*c^2*j - 32*a^2*c^3*j + I*b^3*c^2 - 12*I*a*b*c^3)*x/(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4) + 3*(8*b*c^4*f - 4*b^2*c^3*g + b^3*c^2*h + 4*a*b*c^3*h - b^
5*j + 6*a*b^3*c*j - 2*I*a*b^2*c^2 - 8*I*a^2*c^3)/(b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4))*x + 3*(2*b^2*c^3*f + 8*a*c^4*f - b^3*c^2*g - 4*a*b*c^3*g + 4*a*
b^2*c^2*h - 2*a*b^4*j + 14*a^2*b^2*c*j - 8*a^3*c^2*j - 8*I*a^2*b*c^2)/(b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x - (b^3*c^2*f - 12*a*b*c^3*f + 2*a*b^2*c^
2*g + 8*a^2*c^3*g - 8*a^2*b*c^2*h + 3*a^2*b^3*j - 20*a^3*b*c*j + 16*I*a^3*c
^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^(3/2) - j*log(a
bs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{j\,x^4+i\,x^3+h\,x^2+g\,x+f}{{\left(c\,x^2+b\,x+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2), x)
```

3.366 $\int \frac{f+gx+hx^2+ix^3+jx^4}{\left(a+bx-cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=353

$$\frac{2(ab^2ci + 2ac^2(cg+ai) + ab^3j - bc(c^2f - ach - 3a^2j) + (2c^4f + c^3(bg+2ah) + b^4j + b^2c(bi+4aj) + c^2(b^2}{3c^3\left(b^2+4ac\right)\left(a+bx-cx^2\right)^{3/2}}$$

```
[Out] 2/3*(a*b^2*c*i+2*a*c^2*(a*i+c*g)+a*b^3*j-b*c*(-3*a^2*j-a*c*h+c^2*f)+(2*c^4*
f+c^3*(2*a*h+b*g)+b^4*j+b^2*c*(4*a*j+b*i)+c^2*(2*a^2*j+3*a*b*i+b^2*h))*x)/c
^3/(4*a*c+b^2)/(-c*x^2+b*x+a)^(3/2)-j*arctan(1/2*(-2*c*x+b)/c^(1/2)/(-c*x^2
+b*x+a)^(1/2))/c^(5/2)-2/3*(b^4*c*i+24*a^2*c^3*i+2*b^2*c^2*(3*a*i+2*c*g)+b^
5*j+b^3*c*(10*a*j+c*h)+4*b*c^2*(8*a^2*j-a*c*h+2*c^2*f)-c*(16*c^4*f+8*c^3*(-
a*h+b*g)-4*b^4*j-b^2*c*(28*a*j+b*i)+2*c^2*(-16*a^2*j-6*a*b*i+b^2*h))*x)/c^3
/(4*a*c+b^2)^2/(-c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1674, 12, 635, 210}

$$\frac{2(x(c^2(2a^2j+3abi+b^2h)+b^2c(4aj+bi)+c^3(2ah+bg)+b^4j+2c^4f)-bc(-3a^2j-ach+c^2f)+ab^3j+ab^2ci+2ac^2(ai+cg))}{3c^3\left(4ac+b^2\right)\left(a+bx-cx^2\right)^{3/2}}-\frac{2(-cx(2c^2(-16a^2j-6abi+b^2h)-b^2c(28aj+bi)+8c^3(bg-ah)-4b^4j+16c^4f)+4bc^2(8a^2j-ach+2c^2f)+24a^2c^3i+b^3c(10aj+ch)+2b^2c^2(3ai+2cg)+b^5j+b^4ci)}{3c^3\left(4ac+b^2\right)^2\sqrt{a+bx-cx^2}}-\frac{j\,\text{ArcTan}\left(\frac{b-2cx}{2\sqrt{c}\sqrt{a+bx-cx^2}}\right)}{c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x]
```

```
[Out] (2*(a*b^2*c*i + 2*a*c^2*(c*g + a*i) + a*b^3*j - b*c*(c^2*f - a*c*h - 3*a^2*
j) + (2*c^4*f + c^3*(b*g + 2*a*h) + b^4*j + b^2*c*(b*i + 4*a*j) + c^2*(b^2*
h + 3*a*b*i + 2*a^2*j))*x))/(3*c^3*(b^2 + 4*a*c)*(a + b*x - c*x^2)^(3/2)) -
 (2*(b^4*c*i + 24*a^2*c^3*i + 2*b^2*c^2*(2*c*g + 3*a*i) + b^5*j + b^3*c*(c*
h + 10*a*j) + 4*b*c^2*(2*c^2*f - a*c*h + 8*a^2*j) - c*(16*c^4*f + 8*c^3*(b*
g - a*h) - 4*b^4*j - b^2*c*(b*i + 28*a*j) + 2*c^2*(b^2*h - 6*a*b*i - 16*a^2
*j))*x))/(3*c^3*(b^2 + 4*a*c)^2*Sqrt[a + b*x - c*x^2]) - (j*ArcTan[(b - 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x - c*x^2])])/c^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{f+gx+hx^2+366x^3+jx^4}{\left(a+bx-cx^2\right)^{5/2}}\,dx = -\frac{2\left(c^3\left(bf-\frac{3a^2(244c+bj)}{c^2}-\frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right)-\left(366b^3c+bc^2(109\cdots\right.\right.}{3c^3\left(b^2+4ac\right)(a+b\cdots}$$

$$= -\frac{2\left(c^3\left(bf-\frac{3a^2(244c+bj)}{c^2}-\frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right)-\left(366b^3c+bc^2(109\cdots\right.\right.}{3c^3\left(b^2+4ac\right)(a+b\cdots}$$

$$= -\frac{2\left(c^3\left(bf-\frac{3a^2(244c+bj)}{c^2}-\frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right)-\left(366b^3c+bc^2(109\cdots\right.\right.}{3c^3\left(b^2+4ac\right)(a+b\cdots}$$

$$= -\frac{2\left(c^3\left(bf-\frac{3a^2(244c+bj)}{c^2}-\frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right)-\left(366b^3c+bc^2(109\cdots\right.\right.}{3c^3\left(b^2+4ac\right)(a+b\cdots}$$

$$= -\frac{2\left(c^3\left(bf-\frac{3a^2(244c+bj)}{c^2}-\frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right)-\left(366b^3c+bc^2(109\cdots\right.\right.}{3c^3\left(b^2+4ac\right)(a+b\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.36, size = 319, normalized size = 0.90

$$-\frac{2(3b^5jx^2+b^4(6ajx-4cjx^3)+b^3(3a^2j+18acjx^2+c^2(f+3gx-x^2(3h+ix)))+8c^2(2c^3fx^3+a^3(2i+3jx)-ac^2x(3f+hx^2)-a^2c(g+x^2(3i+4jx)))+4bc(5a^3j+2c^3x^2(-3f+gx)-2a^2c(h-3ix)+3ac^2(f-x(g-hx+ix^2)))+2b^2c(21a^2jx+c^2x(3f+x(-6g+hx))+ac(g+x(-6h+3ix-14jx^2))))}{3c^2\left(b^2+4ac\right)^2(a+x(b-cx))^{3/2}}+\frac{ij\log\left(\frac{i(b-2cx)}{\sqrt{c}}+2\sqrt{a+x(b-cx)}\right)}{c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x]
```

```
[Out] (-2*(3*b^5*j*x^2 + b^4*(6*a*j*x - 4*c*j*x^3) + b^3*(3*a^2*j + 18*a*c*j*x^2
+ c^2*(f + 3*g*x - x^2*(3*h + i*x))) + 8*c^2*(2*c^3*f*x^3 + a^3*(2*i + 3*j*
x) - a*c^2*x*(3*f + h*x^2) - a^2*c*(g + x^2*(3*i + 4*j*x))) + 4*b*c*(5*a^3*
j + 2*c^3*x^2*(-3*f + g*x) - 2*a^2*c*(h - 3*i*x) + 3*a*c^2*(f - x*(g - h*x
+ i*x^2))) + 2*b^2*c*(21*a^2*j*x + c^2*x*(3*f + x*(-6*g + h*x)) + a*c*(g +
x*(-6*h + 3*i*x - 14*j*x^2)))))/(3*c^2*(b^2 + 4*a*c)^2*(a + x*(b - c*x))^(3
/2)) + (I*j*Log[(I*(b - 2*c*x))/Sqrt[c] + 2*Sqrt[a + x*(b - c*x)]])/c^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1146 vs. $2(337) = 674$.
time = 0.17, size = 1147, normalized size = 3.25

method	result	size
default	Expression too large to display	1147

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] j*(1/3*x^3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(x^2/c/(-c*x^2+b*x+a)^(3/2)-1/2*b
/c*(1/2*x/c/(-c*x^2+b*x+a)^(3/2)+1/4*b/c*(1/3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/
c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(
-2*c*x+b)/(-c*x^2+b*x+a)^(1/2)))-1/2*a/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x
^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2)))-2*a
/c*(1/3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2
+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2))))-1/c*
(x/c/(-c*x^2+b*x+a)^(1/2)+1/2*b/c*(1/c/(-c*x^2+b*x+a)^(1/2)+b/c*(-2*c*x+b)/
(-4*a*c-b^2)/(-c*x^2+b*x+a)^(1/2))-1/c^(3/2)*arctan(c^(1/2)*(x-1/2*b/c)/(-c
*x^2+b*x+a)^(1/2))))+i*(x^2/c/(-c*x^2+b*x+a)^(3/2)-1/2*b/c*(1/2*x/c/(-c*x^2
+b*x+a)^(3/2)+1/4*b/c*(1/3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(2/3*(-2*c*x+b)/(
-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b
*x+a)^(1/2)))-1/2*a/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/
3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2)))-2*a/c*(1/3/c/(-c*x^2+b
*x+a)^(3/2)+1/2*b/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*
c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2))))+h*(1/2*x/c/(-c*x^2+b*x+
a)^(3/2)+1/4*b/c*(1/3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(2/3*(-2*c*x+b)/(-4*a*
c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)
^(1/2)))-1/2*a/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(
-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2)))+g*(1/3/c/(-c*x^2+b*x+a)^(3/
2)+1/2*b/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c
-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2)))+f*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-
c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] 1/3*g*(16*b*c*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) - 8*b^2/(sqrt(-c*x
^2 + b*x + a)*(b^2 + 4*a*c)^2) + 2*b*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a
*c)) - b^2/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c) + 1/((-c*x^2 + b*x +
a)^(3/2)*c)) + 2/3*f*(16*c^2*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) - 8
*b*c/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) + 2*c*x/((-c*x^2 + b*x + a)^(
3/2)*(b^2 + 4*a*c)) - b/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c))) + 2/3*h*(
2*(b^2 - 4*a*c)*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) + 2*a*x/((-c*x^2
 + b*x + a)^(3/2)*(b^2 + 4*a*c)) + b^2*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4
*a*c)*c) - (b^2 - 4*a*c)*b/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2*c) + a*b
/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c)) + j*integrate(x^4/((c^2*x^4 -
2*b*c*x^3 + 2*a*b*x + (b^2 - 2*a*c)*x^2 + a^2)*sqrt(-c*x^2 + b*x + a)), x)
- 32/3*I*a*b*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) + 16/3*I*a*b^2/(sqr
t(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2*c) - 1/3*I*b^3*x/((-c*x^2 + b*x + a)^(3
/2)*(b^2 + 4*a*c)*c^2) - 2/3*I*(b^2 - 4*a*c)*b*x/(sqrt(-c*x^2 + b*x + a)*(b
^2 + 4*a*c)^2*c) - 2*I*a*b*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c) + I
*x^2/((-c*x^2 + b*x + a)^(3/2)*c) + 1/3*I*(b^2 - 4*a*c)*b^2/(sqrt(-c*x^2 +
b*x + a)*(b^2 + 4*a*c)^2*c^2) + 1/3*I*a*b^2/((-c*x^2 + b*x + a)^(3/2)*(b^2
+ 4*a*c)*c^2) - 2/3*I*a/((-c*x^2 + b*x + a)^(3/2)*c^2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(339) = 678$.
time = 24.68, size = 1385, normalized size = 3.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="fric
as")
```

```
[Out] [-1/6*(3*((b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 - 2*(b^5*c + 8*a*b^3*c
^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 + 6*a*b^4*c - 32*a^3*c^3)*j*x^2 + 2*(a*b^5
+ 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 + 8*a^3*b^2*c + 16*a^4*c^2)*j)
*sqrt(-c)*log(8*c^2*x^2 - 8*b*c*x + b^2 - 4*sqrt(-c*x^2 + b*x + a)*(2*c*x -
 b)*sqrt(-c) - 4*a*c) - 4*(8*a^2*b*c^3*h - 16*a^3*c^3*i - (16*c^6*f + 8*b*c
^5*g + 2*(b^2*c^4 - 4*a*c^5)*h - (b^3*c^3 + 12*a*b*c^4)*i - 4*(b^4*c^2 + 7*
a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f + 4*b^2*c^4*g + (b^3*c^3 - 4*a
*b*c^4)*h - 2*(a*b^2*c^3 - 4*a^2*c^4)*i - (b^5*c + 6*a*b^3*c^2)*j)*x^2 - (b
```

```
^3*c^3 + 12*a*b*c^4)*f - 2*(a*b^2*c^3 - 4*a^2*c^4)*g - (3*a^2*b^3*c + 20*a^
3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i - 2*(b^2*c^4 - 4*a*c^5)*f - (
b^3*c^3 - 4*a*b*c^4)*g - 2*(a*b^4*c + 7*a^2*b^2*c^2 + 4*a^3*c^3)*j)*x)*sqrt
(-c*x^2 + b*x + a))/(a^2*b^4*c^3 + 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 +
8*a*b^2*c^6 + 16*a^2*c^7)*x^4 - 2*(b^5*c^4 + 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^
3 + (b^6*c^3 + 6*a*b^4*c^4 - 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 + 8*a^2*b^3*c^4
 + 16*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 -
2*(b^5*c + 8*a*b^3*c^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 + 6*a*b^4*c - 32*a^3*c^
3)*j*x^2 + 2*(a*b^5 + 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 + 8*a^3*b^
2*c + 16*a^4*c^2)*j)*sqrt(c)*arctan(1/2*sqrt(-c*x^2 + b*x + a)*(2*c*x - b)*
sqrt(c)/(c^2*x^2 - b*c*x - a*c)) - 2*(8*a^2*b*c^3*h - 16*a^3*c^3*i - (16*c^
6*f + 8*b*c^5*g + 2*(b^2*c^4 - 4*a*c^5)*h - (b^3*c^3 + 12*a*b*c^4)*i - 4*(b
^4*c^2 + 7*a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f + 4*b^2*c^4*g + (b^
3*c^3 - 4*a*b*c^4)*h - 2*(a*b^2*c^3 - 4*a^2*c^4)*i - (b^5*c + 6*a*b^3*c^2)*
j)*x^2 - (b^3*c^3 + 12*a*b*c^4)*f - 2*(a*b^2*c^3 - 4*a^2*c^4)*g - (3*a^2*b^
3*c + 20*a^3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i - 2*(b^2*c^4 - 4*a
*c^5)*f - (b^3*c^3 - 4*a*b*c^4)*g - 2*(a*b^4*c + 7*a^2*b^2*c^2 + 4*a^3*c^3)
*j)*x)*sqrt(-c*x^2 + b*x + a))/(a^2*b^4*c^3 + 8*a^3*b^2*c^4 + 16*a^4*c^5 +
(b^4*c^5 + 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 - 2*(b^5*c^4 + 8*a*b^3*c^5 + 16*a^
2*b*c^6)*x^3 + (b^6*c^3 + 6*a*b^4*c^4 - 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 + 8*
a^2*b^3*c^4 + 16*a^3*b*c^5)*x)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx + hx^2 + ix^3 + jx^4}{(a + bx - cx^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**4+i*x**3+h*x**2+g*x+f)/(-c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Integral((f + g*x + h*x**2 + i*x**3 + j*x**4)/(a + b*x - c*x**2)**(5/2), x)
```

Giac [**A**]
time = 3.44, size = 482, normalized size = 1.37

$$-\frac{2\sqrt{-cx^2+bx+a}\left(\left(\left(\frac{(16c^5f+8bc^4g+2b^2c^3h-8ac^4h-4b^4cj-28ab^2c^2j-32a^2c^3j-ib^3c^2-12iabc^3)x}{b^4c^2+8ab^2c^3+16a^2c^4}-\frac{3(8bc^4f+4b^2c^3g+b^3c^2h-4abc^3h-b^5j-6ab^3cj-2iab^2c^2+8ia^2c^3)}{b^4c^2+8ab^2c^3+16a^2c^4}\right)x+\frac{3(2b^2c^3f-8ac^4f+b^3c^2g-4abc^3g-4ab^2c^2h+2ab^4j+14a^2b^2cj+8a^3c^2j+8ia^2bc^2)}{b^4c^2+8ab^2c^3+16a^2c^4}\right)x+\frac{b^3c^2f+12abc^3f+2ab^2c^2g-8a^2c^3g-8a^2bc^2h+3a^2b^3j+20a^3bcj+16ia^3c^2}{b^4c^2+8ab^2c^3+16a^2c^4}\right)}{3(cx^2-bx-a)^2}-\frac{j\log\left(\left|2\left(\sqrt{-c}x-\sqrt{-cx^2+bx+a}\right)\sqrt{-c}+b\right|\right)}{\sqrt{-c}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="giac
")
```

```
[Out] -2/3*sqrt(-c*x^2 + b*x + a)*((((16*c^5*f + 8*b*c^4*g + 2*b^2*c^3*h - 8*a*c^
4*h - 4*b^4*c*j - 28*a*b^2*c^2*j - 32*a^2*c^3*j - I*b^3*c^2 - 12*I*a*b*c^3)
*x/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4) - 3*(8*b*c^4*f + 4*b^2*c^3*g + b^3*
c^2*h - 4*a*b*c^3*h - b^5*j - 6*a*b^3*c*j - 2*I*a*b^2*c^2 + 8*I*a^2*c^3)/(b
```

```
^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))*x + 3*(2*b^2*c^3*f - 8*a*c^4*f + b^3*c^
2*g - 4*a*b*c^3*g - 4*a*b^2*c^2*h + 2*a*b^4*j + 14*a^2*b^2*c*j + 8*a^3*c^2*
j + 8*I*a^2*b*c^2)/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))*x + (b^3*c^2*f + 1
2*a*b*c^3*f + 2*a*b^2*c^2*g - 8*a^2*c^3*g - 8*a^2*b*c^2*h + 3*a^2*b^3*j + 2
0*a^3*b*c*j + 16*I*a^3*c^2)/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 -
b*x - a)^2 - j*log(abs(2*(sqrt(-c)*x - sqrt(-c*x^2 + b*x + a))*sqrt(-c) + b
))/(sqrt(-c)*c^2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{j\,x^4+i\,x^3+h\,x^2+g\,x+f}{{\left(-c\,x^2+b\,x+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x)
[Out] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2), x)
```

3.367 $\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right)$

Optimal. Leaf size=588

$$\frac{\left(5d^2-2de+3e^2\right)^3\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^{1+m}}{e^{11}(1+m)}-\frac{\left(5d^2-2de+3e^2\right)^2\left(200d^5+169d^4e+108\right.}{e^{11}(2}$$

[Out] (5*d^2-2*d*e+3*e^2)^3*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/e^11/(1+m)-(5*d^2-2*d*e+3*e^2)^2*(200*d^5+169*d^4*e+108*d^3*e^2-20*d^2*e^3+86*d*e^4-15*e^5)*(e*x+d)^(2+m)/e^11/(2+m)+3*(5*d^2-2*d*e+3*e^2)*(1500*d^6+660*d^5*e+792*d^4*e^2+58*d^3*e^3+547*d^2*e^4-156*d*e^5+53*e^6)*(e*x+d)^(3+m)/e^11/(3+m)-2*(30000*d^7+1050*d^6*e+21420*d^5*e^2+1715*d^4*e^3+9990*d^3*e^4-2550*d^2*e^5+2218*d*e^6-287*e^7)*(e*x+d)^(4+m)/e^11/(4+m)+(105000*d^6+3150*d^5*e+53550*d^4*e^2+3430*d^3*e^3+14985*d^2*e^4-2550*d*e^5+1109*e^6)*(e*x+d)^(5+m)/e^11/(5+m)-6*(21000*d^5+525*d^4*e+7140*d^3*e^2+343*d^2*e^3+999*d*e^4-85*e^5)*(e*x+d)^(6+m)/e^11/(6+m)+(105000*d^4+2100*d^3*e+21420*d^2*e^2+686*d*e^3+999*e^4)*(e*x+d)^(7+m)/e^11/(7+m)-2*(30000*d^3+450*d^2*e+3060*d*e^2+49*e^3)*(e*x+d)^(8+m)/e^11/(8+m)+45*(500*d^2+5*d*e+17*e^2)*(e*x+d)^(9+m)/e^11/(9+m)-25*(200*d+e)*(e*x+d)^(10+m)/e^11/(10+m)+500*(e*x+d)^(11+m)/e^11/(11+m)

Rubi [A]
time = 0.23, antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1642}

Antiderivative was successfully verified.

[In] Int[(d + e*x)^m*(3 + 2*x + 5*x^2)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]

[Out] ((5*d^2 - 2*d*e + 3*e^2)^3*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d + e*x)^(1 + m))/(e^11*(1 + m)) - ((5*d^2 - 2*d*e + 3*e^2)^2*(200*d^5 + 169*d^4*e + 108*d^3*e^2 - 20*d^2*e^3 + 86*d*e^4 - 15*e^5)*(d + e*x)^(2 + m))/(e^11*(2 + m)) + (3*(5*d^2 - 2*d*e + 3*e^2)*(1500*d^6 + 660*d^5*e + 792*d^4*e^2 + 58*d^3*e^3 + 547*d^2*e^4 - 156*d*e^5 + 53*e^6)*(d + e*x)^(3 + m))/(e^11*(3 + m)) - (2*(30000*d^7 + 1050*d^6*e + 21420*d^5*e^2 + 1715*d^4*e^3 + 9990*d^3*e^4 - 2550*d^2*e^5 + 2218*d*e^6 - 287*e^7)*(d + e*x)^(4 + m))/(e^11*(4 + m)) + ((105000*d^6 + 3150*d^5*e + 53550*d^4*e^2 + 3430*d^3*e^3 + 14985*d^2*e^4 - 2550*d*e^5 + 1109*e^6)*(d + e*x)^(5 + m))/(e^11*(5 + m)) - (6*(21000*d^5 + 525*d^4*e + 7140*d^3*e^2 + 343*d^2*e^3 + 999*d*e^4 - 85*e^5)*(d + e*x)^(6 + m))/(e^11*(6 + m)) + ((105000*d^4 + 2100*d^3*e + 21420*d^2*e^2 + 686*d*e^3 + 999*e^4)*(d + e*x)^(7 + m))/(e^11*(7 + m)) - (2*(30000*d^3 + 450*d^2*e + 3060*d*e^2 + 49*e^3)*(d + e*x)^(8 + m))/(e^11*(8 + m)) + (45*(500*d^2 + 5*d*e + 17*e^2)*(d + e*x)^(9 + m))/(e^11*(9 + m)) - (25*(200*d +

```
e)*(d + e*x)^(10 + m))/(e^11*(10 + m)) + (500*(d + e*x)^(11 + m))/(e^11*(11
 + m))
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right) dx = \int \left(\frac{\left(5d^2-2de+3e^2\right)^3 \left(4d^4+5d^3e+3d^2e^2\right.}{e^{10}}\right.$$

$$= \frac{\left(5d^2-2de+3e^2\right)^3 \left(4d^4+5d^3e+3d^2e^2-de\right.}{e^{11}(1+m)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2576 vs. $2(588) = 1176$.
time = 2.79, size = 2576, normalized size = 4.38

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] ((d + e*x)^(1 + m)*(1814400000*d^10 - 9072000*d^9*e*(-11 + 200*x + m*(-1 +
200*x)) + 1814400*d^8*e^2*(5*(374 - 11*x + 200*x^2) + 3*m*(119 - 20*x + 500
*x^2) + m^2*(17 - 5*x + 500*x^2)) - 10080*d^7*e^3*(90*(-539 + 3740*x - 110*
x^2 + 2000*x^3) + 30*m^2*(-49 + 2244*x - 210*x^2 + 6000*x^3) + m^3*(-49 + 3
060*x - 450*x^2 + 30000*x^3) + m*(-14651 + 400860*x - 15750*x^2 + 330000*x^
3)) + 720*d^6*e^4*(180*(43956 - 3773*x + 26180*x^2 - 770*x^3 + 14000*x^4) +
 49*m^2*(10989 - 4606*x + 76500*x^2 - 3300*x^3 + 75000*x^4) + m^4*(999 - 68
6*x + 21420*x^2 - 2100*x^3 + 105000*x^4) + 2*m^3*(18981 - 10633*x + 257040*
x^2 - 17850*x^3 + 525000*x^4) + m*(3378618 - 884254*x + 7968240*x^2 - 26670
0*x^3 + 5250000*x^4)) - 720*d^5*e^5*(180*(26180 + 43956*x - 3773*x^2 + 2618
0*x^3 - 770*x^4 + 14000*x^5) + m^5*(85 + 999*x - 343*x^2 + 7140*x^3 - 525*x
^4 + 21000*x^5) + 3*m^4*(1275 + 12987*x - 3773*x^2 + 64260*x^3 - 3675*x^4 +
 105000*x^5) + 3*m^2*(202725 + 1305693*x - 222607*x^2 + 2134860*x^3 - 76125
*x^4 + 1575000*x^5) + m^3*(68425 + 576423*x - 134113*x^2 + 1763580*x^3 - 76
125*x^4 + 1785000*x^5) + 2*m*(1342745 + 5645349*x - 611912*x^2 + 4769520*x^
3 - 150675*x^4 + 2877000*x^5)) + 24*d^4*e^6*(1080*(341572 + 130900*x + 2197
```

80*x^2 - 18865*x^3 + 130900*x^4 - 3850*x^5 + 70000*x^6) + m^6*(1109 + 2550*x + 14985*x^2 - 3430*x^3 + 53550*x^4 - 3150*x^5 + 105000*x^6) + 25*m^4*(47687 + 86700*x + 392607*x^2 - 67228*x^3 + 760410*x^4 - 31500*x^5 + 735000*x^6) + 3*m^5*(18853 + 39100*x + 204795*x^2 - 41160*x^3 + 553350*x^4 - 27300*x^5 + 735000*x^6) + 15*m^3*(886091 + 1353200*x + 5069925*x^2 - 713440*x^3 + 6729450*x^4 - 243600*x^5 + 5145000*x^6) + 12*m*(22642453 + 18494725*x + 38116845*x^2 - 3625510*x^3 + 26792850*x^4 - 822675*x^5 + 15435000*x^6) + 2*m^2*(41323558 + 49404975*x + 143436420*x^2 - 16136435*x^3 + 131840100*x^4 - 4329675*x^5 + 85260000*x^6)) - 12*d^3*e^7*(m^7*(287 + 2218*x + 2550*x^2 + 9990*x^3 - 1715*x^4 + 21420*x^5 - 1050*x^6 + 30000*x^7) + 2160*(220990 + 341572*x + 130900*x^2 + 219780*x^3 - 18865*x^4 + 130900*x^5 - 3850*x^6 + 70000*x^7) + 8*m^6*(2009 + 14417*x + 15300*x^2 + 54945*x^3 - 8575*x^4 + 96390*x^5 - 4200*x^6 + 105000*x^7) + 40*m^4*(124558 + 724177*x + 615825*x^2 + 1758240*x^3 - 217805*x^4 + 1959930*x^5 - 69825*x^6 + 1470000*x^7) + 2*m^5*(190855 + 1248734*x + 1201050*x^2 + 3886110*x^3 - 543655*x^4 + 5462100*x^5 - 213150*x^6 + 4830000*x^7) + 12*m*(37254035 + 106767866*x + 48770450*x^2 + 89420490*x^3 - 8099945*x^4 + 58298100*x^5 - 1760850*x^6 + 32670000*x^7) + 8*m^2*(22157261 + 88589138*x + 52444575*x^2 + 109835055*x^3 - 10787350*x^4 + 81995760*x^5 - 2576175*x^6 + 49245000*x^7) + m^3*(38586863 + 191876962*x + 139405950*x^2 + 343346310*x^3 - 37539635*x^4 + 307355580*x^5 - 10194450*x^6 + 203070000*x^7)) + 6*d^2*e^8*(m^8*(159 + 574*x + 2218*x^2 + 1700*x^3 + 4995*x^4 - 686*x^5 + 7140*x^6 - 300*x^7 + 7500*x^8) + 6*m^7*(1590 + 5453*x + 19962*x^2 + 14450*x^3 + 39960*x^4 - 5145*x^5 + 49980*x^6 - 1950*x^7 + 45000*x^8) + 4320*(244860 + 220990*x + 341572*x^2 + 130900*x^3 + 219780*x^4 - 18865*x^5 + 130900*x^6 - 3850*x^7 + 70000*x^8) + 6*m^6*(41181 + 132594*x + 454690*x^2 + 307700*x^3 + 794205*x^4 - 95354*x^5 + 863940*x^6 - 31500*x^7 + 682500*x^8) + 12*m^5*(300510 + 894005*x + 2830168*x^2 + 1768850*x^3 + 4225770*x^4 - 471625*x^5 + 3998400*x^6 - 137550*x^7 + 2835000*x^8) + 24*m*(52296690 + 77032235*x + 137509346*x^2 + 56624450*x^3 + 99310590*x^4 - 8779085*x^5 + 62225100*x^6 - 1859850*x^7 + 34245000*x^8) + 3*m^4*(10806117 + 29046122*x + 83270374*x^2 + 47401100*x^3 + 104110785*x^4 - 10813418*x^5 + 86415420*x^6 - 2832900*x^7 + 56122500*x^8) + 6*m^3*(30618630 + 71948317*x + 182077838*x^2 + 93086050*x^3 + 187672140*x^4 - 18266465*x^5 + 138894420*x^6 - 4379550*x^7 + 84105000*x^8) + 4*m^2*(160119201 + 312153254*x + 674660150*x^2 + 307319200*x^3 + 573470955*x^4 - 52869334*x^5 + 386281140*x^6 - 11814000*x^7 + 221482500*x^8)) - d*e^9*(m^9*(3 + 2*x + 5*x^2)^2*(15 + 86*x + 20*x^2 + 108*x^3 - 169*x^4 + 200*x^5) + 25920*(103950 + 244860*x + 220990*x^2 + 341572*x^3 + 130900*x^4 + 219780*x^5 - 18865*x^6 + 130900*x^7 - 3850*x^8 + 70000*x^9) + 3*m^8*(2835 + 19398*x + 33866*x^2 + 84284*x^3 + 46750*x^4 + 105894*x^5 - 11662*x^6 + 99960*x^7 - 3525*x^8 + 75000*x^9) + 6*m^7*(39015 + 256626*x + 430500*x^2 + 1029152*x^3 + 548250*x^4 + 1192806*x^5 - 126224*x^6 + 1040400*x^7 - 35325*x^8 + 725000*x^9) + 6*m^6*(615195 + 3853206*x + 6159594*x^2 + 14048812*x^3 + 7152750*x^4 + 14907078*x^5 - 1515374*x^6 + 12038040*x^7 - 395325*x^8 + 7875000*x^9) + 144*m*(28438425 + 96371490*x + 96921335*x^2 + 158003666*x^3 + 62514950*x^4 + 107222670*x^5 - 9345035*x^6 + 65591100*x^7 - 1946475

```
*x^8 + 35645000*x^9) + 3*m^5*(12236805 + 72048942*x + 108594486*x^2 + 23446
4780*x^3 + 113554050*x^4 + 226351422*x^5 - 22132418*x^6 + 170031960*x^7 - 5
425875*x^8 + 105455000*x^9) + 3*m^4*(79518915 + 432260262*x + 605966634*x^2
 + 1227933596*x^3 + 563664750*x^4 + 1075077846*x^5 - 101413438*x^6 + 756597
240*x^7 - 23566725*x^8 + 448875000*x^9) + 12*m^2*(224755965 + 947798682*x +
 1086499918*x^2 + 1899357684*x^3 + 784511750*x^...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5923 vs. $2(588) = 1176$.

time = 0.21, size = 5924, normalized size = 10.07

method	result	size
gosper	Expression too large to display	5924
risch	Expression too large to display	6934

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBO
SE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2267 vs. $2(565) = 1130$.

time = 0.33, size = 2267, normalized size = 3.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 54*(x*e + d)^(m + 1)*e^(-1)/(m + 1) + 135*((m + 1)*x^2*e^2 + d*m*x*e - d^2)
*e^(m*log(x*e + d) - 2)/(m^2 + 3*m + 2) + 477*((m^2 + 3*m + 2)*x^3*e^3 + (m
^2 + m)*d*x^2*e^2 - 2*d^2*m*x*e + 2*d^3)*e^(m*log(x*e + d) - 3)/(m^3 + 6*m^
2 + 11*m + 6) + 574*((m^3 + 6*m^2 + 11*m + 6)*x^4*e^4 + (m^3 + 3*m^2 + 2*m)
*d*x^3*e^3 - 3*(m^2 + m)*d^2*x^2*e^2 + 6*d^3*m*x*e - 6*d^4)*e^(m*log(x*e +
d) - 4)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24) + 1109*((m^4 + 10*m^3 + 35*m^2
+ 50*m + 24)*x^5*e^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*x^4*e^4 - 4*(m^3 + 3*
m^2 + 2*m)*d^2*x^3*e^3 + 12*(m^2 + m)*d^3*x^2*e^2 - 24*d^4*m*x*e + 24*d^5)*
e^(m*log(x*e + d) - 5)/(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120) + 51
0*((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*x^6*e^6 + (m^5 + 10*m^4
+ 35*m^3 + 50*m^2 + 24*m)*d*x^5*e^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*x^
4*e^4 + 20*(m^3 + 3*m^2 + 2*m)*d^3*x^3*e^3 - 60*(m^2 + m)*d^4*x^2*e^2 + 120
*d^5*m*x*e - 120*d^6)*e^(m*log(x*e + d) - 6)/(m^6 + 21*m^5 + 175*m^4 + 735*
m^3 + 1624*m^2 + 1764*m + 720) + 999*((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1
```

624*m^2 + 1764*m + 720)*x^7*e^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^
2 + 120*m)*d*x^6*e^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*x^5*e^
5 + 30*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^3*x^4*e^4 - 120*(m^3 + 3*m^2 + 2*m)*d
^4*x^3*e^3 + 360*(m^2 + m)*d^5*x^2*e^2 - 720*d^6*m*x*e + 720*d^7)*e^(m*log(
x*e + d) - 7)/(m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 1
3068*m + 5040) - 98*((m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*
m^2 + 13068*m + 5040)*x^8*e^8 + (m^7 + 21*m^6 + 175*m^5 + 735*m^4 + 1624*m^
3 + 1764*m^2 + 720*m)*d*x^7*e^7 - 7*(m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*
m^2 + 120*m)*d^2*x^6*e^6 + 42*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^3*x
^5*e^5 - 210*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^4*x^4*e^4 + 840*(m^3 + 3*m^2 +
2*m)*d^5*x^3*e^3 - 2520*(m^2 + m)*d^6*x^2*e^2 + 5040*d^7*m*x*e - 5040*d^8)*
e^(m*log(x*e + d) - 8)/(m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 672
84*m^3 + 118124*m^2 + 109584*m + 40320) + 765*((m^8 + 36*m^7 + 546*m^6 + 45
36*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^2 + 109584*m + 40320)*x^9*e^9 + (
m^8 + 28*m^7 + 322*m^6 + 1960*m^5 + 6769*m^4 + 13132*m^3 + 13068*m^2 + 5040
*m)*d*x^8*e^8 - 8*(m^7 + 21*m^6 + 175*m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 +
720*m)*d^2*x^7*e^7 + 56*(m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m
)*d^3*x^6*e^6 - 336*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^4*x^5*e^5 + 1
680*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^5*x^4*e^4 - 6720*(m^3 + 3*m^2 + 2*m)*d^6
*x^3*e^3 + 20160*(m^2 + m)*d^7*x^2*e^2 - 40320*d^8*m*x*e + 40320*d^9)*e^(m*
log(x*e + d) - 9)/(m^9 + 45*m^8 + 870*m^7 + 9450*m^6 + 63273*m^5 + 269325*m
^4 + 723680*m^3 + 1172700*m^2 + 1026576*m + 362880) - 25*((m^9 + 45*m^8 + 8
70*m^7 + 9450*m^6 + 63273*m^5 + 269325*m^4 + 723680*m^3 + 1172700*m^2 + 102
6576*m + 362880)*x^10*e^10 + (m^9 + 36*m^8 + 546*m^7 + 4536*m^6 + 22449*m^5
+ 67284*m^4 + 118124*m^3 + 109584*m^2 + 40320*m)*d*x^9*e^9 - 9*(m^8 + 28*m
^7 + 322*m^6 + 1960*m^5 + 6769*m^4 + 13132*m^3 + 13068*m^2 + 5040*m)*d^2*x^
8*e^8 + 72*(m^7 + 21*m^6 + 175*m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)
*d^3*x^7*e^7 - 504*(m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^4*
x^6*e^6 + 3024*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^5*x^5*e^5 - 15120*
(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^6*x^4*e^4 + 60480*(m^3 + 3*m^2 + 2*m)*d^7*x^
3*e^3 - 181440*(m^2 + m)*d^8*x^2*e^2 + 362880*d^9*m*x*e - 362880*d^10)*e^(m
*log(x*e + d) - 10)/(m^10 + 55*m^9 + 1320*m^8 + 18150*m^7 + 157773*m^6 + 90
2055*m^5 + 3416930*m^4 + 8409500*m^3 + 12753576*m^2 + 10628640*m + 3628800)
+ 500*((m^10 + 55*m^9 + 1320*m^8 + 18150*m^7 + 157773*m^6 + 902055*m^5 + 3
416930*m^4 + 8409500*m^3 + 12753576*m^2 + 10628640*m + 3628800)*x^11*e^11 +
(m^10 + 45*m^9 + 870*m^8 + 9450*m^7 + 63273*m^6 + 269325*m^5 + 723680*m^4
+ 1172700*m^3 + 1026576*m^2 + 362880*m)*d*x^10*e^10 - 10*(m^9 + 36*m^8 + 54
6*m^7 + 4536*m^6 + 22449*m^5 + 67284*m^4 + 118124*m^3 + 109584*m^2 + 40320*
m)*d^2*x^9*e^9 + 90*(m^8 + 28*m^7 + 322*m^6 + 1960*m^5 + 6769*m^4 + 13132*m
^3 + 13068*m^2 + 5040*m)*d^3*x^8*e^8 - 720*(m^7 + 21*m^6 + 175*m^5 + 735*m^
4 + 1624*m^3 + 1764*m^2 + 720*m)*d^4*x^7*e^7 + 5040*(m^6 + 15*m^5 + 85*m^4
+ 225*m^3 + 274*m^2 + 120*m)*d^5*x^6*e^6 - 30240*(m^5 + 10*m^4 + 35*m^3 + 5
0*m^2 + 24*m)*d^6*x^5*e^5 + 151200*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^7*x^4*e^4
- 604800*(m^3 + 3*m^2 + 2*m)*d^8*x^3*e^3 + 1814400*(m^2 + m)*d^9*x^2*e^2 -
3628800*d^10*m*x*e + 3628800*d^11)*e^(m*log(x*e + d) - 11)/(m^11 + 66*m^10

```
 + 1925*m^9 + 32670*m^8 + 357423*m^7 + 2637558*m^6 + 13339535*m^5 + 4599573
0*m^4 + 105258076*m^3 + 150917976*m^2 + 120543840*m + 39916800)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4176 vs. $2(565) = 1130$.
time = 0.50, size = 4176, normalized size = 7.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] (1814400000*d^11 + (500*(m^10 + 55*m^9 + 1320*m^8 + 18150*m^7 + 157773*m^6
+ 902055*m^5 + 3416930*m^4 + 8409500*m^3 + 12753576*m^2 + 10628640*m + 3628
800)*x^11 - 25*(m^10 + 56*m^9 + 1365*m^8 + 19020*m^7 + 167223*m^6 + 965328*
m^5 + 3686255*m^4 + 9133180*m^3 + 13926276*m^2 + 11655216*m + 3991680)*x^10
 + 765*(m^10 + 57*m^9 + 1412*m^8 + 19962*m^7 + 177765*m^6 + 1037673*m^5 + 4
000478*m^4 + 9991428*m^3 + 15335224*m^2 + 12900960*m + 4435200)*x^9 - 98*(m
^10 + 58*m^9 + 1461*m^8 + 20982*m^7 + 189567*m^6 + 1121022*m^5 + 4371359*m^
4 + 11024858*m^3 + 17059212*m^2 + 14444280*m + 4989600)*x^8 + 999*(m^10 + 5
9*m^9 + 1512*m^8 + 22086*m^7 + 202821*m^6 + 1217811*m^5 + 4814858*m^4 + 122
91724*m^3 + 19216008*m^2 + 16405920*m + 5702400)*x^7 + 510*(m^10 + 60*m^9 +
 1565*m^8 + 23280*m^7 + 217743*m^6 + 1331100*m^5 + 5352935*m^4 + 13878120*m
^3 + 21989356*m^2 + 18981840*m + 6652800)*x^6 + 1109*(m^10 + 61*m^9 + 1620*
m^8 + 24570*m^7 + 234573*m^6 + 1464693*m^5 + 6016070*m^4 + 15915380*m^3 + 2
5681176*m^2 + 22512096*m + 7983360)*x^5 + 574*(m^10 + 62*m^9 + 1677*m^8 + 2
5962*m^7 + 253575*m^6 + 1623258*m^5 + 6846503*m^4 + 18609718*m^3 + 30819204
*m^2 + 27641160*m + 9979200)*x^4 + 477*(m^10 + 63*m^9 + 1736*m^8 + 27462*m^
7 + 275037*m^6 + 1812447*m^5 + 7902194*m^4 + 22289148*m^3 + 38390632*m^2 +
35746080*m + 13305600)*x^3 + 135*(m^10 + 64*m^9 + 1797*m^8 + 29076*m^7 + 29
9271*m^6 + 2039016*m^5 + 9261503*m^4 + 27472724*m^3 + 50312628*m^2 + 502927
20*m + 19958400)*x^2 + 54*(m^10 + 65*m^9 + 1860*m^8 + 30810*m^7 + 326613*m^
6 + 2310945*m^5 + 11028590*m^4 + 34967140*m^3 + 70290936*m^2 + 80627040*m +
 39916800)*x)*e^11 + (54*d*m^10 + 500*(d*m^10 + 45*d*m^9 + 870*d*m^8 + 9450
*d*m^7 + 63273*d*m^6 + 269325*d*m^5 + 723680*d*m^4 + 1172700*d*m^3 + 102657
6*d*m^2 + 362880*d*m)*x^10 + 3510*d*m^9 - 25*(d*m^10 + 47*d*m^9 + 942*d*m^8
 + 10542*d*m^7 + 72345*d*m^6 + 314223*d*m^5 + 858248*d*m^4 + 1408948*d*m^3
+ 1245744*d*m^2 + 443520*d*m)*x^9 + 100440*d*m^8 + 765*(d*m^10 + 49*d*m^9 +
 1020*d*m^8 + 11802*d*m^7 + 83349*d*m^6 + 370881*d*m^5 + 1033430*d*m^4 + 17
23988*d*m^3 + 1543320*d*m^2 + 554400*d*m)*x^8 + 1663740*d*m^7 - 98*(d*m^10
+ 51*d*m^9 + 1104*d*m^8 + 13254*d*m^7 + 96789*d*m^6 + 443499*d*m^5 + 126686
6*d*m^4 + 2156796*d*m^3 + 1961640*d*m^2 + 712800*d*m)*x^7 + 17637102*d*m^6
+ 999*(d*m^10 + 53*d*m^9 + 1194*d*m^8 + 14922*d*m^7 + 113289*d*m^6 + 538077
*d*m^5 + 1586396*d*m^4 + 2773348*d*m^3 + 2575920*d*m^2 + 950400*d*m)*x^6 +
```

```
124791030*d*m^5 + 510*(d*m^10 + 55*d*m^9 + 1290*d*m^8 + 16830*d*m^7 + 13359
3*d*m^6 + 663135*d*m^5 + 2037260*d*m^4 + 3691820*d*m^3 + 3530256*d*m^2 + 13
30560*d*m)*x^5 + 595543860*d*m^4 + 1109*(d*m^10 + 57*d*m^9 + 1392*d*m^8 + 1
9002*d*m^7 + 158565*d*m^6 + 830433*d*m^5 + 2694338*d*m^4 + 5138028*d*m^3 +
5129064*d*m^2 + 1995840*d*m)*x^4 + 1888225560*d*m^3 + 574*(d*m^10 + 59*d*m^
9 + 1500*d*m^8 + 21462*d*m^7 + 189189*d*m^6 + 1055691*d*m^5 + 3679430*d*m^4
 + 7571428*d*m^3 + 8104920*d*m^2 + 3326400*d*m)*x^3 + 3795710544*d*m^2 + 47
7*(d*m^10 + 61*d*m^9 + 1614*d*m^8 + 24234*d*m^7 + 226569*d*m^6 + 1359309*d*
m^5 + 5183576*d*m^4 + 11921996*d*m^3 + 14546640*d*m^2 + 6652800*d*m)*x^2 +
4353860160*d*m + 135*(d*m^10 + 63*d*m^9 + 1734*d*m^8 + 27342*d*m^7 + 271929
*d*m^6 + 1767087*d*m^5 + 7494416*d*m^4 + 19978308*d*m^3 + 30334320*d*m^2 +
19958400*d*m)*x + 2155507200*d)*e^10 - (135*d^2*m^9 + 8505*d^2*m^8 + 5000*(
d^2*m^9 + 36*d^2*m^8 + 546*d^2*m^7 + 4536*d^2*m^6 + 22449*d^2*m^5 + 67284*d
^2*m^4 + 118124*d^2*m^3 + 109584*d^2*m^2 + 40320*d^2*m)*x^9 + 234090*d^2*m^
7 - 225*(d^2*m^9 + 39*d^2*m^8 + 630*d^2*m^7 + 5502*d^2*m^6 + 28329*d^2*m^5
+ 87591*d^2*m^4 + 157520*d^2*m^3 + 148788*d^2*m^2 + 55440*d^2*m)*x^8 + 3691
170*d^2*m^6 + 6120*(d^2*m^9 + 42*d^2*m^8 + 726*d^2*m^7 + 6720*d^2*m^6 + 363
09*d^2*m^5 + 116718*d^2*m^4 + 216404*d^2*m^3 + 209160*d^2*m^2 + 79200*d^2*m
)*x^7 + 36710415*d^2*m^5 - 686*(d^2*m^9 + 45*d^2*m^8 + 834*d^2*m^7 + 8250*d
^2*m^6 + 47289*d^2*m^5 + 159765*d^2*m^4 + 308276*d^2*m^3 + 307140*d^2*m^2 +
 118800*d^2*m)*x^6 + 238556745*d^2*m^4 + 5994*(d^2*m^9 + 48*d^2*m^8 + 954*d
^2*m^7 + 10152*d^2*m^6 + 62529*d^2*m^5 + 225432*d^2*m^4 + 459236*d^2*m^3 +
477168*d^2*m^2 + 190080*d^2*m)*x^5 + 1011746160*d^2*m^3 + 2550*(d^2*m^9 + 5
1*d^2*m^8 + 1086*d^2*m^7 + 12486*d^2*m^6 + 83649*d^2*m^5 + 328539*d^2*m^4 +
 723104*d^2*m^3 + 799404*d^2*m^2 + 332640*d^2*m)*x^4 + 2697071580*d^2*m^2 +
 4436*(d^2*m^9 + 54*d^2*m^8 + 1230*d^2*m^7 + 15312*d^2*m^6 + 112629*d^2*m^5
 + 492546*d^2*m^4 + 1216700*d^2*m^3 + 1487928*d^2*m^2 + 665280*d^2*m)*x^3 +
 4095133200*d^2*m + 1722*(d^2*m^9 + 57*d^2*m^8 + 1386*d^2*m^7 + 18690*d^2*m
^6 + 151809*d^2*m^5 + 752073*d^2*m^4 + 2175284*d^2*m^3 + 3220860*d^2*m^2 +
1663200*d^2*m)*x^2 + 2694384000*d^2 + 954*(d^2*m^9 + 60*d^2*m^8 + 1554*d^2*
m^7 + 22680*d^2*m^6 + 203889*d^2*m^5 + 1155420*d^2*m^4 + 4028156*d^2*m^3 +
7893840*d^2*m^2 + 6652800*d^2*m)*x)*e^9 + 6*(159*d^3*m^8 + 9540*d^3*m^7 + 2
47086*d^3*m^6 + 7500*(d^3*m^8 + 28*d^3*m^7 + 322*d^3*m^6 + 1960*d^3*m^5 + 6
769*d^3*m^4 + 13132*d^3*m^3 + 13068*d^3*m^2 + 5040*d^3*m)*x^8 + 3606120*d^3
*m^5 - 300*(d^3*m^8 + 32*d^3*m^7 + 406*d^3*m^6 ...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 136733 vs. $2(564) = 1128$.

time = 41.41, size = 136733, normalized size = 232.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(5*x**2+2*x+3)**3*(4*x**4-5*x**3+3*x**2+x+2),x)
```

[Out] Piecewise((d**m*(500*x**11/11 - 5*x**10/2 + 85*x**9 - 49*x**8/4 + 999*x**7/
7 + 85*x**6 + 1109*x**5/5 + 287*x**4/2 + 159*x**3 + 135*x**2/2 + 54*x), Eq(
e, 0)), (1260000*d**10*log(d/e + x)/(2520*d**10*e**11 + 25200*d**9*e**12*x
+ 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4
+ 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7
+ 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 3690500
*d**10/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 30
2400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 52
9200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25
200*d*e**20*x**9 + 2520*e**21*x**10) + 12600000*d**9*e*x*log(d/e + x)/(2520
*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e*
*14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e*
*17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*
x**9 + 2520*e**21*x**10) + 35645000*d**9*e*x/(2520*d**10*e**11 + 25200*d**9
*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e*
*15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e*
*18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10)
+ 6300*d**9*e/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x*
*2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x*
*5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x*
*8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 56700000*d**8*e**2*x**2*log(d
/e + x)/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 3
02400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 5
29200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 2
5200*d*e**20*x**9 + 2520*e**21*x**10) + 154102500*d**8*e**2*x**2/(2520*d**1
0*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x
3 + 529200*d6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x
6 + 302400*d3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9
+ 2520*e**21*x**10) + 63000*d**8*e**2*x/(2520*d**10*e**11 + 25200*d**9*e**1
2*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x
4 + 635040*d5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x
7 + 113400*d2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 214
20*d**8*e**2/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**
2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**
5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**
8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 151200000*d**7*e**3*x**3*log(d
/e + x)/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 3
02400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 5
29200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 2
5200*d*e**20*x**9 + 2520*e**21*x**10) + 392040000*d**7*e**3*x**3/(2520*d**1
0*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x
3 + 529200*d6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x
6 + 302400*d3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9
+ 2520*e**21*x**10) + 283500*d**7*e**3*x**2/(2520*d**10*e**11 + 25200*d**9*
e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**

```
15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**
18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) -
 214200*d**7*e**3*x/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e*
*13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e*
*16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e*
*19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 686*d**7*e**3/(2520*d**
10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*
x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*
x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9
 + 2520*e**21*x**10) + 264600000*d**6*e**4*x**4*log(d/e + x)/(2520*d**10*e*
*11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3
+ 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6
+ 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 25
20*e**21*x**10) + 648270000*d**6*e**4*x**4/(2520*d**10*e**11 + 25200*d**9*e
**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**1
5*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**1
8*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) +
756000*d**6*e**4*x**3/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*
e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*
e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*
e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) - 963900*d**6*e**4*x**2
/(2520*d**10*e**11 + 25200*d**9*e**12*x + 11340...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 10960 vs. $2(565) = 1130$.
time = 4.02, size = 10960, normalized size = 18.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] (500*(x*e + d)^m*m^10*x^11*e^11 + 500*(x*e + d)^m*d*m^10*x^10*e^10 - 25*(x*
e + d)^m*m^10*x^10*e^11 + 27500*(x*e + d)^m*m^9*x^11*e^11 - 25*(x*e + d)^m*
d*m^10*x^9*e^10 + 22500*(x*e + d)^m*d*m^9*x^10*e^10 - 5000*(x*e + d)^m*d^2*
m^9*x^9*e^9 + 765*(x*e + d)^m*m^10*x^9*e^11 - 1400*(x*e + d)^m*m^9*x^10*e^1
1 + 660000*(x*e + d)^m*m^8*x^11*e^11 + 765*(x*e + d)^m*d*m^10*x^8*e^10 - 11
75*(x*e + d)^m*d*m^9*x^9*e^10 + 435000*(x*e + d)^m*d*m^8*x^10*e^10 + 225*(x
*e + d)^m*d^2*m^9*x^8*e^9 - 180000*(x*e + d)^m*d^2*m^8*x^9*e^9 + 45000*(x*e
 + d)^m*d^3*m^8*x^8*e^8 - 98*(x*e + d)^m*m^10*x^8*e^11 + 43605*(x*e + d)^m*
m^9*x^9*e^11 - 34125*(x*e + d)^m*m^8*x^10*e^11 + 9075000*(x*e + d)^m*m^7*x^
11*e^11 - 98*(x*e + d)^m*d*m^10*x^7*e^10 + 37485*(x*e + d)^m*d*m^9*x^8*e^10
 - 23550*(x*e + d)^m*d*m^8*x^9*e^10 + 4725000*(x*e + d)^m*d*m^7*x^10*e^10 -
 6120*(x*e + d)^m*d^2*m^9*x^7*e^9 + 8775*(x*e + d)^m*d^2*m^8*x^8*e^9 - 2730
```

000*(x*e + d)^m*d^2*m^7*x^9*e^9 - 1800*(x*e + d)^m*d^3*m^8*x^7*e^8 + 126000 0*(x*e + d)^m*d^3*m^7*x^8*e^8 - 360000*(x*e + d)^m*d^4*m^7*x^7*e^7 + 999*(x *e + d)^m*m^10*x^7*e^11 - 5684*(x*e + d)^m*m^9*x^8*e^11 + 1080180*(x*e + d) ^m*m^8*x^9*e^11 - 475500*(x*e + d)^m*m^7*x^10*e^11 + 78886500*(x*e + d)^m*m ^6*x^11*e^11 + 999*(x*e + d)^m*d*m^10*x^6*e^10 - 4998*(x*e + d)^m*d*m^9*x^7 *e^10 + 780300*(x*e + d)^m*d*m^8*x^8*e^10 - 263550*(x*e + d)^m*d*m^7*x^9*e^ 10 + 31636500*(x*e + d)^m*d*m^6*x^10*e^10 + 686*(x*e + d)^m*d^2*m^9*x^6*e^9 - 257040*(x*e + d)^m*d^2*m^8*x^7*e^9 + 141750*(x*e + d)^m*d^2*m^7*x^8*e^9 - 22680000*(x*e + d)^m*d^2*m^6*x^9*e^9 + 42840*(x*e + d)^m*d^3*m^8*x^6*e^8 - 57600*(x*e + d)^m*d^3*m^7*x^7*e^8 + 14490000*(x*e + d)^m*d^3*m^6*x^8*e^8 + 12600*(x*e + d)^m*d^4*m^7*x^6*e^7 - 7560000*(x*e + d)^m*d^4*m^6*x^7*e^7 + 2520000*(x*e + d)^m*d^5*m^6*x^6*e^6 + 510*(x*e + d)^m*m^10*x^6*e^11 + 5894 1*(x*e + d)^m*m^9*x^7*e^11 - 143178*(x*e + d)^m*m^8*x^8*e^11 + 15270930*(x* e + d)^m*m^7*x^9*e^11 - 4180575*(x*e + d)^m*m^6*x^10*e^11 + 451027500*(x*e + d)^m*m^5*x^11*e^11 + 510*(x*e + d)^m*d*m^10*x^5*e^10 + 52947*(x*e + d)^m* d*m^9*x^6*e^10 - 108192*(x*e + d)^m*d*m^8*x^7*e^10 + 9028530*(x*e + d)^m*d* m^7*x^8*e^10 - 1808625*(x*e + d)^m*d*m^6*x^9*e^10 + 134662500*(x*e + d)^m*d *m^5*x^10*e^10 - 5994*(x*e + d)^m*d^2*m^9*x^5*e^9 + 30870*(x*e + d)^m*d^2*m ^8*x^6*e^9 - 4443120*(x*e + d)^m*d^2*m^7*x^7*e^9 + 1237950*(x*e + d)^m*d^2* m^6*x^8*e^9 - 112245000*(x*e + d)^m*d^2*m^5*x^9*e^9 - 4116*(x*e + d)^m*d^3* m^8*x^5*e^8 + 1542240*(x*e + d)^m*d^3*m^7*x^6*e^8 - 730800*(x*e + d)^m*d^3* m^6*x^7*e^8 + 88200000*(x*e + d)^m*d^3*m^5*x^8*e^8 - 257040*(x*e + d)^m*d^4 *m^7*x^5*e^7 + 327600*(x*e + d)^m*d^4*m^6*x^6*e^7 - 63000000*(x*e + d)^m*d^ 4*m^5*x^7*e^7 - 75600*(x*e + d)^m*d^5*m^6*x^5*e^6 + 37800000*(x*e + d)^m*d^ 5*m^5*x^6*e^6 - 15120000*(x*e + d)^m*d^6*m^5*x^5*e^5 + 1109*(x*e + d)^m*m^1 0*x^5*e^11 + 30600*(x*e + d)^m*m^9*x^6*e^11 + 1510488*(x*e + d)^m*m^8*x^7*e ^11 - 2056236*(x*e + d)^m*m^7*x^8*e^11 + 135990225*(x*e + d)^m*m^6*x^9*e^11 - 24133200*(x*e + d)^m*m^5*x^10*e^11 + 1708465000*(x*e + d)^m*m^4*x^11*e^1 1 + 1109*(x*e + d)^m*d*m^10*x^4*e^10 + 28050*(x*e + d)^m*d*m^9*x^5*e^10 + 1 192806*(x*e + d)^m*d*m^8*x^6*e^10 - 1298892*(x*e + d)^m*d*m^7*x^7*e^10 + 63 761985*(x*e + d)^m*d*m^6*x^8*e^10 - 7855575*(x*e + d)^m*d*m^5*x^9*e^10 + 36 1840000*(x*e + d)^m*d*m^4*x^10*e^10 - 2550*(x*e + d)^m*d^2*m^9*x^4*e^9 - 28 7712*(x*e + d)^m*d^2*m^8*x^5*e^9 + 572124*(x*e + d)^m*d^2*m^7*x^6*e^9 - 411 26400*(x*e + d)^m*d^2*m^6*x^7*e^9 + 6374025*(x*e + d)^m*d^2*m^5*x^8*e^9 - 3 36420000*(x*e + d)^m*d^2*m^4*x^9*e^9 + 29970*(x*e + d)^m*d^3*m^8*x^4*e^8 - 164640*(x*e + d)^m*d^3*m^7*x^5*e^8 + 21848400*(x*e + d)^m*d^3*m^6*x^6*e^8 - 4788000*(x*e + d)^m*d^3*m^5*x^7*e^8 + 304605000*(x*e + d)^m*d^3*m^4*x^8*e^ 8 + 20580*(x*e + d)^m*d^4*m^7*x^4*e^7 - 7968240*(x*e + d)^m*d^4*m^6*x^5*e^7 + 3150000*(x*e + d)^m*d^4*m^5*x^6*e^7 - 264600000*(x*e + d)^m*d^4*m^4*x^7* e^7 + 1285200*(x*e + d)^m*d^5*m^6*x^4*e^6 - 1587600*(x*e + d)^m*d^5*m^5*x^5 *e^6 + 214200000*(x*e + d)^m*d^5*m^4*x^6*e^6 + 378000*(x*e + d)^m*d^6*m^5*x ^4*e^5 - 151200000*(x*e + d)^m*d^6*m^4*x^5*e^5 + 75600000*(x*e + d)^m*d^7*m ^4*x^4*e^4 + 574*(x*e + d)^m*m^10*x^4*e^11 + 67649*(x*e + d)^m*m^9*x^5*e^11 + 798150*(x*e + d)^m*m^8*x^6*e^11 + 22063914*(x*e + d)^m*m^7*x^7*e^11 - 18 577566*(x*e + d)^m*m^6*x^8*e^11 + 793819845*(x*e + d)^m*m^5*x^9*e^11 - 9215

```
6375*(x*e + d)^m*m^4*x^10*e^11 + 4204750000*(x*e + d)^m*m^3*x^11*e^11 + 574
*(x*e + d)^m*d*m^10*x^3*e^10 + 63213*(x*e + d)^m*d*m^9*x^4*e^10 + 657900*(x
*e + d)^m*d*m^8*x^5*e^10 + 14907078*(x*e + d)^m*d*m^7*x^6*e^10 - 9485322*(x
*e + d)^m*d*m^6*x^7*e^10 + 283723965*(x*e + d)^m*d*m^5*x^8*e^10 - 21456200*
(x*e + d)^m*d*m^4*x^9*e^10 + 586350000*(x*e + d)^m*d*m^3*x^10*e^10 - 4436*(
x*e + d)^m*d^2*m^9*x^3*e^9 - 130050*(x*e + d)^m*d^2*m^8*x^4*e^9 - 5718276*(
x*e + d)^m*d^2*m^7*x^5*e^9 + 5659500*(x*e + d)^m*d^2*m^6*x^6*e^9 - 22221108
0*(x*e + d)^m*d^2*m^5*x^7*e^9 + 19707975*(x*e + d)^m*d^2*m^4*x^8*e^9 - 5906
20000*(x*e + d)^m*d^2*m^3*x^9*e^9 + 10200*(x*e + d)^m*d^3*m^8*x^3*e^8 + 131
8680*(x*e + d)^m*d^3*m^7*x^4*e^8 - 2609544*(x*e...
```

Mupad [B]
time = 8.39, size = 2500, normalized size = 4.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m*(2*x + 5*x^2 + 3)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] (500*x^11*(d + e*x)^m*(10628640*m + 12753576*m^2 + 8409500*m^3 + 3416930*m^
4 + 902055*m^5 + 157773*m^6 + 18150*m^7 + 1320*m^8 + 55*m^9 + m^10 + 362880
0))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*
m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39
916800) + ((d + e*x)^m*(2155507200*d*e^10 + 99792000*d^10*e + 1814400000*d^
11 - 2694384000*d^2*e^9 + 6346771200*d^3*e^8 - 5728060800*d^4*e^7 + 8853546
240*d^5*e^6 - 3392928000*d^6*e^5 + 5696697600*d^7*e^4 + 488980800*d^8*e^3 +
 3392928000*d^9*e^2 - 4095133200*d^2*e^9*m + 7530723360*d^3*e^8*m - 5364581
040*d^4*e^7*m + 6521026464*d^5*e^6*m - 1933552800*d^6*e^5*m + 2432604960*d^
7*e^4*m + 147682080*d^8*e^3*m + 647740800*d^9*e^2*m + 3795710544*d*e^10*m^2
 + 1888225560*d*e^10*m^3 + 595543860*d*e^10*m^4 + 124791030*d*e^10*m^5 + 17
637102*d*e^10*m^6 + 1663740*d*e^10*m^7 + 100440*d*e^10*m^8 + 3510*d*e^10*m^
9 + 54*d*e^10*m^10 - 2697071580*d^2*e^9*m^2 + 3842860824*d^3*e^8*m^2 - 2127
097056*d^4*e^7*m^2 + 1983530784*d^5*e^6*m^2 - 437886000*d^6*e^5*m^2 + 38769
1920*d^7*e^4*m^2 + 14817600*d^8*e^3*m^2 + 30844800*d^9*e^2*m^2 - 1011746160
*d^2*e^9*m^3 + 1102270680*d^3*e^8*m^3 - 463042356*d^4*e^7*m^3 + 318992760*d
^5*e^6*m^3 - 49266000*d^6*e^5*m^3 + 27332640*d^7*e^4*m^3 + 493920*d^8*e^3*m
^3 - 238556745*d^2*e^9*m^4 + 194510106*d^3*e^8*m^4 - 59787840*d^4*e^7*m^4 +
 28612200*d^5*e^6*m^4 - 2754000*d^6*e^5*m^4 + 719280*d^7*e^4*m^4 - 36710415
*d^2*e^9*m^5 + 21636720*d^3*e^8*m^5 - 4580520*d^4*e^7*m^5 + 1357416*d^5*e^6
*m^5 - 61200*d^6*e^5*m^5 - 3691170*d^2*e^9*m^6 + 1482516*d^3*e^8*m^6 - 1928
64*d^4*e^7*m^6 + 26616*d^5*e^6*m^6 - 234090*d^2*e^9*m^7 + 57240*d^3*e^8*m^7
 - 3444*d^4*e^7*m^7 - 8505*d^2*e^9*m^8 + 954*d^3*e^8*m^8 - 135*d^2*e^9*m^9
+ 4353860160*d*e^10*m + 9072000*d^10*e*m))/(e^11*(120543840*m + 150917976*m
^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7
 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (x*(d + e*x)^m*(435
```

3860160*e^11*m + 2155507200*e^11 + 3795710544*e^11*m^2 + 1888225560*e^11*m^
3 + 595543860*e^11*m^4 + 124791030*e^11*m^5 + 17637102*e^11*m^6 + 1663740*e
^11*m^7 + 100440*e^11*m^8 + 3510*e^11*m^9 + 54*e^11*m^10 - 6346771200*d^2*e
^9*m + 5728060800*d^3*e^8*m - 8853546240*d^4*e^7*m + 3392928000*d^5*e^6*m -
 5696697600*d^6*e^5*m - 488980800*d^7*e^4*m - 3392928000*d^8*e^3*m - 997920
00*d^9*e^2*m + 4095133200*d*e^10*m^2 + 2697071580*d*e^10*m^3 + 1011746160*d
*e^10*m^4 + 238556745*d*e^10*m^5 + 36710415*d*e^10*m^6 + 3691170*d*e^10*m^7
 + 234090*d*e^10*m^8 + 8505*d*e^10*m^9 + 135*d*e^10*m^10 - 7530723360*d^2*e
^9*m^2 + 5364581040*d^3*e^8*m^2 - 6521026464*d^4*e^7*m^2 + 1933552800*d^5*e
^6*m^2 - 2432604960*d^6*e^5*m^2 - 147682080*d^7*e^4*m^2 - 647740800*d^8*e^3
*m^2 - 9072000*d^9*e^2*m^2 - 3842860824*d^2*e^9*m^3 + 2127097056*d^3*e^8*m^
3 - 1983530784*d^4*e^7*m^3 + 437886000*d^5*e^6*m^3 - 387691920*d^6*e^5*m^3
- 14817600*d^7*e^4*m^3 - 30844800*d^8*e^3*m^3 - 1102270680*d^2*e^9*m^4 + 46
3042356*d^3*e^8*m^4 - 318992760*d^4*e^7*m^4 + 49266000*d^5*e^6*m^4 - 273326
40*d^6*e^5*m^4 - 493920*d^7*e^4*m^4 - 194510106*d^2*e^9*m^5 + 59787840*d^3*
e^8*m^5 - 28612200*d^4*e^7*m^5 + 2754000*d^5*e^6*m^5 - 719280*d^6*e^5*m^5 -
 21636720*d^2*e^9*m^6 + 4580520*d^3*e^8*m^6 - 1357416*d^4*e^7*m^6 + 61200*d
^5*e^6*m^6 - 1482516*d^2*e^9*m^7 + 192864*d^3*e^8*m^7 - 26616*d^4*e^7*m^7 -
 57240*d^2*e^9*m^8 + 3444*d^3*e^8*m^8 - 954*d^2*e^9*m^9 + 2694384000*d*e^10
*m - 1814400000*d^10*e*m))/(e^11*(120543840*m + 150917976*m^2 + 105258076*m
^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1
925*m^9 + 66*m^10 + m^11 + 39916800)) + (x^8*(d + e*x)^m*(13068*m + 13132*m
^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)*(45000*d^3*m - 29
302*e^3*m - 97020*e^3 - 2940*e^3*m^2 - 98*e^3*m^3 + 16065*d*e^2*m^2 + 225*d
^2*e*m^2 + 765*d*e^2*m^3 + 84150*d*e^2*m + 2475*d^2*e*m))/(e^3*(120543840*m
 + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^
6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (3*x^
2*(m + 1)*(d + e*x)^m*(302400000*d^9*m + 1365044400*e^9*m + 898128000*e^9 +
 899023860*e^9*m^2 + 337248720*e^9*m^3 + 79518915*e^9*m^4 + 12236805*e^9*m^
5 + 1230390*e^9*m^6 + 78030*e^9*m^7 + 2835*e^9*m^8 + 45*e^9*m^9 - 954676800
*d^2*e^7*m + 1475591040*d^3*e^6*m - 565488000*d^4*e^5*m + 949449600*d^5*e^4
*m + 81496800*d^6*e^3*m + 565488000*d^7*e^2*m + 1255120560*d*e^8*m^2 + 1512
000*d^8*e*m^2 + 640476804*d*e^8*m^3 + 183711780*d*e^8*m^4 + 32418351*d*e^8*
m^5 + 3606120*d*e^8*m^6 + 247086*d*e^8*m^7 + 9540*d*e^8*m^8 + 159*d*e^8*m^9
 - 894096840*d^2*e^7*m^2 + 1086837744*d^3*e^6*m^2 - 322258800*d^4*e^5*m^2 +
 405434160*d^5*e^4*m^2 + 24613680*d^6*e^3*m^2 + 107956800*d^7*e^2*m^2 - 354
516176*d^2*e^7*m^3 + 330588464*d^3*e^6*m^3 - 72981000*d^4*e^5*m^3 + 6461532
0*d^5*e^4*m^3 + 2469600*d^6*e^3*m^3 + 5140800*d^7*e^2*m^3 - 77173726*d^2*e^
7*m^4 + 53165460*d^3*e^6*m^4 - 8211000*d^4*e^5*m^4 + 4555440*d^5*e^4*m^4 +
82320*d^6*e^3*m^4 - 9964640*d^2*e^7*m^5 + 4768700*d^3*e^6*m^5 - 459000*d^4*
e^5*m^5 + 119880*d^5*e^4*m^5 - 763420*d^2*e^7*m...

3.368 $\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)$

Optimal. Leaf size=432

$$\frac{\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^{1+m}}{e^9(1+m)}-\frac{\left(5d^2-2de+3e^2\right)\left(160d^5+127d^4e+88d^3\right.}{e^9(2}$$

```
[Out] (5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/e
^9/(1+m)-(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*d^3*e^2-4*d^2*e^3+64*d*e
^4-11*e^5)*(e*x+d)^(2+m)/e^9/(2+m)+(2800*d^6+945*d^5*e+1665*d^4*e^2+370*d^3
*e^3+888*d^2*e^4-195*d*e^5+107*e^6)*(e*x+d)^(3+m)/e^9/(3+m)-(5600*d^5+1575*
d^4*e+2220*d^3*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*(e*x+d)^(4+m)/e^9/(4+m)+(7
000*d^4+1575*d^3*e+1665*d^2*e^2+185*d*e^3+148*e^4)*(e*x+d)^(5+m)/e^9/(5+m)-
(5600*d^3+945*d^2*e+666*d*e^2+37*e^3)*(e*x+d)^(6+m)/e^9/(6+m)+(2800*d^2+315
*d*e+111*e^2)*(e*x+d)^(7+m)/e^9/(7+m)-5*(160*d+9*e)*(e*x+d)^(8+m)/e^9/(8+m)
+100*(e*x+d)^(9+m)/e^9/(9+m)
```

Rubi [A]

time = 0.15, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1642}

$$\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{m+7}}{e^9(m+7)}-\frac{\left(5600d^3+945d^2e+666de^2+37e^3\right)(d+ex)^{m+6}}{e^9(m+6)}+\frac{\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^{m+1}}{e^9(m+1)}+\frac{\left(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4\right)(d+ex)^{m+5}}{e^9(m+5)}-\frac{\left(5d^2-2de+3e^2\right)\left(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5\right)(d+ex)^{m+2}}{e^9(m+2)}-\frac{\left(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5\right)(d+ex)^{m+4}}{e^9(m+4)}+\frac{\left(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6\right)(d+ex)^{m+3}}{e^9(m+3)}-\frac{5(160d+9e)(d+ex)^{m+8}}{e^9(m+8)}+\frac{100(d+ex)^{m+9}}{e^9(m+9)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
 + e*x)^(1 + m))/(e^9*(1 + m)) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^
4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x)^(2 + m))/(e^9*(
2 + m)) + ((2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4
 - 195*d*e^5 + 107*e^6)*(d + e*x)^(3 + m))/(e^9*(3 + m)) - ((5600*d^5 + 157
5*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*(d + e*x)^(4 + m
))/(e^9*(4 + m)) + ((7000*d^4 + 1575*d^3*e + 1665*d^2*e^2 + 185*d*e^3 + 148
*e^4)*(d + e*x)^(5 + m))/(e^9*(5 + m)) - ((5600*d^3 + 945*d^2*e + 666*d*e^2
 + 37*e^3)*(d + e*x)^(6 + m))/(e^9*(6 + m)) + ((2800*d^2 + 315*d*e + 111*e^
2)*(d + e*x)^(7 + m))/(e^9*(7 + m)) - (5*(160*d + 9*e)*(d + e*x)^(8 + m))/(
e^9*(8 + m)) + (100*(d + e*x)^(9 + m))/(e^9*(9 + m))
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \int \left(\frac{\left(5d^2-2de+3e^2\right)^2 \left(4d^4+5d^3e+3d^2e^2\right.}{e^8} \right.$$

$$= \frac{\left(5d^2-2de+3e^2\right)^2 \left(4d^4+5d^3e+3d^2e^2-de\right.}{e^9(1+m)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1476 vs. 2(432) = 864.
time = 1.59, size = 1476, normalized size = 3.42

Too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] ((d + e*x)^(1 + m)*(4032000*d^8 - 25200*d^7*e*(-81 + 160*x + m*(-9 + 160*x)
) + 720*d^6*e^2*(7992 - 2835*x + 5600*x^2 + 3*m*(629 - 1050*x + 2800*x^2) +
 m^2*(111 - 315*x + 2800*x^2)) - 120*d^5*e^3*(12*m^2*(-74 + 999*x - 945*x^2
 + 2800*x^3) + 6*(-3108 + 7992*x - 2835*x^2 + 5600*x^3) + m^3*(-37 + 666*x
- 945*x^2 + 5600*x^3) + m*(-7067 + 59274*x - 27405*x^2 + 61600*x^3)) + 24*d
^4*e^4*(m^4*(148 - 185*x + 1665*x^2 - 1575*x^3 + 7000*x^4) + 25*m*(9768 - 5
143*x + 16650*x^2 - 6615*x^3 + 14000*x^4) + 5*m^3*(888 - 925*x + 6660*x^2 -
 4725*x^3 + 14000*x^4) + 6*(74592 - 15540*x + 39960*x^2 - 14175*x^3 + 28000
*x^4) + 5*m^2*(9916 - 7955*x + 41625*x^2 - 20475*x^3 + 49000*x^4)) - 6*d^3*
e^5*(m^5*(65 + 592*x - 370*x^2 + 2220*x^3 - 1575*x^4 + 5600*x^5) + 24*(4095
0 + 74592*x - 15540*x^2 + 39960*x^3 - 14175*x^4 + 28000*x^5) + m^4*(2275 +
18352*x - 9990*x^2 + 51060*x^3 - 29925*x^4 + 84000*x^5) + 5*m^3*(6305 + 432
16*x - 19610*x^2 + 82140*x^3 - 39375*x^4 + 95200*x^5) + 5*m^2*(43225 + 2350
24*x - 83250*x^2 + 277500*x^3 - 114975*x^4 + 252000*x^5) + 2*m*(366405 + 13
83504*x - 350390*x^2 + 992340*x^3 - 373275*x^4 + 767200*x^5)) + 2*d^2*e^6*(
m^6*(107 + 195*x + 888*x^2 - 370*x^3 + 1665*x^4 - 945*x^5 + 2800*x^6) + 3*m
^5*(1391 + 2340*x + 9768*x^2 - 3700*x^3 + 14985*x^4 - 7560*x^5 + 19600*x^6)
 + 72*(89880 + 40950*x + 74592*x^2 - 15540*x^3 + 39960*x^4 - 14175*x^5 + 28
000*x^6) + 15*m^3*(37557 + 49530*x + 160728*x^2 - 47360*x^3 + 151515*x^4 -
62370*x^5 + 137200*x^6) + m^4*(66875 + 101400*x + 379176*x^2 - 128020*x^3 +
 461205*x^4 - 207900*x^5 + 490000*x^6) + 6*m*(1073852 + 857805*x + 1831056*
x^2 - 412550*x^3 + 1112220*x^4 - 407295*x^5 + 823200*x^6) + m^2*(2629418 +
2846805*x + 7675872*x^2 - 1949530*x^3 + 5651010*x^4 - 2172555*x^5 + 4547200
*x^6)) - d*e^7*(m^7*(33 + 214*x + 195*x^2 + 592*x^3 - 185*x^4 + 666*x^5 - 3
15*x^6 + 800*x^7) + 2*m^6*(693 + 4280*x + 3705*x^2 + 10656*x^3 - 3145*x^4 +
 10656*x^5 - 4725*x^6 + 11200*x^7) + 144*(41580 + 89880*x + 40950*x^2 + 745
```

```
92*x^3 - 15540*x^4 + 39960*x^5 - 14175*x^6 + 28000*x^7) + 2*m^5*(12243 + 71
048*x + 57720*x^2 + 155696*x^3 - 43105*x^4 + 137196*x^5 - 57330*x^6 + 12880
0*x^7) + 2*m^4*(117810 + 630230*x + 472875*x^2 + 1182816*x^3 - 305620*x^4 +
 915750*x^5 - 363825*x^6 + 784000*x^7) + 12*m*(663102 + 2152412*x + 1103505
*x^2 + 2129424*x^3 - 459170*x^4 + 1208124*x^5 - 435645*x^6 + 871200*x^7) +
2*m^2*(2209977 + 9072530*x + 5420220*x^2 + 11337984*x^3 - 2568355*x^4 + 698
5674*x^5 - 2579850*x^6 + 5252800*x^7) + m^3*(1332177 + 6385546*x + 4332705*
x^2 + 9939088*x^3 - 2395565*x^4 + 6805854*x^5 - 2595285*x^6 + 5415200*x^7))
 + e^8*(m^8*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4) + m^7*(792
+ 1419*x + 4494*x^2 + 2665*x^3 + 5920*x^4 - 1443*x^5 + 4218*x^6 - 1665*x^7
+ 3600*x^8) + 2*m^6*(7434 + 12936*x + 39804*x^2 + 22945*x^3 + 49580*x^4 - 1
1766*x^5 + 33522*x^6 - 12915*x^7 + 27300*x^8) + 144*(45360 + 41580*x + 8988
0*x^2 + 40950*x^3 + 74592*x^4 - 15540*x^5 + 39960*x^6 - 14175*x^7 + 28000*x
^8) + 2*m^5*(77616 + 130053*x + 386163*x^2 + 215345*x^3 + 451400*x^4 - 1042
29*x^5 + 289821*x^6 - 109305*x^7 + 226800*x^8) + 12*m*(995544 + 1162062*x +
 2691692*x^2 + 1267305*x^3 + 2353200*x^4 - 496466*x^5 + 1288044*x^6 - 45994
5*x^7 + 913200*x^8) + m^4*(983682 + 1567797*x + 4453233*x^2 + 2389985*x^3 +
 4850404*x^4 - 1090353*x^5 + 2965809*x^6 - 1098405*x^7 + 2244900*x^8) + 2*m
^2*(4581036 + 6188589*x + 15529766*x^2 + 7627230*x^3 + 14532120*x^4 - 31193
59*x^5 + 8193798*x^6 - 2953260*x^7 + 5906200*x^8) + m^3*(3864168 + 5752131*
x + 15458076*x^2 + 7946185*x^3 + 15608080*x^4 - 3422907*x^5 + 9134412*x^6 -
 3332385*x^7 + 6728400*x^8))))/(e^9*(1 + m)*(2 + m)*(3 + m)*(4 + m)*(5 + m)
*(6 + m)*(7 + m)*(8 + m)*(9 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3221 vs. $2(432) = 864$.
time = 0.16, size = 3222, normalized size = 7.46

method	result	size
gosper	Expression too large to display	3222
risch	Expression too large to display	3895

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBO
SE)
```

```
[Out] (e*x+d)^(1+m)*(100*e^8*m^8*x^8-45*e^8*m^8*x^7+3600*e^8*m^7*x^8-800*d*e^7*m^
7*x^7+111*e^8*m^8*x^6-1665*e^8*m^7*x^7+54600*e^8*m^6*x^8+315*d*e^7*m^7*x^6-
22400*d*e^7*m^6*x^7-37*e^8*m^8*x^5+4218*e^8*m^7*x^6-25830*e^8*m^6*x^7+45360
0*e^8*m^5*x^8+5600*d^2*e^6*m^6*x^6-666*d*e^7*m^7*x^5+9450*d*e^7*m^6*x^6-257
600*d*e^7*m^5*x^7+148*e^8*m^8*x^4-1443*e^8*m^7*x^5+67044*e^8*m^6*x^6-218610
*e^8*m^5*x^7+2244900*e^8*m^4*x^8-1890*d^2*e^6*m^6*x^5+117600*d^2*e^6*m^5*x^
6+185*d*e^7*m^7*x^4-21312*d*e^7*m^6*x^5+114660*d*e^7*m^5*x^6-1568000*d*e^7*
m^4*x^7+65*e^8*m^8*x^3+5920*e^8*m^7*x^4-23532*e^8*m^6*x^5+579642*e^8*m^5*x^
6-1098405*e^8*m^4*x^7+6728400*e^8*m^3*x^8-33600*d^3*e^5*m^5*x^5+3330*d^2*e^
```

6*m^6*x^4-45360*d^2*e^6*m^5*x^5+980000*d^2*e^6*m^4*x^6-592*d*e^7*m^7*x^3+62
90*d*e^7*m^6*x^4-274392*d*e^7*m^5*x^5+727650*d*e^7*m^4*x^6-5415200*d*e^7*m^
3*x^7+107*e^8*m^8*x^2+2665*e^8*m^7*x^3+99160*e^8*m^6*x^4-208458*e^8*m^5*x^5
+2965809*e^8*m^4*x^6-3332385*e^8*m^3*x^7+11812400*e^8*m^2*x^8+9450*d^3*e^5*
m^5*x^4-504000*d^3*e^5*m^4*x^5-740*d^2*e^6*m^6*x^3+89910*d^2*e^6*m^5*x^4-41
5800*d^2*e^6*m^4*x^5+4116000*d^2*e^6*m^3*x^6-195*d*e^7*m^7*x^2-21312*d*e^7*
m^6*x^3+86210*d*e^7*m^5*x^4-1831500*d*e^7*m^4*x^5+2595285*d*e^7*m^3*x^6-105
05600*d*e^7*m^2*x^7+33*e^8*m^8*x+4494*e^8*m^7*x^2+45890*e^8*m^6*x^3+902800*
e^8*m^5*x^4-1090353*e^8*m^4*x^5+9134412*e^8*m^3*x^6-5906520*e^8*m^2*x^7+109
58400*e^8*m*x^8+168000*d^4*e^4*m^4*x^4-13320*d^3*e^5*m^5*x^3+179550*d^3*e^5
*m^4*x^4-2856000*d^3*e^5*m^3*x^5+1776*d^2*e^6*m^6*x^2-22200*d^2*e^6*m^5*x^3
+922410*d^2*e^6*m^4*x^4-1871100*d^2*e^6*m^3*x^5+9094400*d^2*e^6*m^2*x^6-214
*d*e^7*m^7*x-7410*d*e^7*m^6*x^2-311392*d*e^7*m^5*x^3+611240*d*e^7*m^4*x^4-6
805854*d*e^7*m^3*x^5+5159700*d*e^7*m^2*x^6-10454400*d*e^7*m*x^7+18*e^8*m^8+
1419*e^8*m^7*x+79608*e^8*m^6*x^2+430690*e^8*m^5*x^3+4850404*e^8*m^4*x^4-342
2907*e^8*m^3*x^5+16387596*e^8*m^2*x^6-5519340*e^8*m*x^7+4032000*e^8*x^8-378
00*d^4*e^4*m^4*x^3+1680000*d^4*e^4*m^3*x^4+2220*d^3*e^5*m^5*x^2-306360*d^3*
e^5*m^4*x^3+1181250*d^3*e^5*m^3*x^4-7560000*d^3*e^5*m^2*x^5+390*d^2*e^6*m^6
*x+58608*d^2*e^6*m^5*x^2-256040*d^2*e^6*m^4*x^3+4545450*d^2*e^6*m^3*x^4-434
5110*d^2*e^6*m^2*x^5+9878400*d^2*e^6*m*x^6-33*d*e^7*m^7-8560*d*e^7*m^6*x-11
5440*d*e^7*m^5*x^2-2365632*d*e^7*m^4*x^3+2395565*d*e^7*m^3*x^4-13971348*d*e
^7*m^2*x^5+5227740*d*e^7*m*x^6-4032000*d*e^7*x^7+792*e^8*m^7+25872*e^8*m^6*
x+772326*e^8*m^5*x^2+2389985*e^8*m^4*x^3+15608080*e^8*m^3*x^4-6238718*e^8*m
^2*x^5+15456528*e^8*m*x^6-2041200*e^8*x^7-672000*d^5*e^3*m^3*x^3+39960*d^4*
e^4*m^4*x^2-567000*d^4*e^4*m^3*x^3+5880000*d^4*e^4*m^2*x^4-3552*d^3*e^5*m^5
*x+59940*d^3*e^5*m^4*x^2-2464200*d^3*e^5*m^3*x^3+3449250*d^3*e^5*m^2*x^4-92
06400*d^3*e^5*m*x^5+214*d^2*e^6*m^6+14040*d^2*e^6*m^5*x+758352*d^2*e^6*m^4*
x^2-1420800*d^2*e^6*m^3*x^3+11302020*d^2*e^6*m^2*x^4-4887540*d^2*e^6*m*x^5+
4032000*d^2*e^6*x^6-1386*d*e^7*m^6-142096*d*e^7*m^5*x-945750*d*e^7*m^4*x^2-
9939088*d*e^7*m^3*x^3+5136710*d*e^7*m^2*x^4-14497488*d*e^7*m*x^5+2041200*d*
e^7*x^6+14868*e^8*m^6+260106*e^8*m^5*x+4453233*e^8*m^4*x^2+7946185*e^8*m^3*
x^3+29064240*e^8*m^2*x^4-5957592*e^8*m*x^5+5754240*e^8*x^6+113400*d^5*e^3*m
^3*x^2-4032000*d^5*e^3*m^2*x^3-4440*d^4*e^4*m^4*x+799200*d^4*e^4*m^3*x^2-24
57000*d^4*e^4*m^2*x^3+8400000*d^4*e^4*m*x^4-390*d^3*e^5*m^5-110112*d^3*e^5*
m^4*x+588300*d^3*e^5*m^3*x^2-8325000*d^3*e^5*m^2*x^3+4479300*d^3*e^5*m*x^4-
4032000*d^3*e^5*x^5+8346*d^2*e^6*m^5+202800*d^2*e^6*m^4*x+4821840*d^2*e^6*m
^3*x^2-3899060*d^2*e^6*m^2*x^3+13346640*d^2*e^6*m*x^4-2041200*d^2*e^6*x^5-2
4486*d*e^7*m^5-1260460*d*e^7*m^4*x-4332705*d*e^7*m^3*x^2-22675968*d*e^7*m^2
*x^3+5510040*d*e^7*m*x^4-5754240*d*e^7*x^5+155232*e^8*m^5+1567797*e^8*m^4*x
+15458076*e^8*m^3*x^2+15254460*e^8*m^2*x^3+28238400*e^8*m*x^4-2237760*e^8*x
^5+2016000*d^6*e^2*m^2*x^2-79920*d^5*e^3*m^3*x+1360800*d^5*e^3*m^2*x^2-7392
000*d^5*e^3*m*x^3+3552*d^4*e^4*m^4-111000*d^4*e^4*m^3*x+4995000*d^4*e^4*m^2
*x^2-3969000*d^4*e^4*m*x^3+4032000*d^4*e^4*x^4-13650*d^3*e^5*m^4-1296480*d^
3*e^5*m^3*x+2497500*d^3*e^5*m^2*x^2-11908080*d^3*e^5*m*x^3+2041200*d^3*e^5*
x^4+133750*d^2*e^6*m^4+1485900*d^2*e^6*m^3*x+15351744*d^2*e^6*m^2*x^2-49506

```
00*d^2*e^6*m*x^3+5754240*d^2*e^6*x^4-235620*d*e^7*m^4-6385546*d*e^7*m^3*x-1
0840440*d*e^7*m^2*x^2-25553088*d*e^7*m*x^3+2237760*d*e^7*x^4+983682*e^8*m^4
+5752131*e^8*m^3*x+31059532*e^8*m^2*x^2+15207660*e^8*m*x^3+10741248*e^8*x^4
-226800*d^6*e^2*m^2*x+6048000*d^6*e^2*m*x^2+4440*d^5*e^3*m^3-1438560*d^5*e^
3*m^2*x+3288600*d^5*e^3*m*x^2-4032000*d^5*e^3*x^3+106560*d^4*e^4*m^3-954600
*d^4*e^4*m^2*x+9990000*d^4*e^4*m*x^2-2041200*d^4*e^4*x^3-189150*d^3*e^5*m^3
-7050720*d^3*e^5*m^2*x+4204680*d^3*e^5*m*x^2-5754240*d^3*e^5*x^3+1126710*d^
2*e^6*m^3+5693610*d^2*e^6*m^2*x+21972672*d^2*e^6*m*x^2-2237760*d^2*e^6*x^3-
1332177*d*e^7*m^3-18145060*d*e^7*m^2*x-13242060*d*e^7*m*x^2-10741248*d*e^7*
x^3+3864168*e^8*m^3+12377178*e^8*m^2*x+32300304*e^8*m*x^2+5896800*e^8*x^3-4
032000*d^7*e*m*x+79920*d^6*e^2*m^2-2268000*d^6*e^2*m*x+4032000*d^6*e^2*x^2+
106560*d^5*e^3*m^2-7112880*d^5*e^3*m*x+2041200*d^5*e^3*x^2+1189920*d^4*e^4*
m^2-3085800*d^4*e^4*m*x+5754240*d^4*e^4*x^2-129...
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(418) = 836$.
time = 0.31, size = 1402, normalized size = 3.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 18*(x*e + d)^(m + 1)*e^(-1)/(m + 1) + 33*((m + 1)*x^2*e^2 + d*m*x*e - d^2)*
e^(m*log(x*e + d) - 2)/(m^2 + 3*m + 2) + 107*((m^2 + 3*m + 2)*x^3*e^3 + (m^
2 + m)*d*x^2*e^2 - 2*d^2*m*x*e + 2*d^3)*e^(m*log(x*e + d) - 3)/(m^3 + 6*m^2
 + 11*m + 6) + 65*((m^3 + 6*m^2 + 11*m + 6)*x^4*e^4 + (m^3 + 3*m^2 + 2*m)*d
*x^3*e^3 - 3*(m^2 + m)*d^2*x^2*e^2 + 6*d^3*m*x*e - 6*d^4)*e^(m*log(x*e + d)
 - 4)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24) + 148*((m^4 + 10*m^3 + 35*m^2 + 5
0*m + 24)*x^5*e^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*x^4*e^4 - 4*(m^3 + 3*m^2
 + 2*m)*d^2*x^3*e^3 + 12*(m^2 + m)*d^3*x^2*e^2 - 24*d^4*m*x*e + 24*d^5)*e^(
m*log(x*e + d) - 5)/(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120) - 37*((
m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*x^6*e^6 + (m^5 + 10*m^4 + 35
*m^3 + 50*m^2 + 24*m)*d*x^5*e^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*x^4*e^
4 + 20*(m^3 + 3*m^2 + 2*m)*d^3*x^3*e^3 - 60*(m^2 + m)*d^4*x^2*e^2 + 120*d^5
*m*x*e - 120*d^6)*e^(m*log(x*e + d) - 6)/(m^6 + 21*m^5 + 175*m^4 + 735*m^3
+ 1624*m^2 + 1764*m + 720) + 111*((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*
m^2 + 1764*m + 720)*x^7*e^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 +
120*m)*d*x^6*e^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*x^5*e^5 +
30*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^3*x^4*e^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*x
^3*e^3 + 360*(m^2 + m)*d^5*x^2*e^2 - 720*d^6*m*x*e + 720*d^7)*e^(m*log(x*e
+ d) - 7)/(m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068
*m + 5040) - 45*((m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2
+ 13068*m + 5040)*x^8*e^8 + (m^7 + 21*m^6 + 175*m^5 + 735*m^4 + 1624*m^3 +
```

```
1764*m^2 + 720*m)*d*x^7*e^7 - 7*(m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2
+ 120*m)*d^2*x^6*e^6 + 42*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^3*x^5*e
^5 - 210*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^4*x^4*e^4 + 840*(m^3 + 3*m^2 + 2*m)
*d^5*x^3*e^3 - 2520*(m^2 + m)*d^6*x^2*e^2 + 5040*d^7*m*x*e - 5040*d^8)*e^(m
*log(x*e + d) - 8)/(m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67284*m
^3 + 118124*m^2 + 109584*m + 40320) + 100*((m^8 + 36*m^7 + 546*m^6 + 4536*m
^5 + 22449*m^4 + 67284*m^3 + 118124*m^2 + 109584*m + 40320)*x^9*e^9 + (m^8
+ 28*m^7 + 322*m^6 + 1960*m^5 + 6769*m^4 + 13132*m^3 + 13068*m^2 + 5040*m)*
d*x^8*e^8 - 8*(m^7 + 21*m^6 + 175*m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720
*m)*d^2*x^7*e^7 + 56*(m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^
3*x^6*e^6 - 336*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^4*x^5*e^5 + 1680*
(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^5*x^4*e^4 - 6720*(m^3 + 3*m^2 + 2*m)*d^6*x^3
*e^3 + 20160*(m^2 + m)*d^7*x^2*e^2 - 40320*d^8*m*x*e + 40320*d^9)*e^(m*log(
x*e + d) - 9)/(m^9 + 45*m^8 + 870*m^7 + 9450*m^6 + 63273*m^5 + 269325*m^4 +
 723680*m^3 + 1172700*m^2 + 1026576*m + 362880)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2377 vs. $2(418) = 836$.
time = 0.37, size = 2377, normalized size = 5.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] (4032000*d^9 + (100*(m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67284*
m^3 + 118124*m^2 + 109584*m + 40320)*x^9 - 45*(m^8 + 37*m^7 + 574*m^6 + 485
8*m^5 + 24409*m^4 + 74053*m^3 + 131256*m^2 + 122652*m + 45360)*x^8 + 111*(m
^8 + 38*m^7 + 604*m^6 + 5222*m^5 + 26719*m^4 + 82292*m^3 + 147636*m^2 + 139
248*m + 51840)*x^7 - 37*(m^8 + 39*m^7 + 636*m^6 + 5634*m^5 + 29469*m^4 + 92
511*m^3 + 168614*m^2 + 161016*m + 60480)*x^6 + 148*(m^8 + 40*m^7 + 670*m^6
+ 6100*m^5 + 32773*m^4 + 105460*m^3 + 196380*m^2 + 190800*m + 72576)*x^5 +
65*(m^8 + 41*m^7 + 706*m^6 + 6626*m^5 + 36769*m^4 + 122249*m^3 + 234684*m^2
 + 233964*m + 90720)*x^4 + 107*(m^8 + 42*m^7 + 744*m^6 + 7218*m^5 + 41619*m
^4 + 144468*m^3 + 290276*m^2 + 301872*m + 120960)*x^3 + 33*(m^8 + 43*m^7 +
784*m^6 + 7882*m^5 + 47509*m^4 + 174307*m^3 + 375066*m^2 + 422568*m + 18144
0)*x^2 + 18*(m^8 + 44*m^7 + 826*m^6 + 8624*m^5 + 54649*m^4 + 214676*m^3 + 5
09004*m^2 + 663696*m + 362880)*x)*e^9 + (18*d*m^8 + 100*(d*m^8 + 28*d*m^7 +
 322*d*m^6 + 1960*d*m^5 + 6769*d*m^4 + 13132*d*m^3 + 13068*d*m^2 + 5040*d*m
)*x^8 + 792*d*m^7 - 45*(d*m^8 + 30*d*m^7 + 364*d*m^6 + 2310*d*m^5 + 8239*d*
m^4 + 16380*d*m^3 + 16596*d*m^2 + 6480*d*m)*x^7 + 14868*d*m^6 + 111*(d*m^8
+ 32*d*m^7 + 412*d*m^6 + 2750*d*m^5 + 10219*d*m^4 + 20978*d*m^3 + 21768*d*m
^2 + 8640*d*m)*x^6 + 155232*d*m^5 - 37*(d*m^8 + 34*d*m^7 + 466*d*m^6 + 3304
*d*m^5 + 12949*d*m^4 + 27766*d*m^3 + 29784*d*m^2 + 12096*d*m)*x^5 + 983682*
```

```
d*m^4 + 148*(d*m^8 + 36*d*m^7 + 526*d*m^6 + 3996*d*m^5 + 16789*d*m^4 + 3830
4*d*m^3 + 43164*d*m^2 + 18144*d*m)*x^4 + 3864168*d*m^3 + 65*(d*m^8 + 38*d*m
^7 + 592*d*m^6 + 4850*d*m^5 + 22219*d*m^4 + 55592*d*m^3 + 67908*d*m^2 + 302
40*d*m)*x^3 + 9162072*d*m^2 + 107*(d*m^8 + 40*d*m^7 + 664*d*m^6 + 5890*d*m^
5 + 29839*d*m^4 + 84790*d*m^3 + 120696*d*m^2 + 60480*d*m)*x^2 + 11946528*d*
m + 33*(d*m^8 + 42*d*m^7 + 742*d*m^6 + 7140*d*m^5 + 40369*d*m^4 + 133938*d*
m^3 + 241128*d*m^2 + 181440*d*m)*x + 6531840*d)*e^8 - (33*d^2*m^7 + 1386*d^
2*m^6 + 800*(d^2*m^7 + 21*d^2*m^6 + 175*d^2*m^5 + 735*d^2*m^4 + 1624*d^2*m^
3 + 1764*d^2*m^2 + 720*d^2*m)*x^7 + 24486*d^2*m^5 - 315*(d^2*m^7 + 24*d^2*m
^6 + 220*d^2*m^5 + 990*d^2*m^4 + 2299*d^2*m^3 + 2586*d^2*m^2 + 1080*d^2*m)*
x^6 + 235620*d^2*m^4 + 666*(d^2*m^7 + 27*d^2*m^6 + 277*d^2*m^5 + 1365*d^2*m
^4 + 3394*d^2*m^3 + 4008*d^2*m^2 + 1728*d^2*m)*x^5 + 1332177*d^2*m^3 - 185*
(d^2*m^7 + 30*d^2*m^6 + 346*d^2*m^5 + 1920*d^2*m^4 + 5269*d^2*m^3 + 6690*d^
2*m^2 + 3024*d^2*m)*x^4 + 4419954*d^2*m^2 + 592*(d^2*m^7 + 33*d^2*m^6 + 427
*d^2*m^5 + 2715*d^2*m^4 + 8644*d^2*m^3 + 12372*d^2*m^2 + 6048*d^2*m)*x^3 +
7957224*d^2*m + 195*(d^2*m^7 + 36*d^2*m^6 + 520*d^2*m^5 + 3810*d^2*m^4 + 14
599*d^2*m^3 + 26394*d^2*m^2 + 15120*d^2*m)*x^2 + 5987520*d^2 + 214*(d^2*m^7
 + 39*d^2*m^6 + 625*d^2*m^5 + 5265*d^2*m^4 + 24574*d^2*m^3 + 60216*d^2*m^2
+ 60480*d^2*m)*x)*e^7 + 2*(107*d^3*m^6 + 4173*d^3*m^5 + 66875*d^3*m^4 + 280
0*(d^3*m^6 + 15*d^3*m^5 + 85*d^3*m^4 + 225*d^3*m^3 + 274*d^3*m^2 + 120*d^3*
m)*x^6 + 563355*d^3*m^3 - 945*(d^3*m^6 + 19*d^3*m^5 + 125*d^3*m^4 + 365*d^3
*m^3 + 474*d^3*m^2 + 216*d^3*m)*x^5 + 2629418*d^3*m^2 + 1665*(d^3*m^6 + 23*
d^3*m^5 + 185*d^3*m^4 + 625*d^3*m^3 + 894*d^3*m^2 + 432*d^3*m)*x^4 + 644311
2*d^3*m - 370*(d^3*m^6 + 27*d^3*m^5 + 265*d^3*m^4 + 1125*d^3*m^3 + 1894*d^3
*m^2 + 1008*d^3*m)*x^3 + 6471360*d^3 + 888*(d^3*m^6 + 31*d^3*m^5 + 365*d^3*
m^4 + 1985*d^3*m^3 + 4674*d^3*m^2 + 3024*d^3*m)*x^2 + 195*(d^3*m^6 + 35*d^3
*m^5 + 485*d^3*m^4 + 3325*d^3*m^3 + 11274*d^3*m^2 + 15120*d^3*m)*x)*e^6 - 6
*(65*d^4*m^5 + 2275*d^4*m^4 + 31525*d^4*m^3 + 216125*d^4*m^2 + 5600*(d^4*m^
5 + 10*d^4*m^4 + 35*d^4*m^3 + 50*d^4*m^2 + 24*d^4*m)*x^5 + 732810*d^4*m - 1
575*(d^4*m^5 + 15*d^4*m^4 + 65*d^4*m^3 + 105*d^4*m^2 + 54*d^4*m)*x^4 + 9828
00*d^4 + 2220*(d^4*m^5 + 20*d^4*m^4 + 125*d^4*m^3 + 250*d^4*m^2 + 144*d^4*m
)*x^3 - 370*(d^4*m^5 + 25*d^4*m^4 + 215*d^4*m^3 + 695*d^4*m^2 + 504*d^4*m)*
x^2 + 592*(d^4*m^5 + 30*d^4*m^4 + 335*d^4*m^3 + 1650*d^4*m^2 + 3024*d^4*m)*
x)*e^5 + 24*(148*d^5*m^4 + 4440*d^5*m^3 + 49580*d^5*m^2 + 244200*d^5*m + 44
7552*d^5 + 7000*(d^5*m^4 + 6*d^5*m^3 + 11*d^5*m^2 + 6*d^5*m)*x^4 - 1575*(d^
5*m^4 + 12*d^5*m^3 + 29*d^5*m^2 + 18*d^5*m)*x^3 + 1665*(d^5*m^4 + 18*d^5*m^
3 + 89*d^5*m^2 + 72*d^5*m)*x^2 - 185*(d^5*m^4 + 24*d^5*m^3 + 191*d^5*m^2 +
504*d^5*m)*x)*e^4 + 120*(37*d^6*m^3 + 888*d^6*m^2 + 7067*d^6*m + 18648*d^6
- 5600*(d^6*m^3 + 3*d^6*m^2 + 2*d^6*m)*x^3 + 945*(d^6*m^3 + 10*d^6*m^2 + 9*
d^6*m)*x^2 - 666*(d^6*m^3 + 17*d^6*m^2 + 72*d^6*m)*x)*e^3 + 720*(111*d^7*m^
2 + 1887*d^7*m + 7992*d^7 + 2800*(d^7*m^2 + d^7*m)*x^2 - 315*(d^7*m^2 + 9*d
^7*m)*x)*e^2 - 25200*(160*d^8*m*x - 9*d^8*m - 81*d^8)*e)*(x*e + d)^m*e^(-9)
/(m^9 + 45*m^8 + 870*m^7 + 9450*m^6 + 63273*m^5 + 269325*m^4 + 723680*m^3 +
 1172700*m^2 + 1026576*m + 362880)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 65193 vs. $2(410) = 820$.

time = 15.25, size = 65193, normalized size = 150.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] Piecewise((d**m*(100*x**9/9 - 45*x**8/8 + 111*x**7/7 - 37*x**6/6 + 148*x**5
/5 + 65*x**4/4 + 107*x**3/3 + 33*x**2/2 + 18*x), Eq(e, 0)), (84000*d**8*log
(d/e + x)/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 4704
0*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d
**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 228300*d**8/(840*d**
8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3
+ 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6
720*d*e**16*x**7 + 840*e**17*x**8) + 672000*d**7*e*x*log(d/e + x)/(840*d**8
*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 +
 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 67
20*d*e**16*x**7 + 840*e**17*x**8) + 1742400*d**7*e*x/(840*d**8*e**9 + 6720*
d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e
**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x*
*7 + 840*e**17*x**8) + 4725*d**7*e/(840*d**8*e**9 + 6720*d**7*e**10*x + 235
20*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*
d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**
8) + 2352000*d**6*e**2*x**2*log(d/e + x)/(840*d**8*e**9 + 6720*d**7*e**10*x
 + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 +
47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**
17*x**8) + 5762400*d**6*e**2*x**2/(840*d**8*e**9 + 6720*d**7*e**10*x + 2352
0*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d
**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8
) + 37800*d**6*e**2*x/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11
*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x*
*5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) - 1665*d**
6*e**2/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d
**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2
*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 4704000*d**5*e**3*x**3*
log(d/e + x)/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 4
7040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 2352
0*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 10740800*d**5*e**
3*x**3/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d
**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2
*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 132300*d**5*e**3*x**2/(
840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**1
2*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x
```

```
**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) - 13320*d**5*e**3*x/(840*d**8*e**
9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 588
00*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d
*e**16*x**7 + 840*e**17*x**8) + 185*d**5*e**3/(840*d**8*e**9 + 6720*d**7*e*
*10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x*
*4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 84
0*e**17*x**8) + 5880000*d**4*e**4*x**4*log(d/e + x)/(840*d**8*e**9 + 6720*d
**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e*
*13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**
7 + 840*e**17*x**8) + 12250000*d**4*e**4*x**4/(840*d**8*e**9 + 6720*d**7*e*
*10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x*
*4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 84
0*e**17*x**8) + 264600*d**4*e**4*x**3/(840*d**8*e**9 + 6720*d**7*e**10*x +
23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 470
40*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*
x**8) - 46620*d**4*e**4*x**2/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**
6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e
**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 1
480*d**4*e**4*x/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2
+ 47040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 2
3520*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) - 444*d**4*e**4/
(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**
12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*
x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 4704000*d**3*e**5*x**5*log(d/e
 + x)/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d*
*5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*
e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 8624000*d**3*e**5*x**5/(
840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**1
2*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x
**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 330750*d**3*e**5*x**4/(840*d**8
*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 6223 vs. $2(418) = 836$.
time = 6.55, size = 6223, normalized size = 14.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] (100*(x*e + d)^m*m^8*x^9*e^9 + 100*(x*e + d)^m*d*m^8*x^8*e^8 - 45*(x*e + d)
^m*m^8*x^8*e^9 + 3600*(x*e + d)^m*m^7*x^9*e^9 - 45*(x*e + d)^m*d*m^8*x^7*e^
8 + 2800*(x*e + d)^m*d*m^7*x^8*e^8 - 800*(x*e + d)^m*d^2*m^7*x^7*e^7 + 111*
```

(x*e + d)^m*m^8*x^7*e^9 - 1665*(x*e + d)^m*m^7*x^8*e^9 + 54600*(x*e + d)^m*
m^6*x^9*e^9 + 111*(x*e + d)^m*d*m^8*x^6*e^8 - 1350*(x*e + d)^m*d*m^7*x^7*e^
8 + 32200*(x*e + d)^m*d*m^6*x^8*e^8 + 315*(x*e + d)^m*d^2*m^7*x^6*e^7 - 168
00*(x*e + d)^m*d^2*m^6*x^7*e^7 + 5600*(x*e + d)^m*d^3*m^6*x^6*e^6 - 37*(x*e
 + d)^m*m^8*x^6*e^9 + 4218*(x*e + d)^m*m^7*x^7*e^9 - 25830*(x*e + d)^m*m^6*
x^8*e^9 + 453600*(x*e + d)^m*m^5*x^9*e^9 - 37*(x*e + d)^m*d*m^8*x^5*e^8 + 3
552*(x*e + d)^m*d*m^7*x^6*e^8 - 16380*(x*e + d)^m*d*m^6*x^7*e^8 + 196000*(x
*e + d)^m*d*m^5*x^8*e^8 - 666*(x*e + d)^m*d^2*m^7*x^5*e^7 + 7560*(x*e + d)^
m*d^2*m^6*x^6*e^7 - 140000*(x*e + d)^m*d^2*m^5*x^7*e^7 - 1890*(x*e + d)^m*d
^3*m^6*x^5*e^6 + 84000*(x*e + d)^m*d^3*m^5*x^6*e^6 - 33600*(x*e + d)^m*d^4*
m^5*x^5*e^5 + 148*(x*e + d)^m*m^8*x^5*e^9 - 1443*(x*e + d)^m*m^7*x^6*e^9 +
67044*(x*e + d)^m*m^6*x^7*e^9 - 218610*(x*e + d)^m*m^5*x^8*e^9 + 2244900*(x
*e + d)^m*m^4*x^9*e^9 + 148*(x*e + d)^m*d*m^8*x^4*e^8 - 1258*(x*e + d)^m*d*
m^7*x^5*e^8 + 45732*(x*e + d)^m*d*m^6*x^6*e^8 - 103950*(x*e + d)^m*d*m^5*x^
7*e^8 + 676900*(x*e + d)^m*d*m^4*x^8*e^8 + 185*(x*e + d)^m*d^2*m^7*x^4*e^7
 - 17982*(x*e + d)^m*d^2*m^6*x^5*e^7 + 69300*(x*e + d)^m*d^2*m^5*x^6*e^7 - 5
88000*(x*e + d)^m*d^2*m^4*x^7*e^7 + 3330*(x*e + d)^m*d^3*m^6*x^4*e^6 - 3591
0*(x*e + d)^m*d^3*m^5*x^5*e^6 + 476000*(x*e + d)^m*d^3*m^4*x^6*e^6 + 9450*(
x*e + d)^m*d^4*m^5*x^4*e^5 - 336000*(x*e + d)^m*d^4*m^4*x^5*e^5 + 168000*(x
*e + d)^m*d^5*m^4*x^4*e^4 + 65*(x*e + d)^m*m^8*x^4*e^9 + 5920*(x*e + d)^m*m
^7*x^5*e^9 - 23532*(x*e + d)^m*m^6*x^6*e^9 + 579642*(x*e + d)^m*m^5*x^7*e^9
 - 1098405*(x*e + d)^m*m^4*x^8*e^9 + 6728400*(x*e + d)^m*m^3*x^9*e^9 + 65*(
x*e + d)^m*d*m^8*x^3*e^8 + 5328*(x*e + d)^m*d*m^7*x^4*e^8 - 17242*(x*e + d)
^m*d*m^6*x^5*e^8 + 305250*(x*e + d)^m*d*m^5*x^6*e^8 - 370755*(x*e + d)^m*d*
m^4*x^7*e^8 + 1313200*(x*e + d)^m*d*m^3*x^8*e^8 - 592*(x*e + d)^m*d^2*m^7*x
^3*e^7 + 5550*(x*e + d)^m*d^2*m^6*x^4*e^7 - 184482*(x*e + d)^m*d^2*m^5*x^5*
e^7 + 311850*(x*e + d)^m*d^2*m^4*x^6*e^7 - 1299200*(x*e + d)^m*d^2*m^3*x^7*
e^7 - 740*(x*e + d)^m*d^3*m^6*x^3*e^6 + 76590*(x*e + d)^m*d^3*m^5*x^4*e^6 -
 236250*(x*e + d)^m*d^3*m^4*x^5*e^6 + 1260000*(x*e + d)^m*d^3*m^3*x^6*e^6 -
 13320*(x*e + d)^m*d^4*m^5*x^3*e^5 + 141750*(x*e + d)^m*d^4*m^4*x^4*e^5 - 1
176000*(x*e + d)^m*d^4*m^3*x^5*e^5 - 37800*(x*e + d)^m*d^5*m^4*x^3*e^4 + 10
08000*(x*e + d)^m*d^5*m^3*x^4*e^4 - 672000*(x*e + d)^m*d^6*m^3*x^3*e^3 + 10
7*(x*e + d)^m*m^8*x^3*e^9 + 2665*(x*e + d)^m*m^7*x^4*e^9 + 99160*(x*e + d)^
m*m^6*x^5*e^9 - 208458*(x*e + d)^m*m^5*x^6*e^9 + 2965809*(x*e + d)^m*m^4*x^
7*e^9 - 3332385*(x*e + d)^m*m^3*x^8*e^9 + 11812400*(x*e + d)^m*m^2*x^9*e^9
 + 107*(x*e + d)^m*d*m^8*x^2*e^8 + 2470*(x*e + d)^m*d*m^7*x^3*e^8 + 77848*(x
*e + d)^m*d*m^6*x^4*e^8 - 122248*(x*e + d)^m*d*m^5*x^5*e^8 + 1134309*(x*e +
 d)^m*d*m^4*x^6*e^8 - 737100*(x*e + d)^m*d*m^3*x^7*e^8 + 1306800*(x*e + d)^
m*d*m^2*x^8*e^8 - 195*(x*e + d)^m*d^2*m^7*x^2*e^7 - 19536*(x*e + d)^m*d^2*m
^6*x^3*e^7 + 64010*(x*e + d)^m*d^2*m^5*x^4*e^7 - 909090*(x*e + d)^m*d^2*m^4
*x^5*e^7 + 724185*(x*e + d)^m*d^2*m^3*x^6*e^7 - 1411200*(x*e + d)^m*d^2*m^2
*x^7*e^7 + 1776*(x*e + d)^m*d^3*m^6*x^2*e^6 - 19980*(x*e + d)^m*d^3*m^5*x^3
e^6 + 616050(x*e + d)^m*d^3*m^4*x^4*e^6 - 689850*(x*e + d)^m*d^3*m^3*x^5*
e^6 + 1534400*(x*e + d)^m*d^3*m^2*x^6*e^6 + 2220*(x*e + d)^m*d^4*m^5*x^2*e^
5 - 266400*(x*e + d)^m*d^4*m^4*x^3*e^5 + 614250*(x*e + d)^m*d^4*m^3*x^4*e^5

```
 - 1680000*(x*e + d)^m*d^4*m^2*x^5*e^5 + 39960*(x*e + d)^m*d^5*m^4*x^2*e^4
- 453600*(x*e + d)^m*d^5*m^3*x^3*e^4 + 1848000*(x*e + d)^m*d^5*m^2*x^4*e^4
+ 113400*(x*e + d)^m*d^6*m^3*x^2*e^3 - 2016000*(x*e + d)^m*d^6*m^2*x^3*e^3
+ 2016000*(x*e + d)^m*d^7*m^2*x^2*e^2 + 33*(x*e + d)^m*m^8*x^2*e^9 + 4494*(
x*e + d)^m*m^7*x^3*e^9 + 45890*(x*e + d)^m*m^6*x^4*e^9 + 902800*(x*e + d)^m
*m^5*x^5*e^9 - 1090353*(x*e + d)^m*m^4*x^6*e^9 + 9134412*(x*e + d)^m*m^3*x^
7*e^9 - 5906520*(x*e + d)^m*m^2*x^8*e^9 + 10958400*(x*e + d)^m*m*x^9*e^9 +
33*(x*e + d)^m*d*m^8*x*e^8 + 4280*(x*e + d)^m*d*m^7*x^2*e^8 + 38480*(x*e +
d)^m*d*m^6*x^3*e^8 + 591408*(x*e + d)^m*d*m^5*x^4*e^8 - 479113*(x*e + d)^m*
d*m^4*x^5*e^8 + 2328558*(x*e + d)^m*d*m^3*x^6*e^8 - 746820*(x*e + d)^m*d*m^
2*x^7*e^8 + 504000*(x*e + d)^m*d*m*x^8*e^8 - 214*(x*e + d)^m*d^2*m^7*x*e^7
- 7020*(x*e + d)^m*d^2*m^6*x^2*e^7 - 252784*(x*e + d)^m*d^2*m^5*x^3*e^7 + 3
55200*(x*e + d)^m*d^2*m^4*x^4*e^7 - 2260404*(x*e + d)^m*d^2*m^3*x^5*e^7 + 8
14590*(x*e + d)^m*d^2*m^2*x^6*e^7 - 576000*(x*e + d)^m*d^2*m*x^7*e^7 + 390*
(x*e + d)^m*d^3*m^6*x*e^6 + 55056*(x*e + d)^m*d^3*m^5*x^2*e^6 - 196100*(x*e
 + d)^m*d^3*m^4*x^3*e^6 + 2081250*(x*e + d)^m*d^3*m^3*x^4*e^6 - 895860*(x*e
 + d)^m*d^3*m^2*x^5*e^6 + 672000*(x*e + d)^m*d^3*m*x^6*e^6 - 3552*(x*e + d)
^m*d^4*m^5*x*e^5 + 55500*(x*e + d)^m*d^4*m^4*x^2*e^5 - 1665000*(x*e + d)^m*
d^4*m^3*x^3*e^5 + 992250*(x*e + d)^m*d^4*m^2*x^...
```

Mupad [B]
time = 6.05, size = 2500, normalized size = 5.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] ((d + e*x)^m*(6531840*d*e^8 + 2041200*d^8*e + 4032000*d^9 - 5987520*d^2*e^7
 + 12942720*d^3*e^6 - 5896800*d^4*e^5 + 10741248*d^5*e^4 + 2237760*d^6*e^3
+ 5754240*d^7*e^2 - 7957224*d^2*e^7*m + 12886224*d^3*e^6*m - 4396860*d^4*e^
5*m + 5860800*d^5*e^4*m + 848040*d^6*e^3*m + 1358640*d^7*e^2*m + 9162072*d*
e^8*m^2 + 3864168*d*e^8*m^3 + 983682*d*e^8*m^4 + 155232*d*e^8*m^5 + 14868*d
*e^8*m^6 + 792*d*e^8*m^7 + 18*d*e^8*m^8 - 4419954*d^2*e^7*m^2 + 5258836*d^3
*e^6*m^2 - 1296750*d^4*e^5*m^2 + 1189920*d^5*e^4*m^2 + 106560*d^6*e^3*m^2 +
 79920*d^7*e^2*m^2 - 1332177*d^2*e^7*m^3 + 1126710*d^3*e^6*m^3 - 189150*d^4
*e^5*m^3 + 106560*d^5*e^4*m^3 + 4440*d^6*e^3*m^3 - 235620*d^2*e^7*m^4 + 133
750*d^3*e^6*m^4 - 13650*d^4*e^5*m^4 + 3552*d^5*e^4*m^4 - 24486*d^2*e^7*m^5
+ 8346*d^3*e^6*m^5 - 390*d^4*e^5*m^5 - 1386*d^2*e^7*m^6 + 214*d^3*e^6*m^6 -
 33*d^2*e^7*m^7 + 11946528*d*e^8*m + 226800*d^8*e*m))/(e^9*(1026576*m + 117
2700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^
8 + m^9 + 362880)) + (100*x^9*(d + e*x)^m*(109584*m + 118124*m^2 + 67284*m^
3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320))/(1026576*m + 11
72700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m
^8 + m^9 + 362880) + (x*(d + e*x)^m*(11946528*e^9*m + 6531840*e^9 + 9162072
```

*e^9*m^2 + 3864168*e^9*m^3 + 983682*e^9*m^4 + 155232*e^9*m^5 + 14868*e^9*m^6 + 792*e^9*m^7 + 18*e^9*m^8 - 12942720*d^2*e^7*m + 5896800*d^3*e^6*m - 10741248*d^4*e^5*m - 2237760*d^5*e^4*m - 5754240*d^6*e^3*m - 2041200*d^7*e^2*m + 7957224*d*e^8*m^2 + 4419954*d*e^8*m^3 + 1332177*d*e^8*m^4 + 235620*d*e^8*m^5 + 24486*d*e^8*m^6 + 1386*d*e^8*m^7 + 33*d*e^8*m^8 - 12886224*d^2*e^7*m^2 + 4396860*d^3*e^6*m^2 - 5860800*d^4*e^5*m^2 - 848040*d^5*e^4*m^2 - 1358640*d^6*e^3*m^2 - 226800*d^7*e^2*m^2 - 5258836*d^2*e^7*m^3 + 1296750*d^3*e^6*m^3 - 1189920*d^4*e^5*m^3 - 106560*d^5*e^4*m^3 - 79920*d^6*e^3*m^3 - 1126710*d^2*e^7*m^4 + 189150*d^3*e^6*m^4 - 106560*d^4*e^5*m^4 - 4440*d^5*e^4*m^4 - 133750*d^2*e^7*m^5 + 13650*d^3*e^6*m^5 - 3552*d^4*e^5*m^5 - 8346*d^2*e^7*m^6 + 390*d^3*e^6*m^6 - 214*d^2*e^7*m^7 + 5987520*d*e^8*m - 4032000*d^8*e*m))/(e^9*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) - (x^5*(d + e*x)^m*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)*(33600*d^4*m - 244200*e^4*m - 447552*e^4 - 49580*e^4*m^2 - 4440*e^4*m^3 - 148*e^4*m^4 + 47952*d^2*e^2*m + 7067*d*e^3*m^2 + 1890*d^3*e*m^2 + 888*d*e^3*m^3 + 37*d*e^3*m^4 + 11322*d^2*e^2*m^2 + 666*d^2*e^2*m^3 + 18648*d*e^3*m + 17010*d^3*e*m))/(e^4*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) - (x^7*(d + e*x)^m*(800*d^2*m - 1887*e^2*m - 7992*e^2 - 111*e^2*m^2 + 405*d*e*m + 45*d*e*m^2)*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))/(e^2*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) + (x^6*(d + e*x)^m*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)*(5600*d^3*m - 7067*e^3*m - 18648*e^3 - 888*e^3*m^2 - 37*e^3*m^3 + 1887*d*e^2*m^2 + 315*d^2*e*m^2 + 111*d*e^2*m^3 + 7992*d*e^2*m + 2835*d^2*e*m))/(e^3*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) + (x^4*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6)*(168000*d^5*m + 732810*e^5*m + 982800*e^5 + 216125*e^5*m^2 + 31525*e^5*m^3 + 2275*e^5*m^4 + 65*e^5*m^5 + 93240*d^2*e^3*m + 239760*d^3*e^2*m + 244200*d*e^4*m^2 + 9450*d^4*e*m^2 + 49580*d*e^4*m^3 + 4440*d*e^4*m^4 + 148*d*e^4*m^5 + 35335*d^2*e^3*m^2 + 56610*d^3*e^2*m^2 + 4440*d^2*e^3*m^3 + 3330*d^3*e^2*m^3 + 185*d^2*e^3*m^4 + 447552*d*e^4*m + 85050*d^4*e*m))/(e^5*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) - (5*x^8*(d + e*x)^m*(81*e - 20*d*m + 9*e*m)*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040))/(e*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) + (x^2*(m + 1)*(d + e*x)^m*(2016000*d^7*m + 7957224*e^7*m + 5987520*e^7 + 4419954*e^7*m^2 + 1332177*e^7*m^3 + 235620*e^7*m^4 + 24486*e^7*m^5 + 1386*e^7*m^6 + 33*e^7*m^7 - 2948400*d^2*e^5*m + 5370624*d^3*e^4*m + 1118880*d^4*e^3*m + 2877120*d^5*e^2*m + 6443112*d*e^6*m^2 + 113400*d^6*e*m^2 + 2629418*d*e^6*m^3 + 563355*d*e^6*m^4 + 66875*d*e^6*m^5 + 4173*d*e^6*m^6 + 107*d*e^6*m^7 - 2198430*d^2*e^5*m^2 + 2930400*d^3*e^4*m^2 + 424020*d^4*e^3*m^2 + 679320*d^5*e^2*m^2 - 648375*d^2*e^5*m^3 + 594960*d^3*e^4*m^3 + 53280*d^4*e^3*m^3 + 39960*d^5*e^2*m^3 - 94575*d^2*e^5*m^4 + 53280*d^3*e^4*m^4 + 2220*d^4*e^3*m^4 - 6825*d^2*e^5*m^5 + 1776*d^3*e^4*m^5 - 195*d^2*e^5*m^6 + 6471360

```
*d*e^6*m + 1020600*d^6*e*m))/(e^7*(1026576*m + 1172700*m^2 + 723680*m^3 + 2
69325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) - (x^3
*(d + e*x)^m*(3*m + m^2 + 2)*(672000*d^6*m - 6443112*e^6*m - 6471360*e^6 -
2629418*e^6*m^2 - 563355*e^6*m^3 - 66875*e^6*m^4 - 4173*e^6*m^5 - 107*e^6*m
^6 + 1790208*d^2*e^4*m + 372960*d^3*e^3*m + 959...
```

3.369 $\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right)$

Optimal. Leaf size=292

$$\frac{\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^{1+m}}{e^7(1+m)}-\frac{\left(120d^5+85d^4e+68d^3e^2+12d^2e^3+42d\right.}{e^7(2+m)}$$

```
[Out] (5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/e^7
/(1+m)-(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5)*(e*x+d)^(2+m
)/e^7/(2+m)+(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+21*e^4)*(e*x+d)^(3+m)/e
^7/(3+m)-2*(200*d^3+85*d^2*e+34*d*e^2+2*e^3)*(e*x+d)^(4+m)/e^7/(4+m)+(300*d
^2+85*d*e+17*e^2)*(e*x+d)^(5+m)/e^7/(5+m)-(120*d+17*e)*(e*x+d)^(6+m)/e^7/(6
+m)+20*(e*x+d)^(7+m)/e^7/(7+m)
```

Rubi [A]
time = 0.10, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1642}

$$\frac{\left(300d^2+85de+17e^2\right)(d+ex)^{m+5}}{e^7(m+5)}-\frac{2\left(200d^3+85d^2e+34de^2+2e^3\right)(d+ex)^{m+4}}{e^7(m+4)}+\frac{\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^{m+1}}{e^7(m+1)}+\frac{\left(300d^4+170d^3e+102d^2e^2+12de^3+21e^4\right)(d+ex)^{m+3}}{e^7(m+3)}-\frac{\left(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5\right)(d+ex)^{m+2}}{e^7(m+2)}-\frac{(120d+17e)(d+ex)^{m+6}}{e^7(m+6)}+\frac{20(d+ex)^{m+7}}{e^7(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d +
 e*x)^(1 + m))/(e^7*(1 + m)) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e
^3 + 42*d*e^4 - 7*e^5)*(d + e*x)^(2 + m))/(e^7*(2 + m)) + ((300*d^4 + 170*d
^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*(d + e*x)^(3 + m))/(e^7*(3 + m)) -
(2*(200*d^3 + 85*d^2*e + 34*d*e^2 + 2*e^3)*(d + e*x)^(4 + m))/(e^7*(4 + m))
 + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^(5 + m))/(e^7*(5 + m)) - ((120*d
+ 17*e)*(d + e*x)^(6 + m))/(e^7*(6 + m)) + (20*(d + e*x)^(7 + m))/(e^7*(7 +
 m))
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^m \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx = \int \left(\frac{\left(20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e\right.}{e^6}\right.$$

$$= \frac{\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3e+3d^2e^2-de^3\right.}{e^7(1+m)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 743 vs. 2(292) = 584.
time = 0.34, size = 743, normalized size = 2.54

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((d + e*x)^(1 + m)*(14400*d^6 - 120*d^5*e*(-119 + 120*x + m*(-17 + 120*x))
+ 24*d^4*e^2*(714 - 595*x + 600*x^2 + m^2*(17 - 85*x + 300*x^2) + m*(221 -
680*x + 900*x^2)) - 12*d^3*e^3*(m^3*(-2 + 34*x - 85*x^2 + 200*x^3) + 2*(-21
0 + 714*x - 595*x^2 + 600*x^3) + 2*m^2*(-18 + 238*x - 425*x^2 + 600*x^3) +
m*(-214 + 1870*x - 1955*x^2 + 2200*x^3)) + 2*d^2*e^4*(m^4*(21 - 12*x + 102*
x^2 - 170*x^3 + 300*x^4) + 12*(1470 - 210*x + 714*x^2 - 595*x^3 + 600*x^4)
+ 2*m^3*(231 - 114*x + 816*x^2 - 1105*x^3 + 1500*x^4) + 2*m*(6699 - 1902*x
+ 7752*x^2 - 7055*x^3 + 7500*x^4) + m^2*(3759 - 1500*x + 8466*x^2 - 9010*x^
3 + 10500*x^4)) - d*e^5*(m^5*(7 + 42*x - 12*x^2 + 68*x^3 - 85*x^4 + 120*x^5
) + 24*(735 + 1470*x - 210*x^2 + 714*x^3 - 595*x^4 + 600*x^5) + m^4*(175 +
966*x - 252*x^2 + 1292*x^3 - 1445*x^4 + 1800*x^5) + m^3*(1715 + 8442*x - 19
56*x^2 + 8908*x^3 - 8925*x^4 + 10200*x^5) + 2*m*(9639 + 31038*x - 5064*x^2
+ 18360*x^3 - 15895*x^4 + 16440*x^5) + m^2*(8225 + 34314*x - 6804*x^2 + 272
68*x^3 - 25075*x^4 + 27000*x^5)) + e^6*(m^6*(6 + 7*x + 21*x^2 - 4*x^3 + 17*
x^4 - 17*x^5 + 20*x^6) + m^5*(162 + 182*x + 525*x^2 - 96*x^3 + 391*x^4 - 37
4*x^5 + 420*x^6) + 24*(1260 + 735*x + 1470*x^2 - 210*x^3 + 714*x^4 - 595*x^
5 + 600*x^6) + m^4*(1770 + 1890*x + 5187*x^2 - 904*x^3 + 3519*x^4 - 3230*x^
5 + 3500*x^6) + m^3*(9990 + 9940*x + 25599*x^2 - 4224*x^3 + 15725*x^4 - 139
40*x^5 + 14700*x^6) + 2*m*(24084 + 18459*x + 39858*x^2 - 5904*x^3 + 20502*x
^4 - 17323*x^5 + 17640*x^6) + m^2*(30624 + 27503*x + 65352*x^2 - 10180*x^3
+ 36448*x^4 - 31433*x^5 + 32480*x^6))))/(e^7*(1 + m)*(2 + m)*(3 + m)*(4 + m
)*(5 + m)*(6 + m)*(7 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1219 vs. 2(292) = 584.
time = 0.10, size = 1220, normalized size = 4.18 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE
)
```

```
[Out] d*(6*e^6*m^6-7*d*e^5*m^5+162*e^6*m^5+42*d^2*e^4*m^4-175*d*e^5*m^4+1770*e^6*
m^4+24*d^3*e^3*m^3+924*d^2*e^4*m^3-1715*d*e^5*m^3+9990*e^6*m^3+408*d^4*e^2*
m^2+432*d^3*e^3*m^2+7518*d^2*e^4*m^2-8225*d*e^5*m^2+30624*e^6*m^2+2040*d^5*
e*m+5304*d^4*e^2*m+2568*d^3*e^3*m+26796*d^2*e^4*m-19278*d*e^5*m+48168*e^6*m
+14400*d^6+14280*d^5*e+17136*d^4*e^2+5040*d^3*e^3+35280*d^2*e^4-17640*d*e^5
+30240*e^6)/e^7/(m^7+28*m^6+322*m^5+1960*m^4+6769*m^3+13132*m^2+13068*m+504
```

```
0)*exp(m*ln(e*x+d))+(20*d*m-17*e*m-119*e)/e/(m^2+13*m+42)*x^6*exp(m*ln(e*x+
d))+(17*d*e^2*m^3-4*e^3*m^3+85*d^2*e*m^2+221*d*e^2*m^2-72*e^3*m^2+600*d^3*m
+595*d^2*e*m+714*d*e^2*m-428*e^3*m-840*e^3)/e^3/(m^4+22*m^3+179*m^2+638*m+8
40)*x^4*exp(m*ln(e*x+d))+(21*d*e^4*m^5+7*e^5*m^5+12*d^2*e^3*m^4+462*d*e^4*m
^4+175*e^5*m^4+204*d^3*e^2*m^3+216*d^2*e^3*m^3+3759*d*e^4*m^3+1715*e^5*m^3+
1020*d^4*e*m^2+2652*d^3*e^2*m^2+1284*d^2*e^3*m^2+13398*d*e^4*m^2+8225*e^5*m
^2+7200*d^5*m+7140*d^4*e*m+8568*d^3*e^2*m+2520*d^2*e^3*m+17640*d*e^4*m+1927
8*e^5*m+17640*e^5)/e^5/(m^6+27*m^5+295*m^4+1665*m^3+5104*m^2+8028*m+5040)*x
^2*exp(m*ln(e*x+d))+20/(7+m)*x^7*exp(m*ln(e*x+d))-(17*d*e*m^2-17*e^2*m^2+12
0*d^2*m+119*d*e*m-221*e^2*m-714*e^2)/e^2/(m^3+18*m^2+107*m+210)*x^5*exp(m*l
n(e*x+d))-(4*d*e^3*m^4-21*e^4*m^4+68*d^2*e^2*m^3+72*d*e^3*m^3-462*e^4*m^3+3
40*d^3*e*m^2+884*d^2*e^2*m^2+428*d*e^3*m^2-3759*e^4*m^2+2400*d^4*m+2380*d^3
*e*m+2856*d^2*e^2*m+840*d*e^3*m-13398*e^4*m-17640*e^4)/e^4/(m^5+25*m^4+245*
m^3+1175*m^2+2754*m+2520)*x^3*exp(m*ln(e*x+d))-(-7*d*e^5*m^6-6*e^6*m^6+42*d
^2*e^4*m^5-175*d*e^5*m^5-162*e^6*m^5+24*d^3*e^3*m^4+924*d^2*e^4*m^4-1715*d*
e^5*m^4-1770*e^6*m^4+408*d^4*e^2*m^3+432*d^3*e^3*m^3+7518*d^2*e^4*m^3-8225*
d*e^5*m^3-9990*e^6*m^3+2040*d^5*e*m^2+5304*d^4*e^2*m^2+2568*d^3*e^3*m^2+267
96*d^2*e^4*m^2-19278*d*e^5*m^2-30624*e^6*m^2+14400*d^6*m+14280*d^5*e*m+1713
6*d^4*e^2*m+5040*d^3*e^3*m+35280*d^2*e^4*m-17640*d*e^5*m-48168*e^6*m-30240*
e^6)/e^6/(m^7+28*m^6+322*m^5+1960*m^4+6769*m^3+13132*m^2+13068*m+5040)*x*ex
p(m*ln(e*x+d))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 785 vs. $2(285) = 570$.
time = 0.32, size = 785, normalized size = 2.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 6*(x*e + d)^(m + 1)*e^(-1)/(m + 1) + 7*((m + 1)*x^2*e^2 + d*m*x*e - d^2)*e^
(m*log(x*e + d) - 2)/(m^2 + 3*m + 2) + 21*((m^2 + 3*m + 2)*x^3*e^3 + (m^2 +
 m)*d*x^2*e^2 - 2*d^2*m*x*e + 2*d^3)*e^(m*log(x*e + d) - 3)/(m^3 + 6*m^2 +
11*m + 6) - 4*((m^3 + 6*m^2 + 11*m + 6)*x^4*e^4 + (m^3 + 3*m^2 + 2*m)*d*x^3
*e^3 - 3*(m^2 + m)*d^2*x^2*e^2 + 6*d^3*m*x*e - 6*d^4)*e^(m*log(x*e + d) - 4
)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24) + 17*((m^4 + 10*m^3 + 35*m^2 + 50*m +
 24)*x^5*e^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*x^4*e^4 - 4*(m^3 + 3*m^2 + 2*
m)*d^2*x^3*e^3 + 12*(m^2 + m)*d^3*x^2*e^2 - 24*d^4*m*x*e + 24*d^5)*e^(m*log
(x*e + d) - 5)/(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120) - 17*((m^5 +
 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*x^6*e^6 + (m^5 + 10*m^4 + 35*m^3
+ 50*m^2 + 24*m)*d*x^5*e^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*x^4*e^4 + 2
0*(m^3 + 3*m^2 + 2*m)*d^3*x^3*e^3 - 60*(m^2 + m)*d^4*x^2*e^2 + 120*d^5*m*x*
e - 120*d^6)*e^(m*log(x*e + d) - 6)/(m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 162
```

```
4*m^2 + 1764*m + 720) + 20*((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 +
1764*m + 720)*x^7*e^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)
*d*x^6*e^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*x^5*e^5 + 30*(m^
4 + 6*m^3 + 11*m^2 + 6*m)*d^3*x^4*e^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*x^3*e^3
 + 360*(m^2 + m)*d^5*x^2*e^2 - 720*d^6*m*x*e + 720*d^7)*e^(m*log(x*e + d) -
 7)/(m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5
040)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. $2(285) = 570$.
time = 0.35, size = 1190, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] (14400*d^7 + (20*(m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 72
0)*x^7 - 17*(m^6 + 22*m^5 + 190*m^4 + 820*m^3 + 1849*m^2 + 2038*m + 840)*x^
6 + 17*(m^6 + 23*m^5 + 207*m^4 + 925*m^3 + 2144*m^2 + 2412*m + 1008)*x^5 -
4*(m^6 + 24*m^5 + 226*m^4 + 1056*m^3 + 2545*m^2 + 2952*m + 1260)*x^4 + 21*(
m^6 + 25*m^5 + 247*m^4 + 1219*m^3 + 3112*m^2 + 3796*m + 1680)*x^3 + 7*(m^6
+ 26*m^5 + 270*m^4 + 1420*m^3 + 3929*m^2 + 5274*m + 2520)*x^2 + 6*(m^6 + 27
*m^5 + 295*m^4 + 1665*m^3 + 5104*m^2 + 8028*m + 5040)*x)*e^7 + (6*d*m^6 + 2
0*(d*m^6 + 15*d*m^5 + 85*d*m^4 + 225*d*m^3 + 274*d*m^2 + 120*d*m)*x^6 + 162
*d*m^5 - 17*(d*m^6 + 17*d*m^5 + 105*d*m^4 + 295*d*m^3 + 374*d*m^2 + 168*d*m
)*x^5 + 1770*d*m^4 + 17*(d*m^6 + 19*d*m^5 + 131*d*m^4 + 401*d*m^3 + 540*d*m
^2 + 252*d*m)*x^4 + 9990*d*m^3 - 4*(d*m^6 + 21*d*m^5 + 163*d*m^4 + 567*d*m^
3 + 844*d*m^2 + 420*d*m)*x^3 + 30624*d*m^2 + 21*(d*m^6 + 23*d*m^5 + 201*d*m
^4 + 817*d*m^3 + 1478*d*m^2 + 840*d*m)*x^2 + 48168*d*m + 7*(d*m^6 + 25*d*m^
5 + 245*d*m^4 + 1175*d*m^3 + 2754*d*m^2 + 2520*d*m)*x + 30240*d)*e^6 - (7*d
^2*m^5 + 175*d^2*m^4 + 120*(d^2*m^5 + 10*d^2*m^4 + 35*d^2*m^3 + 50*d^2*m^2
+ 24*d^2*m)*x^5 + 1715*d^2*m^3 - 85*(d^2*m^5 + 13*d^2*m^4 + 53*d^2*m^3 + 83
*d^2*m^2 + 42*d^2*m)*x^4 + 8225*d^2*m^2 + 68*(d^2*m^5 + 16*d^2*m^4 + 83*d^2
*m^3 + 152*d^2*m^2 + 84*d^2*m)*x^3 + 19278*d^2*m - 12*(d^2*m^5 + 19*d^2*m^4
 + 125*d^2*m^3 + 317*d^2*m^2 + 210*d^2*m)*x^2 + 17640*d^2 + 42*(d^2*m^5 + 2
2*d^2*m^4 + 179*d^2*m^3 + 638*d^2*m^2 + 840*d^2*m)*x)*e^5 + 2*(21*d^3*m^4 +
 462*d^3*m^3 + 3759*d^3*m^2 + 300*(d^3*m^4 + 6*d^3*m^3 + 11*d^3*m^2 + 6*d^3
*m)*x^4 + 13398*d^3*m - 170*(d^3*m^4 + 10*d^3*m^3 + 23*d^3*m^2 + 14*d^3*m)*
x^3 + 17640*d^3 + 102*(d^3*m^4 + 14*d^3*m^3 + 55*d^3*m^2 + 42*d^3*m)*x^2 -
12*(d^3*m^4 + 18*d^3*m^3 + 107*d^3*m^2 + 210*d^3*m)*x)*e^4 + 12*(2*d^4*m^3
+ 36*d^4*m^2 + 214*d^4*m + 420*d^4 - 200*(d^4*m^3 + 3*d^4*m^2 + 2*d^4*m)*x^
3 + 85*(d^4*m^3 + 8*d^4*m^2 + 7*d^4*m)*x^2 - 34*(d^4*m^3 + 13*d^4*m^2 + 42*
d^4*m)*x)*e^3 + 24*(17*d^5*m^2 + 221*d^5*m + 714*d^5 + 300*(d^5*m^2 + d^5*m
```

```
)*x^2 - 85*(d^5*m^2 + 7*d^5*m)*x)*e^2 - 120*(120*d^6*m*x - 17*d^6*m - 119*d
^6)*e)*(x*e + d)^m*e^(-7)/(m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 1
3132*m^2 + 13068*m + 5040)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26165 vs. $2(272) = 544$.
time = 5.38, size = 26165, normalized size = 89.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] Piecewise((d**m*(20*x**7/7 - 17*x**6/6 + 17*x**5/5 - x**4 + 7*x**3 + 7*x**2
/2 + 6*x), Eq(e, 0)), (1200*d**6*log(d/e + x)/(60*d**6*e**7 + 360*d**5*e**8
*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*
d*e**12*x**5 + 60*e**13*x**6) + 2940*d**6/(60*d**6*e**7 + 360*d**5*e**8*x +
 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e*
*12*x**5 + 60*e**13*x**6) + 7200*d**5*e*x*log(d/e + x)/(60*d**6*e**7 + 360*
d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x*
*4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 16440*d**5*e*x/(60*d**6*e**7 + 360
*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x
**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 170*d**5*e/(60*d**6*e**7 + 360*d*
*5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4
 + 360*d*e**12*x**5 + 60*e**13*x**6) + 18000*d**4*e**2*x**2*log(d/e + x)/(6
0*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 +
 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 37500*d**4*e**2*
x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10
*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 1020*d**4
*e**2*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e*
*10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 34*d**
4*e**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**
10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 24000*d
**3*e**3*x**3*log(d/e + x)/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*
x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e
**13*x**6) + 44000*d**3*e**3*x**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**
4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5
 + 60*e**13*x**6) + 2550*d**3*e**3*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 9
00*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**1
2*x**5 + 60*e**13*x**6) - 204*d**3*e**3*x/(60*d**6*e**7 + 360*d**5*e**8*x +
 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e*
*12*x**5 + 60*e**13*x**6) + 4*d**3*e**3/(60*d**6*e**7 + 360*d**5*e**8*x + 9
00*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**1
2*x**5 + 60*e**13*x**6) + 18000*d**2*e**4*x**4*log(d/e + x)/(60*d**6*e**7 +
 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**
```

```
11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 27000*d**2*e**4*x**4/(60*d**6
*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d
**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 3400*d**2*e**4*x**3/(6
0*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 +
 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 510*d**2*e**4*x*
*2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x
**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 24*d**2*e**
4*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*
x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 21*d**2*e*
*4/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x
**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 7200*d*e**5
*x**5*log(d/e + x)/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1
200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**
6) + 7200*d*e**5*x**5/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2
+ 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*
x**6) + 2550*d*e**5*x**4/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x*
*2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**
13*x**6) - 680*d*e**5*x**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*
x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e
**13*x**6) + 60*d*e**5*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9
*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*
e**13*x**6) - 126*d*e**5*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*
x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e
**13*x**6) - 14*d*e**5/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2
 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13
*x**6) + 1200*e**6*x**6*log(d/e + x)/(60*d**6*e**7 + 360*d**5*e**8*x + 900*
d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x
**5 + 60*e**13*x**6) + 1020*e**6*x**5/(60*d**6*e**7 + 360*d**5*e**8*x + 900
*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*
x**5 + 60*e**13*x**6) - 510*e**6*x**4/(60*d**6*e**7 + 360*d**5*e**8*x + 900
*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*
x**5 + 60*e**13*x**6) + 80*e**6*x**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900*
d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x
**5 + 60*e**13*x**6) - 315*e**6*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*
d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3098 vs. $2(285) = 570$.
time = 3.89, size = 3098, normalized size = 10.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

[Out] (20*(x*e + d)^m*m^6*x^7*e^7 + 20*(x*e + d)^m*d*m^6*x^6*e^6 - 17*(x*e + d)^m
*m^6*x^6*e^7 + 420*(x*e + d)^m*m^5*x^7*e^7 - 17*(x*e + d)^m*d*m^6*x^5*e^6 +
 300*(x*e + d)^m*d*m^5*x^6*e^6 - 120*(x*e + d)^m*d^2*m^5*x^5*e^5 + 17*(x*e
+ d)^m*m^6*x^5*e^7 - 374*(x*e + d)^m*m^5*x^6*e^7 + 3500*(x*e + d)^m*m^4*x^7
e^7 + 17(x*e + d)^m*d*m^6*x^4*e^6 - 289*(x*e + d)^m*d*m^5*x^5*e^6 + 1700*
(x*e + d)^m*d*m^4*x^6*e^6 + 85*(x*e + d)^m*d^2*m^5*x^4*e^5 - 1200*(x*e + d)
^m*d^2*m^4*x^5*e^5 + 600*(x*e + d)^m*d^3*m^4*x^4*e^4 - 4*(x*e + d)^m*m^6*x^
4*e^7 + 391*(x*e + d)^m*m^5*x^5*e^7 - 3230*(x*e + d)^m*m^4*x^6*e^7 + 14700*
(x*e + d)^m*m^3*x^7*e^7 - 4*(x*e + d)^m*d*m^6*x^3*e^6 + 323*(x*e + d)^m*d*m
^5*x^4*e^6 - 1785*(x*e + d)^m*d*m^4*x^5*e^6 + 4500*(x*e + d)^m*d*m^3*x^6*e^
6 - 68*(x*e + d)^m*d^2*m^5*x^3*e^5 + 1105*(x*e + d)^m*d^2*m^4*x^4*e^5 - 420
0*(x*e + d)^m*d^2*m^3*x^5*e^5 - 340*(x*e + d)^m*d^3*m^4*x^3*e^4 + 3600*(x*e
 + d)^m*d^3*m^3*x^4*e^4 - 2400*(x*e + d)^m*d^4*m^3*x^3*e^3 + 21*(x*e + d)^m
*m^6*x^3*e^7 - 96*(x*e + d)^m*m^5*x^4*e^7 + 3519*(x*e + d)^m*m^4*x^5*e^7 -
13940*(x*e + d)^m*m^3*x^6*e^7 + 32480*(x*e + d)^m*m^2*x^7*e^7 + 21*(x*e + d
)^m*d*m^6*x^2*e^6 - 84*(x*e + d)^m*d*m^5*x^3*e^6 + 2227*(x*e + d)^m*d*m^4*x
^4*e^6 - 5015*(x*e + d)^m*d*m^3*x^5*e^6 + 5480*(x*e + d)^m*d*m^2*x^6*e^6 +
12*(x*e + d)^m*d^2*m^5*x^2*e^5 - 1088*(x*e + d)^m*d^2*m^4*x^3*e^5 + 4505*(x
*e + d)^m*d^2*m^3*x^4*e^5 - 6000*(x*e + d)^m*d^2*m^2*x^5*e^5 + 204*(x*e + d
)^m*d^3*m^4*x^2*e^4 - 3400*(x*e + d)^m*d^3*m^3*x^3*e^4 + 6600*(x*e + d)^m*d
^3*m^2*x^4*e^4 + 1020*(x*e + d)^m*d^4*m^3*x^2*e^3 - 7200*(x*e + d)^m*d^4*m^
2*x^3*e^3 + 7200*(x*e + d)^m*d^5*m^2*x^2*e^2 + 7*(x*e + d)^m*m^6*x^2*e^7 +
525*(x*e + d)^m*m^5*x^3*e^7 - 904*(x*e + d)^m*m^4*x^4*e^7 + 15725*(x*e + d)
^m*m^3*x^5*e^7 - 31433*(x*e + d)^m*m^2*x^6*e^7 + 35280*(x*e + d)^m*m*x^7*e^
7 + 7*(x*e + d)^m*d*m^6*x*e^6 + 483*(x*e + d)^m*d*m^5*x^2*e^6 - 652*(x*e +
d)^m*d*m^4*x^3*e^6 + 6817*(x*e + d)^m*d*m^3*x^4*e^6 - 6358*(x*e + d)^m*d*m^
2*x^5*e^6 + 2400*(x*e + d)^m*d*m*x^6*e^6 - 42*(x*e + d)^m*d^2*m^5*x*e^5 + 2
28*(x*e + d)^m*d^2*m^4*x^2*e^5 - 5644*(x*e + d)^m*d^2*m^3*x^3*e^5 + 7055*(x
*e + d)^m*d^2*m^2*x^4*e^5 - 2880*(x*e + d)^m*d^2*m*x^5*e^5 - 24*(x*e + d)^m
*d^3*m^4*x*e^4 + 2856*(x*e + d)^m*d^3*m^3*x^2*e^4 - 7820*(x*e + d)^m*d^3*m^
2*x^3*e^4 + 3600*(x*e + d)^m*d^3*m*x^4*e^4 - 408*(x*e + d)^m*d^4*m^3*x*e^3
+ 8160*(x*e + d)^m*d^4*m^2*x^2*e^3 - 4800*(x*e + d)^m*d^4*m*x^3*e^3 - 2040*
(x*e + d)^m*d^5*m^2*x*e^2 + 7200*(x*e + d)^m*d^5*m*x^2*e^2 - 14400*(x*e + d
)^m*d^6*m*x*e + 6*(x*e + d)^m*m^6*x*e^7 + 182*(x*e + d)^m*m^5*x^2*e^7 + 518
7*(x*e + d)^m*m^4*x^3*e^7 - 4224*(x*e + d)^m*m^3*x^4*e^7 + 36448*(x*e + d)^
m*m^2*x^5*e^7 - 34646*(x*e + d)^m*m*x^6*e^7 + 14400*(x*e + d)^m*x^7*e^7 + 6
*(x*e + d)^m*d*m^6*e^6 + 175*(x*e + d)^m*d*m^5*x*e^6 + 4221*(x*e + d)^m*d*m
^4*x^2*e^6 - 2268*(x*e + d)^m*d*m^3*x^3*e^6 + 9180*(x*e + d)^m*d*m^2*x^4*e^
6 - 2856*(x*e + d)^m*d*m*x^5*e^6 - 7*(x*e + d)^m*d^2*m^5*e^5 - 924*(x*e + d
)^m*d^2*m^4*x*e^5 + 1500*(x*e + d)^m*d^2*m^3*x^2*e^5 - 10336*(x*e + d)^m*d^
2*m^2*x^3*e^5 + 3570*(x*e + d)^m*d^2*m*x^4*e^5 + 42*(x*e + d)^m*d^3*m^4*e^4
 - 432*(x*e + d)^m*d^3*m^3*x*e^4 + 11220*(x*e + d)^m*d^3*m^2*x^2*e^4 - 4760
*(x*e + d)^m*d^3*m*x^3*e^4 + 24*(x*e + d)^m*d^4*m^3*e^3 - 5304*(x*e + d)^m*
d^4*m^2*x*e^3 + 7140*(x*e + d)^m*d^4*m*x^2*e^3 + 408*(x*e + d)^m*d^5*m^2*e^
2 - 14280*(x*e + d)^m*d^5*m*x*e^2 + 2040*(x*e + d)^m*d^6*m*e + 14400*(x*e +

```
 d)^m*d^7 + 162*(x*e + d)^m*m^5*x*e^7 + 1890*(x*e + d)^m*m^4*x^2*e^7 + 2559
9*(x*e + d)^m*m^3*x^3*e^7 - 10180*(x*e + d)^m*m^2*x^4*e^7 + 41004*(x*e + d)
^m*m*x^5*e^7 - 14280*(x*e + d)^m*x^6*e^7 + 162*(x*e + d)^m*d*m^5*e^6 + 1715
*(x*e + d)^m*d*m^4*x*e^6 + 17157*(x*e + d)^m*d*m^3*x^2*e^6 - 3376*(x*e + d)
^m*d*m^2*x^3*e^6 + 4284*(x*e + d)^m*d*m*x^4*e^6 - 175*(x*e + d)^m*d^2*m^4*e
^5 - 7518*(x*e + d)^m*d^2*m^3*x*e^5 + 3804*(x*e + d)^m*d^2*m^2*x^2*e^5 - 57
12*(x*e + d)^m*d^2*m*x^3*e^5 + 924*(x*e + d)^m*d^3*m^3*e^4 - 2568*(x*e + d)
^m*d^3*m^2*x*e^4 + 8568*(x*e + d)^m*d^3*m*x^2*e^4 + 432*(x*e + d)^m*d^4*m^2
*e^3 - 17136*(x*e + d)^m*d^4*m*x*e^3 + 5304*(x*e + d)^m*d^5*m*e^2 + 14280*(
x*e + d)^m*d^6*e + 1770*(x*e + d)^m*m^4*x*e^7 + 9940*(x*e + d)^m*m^3*x^2*e^
7 + 65352*(x*e + d)^m*m^2*x^3*e^7 - 11808*(x*e + d)^m*m*x^4*e^7 + 17136*(x*
e + d)^m*x^5*e^7 + 1770*(x*e + d)^m*d*m^4*e^6 + 8225*(x*e + d)^m*d*m^3*x*e^
6 + 31038*(x*e + d)^m*d*m^2*x^2*e^6 - 1680*(x*e + d)^m*d*m*x^3*e^6 - 1715*(
x*e + d)^m*d^2*m^3*e^5 - 26796*(x*e + d)^m*d^2*m^2*x*e^5 + 2520*(x*e + d)^m
*d^2*m*x^2*e^5 + 7518*(x*e + d)^m*d^3*m^2*e^4 - 5040*(x*e + d)^m*d^3*m*x*e^
4 + 2568*(x*e + d)^m*d^4*m*e^3 + 17136*(x*e + d)^m*d^5*e^2 + 9990*(x*e + d)
^m*m^3*x*e^7 + 27503*(x*e + d)^m*m^2*x^2*e^7 + 79716*(x*e + d)^m*m*x^3*e^7
- 5040*(x*e + d)^m*x^4*e^7 + 9990*(x*e + d)^m*d*m^3*e^6 + 19278*(x*e + d)^m
*d*m^2*x*e^6 + 17640*(x*e + d)^m*d*m*x^2*e^6 - 8225*(x*e + d)^m*d^2*m^2*e^5
 - 35280*(x*e + d)^m*d^2*m*x*e^5 + 26796*(x*e + d)^m*d^3*m*e^4 + 5040*(x*e
+ d)^m*d^4*e^3 + 30624*(x*e + d)^m*m^2*x*e^7 + 36918*(x*e + d)^m*m*x^2*e^7
+ 35280*(x*e + d)^m*x^3*e^7 + 30624*(x*e + d)^m...
```

Mupad [B]
time = 5.09, size = 1425, normalized size = 4.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] ((d + e*x)^m*(30240*d*e^6 + 14280*d^6*e + 14400*d^7 - 17640*d^2*e^5 + 35280
*d^3*e^4 + 5040*d^4*e^3 + 17136*d^5*e^2 - 19278*d^2*e^5*m + 26796*d^3*e^4*m
 + 2568*d^4*e^3*m + 5304*d^5*e^2*m + 30624*d*e^6*m^2 + 9990*d*e^6*m^3 + 177
0*d*e^6*m^4 + 162*d*e^6*m^5 + 6*d*e^6*m^6 - 8225*d^2*e^5*m^2 + 7518*d^3*e^4
*m^2 + 432*d^4*e^3*m^2 + 408*d^5*e^2*m^2 - 1715*d^2*e^5*m^3 + 924*d^3*e^4*m
^3 + 24*d^4*e^3*m^3 - 175*d^2*e^5*m^4 + 42*d^3*e^4*m^4 - 7*d^2*e^5*m^5 + 48
168*d*e^6*m + 2040*d^6*e*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^
4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (20*x^7*(d + e*x)^m*(1764*m + 1624*m^
2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))/(13068*m + 13132*m^2 + 6769*m^
3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) - (x*(d + e*x)^m*(35280*d^2*e
^5*m - 30240*e^7 - 30624*e^7*m^2 - 9990*e^7*m^3 - 1770*e^7*m^4 - 162*e^7*m^
5 - 6*e^7*m^6 - 48168*e^7*m + 5040*d^3*e^4*m + 17136*d^4*e^3*m + 14280*d^5*
e^2*m - 19278*d*e^6*m^2 - 8225*d*e^6*m^3 - 1715*d*e^6*m^4 - 175*d*e^6*m^5 -
 7*d*e^6*m^6 + 26796*d^2*e^5*m^2 + 2568*d^3*e^4*m^2 + 5304*d^4*e^3*m^2 + 20
```

40*d^5*e^2*m^2 + 7518*d^2*e^5*m^3 + 432*d^3*e^4*m^3 + 408*d^4*e^3*m^3 + 924*d^2*e^5*m^4 + 24*d^3*e^4*m^4 + 42*d^2*e^5*m^5 - 17640*d*e^6*m + 14400*d^6*e*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) - (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(2400*d^4*m - 13398*e^4*m - 17640*e^4 - 3759*e^4*m^2 - 462*e^4*m^3 - 21*e^4*m^4 + 2856*d^2*e^2*m + 428*d*e^3*m^2 + 340*d^3*e*m^2 + 72*d*e^3*m^3 + 4*d*e^3*m^4 + 884*d^2*e^2*m^2 + 68*d^2*e^2*m^3 + 840*d*e^3*m + 2380*d^3*e*m))/(e^4*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) - (x^5*(d + e*x)^m*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)*(120*d^2*m - 221*e^2*m - 714*e^2 - 17*e^2*m^2 + 119*d*e*m + 17*d*e*m^2))/(e^2*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x^4*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6)*(600*d^3*m - 428*e^3*m - 840*e^3 - 72*e^3*m^2 - 4*e^3*m^3 + 221*d*e^2*m^2 + 85*d^2*e*m^2 + 17*d*e^2*m^3 + 714*d*e^2*m + 595*d^2*e*m))/(e^3*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x^2*(m + 1)*(d + e*x)^m*(7200*d^5*m + 19278*e^5*m + 17640*e^5 + 8225*e^5*m^2 + 1715*e^5*m^3 + 175*e^5*m^4 + 7*e^5*m^5 + 2520*d^2*e^3*m + 8568*d^3*e^2*m + 13398*d*e^4*m^2 + 1020*d^4*e*m^2 + 3759*d*e^4*m^3 + 462*d*e^4*m^4 + 21*d*e^4*m^5 + 1284*d^2*e^3*m^2 + 2652*d^3*e^2*m^2 + 216*d^2*e^3*m^3 + 204*d^3*e^2*m^3 + 12*d^2*e^3*m^4 + 17640*d*e^4*m + 7140*d^4*e*m))/(e^5*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) - (x^6*(d + e*x)^m*(119*e - 20*d*m + 17*e*m)*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120))/(e*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040))

3.370 $\int \frac{(d+ex)^m(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

Optimal. Leaf size=255

$$\frac{(100d^2+165de+81e^2)(d+ex)^{1+m}}{125e^3(1+m)}-\frac{(40d+33e)(d+ex)^{2+m}}{25e^3(2+m)}+\frac{4(d+ex)^{3+m}}{5e^3(3+m)}-\frac{\left(6412i-423\sqrt{14}\right)(d+ex}{3500\left(5id-\right.}$$

```
[Out] 1/125*(100*d^2+165*d*e+81*e^2)*(e*x+d)^(1+m)/e^3/(1+m)-1/25*(40*d+33*e)*(e*
x+d)^(2+m)/e^3/(2+m)+4/5*(e*x+d)^(3+m)/e^3/(3+m)-1/3500*(e*x+d)^(1+m)*hyper
geom([1, 1+m],[2+m],5*(e*x+d)/(5*d-e*(1+I*14^(1/2))))*(6412*I+423*14^(1/2))
/(1+m)/(5*I*d-e*(I-14^(1/2)))-1/3500*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m]
,5*(e*x+d)/(5*d-e+I*14^(1/2)*e))*(6412*I-423*14^(1/2))/(1+m)/(5*I*d-e*(I+14
^(1/2)))
```

Rubi [A]
time = 0.34, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1642, 70}

$$\frac{(100d^2+165de+81e^2)(d+ex)^{m+1}}{125e^3(m+1)}-\frac{(40d+33e)(d+ex)^{m+2}}{25e^3(m+2)}+\frac{4(d+ex)^{m+3}}{5e^3(m+3)}-\frac{\left(-423\sqrt{14}+6412i\right)(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{5(d+ex)}{5d+i\sqrt{14}e-e}\right)}{3500(m+1)\left(5id-\left(\sqrt{14}+i\right)e\right)}-\frac{\left(423\sqrt{14}+6412i\right)(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{5(d+ex)}{5d-\left(1+i\sqrt{14}\right)e}\right)}{3500(m+1)\left(5id-\left(-\sqrt{14}+i\right)e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] ((100*d^2 + 165*d*e + 81*e^2)*(d + e*x)^(1 + m))/(125*e^3*(1 + m)) - ((40*d
 + 33*e)*(d + e*x)^(2 + m))/(25*e^3*(2 + m)) + (4*(d + e*x)^(3 + m))/(5*e^3
*(3 + m)) - ((6412*I - 423*Sqrt[14])*(d + e*x)^(1 + m)*Hypergeometric2F1[1,
 1 + m, 2 + m, (5*(d + e*x))/(5*d - e + I*Sqrt[14]*e)])/(3500*((5*I)*d - (I
 + Sqrt[14])*e)*(1 + m)) - ((6412*I + 423*Sqrt[14])*(d + e*x)^(1 + m)*Hyper
geometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - (1 + I*Sqrt[14])*e)])/(3
500*((5*I)*d - (I - Sqrt[14])*e)*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2} \, dx = \int \left(\frac{(100d^2+165de+81e^2)(d+ex)^m}{125e^2} + \frac{\left(\frac{458}{125}+\frac{423i}{125\sqrt{14}}\right)(}{2-2i\sqrt{14}+} \right.$$

$$= \frac{(100d^2+165de+81e^2)(d+ex)^{1+m}}{125e^3(1+m)} - \frac{(40d+33e)(d+ex)^{2}}{25e^3(2+m)}$$

$$= \frac{(100d^2+165de+81e^2)(d+ex)^{1+m}}{125e^3(1+m)} - \frac{(40d+33e)(d+ex)^{2}}{25e^3(2+m)}$$

Mathematica [A]
time = 0.62, size = 221, normalized size = 0.87

$$\frac{(d+ex)^{1+m}\left(\frac{28(100d^2+165de+81e^2)}{e^3(1+m)} - \frac{140(40d+33e)(d+ex)}{e^3(2+m)} + \frac{2800(d+ex)^2}{e^3(3+m)} - \frac{\left(6412i+423\sqrt{14}\right) {}_2F_1\left(1,1+m;2+m;\frac{5(d+ex)}{5d+\left(-1-i\sqrt{14}\right)e}\right)}{\left(5id+\left(-i+\sqrt{14}\right)e\right)(1+m)} - \frac{\left(-6412i+423\sqrt{14}\right) {}_2F_1\left(1,1+m;2+m;\frac{5(d+ex)}{5d+i\left(i+\sqrt{14}\right)e}\right)}{\left(-5id+\left(i+\sqrt{14}\right)e\right)(1+m)}\right)}{3500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x
]
```

```
[Out] ((d + e*x)^(1 + m)*((28*(100*d^2 + 165*d*e + 81*e^2))/(e^3*(1 + m)) - (140*
(40*d + 33*e)*(d + e*x))/(e^3*(2 + m)) + (2800*(d + e*x)^2)/(e^3*(3 + m)) -
 ((6412*I + 423*Sqrt[14])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/
(5*d + (-1 - I*Sqrt[14])*e)])/(((5*I)*d + (-I + Sqrt[14])*e)*(1 + m)) - ((-
6412*I + 423*Sqrt[14])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*
d + I*(I + Sqrt[14])*e)])/(((-5*I)*d + (I + Sqrt[14])*e)*(1 + m))))/3500
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m (4x^4-5x^3+3x^2+x+2)}{5x^2+2x+3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

```
[Out] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(x*e + d)^m/(5*x^2 + 2*x + 3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] integral((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(x*e + d)^m/(5*x^2 + 2*x + 3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(x*e + d)^m/(5*x^2 + 2*x + 3), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^m\,\left(4\,x^4-5\,x^3+3\,x^2+x+2\right)}{5\,x^2+2\,x+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
[Out] int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3), x)
```

3.371 $\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

Optimal. Leaf size=377

$$\frac{4(d+ex)^{1+m}}{25e(1+m)}-\frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\,(5d^2-2de+3e^2)\,(3+2x+5x^2)}+\frac{\left(80360d^2-32144de+48216e^2+i\sqrt{14}\,(65\right.}{}$$

```
[Out] 4/25*(e*x+d)^(1+m)/e/(1+m)-1/700*(1367*d-293*e+(423*d-1367*e)*x)*(e*x+d)^(1
+m)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)+1/19600*(e*x+d)^(1+m)*hypergeom([1, 1
+m],[2+m],5*(e*x+d)/(5*d-e*(1+I*14^(1/2))))*(80360*d^2-32144*d*e+48216*e^2-
5922*d*e*m+19138*e^2*m-I*(6565*d^2-2*d*e*(1313-3206*m)+e^2*(3939-98*m))*14^
(1/2))/(5*d^2-2*d*e+3*e^2)/(1+m)/(5*d-e*(1+I*14^(1/2)))+1/19600*(e*x+d)^(1+
m)*hypergeom([1, 1+m],[2+m],5*(e*x+d)/(5*d-e+I*14^(1/2)*e))*(80360*d^2-3214
4*d*e+48216*e^2-5922*d*e*m+19138*e^2*m+I*(6565*d^2-2*d*e*(1313-3206*m)+e^2*
(3939-98*m))*14^(1/2))/(5*d^2-2*d*e+3*e^2)/(1+m)/(5*d+I*e*(I+14^(1/2)))
```

Rubi [A]

time = 0.60, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {1662, 1642, 70}

$$\frac{\left(i\sqrt{14}\,(6565d^2-2de(1313-3206m)+e^2(3939-98m))+80360d^2-5922dem-32144de+19138e^2m+48216e^2\right)(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{5(d+ex)}{5d+i\sqrt{14}\,e-e}\right)}{19600(m+1)\left(5d+i\left(\sqrt{14}+i\right)e\right)(5d^2-2de+3e^2)}+\frac{\left(-i\sqrt{14}\,(6565d^2-2de(1313-3206m)+e^2(3939-98m))+80360d^2-5922dem-32144de+19138e^2m+48216e^2\right)(d+ex)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{5(d+ex)}{5d-\left(1+i\sqrt{14}\right)e}\right)}{19600(m+1)\left(5d-\left(1+i\sqrt{14}\right)e\right)(5d^2-2de+3e^2)}-\frac{(x(423d-1367e)+1367d-293e)(d+ex)^{m+1}}{700\,(5x^2+2x+3)\,(5d^2-2de+3e^2)}+\frac{4(d+ex)^{m+1}}{25e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] (4*(d + e*x)^(1 + m))/(25*e*(1 + m)) - ((1367*d - 293*e + (423*d - 1367*e)*
x)*(d + e*x)^(1 + m))/(700*(5*d^2 - 2*d*e + 3*e^2)*(3 + 2*x + 5*x^2)) + ((8
0360*d^2 - 32144*d*e + 48216*e^2 + I*Sqrt[14]*(6565*d^2 - 2*d*e*(1313 - 320
6*m) + e^2*(3939 - 98*m)) - 5922*d*e*m + 19138*e^2*m)*(d + e*x)^(1 + m)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - e + I*Sqrt[14]*e)])/(1
9600*(5*d + I*(I + Sqrt[14])*e)*(5*d^2 - 2*d*e + 3*e^2)*(1 + m)) + ((80360*
d^2 - 32144*d*e + 48216*e^2 - I*Sqrt[14]*(6565*d^2 - 2*d*e*(1313 - 3206*m)
+ e^2*(3939 - 98*m)) - 5922*d*e*m + 19138*e^2*m)*(d + e*x)^(1 + m)*Hypergeo
metric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - (1 + I*Sqrt[14])*e)])/(1960
0*(5*d - (1 + I*Sqrt[14])*e)*(5*d^2 - 2*d*e + 3*e^2)*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1662

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1)*((f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e
 + a*e^2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^m\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx = -\frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{\int \frac{(d+ex)^m(}{}}{}$$

$$= -\frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{\int\left(\frac{224}{25}(5c\right.}{}$$

$$= \frac{4(d+ex)^{1+m}}{25e(1+m)} - \frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}$$

$$= \frac{4(d+ex)^{1+m}}{25e(1+m)} - \frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}$$

Mathematica [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2
,x]
```

```
[Out] Integrate[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2
, x]
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m\left(4x^4-5x^3+3x^2+x+2\right)}{\left(5x^2+2x+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

```
[Out] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(x*e + d)^m/(5*x^2 + 2*x + 3)^2,
x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] integral((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(x*e + d)^m/(25*x^4 + 20*x^3 + 34*
x^2 + 12*x + 9), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(x*e + d)^m/(5*x^2 + 2*x + 3)^2,
x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^m\,\left(4\,x^4-5\,x^3+3\,x^2+x+2\right)}{{\left(5\,x^2+2\,x+3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
[Out] int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2, x)
```

3.372 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx+cx^2)^3}\,dx$

Optimal. Leaf size=528

$$-\frac{ab^3ch+bc^2(c^2d+acf-3a^2h)-ab^4i-ab^2c(cg-4ai)-2ac^2(c^2e-acg+a^2i)+(2c^5d-c^4(be+2af)+}{2c^4\,(b^2-4ac)\,(a+bx+cx^2)^2}$$

```
[Out] 1/2*(-a*b^3*c*h-b*c^2*(-3*a^2*h+a*c*f+c^2*d)+a*b^4*i+a*b^2*c*(-4*a*i+c*g)+2
*a*c^2*(a^2*i-a*c*g+c^2*e)-(2*c^5*d-c^4*(2*a*f+b*e)+c^3*(2*a^2*h+3*a*b*g+b^
2*f)-b^5*i+b^3*c*(5*a*i+b*h)-b*c^2*(5*a^2*i+4*a*b*h+b^2*g))*x)/c^4/(-4*a*c+
b^2)/(c*x^2+b*x+a)^2+1/2*(b^5*c*h+b^3*c^2*(-8*a*h+c*f)+2*b*c^3*(11*a^2*h+a*
c*f+3*c^2*d)-b^6*i-b^4*c*(-11*a*i+c*g)-16*a^2*c^3*(-2*a*i+c*g)-b^2*c^2*(39*
a^2*i-5*a*c*g+3*c^2*e)+2*c*(6*c^5*d-c^4*(-2*a*f+3*b*e)+c^3*(-10*a^2*h-3*a*b
*g+b^2*f)+2*b^5*i-b^3*c*(15*a*i+b*h)+a*b*c^2*(25*a*i+8*b*h))*x)/c^4/(-4*a*c
+b^2)^2/(c*x^2+b*x+a)-(12*c^5*d-c^4*(-4*a*f+6*b*e)+2*c^3*(6*a^2*h-3*a*b*g+b
^2*f)-b^5*i+10*a*b^3*c*i-30*a^2*b*c^2*i)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/
2))/c^3/(-4*a*c+b^2)^(5/2)+1/2*i*ln(c*x^2+b*x+a)/c^3
```

Rubi [A]
time = 0.84, antiderivative size = 528, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {1674, 648, 632, 212, 642}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2c^3(6a^2h-3abg+b^2f)-30a^2bc^2i+10ab^3ci-c^4(6be-4af)+b^5(-i)+12c^5d\right)}{c^3\,(b^2-4ac)^{5/2}}-\frac{x\left(c^3(2a^2h+3abg+b^2f)-bc^2(5a^2i+4abh+b^2g)+b^3c(5ai+bh)-c^4(2af+be)+b^5(-i)+2c^5d\right)+bc^2(-3a^2h+acf+c^2d)-2ac^2(a^2i-acg+c^2e)-ab^4i+ab^3ch-ab^2c(cg-4ai)}{2c^4\,(b^2-4ac)\,(a+bx+cx^2)^2}+\frac{-b^2c^2(39a^2i-5acg+3c^2e)+2cx\left(c^3(-10a^2h-3abg+b^2f)-b^3c(15ai+bh)-c^4(3be-2af)+abc^2(25ai+8bh)+2b^5i+6c^5d\right)+2bc^3(11a^2h+acf+3c^2d)-16a^2c^3(cg-2ai)-b^4c(cg-11ai)+b^3c^2(cf-8ah)+b^6(-i)+b^5ch}{2c^4\,(b^2-4ac)^2\,(a+bx+cx^2)}+\frac{i\log\left(a+bx+cx^2\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -1/2*(a*b^3*c*h + b*c^2*(c^2*d + a*c*f - 3*a^2*h) - a*b^4*i - a*b^2*c*(c*g
- 4*a*i) - 2*a*c^2*(c^2*e - a*c*g + a^2*i) + (2*c^5*d - c^4*(b*e + 2*a*f) +
 c^3*(b^2*f + 3*a*b*g + 2*a^2*h) - b^5*i + b^3*c*(b*h + 5*a*i) - b*c^2*(b^2
*g + 4*a*b*h + 5*a^2*i))*x)/(c^4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (b^5*
c*h + b^3*c^2*(c*f - 8*a*h) + 2*b*c^3*(3*c^2*d + a*c*f + 11*a^2*h) - b^6*i
- b^4*c*(c*g - 11*a*i) - 16*a^2*c^3*(c*g - 2*a*i) - b^2*c^2*(3*c^2*e - 5*a*
c*g + 39*a^2*i) + 2*c*(6*c^5*d - c^4*(3*b*e - 2*a*f) + c^3*(b^2*f - 3*a*b*g
 - 10*a^2*h) + 2*b^5*i - b^3*c*(b*h + 15*a*i) + a*b*c^2*(8*b*h + 25*a*i))*x
)/(2*c^4*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - ((12*c^5*d - c^4*(6*b*e - 4*a
*f) + 2*c^3*(b^2*f - 3*a*b*g + 6*a^2*h) - b^5*i + 10*a*b^3*c*i - 30*a^2*b*c
^2*i)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + (
i*Log[a + b*x + c*x^2])/(2*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+372x^5}{(a+bx+cx^2)^3}\,dx = \frac{744a^3c^2-bc^4d-a^2c(1488b^2+2c^2g-3bch)+a(372b^4+2c^4e}{}$$

$$= \frac{744a^3c^2-bc^4d-a^2c(1488b^2+2c^2g-3bch)+a(372b^4+2c^4e}{}$$

$$= \frac{744a^3c^2-bc^4d-a^2c(1488b^2+2c^2g-3bch)+a(372b^4+2c^4e}{}$$

$$= \frac{744a^3c^2-bc^4d-a^2c(1488b^2+2c^2g-3bch)+a(372b^4+2c^4e}{}$$

$$= \frac{744a^3c^2-bc^4d-a^2c(1488b^2+2c^2g-3bch)+a(372b^4+2c^4e}{}$$

Mathematica [A]
time = 0.69, size = 488, normalized size = 0.92

$$\frac{\frac{b^5ix+b^4(ai-chx)+2c^2(a^3i-c^3dx+ac^2(e+fx)-a^2c(g+hx))+b^2c(-4a^2i-c^2fx+ac(g+4hx))+b^3c(cgx-a(h+5ix))+bc^2(c^2(-d+ex)-ac(f+3gx)+a^2(3h+5ix))}{(b^2-4ac)(a+x(b+cx))^2}+\frac{-b^6i+b^5c(h+4ix)+b^3c^2(cf-8ah-30aix)-b^4c(-11ai+c(g+2hx))+4c^3(8a^3i+3c^3dx+ac^2fx-a^2c(4g+5hx))+b^2c^2(-39a^2i+c^2(-3e+2fx)+ac(5g+16hx))+2bc^3(3c^2(d-ex)+ac(f-3gx)+a^2(11h+25ix))}{(b^2-4ac)^2(a+x(b+cx))}+\frac{2c(12c^5d+c^4(-6be+4af)+2c^3(b^2f-3abg+6a^2h)-b^5i+10ab^3ci-30a^2bc^2i)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}+ci\log(a+x(b+cx))}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x]
```

```
[Out] ((b^5*i*x + b^4*(a*i - c*h*x) + 2*c^2*(a^3*i - c^3*d*x + a*c^2*(e + f*x) -
a^2*c*(g + h*x)) + b^2*c*(-4*a^2*i - c^2*f*x + a*c*(g + 4*h*x)) + b^3*c*(c*
g*x - a*(h + 5*i*x)) + b*c^2*(c^2*(-d + e*x) - a*c*(f + 3*g*x) + a^2*(3*h +
 5*i*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (-(b^6*i) + b^5*c*(h + 4*i*
x) + b^3*c^2*(c*f - 8*a*h - 30*a*i*x) - b^4*c*(-11*a*i + c*(g + 2*h*x)) + 4
*c^3*(8*a^3*i + 3*c^3*d*x + a*c^2*f*x - a^2*c*(4*g + 5*h*x)) + b^2*c^2*(-39
*a^2*i + c^2*(-3*e + 2*f*x) + a*c*(5*g + 16*h*x)) + 2*b*c^3*(3*c^2*(d - e*x
) + a*c*(f - 3*g*x) + a^2*(11*h + 25*i*x)))/((b^2 - 4*a*c)^2*(a + x*(b + c*
x))) + (2*c*(12*c^5*d + c^4*(-6*b*e + 4*a*f) + 2*c^3*(b^2*f - 3*a*b*g + 6*a
^2*h) - b^5*i + 10*a*b^3*c*i - 30*a^2*b*c^2*i)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*i*Log[a + x*(b + c*x)])/(2*c^4)
```

Maple [A]
time = 0.24, size = 769, normalized size = 1.46

method	result

default	$\frac{\frac{\left(25a^2 b c^2 i-10a^2 c^3 h-15a b^3 c i+8a b^2 c^2 h-3ab c^3 g+2a c^4 f+2b^5 i-b^4 c h+b^2 c^3 f-3b c^4 e+6c^5 d\right)x^3}{c^2\left(16a^2 c^2-8a b^2 c+b^4\right)}+\frac{\left(32a^3 c^3 i+11a^2 b^2 c^2 i+2a^2 b c^3 h-16a^2 c^4 g-19a b^4\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] ((25*a^2*b*c^2*i-10*a^2*c^3*h-15*a*b^3*c*i+8*a*b^2*c^2*h-3*a*b*c^3*g+2*a*c^
4*f+2*b^5*i-b^4*c*h+b^2*c^3*f-3*b*c^4*e+6*c^5*d)/c^2/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^3+1/2*(32*a^3*c^3*i+11*a^2*b^2*c^2*i+2*a^2*b*c^3*h-16*a^2*c^4*g-19*a
*b^4*c*i+8*a*b^3*c^2*h-a*b^2*c^3*g+6*a*b*c^4*f+3*b^6*i-b^5*c*h-b^4*c^2*g+3*
b^3*c^3*f-9*b^2*c^4*e+18*b*c^5*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x^2+(31*a^
3*b*c^2*i-6*a^3*c^3*h-22*a^2*b^3*c*i+10*a^2*b^2*c^2*h-5*a^2*b*c^3*g-2*a^2*c
^4*f+3*a*b^5*i-a*b^4*c*h-a*b^3*c^2*g+5*a*b^2*c^3*f-5*a*b*c^4*e+10*a*c^5*d-b
^3*c^3*e+2*b^2*c^4*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x+1/2/c^3*(24*a^4*c^2*
i-21*a^3*b^2*c*i+10*a^3*b*c^2*h-8*a^3*c^3*g+3*a^2*b^4*i-a^2*b^3*c*h-a^2*b^2
*c^2*g+6*a^2*b*c^3*f-8*a^2*c^4*e-a*b^2*c^3*e+10*a*b*c^4*d-b^3*c^3*d)/(16*a^
2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+1/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2
*(16*a^2*c^2*i-8*a*b^2*c*i+b^4*i)/c*ln(c*x^2+b*x+a)+2*(-7*a^2*b*i*c+6*a^2*c
^2*h+a*b^3*i-3*a*b*c^2*g+2*a*c^3*f+f*c^2*b^2-3*b*c^3*e+6*c^4*d-1/2*(16*a^2*
c^2*i-8*a*b^2*c*i+b^4*i)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2
)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3708 vs. 2(505) = 1010.
time = 0.43, size = 3708, normalized size = 7.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="fri
cas")

[Out] 1/2*(6*a^2*b*c^3*f + 3*I*a^2*b^4 - 21*I*a^3*b^2*c + 24*I*a^4*c^2 + 2*(6*c^6
*d - 3*a*b*c^4*g + 2*I*b^5*c - 15*I*a*b^3*c^2 + 25*I*a^2*b*c^3 + (b^2*c^4 +
 2*a*c^5)*f - (b^4*c^2 - 8*a*b^2*c^3 + 10*a^2*c^4)*h)*x^3 + (18*b*c^5*d + 3
*I*b^6 - 19*I*a*b^4*c + 11*I*a^2*b^2*c^2 + 32*I*a^3*c^3 + 3*(b^3*c^3 + 2*a*
b*c^4)*f - (b^4*c^2 + a*b^2*c^3 + 16*a^2*c^4)*g - (b^5*c - 8*a*b^3*c^2 - 2*
a^2*b*c^3)*h)*x^2 - (b^3*c^3 - 10*a*b*c^4)*d - (a^2*b^2*c^2 + 8*a^3*c^3)*g
- (a^2*b^3*c - 10*a^3*b*c^2)*h - 2*(-3*I*a*b^5 + 22*I*a^2*b^3*c - 31*I*a^3*
b*c^2 - 2*(b^2*c^4 + 5*a*c^5)*d - (5*a*b^2*c^3 - 2*a^2*c^4)*f + (a*b^3*c^2
+ 5*a^2*b*c^3)*g + (a*b^4*c - 10*a^2*b^2*c^2 + 6*a^3*c^3)*h)*x - (6*b*c^5*x
^3 + 9*b^2*c^4*x^2 + a*b^2*c^3 + 8*a^2*c^4 + 2*(b^3*c^3 + 5*a*b*c^4)*x)*e +
 (I*a^2*b^4 - 8*I*a^3*b^2*c + 16*I*a^4*c^2 + (I*b^4*c^2 - 8*I*a*b^2*c^3 + 1
6*I*a^2*c^4)*x^4 - 2*(-I*b^5*c + 8*I*a*b^3*c^2 - 16*I*a^2*b*c^3)*x^3 + (I*b
^6 - 6*I*a*b^4*c + 32*I*a^3*c^3)*x^2 - 2*(-I*a*b^5 + 8*I*a^2*b^3*c - 16*I*a
^3*b*c^2)*x - (a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*b^
2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^3 + (b
^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*
a^3*b*c^5)*x)*sqrt((144*c^10*d^2 + 36*a^2*b^2*c^6*g^2 + 144*a^4*c^6*h^2 + 3
6*b^2*c^8*e^2 - b^10 + 20*a*b^8*c - 160*a^2*b^6*c^2 + 600*a^3*b^4*c^3 - 900
*a^4*b^2*c^4 + 4*(b^4*c^6 + 4*a*b^2*c^7 + 4*a^2*c^8)*f^2 + 24*(-I*b^5*c^5 +
 10*I*a*b^3*c^6 - 30*I*a^2*b*c^7)*d + 4*(-I*b^7*c^3 + 8*I*a*b^5*c^4 - 10*I*
a^2*b^3*c^5 - 60*I*a^3*b*c^6 + 12*(b^2*c^8 + 2*a*c^9)*d)*f - 12*(12*a*b*c^8
*d - I*a*b^6*c^3 + 10*I*a^2*b^4*c^4 - 30*I*a^3*b^2*c^5 + 2*(a*b^3*c^6 + 2*a
^2*b*c^7)*f)*g + 24*(12*a^2*c^8*d - 6*a^3*b*c^6*g - I*a^2*b^5*c^3 + 10*I*a^
3*b^3*c^4 - 30*I*a^4*b*c^5 + 2*(a^2*b^2*c^6 + 2*a^3*c^7)*f)*h - 12*(12*b*c^
9*d - 6*a*b^2*c^7*g + 12*a^2*b*c^7*h - I*b^6*c^4 + 10*I*a*b^4*c^5 - 30*I*a^
2*b^2*c^6 + 2*(b^3*c^7 + 2*a*b*c^8)*f)*e)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^
2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*log(1/2*
(12*b*c^5*d - 6*a*b^2*c^3*g + 12*a^2*b*c^3*h - I*b^6 + 10*I*a*b^4*c - 30*I*
a^2*b^2*c^2 + 2*(b^3*c^3 + 2*a*b*c^4)*f + 2*(12*c^6*d - 6*a*b*c^4*g + 12*a^
2*c^4*h - I*b^5*c + 10*I*a*b^3*c^2 - 30*I*a^2*b*c^3 + 2*(b^2*c^4 + 2*a*c^5)
*f)*x - 6*(2*b*c^5*x + b^2*c^4)*e + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^
5 - 64*a^3*c^6)*sqrt((144*c^10*d^2 + 36*a^2*b^2*c^6*g^2 + 144*a^4*c^6*h^2 +
 36*b^2*c^8*e^2 - b^10 + 20*a*b^8*c - 160*a^2*b^6*c^2 + 600*a^3*b^4*c^3 - 9
00*a^4*b^2*c^4 + 4*(b^4*c^6 + 4*a*b^2*c^7 + 4*a^2*c^8)*f^2 + 24*(-I*b^5*c^5
 + 10*I*a*b^3*c^6 - 30*I*a^2*b*c^7)*d + 4*(-I*b^7*c^3 + 8*I*a*b^5*c^4 - 10*
I*a^2*b^3*c^5 - 60*I*a^3*b*c^6 + 12*(b^2*c^8 + 2*a*c^9)*d)*f - 12*(12*a*b*c
^8*d - I*a*b^6*c^3 + 10*I*a^2*b^4*c^4 - 30*I*a^3*b^2*c^5 + 2*(a*b^3*c^6 + 2
*a^2*b*c^7)*f)*g + 24*(12*a^2*c^8*d - 6*a^3*b*c^6*g - I*a^2*b^5*c^3 + 10*I*
a^3*b^3*c^4 - 30*I*a^4*b*c^5 + 2*(a^2*b^2*c^6 + 2*a^3*c^7)*f)*h - 12*(12*b*
c^9*d - 6*a*b^2*c^7*g + 12*a^2*b*c^7*h - I*b^6*c^4 + 10*I*a*b^4*c^5 - 30*I*
```

```
a^2*b^2*c^6 + 2*(b^3*c^7 + 2*a*b*c^8)*f)*e)/(b^10*c^6 - 20*a*b^8*c^7 + 160*
a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(12*c^
6*d - 6*a*b*c^4*g + 12*a^2*c^4*h - 6*b*c^5*e - I*b^5*c + 10*I*a*b^3*c^2 - 3
0*I*a^2*b*c^3 + 2*(b^2*c^4 + 2*a*c^5)*f)) + (I*a^2*b^4 - 8*I*a^3*b^2*c + 16
*I*a^4*c^2 + (I*b^4*c^2 - 8*I*a*b^2*c^3 + 16*I*a^2*c^4)*x^4 - 2*(-I*b^5*c +
 8*I*a*b^3*c^2 - 16*I*a^2*b*c^3)*x^3 + (I*b^6 - 6*I*a*b^4*c + 32*I*a^3*c^3)
*x^2 - 2*(-I*a*b^5 + 8*I*a^2*b^3*c - 16*I*a^3*b*c^2)*x + (a^2*b^4*c^3 - 8*a
^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5
*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^
6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*x)*sqrt((144*c^10*d^2
 + 36*a^2*b^2*c^6*g^2 + 144*a^4*c^6*h^2 + 36*b^2*c^8*e^2 - b^10 + 20*a*b^8*
c - 160*a^2*b^6*c^2 + 600*a^3*b^4*c^3 - 900*a^4*b^2*c^4 + 4*(b^4*c^6 + 4*a*
b^2*c^7 + 4*a^2*c^8)*f^2 + 24*(-I*b^5*c^5 + 10*I*a*b^3*c^6 - 30*I*a^2*b*c^7
)*d + 4*(-I*b^7*c^3 + 8*I*a*b^5*c^4 - 10*I*a^2*b^3*c^5 - 60*I*a^3*b*c^6 + 1
2*(b^2*c^8 + 2*a*c^9)*d)*f - 12*(12*a*b*c^8*d - I*a*b^6*c^3 + 10*I*a^2*b^4*
c^4 - 30*I*a^3*b^2*c^5 + 2*(a*b^3*c^6 + 2*a^2*b*c^7)*f)*g + 24*(12*a^2*c^8*
d - 6*a^3*b*c^6*g - I*a^2*b^5*c^3 + 10*I*a^3*b^3*c^4 - 30*I*a^4*b*c^5 + 2*(
a^2*b^2*c^6 + 2*a^3*c^7)*f)*h - 12*(12*b*c^9*d - 6*a*b^2*c^7*g + 12*a^2*b*c
^7*h - I*b^6*c^4 + 10*I*a*b^4*c^5 - 30*I*a^2*b^2*c^6 + 2*(b^3*c^7 + 2*a*b*c
^8)*f)*e)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 12
80*a^4*b^2*c^10 - 1024*a^5*c^11)))*log(1/2*(12*b*c^5*d - 6*a*b^2*c^3*g + 12
*a^2*b*c^3*h - I*b^6 + 10*I*a*b^4*c - 30*I*a^2*b^2*c^2 + 2*(b^3*c^3 + 2*a*b
*c^4)*f + 2*(12*c^6*d - 6*a*b*c^4*g + 12*a^2*c^4*h - I*b^5*c + 10*I*a*b^3*c
^2 - 30*I*a^2*b*c^3 + 2*(b^2*c^4 + 2*a*c^5)*f)*x - 6*(2*b*c^5*x + b^2*c^4)*
e - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((144*c^10*d
^2 + 36*a^2*b^2*c^6*g^2 + 144*a^4*c^6*h^2 + 36*b^2*c^8*e^2 - b^10 + 20*a*b^
8*c - 160*a^2*b^6*c^2 + 600*a^3*b^4*c^3 - 900*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.37, size = 635, normalized size = 1.20

$$\frac{\left(12i\,c^{5}d+2i\,b^{2}c^{3}f+4i\,ac^{4}f-6i\,abc^{3}g+12i\,a^{2}c^{3}h-6i\,bc^{4}e+b^{5}-10\,ab^{3}c+30\,a^{2}bc^{2}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(i\,b^{4}c^{3}-8i\,ab^{2}c^{4}+16i\,a^{2}c^{5}\right)\sqrt{-b^{2}+4\,ac}}+\frac{i\log\left(cx^{2}+bx+a\right)}{2\,c^{3}}-\frac{b^{3}c^{3}d-10\,abc^{4}d-6\,a^{2}bc^{3}f+a^{2}b^{2}c^{2}g+8\,a^{3}c^{3}g+a^{2}b^{3}ch-10\,a^{3}bc^{2}h+ab^{2}c^{3}e+8\,a^{2}c^{4}e-3i\,a^{2}b^{4}+21i\,a^{3}b^{2}c-24i\,a^{4}c^{2}-2\left(6\,c^{6}d+b^{2}c^{4}f+2\,ac^{5}f-3\,abc^{4}g-b^{4}c^{2}h+8\,ab^{2}c^{3}h-10\,a^{2}c^{4}h-3\,bc^{5}e+2i\,b^{5}c-15i\,ab^{3}c^{2}+25i\,a^{2}bc^{3}\right)x^{3}-\left(18\,bc^{5}d+3\,b^{3}c^{3}f+6\,abc^{4}f-b^{4}c^{2}g-ab^{2}c^{3}g-16\,a^{2}c^{4}g-b^{5}ch+8\,ab^{3}c^{2}h+2\,a^{2}bc^{3}h-9\,b^{2}c^{4}e+3i\,b^{6}-19i\,ab^{4}c+11i\,a^{2}b^{2}c^{2}+32i\,a^{3}c^{3}\right)x^{2}-2\left(2\,b^{2}c^{4}d+10\,ac^{5}d+5\,ab^{2}c^{3}f-2\,a^{2}c^{4}f-ab^{3}c^{2}g-5\,a^{2}bc^{3}g-ab^{4}ch+10\,a^{2}b^{2}c^{2}h-6\,a^{3}c^{3}h-b^{3}c^{3}e-5\,abc^{4}e+3i\,ab^{5}-22i\,a^{2}b^{3}c+31i\,a^{3}bc^{2}\right)x}{2\left(cx^{2}+bx+a\right)^{2}\left(b^{2}-4\,ac\right)^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="gia
c")
```

```
[Out] (12*I*c^5*d + 2*I*b^2*c^3*f + 4*I*a*c^4*f - 6*I*a*b*c^3*g + 12*I*a^2*c^3*h
- 6*I*b*c^4*e + b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b
^2 + 4*a*c))/((I*b^4*c^3 - 8*I*a*b^2*c^4 + 16*I*a^2*c^5)*sqrt(-b^2 + 4*a*c)
) + 1/2*I*log(c*x^2 + b*x + a)/c^3 - 1/2*(b^3*c^3*d - 10*a*b*c^4*d - 6*a^2*
b*c^3*f + a^2*b^2*c^2*g + 8*a^3*c^3*g + a^2*b^3*c*h - 10*a^3*b*c^2*h + a*b^
2*c^3*e + 8*a^2*c^4*e - 3*I*a^2*b^4 + 21*I*a^3*b^2*c - 24*I*a^4*c^2 - 2*(6*
c^6*d + b^2*c^4*f + 2*a*c^5*f - 3*a*b*c^4*g - b^4*c^2*h + 8*a*b^2*c^3*h - 1
0*a^2*c^4*h - 3*b*c^5*e + 2*I*b^5*c - 15*I*a*b^3*c^2 + 25*I*a^2*b*c^3)*x^3
- (18*b*c^5*d + 3*b^3*c^3*f + 6*a*b*c^4*f - b^4*c^2*g - a*b^2*c^3*g - 16*a^
2*c^4*g - b^5*c*h + 8*a*b^3*c^2*h + 2*a^2*b*c^3*h - 9*b^2*c^4*e + 3*I*b^6 -
 19*I*a*b^4*c + 11*I*a^2*b^2*c^2 + 32*I*a^3*c^3)*x^2 - 2*(2*b^2*c^4*d + 10*
a*c^5*d + 5*a*b^2*c^3*f - 2*a^2*c^4*f - a*b^3*c^2*g - 5*a^2*b*c^3*g - a*b^4
*c*h + 10*a^2*b^2*c^2*h - 6*a^3*c^3*h - b^3*c^3*e - 5*a*b*c^4*e + 3*I*a*b^5
 - 22*I*a^2*b^3*c + 31*I*a^3*b*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2
*c^3)
```

Mupad [B]
time = 6.17, size = 1027, normalized size = 1.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x)
```

```
[Out] (atan((x*(32*a^2*c^5*(4*a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) -
16*a*b^2*c^4*(4*a*c - b^2)^(5/2)))/(c^2*(4*a*c - b^2)^5) + ((32*a^2*c^5*(4*
a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) - 16*a*b^2*c^4*(4*a*c - b^
2)^(5/2))*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4))/(2*c^5*(4*a*c - b^2)^5*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(12*c^5*d - b^5*i + 2*b^2*c^3*f + 12*a^2*c^3
*h + 4*a*c^4*f - 6*b*c^4*e - 6*a*b*c^3*g + 10*a*b^3*c*i - 30*a^2*b*c^2*i))/
(c^3*(4*a*c - b^2)^(5/2)) - (log(a + b*x + c*x^2)*(b^10*i - 1024*a^5*c^5*i
+ 160*a^2*b^6*c^2*i - 640*a^3*b^4*c^3*i + 1280*a^4*b^2*c^4*i - 20*a*b^8*c*i
))/(2*(1024*a^5*c^8 - b^10*c^3 + 20*a*b^8*c^4 - 160*a^2*b^6*c^5 + 640*a^3*b
^4*c^6 - 1280*a^4*b^2*c^7)) - ((8*a^2*c^4*e + b^3*c^3*d + 8*a^3*c^3*g - 3*a
^2*b^4*i - 24*a^4*c^2*i + a^2*b^2*c^2*g - 10*a*b*c^4*d + a*b^2*c^3*e - 6*a^
2*b*c^3*f + a^2*b^3*c*h - 10*a^3*b*c^2*h + 21*a^3*b^2*c*i)/(2*c^3*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)) - (x^2*(3*b^6*i - 9*b^2*c^4*e - 16*a^2*c^4*g + 3*b^3
*c^3*f - b^4*c^2*g + 32*a^3*c^3*i + 18*b*c^5*d - b^5*c*h + 11*a^2*b^2*c^2*i
 + 6*a*b*c^4*f - 19*a*b^4*c*i - a*b^2*c^3*g + 8*a*b^3*c^2*h + 2*a^2*b*c^3*h
))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(2*a^2*c^4*f - 2*b^2*c^4*d +
 b^3*c^3*e + 6*a^3*c^3*h - 10*a*c^5*d - 3*a*b^5*i - 10*a^2*b^2*c^2*h + 5*a*
b*c^4*e + a*b^4*c*h - 5*a*b^2*c^3*f + a*b^3*c^2*g + 5*a^2*b*c^3*g + 22*a^2*
b^3*c*i - 31*a^3*b*c^2*i))/(c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^3*(6*c
^5*d + 2*b^5*i + b^2*c^3*f - 10*a^2*c^3*h + 2*a*c^4*f - 3*b*c^4*e - b^4*c*h
 - 3*a*b*c^3*g - 15*a*b^3*c*i + 8*a*b^2*c^2*h + 25*a^2*b*c^2*i))/(c^2*(b^4
```

```
+ 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x +
2*b*c*x^3)
```

3.373 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx$

Optimal. Leaf size=765

$$\frac{(c^6f-c^5(bg+ah)+c^4(b^2h+2abj+a^2k)+b^6m-b^4c(bl+5am)+b^2c^2(b^2k+4abl+6a^2m)-c^3(b^3j+3a}{c^7}$$

```
[Out] (c^6*f-c^5*(a*h+b*g)+c^4*(a^2*k+2*a*b*j+b^2*h)+b^6*m-b^4*c*(5*a*m+b*l)+b^2*
c^2*(6*a^2*m+4*a*b*l+b^2*k)-c^3*(a^3*m+3*a^2*b*l+3*a*b^2*k+b^3*j))*x/c^7+1/
2*(c^5*g-c^4*(a*j+b*h)+c^3*(a^2*l+2*a*b*k+b^2*j)-b^5*m+b^3*c*(4*a*m+b*l)-b*
c^2*(3*a^2*m+3*a*b*l+b^2*k))*x^2/c^6+1/3*(c^4*h-c^3*(a*k+b*j)+b^4*m-b^2*c*(
3*a*m+b*l)+c^2*(a^2*m+2*a*b*l+b^2*k))*x^3/c^5+1/4*(c^3*j-c^2*(a*l+b*k)-b^3*
m+b*c*(2*a*m+b*l))*x^4/c^4+1/5*(c^2*k+b^2*m-c*(a*m+b*l))*x^5/c^3+1/6*(-b*m+
c*l)*x^6/c^2+1/7*m*x^7/c+1/2*(c^7*e-c^6*(a*g+b*f)+c^5*(a^2*j+2*a*b*h+b^2*g)
-c^4*(a^3*l+3*a^2*b*k+3*a*b^2*j+b^3*h)-b^7*m+b^5*c*(6*a*m+b*l)-b^3*c^2*(10*
a^2*m+5*a*b*l+b^2*k)+b*c^3*(4*a^3*m+6*a^2*b*l+4*a*b^2*k+b^3*j))*ln(c*x^2+b*
x+a)/c^8-(2*c^8*d-c^7*(2*a*f+b*e)+c^6*(2*a^2*h+3*a*b*g+b^2*f)-c^5*(2*a^3*k+
5*a^2*b*j+4*a*b^2*h+b^3*g)+b^8*m-b^6*c*(8*a*m+b*l)+b^4*c^2*(20*a^2*m+7*a*b*
l+b^2*k)-b^2*c^3*(16*a^3*m+14*a^2*b*l+6*a*b^2*k+b^3*j)+c^4*(2*a^4*m+7*a^3*b
*l+9*a^2*b^2*k+5*a*b^3*j+b^4*h))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^8/
(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 4.67, antiderivative size = 765, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1671, 648, 632, 212, 642}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
b*x + c*x^2),x]
```

```
[Out] ((c^6*f - c^5*(b*g + a*h) + c^4*(b^2*h + 2*a*b*j + a^2*k) + b^6*m - b^4*c*(
b*l + 5*a*m) + b^2*c^2*(b^2*k + 4*a*b*l + 6*a^2*m) - c^3*(b^3*j + 3*a*b^2*k
 + 3*a^2*b*l + a^3*m))*x)/c^7 + ((c^5*g - c^4*(b*h + a*j) + c^3*(b^2*j + 2*
a*b*k + a^2*l) - b^5*m + b^3*c*(b*l + 4*a*m) - b*c^2*(b^2*k + 3*a*b*l + 3*a
^2*m))*x^2)/(2*c^6) + ((c^4*h - c^3*(b*j + a*k) + b^4*m - b^2*c*(b*l + 3*a*
m) + c^2*(b^2*k + 2*a*b*l + a^2*m))*x^3)/(3*c^5) + ((c^3*j - c^2*(b*k + a*l
) - b^3*m + b*c*(b*l + 2*a*m))*x^4)/(4*c^4) + ((c^2*k + b^2*m - c*(b*l + a*
m))*x^5)/(5*c^3) + ((c*l - b*m)*x^6)/(6*c^2) + (m*x^7)/(7*c) - ((2*c^8*d -
c^7*(b*e + 2*a*f) + c^6*(b^2*f + 3*a*b*g + 2*a^2*h) - c^5*(b^3*g + 4*a*b^2*
h + 5*a^2*b*j + 2*a^3*k) + b^8*m - b^6*c*(b*l + 8*a*m) + b^4*c^2*(b^2*k + 7
*a*b*l + 20*a^2*m) - b^2*c^3*(b^3*j + 6*a*b^2*k + 14*a^2*b*l + 16*a^3*m) +
```

```
c^4*(b^4*h + 5*a*b^3*j + 9*a^2*b^2*k + 7*a^3*b*l + 2*a^4*m))*ArcTanh[(b + 2
*c*x)/Sqrt[b^2 - 4*a*c]])/(c^8*Sqrt[b^2 - 4*a*c]) + ((c^7*e - c^6*(b*f + a*
g) + c^5*(b^2*g + 2*a*b*h + a^2*j) - c^4*(b^3*h + 3*a*b^2*j + 3*a^2*b*k + a
^3*l) - b^7*m + b^5*c*(b*l + 6*a*m) - b^3*c^2*(b^2*k + 5*a*b*l + 10*a^2*m)
+ b*c^3*(b^3*j + 4*a*b^2*k + 6*a^2*b*l + 4*a^3*m))*Log[a + b*x + c*x^2])/(2
*c^8)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx = \int \Bigg(\frac{c^6f - c^5(bg+ah) + c^4(b^2h+2abj+a^2k}{}$$

$$= \frac{(c^6f - c^5(bg+ah) + c^4(b^2h+2abj+a^2k) +}{}$$

$$= \frac{(c^6f - c^5(bg+ah) + c^4(b^2h+2abj+a^2k) +}{}$$

$$= \frac{(c^6f - c^5(bg+ah) + c^4(b^2h+2abj+a^2k) +}{}$$

$$= \frac{(c^6f - c^5(bg+ah) + c^4(b^2h+2abj+a^2k) +}{}$$

Mathematica [A]
time = 0.47, size = 754, normalized size = 0.99

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)
/(a + b*x + c*x^2),x]
```

```
[Out] (420*c*(c^6*f - c^5*(b*g + a*h) + c^4*(b^2*h + 2*a*b*j + a^2*k) + b^6*m - b
^4*c*(b*l + 5*a*m) + b^2*c^2*(b^2*k + 4*a*b*l + 6*a^2*m) - c^3*(b^3*j + 3*a
*b^2*k + 3*a^2*b*l + a^3*m))*x + 210*c^2*(c^5*g - c^4*(b*h + a*j) + c^3*(b^
2*j + 2*a*b*k + a^2*l) - b^5*m + b^3*c*(b*l + 4*a*m) - b*c^2*(b^2*k + 3*a*b
*l + 3*a^2*m))*x^2 + 140*c^3*(c^4*h - c^3*(b*j + a*k) + b^4*m - b^2*c*(b*l
+ 3*a*m) + c^2*(b^2*k + 2*a*b*l + a^2*m))*x^3 + 105*c^4*(c^3*j - c^2*(b*k +
 a*l) - b^3*m + b*c*(b*l + 2*a*m))*x^4 + 84*c^5*(c^2*k + b^2*m - c*(b*l + a
*m))*x^5 + 70*c^6*(c*l - b*m)*x^6 + 60*c^7*m*x^7 + (420*(2*c^8*d - c^7*(b*e
 + 2*a*f) + c^6*(b^2*f + 3*a*b*g + 2*a^2*h) - c^5*(b^3*g + 4*a*b^2*h + 5*a^
2*b*j + 2*a^3*k) + b^8*m - b^6*c*(b*l + 8*a*m) + b^4*c^2*(b^2*k + 7*a*b*l +
 20*a^2*m) - b^2*c^3*(b^3*j + 6*a*b^2*k + 14*a^2*b*l + 16*a^3*m) + c^4*(b^4
*h + 5*a*b^3*j + 9*a^2*b^2*k + 7*a^3*b*l + 2*a^4*m))*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 210*(c^7*e - c^6*(b*f + a*g) + c^5*(
b^2*g + 2*a*b*h + a^2*j) - c^4*(b^3*h + 3*a*b^2*j + 3*a^2*b*k + a^3*l) - b^
7*m + b^5*c*(b*l + 6*a*m) - b^3*c^2*(b^2*k + 5*a*b*l + 10*a^2*m) + b*c^3*(b
^3*j + 4*a*b^2*k + 6*a^2*b*l + 4*a^3*m))*Log[a + x*(b + c*x)])/(420*c^8)
```

Maple [A]
time = 0.29, size = 1086, normalized size = 1.42

method	result
default	$-\frac{a\,b^2c^3m\,x^3-\frac{1}{2}ab\,c^4m\,x^4-b^4c^2kx+b^3c^3jx-b^2c^4hx+b\,c^5gx+\frac{1}{2}a\,c^5j\,x^2+\frac{1}{2}b^5cm\,x^2-\frac{1}{2}b^4c^2l\,x^2+\frac{1}{2}b^3c^3k\,x^2-\frac{1}{2}b^2c^4j\,x^2+\frac{1}{2}b\,c^5h\,x^2+a}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -1/c^7*(a*b^2*c^3*m*x^3-1/2*a*b*c^4*m*x^4-b^4*c^2*k*x+b^3*c^3*j*x-b^2*c^4*h
*x+b*c^5*g*x+1/2*a*c^5*j*x^2+1/2*b^5*c*m*x^2-1/2*b^4*c^2*l*x^2+1/2*b^3*c^3*
k*x^2-1/2*b^2*c^4*j*x^2+1/2*b*c^5*h*x^2+a^3*c^3*m*x-a^2*c^4*k*x+a*c^5*h*x+b
^5*c*l*x-1/3*b^4*c^2*m*x^3+1/3*b^3*c^3*l*x^3-1/3*b^2*c^4*k*x^3+1/3*b*c^5*j*
x^3-1/2*a^2*c^4*l*x^2-1/4*b^2*c^4*l*x^4+1/4*b*c^5*k*x^4-1/3*a^2*c^4*m*x^3+1
/3*a*c^5*k*x^3+1/4*b^3*c^3*m*x^4+1/5*b*c^5*l*x^5+1/4*a*c^5*l*x^4+1/5*a*c^5*
m*x^5-1/5*b^2*c^4*m*x^5+1/6*b*c^5*m*x^6-b^6*m*x-c^6*f*x-1/2*c^6*g*x^2-1/3*c
^6*h*x^3-1/4*c^6*j*x^4-1/6*c^6*l*x^6-1/5*c^6*k*x^5-1/7*m*x^7*c^6+5*a*b^4*c*
m*x-4*a*b^3*c^2*l*x+3*a*b^2*c^3*k*x-2*a*b*c^4*j*x-6*a^2*b^2*c^2*m*x+3*a^2*b
*c^3*l*x-a*b*c^4*k*x^2-2*a*b^3*c^2*m*x^2+3/2*a*b^2*c^3*l*x^2+3/2*a^2*b*c^3*
m*x^2-2/3*a*b*c^4*l*x^3)+1/c^7*(1/2*(4*a^3*b*c^3*m-a^3*c^4*l-10*a^2*b^3*c^2
*m+6*a^2*b^2*c^3*l-3*a^2*b*c^4*k+a^2*c^5*j+6*a*b^5*c*m-5*a*b^4*c^2*l+4*a*b^
3*c^3*k-3*a*b^2*c^4*j+2*a*b*c^5*h-a*c^6*g-b^7*m+b^6*c*l-b^5*c^2*k+b^4*c^3*j
-b^3*c^4*h+b^2*c^5*g-b*c^6*f+c^7*e)/c*ln(c*x^2+b*x+a)+2*(a^4*c^3*m-6*a^3*b^
2*c^2*m+3*a^3*b*c^3*l-a^3*c^4*k+5*a^2*b^4*c*m-4*a^2*b^3*c^2*l+3*a^2*b^2*c^3
*k-2*a^2*b*c^4*j+a^2*c^5*h-a*b^6*m+a*b^5*c*l-a*b^4*c^2*k+a*b^3*c^3*j-a*b^2*
c^4*h+a*b*c^5*g-a*c^6*f+c^7*d-1/2*(4*a^3*b*c^3*m-a^3*c^4*l-10*a^2*b^3*c^2*m
+6*a^2*b^2*c^3*l-3*a^2*b*c^4*k+a^2*c^5*j+6*a*b^5*c*m-5*a*b^4*c^2*l+4*a*b^3*
c^3*k-3*a*b^2*c^4*j+2*a*b*c^5*h-a*c^6*g-b^7*m+b^6*c*l-b^5*c^2*k+b^4*c^3*j-b
^3*c^4*h+b^2*c^5*g-b*c^6*f+c^7*e)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(
4*a*c-b^2)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.64, size = 2646, normalized size = 3.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x
, algorithm="fricas")
```

```
[Out] [1/420*(60*(b^2*c^7 - 4*a*c^8)*m*x^7 + 70*((b^2*c^7 - 4*a*c^8)*l - (b^3*c^6
 - 4*a*b*c^7)*m)*x^6 + 84*((b^2*c^7 - 4*a*c^8)*k - (b^3*c^6 - 4*a*b*c^7)*l
+ (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*m)*x^5 + 105*((b^2*c^7 - 4*a*c^8)*j -
 (b^3*c^6 - 4*a*b*c^7)*k + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*l - (b^5*c^4
 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*m)*x^4 + 140*((b^2*c^7 - 4*a*c^8)*h - (b^3*c^
6 - 4*a*b*c^7)*j + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*k - (b^5*c^4 - 6*a*b
^3*c^5 + 8*a^2*b*c^6)*l + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c
^6)*m)*x^3 + 210*((b^2*c^7 - 4*a*c^8)*g - (b^3*c^6 - 4*a*b*c^7)*h + (b^4*c^
5 - 5*a*b^2*c^6 + 4*a^2*c^7)*j - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*k +
(b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*l - (b^7*c^2 - 8*a*b^5
*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*m)*x^2 - 210*(2*c^8*d - b*c^7*e + (b^
2*c^6 - 2*a*c^7)*f - (b^3*c^5 - 3*a*b*c^6)*g + (b^4*c^4 - 4*a*b^2*c^5 + 2*a
^2*c^6)*h - (b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5)*j + (b^6*c^2 - 6*a*b^4*c^
3 + 9*a^2*b^2*c^4 - 2*a^3*c^5)*k - (b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 -
7*a^3*b*c^4)*l + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 2*a^4
*c^4)*m)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 420*((b^2*c^7 - 4*a*c^8)*f - (
b^3*c^6 - 4*a*b*c^7)*g + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*h - (b^5*c^4 -
 6*a*b^3*c^5 + 8*a^2*b*c^6)*j + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4
*a^3*c^6)*k - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*l + (
b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a^3*b^2*c^4 + 4*a^4*c^5)*m)*x - 2
10*((b^3*c^6 - 4*a*b*c^7)*f - (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*g + (b^5*
c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*h - (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^
5 - 4*a^3*c^6)*j + (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*
k - (b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a^3*b^2*c^4 + 4*a^4*c^5)*l +
 (b^9 - 10*a*b^7*c + 34*a^2*b^5*c^2 - 44*a^3*b^3*c^3 + 16*a^4*b*c^4)*m - (b
^2*c^7 - 4*a*c^8)*e)*log(c*x^2 + b*x + a))/(b^2*c^8 - 4*a*c^9), 1/420*(60*(
b^2*c^7 - 4*a*c^8)*m*x^7 + 70*((b^2*c^7 - 4*a*c^8)*l - (b^3*c^6 - 4*a*b*c^7
)*m)*x^6 + 84*((b^2*c^7 - 4*a*c^8)*k - (b^3*c^6 - 4*a*b*c^7)*l + (b^4*c^5 -
 5*a*b^2*c^6 + 4*a^2*c^7)*m)*x^5 + 105*((b^2*c^7 - 4*a*c^8)*j - (b^3*c^6 -
4*a*b*c^7)*k + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*l - (b^5*c^4 - 6*a*b^3*c
^5 + 8*a^2*b*c^6)*m)*x^4 + 140*((b^2*c^7 - 4*a*c^8)*h - (b^3*c^6 - 4*a*b*c^
7)*j + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*k - (b^5*c^4 - 6*a*b^3*c^5 + 8*a
```

```
^2*b*c^6)*l + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*m)*x^3 +
 210*((b^2*c^7 - 4*a*c^8)*g - (b^3*c^6 - 4*a*b*c^7)*h + (b^4*c^5 - 5*a*b^2*
c^6 + 4*a^2*c^7)*j - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*k + (b^6*c^3 - 7
*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*l - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^
2*b^3*c^4 - 12*a^3*b*c^5)*m)*x^2 - 420*(2*c^8*d - b*c^7*e + (b^2*c^6 - 2*a*
c^7)*f - (b^3*c^5 - 3*a*b*c^6)*g + (b^4*c^4 - 4*a*b^2*c^5 + 2*a^2*c^6)*h -
(b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5)*j + (b^6*c^2 - 6*a*b^4*c^3 + 9*a^2*b^
2*c^4 - 2*a^3*c^5)*k - (b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 - 7*a^3*b*c^4)
*l + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 2*a^4*c^4)*m)*sqr
t(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 420
*((b^2*c^7 - 4*a*c^8)*f - (b^3*c^6 - 4*a*b*c^7)*g + (b^4*c^5 - 5*a*b^2*c^6
+ 4*a^2*c^7)*h - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*j + (b^6*c^3 - 7*a*b
^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*k - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*b^
3*c^4 - 12*a^3*b*c^5)*l + (b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a^3*b^
2*c^4 + 4*a^4*c^5)*m)*x - 210*((b^3*c^6 - 4*a*b*c^7)*f - (b^4*c^5 - 5*a*b^2
*c^6 + 4*a^2*c^7)*g + (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*h - (b^6*c^3 -
7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*j + (b^7*c^2 - 8*a*b^5*c^3 + 19*a
^2*b^3*c^4 - 12*a^3*b*c^5)*k - (b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a
^3*b^2*c^4 + 4*a^4*c^5)*l + (b^9 - 10*a*b^7*c + 34*a^2*b^5*c^2 - 44*a^3*b^3
*c^3 + 16*a^4*b*c^4)*m - (b^2*c^7 - 4*a*c^8)*e)*log(c*x^2 + b*x + a))/(b^2*
c^8 - 4*a*c^9)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**2+
b*x+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.75, size = 982, normalized size = 1.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x
, algorithm="giac")
```

```
[Out] 1/420*(60*c^6*m*x^7 + 70*c^6*l*x^6 - 70*b*c^5*m*x^6 + 84*c^6*k*x^5 - 84*b*c
^5*l*x^5 + 84*b^2*c^4*m*x^5 - 84*a*c^5*m*x^5 + 105*c^6*j*x^4 - 105*b*c^5*k*
x^4 + 105*b^2*c^4*l*x^4 - 105*a*c^5*l*x^4 - 105*b^3*c^3*m*x^4 + 210*a*b*c^4
*m*x^4 + 140*c^6*h*x^3 - 140*b*c^5*j*x^3 + 140*b^2*c^4*k*x^3 - 140*a*c^5*k*
```

```
x^3 - 140*b^3*c^3*l*x^3 + 280*a*b*c^4*l*x^3 + 140*b^4*c^2*m*x^3 - 420*a*b^2
*c^3*m*x^3 + 140*a^2*c^4*m*x^3 + 210*c^6*g*x^2 - 210*b*c^5*h*x^2 + 210*b^2*
c^4*j*x^2 - 210*a*c^5*j*x^2 - 210*b^3*c^3*k*x^2 + 420*a*b*c^4*k*x^2 + 210*b
^4*c^2*l*x^2 - 630*a*b^2*c^3*l*x^2 + 210*a^2*c^4*l*x^2 - 210*b^5*c*m*x^2 +
840*a*b^3*c^2*m*x^2 - 630*a^2*b*c^3*m*x^2 + 420*c^6*f*x - 420*b*c^5*g*x + 4
20*b^2*c^4*h*x - 420*a*c^5*h*x - 420*b^3*c^3*j*x + 840*a*b*c^4*j*x + 420*b^
4*c^2*k*x - 1260*a*b^2*c^3*k*x + 420*a^2*c^4*k*x - 420*b^5*c*l*x + 1680*a*b
^3*c^2*l*x - 1260*a^2*b*c^3*l*x + 420*b^6*m*x - 2100*a*b^4*c*m*x + 2520*a^2
*b^2*c^2*m*x - 420*a^3*c^3*m*x)/c^7 - 1/2*(b*c^6*f - b^2*c^5*g + a*c^6*g +
b^3*c^4*h - 2*a*b*c^5*h - b^4*c^3*j + 3*a*b^2*c^4*j - a^2*c^5*j + b^5*c^2*k
 - 4*a*b^3*c^3*k + 3*a^2*b*c^4*k - b^6*c*l + 5*a*b^4*c^2*l - 6*a^2*b^2*c^3*
l + a^3*c^4*l + b^7*m - 6*a*b^5*c*m + 10*a^2*b^3*c^2*m - 4*a^3*b*c^3*m - c^
7*e)*log(c*x^2 + b*x + a)/c^8 + (2*c^8*d + b^2*c^6*f - 2*a*c^7*f - b^3*c^5*
g + 3*a*b*c^6*g + b^4*c^4*h - 4*a*b^2*c^5*h + 2*a^2*c^6*h - b^5*c^3*j + 5*a
*b^3*c^4*j - 5*a^2*b*c^5*j + b^6*c^2*k - 6*a*b^4*c^3*k + 9*a^2*b^2*c^4*k -
2*a^3*c^5*k - b^7*c*l + 7*a*b^5*c^2*l - 14*a^2*b^3*c^3*l + 7*a^3*b*c^4*l +
b^8*m - 8*a*b^6*c*m + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 2*a^4*c^4*m - b
*c^7*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^8)
```

Mupad [B]
time = 7.26, size = 2779, normalized size = 3.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
b*x + c*x^2),x)
```

```
[Out] x^6*(l/(6*c) - (b*m)/(6*c^2)) + x*(f/c + (b*((a*(j/c - (a*(l/c - (b*m)/c^2)
)/c + (b*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/c - g/c + (b*(h/c
 - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l/c - (b*m)/c^2))/c - k/c +
(a*m)/c^2))/c))/c + (a*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/c))
/c - (a*(h/c - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l/c - (b*m)/c^2)
)/c - k/c + (a*m)/c^2))/c))/c + (a*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c
^2))/c))/c) + x^4*(j/(4*c) - (a*(l/c - (b*m)/c^2))/(4*c) + (b*((b*(l/c - (b
*m)/c^2))/c - k/c + (a*m)/c^2))/(4*c)) - x^2*((a*(j/c - (a*(l/c - (b*m)/c^2
))/c + (b*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/(2*c) - g/(2*c)
+ (b*(h/c - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l/c - (b*m)/c^2))/c
 - k/c + (a*m)/c^2))/c))/c + (a*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2)
)/c))/(2*c)) + x^3*(h/(3*c) - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l
/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/(3*c) + (a*((b*(l/c - (b*m)/c^2)
)/c - k/c + (a*m)/c^2))/(3*c)) - x^5*((b*(l/c - (b*m)/c^2))/(5*c) - k/(5*c)
 + (a*m)/(5*c^2)) + (log((2*c^9*x*(-(2*c^8*d + b^8*m + b^2*c^6*f + 2*a^2*c^
6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b^6*c^2*k + 2*a^4*c
^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^2*c^4*k - 14*a^2*b^3*c^3*l +
```

20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^6*g - 8*a*b^6*c*m - 4*a*b^2*
c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4*c^3*k + 7*a*b^5*c^2*l + 7*a
^3*b*c^4*l)^2/(c^16*(4*a*c - b^2)))^(1/2) - b^8*m - 2*c^8*d - b^2*c^6*f - 2
*a^2*c^6*h + b^3*c^5*g - b^4*c^4*h + 2*a^3*c^5*k + b^5*c^3*j - b^6*c^2*k -
2*a^4*c^4*m + 2*a*c^7*f + b*c^7*e + b^7*c*l + b*c^8*(-(2*c^8*d + b^8*m + b^
2*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b
^6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^2*c^4*k -
14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^6*g - 8*a*
b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4*c^3*k + 7
*a*b^5*c^2*l + 7*a^3*b*c^4*l)^2/(c^16*(4*a*c - b^2)))^(1/2) - 9*a^2*b^2*c^4
*k + 14*a^2*b^3*c^3*l - 20*a^2*b^4*c^2*m + 16*a^3*b^2*c^3*m - 3*a*b*c^6*g +
8*a*b^6*c*m + 4*a*b^2*c^5*h - 5*a*b^3*c^4*j + 5*a^2*b*c^5*j + 6*a*b^4*c^3*
k - 7*a*b^5*c^2*l - 7*a^3*b*c^4*l)*(2*c^8*d + b^8*m + 2*c^9*x*(-(2*c^8*d +
b^8*m + b^2*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5
*c^3*j + b^6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^
2*c^4*k - 14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^
6*g - 8*a*b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4
*c^3*k + 7*a*b^5*c^2*l + 7*a^3*b*c^4*l)^2/(c^16*(4*a*c - b^2)))^(1/2) + b^2
*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b^
6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + b*c^8*(-(2*c^8*d +
b^8*m + b^2*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5
*c^3*j + b^6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^
2*c^4*k - 14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^
6*g - 8*a*b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4
*c^3*k + 7*a*b^5*c^2*l + 7*a^3*b*c^4*l)^2/(c^16*(4*a*c - b^2)))^(1/2) + 9*a
^2*b^2*c^4*k - 14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a
*b*c^6*g - 8*a*b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*
a*b^4*c^3*k + 7*a*b^5*c^2*l + 7*a^3*b*c^4*l))*(b^9*m - b^2*c^7*e - 4*a^2*c^
7*g + b^3*c^6*f - b^4*c^5*g + b^5*c^4*h + 4*a^3*c^6*j - b^6*c^3*j - 4*a^4*c
^5*l + b^7*c^2*k + 4*a*c^8*e - b^8*c*l - 13*a^2*b^2*c^5*j + 19*a^2*b^3*c^4*
k - 26*a^2*b^4*c^3*l + 25*a^3*b^2*c^4*l + 34*a^2*b^5*c^2*m - 44*a^3*b^3*c^3
*m - 4*a*b*c^7*f - 10*a*b^7*c*m + 5*a*b^2*c^6*g - 6*a*b^3*c^5*h + 8*a^2*b*c
^6*h + 7*a*b^4*c^4*j - 8*a*b^5*c^3*k - 12*a^3*b*c^5*k + 9*a*b^6*c^2*l + 16*
a^4*b*c^4*m))/(2*(4*a*c^9 - b^2*c^8)) + (m*x^7)/(7*c) + (atan(b/(4*a*c - b^
2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*(2*c^8*d + b^8*m + b^2*c^6*f + 2*a^
2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b^6*c^2*k + 2*a
^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^2*c^4*k - 14*a^2*b^3*c^3
*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^6*g - 8*a*b^6*c*m - 4*a*
b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4*c^3*k + 7*a*b^5*c^2*l +
7*a^3*b*c^4*l))/(c^8*(4*a*c - b^2)^(1/2))

3.374 $\int (1+4x-7x^2)^3 (2+5x+x^2) \sqrt{3+2x+5x^2}\, dx$

<u>**Optimal**</u>. Leaf size=208

$$-\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000}-\frac{1968340667(3+2x+5x^2)^{3/2}}{131250000}+\frac{1045360143x(3+2x+5x^2)^{3/2}}{43750000}+9806$$

```
[Out] -1968340667/131250000*(5*x^2+2*x+3)^(3/2)+1045360143/43750000*x*(5*x^2+2*x+
3)^(3/2)+98060877/4375000*x^2*(5*x^2+2*x+3)^(3/2)-90960857/1575000*x^3*(5*x
^2+2*x+3)^(3/2)-888751/105000*x^4*(5*x^2+2*x+3)^(3/2)+190939/3000*x^5*(5*x^
2+2*x+3)^(3/2)-50519/2250*x^6*(5*x^2+2*x+3)^(3/2)-343/50*x^7*(5*x^2+2*x+3)^
(3/2)-540119881/78125000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-77159983/31
250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

<u>**Rubi**</u> [A]
time = 0.22, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1675, 654, 626, 633, 221}

$$\frac{98060877(5x^2+2x+3)^{3/2}x^2}{4375000}+\frac{1045360143(5x^2+2x+3)^{3/2}x}{43750000}-\frac{1968340667(5x^2+2x+3)^{3/2}}{131250000}-\frac{77159983(5x+1)\sqrt{5x^2+2x+3}}{31250000}-\frac{343}{50}(5x^2+2x+3)^{3/2}x^7-\frac{50519(5x^2+2x+3)^{3/2}x^6}{2250}+\frac{190939(5x^2+2x+3)^{3/2}x^5}{3000}-\frac{888751(5x^2+2x+3)^{3/2}x^4}{105000}-\frac{90960857(5x^2+2x+3)^{3/2}x^3}{1575000}-\frac{540119881\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{15625000\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-77159983*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/31250000 - (1968340667*(3 + 2*x
 + 5*x^2)^(3/2))/131250000 + (1045360143*x*(3 + 2*x + 5*x^2)^(3/2))/4375000
0 + (98060877*x^2*(3 + 2*x + 5*x^2)^(3/2))/4375000 - (90960857*x^3*(3 + 2*x
 + 5*x^2)^(3/2))/1575000 - (888751*x^4*(3 + 2*x + 5*x^2)^(3/2))/105000 + (1
90939*x^5*(3 + 2*x + 5*x^2)^(3/2))/3000 - (50519*x^6*(3 + 2*x + 5*x^2)^(3/2
))/2250 - (343*x^7*(3 + 2*x + 5*x^2)^(3/2))/50 - (540119881*ArcSinh[(1 + 5*
x)/Sqrt[14]])/(15625000*Sqrt[5])
```

<u>Rule 221</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

<u>Rule 626</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

<u>Rule 633</u>

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx = -\frac{343}{50} x^7 \left(3+2x+5x^2\right)^{3/2} + \frac{1}{50} \int \sqrt{3+2x+5x^2}$$

$$= -\frac{50519x^6(3+2x+5x^2)^{3/2}}{2250} - \frac{343}{50} x^7 \left(3+2x+5x^2\right.$$

$$= \frac{190939x^5(3+2x+5x^2)^{3/2}}{3000} - \frac{50519x^6(3+2x+5x}{2250}$$

$$= -\frac{888751x^4(3+2x+5x^2)^{3/2}}{105000} + \frac{190939x^5(3+2x+}{3000}$$

$$= -\frac{90960857x^3(3+2x+5x^2)^{3/2}}{1575000} - \frac{888751x^4(3+2x}{105000}$$

$$= \frac{98060877x^2(3+2x+5x^2)^{3/2}}{4375000} - \frac{90960857x^3(3+2x}{1575000}$$

$$= \frac{1045360143x(3+2x+5x^2)^{3/2}}{43750000} + \frac{98060877x^2(3+}{437500}$$

$$= -\frac{1968340667(3+2x+5x^2)^{3/2}}{131250000} + \frac{1045360143x(3+}{43750}$$

$$= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000} - \frac{1968340667}{131}$$

$$= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000} - \frac{1968340667}{131}$$

$$= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000} - \frac{1968340667}{131}$$

Mathematica [A]
time = 0.73, size = 99, normalized size = 0.48

$$\frac{\sqrt{3+2x+5x^2}\left(-93436408944+57768004650x+78839046795x^2-17642392275x^3-56757413000x^4-225922362500x^5+34674656250x^6+497593468750x^7-248031875000x^8-67528125000x^9\right)}{1968750000}+\frac{540119881\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{15625000\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-93436408944 + 57768004650*x + 78839046795*x^2 - 17
642392275*x^3 - 56757413000*x^4 - 225922362500*x^5 + 34674656250*x^6 + 4975
93468750*x^7 - 248031875000*x^8 - 67528125000*x^9))/1968750000 + (540119881
*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(15625000*Sqrt[5])
```

Maple [A]
time = 0.20, size = 166, normalized size = 0.80

method	result
risch	$-\frac{(67528125000x^9+248031875000x^8-497593468750x^7-34674656250x^6+225922362500x^5+56757413000x^4+17642392275x^3-78839}{1968750000}$
trager	$\left(-\frac{343}{10}x^9-\frac{56693}{450}x^8+\frac{2274713}{9000}x^7+\frac{369863}{21000}x^6-\frac{18073789}{157500}x^5-\frac{56757413}{1968750}x^4-\frac{235231897}{26250000}x^3+\frac{5255936453}{131250000}x^2+\right.$
default	$-\frac{540119881\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\ \left(x+\frac{1}{5}\right)}{14}\right)}{78125000}-\frac{77159983(10x+2)\sqrt{5x^2+2x+3}}{62500000}-\frac{1968340667(5x^2+2x+3)^{\frac{3}{2}}}{131250000}+\frac{10453}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -540119881/78125000*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-77159983/6250000
0*(10*x+2)*(5*x^2+2*x+3)^(1/2)-1968340667/131250000*(5*x^2+2*x+3)^(3/2)+104
5360143/43750000*x*(5*x^2+2*x+3)^(3/2)+98060877/4375000*x^2*(5*x^2+2*x+3)^(
3/2)-90960857/1575000*x^3*(5*x^2+2*x+3)^(3/2)-888751/105000*x^4*(5*x^2+2*x+
3)^(3/2)+190939/3000*x^5*(5*x^2+2*x+3)^(3/2)-50519/2250*x^6*(5*x^2+2*x+3)^(
3/2)-343/50*x^7*(5*x^2+2*x+3)^(3/2)
```

Maxima [A]
time = 0.50, size = 177, normalized size = 0.85

$$-\frac{343}{50}\,{\left(5\,x^{2}+2\,x+3\right)}^{\frac{3}{2}}x^{7}-\frac{50519}{2250}\,{\left(5\,x^{2}+2\,x+3\right)}^{\frac{3}{2}}x^{6}+\frac{190939}{3000}\,{\left(5\,x^{2}+2\,x+3\right)}^{\frac{3}{2}}x^{5}-\frac{888751}{105000}\,{\left(5\,x^{2}+2\,x+3\right)}^{\frac{3}{2}}x^{4}-\frac{90960857}{1575000}\,{\left(5\,x^{2}+2\,x+3\right)}^{\frac{3}{2}}x^{3}+\frac{98060877}{4375000}\,{\left(5\,x^{2}+2\,x+3\right)}^{\frac{3}{2}}x^{2}+\frac{1045360143}{43750000}\,{\left(5\,x^{2}+2\,x+3\right)}^{\frac{3}{2}}x-\frac{1968340667}{131250000}\,{\left(5\,x^{2}+2\,x+3\right)}^{\frac{3}{2}}-\frac{77159983}{6250000}\,\sqrt{5\,x^{2}+2\,x+3}\,x-\frac{540119881}{78125000}\,\sqrt{5}\,\operatorname{arsinh}\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)-\frac{77159983}{31250000}\,\sqrt{5\,x^{2}+2\,x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -343/50*(5*x^2 + 2*x + 3)^(3/2)*x^7 - 50519/2250*(5*x^2 + 2*x + 3)^(3/2)*x^
6 + 190939/3000*(5*x^2 + 2*x + 3)^(3/2)*x^5 - 888751/105000*(5*x^2 + 2*x +
3)^(3/2)*x^4 - 90960857/1575000*(5*x^2 + 2*x + 3)^(3/2)*x^3 + 98060877/4375
000*(5*x^2 + 2*x + 3)^(3/2)*x^2 + 1045360143/43750000*(5*x^2 + 2*x + 3)^(3/
2)*x - 1968340667/131250000*(5*x^2 + 2*x + 3)^(3/2) - 77159983/6250000*sqrt
(5*x^2 + 2*x + 3)*x - 540119881/78125000*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x
 + 1)) - 77159983/31250000*sqrt(5*x^2 + 2*x + 3)
```

Fricas [A]
time = 0.36, size = 97, normalized size = 0.47

$$-\frac{1}{1968750000}\,{\left(67528125000\,x^{9}+248031875000\,x^{8}-497593468750\,x^{7}-34674656250\,x^{6}+225922362500\,x^{5}+56757413000\,x^{4}+17642392275\,x^{3}-78839046795\,x^{2}-57768004650\,x+93436408944\right)}\sqrt{5\,x^{2}+2\,x+3}+\frac{540119881}{156250000}\,\sqrt{5}\,\log\left(\sqrt{5}\,\sqrt{5\,x^{2}+2\,x+3}\,(5\,x+1)-25\,x^{2}-10\,x-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/1968750000*(67528125000*x^9 + 248031875000*x^8 - 497593468750*x^7 - 3467
4656250*x^6 + 225922362500*x^5 + 56757413000*x^4 + 17642392275*x^3 - 788390
46795*x^2 - 57768004650*x + 93436408944)*sqrt(5*x^2 + 2*x + 3) + 540119881/
156250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10
*x - 8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-29x\sqrt{5x^2+2x+3}\right)dx-\int\left(-115x^2\sqrt{5x^2+2x+3}\right)dx-\int 61x^3\sqrt{5x^2+2x+3}\,dx-\int 871x^4\sqrt{5x^2+2x+3}\,dx-\int\left(-127x^5\sqrt{5x^2+2x+3}\right)dx-\int\left(-2065x^6\sqrt{5x^2+2x+3}\right)dx-\int 1127x^7\sqrt{5x^2+2x+3}\,dx-\int 343x^8\sqrt{5x^2+2x+3}\,dx-\int\left(-2\sqrt{5x^2+2x+3}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(-29*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-115*x**2*sqrt(5*x**2
 + 2*x + 3), x) - Integral(61*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(87
1*x**4*sqrt(5*x**2 + 2*x + 3), x) - Integral(-127*x**5*sqrt(5*x**2 + 2*x +
3), x) - Integral(-2065*x**6*sqrt(5*x**2 + 2*x + 3), x) - Integral(1127*x**
7*sqrt(5*x**2 + 2*x + 3), x) - Integral(343*x**8*sqrt(5*x**2 + 2*x + 3), x)
 - Integral(-2*sqrt(5*x**2 + 2*x + 3), x)
```

Giac [A]
time = 2.86, size = 92, normalized size = 0.44

$$-\frac{1}{1968750000}\left(5\left(\left(5\left(10\left(25\left(5\left(49\left(140\left(315x+1157\right)x-324959\right)x-1109589\right)x+36147578\right)x+227029652\right)x+705695691\right)x-15767809359\right)x-11553600930\right)x+93436408944\right)\sqrt{5x^2+2x+3}+\frac{540119881}{78125000}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/1968750000*(5*((5*(10*(25*(5*(49*(140*(315*x + 1157)*x - 324959)*x - 110
9589)*x + 36147578)*x + 227029652)*x + 705695691)*x - 15767809359)*x - 1155
3600930)*x + 93436408944)*sqrt(5*x^2 + 2*x + 3) + 540119881/78125000*sqrt(5
)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [B]
time = 6.31, size = 221, normalized size = 1.06

$$\frac{98060877x^2(5x^2+2x+3)^{3/2}}{4375000}-\frac{90960857x^3(5x^2+2x+3)^{3/2}}{1575000}-\frac{888751x^4(5x^2+2x+3)^{3/2}}{105000}+\frac{190939x^5(5x^2+2x+3)^{3/2}}{3000}-\frac{50519x^6(5x^2+2x+3)^{3/2}}{2250}-\frac{343x^7(5x^2+2x+3)^{3/2}}{50}-\frac{3048580429\sqrt{5}\ln\left(\sqrt{5x^2+2x+3}+\frac{\sqrt{5}(5x+1)}{5}\right)}{156250000}-\frac{3048580429\left(\frac{x}{2}+\frac{1}{10}\right)\sqrt{5x^2+2x+3}}{43750000}-\frac{1968340667\sqrt{5x^2+2x+3}(200x^2+20x+108)}{5250000000}+\frac{1045360143x(5x^2+2x+3)^{3/2}}{43750000}+\frac{1968340667\sqrt{5}\ln\left(2\sqrt{5x^2+2x+3}+\frac{\sqrt{5}(10x+2)}{5}\right)}{156250000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3,x)
```

```
[Out] (98060877*x^2*(2*x + 5*x^2 + 3)^(3/2))/4375000 - (90960857*x^3*(2*x + 5*x^2
 + 3)^(3/2))/1575000 - (888751*x^4*(2*x + 5*x^2 + 3)^(3/2))/105000 + (19093
9*x^5*(2*x + 5*x^2 + 3)^(3/2))/3000 - (50519*x^6*(2*x + 5*x^2 + 3)^(3/2))/2
250 - (343*x^7*(2*x + 5*x^2 + 3)^(3/2))/50 - (3048580429*5^(1/2)*log((2*x +
 5*x^2 + 3)^(1/2) + (5^(1/2)*(5*x + 1))/5))/156250000 - (3048580429*(x/2 +
1/10)*(2*x + 5*x^2 + 3)^(1/2))/43750000 - (1968340667*(2*x + 5*x^2 + 3)^(1/
2)*(20*x + 200*x^2 + 108))/5250000000 + (1045360143*x*(2*x + 5*x^2 + 3)^(3/
2))/43750000 + (1968340667*5^(1/2)*log(2*(2*x + 5*x^2 + 3)^(1/2) + (5^(1/2)
*(10*x + 2))/5))/156250000
```

3.375 $\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx$

Optimal. Leaf size=166

$$-\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000}+\frac{198439\left(3+2x+5x^2\right)^{3/2}}{750000}+\frac{1781669x\left(3+2x+5x^2\right)^{3/2}}{250000}-\frac{77509x^2(3+2}{2500}$$

```
[Out] 198439/750000*(5*x^2+2*x+3)^(3/2)+1781669/250000*x*(5*x^2+2*x+3)^(3/2)-7750
9/25000*x^2*(5*x^2+2*x+3)^(3/2)-25277/3000*x^3*(5*x^2+2*x+3)^(3/2)+989/200*
x^4*(5*x^2+2*x+3)^(3/2)+49/40*x^5*(5*x^2+2*x+3)^(3/2)-17652061/3125000*arcs
inh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-2521723/1250000*(1+5*x)*(5*x^2+2*x+3)^(1
/2)
```

Rubi [A]
time = 0.13, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1675, 654, 626, 633, 221}

$$-\frac{77509\left(5x^2+2x+3\right)^{3/2}x^2}{25000}+\frac{1781669\left(5x^2+2x+3\right)^{3/2}x}{250000}+\frac{198439\left(5x^2+2x+3\right)^{3/2}}{750000}-\frac{2521723(5x+1)\sqrt{5x^2+2x+3}}{1250000}+\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5+\frac{989}{200}\left(5x^2+2x+3\right)^{3/2}x^4-\frac{25277\left(5x^2+2x+3\right)^{3/2}x^3}{3000}-\frac{17652061\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{625000\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-2521723*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/1250000 + (198439*(3 + 2*x + 5*x
^2)^(3/2))/750000 + (1781669*x*(3 + 2*x + 5*x^2)^(3/2))/250000 - (77509*x^2
*(3 + 2*x + 5*x^2)^(3/2))/25000 - (25277*x^3*(3 + 2*x + 5*x^2)^(3/2))/3000
+ (989*x^4*(3 + 2*x + 5*x^2)^(3/2))/200 + (49*x^5*(3 + 2*x + 5*x^2)^(3/2))/
40 - (17652061*ArcSinh[(1 + 5*x)/Sqrt[14]])/(625000*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx = \frac{49}{40} x^5 \left(3+2x+5x^2\right)^{3/2} + \frac{1}{40} \int \sqrt{3+2x+5x^2}\, (80$$

$$= \frac{989}{200} x^4 \left(3+2x+5x^2\right)^{3/2} + \frac{49}{40} x^5 \left(3+2x+5x^2\right)^{3/2} -$$

$$= -\frac{25277x^3(3+2x+5x^2)^{3/2}}{3000} + \frac{989}{200} x^4 \left(3+2x+5x^2\right.$$

$$= -\frac{77509x^2(3+2x+5x^2)^{3/2}}{25000} - \frac{25277x^3(3+2x+5x}{3000}$$

$$= \frac{1781669x(3+2x+5x^2)^{3/2}}{250000} - \frac{77509x^2(3+2x+5x}{25000}$$

$$= \frac{198439(3+2x+5x^2)^{3/2}}{750000} + \frac{1781669x(3+2x+5x^2}{250000}$$

$$= -\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000} + \frac{198439(3+2}{7500}$$

$$= -\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000} + \frac{198439(3+2}{7500}$$

$$= -\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000} + \frac{198439(3+2}{7500}$$

Mathematica [A]
time = 0.51, size = 89, normalized size = 0.54

$$\frac{\sqrt{3+2x+5x^2}\,(-4588584+44333650x+23531995x^2+15583725x^3-65693000x^4-107112500x^5+101906250x^6+22968750x^7)}{3750000} + \frac{17652061 \log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{625000\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-4588584 + 44333650*x + 23531995*x^2 + 15583725*x^3
 - 65693000*x^4 - 107112500*x^5 + 101906250*x^6 + 22968750*x^7))/3750000 +
(17652061*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(625000*Sqrt[5])
```

Maple [A]
time = 0.17, size = 132, normalized size = 0.80

method	result

risch	$\frac{(22968750x^7+101906250x^6-107112500x^5-65693000x^4+15583725x^3+23531995x^2+44333650x-4588584)\sqrt{5x^2+2x+3}}{3750000} -$
trager	$\left(\frac{49}{8}x^7+\frac{1087}{40}x^6-\frac{8569}{300}x^5-\frac{65693}{3750}x^4+\frac{207783}{50000}x^3+\frac{4706399}{750000}x^2+\frac{886673}{75000}x-\frac{191191}{156250}\right)\sqrt{5x^2+2x+3} - \frac{17\cdots}{}$
default	$-\frac{17652061\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,\left(x+\frac{1}{5}\right)}{14}\right)}{3125000}-\frac{2521723(10x+2)\sqrt{5x^2+2x+3}}{2500000}+\frac{198439(5x^2+2x+3)^{\frac{3}{2}}}{750000}+\frac{1781669x(5x\cdots}{2500\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -17652061/3125000*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-2521723/2500000*(1
0*x+2)*(5*x^2+2*x+3)^(1/2)+198439/750000*(5*x^2+2*x+3)^(3/2)+1781669/250000
*x*(5*x^2+2*x+3)^(3/2)-77509/25000*x^2*(5*x^2+2*x+3)^(3/2)-25277/3000*x^3*(
5*x^2+2*x+3)^(3/2)+989/200*x^4*(5*x^2+2*x+3)^(3/2)+49/40*x^5*(5*x^2+2*x+3)^
(3/2)
```

Maxima [A]
time = 0.49, size = 143, normalized size = 0.86

$$\frac{49}{40}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^5+\frac{989}{200}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^4-\frac{25277}{3000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3-\frac{77509}{25000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2+\frac{1781669}{250000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x+\frac{198439}{750000}\left(5x^2+2x+3\right)^{\frac{3}{2}}-\frac{2521723}{250000}\sqrt{5x^2+2x+3}\,x-\frac{17652061}{3125000}\sqrt{5}\,\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}\,(5x+1)\right)-\frac{2521723}{1250000}\sqrt{5x^2+2x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] 49/40*(5*x^2 + 2*x + 3)^(3/2)*x^5 + 989/200*(5*x^2 + 2*x + 3)^(3/2)*x^4 - 2
5277/3000*(5*x^2 + 2*x + 3)^(3/2)*x^3 - 77509/25000*(5*x^2 + 2*x + 3)^(3/2)
*x^2 + 1781669/250000*(5*x^2 + 2*x + 3)^(3/2)*x + 198439/750000*(5*x^2 + 2*
x + 3)^(3/2) - 2521723/250000*sqrt(5*x^2 + 2*x + 3)*x - 17652061/3125000*sq
rt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 2521723/1250000*sqrt(5*x^2 + 2*x +
 3)
```

Fricas [A]
time = 0.37, size = 87, normalized size = 0.52

$$\frac{1}{3750000}\left(22968750x^7+101906250x^6-107112500x^5-65693000x^4+15583725x^3+23531995x^2+44333650x-4588584\right)\sqrt{5x^2+2x+3}+\frac{17652061}{6250000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5x^2+2x+3}\,(5x+1)-25x^2-10x-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/3750000*(22968750*x^7 + 101906250*x^6 - 107112500*x^5 - 65693000*x^4 + 15
583725*x^3 + 23531995*x^2 + 44333650*x - 4588584)*sqrt(5*x^2 + 2*x + 3) + 1
7652061/6250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^
2 - 10*x - 8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2 + 5x + 2\right) \sqrt{5x^2 + 2x + 3} \left(7x^2 - 4x - 1\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)*sqrt(5*x**2 + 2*x + 3)*(7*x**2 - 4*x - 1)**2, x)
```

Giac [A]
time = 2.74, size = 82, normalized size = 0.49

$$\frac{1}{3750000} \left(5 \left(\left(5 \left(10 \left(25 \left(15 \left(245 x + 1087\right) x - 17138\right) x - 262772\right) x + 623349\right) x + 4706399\right) x + 8866730\right) x - 4588584\right) \sqrt{5 x^2 + 2 x + 3} + \frac{17652061}{3125000} \sqrt{5} \log\left(-\sqrt{5} \left(\sqrt{5} x - \sqrt{5 x^2 + 2 x + 3}\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/3750000*(5*((5*(10*(25*(15*(245*x + 1087)*x - 17138)*x - 262772)*x + 6233
49)*x + 4706399)*x + 8866730)*x - 4588584)*sqrt(5*x^2 + 2*x + 3) + 17652061
/3125000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [B]
time = 6.01, size = 187, normalized size = 1.13

$$\frac{989 x^4 \left(5 x^2 + 2 x + 3\right)^{3/2}}{200} - \frac{25277 x^3 \left(5 x^2 + 2 x + 3\right)^{3/2}}{3000} - \frac{77509 x^2 \left(5 x^2 + 2 x + 3\right)^{3/2}}{25000} + \frac{49 x^5 \left(5 x^2 + 2 x + 3\right)^{3/2}}{40} - \frac{33915049 \sqrt{5} \ln\left(\sqrt{5 x^2 + 2 x + 3} + \frac{\sqrt{5} \left(5 x + 1\right)}{5}\right)}{6250000} - \frac{4845007 \left(\frac{x}{2} + \frac{1}{10}\right) \sqrt{5 x^2 + 2 x + 3}}{250000} + \frac{198439 \sqrt{5 x^2 + 2 x + 3} \left(200 x^2 + 20 x + 108\right)}{30000000} + \frac{1781669 x \left(5 x^2 + 2 x + 3\right)^{3/2}}{250000} - \frac{1389073 \sqrt{5} \ln\left(2 \sqrt{5 x^2 + 2 x + 3} + \frac{\sqrt{5} \left(10 x + 2\right)}{5}\right)}{6250000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2,x)
```

```
[Out] (989*x^4*(2*x + 5*x^2 + 3)^(3/2))/200 - (25277*x^3*(2*x + 5*x^2 + 3)^(3/2))
/3000 - (77509*x^2*(2*x + 5*x^2 + 3)^(3/2))/25000 + (49*x^5*(2*x + 5*x^2 +
3)^(3/2))/40 - (33915049*5^(1/2)*log((2*x + 5*x^2 + 3)^(1/2) + (5^(1/2)*(5*
x + 1))/5))/6250000 - (4845007*(x/2 + 1/10)*(2*x + 5*x^2 + 3)^(1/2))/250000
 + (198439*(2*x + 5*x^2 + 3)^(1/2)*(20*x + 200*x^2 + 108))/30000000 + (1781
669*x*(2*x + 5*x^2 + 3)^(3/2))/250000 - (1389073*5^(1/2)*log(2*(2*x + 5*x^2
 + 3)^(1/2) + (5^(1/2)*(10*x + 2))/5))/6250000
```

3.376 $\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx$

Optimal. Leaf size=124

$$-\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500}+\frac{7819(3+2x+5x^2)^{3/2}}{7500}+\frac{2149x(3+2x+5x^2)^{3/2}}{2500}-\frac{289}{250}x^2(3+2x+5x^2)^{3/2}-$$

```
[Out] 7819/7500*(5*x^2+2*x+3)^(3/2)+2149/2500*x*(5*x^2+2*x+3)^(3/2)-289/250*x^2*(
5*x^2+2*x+3)^(3/2)-7/30*x^3*(5*x^2+2*x+3)^(3/2)-32431/31250*arcsinh(1/14*(1
+5*x)*14^(1/2))*5^(1/2)-4633/12500*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1675, 654, 626, 633, 221}

$$-\frac{289}{250}(5x^2+2x+3)^{3/2}x^2+\frac{2149(5x^2+2x+3)^{3/2}x}{2500}+\frac{7819(5x^2+2x+3)^{3/2}}{7500}-\frac{4633(5x+1)\sqrt{5x^2+2x+3}}{12500}-\frac{7}{30}(5x^2+2x+3)^{3/2}x^3-\frac{32431\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{6250\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-4633*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/12500 + (7819*(3 + 2*x + 5*x^2)^(3/
2))/7500 + (2149*x*(3 + 2*x + 5*x^2)^(3/2))/2500 - (289*x^2*(3 + 2*x + 5*x^
2)^(3/2))/250 - (7*x^3*(3 + 2*x + 5*x^2)^(3/2))/30 - (32431*ArcSinh[(1 + 5*
x)/Sqrt[14]])/(6250*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx = -\frac{7}{30}x^3\left(3+2x+5x^2\right)^{3/2}+\frac{1}{30}\int \sqrt{3+2x+5x^2}\,(6$$

$$= -\frac{289}{250}x^2\left(3+2x+5x^2\right)^{3/2}-\frac{7}{30}x^3\left(3+2x+5x^2\right)^{3/2}$$

$$= \frac{2149x(3+2x+5x^2)^{3/2}}{2500}-\frac{289}{250}x^2\left(3+2x+5x^2\right)^{3/2}$$

$$= \frac{7819(3+2x+5x^2)^{3/2}}{7500}+\frac{2149x(3+2x+5x^2)^{3/2}}{2500}-$$

$$= -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500}+\frac{7819(3+2x+5x}{7500}$$

$$= -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500}+\frac{7819(3+2x+5x}{7500}$$

$$= -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500}+\frac{7819(3+2x+5x}{7500}$$

Mathematica [A]

time = 0.36, size = 79, normalized size = 0.64

$$\frac{\sqrt{3+2x+5x^2}\,(103386+105400x+129895x^2+48225x^3-234250x^4-43750x^5)}{37500}+\frac{32431\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{6250\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(103386 + 105400*x + 129895*x^2 + 48225*x^3 - 234250
*x^4 - 43750*x^5))/37500 + (32431*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x
^2]])/(6250*Sqrt[5])
```

Maple [A]
time = 0.14, size = 98, normalized size = 0.79

method	result
risch	$-\frac{(43750x^5+234250x^4-48225x^3-129895x^2-105400x-103386)\sqrt{5x^2+2x+3}}{37500}-\frac{32431\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,(x+\frac{1}{5})}{14}\right)}{31250}$
trager	$\left(-\frac{7}{6}x^5-\frac{937}{150}x^4+\frac{643}{500}x^3+\frac{25979}{7500}x^2+\frac{1054}{375}x+\frac{17231}{6250}\right)\sqrt{5x^2+2x+3}+\frac{32431\,\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(-5\,\mathrm{Root}\right.}{}$
default	$-\frac{7x^3(5x^2+2x+3)^{\frac{3}{2}}}{30}-\frac{289x^2(5x^2+2x+3)^{\frac{3}{2}}}{250}+\frac{2149x(5x^2+2x+3)^{\frac{3}{2}}}{2500}+\frac{7819(5x^2+2x+3)^{\frac{3}{2}}}{7500}-\frac{4633(10x+2)\sqrt{5x^2+2x+}}{25000}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -7/30*x^3*(5*x^2+2*x+3)^(3/2)-289/250*x^2*(5*x^2+2*x+3)^(3/2)+2149/2500*x*(
5*x^2+2*x+3)^(3/2)+7819/7500*(5*x^2+2*x+3)^(3/2)-4633/25000*(10*x+2)*(5*x^2
+2*x+3)^(1/2)-32431/31250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Maxima [A]
time = 0.49, size = 109, normalized size = 0.88

$$-\frac{7}{30}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3-\frac{289}{250}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2+\frac{2149}{2500}\left(5x^2+2x+3\right)^{\frac{3}{2}}x+\frac{7819}{7500}\left(5x^2+2x+3\right)^{\frac{3}{2}}-\frac{4633}{2500}\sqrt{5x^2+2x+3}\,x-\frac{32431}{31250}\sqrt{5}\,\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}\,(5x+1)\right)-\frac{4633}{12500}\sqrt{5x^2+2x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -7/30*(5*x^2 + 2*x + 3)^(3/2)*x^3 - 289/250*(5*x^2 + 2*x + 3)^(3/2)*x^2 + 2
149/2500*(5*x^2 + 2*x + 3)^(3/2)*x + 7819/7500*(5*x^2 + 2*x + 3)^(3/2) - 46
33/2500*sqrt(5*x^2 + 2*x + 3)*x - 32431/31250*sqrt(5)*arcsinh(1/14*sqrt(14)
*(5*x + 1)) - 4633/12500*sqrt(5*x^2 + 2*x + 3)
```

Fricas [A]
time = 0.34, size = 77, normalized size = 0.62

$$-\frac{1}{37500}\left(43750\,x^5+234250\,x^4-48225\,x^3-129895\,x^2-105400\,x-103386\right)\sqrt{5\,x^2+2\,x+3}+\frac{32431}{62500}\sqrt{5}\,\log\left(\sqrt{5}\,\sqrt{5\,x^2+2\,x+3}\,(5\,x+1)-25\,x^2-10\,x-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="fric
as")
```

```
[Out] -1/37500*(43750*x^5 + 234250*x^4 - 48225*x^3 - 129895*x^2 - 105400*x - 1033
86)*sqrt(5*x^2 + 2*x + 3) + 32431/62500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*
x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-13x\sqrt{5x^2+2x+3}\right)dx - \int \left(-7x^2\sqrt{5x^2+2x+3}\right)dx - \int 31x^3\sqrt{5x^2+2x+3}\,dx - \int 7x^4\sqrt{5x^2+2x+3}\,dx - \int \left(-2\sqrt{5x^2+2x+3}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(-13*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-7*x**2*sqrt(5*x**2 +
 2*x + 3), x) - Integral(31*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(7*x*
*4*sqrt(5*x**2 + 2*x + 3), x) - Integral(-2*sqrt(5*x**2 + 2*x + 3), x)
```

Giac [A]
time = 3.04, size = 72, normalized size = 0.58

$$-\frac{1}{37500}\,(5\,((5\,(10\,(175\,x+937)x-1929)x-25979)x-21080)x-103386)\sqrt{5\,x^2+2\,x+3}+\frac{32431}{31250}\,\sqrt{5}\,\log\left(-\sqrt{5}\,\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/37500*(5*((5*(10*(175*x + 937)*x - 1929)*x - 25979)*x - 21080)*x - 10338
6)*sqrt(5*x^2 + 2*x + 3) + 32431/31250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sq
rt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [B]
time = 5.38, size = 153, normalized size = 1.23

$$\frac{7819\,\sqrt{5\,x^2+2\,x+3}\,\left(200\,x^2+20\,x+108\right)}{300000}-\frac{7\,x^3\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{30}-\frac{10129\,\sqrt{5}\,\ln\left(\sqrt{5\,x^2+2\,x+3}+\frac{\sqrt{5}\,\left(5\,x+1\right)}{5}\right)}{62500}-\frac{1447\,\left(\frac{x}{2}+\frac{1}{10}\right)\,\sqrt{5\,x^2+2\,x+3}}{2500}-\frac{289\,x^2\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{250}+\frac{2149\,x\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{2500}-\frac{54733\,\sqrt{5}\,\ln\left(2\,\sqrt{5\,x^2+2\,x+3}+\frac{\sqrt{5}\,\left(10\,x+2\right)}{5}\right)}{62500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1),x)
```

```
[Out] (7819*(2*x + 5*x^2 + 3)^(1/2)*(20*x + 200*x^2 + 108))/300000 - (7*x^3*(2*x
+ 5*x^2 + 3)^(3/2))/30 - (10129*5^(1/2)*log((2*x + 5*x^2 + 3)^(1/2) + (5^(1
/2)*(5*x + 1))/5))/62500 - (1447*(x/2 + 1/10)*(2*x + 5*x^2 + 3)^(1/2))/2500
 - (289*x^2*(2*x + 5*x^2 + 3)^(3/2))/250 + (2149*x*(2*x + 5*x^2 + 3)^(3/2))
/2500 - (54733*5^(1/2)*log(2*(2*x + 5*x^2 + 3)^(1/2) + (5^(1/2)*(10*x + 2))
/5))/62500
```

3.377 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx$

Optimal. Leaf size=187

$$-\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2}-\frac{8233\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1715\sqrt{5}}-\frac{3}{343}\sqrt{\frac{1}{11}\left(497041-146555\sqrt{11}\right)}\tanh^{-1}\left(\frac{2}{\sqrt{2}}\right.$$

[Out] -8233/8575*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-1/490*(397+35*x)*(5*x^2+2*x+3)^(1/2)-3/3773*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(5467451-1612105*11^(1/2))^(1/2)+3/3773*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(5467451+1612105*11^(1/2))^(1/2)

Rubi [A]

time = 0.23, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1080, 1090, 633, 221, 1046, 738, 212}

$$-\frac{1}{490}\sqrt{5x^2+2x+3}\,(35x+397)-\frac{3}{343}\sqrt{\frac{1}{11}\left(497041-146555\sqrt{11}\right)}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)+\frac{3}{343}\sqrt{\frac{1}{11}\left(497041+146555\sqrt{11}\right)}\tanh^{-1}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)-\frac{8233\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{1715\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2),x]

[Out] -1/490*((397 + 35*x)*Sqrt[3 + 2*x + 5*x^2]) - (8233*ArcSinh[(1 + 5*x)/Sqrt[14]])/(1715*Sqrt[5]) - (3*Sqrt[(497041 - 146555*Sqrt[11])/11]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/343 + (3*Sqrt[(497041 + 146555*Sqrt[11])/11]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/343

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p
 + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b
*x + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
```

```
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx = -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{1}{490}\int \frac{-3442-13408x-164\ldots}{(1+4x-7x^2)\sqrt{3+2x+\ldots}}$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} + \frac{\int \frac{40560+159720x}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx}{3430} - \ldots$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{8233\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2\ldots\right.}{3430\sqrt{70}}$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{8233\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1715\sqrt{5}} - \frac{\left(24\left(14\ldots\right.\right.}{\ldots}$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{8233\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1715\sqrt{5}} - \frac{3\sqrt{546\ldots}}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.35, size = 234, normalized size = 1.25

$$\frac{1}{490}(-397-35x)\sqrt{3+2x+5x^2} + \frac{8233\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{1715\sqrt{5}} + \frac{6}{343}\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{3317\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+676\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1-1331\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1^2}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2),x]
```

```
[Out] ((-397 - 35*x)*Sqrt[3 + 2*x + 5*x^2])/490 + (8233*Log[-1 - 5*x + Sqrt[5]*Sq
rt[3 + 2*x + 5*x^2]])/(1715*Sqrt[5]) + (6*RootSum[83 - 16*Sqrt[5]*#1 - 70*#
1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (3317*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5
*x^2] - #1] + 676*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1
 - 1331*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] -
35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/343
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(134) = 268.
time = 0.80, size = 403, normalized size = 2.16 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/140*(10*x+2)*(5*x^2+2*x+3)^(1/2)-1/25*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1
/5))-3/154*(61+13*11^(1/2))*11^(1/2)*(1/49*(245*(x-2/7-1/7*11^(1/2))^2+49*(
34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/70*(34/7+
10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10
/7*11^(1/2))^2)^(1/2)*(x+1/5))-(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1
/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^
(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*1
1^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-3/154*(-61+13*11^(1/
2))*11^(1/2)*(1/49*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2
/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/70*(34/7-10/7*11^(1/2))*5^(1/2)*a
rcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x
+1/5))-(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49
-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2)
)^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(
1/2))+250-34*11^(1/2))^(1/2)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(133) = 266$.
time = 0.53, size = 500, normalized size = 2.67

$$\frac{1}{188650}\sqrt{11}\left(975\sqrt{11}\sqrt{2}\sqrt{17\sqrt{11}+125}\,\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7\left|14x-2\sqrt{11}-4\right|}+\frac{17\sqrt{7}\sqrt{2}x}{7\left|14x-2\sqrt{11}-4\right|}+\frac{\sqrt{11}\sqrt{7}\sqrt{2}}{7\left|14x-2\sqrt{11}-4\right|}+\frac{23\sqrt{7}\sqrt{2}}{7\left|14x-2\sqrt{11}-4\right|}\right)-1225\sqrt{11}\sqrt{5x^2+2x+3}\,x-16466\sqrt{11}\sqrt{5}\,\operatorname{arsinh}\left(\frac{5}{14}\sqrt{7}\sqrt{2}x+\frac{1}{14}\sqrt{7}\sqrt{2}\right)-6825\sqrt{11}\sqrt{-\frac{34}{49}\sqrt{11}+\frac{250}{49}}\,\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7\left|14x+2\sqrt{11}-4\right|}-\frac{17\sqrt{7}\sqrt{2}x}{7\left|14x+2\sqrt{11}-4\right|}+\frac{\sqrt{11}\sqrt{7}\sqrt{2}}{7\left|14x+2\sqrt{11}-4\right|}-\frac{23\sqrt{7}\sqrt{2}}{7\left|14x+2\sqrt{11}-4\right|}\right)+4575\sqrt{2}\sqrt{17\sqrt{11}+125}\,\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7\left|14x-2\sqrt{11}-4\right|}+\frac{17\sqrt{7}\sqrt{2}x}{7\left|14x-2\sqrt{11}-4\right|}+\frac{\sqrt{11}\sqrt{7}\sqrt{2}}{7\left|14x-2\sqrt{11}-4\right|}+\frac{23\sqrt{7}\sqrt{2}}{7\left|14x-2\sqrt{11}-4\right|}\right)+32025\sqrt{-\frac{34}{49}\sqrt{11}+\frac{250}{49}}\,\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7\left|14x+2\sqrt{11}-4\right|}-\frac{17\sqrt{7}\sqrt{2}x}{7\left|14x+2\sqrt{11}-4\right|}+\frac{\sqrt{11}\sqrt{7}\sqrt{2}}{7\left|14x+2\sqrt{11}-4\right|}-\frac{23\sqrt{7}\sqrt{2}}{7\left|14x+2\sqrt{11}-4\right|}\right)-13895\sqrt{11}\sqrt{5x^2+2x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="maxi
ma")
```

```
[Out] 1/188650*sqrt(11)*(975*sqrt(11)*sqrt(2)*sqrt(17*sqrt(11) + 125)*arcsinh(5/7
*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(
2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2
*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) - 1225*sq
rt(11)*sqrt(5*x^2 + 2*x + 3)*x - 16466*sqrt(11)*sqrt(5)*arcsinh(5/14*sqrt(7
)*sqrt(2)*x + 1/14*sqrt(7)*sqrt(2)) - 6825*sqrt(11)*sqrt(-34/49*sqrt(11) +
250/49)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) -
 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*s
qrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(
11) - 4)) + 4575*sqrt(2)*sqrt(17*sqrt(11) + 125)*arcsinh(5/7*sqrt(11)*sqrt(
7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x -
 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)
+ 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) + 32025*sqrt(-34/49*sqrt
(11) + 250/49)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11)
 - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sq
rt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x +
2*sqrt(11) - 4)) - 13895*sqrt(11)*sqrt(5*x^2 + 2*x + 3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(133) = 266$.

time = 0.36, size = 304, normalized size = 1.63

$$\frac{3}{7546}\,\sqrt{11}\sqrt{146555\,\sqrt{11}+497041}\log\left(\frac{6\left(\sqrt{5\,x^{2}+2\,x+3}\sqrt{146555\,\sqrt{11}+497041}\left(87\,\sqrt{11}-265\right)+6517\,\sqrt{11}\left(x+3\right)+19551\,x-32585\right)}{x}\right)-\frac{3}{7546}\,\sqrt{11}\sqrt{146555\,\sqrt{11}+497041}\log\left(-\frac{6\left(\sqrt{5\,x^{2}+2\,x+3}\sqrt{146555\,\sqrt{11}+497041}\left(87\,\sqrt{11}-265\right)-6517\,\sqrt{11}\left(x+3\right)-19551\,x+32585\right)}{x}\right)-\frac{1}{15092}\,\sqrt{11}\sqrt{-5275980\,\sqrt{11}+17893476}\log\left(-\frac{\sqrt{5\,x^{2}+2\,x+3}\left(87\,\sqrt{11}+265\right)\sqrt{-5275980\,\sqrt{11}+17893476}+39102\,\sqrt{11}\left(x+3\right)-117306\,x+195510}{x}\right)+\frac{1}{15092}\,\sqrt{11}\sqrt{-5275980\,\sqrt{11}+17893476}\log\left(\frac{\sqrt{5\,x^{2}+2\,x+3}\left(87\,\sqrt{11}+265\right)\sqrt{-5275980\,\sqrt{11}+17893476}-39102\,\sqrt{11}\left(x+3\right)+117306\,x-195510}{x}\right)-\frac{1}{490}\,\sqrt{5\,x^{2}+2\,x+3}\left(35\,x+397\right)+\frac{8233}{17150}\,\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^{2}+2\,x+3}\left(5\,x+1\right)-25\,x^{2}-10\,x-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="fric
as")
```

```
[Out] 3/7546*sqrt(11)*sqrt(146555*sqrt(11) + 497041)*log(6*(sqrt(5*x^2 + 2*x + 3)
*sqrt(146555*sqrt(11) + 497041)*(87*sqrt(11) - 265) + 6517*sqrt(11)*(x + 3)
 + 19551*x - 32585)/x) - 3/7546*sqrt(11)*sqrt(146555*sqrt(11) + 497041)*log
(-6*(sqrt(5*x^2 + 2*x + 3)*sqrt(146555*sqrt(11) + 497041)*(87*sqrt(11) - 26
5) - 6517*sqrt(11)*(x + 3) - 19551*x + 32585)/x) - 1/15092*sqrt(11)*sqrt(-5
275980*sqrt(11) + 17893476)*log(-(sqrt(5*x^2 + 2*x + 3)*(87*sqrt(11) + 265)
*sqrt(-5275980*sqrt(11) + 17893476) + 39102*sqrt(11)*(x + 3) - 117306*x + 1
95510)/x) + 1/15092*sqrt(11)*sqrt(-5275980*sqrt(11) + 17893476)*log((sqrt(5
*x^2 + 2*x + 3)*(87*sqrt(11) + 265)*sqrt(-5275980*sqrt(11) + 17893476) - 39
102*sqrt(11)*(x + 3) + 117306*x - 195510)/x) - 1/490*sqrt(5*x^2 + 2*x + 3)*
(35*x + 397) + 8233/17150*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x +
1) - 25*x^2 - 10*x - 8)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx-\int \frac{5x\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx-\int \frac{x^2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1),x)
```

```
[Out] -Integral(2*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(5*x*sq
rt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(x**2*sqrt(5*x**2 + 2
*x + 3)/(7*x**2 - 4*x - 1), x)
```

Giac [A]

time = 3.35, size = 144, normalized size = 0.77

$$-\frac{1}{490}\,\sqrt{5\,x^{2}+2\,x+3}\left(35\,x+397\right)+\frac{8233}{8575}\,\sqrt{5}\log\left(-5\,\sqrt{5}x-\sqrt{5}+5\,\sqrt{5\,x^{2}+2\,x+3}\right)+2.61475869687464\log\left(-\sqrt{5}x+\sqrt{5\,x^{2}+2\,x+3}+4.41924736459000\right)-0.276245077121866\log\left(-\sqrt{5}x+\sqrt{5\,x^{2}+2\,x+3}+1.25295163054000\right)-2.61475869687464\log\left(-\sqrt{5}x+\sqrt{5\,x^{2}+2\,x+3}-1.02258038113000\right)+0.276245077121866\log\left(-\sqrt{5}x+\sqrt{5\,x^{2}+2\,x+3}-2.09411235400000\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="giac
")
```

```
[Out] -1/490*sqrt(5*x^2 + 2*x + 3)*(35*x + 397) + 8233/8575*sqrt(5)*log(-5*sqrt(5
)*x - sqrt(5) + 5*sqrt(5*x^2 + 2*x + 3)) + 2.61475869687464*log(-sqrt(5)*x
+ sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.276245077121866*log(-sqrt(5
)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 2.61475869687464*log(-sqr
t(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.276245077121866*log(
-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^2+5\,x+2)\,\sqrt{5\,x^2+2\,x+3}}{-7\,x^2+4\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1),x)
```

```
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1), x)
```

3.378 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx$

Optimal. Leaf size=199

$$\frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\,(1+4x-7x^2)}+\frac{1}{49}\sqrt{5}\,\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)-\frac{\sqrt{\frac{325022311+39132731\sqrt{11}}{1397}}\,\tanh^{-1}\left(\frac{23-}{\sqrt{2\left(125-\right.}}\right.}{2156}$$

```
[Out] 1/49*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)+3/154*(3+61*x)*(5*x^2+2*x+3)^(1
/2)/(-7*x^2+4*x+1)+1/3011932*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2
+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(454056168467-54668425207*11^(1/2))^
(1/2)-1/3011932*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)
/(250-34*11^(1/2))^(1/2))*(454056168467+54668425207*11^(1/2))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1068, 1090, 633, 221, 1046, 738, 212}

$$\frac{3\sqrt{5x^2+2x+3}\,(61x+3)}{154\,(-7x^2+4x+1)}-\frac{\sqrt{\frac{325022311+39132731\sqrt{11}}{1397}}\,\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\,\sqrt{5x^2+2x+3}}\right)}{2156}+\frac{\sqrt{\frac{325022311-39132731\sqrt{11}}{1397}}\,\tanh^{-1}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\,\sqrt{5x^2+2x+3}}\right)}{2156}+\frac{1}{49}\sqrt{5}\,\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] (3*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(154*(1 + 4*x - 7*x^2)) + (Sqrt[5]*Arc
Sinh[(1 + 5*x)/Sqrt[14]])/49 - (Sqrt[(325022311 + 39132731*Sqrt[11])/1397]*
ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*
Sqrt[3 + 2*x + 5*x^2])])/2156 + (Sqrt[(325022311 - 39132731*Sqrt[11])/1397]
*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]
*Sqrt[3 + 2*x + 5*x^2])])/2156
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(A*b*c - 2*
a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p
 + 1)*((d + e*x + f*x^2)^q/(c*(b^2 - 4*a*c)*(p + 1))), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx = \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\,(1+4x-7x^2)} - \frac{1}{308}\int \frac{-948-188x+220x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\,(1+4x-7x^2)} + \frac{\int \frac{6416+436x}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx}{2156} + \frac{5}{49}\int$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\,(1+4x-7x^2)} + \frac{1}{98}\sqrt{\frac{5}{14}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2\right.$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\,(1+4x-7x^2)} + \frac{1}{49}\sqrt{5}\,\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right) - \frac{\left(2\left(1199-\right.\right.}{}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\,(1+4x-7x^2)} + \frac{1}{49}\sqrt{5}\,\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right) - \frac{\sqrt{\frac{3250223}{}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.52, size = 427, normalized size = 2.15

$$\frac{-\frac{5145(3+61x)\sqrt{3+2x+5x^2}}{-1-4x+7x^2} - 5390\sqrt{5}\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right) - 55\,\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{-314239\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+28462\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1-11221\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1^2}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\right] - 6\sqrt{5}\,\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{599633\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)-391895\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1+21462\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1^2}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\right]}{264110}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] ((-5145*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(-1 - 4*x + 7*x^2) - 5390*Sqrt[5]
*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]] - 55*RootSum[83 - 16*Sqrt[5]
*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-314239*Log[-(Sqrt[5]*x) + Sqr
t[3 + 2*x + 5*x^2] - #1] + 28462*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x +
5*x^2] - #1]*#1 - 11221*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2
)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] - 6*Sqrt[5]*RootSum[83
 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (599633*Sqrt[5]*Lo
g[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 391895*Log[-(Sqrt[5]*x) + Sq
rt[3 + 2*x + 5*x^2] - #1]*#1 + 21462*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*
x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/
264110
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1083 vs. $2(146) = 292$.
time = 0.79, size = 1084, normalized size = 5.45

method	result
risch	$-\frac{3(3+61x)\sqrt{5x^2+2x+3}}{154(7x^2-4x-1)}+\frac{\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,\left(x+\frac{1}{5}\right)}{14}\right)}{49}+\frac{\left(-11446+109\sqrt{11}\right)\sqrt{11}\,\operatorname{arctanh}\left(\frac{}{\sqrt{250-}}\right.}{}$
trager	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x,method=_RETURNVERBOS
E)
```

```
[Out] (183/44+39/44*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))
*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49
+34/49*11^(1/2))^(3/2)+1/98*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1
/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))
+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(
250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49
+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2
)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(
x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*1
1^(1/2))^(1/2)))+10/49/(250/49+34/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7
*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2
))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arc
sinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1
/5))))-161/484*11^(1/2)*(1/49*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^
(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/70*(34/7+10/7*11^(1/2)
)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^
2)^(1/2)*(x+1/5))-(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(4
9/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+
34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2
/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))+(183/44-39/44*11^(1/2))*(-1/49/(2
50/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-
10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/98*(34/7
-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49
*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/
7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-
10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2)
```

```
)^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7
*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10
/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))+10/49/(250/49-34
/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))
*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11
^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/
2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))))+161/484*11^(1/2)*(1/49*(24
5*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-3
4*11^(1/2))^(1/2)+1/70*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49
-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-(250/49-34/49*1
1^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-
10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/
7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))
^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(5*x^2 + 2*x + 3)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(145) = 290$.
time = 0.38, size = 378, normalized size = 1.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="fr
icas")
```

```
[Out] -1/6023864*(sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(39132731*sqrt(11) + 325022311
)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(39132731*sqrt(11) + 325022311
)*(16943*sqrt(11) + 235367) + 26119953475*sqrt(11)*(x + 3) - 78359860425*x
+ 130599767375)/x) - sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(39132731*sqrt(11) +
325022311)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(39132731*sqrt(11) + 3
25022311)*(16943*sqrt(11) + 235367) - 26119953475*sqrt(11)*(x + 3) + 783598
60425*x - 130599767375)/x) + sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(-39132731*sq
rt(11) + 325022311)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(16943*sqrt(11) -
 235367)*sqrt(-39132731*sqrt(11) + 325022311) + 26119953475*sqrt(11)*(x + 3
) + 78359860425*x - 130599767375)/x) - sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(-3
```

```
9132731*sqrt(11) + 325022311)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(16943
*sqrt(11) - 235367)*sqrt(-39132731*sqrt(11) + 325022311) - 26119953475*sqrt
(11)*(x + 3) - 78359860425*x + 130599767375)/x) - 61468*sqrt(5)*(7*x^2 - 4*
x - 1)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) +
117348*sqrt(5*x^2 + 2*x + 3)*(61*x + 3))/(7*x^2 - 4*x - 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2+5x+2)\sqrt{5x^2+2x+3}}{(7x^2-4x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1)**2,x)
[Out] Integral((x**2 + 5*x + 2)*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="gi
ac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{184473632,[8]%%%}+%%%{%%{[421654016,0]:[1,0,-5]%%},[7]%%%}+%%
%{-248
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5\,x+2\right)\,\sqrt{5\,x^2+2\,x+3}}{{\left(-7\,x^2+4\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^2,x)
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^2, x)
```

3.379 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx$

Optimal. Leaf size=213

$$\frac{3(3+61x)\sqrt{3+2x+5x^2}}{308\,(1+4x-7x^2)^2}-\frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104\,(1+4x-7x^2)}-\frac{\sqrt{\frac{6492253020949-11879169071\sqrt{11}}{1397}}}{}$$

```
[Out] 3/308*(3+61*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2-1/1721104*(272941-81311
3*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)-1/686966368*arctanh((23+x*(17-5*11^
(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(906967747026
5753-16595199192187*11^(1/2))^(1/2)+1/686966368*arctanh((23+11^(1/2)+x*(17+
5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(9069677470265753
+16595199192187*11^(1/2))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1068, 1074, 1046, 738, 212}

$$-\frac{\sqrt{5x^2+2x+3}\,(272941-813113x)}{1721104\,(-7x^2+4x+1)}+\frac{3(61x+3)\sqrt{5x^2+2x+3}}{308\,(-7x^2+4x+1)^2}-\frac{\sqrt{\frac{6492253020949-11879169071\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{491744}+\frac{\sqrt{\frac{6492253020949+11879169071\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{491744}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] (3*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(308*(1 + 4*x - 7*x^2)^2) - ((272941 -
 813113*x)*Sqrt[3 + 2*x + 5*x^2])/(1721104*(1 + 4*x - 7*x^2)) - (Sqrt[(6492
253020949 - 11879169071*Sqrt[11])/1397]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sq
rt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/491744 + (
Sqrt[(6492253020949 + 11879169071*Sqrt[11])/1397]*ArcTanh[(23 + Sqrt[11] +
(17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/
491744
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(A*b*c - 2*
a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p
 + 1)*((d + e*x + f*x^2)^q/(c*(b^2 - 4*a*c)*(p + 1))), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
```

```
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx = \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308\,(1+4x-7x^2)^2} - \frac{1}{616}\int \frac{-3012-1564x-3220x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308\,(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104\,(1+4x-7x^2)} +$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308\,(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104\,(1+4x-7x^2)} +$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308\,(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104\,(1+4x-7x^2)} +$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308\,(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104\,(1+4x-7x^2)} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.72, size = 602, normalized size = 2.83

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] ((5764801*Sqrt[3 + 2*x + 5*x^2]*(-31807 + 106279*x + 737577*x^2 - 813113*x^
3))/(1 + 4*x - 7*x^2)^2 - 60545521580434*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1
^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] -
 #1]/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] + 20661853520*RootS
um[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-465*Log[-(S
qrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 7*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt
[3 + 2*x + 5*x^2] - #1]*#1)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3)
```

```
& ] + 22*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & ,
 (3751778663030*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 +
 2597308755559*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqr
t[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] - 6*RootSum[83 - 16*Sqrt[5]*#1
- 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-1164877805727l*Log[-(Sqrt[5]*x) +
 Sqrt[3 + 2*x + 5*x^2] - #1] + 13372446682211*Sqrt[5]*Log[-(Sqrt[5]*x) + Sq
rt[3 + 2*x + 5*x^2] - #1]*#1 + 9645047011740*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*
x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/
1417403151472
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2341 vs. $2(161) = 322$.

time = 0.73, size = 2342, normalized size = 11.00

method	result
risch	$-\frac{(813113x^3-737577x^2-106279x+31807)\sqrt{5x^2+2x+3}}{245872(7x^2-4x-1)^2}+\frac{\left(-1740003+126542\sqrt{11}\right)\sqrt{11}\ \text{arctanh}\left(\frac{}{\sqrt{250-3}}\right.}{}$
trager	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x,method=_RETURNVERBOS
E)
```

```
[Out] -21/968*(-61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49-34/49*11^(1/2))/(x-2/7+1
/7*11^(1/2))^2*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11
^(1/2))+250/49-34/49*11^(1/2))^(3/2)-1/1372*(34/7-10/7*11^(1/2))/(250/49-34
/49*11^(1/2))*(-1/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/
7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/
2))^(3/2)+1/2*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7
+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/
2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*1
1^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2
))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*1
1^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(
1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)
))+10/(250/49-34/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/
7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5
000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250
/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))))+5/686/(250
```

/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2)
)*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(
1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1
/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2
*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*
11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+
1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))-(-3535/1936-273/1936*11^(1/2))*(-1/
49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7*11^(1/2))^2+(
34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)+1/98*
(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))
^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10
*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(
34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^
(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/
7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34
/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))+10/49/(250/
49+34/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(
1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/
49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*1
1^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))))-21/968*(61+13*11^(1/2
))*11^(1/2)*(-1/686/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))^2*(5*(x-2/
7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11
^(1/2))^(3/2)-1/1372*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(-1/(250/
49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/
7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)+1/2*(34/7+10/
7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34
/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10
/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7
11^(1/2))^2)^(1/2)(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1
/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^
(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*1
1^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))+10/(250/49+34/49*11^
(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7
-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-
(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20
*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))))+5/686/(250/49+34/49*11^(1/2))*(1/
7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+
250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(2
50/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+
34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)
+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x
-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11
^(1/2))^(1/2))))-3535/21296*11^(1/2)*(1/49*(245*(x-2/7-1/7*11^(1/2))^2+49*(
34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/70*(34/7+
10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10

```
/7*11^(1/2))^2)^(1/2)*(x+1/5))-(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1
/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^
(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*1
1^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-(-3535/1936+273/1936
*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/
7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="ma
xima")
```

```
[Out] -integrate(sqrt(5*x^2 + 2*x + 3)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(160) = 320.
time = 0.40, size = 390, normalized size = 1.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="fr
icas")
```

```
[Out] -1/1373932736*(sqrt(1397)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(11879169
071*sqrt(11) + 6492253020949)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(11
879169071*sqrt(11) + 6492253020949)*(4822219*sqrt(11) - 37335441) + 5690716
98870455*sqrt(11)*(x + 3) + 1707215096611365*x - 2845358494352275)/x) - sqr
t(1397)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(11879169071*sqrt(11) + 649
2253020949)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(11879169071*sqrt(11
) + 6492253020949)*(4822219*sqrt(11) - 37335441) - 569071698870455*sqrt(11)
*(x + 3) - 1707215096611365*x + 2845358494352275)/x) + sqrt(1397)*(49*x^4 -
 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-11879169071*sqrt(11) + 6492253020949)*log(
-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(4822219*sqrt(11) + 37335441)*sqrt(-1187
9169071*sqrt(11) + 6492253020949) + 569071698870455*sqrt(11)*(x + 3) - 1707
215096611365*x + 2845358494352275)/x) - sqrt(1397)*(49*x^4 - 56*x^3 + 2*x^2
 + 8*x + 1)*sqrt(-11879169071*sqrt(11) + 6492253020949)*log((sqrt(1397)*sqr
t(5*x^2 + 2*x + 3)*(4822219*sqrt(11) + 37335441)*sqrt(-11879169071*sqrt(11)
 + 6492253020949) - 569071698870455*sqrt(11)*(x + 3) + 1707215096611365*x -
 2845358494352275)/x) + 5588*(813113*x^3 - 737577*x^2 - 106279*x + 31807)*s
qrt(5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx-\int \frac{5x\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx-\int \frac{x^2\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1)**3,x)
```

```
[Out] -Integral(2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x*
*3 - 27*x**2 - 12*x - 1), x) - Integral(5*x*sqrt(5*x**2 + 2*x + 3)/(343*x**
6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral(x**
2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x*
*2 - 12*x - 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(160) = 320.
time = 4.06, size = 378, normalized size = 1.77

$$\frac{6200558\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^7-835775\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^6-190947036\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^5-92732607\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^4+816321374\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^3+419437335\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^2-765111048\sqrt{5}x-376983161\sqrt{5}+765111048\sqrt{5x^2+2x+3}}{430276\left(7\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^4-8\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^3-70\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^2+16\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)+83\right)^2}+0.139051039089329\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}+4.41924736459000\right)-0.138209741946100\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}+1.25295163054000\right)-0.139051039089329\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}-1.02258038113000\right)+0.138209741946100\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}-2.09411235400000\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="gi
ac")
```

```
[Out] 1/430276*(6200558*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 - 835775*sqrt(5)*(s
qrt(5)*x - sqrt(5*x^2 + 2*x + 3))^6 - 190947036*(sqrt(5)*x - sqrt(5*x^2 + 2
*x + 3))^5 - 92732607*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 + 81632
1374*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 419437335*sqrt(5)*(sqrt(5)*x -
 sqrt(5*x^2 + 2*x + 3))^2 - 765111048*sqrt(5)*x - 376983161*sqrt(5) + 76511
1048*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sq
rt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 +
2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83)^2 + 0.13
9051039089329*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) -
0.138209741946100*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000
) - 0.139051039089329*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.0225803811
3000) + 0.138209741946100*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.094112
35400000)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^2+5\,x+2\right)\,\sqrt{5\,x^2+2\,x+3}}{{\left(-7\,x^2+4\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^3,x)
```

```
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^3, x)
```

3.380 $\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx$

Optimal. Leaf size=231

$$-\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000}-\frac{22840599(1+5x)\left(3+2x+5x^2\right)^{3/2}}{62500000}-\frac{6133820867\left(3+2x+5x^2\right)^{5/2}}{1203125000}$$

```
[Out] -22840599/62500000*(1+5*x)*(5*x^2+2*x+3)^(3/2)-6133820867/1203125000*(5*x^2
+2*x+3)^(5/2)+837379699/72187500*x*(5*x^2+2*x+3)^(5/2)+2173004363/173250000
*x^2*(5*x^2+2*x+3)^(5/2)-190236913/4950000*x^3*(5*x^2+2*x+3)^(5/2)-796559/1
23750*x^4*(5*x^2+2*x+3)^(5/2)+1031177/20625*x^5*(5*x^2+2*x+3)^(5/2)-61103/3
300*x^6*(5*x^2+2*x+3)^(5/2)-343/60*x^7*(5*x^2+2*x+3)^(5/2)-3357568053/78125
0000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-479652579/312500000*(1+5*x)*(5*
x^2+2*x+3)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1675, 654, 626, 633, 221}

$$\frac{2173004363\left(5x^2+2x+3\right)^{5/2}x^2}{173250000}+\frac{837379699\left(5x^2+2x+3\right)^{5/2}x}{72187500}-\frac{6133820867\left(5x^2+2x+3\right)^{5/2}}{1203125000}-\frac{22840599(5x+1)\left(5x^2+2x+3\right)^{3/2}}{62500000}-\frac{479652579(5x+1)\sqrt{5x^2+2x+3}}{312500000}-\frac{343}{60}\left(5x^2+2x+3\right)^{5/2}x^7-\frac{61103\left(5x^2+2x+3\right)^{5/2}x^6}{3300}+\frac{1031177\left(5x^2+2x+3\right)^{5/2}x^5}{20625}-\frac{796559\left(5x^2+2x+3\right)^{5/2}x^4}{123750}-\frac{190236913\left(5x^2+2x+3\right)^{5/2}x^3}{4950000}-\frac{3357568053\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{156250000\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-479652579*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/312500000 - (22840599*(1 + 5*x
)*(3 + 2*x + 5*x^2)^(3/2))/62500000 - (6133820867*(3 + 2*x + 5*x^2)^(5/2))/
1203125000 + (837379699*x*(3 + 2*x + 5*x^2)^(5/2))/72187500 + (2173004363*x
^2*(3 + 2*x + 5*x^2)^(5/2))/173250000 - (190236913*x^3*(3 + 2*x + 5*x^2)^(5
/2))/4950000 - (796559*x^4*(3 + 2*x + 5*x^2)^(5/2))/123750 + (1031177*x^5*(
3 + 2*x + 5*x^2)^(5/2))/20625 - (61103*x^6*(3 + 2*x + 5*x^2)^(5/2))/3300 -
(343*x^7*(3 + 2*x + 5*x^2)^(5/2))/60 - (3357568053*ArcSinh[(1 + 5*x)/Sqrt[1
4]])/(156250000*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx = -\frac{343}{60}x^7\left(3+2x+5x^2\right)^{5/2} + \frac{1}{60}\int \left(3+2x+5x^2\right.$$

$$= -\frac{61103x^6(3+2x+5x^2)^{5/2}}{3300} - \frac{343}{60}x^7\left(3+2x+5\right.$$

$$= \frac{1031177x^5(3+2x+5x^2)^{5/2}}{20625} - \frac{61103x^6(3+2x+}{3300}$$

$$= -\frac{796559x^4(3+2x+5x^2)^{5/2}}{123750} + \frac{1031177x^5(3+2}{20625}$$

$$= -\frac{190236913x^3(3+2x+5x^2)^{5/2}}{4950000} - \frac{796559x^4(3+}{123}$$

$$= \frac{2173004363x^2(3+2x+5x^2)^{5/2}}{173250000} - \frac{190236913x^3(}{495}$$

$$= \frac{837379699x(3+2x+5x^2)^{5/2}}{72187500} + \frac{2173004363x^2(3}{17325}$$

$$= -\frac{6133820867(3+2x+5x^2)^{5/2}}{1203125000} + \frac{837379699x(3}{7218}$$

$$= -\frac{22840599(1+5x)\left(3+2x+5x^2\right)^{3/2}}{62500000} - \frac{6133820}{}$$

$$= -\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000} - \frac{22840599}{}$$

$$= -\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000} - \frac{22840599}{}$$

$$= -\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000} - \frac{22840599}{}$$

Mathematica [A]
time = 0.89, size = 109, normalized size = 0.47

$$\frac{\sqrt{3+2x+5x^2}\left(-10506617068392+6352777129950x+15865844408685x^2+19041688239675x^3+2573089891000x^4-85130334087500x^5-52106830406250x^6+72918247281250x^7+30505457500000x^8+148393743750000x^9-125007421875000x^{10}-30950390625000x^{11}\right)}{216562500000}+\frac{3357568053\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{156250000\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-10506617068392 + 6352777129950*x + 15865844408685*
x^2 + 19041688239675*x^3 + 2573089891000*x^4 - 85130334087500*x^5 - 5210683
```

```
0406250*x^6 + 72918247281250*x^7 + 30505457500000*x^8 + 148393743750000*x^9
 - 125007421875000*x^10 - 30950390625000*x^11))/216562500000 + (3357568053*
Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(156250000*Sqrt[5])
```

Maple [A]
time = 0.18, size = 185, normalized size = 0.80

method	result
risch	$-\frac{(30950390625000x^{11}+125007421875000x^{10}-148393743750000x^{9}-30505457500000x^{8}-72918247281250x^{7}+52106830406250x^{6}+8}{216}$
trager	$\left(-\frac{1715}{12}x^{11}-\frac{76195}{132}x^{10}+\frac{376873}{550}x^{9}+\frac{1743169}{12375}x^{8}+\frac{333340559}{990000}x^{7}-\frac{555806191}{2310000}x^{6}-\frac{6810426727}{17325000}x^{5}+\frac{2573089891}{216562500}x\right.$
default	$-\frac{3357568053\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,\left(x+\frac{1}{5}\right)}{14}\right)}{781250000}-\frac{479652579(10x+2)\sqrt{5x^{2}+2x+3}}{625000000}-\frac{22840599(10x+2)(5x^{2}+2x+3)^{\frac{3}{2}}}{125000000}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -3357568053/781250000*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-479652579/6250
00000*(10*x+2)*(5*x^2+2*x+3)^(1/2)-22840599/125000000*(10*x+2)*(5*x^2+2*x+3
)^(3/2)-6133820867/1203125000*(5*x^2+2*x+3)^(5/2)+837379699/72187500*x*(5*x
^2+2*x+3)^(5/2)+2173004363/173250000*x^2*(5*x^2+2*x+3)^(5/2)-190236913/4950
000*x^3*(5*x^2+2*x+3)^(5/2)-796559/123750*x^4*(5*x^2+2*x+3)^(5/2)+1031177/2
0625*x^5*(5*x^2+2*x+3)^(5/2)-61103/3300*x^6*(5*x^2+2*x+3)^(5/2)-343/60*x^7*
(5*x^2+2*x+3)^(5/2)
```

Maxima [A]
time = 0.50, size = 206, normalized size = 0.89

$$-\frac{343}{60}(5x^2+2x+3)^{\frac{5}{2}}x^7-\frac{61103}{3300}(5x^2+2x+3)^{\frac{5}{2}}x^6+\frac{1031177}{20625}(5x^2+2x+3)^{\frac{5}{2}}x^5-\frac{796559}{123750}(5x^2+2x+3)^{\frac{5}{2}}x^4-\frac{190236913}{4950000}(5x^2+2x+3)^{\frac{5}{2}}x^3+\frac{2173004363}{173250000}(5x^2+2x+3)^{\frac{5}{2}}x^2+\frac{837379699}{72187500}(5x^2+2x+3)^{\frac{5}{2}}x-\frac{6133820867}{1203125000}(5x^2+2x+3)^{\frac{5}{2}}-\frac{22840599}{12500000}(5x^2+2x+3)^{\frac{3}{2}}x-\frac{22840599}{62500000}(5x^2+2x+3)^{\frac{3}{2}}-\frac{479652579}{62500000}\sqrt{5x^2+2x+3}\,x-\frac{3357568053}{781250000}\sqrt{5}\,\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right)-\frac{479652579}{312500000}\sqrt{5x^2+2x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] -343/60*(5*x^2 + 2*x + 3)^(5/2)*x^7 - 61103/3300*(5*x^2 + 2*x + 3)^(5/2)*x^
6 + 1031177/20625*(5*x^2 + 2*x + 3)^(5/2)*x^5 - 796559/123750*(5*x^2 + 2*x
+ 3)^(5/2)*x^4 - 190236913/4950000*(5*x^2 + 2*x + 3)^(5/2)*x^3 + 2173004363
/173250000*(5*x^2 + 2*x + 3)^(5/2)*x^2 + 837379699/72187500*(5*x^2 + 2*x +
3)^(5/2)*x - 6133820867/1203125000*(5*x^2 + 2*x + 3)^(5/2) - 22840599/12500
000*(5*x^2 + 2*x + 3)^(3/2)*x - 22840599/62500000*(5*x^2 + 2*x + 3)^(3/2) -
 479652579/62500000*sqrt(5*x^2 + 2*x + 3)*x - 3357568053/781250000*sqrt(5)*
```

```
arcsinh(1/14*sqrt(14)*(5*x + 1)) - 479652579/312500000*sqrt(5*x^2 + 2*x + 3
)
```

Fricas [A]
time = 0.36, size = 107, normalized size = 0.46

$$-\frac{1}{216562500000}\left(30950390625000\,x^{11}+125007421875000\,x^{10}-148393743750000\,x^{9}-30505457500000\,x^{8}-72918247281250\,x^{7}+52106830406250\,x^{6}+85130334087500\,x^{5}-2573089891000\,x^{4}-19041688239675\,x^{3}-15865844408685\,x^{2}-6352777129950\,x+10506617068392\right)\sqrt{5\,x^{2}+2\,x+3}+\frac{3357568053}{1562500000}\,\sqrt{5}\,\log\left(\sqrt{5}\,\sqrt{5\,x^{2}+2\,x+3}\,(5\,x+1)-25\,x^{2}-10\,x-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/216562500000*(30950390625000*x^11 + 125007421875000*x^10 - 1483937437500
00*x^9 - 30505457500000*x^8 - 72918247281250*x^7 + 52106830406250*x^6 + 851
30334087500*x^5 - 2573089891000*x^4 - 19041688239675*x^3 - 15865844408685*x
^2 - 6352777129950*x + 10506617068392)*sqrt(5*x^2 + 2*x + 3) + 3357568053/1
562500000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10
*x - 8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-91x\sqrt{5x^2+2x+3}\right)dx-\int\left(-413x^2\sqrt{5x^2+2x+3}\right)dx-\int\left(-192x^3\sqrt{5x^2+2x+3}\right)dx-\int 2160x^4\sqrt{5x^2+2x+3}\,dx-\int 1666x^5\sqrt{5x^2+2x+3}\,dx-\int\left(-2094x^6\sqrt{5x^2+2x+3}\right)dx-\int\left(-1384x^7\sqrt{5x^2+2x+3}\right)dx-\int\left(-7042x^8\sqrt{5x^2+2x+3}\right)dx-\int 6321x^9\sqrt{5x^2+2x+3}\,dx-\int 1715x^{10}\sqrt{5x^2+2x+3}\,dx-\int\left(-6\sqrt{5x^2+2x+3}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-91*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-413*x**2*sqrt(5*x**2
 + 2*x + 3), x) - Integral(-192*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(
2160*x**4*sqrt(5*x**2 + 2*x + 3), x) - Integral(1666*x**5*sqrt(5*x**2 + 2*x
 + 3), x) - Integral(-2094*x**6*sqrt(5*x**2 + 2*x + 3), x) - Integral(-1384
*x**7*sqrt(5*x**2 + 2*x + 3), x) - Integral(-7042*x**8*sqrt(5*x**2 + 2*x +
3), x) - Integral(6321*x**9*sqrt(5*x**2 + 2*x + 3), x) - Integral(1715*x**1
0*sqrt(5*x**2 + 2*x + 3), x) - Integral(-6*sqrt(5*x**2 + 2*x + 3), x)
```

Giac [A]
time = 3.34, size = 102, normalized size = 0.44

$$-\frac{1}{216562500000}\left(5\left(\left(5\left(10\left(25\left(5\left(7\left(20\left(105\left(875\left(77\,x+311\right)x-323034\right)x-6972676\right)x-333340559\right)x+1667418573\right)x+13620853454\right)x-10292359564\right)x-761667529587\right)x-3173168881737\right)x-1270555425990\right)x+10506617068392\right)\sqrt{5\,x^{2}+2\,x+3}+\frac{3357568053}{781250000}\,\sqrt{5}\,\log\left(-\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^{2}+2\,x+3}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/216562500000*(5*((5*(10*(25*(5*(7*(20*(105*(875*(77*x + 311)*x - 323034)
*x - 6972676)*x - 333340559)*x + 1667418573)*x + 13620853454)*x - 102923595
```

```
64)*x - 761667529587)*x - 3173168881737)*x - 1270555425990)*x + 10506617068
392)*sqrt(5*x^2 + 2*x + 3) + 3357568053/781250000*sqrt(5)*log(-sqrt(5)*(sqr
t(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(x^2+5\,x+2\right)\,{\left(5\,x^2+2\,x+3\right)}^{3/2}\,{\left(-7\,x^2+4\,x+1\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3,x)
[Out] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3, x)
```

3.381 $\int (1+4x-7x^2)^2 (2+5x+x^2) (3+2x+5x^2)^{3/2} \, dx$

Optimal. Leaf size=189

$$-\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000}-\frac{690561(1+5x)\left(3+2x+5x^2\right)^{3/2}}{1250000}+\frac{505667(3+2x+5x^2)^{5/2}}{2187500}+\frac{86721x}{}$$

```
[Out] -690561/1250000*(1+5*x)*(5*x^2+2*x+3)^(3/2)+505667/2187500*(5*x^2+2*x+3)^(5
/2)+86721/21875*x*(5*x^2+2*x+3)^(5/2)-219271/105000*x^2*(5*x^2+2*x+3)^(5/2)
-18379/3000*x^3*(5*x^2+2*x+3)^(5/2)+581/150*x^4*(5*x^2+2*x+3)^(5/2)+49/50*x
^5*(5*x^2+2*x+3)^(5/2)-101512467/15625000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^
(1/2)-14501781/6250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1675, 654, 626, 633, 221}

$$-\frac{219271(5x^2+2x+3)^{5/2}x^2}{105000}+\frac{86721(5x^2+2x+3)^{5/2}x}{21875}+\frac{505667(5x^2+2x+3)^{5/2}}{2187500}-\frac{690561(5x+1)\left(5x^2+2x+3\right)^{3/2}}{1250000}-\frac{14501781(5x+1)\sqrt{5x^2+2x+3}}{6250000}+\frac{49}{50}(5x^2+2x+3)^{5/2}x^5+\frac{581}{150}(5x^2+2x+3)^{5/2}x^4-\frac{18379(5x^2+2x+3)^{5/2}x^3}{3000}-\frac{101512467\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{3125000\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-14501781*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/6250000 - (690561*(1 + 5*x)*(3
+ 2*x + 5*x^2)^(3/2))/1250000 + (505667*(3 + 2*x + 5*x^2)^(5/2))/2187500 +
(86721*x*(3 + 2*x + 5*x^2)^(5/2))/21875 - (219271*x^2*(3 + 2*x + 5*x^2)^(5/
2))/105000 - (18379*x^3*(3 + 2*x + 5*x^2)^(5/2))/3000 + (581*x^4*(3 + 2*x +
 5*x^2)^(5/2))/150 + (49*x^5*(3 + 2*x + 5*x^2)^(5/2))/50 - (101512467*ArcSi
nh[(1 + 5*x)/Sqrt[14]])/(3125000*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx = \frac{49}{50}x^5\left(3+2x+5x^2\right)^{5/2} + \frac{1}{50}\int \left(3+2x+5x^2\right)^{3/2}$$

$$= \frac{581}{150}x^4\left(3+2x+5x^2\right)^{5/2} + \frac{49}{50}x^5\left(3+2x+5x^2\right)^{5/}$$

$$= -\frac{18379x^3(3+2x+5x^2)^{5/2}}{3000} + \frac{581}{150}x^4\left(3+2x+5\right.$$

$$= -\frac{219271x^2(3+2x+5x^2)^{5/2}}{105000} - \frac{18379x^3(3+2x}{3000}$$

$$= \frac{86721x(3+2x+5x^2)^{5/2}}{21875} - \frac{219271x^2(3+2x+5}{105000}$$

$$= \frac{505667(3+2x+5x^2)^{5/2}}{2187500} + \frac{86721x(3+2x+5x^2}{21875}$$

$$= -\frac{690561(1+5x)\left(3+2x+5x^2\right)^{3/2}}{1250000} + \frac{505667(3}{21}$$

$$= -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000} - \frac{690561(1}{}$$

$$= -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000} - \frac{690561(1}{}$$

$$= -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000} - \frac{690561(1}{}$$

Mathematica [A]
time = 0.71, size = 99, normalized size = 0.52

$$\frac{\sqrt{3+2x+5x^2}\,(-249003936+2291675850x+3721040355x^2+5959365525x^3-3227597000x^4-12554262500x^5-4105593750x^6-5561281250x^7+15281875000x^8+3215625000x^9)}{131250000} + \frac{101512467\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{3125000\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-249003936 + 2291675850*x + 3721040355*x^2 + 595936
5525*x^3 - 3227597000*x^4 - 12554262500*x^5 - 4105593750*x^6 - 5561281250*x
^7 + 15281875000*x^8 + 3215625000*x^9))/131250000 + (101512467*Log[-1 - 5*x
 + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(3125000*Sqrt[5])
```

Maple [A]
time = 0.18, size = 151, normalized size = 0.80

method	result
risch	$\frac{(3215625000x^9+15281875000x^8-5561281250x^7-4105593750x^6-12554262500x^5-3227597000x^4+5959365525x^3+3721040355x^2+22}{131250000}$
trager	$\left(\frac{49}{2}x^9+\frac{3493}{30}x^8-\frac{25423}{600}x^7-\frac{43793}{1400}x^6-\frac{1004341}{10500}x^5-\frac{3227597}{131250}x^4+\frac{79458207}{1750000}x^3+\frac{248069357}{8750000}x^2+\frac{15277839}{875000}x-\right.$
default	$-\frac{101512467\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\,\left(x+\frac{1}{5}\right)}{14}\right)}{15625000}-\frac{14501781(10x+2)\sqrt{5x^2+2x+3}}{12500000}-\frac{690561(10x+2)\left(5x^2+2x+3\right)^{\frac{3}{2}}}{2500000}+50$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -101512467/15625000*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-14501781/1250000
0*(10*x+2)*(5*x^2+2*x+3)^(1/2)-690561/2500000*(10*x+2)*(5*x^2+2*x+3)^(3/2)+
505667/2187500*(5*x^2+2*x+3)^(5/2)+86721/21875*x*(5*x^2+2*x+3)^(5/2)-219271
/105000*x^2*(5*x^2+2*x+3)^(5/2)-18379/3000*x^3*(5*x^2+2*x+3)^(5/2)+581/150*
x^4*(5*x^2+2*x+3)^(5/2)+49/50*x^5*(5*x^2+2*x+3)^(5/2)
```

Maxima [A]
time = 0.50, size = 172, normalized size = 0.91

$$\frac{49}{50}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^5+\frac{581}{150}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^4-\frac{18379}{3000}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^3-\frac{219271}{105000}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^2+\frac{86721}{21875}\left(5x^2+2x+3\right)^{\frac{5}{2}}x+\frac{505667}{2187500}\left(5x^2+2x+3\right)^{\frac{5}{2}}-\frac{690561}{250000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x-\frac{690561}{1250000}\left(5x^2+2x+3\right)^{\frac{3}{2}}-\frac{14501781}{1250000}\sqrt{5x^2+2x+3}\,x-\frac{101512467}{15625000}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}\,(5x+1)\right)-\frac{14501781}{6250000}\sqrt{5x^2+2x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] 49/50*(5*x^2 + 2*x + 3)^(5/2)*x^5 + 581/150*(5*x^2 + 2*x + 3)^(5/2)*x^4 - 1
8379/3000*(5*x^2 + 2*x + 3)^(5/2)*x^3 - 219271/105000*(5*x^2 + 2*x + 3)^(5/
2)*x^2 + 86721/21875*(5*x^2 + 2*x + 3)^(5/2)*x + 505667/2187500*(5*x^2 + 2*
x + 3)^(5/2) - 690561/250000*(5*x^2 + 2*x + 3)^(3/2)*x - 690561/1250000*(5*
x^2 + 2*x + 3)^(3/2) - 14501781/1250000*sqrt(5*x^2 + 2*x + 3)*x - 101512467
/15625000*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 14501781/6250000*sqrt(
5*x^2 + 2*x + 3)
```

Fricas [A]
time = 0.37, size = 97, normalized size = 0.51

$$\frac{1}{131250000}\left(3215625000x^9+15281875000x^8-5561281250x^7-4105593750x^6-12554262500x^5-3227597000x^4+5959365525x^3+3721040355x^2+2291675850x-249003936\right)\sqrt{5x^2+2x+3}+\frac{101512467}{31250000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5x^2+2x+3}\,(5x+1)-25x^2-10x-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/131250000*(3215625000*x^9 + 15281875000*x^8 - 5561281250*x^7 - 4105593750
*x^6 - 12554262500*x^5 - 3227597000*x^4 + 5959365525*x^3 + 3721040355*x^2 +
 2291675850*x - 249003936)*sqrt(5*x^2 + 2*x + 3) + 101512467/31250000*sqrt(
5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+5x+2\right)\left(5x^2+2x+3\right)^{\frac{3}{2}}\left(7x^2-4x-1\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)*(5*x**2 + 2*x + 3)**(3/2)*(7*x**2 - 4*x - 1)**2,
x)
```

Giac [A]
time = 5.17, size = 92, normalized size = 0.49

$$\frac{1}{131250000}\,(5\,((5\,(10\,(25\,(5\,(7\,(140\,(105\,x+499)x-25423)x-131379)x-2008682)x-12910388)x+238374621)x+744208071)x+458335170)x-249003936)\sqrt{5\,x^2+2\,x+3}+\frac{101512467}{15625000}\,\sqrt{5}\,\log\left(-\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/131250000*(5*((5*(10*(25*(5*(7*(140*(105*x + 499)*x - 25423)*x - 131379)*
x - 2008682)*x - 12910388)*x + 238374621)*x + 744208071)*x + 458335170)*x -
 249003936)*sqrt(5*x^2 + 2*x + 3) + 101512467/15625000*sqrt(5)*log(-sqrt(5)
*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^2+5\,x+2\right)\,{\left(5\,x^2+2\,x+3\right)}^{3/2}\,{\left(-7\,x^2+4\,x+1\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2,x)
```

```
[Out] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2, x)
```

3.382 $\int (1+4x-7x^2)(2+5x+x^2)(3+2x+5x^2)^{3/2}\,dx$

Optimal. Leaf size=147

$$-\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x)(3+2x+5x^2)^{3/2}}{150000}+\frac{149509(3+2x+5x^2)^{5/2}}{262500}+\frac{2809x(3+2}{52}$$

```
[Out] -18397/150000*(1+5*x)*(5*x^2+2*x+3)^(3/2)+149509/262500*(5*x^2+2*x+3)^(5/2)
+2809/5250*x*(5*x^2+2*x+3)^(5/2)-1163/1400*x^2*(5*x^2+2*x+3)^(5/2)-7/40*x^3
*(5*x^2+2*x+3)^(5/2)-901453/625000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-1
28779/250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1675, 654, 626, 633, 221}

$$-\frac{1163(5x^2+2x+3)^{5/2}x^2}{1400}+\frac{2809(5x^2+2x+3)^{5/2}x}{5250}+\frac{149509(5x^2+2x+3)^{5/2}}{262500}-\frac{18397(5x+1)(5x^2+2x+3)^{3/2}}{150000}-\frac{128779(5x+1)\sqrt{5x^2+2x+3}}{250000}-\frac{7}{40}(5x^2+2x+3)^{5/2}x^3-\frac{901453\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{125000\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-128779*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/250000 - (18397*(1 + 5*x)*(3 + 2*
x + 5*x^2)^(3/2))/150000 + (149509*(3 + 2*x + 5*x^2)^(5/2))/262500 + (2809*
x*(3 + 2*x + 5*x^2)^(5/2))/5250 - (1163*x^2*(3 + 2*x + 5*x^2)^(5/2))/1400 -
 (7*x^3*(3 + 2*x + 5*x^2)^(5/2))/40 - (901453*ArcSinh[(1 + 5*x)/Sqrt[14]])/
(125000*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx = -\frac{7}{40}x^3\left(3+2x+5x^2\right)^{5/2}+\frac{1}{40}\int \left(3+2x+5x^2\right)^{3/2}$$

$$= -\frac{1163x^2\left(3+2x+5x^2\right)^{5/2}}{1400}-\frac{7}{40}x^3\left(3+2x+5x^2\right)$$

$$= \frac{2809x\left(3+2x+5x^2\right)^{5/2}}{5250}-\frac{1163x^2\left(3+2x+5x^2\right)^{5/2}}{1400}$$

$$= \frac{149509\left(3+2x+5x^2\right)^{5/2}}{262500}+\frac{2809x\left(3+2x+5x^2\right)^{5/2}}{5250}$$

$$= -\frac{18397(1+5x)\left(3+2x+5x^2\right)^{3/2}}{150000}+\frac{149509(3+}{262}$$

$$= -\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x}{1}$$

$$= -\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x}{1}$$

$$= -\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x}{1}$$

Mathematica [A]
time = 0.52, size = 89, normalized size = 0.61

$$\frac{\sqrt{3+2x+5x^2}\,(22275576+36695150x+86464445x^2+78608475x^3-28373000x^4-48237500x^5-127406250x^6-22968750x^7)}{5250000}+\frac{901453\log\left(-1-5x+\sqrt{5}\,\sqrt{3+2x+5x^2}\right)}{125000\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(22275576 + 36695150*x + 86464445*x^2 + 78608475*x^3
 - 28373000*x^4 - 48237500*x^5 - 127406250*x^6 - 22968750*x^7))/5250000 + (
901453*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(125000*Sqrt[5])
```

Maple [A]
time = 0.15, size = 117, normalized size = 0.80

method	result
risch	$-\frac{(22968750x^7+127406250x^6+48237500x^5+28373000x^4-78608475x^3-86464445x^2-36695150x-22275576)\sqrt{5x^2+2x+3}}{5250000}$
trager	$\left(-\frac{35}{8}x^7-\frac{1359}{56}x^6-\frac{3859}{420}x^5-\frac{28373}{5250}x^4+\frac{1048113}{70000}x^3+\frac{17292889}{1050000}x^2+\frac{733903}{105000}x+\frac{928149}{218750}\right)\sqrt{5x^2+2x+3}$
default	$-\frac{901453\sqrt{5}\ \mathrm{arcsinh}\left(\frac{5\sqrt{14}\,\left(x+\frac{1}{5}\right)}{14}\right)}{625000}-\frac{128779(10x+2)\sqrt{5x^2+2x+3}}{500000}-\frac{18397(10x+2)\left(5x^2+2x+3\right)^{\frac{3}{2}}}{300000}+\frac{149509(5}{26}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -901453/625000*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-128779/500000*(10*x+2
)*(5*x^2+2*x+3)^(1/2)-18397/300000*(10*x+2)*(5*x^2+2*x+3)^(3/2)+149509/2625
00*(5*x^2+2*x+3)^(5/2)+2809/5250*x*(5*x^2+2*x+3)^(5/2)-1163/1400*x^2*(5*x^2
+2*x+3)^(5/2)-7/40*x^3*(5*x^2+2*x+3)^(5/2)
```

Maxima [A]
time = 0.49, size = 138, normalized size = 0.94

$$-\frac{7}{40}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^3-\frac{1163}{1400}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^2+\frac{2809}{5250}\left(5x^2+2x+3\right)^{\frac{5}{2}}x+\frac{149509}{262500}\left(5x^2+2x+3\right)^{\frac{5}{2}}-\frac{18397}{30000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x-\frac{18397}{150000}\left(5x^2+2x+3\right)^{\frac{3}{2}}-\frac{128779}{50000}\sqrt{5x^2+2x+3}\,x-\frac{901453}{625000}\sqrt{5}\ \mathrm{arsinh}\left(\frac{1}{14}\sqrt{14}\,(5x+1)\right)-\frac{128779}{250000}\sqrt{5x^2+2x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -7/40*(5*x^2 + 2*x + 3)^(5/2)*x^3 - 1163/1400*(5*x^2 + 2*x + 3)^(5/2)*x^2 +
 2809/5250*(5*x^2 + 2*x + 3)^(5/2)*x + 149509/262500*(5*x^2 + 2*x + 3)^(5/2
) - 18397/30000*(5*x^2 + 2*x + 3)^(3/2)*x - 18397/150000*(5*x^2 + 2*x + 3)^
```

```
(3/2) - 128779/50000*sqrt(5*x^2 + 2*x + 3)*x - 901453/625000*sqrt(5)*arcsin
h(1/14*sqrt(14)*(5*x + 1)) - 128779/250000*sqrt(5*x^2 + 2*x + 3)
```

Fricas [A]
time = 0.37, size = 87, normalized size = 0.59

$$-\frac{1}{5250000}\left(22968750\,x^7+127406250\,x^6+48237500\,x^5+28373000\,x^4-78608475\,x^3-86464445\,x^2-36695150\,x-22275576\right)\sqrt{5\,x^2+2\,x+3}+\frac{901453}{1250000}\,\sqrt{5}\,\log\left(\sqrt{5}\,\sqrt{5\,x^2+2\,x+3}\,(5\,x+1)-25\,x^2-10\,x-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/5250000*(22968750*x^7 + 127406250*x^6 + 48237500*x^5 + 28373000*x^4 - 78
608475*x^3 - 86464445*x^2 - 36695150*x - 22275576)*sqrt(5*x^2 + 2*x + 3) +
901453/1250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2
 - 10*x - 8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-43x\sqrt{5x^2+2x+3}\right)dx-\int\left(-57x^2\sqrt{5x^2+2x+3}\right)dx-\int 14x^3\sqrt{5x^2+2x+3}\,dx-\int 48x^4\sqrt{5x^2+2x+3}\,dx-\int 169x^5\sqrt{5x^2+2x+3}\,dx-\int 35x^6\sqrt{5x^2+2x+3}\,dx-\int\left(-6\sqrt{5x^2+2x+3}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-43*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-57*x**2*sqrt(5*x**2
+ 2*x + 3), x) - Integral(14*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(48*
x**4*sqrt(5*x**2 + 2*x + 3), x) - Integral(169*x**5*sqrt(5*x**2 + 2*x + 3),
 x) - Integral(35*x**6*sqrt(5*x**2 + 2*x + 3), x) - Integral(-6*sqrt(5*x**2
 + 2*x + 3), x)
```

Giac [A]
time = 4.23, size = 82, normalized size = 0.56

$$-\frac{1}{5250000}\left(5\left(\left(5\left(10\left(25\left(15\left(245\,x+1359\right)x+7718\right)x+113492\right)x-3144339\right)x-17292889\right)x-7339030\right)x-22275576\right)\sqrt{5\,x^2+2\,x+3}+\frac{901453}{625000}\,\sqrt{5}\,\log\left(-\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="giac
")
```

```
[Out] -1/5250000*(5*((5*(10*(25*(15*(245*x + 1359)*x + 7718)*x + 113492)*x - 3144
339)*x - 17292889)*x - 7339030)*x - 22275576)*sqrt(5*x^2 + 2*x + 3) + 90145
3/625000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^2+5\,x+2\right)\,{\left(5\,x^2+2\,x+3\right)}^{3/2}\,\left(-7\,x^2+4\,x+1\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1),x)
[Out] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1), x)
```

3.383 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2} dx$

Optimal. Leaf size=210

$$-\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100}-\frac{1}{980}(267+35x)\left(3+2x+5x^2\right)^{3/2}-\frac{34425687\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{840350\sqrt{5}}-\,6\,\sqrt{}$$

[Out] -1/980*(267+35*x)*(5*x^2+2*x+3)^(3/2)-34425687/4201750*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-3/240100*(571621+196105*x)*(5*x^2+2*x+3)^(1/2)-6/184877*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(178175857354-53550689170*11^(1/2))^(1/2)+6/184877*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(178175857354+53550689170*11^(1/2))^(1/2)

Rubi [A]
time = 0.20, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1080, 1090, 633, 221, 1046, 738, 212}

$$-\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}-\frac{3(196105x+571621)\sqrt{5x^2+2x+3}}{240100}-\frac{6\sqrt{\frac{2}{11}\left(8098902607-2434122235\sqrt{11}\right)}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{16807}+\frac{6\sqrt{\frac{2}{11}\left(8098902607+2434122235\sqrt{11}\right)}\tanh^{-1}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{16807}-\frac{34425687\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{840350\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2),x]

[Out] (-3*(571621 + 196105*x)*Sqrt[3 + 2*x + 5*x^2])/240100 - ((267 + 35*x)*(3 + 2*x + 5*x^2)^(3/2))/980 - (34425687*ArcSinh[(1 + 5*x)/Sqrt[14]])/(840350*Sqrt[5]) - (6*Sqrt[(2*(8098902607 - 2434122235*Sqrt[11]))/11]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/16807 + (6*Sqrt[(2*(8098902607 + 2434122235*Sqrt[11]))/11]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/16807

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p
 + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b
*x + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
```

```
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx = -\frac{1}{980}(267+35x)\left(3+2x+5x^2\right)^{3/2} - \frac{\int \frac{(-20358-79272x-100854x^2)\sqrt{3}}{1+4x-7x^2}}{2940}$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)(3+2x$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)(3+2x$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)(3+2x$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)(3+2x$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)(3+2x$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.56, size = 254, normalized size = 1.21

$$\frac{\sqrt{3+2x+5x^2}\,(-1911108-744870x-344225x^2-42875x^3)}{240100}+\frac{34425687\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{840350\sqrt{5}}-\frac{12\,\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&,\frac{-648783\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)-533850\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1+251851\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1^2}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\right]}{16807\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2),x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-1911108 - 744870*x - 344225*x^2 - 42875*x^3))/2401
00 + (34425687*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(840350*Sqrt[
5]) - (12*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 &
, (-648783*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 533850*
```

```
Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 251851*Sqrt[5]*Log[-(Sq
rt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5
]*#1^2 + 7*#1^3) & ])/(16807*Sqrt[5])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(153) = 306$.

time = 0.79, size = 730, normalized size = 3.48

method	result
risch	$-\frac{(42875x^3+344225x^2+744870x+1911108)\sqrt{5x^2+2x+3}}{240100} - \frac{34425687\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,(x+\frac{1}{5})}{14}\right)}{4201750} + \frac{12\left(877397+2\right.}{}$
trager	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/280*(10*x+2)*(5*x^2+2*x+3)^(3/2)-3/200*(10*x+2)*(5*x^2+2*x+3)^(1/2)-21/2
50*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-3/154*(61+13*11^(1/2))*11^(1/2)*(
1/21*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+25
0/49+34/49*11^(1/2))^(3/2)+1/14*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2
/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*1
1^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/
2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2
)*(x+1/5)))+1/7*(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49
*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/
7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+
10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2)
)^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7
*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10
/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))))-3/154*(-61+13*1
1^(1/2))*11^(1/2)*(1/21*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2
/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/14*(34/7-10/7*11^(1/2))*(1/
20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1
/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7
*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/
7*11^(1/2))^2)^(1/2)*(x+1/5)))+1/7*(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7
+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/
2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*1
1^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2
```

))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*1
1^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(
1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)
)))

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(152) = 304$.
time = 0.57, size = 535, normalized size = 2.55

$$\frac{1}{92438500}\sqrt{11}\left(19500\sqrt{11}\sqrt{2}\left(17\sqrt{11}+125\right)^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7|14x-2\sqrt{11}-4|}+\frac{17\sqrt{7}\sqrt{2}x}{7|14x-2\sqrt{11}-4|}+\frac{\sqrt{11}\sqrt{7}\sqrt{2}}{7|14x-2\sqrt{11}-4|}+\frac{23\sqrt{7}\sqrt{2}}{7|14x-2\sqrt{11}-4|}\right)-300125\sqrt{11}\left(5x^2+2x+3\right)^{\frac{3}{2}}x-3344250\sqrt{11}\left(-\frac{34}{49}\sqrt{11}+\frac{250}{49}\right)^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7|14x+2\sqrt{11}-4|}-\frac{17\sqrt{7}\sqrt{2}x}{7|14x+2\sqrt{11}-4|}+\frac{\sqrt{11}\sqrt{7}\sqrt{2}}{7|14x+2\sqrt{11}-4|}-\frac{23\sqrt{7}\sqrt{2}}{7|14x+2\sqrt{11}-4|}\right)+91500\sqrt{2}\left(17\sqrt{11}+125\right)^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7|14x-2\sqrt{11}-4|}+\frac{17\sqrt{7}\sqrt{2}x}{7|14x-2\sqrt{11}-4|}+\frac{\sqrt{11}\sqrt{7}\sqrt{2}}{7|14x-2\sqrt{11}-4|}+\frac{23\sqrt{7}\sqrt{2}}{7|14x-2\sqrt{11}-4|}\right)+15692250\left(-\frac{34}{49}\sqrt{11}+\frac{250}{49}\right)^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7|14x+2\sqrt{11}-4|}-\frac{17\sqrt{7}\sqrt{2}x}{7|14x+2\sqrt{11}-4|}+\frac{\sqrt{11}\sqrt{7}\sqrt{2}}{7|14x+2\sqrt{11}-4|}-\frac{23\sqrt{7}\sqrt{2}}{7|14x+2\sqrt{11}-4|}\right)-2289525\sqrt{11}\left(5x^2+2x+3\right)^{\frac{3}{2}}-20591025\sqrt{11}\sqrt{5x^2+2x+3}\,x-68851374\sqrt{11}\sqrt{5}\operatorname{arsinh}\left(\frac{5}{14}\sqrt{7}\sqrt{2}x+\frac{1}{14}\sqrt{7}\sqrt{2}\right)-60020205\sqrt{11}\sqrt{5x^2+2x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="maxi
ma")
```

```
[Out] 1/92438500*sqrt(11)*(19500*sqrt(11)*sqrt(2)*(17*sqrt(11) + 125)^(3/2)*arcsi
nh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)
*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14
*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) - 3
00125*sqrt(11)*(5*x^2 + 2*x + 3)^(3/2)*x - 3344250*sqrt(11)*(-34/49*sqrt(11
) + 250/49)^(3/2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(
11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)
*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x
 + 2*sqrt(11) - 4)) + 91500*sqrt(2)*(17*sqrt(11) + 125)^(3/2)*arcsinh(5/7*s
qrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)
*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*s
qrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) + 15692250*
(-34/49*sqrt(11) + 250/49)^(3/2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs
(14*x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4)
 + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*s
qrt(2)/abs(14*x + 2*sqrt(11) - 4)) - 2289525*sqrt(11)*(5*x^2 + 2*x + 3)^(3/
2) - 20591025*sqrt(11)*sqrt(5*x^2 + 2*x + 3)*x - 68851374*sqrt(11)*sqrt(5)*
arcsinh(5/14*sqrt(7)*sqrt(2)*x + 1/14*sqrt(7)*sqrt(2)) - 60020205*sqrt(11)*
sqrt(5*x^2 + 2*x + 3))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(152) = 304$.
time = 0.39, size = 326, normalized size = 1.55

$$\frac{3}{184877}\sqrt{11}\sqrt{2}\sqrt{2434122235\sqrt{11}+8098902607}\log\left(\frac{12\left(\sqrt{2}\sqrt{5x^2+2x+3}\sqrt{2434122235\sqrt{11}+8098902607}\left(7690\sqrt{11}-24697\right)+40555291\sqrt{11}(x+3)+121665873x-202776455\right)}{x}\right)-\frac{3}{184877}\sqrt{11}\sqrt{2}\sqrt{2434122235\sqrt{11}+8098902607}\log\left(-\frac{12\left(\sqrt{2}\sqrt{5x^2+2x+3}\sqrt{2434122235\sqrt{11}+8098902607}\left(7690\sqrt{11}-24697\right)-40555291\sqrt{11}(x+3)-121665873x+202776455\right)}{x}\right)-\frac{1}{739508}\sqrt{11}\sqrt{-7010272033680\sqrt{11}+23324833950816}\log\left(-\frac{\sqrt{5x^2+2x+3}\left(7690\sqrt{11}+24697\right)\sqrt{-7010272033680\sqrt{11}+23324833950816}+486663492\sqrt{11}(x+3)-1459990476x+2433317460}{x}\right)+\frac{1}{739508}\sqrt{11}\sqrt{-7010272033680\sqrt{11}+23324833950816}\log\left(\frac{\sqrt{5x^2+2x+3}\left(7690\sqrt{11}+24697\right)\sqrt{-7010272033680\sqrt{11}+23324833950816}-486663492\sqrt{11}(x+3)+1459990476x-2433317460}{x}\right)-\frac{1}{240100}\left(42875x^3+344225x^2+744870x+1911108\right)\sqrt{5x^2+2x+3}+\frac{34425687}{8403500}\sqrt{5}\log\left(\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="fric
as")
```

```
[Out] 3/184877*sqrt(11)*sqrt(2)*sqrt(2434122235*sqrt(11) + 8098902607)*log(12*(sq
rt(2)*sqrt(5*x^2 + 2*x + 3)*sqrt(2434122235*sqrt(11) + 8098902607)*(7690*sq
rt(11) - 24697) + 40555291*sqrt(11)*(x + 3) + 121665873*x - 202776455)/x) -
 3/184877*sqrt(11)*sqrt(2)*sqrt(2434122235*sqrt(11) + 8098902607)*log(-12*(
sqrt(2)*sqrt(5*x^2 + 2*x + 3)*sqrt(2434122235*sqrt(11) + 8098902607)*(7690*
sqrt(11) - 24697) - 40555291*sqrt(11)*(x + 3) - 121665873*x + 202776455)/x)
 - 1/739508*sqrt(11)*sqrt(-701027203680*sqrt(11) + 2332483950816)*log(-(sqr
t(5*x^2 + 2*x + 3)*(7690*sqrt(11) + 24697)*sqrt(-701027203680*sqrt(11) + 23
32483950816) + 486663492*sqrt(11)*(x + 3) - 1459990476*x + 2433317460)/x) +
 1/739508*sqrt(11)*sqrt(-701027203680*sqrt(11) + 2332483950816)*log((sqrt(5
*x^2 + 2*x + 3)*(7690*sqrt(11) + 24697)*sqrt(-701027203680*sqrt(11) + 23324
83950816) - 486663492*sqrt(11)*(x + 3) + 1459990476*x - 2433317460)/x) - 1/
240100*(42875*x^3 + 344225*x^2 + 744870*x + 1911108)*sqrt(5*x^2 + 2*x + 3)
+ 34425687/8403500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25
*x^2 - 10*x - 8)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{6\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{19x\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{23x^2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{27x^3\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{5x^4\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1),x)
```

```
[Out] -Integral(6*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(19*x*s
qrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(23*x**2*sqrt(5*x**2
 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(27*x**3*sqrt(5*x**2 + 2*x + 3
)/(7*x**2 - 4*x - 1), x) - Integral(5*x**4*sqrt(5*x**2 + 2*x + 3)/(7*x**2 -
 4*x - 1), x)
```

Giac [**A**]
time = 4.98, size = 154, normalized size = 0.73

$$-\frac{1}{240100}\,(35\,(35\,(35\,x+281)x+21282)x+1911108)\sqrt{5\,x^2+2\,x+3}+\frac{34425687}{4201750}\,\sqrt{5}\,\log\left(-5\,\sqrt{5}\,x-\sqrt{5}+5\,\sqrt{5\,x^2+2\,x+3}\right)+19.3580321168561\,\log\left(-\sqrt{5}\,x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)-0.773682164624264\,\log\left(-\sqrt{5}\,x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)-19.3580321168561\,\log\left(-\sqrt{5}\,x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)+0.773682164625454\,\log\left(-\sqrt{5}\,x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="giac
")
```

```
[Out] -1/240100*(35*(35*(35*x + 281)*x + 21282)*x + 1911108)*sqrt(5*x^2 + 2*x + 3
) + 34425687/4201750*sqrt(5)*log(-5*sqrt(5)*x - sqrt(5) + 5*sqrt(5*x^2 + 2*
x + 3)) + 19.3580321168561*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924
736459000) - 0.773682164624264*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.2
5295163054000) - 19.3580321168561*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) -
1.02258038113000) + 0.773682164625454*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3
) - 2.09411235400000)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x^2+5x+2)\,(5x^2+2x+3)^{3/2}}{-7x^2+4x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1),x)
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1), x)
```

3.384 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2} \, dx$

Optimal. Leaf size=222

$$\frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773}+\frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)}+\frac{16691\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{2401\sqrt{5}}-\frac{\sqrt{\frac{1}{22}\left(5217540031\right.}}{}$$

```
[Out] 3/154*(3+61*x)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)+16691/12005*arcsinh(1/14*
(1+5*x)*14^(1/2))*5^(1/2)+1/3773*(5826+3395*x)*(5*x^2+2*x+3)^(1/2)-1/581042
*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/
2))^(1/2))*(1147858806842-289418283682*11^(1/2))^(1/2)-1/581042*arctanh((23
+11^(1/2)+x*(17+5*11^(1/2))))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(
1147858806842+289418283682*11^(1/2))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1068, 1080, 1090, 633, 221, 1046, 738, 212}

$$\frac{3(61x+3)(5x^2+2x+3)^{3/2}}{154(-7x^2+4x+1)}+\frac{(3395x+5826)\sqrt{5x^2+2x+3}}{3773}-\frac{\sqrt{\frac{1}{22}\left(52175400311-13155376531\sqrt{11}\right)}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{26411}-\frac{\sqrt{\frac{1}{22}\left(52175400311+13155376531\sqrt{11}\right)}\tanh^{-1}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{26411}+\frac{16691\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{2401\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] ((5826 + 3395*x)*Sqrt[3 + 2*x + 5*x^2])/3773 + (3*(3 + 61*x)*(3 + 2*x + 5*x
^2)^(3/2))/(154*(1 + 4*x - 7*x^2)) + (16691*ArcSinh[(1 + 5*x)/Sqrt[14]])/(2
401*Sqrt[5]) - (Sqrt[(52175400311 - 13155376531*Sqrt[11])/22]*ArcTanh[(23 -
 Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x
+ 5*x^2])])/26411 - (Sqrt[(52175400311 + 13155376531*Sqrt[11])/22]*ArcTanh[
(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 +
 2*x + 5*x^2])])/26411
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(A*b*c - 2*
a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p
 + 1)*((d + e*x + f*x^2)^q/(c*(b^2 - 4*a*c)*(p + 1))), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1080

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p
 + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b
*x + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
```

```
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \frac{3(3+61x)\,(3+2x+5x^2)^{3/2}}{154\,(1+4x-7x^2)} - \frac{1}{308}\int \frac{\sqrt{3+2x+5x^2}\,(-912+724}{1+4x-7x^2}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)\,(3+2x+5x^2)^{3/2}}{154\,(1+4x-7x^2)} +$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)\,(3+2x+5x^2)^{3/2}}{154\,(1+4x-7x^2)} -$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)\,(3+2x+5x^2)^{3/2}}{154\,(1+4x-7x^2)} +$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)\,(3+2x+5x^2)^{3/2}}{154\,(1+4x-7x^2)} +$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)\,(3+2x+5x^2)^{3/2}}{154\,(1+4x-7x^2)} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.70, size = 447, normalized size = 2.01

$$\frac{\frac{1715\sqrt{3+2x+5x^2}\left(-12975-81181x+34265x^2+2695x^3\right)}{-1-4x+7x^2}-17992898\sqrt{5}\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)+44\sqrt{5}\,\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&,\frac{25954129\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)-19416530\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1+2717099\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1^2}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\right]-6\sqrt{5}\,\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&,\frac{225782939\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)-137400830\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1+7775369\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1^2}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\right]}{12941390}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] ((1715*Sqrt[3 + 2*x + 5*x^2]*(-12975 - 81181*x + 34265*x^2 + 2695*x^3))/(-1
 - 4*x + 7*x^2) - 17992898*Sqrt[5]*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*
x^2]] + 44*Sqrt[5]*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 +
7*#1^4 & , (25954129*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]
 - 19416530*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 2717099*Sqr
t[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*
#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] - 6*Sqrt[5]*RootSum[83 - 16*Sqrt[5]*#1 -
70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (225782939*Sqrt[5]*Log[-(Sqrt[5]*x) +
 Sqrt[3 + 2*x + 5*x^2] - #1] - 137400830*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x +
5*x^2] - #1]*#1 + 7775369*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2]
- #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/12941390
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1827 vs. $2(165) = 330$.
time = 0.78, size = 1828, normalized size = 8.23

method	result
risch	$\frac{\left(2695x^3+34265x^2-81181x-12975\right)\sqrt{5x^2+2x+3}}{52822x^2-30184x-7546}+\frac{16691\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,\left(x+\frac{1}{5}\right)}{14}\right)}{12005}-\frac{\left(1701489+743879\sqrt{11}\right.}{}$
trager	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x,method=_RETURNVERBOS
E)
```

```
[Out] (183/44+39/44*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))
*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49
+34/49*11^(1/2))^(5/2)+3/98*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1
/3*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/
49+34/49*11^(1/2))^(3/2)+1/2*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-
```

1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(
1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*
arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(
x+1/5)))+(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+
10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*
11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11
^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)
arctanh(49/2(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/
2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(
1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))))+20/49/(250/49+34/49*11
^(1/2))*(1/40*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/
7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)+3/80*(5000/49+680/49*11^(1/2)-
(34/7+10/7*11^(1/2))^2)*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7
11^(1/2))(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49
+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34
/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))))+(183/44-39/44*1
1^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*
11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2)
)^(5/2)+3/98*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/3*(5*(x-2/7+1/
7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/
2))^(3/2)+1/2*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2
+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/2
00*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)
/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/4
9-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*
(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/
2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2
)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(
500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11
^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/
7*11^(1/2))+250-34*11^(1/2))^(1/2))))+20/49/(250/49-34/49*11^(1/2))*(1/40*(
10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))
+250/49-34/49*11^(1/2))^(3/2)+3/80*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(
1/2))^2)*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2
/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2
)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/
20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))))-161/484*11^(1/2)*(1/21*(5*(x-2
/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*1
1^(1/2))^(3/2)+1/14*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1
/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/
2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5
^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))+
1/7*(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*
11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1
/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2

```
))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arct
anh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/
(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))
*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))))+161/484*11^(1/2)*(1/21*(5*(
x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/4
9*11^(1/2))^(3/2)+1/14*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11
^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^
(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsin
h(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)
))+1/7*(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10
/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11
^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(
1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*a
rctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)
))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/
2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="ma
xima")
```

```
[Out] integrate((5*x^2 + 2*x + 3)^(3/2)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^2, x)
```

<u>Fricas</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(164) = 328$.
time = 0.38, size = 378, normalized size = 1.70

$$\frac{5\sqrt{11}(7x^2-4x-1)\sqrt{26310753062\sqrt{11}+104350800622}\log\left(\frac{\sqrt{5x^2+2x+3}\sqrt{26310753062\sqrt{11}+104350800622}(16206\sqrt{11}-68441)+1795191685\sqrt{11}(x+3)+5385575055x-8975958425}{x}\right)-5\sqrt{11}(7x^2-4x-1)\sqrt{26310753062\sqrt{11}+104350800622}\log\left(-\frac{\sqrt{5x^2+2x+3}\sqrt{26310753062\sqrt{11}+104350800622}(16206\sqrt{11}-68441)-1795191685\sqrt{11}(x+3)-5385575055x+8975958425}{x}\right)-5\sqrt{11}(7x^2-4x-1)\sqrt{-26310753062\sqrt{11}+104350800622}\log\left(-\frac{\sqrt{5x^2+2x+3}(16206\sqrt{11}+68441)\sqrt{-26310753062\sqrt{11}+104350800622}+1795191685\sqrt{11}(x+3)-5385575055x+8975958425}{x}\right)+5\sqrt{11}(7x^2-4x-1)\sqrt{-26310753062\sqrt{11}+104350800622}\log\left(\frac{\sqrt{5x^2+2x+3}(16206\sqrt{11}+68441)\sqrt{-26310753062\sqrt{11}+104350800622}-1795191685\sqrt{11}(x+3)+5385575055x-8975958425}{x}\right)+4039222\sqrt{5}(7x^2-4x-1)\log\left(-\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)+770(2695x^3+34265x^2-81181x-12975)\sqrt{5x^2+2x+3}}{5810420(7x^2-4x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="fr
icas")
```

```
[Out] 1/5810420*(5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(26310753062*sqrt(11) + 1043508
00622)*log((sqrt(5*x^2 + 2*x + 3)*sqrt(26310753062*sqrt(11) + 104350800622)
*(16206*sqrt(11) - 68441) + 1795191685*sqrt(11)*(x + 3) + 5385575055*x - 89
75958425)/x) - 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(26310753062*sqrt(11) + 104
350800622)*log(-(sqrt(5*x^2 + 2*x + 3)*sqrt(26310753062*sqrt(11) + 10435080
0622)*(16206*sqrt(11) - 68441) - 1795191685*sqrt(11)*(x + 3) - 5385575055*x
 + 8975958425)/x) - 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(-26310753062*sqrt(11)
 + 104350800622)*log(-(sqrt(5*x^2 + 2*x + 3)*(16206*sqrt(11) + 68441)*sqrt(
```

```
-26310753062*sqrt(11) + 104350800622) + 1795191685*sqrt(11)*(x + 3) - 53855
75055*x + 8975958425)/x) + 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(-26310753062*s
qrt(11) + 104350800622)*log((sqrt(5*x^2 + 2*x + 3)*(16206*sqrt(11) + 68441)
*sqrt(-26310753062*sqrt(11) + 104350800622) - 1795191685*sqrt(11)*(x + 3) +
 5385575055*x - 8975958425)/x) + 4039222*sqrt(5)*(7*x^2 - 4*x - 1)*log(-sqr
t(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) + 770*(2695*x^3 +
 34265*x^2 - 81181*x - 12975)*sqrt(5*x^2 + 2*x + 3))/(7*x^2 - 4*x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2+5x+2)(5x^2+2x+3)^{\frac{3}{2}}}{(7x^2-4x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1)**2,x)
```

```
[Out] Integral((x**2 + 5*x + 2)*(5*x**2 + 2*x + 3)**(3/2)/(7*x**2 - 4*x - 1)**2,
x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{63274455776,[8]%%%}+%%%{%%{[144627327488,0]:[1,0,-5]%%},[7]%%
%}+%%%
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(x^2+5\,x+2)\,(5\,x^2+2\,x+3)^{3/2}}{(-7\,x^2+4\,x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^2,x)
```

```
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^2, x)
```

3.385 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx$

Optimal. Leaf size=234

$$-\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716\,(1+4x-7x^2)}+\frac{3(3+61x)\,(3+2x+5x^2)^{3/2}}{308\,(1+4x-7x^2)^2}-\frac{5}{343}\sqrt{5}\,\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)-\frac{\sqrt{\frac{6229419}{}}}{}$$

```
[Out] 3/308*(3+61*x)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2-5/343*arcsinh(1/14*(1+5
*x)*14^(1/2))*5^(1/2)-1/23716*(9495-37088*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*
x+1)-1/927675056*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2
)/(250-34*11^(1/2))^(1/2))*(17404998711697774-5826721433301670*11^(1/2))^(
1/2)+1/927675056*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2
)/(250+34*11^(1/2))^(1/2))*(17404998711697774+5826721433301670*11^(1/2))^(
1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1068, 1090, 633, 221, 1046, 738, 212}

$$\frac{3(61x+3)\,(5x^2+2x+3)^{3/2}}{308\,(-7x^2+4x+1)^2}-\frac{(9495-37088x)\sqrt{5x^2+2x+3}}{23716\,(-7x^2+4x+1)}-\frac{\sqrt{\frac{62294197250171-2085440742055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{332024}+\frac{\sqrt{\frac{62294197250171+2085440742055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{332024}-\frac{5}{343}\sqrt{5}\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] -1/23716*((9495 - 37088*x)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2) + (3*(3
 + 61*x)*(3 + 2*x + 5*x^2)^(3/2))/(308*(1 + 4*x - 7*x^2)^2) - (5*Sqrt[5]*Ar
cSinh[(1 + 5*x)/Sqrt[14]])/343 - (Sqrt[(62294197250171 - 2085440742055*Sqrt
[11])/2794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17
*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/332024 + (Sqrt[(62294197250171 + 20854
40742055*Sqrt[11])/2794]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqr
t[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/332024
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1068

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(A*b*c - 2*
a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p
 + 1)*((d + e*x + f*x^2)^q/(c*(b^2 - 4*a*c)*(p + 1))), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
```

```
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx = \frac{3(3+61x)\,(3+2x+5x^2)^{3/2}}{308\,(1+4x-7x^2)^2} - \frac{1}{616}\int \frac{\sqrt{3+2x+5x^2}\,(-2976-\ldots}{(1+4x-7x^2)\ldots}$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716\,(1+4x-7x^2)} + \frac{3(3+61x)\,(3+2x+5x^2)^{3/2}}{308\,(1+4x-7x^2)^2}\ldots$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716\,(1+4x-7x^2)} + \frac{3(3+61x)\,(3+2x+5x^2)^{3/2}}{308\,(1+4x-7x^2)^2}\ldots$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716\,(1+4x-7x^2)} + \frac{3(3+61x)\,(3+2x+5x^2)^{3/2}}{308\,(1+4x-7x^2)^2}\ldots$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716\,(1+4x-7x^2)} + \frac{3(3+61x)\,(3+2x+5x^2)^{3/2}}{308\,(1+4x-7x^2)^2}\ldots$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716\,(1+4x-7x^2)} + \frac{3(3+61x)\,(3+2x+5x^2)^{3/2}}{308\,(1+4x-7x^2)^2}\ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.81, size = 636, normalized size = 2.72

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-7416 + 42767*x + 246464*x^2 - 189161*x^3))/(23716*
(-1 - 4*x + 7*x^2)^2) + (5*Sqrt[5]*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*
x^2]])/343 - RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4
 & , (4506829*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 1320
```

```
270*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 64435*Sqrt[5]*Log[-
(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqr
t[5]*#1^2 + 7*#1^3) & ]/(33614*Sqrt[5]) + RootSum[83 - 16*Sqrt[5]*#1 - 70*#
1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-16323208013227*Sqrt[5]*Log[-(Sqrt[5]*x)
 + Sqrt[3 + 2*x + 5*x^2] - #1] + 151120773150070*Log[-(Sqrt[5]*x) + Sqrt[3
+ 2*x + 5*x^2] - #1]*#1 + 21832390993791*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3
+ 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) &
 ]/(71748713246*Sqrt[5]) - (3*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt
[5]*#1^3 + 7*#1^4 & , (-4192656948824863*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3
+ 2*x + 5*x^2] - #1] + 24518831643829090*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x +
5*x^2] - #1]*#1 + 3523608887504055*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x
+ 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/(3
4726377211064*Sqrt[5])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3827 vs. $2(177) = 354$.

time = 0.80, size = 3828, normalized size = 16.36

method	result
risch	$-\frac{(189161x^3-246464x^2-42767x+7416)\sqrt{5x^2+2x+3}}{23716(7x^2-4x-1)^2}-\frac{5\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,(x+\frac{1}{5})}{14}\right)}{343}+\frac{\left(7706073+674221\sqrt{11}\right.}{}$
trager	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x,method=_RETURNVERBOS
E)
```

```
[Out] -21/968*(61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49+34/49*11^(1/2))/(x-2/7-1/
7*11^(1/2))^2*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^
(1/2))+250/49+34/49*11^(1/2))^(5/2)+1/1372*(34/7+10/7*11^(1/2))/(250/49+34/
49*11^(1/2))*(-1/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7
*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2
))^(5/2)+3/2*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/3*(5*(x-2/7-1/
7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/
2))^(3/2)+1/2*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2
+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/2
00*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)
/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/4
9+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*
```

(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/
2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2
)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(
500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11
^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/
7*11^(1/2))+250+34*11^(1/2))^(1/2))))+20/(250/49+34/49*11^(1/2))*(1/40*(10*
x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+25
0/49+34/49*11^(1/2))^(3/2)+3/80*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2
))^2)*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-
1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(
34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*
(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))))+15/686/(250/49+34/49*11^(1/2))*(1
/3*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/
49+34/49*11^(1/2))^(3/2)+1/2*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-
1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(
1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*
arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(
x+1/5)))+(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+
10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*
11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11
^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)
arctanh(49/2(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/
2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(
1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))))-21/968*(-61+13*11^(1/
2))*11^(1/2)*(-1/686/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))^2*(5*(x-2
/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*1
1^(1/2))^(5/2)+1/1372*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(-1/(250
/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10
/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(5/2)+3/2*(34/7-10
/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/3*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-1
0/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/2*(34/7-1
0/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))
*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11
^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/
2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/49-34/49*11^(1/2))*(1/
7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+
250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(2
50/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-
34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)
+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x
-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11
^(1/2))^(1/2))))+20/(250/49-34/49*11^(1/2))*(1/40*(10*x+2)*(5*(x-2/7+1/7*11
^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^
(3/2)+3/80*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*(1/20*(10*x+2)*
(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-

```
34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2
)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^
2)^(1/2)*(x+1/5)))))+15/686/(250/49-34/49*11^(1/2))*(1/3*(5*(x-2/7+1/7*11^(
1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3
/2)+1/2*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7
-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(50
00/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/
49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/49-34/4
9*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7
+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="ma
xima")
```

```
[Out] -integrate((5*x^2 + 2*x + 3)^(3/2)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(176) = 352$.
time = 0.44, size = 447, normalized size = 1.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="fr
icas")
```

```
[Out] -1/1855350112*(sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(20854407
42055*sqrt(11) + 62294197250171)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt
(2085440742055*sqrt(11) + 62294197250171)*(11840590*sqrt(11) - 83479737) +
5426671202560069*sqrt(11)*(x + 3) + 16280013607680207*x - 27133356012800345
)/x) - sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(2085440742055*sq
rt(11) + 62294197250171)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(208544
0742055*sqrt(11) + 62294197250171)*(11840590*sqrt(11) - 83479737) - 5426671
202560069*sqrt(11)*(x + 3) - 16280013607680207*x + 27133356012800345)/x) +
sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-2085440742055*sqrt(11)
 + 62294197250171)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(11840590*sqrt(11
) + 83479737)*sqrt(-2085440742055*sqrt(11) + 62294197250171) + 542667120256
0069*sqrt(11)*(x + 3) - 16280013607680207*x + 27133356012800345)/x) - sqrt(
2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-2085440742055*sqrt(11) + 62
294197250171)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(11840590*sqrt(11) + 83
```

```
479737)*sqrt(-2085440742055*sqrt(11) + 62294197250171) - 5426671202560069*s
qrt(11)*(x + 3) + 16280013607680207*x - 27133356012800345)/x) - 13522960*sq
rt(5)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)
*(5*x + 1) - 25*x^2 - 10*x - 8) + 78232*(189161*x^3 - 246464*x^2 - 42767*x
+ 7416)*sqrt(5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{6\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx-\int \frac{19x\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx-\int \frac{23x^2\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx-\int \frac{27x^3\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx-\int \frac{5x^4\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1)**3,x)
```

```
[Out] -Integral(6*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x*
*3 - 27*x**2 - 12*x - 1), x) - Integral(19*x*sqrt(5*x**2 + 2*x + 3)/(343*x*
*6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral(23
*x**2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 2
7*x**2 - 12*x - 1), x) - Integral(27*x**3*sqrt(5*x**2 + 2*x + 3)/(343*x**6
- 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral(5*x**
4*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x*
*2 - 12*x - 1), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-1771684761728,[12]%%%}+%%%{%%{[-6074347754496,0]:[1,0,-5]%%}
,[11]%
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^2+5\,x+2\right)\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{{\left(-7\,x^2+4\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^3,x)
```

```
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^3, x)
```

3.386 $\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

Optimal. Leaf size=185

$$-\frac{16515809\sqrt{3+2x+5x^2}}{156250}+\frac{5793077x\sqrt{3+2x+5x^2}}{75000}+\frac{40722851x^2\sqrt{3+2x+5x^2}}{750000}-\frac{5160533x^3\sqrt{3+2x+}}{50000}$$

```
[Out] -77513689/3125000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-16515809/156250*(5
*x^2+2*x+3)^(1/2)+5793077/75000*x*(5*x^2+2*x+3)^(1/2)+40722851/750000*x^2*(
5*x^2+2*x+3)^(1/2)-5160533/50000*x^3*(5*x^2+2*x+3)^(1/2)-47807/3750*x^4*(5*
x^2+2*x+3)^(1/2)+26159/300*x^5*(5*x^2+2*x+3)^(1/2)-1141/40*x^6*(5*x^2+2*x+3
)^(1/2)-343/40*x^7*(5*x^2+2*x+3)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1675, 654, 633, 221}

$$\frac{40722851\sqrt{5x^2+2x+3}\,x^2}{750000}+\frac{5793077\sqrt{5x^2+2x+3}\,x}{75000}-\frac{16515809\sqrt{5x^2+2x+3}}{156250}-\frac{343}{40}\sqrt{5x^2+2x+3}\,x^7-\frac{1141}{40}\sqrt{5x^2+2x+3}\,x^6+\frac{26159}{300}\sqrt{5x^2+2x+3}\,x^5-\frac{47807\sqrt{5x^2+2x+3}\,x^4}{3750}-\frac{5160533\sqrt{5x^2+2x+3}\,x^3}{50000}-\frac{77513689\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{625000\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-16515809*Sqrt[3 + 2*x + 5*x^2])/156250 + (5793077*x*Sqrt[3 + 2*x + 5*x^2]
)/75000 + (40722851*x^2*Sqrt[3 + 2*x + 5*x^2])/750000 - (5160533*x^3*Sqrt[3
 + 2*x + 5*x^2])/50000 - (47807*x^4*Sqrt[3 + 2*x + 5*x^2])/3750 + (26159*x^
5*Sqrt[3 + 2*x + 5*x^2])/300 - (1141*x^6*Sqrt[3 + 2*x + 5*x^2])/40 - (343*x
^7*Sqrt[3 + 2*x + 5*x^2])/40 - (77513689*ArcSinh[(1 + 5*x)/Sqrt[14]])/(6250
00*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{343}{40}x^7\sqrt{3+2x+5x^2} + \frac{1}{40}\int \frac{80+1160x+4600x^2-2440x^3-3}{\sqrt{3+}}$$

$$= -\frac{1141}{40}x^6\sqrt{3+2x+5x^2} - \frac{343}{40}x^7\sqrt{3+2x+5x^2} + \frac{\int \frac{2800+40600x+1}{}}{}$$

$$= \frac{26159}{300}x^5\sqrt{3+2x+5x^2} - \frac{1141}{40}x^6\sqrt{3+2x+5x^2} - \frac{343}{40}x^7\sqrt{3+2}$$

$$= -\frac{47807x^4\sqrt{3+2x+5x^2}}{3750} + \frac{26159}{300}x^5\sqrt{3+2x+5x^2} - \frac{1141}{40}x^6\sqrt{3}$$

$$= -\frac{5160533x^3\sqrt{3+2x+5x^2}}{50000} - \frac{47807x^4\sqrt{3+2x+5x^2}}{3750} + \frac{26159}{300}x^5$$

$$= \frac{40722851x^2\sqrt{3+2x+5x^2}}{750000} - \frac{5160533x^3\sqrt{3+2x+5x^2}}{50000} - \frac{47807x}{}$$

$$= \frac{5793077x\sqrt{3+2x+5x^2}}{75000} + \frac{40722851x^2\sqrt{3+2x+5x^2}}{750000} - \frac{516053}{}$$

$$= -\frac{16515809\sqrt{3+2x+5x^2}}{156250} + \frac{5793077x\sqrt{3+2x+5x^2}}{75000} + \frac{4072285}{}$$

$$= -\frac{16515809\sqrt{3+2x+5x^2}}{156250} + \frac{5793077x\sqrt{3+2x+5x^2}}{75000} + \frac{4072285}{}$$

$$= -\frac{16515809\sqrt{3+2x+5x^2}}{156250} + \frac{5793077x\sqrt{3+2x+5x^2}}{75000} + \frac{4072285}{}$$

Mathematica [A]
time = 0.61, size = 89, normalized size = 0.48

$$\frac{\sqrt{3+2x+5x^2}\,(-396379416+289653850x+203614255x^2-387039975x^3-47807000x^4+326987500x^5-106968750x^6-32156250x^7)}{3750000}+\frac{77513689\log\left(-1-5x+\sqrt{5}\,\sqrt{3+2x+5x^2}\right)}{625000\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-396379416 + 289653850*x + 203614255*x^2 - 38703997
5*x^3 - 47807000*x^4 + 326987500*x^5 - 106968750*x^6 - 32156250*x^7))/37500
00 + (77513689*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(625000*Sqrt[
5])
```

Maple [A]
time = 0.17, size = 147, normalized size = 0.79

method	result
risch	$-\frac{(32156250x^7+106968750x^6-326987500x^5+47807000x^4+387039975x^3-203614255x^2-289653850x+396379416)\sqrt{5x^2+2x+}}{3750000}$
trager	$\left(-\frac{343}{40}x^7-\frac{1141}{40}x^6+\frac{26159}{300}x^5-\frac{47807}{3750}x^4-\frac{5160533}{50000}x^3+\frac{40722851}{750000}x^2+\frac{5793077}{75000}x-\frac{16515809}{156250}\right)\sqrt{5x^2+2x+}$
default	$-\frac{343x^7\sqrt{5x^2+2x+3}}{40}-\frac{47807x^4\sqrt{5x^2+2x+3}}{3750}+\frac{26159x^5\sqrt{5x^2+2x+3}}{300}-\frac{1141x^6\sqrt{5x^2+2x+3}}{40}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -343/40*x^7*(5*x^2+2*x+3)^(1/2)-47807/3750*x^4*(5*x^2+2*x+3)^(1/2)+26159/30
0*x^5*(5*x^2+2*x+3)^(1/2)-1141/40*x^6*(5*x^2+2*x+3)^(1/2)+5793077/75000*x*(
5*x^2+2*x+3)^(1/2)+40722851/750000*x^2*(5*x^2+2*x+3)^(1/2)-5160533/50000*x^
3*(5*x^2+2*x+3)^(1/2)-16515809/156250*(5*x^2+2*x+3)^(1/2)-77513689/3125000*
5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Maxima [A]
time = 0.51, size = 148, normalized size = 0.80

$$-\frac{343}{40}\sqrt{5x^2+2x+3}\,x^7-\frac{1141}{40}\sqrt{5x^2+2x+3}\,x^6+\frac{26159}{300}\sqrt{5x^2+2x+3}\,x^5-\frac{47807}{3750}\sqrt{5x^2+2x+3}\,x^4-\frac{5160533}{50000}\sqrt{5x^2+2x+3}\,x^3+\frac{40722851}{750000}\sqrt{5x^2+2x+3}\,x^2+\frac{5793077}{75000}\sqrt{5x^2+2x+3}\,x-\frac{77513689}{3125000}\sqrt{5}\,\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}\,(5x+1)\right)-\frac{16515809}{156250}\sqrt{5x^2+2x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -343/40*sqrt(5*x^2 + 2*x + 3)*x^7 - 1141/40*sqrt(5*x^2 + 2*x + 3)*x^6 + 261
59/300*sqrt(5*x^2 + 2*x + 3)*x^5 - 47807/3750*sqrt(5*x^2 + 2*x + 3)*x^4 - 5
160533/50000*sqrt(5*x^2 + 2*x + 3)*x^3 + 40722851/750000*sqrt(5*x^2 + 2*x +
 3)*x^2 + 5793077/75000*sqrt(5*x^2 + 2*x + 3)*x - 77513689/3125000*sqrt(5)*
arcsinh(1/14*sqrt(14)*(5*x + 1)) - 16515809/156250*sqrt(5*x^2 + 2*x + 3)
```

Fricas [A]
time = 0.35, size = 87, normalized size = 0.47

$$-\frac{1}{3750000}\left(32156250\,x^7+106968750\,x^6-326987500\,x^5+47807000\,x^4+387039975\,x^3-203614255\,x^2-289653850\,x+396379416\right)\sqrt{5\,x^2+2\,x+3}+\frac{77513689}{6250000}\,\sqrt{5}\,\log\left(\sqrt{5}\,\sqrt{5\,x^2+2\,x+3}\,(5\,x+1)-25\,x^2-10\,x-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/3750000*(32156250*x^7 + 106968750*x^6 - 326987500*x^5 + 47807000*x^4 + 3
87039975*x^3 - 203614255*x^2 - 289653850*x + 396379416)*sqrt(5*x^2 + 2*x +
3) + 77513689/6250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) -
 25*x^2 - 10*x - 8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{29x}{\sqrt{5x^2+2x+3}}\right)dx-\int\left(-\frac{115x^2}{\sqrt{5x^2+2x+3}}\right)dx-\int\frac{61x^3}{\sqrt{5x^2+2x+3}}\,dx-\int\frac{871x^4}{\sqrt{5x^2+2x+3}}\,dx-\int\left(-\frac{127x^5}{\sqrt{5x^2+2x+3}}\right)dx-\int\left(-\frac{2065x^6}{\sqrt{5x^2+2x+3}}\right)dx-\int\frac{1127x^7}{\sqrt{5x^2+2x+3}}\,dx-\int\frac{343x^8}{\sqrt{5x^2+2x+3}}\,dx-\int\left(-\frac{2}{\sqrt{5x^2+2x+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(-29*x/sqrt(5*x**2 + 2*x + 3), x) - Integral(-115*x**2/sqrt(5*x**2
 + 2*x + 3), x) - Integral(61*x**3/sqrt(5*x**2 + 2*x + 3), x) - Integral(87
1*x**4/sqrt(5*x**2 + 2*x + 3), x) - Integral(-127*x**5/sqrt(5*x**2 + 2*x +
3), x) - Integral(-2065*x**6/sqrt(5*x**2 + 2*x + 3), x) - Integral(1127*x**
7/sqrt(5*x**2 + 2*x + 3), x) - Integral(343*x**8/sqrt(5*x**2 + 2*x + 3), x)
 - Integral(-2/sqrt(5*x**2 + 2*x + 3), x)
```

Giac [A]
time = 4.15, size = 82, normalized size = 0.44

$$-\frac{1}{3750000}\left(5\left(\left(5\left(10\left(175\left(15\left(49\,x+163\right)x-7474\right)x+191228\right)x+15481599\right)x-40722851\right)x-57930770\right)x+396379416\right)\sqrt{5\,x^2+2\,x+3}+\frac{77513689}{3125000}\,\sqrt{5}\,\log\left(-\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/3750000*(5*((5*(10*(175*(15*(49*x + 163)*x - 7474)*x + 191228)*x + 15481
599)*x - 40722851)*x - 57930770)*x + 396379416)*sqrt(5*x^2 + 2*x + 3) + 775
13689/3125000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5\,x+2\right)\,{\left(-7\,x^2+4\,x+1\right)}^3}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(1/2),x)
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(1/2), x)
```

3.387 $\int \frac{(1+4x-7x^2)^2 (2+5x+x^2)}{\sqrt{3+2x+5x^2}} \, dx$

Optimal. Leaf size=143

$$-\frac{22053\sqrt{3+2x+5x^2}}{31250}+\frac{36073x\sqrt{3+2x+5x^2}}{1875}-\frac{207427x^2\sqrt{3+2x+5x^2}}{37500}-\frac{33259x^3\sqrt{3+2x+5x^2}}{2500}+\frac{5}{7}$$

```
[Out] -1719097/156250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-22053/31250*(5*x^2+2
*x+3)^(1/2)+36073/1875*x*(5*x^2+2*x+3)^(1/2)-207427/37500*x^2*(5*x^2+2*x+3)
^(1/2)-33259/2500*x^3*(5*x^2+2*x+3)^(1/2)+5131/750*x^4*(5*x^2+2*x+3)^(1/2)+
49/30*x^5*(5*x^2+2*x+3)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1675, 654, 633, 221}

$$-\frac{207427\sqrt{5x^2+2x+3}\,x^2}{37500}+\frac{36073\sqrt{5x^2+2x+3}\,x}{1875}-\frac{22053\sqrt{5x^2+2x+3}}{31250}+\frac{49}{30}\sqrt{5x^2+2x+3}\,x^5+\frac{5131}{750}\sqrt{5x^2+2x+3}\,x^4-\frac{33259\sqrt{5x^2+2x+3}\,x^3}{2500}-\frac{1719097\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{31250\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-22053*Sqrt[3 + 2*x + 5*x^2])/31250 + (36073*x*Sqrt[3 + 2*x + 5*x^2])/1875
 - (207427*x^2*Sqrt[3 + 2*x + 5*x^2])/37500 - (33259*x^3*Sqrt[3 + 2*x + 5*x
^2])/2500 + (5131*x^4*Sqrt[3 + 2*x + 5*x^2])/750 + (49*x^5*Sqrt[3 + 2*x + 5
*x^2])/30 - (1719097*ArcSinh[(1 + 5*x)/Sqrt[14]])/(31250*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \frac{49}{30}x^5\sqrt{3+2x+5x^2} + \frac{1}{30}\int \frac{60+630x+1350x^2-2820x^3-6135x}{\sqrt{3+2x+5x^2}}$$

$$= \frac{5131}{750}x^4\sqrt{3+2x+5x^2} + \frac{49}{30}x^5\sqrt{3+2x+5x^2} + \frac{1}{750}\int \frac{1500+1575}{}$$

$$= -\frac{33259x^3\sqrt{3+2x+5x^2}}{2500} + \frac{5131}{750}x^4\sqrt{3+2x+5x^2} + \frac{49}{30}x^5\sqrt{3+2x}$$

$$= -\frac{207427x^2\sqrt{3+2x+5x^2}}{37500} - \frac{33259x^3\sqrt{3+2x+5x^2}}{2500} + \frac{5131}{750}x^4\sqrt{3}$$

$$= \frac{36073x\sqrt{3+2x+5x^2}}{1875} - \frac{207427x^2\sqrt{3+2x+5x^2}}{37500} - \frac{33259x^3\sqrt{3+}}{2500}$$

$$= -\frac{22053\sqrt{3+2x+5x^2}}{31250} + \frac{36073x\sqrt{3+2x+5x^2}}{1875} - \frac{207427x^2\sqrt{3+}}{37500}$$

$$= -\frac{22053\sqrt{3+2x+5x^2}}{31250} + \frac{36073x\sqrt{3+2x+5x^2}}{1875} - \frac{207427x^2\sqrt{3+}}{37500}$$

$$= -\frac{22053\sqrt{3+2x+5x^2}}{31250} + \frac{36073x\sqrt{3+2x+5x^2}}{1875} - \frac{207427x^2\sqrt{3+}}{37500}$$

Mathematica [A]
time = 0.44, size = 79, normalized size = 0.55

$$\frac{\sqrt{3+2x+5x^2}\,(-132318+3607300x-1037135x^2-2494425x^3+1282750x^4+306250x^5)}{187500}+\frac{1719097\log\left(-1-5x+\sqrt{5}\,\sqrt{3+2x+5x^2}\right)}{31250\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (Sqrt[3 + 2*x + 5*x^2]*(-132318 + 3607300*x - 1037135*x^2 - 2494425*x^3 + 1
282750*x^4 + 306250*x^5))/187500 + (1719097*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 +
 2*x + 5*x^2]])/(31250*Sqrt[5])
```

Maple [A]
time = 0.17, size = 113, normalized size = 0.79

method	result
risch	$\frac{(306250x^5+1282750x^4-2494425x^3-1037135x^2+3607300x-132318)\sqrt{5x^2+2x+3}}{187500} - \frac{1719097\sqrt{5}\ \mathrm{arcsinh}\left(\frac{5\sqrt{14}\,(x}{14}\right.}{156250}$
trager	$\left(\frac{49}{30}x^5+\frac{5131}{750}x^4-\frac{33259}{2500}x^3-\frac{207427}{37500}x^2+\frac{36073}{1875}x-\frac{22053}{31250}\right)\sqrt{5x^2+2x+3} - \frac{1719097\,\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(\right.}{}$
default	$\frac{5131x^4\sqrt{5x^2+2x+3}}{750}+\frac{49x^5\sqrt{5x^2+2x+3}}{30}+\frac{36073x\sqrt{5x^2+2x+3}}{1875}-\frac{207427x^2\sqrt{5x^2+2x+3}}{37500}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 5131/750*x^4*(5*x^2+2*x+3)^(1/2)+49/30*x^5*(5*x^2+2*x+3)^(1/2)+36073/1875*x
*(5*x^2+2*x+3)^(1/2)-207427/37500*x^2*(5*x^2+2*x+3)^(1/2)-33259/2500*x^3*(5
*x^2+2*x+3)^(1/2)-22053/31250*(5*x^2+2*x+3)^(1/2)-1719097/156250*5^(1/2)*ar
csinh(5/14*14^(1/2)*(x+1/5))
```

Maxima [A]
time = 0.51, size = 114, normalized size = 0.80

$$\frac{49}{30}\sqrt{5x^2+2x+3}\,x^5+\frac{5131}{750}\sqrt{5x^2+2x+3}\,x^4-\frac{33259}{2500}\sqrt{5x^2+2x+3}\,x^3-\frac{207427}{37500}\sqrt{5x^2+2x+3}\,x^2+\frac{36073}{1875}\sqrt{5x^2+2x+3}\,x-\frac{1719097}{156250}\sqrt{5}\ \mathrm{arsinh}\left(\frac{1}{14}\sqrt{14}\,(5x+1)\right)-\frac{22053}{31250}\sqrt{5x^2+2x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] 49/30*sqrt(5*x^2 + 2*x + 3)*x^5 + 5131/750*sqrt(5*x^2 + 2*x + 3)*x^4 - 3325
9/2500*sqrt(5*x^2 + 2*x + 3)*x^3 - 207427/37500*sqrt(5*x^2 + 2*x + 3)*x^2 +
 36073/1875*sqrt(5*x^2 + 2*x + 3)*x - 1719097/156250*sqrt(5)*arcsinh(1/14*s
qrt(14)*(5*x + 1)) - 22053/31250*sqrt(5*x^2 + 2*x + 3)
```

Fricas [A]
time = 0.38, size = 77, normalized size = 0.54

$$\frac{1}{187500}\left(306250\,x^5+1282750\,x^4-2494425\,x^3-1037135\,x^2+3607300\,x-132318\right)\sqrt{5\,x^2+2\,x+3}+\frac{1719097}{312500}\sqrt{5}\ \log\left(\sqrt{5}\ \sqrt{5\,x^2+2\,x+3}\,(5\,x+1)-25\,x^2-10\,x-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/187500*(306250*x^5 + 1282750*x^4 - 2494425*x^3 - 1037135*x^2 + 3607300*x
- 132318)*sqrt(5*x^2 + 2*x + 3) + 1719097/312500*sqrt(5)*log(sqrt(5)*sqrt(5
*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2+5x+2)(7x^2-4x-1)^2}{\sqrt{5x^2+2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)*(7*x**2 - 4*x - 1)**2/sqrt(5*x**2 + 2*x + 3), x)
```

Giac [A]
time = 4.56, size = 72, normalized size = 0.50

$$\frac{1}{187500}\,(5\,((5\,(70\,(175\,x+733)x-99777)x-207427)x+721460)x-132318)\sqrt{5\,x^2+2\,x+3}+\frac{1719097}{156250}\sqrt{5}\,\log\left(-\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/187500*(5*((5*(70*(175*x + 733)*x - 99777)*x - 207427)*x + 721460)*x - 13
2318)*sqrt(5*x^2 + 2*x + 3) + 1719097/156250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*
x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^2+5\,x+2)\,(-7\,x^2+4\,x+1)^2}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(1/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(1/2), x)
```

3.388 $\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

Optimal. Leaf size=101

$$\frac{463}{125}\sqrt{3+2x+5x^2}+\frac{59}{30}x\sqrt{3+2x+5x^2}-\frac{571}{300}x^2\sqrt{3+2x+5x^2}-\frac{7}{20}x^3\sqrt{3+2x+5x^2}-\frac{1901\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{250\sqrt{5}}$$

```
[Out] -1901/1250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)+463/125*(5*x^2+2*x+3)^(1/
2)+59/30*x*(5*x^2+2*x+3)^(1/2)-571/300*x^2*(5*x^2+2*x+3)^(1/2)-7/20*x^3*(5*
x^2+2*x+3)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1675, 654, 633, 221}

$$-\frac{571}{300}\sqrt{5x^2+2x+3}\,x^2+\frac{59}{30}\sqrt{5x^2+2x+3}\,x+\frac{463}{125}\sqrt{5x^2+2x+3}-\frac{7}{20}\sqrt{5x^2+2x+3}\,x^3-\frac{1901\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{250\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (463*Sqrt[3 + 2*x + 5*x^2])/125 + (59*x*Sqrt[3 + 2*x + 5*x^2])/30 - (571*x^
2*Sqrt[3 + 2*x + 5*x^2])/300 - (7*x^3*Sqrt[3 + 2*x + 5*x^2])/20 - (1901*Arc
Sinh[(1 + 5*x)/Sqrt[14]])/(250*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{7}{20}x^3\sqrt{3+2x+5x^2} + \frac{1}{20}\int \frac{40+260x+203x^2-571x^3}{\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{571}{300}x^2\sqrt{3+2x+5x^2} - \frac{7}{20}x^3\sqrt{3+2x+5x^2} + \frac{1}{300}\int \frac{600+7326x}{\sqrt{3+2x}}$$

$$= \frac{59}{30}x\sqrt{3+2x+5x^2} - \frac{571}{300}x^2\sqrt{3+2x+5x^2} - \frac{7}{20}x^3\sqrt{3+2x+5x^2}$$

$$= \frac{463}{125}\sqrt{3+2x+5x^2} + \frac{59}{30}x\sqrt{3+2x+5x^2} - \frac{571}{300}x^2\sqrt{3+2x+5x^2} -$$

$$= \frac{463}{125}\sqrt{3+2x+5x^2} + \frac{59}{30}x\sqrt{3+2x+5x^2} - \frac{571}{300}x^2\sqrt{3+2x+5x^2} -$$

$$= \frac{463}{125}\sqrt{3+2x+5x^2} + \frac{59}{30}x\sqrt{3+2x+5x^2} - \frac{571}{300}x^2\sqrt{3+2x+5x^2} -$$

Mathematica [A]
time = 0.31, size = 69, normalized size = 0.68

$$\frac{\sqrt{3+2x+5x^2}\,(5556+2950x-2855x^2-525x^3)}{1500} + \frac{1901\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{250\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
[Out] (Sqrt[3 + 2*x + 5*x^2]*(5556 + 2950*x - 2855*x^2 - 525*x^3))/1500 + (1901*L
og[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(250*Sqrt[5])
```

Maple [A]
time = 0.14, size = 79, normalized size = 0.78

method	result
risch	$-\frac{(525x^3+2855x^2-2950x-5556)\sqrt{5x^2+2x+3}}{1500}-\frac{1901\sqrt{5}\ \mathrm{arcsinh}\left(\frac{5\sqrt{14}\,(x+\frac{1}{5})}{14}\right)}{1250}$
trager	$\left(-\frac{7}{20}x^3-\frac{571}{300}x^2+\frac{59}{30}x+\frac{463}{125}\right)\sqrt{5x^2+2x+3}-\frac{1901\,\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(5\,\mathrm{RootOf}\left(\_Z^2-5\right)x+\mathrm{RootOf}\left(\_Z\right.\right.}{1250}$
default	$-\frac{7x^3\sqrt{5x^2+2x+3}}{20}-\frac{571x^2\sqrt{5x^2+2x+3}}{300}+\frac{59x\sqrt{5x^2+2x+3}}{30}+\frac{463\sqrt{5x^2+2x+3}}{125}-\frac{1901\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -7/20*x^3*(5*x^2+2*x+3)^(1/2)-571/300*x^2*(5*x^2+2*x+3)^(1/2)+59/30*x*(5*x^
2+2*x+3)^(1/2)+463/125*(5*x^2+2*x+3)^(1/2)-1901/1250*5^(1/2)*arcsinh(5/14*1
4^(1/2)*(x+1/5))
```

Maxima [A]
time = 0.50, size = 80, normalized size = 0.79

$$-\frac{7}{20}\sqrt{5x^2+2x+3}\,x^3-\frac{571}{300}\sqrt{5x^2+2x+3}\,x^2+\frac{59}{30}\sqrt{5x^2+2x+3}\,x-\frac{1901}{1250}\sqrt{5}\ \mathrm{arsinh}\left(\frac{1}{14}\sqrt{14}\,(5x+1)\right)+\frac{463}{125}\sqrt{5x^2+2x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -7/20*sqrt(5*x^2 + 2*x + 3)*x^3 - 571/300*sqrt(5*x^2 + 2*x + 3)*x^2 + 59/30
*sqrt(5*x^2 + 2*x + 3)*x - 1901/1250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1
)) + 463/125*sqrt(5*x^2 + 2*x + 3)
```

Fricas [A]
time = 0.35, size = 67, normalized size = 0.66

$$-\frac{1}{1500}\left(525x^3+2855x^2-2950x-5556\right)\sqrt{5x^2+2x+3}+\frac{1901}{2500}\sqrt{5}\ \log\left(\sqrt{5}\sqrt{5x^2+2x+3}\,(5x+1)-25x^2-10x-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fric
as")
```

```
[Out] -1/1500*(525*x^3 + 2855*x^2 - 2950*x - 5556)*sqrt(5*x^2 + 2*x + 3) + 1901/2
500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{13x}{\sqrt{5x^2+2x+3}}\right) dx - \int \left(-\frac{7x^2}{\sqrt{5x^2+2x+3}}\right) dx - \int \frac{31x^3}{\sqrt{5x^2+2x+3}} dx - \int \frac{7x^4}{\sqrt{5x^2+2x+3}} dx - \int \left(-\frac{2}{\sqrt{5x^2+2x+3}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(-13*x/sqrt(5*x**2 + 2*x + 3), x) - Integral(-7*x**2/sqrt(5*x**2 +
 2*x + 3), x) - Integral(31*x**3/sqrt(5*x**2 + 2*x + 3), x) - Integral(7*x*
*4/sqrt(5*x**2 + 2*x + 3), x) - Integral(-2/sqrt(5*x**2 + 2*x + 3), x)
```

Giac [A]
time = 4.16, size = 62, normalized size = 0.61

$$-\frac{1}{1500}\left(5\left((105x+571)x-590\right)x-5556\right)\sqrt{5x^2+2x+3}+\frac{1901}{1250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/1500*(5*((105*x + 571)*x - 590)*x - 5556)*sqrt(5*x^2 + 2*x + 3) + 1901/1
250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5x+2\right)\left(-7x^2+4x+1\right)}{\sqrt{5x^2+2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(1/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(1/2), x)
```

3.389 $\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx$

Optimal. Leaf size=164

$$-\frac{\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{7\sqrt{5}}-\frac{3}{14}\sqrt{\frac{4091-1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)+\frac{3}{14}\sqrt{\frac{4091}{}}$$

```
[Out] -1/35*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-3/39116*arctanh((23+x*(17-5*11
^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(11430254-29
47670*11^(1/2))^(1/2)+3/39116*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^
2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(11430254+2947670*11^(1/2))^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1090, 633, 221, 1046, 738, 212}

$$-\frac{3}{14}\sqrt{\frac{4091-1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)+\frac{3}{14}\sqrt{\frac{4091+1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)-\frac{\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{7\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] -1/7*ArcSinh[(1 + 5*x)/Sqrt[14]]/Sqrt[5] - (3*Sqrt[(4091 - 1055*Sqrt[11])/2
794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[1
1])]*Sqrt[3 + 2*x + 5*x^2])])/14 + (3*Sqrt[(4091 + 1055*Sqrt[11])/2794]*Arc
Tanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqr
t[3 + 2*x + 5*x^2])])/14
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx = -\left(\frac{1}{7}\int \frac{1}{\sqrt{3+2x+5x^2}}\,dx\right) - \frac{1}{7}\int \frac{-15-39x}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right)}{14\sqrt{70}} + \frac{1}{77}\left(3\left(143-61\sqrt{11}\right)\right)$$

$$= -\frac{\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{7\sqrt{5}} - \frac{1}{77}\left(6\left(143-61\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{}{2352+11}\right.$$

$$= -\frac{\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{7\sqrt{5}} - \frac{3}{14}\sqrt{\frac{4091-1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{23-}{\sqrt{2\left(125-\right.}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.30, size = 211, normalized size = 1.29

$$\frac{\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{7\sqrt{5}} + \frac{3}{14}\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{29\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+10\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1-13\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1^2}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]]/(7*Sqrt[5]) + (3*RootSum[83 -
 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (29*Log[-(Sqrt[5]*x)
 + Sqrt[3 + 2*x + 5*x^2] - #1] + 10*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x
 + 5*x^2] - #1]*#1 - 13*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2
)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/14
```

Maple [A]
time = 0.77, size = 204, normalized size = 1.24

method	result

default	$-\frac{\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,\left(x+\frac{1}{5}\right)}{14}\right)}{35}+\frac{3\left(61+13\sqrt{11}\right)\sqrt{11}\,\operatorname{arctanh}\left(\frac{250+34\sqrt{11}}{\sqrt{250+34\sqrt{11}}\,\sqrt{245\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right.}}\right.}{154\sqrt{250+3}}$
trager	$\operatorname{RootOf}\left(24095456\_Z^4-3240072\_Z^2+29241\right)\ln\left(\frac{16819145472867584x\operatorname{RootOf}\left(24095456\_Z^4-3240072\_Z^2\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/35*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+3/154*(61+13*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))+3/154*(-61+13*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(114) = 228$.
time = 0.52, size = 465, normalized size = 2.84

$$-\frac{1}{10780}\sqrt{11}\left(28\sqrt{11}\sqrt{5}\operatorname{arsinh}\left(\frac{5}{14}\sqrt{7}\sqrt{2}x+\frac{1}{14}\sqrt{7}\sqrt{2}\right)-\frac{1365\sqrt{11}\sqrt{2}\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7|14x-2\sqrt{11}-4|}+\frac{17\sqrt{7}\sqrt{2}x}{7|14x-2\sqrt{11}-4|}+\frac{\sqrt{11}\sqrt{7}\sqrt{2}}{7|14x-2\sqrt{11}-4|}+\frac{23\sqrt{7}\sqrt{2}}{7|14x-2\sqrt{11}-4|}\right)}{\sqrt{17\sqrt{11}+125}}+\frac{390\sqrt{11}\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7|14x+2\sqrt{11}-4|}-\frac{17\sqrt{7}\sqrt{2}x}{7|14x+2\sqrt{11}-4|}+\frac{\sqrt{11}\sqrt{7}\sqrt{2}}{7|14x+2\sqrt{11}-4|}-\frac{23\sqrt{7}\sqrt{2}}{7|14x+2\sqrt{11}-4|}\right)}{\sqrt{-\frac{34}{49}\sqrt{11}+\frac{250}{49}}}-\frac{6405\sqrt{2}\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7|14x-2\sqrt{11}-4|}+\frac{17\sqrt{7}\sqrt{2}x}{7|14x-2\sqrt{11}-4|}+\frac{\sqrt{11}\sqrt{7}\sqrt{2}}{7|14x-2\sqrt{11}-4|}+\frac{23\sqrt{7}\sqrt{2}}{7|14x-2\sqrt{11}-4|}\right)}{\sqrt{17\sqrt{11}+125}}-\frac{1830\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7|14x+2\sqrt{11}-4|}-\frac{17\sqrt{7}\sqrt{2}x}{7|14x+2\sqrt{11}-4|}+\frac{\sqrt{11}\sqrt{7}\sqrt{2}}{7|14x+2\sqrt{11}-4|}-\frac{23\sqrt{7}\sqrt{2}}{7|14x+2\sqrt{11}-4|}\right)}{\sqrt{-\frac{34}{49}\sqrt{11}+\frac{250}{49}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxima")

[Out] -1/10780*sqrt(11)*(28*sqrt(11)*sqrt(5)*arcsinh(5/14*sqrt(7)*sqrt(2)*x + 1/14*sqrt(7)*sqrt(2)) - 1365*sqrt(11)*sqrt(2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/sqrt(17*sqrt(11) + 125) + 390*sqrt(11)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4))/sqrt(-34/49*sqrt(11) + 250/49) - 6405*sqrt(2)*arcsinh(5/7*sqrt(1

```
1)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/ab
s(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(1
1) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/sqrt(17*sqrt(11)
 + 125) - 1830*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11)
 - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sq
rt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x +
2*sqrt(11) - 4))/sqrt(-34/49*sqrt(11) + 250/49))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(114) = 228$.
time = 0.38, size = 297, normalized size = 1.81

$$-\frac{3}{78232}\sqrt{2794}\sqrt{1055\sqrt{11}+4091}\log\left(\frac{3\left(\sqrt{2794}\sqrt{5x^2+2x+3}\sqrt{1055\sqrt{11}+4091}\left(172\sqrt{11}-715\right)+185801\sqrt{11}(x+3)+557403x-929005\right)}{x}\right)+\frac{3}{78232}\sqrt{2794}\sqrt{1055\sqrt{11}+4091}\log\left(-\frac{3\left(\sqrt{2794}\sqrt{5x^2+2x+3}\sqrt{1055\sqrt{11}+4091}\left(172\sqrt{11}-715\right)-185801\sqrt{11}(x+3)-557403x+929005\right)}{x}\right)-\frac{1}{78232}\sqrt{2794}\sqrt{-9495\sqrt{11}+36819}\log\left(-\frac{\sqrt{2794}\sqrt{5x^2+2x+3}\left(172\sqrt{11}+715\right)\sqrt{-9495\sqrt{11}+36819}+557403\sqrt{11}(x+3)-1672209x+2787015}{x}\right)+\frac{1}{78232}\sqrt{2794}\sqrt{-9495\sqrt{11}+36819}\log\left(\frac{\sqrt{2794}\sqrt{5x^2+2x+3}\left(172\sqrt{11}+715\right)\sqrt{-9495\sqrt{11}+36819}-557403\sqrt{11}(x+3)+1672209x-2787015}{x}\right)+\frac{1}{70}\sqrt{5}\log\left(\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="fric
as")
```

```
[Out] -3/78232*sqrt(2794)*sqrt(1055*sqrt(11) + 4091)*log(3*(sqrt(2794)*sqrt(5*x^2
 + 2*x + 3)*sqrt(1055*sqrt(11) + 4091)*(172*sqrt(11) - 715) + 185801*sqrt(1
1)*(x + 3) + 557403*x - 929005)/x) + 3/78232*sqrt(2794)*sqrt(1055*sqrt(11)
+ 4091)*log(-3*(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(1055*sqrt(11) + 4091)
*(172*sqrt(11) - 715) - 185801*sqrt(11)*(x + 3) - 557403*x + 929005)/x) - 1
/78232*sqrt(2794)*sqrt(-9495*sqrt(11) + 36819)*log(-(sqrt(2794)*sqrt(5*x^2
+ 2*x + 3)*(172*sqrt(11) + 715)*sqrt(-9495*sqrt(11) + 36819) + 557403*sqrt(
11)*(x + 3) - 1672209*x + 2787015)/x) + 1/78232*sqrt(2794)*sqrt(-9495*sqrt(
11) + 36819)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(172*sqrt(11) + 715)*sqr
t(-9495*sqrt(11) + 36819) - 557403*sqrt(11)*(x + 3) + 1672209*x - 2787015)/
x) + 1/70*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10
*x - 8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{5x}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{5x^2+2x+3}}\,dx-\int\frac{x^2}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{5x^2+2x+3}}\,dx-\int\frac{2}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{5x^2+2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(5*x/(7*x**2*sqrt(5*x**2 + 2*x + 3) - 4*x*sqrt(5*x**2 + 2*x + 3) -
 sqrt(5*x**2 + 2*x + 3)), x) - Integral(x**2/(7*x**2*sqrt(5*x**2 + 2*x + 3)
 - 4*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(7
*x**2*sqrt(5*x**2 + 2*x + 3) - 4*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2
*x + 3)), x)
```

Giac [A]
time = 3.93, size = 125, normalized size = 0.76

$$\frac{1}{35}\,\sqrt{5}\log\left(-5\,\sqrt{5}x-\sqrt{5}+5\,\sqrt{5\,x^{2}+2\,x+3}\right)+0.353184817631429\,\log\left(-\sqrt{5}x+\sqrt{5\,x^{2}+2\,x+3}+4.41924736459000\right)-0.0986339689905714\,\log\left(-\sqrt{5}x+\sqrt{5\,x^{2}+2\,x+3}+1.25295163054000\right)-0.353184817631429\,\log\left(-\sqrt{5}x+\sqrt{5\,x^{2}+2\,x+3}-1.02258038113000\right)+0.0986339689905714\,\log\left(-\sqrt{5}x+\sqrt{5\,x^{2}+2\,x+3}-2.09411235400000\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac
")
```

```
[Out] 1/35*sqrt(5)*log(-5*sqrt(5)*x - sqrt(5) + 5*sqrt(5*x^2 + 2*x + 3)) + 0.3531
84817631429*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.
0986339689905714*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000)
 - 0.353184817631429*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113
000) + 0.0986339689905714*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.094112
35400000)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+5\,x+2}{\sqrt{5\,x^2+2\,x+3}\,\left(-7\,x^2+4\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)),x)
```

```
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)), x)
```

3.390 $\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx$

Optimal. Leaf size=178

$$-\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\,(1+4x-7x^2)}-\frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}}\,\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\,\sqrt{3+2x}}\right.}{11176}$$

```
[Out] -3/5588*(40-371*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)+1/31225744*arctanh((2
3+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*
(8459955268270-39215692714*11^(1/2))^(1/2)-1/31225744*arctanh((23+x*(17-5*1
1^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(8459955268
270+39215692714*11^(1/2))^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1074, 1046, 738, 212}

$$-\frac{3\sqrt{5x^2+2x+3}\,(40-371x)}{5588\,(-7x^2+4x+1)}-\frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}}\,\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\,\sqrt{5x^2+2x+3}}\right)}{11176}+\frac{\sqrt{\frac{3027900955-14035681\sqrt{11}}{2794}}\,\tanh^{-1}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\,\sqrt{5x^2+2x+3}}\right)}{11176}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] (-3*(40 - 371*x)*Sqrt[3 + 2*x + 5*x^2])/(5588*(1 + 4*x - 7*x^2)) - (Sqrt[(3
027900955 + 14035681*Sqrt[11])/2794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[
11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/11176 + (Sqrt
[(3027900955 - 14035681*Sqrt[11])/2794]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sq
rt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/11176
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx = -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\,(1+4x-7x^2)} - \frac{\int \frac{-52136-29544x}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx}{44704}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\,(1+4x-7x^2)} - \frac{\left(-40623+53005\sqrt{11}\right)\int \frac{}{(4-}}{614}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\,(1+4x-7x^2)} - \frac{\left(40623-53005\sqrt{11}\right)\text{Subst}\Big(}{}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\,(1+4x-7x^2)} - \frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.44, size = 352, normalized size = 1.98

$$-\frac{3(-40+371x)\sqrt{3+2x+5x^2}}{5588\,(-1-4x+7x^2)} - \frac{1}{49}\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{-397\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+7\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\right] + \frac{3\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{-1510889\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+238966\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1-60319\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1^2}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\right]}{547624}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] (-3*(-40 + 371*x)*Sqrt[3 + 2*x + 5*x^2])/(5588*(-1 - 4*x + 7*x^2)) - RootSu
m[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-397*Log[-(Sq
rt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 7*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[
3 + 2*x + 5*x^2] - #1]*#1)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) &
 ]/49 + (3*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 &
 , (-1510889*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 238966*Sqrt[5
]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 - 60319*Log[-(Sqrt[5]*x
) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2
+ 7*#1^3) & ])/547624
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(130) = 260$.
time = 0.70, size = 510, normalized size = 2.87

method	result

risch	$-\frac{3(-40+371x)\sqrt{5x^2+2x+3}}{5588(7x^2-4x-1)}+\frac{\left(53005+3693\sqrt{11}\right)\sqrt{11}\ \operatorname{arctanh}\left(\frac{250+34\cdots}{\sqrt{250+34\sqrt{11}}\ \sqrt{245\left(x-\frac{2}{7}-\cdots\right.}}\right.}{122936\sqrt{250\cdots}}$
trager	Expression too large to display
default	$\left(\frac{183}{44}-\frac{39\sqrt{11}}{44}\right)\left(-\frac{\sqrt{5\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)+\frac{250}{49}-\frac{34\sqrt{11}}{49}\cdots}}{49\left(\frac{250}{49}-\frac{34\sqrt{11}}{49}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] (183/44-39/44*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))
*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49
-34/49*11^(1/2))^(1/2)+1/14*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))/(2
50-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/
2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))
^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))+(1
83/44+39/44*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(
5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+3
4/49*11^(1/2))^(1/2)+1/14*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))/(250
+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2)
)*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2
+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))+161/
484*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(3
4/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/
7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1
/2))^(1/2))-161/484*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-6
8/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^
(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/
2))+250-34*11^(1/2))^(1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^2*sqrt(5*x^2 + 2*x + 3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(129) = 258$.
time = 0.37, size = 330, normalized size = 1.85

$$-\frac{\sqrt{2794}\,(7x^2-4x-1)\sqrt{14035681\sqrt{11}+3027900955}\,\log\left(-\frac{\sqrt{2794}\sqrt{5x^2+2x+3}\sqrt{14035681\sqrt{11}+3027900955}\,(71796\sqrt{11}+567523)+265381033753\sqrt{11}\,(x+3)-796143101259x+1326905168765}{x}\right)-\sqrt{2794}\,(7x^2-4x-1)\sqrt{14035681\sqrt{11}+3027900955}\,\log\left(\frac{\sqrt{2794}\sqrt{5x^2+2x+3}\sqrt{14035681\sqrt{11}+3027900955}\,(71796\sqrt{11}+567523)-265381033753\sqrt{11}\,(x+3)+796143101259x-1326905168765}{x}\right)+\sqrt{2794}\,(7x^2-4x-1)\sqrt{-14035681\sqrt{11}+3027900955}\,\log\left(\frac{\sqrt{2794}\sqrt{5x^2+2x+3}\,(71796\sqrt{11}-567523)\sqrt{-14035681\sqrt{11}+3027900955}+265381033753\sqrt{11}\,(x+3)+796143101259x-1326905168765}{x}\right)-\sqrt{2794}\,(7x^2-4x-1)\sqrt{-14035681\sqrt{11}+3027900955}\,\log\left(-\frac{\sqrt{2794}\sqrt{5x^2+2x+3}\,(71796\sqrt{11}-567523)\sqrt{-14035681\sqrt{11}+3027900955}-265381033753\sqrt{11}\,(x+3)-796143101259x+1326905168765}{x}\right)+33528\sqrt{5x^2+2x+3}\,(371x-40)}{62451488\,(7x^2-4x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/62451488*(sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(14035681*sqrt(11) + 30279009
55)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(14035681*sqrt(11) + 3027900
955)*(71796*sqrt(11) + 567523) + 265381033753*sqrt(11)*(x + 3) - 7961431012
59*x + 1326905168765)/x) - sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(14035681*sqrt(
11) + 3027900955)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(14035681*sqrt(
11) + 3027900955)*(71796*sqrt(11) + 567523) - 265381033753*sqrt(11)*(x + 3)
 + 796143101259*x - 1326905168765)/x) + sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(-
14035681*sqrt(11) + 3027900955)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(7179
6*sqrt(11) - 567523)*sqrt(-14035681*sqrt(11) + 3027900955) + 265381033753*s
qrt(11)*(x + 3) + 796143101259*x - 1326905168765)/x) - sqrt(2794)*(7*x^2 -
4*x - 1)*sqrt(-14035681*sqrt(11) + 3027900955)*log(-(sqrt(2794)*sqrt(5*x^2
+ 2*x + 3)*(71796*sqrt(11) - 567523)*sqrt(-14035681*sqrt(11) + 3027900955)
- 265381033753*sqrt(11)*(x + 3) - 796143101259*x + 1326905168765)/x) + 3352
8*sqrt(5*x^2 + 2*x + 3)*(371*x - 40))/(7*x^2 - 4*x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+5x+2}{\sqrt{5x^2+2x+3}\,\left(7x^2-4x-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**2/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)/(sqrt(5*x**2 + 2*x + 3)*(7*x**2 - 4*x - 1)**2), x
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(129) = 258$.
time = 3.40, size = 276, normalized size = 1.55

$$\frac{3\left(1231\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^3+1735\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^2-3913\sqrt{5}x-3989\sqrt{5}+3913\sqrt{5x^2+2x+3}\right)}{2794\left(7\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^4-8\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^3-70\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^2+16\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)+83\right)}+0.0924287071106453\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}+4.41924736459000\right)-0.0938608034604765\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}+1.25295163054000\right)-0.0924287071106453\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}-1.02258038113000\right)+0.0938608034604765\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}-2.09411235400000\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 3/2794*(1231*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 1735*sqrt(5)*(sqrt(5)*
x - sqrt(5*x^2 + 2*x + 3))^2 - 3913*sqrt(5)*x - 3989*sqrt(5) + 3913*sqrt(5*
x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sqrt(5)*(sqrt(
5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2
+ 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83) + 0.0924287071106453
*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.09386080346
04765*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0.092428
7071106453*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.0
938608034604765*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+5\,x+2}{\sqrt{5\,x^2+2\,x+3}\,{\left(-7\,x^2+4\,x+1\right)}^2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2),x)
```

```
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2), x)
```

3.391 $\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx$

Optimal. Leaf size=227

$$-\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176(1+4x-7x^2)^2}-\frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488(1+4x-7x^2)}-\frac{7\left(39370231-2538725\sqrt{11}\right)\text{ta}}{12490297}$$

```
[Out] -3/11176*(40-371*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2-7/62451488*(409769
-1189370*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)-7/124902976*arctanh((23+x*(1
7-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(39370
231-2538725*11^(1/2))/(2750-374*11^(1/2))^(1/2)+7/124902976*arctanh((23+11^
(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(3937
0231+2538725*11^(1/2))/(2750+374*11^(1/2))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1074, 1046, 738, 212}

$$-\frac{7\sqrt{5x^2+2x+3}(409769-1189370x)}{62451488(-7x^2+4x+1)}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176(-7x^2+4x+1)^2}-\frac{7\left(39370231-2538725\sqrt{11}\right)\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{124902976\sqrt{22\left(125-17\sqrt{11}\right)}}+\frac{7\left(39370231+2538725\sqrt{11}\right)\tanh^{-1}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{124902976\sqrt{22\left(125+17\sqrt{11}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] (-3*(40 - 371*x)*Sqrt[3 + 2*x + 5*x^2])/(11176*(1 + 4*x - 7*x^2)^2) - (7*(4
09769 - 1189370*x)*Sqrt[3 + 2*x + 5*x^2])/(62451488*(1 + 4*x - 7*x^2)) - (7
*(39370231 - 2538725*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x
)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(124902976*Sqrt[22*
(125 - 17*Sqrt[11])]) + (7*(39370231 + 2538725*Sqrt[11])*ArcTanh[(23 + Sqrt
[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x
^2])])/(124902976*Sqrt[22*(125 + 17*Sqrt[11])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx = -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\,(1+4x-7x^2)^2} - \frac{\int \frac{-130024-81000x-89040x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx}{89408}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\,(1+4x-7x^2)^2} - \frac{7(409769-1189370x)\sqrt{3+2x}}{62451488\,(1+4x-7x^2)}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\,(1+4x-7x^2)^2} - \frac{7(409769-1189370x)\sqrt{3+2x}}{62451488\,(1+4x-7x^2)}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\,(1+4x-7x^2)^2} - \frac{7(409769-1189370x)\sqrt{3+2x}}{62451488\,(1+4x-7x^2)}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\,(1+4x-7x^2)^2} - \frac{7(409769-1189370x)\sqrt{3+2x}}{62451488\,(1+4x-7x^2)}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.67, size = 433, normalized size = 1.91

$$\frac{\frac{235298\sqrt{3+2x+5x^2}(-3538943+3071502x+53381041x^2-58279130x^3)}{(1+4x-7x^2)^2} - 1796775175713\,\mathrm{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\right] + 11176\,\mathrm{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{10486671792\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1+6928653865\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1^2}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\right] - 3\,\mathrm{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{36376673721218\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1+26508461599305\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1^2}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\right]}{14694710223424}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] ((235298*Sqrt[3 + 2*x + 5*x^2]*(-3538943 + 3071502*x + 53381041*x^2 - 58279
130*x^3))/(1 + 4*x - 7*x^2)^2 - 1796775175713*RootSum[83 - 16*Sqrt[5]*#1 -
70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x
^2] - #1]/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] + 11176*RootSu
m[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (10486671792*S
qrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 6928653865*Log[-
(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqr
t[5]*#1^2 + 7*#1^3) & ] - 3*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5
]*#1^3 + 7*#1^4 & , (36376673721218*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x
 + 5*x^2] - #1]*#1 + 26508461599305*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2
] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/146947102
23424
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1193 vs. $2(175) = 350$.
time = 0.74, size = 1194, normalized size = 5.26

method	result
risch	$-\frac{(58279130x^3-53381041x^2-3071502x+3538943)\sqrt{5x^2+2x+3}}{62451488(7x^2-4x-1)^2}+\frac{7\left(39370231+2538725\sqrt{11}\right)\sqrt{11}\ \mathrm{arctanh}\left(\frac{}{\sqrt{25}}\right.}{}$
trager	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -21/968*(-61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49-34/49*11^(1/2))/(x-2/7+1
/7*11^(1/2))^2*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11
^(1/2))+250/49-34/49*11^(1/2))^(1/2)-3/1372*(34/7-10/7*11^(1/2))/(250/49-34
/49*11^(1/2))*(-1/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/
7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/
2))^(1/2)+7/2*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))/(250-34*11^(1/2)
)^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7
*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10
/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))+5/98/(250/49-34/
49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(3
4/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/
7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1
/2))^(1/2)))-(-3535/1936+273/1936*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/
(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+
1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/14*(34/7-10/7*11^(1/2))/(250/4
9-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/
2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*
(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*
11^(1/2))^(1/2)))-21/968*(61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49+34/49*11
^(1/2))/(x-2/7-1/7*11^(1/2))^2*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2
))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-3/1372*(34/7+10/7*11^(
1/2))/(250/49+34/49*11^(1/2))*(-1/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/
2))*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250
/49+34/49*11^(1/2))^(1/2)+7/2*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))/
(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(
1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2
```

```
))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))+
5/98/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+6
8/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^
(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/
2))+250+34*11^(1/2))^(1/2)))+3535/21296*11^(1/2)/(250+34*11^(1/2))^(1/2)*ar
ctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))
)/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2
))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))-(-3535/1936-273/1936*11^(1/
2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7*11^(1
/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/
2)+1/14*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2
)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1
/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^
(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-3535/21296*11^(1/2)/(2
50-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/
2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))
^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^3*sqrt(5*x^2 + 2*x + 3)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(174) = 348$.
time = 0.39, size = 390, normalized size = 1.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/697957829888*(sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(128397
3697005131*sqrt(11) + 82616280769148425)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x
+ 3)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*(358684877*sqrt(11
) + 2940638404) + 723215097220611079 7*sqrt(11)*(x + 3) - 216964529166183323
91*x + 36160754861030553985)/x) - sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x
 + 1)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*log((sqrt(2794)*s
qrt(5*x^2 + 2*x + 3)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*(3
```

```
58684877*sqrt(11) + 2940638404) - 7232150972206110797*sqrt(11)*(x + 3) + 21
696452916618332391*x - 36160754861030553985)/x) + sqrt(2794)*(49*x^4 - 56*x
^3 + 2*x^2 + 8*x + 1)*sqrt(-1283973697005131*sqrt(11) + 82616280769148425)*
log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(358684877*sqrt(11) - 2940638404)*sqr
t(-1283973697005131*sqrt(11) + 82616280769148425) + 7232150972206110797*sqr
t(11)*(x + 3) + 21696452916618332391*x - 36160754861030553985)/x) - sqrt(27
94)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-1283973697005131*sqrt(11) + 8
2616280769148425)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(358684877*sqrt(11
) - 2940638404)*sqrt(-1283973697005131*sqrt(11) + 82616280769148425) - 7232
150972206110797*sqrt(11)*(x + 3) - 21696452916618332391*x + 361607548610305
53985)/x) + 11176*(58279130*x^3 - 53381041*x^2 - 3071502*x + 3538943)*sqrt(
5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{5x}{343x^6\sqrt{5x^2+2x+3} - 588x^5\sqrt{5x^2+2x+3} + 189x^4\sqrt{5x^2+2x+3} + 104x^3\sqrt{5x^2+2x+3} - 27x^2\sqrt{5x^2+2x+3} - 12x\sqrt{5x^2+2x+3} - \sqrt{5x^2+2x+3}}\,dx - \int \frac{x^2}{343x^6\sqrt{5x^2+2x+3} - 588x^5\sqrt{5x^2+2x+3} + 189x^4\sqrt{5x^2+2x+3} + 104x^3\sqrt{5x^2+2x+3} - 27x^2\sqrt{5x^2+2x+3} - 12x\sqrt{5x^2+2x+3} - \sqrt{5x^2+2x+3}}\,dx - \int \frac{2}{343x^6\sqrt{5x^2+2x+3} - 588x^5\sqrt{5x^2+2x+3} + 189x^4\sqrt{5x^2+2x+3} + 104x^3\sqrt{5x^2+2x+3} - 27x^2\sqrt{5x^2+2x+3} - 12x\sqrt{5x^2+2x+3} - \sqrt{5x^2+2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**3/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(5*x/(343*x**6*sqrt(5*x**2 + 2*x + 3) - 588*x**5*sqrt(5*x**2 + 2*x
 + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3) + 104*x**3*sqrt(5*x**2 + 2*x + 3) -
 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2
 + 2*x + 3)), x) - Integral(x**2/(343*x**6*sqrt(5*x**2 + 2*x + 3) - 588*x**
5*sqrt(5*x**2 + 2*x + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3) + 104*x**3*sqrt(
5*x**2 + 2*x + 3) - 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*sqrt(5*x**2 + 2*x
 + 3) - sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(343*x**6*sqrt(5*x**2 + 2*
x + 3) - 588*x**5*sqrt(5*x**2 + 2*x + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3)
+ 104*x**3*sqrt(5*x**2 + 2*x + 3) - 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*s
qrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(174) = 348.
time = 3.78, size = 378, normalized size = 1.67

$$\frac{124397525\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^7+26796567\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^6-3595807617\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^5-1719888775\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^4+17096132999\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^3+8328401413\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^2-16383202915\sqrt{5}x-7800623485\sqrt{5}+16383202915\sqrt{5x^2+2x+3}}{31225744\left(7\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^4-8\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^3-70\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^2+16\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)+83\right)^2}+0.0423989586659649\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}+4.41924736459000\right)-0.0446437606656958\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}+1.25295163054000\right)-0.0423989586659649\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}-1.02258038113000\right)+0.0446437606656958\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}-2.09411235400000\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/31225744*(124397525*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 + 26796567*sqrt
(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^6 - 3595807617*(sqrt(5)*x - sqrt(5*
x^2 + 2*x + 3))^5 - 1719888775*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^
4 + 17096132999*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 8328401413*sqrt(5)*
```

```
(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 - 16383202915*sqrt(5)*x - 7800623485*
sqrt(5) + 16383202915*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2
*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*
x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3
)) + 83)^2 + 0.0423989586659649*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.
41924736459000) - 0.0446437606656958*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3)
 + 1.25295163054000) - 0.0423989586659649*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x
 + 3) - 1.02258038113000) + 0.0446437606656958*log(-sqrt(5)*x + sqrt(5*x^2
+ 2*x + 3) - 2.09411235400000)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2+5\,x+2}{\sqrt{5\,x^2+2\,x+3}\,\left(-7\,x^2+4\,x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3),x)
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3), x)
```

3.392 $\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

Optimal. Leaf size=166

$$\frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}}+\frac{15715799\sqrt{3+2x+5x^2}}{156250}-\frac{3192602x\sqrt{3+2x+5x^2}}{46875}-\frac{2583293x^2\sqrt{3+2x+5x^2}}{187500}$$

```
[Out] 50047657/781250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)+16/546875*(6122807-5
338217*x)/(5*x^2+2*x+3)^(1/2)+15715799/156250*(5*x^2+2*x+3)^(1/2)-3192602/4
6875*x*(5*x^2+2*x+3)^(1/2)-2583293/187500*x^2*(5*x^2+2*x+3)^(1/2)+393659/12
500*x^3*(5*x^2+2*x+3)^(1/2)-25921/3750*x^4*(5*x^2+2*x+3)^(1/2)-343/150*x^5*
(5*x^2+2*x+3)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$,
Rules used = {1674, 1675, 654, 633, 221}

$$-\frac{2583293\sqrt{5x^2+2x+3}\,x^2}{187500}-\frac{3192602\sqrt{5x^2+2x+3}\,x}{46875}+\frac{15715799\sqrt{5x^2+2x+3}}{156250}+\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}-\frac{343}{150}\sqrt{5x^2+2x+3}\,x^5-\frac{25921\sqrt{5x^2+2x+3}\,x^4}{3750}+\frac{393659\sqrt{5x^2+2x+3}\,x^3}{12500}+\frac{50047657\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{156250\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (16*(6122807 - 5338217*x))/(546875*Sqrt[3 + 2*x + 5*x^2]) + (15715799*Sqrt[
3 + 2*x + 5*x^2])/156250 - (3192602*x*Sqrt[3 + 2*x + 5*x^2])/46875 - (25832
93*x^2*Sqrt[3 + 2*x + 5*x^2])/187500 + (393659*x^3*Sqrt[3 + 2*x + 5*x^2])/1
2500 - (25921*x^4*Sqrt[3 + 2*x + 5*x^2])/3750 - (343*x^5*Sqrt[3 + 2*x + 5*x
^2])/150 + (50047657*ArcSinh[(1 + 5*x)/Sqrt[14]])/(156250*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(1+4x-7x^2)^3 (2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{1}{28}\int \frac{\frac{473724104}{78125} + \frac{94462228x}{15625} - \frac{40822404x^2}{3125} - \cdots}{\sqrt{3+2\cdots}}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} - \frac{343}{150}x^5\sqrt{3+2x+5x^2} + \frac{1}{840}\int \frac{\frac{2842344624\cdots}{15625}\cdots}{\cdots}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} - \frac{25921x^4\sqrt{3+2x+5x^2}}{3750} - \frac{343}{150}x^5\sqrt{3+2\cdots}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{393659x^3\sqrt{3+2x+5x^2}}{12500} - \frac{25921x^4\sqrt{3\cdots}}{37\cdots}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} - \frac{2583293x^2\sqrt{3+2x+5x^2}}{187500} + \frac{393659x^3\sqrt{\cdots}}{1\cdots}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} - \frac{3192602x\sqrt{3+2x+5x^2}}{46875} - \frac{2583293x^2\sqrt{\cdots}}{18\cdots}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{15715799\sqrt{3+2x+5x^2}}{156250} - \frac{3192602x\sqrt{\cdots}}{46\cdots}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{15715799\sqrt{3+2x+5x^2}}{156250} - \frac{3192602x\sqrt{\cdots}}{46\cdots}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{15715799\sqrt{3+2x+5x^2}}{156250} - \frac{3192602x\sqrt{\cdots}}{46\cdots}$$

Mathematica [A]
time = 0.54, size = 89, normalized size = 0.54

$$\frac{3155769618 - 1045703388x + 2135143465x^2 - 1795638985x^3 - 174819575x^4 + 897612625x^5 - 256821250x^6 - 75031250x^7}{6562500\sqrt{3+2x+5x^2}} - \frac{50047657\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{156250\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (3155769618 - 1045703388*x + 2135143465*x^2 - 1795638985*x^3 - 174819575*x^
4 + 897612625*x^5 - 256821250*x^6 - 75031250*x^7)/(6562500*Sqrt[3 + 2*x + 5
*x^2]) - (50047657*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(156250*S
qrt[5])
```

Maple [A]
time = 0.19, size = 166, normalized size = 1.00

method	result
risch	$-\frac{750312 50x^7+256821250x^6-897612625x^5+174819575x^4+1795638985x^3-2135143465x^2+1045703388x-3155769618}{6562500\sqrt{5x^2+2x+3}}+\frac{500476\ldots}{}$
trager	$-\frac{75031250x^7+256821250x^6-897612625x^5+174819575x^4+1795638985x^3-2135143465x^2+1045703388x-3155769618}{6562500\sqrt{5x^2+2x+3}}-\frac{500476\ldots}{}$
default	$\frac{175268451}{390625\sqrt{5x^2+2x+3}}+\frac{1025843x^5}{7500\sqrt{5x^2+2x+3}}-\frac{998969x^4}{37500\sqrt{5x^2+2x+3}}+\frac{50047657\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}}{14}\ldots\right.}{781250}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 175268451/390625/(5*x^2+2*x+3)^(1/2)+1025843/7500*x^5/(5*x^2+2*x+3)^(1/2)-9
98969/37500*x^4/(5*x^2+2*x+3)^(1/2)+50047657/781250*5^(1/2)*arcsinh(5/14*14
^(1/2)*(x+1/5))+176049701/10937500*(10*x+2)/(5*x^2+2*x+3)^(1/2)-343/30*x^7/
(5*x^2+2*x+3)^(1/2)-29351/750*x^6/(5*x^2+2*x+3)^(1/2)-51303971/187500*x^3/(
5*x^2+2*x+3)^(1/2)+61004099/187500*x^2/(5*x^2+2*x+3)^(1/2)-50047657/156250*
x/(5*x^2+2*x+3)^(1/2)
```

Maxima [A]
time = 0.50, size = 148, normalized size = 0.89

$$-\frac{343\,x^{7}}{30\,\sqrt{5\,x^{2}+2\,x+3}}-\frac{29351\,x^{6}}{750\,\sqrt{5\,x^{2}+2\,x+3}}+\frac{1025843\,x^{5}}{7500\,\sqrt{5\,x^{2}+2\,x+3}}-\frac{998969\,x^{4}}{37500\,\sqrt{5\,x^{2}+2\,x+3}}-\frac{51303971\,x^{3}}{187500\,\sqrt{5\,x^{2}+2\,x+3}}+\frac{61004099\,x^{2}}{187500\,\sqrt{5\,x^{2}+2\,x+3}}+\frac{50047657}{781250}\,\sqrt{5}\,\operatorname{arsinh}\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)-\frac{87141949\,x}{546875\,\sqrt{5\,x^{2}+2\,x+3}}+\frac{525961603}{1093750\,\sqrt{5\,x^{2}+2\,x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] -343/30*x^7/sqrt(5*x^2 + 2*x + 3) - 29351/750*x^6/sqrt(5*x^2 + 2*x + 3) + 1
025843/7500*x^5/sqrt(5*x^2 + 2*x + 3) - 998969/37500*x^4/sqrt(5*x^2 + 2*x +
 3) - 51303971/187500*x^3/sqrt(5*x^2 + 2*x + 3) + 61004099/187500*x^2/sqrt(
5*x^2 + 2*x + 3) + 50047657/781250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1))
 - 87141949/546875*x/sqrt(5*x^2 + 2*x + 3) + 525961603/1093750/sqrt(5*x^2 +
 2*x + 3)
```

Fricas [A]
time = 0.36, size = 112, normalized size = 0.67

$$\frac{1051000797\,\sqrt{5}\,(5\,x^{2}+2\,x+3)\log\left(-\sqrt{5}\,\sqrt{5\,x^{2}+2\,x+3}\,(5\,x+1)-25\,x^{2}-10\,x-8\right)-5\,(75031250\,x^{7}+256821250\,x^{6}-897612625\,x^{5}+174819575\,x^{4}+1795638985\,x^{3}-2135143465\,x^{2}+1045703388\,x-3155769618)\sqrt{5\,x^{2}+2\,x+3}}{32812500\,(5\,x^{2}+2\,x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/32812500*(1051000797*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 +
2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) - 5*(75031250*x^7 + 256821250*x^6 -
 897612625*x^5 + 174819575*x^4 + 1795638985*x^3 - 2135143465*x^2 + 10457033
88*x - 3155769618)*sqrt(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{29x}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx-\int\left(-\frac{115x^2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx-\int\frac{61x^3}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx-\int\frac{871x^4}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx-\int\left(-\frac{127x^5}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx-\int\left(-\frac{2065x^6}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx-\int\frac{1127x^7}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx-\int\frac{343x^8}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx-\int\left(-\frac{2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-29*x/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
 + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-115*x**2/(5*x**2*sqrt(5*x**2 +
 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - In
tegral(61*x**3/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
+ 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(871*x**4/(5*x**2*sqrt(5*x**2 + 2
*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Inte
gral(-127*x**5/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
+ 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-2065*x**6/(5*x**2*sqrt(5*x**2 +
 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - In
tegral(1127*x**7/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3
) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(343*x**8/(5*x**2*sqrt(5*x**2 +
 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - In
tegral(-2/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*s
qrt(5*x**2 + 2*x + 3)), x)
```

Giac [A]
time = 3.47, size = 81, normalized size = 0.49

$$-\frac{50047657}{781250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)-\frac{(35\,((5\,(35\,(70\,(175\,x+599)x-146549)x+998969)x+51303971)x-61004099)x+1045703388)x-3155769618}{6562500\,\sqrt{5\,x^2+2\,x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] -50047657/781250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) -
 1) - 1/6562500*((35*((5*(35*(70*(175*x + 599)*x - 146549)*x + 998969)*x +
51303971)*x - 61004099)*x + 1045703388)*x - 3155769618)/sqrt(5*x^2 + 2*x +
3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^2+5x+2)\,(-7x^2+4x+1)^3}{(5x^2+2x+3)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(3/2),x)
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(3/2), x)
```

3.393 $\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}}-\frac{5086\sqrt{3+2x+5x^2}}{3125}-\frac{8749x\sqrt{3+2x+5x^2}}{1250}+\frac{203}{100}x^2\sqrt{3+2x+5x^2}+\frac{49}{100}x^3\sqrt{3+$$

```
[Out] 89583/6250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-8/21875*(12983+136602*x)/
(5*x^2+2*x+3)^(1/2)-5086/3125*(5*x^2+2*x+3)^(1/2)-8749/1250*x*(5*x^2+2*x+3)
^(1/2)+203/100*x^2*(5*x^2+2*x+3)^(1/2)+49/100*x^3*(5*x^2+2*x+3)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1674, 1675, 654, 633, 221}

$$\frac{203}{100}\sqrt{5x^2+2x+3}\,x^2-\frac{8749\sqrt{5x^2+2x+3}\,x}{1250}-\frac{5086\sqrt{5x^2+2x+3}}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}+\frac{49}{100}\sqrt{5x^2+2x+3}\,x^3+\frac{89583\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{1250\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-8*(12983 + 136602*x))/(21875*Sqrt[3 + 2*x + 5*x^2]) - (5086*Sqrt[3 + 2*x
+ 5*x^2])/3125 - (8749*x*Sqrt[3 + 2*x + 5*x^2])/1250 + (203*x^2*Sqrt[3 + 2*
x + 5*x^2])/100 + (49*x^3*Sqrt[3 + 2*x + 5*x^2])/100 + (89583*ArcSinh[(1 +
5*x)/Sqrt[14]])/(1250*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} + \frac{1}{28}\int \frac{\frac{4291112}{3125} - \frac{296716x}{625} - \frac{194012x^2}{125} + \frac{23716x^3}{25}}{\sqrt{3+2x+5x^2}}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} + \frac{49}{100}x^3\sqrt{3+2x+5x^2} + \frac{1}{560}\int \frac{\frac{17164448}{625} -}{}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} + \frac{203}{100}x^2\sqrt{3+2x+5x^2} + \frac{49}{100}x^3\sqrt{3+2x+}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} - \frac{8749x\sqrt{3+2x+5x^2}}{1250} + \frac{203}{100}x^2\sqrt{3+2x+}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} - \frac{5086\sqrt{3+2x+5x^2}}{3125} - \frac{8749x\sqrt{3+2x+}}{1250}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} - \frac{5086\sqrt{3+2x+5x^2}}{3125} - \frac{8749x\sqrt{3+2x+}}{1250}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} - \frac{5086\sqrt{3+2x+5x^2}}{3125} - \frac{8749x\sqrt{3+2x+}}{1250}$$

Mathematica [A]
time = 0.63, size = 79, normalized size = 0.64

$$\frac{-168536-1298674x-280805x^2-515655x^3+194775x^4+42875x^5}{17500\sqrt{3+2x+5x^2}} - \frac{89583\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{1250\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-168536 - 1298674*x - 280805*x^2 - 515655*x^3 + 194775*x^4 + 42875*x^5)/(1
7500*Sqrt[3 + 2*x + 5*x^2]) - (89583*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x +
5*x^2]])/(1250*Sqrt[5])
```

Maple [A]
time = 0.19, size = 132, normalized size = 1.06

method	result

risch	$\frac{42875x^5+194775x^4-515655x^3-280805x^2-1298674x-168536}{17500\sqrt{5x^2+2x+3}}+\frac{89583\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,\left(x+\frac{1}{5}\right)}{14}\right)}{6250}$
trager	$\frac{42875x^5+194775x^4-515655x^3-280805x^2-1298674x-168536}{17500\sqrt{5x^2+2x+3}}+\frac{89583\,\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(5\,\mathrm{RootOf}\left(\_Z^2-5\right)x+\mathrm{RootOf}\left(\_\right.\right.}{6250}$
default	$-\frac{28506}{3125\sqrt{5x^2+2x+3}}+\frac{49x^5}{20\sqrt{5x^2+2x+3}}+\frac{1113x^4}{100\sqrt{5x^2+2x+3}}+\frac{89583\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,\left(x+\frac{1}{5}\right)}{14}\right)}{6250}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -28506/3125/(5*x^2+2*x+3)^(1/2)+49/20*x^5/(5*x^2+2*x+3)^(1/2)+1113/100*x^4/
(5*x^2+2*x+3)^(1/2)+89583/6250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-5564/
21875*(10*x+2)/(5*x^2+2*x+3)^(1/2)-14733/500*x^3/(5*x^2+2*x+3)^(1/2)-8023/5
00*x^2/(5*x^2+2*x+3)^(1/2)-89583/1250*x/(5*x^2+2*x+3)^(1/2)
```

Maxima [A]
time = 0.50, size = 114, normalized size = 0.92

$$\frac{49\,x^5}{20\,\sqrt{5\,x^2+2\,x+3}}+\frac{1113\,x^4}{100\,\sqrt{5\,x^2+2\,x+3}}-\frac{14733\,x^3}{500\,\sqrt{5\,x^2+2\,x+3}}-\frac{8023\,x^2}{500\,\sqrt{5\,x^2+2\,x+3}}+\frac{89583}{6250}\,\sqrt{5}\,\operatorname{arsinh}\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)-\frac{649337\,x}{8750\,\sqrt{5\,x^2+2\,x+3}}-\frac{42134}{4375\,\sqrt{5\,x^2+2\,x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] 49/20*x^5/sqrt(5*x^2 + 2*x + 3) + 1113/100*x^4/sqrt(5*x^2 + 2*x + 3) - 1473
3/500*x^3/sqrt(5*x^2 + 2*x + 3) - 8023/500*x^2/sqrt(5*x^2 + 2*x + 3) + 8958
3/6250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 649337/8750*x/sqrt(5*x^2
+ 2*x + 3) - 42134/4375/sqrt(5*x^2 + 2*x + 3)
```

Fricas [A]
time = 0.35, size = 102, normalized size = 0.82

$$\frac{627081\,\sqrt{5}\,(5\,x^2+2\,x+3)\log\left(-\sqrt{5}\,\sqrt{5\,x^2+2\,x+3}\,(5\,x+1)-25\,x^2-10\,x-8\right)+5\,(42875\,x^5+194775\,x^4-515655\,x^3-280805\,x^2-1298674\,x-168536)\sqrt{5\,x^2+2\,x+3}}{87500\,(5\,x^2+2\,x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/87500*(627081*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3
)*(5*x + 1) - 25*x^2 - 10*x - 8) + 5*(42875*x^5 + 194775*x^4 - 515655*x^3 -
 280805*x^2 - 1298674*x - 168536)*sqrt(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2+5x+2)(7x^2-4x-1)^2}{(5x^2+2x+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)*(7*x**2 - 4*x - 1)**2/(5*x**2 + 2*x + 3)**(3/2),
x)
```

Giac [A]
time = 3.70, size = 71, normalized size = 0.57

$$-\frac{89583}{6250}\sqrt{5}\,\log\left(-\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)+\frac{(35\,((35\,(35\,(35\,x+159)x-14733)x-8023)x-1298674)x-168536}{17500\,\sqrt{5\,x^2+2\,x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] -89583/6250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1) +
 1/17500*((35*((35*(35*x + 159)*x - 14733)*x - 8023)*x - 1298674)*x - 16853
6)/sqrt(5*x^2 + 2*x + 3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5\,x+2\right)\,\left(-7\,x^2+4\,x+1\right)^2}{\left(5\,x^2+2\,x+3\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(3/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(3/2), x)
```

3.394 $\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}}-\frac{261}{250}\sqrt{3+2x+5x^2}-\frac{7}{50}x\sqrt{3+2x+5x^2}+\frac{149\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{25\sqrt{5}}$$

```
[Out] 149/125*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-2/875*(2321+2449*x)/(5*x^2+2
*x+3)^(1/2)-261/250*(5*x^2+2*x+3)^(1/2)-7/50*x*(5*x^2+2*x+3)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1674, 1675, 654, 633, 221}

$$-\frac{7}{50}\sqrt{5x^2+2x+3}\,x-\frac{261}{250}\sqrt{5x^2+2x+3}-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}+\frac{149\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-2*(2321 + 2449*x))/(875*Sqrt[3 + 2*x + 5*x^2]) - (261*Sqrt[3 + 2*x + 5*x^
2])/250 - (7*x*Sqrt[3 + 2*x + 5*x^2])/50 + (149*ArcSinh[(1 + 5*x)/Sqrt[14]]
)/(25*Sqrt[5])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}} + \frac{1}{28}\int \frac{\frac{15736}{125}-\frac{3948x}{25}-\frac{196x^2}{5}}{\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}} - \frac{7}{50}x\sqrt{3+2x+5x^2} + \frac{1}{280}\int \frac{\frac{34412}{25}-\frac{7308x}{5}}{\sqrt{3+2x+5x^2}}$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}} - \frac{261}{250}\sqrt{3+2x+5x^2} - \frac{7}{50}x\sqrt{3+2x+5x^2} +$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}} - \frac{261}{250}\sqrt{3+2x+5x^2} - \frac{7}{50}x\sqrt{3+2x+5x^2} +$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}} - \frac{261}{250}\sqrt{3+2x+5x^2} - \frac{7}{50}x\sqrt{3+2x+5x^2} +$$

Mathematica [**A**]
time = 0.42, size = 69, normalized size = 0.84

$$\frac{-2953-2837x-1925x^2-245x^3}{350\sqrt{3+2x+5x^2}} - \frac{149\log\left(-1-5x+\sqrt{5}\,\sqrt{3+2x+5x^2}\right)}{25\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-2953 - 2837*x - 1925*x^2 - 245*x^3)/(350*Sqrt[3 + 2*x + 5*x^2]) - (149*Lo
g[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(25*Sqrt[5])
```

Maple [A]
time = 0.16, size = 98, normalized size = 1.20

method	result
risch	$-\frac{245x^3+1925x^2+2837x+2953}{350\sqrt{5x^2+2x+3}}+\frac{149\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,\left(x+\frac{1}{5}\right)}{14}\right)}{125}$
trager	$-\frac{245x^3+1925x^2+2837x+2953}{350\sqrt{5x^2+2x+3}}+\frac{149\,\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(5\,\mathrm{RootOf}\left(\_Z^2-5\right)x+\mathrm{RootOf}\left(\_Z^2-5\right)+5\sqrt{5x^2+2x+3}\right.}{125}$
default	$-\frac{7x^3}{10\sqrt{5x^2+2x+3}}-\frac{11x^2}{2\sqrt{5x^2+2x+3}}-\frac{149x}{25\sqrt{5x^2+2x+3}}-\frac{1001}{125\sqrt{5x^2+2x+3}}-\frac{751(10}{3500\sqrt{5x^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -7/10*x^3/(5*x^2+2*x+3)^(1/2)-11/2*x^2/(5*x^2+2*x+3)^(1/2)-149/25*x/(5*x^2+
2*x+3)^(1/2)-1001/125/(5*x^2+2*x+3)^(1/2)-751/3500*(10*x+2)/(5*x^2+2*x+3)^(
1/2)+149/125*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Maxima [A]
time = 0.49, size = 80, normalized size = 0.98

$$-\frac{7\,x^3}{10\,\sqrt{5\,x^2+2\,x+3}}-\frac{11\,x^2}{2\,\sqrt{5\,x^2+2\,x+3}}+\frac{149}{125}\,\sqrt{5}\,\operatorname{arsinh}\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)-\frac{2837\,x}{350\,\sqrt{5\,x^2+2\,x+3}}-\frac{2953}{350\,\sqrt{5\,x^2+2\,x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -7/10*x^3/sqrt(5*x^2 + 2*x + 3) - 11/2*x^2/sqrt(5*x^2 + 2*x + 3) + 149/125*
sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 2837/350*x/sqrt(5*x^2 + 2*x + 3)
 - 2953/350/sqrt(5*x^2 + 2*x + 3)
```

Fricas [A]
time = 0.36, size = 92, normalized size = 1.12

$$\frac{1043\,\sqrt{5}\,(5\,x^2+2\,x+3)\log\left(-\sqrt{5}\,\sqrt{5\,x^2+2\,x+3}\,(5\,x+1)-25\,x^2-10\,x-8\right)-5\,(245\,x^3+1925\,x^2+2837\,x+2953)\sqrt{5\,x^2+2\,x+3}}{1750\,(5\,x^2+2\,x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/1750*(1043*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(
5*x + 1) - 25*x^2 - 10*x - 8) - 5*(245*x^3 + 1925*x^2 + 2837*x + 2953)*sqrt
(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{13x}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx-\int\left(-\frac{7x^2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx-\int\frac{31x^3}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx-\int\frac{7x^4}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx-\int\left(-\frac{2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-13*x/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
 + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-7*x**2/(5*x**2*sqrt(5*x**2 + 2
*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Inte
gral(31*x**3/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) +
3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(7*x**4/(5*x**2*sqrt(5*x**2 + 2*x +
 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral
(-2/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*
x**2 + 2*x + 3)), x)
```

Giac [A]
time = 4.90, size = 62, normalized size = 0.76

$$-\frac{149}{125}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)-\frac{(35(7x+55)x+2837)x+2953}{350\sqrt{5x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="giac
")
```

```
[Out] -149/125*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1) - 1/
350*((35*(7*x + 55)*x + 2837)*x + 2953)/sqrt(5*x^2 + 2*x + 3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(x^2+5x+2)(-7x^2+4x+1)}{(5x^2+2x+3)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(3/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(3/2), x)
```

3.395 $\int \frac{2+5x+x^2}{(1+4x-7x^2)\left(3+2x+5x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{131-605x}{3556\sqrt{3+2x+5x^2}}-\frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{1016}+\ldots$$

```
[Out] 1/3556*(-131+605*x)/(5*x^2+2*x+3)^(1/2)-3/1419352*arctanh((23+x*(17-5*11^(1
/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(393525121-3494
5955*11^(1/2))^(1/2)+3/1419352*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x
^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(393525121+34945955*11^(1/2))^(1/2
)
```

Rubi [A]
time = 0.13, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1074, 1046, 738, 212}

$$-\frac{131-605x}{3556\sqrt{5x^2+2x+3}}-\frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{1016}+\frac{3\sqrt{\frac{281693+25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{1016}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] -1/3556*(131 - 605*x)/Sqrt[3 + 2*x + 5*x^2] - (3*Sqrt[(281693 - 25015*Sqrt[
11])/1397]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*
Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/1016 + (3*Sqrt[(281693 + 25015*Sqrt[11]
)/1397]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqr
t[11])]*Sqrt[3 + 2*x + 5*x^2])])/1016
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/((((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\,(3+2x+5x^2)^{3/2}}\,dx = -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} + \frac{\int \frac{13776+14112x}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx}{28448}$$

$$= -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} + \frac{\left(21\left(66-53\sqrt{11}\right)\right)\int \frac{}{\left(4-2\sqrt{11}-14x\right)}}{2794}$$

$$= -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} - \frac{\left(21\left(66-53\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{}{2352+112}\right.}{}$$

$$= -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} - \frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}}\,\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{101}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.40, size = 199, normalized size = 1.20

$$\frac{-131+605x}{3556\sqrt{3+2x+5x^2}}+\frac{3}{254}\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&,\frac{22\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+41\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1-21\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1^2}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] (-131 + 605*x)/(3556*Sqrt[3 + 2*x + 5*x^2]) + (3*RootSum[83 - 16*Sqrt[5]*#1
 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (22*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*
x + 5*x^2] - #1] + 41*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1
]*#1 - 21*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5]
- 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/254
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(118) = 236$.
time = 0.69, size = 489, normalized size = 2.95 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/196*(10*x+2)/(5*x^2+2*x+3)^(1/2)-3/154*(61+13*11^(1/2))*11^(1/2)*(1/7/(2
50/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7
-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-1/7*(34/7+10/7*11^(1/2))/(250/4
```

```
9+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)
/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49
+34/49*11^(1/2))^(1/2)-1/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*ar
ctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))
)/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2
))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-3/154*(-61+13*11^(1/2))*11
^(1/2)*(1/7/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11
^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/7*(34/7-10/7*11
^(1/2))/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/
7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11
^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/(250/49-34/49*11^(1/2))/(250-34*11^(
1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7
+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/
7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 777 vs. $2(117) = 234$.
time = 0.54, size = 777, normalized size = 4.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -1/4312*sqrt(11)*(20*sqrt(11)*x/sqrt(5*x^2 + 2*x + 3) - 7890*sqrt(11)*x/(17
*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x + 3)) + 7890*sqrt(11
)*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x + 3)) - 13377
*sqrt(11)*sqrt(2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(
11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)
*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x
 - 2*sqrt(11) - 4))/(17*sqrt(11) + 125)^(3/2) + 4*sqrt(11)/sqrt(5*x^2 + 2*x
 + 3) - 26280*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x +
 3)) - 26280*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x +
3)) + 156*sqrt(11)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt
(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11
)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*
x + 2*sqrt(11) - 4))/(-34/49*sqrt(11) + 250/49)^(3/2) - 62769*sqrt(2)*arcsi
nh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)
*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14
*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/(17
*sqrt(11) + 125)^(3/2) + 2244*sqrt(11)/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) +
 125*sqrt(5*x^2 + 2*x + 3)) - 2244*sqrt(11)/(17*sqrt(11)*sqrt(5*x^2 + 2*x +
 3) - 125*sqrt(5*x^2 + 2*x + 3)) - 732*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)
*x/abs(14*x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11
```

```
) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqr
t(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4))/(-34/49*sqrt(11) + 250/49)^(3/2) +
 12678/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x + 3)) + 12
678/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x + 3)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(117) = 234$.
time = 0.39, size = 333, normalized size = 2.01

$$-\frac{21 \sqrt{1397} (5x^2+2x+3) \sqrt{25015\sqrt{11}+281693} \log\left(\frac{3\left(\sqrt{1397}\sqrt{5x^2+2x+3}\sqrt{25015\sqrt{11}+281693}\left(1335\sqrt{11}-8173\right)+23596727\sqrt{11}(x+3)+70790181x-117983635\right)}{x}\right) - 21\sqrt{1397}(5x^2+2x+3)\sqrt{25015\sqrt{11}+281693}\log\left(-\frac{3\left(\sqrt{1397}\sqrt{5x^2+2x+3}\sqrt{25015\sqrt{11}+281693}\left(1335\sqrt{11}-8173\right)-23596727\sqrt{11}(x+3)-70790181x+117983635\right)}{x}\right) + 7\sqrt{1397}(5x^2+2x+3)\sqrt{-225135\sqrt{11}+2535237}\log\left(-\frac{\sqrt{1397}\sqrt{5x^2+2x+3}\left(1335\sqrt{11}+8173\right)\sqrt{-225135\sqrt{11}+2535237}+70790181\sqrt{11}(x+3)-212370543x+353950905}{x}\right) - 7\sqrt{1397}(5x^2+2x+3)\sqrt{-225135\sqrt{11}+2535237}\log\left(\frac{\sqrt{1397}\sqrt{5x^2+2x+3}\left(1335\sqrt{11}+8173\right)\sqrt{-225135\sqrt{11}+2535237}-70790181\sqrt{11}(x+3)+212370543x-353950905}{x}\right) - 5588\sqrt{5x^2+2x+3}(605x-131)}{19870928(5x^2+2x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/19870928*(21*sqrt(1397)*(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*
log(3*(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*(1335
*sqrt(11) - 8173) + 23596727*sqrt(11)*(x + 3) + 70790181*x - 117983635)/x)
- 21*sqrt(1397)*(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*log(-3*(sqr
t(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*(1335*sqrt(11)
- 8173) - 23596727*sqrt(11)*(x + 3) - 70790181*x + 117983635)/x) + 7*sqrt(1
397)*(5*x^2 + 2*x + 3)*sqrt(-225135*sqrt(11) + 2535237)*log(-(sqrt(1397)*sq
rt(5*x^2 + 2*x + 3)*(1335*sqrt(11) + 8173)*sqrt(-225135*sqrt(11) + 2535237)
 + 70790181*sqrt(11)*(x + 3) - 212370543*x + 353950905)/x) - 7*sqrt(1397)*(
5*x^2 + 2*x + 3)*sqrt(-225135*sqrt(11) + 2535237)*log((sqrt(1397)*sqrt(5*x^
2 + 2*x + 3)*(1335*sqrt(11) + 8173)*sqrt(-225135*sqrt(11) + 2535237) - 7079
0181*sqrt(11)*(x + 3) + 212370543*x - 353950905)/x) - 5588*sqrt(5*x^2 + 2*x
 + 3)*(605*x - 131))/(5*x^2 + 2*x + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{5x}{35x^4\sqrt{5x^2+2x+3}-6x^3\sqrt{5x^2+2x+3}+8x^2\sqrt{5x^2+2x+3}-14x\sqrt{5x^2+2x+3}-3\sqrt{5x^2+2x+3}}\,dx-\int \frac{x^2}{35x^4\sqrt{5x^2+2x+3}-6x^3\sqrt{5x^2+2x+3}+8x^2\sqrt{5x^2+2x+3}-14x\sqrt{5x^2+2x+3}-3\sqrt{5x^2+2x+3}}\,dx-\int \frac{2}{35x^4\sqrt{5x^2+2x+3}-6x^3\sqrt{5x^2+2x+3}+8x^2\sqrt{5x^2+2x+3}-14x\sqrt{5x^2+2x+3}-3\sqrt{5x^2+2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(5*x/(35*x**4*sqrt(5*x**2 + 2*x + 3) - 6*x**3*sqrt(5*x**2 + 2*x +
3) + 8*x**2*sqrt(5*x**2 + 2*x + 3) - 14*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5
*x**2 + 2*x + 3)), x) - Integral(x**2/(35*x**4*sqrt(5*x**2 + 2*x + 3) - 6*x
**3*sqrt(5*x**2 + 2*x + 3) + 8*x**2*sqrt(5*x**2 + 2*x + 3) - 14*x*sqrt(5*x*
*2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(35*x**4*sqrt(5*
x**2 + 2*x + 3) - 6*x**3*sqrt(5*x**2 + 2*x + 3) + 8*x**2*sqrt(5*x**2 + 2*x
+ 3) - 14*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x)
```

Giac [A]
time = 4.12, size = 112, normalized size = 0.67

$$\frac{605\,x-131}{3556\,\sqrt{5\,x^2+2\,x+3}}+0.0477059376663667\,\log\left(-\sqrt{5}\,x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)-0.0352174957838020\,\log\left(-\sqrt{5}\,x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)-0.0477059376663667\,\log\left(-\sqrt{5}\,x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)+0.0352174957838020\,\log\left(-\sqrt{5}\,x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="giac
")
```

```
[Out] 1/3556*(605*x - 131)/sqrt(5*x^2 + 2*x + 3) + 0.0477059376663667*log(-sqrt(5
)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.0352174957838020*log(-s
qrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0.0477059376663667*l
og(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.0352174957838
020*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+5\,x+2}{{\left(5\,x^2+2\,x+3\right)}^{3/2}\,\left(-7\,x^2+4\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)),x)
```

```
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)), x)
```

3.396 $\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\left(3+2x+5x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=215

$$-\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}}-\frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}-\frac{7\left(541543-5144\sqrt{11}\right)\tanh^{-1}\left(\frac{\quad}{\sqrt{2}\ \cdots}\right.}{2838704\sqrt{22\left(12\cdots\right.}}$$

```
[Out] 1/19870928*(-76567-22755*x)/(5*x^2+2*x+3)^(1/2)-3/5588*(40-371*x)/(-7*x^2+4
*x+1)/(5*x^2+2*x+3)^(1/2)-7/2838704*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))
/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(541543-5144*11^(1/2))/(2750-
374*11^(1/2))^(1/2)+7/2838704*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^
2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(541543+5144*11^(1/2))/(2750+374*11
^(1/2))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1074, 1046, 738, 212}

$$-\frac{3(40-371x)}{5588\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}-\frac{22755x+76567}{19870928\sqrt{5x^2+2x+3}}-\frac{7\left(541543-5144\sqrt{11}\right)\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{2838704\sqrt{22\left(125-17\sqrt{11}\right)}}+\frac{7\left(541543+5144\sqrt{11}\right)\tanh^{-1}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{2838704\sqrt{22\left(125+17\sqrt{11}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] -1/19870928*(76567 + 22755*x)/Sqrt[3 + 2*x + 5*x^2] - (3*(40 - 371*x))/(558
8*(1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]) - (7*(541543 - 5144*Sqrt[11])*Ar
cTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sq
rt[3 + 2*x + 5*x^2])])/(2838704*Sqrt[22*(125 - 17*Sqrt[11])]) + (7*(541543
+ 5144*Sqrt[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125
 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(2838704*Sqrt[22*(125 + 17*Sqrt[1
1])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\left(3+2x+5x^2\right)^{3/2}}\,dx = -\frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} - \frac{\int \frac{-50216-37752x-89040x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/}}}{44704}$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5}}$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5}}$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5}}$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.70, size = 416, normalized size = 1.93

$$\frac{503287-3628805x-444949x^2-159285x^3}{19870928\sqrt{3+2x+5x^2}\left(-1-4x+7x^2\right)}+\frac{\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&,\frac{116685\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+205710\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1+8351\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1^2}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\right]}{258064\sqrt{5}}-\frac{3\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&,\frac{746007\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)-1016580\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1+42623\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1^2}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\right]}{2838704\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] (503287 - 3628805*x - 444949*x^2 - 159285*x^3)/(19870928*Sqrt[3 + 2*x + 5*x
^2]*(-1 - 4*x + 7*x^2)) + RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*
#1^3 + 7*#1^4 & , (116685*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2]
- #1] + 205710*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 8351*Sqr
t[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*
#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ]/(258064*Sqrt[5]) - (3*RootSum[83 - 16*Sqr
t[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (746007*Sqrt[5]*Log[-(Sqrt[
5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 1016580*Log[-(Sqrt[5]*x) + Sqrt[3 + 2
*x + 5*x^2] - #1]*#1 + 42623*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^
2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/(2838704
*Sqrt[5])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1213 vs. $2(163) = 326$.
time = 0.75, size = 1214, normalized size = 5.65

method	result
risch	$-\frac{159285x^3+444949x^2+3628805x-503287}{19870928(7x^2-4x-1)\sqrt{5x^2+2x+3}} + \frac{7\left(541543+5144\sqrt{11}\right)\sqrt{11}\ \operatorname{arctanh}\left(\frac{\quad}{\sqrt{250+34\sqrt{11}}\ \sqrt{245\left(x \ldots\right.}}\right.}{312257\ldots}$
trager	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] (183/44-39/44*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))
/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49
-34/49*11^(1/2))^(1/2)-3/98*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1
/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-
2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-(34/7-10/7*11^(1/2))/(250/49
-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/
(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-
34/49*11^(1/2))^(1/2)-7/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arc
tanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))
/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2)
)*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))-20/49/(250/49-34/49*11^(1/2
))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*
11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2)
)^(1/2))-161/484*11^(1/2)*(1/7/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/
2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2
)-1/7*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49
*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(
1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-1/(250/49+34/49*11^
(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10
/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*
11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(
1/2)))+(183/44+39/44*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11
^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))
+250/49+34/49*11^(1/2))^(1/2)-3/98*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1
/2))*(1/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/
2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-(34/7+10/7*11^(1/2))/
```

```
(250/49+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/
2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+
250/49+34/49*11^(1/2))^(1/2)-7/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1
/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^
(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*1
1^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-20/49/(250/49+34/49*
11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2
/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*1
1^(1/2))^(1/2))+161/484*11^(1/2)*(1/7/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7
*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2
))^(1/2)-1/7*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49
-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10
/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/(250/49-34
/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(
34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2
/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(
1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^2*(5*x^2 + 2*x + 3)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(162) = 324$.
time = 0.36, size = 392, normalized size = 1.82

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/111038745664*(7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(4294
093814065*sqrt(11) + 35653135368317)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)
*sqrt(4294093814065*sqrt(11) + 35653135368317)*(5609479*sqrt(11) + 77949905
) + 2865029444171587*sqrt(11)*(x + 3) - 8595088332514761*x + 14325147220857
935)/x) - 7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(42940938140
65*sqrt(11) + 35653135368317)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(42
94093814065*sqrt(11) + 35653135368317)*(5609479*sqrt(11) + 77949905) - 2865
```

```
029444171587*sqrt(11)*(x + 3) + 8595088332514761*x - 14325147220857935)/x)
+ 7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(-4294093814065*sqrt
(11) + 35653135368317)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(5609479*sqrt(
11) - 77949905)*sqrt(-4294093814065*sqrt(11) + 35653135368317) + 2865029444
171587*sqrt(11)*(x + 3) + 8595088332514761*x - 14325147220857935)/x) - 7*sq
rt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(-4294093814065*sqrt(11) +
 35653135368317)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(5609479*sqrt(11) -
 77949905)*sqrt(-4294093814065*sqrt(11) + 35653135368317) - 286502944417158
7*sqrt(11)*(x + 3) - 8595088332514761*x + 14325147220857935)/x) + 5588*(159
285*x^3 + 444949*x^2 + 3628805*x - 503287)*sqrt(5*x^2 + 2*x + 3))/(35*x^4 -
 6*x^3 + 8*x^2 - 14*x - 3)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+5x+2}{\left(5x^2+2x+3\right)^{\frac{3}{2}}\left(7x^2-4x-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**2/(5*x**2+2*x+3)**(3/2),x)
[Out] Integral((x**2 + 5*x + 2)/((5*x**2 + 2*x + 3)**(3/2)*(7*x**2 - 4*x - 1)**2)
, x)
```

Giac [A]

time = 4.37, size = 295, normalized size = 1.37

$$\frac{25230\,x+13397}{903224\,\sqrt{5\,x^2+2\,x+3}}+\frac{3\left(42623\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^3+77302\,\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^2-275511\,\sqrt{5}\,x-219860\,\sqrt{5}+275511\,\sqrt{5\,x^2+2\,x+3}\right)}{709676\left(7\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^4-8\,\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^3-70\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^2+16\,\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)+83\right)}+0.0218058276254033\,\log\left(-\sqrt{5}\,x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)-0.0332874364433911\,\log\left(-\sqrt{5}\,x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)-0.0218058276254033\,\log\left(-\sqrt{5}\,x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)+0.0332874364433911\,\log\left(-\sqrt{5}\,x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
[Out] 1/903224*(25230*x + 13397)/sqrt(5*x^2 + 2*x + 3) + 3/709676*(42623*(sqrt(5)
*x - sqrt(5*x^2 + 2*x + 3))^3 + 77302*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x
 + 3))^2 - 275511*sqrt(5)*x - 219860*sqrt(5) + 275511*sqrt(5*x^2 + 2*x + 3)
)/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqrt(5*
x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(
sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83) + 0.0218058276254033*log(-sqrt(5)*
x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.0332874364433911*log(-sqr
t(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0.0218058276254033*log
(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.033287436443391
1*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2+5\,x+2}{{\left(5\,x^2+2\,x+3\right)}^{3/2}\,{\left(-7\,x^2+4\,x+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2),x)
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2), x)
```

3.397 $\int \frac{2+5x+x^2}{(1+4x-7x^2)^3 (3+2x+5x^2)^{3/2}} \, dx$

Optimal. Leaf size=250

$$-\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}}-\frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}}-\frac{2701733-9148874x}{62451488\left(1+4x-7x^2\right)\sqrt{3+2x}}$$

```
[Out] -5/222077491328*(461370781+1118731375*x)/(5*x^2+2*x+3)^(1/2)-3/11176*(40-37
1*x)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2)+1/62451488*(-2701733+9148874*x)/(
-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2)-7/31725355904*arctanh((23+x*(17-5*11^(1/2
))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(2792860024-84865
895*11^(1/2))/(2750-374*11^(1/2))^(1/2)+7/31725355904*arctanh((23+11^(1/2)+
x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(2792860024
+84865895*11^(1/2))/(2750+374*11^(1/2))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1074, 1046, 738, 212}

$$-\frac{2701733-9148874x}{62451488\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}-\frac{5(1118731375x+461370781)}{222077491328\sqrt{5x^2+2x+3}}-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}-\frac{7\left(2792860024-84865895\sqrt{11}\right)\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{31725355904\sqrt{22\left(125-17\sqrt{11}\right)}}+\frac{7\left(2792860024+84865895\sqrt{11}\right)\tanh^{-1}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{31725355904\sqrt{22\left(125+17\sqrt{11}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] (-5*(461370781 + 1118731375*x))/(222077491328*Sqrt[3 + 2*x + 5*x^2]) - (3*(
40 - 371*x))/(11176*(1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]) - (2701733 -
 9148874*x)/(62451488*(1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]) - (7*(279286
0024 - 84865895*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sq
rt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(31725355904*Sqrt[22*(12
5 - 17*Sqrt[11])]) + (7*(2792860024 + 84865895*Sqrt[11])*ArcTanh[(23 + Sqrt
[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x
^2])])/(31725355904*Sqrt[22*(125 + 17*Sqrt[11])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\,(3+2x+5x^2)^{3/2}}\,dx = -\frac{3(40-371x)}{11176\,(1+4x-7x^2)^2\,\sqrt{3+2x+5x^2}} - \frac{\int \frac{-128104-89208x-178080}{(1+4x-7x^2)^2(3+2x+5x^2)}}{89408}$$

$$= -\frac{3(40-371x)}{11176\,(1+4x-7x^2)^2\,\sqrt{3+2x+5x^2}} - \frac{2701733-}{62451488\,(1+4x-7}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\,(1+4x-7x^2)^2\,\sqrt{3+2}}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\,(1+4x-7x^2)^2\,\sqrt{3+2}}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\,(1+4x-7x^2)^2\,\sqrt{3+2}}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\,(1+4x-7x^2)^2\,\sqrt{3+2}}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.96, size = 607, normalized size = 2.43

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] ((-1715*(14298727813 + 7828199499*x - 148022158802*x^2 + 109737266678*x^3 -
 200208943655*x^4 + 274089186875*x^5))/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x +
5*x^2]) + 2324168*Sqrt[5]*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*
#1^3 + 7*#1^4 & , (-4989740*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2
] - #1] + 3790865*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 40044
9*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5]
- 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] + 22*Sqrt[5]*RootSum[83 - 16*Sqrt[5]
*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-3200991286865*Sqrt[5]*Log[-(S
qrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 18470877323690*Log[-(Sqrt[5]*x) +
```

```
Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 2296522946389*Sqrt[5]*Log[-(Sqrt[5]*x) +
Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*
#1^3) & ] - 9*Sqrt[5]*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3
 + 7*#1^4 & , (-8189062651053*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x
^2] - #1] + 39132066594240*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#
1 + 5875617407695*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1
^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/380862897627520
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2599 vs. $2(194) = 388$.
time = 0.74, size = 2600, normalized size = 10.40

method	result
risch	$-\frac{274089186875x^5-200208943655x^4+109737266678x^3-148022158802x^2+7828199499x+14298727813}{222077491328(7x^2-4x-1)^2\sqrt{5x^2+2x+3}}+\frac{7\left(2792860024+848658\right.}{}$
trager	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -21/968*(-61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49-34/49*11^(1/2))/(x-2/7+1
/7*11^(1/2))^2/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11
^(1/2))+250/49-34/49*11^(1/2))^(1/2)-5/1372*(34/7-10/7*11^(1/2))/(250/49-34
/49*11^(1/2))*(-1/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))/(5*(x-2/7+1/
7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/
2))^(1/2)-3/2*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/(250/49-34/49
*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/
2))+250/49-34/49*11^(1/2))^(1/2)-(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2
))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*
11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2)
)^(1/2)-7/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500
/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1
/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*1
1^(1/2))+250-34*11^(1/2))^(1/2)))-20/(250/49-34/49*11^(1/2))*(10*x+2)/(5000
/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7
-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2))-15/686/(
250/49-34/49*11^(1/2))*(1/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2
+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-(34
```

/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)
-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-
2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-7/(250/49-34/49*11^(1/2))/(2
50-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/
2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))
^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))-(
-3535/1936+273/1936*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^
(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+
250/49-34/49*11^(1/2))^(1/2)-3/98*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/
2))*(1/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2
))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-(34/7-10/7*11^(1/2))/(
250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2
))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+2
50/49-34/49*11^(1/2))^(1/2)-7/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/
2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(
1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11
^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))-20/49/(250/49-34/49*1
1^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/
7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11
^(1/2))^(1/2))-3535/21296*11^(1/2)*(1/7/(250/49+34/49*11^(1/2))/(5*(x-2/7-1
/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1
/2))^(1/2)-1/7*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(10*x+2)/(5000/
49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+
10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-1/(250/49+
34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)
+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x
-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11
^(1/2))^(1/2)))-21/968*(61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49+34/49*11^(
1/2))/(x-2/7-1/7*11^(1/2))^2/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))
*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-5/1372*(34/7+10/7*11^(1/
2))/(250/49+34/49*11^(1/2))*(-1/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2)
)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/4
9+34/49*11^(1/2))^(1/2)-3/2*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1
/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-
2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-(34/7+10/7*11^(1/2))/(250/49
+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/
(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+
34/49*11^(1/2))^(1/2)-7/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arc
tanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))
/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2)
)*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-20/(250/49+34/49*11^(1/2))*
(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^
(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(
1/2))-15/686/(250/49+34/49*11^(1/2))*(1/(250/49+34/49*11^(1/2))/(5*(x-2/7-1
/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1

```
/2))^(1/2)-(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(10*x+2)/(5000/49+6
80/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7
*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-7/(250/49+34/4
9*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] -integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^3*(5*x^2 + 2*x + 3)^(3/2)), x
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(193) = 386.
time = 0.49, size = 452, normalized size = 1.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/1240969021540864*(7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 2
7*x^2 + 26*x + 3)*sqrt(74693314710639641467*sqrt(11) + 896266498377233657785
5)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(74693314710639641467*sqrt(11
) + 896266498377233657855)*(37271563201*sqrt(11) + 407780707037) + 75502120
686844055144479*sqrt(11)*(x + 3) - 226506362060532165433437*x + 37751060343
4220275722395)/x) - 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27
*x^2 + 26*x + 3)*sqrt(74693314710639641467*sqrt(11) + 896266498377233657855
)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(74693314710639641467*sqrt(11)
+ 896266498377233657855)*(37271563201*sqrt(11) + 407780707037) - 7550212068
6844055144479*sqrt(11)*(x + 3) + 226506362060532165433437*x - 3775106034342
20275722395)/x) + 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x
^2 + 26*x + 3)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233657855)
*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(37271563201*sqrt(11) - 407780707037
)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233657855) + 7550212068
6844055144479*sqrt(11)*(x + 3) + 226506362060532165433437*x - 3775106034342
20275722395)/x) - 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x
^2 + 26*x + 3)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233657855)
*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(37271563201*sqrt(11) - 40778070703
7)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233657855) - 755021206
```

```
86844055144479*sqrt(11)*(x + 3) - 2265063620605321654334 37*x + 377510603434
220275722395)/x) + 5588*(274089186875*x^5 - 200208943655*x^4 + 109737266678
*x^3 - 148022158802*x^2 + 7828199499*x + 14298727813)*sqrt(5*x^2 + 2*x + 3)
)/(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x^2 + 26*x + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{5x}{1715x^{8}\sqrt{5x^{2}+2x+3}-2254x^{7}\sqrt{5x^{2}+2x+3}+798x^{6}\sqrt{5x^{2}+2x+3}-866x^{5}\sqrt{5x^{2}+2x+3}+640x^{4}\sqrt{5x^{2}+2x+3}+198x^{3}\sqrt{5x^{2}+2x+3}-110x^{2}\sqrt{5x^{2}+2x+3}-38x\sqrt{5x^{2}+2x+3}-3\sqrt{5x^{2}+2x+3}}\,dx-\int \frac{x^{2}}{1715x^{8}\sqrt{5x^{2}+2x+3}-2254x^{7}\sqrt{5x^{2}+2x+3}+798x^{6}\sqrt{5x^{2}+2x+3}-866x^{5}\sqrt{5x^{2}+2x+3}+640x^{4}\sqrt{5x^{2}+2x+3}+198x^{3}\sqrt{5x^{2}+2x+3}-110x^{2}\sqrt{5x^{2}+2x+3}-38x\sqrt{5x^{2}+2x+3}-3\sqrt{5x^{2}+2x+3}}\,dx-\int \frac{2}{1715x^{8}\sqrt{5x^{2}+2x+3}-2254x^{7}\sqrt{5x^{2}+2x+3}+798x^{6}\sqrt{5x^{2}+2x+3}-866x^{5}\sqrt{5x^{2}+2x+3}+640x^{4}\sqrt{5x^{2}+2x+3}+198x^{3}\sqrt{5x^{2}+2x+3}-110x^{2}\sqrt{5x^{2}+2x+3}-38x\sqrt{5x^{2}+2x+3}-3\sqrt{5x^{2}+2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**3/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(5*x/(1715*x**8*sqrt(5*x**2 + 2*x + 3) - 2254*x**7*sqrt(5*x**2 + 2
*x + 3) + 798*x**6*sqrt(5*x**2 + 2*x + 3) - 866*x**5*sqrt(5*x**2 + 2*x + 3)
 + 640*x**4*sqrt(5*x**2 + 2*x + 3) + 198*x**3*sqrt(5*x**2 + 2*x + 3) - 110*
x**2*sqrt(5*x**2 + 2*x + 3) - 38*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5*x**2 +
 2*x + 3)), x) - Integral(x**2/(1715*x**8*sqrt(5*x**2 + 2*x + 3) - 2254*x**
7*sqrt(5*x**2 + 2*x + 3) + 798*x**6*sqrt(5*x**2 + 2*x + 3) - 866*x**5*sqrt(
5*x**2 + 2*x + 3) + 640*x**4*sqrt(5*x**2 + 2*x + 3) + 198*x**3*sqrt(5*x**2
+ 2*x + 3) - 110*x**2*sqrt(5*x**2 + 2*x + 3) - 38*x*sqrt(5*x**2 + 2*x + 3)
- 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(1715*x**8*sqrt(5*x**2 + 2*x +
 3) - 2254*x**7*sqrt(5*x**2 + 2*x + 3) + 798*x**6*sqrt(5*x**2 + 2*x + 3) -
866*x**5*sqrt(5*x**2 + 2*x + 3) + 640*x**4*sqrt(5*x**2 + 2*x + 3) + 198*x**
3*sqrt(5*x**2 + 2*x + 3) - 110*x**2*sqrt(5*x**2 + 2*x + 3) - 38*x*sqrt(5*x*
*2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(193) = 386.
time = 4.13, size = 397, normalized size = 1.59

$$\frac{501205\,x+1702037}{458837792\,\sqrt{5x^{2}+2x+3}}+\frac{6871871279\left(\sqrt{5}x-\sqrt{5x^{2}+2x+3}\right)^{7}+4012856750\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^{2}+2x+3}\right)^{6}-223088535693\left(\sqrt{5}x-\sqrt{5x^{2}+2x+3}\right)^{5}-100577598176\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^{2}+2x+3}\right)^{4}+1255097956673\left(\sqrt{5}x-\sqrt{5x^{2}+2x+3}\right)^{3}+566810398070\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^{2}+2x+3}\right)^{2}-1246245909011\sqrt{5}x-561299654796\sqrt{5}+1246245909011\sqrt{5x^{2}+2x+3}}{7931338976\left(7\left(\sqrt{5}x-\sqrt{5x^{2}+2x+3}\right)^{4}-8\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^{2}+2x+3}\right)^{3}-70\left(\sqrt{5}x-\sqrt{5x^{2}+2x+3}\right)^{2}+16\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^{2}+2x+3}\right)+83\right)^{2}}+0.0107382277384513\log\left(-\sqrt{5}x+\sqrt{5x^{2}+2x+3}+4.41924736459000\right)-0.0142619066316905\log\left(-\sqrt{5}x+\sqrt{5x^{2}+2x+3}+1.25295163054000\right)-0.0107382277384513\log\left(-\sqrt{5}x+\sqrt{5x^{2}+2x+3}-1.02258038113000\right)+0.0142619066316905\log\left(-\sqrt{5}x+\sqrt{5x^{2}+2x+3}-2.09411235400000\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/458837792*(501205*x + 1702037)/sqrt(5*x^2 + 2*x + 3) + 1/7931338976*(6871
871279*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 + 4012856750*sqrt(5)*(sqrt(5)*
x - sqrt(5*x^2 + 2*x + 3))^6 - 223088535693*(sqrt(5)*x - sqrt(5*x^2 + 2*x +
 3))^5 - 100577598176*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 + 12550
97956673*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 566810398070*sqrt(5)*(sqrt
(5)*x - sqrt(5*x^2 + 2*x + 3))^2 - 1246245909011*sqrt(5)*x - 561299654796*s
qrt(5) + 1246245909011*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 +
2*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)
*x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x +
```

```
3)) + 83)^2 + 0.0107382277384513*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4
.41924736459000) - 0.0142619066316905*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3
) + 1.25295163054000) - 0.0107382277384513*log(-sqrt(5)*x + sqrt(5*x^2 + 2*
x + 3) - 1.02258038113000) + 0.0142619066316905*log(-sqrt(5)*x + sqrt(5*x^2
 + 2*x + 3) - 2.09411235400000)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2+5\,x+2}{{\left(5\,x^2+2\,x+3\right)}^{3/2}\,{\left(-7\,x^2+4\,x+1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3),x)
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3), x)
```

3.398 $\int (a + cx^2)^p (A + Cx^2) (d + fx^2)^q \, dx$

Optimal. Leaf size=166

$$Ax(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}(d+fx^2)^q\left(1+\frac{fx^2}{d}\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3(a+cx^2)^p\left(1\right.$$

```
[Out] A*x*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(1/2,-p,-q,3/2,-c*x^2/a,-f*x^2/d)/((c*x
^2/a+1)^p)/((1+f*x^2/d)^q)+1/3*C*x^3*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(3/2,-
p,-q,5/2,-c*x^2/a,-f*x^2/d)/((c*x^2/a+1)^p)/((1+f*x^2/d)^q)
```

Rubi [A]
time = 0.09, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {545, 441, 440, 525, 524}

$$Ax(a+cx^2)^p\left(\frac{cx^2}{a}+1\right)^{-p}(d+fx^2)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3(a+cx^2)^p\left(\frac{cx^2}{a}+1\right)^{-p}(d+fx^2)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^p*(A + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (C*x^3*(a + c*x^2)^p*
(d + f*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*(1
 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \left(a+cx^2\right)^p \left(A+Cx^2\right) \left(d+fx^2\right)^q \, dx = A\int \left(a+cx^2\right)^p \left(d+fx^2\right)^q \, dx + C\int x^2\left(a+cx^2\right)^p \left(d+fx^2\right)^q \, d$$

$$= \left(A\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p}\right)\int \left(1+\frac{cx^2}{a}\right)^p \left(d+fx^2\right)^q \, dx + \Big($$

$$= \left(A\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(d+fx^2\right)^q \left(1+\frac{fx^2}{d}\right)^{-q}\right)\int \Big(1+$$

$$= Ax\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(d+fx^2\right)^q \left(1+\frac{fx^2}{d}\right)^{-q} F_1\left(\frac{1}{2};-p,\right.$$

Mathematica [A]
time = 0.30, size = 242, normalized size = 1.46

$$\frac{1}{3}x\left(a+cx^2\right)^p\left(d+fx^2\right)^q\left(\frac{9aAdF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{3adF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+2x^2\left(cdpF_1\left(\frac{3}{2};1-p,-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+afqF_1\left(\frac{3}{2};-p,1-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)\right)}+Cx^2\left(1+\frac{cx^2}{a}\right)^{-p}\left(1+\frac{fx^2}{d}\right)^{-q}F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*x^2)^p*(A + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (x*(a + c*x^2)^p*(d + f*x^2)^q*((9*a*A*d*AppellF1[1/2, -p, -q, 3/2, -((c*x^
2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -((f*
x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -q, 5/2, -((c*x^2)/a), -((f*x^
2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2, -((c*x^2)/a), -((f*x^2)/d)]))
```

```
+ (C*x^2*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/((1 + (c*x
^2)/a)^p*(1 + (f*x^2)/d)^q)))/3
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(c\,x^{2}+a\right)^{p} \left(C\,x^{2}+A\right) \left(f\,x^{2}+d\right)^{q} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x)
[Out] int((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="maxima")
[Out] integrate((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="fricas")
[Out] integral((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**p*(C*x**2+A)*(f*x**2+d)**q,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\,x^2+A\right)\,{\left(c\,x^2+a\right)}^p\,{\left(f\,x^2+d\right)}^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + c*x^2)^p*(d + f*x^2)^q,x)
```

```
[Out] int((A + C*x^2)*(a + c*x^2)^p*(d + f*x^2)^q, x)
```

3.399 $\int (A+Bx)\left(a+cx^2\right)^p \left(d+fx^2\right)^q \, dx$

Optimal. Leaf size=167

$$Ax\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d+fx^2\right)^q\left(1+\frac{fx^2}{d}\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{B\left(a+cx^2\right)^{1+p}\left(d+\right.}{}$$

```
[Out] A*x*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(1/2,-p,-q,3/2,-c*x^2/a,-f*x^2/d)/((c*x
^2/a+1)^p)/((1+f*x^2/d)^q)+1/2*B*(c*x^2+a)^(1+p)*(f*x^2+d)^q*hypergeom([-q,
 1+p],[2+p],-f*(c*x^2+a)/(-a*f+c*d))/c/(1+p)/((c*(f*x^2+d)/(-a*f+c*d))^q)
```

Rubi [A]

time = 0.10, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1024, 441, 440, 455, 72, 71}

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{B\left(a+cx^2\right)^{p+1}\left(d+fx^2\right)^q\left(\frac{c\left(d+fx^2\right)}{cd-af}\right)^{-q}{}_2F_1\left(p+1,-q;p+2;-\frac{f\left(cx^2+a\right)}{cd-af}\right)}{2c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)*(a + c*x^2)^p*(d + f*x^2)^q,x]
```

```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (B*(a + c*x^2)^(1 + p
)*(d + f*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((f*(a + c*x^2))/(c*d
- a*f))])/(2*c*(1 + p)*((c*(d + f*x^2))/(c*d - a*f))^q)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\begin{aligned}\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx &= A\int \left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx+B\int x\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx\\ &= \frac{1}{2}B\text{Subst}\left(\int (a+cx)^p(d+fx)^q\,dx,x,x^2\right)+\left(A\left(a+cx^2\right)^p\left(1+\right.\right.\\ &= \frac{1}{2}\left(B\left(d+fx^2\right)^q\left(\frac{c\left(d+fx^2\right)}{cd-af}\right)^{-q}\right)\text{Subst}\left(\int (a+cx)^p\left(\frac{cd}{cd-af}\right.\right.\\ &= Ax\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d+fx^2\right)^q\left(1+\frac{fx^2}{d}\right)^{-q}F_1\left(\frac{1}{2};-p,-\right.\end{aligned}$$

Mathematica [A]
time = 0.24, size = 236, normalized size = 1.41

$$\frac{1}{2}x\left(a+cx^2\right)^p\left(d+fx^2\right)^q\left(Bx\left(1+\frac{cx^2}{a}\right)^{-p}\left(1+\frac{fx^2}{d}\right)^{-q}F_1\left(1;-p,-q;2;-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{6aAdF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{3adF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+2x^2\left(cdpF_1\left(\frac{3}{2};1-p,-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+afqF_1\left(\frac{3}{2};-p,1-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)\right)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*x)*(a + c*x^2)^p*(d + f*x^2)^q,x]
```

```
[Out] (x*(a + c*x^2)^p*(d + f*x^2)^q*((B*x*AppellF1[1, -p, -q, 2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (6*a*A*d*AppellF1[1/2
, -p, -q, 3/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p, -q, 3/
2, -((c*x^2)/a), -((f*x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -q, 5/2,
 -((c*x^2)/a), -((f*x^2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2, -((c*x^2
)/a), -((f*x^2)/d)]))))/2
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int (Bx+A)\left(c\,x^{2}+a\right)^{p}\left(f\,x^{2}+d\right)^{q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x)
```

```
[Out] int((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+a)**p*(f*x**2+d)**q,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x^2+a)^p\,(f\,x^2+d)^q\,(A+B\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x),x)
```

```
[Out] int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x), x)
```

3.400 $\int (a+cx^2)^p (A+Bx+Cx^2) (d+fx^2)^q \, dx$

Optimal. Leaf size=252

$$Ax(a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} (d+fx^2)^q \left(1+\frac{fx^2}{d}\right)^{-q} F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3(a+cx^2)^p \left(1\right.$$

```
[Out] A*x*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(1/2,-p,-q,3/2,-c*x^2/a,-f*x^2/d)/((c*x
^2/a+1)^p)/((1+f*x^2/d)^q)+1/3*C*x^3*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(3/2,-
p,-q,5/2,-c*x^2/a,-f*x^2/d)/((c*x^2/a+1)^p)/((1+f*x^2/d)^q)+1/2*B*(c*x^2+a)
^(1+p)*(f*x^2+d)^q*hypergeom([-q, 1+p],[2+p],-f*(c*x^2+a)/(-a*f+c*d))/c/(1+
p)/((c*(f*x^2+d)/(-a*f+c*d))^q)
```

Rubi [A]
time = 0.28, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {6874, 441, 440, 455, 72, 71, 525, 524}

$$Ax(a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} (d+fx^2)^q \left(\frac{fx^2}{d}+1\right)^{-q} F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3(a+cx^2)^p \left(\frac{cx^2}{a}+1\right)^{-p} (d+fx^2)^q \left(\frac{fx^2}{d}+1\right)^{-q} F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{B(a+cx^2)^{p+1}(d+fx^2)^q \left(\frac{c(d+fx^2)}{cd-af}\right)^{-q} {}_2F_1\left(p+1,-q;p+2;-\frac{f(cx^2+a)}{cd-af}\right)}{2c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^p*(A + B*x + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (C*x^3*(a + c*x^2)^p*
(d + f*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*(1
 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (B*(a + c*x^2)^(1 + p)*(d + f*x^2)^q*H
ypergeometric2F1[1 + p, -q, 2 + p, -((f*(a + c*x^2))/(c*d - a*f))])/(2*c*(1
 + p)*((c*(d + f*x^2))/(c*d - a*f))^q)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+cx^2\right)^p \left(A+Bx+Cx^2\right) \left(d+fx^2\right)^q \, dx &= \int \left(A\left(a+cx^2\right)^p \left(d+fx^2\right)^q + Bx\left(a+cx^2\right)^p \left(d+fx^2\right)^q + C \\
&= A\int \left(a+cx^2\right)^p \left(d+fx^2\right)^q \, dx + B\int x\left(a+cx^2\right)^p \left(d+fx^2 \right. \\
&= \frac{1}{2}B\text{Subst}\left(\int (a+cx)^p(d+fx)^q \, dx, x, x^2\right) + \left(A\left(a+cx^2\right)^p\right. \\
&= \frac{1}{2}\left(B\left(d+fx^2\right)^q \left(\frac{c\left(d+fx^2\right)}{cd-af}\right)^{-q}\right) \text{Subst}\left(\int (a+cx)^p \left(\frac{\ }{cd}\right.\right. \\
&= Ax\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(d+fx^2\right)^q \left(1+\frac{fx^2}{d}\right)^{-q} F_1\left(\frac{1}{2}\right.
\end{aligned}
$$

Mathematica [A]
time = 0.41, size = 302, normalized size = 1.20

$$\frac{1}{6}x\left(a+cx^2\right)^p \left(d+fx^2\right)^q \left(3Bx\left(1+\frac{cx^2}{a}\right)^{-p}\left(1+\frac{fx^2}{d}\right)^{-q}F_1\left(1;-p,-q;2;-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{18aAdF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{3adF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+2x^2\left(cdpF_1\left(\frac{3}{2};1-p,-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+afqF_1\left(\frac{3}{2};-p,1-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)\right)}+2Cx^2\left(1+\frac{cx^2}{a}\right)^{-p}\left(1+\frac{fx^2}{d}\right)^{-q}F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*x^2)^p*(A + B*x + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (x*(a + c*x^2)^p*(d + f*x^2)^q*((3*B*x*AppellF1[1, -p, -q, 2, -((c*x^2)/a),
 -((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (18*a*A*d*AppellF1[
1/2, -p, -q, 3/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p, -q,
 3/2, -((c*x^2)/a), -((f*x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -q, 5
/2, -((c*x^2)/a), -((f*x^2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2, -((c*
x^2)/a), -((f*x^2)/d)])) + (2*C*x^2*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a)
, -((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q)))/6
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(c\,x^2+a\right)^p \left(C\,x^2+Bx+A\right) \left(f\,x^2+d\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x)
```

```
[Out] int((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**p*(C*x**2+B*x+A)*(f*x**2+d)**q,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,x^2+a\right)}^p\,{\left(f\,x^2+d\right)}^q\,\left(C\,x^2+B\,x+A\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x + C*x^2),x)
```

```
[Out] int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x + C*x^2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```